Charm theory

Hadronic physics of CP-violation

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- Burt Richter and Sam Ting discovered J/ψ state in November of 1974

The Arrival of Charm[1]

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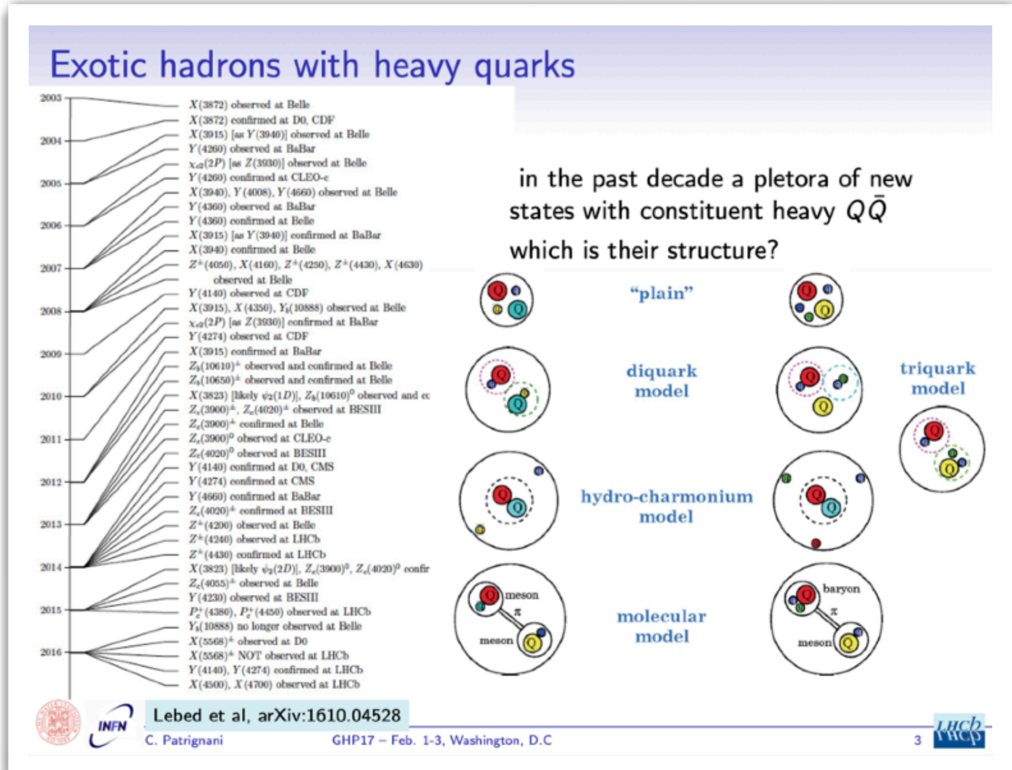
Abstract. Some of the theoretical motivations and experimental developments leading to the discovery of charm are recalled.

hep-ph/9811359

- At 50, charm quark continue to churn out surprises!

- At 50, charm quark continue to churn out surprises!
  - spectroscopy:



  - D-mixing and CP-violation in decays (soon: in mixing?)

- Interpretation of the results of observations depends on our understanding of low-energy hadronic physics

- How can CP-violation be observed in charm system?
  - can be observed by comparing CP-conjugated decay rates in various ways, both with and w/out time dependence

$$a_{\rm CP}(f) = \frac{\Gamma(D \to f) - \Gamma(\overline{D} \to \overline{f})}{\Gamma(D \to f) + \Gamma(\overline{D} \to \overline{f})},$$

  - can manifest itself in charm ΔC=1 transitions (direct CP-violation)

$$\Gamma(D \to f) \neq \Gamma(CP[D] \to CP[f])$$

dCPV

  - or in ΔC=2 transitions (indirect CP-violation): mixing $\left|D_{1,2}\right\rangle = p\left|D^0\right\rangle \pm q\left|\overline{D^0}\right\rangle$

$$R_m^2 = |q/p|^2 = \left|\frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - (i/2)\Delta\Gamma}\right|^2 = 1 + A_m \neq 1$$

CPVmix

  - or in the interference b/w decays (ΔC=1) and mixing (ΔC=2)

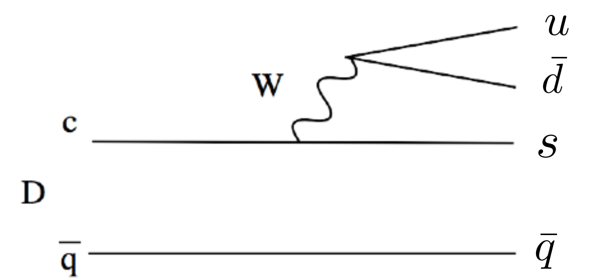
$$\lambda_f = \frac{q}{p}\frac{\overline{A_f}}{A_f} = R_m e^{i(\phi+\delta)}\left|\frac{\overline{A_f}}{A_f}\right|$$

CPVint

Amplitudes?

★ Can be classified by SM CKM suppression of tree amplitude ($V_{us} \sim \lambda$)

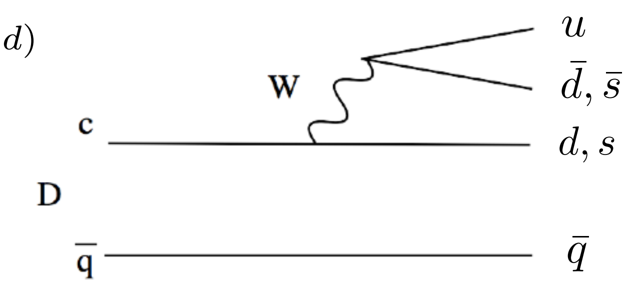
$V_{cs}V_{ud}^*$



★ Cabibbo-favored (CF: λ^0) decay

- originates from $c \to s\, u\bar{d}$
- examples: $D^0 \to K^-\pi^+$

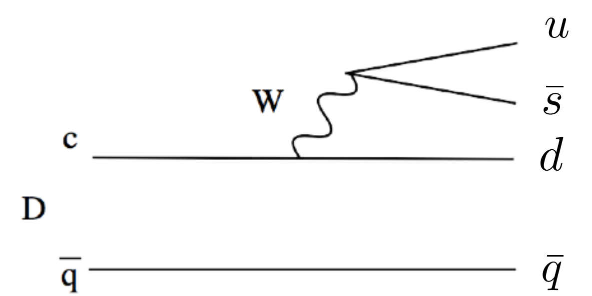
$V_{cs(d)}V_{us(d)}^*$



★ Singly Cabibbo-suppressed (SCS: λ^1) decay

- originates from $c \to q\, u\bar{q}$
- examples: $D^0 \to \pi\pi$ and $D^0 \to KK$

$V_{cd}V_{us}^*$



★ Doubly Cabibbo-suppressed (DCS: λ^2) decay

- originates from $c \to d\, u\bar{s}$
- examples: $D^0 \to K^+\pi^-$

★ We shall concentrate on SCS decays. This is because of the

Jedrzej Biesiada

★IDEA: consider the DIFFERENCE of decay rate asymmetries: D →ππ vs D → KK! For each final state the asymmetry

D⁰: no neutrals in the final state!

$$a_f = \frac{\Gamma(D\to f) - \Gamma(\overline{D}\to \overline{f})}{\Gamma(D\to f) + \Gamma(\overline{D}\to \overline{f})} \quad \Rightarrow \quad a_f = a_f^d + a_f^m + a_f^i$$

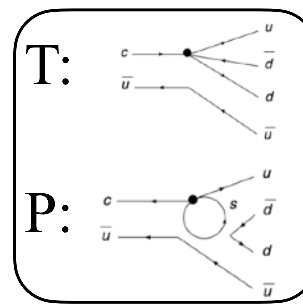
direct, mixing, interference

★ A reason: $a^m_{KK}=a^m_{\pi\pi}$ and $a^i_{KK}=a^i_{\pi\pi}$ (for CP-eigenstate final states), so, ideally, mixing asymmetries cancel ($r_f=P_f/A_f$)!

$$a_f^d = 2 r_f \sin\phi_f \sin\delta_f$$

★ ... and the resulting DCPV asymmetry is $\Delta a_{CP} = a^d_{KK} - a^d_{\pi\pi} \approx 2 a^d_{KK}$ (double!)

$$A_{KK} = \frac{G_F}{\sqrt{2}}\lambda\left[(T+E+P_{sd}) + a\lambda^4 e^{-i\gamma}P_{bd}\right]$$
$$A_{\pi\pi} = \frac{G_F}{\sqrt{2}}\lambda\left[(-(T+E)+P_{sd}) + a\lambda^4 e^{-i\gamma}P_{bd}\right]$$



★ ... so it is doubled in the limit of $SU(3)_F$ symmetry

SU(3) is badly broken in D-decays

- Experimental results
  - Result 1: an observation of CP-violation in the difference…

$$\Delta a_{CP}^{dir} = a_{CP}(K^- K^+) - a_{CP}(\pi^- \pi^+) = (-15.4 \pm 2.9) \times 10^{-4}$$

LHCb 2019

  - Result 2: the individual CPV asymmetry in $D^0 \to K^+ K^-$ channel

$$a_{CP}(K^- K^+) = (7.7 \pm 5.7) \times 10^{-4}$$

LHCb 2022
2209.03179v2

  - Result 3: LHCb combined the above results to obtain the CPV asymmetry in $D^0 \to \pi^+ \pi^-$ channel

$$a_{CP}(\pi^- \pi^+) = (23.2 \pm 6.1) \times 10^{-4}$$

LHCb 2022
2209.03179v2

  - Wishlist: obtain the CPV asymmetries in $D^0 \to K^+ K^-$ and $D^0 \to \pi^+ \pi^-$ channels independently to check consistency of Δa_{CP}^{dir}
- Need confirmation from other experiments (Belle II)
- What do those results mean? New Physics? Standard Model?

Implications of experimental observations

- Check SU(3) symmetry: only need U-spin (interchange $s \leftrightarrow d$)
  - Branching ratios: $\Gamma(D^0 \to K^+K^-) = \Gamma(D^0 \to \pi^+\pi^-)$

$$\frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-)} = 2.81 \pm 0.06$$

  - CPV asymmetries: $a_{CP}(D^0 \to \pi^+\pi^-) = -\, a_{CP}(D^0 \to K^+K^-)$

$$\frac{a_{CP}(D^0 \to \pi^+\pi^-)}{a_{CP}(D^0 \to K^+K^-)} = 3.01^{+0.95}_{-5.95}$$

- In both cases: appearance of badly-broken symmetry. Also: wrong sign!

- U-spin sum rule: $$\frac{a_{CP}(D^0 \to \pi^+\pi^-)}{a_{CP}(D^0 \to K^+K^-)} \frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-)} = -1$$

… but it appears that experimentally $= +\,0.93^{+0.62}_{-0.41}$

S. Schacht, JHEP 03 (2023) 205

ΔA_{CP} within the Standard Model and beyond

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Implications on the first observation of charm CPV at LHCb

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The Emergence of the $\Delta U = 0$ Rule in Charm Physics

Yuval Grossman[*] and Stefan Schacht[†]

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Revisiting CP violation in $D \to PP$ and VP decays

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2a. Calculating CP-asymmetries?

