

## $B$-Physics results from Belle \& Belle II



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## Belle \& Belle II Detectors



- Asymmetric $e^{+}(3.5 \mathrm{GeV})-e^{-}(8 \mathrm{GeV})$ collider
- Collected total $1 \mathrm{ab}^{-1}$ of data
- Data taken from 1999 to 2010
- Collected $711 \mathrm{fb}^{-1}$ at $\Upsilon(4 S)$ resonance

- Asymmetric $e^{+}(4 \mathrm{GeV})$ - $e^{-}(7 \mathrm{GeV})$ collider
- Recorded $424 \mathrm{fb}^{-1}$ of data: ~ equivalent to BaBar and $1 / 2$ of Belle data sample during Run I
- Data taken between 2019-2022
- Run I data at $\Upsilon(4 S)$ resonance: $362 \mathrm{fb}^{-1}$
- Run II has started from early 2024
- Aims to collect many-ab-1 of data


## Advantages of Belle \& Belle II

- Clean environment: Average 11 tracks per event compared to hundreds of tracks for hadron colliders

- Good particle identification and performance
- Similar sensitivity for $\mu$ and $e$
- High photon detection efficiency
- Good $\pi^{0}$ mass resolution

- Dealing with missing energy:
- fully reconstruct the partner $B$ meson: full event interpretation
- Identify invisible particle using, $M_{\text {miss }}^{2}=\left(p_{e^{+} e^{-}}-p_{\text {visible }}\right)^{2}$

Belle II Preliminary $\int c d t=62.8 \mathrm{fb}^{-1}$

- Unique for $B$ factories:
- channels with $\pi^{0}, \gamma, \eta^{\left({ }^{\prime}\right)}, K_{L} \ldots$
- final states with one or more missing $\nu$


## Physics Programs



## Rare decays and Lepton flavor Universality




- Involves $b \rightarrow \boldsymbol{s} \gamma$ transition
- SM $\mathcal{B}$ predictions have large theoretical uncertainties ( $\sim 20 \%$ ) related to form factors [JHEP 04 (2017) 027]

- Observables;
- Branching fraction, $\mathcal{B}$
- $C P$ violation asymmetry

$$
\mathcal{A}_{C P}=\frac{\Gamma\left(\bar{B} \rightarrow \bar{K}^{*} \gamma\right)-\Gamma\left(B \rightarrow K^{*} \gamma\right)}{\Gamma\left(\bar{B} \rightarrow \bar{K}^{*} \gamma\right)+\Gamma\left(B \rightarrow K^{*} \gamma\right)}
$$

$$
\Delta \mathcal{A}_{C P}=\mathcal{A}_{C P}\left(B^{0} \rightarrow K^{* 0} \gamma\right)-\mathcal{A}_{C P}\left(B^{+} \rightarrow K^{*+} \gamma\right)
$$

- Isospin asymmetry

$$
\Delta_{0+}=\frac{\Gamma\left(B^{0} \rightarrow K^{* 0} \gamma\right)-\Gamma\left(B^{+} \rightarrow K^{*+} \gamma\right)}{\Gamma\left(B^{0} \rightarrow K^{* 0} \gamma\right)+\Gamma\left(B^{+} \rightarrow K^{*+} \gamma\right)}
$$

- $\mathcal{A}_{C P}$ and $\Delta_{0+}$ are theoretically clean due to cancellation of form factor contributions in ratio
- Belle [PRL 119 (2017) 191802] has observed evidence of isospin violation at $3.1 \sigma$ using $711 \mathrm{fb}^{-1}$
- Used Run I data sample of Belle II
- Measured $\mathcal{A}_{C P}, \Delta \mathcal{A}_{C P}, \Delta_{0+}$ in addition to $\mathcal{B}$

$$
\begin{aligned}
& K^{* 0} \rightarrow K^{+} \pi^{-}, K_{S}^{0} \pi^{0} \\
& K^{*+} \rightarrow K^{+} \pi^{0}, K_{S}^{0} \pi^{+}
\end{aligned}
$$

- 2D fit of $M_{\mathrm{bc}}\left(\sqrt{\left(E_{\text {beam }} / c^{2}\right)^{2}-\left(p_{B} / c\right)^{2}}\right)$ and $\Delta E\left(E_{B}-E_{\text {beam }}\right)$ to extract signal

$$
\begin{aligned}
\mathcal{B}\left(B^{0} \rightarrow K^{* 0} \gamma\right) & =(4.16 \pm 0.10 \pm 0.11) \times 10^{-5} \\
\mathcal{B}\left(B^{+} \rightarrow K^{*+} \gamma\right) & =(4.04 \pm 0.13 \pm 0.13) \times 10^{-5} \\
\mathcal{B}\left(B \rightarrow K^{*} \gamma\right) & =(4.12 \pm 0.08 \pm 0.11) \times 10^{-5}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{C P}\left(B^{0} \rightarrow K^{* 0} \gamma\right) & =(-3.2 \pm 2.4 \pm 0.4) \% \\
\mathcal{A}_{C P}\left(B^{+} \rightarrow K^{*+} \gamma\right) & =(-1.0 \pm 3.0 \pm 0.6) \% \\
\mathcal{A}_{C P}\left(B \rightarrow K^{*} \gamma\right) & =(-2.3 \pm 1.9 \pm 0.3) \%
\end{aligned}
$$

$$
\begin{gathered}
\Delta \mathcal{A}_{C P}=(2.2 \pm 3.8 \pm 0.7) \% \\
\Delta_{0+}=(5.1 \pm 2.0 \pm 1.0 \pm 1.1) \%
\end{gathered}
$$

- Results are consistent with WA and SM
- Similar sensitivity to Belle [PRL 119 (2017) 191802] due to improved $\Delta E$ resolution and $K_{S}^{0}$ identification efficiency
- Dominant systematic for $\Delta_{0+}$ is $f^{+-}=\Gamma\left(\Upsilon(4 S) \rightarrow B^{+} B^{-}\right)$or $f^{00}=\Gamma\left(\Upsilon(4 S) \rightarrow B^{0} \overline{B^{0}}\right)$


## $B \rightarrow(\eta, \omega, \pi, \rho) \ell^{+} \ell^{-}$at Belle

- Involves $b \rightarrow d \ell^{+} \ell^{-}$transition
- $\mathcal{B}$ for $b \rightarrow d \ell^{+} \ell^{-}$is more sensitive to new physics than $b \rightarrow s \ell^{+} \ell^{-}$as SM $\mathcal{B}$ is suppressed by a factor of $\left|V_{t d} / V_{t s}\right|^{2} \sim 0.04$
- Typical $\mathcal{B}$ in SM is $\mathcal{O}\left(10^{-8}\right)$ or smaller [PRD 86 (2012)
 114025]
- LHCb [JHEP 10 (2015) 034, PLB 743 (2015) 46]

$$
\begin{aligned}
\mathcal{B}\left(B^{+} \rightarrow \pi^{+} \mu^{+} \mu^{-}\right) & =(1.78 \pm 0.23) \times 10^{-8} \\
\mathcal{B}\left(B^{0} \rightarrow \rho^{0} \mu^{+} \mu^{-}\right) & =(1.98 \pm 0.53) \times 10^{-8} \\
\mathcal{B}\left(B^{0} \rightarrow \pi^{+} \pi^{-} \mu^{+} \mu^{-}\right) & =(2.11 \pm 0.52) \times 10^{-8}
\end{aligned}
$$

