

# An axion-like particle explanation of $B \rightarrow \pi K$ puzzle and $B^+ \rightarrow K^+ \nu \bar{\nu}$ excess

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In preparation 2406 . xxxx

Chennai Mathematical Institute

- $B \rightarrow \pi K$  decays
- $B \rightarrow K \nu \bar{\nu}$  decays
- ALP in  $B$  decays
- Conclusion

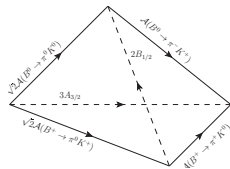
- The four  $B \rightarrow \pi K$  decay amplitudes are related by isospin,
- $A_{1/2}$ ,  $A_{3/2}$  and  $B_{1/2}$  are isospin amplitudes corresponding to  $\Delta I = 1$  and  $\Delta I = 0$  part of the effective Hamiltonian.

$$A(B^+ \rightarrow \pi^+ K^0) = B_{1/2} + A_{1/2} + A_{3/2},$$

$$A(B^+ \rightarrow \pi^0 K^+) = -\frac{1}{\sqrt{2}} (B_{1/2} + A_{1/2}) + \sqrt{2}A_{3/2}$$

$$A(B^0 \rightarrow \pi^- K^+) = -B_{1/2} + A_{1/2} + A_{3/2},$$

$$A(B^0 \rightarrow \pi^0 K^0) = \frac{1}{\sqrt{2}} (B_{1/2} - A_{1/2}) + \sqrt{2}A_{3/2}.$$



- Isospin relation

$$\sqrt{2}\mathcal{A}(B^0 \rightarrow \pi^0 K^0) - \sqrt{2}\mathcal{A}(B^+ \rightarrow \pi^0 K^+) = \mathcal{A}(B^+ \rightarrow \pi^+ K^0) - \mathcal{A}(B^0 \rightarrow \pi^- K^+)$$

- $B \rightarrow \pi K$ : hadronic weak decays with  $|\Delta S|=1$ ,  $\Delta S$  being change in strangeness,
- Underlying quark level transition:  $b \rightarrow u\bar{u}s$ , relevant energy scale  $\sim \mathcal{O}(m_b) \ll m_W$
- The decay mediated by dimension-6 effective Hamiltonian consists of **tree**, **QCD penguin** and **electroweak penguin** four-fermion operators,

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \left[ \lambda_u \left( C_1 (\bar{b}_\alpha u_\beta)_{V-A} (\bar{u}_\beta s_\alpha)_{V-A} + C_2 (\bar{b}_\alpha u_\alpha)_{V-A} (\bar{u}_\beta s_\beta)_{V-A} \right) - \lambda_t \sum_{i=3}^6 C_i Q_i - \lambda_t \sum_{i=7}^{10} C_i Q_i \right]$$

Buras+, Rev. Mod. Phys '96

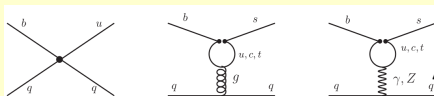
$$Q_{3,5} = (\bar{b}_\alpha s_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V\mp A}$$

$$Q_{4,6} = (\bar{b}_\alpha s_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V\mp A}$$

$$Q_{7,9} = \frac{3}{2} (\bar{b}_\alpha s_\alpha)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\beta)_{V\pm A}$$

$$Q_{8,10} = \frac{3}{2} (\bar{b}_\alpha s_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V\pm A}$$