- Effective Hamiltonian for singly Cabibbo-suppressed (SCS) decays
  - drop all "penguin" operators (Q_i for $i \geq 3$) as C_i are small, $\lambda_q = V_{uq} V^*_{cq}$,

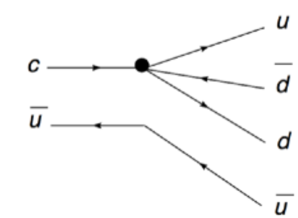
$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\sum_{q=d,s} \lambda_q \left(C_1 \mathcal{Q}_1^q + C_2 \mathcal{Q}_2^q \right) - \lambda_b \sum_{i=3,\ldots,6,8g} C_i \mathcal{Q}_i \right]$$

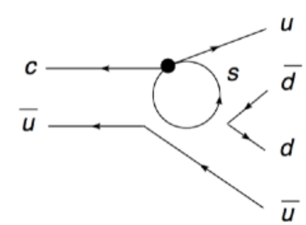
$$\mathcal{Q}_1^q = (\bar{u}\Gamma_\mu q)(\bar{q}\Gamma^\mu c), \qquad \mathcal{Q}_2^q = (\bar{q}\Gamma_\mu q)(\bar{u}\Gamma^\mu c)$$

  - recall that $\sum_{q=d,s,b} \lambda_q = 0$ or $\lambda_d = -(\lambda_s + \lambda_b)$ and $\mathcal{O}^q \equiv \frac{G_F}{\sqrt{2}} \sum_{i=1,2} C_i \mathcal{Q}_i^q$, with $q = d, s$.

without QCD

with QCD





Amplitudes?

How to compute decay amplitudes?

- A_{CP}: need to compute/fit/derive hadronic decay amplitudes
  - matrix elements of 4-fermion operators (factorization?)

$$A_{\pi\pi} = \langle \pi^+\pi^- | \mathcal{H} | D^0 \rangle$$

$$= \frac{G_F}{\sqrt{2}} V_{ud} V_{cd}^* \langle \pi^+\pi^- | \left(\bar{u}d\right)_L \left(\bar{d}c\right)_L | D^0 \rangle$$

$$\sim \frac{G_F}{\sqrt{2}} V_{ud} V_{cd}^* \langle \pi^+ | \left(\bar{u}d\right)_L 0 \rangle \langle \pi^- | (\bar{d}c)_L | D^0 \rangle$$

$$\sim\sim \frac{G_F}{\sqrt{2}} V_{ud} V_{cd}^* \; f_\pi F_{D\to\pi} m_D^2$$

No imaginary part?

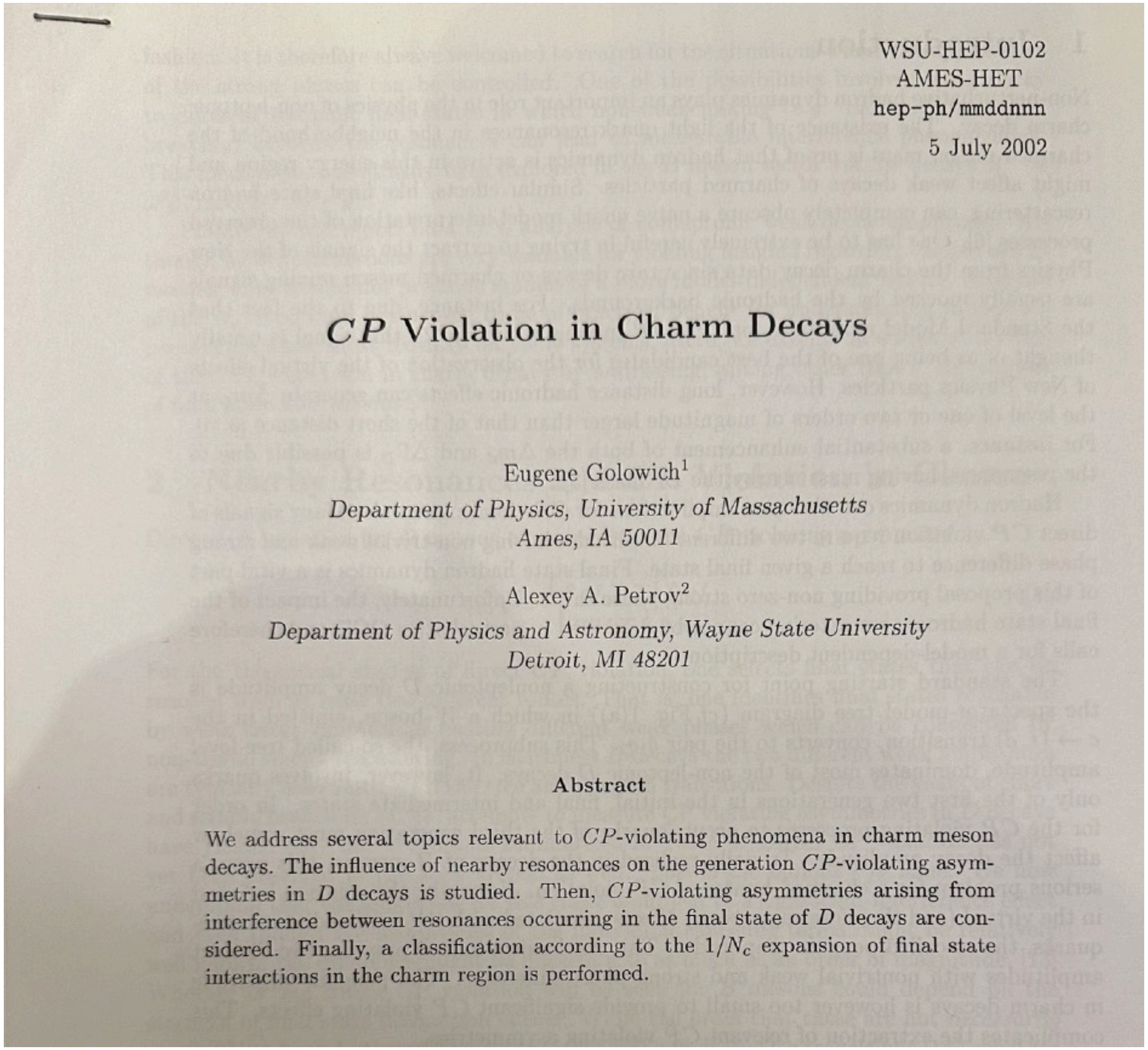
  - need a better approach (but can retain some elements)! Recall $R_{DCS/CF}$

Falk, Nir, AAP
JHEP 12 (1999) 019

$$A_R = \frac{g_H}{m_D^2 - m_{K_H}^2 + i\Gamma_{K_H} m_D} \langle K_H | H_{eff} | D^0 \rangle \,,$$

$$B_R = \frac{g_H}{m_D^2 - m_{K_H}^2 + i\Gamma_{K_H} m_D} \langle K_H | H_{eff} | \overline{D}^0 \rangle \,,$$

Resonances? FSI? Both?



WSU-HEP-0102
AMES-HET
hep-ph/mmddnnn
5 July 2002

CP Violation in Charm Decays

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Abstract

We address several topics relevant to CP-violating phenomena in charm meson decays. The influence of nearby resonances on the generation CP-violating asymmetries in D decays is studied. Then, CP-violating asymmetries arising from interference between resonances occurring in the final state of D decays are considered. Finally, a classification according to the $1/N_c$ expansion of final state interactions in the charm region is performed.

- I was going through a pile of old papers…

Resonance enhancement of decay amplitudes

- A_{CP}: need to compute/fit/derive hadronic decay amplitudes
  - parameterize $D \to KK$ and $D \to \pi\pi$ decay amplitudes
  - use isospin decomposition, as possible nearby resonances are classified according to isospin, etc.

Schacht, Soni
PLB 825 (2022) 136855

$$A(D^0 \to \pi^+\pi^-) = \frac{1}{\sqrt{6}}\lambda_{sd}A^{\pi\pi}_{\frac{3}{2},2} + \frac{1}{\sqrt{3}}\left(\lambda_{sd}A^{\pi\pi}_{\frac{1}{2},0} - \frac{\lambda_b}{2}B^{\pi\pi}_{\frac{1}{2},0}\right)$$

$$A(D^0 \to K^+K^-) = \frac{1}{2}\lambda_{sd}A^{KK}_{\frac{3}{2},1} + \frac{1}{2}\left(\lambda_{sd}A^{KK}_{\frac{1}{2},1} - \frac{\lambda_b}{2}B^{KK}_{\frac{1}{2},1}\right) + \frac{1}{2}\left(\lambda_{sd}A^{KK}_{\frac{1}{2},0} - \frac{\lambda_b}{2}B^{KK}_{\frac{1}{2},0}\right)$$

… and similarly for other D-decays, where $\lambda_{sd} = (\lambda_s - \lambda_d)/2$ and $A^{ff}_{\Delta I\, I}$ ($B^{ff}_{\Delta I\, I}$) are CP-even (CP-odd)

- Resonance enhancement of decay amplitudes (model)
  - choose model and resonances that provide enhancement (I=0): f_0 states

$$A^{ff}_{\frac{1}{2},0} = g_{f_0\to ff}M^{sd}_{f_0}R(m_{f_0}, \Gamma_{f_0}, m_D, \ldots)$$

$$B^{ff}_{\frac{1}{2},0} = g_{f_0\to ff}M^{b}_{f_0}R(m_{f_0}, \Gamma_{f_0}, m_D, \ldots)$$

- Resonance enhancement of decay amplitudes (model)
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$$A^{ff}_{\frac{1}{2},0} = g_{f_0\to ff} M^{sd}_{f_0} R(m_{f_0}, \Gamma_{f_0}, m_D, ...)$$

$$B^{ff}_{\frac{1}{2},0} = g_{f_0\to ff} M^{b}_{f_0} R(m_{f_0}, \Gamma_{f_0}, m_D, ...)$$

possible interference among different f_0 states

  - ...where $g_{f_0\to ff}$ describes f_0 coupling to KK or $\pi\pi$ and

$$M^{sd}_{f_0} = \langle f_0 | \mathcal{O}^{\Delta I=1/2}_{sd} | D^0 \rangle \qquad M^{b}_{f_0} = \langle f_0 | \mathcal{O}^{\Delta I=1/2}_{b} | D^0 \rangle$$