- Search for $e$ channels in addition to $\mu$ channels: tests LFU in $b \rightarrow d \ell^{+} \ell^{-}$
- Used $711 \mathrm{fb}^{-1}$ data sample of Belle

$$
\begin{aligned}
B^{ \pm, 0} & \rightarrow\left(\eta, \omega, \pi^{ \pm, 0}, \rho^{ \pm, 0}\right) e e \\
B^{ \pm, 0} & \rightarrow\left(\eta, \omega, \pi^{0}, \rho^{ \pm}\right) \mu \mu
\end{aligned}
$$

| channel | $\mathcal{B}^{\text {UL }}\left(10^{-8}\right)$ |
| :---: | :---: |
| $B^{0} \rightarrow \eta e^{+} e^{-}$ | $<10.5$ |
| $B^{0} \rightarrow \eta \mu^{+} \mu^{-}$ | $<9.4$ |
| $B^{0} \rightarrow \eta \ell^{+} \ell^{-}$ | < 4.8 |
| $B^{\overline{0}} \rightarrow \bar{\omega} \bar{e}^{+} \bar{e}^{-}$ | $<30.7$ |
| $B^{0} \rightarrow \omega \mu^{+} \mu^{-}$ | < 24.9 |
| $B^{0} \rightarrow \omega \ell^{+} \ell^{-}$ | < 22.0 |
| $B^{\overline{0}} \rightarrow \bar{\pi}^{0} \bar{e}^{+} e^{-}$ | $<\overline{7.9}$ |
| $B^{0} \rightarrow \pi^{0} \mu^{+} \mu^{-}$ | $<5.9$ |
| $B^{0} \rightarrow \pi^{0} \ell^{+} \ell^{-}$ | $<3.8$ |
| $\bar{B}^{+} \xrightarrow{-} \bar{\pi}^{+} e^{+} e^{-}$ | < 5.4 |
| $B^{\overline{0}} \xrightarrow{-} \bar{\rho}^{0} \bar{e}^{+}{ }^{-} e^{-}$ | $<4 \overline{5} .5$ |
| $\bar{B}^{+} \xrightarrow{-} \bar{\rho}^{+} e^{+} e^{-}$ | < 46.7 |
| $B^{+} \rightarrow \rho^{+} \mu^{+} \mu^{-}$ | < 38.1 |
| $B^{+} \rightarrow \rho^{+} \ell^{+} \ell^{-}$ | < 18.9 |

- 2D $M_{\mathrm{bc}}$ and $\Delta E$ fit to extract signal yield




- World's best limits for $B^{ \pm, 0} \rightarrow\left(\eta, \omega, \pi^{ \pm, 0}, \rho^{ \pm, 0}\right) e e$ and $B^{ \pm, 0} \rightarrow\left(\eta, \omega, \pi^{0}, \rho^{ \pm}\right) \mu \mu$
- World's first limits for $B^{0} \rightarrow \omega \ell^{+} \ell^{-}, B^{+} \rightarrow \rho^{+} \ell^{+} \ell^{-}$, and $B^{0} \rightarrow \rho^{0} e^{+} e^{-}$
- No LFU $\left(b \rightarrow d \mu^{+} \mu^{-} / b \rightarrow d e^{+} e^{-}\right)$in $b \rightarrow d \ell^{+} \ell^{-}$transitions
- No b-d vertex
- FCNC process through loop diagram as quark emits and reabsorbs a $W^{-}$boson

- $B^{0} \rightarrow \gamma \gamma$ is suppressed by a factor of $\left|V_{t d}^{2}\right| /\left|V_{t s}^{2}\right| \sim 0.04$ compared to $B_{s} \rightarrow \gamma \gamma$ decay
- $\operatorname{SM~} \mathcal{B}=\left(1.4_{-0.8}^{+1.4}\right) \times 10^{-8}$ [JHEP 12 (2020) 169], significant long distance contribution
- Best UL of $<3.2 \times 10^{-7}$ by BaBar [PRD 83 (2011) 032006] at $90 \% \mathrm{CL}$ with $426 \mathrm{fb}^{-1}$ data
- Used $1.1 \mathrm{ab}^{-1}$ of data sample: $694 \mathrm{fb}^{-1}($ Belle $)+362 \mathrm{fb}^{-1}$ (Belle II)
- 3D simultaneous fit between Belle and Belle II using $M_{\mathrm{bc}}, \Delta E$, and $C_{\mathrm{BDT}}^{\prime}$ (transformed BDT for continuum suppression) to extract signal yield

- $N_{\text {sig }}=11.0_{-5.5}^{+6.5}$ having a significance of $2.5 \sigma$

$$
\mathcal{B}^{\mathrm{UL}}\left(B^{0} \rightarrow \gamma \gamma\right)<6.4 \times 10^{-8} \text { at } 90 \% \mathrm{CL}
$$

- Most stringent UL: $5 \times$ better than best limit from BaBar [PRD 83 (2011) 032006]
- Result is not too far from SM expectation


## $B^{+} \rightarrow K^{+} \nu \bar{\nu}$ at Belle II

- $\mathcal{B}_{\mathrm{SM}}\left(B^{+} \rightarrow K^{+} \nu \bar{\nu}\right)=(5.58 \pm 0.37) \times 10^{-6}[\mathrm{PRD}$ 107 (2023) 119903]
- Experimentally challenging due to two neutrinos
- Best limit of $<1.6 \times 10^{-5}$ at $90 \%$ CL from BaBar [PRD 87 (2013) 112005]

- Used Run I data sample of Belle II


Inclusive Tag


- ITA has low purity but high efficiency and HTA has high purity and low efficiency
- ITA: signal efficiency $=8 \%$; purity $=0.9 \%$
- HTA: signal efficiency $=0.4 \%$; purity $=3.5 \%$
- Parameter of interest: signal strength $=\mu=\frac{\mathcal{B}\left(B^{+} \rightarrow K^{+} \nu \bar{\nu}\right)}{\mathcal{B}_{\mathrm{SM}}\left(B^{+} \rightarrow K^{+} \nu \bar{\nu}\right)}$


## $B^{+} \rightarrow K^{+} \nu \bar{\nu}$ at Belle II

- Binned fit to extract $\mu$ :
- ITA: Classifier 2 output $\left[\eta\left(\mathrm{BDT}_{2}\right)\right]$ and $q^{2}$ [mass square of neutrino pair]
- HTA: Classifier output [ $\eta$ (BDTh $)$ ]

- $\mu=5.4 \pm 1.0 \pm 1.1$
- $\mu=2.2_{-1.7-1.1}^{+1.8+1.6}$
- $\mathcal{B}=(2.7 \pm 0.5 \pm 0.5) \times 10^{-5}$
- $\mathcal{B}=\left(1.1_{-0.8-0.5}^{+0.9+0.8}\right) \times 10^{-5}$
- $3.5 \sigma$ significance w.r.t bkg-only hypo
- $1.1 \sigma$ significance w.r.t bkg-only hypo
- $2.9 \sigma$ departure from SM


