Lattice QCD for Heavy Flavors

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Outline

- Motivation & Review of Lattice QCD
- Leptonic decays
- Semi-leptonic decays of B and D mesons
- Neutral B-meson mixing
- Summary

Enormous lattice literature on heavy quarks.

Impossible to be comprehensive.

Talk is unavoidably biased, focusing attention on recent published results and a few selected topics

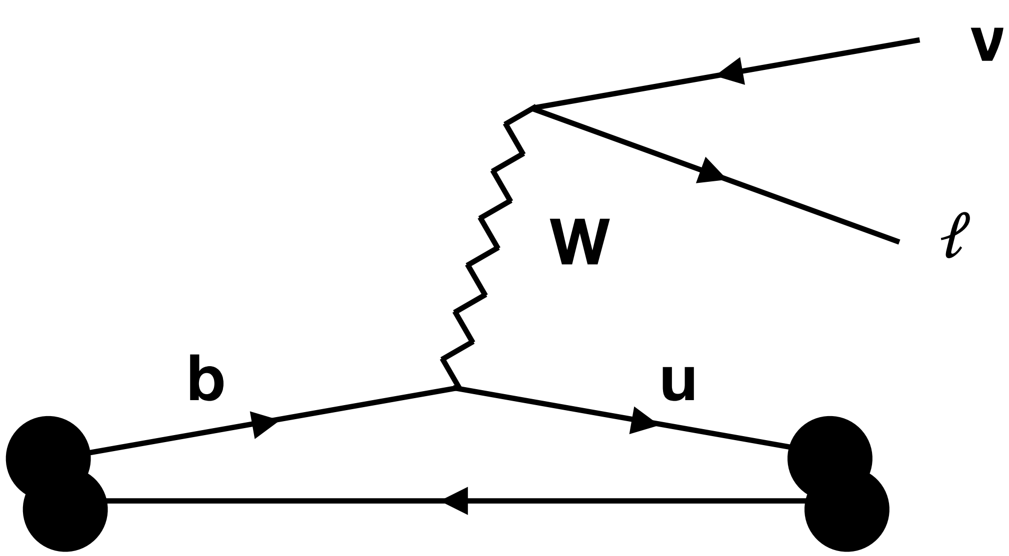
Apologies for all omissions

Context & Motivation

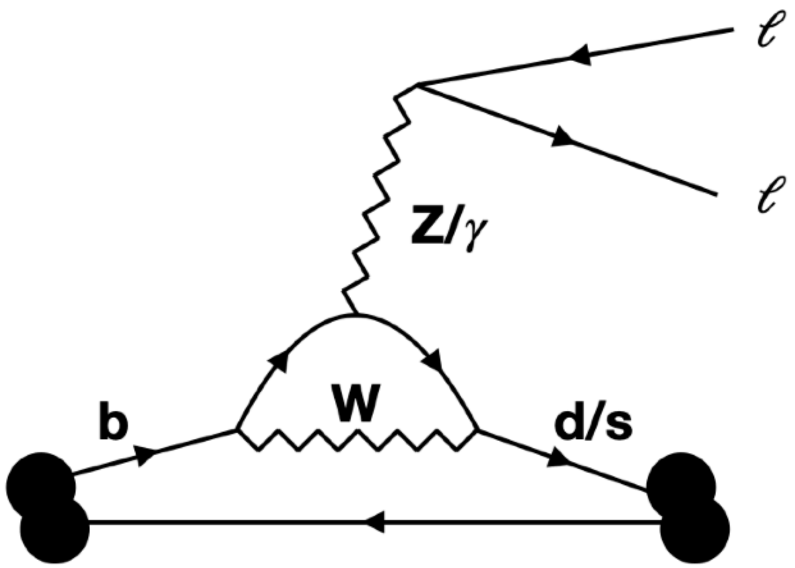
Quark Flavor and Lattice QCD

Two complementary roles

$$d\Gamma = \begin{pmatrix}\text{CKM}\\ \text{factor}\end{pmatrix}\begin{pmatrix}\text{kinematic}\\ \text{factor}\end{pmatrix}\begin{pmatrix}\text{QCD}\\ \text{factor}\end{pmatrix} + \begin{bmatrix}\text{BSM}\\ \text{term}\end{bmatrix}$$



Determine CKM matrix elements via tree-level decays



Test the CKM paradigm of the SM via rare decays

Quark Flavor and Lattice QCD

Accessing the CKM Matrix

“Gold-plated processes” $\iff$

Single-hadron initial state.

Zero- or one-hadron final state.

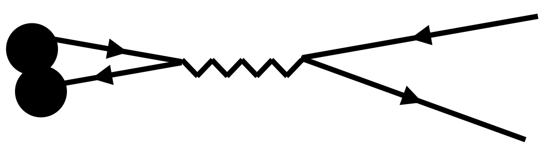
All hadrons stable under QCD.

***Nota bene*: Different lattice QCD formalism for exclusive multi-hadron or inclusive final states**

Quark Flavor and Lattice QCD

Tree level: CKM Matrix Elements

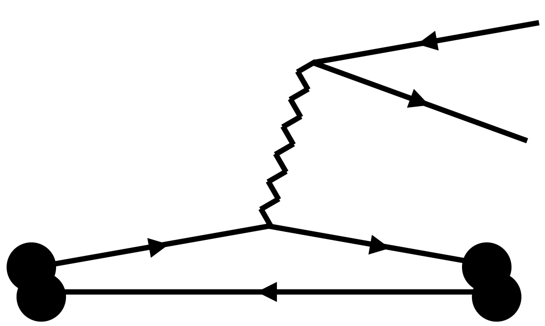
Leptonic decays



(Decay constants)

$$\langle 0|\, A^{\mu}\, |H(P)\rangle = i f_H\, p^{\mu}$$

Semi-leptonic decays

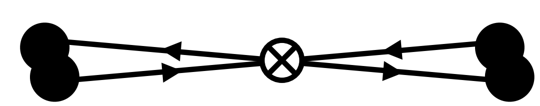


(Form factors)

$$f_J(p) \propto \langle \text{final}|\, J(p)\, |\text{initial}\rangle$$

$\mathbf{V_{ud}}$	$\mathbf{V_{us}}$	$\mathbf{V_{ub}}$
$\pi \to \ell\nu$	$K \to \ell\nu$ $K \to \pi\ell\nu$	$B \to \ell\nu$ $B \to \pi\ell\nu$ $\Lambda_b \to p\ell\nu$
$\mathbf{V_{cd}}$	$\mathbf{V_{cs}}$	$\mathbf{V_{cb}}$
$D \to \ell\nu$ $D \to \pi\ell\nu$ $D_s \to K\ell\nu$	$D_s \to \ell\nu$ $D \to K\ell\nu$	$B \to D\ell\nu$ $B \to D^{\star}\ell\nu$ $\Lambda_b \to \Lambda_c\ell\nu$
$\mathbf{V_{td}}$	$\mathbf{V_{ts}}$	$\mathbf{V_{tb}}$
$\langle B_d\vert \bar{B}_d\rangle\rangle$	$\langle B_s\vert \bar{B}_s\rangle\rangle$	

Neutral-meson mixing



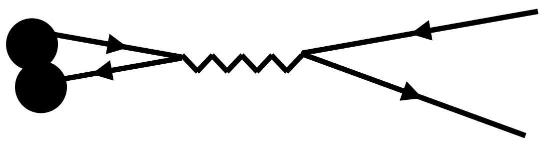
(Matrix elements)

$$\langle \bar{B}^0|\mathcal{H}_{\text{eff}}|B^0\rangle$$

Quark Flavor and Lattice QCD

Loop level: Flavor-Changing Neutral Currents

Leptonic decays



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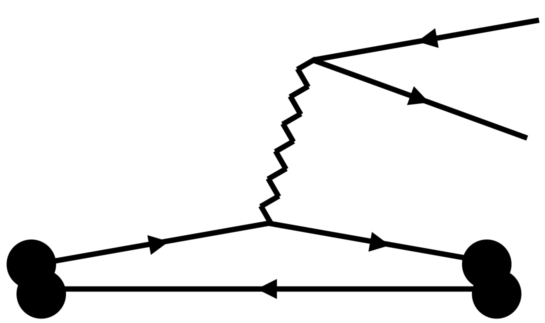
Semi-leptonic decays



(Form factors)

$$f_J(p) \propto \langle \text{final} | \, J(p) \, | \text{initial} \rangle$$

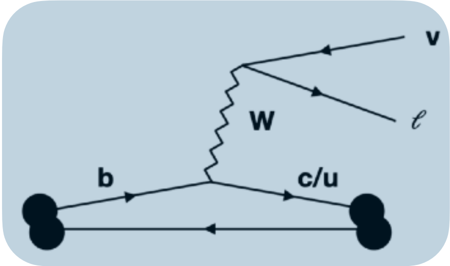
$B_s \to \ell^+ \ell^-$

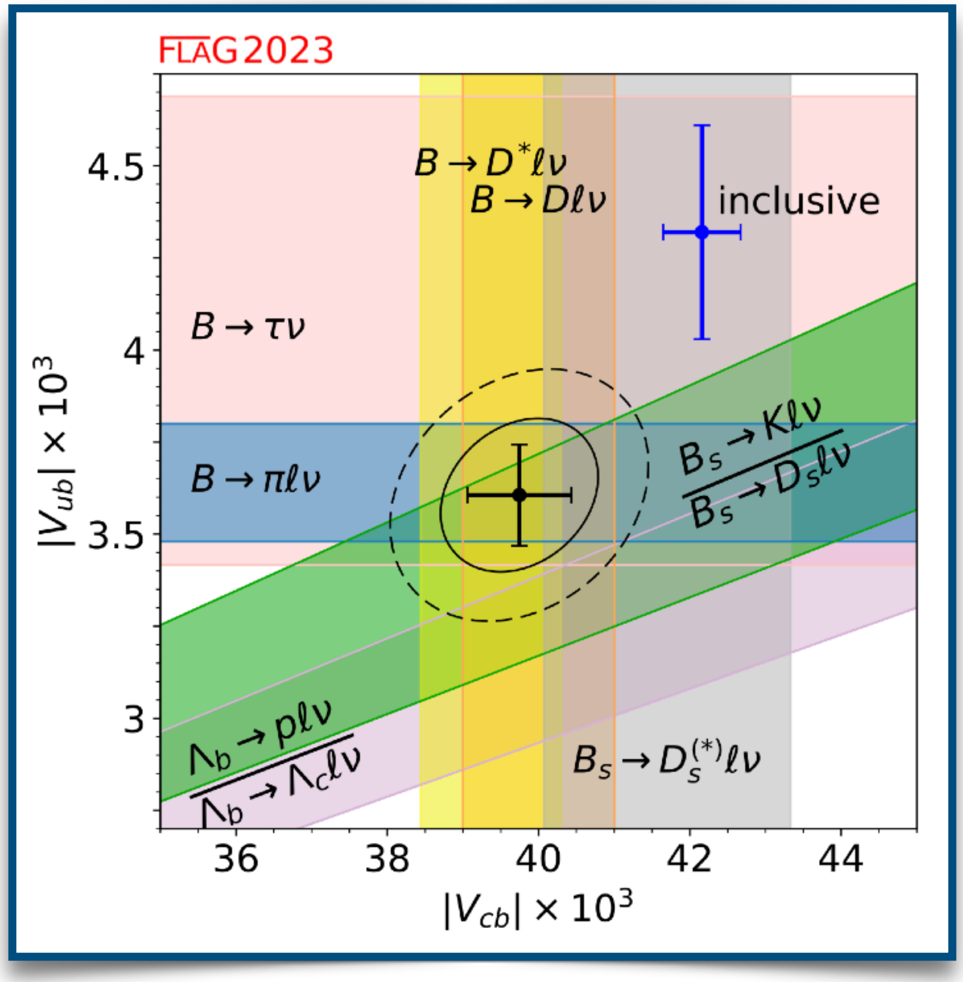
$B \to K \ell \nu$

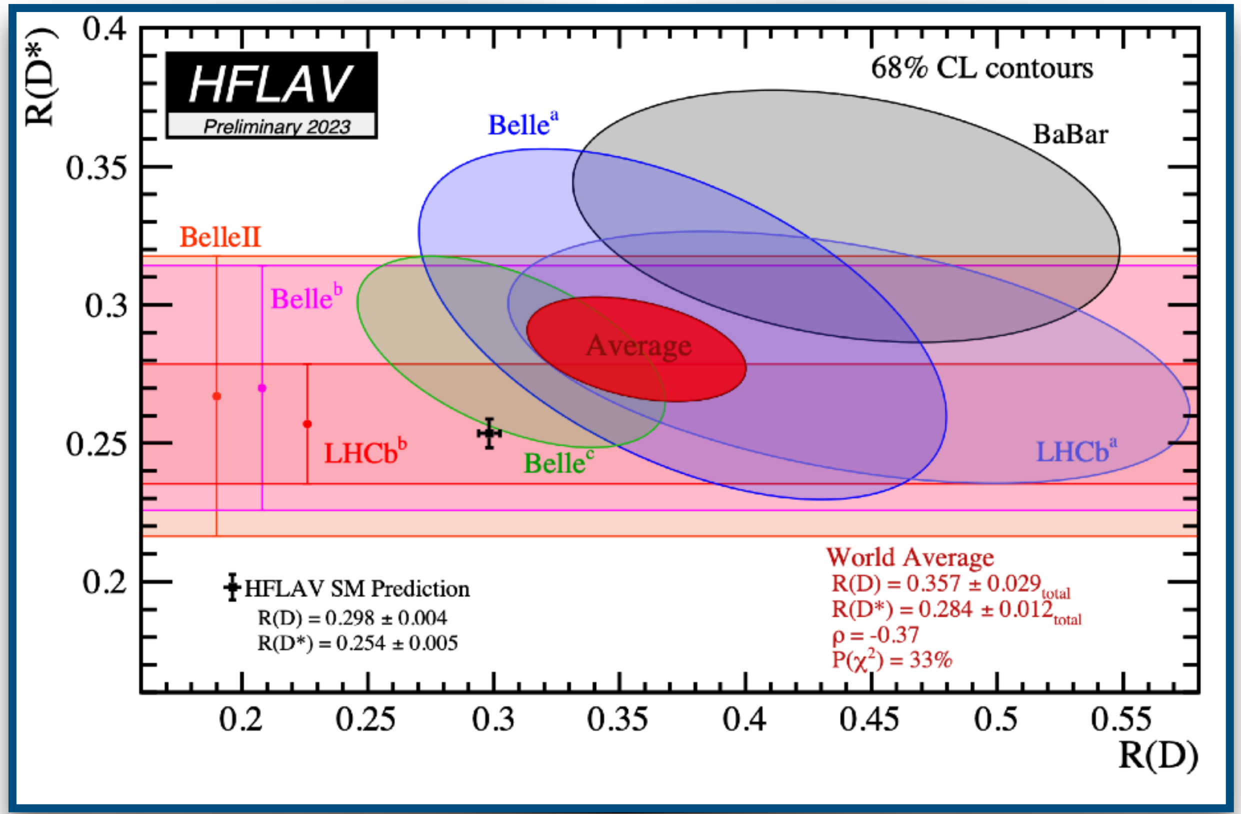
$B \to K^{\star} \ell \nu$

$\Lambda_B \to \Lambda \ell \nu$

Tensions: Trees and Loops





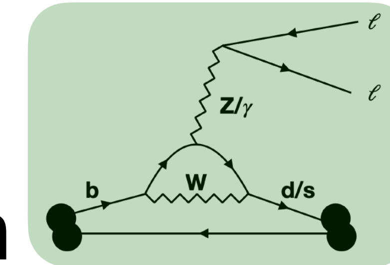


- **Tree level**: Exclusive (LQCD) vs Inclusive (OPE+HQE) determinations of CKM matrix elements
  - $|V_{cb}|$ from $B \to D^\star \ell \nu$, $B \to D\ell\nu$
  - $|V_{ub}|$ from $B \to \pi\ell\nu$

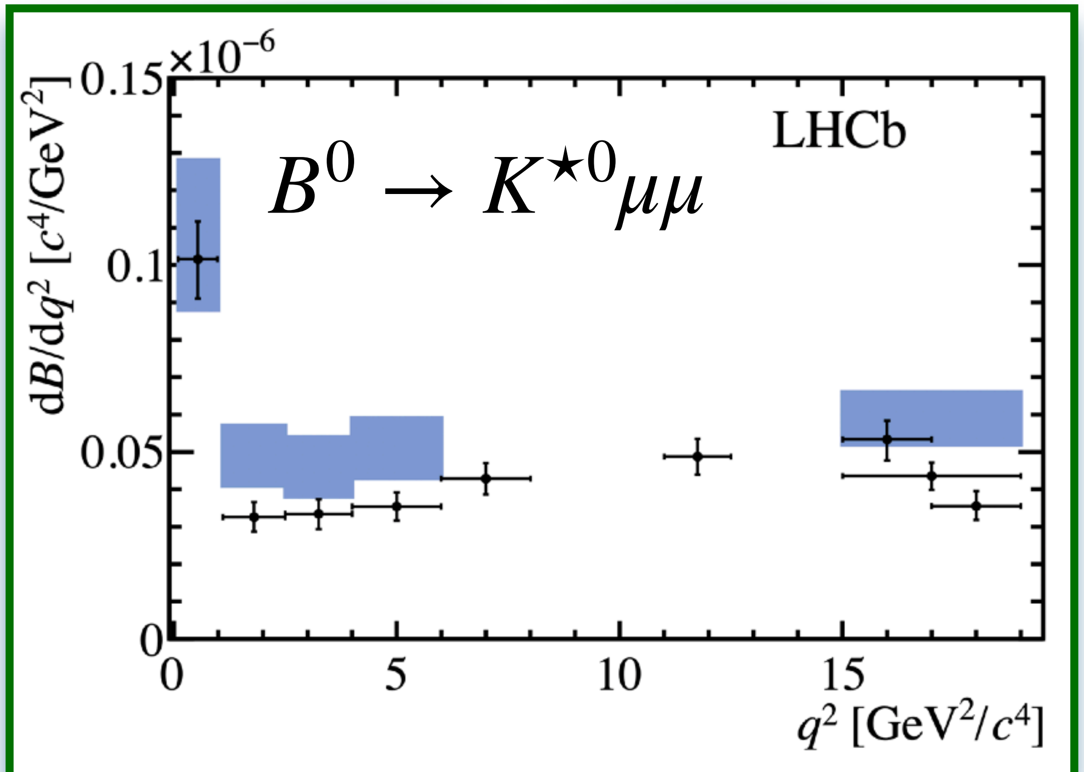
- **Tree level**: Lepton Flavor Universality: $R(D)$, $R(D^\star)$

$$R(D) = \frac{\mathscr{B}(B \to D\tau\bar{\nu})}{\mathscr{B}(B \to D\mu\bar{\nu})}$$

Tensions: Trees and Loops

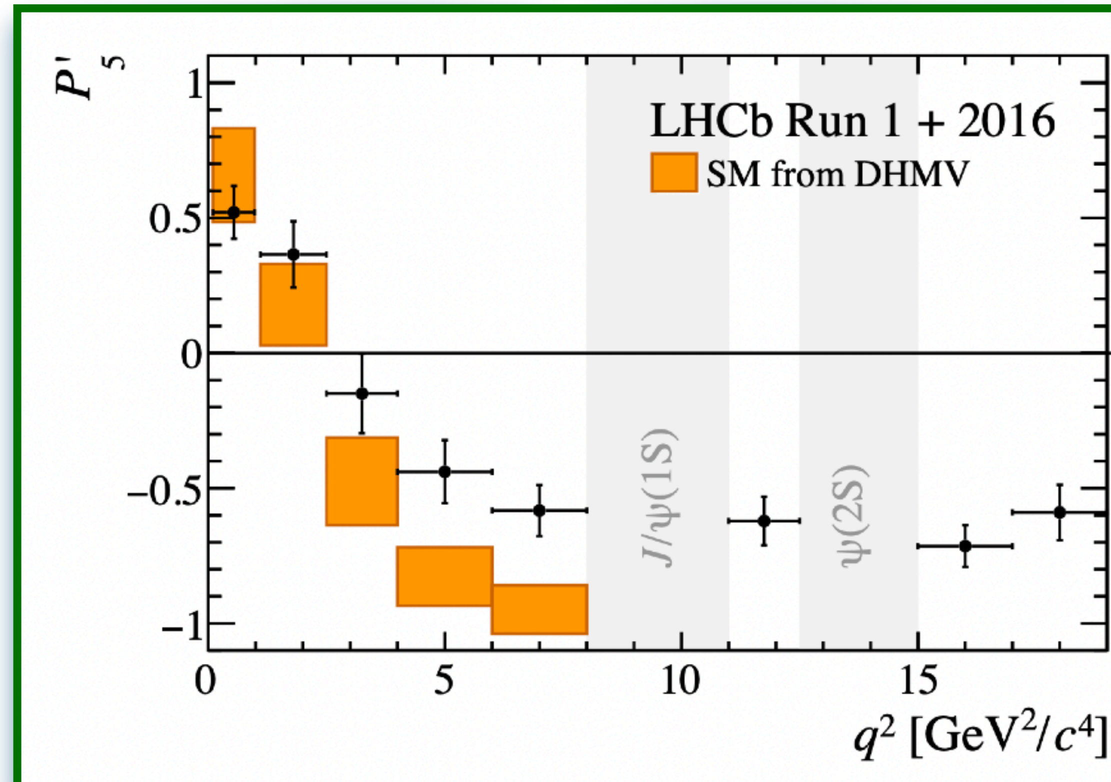


Branching fractions



- **Loop level**: $b \to s\ell\ell$ FCNC branching fractions:
  - $B^0 \to K^{\star 0}\mu\mu$, $B_s^0 \to \varphi\mu\mu$, $\Lambda_b^0 \to \Lambda^0\mu\mu$, $B^+ \to K^+\mu\mu$, $B^0 \to K^0\mu\mu$, $B^+ \to K^{\star +}\mu\mu$

Angular distribution



- **Loop level**: $b \to s\ell\ell$ FCNC angular observables
  - $B^0 \to K^{\star 0}\mu\mu$, $B^+ \to K^{\star +}\mu\mu$, $B_s^0 \to \varphi\mu\mu$

LHCb *JHEP* 11 (2016) 047
LHCb *JHEP* 04 (2017) 142
LHCb PRL 125 (2020) 011802
LHCb *JHEP 09 (2015) 179*
LHCb *PRL* 127 (2021) 15, 151801
LHCb JHEP 11 (2021) 043

Lattice QCD with Heavy Quarks

Lattice QCD

- Lattice QCD gives complete non-perturbative definition to the strong interactions
- This framework gives:

$$\mathcal{Z} = \int \mathcal{D}[\text{fields}]e^{-S_E[\text{fields}]}$$

- **Fundamental approximations:**
  - UV cutoff: lattice spacing *a* [target: a $\ll$ physical scales]
  - IR cutoff: finite spacetime volume $V = L^3 \times T$ [target: $1 \ll m_\pi L$]

- **Approximations of convenience:**
  - Often: Heavier-than-physical pions: $(m_\pi)^{\text{lattice}} > (m_\pi)^{\text{PDG}}$
  - Often: Isospin limit $m_u = m_d$
  - Often: QCD interactions only, no QED
  - Often: lighter-than-physical or static heavy quarks

Lattice QCD is systematically improvable

- All approximations admit theoretical descriptions via EFT
  - Cutoff dependence $\Longleftrightarrow$ Symanzik effective theory
  - Finite-volume dependence $\Longleftrightarrow$ Finite-volume χPT
  - Chiral extrapolation / interpolation $\Longleftrightarrow$ χPT
  - Heavy quark extrapolation / interpolation $\Longleftrightarrow$ HQET, NRQCD, etc…
  - QED, isospin breaking $\Longleftrightarrow$ perturbative expansion of path integral
- Careful treatment of all systematic effects is key to modern high-precision lattice QCD
- Technical advances in controlling these systematics have been drivers of progress in lattice QCD, especially in heavy-quark physics

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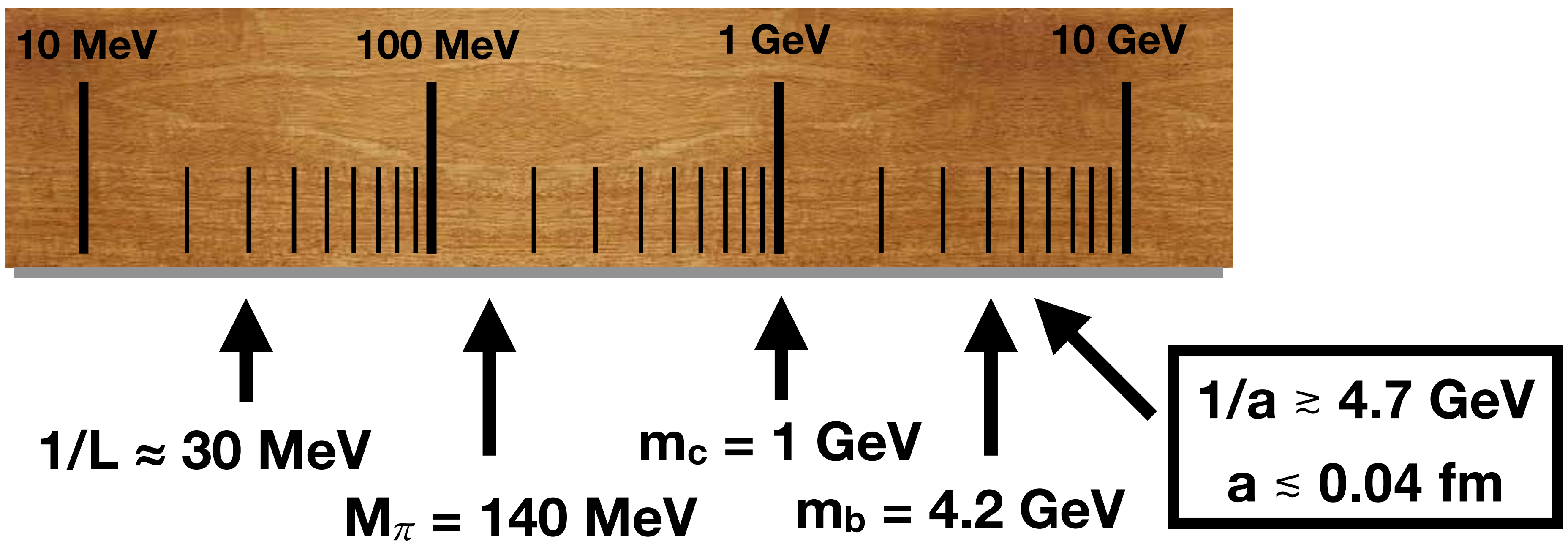
Lattice QCD with Heavy Quarks

A challenging multi-scale problem



Heavy quarks are hard: lattice artifacts grow like powers $(am_h)^n$ — especially tricky for masses near or above the cutoff

$$\frac{1}{L} \ll M_\pi \ll m_h \ll \frac{1}{a}$$

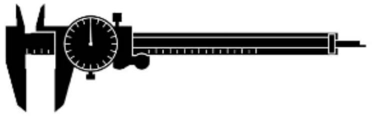
Lattice QCD with Heavy Quarks

A challenging multi-scale problem

Solutions to the cutoff challenge?

