Nuclear Matrix Elements for Neutrinoless Double-Beta Decay

J. Engel

June 10, 2024

A Strange Time for Fundamental Physics

Lots problems; solutions hard to come by

Big Open Questions

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- Nature of dark energy and matter
- Orders of magnitude difference between masses of neutrinos and masses of other particles
- Excess of matter over antimatter

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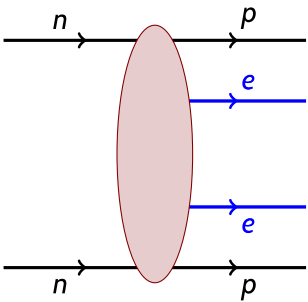
Neutrinoless double-beta decay may help.

If $0\nu\beta\beta$ Decay is Observed ...



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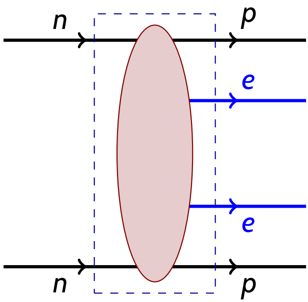

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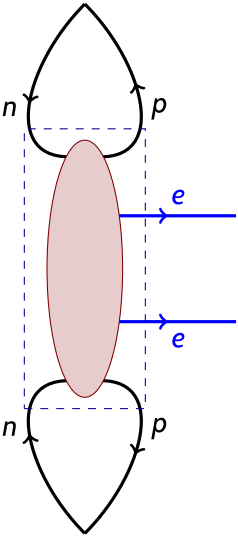

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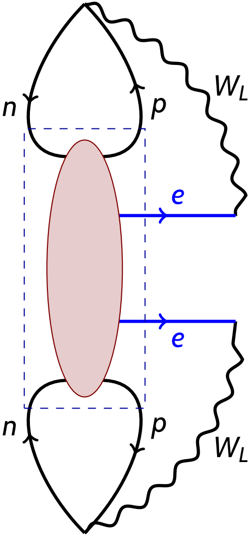

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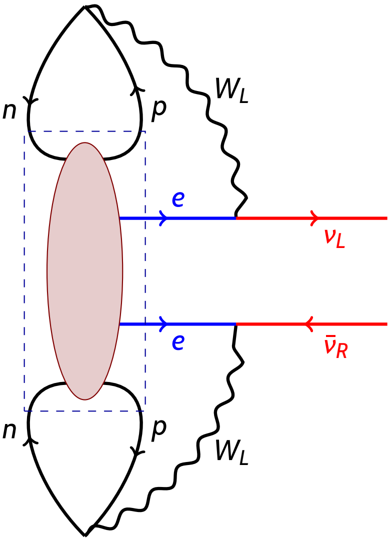

Majorana neutrino propagator

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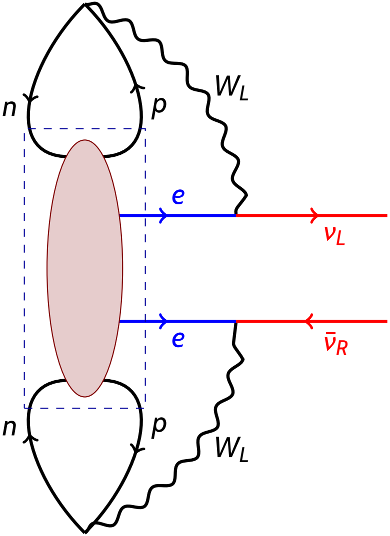


Majorana neutrino propagator

...neutrinos are Majorana particles and lepton number is violated, no matter what physics is responsible.

Explicit Neutrino Physics

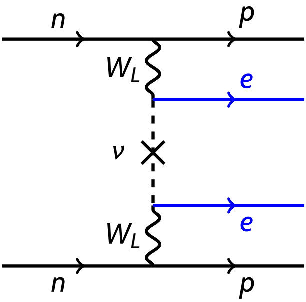


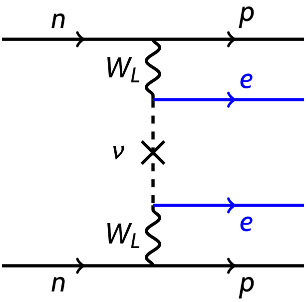
Diagram is proportional to effective "Majorana mass" of light neutrinos,

$$\bar{m}_\nu = \sum_i U_{ei}^2 m_i$$

U_{ei} is amount of mixing of electron flavor with i^{th} mass eigenstate.