## $B^{+} \rightarrow K^{+} \nu \bar{\nu}$ at Belle II




- Excluded common events from ITA sample
- Correlation between common systematic uncertainties are included
- $\mu=4.6 \pm 1.0 \pm 0.9$
- $\mathcal{B}=\left(2.3 \pm 0.5_{-0.4}^{+0.5}\right) \times 10^{-5}$
- First evidence of $B^{+} \rightarrow K^{+} \nu \bar{\nu}$ process
- Results are in agreement with all previous measurements
- $3.5 \sigma$ significance w.r.t bkg-only hypo
- $2.7 \sigma$ departure from SM
- Semileptonic $B \rightarrow X \tau(\ell) \nu$ is inclusive $b \rightarrow c \tau(\ell) \nu$ transition
- Test LFU in charged-current weak interaction by measuring tau-to-light-lepton ratio

$$
R\left(X_{\tau / \ell}\right)=\frac{\mathcal{B}(B \rightarrow X \tau \nu)}{\mathcal{B}(B \rightarrow X \ell \nu)}
$$

- Experimentally LFU in exclusive $B \rightarrow D^{(*)} \tau(\ell) \nu$ has a tension of $3.3 \sigma$

- Used $189 \mathrm{fb}^{-1}$ data sample of Belle II
- Tag-side $B$ is fully reconstructed hadronically
- Hadronic system $X$ is reconstructed using remaining tracks and energy deposits in the calorimeter
- Inclusive decay with $\tau$ is challenging due to larger background from less constrained $X$ system


## $R\left(X_{\tau / \ell}\right)$ at Belle II

- 2D fit to $p_{\ell}^{B}$ (in the $B_{\text {sig }}$ rest frame) and $M_{\text {miss }}^{2}$ (mass squared for undetected neutrinos) to extract signal yield
- Simultaneous extraction of signal yields for $B \rightarrow X \tau \nu$ and $B \rightarrow X \ell \nu$

$R\left(X_{\tau / e}\right)=0.232 \pm 0.020 \pm 0.037$
$R\left(X_{\tau / \mu}\right)=0.222 \pm 0.027 \pm 0.050$
$R\left(X_{\tau / \ell}\right)=0.228 \pm 0.016 \pm 0.036$
- $R\left(X_{\tau / \ell}\right)$ result is consistent with SM predictions of $0.223 \pm 0.005$ [JHEP 11 (2022) 007]
- First measurement at $B$-factories with $\Upsilon(4 S)$
- Result is also consistent with $R\left(D^{(*)}\right)$


## CKM and CP Violation



- $\left|V_{u b}\right|$ is important to constrain CKM unitarity triangle and test SM

$$
V_{\mathrm{CKM}}=\left(\begin{array}{lll}
V_{u d} & V_{u s} & V_{u b} \\
V_{c d} & V_{c s} & V_{c b} \\
V_{t d} & V_{t s} & V_{t b}
\end{array}\right)
$$

$\mathcal{B} \propto\left|V_{u b}\right|^{2} \times f, f=$ form factor (LCSR, LQCD)

- Long-standing $V_{x b}$-puzzle between inclusive and exclusive decays
- $\left|V_{u b}\right|$ for inclusive and exclusive differ by $2.5 \sigma$ experimentally
Exclusive : $B \rightarrow \pi \ell \nu, B \rightarrow \rho \ell \nu$
Inclusive: $B \rightarrow X_{u} \ell \nu$
- Used Run I data sample of Belle II
- Untagged analysis

$$
\begin{aligned}
& B^{0} \rightarrow \pi^{-} \ell^{+} \nu_{\ell} \\
& B^{+} \rightarrow \rho^{0} \ell^{+} \nu_{\ell}
\end{aligned}
$$

- Sum of measured exclusive $\mathcal{B}$ is $\sim 20 \%$ of inclusive $\mathcal{B}\left(B \rightarrow X_{u} \ell \nu\right)$




- Simultaneous $B^{0} \rightarrow \pi^{-} \ell^{+} \nu_{\ell}$ and $B^{+} \rightarrow \rho^{0} \ell^{+} \nu_{\ell}$ signal yields extraction with binned 3D fit to $M_{\mathrm{bc}}, \Delta E$, and $q^{2}=\left(p_{B}-p_{\pi^{-}, \rho^{0}}\right)^{2}$



$$
\begin{aligned}
& \mathcal{B}\left(B^{0} \rightarrow \pi^{-} \ell^{+} \nu_{\ell}\right)=(1.516 \pm 0.042 \pm 0.059) \times 10^{-4} \\
& \mathcal{B}\left(B^{+} \rightarrow \rho^{0} \ell^{+} \nu_{\ell}\right)=(1.625 \pm 0.079 \pm 0.180) \times 10^{-4}
\end{aligned}
$$

$$
\begin{aligned}
& \left|V_{u b}\right|_{B \rightarrow \pi \ell \nu}=(3.73 \pm 0.07 \pm 0.07 \pm 0.16) \times 10^{-3} \\
& \left|V_{u b}\right|_{B \rightarrow \rho \ell \nu}=(3.19 \pm 0.12 \pm 0.17 \pm 0.26) \times 10^{-3}
\end{aligned}
$$

- $\mathcal{B}$ results are consistent with WA
- $\left|V_{u b}\right|$ results are consistent with previous exclusive measurements [PRD 107 (2023) 052008, PRD 104 (2021) 034032], and theoretical uncertainty dominated


## Time dependent $C P$ violation

- Flagship measurement for $B$-factories

- Measurement relies on ability to identify flavor of tag side $B$
- Graphical neutral network approach for flavor tagging has improved the tagging efficiency by $\sim 18 \%$ at Belle II [arXiv:2402.17260]
- Gluonic penguin with $b \rightarrow s q \bar{q}$ transition, $q=u, d$, or $s$
- Golden mode: Relatively large $\mathcal{B}$ and limited contribution from tree amplitudes
- In SM: $S \approx \sin 2 \phi_{1}$ by $0.01 \pm 0.01$ and $C \approx 0$
- Used Run I data sample of Belle II

- Fit to $M_{\mathrm{bc}}, \Delta E, C_{\mathrm{BDT}}, \Delta t$, and $q_{\mathrm{tag}}$ (tag-flavor)


$C=-0.19 \pm 0.08 \pm 0.03$
$S=0.67 \pm 0.10 \pm 0.04$
- Results in agreement with WA
- Precision comparable with Belle [JHEP 10 (2014) 165] and BaBar [PRD 79 (2009) 052003] is spite of small data set
- Radiative penguin with $b \rightarrow s \gamma$ transition
- Used Run I data sample of Belle II
- Challenging to get $B^{0}$ vertex without prompt tracks


## Res:

$$
\begin{gathered}
M_{K_{S}^{0} \pi^{0}} \in(0.8,1.0) \mathrm{GeV} / c^{2} \text { i.e., } K^{* 0} \rightarrow K_{S}^{0} \pi^{0} \\
M_{K_{S}^{0} \pi^{0}} \in\left(0.6, \frac{\text { Non-Res: }}{0.8) \cup(1.0,1.8) \mathrm{GeV} / c^{2}}\right.
\end{gathered}
$$