1. Use an “effective theory” for heavy quarks (b, sometimes c)
   - “FNAL interpretation,” NRQCD, RHQ, Oktay-Kronfeld
   - Good: Solves problem with artifacts (am_h)
   - No free lunch: EFTs require matching and/or parameter tuning, which introduces systematic effects
   - (1-3)% total errors

2. Use highly-improved relativistic light-quark action on fine lattices
   - Good: advantageous renormalization, continuum limit
   - No free lunch: simulations still need $am_h < 1$ and often an extrapolation to the physical bottom mass
   - (< 1)% total errors possible



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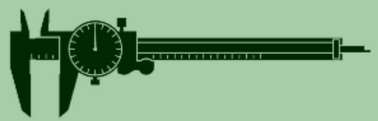
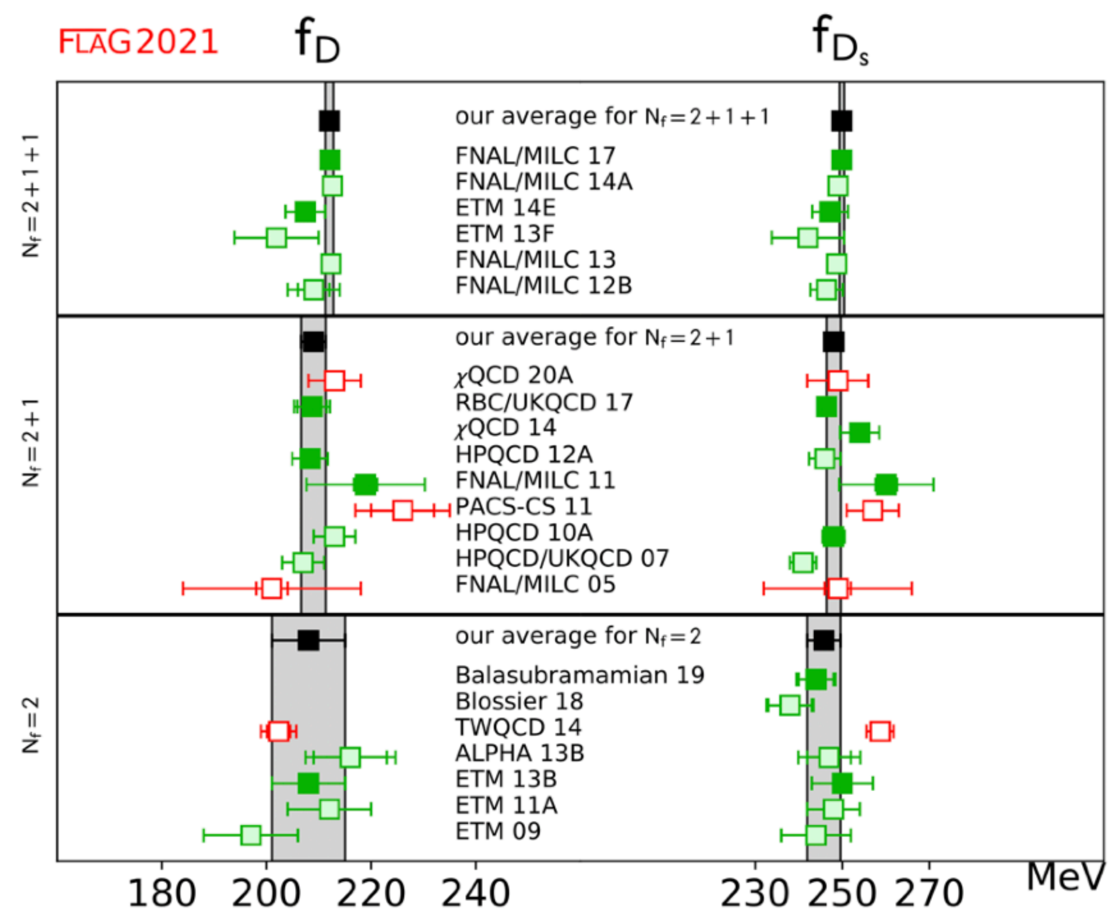


Leptonic Decays

An invitation to precision in lattice QCD

FLAG Review 21
Y. Aoki et al.
EPJC 82 (2022) 10, 869
arXiv: 2111.09849

- Sub-percent precision for $f_{D_{(s)}}$ and $f_{B_{(s)}}$
  - LQCD precision is below existing/expected experimental uncertainties
  - Complementary calculations and discretizations bolster confidence in results
- "Pure QCD problem is solved"
  - Further improvement: systematic inclusion of QED, isospin breaking

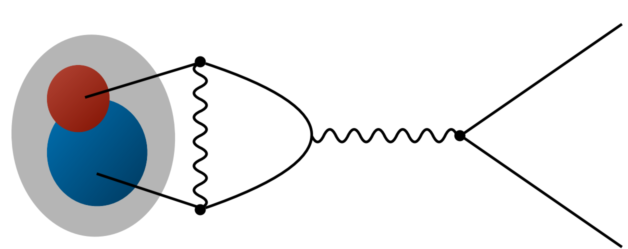


SM prediction for rare leptonic decay rate

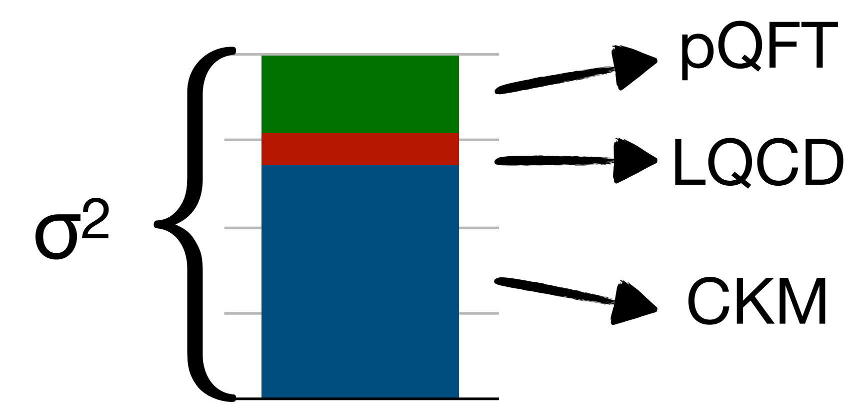
Beneke et al, arXiv:1908.07011, JHEP 2019

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = 3.660(38) \times 10^{-9}$$

B_s



Lattice QCD value for f_{B_s} is now a sub-dominant source of uncertainty



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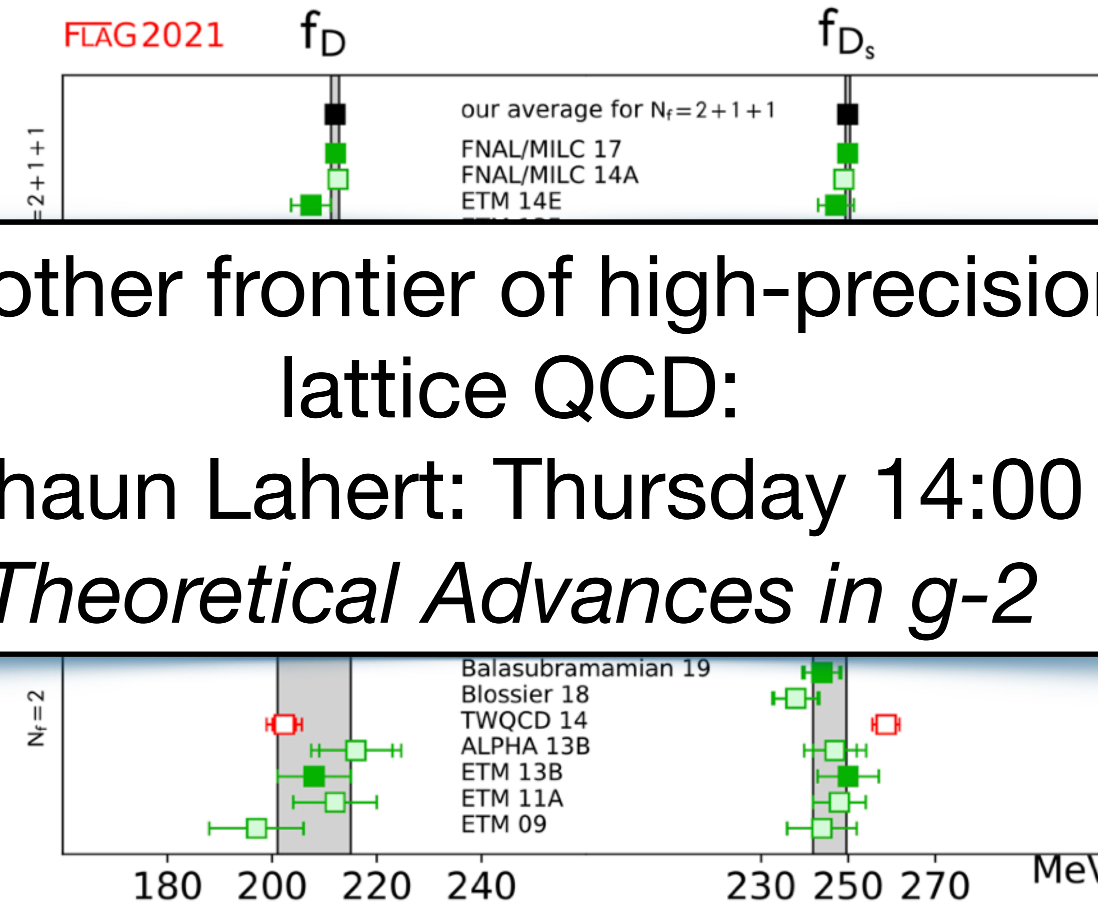

Another frontier of high-precision lattice QCD:
Shaun Lahert: Thursday 14:00
Theoretical Advances in g-2

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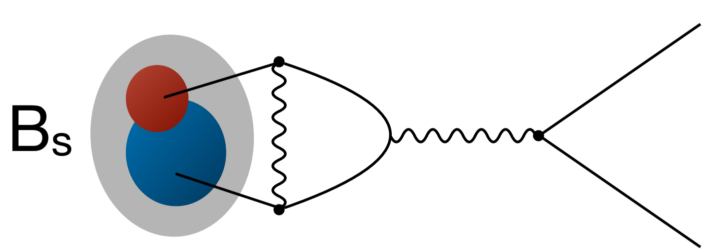

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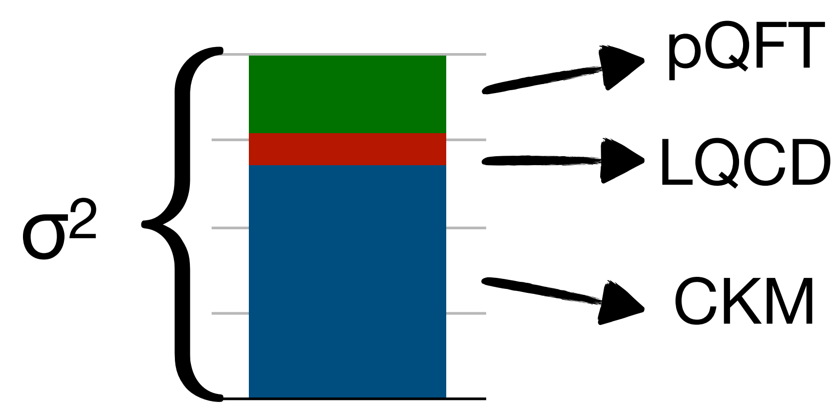
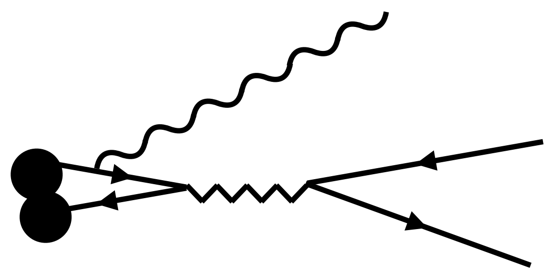


Leptonic Decays

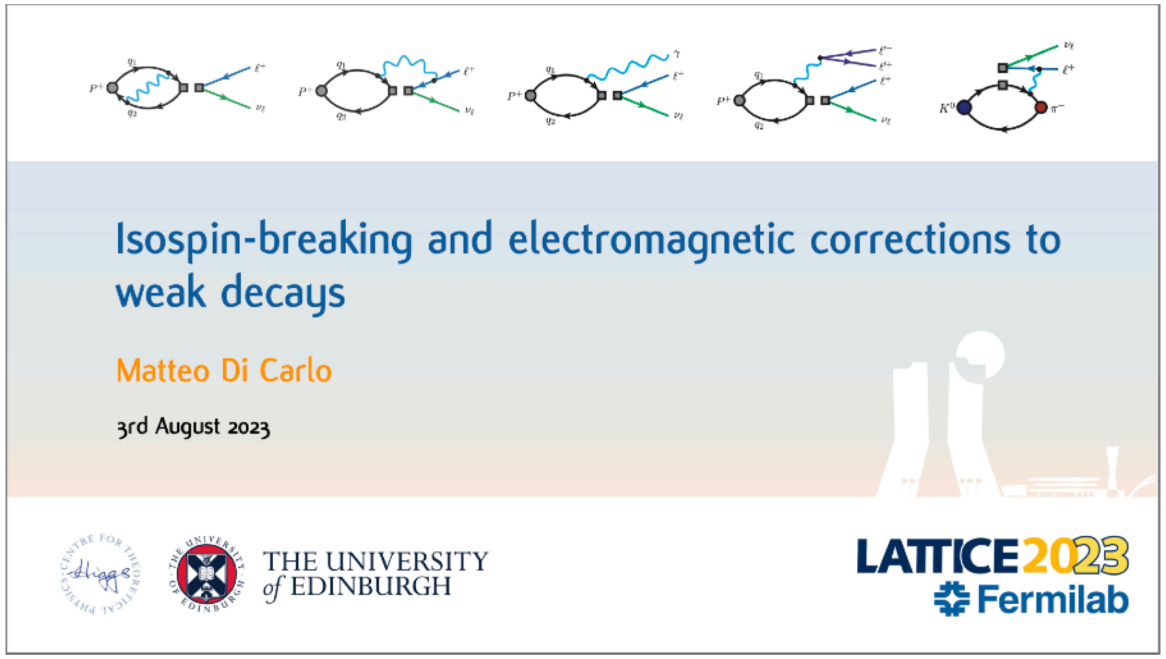
An invitation to precision in lattice QCD

- Isospin / QED corrections to weak decays have been considered by the lattice community since ≈ 2015



$$\epsilon_\mu(k) \int d^4y e^{iky} \mathcal{T} \langle \varnothing \,|\, j_W^\alpha(0) j_{EM}^\mu(y) \,|\, P(\mathbf{p}) \rangle$$

Structure-dependent form factors: qualitatively new element for leptonic decays.



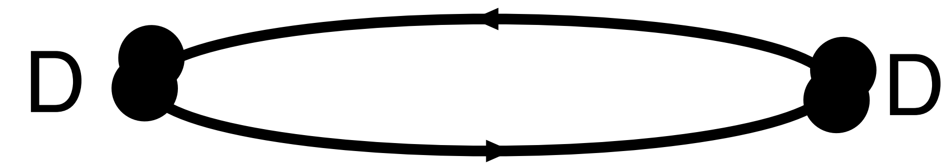
Matteo Di Carlo
Plenary Review @ Lattice 2023
Includes discussion and references to literature, recent work reported at Lattice 2023

Semileptonic decays: $H \to L\ell\nu$

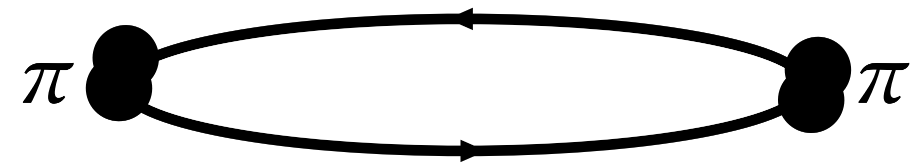
Anatomy of a calculation: correlation functions

- Hadron masses $\Longleftrightarrow$ QCD 2pt functions
- Matrix elements $\Longleftrightarrow$ QCD 3pt functions
- For concreteness: consider $D \to \pi\ell\nu$

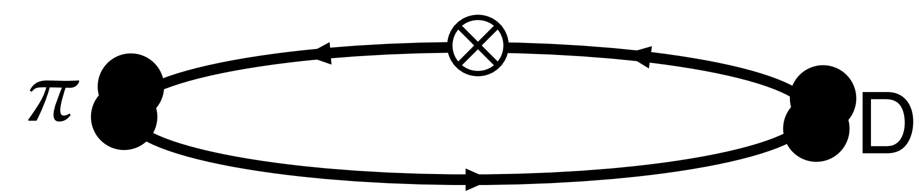
$$C_D(t) = \sum_{\boldsymbol{x}} \langle \mathcal{O}_D(0,\mathbf{0})\mathcal{O}_D(t,\boldsymbol{x})\rangle$$



$$C_\pi(t,\boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \langle \mathcal{O}_\pi(0,\mathbf{0})\mathcal{O}_\pi(t,\boldsymbol{x})\rangle$$



$$C_3(t,T,\boldsymbol{p}) = \sum_{\boldsymbol{x},\boldsymbol{y}} e^{i\boldsymbol{p}\cdot\boldsymbol{y}} \langle \mathcal{O}_\pi(0,\mathbf{0})J(t,\boldsymbol{y})\mathcal{O}_D(T,\boldsymbol{x})\rangle$$



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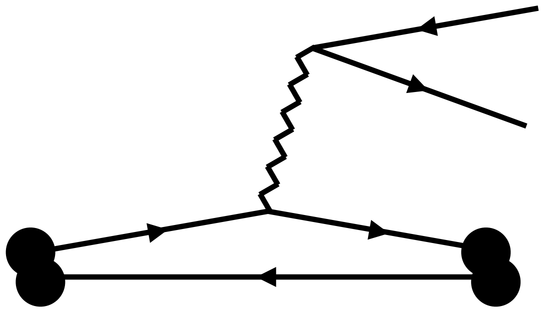
$$C_3(t,T,\boldsymbol{p}) = \sum_{\boldsymbol{x},\boldsymbol{y}} e^{i\boldsymbol{p}\cdot\boldsymbol{y}} \langle \mathcal{O}_\pi(0,\mathbf{0})J(t,\boldsymbol{y})\mathcal{O}_D(T,\boldsymbol{x})\rangle$$

$$\longrightarrow \langle 0|\,\mathcal{O}_\pi\,|\pi\rangle \langle \pi|\,J\,|D\rangle \langle D|\,\mathcal{O}_D\,|0\rangle\, e^{-E_\pi t} e^{M_D(T-t)}$$

Matrix elements ⇒ Form factors

Semileptonic Decays

of D and B mesons



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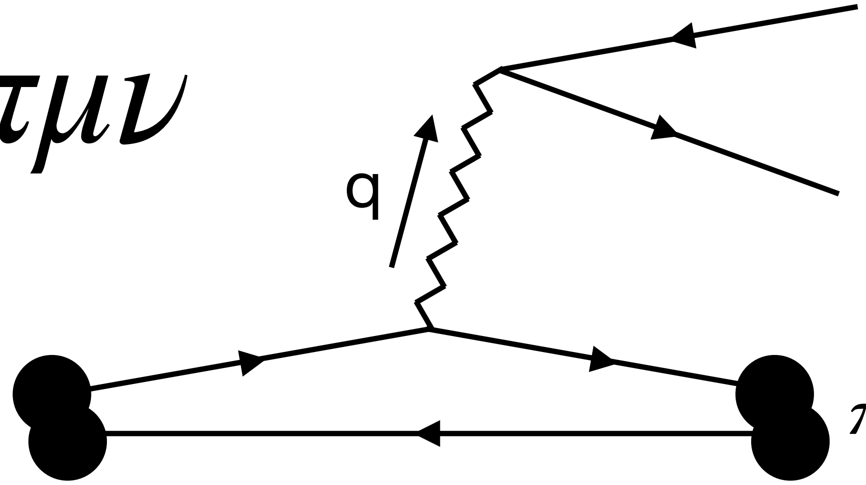
$\langle B_d \vert \bar{B}_d \rangle)$	$\langle B_s \vert \bar{B}_s \rangle)$	

Semileptonic decays: $D \to \pi\mu\nu$

Theoretical preliminaries



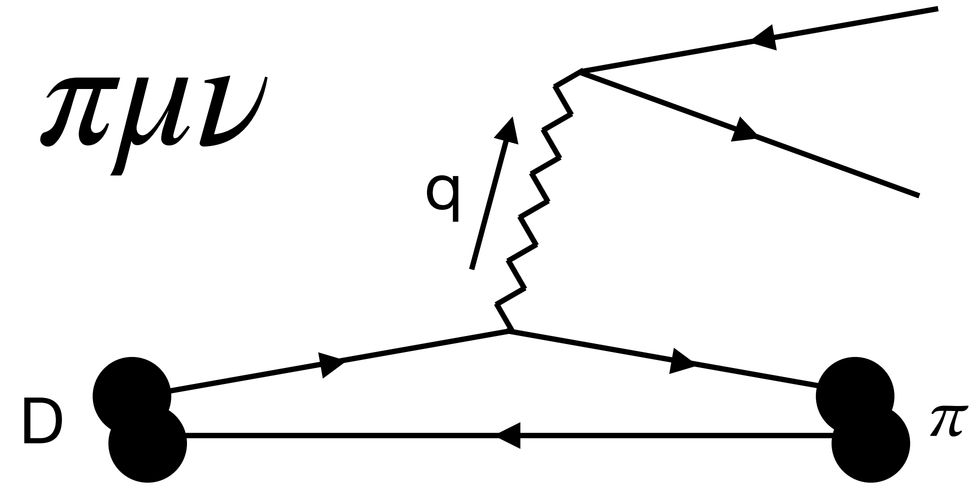
$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 \eta_{EW}^2}{24\pi^3} \left|V_{cd}\right|^2 (1-\epsilon)^2 (1+\delta_{\mathrm{EM}}) \times$$
$$\left[|\mathbf{p}|^3 \left(1+\frac{\epsilon}{2}\right) \left|f_+(q^2)\right|^2 + |\mathbf{p}|\, M_D^2 \left(1 - \frac{M_\pi^2}{M_D^2}\right)^2 \frac{3\epsilon}{8} \left|f_0(q^2)\right|^2 \right]$$

: measured decay rate

$$\epsilon = m_\mu^2/q^2 \ll 1$$

Semileptonic decays: $D \to \pi\mu\nu$

Theoretical preliminaries



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: measured decay rate

: (non-perturbative) hadronic form factors

Semileptonic decays: $D \to \pi\mu\nu$

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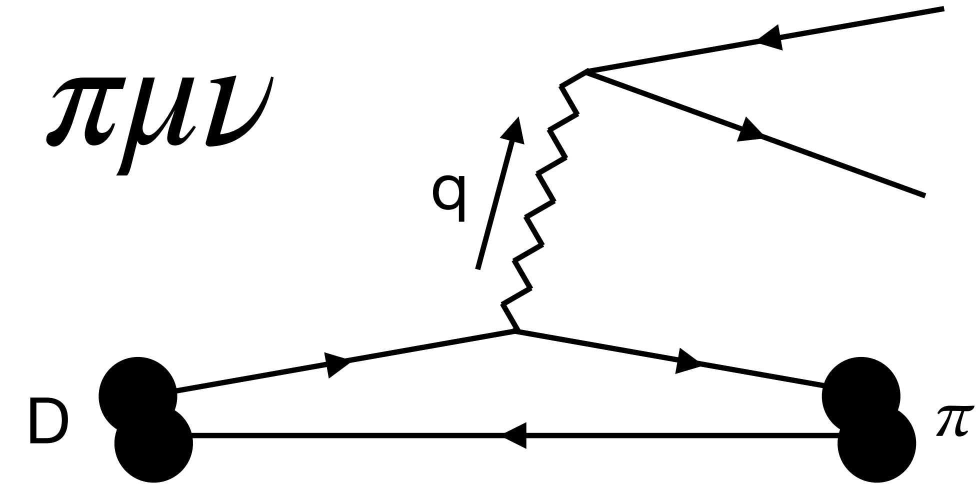

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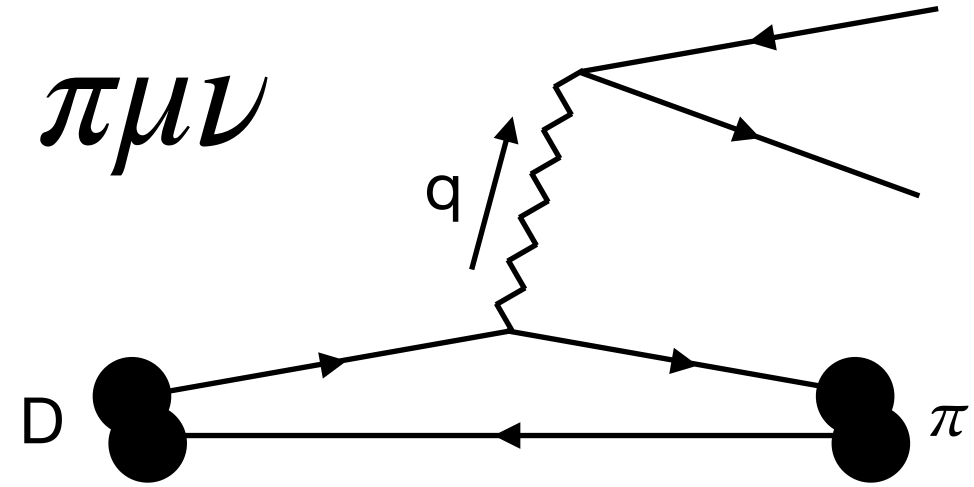
: measured decay rate (blue)

: (non-perturbative) hadronic form factors (green)

: kinematic factors (yellow)

Semileptonic decays: $D \to \pi\mu\nu$

Theoretical preliminaries



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$$\epsilon = m_\mu^2/q^2 \ll 1$$