Scheme	$\Lambda_{\overline{MS}}^{(5)} = 140 \text{ MeV}$			$\Lambda_{\overline{MS}}^{(5)} = 225 \text{ MeV}$			$\Lambda_{\overline{MS}}^{(5)} = 310 \text{ MeV}$		
	LO	NDR	HV	LO	NDR	HV	LO	NDR	HV
$C_1$	-0.273	-0.165	-0.202	-0.308	-0.185	-0.228	-0.339	-0.203	-0.251
$C_2$	1.125	1.072	1.091	1.144	1.082	1.105	1.161	1.092	1.117
$C_3$	0.013	0.013	0.012	0.014	0.014	0.013	0.016	0.016	0.015
$C_4$	-0.027	-0.031	-0.026	-0.030	-0.035	-0.029	-0.033	-0.039	-0.033
$C_5$	0.008	0.008	0.008	0.009	0.009	0.009	0.009	0.009	0.010
$C_6$	-0.033	-0.036	-0.029	-0.038	-0.041	-0.033	-0.043	-0.046	-0.037
$C_7/\alpha$	0.042	-0.003	0.006	0.045	-0.002	0.005	0.047	-0.001	0.005
$C_8/\alpha$	0.041	0.047	0.052	0.048	0.054	0.060	0.054	0.061	0.067
$C_9/\alpha$	-1.264	-1.279	-1.269	-1.280	-1.292	-1.283	-1.294	-1.303	-1.296
$C_{10}/\alpha$	0.291	0.234	0.237	0.328	0.263	0.266	0.360	0.288	0.291



- In terms of topological flavor-flow amplitudes:

$$\mathcal{A}^{-+} = -\lambda_u (P_{uc} + T) - \lambda_t \left( P_{tc} + \frac{2}{3} P_{EW}^c \right),$$

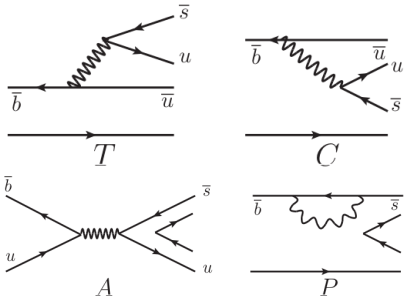
$$\mathcal{A}^{+0} = \lambda_u (P_{uc} + A) + \lambda_t \left( P_{tc} - \frac{1}{3} P_{EW}^c \right),$$

$$\sqrt{2} \mathcal{A}^{00} = \lambda_u (P_{uc} - C) + \lambda_t \left( P_{tc} - P_{EW} - \frac{1}{3} P_{EW}^c \right),$$

$$\sqrt{2} \mathcal{A}^{0+} = -\lambda_u (P_{uc} + T + C + A) - \lambda_t \left( P_{tc} + P_{EW} + \frac{2}{3} P_{EW}^c \right),$$

Gronau+, PRD '94, Fleischer+ PLB '96, PRD '98, Neubert+ PLB '98, Gronau PLB '05,...

- T**: Color-allowed tree
- C**: Color-suppressed tree
- A**: Annihilation
- P**: QCD penguin
- $P_{EW}$  &  $P_{EW}^c$** : Color-allowed and suppressed EW penguin



- In the SM, the relative importance of the flavor flow topologies

$$|\lambda_t P_{tc}| > |\lambda_u T| > |\lambda_u C| > |\lambda_u A|, |\lambda_u P_{uc}|, \quad \text{Gronau+, PRD '95}$$

suppression factor of the order of  $\lambda \approx \sin \theta_C = 0.22$ ,  $\theta_C$ : Cabibbo angle.

- Particularly important ratio  $|C/T| \sim \lambda$ . *Beneke+, Nucl. Phys. B '01*
- A and  $P_{uc}$  expected to be subdominant. Can be neglected at  $\mathcal{O}(\lambda^2)$ .
- The  $SU(3)$ -flavor symmetry is used to establish a relation between the electroweak penguin amplitudes and tree amplitudes.
- In SM, both  $P_{EW}/T$  and  $P_{EW}^C/C$  are approximately the same. Given by a common ratio  $\kappa$ ,

$$\kappa = -\frac{3}{2} \frac{C_9 + C_{10}}{C_1 + C_2} \simeq -\frac{3}{2} \frac{C_9 - C_{10}}{C_1 - C_2} \simeq 0.0135 \pm 0.0012$$