  - there are nearby f_0 resonances

Schacht, Soni
PLB 825 (2022) 136855

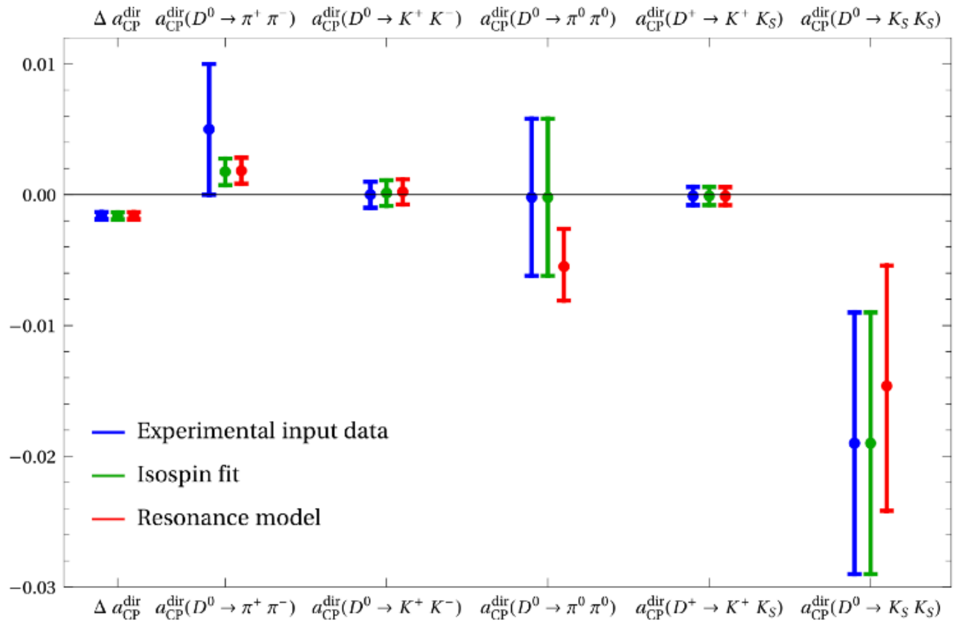
Employed experimental data for scalar unflavored resonances close to the D^0 mass.

Resonance	$I^G(J^{PC})$	mass m [MeV]	Γ [MeV]	Ref.
$f_0(1710)$	$0^+(0^{++})$	1704 ± 12	123 ± 18	[5]
$f_0(1790)$	$0^+(0^{++})$	1790^{+40}_{-30}	270^{+60}_{-30}	[53,54]

Note: other f_0 states? E.g., $f_0(2020)$: $m_{f_0(2020)} = 1982^{+54.1}_{-3.0}$ MeV, $\Gamma_{f_0(2020)} = 436 \pm 50$ MeV

Resonance enhancement of decay amplitudes

- Resonance enhancement of decay amplitudes (model)



Schacht, Soni
PLB 825 (2022) 136855

- Note: compatibility of the result depends on how many resonances are included in the fit

Why resonances if we have data?

- Resonance enhancement of decay amplitudes is a model!
  - there is ample experimental data on $\pi\pi(KK)$ scattering at $s \approx m_D^2$!
  - coupled-channel unitarity

$$S = \left(\begin{array}{c|ccc} D\to D & D\to\pi\pi & D\to KK & \cdots \\ \hline \pi\pi\to D & \pi\pi\to\pi\pi & \pi\pi\to KK & \cdots \\ KK\to D & KK\to\pi\pi & KK\to KK & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{array}\right) \equiv \begin{pmatrix} 1 & -i(T)^T \\ -i\,\mathrm{CP}(T) & S_S \end{pmatrix},$$

Franco, Mishima, Silvestrini
JHEP 05 (2012) 140
Pich, Solomonidia, Silva
arXiv: 2305.11951

  - two-channel approximation ($\pi\pi, KK$)

$$\begin{pmatrix} \mathcal{A}_0^\pi \\ \mathcal{A}_0^K \end{pmatrix} = \begin{pmatrix} \eta\, e^{2i\delta_1} & \pm i\sqrt{1-\eta^2}\, e^{i(\delta_1+\delta_2)} \\ \pm i\sqrt{1-\eta^2}\, e^{i(\delta_1+\delta_2)} & \eta\, e^{2i\delta_2} \end{pmatrix} \begin{pmatrix} (\mathcal{A}_0^\pi)^* \\ (\mathcal{A}_0^K)^* \end{pmatrix}$$

Note 1: inelasticity η and strong phases $\delta_{1,2}$ can be obtained from the low-energy experiments

Note 2: some other two-body ($\eta\eta$) and multi-body (4π) intermediate states have large branching ratios: could potentially change predictions in a two-body approximation!

- Idea: expand all charm decay amplitudes in terms of a universal set
  - need to select a basis: flavor SU(3), unbroken (for now)

$$\mathcal{H}_{\rm CF} = \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* (\bar{u}d)(\bar{s}c) + {\rm h.c.}$$

- Light quarks transform as triplets: $\bar{3} \times 3 \times \bar{3} = \overline{15} + 6 + \bar{3} + \bar{3}$
  - concentrate on CF decays for now: only $\overline{15}$ and 6 contribute

$$\mathcal{H}_{\rm CF} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \left(A\, \mathcal{O}^{(\overline{\mathbf{15}})}_{\frac{2}{3},1,-1} + C\, \mathcal{O}^{(\mathbf{6})}_{\frac{2}{3},1,-1} \right) + {\rm h.c.}$$

- ME: need to couple $\mathscr{H}_{CF}$ to the initial (triplet) and final 8×8 states
  - final state: $[(\mathbf{8}+\mathbf{1}) \times (\mathbf{8}+\mathbf{1})]_{PP} = (\mathbf{8} \times \mathbf{8})_{\rm sym} + (\mathbf{8} \times \mathbf{1}) + \mathbf{1}\,,$
    $= \mathbf{27} + \mathbf{8}_{\mathbf{8}\times\mathbf{8}} + \mathbf{8}_{\mathbf{8}\times\mathbf{1}} + \mathbf{1}_{\mathbf{8}\times\mathbf{8}} + \mathbf{1}$
  - initial state: $|\mathscr{H}|D\rangle$ contains 27 (from $\overline{15} \times 3$) and 8 (from $\overline{15} \times 3$ and 6×3)
- Basis of reduced ME: $A_{27} = \left\langle \mathbf{27} | \mathcal{O}^{\overline{\mathbf{15}}} | \mathbf{3} \right\rangle\,, \quad A_8 = \left\langle \mathbf{8} | \mathcal{O}^{\overline{\mathbf{15}}} | \mathbf{3} \right\rangle\,, \quad C_8 = \left\langle \mathbf{8} | \mathcal{O}^{\mathbf{6}} | \mathbf{3} \right\rangle$

- Select a basis, expand decay amps (CF decays only), include η and η'
  - assume mixing angle $\theta = \arcsin(1/3)$, but can do independent fit

Decay	$SU(3)_F$ Amplitude
$D^0 \to K^- \pi^+$	$\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \ \frac{1}{5} \left(\sqrt{2} A_{27} + \sqrt{2} A_8 - \sqrt{5} C_8 \right)$
$D^0 \to \overline{K}^0 \pi^0$	$\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \ \frac{1}{10} \left(3 A_{27} - 2 A_8 + \sqrt{10} C_8 \right)$
$D^0 \to \overline{K}^0 \eta$	$\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \ \frac{1}{15\sqrt{3}} \left(3\sqrt{2} A_{27} + \sqrt{2}(\sqrt{5}-2) A_8 - \sqrt{5}(\sqrt{5}-2) C_8 \right)$
$D^0 \to \overline{K}^0 \eta'$	$\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \ \frac{1}{30\sqrt{3}} \left(3 A_{27} - 2(1+4\sqrt{5}) A_8 + \sqrt{10}(1+4\sqrt{5}) C_8 \right)$
$D^+ \to \overline{K}^0 \pi^+$	$\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \ \frac{1}{\sqrt{2}} A_{27}$
$D_s^+ \to \overline{K}^0 K^+$	$\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \ \frac{1}{5} \left(\sqrt{2} A_{27} + \sqrt{2} A_8 + \sqrt{5} C_8 \right)$
$D_s^+ \to \pi^+ \eta$	$\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \ \frac{1}{15\sqrt{3}} \left(6\sqrt{2} A_{27} - \sqrt{2}(4+\sqrt{5}) A_8 - \sqrt{5}(4+\sqrt{5}) C_8 \right)$
$D_s^+ \to \pi^+ \eta'$	$\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \ \frac{1}{15\sqrt{3}} \left(3 A_{27} + 2(2\sqrt{5}-1) A_8 + \sqrt{10}(2\sqrt{5}-1) C_8 \right)$

  - there are 8 decays and 5 parameters: $|\mathcal{A}_{D\to PP}| = \sqrt{\frac{8\pi\hbar\, m_D^2\, \mathcal{B}_{D\to PP}}{\tau_D\, p^*}}$

Amplitudes: flavor SU(3) analysis

- Fit to experimental data...

Meson	Decay	Branching Ratio (%)
D^0	$K^-\pi^+$	3.950 ± 0.031
	$\overline{K}^0\pi^0$	2.480 ± 0.044
	$\overline{K}^0\eta$	1.018 ± 0.012
	$\overline{K}^0\eta'$	1.898 ± 0.064
D^+	$\overline{K}^0\pi^+$	3.124 ± 0.062
D_s^+	$\overline{K}^0 K^+$	2.95 ± 0.14
	$\pi^+\eta$	1.70 ± 0.09
	$\pi^+\eta'$	3.94 ± 0.25

- ... yields poor fit results

$$\chi^2_{\rm min}/{\rm dof} = 7477/3,$$

$$A_{27} = (0.279 \pm 0.002)\ {\rm GeV}^3,$$

$$A_8 = (0.840 \pm 0.008)\ e^{(59\pm1)^\circ i}\ {\rm GeV}^3,$$

$$C_8 = (0.17 \pm 0.02)\ e^{(-58\pm2)^\circ i}\ {\rm GeV}^3.$$

- What can be done to improve the fit?