If mass hierarchy is inverted, or if it's normal and neutrinos are heavy enough, coming generation of experiments should be able to see the decay.

Explicit Neutrino Physics



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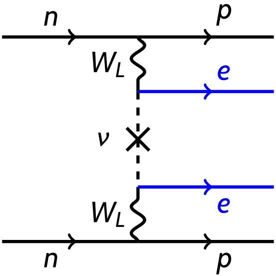
Whatever the hierarchy, the Majorana mass must come from somewhere, and the Standard Model by itself doesn't allow it.

Its presence implies new particles, which would make the low masses of neutrinos natural, and could also change $0\nu\beta\beta$ rate.

New Physics Can Contribute Directly to $\beta\beta$ Decay



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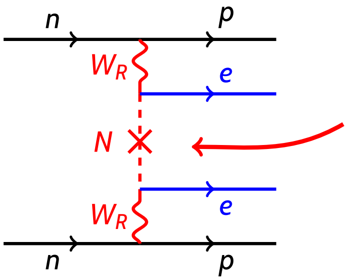

Exchange of heavy right-handed neutrino in left-right symmetric model.

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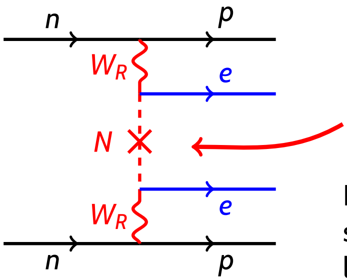
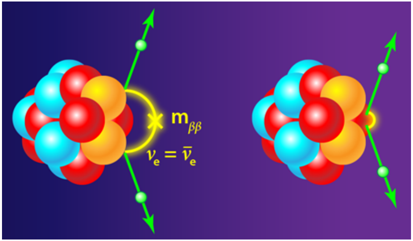


Exchange of heavy right-handed neutrino in left-right symmetric model.

Heavy-particle exchange can occur at the same rate as light-ν exchange (or even a larger rate) if $m_N \approx m_{W_R} \approx 1\,\text{TeV}$.

The Problem



Rate depends on squares of unknown nuclear matrix elements. We need to compute them and assign a believable (and not too large) uncertainty so that experimentalists can better

1. plan their experiments
2. draw conclusions from their results

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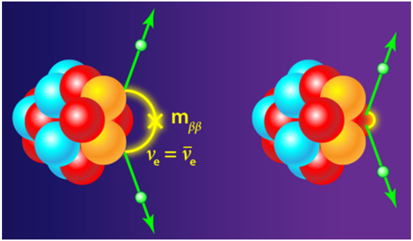

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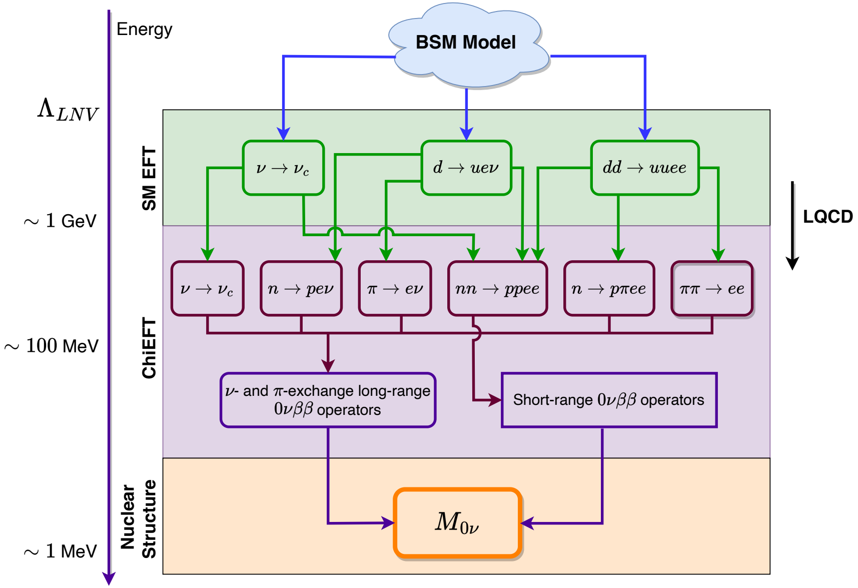
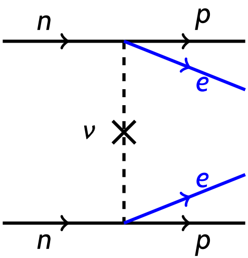
May soon be an NSF theory research hub to address the problem.