: **measured decay rate**

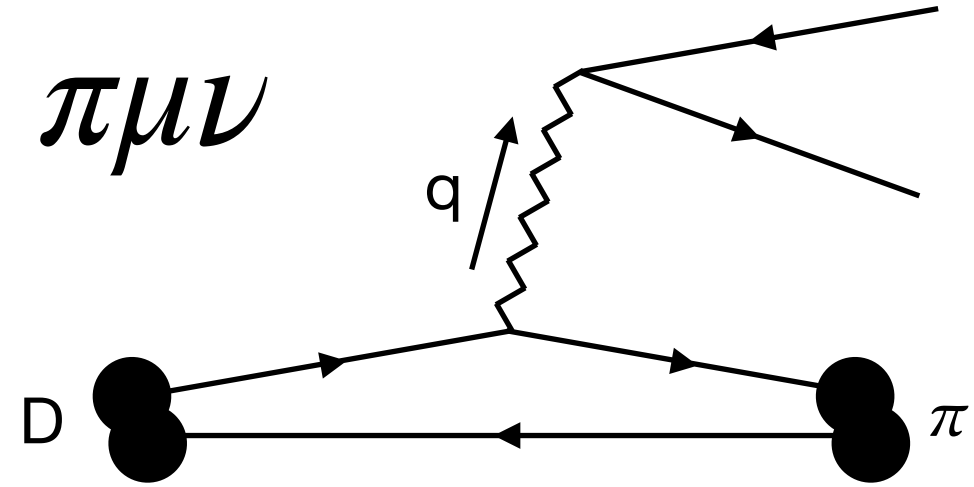
: **(non-perturbative) hadronic form factors**

: **kinematic factors**

: **perturbative corrections**

Semileptonic decays: $D \to \pi\mu\nu$

Theoretical preliminaries



$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 \eta_{EW}^2}{24\pi^3} |V_{cd}|^2 (1-\epsilon)^2 (1+\delta_{\mathrm{EM}}) \times \left[|\mathbf{p}|^3 \left(1+\frac{\epsilon}{2}\right) |f_+(q^2)|^2 + |\mathbf{p}| M_D^2 \left(1-\frac{M_\pi^2}{M_D^2}\right)^2 \frac{3\epsilon}{8} |f_0(q^2)|^2 \right]$$

$$\epsilon = m_\mu^2/q^2 \ll 1$$

- (blue) **: measured decay rate**
- (green) **: (non-perturbative) hadronic form factors**
- (yellow) **: kinematic factors**
- (red) **: perturbative corrections**

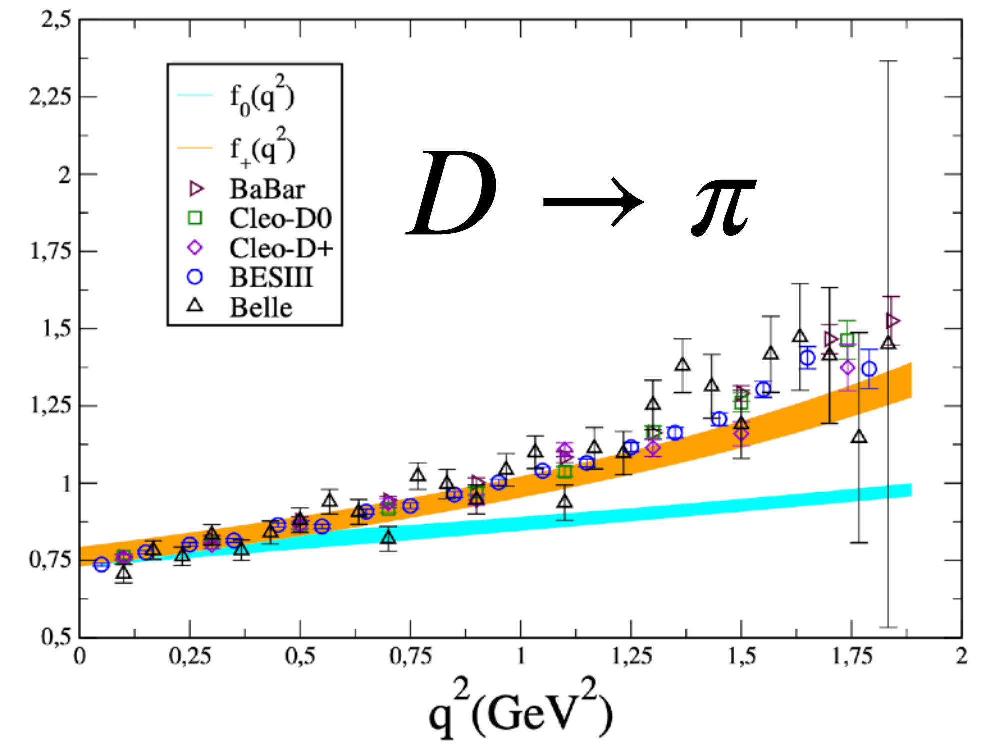
At O(1%) precision, all sectors of SM become important: QCD, QED, EW

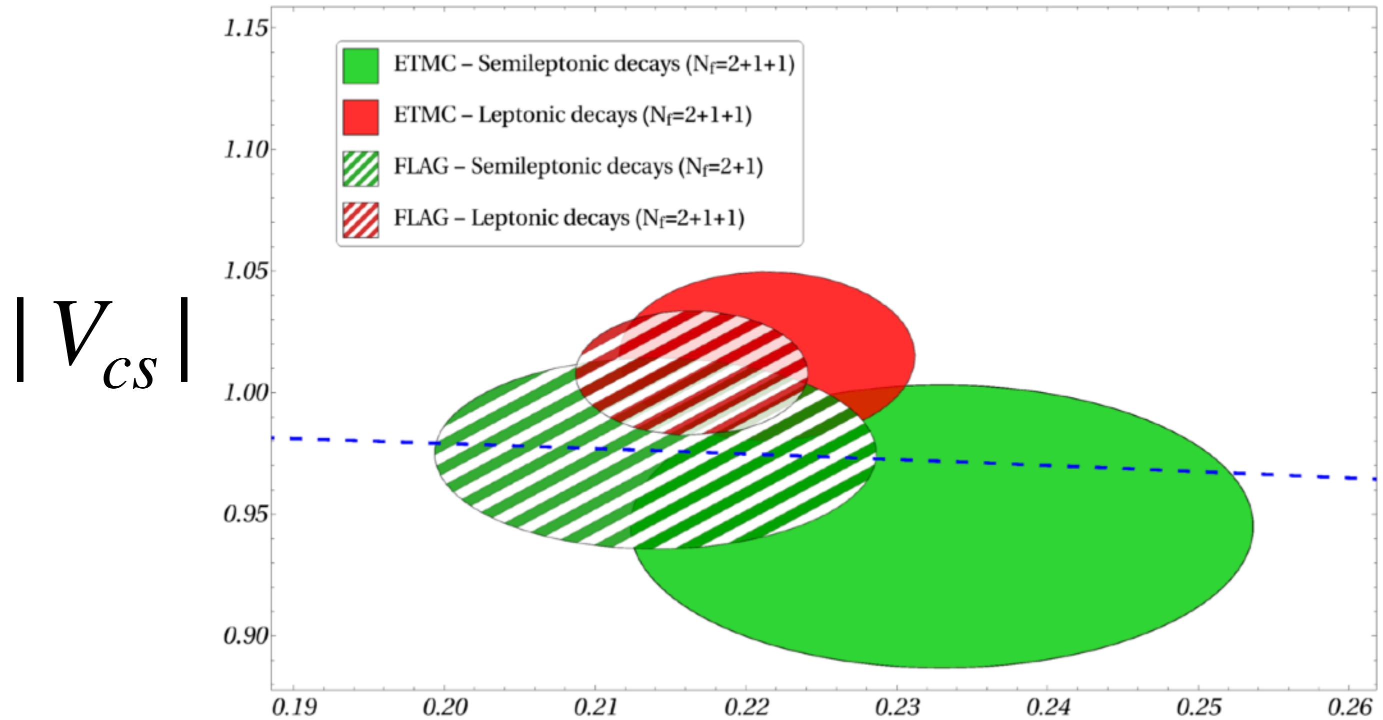
D-meson Semileptonic Decays

ETMC
PRD 96 (2017) 5, 054514
arXiv:1706.03017

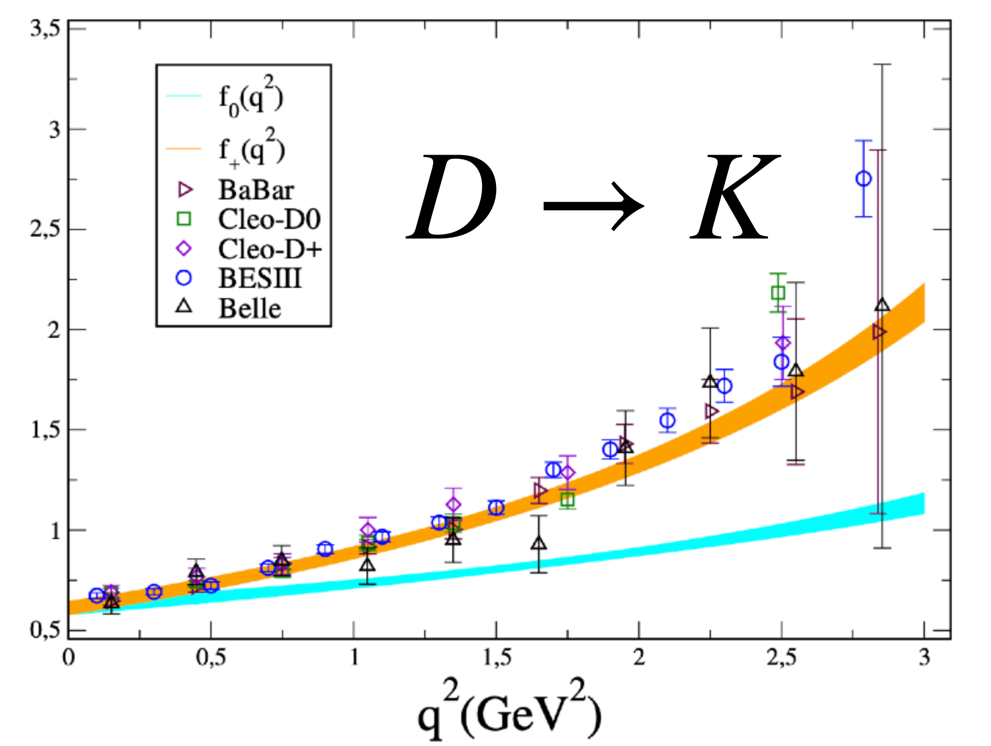
$D \to K/\pi \ell \nu$ and $|V_{cd}|$, $|V_{cs}|$

- (N_f=2+1+1)ETMC Wilson twisted mass ensembles
- Lattice spacings: $a \in \{0.09, 0.08, 0.06\}$ fm
- $M_\pi \simeq 210 - 450$ MeV
- $\approx 4 - 6\,\%$ precision for $f_{+/0}(0)$
- $|V_{cd}| = 0.2330(133)^{\mathrm{LQCD}}(31)^{\mathrm{EXP}}$ [≈6%]
- $|V_{cs}| = 0.945(38)^{\mathrm{LQCD}}(4)^{\mathrm{EXP}}$ [≈4%]





$|V_{cd}|$

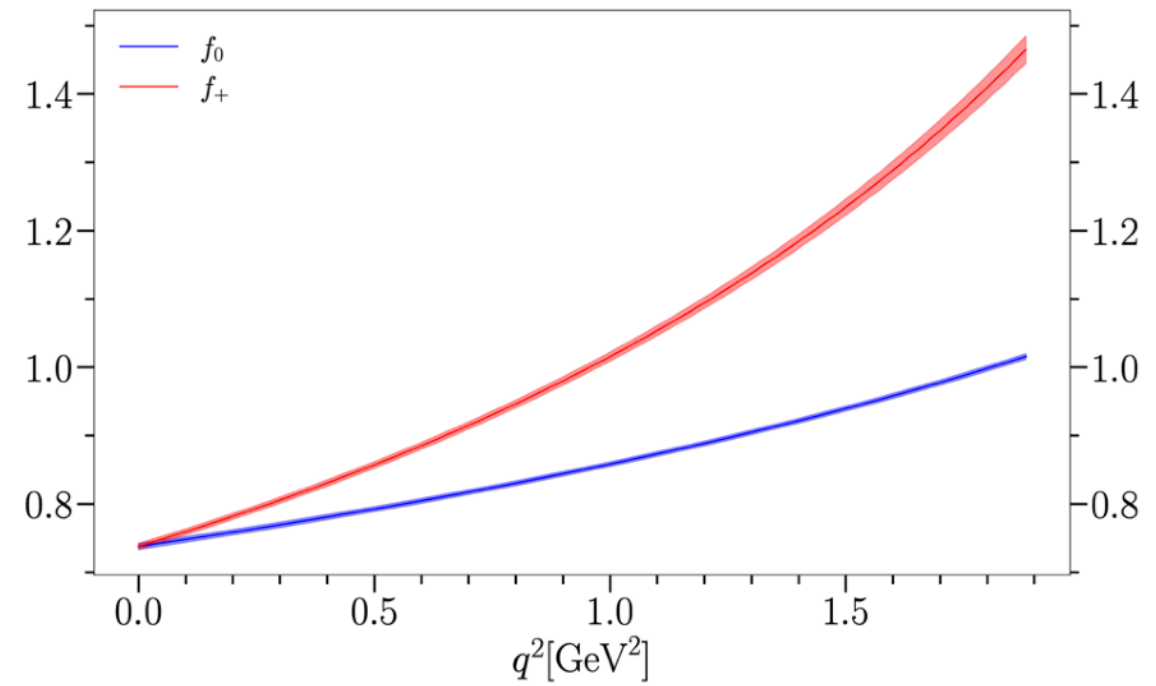


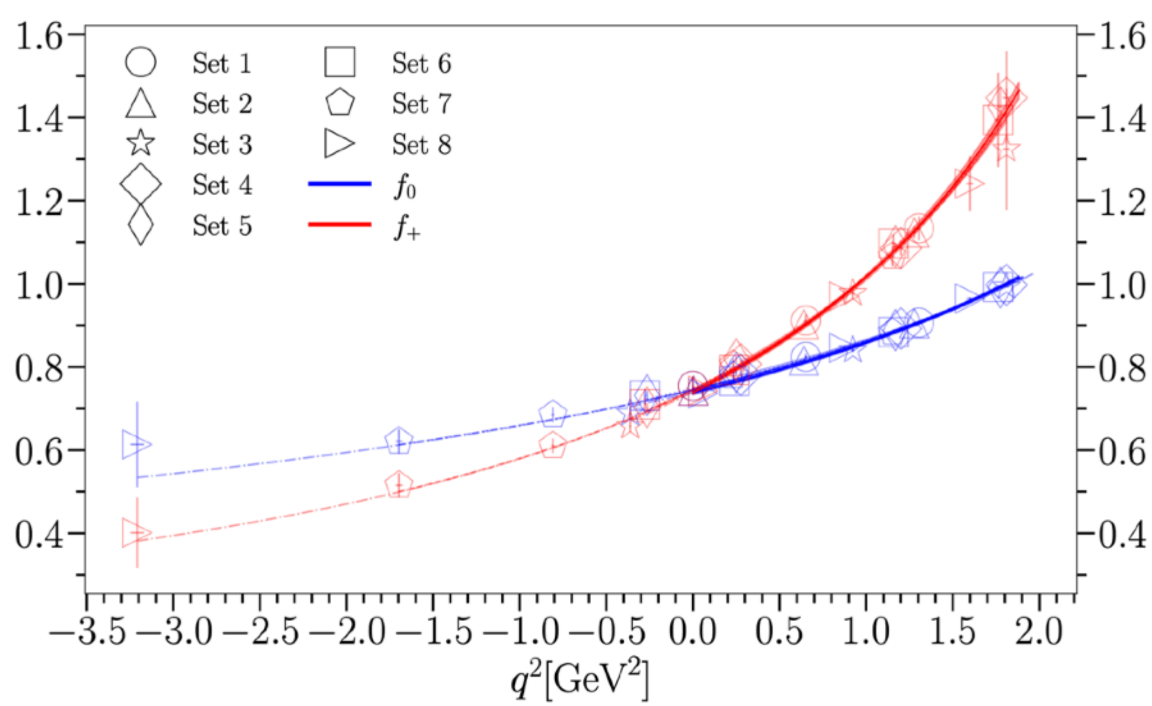
D-meson Semileptonic Decays

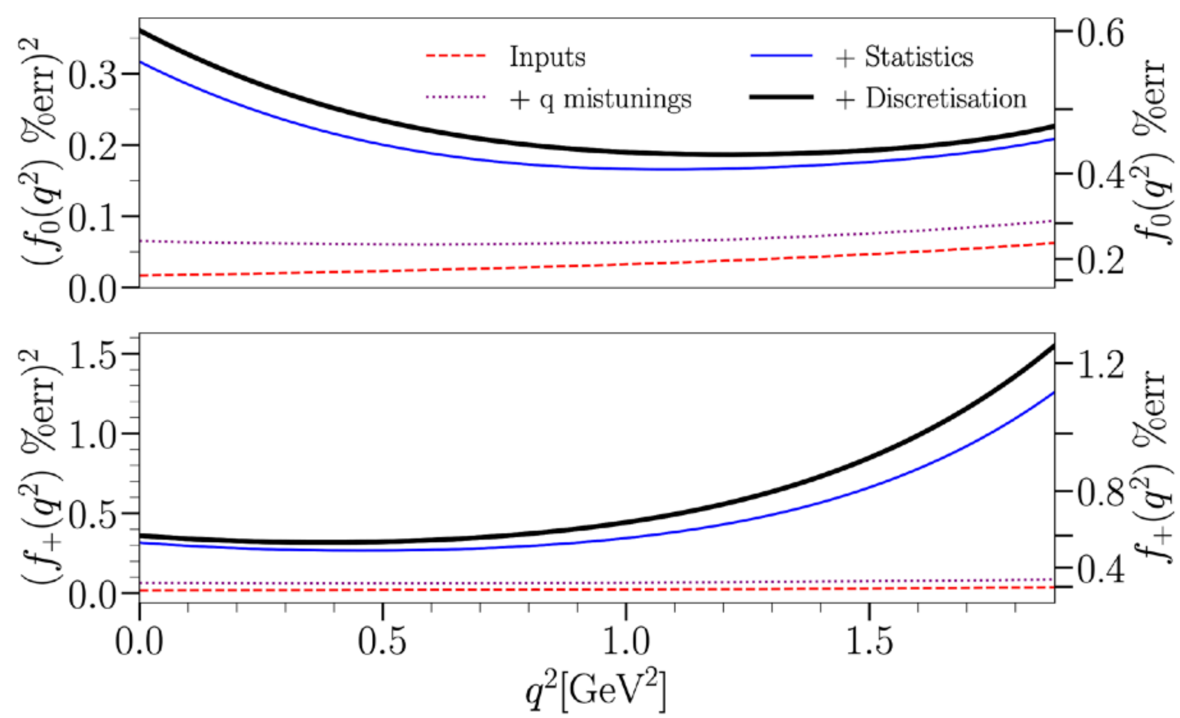
HPQCD
PRD 104 (2021) 3, 034505
arXiv:2104.09883

$D \to K\ell\nu$ and $|V_{cs}|$

- (N_f=2+1+1) MILC HISQ ensembles
- Lattice spacings: $a \in \{0.045 - 0.15\}$ fm
- $M_\pi \simeq 135 - 320$ MeV
- Valence: heavy HISQ
- Chiral-continuum analysis via "modified z-expansion"
- $\lesssim 1\,\%$ precision for $f_{+/0}(0)$
- $|V_{cs}| = 0.9663(53)^{\mathrm{LQCD}}(39)^{\mathrm{EXP}}(19)^{\mathrm{EW}}(40)^{\mathrm{EM}}$ [≈1%]





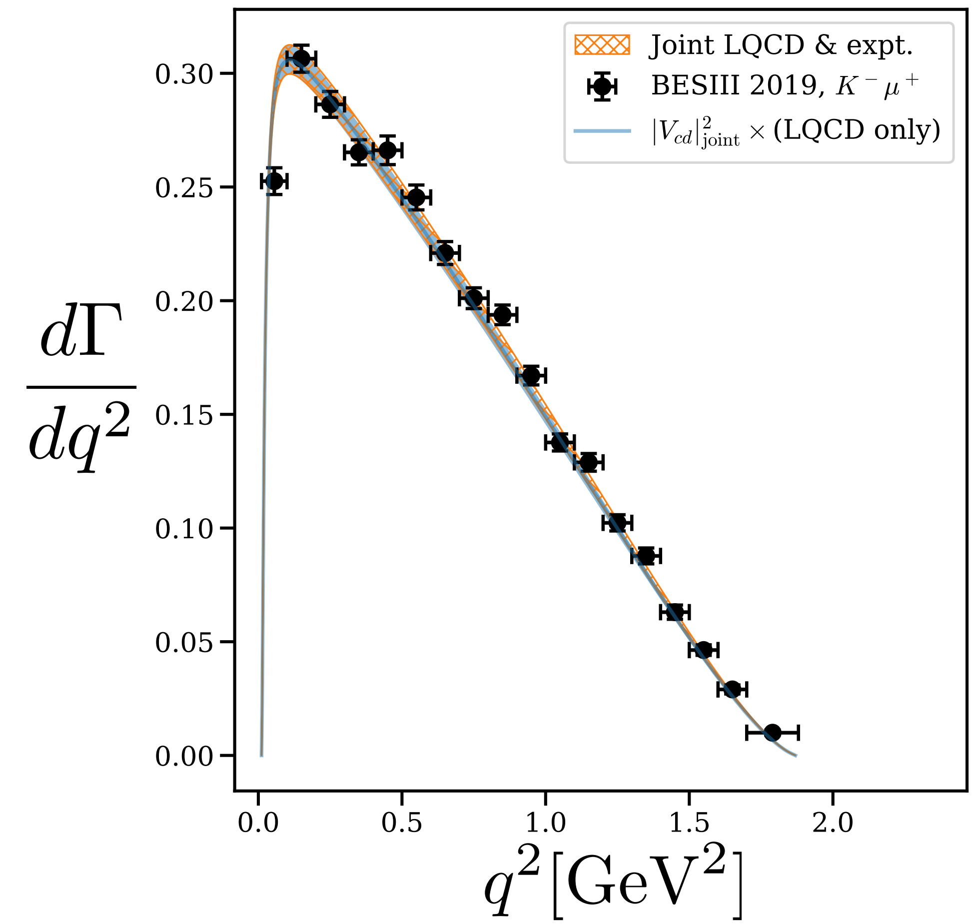


D-meson Semileptonic Decays

Fermilab-MILC [WJ]
PRD 107 (2023) 9, 094516
arXiv:2212.12648

$D_{(s)} \to K/\pi \ell \nu$

- (N_f=2+1+1) MILC HISQ ensembles
- Lattice spacings: [0.045 - 0.12] fm
- HISQ treatment for all quarks: u, d, s, c
- Enabling technology: non-perturbative renormalization via Ward identity: "PCVC"
- All results from a **blinded analysis**



✓ First time that LQCD and experimental errors are commensurate for $D \to \pi \ell \nu$

✓ Consistent with a percent-level extraction of $|V_{cs}|$ from HPQCD in 2021 [arXiv:2104.09883]

Percent-level total precision

$$|V_{cd}|^{D\to\pi} = 0.2338(11)^{\mathrm{Expt}}(15)^{\mathrm{LQCD}}[22]^{\mathrm{EW/QED/SIB}}$$

$$|V_{cs}|^{D\to K} = 0.9589(23)^{\mathrm{Expt}}(40)^{\mathrm{LQCD}}[96]^{\mathrm{EW/QED/SIB}}$$

Measure: Expt.

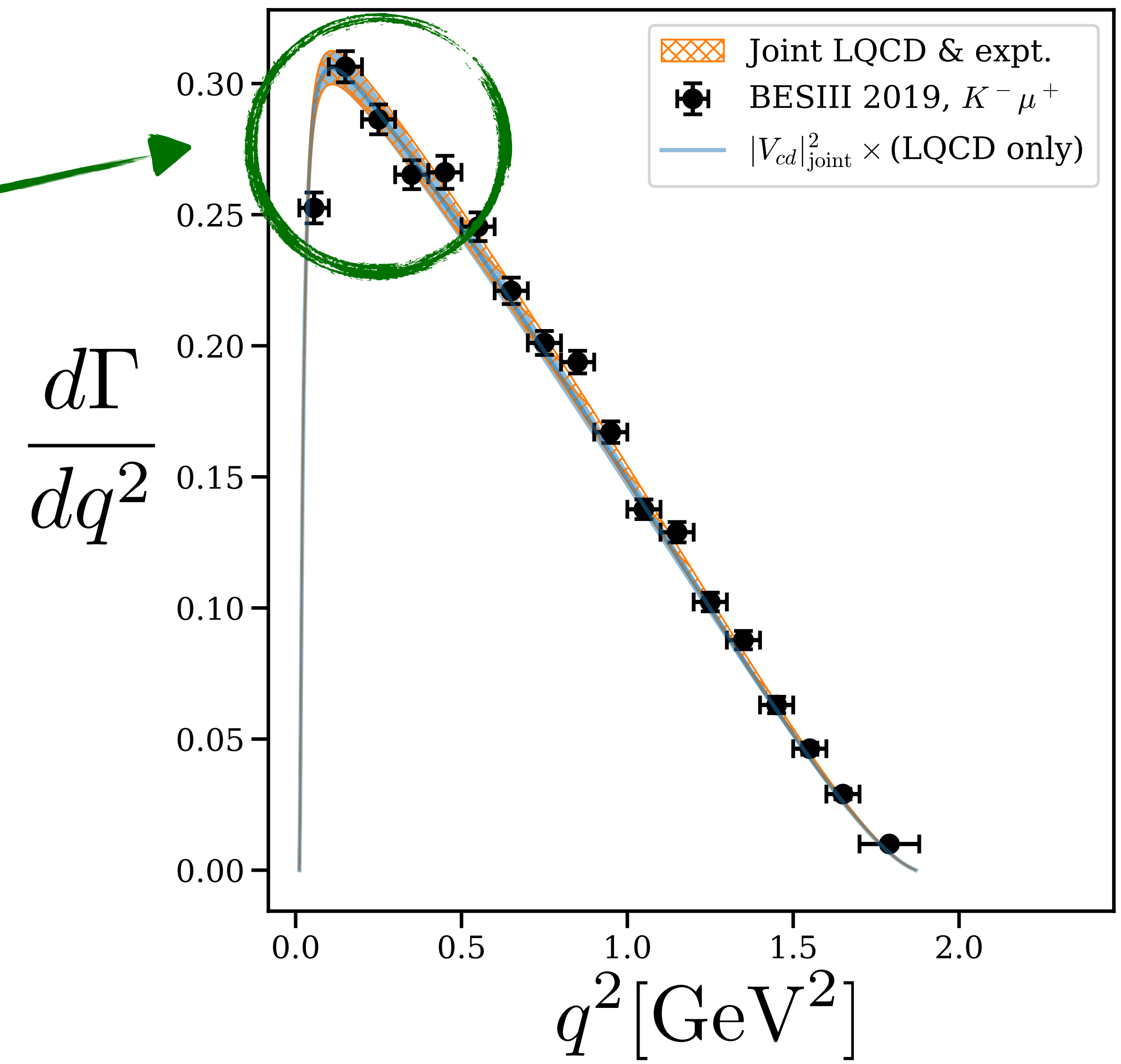
Calculate: LQCD

D-meson Semileptonic Decays

Fermilab-MILC [WJ]
PRD 107 (2023) 9, 094516
arXiv:2212.12648

$D_{(s)} \to K/\pi \ell \nu$ for $|V_{cs}|$ and $|V_{cd}|$

"The full Standard Model on proud display."