B. Bhattacharya, A. Datta, AAP, J. Waite, JHEP 10 (2021) 024

- How to improve the quality of fit?
  - drop some of the assumptions [not enough data]: include SCS decays
    - new reduced MEs ($\mathscr{H}$ now contains triplets)

Recent: B. Bhattacharya, A. Datta, AAP, J. Waite, JHEP 10 (2021) 024

  - take into account SU(3) breaking [not enough data]: include SCS decays
    - new reduced MEs $\Delta \mathcal{L}_{QCD} = -m_s \bar{\psi} \lambda^8 \psi$

Pirtskhalava, Uttayarat (2012)
Hiller, Jung, Schacht (2013), …

$$\mathcal{H} = (\bar{3} + 6 + \overline{15}) \times (1 + \epsilon\, 8 + \mathcal{O}(\epsilon^2))$$
$$\supset \bar{3} + 6 + \overline{15} + \epsilon \Big(\bar{3}_i + 6_i + \overline{15}_1 + \overline{15}_2 + \overline{15}_3^1 + \overline{15}_3^2 + \overline{24}_3 + \overline{42}_3 + \ldots \Big),$$

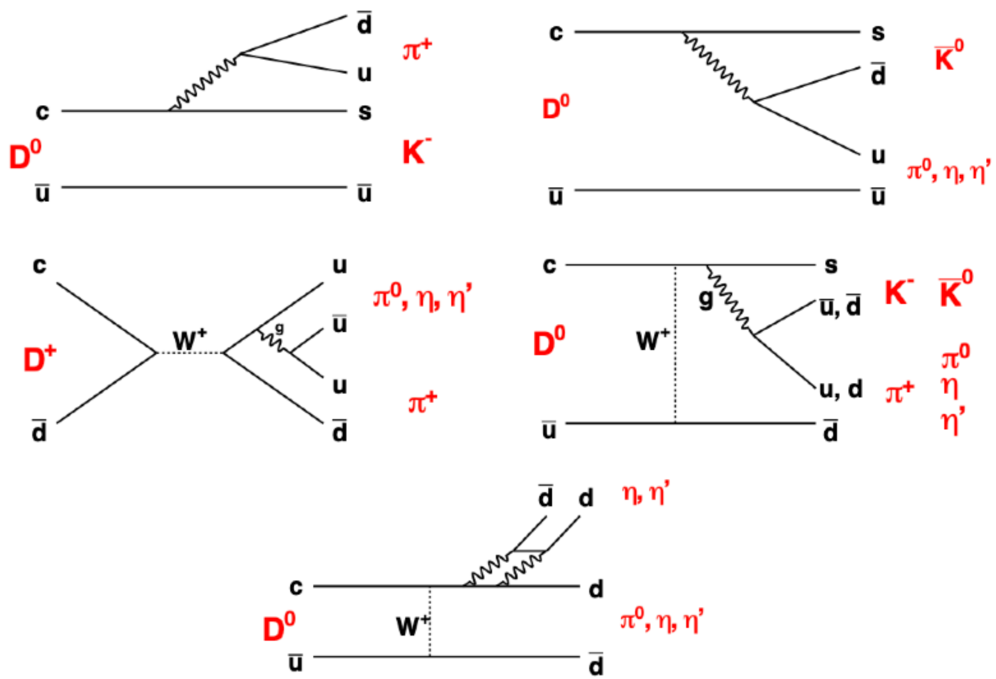
  - there are now 13 parameters (no η/η'):

$$\langle 1|3_{(i)}|3\rangle = G_{(i)}, \quad \langle 8|3_{(i)}|3\rangle = F_{(i)}, \quad \langle 8|6_{(i)}|3\rangle = S_{(i)}, \quad \langle 8|\overline{15}_{(i)}^{(\alpha)}|3\rangle = E_{(i)}^{(\alpha)},$$
$$\langle 27|\overline{15}_{(i)}^{(\alpha)}|3\rangle = T_{(i)}^{(\alpha)}, \quad \langle 27|\overline{24}_{(i)}|3\rangle = H_{(i)}, \quad \langle 27|\overline{42}_{(i)}|3\rangle = J_{(i)}.$$

  - need to assume dominance of some MEs over others…

- Topological flavor-flow diagrams could be used to deal with hadronic uncertainties

B. Bhattacharya, A. Datta, AAP, J. Waite, 2107.13564
Bhattacharya, Rosner, …



T and C

A and E

SE

- Fit many decay modes, assume SM weak phase!

- Select a basis, expand decay amps (CF decays only), include η and η'
  - assume mixing angle $\theta = \arcsin(1/3)$, but can do independent fit
  - there are 8 decays and 7 parameters: $|\mathcal{A}_{D\to PP}| = \sqrt{\frac{8\pi\hbar m_D^2 \mathcal{B}_{D\to PP}}{\tau_D\, p^*}}$

Decay	Diagrammatic Amplitude
$D^0 \to K^-\pi^+$	$\frac{G_F}{\sqrt{2}} V_{ud}V_{cs}^*\ (T+E)$
$D^0 \to \overline{K}^0\pi^0$	$\frac{G_F}{\sqrt{2}} V_{ud}V_{cs}^*\ \frac{1}{\sqrt{2}}(C-E)$
$D^0 \to \overline{K}^0\eta$	$\frac{G_F}{\sqrt{2}} V_{ud}V_{cs}^*\ \frac{1}{\sqrt{3}}C$
$D^0 \to \overline{K}^0\eta'$	$\frac{G_F}{\sqrt{2}} V_{ud}V_{cs}^*\ \left(-\frac{1}{\sqrt{6}}\right)(C+3E)$
$D^+ \to \overline{K}^0\pi^+$	$\frac{G_F}{\sqrt{2}} V_{ud}V_{cs}^*\ (C+T)$
$D_s^+ \to \overline{K}^0K^+$	$\frac{G_F}{\sqrt{2}} V_{ud}V_{cs}^*\ (C+A)$
$D_s^+ \to \pi^+\eta$	$\frac{G_F}{\sqrt{2}} V_{ud}V_{cs}^*\ \frac{1}{\sqrt{3}}(T-2A)$
$D_s^+ \to \pi^+\eta'$	$\frac{G_F}{\sqrt{2}} V_{ud}V_{cs}^*\ \frac{2}{\sqrt{6}}(T+A)$

- Fit to experimental data…

$$\begin{aligned}
\chi^2_{\rm min}/{\rm dof} &= 1.36/1\,,\\
T &= (0.366 \pm 0.003)\,{\rm GeV}^3\,,\\
C &= (0.298 \pm 0.002)\,e^{i(-151.0\pm0.4)^\circ}\,{\rm GeV}^3\,,\\
E &= (0.201 \pm 0.004)\,e^{i(119.3\pm0.8)^\circ}\,{\rm GeV}^3\,,\\
A &= (0.04 \pm 0.01)\,e^{i(63\pm9)^\circ}\,{\rm GeV}^3\,.
\end{aligned}$$

- … appears to be excellent! There are still issues for SCS decays…

- All SCS decays can be written in terms of the set of flavor flow diagrams
  - provided SU(3)-breaking is accounted for "phenomenologically"

	Mode	Representation
D^0	$\pi^+\pi^-$	$\lambda_d(0.96T + E_d) + \lambda_p(P_p + PE_p + PA_p)$
	$\pi^0\pi^0$	$\frac{1}{\sqrt{2}}\lambda_d(-0.78C + E_d) + \frac{1}{\sqrt{2}}\lambda_p(P_p + PE_p + PA_p)$
	$\pi^0\eta$	$-\lambda_d(E_d)\cos\phi - \frac{1}{\sqrt{2}}\lambda_s(1.28C)\sin\phi + \lambda_p(P_p + PE_p)\cos\phi$
	$\pi^0\eta'$	$-\lambda_d(E_d)\sin\phi + \frac{1}{\sqrt{2}}\lambda_s(1.28C)\cos\phi + \lambda_p(P_p + PE_p)\sin\phi$
	$\eta\eta$	$\frac{1}{\sqrt{2}}\lambda_d(0.78C + E_d)\cos^2\phi + \lambda_s(-\frac{1}{2}1.08C\sin 2\phi + \sqrt{2}\,E_s\sin^2\phi) + \frac{1}{\sqrt{2}}\lambda_p(P_p + PE_p + PA_p)\cos^2\phi$
	$\eta\eta'$	$\frac{1}{2}\lambda_d(0.78C + E_d)\sin 2\phi + \lambda_s(\frac{1}{\sqrt{2}}1.08C\cos 2\phi - E_s\sin 2\phi) + \frac{1}{2}\lambda_p(P_p + PE_p + PA_p)\sin 2\phi$
	K^+K^-	$\lambda_s(1.27T + E_s) + \lambda_p(P_p + PE_p + PA_p)$
	$K^0\overline{K}^0$	$\lambda_d(E_d) + \lambda_s(E_s) + 2\lambda_p(PA_p)$
D^+	$\pi^+\pi^0$	$\frac{1}{\sqrt{2}}\lambda_d(0.97T + 0.78C)$
	$\pi^+\eta$	$\frac{1}{\sqrt{2}}\lambda_d(0.82T + 0.93C + 1.19A)\cos\phi - \lambda_s(1.28C)\sin\phi + \sqrt{2}\lambda_p(P_p + PE_p)\cos\phi$
	$\pi^+\eta'$	$\frac{1}{\sqrt{2}}\lambda_d(0.82T + 0.93C + 1.61A)\sin\phi + \lambda_s(1.28C)\cos\phi + \sqrt{2}\lambda_p(P_p + PE_p)\sin\phi$
	$K^+\overline{K}^0$	$\lambda_d(0.85A) + \lambda_s(1.28T) + \lambda_p(P_p + PE_p)$
D_s^+	π^+K^0	$\lambda_d(1.00T) + \lambda_s(0.84A) + \lambda_p(P_p + PE_p)$
	π^0K^+	$\frac{1}{\sqrt{2}}[-\lambda_d(0.81C) + \lambda_s(0.84A) + \lambda_p(P_p + PE_p)]$
	$K^+\eta$	$\frac{1}{\sqrt{2}}\lambda_p[0.92C\delta_{pd} + 1.14A\delta_{ps} + P_p + PE_p]\cos\phi - \lambda_p[(1.31T + 1.27C + 1.14A)\delta_{ps} + P_p + PE_p]\sin\phi$
	$K^+\eta'$	$\frac{1}{\sqrt{2}}\lambda_p[0.92C\delta_{pd} + 1.14A\delta_{ps} + P_p + PE_p]\sin\phi + \lambda_p[(1.31T + 1.27C + 1.14A)\delta_{ps} + P_p + PE_p]\cos\phi$

H.-Y. Cheng, C.W. Chiang
Phys.Rev.D 100 (2019) 9, 093002

- Fits to experimental data in SCS and CF results in

Decay Mode	$\mathcal{B}_{\rm SU(3)}$	$\mathcal{B}_{\rm SU(3)-breaking}$	$\mathcal{B}_{\rm expt}$
$D^0 \to \pi^+\pi^-$	2.28 ± 0.02	1.47 ± 0.02	1.455 ± 0.024
$D^0 \to \pi^0\pi^0$	1.50 ± 0.03	0.82 ± 0.02	0.826 ± 0.025
$D^0 \to \pi^0\eta$	0.83 ± 0.02	0.92 ± 0.02	0.63 ± 0.06
$D^0 \to \pi^0\eta'$	0.75 ± 0.02	1.36 ± 0.03	0.92 ± 0.10
$D^0 \to \eta\eta$	1.52 ± 0.03	1.82 ± 0.04	2.11 ± 0.19
	1.52 ± 0.03	2.11 ± 0.04	
$D^0 \to \eta\eta'$	1.28 ± 0.05	0.69 ± 0.03	1.01 ± 0.19
	1.28 ± 0.05	1.63 ± 0.08	
$D^0 \to K^+K^-$	1.91 ± 0.02	4.03 ± 0.03	4.08 ± 0.06
	1.91 ± 0.02	4.05 ± 0.05	
$D^0 \to K_SK_S$	0	0.141 ± 0.007	0.141 ± 0.005
	0	0.141 ± 0.007	
$D^+ \to \pi^+\pi^0$	0.89 ± 0.02	0.93 ± 0.02	1.247 ± 0.033
$D^+ \to \pi^+\eta$	1.90 ± 0.16	4.08 ± 0.16	3.77 ± 0.09
$D^+ \to \pi^+\eta'$	4.21 ± 0.12	4.69 ± 0.08	4.97 ± 0.19
$D^+ \to K^+K_S$	2.29 ± 0.09	4.25 ± 0.10	3.04 ± 0.09
$D_s^+ \to \pi^+K_S$	1.20 ± 0.04	1.27 ± 0.04	1.22 ± 0.06
$D_s^+ \to \pi^0K^+$	0.86 ± 0.04	0.56 ± 0.02	0.63 ± 0.21
$D_s^+ \to K^+\eta$	0.91 ± 0.03	0.86 ± 0.03	1.77 ± 0.35
$D_s^+ \to K^+\eta'$	1.23 ± 0.06	1.49 ± 0.08	1.8 ± 0.6