- Ignore diagrams of  $\mathcal{O}(\lambda^2)$  and beyond.  $B \rightarrow \pi K$  amplitudes reduce to

$$\begin{aligned} \mathcal{A}^{-+} &= -\lambda_u T - \lambda_t P_{tc}, \\ \mathcal{A}^{+0} &= \lambda_t P_{tc}, \quad \text{Buras+, EPJC '03, PRL '04, London+, PRD '05} \\ \sqrt{2}\mathcal{A}^{00} &= \lambda_t (P_{tc} - P_{EW}), \\ \sqrt{2}\mathcal{A}^{0+} &= -\lambda_u T - \lambda_t (P_{tc} + P_{EW}). \end{aligned}$$

- Direct CP asymmetries in  $B^0 \rightarrow \pi^- K^+$  and  $B^+ \rightarrow \pi^0 K^+$  due to  $T$ - $P_{tc}$  interference,
  - a non-zero relative strong phase,
  - weak phase difference between  $T$  and  $P_{tc}$ .
- $P_{EW}$  and  $T$  carry the same strong phase, related by  $\kappa$ , which is real in SM.
- $P_{EW}$ - $T$  interference for  $B^+ \rightarrow \pi^0 K^+$  does not contribute to  $A_{CP}$ .
- Expect a simplified relation,

$$A_{CP}(B^0 \rightarrow \pi^- K^+) = A_{CP}(B^+ \rightarrow \pi^0 K^+). \quad \text{Gronau and Rosner, PRD '99}$$

- A measurable quantity sensitive to isospin violation encoded in the observable  $\Delta_4$ :

$$\Delta_4 = A_{CP}(\pi^- K^+) + A_{CP}(\pi^+ K^0) \frac{\mathcal{B}(\pi^+ K^0) \tau_0}{\mathcal{B}(\pi^- K^+) \tau_+} - A_{CP}(\pi^0 K^+) \frac{2\mathcal{B}(\pi^0 K^+) \tau_0}{\mathcal{B}(\pi^- K^+) \tau_+} - A_{CP}(\pi^0 K^0) \frac{2\mathcal{B}(\pi^0 K^0)}{\mathcal{B}(\pi^- K^+)}$$

*Gronau+, PLB '05, PRD '06*

- $\Delta_4 = 0$  holds up to a few percent in the SM.



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## Measurements

$$\Delta_4 = -0.270 \pm 0.132 \pm 0.060 \quad \text{Belle 2012, PRD 2013}$$

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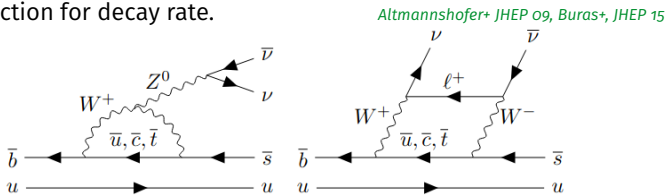
$$\Delta_4 = -0.03 \pm 0.13 \pm 0.04 \quad \text{Belle II 2023, PRD 2024}$$

- Also  $\Delta A_{CP} = A_{CP}(B^+ \rightarrow \pi^0 K^+) - A_{CP}(B^0 \rightarrow \pi^- K^+) = 0.108 \pm 0.017$ ,  
*LHCb, PRL 21*
- Requires large  $\frac{C}{T} \Rightarrow$  "Naive  $B \rightarrow \pi K$  puzzle".

*Datta+, JHEP 17, PRDL 21, Fleischer+ PLB 18*

# $B^+ \rightarrow K^+ \nu \bar{\nu}$ decay

- Semileptonic FCNC decay with negligible hadronic uncertainty. Accurate SM prediction for decay rate.