Suppressed contributions
$\propto (m_\mu^2/q^2)\,|f_0|^2$

✓ Validation of Standard Model prediction

✓ Enables precise extraction of CKM matrix elements

D-meson Decays

Does theory track experiment?

Fermilab-MILC [WJ]
PRD 107 (2023) 9, 094516
arXiv:2212.12648

1. Do theory and experiment “agree?”
2. Does theory provide good predictions for well-measured physical quantities?

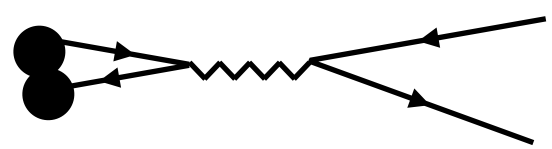
   New: Can now leverage experimentally abundant semileptonic decays to their full potential

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1 \checkmark$$

Leptonic decays

“Simple but rare.”

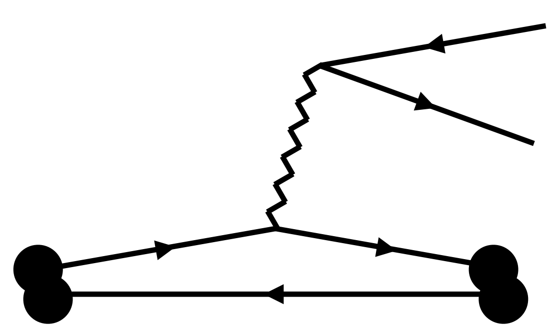
$\mathscr{B}(D^+ \to e^+\nu) \approx 10^{-5}$



Semileptonic decays

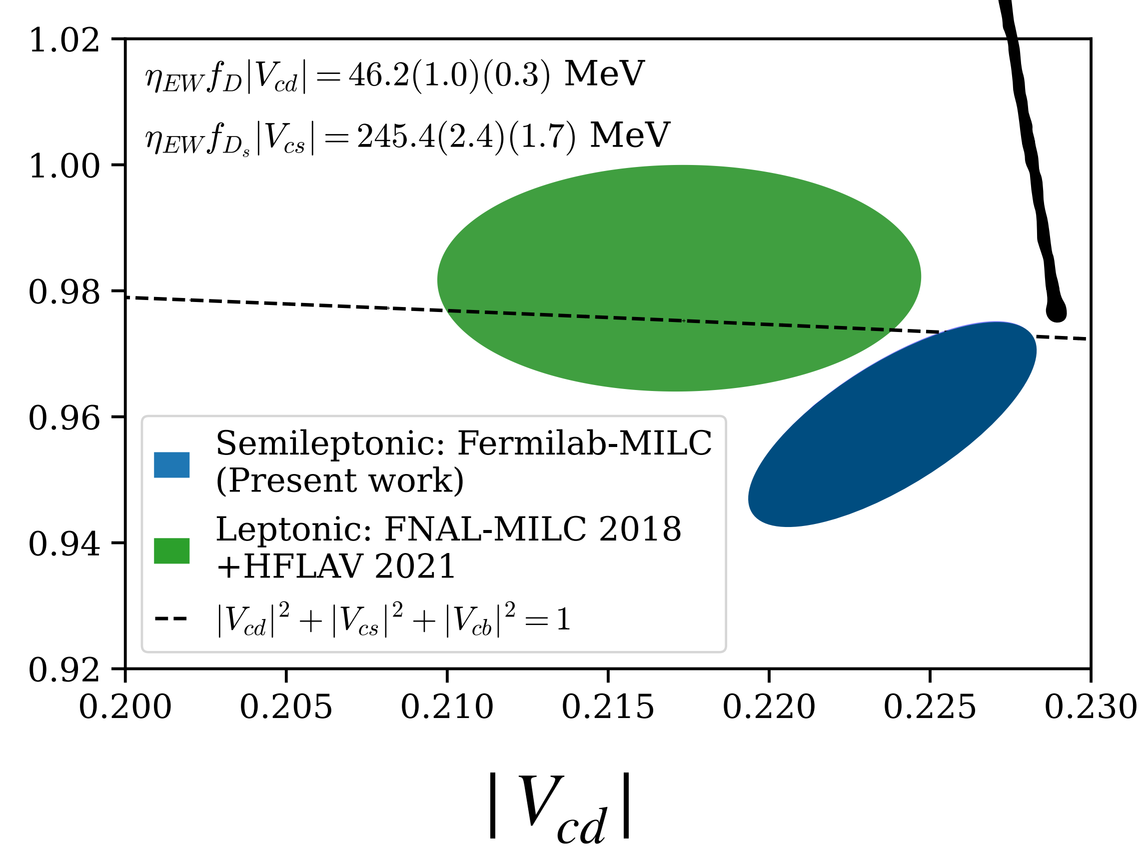
“Abundant but tougher theoretically.”

$\mathscr{B}(D^+ \to \bar{K}^0 e^+\nu) \approx 10\,\%$



$|V_{cs}|$

Blue ellipse < **Green Ellipse**



B-meson Semileptonic Decays

Measure: Expt.

Calculate: LQCD

Vector final state: $B \to D^{\star}\ell\nu$

$$\frac{d\Gamma}{dw} \propto (\text{kinematics}) \times |V_{\text{cb}}|^2 \mathcal{F}(w)^2$$

In terms of "recoil":

$$w = v_B \cdot v_D$$

Hadronic form factors $\mathcal{F}(w)$ $\Longleftrightarrow$ $\langle D^{\star} | V^{\mu}(q) | B \rangle$, $\langle D^{\star} | A^{\mu}(q) | B \rangle$

Improved theory calculations are needed to keep pace with experiment

$$\left|V_{cb}^{\text{excl.}}\right| = (39.4 \pm 0.8) \times 10^{-3}$$

- Combined precision for $B \to D^{(*)}$ ~ 2%
- Commensurate errors from theory/expt
- LHCb, e.g., expects 1% errors in near future

A.X. El-Khadra and P. Urquijo
PDG 2021: 76. Semileptonic b-Hadron Decays, Determination of V_{cb}, V_{ub}

B-meson Semileptonic Decays

Measure: Expt.

Calculate: LQCD

Vector final state: $B \to D^\star \ell \nu$

$$\frac{d\Gamma}{dw} \propto (\text{kinematics}) \times |V_{cb}|^2 \mathcal{F}(w)^2$$

- Historically: LQCD calculations limited to $w = 1$, i.e., hadrons at rest
- Big advance since 2021: three calculations now compute the kinematic dependence of form factors

Fermilab-MILC
A. Bazavov et al.
EPJC 82 (2022) 12, 1141
arXiv:2105.14019

Valence b/c quark:
Anisotropic Wilson
(Fermilab interpretation)

Sea quarks: asqtad

HPQCD
J. Harrison and C. Davies
PRD 109 (2024) 9, 094515
arXiv:2304.03137

Sea and valence quarks:
HISQ

JLQCD
Y. Aoki et al.
PRD 109 (2024) 7, 074503
arXiv:2306.05657

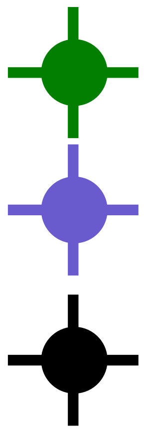
Sea and valence quarks:
Möbius Domain Wall

$B \to D^{\star}\ell\nu$: Comparison of Recent Results

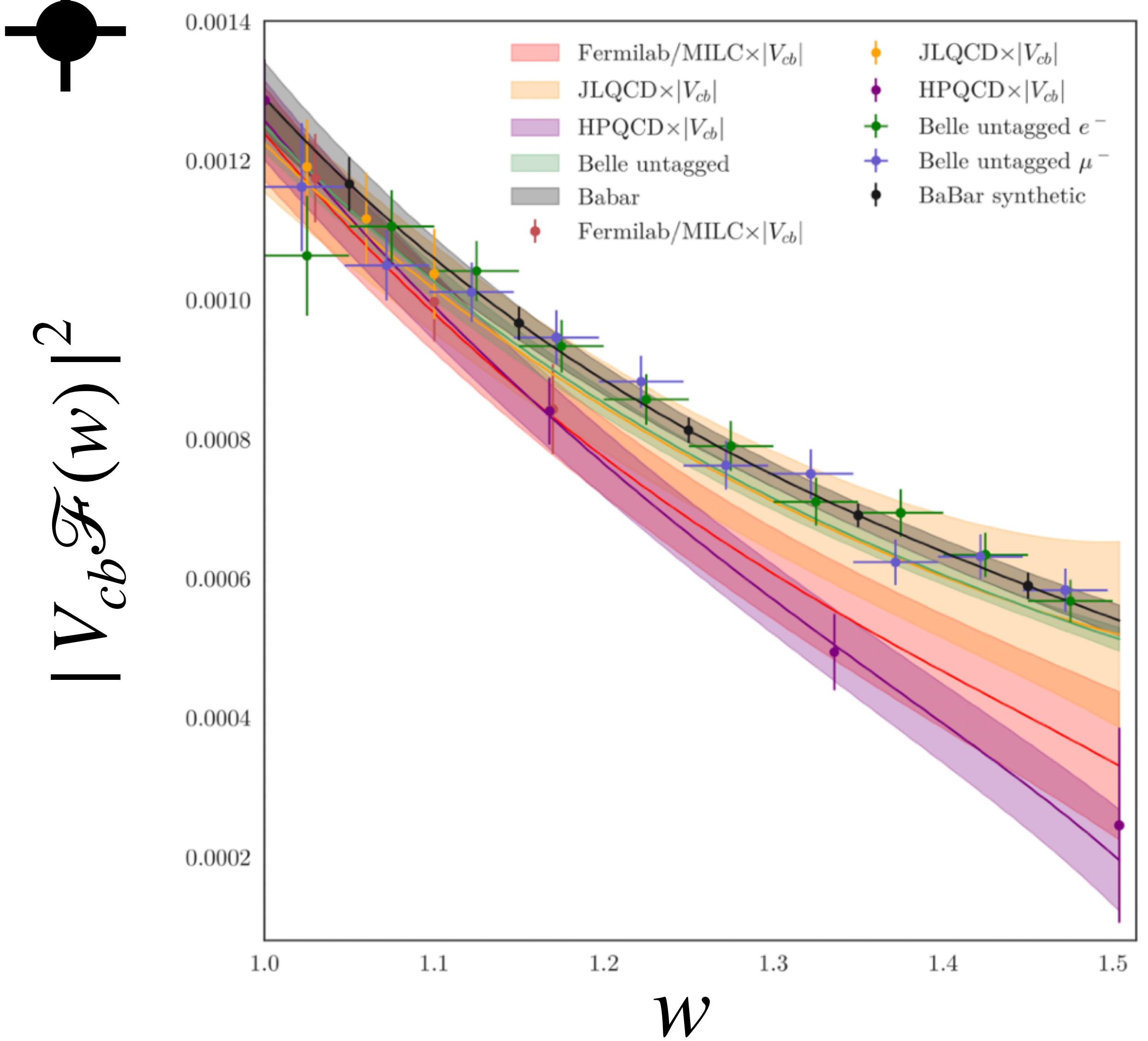
Figure: A. Vaquero
CKM2023 Workshop
[link here]

Lattice QCD
- **Fermilab-MILC** arXiv:2105.14019
- **JLQCD** arXiv:2306.05657
- **HPQCD** arXiv:2304.03137

Experiment (Belle/BaBar)



- Reasonable statistical consistency ($\approx 1-2\sigma$) between LQCD calculations
- Improved precision is required in near term to keep pace with experiment, demonstrate full systematic control
- Similar conclusions regarding $|V_{cb}|$ and $R(D^{\star})$

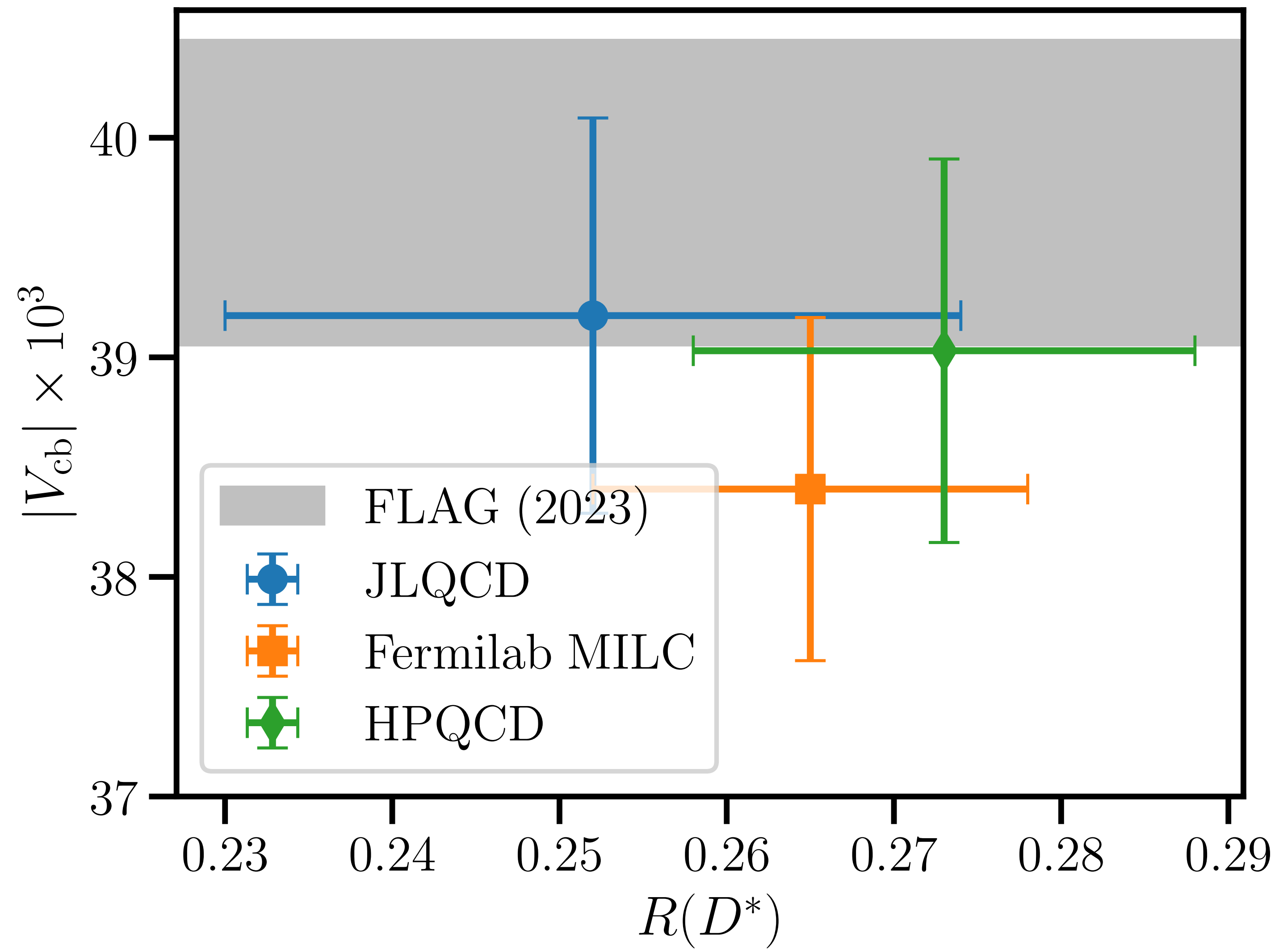


$B \to D^{\star}\ell\nu$: Comparison of Recent Results

Figure: A. Vaquero
CKM2023 Workshop
[link here]

Lattice QCD

- **Fermilab-MILC** arXiv:2105.14019
- **JLQCD** arXiv:2306.05657
- **HPQCD** arXiv:2304.03137

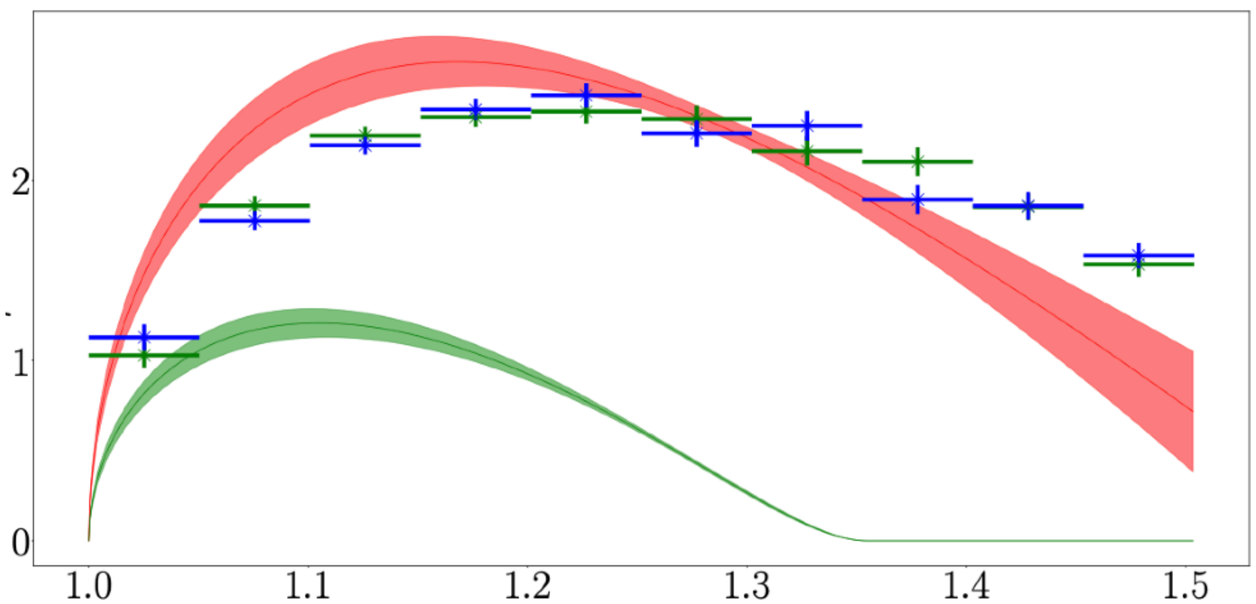


- Reasonable statistical consistency ($\approx 1-2\sigma$) between LQCD calculations
- Improved precision is required in near term to keep pace with experiment, demonstrate full systematic control
- Similar conclusions regarding $|V_{cb}|$ and $R(D^{\star})$

$B \to D^\star \ell \nu$: Comparison of Recent Results

HPQCD
J. Harrison and C. Davies
PRD 109 (2024) 9, 094515
arXiv:2304.03137

$$\frac{d\Gamma}{dw}$$



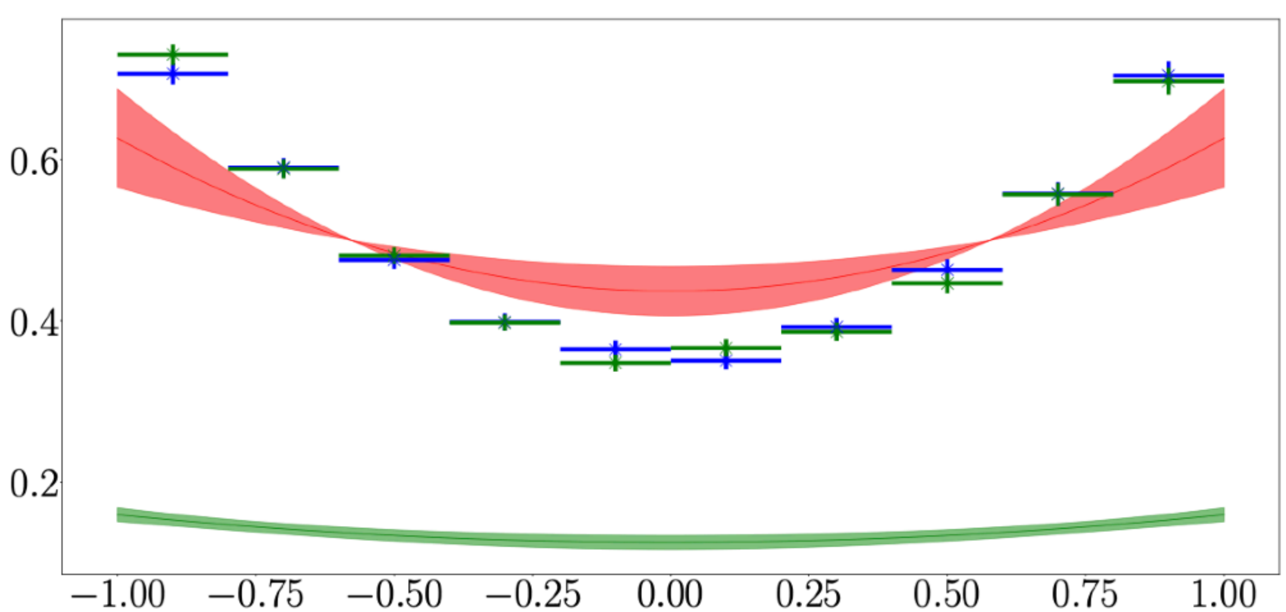
w

$B \to D^\star \ell \nu$

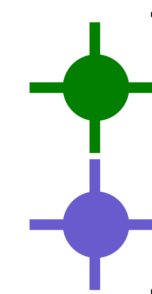
$B \to D^\star \tau \nu$

HPQCD

$$\frac{d\Gamma}{d\cos\theta_{D^\star}}$$



$\cos\theta_{D^\star}$

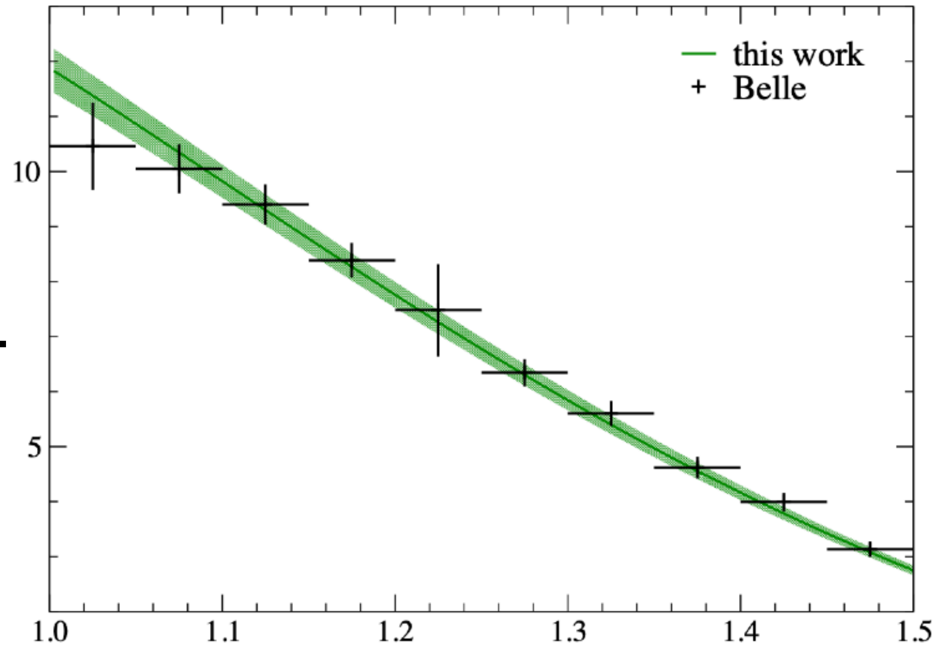


Belle

$B \to D^\star \ell \nu$: Comparison of Recent Results

JLQCD
Y. Aoki et al.
PRD 109 (2024) 7, 074503
arXiv:2306.05657

$$\frac{1}{\sqrt{w^2-1}}\frac{d\Gamma}{dw}$$

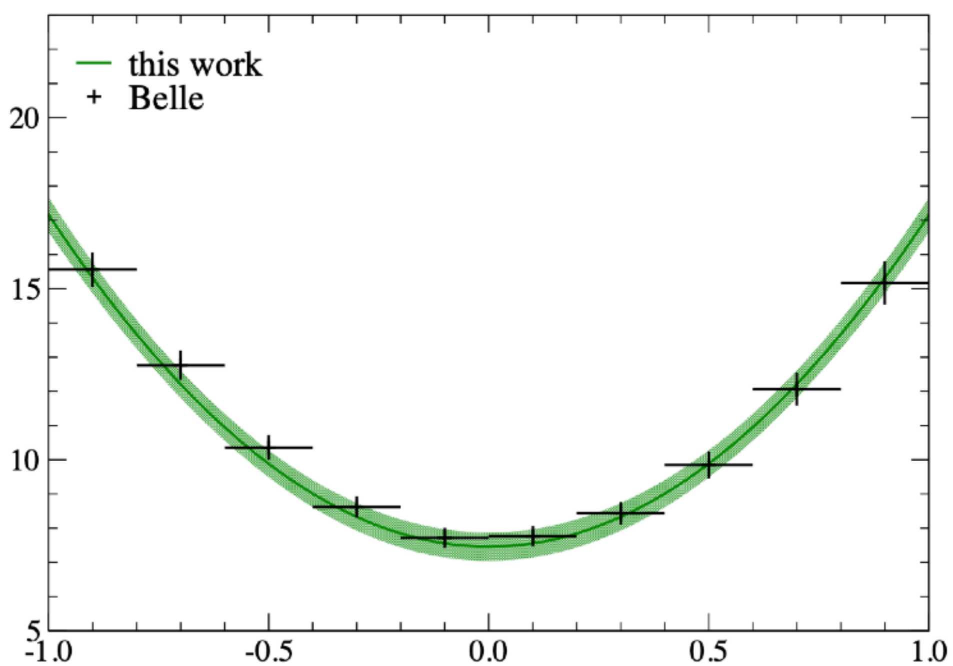


w

$B \to D^\star \ell \nu$ } JLQCD

} Belle

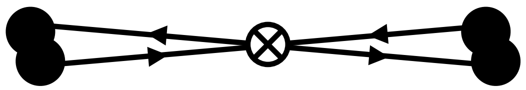
$$\frac{d\Gamma}{d\cos\theta_\nu}$$



$\cos\theta_\nu$

Mixing of Neutral

B-mesons



$\mathbf{V_{ud}}$	$\mathbf{V_{us}}$	$\mathbf{V_{ub}}$
$\pi \to \ell\nu$	$K \to \ell\nu$ $K \to \pi\ell\nu$	$B \to \ell\nu$ $B \to \pi\ell\nu$ $\Lambda_b \to p\ell\nu$
$\mathbf{V_{cd}}$	$\mathbf{V_{cs}}$	$\mathbf{V_{cb}}$
$D \to \ell\nu$ $D \to \pi\ell\nu$ $D_s \to K\ell\nu$	$D_s \to \ell\nu$ $D \to K\ell\nu$	$B \to D\ell\nu$ $B \to D^\star\ell\nu$ $\Lambda_b \to \Lambda_c\ell\nu$
$\mathbf{V_{td}}$	$\mathbf{V_{ts}}$	$\mathbf{V_{tb}}$
$\langle B_d \vert \bar{B}_d \rangle$	$\langle B_s \vert \bar{B}_s \rangle$	

Neutral B-meson Mixing

State of the art circa 2021

Measure: Expt.