H.-Y. Cheng, C.W. Chiang
Phys.Rev.D 100 (2019) 9, 093002
but see also:
B. Bhattacharya, A. Datta, AAP,
J. Waite, JHEP 10 (2021) 024

- Individual asymmetries:

$$a_{CP}^{\rm dir}(\pi^+\pi^-) = (0.80 \pm 0.22) \times 10^{-3},$$

$$a_{CP}^{\rm dir}(K^+K^-) = \begin{cases} (-0.33 \pm 0.14) \times 10^{-3} & \text{Solution I,} \\ (-0.44 \pm 0.12) \times 10^{-3} & \text{Solution II.} \end{cases}$$

Consistent with experiment?

- Asymmetry differences

$$\Delta a_{CP}^{\rm dir} = \begin{cases} (-1.14 \pm 0.26) \times 10^{-3} & \text{Solution I,} \\ (-1.25 \pm 0.25) \times 10^{-3} & \text{Solution II.} \end{cases}$$

Consistent with Standard Model?

- Some of the results can be obtained using U-spin analysis
  - note: while the analysis could be simpler, U-spin-breaking effects are expected to be as large as in general SU(3) analysis (minus E/M effects)

$$A(D^0 \to \pi^+\pi^-) = -\lambda_{sd}\left(t_0 + s_1 + \frac{1}{2}t_2\right) - \lambda_b\left(p_0 - \frac{1}{2}p_1\right)$$
$$-\lambda_{sd}t_0\left(1 + \tilde{s}_1 + \frac{1}{2}\tilde{t}_2\right) - \lambda_b\left(\tilde{p}_0 - \frac{1}{2}\tilde{p}_1\right)$$

Schacht JHEP03(2023)205

$$A(D^0 \to K^+K^-) = \lambda_{sd}\left(t_0 - s_1 + \frac{1}{2}t_2\right) - \lambda_b\left(p_0 + \frac{1}{2}p_1\right)$$
$$\lambda_{sd}t_0\left(1 - \tilde{s}_1 + \frac{1}{2}\tilde{t}_2\right) - \lambda_b\left(\tilde{p}_0 + \frac{1}{2}\tilde{p}_1\right)$$

… and similarly for other D-decays, with $\lambda_{sd} = (\lambda_s - \lambda_d)/2$, including subleading $\Delta U = 1$ contributions

- fitting to several branching ratios and A_{CP} for $\pi^+\pi^-$ and K^+K^-…

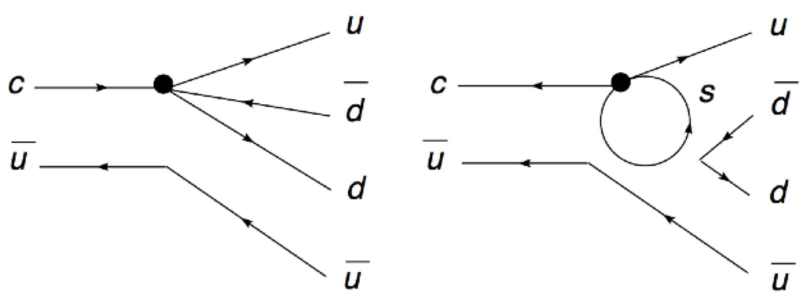
$$\frac{|\tilde{p}_1|}{2\,|\tilde{p}_0|} = 173^{+85}_{-74}\%$$

U-spin anomaly?

★ These asymmetries are notoriously difficult to compute

★ In the Standard Model

- need to estimate size of penguin/penguin contractions vs. tree



- unknown penguin contributions
  - SU(3) analysis: some ME are enhanced?

    Golden & Grinstein PLB 222 (1989) 501; Pirtshalava & Uttayarat 1112.5451
  - could expect large $1/m_c$ corrections (E/PE/PA/…)

    Isidori et al PLB 711 (2012) 46; Brod et al 1111.5000
  - flavor-flow diagrams

    Broad et al 1203.6659; Bhattacharya et al PRD 85 (2012) 054014; Cheng & Chiang 1205.0580; 1909.03063; Gronau, Rosner

★ General comments on SU(3)/flavor flow — type analyses

- fit both SM and (possible) NP parts of the amplitudes: can one claim SM-only?
- many parameters: can one claim $O(10^{-4})$ precision if rates are known to $O(10^{-2})$?

★ Need direct calculations of amplitudes/CPV-asymmetries

- QCD sum rule calculations of Δa_{CP} Khodjamirian, AAP
- SU(3) breaking analyses of $D \to PV, VV$
- constant (but slow) lattice QCD progress in $D \to \pi\pi, \pi\pi\pi$ Hansen, Sharpe

- Recipe for calculation of CPV asymmetry
  - prepare decay amplitudes (and using $\lambda_d = -(\lambda_s + \lambda_b)$)

$$A(D^0 \to \pi^-\pi^+) = \lambda_d \langle \pi^-\pi^+ | \mathcal{O}^d | D^0 \rangle + \lambda_s \langle \pi^-\pi^+ | \mathcal{O}^s | D^0 \rangle$$
$$A(D^0 \to K^-K^+) = \lambda_s \langle K^-K^+ | \mathcal{O}^s | D^0 \rangle + \lambda_d \langle K^-K^+ | \mathcal{O}^d | D^0 \rangle$$

  - add and subtract $\lambda_b \langle \pi^-\pi^+ | \mathcal{O}^s | D^0 \rangle$, put in a new form

$$A(D^0 \to \pi^-\pi^+) = -\lambda_s \mathcal{A}_{\pi\pi} \left[1 + \frac{\lambda_b}{\lambda_s} \left(1 + r_\pi \exp(i\delta_\pi)\right) \right]$$
$$A(D^0 \to K^-K^+) = \lambda_s \mathcal{A}_{KK} \left[1 - \frac{\lambda_b}{\lambda_s} r_K \exp(i\delta_K) \right]$$

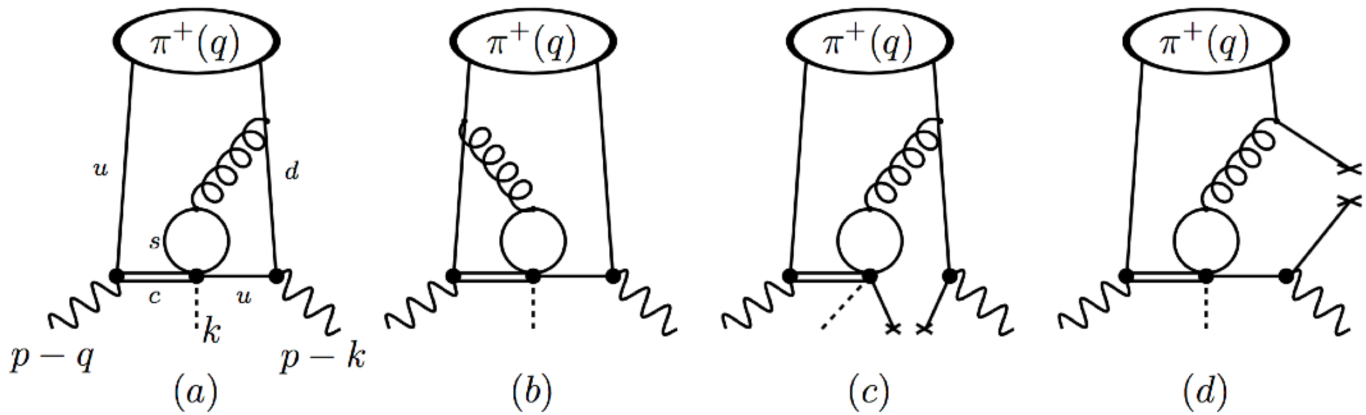
  - define things we cannot compute (extract from branching ratios)

$$\mathcal{A}_{\pi\pi} = \langle \pi^-\pi^+ | \mathcal{O}^d | D^0 \rangle - \langle \pi^-\pi^+ | \mathcal{O}^s | D^0 \rangle$$
$$\mathcal{A}_{KK} = \langle K^-K^+ | \mathcal{O}^s | D^0 \rangle - \langle K^-K^+ | \mathcal{O}^d | D^0 \rangle$$

  - … and things we can $\mathcal{P}^s_{\pi\pi} = \langle \pi^-\pi^+ | \mathcal{O}^s | D^0 \rangle$, $\mathcal{P}^d_{KK} = \langle K^-K^+ | \mathcal{O}^d | D^0 \rangle$

$$r_\pi = \left| \frac{\mathcal{P}^s_{\pi\pi}}{\mathcal{A}_{\pi\pi}} \right|, \quad r_K = \left| \frac{\mathcal{P}^d_{KK}}{\mathcal{A}_{KK}} \right|$$

- Evaluate (leading) diagrams contributing to the correlation function
  - calculate OPE in terms of known LC DAs — Khodjamirian, AAP: PLB774 (2017) 235



  - extract $\mathcal{A}_{\pi\pi}$ and $\mathcal{A}_{KK}$ amplitudes from measured branch. fractions

$$|\mathcal{A}_{\pi\pi}| \simeq \lambda_s^{-1} \left|A(D \to \pi^-\pi^+)\right| = (2.10 \pm 0.02) \times 10^{-6}\ \mathrm{GeV}\,,$$

$$|\mathcal{A}_{KK}| \simeq \lambda_s^{-1} \left|A(D \to K^-K^+)\right| = (3.80 \pm 0.03) \times 10^{-6}\ \mathrm{GeV}\,.$$

- As a result… $\langle \pi^+\pi^-|\widetilde{\mathcal{Q}}_2^s|D^0\rangle = (9.50 \pm 1.13)\times 10^{-3}\exp[i(-97.5^o \pm 11.6)]\,\mathrm{GeV}^3$

$$\langle K^+K^-|\widetilde{\mathcal{Q}}_2^d|D^0\rangle = (13.9 \pm 2.70)\times 10^{-3}\exp[i(-71.6^o \pm 29.5)]\,\mathrm{GeV}^3$$

- Thus, $r_\pi = \dfrac{|\mathcal{P}^s_{\pi\pi}|}{|\mathcal{A}_{\pi\pi}|} = 0.093 \pm 0.011\,, \qquad r_K = \dfrac{|\mathcal{P}^d_{KK}|}{|\mathcal{A}_{KK}|} = 0.075 \pm 0.015$

  and with $\Delta a_{CP}^{dir} = -2r_b \sin\gamma(r_K \sin\delta_K + r_\pi \sin\delta_\pi)$

- Phases of $r_{\pi\pi(KK)}$ are given by the phases of $\mathcal{P}^{s(d)}_{\pi\pi(KK)}$?