- $M_{\mathrm{bc}}$ and $\Delta E$ followed by $\Delta t$ fit to extract signal yield

$$
\begin{aligned}
& S=0.00_{-0.26}^{+0.27} \pm 0.03 \\
& C=0.10 \pm 0.13 \pm 0.03
\end{aligned}
$$

$$
S=0.04_{-0.44}^{+0.45} \pm 0.10
$$

$$
C=-0.06 \pm 0.25 \pm 0.08
$$

- Results have improved precision compared to Belle [PRD 74 (2006) 111104] and BaBar [PRD 78 (2008) 071102]
- Results for $S$ are most precise due to better $K_{S}^{0}$ identification
- Tree-level $b \rightarrow u$ processes allow extraction of $\phi_{2}$ or $\alpha$ (least known CKM angle)
- Theoretical $\mathcal{B}$ [PLB 794 (2008) 154, PRD 90 (2014) 014029] is $5 \times$ smaller than experimental results as amplitude calculation is challenging involving low-energy, non-perturbative gluon exchanges
- Experimentally challenging: no tracks, $\gamma$ trajectory and energy less precise than tracks
- Used Run I data sample of Belle II



- Fit to $M_{\mathrm{bc}}, \Delta E, C$, and $w$ (wrong tag probability)

$$
\mathcal{B}\left(B^{0} \rightarrow \pi^{0} \pi^{0}\right)=(1.26 \pm 0.20 \pm 0.12) \times 10^{-6}
$$

$\mathcal{A}_{\mathrm{CP}}\left(B^{0} \rightarrow \pi^{0} \pi^{0}\right)=0.06 \pm 0.30 \pm 0.05$

- Results in agreement with WA
- Results have superior or comparable precision with Belle [PRD 96 (2017) 032007] and BaBar [PRD 87 (2013) 052009] in spite of small data set


## Conclusions

- Exploited full data of Belle or/and Run 1 data of Belle II
- Belle and Belle II have provided many world's leading measurements, best upper limits, and evidence
- Run II data taking of Belle II has started from early 2024: collecting quality physics data
- Waiting to enter $10^{35}$ luminosity era
- Many new results are on their way

A long way to go ... Stay tuned ...



## Backup slides

- Angular coefficiencts from differential decay rate of exclusive semileptonic $\bar{B} \rightarrow D^{*} \ell \bar{\nu}_{\ell}$
- $\left|V_{c b}\right|$ from angular coefficients of $\bar{B} \rightarrow D^{*} \ell \bar{\nu}_{\ell}$
- Angular coefficients also allow determination of form factors for $B \rightarrow D^{*}$ decay, test sensitive to BSM effects and LFU

- Differential decay rate can be decomposed in a basis of angular functions with 12 coefficients, $\hat{J}_{i}$, all dependent on $w$

$$
\begin{aligned}
\frac{\mathrm{d} \Gamma\left(\bar{B} \rightarrow D^{*} \ell \bar{\nu}_{\ell}\right)}{\mathrm{d} w \mathrm{~d} \cos \theta_{\ell} \operatorname{dos} \theta_{\mathrm{V}} \mathrm{~d} \chi}= & \frac{2 G_{\mathrm{F}}^{2} \eta_{\mathrm{EW}}^{2}\left|V_{\mathrm{cb}}\right|^{2} m_{B}^{4} m_{\mathrm{D}}}{2 \pi^{4}} \times\left(J_{1 s} \sin ^{2} \theta_{\mathrm{V}}+J_{1 c} \cos ^{2} \theta_{\mathrm{V}}\right. \\
& +\left(J_{2 s} \sin ^{2} \theta_{\mathrm{V}}+J_{2 c} \cos ^{2} \theta_{\mathrm{V}}\right) \cos 2 \theta_{\ell}+J_{3} \sin ^{2} \theta_{\mathrm{V}} \sin ^{2} \theta_{\ell} \cos 2 \chi \\
& +J_{4} \sin 2 \theta_{V} \sin 2 \theta_{\ell} \cos \chi+J_{5} \sin 2 \theta_{V} \sin \theta_{\ell} \cos \chi+\left(J_{6 s} \sin ^{2} \theta_{\mathrm{V}}+J_{6 c} \cos ^{2} \theta_{\mathrm{V}}\right) \cos \theta_{\ell} \\
& \left.+J_{7} \sin 2 \theta_{V} \sin \theta_{\ell} \sin \chi+J_{8} \sin 2 \theta_{V} \sin 2 \theta_{\ell} \sin \chi+J_{9} \sin ^{2} \theta_{\mathrm{V}} \sin ^{2} \theta_{\ell} \sin 2 \chi\right) \\
& \text { Hadronic recoil energy }(w)=\frac{m_{B}^{2}+m_{D^{*}}^{2}-q^{2}}{2 m_{B} m_{D^{*}}}
\end{aligned}
$$

- Used full $711 \mathrm{fb}^{-1}$ data sample of Belle
- Hadronic $B$ tagging

$$
\begin{aligned}
& B^{+} \rightarrow D^{* 0} \ell^{+} \nu, D^{* 0} \rightarrow D^{+} \pi^{0} \\
& B^{0} \rightarrow D^{*+} \ell^{-} \nu, D^{*+} \rightarrow D^{0} \pi^{+}
\end{aligned}
$$

- $12 \hat{J}_{i}$ coefficients in 4 bins of $w$


## $\left|V_{c b}\right|$ at Belle

[arXiv:2310.20286]


- Coefficients are in good agreement with the fit using BGL [NPB 461 (1996) 493, PRD 56 (1997) 6895] and CLN [NPB 530 (1998) 153] form-factor parametrizations
- Coefficients are consistent with the SM predictions

$$
\begin{aligned}
& \left|V_{c b}\right|=(41.0 \pm 0.3 \pm 0.4 \pm 0.5) \times 10^{-3}(\mathrm{BGL}) \\
& \left|V_{c b}\right|=(40.9 \pm 0.3 \pm 0.4 \pm 0.4) \times 10^{-3}(\mathrm{CLN})
\end{aligned}
$$

- Similar values of $\left|V_{c b}\right|$ using CLN and BGL form factors

This work
( $41.0 \pm 0.7$ ) $\times 10^{-3}$


- Closing the gap with inclusive $\left|V_{c b}\right|$ measurement

- No significant deviation from SM in LFU

- Design peak luminosity of $6.5 \times 10^{35} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}$ (30 times that of KEKB) to be achieved by;
- reducing beam size by 20 times
- increasing beam current by 1.5 times