Calculate: LQCD

$$\Delta M_{B_q} \propto G_F^2 m_W^2 M_{B_q} |V_{tq}|^2 |V_{tb}|^2 f_{B_q}^2 \hat{B}_{B_q}$$

Bag parameters

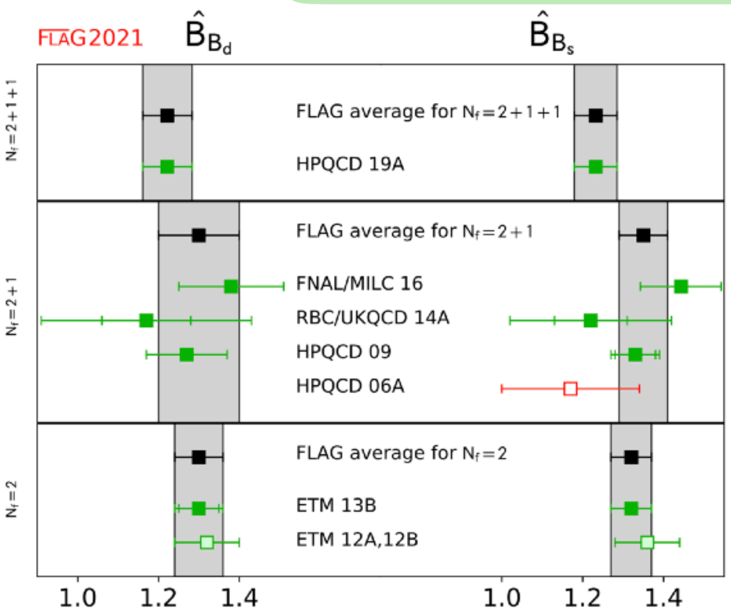
$$B_{B_q}(\mu) = \frac{\langle \bar{B}_q^0 | Q(\mu) | B_q^0 \rangle}{\frac{8}{3} f_{B_q}^2 M_B^2}$$

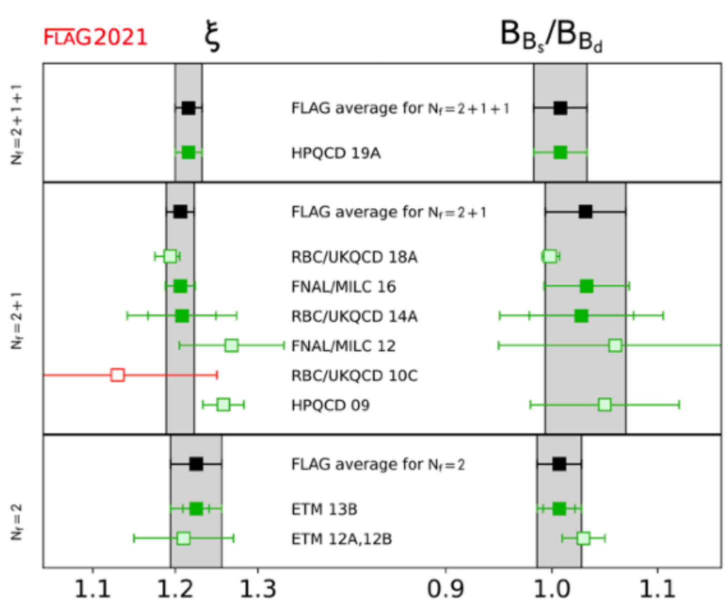
SU(3)-breaking ratio

$$\xi^2 = \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}}$$

$$|V_{td}/V_{ts}| = 0.2053(0.0004)^{\text{EXP}}(0.0029)^{\text{LQCD}}$$

- Experiments measure ΔM_{B_q} very precisely
- Computed values for ΔM_{B_q} are proportional to "bag parameters
  - Known to ≈ 5-10%
- Ratio $|V_{td}/V_{ts}|$ constrained by "SU(3)-breaking ratio"
  - Known to 1-2%
- Improved theoretical calculations are timely





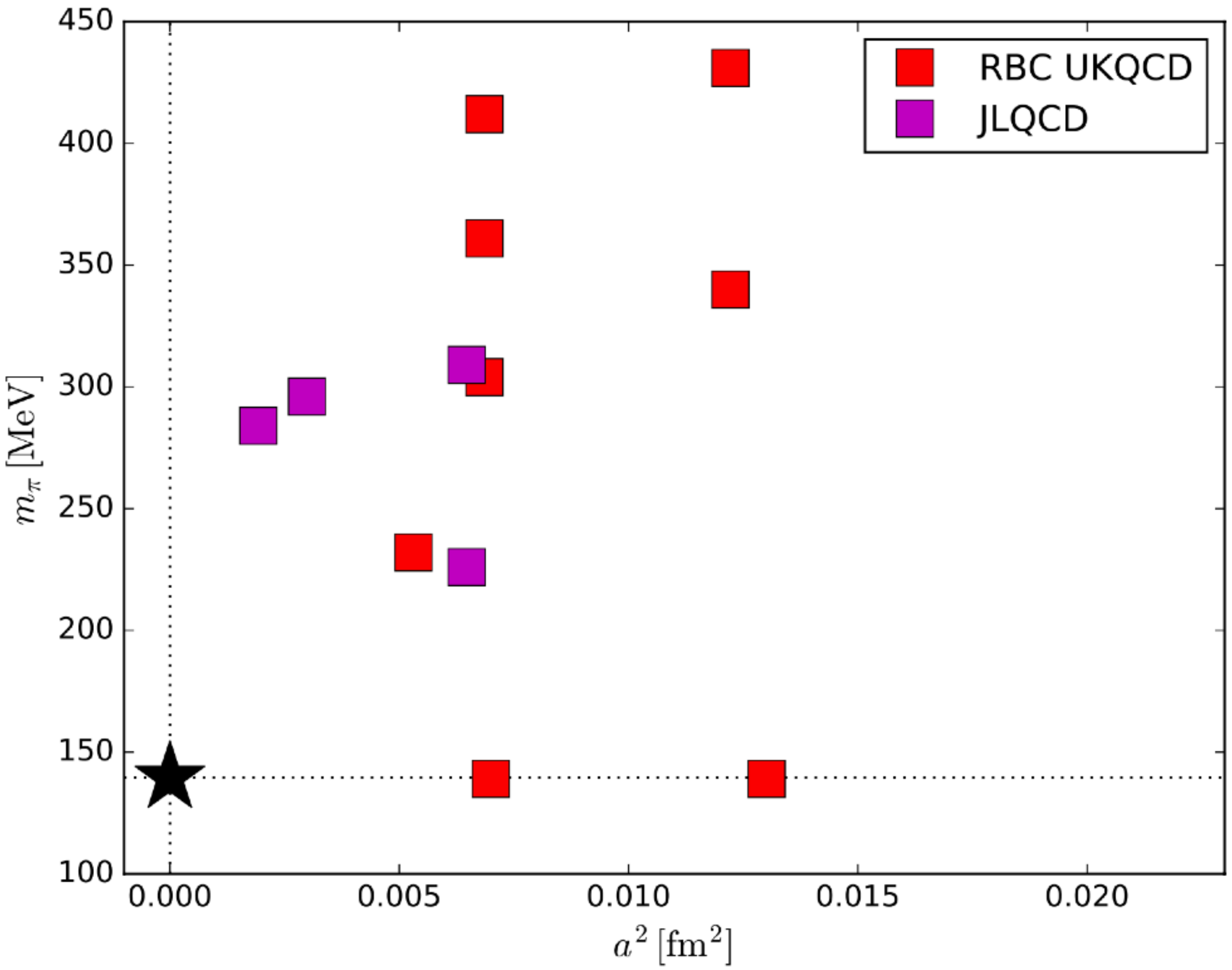
FLAG Review 21
Y. Aoki et al.
EPJC 82 (2022) 10, 869
arXiv: 2111.09849

Neutral B-meson mixing

RBC/UKQCD/JLQCD
Lattice 2021
arXiv:2111.11287
+ recent update @ Lattice 2023

- Work in progress
- (N_f=2+1) RBC/UKQCD and JLQCD DW ensembles
- 15 ensembles with lattice spacings: $1/a \in [1.7, 4.5]\,\mathrm{GeV}$
- $M_\pi \in [140, 360)\,\mathrm{MeV}$
- Valence b: domain wall
- All-domain wall setup → block-diagonal physical renormalization pattern
- Aiming for percent-level uncertainties

$$Z_{ij} = \begin{pmatrix} * & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 \\ 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & * & * \end{pmatrix}$$



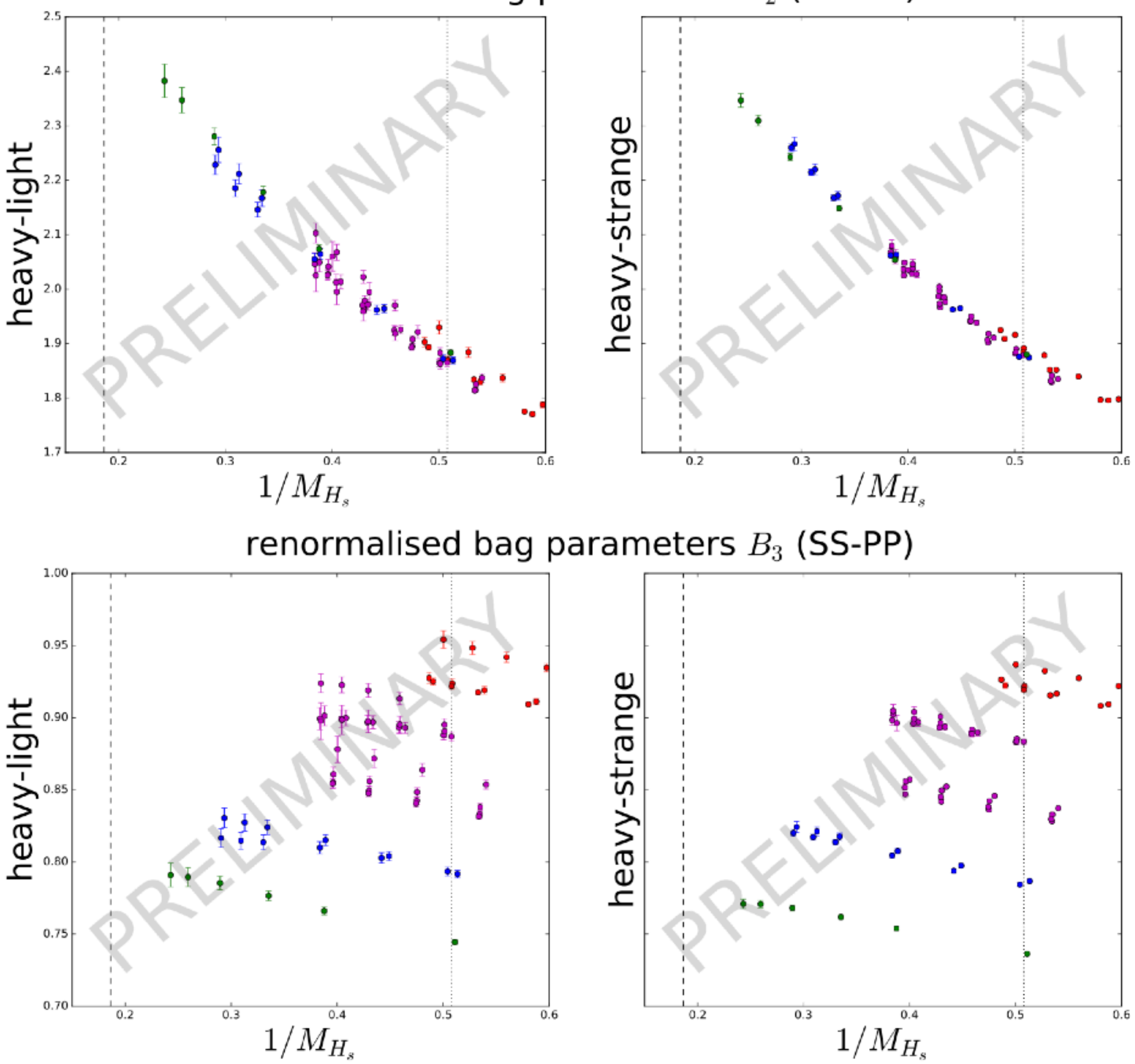
Neutral B-meson mixing

RBC/UKQCD/JLQCD
Lattice 2021
arXiv:2111.11287
+ recent update @ Lattice 2023

RBC/UKQCD/JLQCD IV – individual bag parameters

renormalised bag parameters B_2 (VV-AA)

renormalised bag parameters B_3 (SS-PP)



- Mild discretisation effects
- Seemingly mild behaviour with a, M_π, m_s, V

- Large discretisation effects
- More noticeable chiral effects

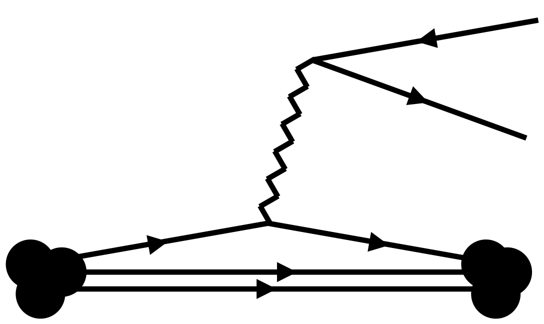
25 / 28 J. Tobias Tsang (CERN) Status and Outlook for B-mixing and the ξ parameter

Summary & Outlook

- **Lattice QCD calculations have achieved:**
  - Sub-percent precision for leptonic decays
  - Percent-level precision for D-meson semileptonic decays
  - Coming soon: Percent-level precision for B-meson semileptonic decay
  - Coming soon: Percent-level precision for B-meson mixing observables
- **Enabling "technologies" for high precision include:**
  - Ensembles with physical mass pions: $M_\pi \approx 140~\mathrm{MeV}$
  - Relativistic light-quark action(s) for charm and bottom: absolutely normalized currents
  - Highly improved actions: reduced discretization effects for charm and bottom
- **Precise LQCD + latest experimental results give:**
  - CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$ at O(1%)
  - Improved tests of CKM unitarity
  - New perspectives on the b anomalies

Backup

Semileptonic Decays
of D-baryons



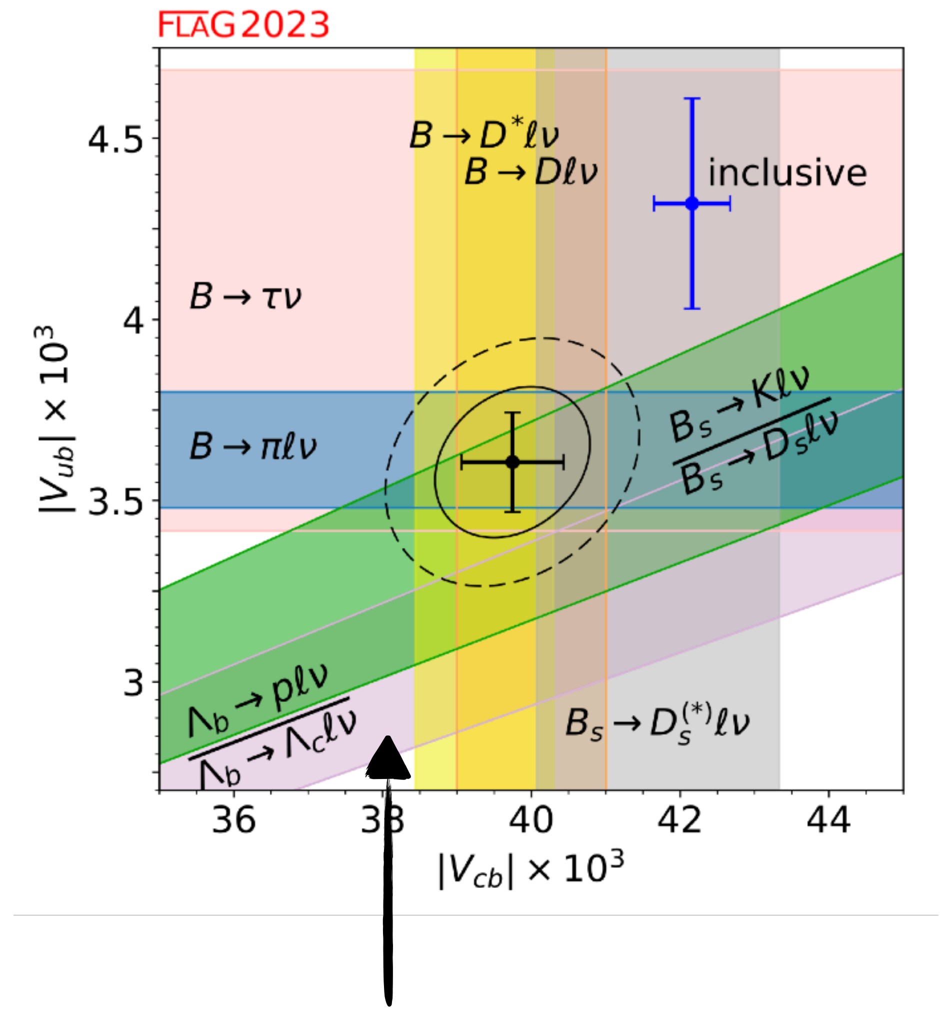
$\mathbf{V_{ud}}$	$\mathbf{V_{us}}$	$\mathbf{V_{ub}}$
$\pi \to \ell\nu$	$K \to \ell\nu$	$B \to \ell\nu$
	$K \to \pi\ell\nu$	$B \to \pi\ell\nu$
		$\Lambda_b \to p\ell\nu$
$\mathbf{V_{cd}}$	$\mathbf{V_{cs}}$	$\mathbf{V_{cb}}$
$D \to \ell\nu$	$D_s \to \ell\nu$	$B \to D\ell\nu$
$D \to \pi\ell\nu$	$D \to K\ell\nu$	$B \to D^\star\ell\nu$
$D_s \to K\ell\nu$	$\Lambda_c \to \Lambda\ell\nu$	$\Lambda_b \to \Lambda_c\ell\nu$
$\Lambda_c \to N\ell\nu$	$\Xi_c \to \Xi\ell\nu$	
$\mathbf{V_{td}}$	$\mathbf{V_{ts}}$	$\mathbf{V_{tb}}$
$\langle B_d \vert \bar{B}_d \rangle)$	$\langle B_s \vert \bar{B}_s \rangle)$	

B-baryon semileptonic decays

Next-generation calculations $\Lambda_b \to p, \Lambda, \Lambda_c$

S. Meinel
PoS LATTICE2023 (2024) 275
arXiv:2309.01821

- Work in progress
- New ensembles for improved systematic control
  - One with physical M_π
  - One with larger volume
  - One with $a \approx 0.07$ fm
- EFT treatment charm and bottom quark
  - Anisotripic clover action, tuned to match dispersion relations
  - Improved tuning of heavy quark masses
- "Mostly non-perturbative" renormalization
  - Fully non-perturbative renormalization may be possible



B-baryon decays

$$\frac{\Lambda_b \to \Lambda \ell \nu}{\Lambda_b \to \Lambda_c \ell \nu}$$

D-baryon semileptonic decays

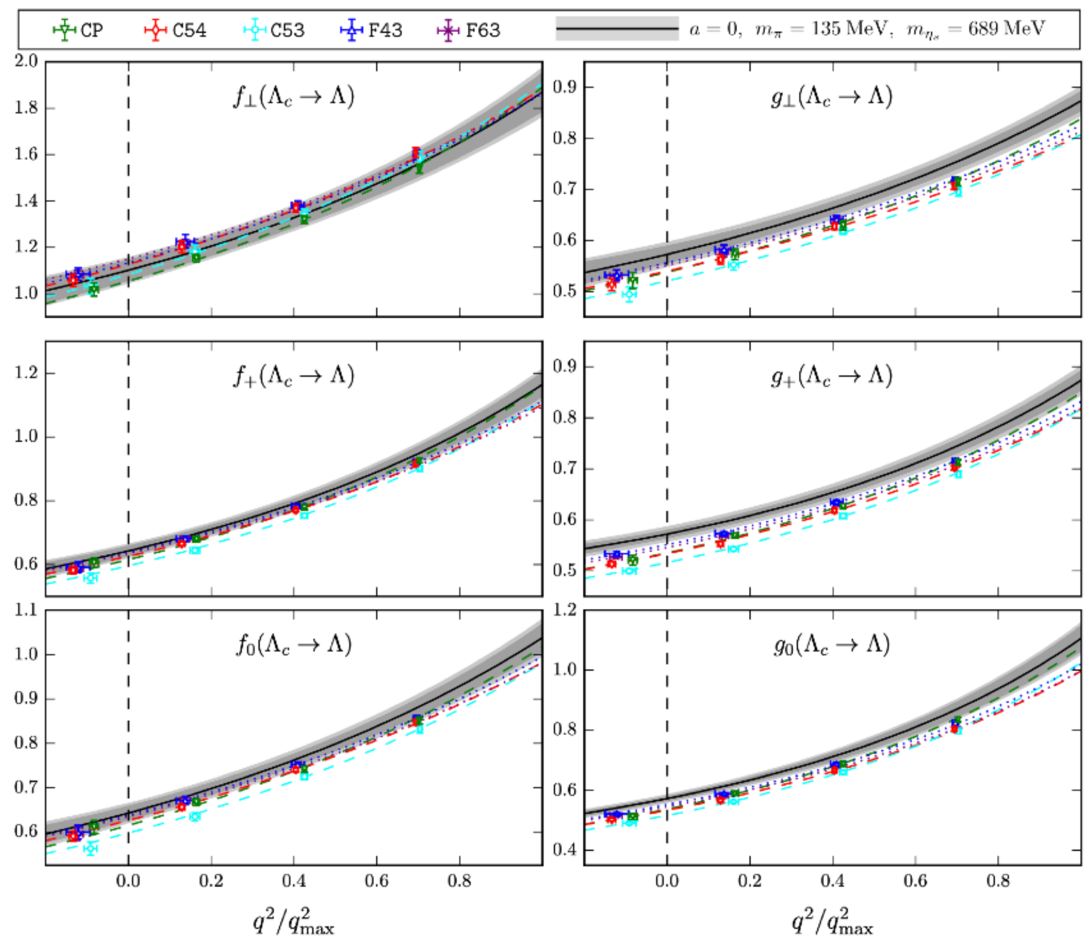
S. Meinel

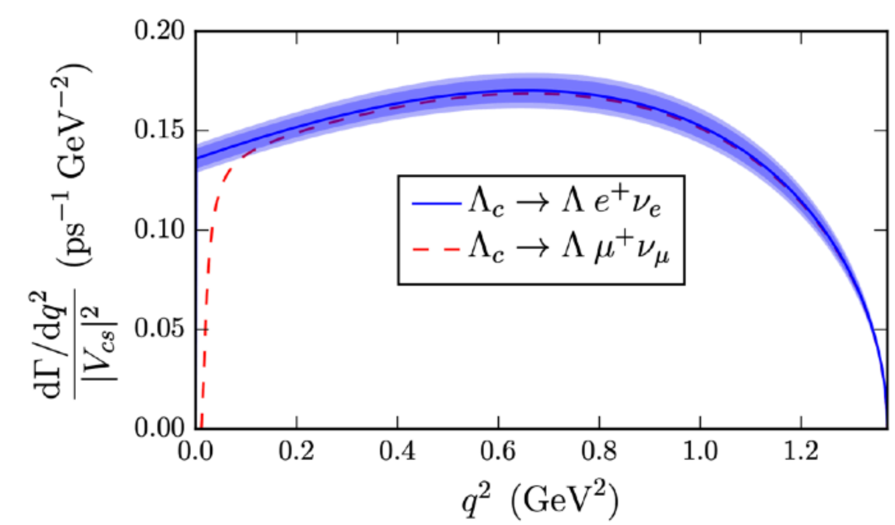
PRL 118 (2017) 8, 082001
arXiv:1611.09696

$\Lambda_c \to \Lambda \ell \nu$

- 5x ensembles, $N_f = 2+1$ domain wall fermions
  - $a \in \{0.09, 0.11\}$ fm
  - $M_\pi \in \{139 - 350\}$ MeV
- Valence charm: Columbia RHQ (clover action, tuned to give J/ψ dispersion relation)
- "Mostly non-perturbative" renormalization
- First-ever determination of $|V_{cs}|$ [≈6%] from baryon decays when combined with measurements from BESIII

$$|V_{cs}| = \begin{cases} 0.951(24)_{\mathrm{LQCD}}(14)_{\tau_{\Lambda_c}}(56)_{\mathcal{B}}, & \ell = e, \\ 0.947(24)_{\mathrm{LQCD}}(14)_{\tau_{\Lambda_c}}(72)_{\mathcal{B}}, & \ell = \mu, \\ 0.949(24)_{\mathrm{LQCD}}(14)_{\tau_{\Lambda_c}}(49)_{\mathcal{B}}, & \ell = e, \mu, \end{cases}$$