No:

$$\left|a_{CP}^{dir}(\pi^-\pi^+)\right| < 0.012 \pm 0.001\%,$$
$$\left|a_{CP}^{dir}(K^-K^+)\right| < 0.009 \pm 0.002\%,$$
$$\left|\Delta a_{CP}^{dir}\right| < 0.020 \pm 0.003\%\,.$$

Yes:

$$a_{CP}^{dir}(\pi^-\pi^+) = -0.011 \pm 0.001\%,$$
$$a_{CP}^{dir}(K^-K^+) = 0.009 \pm 0.002\%.$$
$$\Delta a_{CP}^{dir} = 0.020 \pm 0.003\%\,.$$

Khodjamirian, AAP: PLB774 (2017) 235

- … seems to be too small to explain the experimental results?

- Experimental results
  - Result 1: an observation of CP-violation in the difference…

$$\Delta a_{CP}^{dir} = a_{CP}(K^- K^+) - a_{CP}(\pi^- \pi^+) = (-15.4 \pm 2.9) \times 10^{-4}$$

LHCb 2019

  - Result 2: the individual CPV asymmetry in $D^0 \to K^+ K^-$ channel

$$a_{CP}(K^- K^+) = (7.7 \pm 5.7) \times 10^{-4}$$

LHCb 2022
2209.03179v2

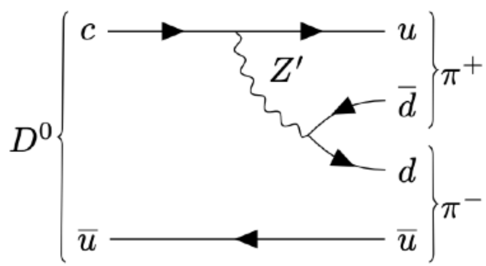
  - Result 3: LHCb combined the above results to obtain the CPV asymmetry in $D^0 \to \pi^+ \pi^-$ channel

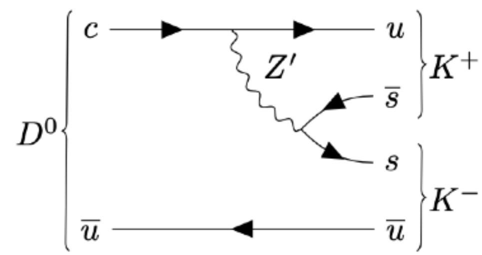
$$a_{CP}(\pi^- \pi^+) = (23.2 \pm 6.1) \times 10^{-4}$$

LHCb 2022
2209.03179v2

- Unaccounted for hadronic effects? New Physics? Experiment?

- Can New Physics explain CPV data in $\pi\pi$ and KK?
  - note: large $\Delta U = 1$ contributions: any New Physics operators contributing to $\Delta U = 1$ contributions? Yes, in models with generation-dependent couplings.





Two is better than one: The *U*-spin-CP anomaly in charm

Rigo Bause,[1,*] Hector Gisbert,[1,†] Gudrun Hiller,[1,‡] Tim Höhne,[1,§] Daniel F. Litim,[2,¶] and Tom Steudtner[1,**]
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Things to take home

- Computation of charm decay amplitudes and A_{CP} is a difficult task
  - no obvious model-independent/perturbative technique
  - SU(3)/flavor flow fits need theory input/better exp data
- Computation of charm mixing amplitudes is a difficult task
  - no dominant heavy dof, as in beauty decays
  - light dofs give no contribution in the flavor SU(3) limit
  - "hadronic" techniques need to sum over large number of intermediate states, AND cannot use current experimental data on D-decays
  - "hadronic" techniques currently neglect some sources of SU(3) breaking
- Philosophy: does exclusive approach to mixing constitute a prediction?

"Charm physics"
Eur. Phys. J. ST 233 (2024) 2, 439-456

Things to take home

- More philosophy: CP-violation in $D \to \pi\pi/KK$

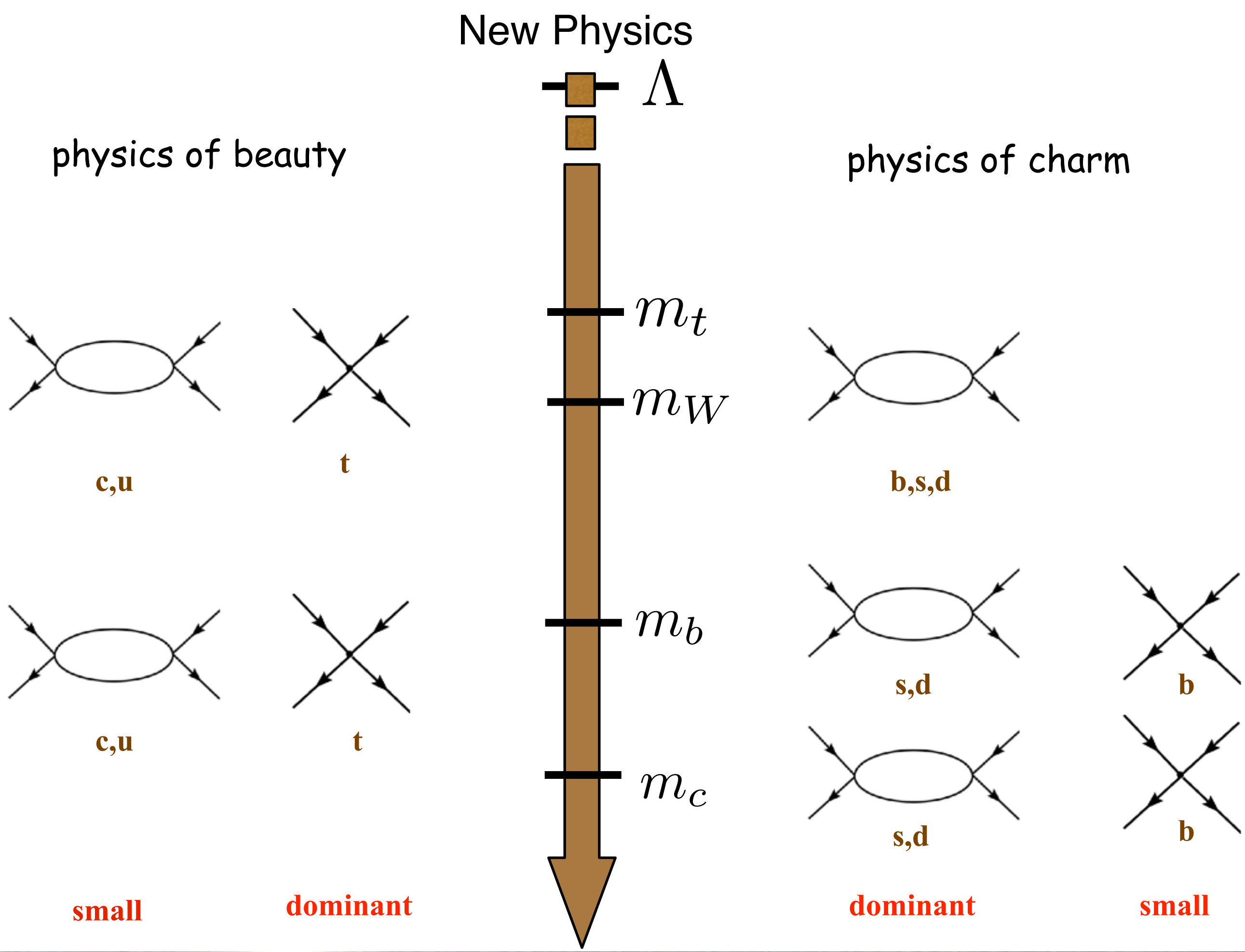
Theory ✗ **Experiment** ✗ Not a very interesting case…	**Theory** ✗ **Experiment** ✓ SM wins again?
Theory ✓ **Experiment** ✗ SM wins again!	**Theory** ✓ **Experiment** ✓ New Physics!

Chala, Lenz, Rusov, Scholtz: JHEP 1907 (2019) 161
Lenz, Piscoppo, Rusov: JHEP 03 (2024) 151


EXTRA!

★ Main goal of the exercise: understand physics at the most fundamental scale

★ It is important to understand relevant energy scales for the problem at hand



★ How can one tell that a process is dominated by long-distance or short-distance?

★ To start, mass and lifetime differences of mass eigenstates...

$$x_D = \frac{M_2 - M_1}{\Gamma_D}, \; y_D = \frac{\Gamma_2 - \Gamma_1}{2\Gamma_D}$$

★ ...can be calculated as real and imaginary parts of a correlation function

$$y_{\rm D} = \frac{1}{2M_{\rm D}\Gamma_{\rm D}} \,{\rm Im}\, \langle \overline{D^0}|\, i \int {\rm d}^4x\, T\Big\{\mathcal{H}_w^{|\Delta C|=1}(x)\, \mathcal{H}_w^{|\Delta C|=1}(0)\Big\}|D^0\rangle$$

bi-local time-ordered product

$$x_{\rm D} = \frac{1}{2M_{\rm D}\Gamma_{\rm D}} \,{\rm Re}\, \Big[2\langle \overline{D^0}|H^{|\Delta C|=2}\,|D^0\rangle + \langle \overline{D^0}|\, i \int {\rm d}^4x\, T\Big\{\mathcal{H}_w^{|\Delta C|=1}(x)\, \mathcal{H}_w^{|\Delta C|=1}(0)\Big\}|D^0\rangle\Big]$$

local operator
(b-quark, NP): small?

bi-local time-ordered product

★ ... or can be written in terms of hadronic degrees of freedom...

$$y = \frac{1}{2\Gamma}\sum_n \rho_n \left[\langle D^0|H_W^{\Delta C=1}|n\rangle\langle n|H_W^{\Delta C=1}|\overline{D}^0\rangle + \langle \overline{D}^0|H_W^{\Delta C=1}|n\rangle\langle n|H_W^{\Delta C=1}|D^0\rangle\right]$$

Inclusive approach to mixing: quark-hadron duality

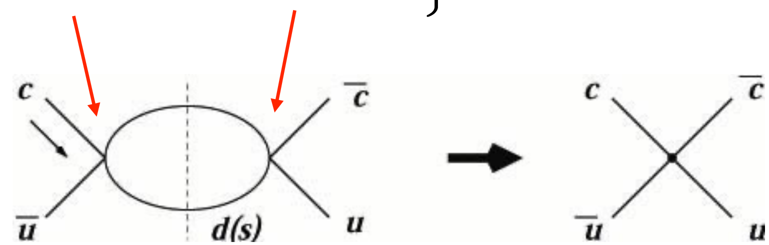
★ How can one tell that a process is dominated by long-distance or short-distance?