## $B \rightarrow K^{*} \gamma$ at Belle II

| Source | $K^{* 0}\left[K^{+} \pi^{-}\right] \gamma$ | $K^{* 0}\left[K_{\mathrm{S}}^{0} \pi^{0}\right] \gamma$ | $K^{*+}\left[K^{+} \pi^{0}\right] \gamma$ | $K^{*+}\left[K_{\mathrm{S}}^{0} \pi^{+}\right] \gamma$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $B$ counting | 1.5 | 1.5 | 1.5 | 1.5 |  |  |  |  |
| $f^{ \pm} / f^{00}$ | 1.6 | 1.6 | 1.6 | 1.6 |  |  |  |  |
| $\gamma$ selection | 0.9 | 0.9 | 0.9 | 0.9 |  |  |  |  |
| $\pi^{0}$ veto | 0.7 | 0.7 | 0.7 | 0.7 |  |  |  |  |
| $\eta$ veto | 0.2 | 0.2 | 0.2 | 0.2 |  |  |  |  |
| Tracking efficiency | 0.5 | 0.5 | 0.2 | 0.7 |  |  |  |  |
| $\pi^{+}$selection | 0.2 | - | - | 0.2 |  |  |  |  |
| $K^{+}$selection | 0.4 | - | 0.4 | - |  |  |  |  |
| $K_{\mathrm{S}}^{0}$ reconstruction | - | 1.4 | - | 1.4 |  |  |  |  |
| $\pi^{0}$ reconstruction | - | 3.9 | 3.9 | - | Table 2. Systematic | uncertainties (\%) | for $\mathcal{A}_{C P}$ measur | ements. |
| $\chi^{2}$ requirement | 0.2 | 1.0 | 0.2 | 1.0 | Source | $K^{* 0}\left[K^{+} \pi^{-}\right] \gamma$ | $K^{*+}\left[K^{+} \pi^{0}\right] \gamma$ | $K^{*+}\left[K_{\mathrm{S}}^{0} \pi^{+}\right] \gamma$ |
| CSBDT requirement | 0.3 | 0.4 | 0.4 | 0.3 | Fit bias | 0.1 | 0.2 | 0.2 |
| Best candidate selection | 0.1 | 1.0 | 0.6 | 0.2 | Signal PDF model | 0.1 | 0.1 | 0.1 |
| Fit bias | 0.1 | 0.9 | 0.5 | 0.2 | KDE PDF model | 0.1 | 0.4 | 0.2 |
| Signal PDF model | 0.1 | 0.4 | 0.3 | 0.2 | Best candidate selection | 0.1 | 0.5 | 0.2 |
| KDE PDF model | 0.1 | 0.8 | 0.6 | 0.2 | $K^{+}$asymmetry | - | 0.6 | - |
| Simulation sample size | 0.2 | 0.8 | 0.4 | 0.5 | $\pi^{+}$asymmetry | - | - | 0.6 |
| Self-crossfeed fraction | - | 1.0 | 1.0 | - | $K^{+} \pi^{-}$asymmetry | 0.3 | - | - |
| Total | 2.6 | 5.4 | 4.9 | 3.2 | Total | 0.4 | 0.9 | 0.7 |

- Dominate systematics from number of $B \bar{B}$ events and $f^{ \pm}$or $f^{00}$
- $A_{\mathrm{CP}}$ measurement dominate uncertainty is coming from interaction of charged hadrons with detector material which give rise to asymmetries in track reconstruction efficiency


## $b \rightarrow$ dll

- Used a classifier to suppress backgrounds from continuum ( $e^{+} e^{-} \rightarrow q \bar{q}, q \in u, d, s, c$ ) and generic $B(B \bar{B})$
- Peaking backgrounds are either vetoed or included in the fit of signal yield extraction

| channel | background source |
| :---: | :---: |
| $B^{0} \rightarrow \eta \mu \mu$ | $B^{0} \rightarrow \eta K \pi$ |
| $B^{0} \rightarrow \pi^{0} \mu \mu$ | $B^{0} \rightarrow \pi^{0} K \pi \& B^{0} \rightarrow \pi^{0} K K$ |
| $B^{+} \rightarrow \pi^{+} e e$ | $B^{+} \rightarrow K^{+} e e$ |
| $B^{0} \rightarrow \rho^{0} e e$ | $J / \psi \rightarrow e \& K^{*} \rightarrow K \leftrightarrow e$ |
| $B^{+} \rightarrow \rho^{+} \mu \mu$ | $J / \psi \rightarrow \mu \& K^{*} \rightarrow K \leftrightarrow \mu, \rho^{+} \overline{D^{0}}\left(K^{+} \pi^{-}\right)$ |


| source | $\eta e e$ | $\eta \mu \mu$ | $\omega e e$ | $\omega \mu \mu$ | $\pi^{0} e e$ | $\pi^{0} \mu \mu$ | $\pi^{+} e e$ | $\rho^{0} e e$ | $\rho^{0} \mu \mu$ | $\rho^{+} e e$ | $\rho^{+} \mu \mu$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mu$ | - | 0.6 | - | 0.6 | - | 0.6 | - | - | 0.6 | - | 0.6 |
| e | 0.8 | - | 0.8 | - | 0.8 | - | 0.8 | 0.8 | - | 0.8 | - |
| $\pi^{+}$ | 1.0 | 1.0 | 1.0 | 1.0 | - | - | 0.5 | 1.0 | 1.0 | 0.5 | 0.5 |
| $\pi^{0}$ | 2.3 | 2.3 | 2.3 | 2.3 | 2.3 | 2.3 | - | - | - | 2.3 | 2.3 |
| $\gamma$ | 4.0 | 4.0 | - | - | - | - | - | - | - | - | - |
| FastBDT | 7.1 | 6.6 | 7.1 | 6.6 | 7.1 | 6.6 | 1.4 | 1.4 | 0.8 | 7.1 | 6.6 |
| MC statistics | 0.48 | 0.37 | 0.73 | 0.53 | 0.34 | 0.24 | 0.24 | 0.53 | 0.34 | 0.80 | 0.54 |
| decay model | 0.57 | 0.45 | 0.75 | 0.69 | 0.49 | 0.76 | 0.40 | 0.66 | 0.51 | 0.81 | 0.52 |
| mass window | 1.05 | 1.05 | 1.21 | 1.21 | - | - | - | 3.03 | 3.03 | 3.03 | 3.03 |
| BCS | 0.03 | 0.11 | 0.15 | 0.43 | 0.21 | 0.23 | 0.11 | 0.02 | 1.09 | 0.6 | 0.5 |
| Tracking | $0.7-1.4$ | $0.7-1.4$ | 1.4 | 1.4 | 0.7 | 0.7 | 1.05 | 1.4 | 1.4 | 1.05 | 1.05 |
| PDF shape | 0.04 | 0.04 | 0.43 | 0.07 | 0.10 | 0.09 | 0.50 | 0.20 | 0.06 | 0.34 | 0.32 |
| $f^{+-/ 00}$ | 2.45 | 2.45 | 2.45 | 2.45 | 2.45 | 2.45 | 2.45 | 2.45 | 2.45 | 2.45 | 2.45 |
| $N_{B \bar{B}}$ | 1.4 | 1.4 | 1.4 | 1.4 | 1.4 | 1.4 | 1.4 | 1.4 | 1.4 | 1.4 | 1.4 |
| Total | 9.35 | 8.95 | 8.37 | 7.91 | 8.07 | 7.64 | 3.56 | 4.80 | 4.76 | 8.75 | 8.29 |

TABLE XVII: Systematic uncertainties for $b \rightarrow d \ell \ell$ decay channels. The uncertainties are shown in \%.