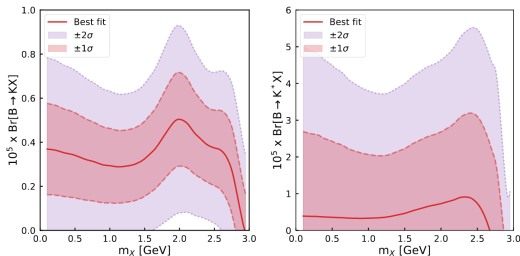


- $Br(B^+ \rightarrow K^+ \nu \bar{\nu})|_{\text{SM}} = (5.58 \pm 0.37) \times 10^{-6}$  C. Davies+, PoS LATTICE2022, 421 (2023)
- Experimentally the final state neutrinos are not reconstructed, signal looks identical to  $B^+ \rightarrow K^+ \cancel{E}$ .
- The measured branching ratio in Belle II Belle-II, PRD '24

$$Br(B^+ \rightarrow K^+ \nu \bar{\nu})|_{\text{exp}} = (2.3 \pm 0.5^{+0.5}_{-0.4}) \times 10^{-5}$$

- A deviation of  $2.7\sigma$  from the SM expectation.
- Several interpretations  $\dots$  McKeen+ '23, He+ PRD '24, Fridell+ '24, Altmannshofer+ PRD '24  $\dots$

# $B^+ \rightarrow K^+ \nu \bar{\nu}$ decay



Altmannshofer+ PRD '24

Fit of  $Br(B \rightarrow KX)$  from Belle II and BaBar as a function  $m_X$ ,

- The  $B^+ \rightarrow K^+ \nu \bar{\nu}$  events may contain  $B^+ \rightarrow K^+ a$  decays, where  $a$  is a long-lived axion-like particle.
- **Assumption:**  $a$  decays dominantly to two photons. Agnostic about the origin of  $a\gamma\gamma$  coupling.
- Can mimic the signal for  $\pi^0 \rightarrow \gamma\gamma$ , therefore challenging to distinguish from actual  $\pi^0$  decays if  $m_{\pi^0} \simeq m_a$ .
- Most of the  $a$  decays happen outside the Belle-II detector volume.

## Interplay of $B^+ \rightarrow K^+ \nu \bar{\nu}$ and $B^+ \rightarrow K^+ \pi^0$ decays

- The ALP originating in  $B^+ \rightarrow aK^+$  can get misidentified as a  $B^+ \rightarrow \pi^0 K^+$  if  $a \rightarrow \gamma\gamma$  decay happens within the detector.
- Since the signals are indistinguishable,

$$\Gamma(B^+ \rightarrow \pi^0 K^+) |_{\text{exp}} = \Gamma(B^+ \rightarrow K^+ \pi^0) + \Gamma(B^+ \rightarrow K^+ a^0)$$

$$\Gamma(B^0 \rightarrow \pi^0 K^0) |_{\text{exp}} = \Gamma(B^0 \rightarrow K^0 \pi^0) + \Gamma(B^0 \rightarrow K^0 a^0)$$

- Effective Hamiltonian for  $b \rightarrow sa$  decay:

$$\mathcal{L}_{\text{FCNC}} \supset \bar{s}(h_{sb}^S + h_{sb}^P \gamma_5) b a + \text{h.c.} \quad \text{Dolan+ JHEP '17, Camalich+ PRD '20, Bauer+ JHEP '22}$$

- Decay rate for  $B \rightarrow aK$  : ( $f_0^B$ :  $B$ -meson decay constant)

$$\text{Br}(B \rightarrow aK) = \tau_B \frac{p_K}{8\pi m_B^2} \frac{(m_B^2 - m_K^2)^2}{(m_b - m_s)^2} |f_0^B|^2 |h_{sb}^S|^2 \quad \text{Ferber+ JHEP '23, Bruggisser+ JHEP '24}$$

# Estimate of $B \rightarrow aK$ decay rate

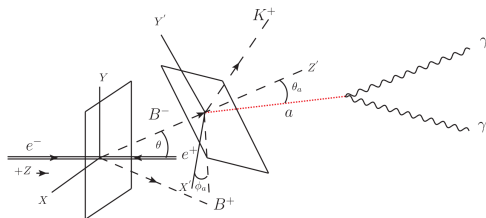
Scenario	$p$ -value	Fit parameter	Fit value
I (SM fit)	0.46	$P_{tc}$	$-0.147 \pm 0.001$
		$\kappa$	$0.013 \pm 0.007$
		$ T $	$1.3 \pm 0.7$
		$ C $	$0.36 \pm 0.10$
		$\delta_T$	$0.19 \pm 0.12$
		$\delta_C$	$4.38 \pm 0.67$
II (SM + ALP)	0.76	$P_{tc}$	$-0.148 \pm 0.001$
		$\kappa$	$0.014 \pm 0.005$
		$ T $	$1.21 \pm 0.44$
		$ C $	$0.56 \pm 0.14$
		$\delta_T$	$3.35 \pm 0.08$
		$\delta_C$	$0.66 \pm 0.21$
		$\Delta$	$0.00021 \pm 0.00009$