$$y = \frac{1}{2\Gamma}\sum_n \rho_n \left[\langle D^0|H_W^{\Delta C=1}|n\rangle\langle n|H_W^{\Delta C=1}|\overline{D}^0\rangle + \langle \overline{D}^0|H_W^{\Delta C=1}|n\rangle\langle n|H_W^{\Delta C=1}|D^0\rangle\right]$$

★ It is important to remember that the expansion parameter is $1/E_{released}$

$$y_{\mathrm{D}} = \frac{1}{2M_{\mathrm{D}}\Gamma_{\mathrm{D}}}\,\mathrm{Im}\,\langle\overline{D^0}|\,i\int \mathrm{d}^4x\,T\left\{\mathcal{H}_w^{|\Delta C|=1}(x)\,\mathcal{H}_w^{|\Delta C|=1}(0)\right\}|D^0\rangle$$

OPE-leading contribution:



★ In the heavy-quark limit $m_c \to \infty$ we have $m_c \gg \sum m_{\text{intermediate quarks}}$, so $E_{released} \sim m_c$

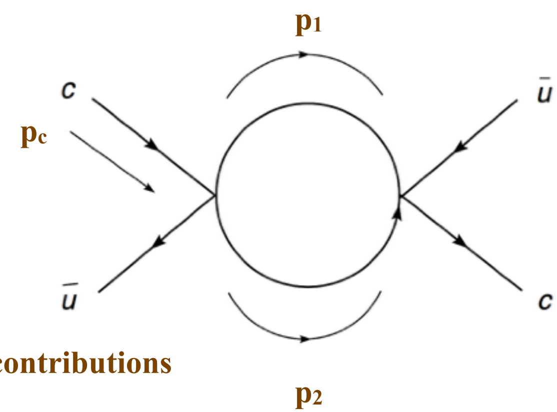
- the situation is similar to B-physics, where it is "short-distance" dominated
- one can consistently compute pQCD and 1/m corrections

★ But wait, m_c is NOT infinitely large! What happens for finite m_c???

- how is large momentum routed in the diagrams?
- are there important hadronization (threshold) effects?

★ How can one tell that a process is dominated by long-distance or short-distance?

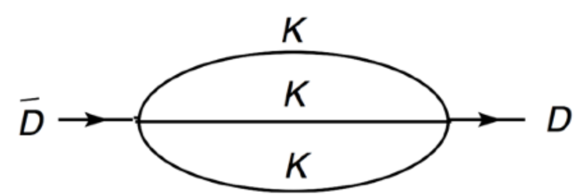
★ Let's look at how the momentum is routed in a leading-order diagram
- injected momentum is $p_c \sim m_c$
- thus, $p_1 \sim p_2 \sim m_c/2 \sim O(\Lambda_{QCD})$?



Still OK with OPE, signals large nonperturbative contributions

★ For a particular example of the lifetime difference, have hadronic intermediate states

-let's use an example of KKK intermediate state
- in this example, $E_{released} \sim m_D - 3\ m_K \sim O(\Lambda_{QCD})$



★ Similar threshold effects exist in B-mixing calculations
- but $m_b \gg \sum m_{intermediate\ quarks}$, so $E_{released} \sim m_b$ (almost) always
- quark-hadron duality takes care of the rest!

Let's saturate correlators by hadronic states

Exclusive approach to mixing: use data?

★ LD calculation: saturate the correlator by hadronic states, e.g.

$$y = \frac{1}{2\Gamma} \sum_n \rho_n \left[\langle D^0 | H_W^{\Delta C=1} | n \rangle \langle n | H_W^{\Delta C=1} | \overline{D}^0 \rangle + \langle \overline{D}^0 | H_W^{\Delta C=1} | n \rangle \langle n | H_W^{\Delta C=1} | D^0 \rangle \right]$$

… with n being all states to which D^0 and $\overline{D}^0$ can decay. Consider ππ, πK, KK intermediate states as an example…

J. Donoghue et. al.
L. Wolfenstein
P. Colangelo et. al.

H.Y. Cheng and C. Chiang

$$y_2 = Br(D^0 \to K^+K^-) + Br(D^0 \to \pi^+\pi^-) - 2\cos\delta \sqrt{Br(D^0 \to K^+\pi^-) Br(D^0 \to \pi^+K^-)}$$

cancellation expected

If every Br is known up to O(1%) ➡ the result is expected to be O(1%)!

The result here is a series of large numbers with alternating signs, SU(3) forces 0

If experimental data on Br is used, are we only sensitive to exit. uncertainties?

★ Need to "repackage" the analysis: look at complete multiplet contribution

$$y = \sum_{F_R} y_{F,R}\, Br(D^0 \to F_R) = \sum_{F_R} y_{F,R} \frac{1}{\Gamma} \sum_{n \in F_R} \Gamma(D^0 \to n)$$

Falk, Grossman, Ligeti, Nir. A.A.P. Phys.Rev. D69, 114021, 2004
Falk, Grossman, Ligeti, and A.A.P. Phys.Rev. D65, 054034, 2002

Exclusive approach to mixing: use data!

★ What if we insist on using experimental data anyway?

★ Ex., one can employ Factorizaton-Assisted Topological Amplitudes

in units of 10^{-3}

Modes	$\mathcal{B}$(exp)	$\mathcal{B}$(FAT)	Modes	$\mathcal{B}$(exp)	$\mathcal{B}$(FAT)	Modes	$\mathcal{B}$(exp)	$\mathcal{B}$(FAT)
$\pi^0\overline{K}^0$	24.0 ± 0.8	24.2 ± 0.8	$\pi^0\overline{K}^{*0}$	37.5 ± 2.9	35.9 ± 2.2	$\overline{K}^0\rho^0$	$12.8^{+1.4}_{-1.6}$	13.5 ± 1.4
π^+K^-	39.3 ± 0.4	39.2 ± 0.4	π^+K^{*-}	54.3 ± 4.4	62.5 ± 2.7	$K^-\rho^+$	111.0 ± 9.0	105.0 ± 5.2
$\eta\overline{K}^0$	9.70 ± 0.6	9.6 ± 0.6	$\eta\overline{K}^{*0}$	9.6 ± 3.0	6.1 ± 1.0	$\overline{K}^0\omega$	22.2 ± 1.2	22.3 ± 1.1
$\eta'\overline{K}^0$	19.0 ± 1.0	19.5 ± 1.0	$\eta'\overline{K}^{*0}$	< 1.10	0.19 ± 0.01	$\overline{K}^0\phi$	$8.47^{+0.66}_{-0.34}$	8.2 ± 0.6
$\pi^+\pi^-$	1.421 ± 0.025	1.44 ± 0.02	$\pi^+\rho^-$	5.09 ± 0.34	4.5 ± 0.2	$\pi^-\rho^+$	10.0 ± 0.6	9.2 ± 0.3
K^+K^-	4.01 ± 0.07	4.05 ± 0.07	K^+K^{*-}	1.62 ± 0.15	1.8 ± 0.1	K^-K^{*+}	4.50 ± 0.30	4.3 ± 0.2
$K^0\overline{K}^0$	0.36 ± 0.08	0.29 ± 0.07	$K^0\overline{K}^{*0}$	0.18 ± 0.04	0.19 ± 0.03	$\overline{K}^0K^{*0}$	0.21 ± 0.04	0.19 ± 0.03
$\pi^0\eta$	0.69 ± 0.07	0.74 ± 0.03	$\eta\rho^0$		1.4 ± 0.2	$\pi^0\omega$	0.117 ± 0.035	0.10 ± 0.03
$\pi^0\eta'$	0.91 ± 0.14	1.08 ± 0.05	$\eta'\rho^0$		0.25 ± 0.01	$\pi^0\phi$	1.35 ± 0.10	1.4 ± 0.1
$\eta\eta$	1.70 ± 0.20	1.86 ± 0.06	$\eta\omega$	2.21 ± 0.23	2.0 ± 0.1	$\eta\phi$	0.14 ± 0.05	0.18 ± 0.04
$\eta\eta'$	1.07 ± 0.26	1.05 ± 0.08	$\eta'\omega$		0.044 ± 0.004			
$\pi^0\pi^0$	0.826 ± 0.035	0.78 ± 0.03	$\pi^0\rho^0$	3.82 ± 0.29	4.1 ± 0.2			
π^0K^0		0.069 ± 0.002	π^0K^{*0}		0.103 ± 0.006	$K^0\rho^0$		0.039 ± 0.004
π^-K^+	0.133 ± 0.009	0.133 ± 0.001	π^-K^{*+}	$0.345^{+0.180}_{-0.102}$	0.40 ± 0.02	$K^+\rho^-$		0.144 ± 0.009
ηK^0		0.027 ± 0.002	ηK^{*0}		0.017 ± 0.003	$K^0\omega$		0.064 ± 0.003
$\eta'K^0$		0.056 ± 0.003	$\eta'K^{*0}$		0.00055 ± 0.00004	$K^0\phi$		0.024 ± 0.002

Jiang, Yu, Qin, Li, and Lu, 2017

★ … but it appears to yield a smaller result, $y_{PP+PV} = (0.21 \pm 0.07)\%$.

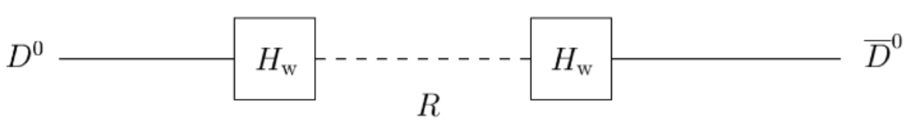
★ Exclusive approach to $D^0 - \bar{D}^0$ mixing: use data!

★ Possible additional contributions?

- each intermediate state has a finite width, i.e. is not a proper asymptotic state
- within each multiplet widths experience (incomplete) SU(3) cancelations
- this effect already happens for the simplest intermediate states!

★ Consider, for illustration, a set of single-particle intermediate states:

$$-\Sigma_{p_D}(p_D)\Big|_{\text{tot}}^{\text{res}} = \frac{1}{2m_D}\sum_R Re \frac{\langle D_L|\mathcal{H}_W|R\rangle\langle R|\mathcal{H}_W^\dagger|D_L\rangle}{m_D^2 - m_R^2 + i\Gamma_R m_D} \quad - \quad (D_L \to D_S)$$



$$\Delta m_D\Big|_{\text{R}}^{\text{res}} \propto \frac{m_D^2 - m_R^2}{(m_D^2 - m_R^2)^2 + \Gamma_R^2 m_D^2}$$

$$\Delta\Gamma_D\Big|_{\text{R}}^{\text{res}} \propto -\frac{\Gamma_R m_D}{(m_D^2 - m_R^2)^2 + \Gamma_R^2 m_D^2}$$

★ Each resonance contributes to $\Delta\Gamma$ only because of its finite width!