## $b \rightarrow$ dll results

| channel | $\mathcal{B}^{\mathrm{UL}}\left(10^{-8}\right)$ | $\mathcal{B}\left(10^{-8}\right)$ |
| :---: | :---: | :---: |
| $B^{0} \rightarrow \eta e^{+} e^{-}$ | $<10.5$ | $0.0_{-3.4}^{+4.9} \pm 0.1$ |
| $B^{0} \rightarrow \eta \mu^{+} \mu^{-}$ | $<9.4$ | $1.9_{-2.5}^{+3.4} \pm 0.2$ |
| $B^{0} \rightarrow \eta \ell^{+} \ell^{-}$ | $<4.8$ | $1.3_{-2.2}^{+2.8} \pm 0.1$ |
| $B^{0} \rightarrow \omega e^{+} e^{-}$ | $<30.7$ | $-2.1_{-20.8}^{+26.5} \pm 0.2$ |
| $B^{0} \rightarrow \omega \mu^{+} \mu^{-}$ | $<24.9$ | $7.7_{-7.8}^{+10.5} \pm 0.6$ |
| $B^{0} \rightarrow \omega \ell^{+} \ell^{-}$ | $<22.0$ | $6.4_{-7.8}^{+10.7} \pm 0.5$ |
| $B^{0} \rightarrow \pi^{0} e^{+} e^{-}$ | $<7.9$ | $-5.8_{-2.8}^{+3.6} \pm 0.5$ |
| $B^{0} \rightarrow \pi^{0} \mu^{+} \mu^{-}$ | $<5.9$ | $-0.4_{-2.6}^{+3.5} \pm 0.1$ |
| $B^{0} \rightarrow \pi^{0} \ell^{+} \ell^{-}$ | $<3.8$ | $-2.3_{-1.5}^{+2.1} \pm 0.2$ |
| $B^{+} \rightarrow \pi^{+} e^{+} e^{-}$ | $<5.4$ | $0.1_{-1.8}^{+2.7} \pm 0.1$ |
| $B^{0} \rightarrow \rho^{0} e^{+} e^{-}$ | $<45.5$ | $23.6_{-11.2}^{+14.6} \pm 1.1$ |
| $B^{+} \rightarrow \rho^{+} e^{+} e^{-}$ | $<46.7$ | $-38.2_{-17.2}^{+24.5} \pm 3.4$ |
| $B^{+} \rightarrow \rho^{+} \mu^{+} \mu^{-}$ | $<38.1$ | $13.0_{-13.3}^{+17.5} \pm 1.1$ |
| $B^{+} \rightarrow \rho^{+} \ell^{+} \ell^{-}$ | $<18.9$ | $2.5_{-11.8}^{+14.6} \pm 0.2$ |

## $B^{0} \rightarrow \gamma \gamma$ at Belle \& Belle II

- Signal reconstruction by two back-to-back highly energetic photons, with $E_{\gamma}^{*} \in[1.4-3.4] \mathrm{GeV}$
- To reject background from Bhabha scattering or $e^{+} e^{-} \rightarrow \gamma \gamma$, timing requirement is applied
- Classifier is used to distinguish photon from $K_{L}^{0}$ showers
- Used classifiers to reject background from continuum, $\pi^{0} \rightarrow \gamma \gamma$ and $\eta \rightarrow \gamma \gamma$

TABLE II. Summary of multiplicative systematic uncertainties.

| Source | Belle <br> $(\%)$ | Belle II <br> $(\%)$ |
| :--- | :--- | :--- |
| Photon detection efficiency | 4.0 | 2.7 |
| MC statistics | 0.4 | 0.3 |
| Number of $B \bar{B}$ pairs | 1.3 | 1.5 |
| $f^{00}$ | 2.5 | 2.5 |
| $C_{\text {BDT requirement }}$ | 0.4 | 0.9 |
| $\pi^{0} / \eta$ veto | 0.4 | 0.6 |
| Timing requirement efficiency | 2.8 | - |
| Total (sum in quadrature) | 5.7 | 4.1 |

- Photon detection systematic is dominate, obtained using recoil technique in radiative Bhabha events $e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma$ in Belle, and $e^{+} e^{-} \rightarrow \mu^{+} \mu^{-} \gamma$ in Belle II


## $B \rightarrow K \nu \bar{\nu}$ at Belle II

- ITA: low purity ( $0.8 \%$ ) and high efficiency (8\%)
- HTA: high purity (3.5\%) and low efficiency (0.4\%)
- Inclusive properties of tag- $B$ is used to suppress bkg from signal side
- Background suppression:
- ITA: 2 consecutive classifiers to suppress continuum and generic $B$ backgrounds
- HTA: 1 classifier to suppress continuum and generic $B$ backgrounds


## $B^{+} \rightarrow K^{+} \nu \bar{\nu}$ ITA result validation

- Trained two consecutive BDTs and signal efficiency was checked with $B^{+} \rightarrow J / \psi K^{+}$decays:
- Remove $J / \psi$ and correct $K^{+}$kinematics to match $K^{+} \nu \bar{\nu}$

- Closure test with measurement of $\mathcal{B}\left(B^{+} \rightarrow \pi^{+} K_{S}^{0}\right)=(2.5 \pm 0.5) \times 10^{-5}$
- Result is compatible with PDG value of $(2.38 \pm 0.08) \times 10^{-5}$
- Controlled background using
- Off-resonance data for continuum background
- Background from charmless hadronic $B$ decays with $K_{L}^{0}$ or neutrons i.e., $B^{+} \rightarrow K^{+} K^{0} \overline{K^{0}}$, $B^{+} \rightarrow K^{+} n \bar{n}$, and $B \rightarrow K^{*} \nu \bar{\nu}$, are considered.
- $B^{+} \rightarrow K^{+} K^{0} \overline{K^{0}}$ bkg is validated by reconstructing $B^{+} \rightarrow K^{+} K_{S}^{0} K_{S}^{0}$ and $B^{0} \rightarrow K_{S}^{0} K^{+} K^{-}$decays
- $B \rightarrow D\left(\rightarrow K^{+} X\right) \ell^{-} \nu_{\ell}$ background is studied by combining $K^{+}$with other charged tracks in the event
- Used pion-enhanced sideband for misidentification study


## $B \rightarrow K \nu \bar{\nu}$ at Belle II

| Source | Correction | Uncertainty type | Uncertainty size | Impact on $\mu$ |
| :---: | :---: | :---: | :---: | :---: |
| Normalization of continuum and $B \bar{B}$ background | - | Global, 7 NP | 50\% | 0.87 |
| Leading $B$-decays branching fractions | - | Shape, 5 NP | $O(1 \%)$ | 0.22 |
| Branching fraction for $B^{+} \rightarrow K^{+} K_{L}^{0} K_{L}^{0}$ | $q^{2}$ dependent $O(100 \%)$ | Shape, 1 NP | 20\% | 0.48 |
| $p$-wave component for $B^{+} \rightarrow K^{+} K_{S}^{0} K_{L}^{0}$ | $q^{2}$ dependent $O(100 \%)$ | Shape, 1 NP | 30\% | 0.02 |
| Branching fraction for $B \rightarrow D^{(* *)}$ | - | Shape, 1 NP | 50\% | 0.41 |
| Branching fraction for $B^{+} \rightarrow n \bar{n} K^{+}$ | $q^{2}$ dependent $O(100 \%)$ | Shape, 1 NP | 100\% | 0.20 |
| Branching fraction for $D \rightarrow K_{L} X$ | +30\% | Shape, 1 NP | 10\% | 0.14 |
| Continuum background modeling, $\mathrm{BDT}_{c}$ | Multivariate $O(10 \%)$ | Shape, 1 NP | 100\% of correction | 0.01 |
| Integrated luminosity | - | Global, 1 NP | 1\% | $<0.01$ |
| Number of $B \bar{B}$ | - | Global, 1 NP | 1.5\% | 0.02 |
| Off-resonance sample normalization | - | Global, 1 NP | 5\% | $<0.01$ |
| Track finding efficiency | - | Shape, 1 NP | 0.3\% | 0.20 |
| Signal kaon PID | $p, \theta$ dependent $O(10-100 \%)$ | Shape, 7 NP | $O(1 \%)$ | 0.07 |
| Photon energy scale | - | Shape, 1 NP | 0.5\% | 0.07 |
| Hadronic energy scale | -10\% | Shape, 1 NP | 10\% | 0.35 |
| $K_{L}^{0}$ efficiency in ECL | -17\% | Shape, 1 NP | 8\% | 0.21 |
| Signal SM form factors | $q^{2}$ dependent $O(1 \%)$ | Shape, 3 NP | $O(1 \%)$ | 0.02 |
| Global signal efficiency | - | Global, 1 NP | $3 \%$ | 0.03 |
| MC statistics | - | Shape, 156 NP | $O(1 \%)$ | 0.52 |