$$Br(B^+ \rightarrow aK^+) = 1.12_{-0.75}^{+1.16} \times 10^{-7} (\text{fit})$$

# Condition on the lifetime of ALP

The photon-ALP effective Lagrangian,

$$\mathcal{L}_{a\gamma\gamma} = -\frac{g_{a\gamma\gamma}}{4} a F^{\mu\nu} \tilde{F}_{\mu\nu}$$



The decay probability,

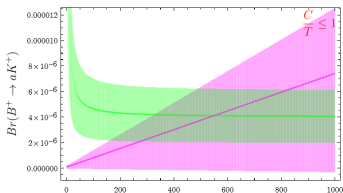
$$f_L(m_B, m_K, m_a = m_{\pi^0}, l_{\max}) = \int_0^{\pi/2} \sin \theta_a d\theta_a \left( 1 - \exp\left(-\frac{m_a l_{\max}}{c\tau_0 |p_L^{\text{lab}}|}\right) \right)$$

where  $l_{\max}$  is the maximum distance from the primary decay vertex up to which ALP decay products can be resolved.

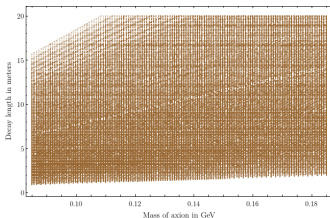
- By assumption,  $\Gamma = \frac{1}{\tau_0} = \frac{g_{a\gamma\gamma}^2}{64\pi m_a^3}$
- Allow for a variation of  $m_a$  around  $m_{\pi^0}$ ,

$$N_a(B^+ \rightarrow \pi^0 K^+) |_{\text{fake}} = N_B \text{Br}(B^+ \rightarrow aK^+) f_L$$

$$N_a(B^+ \rightarrow K^+ \cancel{E}) = N_B \text{Br}(B^+ \rightarrow aK^+) (1 - f_L)$$



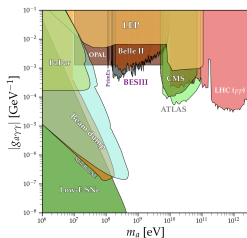
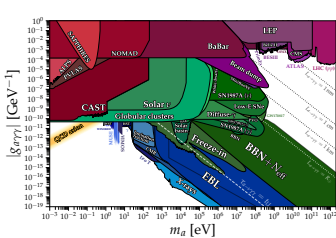
ALP effective lifetime in its rest frame (in centimeters)



- $c\tau_0 \geq 1.6 \text{ m}$
- $\text{Br}(B^+ \rightarrow aK^+) = 4.13_{-2.09}^{+2.09} \times 10^{-6}$



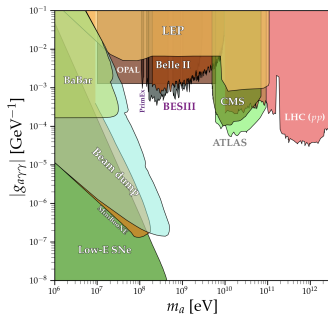
# Existing bounds on $g_{a\gamma\gamma}$



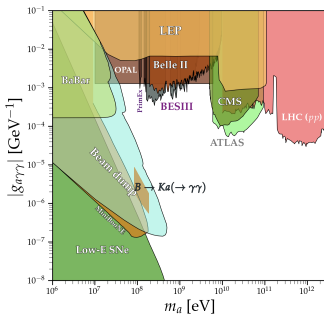
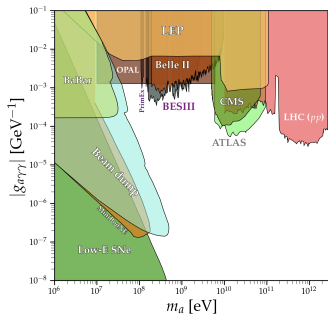
AxionLimits by C O'hare

- The relevant  $g_{a\gamma\gamma} - m_a$  parameter space to be probed in Beam-dump experiments like SHIP.
- Decay volume of the order of several meters.