How We Understand Things: A Tower of EFTs



Example: Light-ν Exchange at Leading Order



Usual long-range exchange

Most calculations so far include only this.

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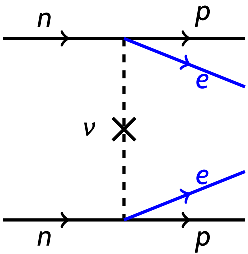

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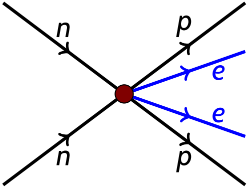

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Contact counter term from short short-range exchange at energies beyond breakdown scale of EFT.

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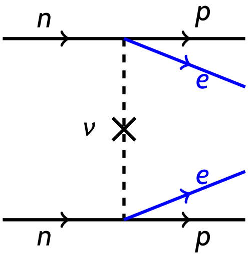

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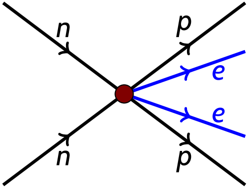

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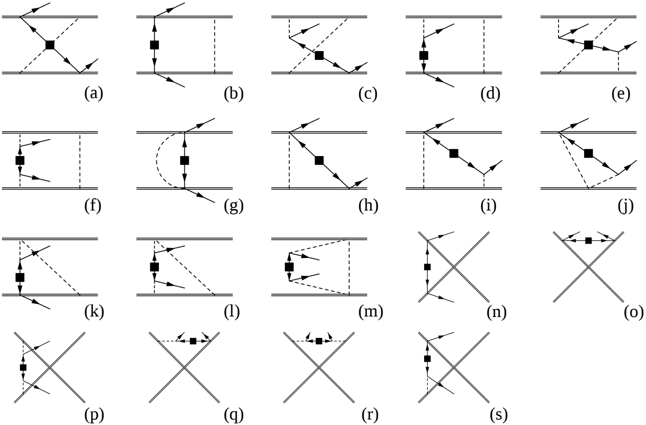
Contact counter term from short short-range exchange at energies beyond breakdown scale of EFT.

At present, coefficient of contact term estimated from large-N QCD or via sum rules.

Can estimates be improved?

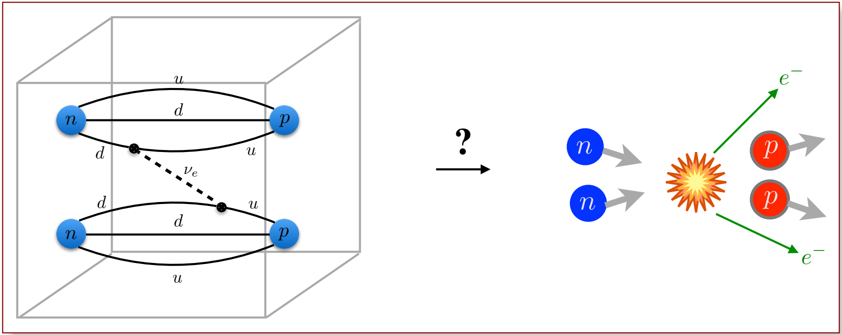
Corrections of order 10% to standard diagram:



Many of the coefficients are still undetermined. Techniques used to estimate leading-order contact should be extended.

Lattice QCD

Determining chiral-EFT coefficients



Chiral-EFT $0\nu\beta\beta$ operator from LQCD (courtesy of Z. Davoudi).

Recent progress in NN systems at large pion mass, $0\nu\beta\beta$ decay involving exchange of neutrinos/heavy particles between pions, steps towards more general calculations.

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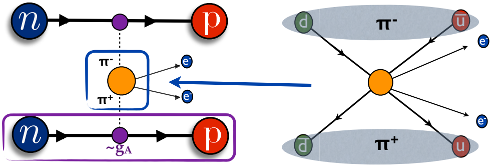

From A. Nicholson

Important LQCD Steps

Determining chiral-EFT coefficients

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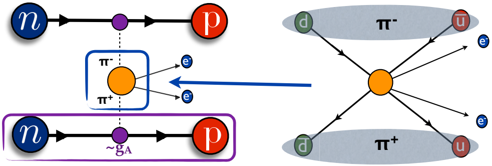