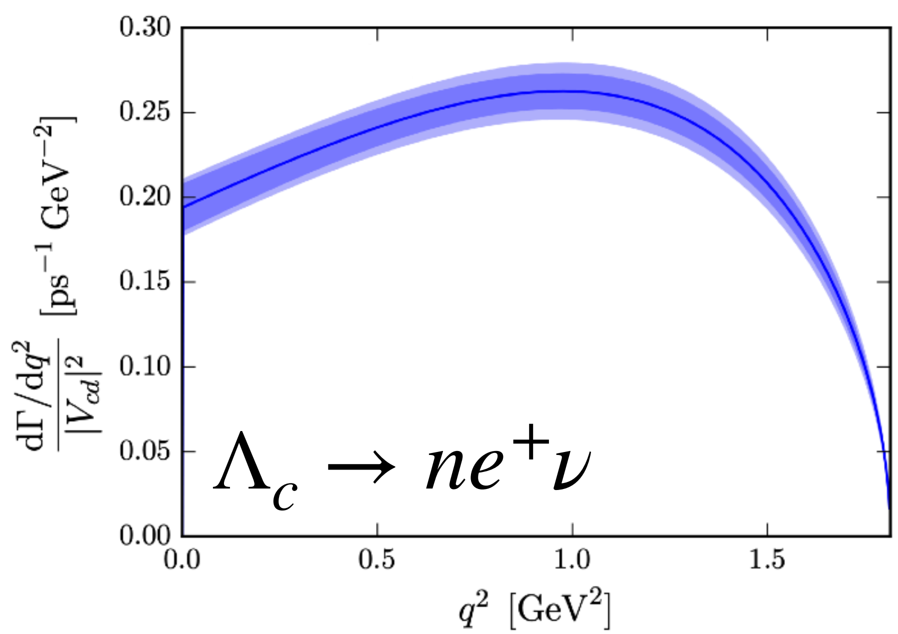


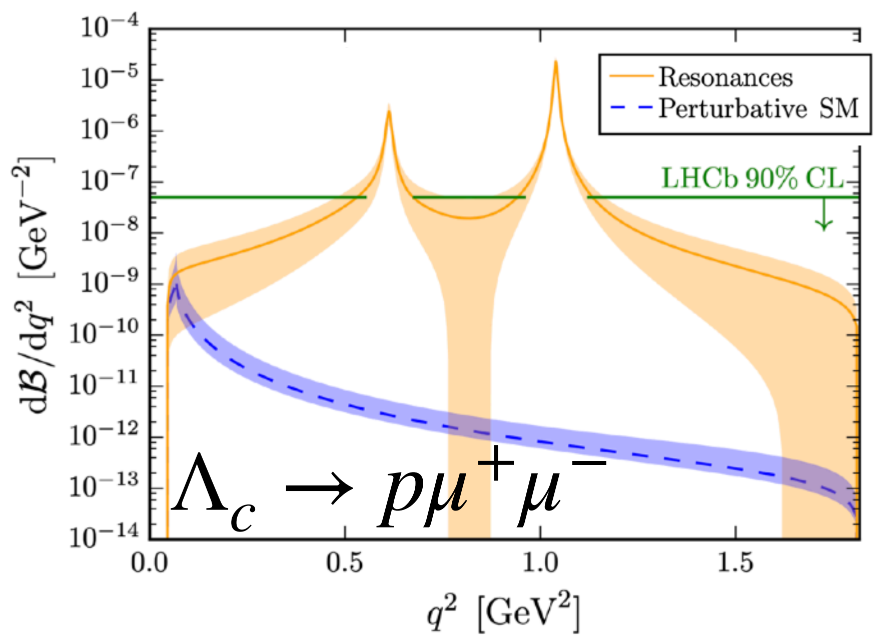
D-baryon semileptonic decays

S. Meinel
PRD 97 (2018) 3, 034511
arXiv:1712.05783

$\Lambda_c \to N$ form factors

- Isospin limit: same form factors for $\Lambda_c \to p^+$, $\Lambda_c \to n$
- 6x ensembles, $N_f = 2+1$ domain wall fermions
  - $a \in \{0.09, 0.11\}$ fm
  - $M_\pi \in \{240 - 350\}$ MeV
- Valence charm: Columbia RHQ
- “Mostly non-perturbative” renormalization
- SM predictions for charged-current $\Lambda_c \to n\ell^+\nu$ rates [≈6.4%]
  - $\Gamma(\Lambda_c \to ne^+\nu)/|V_{cd}|^2 = (0.405 \pm 0.016_{\text{stat}} \pm 0.020_{\text{syst}})\ \text{ps}^{-1}$
  - Tough to measure experimentally (n and ν in final state)
  - Results larger by factor of ≈1.5–2 compared to other calculations [quark models, sum rules, SU(3)]
- Rare neutral-current decay:
  - LHCb 2018: $\mathcal{B}(\Lambda_c \to p^+\mu^+\mu^-) < 7.7 \times 10^{-8}$ [90%]
  - Comparison to LQCD with additional assumptions
    - SM Wilson coefficients at NLO
    - Breit-Wigner model for intermediate $\phi/\omega/\rho$





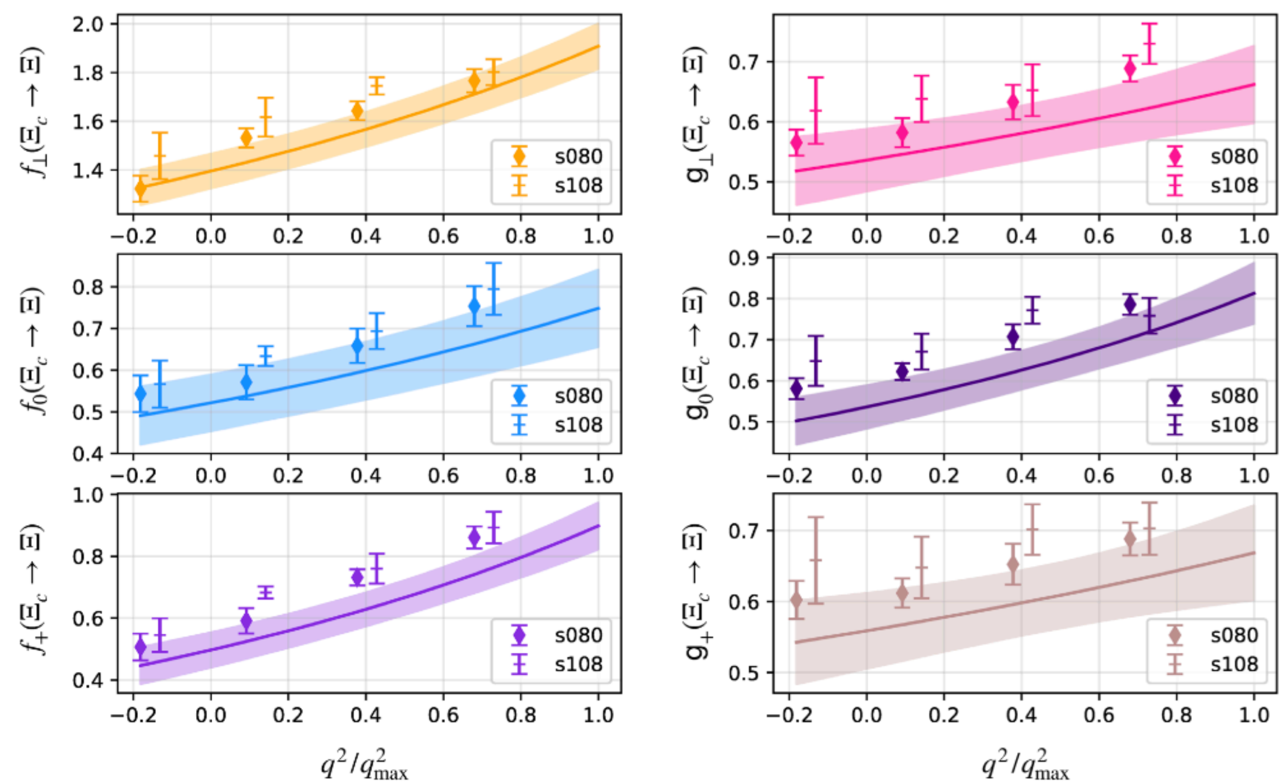
LHCb
PRD 97 (2018) 9, 091101
arXiv:1712.07938

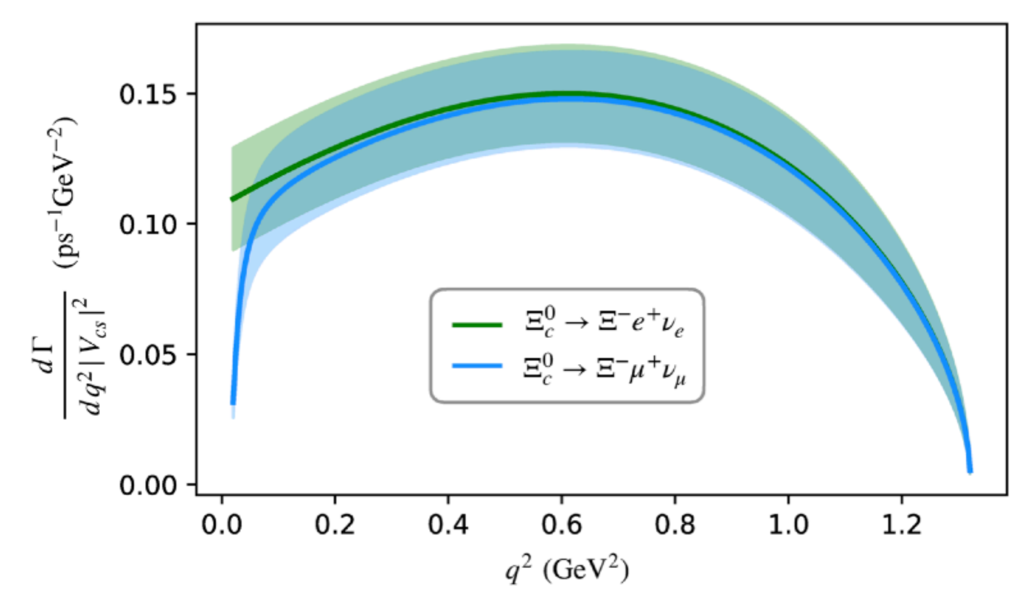
D-baryon semileptonic decays

Q.-A. Zhang et al.
Chin.Phys.C 46 (2022) 1, 011002
arXiv:2103.07064

$\Xi_c \to \Xi \ell \nu$ form factors

- 2x ensembles with N_f=2+1 Wilson clover quarks
  - $a \in \{0.11, 0.08\}$ fm
  - $M_\pi \approx 300$ MeV
- Continuum extrapolation is given
- No chiral extrapolation to physical pion mass
- Extractions of $|V_{cs}|$:
  - Using ALICE branching-fraction measurements: $|V_{cs}| = 0.983(0.060)^{\text{stat}}(0.065)^{\text{syst}}(0.167)^{\text{exp}}$ [≈19%]
  - Using Belle branching-fraction measurements $|V_{cs}| = 0.834(0.051)^{\text{stat}}(0.056)^{\text{syst}}(0.127)^{\text{exp}}$ [≈18%]

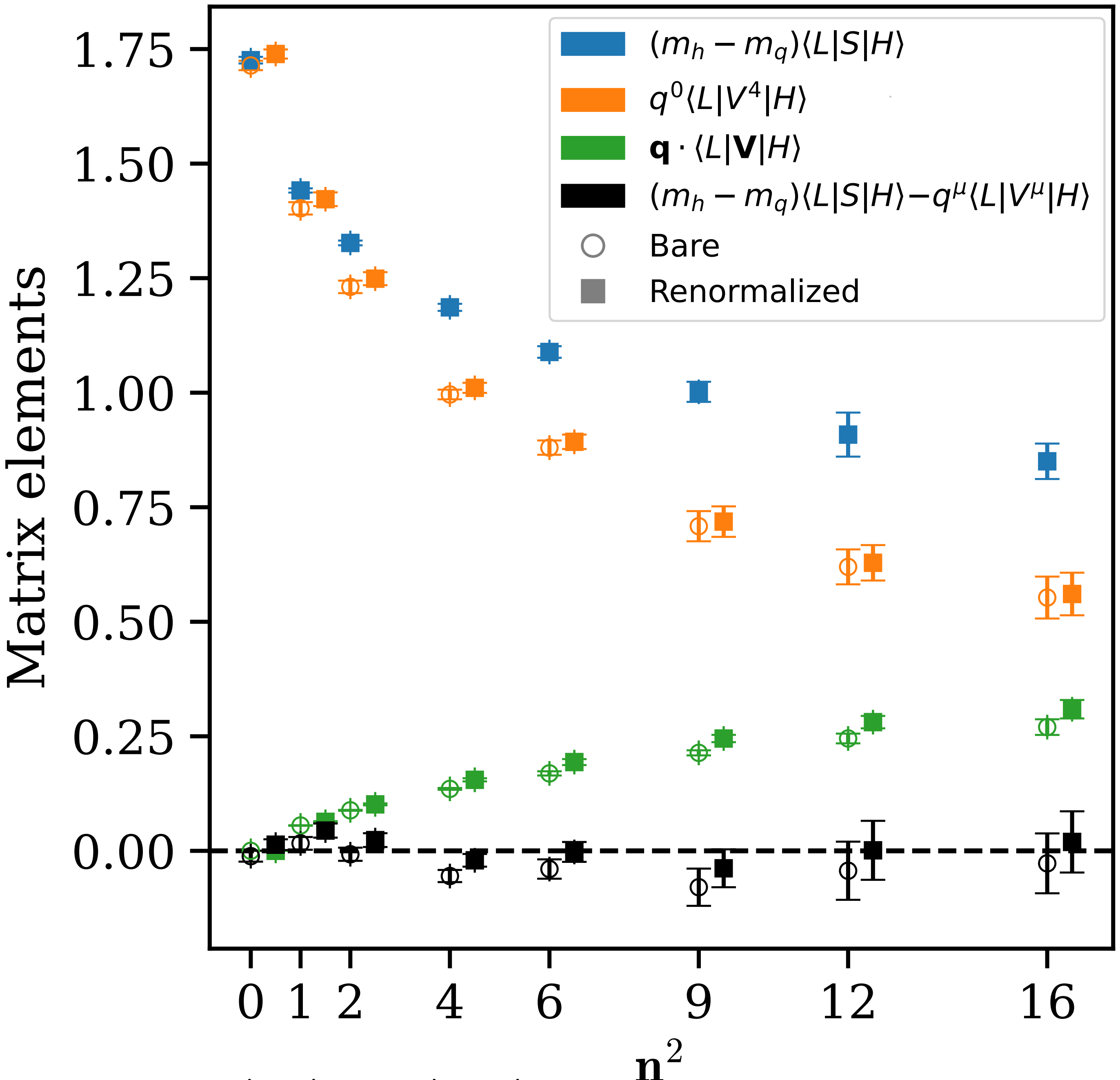




Renormalization semileptonic decays

Example $D \to \pi \ell \nu$

- Recall $\mathcal{J} = Z_J J$
- PVCV: $\partial_\mu \mathcal{V}^\mu = (m_1 - m_2)\mathcal{S}$
- For the HISQ action, the local scalar density is absolutely normalized.
- Imposing PCVC in a global fit gives values for Z_{V_0} and Z_{V_i}
- In terms of D→π matrix elements, PCVC reads:



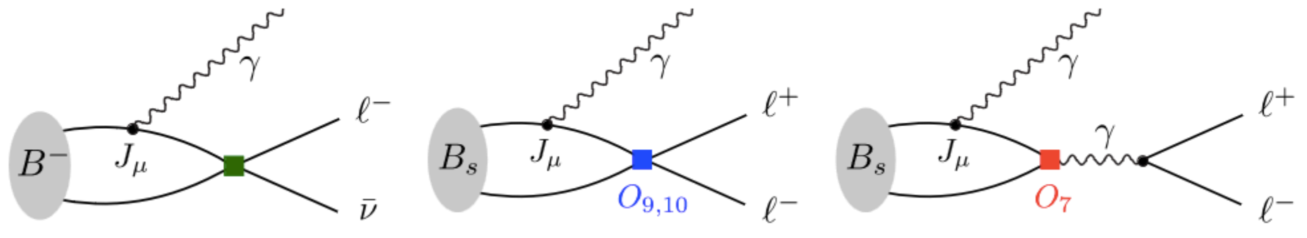
$$Z_{V^0}(M_D - E_\pi)\left\langle \pi \right| V^0 \left| D \right\rangle + Z_{V^i}\,\boldsymbol{q}\cdot\left\langle \pi \right| \boldsymbol{V} \left| D \right\rangle$$
$$= (m_c - m_d)\left\langle \pi \right| S \left| D \right\rangle$$

Radiative Leptonic Decays

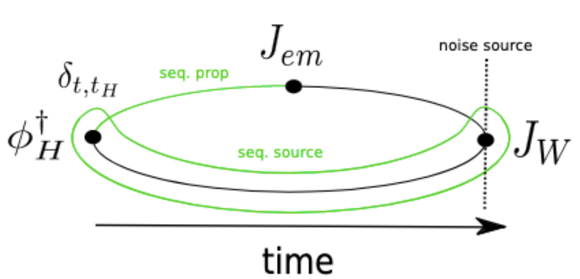
Kane, Lehner, Meinel, Soni
Lattice 2019
arXiv:1907.00279
Kane, Giusti, Lehner, Meinel, Soni
Lattice 2021
arXiv:2110.13196

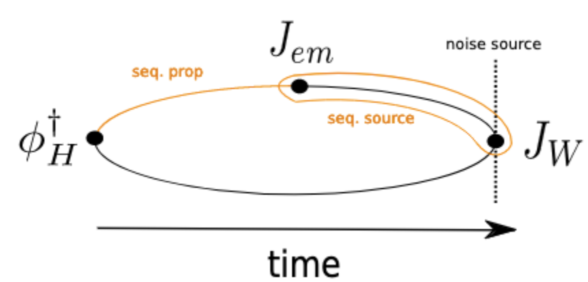
$D_s \to \ell\nu\gamma, K \to \ell\nu\gamma$

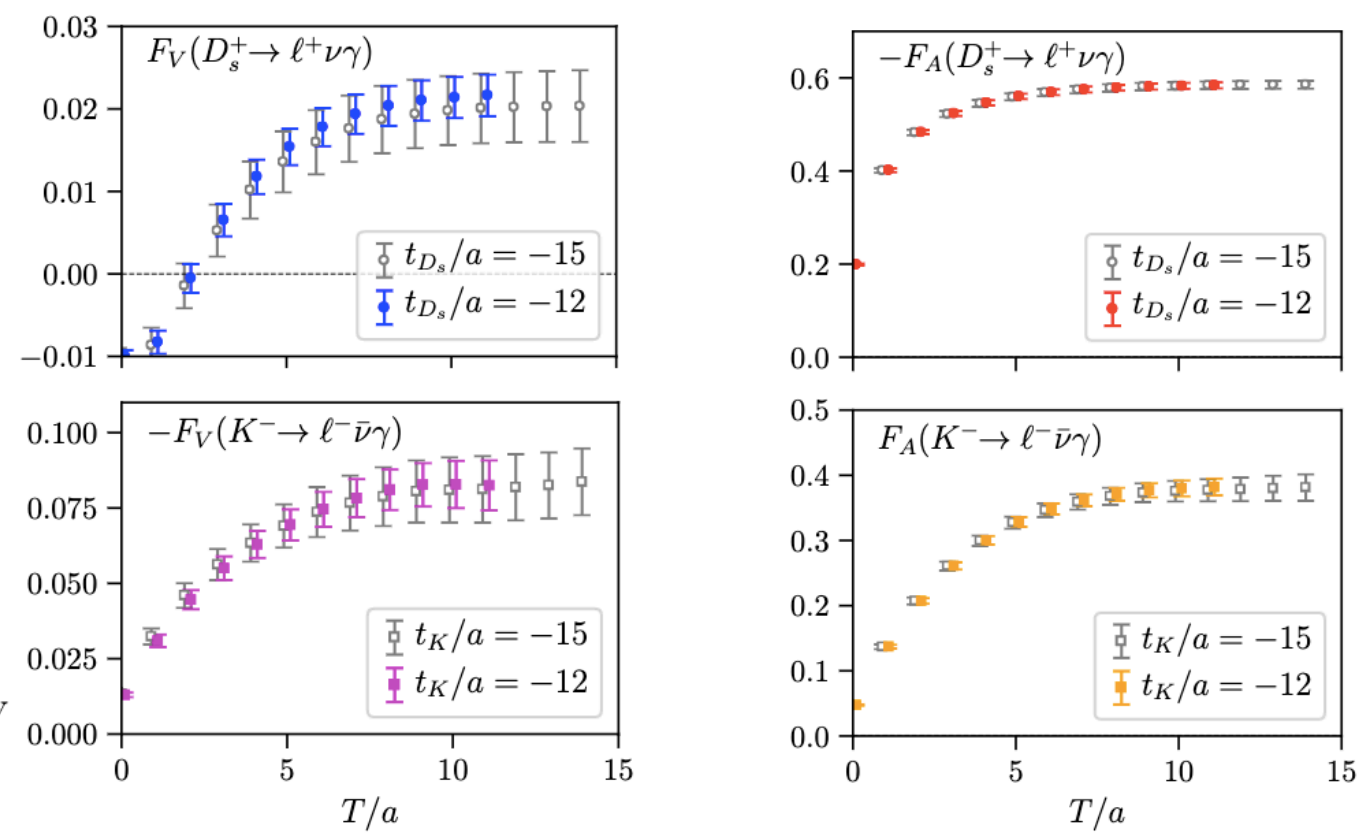
- Radiative decays probe weak interaction and hadronic structure
- Example: $B \to \ell\nu\gamma$ is sensitive to the LCDA parameter λ_B
- Radiative leptonic decays probe all Wilson coefficients in the Weak effective Hamiltonian
- Exploratory calculations developing methods



$$T_{\mu\nu} = -i\int d^4x e^{ip_\gamma\cdot x}\langle 0|T\{J_\mu(x)J_\nu^{\text{weak}}(0)\}|H(\mathbf{p})\rangle$$

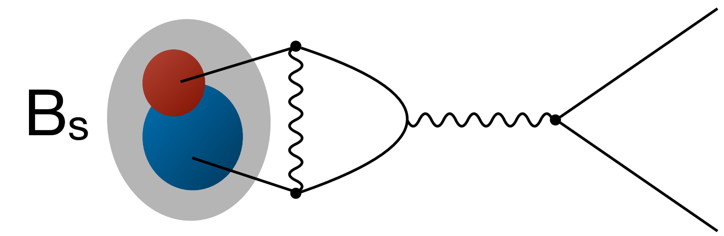






Rare Decay $B_s \to \mu^+\mu^-$

Uncertainty Breakdown

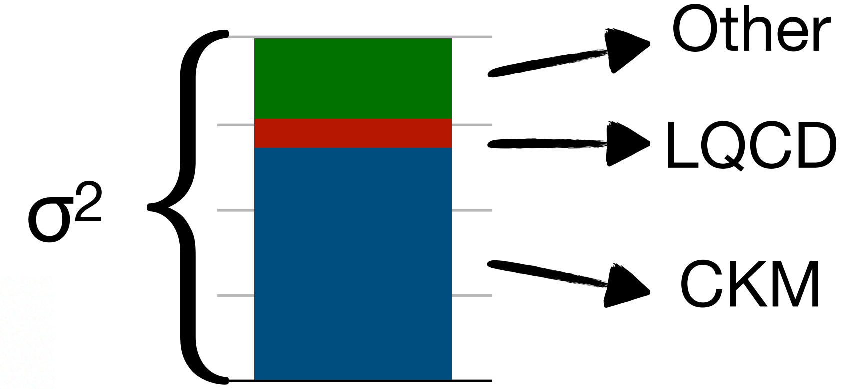


SM prediction for rare leptonic decay rate

Beneke et al, arXiv:1908.07011, JHEP 2019

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = 3.660(38) \times 10^{-9}$$

$$\overline{\mathrm{Br}}^{(0)}_{s\mu} = \begin{pmatrix} 3.599 \\ 3.660 \end{pmatrix} \Big[1 + \begin{pmatrix} 0.032 \\ 0.011 \end{pmatrix}_{f_{B_s}} + 0.031|_{\mathrm{CKM}} + 0.011|_{m_t} + 0.006|_{\mathrm{pmr}} + 0.012|_{\text{non-pmr}} \, {}^{+0.003}_{-0.005}|_{\mathrm{LCDA}} \Big] \cdot 10^{-9}$$



Lattice QCD value for f_{B_s} is now a sub-dominant source of uncertainty

- Parametric uncertainties
  - Long distance (f_{B_s}) and short distance (CKM, m_t)
- Non-QED parametric (Γ_q, α_s)
- Non-QED non-parametric (μ_W, μ_b, and higher order)
- QED parametric: B-meson LCDA parameters (λ_B, $\sigma_{1,2}$)

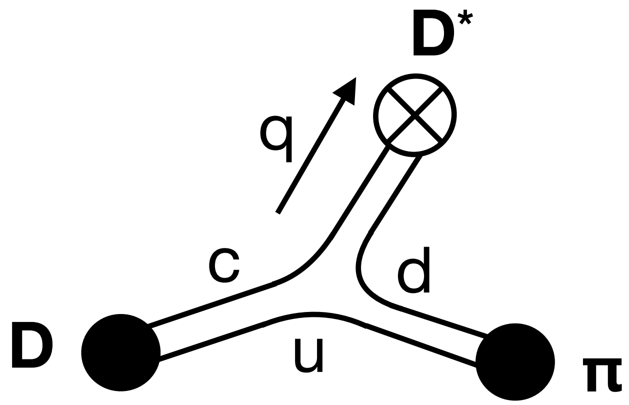
Chiral-continuum analysis

Heavy-meson rooted staggered chiral perturbation theory

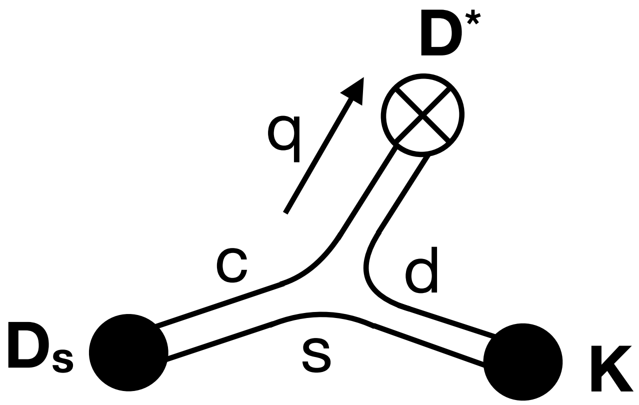
- With simulations at and above the physical pion mass, the chiral fits are ***interpolations,*** not extrapolations

- The shape of the form factors can be modeled with EFT combining:
    - Chiral symmetry $$\Sigma = \exp(2i\phi/f)$$
    - HQET spin symmetry $$H^a = \frac{1+\not{v}}{2}\left[P^{*a}_{\mu}(v)\gamma^{\mu} - P^a(v)\gamma_5\right]$$
    - Light-quark discretization effects $$\frac{1}{16}\sum_{\text{tastes }\xi} M_\xi^2 \log\left(\frac{M_\xi^2}{\Lambda^2}\right)$$

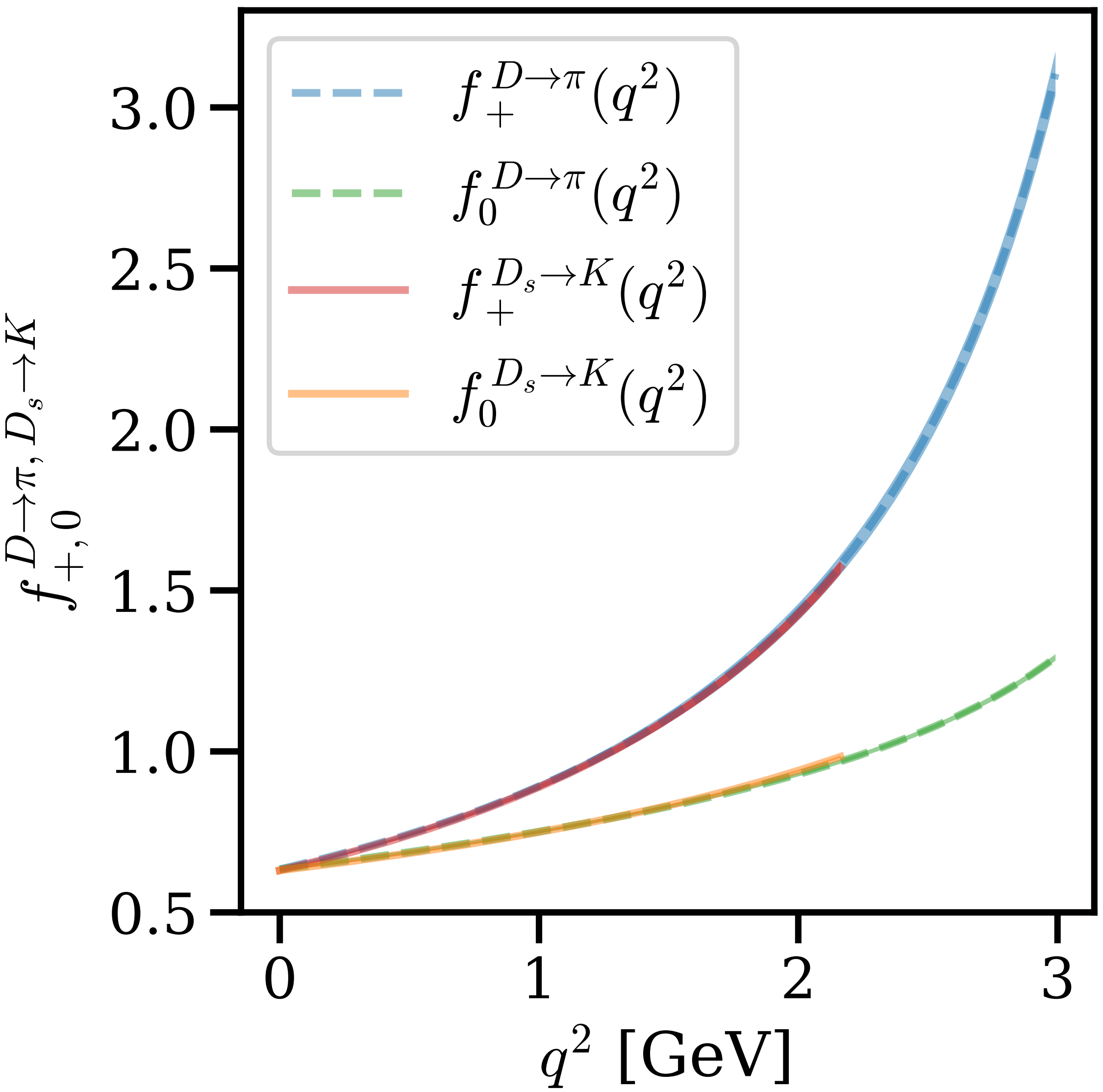
Spectator dependence: D→π vs D_s→K



$$\sim \frac{1}{M_{D^*}^2 - q^2} \sim$$



- D→π and D_s→K only differ by the mass of the spectator quark
- Vector and scalar form factors agree at ≲ 2% level throughout the kinematic range
- Older unpublished results by HPQCD are consistent with our findings



Experimental Motivation: CKM Unitarity

First-row unitarity?