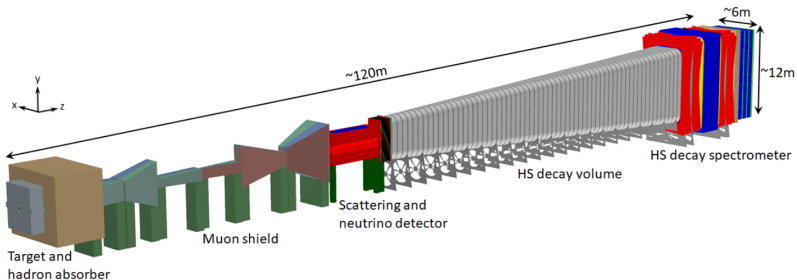
$$B^+ \rightarrow a(\rightarrow \gamma\gamma)K^+ \dots$$



$$B^+ \rightarrow a(\rightarrow \gamma\gamma)K^+ \dots$$



# Production at proposed SHiP experiment (Preliminary!)

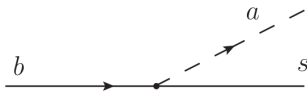


- Indirect production of ALP through  $B \rightarrow a K$  decays,
- Primakoff production of ALP through coherent scattering with target nucleus  $p + N \rightarrow p + N + a$

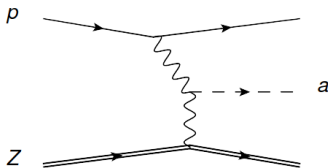
In both cases,  $a$  decays to  $\gamma\gamma$ .

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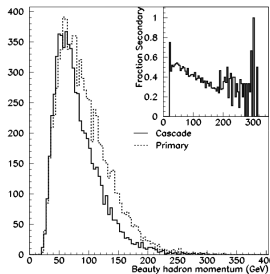
# $B \rightarrow aK$ production (Preliminary!)

- 400 GeV proton beam on Molybdenum target,  $E_{CM} \sim 27.4$  GeV
- $N_{PoT} = 2 \times 10^{20}$ ,  $\sigma_{pN} \simeq 10.7$  mbarn
- $\sigma_{bb} = 1.7$  nanobarn
- $N(B \rightarrow aK) = N_{PoT} \times \frac{\sigma_{bb}}{\sigma_{pN}} \times Br(B \rightarrow aK) = 3.9 \times 10^8$

$$N_{det} = N_{B \rightarrow aK} \epsilon_{det} \times \left( \exp\left[-\frac{l_1}{\gamma\beta c\tau_0}\right] - \exp\left[-\frac{l_2}{\gamma\beta c\tau_0}\right] \right)$$

- $l_1 = 70$  m,  $l_2 = 120$  m

$$N_{det} \sim \mathcal{O}(110 - 330) \quad \text{Preliminary!}$$



SHiP-NOTE, T Ruf+ 15, Mangano+  
Nucl.Phys.B 93

- Calculated using the Weizsäcker-Williams approximation
- Coherent scattering  $\propto Z^2 F(|t|^2)$ ,  $Z$ : atomic number of the target,  $F(t)$ : charge form factor
- ALPs produced typically have a small transverse momentum
- Production strongly peaked in the forward direction,  $\theta_a^{\max} = \frac{m_a^2}{2E_\gamma^2}$
- $E_\gamma \simeq E_a$
- Momentum transfer  $t = (p_\gamma - p_a)^2 = -\frac{m_a^4}{4E_a^2} - p_t^2 + 2E_a p_t \theta_a - E_a^2 \theta_a^2$
- Only those ALPs captured by the detector which has  $\theta_a^{\max} \lesssim \frac{r}{L}$ ,  
 $\theta_a^{\min} > \frac{2L}{d_{\min}} = 5 \text{ mrad}$
- Opening angle of ALP decay products  $\theta_\gamma = \frac{2}{\gamma}$