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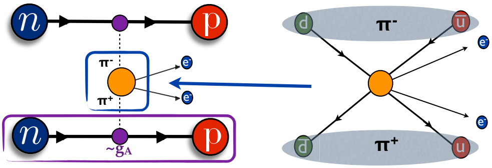

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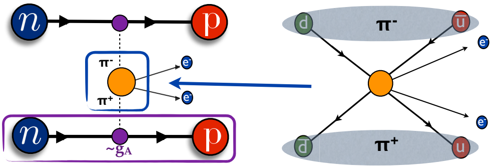

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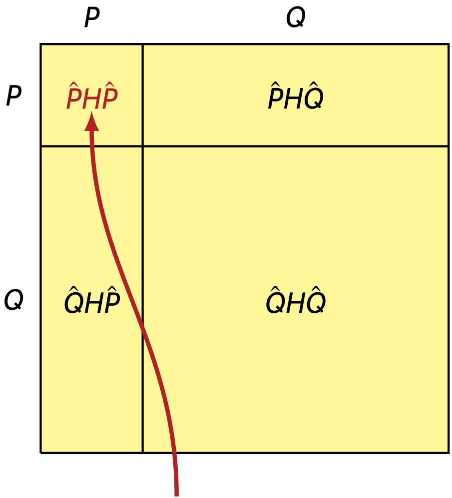
For general case, still need to:

- Improve NN calculations, move to lower pion mass.
- Extract $0\nu\beta\beta$ decay amplitude.
- Develop formalism for matching LQCD amplitudes to chiral EFT couplings.

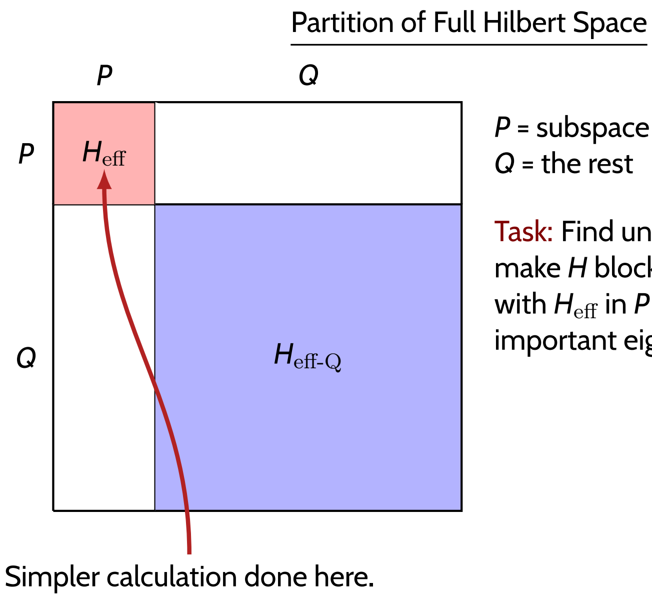
Partition of Full Hilbert Space



Simpler calculation done here.

P = subspace you want
Q = the rest

Task: Find unitary transformation to make *H* block-diagonal in *P* and *Q*, with $H_{\rm eff}$ in *P* reproducing most important eigenvalues.



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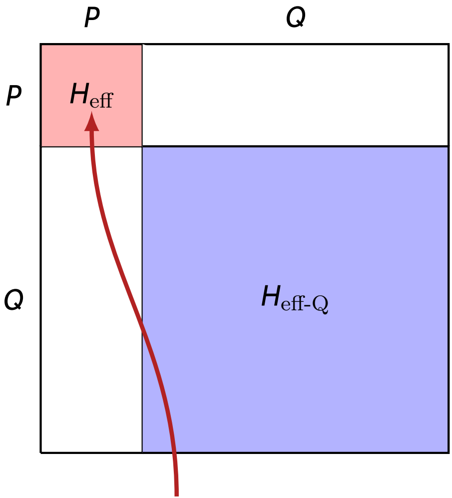

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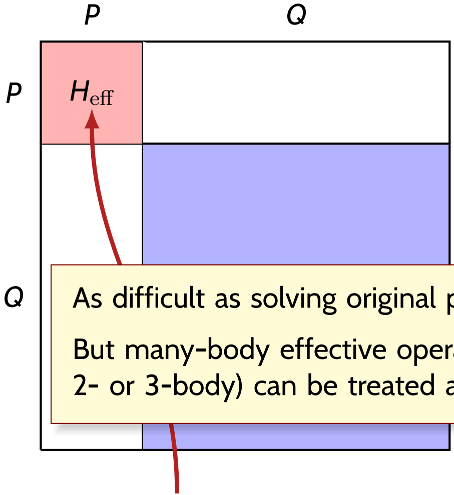
Must must apply same unitary transformation to transition operator.