- PDG 2022: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(6)^{|V_{ud}|^2}(4)^{|V_{us}|^2}$
- Quoted value has 2σ tension with unity, using as inputs
  - $|V_{ud}|$ from super-allowed $0^+ \to 0^+ \; \beta$ decays
  - $|V_{us}|$ from semileptonic decay: $K_{\ell 3} \equiv K \to \pi \ell \nu$
  - Tension increases to ≈3σ if nuclear-structure uncertainties from $|V_{ud}|$ are ignored
- Similar ≈2-3σ tension if $|V_{us}|/|V_{ud}|$ taken from ratio of leptonic decays K_{l2}/π_{l2}

- Historically, similarly precise tests of second-row unitarity have been limited by experimental and theoretical precisions.
- Today's talk: recent progress in the second row via semileptonic decays

E. Blucher and W.J. Marciano
PDG 2022: 67. V_{ud}, V_{us}, the Cabibbo Angle, and CKM, Unitarity

Lattice QCD: particle masses

- 2-point correlation functions encode particle masses
- Analogy with condensed matter
  - Correlation length $\lambda \longleftrightarrow$ Particle mass 1/m

$$\langle (\bar{q}\, q)_t (\bar{q}\, q)_0 \rangle \sim \exp(-mt)$$

Lattice QCD: particle masses

- Hadronic spectrum $\longleftrightarrow$ QCD 2pt correlation functions

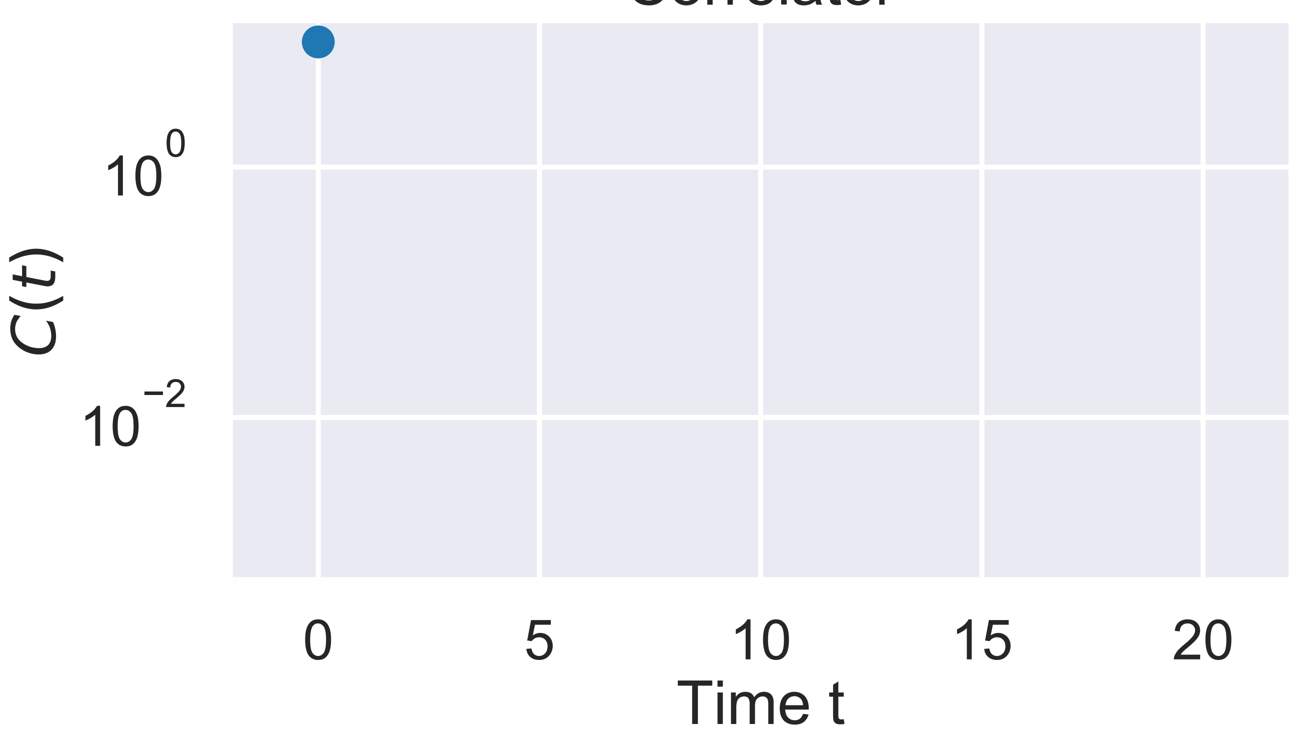
$$
\begin{aligned}
\langle O(t) O(0) \rangle &= \langle 0 | \, e^{Ht} O(0) e^{-Ht} O(0) \, | 0 \rangle \\
&= \sum_n e^{-E_n t} \langle 0 | \, O(0) \, | n \rangle \langle n | \, O(0) \, | 0 \rangle \\
&= \sum_n e^{-E_n t} \, | \langle 0 | \, O(0) \, | n \rangle |^2 \\
&= \sum_n |Z_n|^2 e^{-E_n t}
\end{aligned}
$$

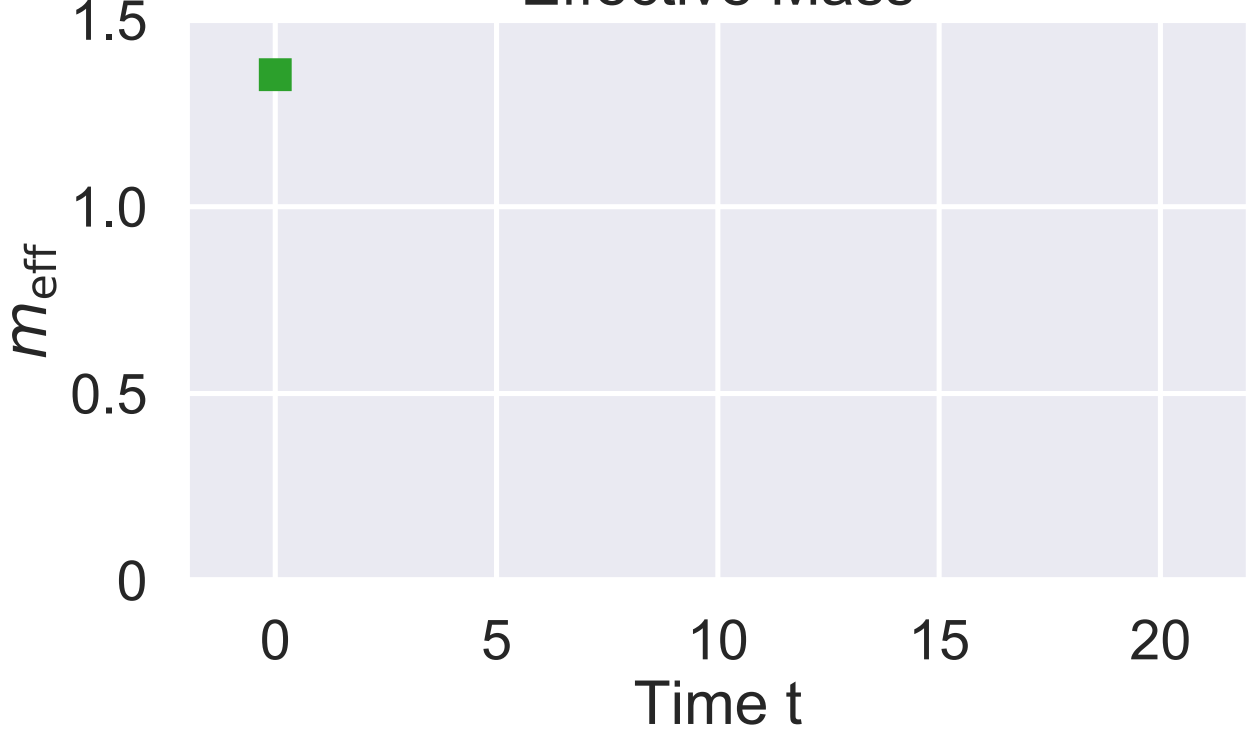
"Operators couple to an infinite tower of states."

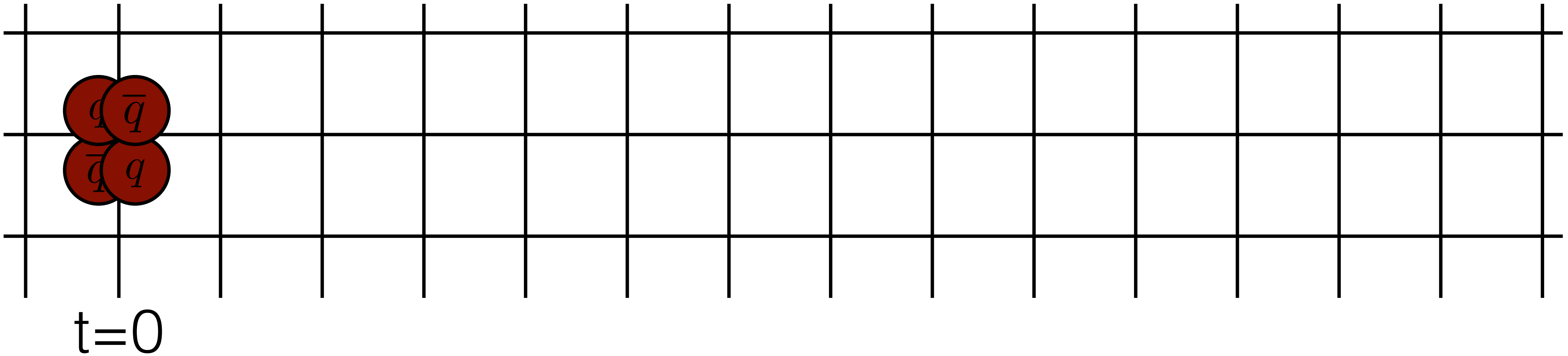
$$
m_{\text{eff}}(t) = \log C(t) / C(t+1) \stackrel{t \to \infty}{=} m_0
$$

"The ground state asymptotically dominates the Euclidean 2pt function."