$$\frac{d\sigma(\gamma N \rightarrow aN)}{d \cos \theta_a} = \frac{\alpha g_{\alpha\gamma}^2 Z^2 F(|t|^2)}{16} \left[ -\frac{(t - m_a^2)^2 + 4E_\gamma^2 t}{t^2} + \frac{2E_\gamma(m_a^2 - t)}{tm_N} - \frac{2E_\gamma((t - m_a^2)^2 + 4E_\gamma t)}{2tm_N} \right]$$

- Total number of detectable events,

$$N_{\text{det}} = \frac{N_{\text{PoT}}}{\sigma_{pN}} \frac{1}{2\pi E_{\text{beam}}} \int d \cos \theta_a dE_a dp_t^2 \gamma_p(E_a/E_{\text{beam}}, p_t^2) \frac{d\sigma(\gamma N \rightarrow aN)}{d \cos \theta_a}, \quad \theta_a \in [0, \min[\frac{m_a^2}{2E_a^2}, \theta_{\text{det}}]]$$

$$\left( \exp\left[-\frac{l_1}{\gamma(E_a)\beta(E_a)c\tau_0}\right] - \exp\left[-\frac{l_2}{\gamma(E_a)\beta(E_a)c\tau_0}\right] \right) \epsilon_{\text{det}}$$

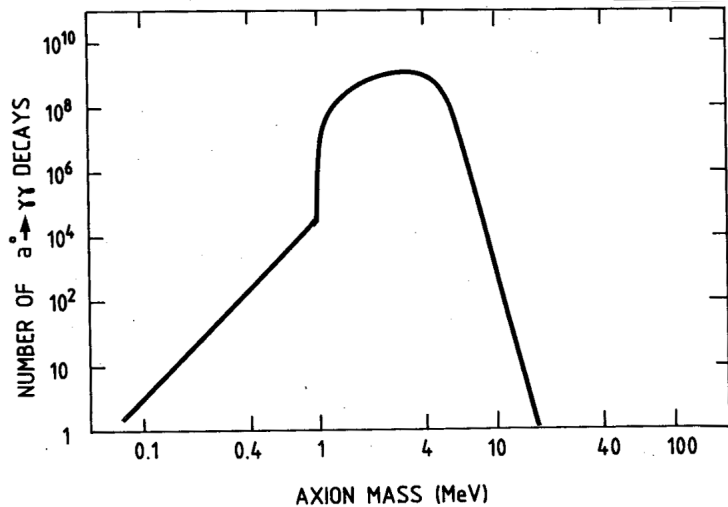
- $N_{\text{det}}^{\text{Primakoff}} \simeq 5$  (Preliminary!)
- Use more realistic angular acceptance ( $\epsilon_{\text{det}}$ ) for improved prediction (In progress)



- ALPs with MeV-to-GeV scale mass dominantly coupled to photons can be probed in collider experiments.
- Long lived ALPs produced in  $B$ -decays contribute to the measured  $B \rightarrow K\nu\bar{\nu}$  decay rate at Belle-II if the ALPs decay outside the detector.
- Provides a simple solution to the  $B \rightarrow \pi K$  puzzle if a tiny fraction of the ALPs decay to two photons within the detector.
- The  $g_{a\gamma\gamma} - m_a$  parameter space is within the sensitivity reach of the upcoming beam dump experiments.
- An additional invisible decay width of the ALP will move the preferred region to larger values of  $g_{a\gamma\gamma}$ .
- The inferred  $bsa$  coupling is well below the constraints coming from  $B_s$  meson mixing.

Thank You

# Number of $a \rightarrow \gamma\gamma$ events in CHARM experiment



# Reach of NA64 (PRL, 081801 (2020))