Finite width effects: one-body contributions

★ Multiplet effects for (single-particle) intermediate states

- in this simple example: heavy pion, kaon and eta/eta'
- each single-particle intermediate state has a rather large width

$$\Delta\Gamma_D\Big|^{\rm res}_{\rm octet} = \Delta\Gamma_D^{(K_H)} - \frac{1}{4}\Delta\Gamma_D^{(\pi_H)} - \frac{3\cos^2\theta_{\rm H}}{4}\Delta\Gamma_D^{(\eta_H)} - \frac{1\sin^2\theta_{\rm H}}{4}\Delta\Gamma_D^{(\eta'_H)}$$

- where for each state $\Delta\Gamma_D^{\rm res} = -C f_R^2 \dfrac{\mu_R\gamma_R}{(1-\mu_R)^2+\gamma_R^2}$ with $\mu_R = m_R^2/m_D^2$, $\gamma_R = \Gamma_R/m_D$
- … and a model calculation gives $C \equiv 2m_D(G_F a_2 f_D \xi_d/\sqrt{2})^2$
- SU(3) forces cancellations: a new SU(3) breaking effect due to widths!

Table: Magnitudes of Pseudoscalar Resonance Contributions.

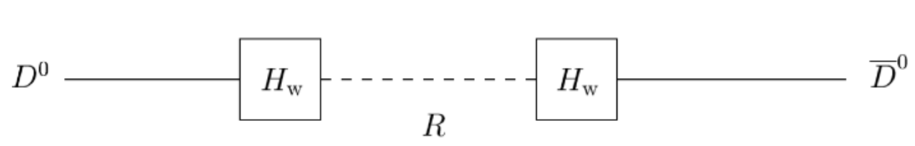
Resonance	$\lvert\Delta m_D\rvert \times 10^{-16}$ (GeV)	$\lvert\Delta\Gamma_D\rvert \times 10^{-16}$ (GeV)
$K(1460)$	$\sim 1.24\ (f_{K(1460)}/0.025)^2$	$\sim 0.88\ (f_{K(1460)}/0.025)^2$
$\eta(1760)$	$(0.77 \pm 0.27)\ (f_{\eta(1760)}/0.01)^2$	$(0.43 \pm 0.53)\ (f_{\eta(1760)}/0.01)^2$
$\pi(1800)$	$(0.13 \pm 0.06)\ (f_{\pi(1800)}/0.01)^2$	$(0.41 \pm 0.11)\ (f_{\pi(1800)}/0.01)^2$
$K(1830)$	$\sim 0.29\ (f_{K(1830)}/0.01)^2$	$\sim 1.86\ (f_{K(1830)}/0.01)^2$

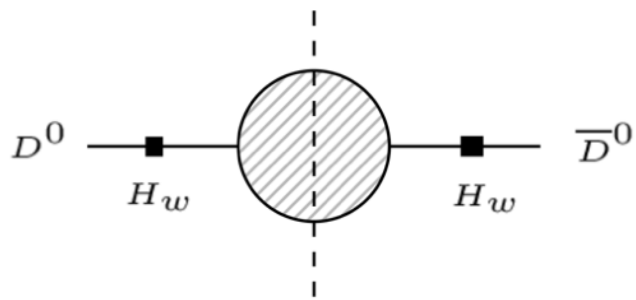
E. Golowich and A.A.P.
PLB427 (1998) 172-178

Finite width effects: one-body contributions

★ Let us take another look at those one-body contributions

- the width of each excited light quark state $\Gamma_R = \Gamma(R \to P_1 P_2) + \Gamma(R \to P_1 P_2 P_3) + \ldots$
- … which is equivalent to accounting for resonant FSI in 2-body intermediate state!





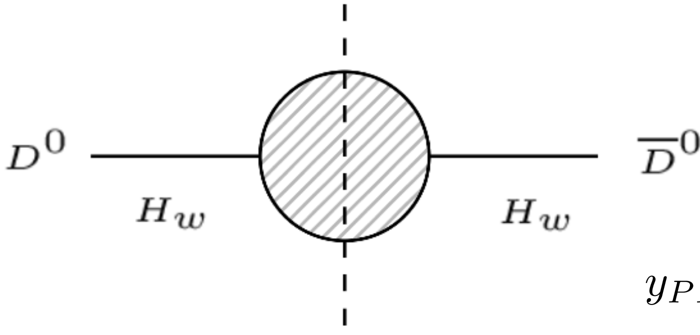
Since we shall be using experimental data to compute 2-body contributions, this effect will be taken into account automatically!

It's consistent to omit 1-body IntSt if experimental data is used

Finite width effects: two-body contributions

★ Let us apply similar logic to two-body contributions

- consider contributions from the stable (wrt strong interactions) octet of pions, kaons, etas



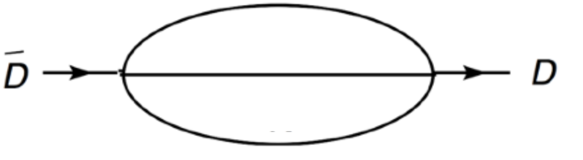
$$y_{PP} = (0.1 - 20) \times 10^{-4}$$

Falls short of the experimentally observed value of y

★ What about other two body contributions (PV, SP, SS, etc.)?

- can use similar techniques to evaluate contribution to mixing as above 2BIS…
- … but V, P', S states are not good asymptotic states!
- we get new SU(3)-breaking contribution from the widths of those states!

Since we are to use experimental data, use Dalitz plot analyses to get at these contributions



A.A.P.. arXiv:even.tually [hep-ph]

Experimental analysis from LHCb

★ Since we are comparing rates for D^0 and anti-D^0: need to tag the flavor at production

$$D^{*+} \to D^0 \pi_s^+$$

"D*-trick" -- tag the charge of the slow pion (or muon for D's produced in B-decays)

★ The difference Δa_{CP} is also preferable experimentally, as

$$a_f^{\rm raw} = a_f^{CP} + a_f^{\rm detect,\ D} + a_D^{\rm detect,\ \pi_s} + a_{D*}^{\rm prod}$$

physics; detection asymmetry of D^0; detection asymmetry of soft pion; production asymmetry of D^{*+}

★ D* production asymmetry and soft pion asymmetries are the same for KK and ππ final states-- they cancel in Δa_{CP}!

★ Integrate over time,

$$a_{CP,\ f} = \int_0^\infty a_{CP}(f;t) D(t) dt = a_f^d + \frac{\langle t \rangle}{\tau} a_f^{ind}$$

$D(t)$: distribution of proper decay time

★ Viola! Report observation!

Khodjamirian, NPB 605 (2001) 558

- Use modified light-cone QCD Sum Rule (LCSR) method
  - start with the correlation function ($j_5^{(D)} = i m_c \bar{c}\gamma_5 u$ and $j_{\alpha 5}^{(\pi)} = \bar{d}\gamma_\alpha\gamma_5 u$)

$$F_\alpha(p,q,k) = i^2 \int d^4x e^{-i(p-q)x} \int d^4y e^{i(p-k)y} \langle 0|\, T\left\{ j_{\alpha 5}^{(\pi)}(y) \mathcal{Q}_1^s(0) j_5^{(D)}(x) \right\} |\pi^+(q)\rangle$$
$$= (p-k)_\alpha F((p-k)^2, (p-q)^2, P^2) + \dots,$$

  - use dispersion relation in (p-k) and (p-q), perform Borel transform, extract matrix element:

Khodjamirian, Mannel, Melic, PLB571 (2003) 75

$$\langle \pi^-(-q)\pi^+(p)|\mathcal{Q}_1^s|D^0(p-q)\rangle = \frac{-i}{\pi^2 f_\pi f_D m_D^2} \int_0^{s_0^\pi} ds\, e^{-s/M_1^2} \int_{m_c^2}^{s_0^D} ds' e^{(m_D^2 - s')/M_2^2} \mathrm{Im}_{s'} \mathrm{Im}_s F(s, s', m_D^2)$$

  - perform LC expansion of F(s, s' m_D^2) to get $\mathcal{P}_{\pi\pi}^s$
  - note that $C_1 \mathcal{Q}_1^s + C_2 \mathcal{Q}_2^s = 2C_1 \widetilde{\mathcal{Q}}_2^s + \left(\frac{C_1}{3} + C_2\right) \mathcal{Q}_2^s$ with $\widetilde{\mathcal{Q}}_2^s = \left(\bar{s}\Gamma_\mu \frac{\lambda^a}{2} s\right)\left(\bar{u}\Gamma^\mu \frac{\lambda^a}{2} c\right)$

thus $\mathcal{P}_{\pi\pi}^s = \frac{2G_F}{\sqrt{2}} C_1 \langle \pi^+\pi^- | \widetilde{\mathcal{Q}}_2^s | D^0 \rangle$

Error budget: parameter uncertainties

Parameter values and references	Parameter rescaled to $\mu = 1.5$ GeV
$\alpha_s(m_Z) = 0.1181 \pm 0.0011$ [6]	0.351
$\bar{m}_c(\bar{m}_c) = 1.27 \pm 0.03$ GeV [6]	1.19 GeV
$\bar{m}_s(2\,\text{GeV}) = 96^{+8}_{-4}\,\text{MeV}$ [6]	105 MeV
$\langle\bar{q}q\rangle(2\,\text{GeV}) = (-276^{+12}_{-10}\,\text{MeV})^3$ [6]	$(-268\,\text{MeV})^3$
$\langle\bar{s}s\rangle = (0.8 \pm 0.3)\langle\bar{q}q\rangle$ [21]	$(-249\ \text{MeV})^3$
$a_2^\pi(1\,\text{GeV}) = 0.17 \pm 0.08$ [22]	0.14
$a_4^\pi(1\,\text{GeV}) = 0.06 \pm 0.10$ [22]	0.045
$\mu_\pi(2\,\text{GeV}) = 2.48 \pm 0.30\,\text{GeV}$ [6]	2.26 GeV
$f_{3\pi}(1\,\text{GeV}) = 0.0045 \pm 0.015\ \text{GeV}^2$ [19]	$0.0036\,\text{GeV}^2$
$\omega_{3\pi}(1\,\text{GeV}) = -1.5 \pm 0.7$ [19]	-1.1
$a_1^K(1\,\text{GeV}) = 0.10 \pm 0.04$ [23]	0.09
$a_2^K(1\,\text{GeV}) = 0.25 \pm 0.15$ [19]	0.21
$\mu_K(2\,\text{GeV}) = 2.47^{+0.19}_{-0.10}$ GeV [6]	2.25
$f_{3K} = f_{3\pi}$	$0.0036\,\text{GeV}^2$
$\omega_{3K}(1\,\text{GeV}) = -1.2 \pm 0.7$ [19]	-0.99
$\lambda_{3K}(1\,\text{GeV}) = 1.6 \pm 0.4$ [19]	1.5