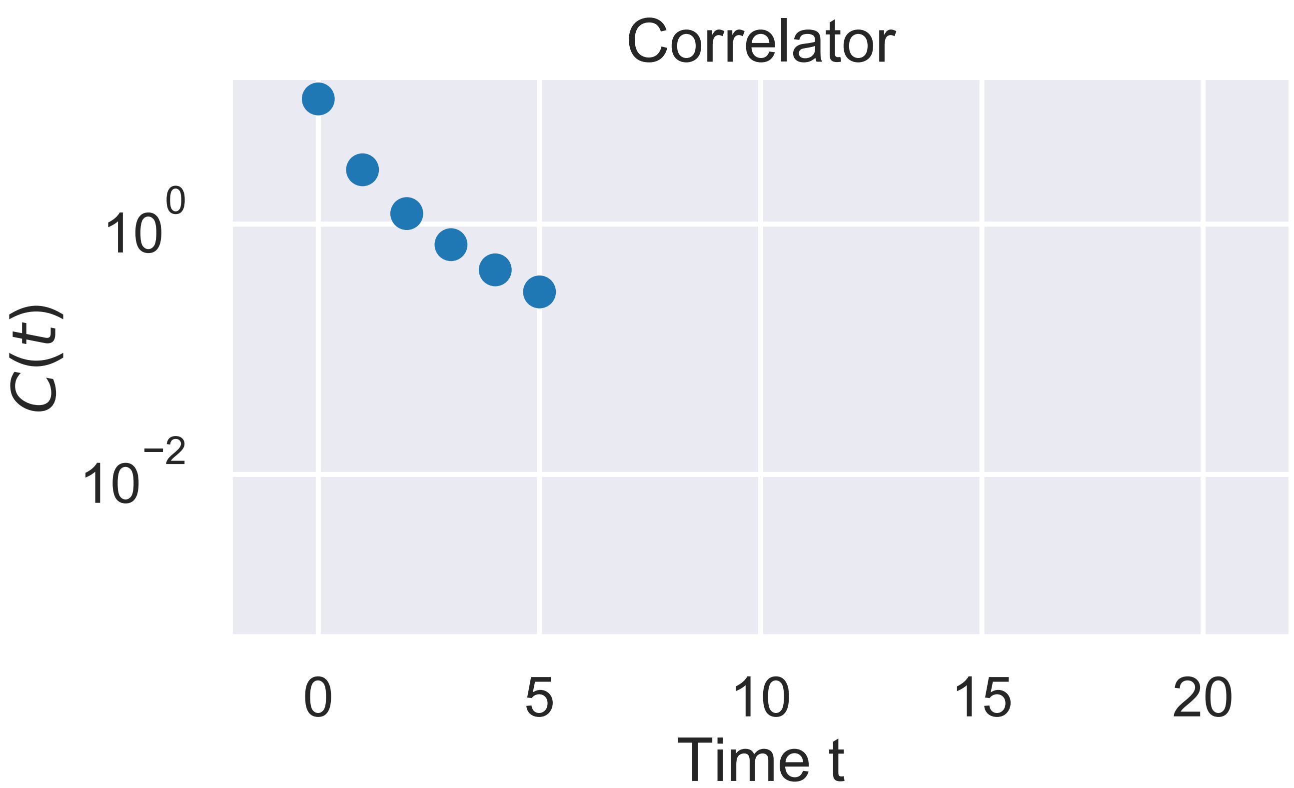
Lattice QCD: particle masses

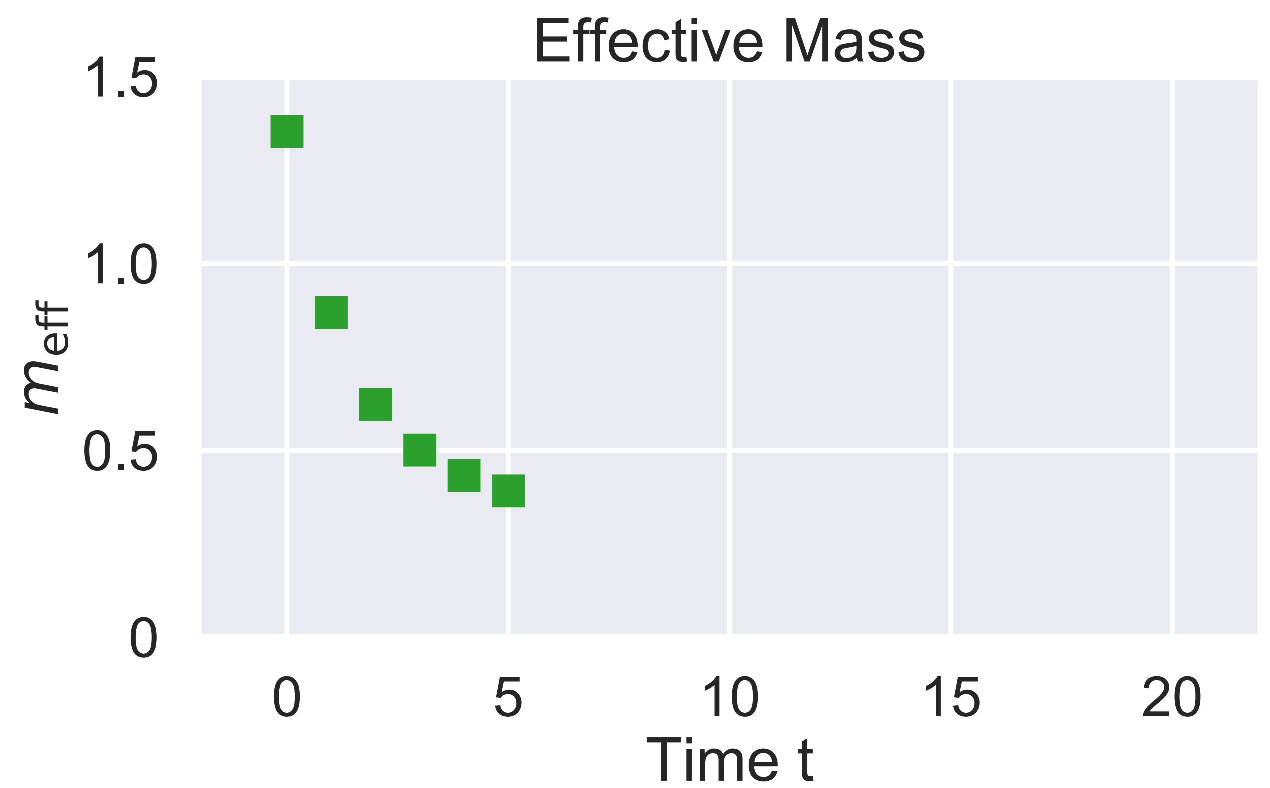


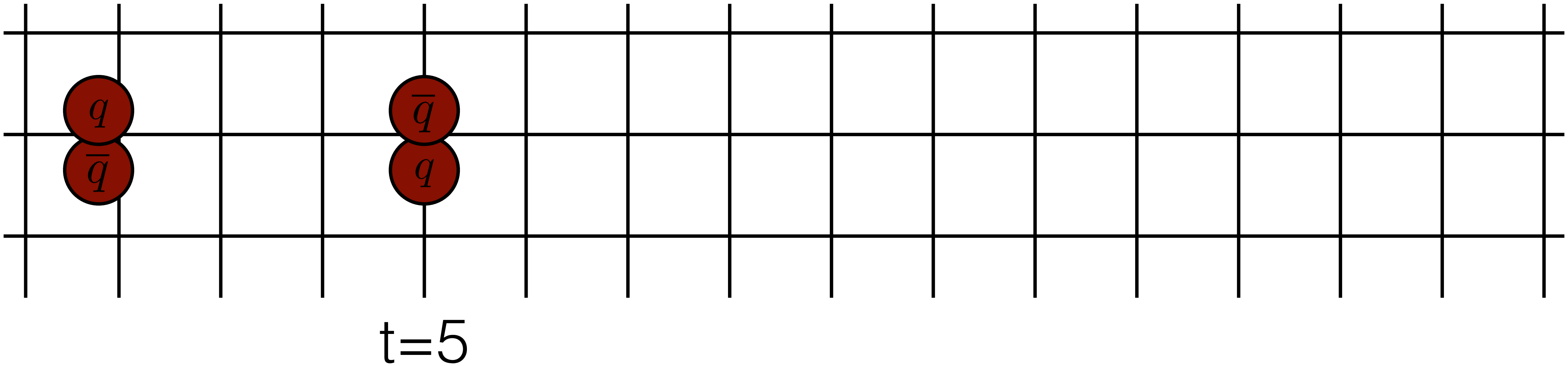




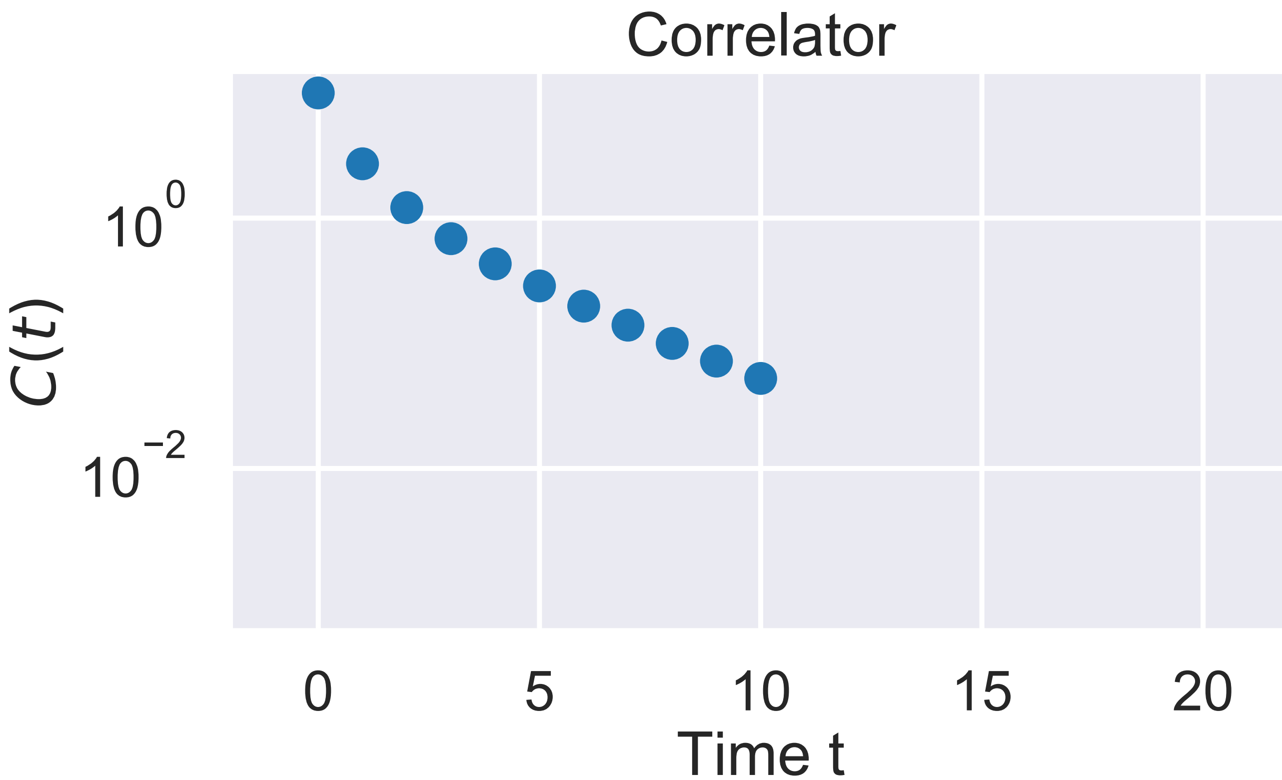
Lattice QCD: particle masses

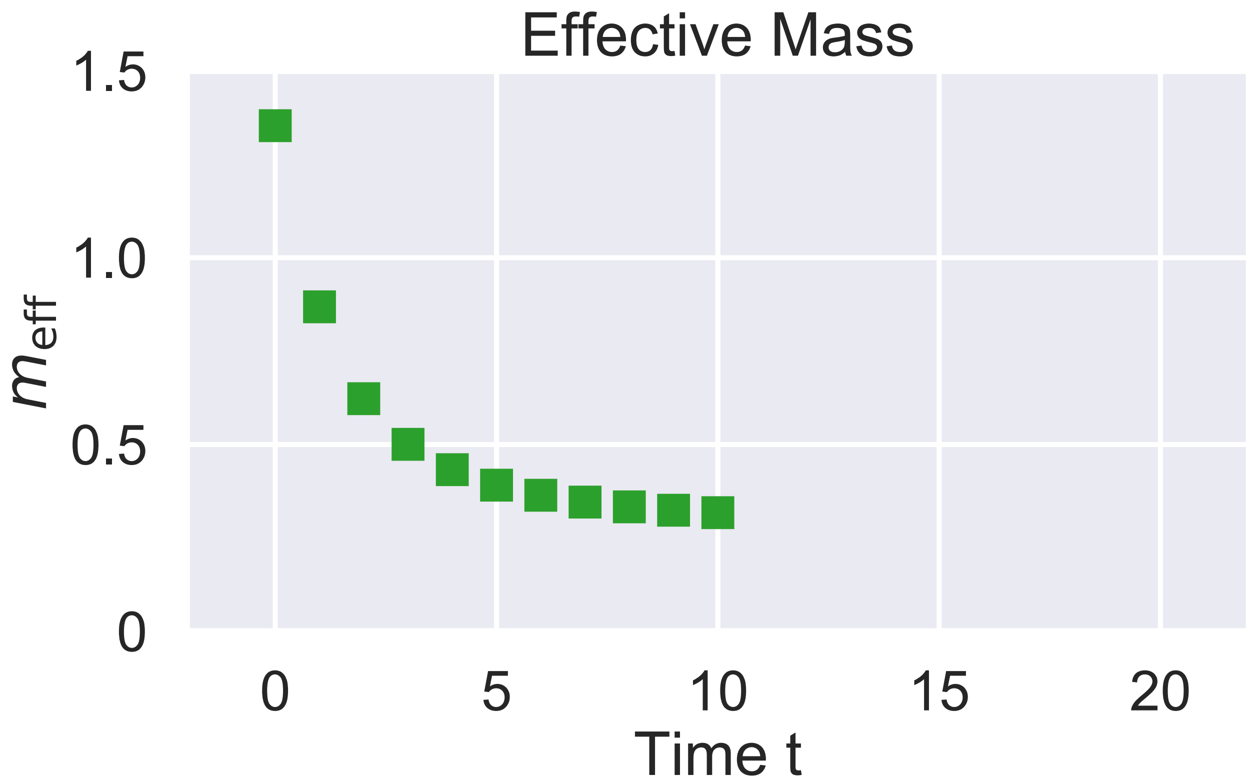


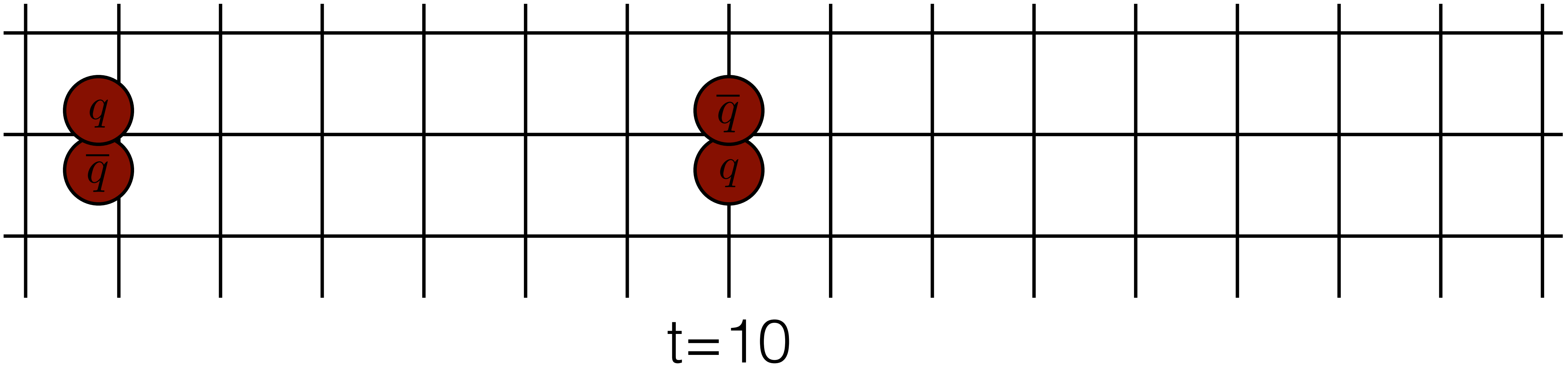




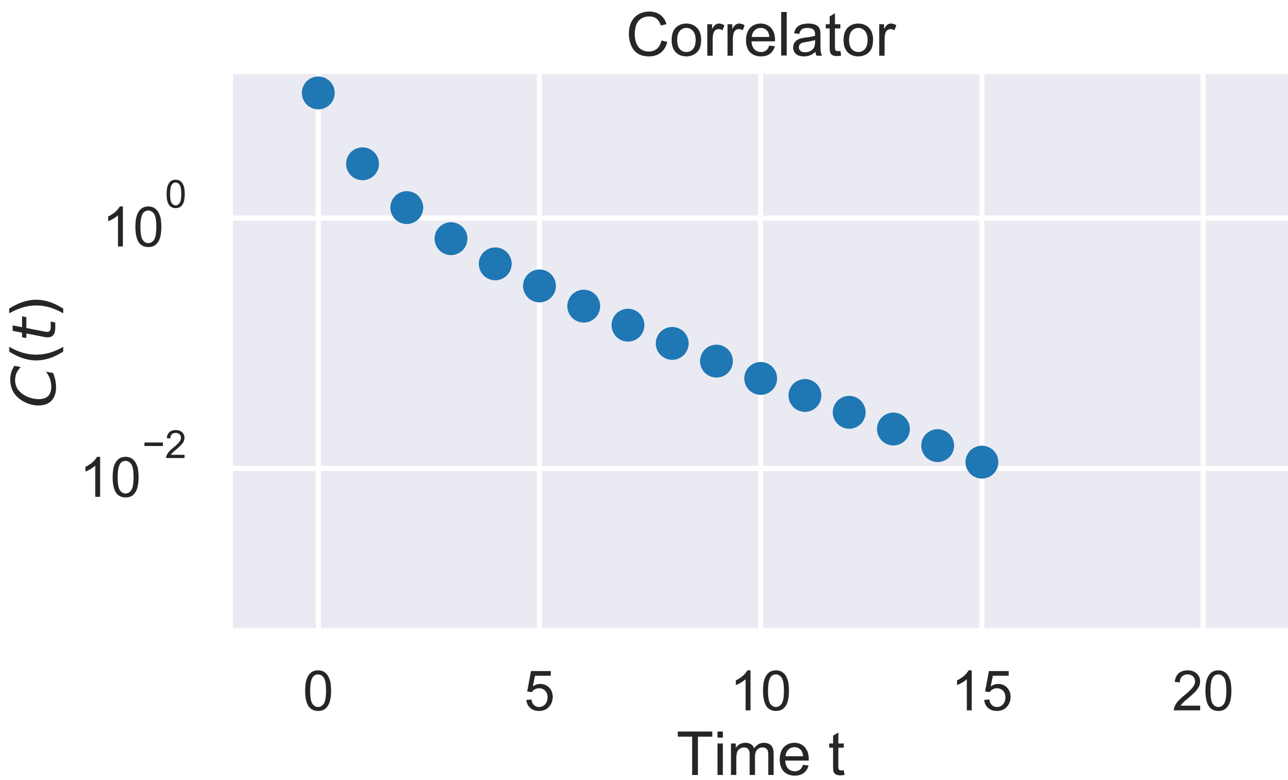
Lattice QCD: particle masses

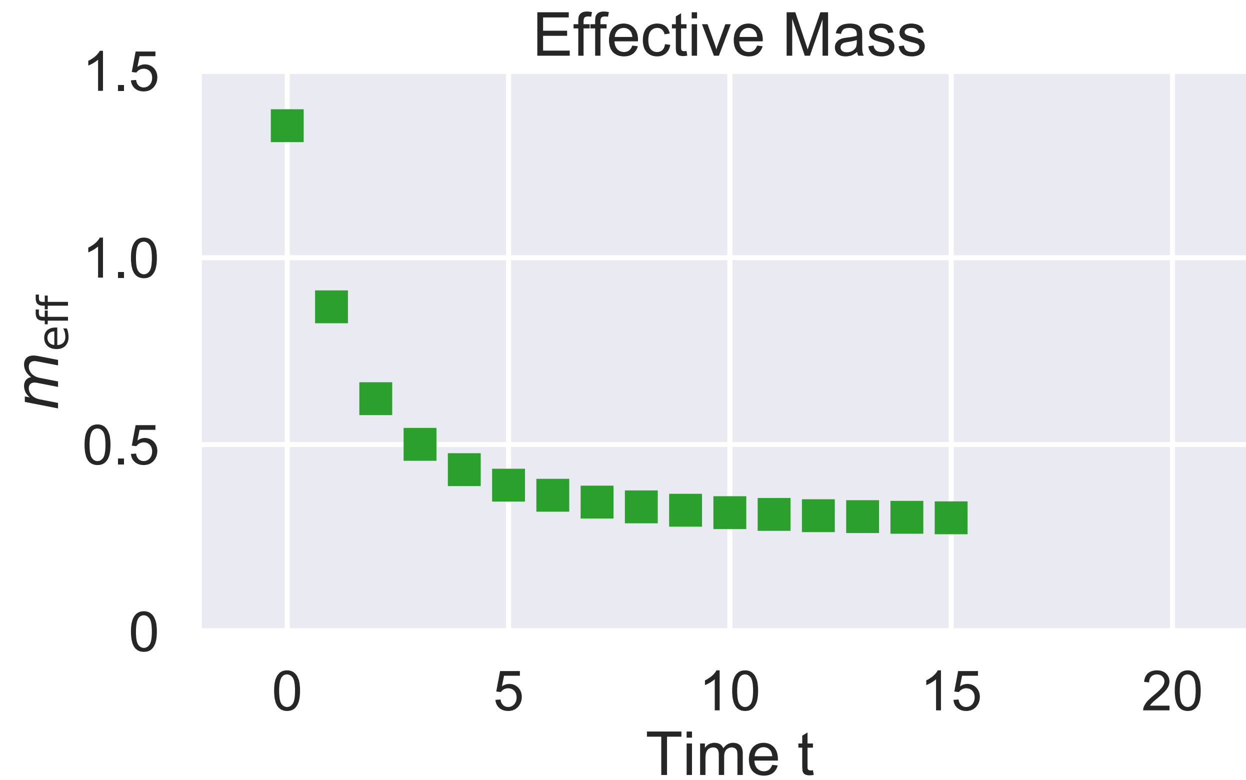


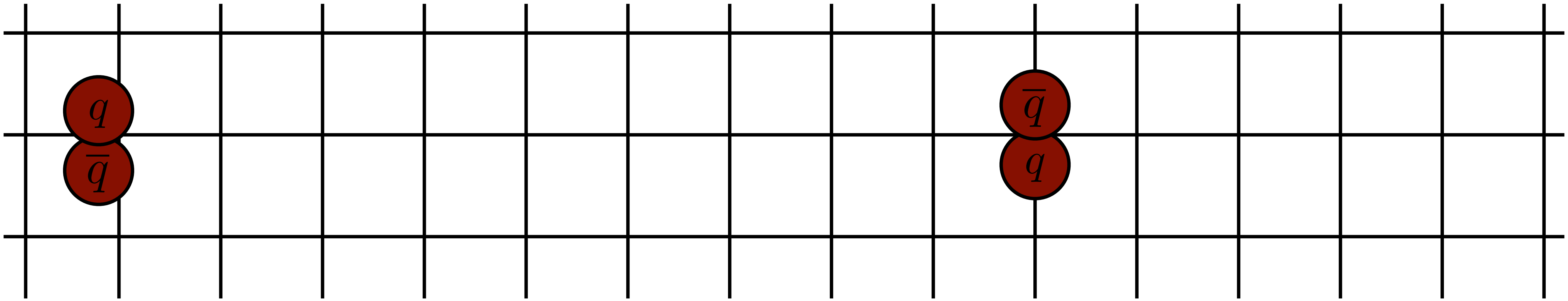




Lattice QCD: particle masses

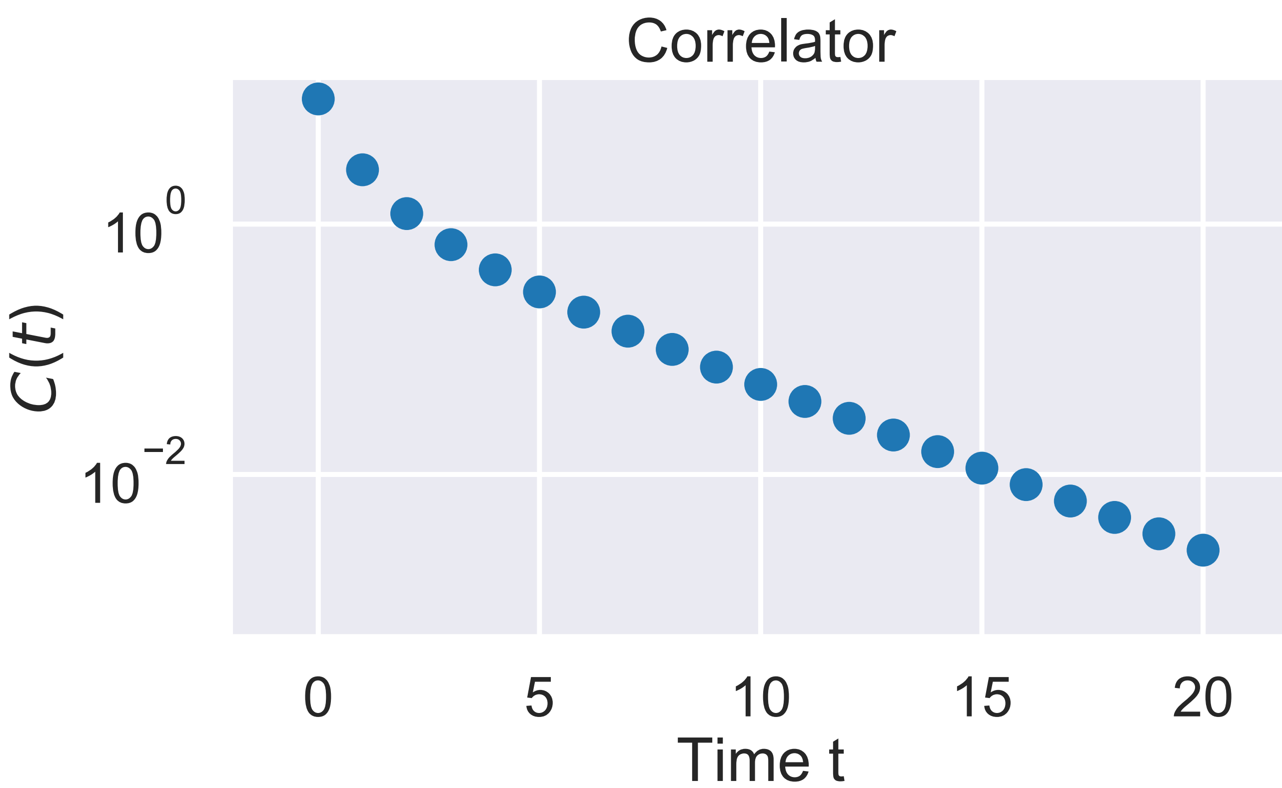


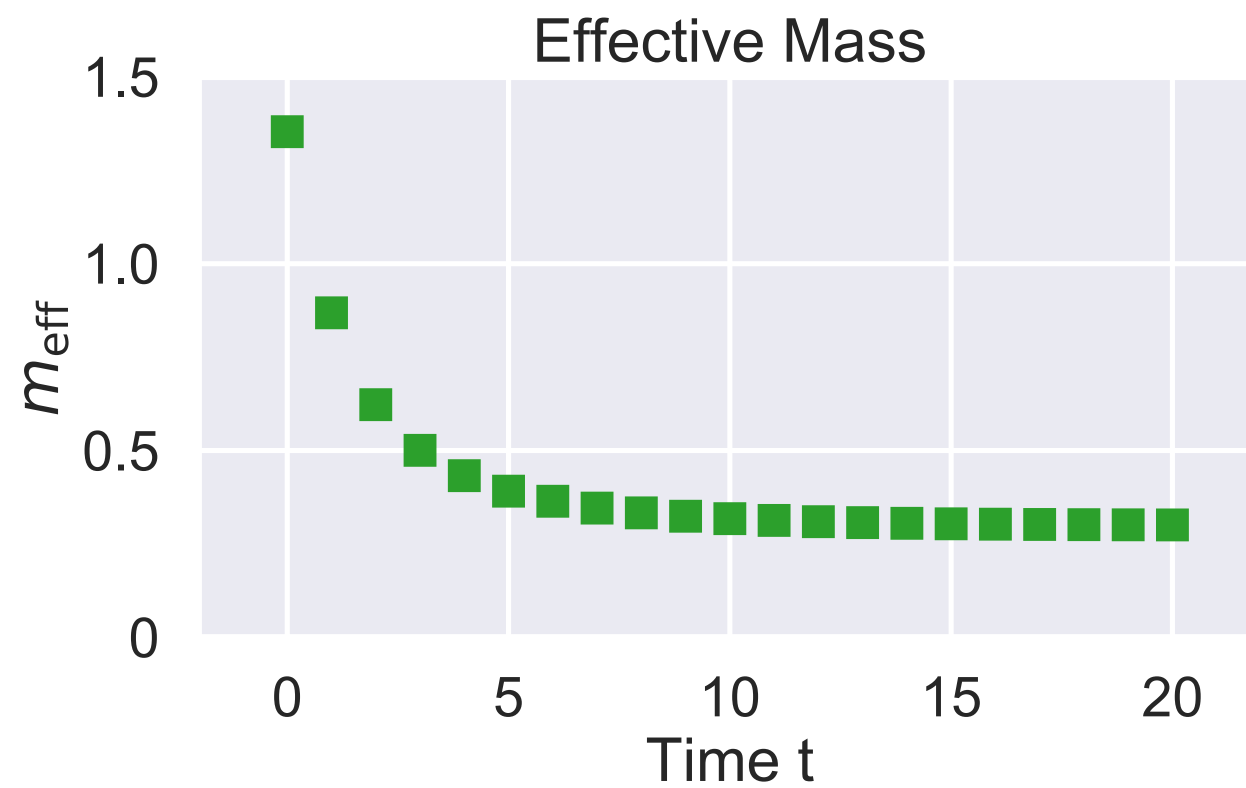


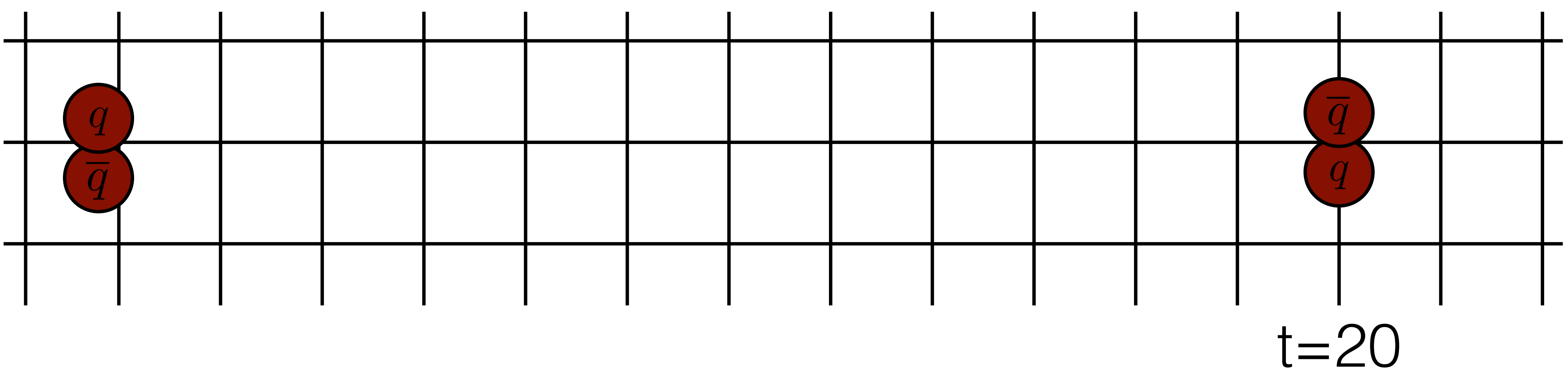


t=15

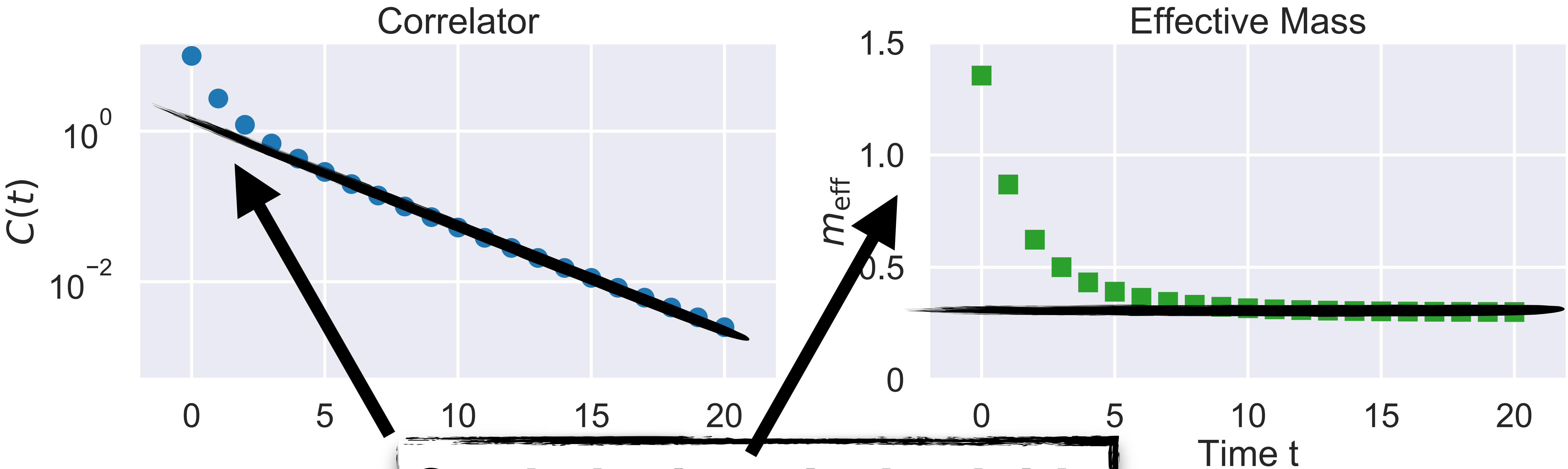
Lattice QCD: particle masses



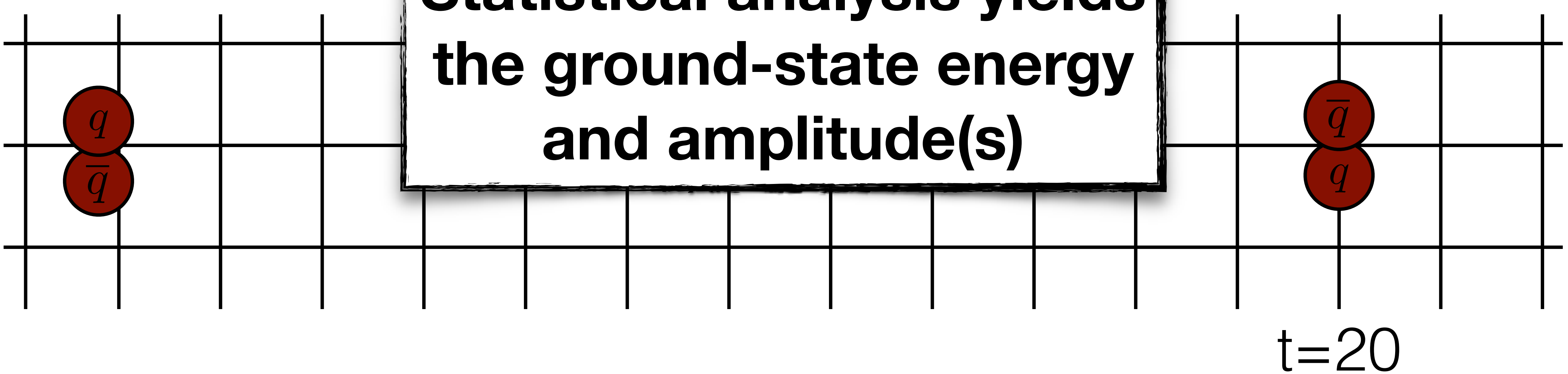




Lattice QCD: particle masses



Statistical analysis yields the ground-state energy and amplitude(s)



D-meson Semileptonic Decays

Lepton Flavor Universality Ratios

Fermilab-MILC [WJ]
PRD 107 (2023) 9, 094516
arXiv:2212.12648

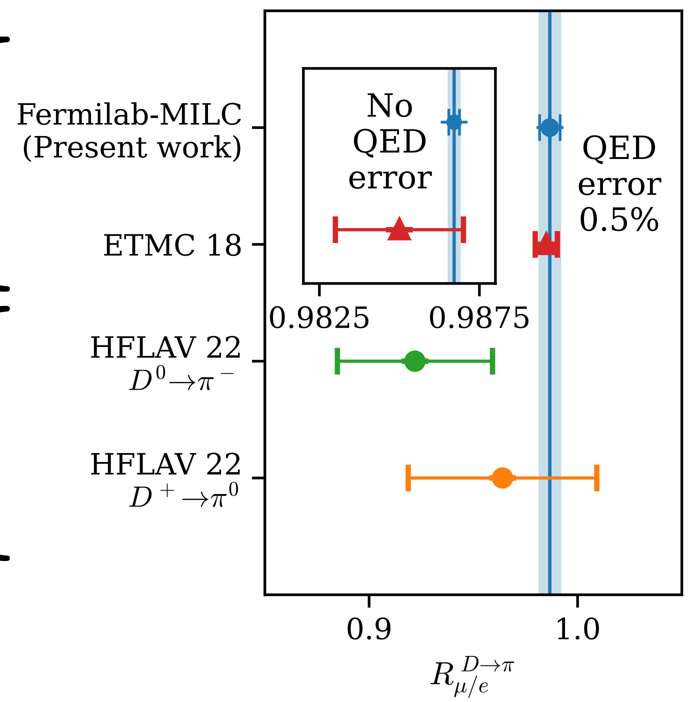
$$R_{\mu/e}^{H\to L} \equiv \frac{\mathscr{B}(H \to L\mu\nu)}{\mathscr{B}(H \to Le\nu)}$$

- CKM factors cancel in the ratio
  → pure theoretical SM predictions are available
- Theoretical uncertainties cancel in the ratio
  → lattice QCD gives very precise results

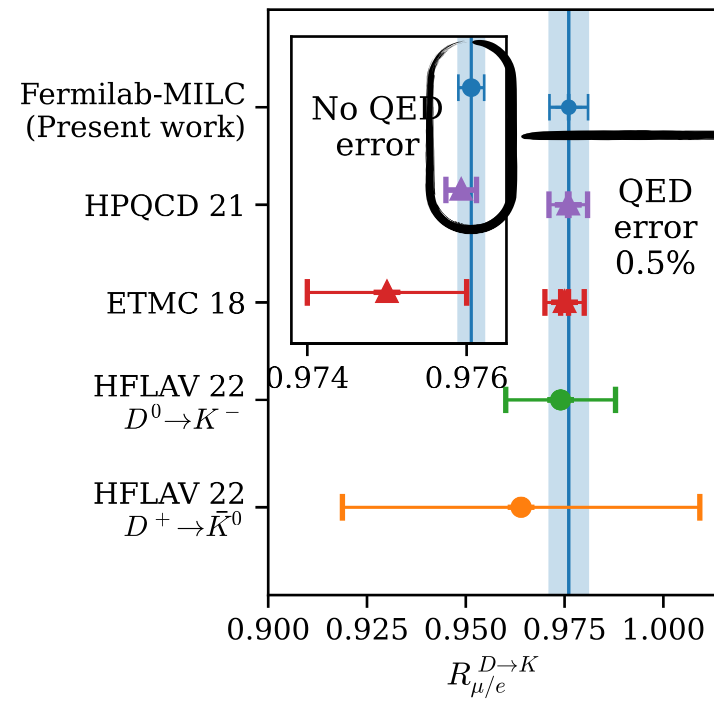
LQCD

Experiment

$D \to \pi$



$D \to K$



LQCD
agreement
at ≈ 0.01%