Ab Initio Nuclear-Structure for Heavy Nuclei

Partition of Full Hilbert Space



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…nvalues.

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operator.

As difficult as solving original problem.

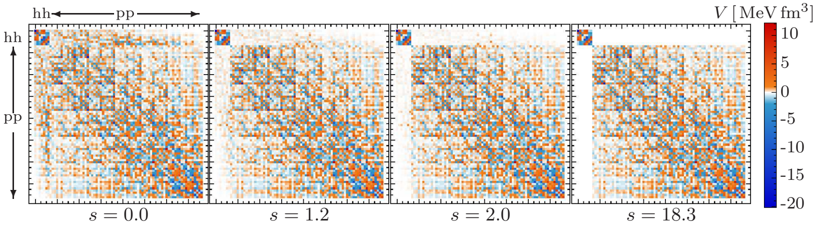
But many-body effective operators (beyond 2- or 3-body) can be treated approximately.

Simpler calculation done here.

In-Medium Similarity Renormalization Group

One way to determine the transformation

Flow equation for effective Hamiltonian.
Gradually decouples selected set of states.



from H. Hergert

Trick is to keep all 1- and 2-body terms in effective Hamiltonian at each step (**IMSRG-2**, also includes "coherent" 3, 4-body ...terms).

If selected set is one state, end up with ground-state energy. If it's a valence space, get effective shell-model interaction and operators.

Ground state in closed-shell nucleus:

$$|\Psi_0\rangle = e^T\,|\varphi_0\rangle \qquad T = \sum_{i,m} t_i^m a_m^\dagger a_i + \sum_{ij,mn} \frac{1}{4} t_{ij}^{mn} a_m^\dagger a_n^\dagger a_i a_j + \ldots$$

Slater determinant

$m,n>F \quad i,j<F$

Here the Hamiltonian is transformed in a non-unitary way:

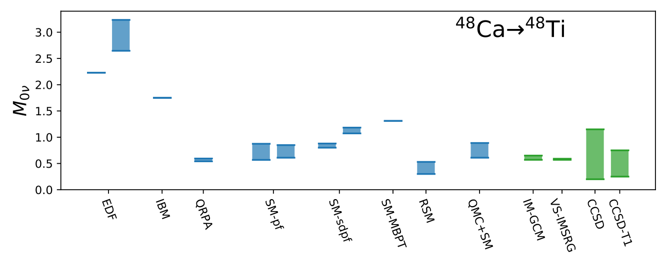
$$H \longrightarrow \tilde{H} \equiv e^{-T} H e^T$$

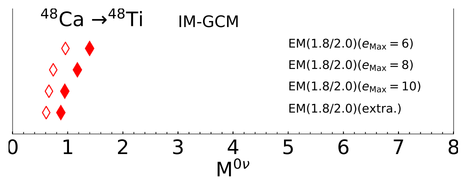
so that $|\varphi_0\rangle$ is its ground state. Must solve algebraic equations for the t's.

Excited states, states in closed-shell + a few nucleons, constructed from simple excitations of $|\varphi_0\rangle$.

Ab initio results are leading order in χEFT

Comparison of all methods: no contact



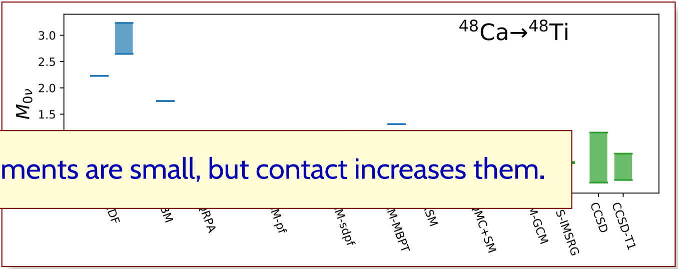


Effect of contact in “IM-GCM”

Ab Initio Light-ν-Exchange Matrix Elements for ^{48}Ca

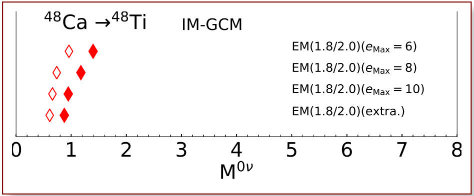
In-Medium Generator Coordinate Method and Coupled Clusters

Ab initio results are leading order in χEFT



Comparison of all me... no c...