## $R\left(X_{\tau / \ell}\right)$ at Belle II

- Main backgrounds from hadrons misidentiied as leptons and leptons originating from charmed hadrons
- Suppressed $\mu$ fakes from $\pi$ or $K$ by rejecting $\omega \rightarrow \pi^{+} \pi^{-} \pi^{0}, K^{* 0} \rightarrow \pi^{-} K^{+}, D^{0} \rightarrow K^{-} \pi^{-} \pi^{+} \pi^{+}$, $D^{+} \rightarrow \pi^{+} \pi^{+} \pi^{-}+\left[\pi^{0}\right.$ or $\left.\pi^{+} \pi^{-}\right]$, and $D^{+} \rightarrow K^{-} \pi^{+} \pi^{+}\left(\pi^{0}\right)$
- $B \rightarrow X_{c} \rightarrow \ell$ background modelling: reweight $B \rightarrow X \tau(\ell) \nu$ events by taking exp-to-sim ratio in $M_{X}$ bin using high $p_{\ell}^{B}$ sample



## $R\left(X_{\tau / \ell}\right)$ at Belle II

| Source | Uncertainty [\%] |  |  |
| :--- | :---: | :---: | :---: |
|  | $e$ | $\mu$ | $\ell$ |
| Experimental sample size | 8.8 | 12.0 | 7.1 |
| Simulation sample size | 6.7 | 10.6 | 5.7 |
| Tracking efficiency | 2.9 | 3.3 | 3.0 |
| Lepton identification | 2.8 | 5.2 | 2.4 |
| $X_{c} \ell \nu$ reweighting | 7.3 | 6.8 | 7.1 |
| $B \bar{B}$ background reweighting | 5.8 | 11.5 | 5.7 |
| $X \ell \nu$ branching fractions | 7.0 | 10.0 | 7.7 |
| $X \tau \nu$ branching fractions | 1.0 | 1.0 | 1.0 |
| $X_{c} \tau(\ell) \nu$ form factors | 7.4 | 8.9 | 7.8 |
| Total | 18.1 | 25.6 | 17.3 |

- Dominated systematic uncertainties are experimental and simulated sample sizes as normalization
- Control sample reweighting procedure: $B \bar{B}$ background shapes uncertainties are associated with simulation reweighting
- Background from $B \rightarrow X_{c} \ell \nu$ is suppressed by applying $p_{\ell}^{*}>1.0$ or $>1.4 \mathrm{GeV} / \mathrm{c}^{2}$ for $B^{0} \rightarrow \pi^{-} \ell^{+} \nu_{\ell}$ and $B^{+} \rightarrow \rho^{0} \ell^{+} \nu_{\ell}$

| $B^{0} \rightarrow \pi^{-} \ell^{+} \nu_{\ell}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Source | $q 1$ | $q 2$ | q3 | $q 4$ | $q 5$ | $q 6$ | $q 7$ | $q 8$ | $q 9$ | $q 10$ | $q 11$ | $q 12$ | $q 13$ |
| Detector effects | 2.0 | 0.9 | 1.1 | 1.0 | 1.0 | 1.1 | 1.1 | 1.0 | 0.9 | 1.2 | 2.3 | 4.1 | 5.8 |
| Beam energy | 0.6 | 0.8 | 0.7 | 0.8 | 0.7 | 0.6 | 0.6 | 0.6 | 0.5 | 0.5 | 0.5 | 0.6 | 0.7 |
| Simulated sample size | 4.7 | 3.8 | 3.3 | 3.2 | 3.2 | 2.9 | 3.8 | 3.7 | 4.0 | 4.5 | 5.9 | 8.0 | 13.6 |
| BDT efficiency | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 |
| Physics constraints | 2.9 | 2.9 | 2.9 | 2.9 | 2.9 | 2.9 | 2.9 | 2.9 | 2.9 | 2.9 | 2.9 | 2.9 | 2.9 |
| Signal model | 0.1 | 0.1 | 0.2 | 0.1 | 0.0 | 0.2 | 0.2 | 0.4 | 0.3 | 0.8 | 0.9 | 0.2 | 4.9 |
| $\rho$ lineshape | 0.1 | 0.1 | 0.3 | 0.3 | 0.2 | 0.1 | 0.3 | 0.1 | 0.3 | 0.1 | 0.2 | 0.2 | 0.6 |
| Nonres. $B \rightarrow \pi \pi \ell \nu_{\ell}$ | 0.5 | 0.6 | 0.4 | 0.4 | 0.5 | 1.0 | 1.2 | 1.0 | 0.8 | 1.8 | 1.2 | 2.3 | 14.3 |
| DFN parameters | 0.8 | 0.4 | 1.5 | 1.6 | 1.4 | 1.7 | 1.2 | 0.1 | 0.7 | 1.2 | 2.9 | 3.5 | 3.7 |
| $B \rightarrow X_{u} \ell \nu_{\ell}$ model | 0.2 | 0.4 | 0.3 | 0.4 | 0.2 | 0.9 | 1.1 | 1.2 | 1.0 | 1.3 | 1.6 | 0.7 | 8.7 |
| $B \rightarrow X_{c} \ell \nu_{\ell}$ model | 1.4 | 2.0 | 1.7 | 1.3 | 1.3 | 1.4 | 1.8 | 1.6 | 1.3 | 1.4 | 1.1 | 0.5 | 1.7 |
| Continuum | 15.1 | 11.3 | 7.6 | 7.1 | 5.8 | 5.7 | 8.1 | 8.3 | 9.6 | 10.4 | 14.5 | 23.8 | 34.4 |
| Total systematic | 16.4 | 12.6 | 9.3 | 8.7 | 7.7 | 7.7 | 10.0 | 9.9 | 11.1 | 12.2 | 16.6 | 26.0 | 41.6 |
| Statistical | 11.0 | 8.8 | 7.9 | 7.0 | 7.5 | 6.4 | 7.9 | 7.7 | 9.1 | 10.7 | 9.6 | 14.6 | 22.6 |
| Total | 19.7 | 15.4 | 12.2 | 11.2 | 10.7 | 10.0 | 12.7 | 12.6 | 14.4 | 16.3 | 19.1 | 29.8 | 47.3 |