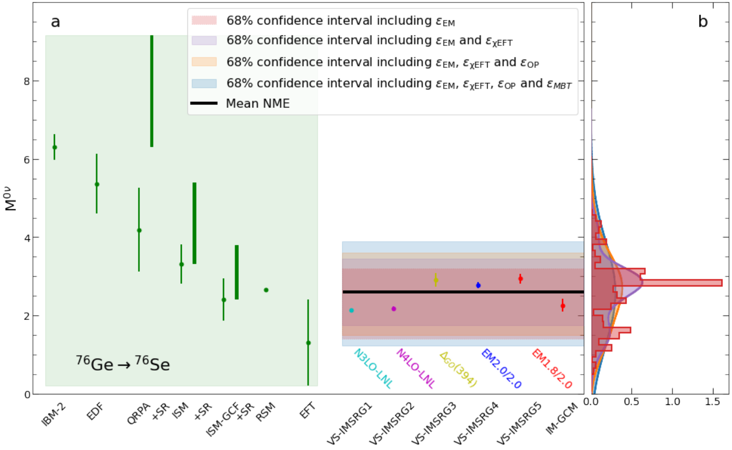
Matrix elements are small, but contact increases them.



Effect of contact in “IM-GCM”

^{76}Ge with Two Versions of In-Medium Similarity Renormalization Group

A. Belley et al. PRL **132**, 182502 (2024)



Matrix elements are on the small side (compared to phenomenological ones) but still quite uncertain.

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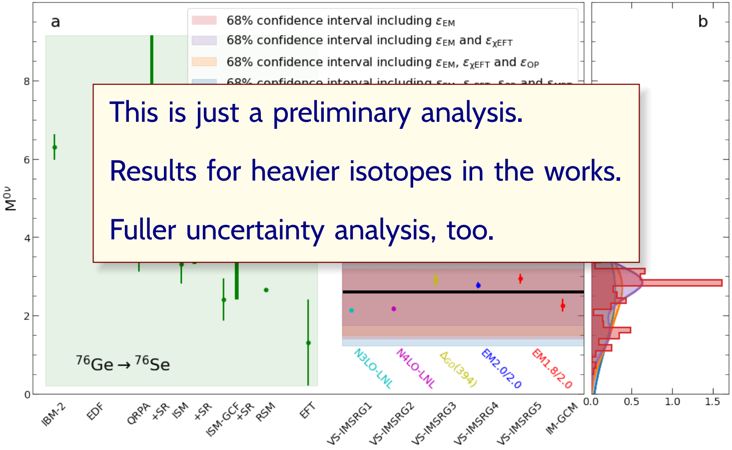

This is just a preliminary analysis.

Results for heavier isotopes in the works.

Fuller uncertainty analysis, too.

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Important Nuclear-Structure Steps

Still need:

- Next Order in truncation schemes.
- More general treatment of nuclear deformation.
- Faster codes.

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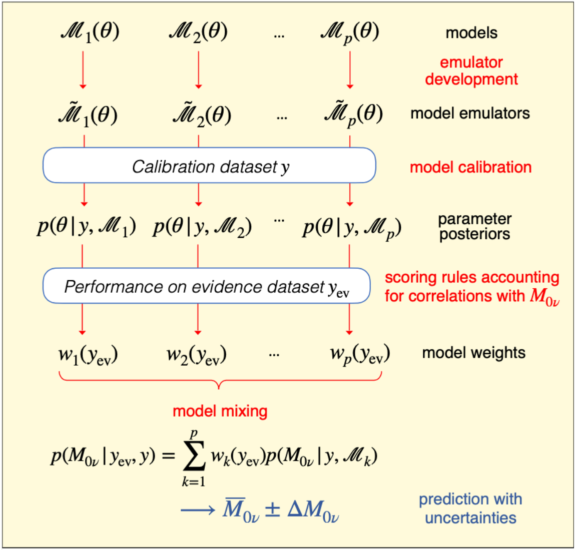
These will require more intensive and sophisticated supercomputer use, and more personnel.

A Plan for Uncertainty Quantification



- Repeat for all nuclei of 0νββ interest
- Repeat with calibration dataset refined for 0νββ

Important UQ Steps

Will need:

- Development of good calculation emulators.
- Collaboration between physicists and statisticians to develop model-mixing procedure.

Well …

It’s an interesting time to be alive.

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Thanks for listening!