| $B^{+} \rightarrow \rho^{0} \ell^{+} \nu_{\ell}$ |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Source | $q 1$ | $q 2$ | $q 3$ | $q 4$ | $q 5$ | $q 6$ | $q 7$ | $q 8$ | $q 9$ | $q 10$ |
| Detector effects | 2.8 | 2.0 | 1.6 | 1.1 | 1.7 | 1.9 | 2.4 | 1.4 | 1.4 | 1.6 |
| Beam energy | 2.1 | 1.9 | 1.9 | 1.5 | 1.3 | 1.1 | 1.0 | 0.9 | 0.8 | 0.5 |
| Simulated sample size | 14.1 | 7.8 | 7.4 | 6.3 | 6.3 | 5.2 | 6.4 | 5.6 | 6.2 | 7.3 |
| BDT efficiency | 1.6 | 1.6 | 1.6 | 1.6 | 1.6 | 1.6 | 1.6 | 1.6 | 1.6 | 1.6 |
| Physics constraints | 2.8 | 2.8 | 2.8 | 2.8 | 2.8 | 2.8 | 2.8 | 2.8 | 2.8 | 2.8 |
| Signal model | 0.7 | 0.2 | 0.2 | 0.2 | 0.3 | 0.4 | 0.5 | 0.3 | 1.8 | 2.4 |
| $\rho$ lineshape | 1.7 | 1.6 | 2.0 | 1.0 | 1.9 | 1.8 | 1.4 | 0.9 | 1.6 | 1.7 |
| Nonres. $B \rightarrow \pi \pi \ell \nu_{\ell}$ | 5.6 | 6.3 | 6.7 | 8.6 | 9.3 | 10.7 | 10.1 | 7.0 | 7.8 | 11.8 |
| DFN parameters | 3.6 | 5.5 | 4.1 | 3.5 | 1.1 | 1.2 | 2.7 | 1.7 | 1.9 | 2.3 |
| $B \rightarrow X_{u} \ell \nu_{\ell}$ model | 1.7 | 3.0 | 3.8 | 5.0 | 5.8 | 6.1 | 6.3 | 1.9 | 7.2 | 12.4 |
| $B \rightarrow X_{c} \ell \nu_{\ell}$ model | 1.8 | 1.9 | 1.7 | 1.1 | 1.4 | 1.7 | 0.9 | 0.9 | 1.9 | 2.6 |
| Continuum | 31.5 | 24.3 | 17.0 | 19.6 | 13.2 | 14.8 | 16.0 | 16.6 | 15.2 | 18.7 |
| Total systematic | 35.6 | 27.5 | 21.0 | 23.5 | 18.8 | 20.5 | 21.6 | 19.4 | 20.2 | 27.0 |
| Statistical | 30.0 | 17.5 | 20.8 | 14.4 | 12.4 | 13.6 | 14.1 | 10.4 | 12.2 | 11.8 |
| Total | 46.6 | 32.6 | 29.6 | 27.6 | 22.6 | 24.6 | 25.8 | 22.0 | 23.6 | 29.5 |

- Continuum reweighting is dominate systematic and limited by off-resonance sample size (42 fb ${ }^{-1}$ )


## $B^{0} \rightarrow \eta^{\prime} K_{S}^{0}, \quad B^{0} \rightarrow K_{S}^{0} \pi^{0} \gamma$, and $B^{0} \rightarrow \pi^{0} \pi^{0}$

Table II: Summary of systematic uncertainties for $C_{\eta^{\prime} K_{S}^{0}}$ and $S_{\eta^{\prime} K_{S}^{0}}$.

| Source | $C_{\eta^{\prime} K_{S}^{0}}$ | $S_{\eta^{\prime} K_{S}^{0}}$ |
| :--- | ---: | ---: |
| Signal and continuum yields | $<0.001$ | 0.002 |
| SxF and $B \bar{B}$ yields | $<0.001$ | 0.006 |
| $C_{\text {BDT }}$ mismodeling | 0.004 | 0.010 |
| Signal and background modeling | 0.020 | 0.014 |
| Observable correlations | 0.008 | 0.001 |
| $\Delta t$ resolution fixed parameters | 0.005 | 0.009 |
| $\Delta t$ resolution model | 0.004 | 0.019 |
| Flavor tagging | 0.007 | 0.004 |
| $\tau_{B^{0} \text { and } \Delta m_{d}} \quad<0.001$ | 0.002 |  |
| Fit bias | 0.003 | 0.002 |
| Tracker misalignment | 0.004 | 0.006 |
| Momentum scale | 0.001 | 0.001 |
| Beam spot | 0.002 | 0.002 |
| $B$-meson motion in the $\Upsilon(4 S)$ frame | $<0.001$ | 0.017 |
| Tag-side interference | 0.005 | 0.011 |
| $B \bar{B}$ background asymmetry | 0.008 | 0.006 |
| Candidate selection | 0.007 | 0.009 |
| Total | 0.027 | 0.037 |


| Source | $K^{* 0} \gamma$ |  | $K_{S}^{0} \pi^{0} \gamma$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $S$ | $C$ | $S$ | C |
| $E$ and $p$ scales | $\pm 0.017$ | $\pm 0.015$ | $\pm 0.083$ | $\pm 0.047$ |
| Vertex measurement | $\pm 0.021$ | $\pm 0.009$ | $\pm 0.023$ | 士0.036 |
| Flavor tagging | $\pm 0.005$ | ${ }_{-0.009}^{+0.012}$ | ${ }_{-0.009}^{+0.008}$ | ${ }_{-0.009}^{+0.013}$ |
| Signal modeling | $\pm 0.003$ | $\pm 0.003$ | $\pm 0.032$ | $\pm 0.013$ |
| $\Delta t$ resolution function | $\pm 0.014$ | $\pm 0.009$ | $\pm 0.031$ | $\pm 0.013$ |
| $\tau_{B^{0}}$ and $\Delta m_{d}$ | $<0.001$ | $<0.001$ | $\pm 0.003$ | < 0.001 |
| $B \bar{B}$ background asym. | ${ }_{-0.008}^{+0.007}$ | $\pm 0.011$ | ${ }_{-0.026}^{+0.030}$ | ${ }^{+0.049}$ |
| Tag-side interference | $<0.001$ | -0.002 | +0.001 | +0.001 |
| Total | $\pm 0.032$ | ${ }_{-0.025}^{+0.026}$ | ${ }_{-0.100}^{+0.102}$ | $\pm 0.080$ |
| Source |  |  | $\mathcal{B}$ | $\mathcal{A}_{C P}$ |
| $\pi^{0}$ efficiency |  |  | 8.6 \% |  |
| $\Upsilon(4 S)$ branching frac | tions ( $1+$ | $\left.f^{+-} / f^{00}\right)$ | 2.5 \% |  |
| Continuum-suppressi | n efficienc |  | 1.9 \% |  |
| $B \bar{B}$-background mod |  |  | 1.7 \% | 0.034 |
| Sample size $N_{B \bar{B}}$ |  |  | $1.5 \%$ |  |
| Signal model |  |  | 1.2 \% | 0.021 |
| Continuum-backgrou | d model |  | 0.9 \% | 0.025 |
| Wrong-tag probability | calibrati |  | n/a | 0.008 |
| Total systematic unce | rtainty |  | 9.6 \% | 0.048 |
| Statistical uncertainty |  |  | 15.9 \% | 0.303 |

- For $B^{0} \rightarrow K_{S}^{0} \pi^{0} \gamma$ dominate systematic from vertex measurement: as main challenge was to find $B^{0}$ vertex without prompt tracks
- Used $K_{S}^{0} \rightarrow \pi^{+} \pi^{-}$information and beamspot constraint

