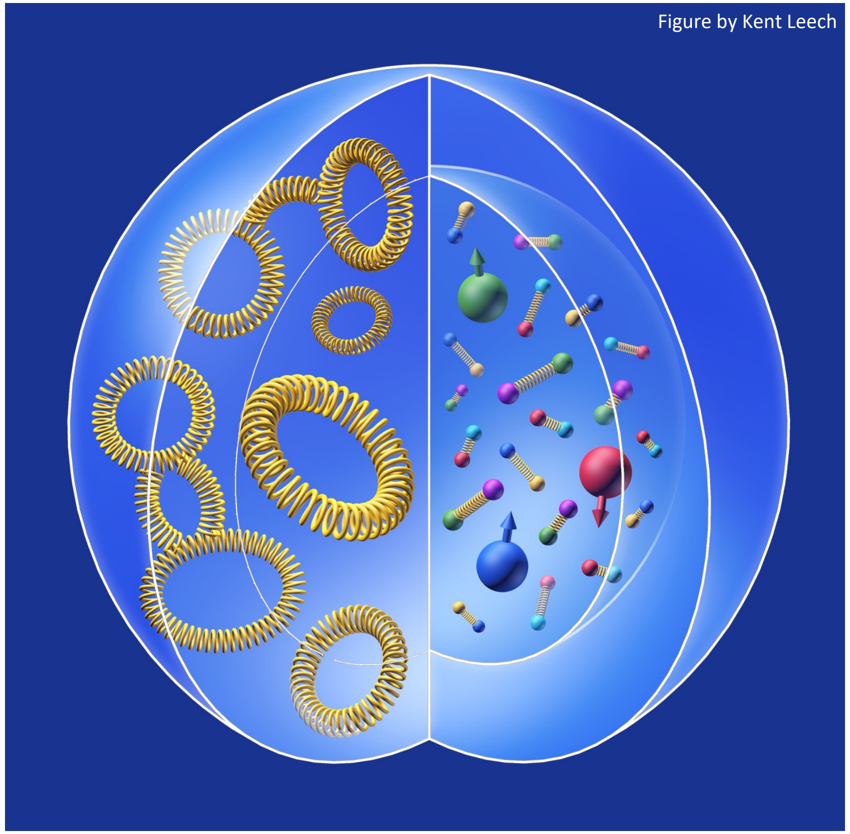
Gravitational form factors on the lattice



Fermilab Theory Seminar

June 20, 2024

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Outline

Gravitational structure of hadrons

- Gravitational form factors (GFFs)?
- Why are GFFs interesting?

GFFs on the lattice

- Overview of calculation

Results

- GFFs of pion, proton (w/ flavor decomp)
- Experimental comparison
- Mechanical densities & radii

Glueball GFFs

- Very prelim results

2307.11707

Gravitational form factors of the pion from lattice QCD

Daniel C. Hackett, Patrick R. Oare, Dimitra A. Pefkou, and Phiala E. Shanahan
Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, U.S.A.

The two gravitational form factors of the pion, $A^\pi(t)$ and $D^\pi(t)$, are computed as functions of the momentum transfer squared t in the kinematic region $0 \leq -t < 2$ GeV^2 on a lattice QCD ensemble with quark masses corresponding to a close-to-physical pion mass $m_\pi \approx 170$ MeV and $N_f = 2+1$ quark flavors. The flavor decomposition of these form factors into gluon, up/down light-quark, and strange quark contributions is presented in the $\overline{\text{MS}}$ scheme at energy scale $\mu = 2$ GeV, with renormalization factors computed non-perturbatively via the RI-MOM scheme. Using monopole and z-expansion fits to the gravitational form factors, we obtain estimates for the pion momentum fraction and D-term that are consistent with the momentum fraction sum rule and the next-to-leading order chiral perturbation theory prediction for $D^\pi(0)$.

2310.08484

Gravitational form factors of the proton from lattice QCD

Daniel C. Hackett,[1,2] Dimitra A. Pefkou,[3,2] and Phiala E. Shanahan[2]
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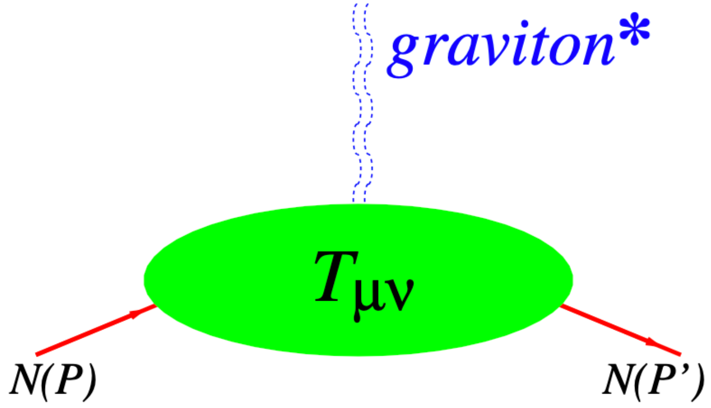
The gravitational form factors (GFFs) of a hadron encode fundamental aspects of its structure, including its shape and size as defined from e.g., its energy density. This work presents a determination of the flavor decomposition of the GFFs of the proton from lattice QCD, in the kinematic region $0 \leq -t \leq 2$ GeV^2. The decomposition into up-, down-, strange-quark, and gluon contributions provides first-principles constraints on the role of each constituent in generating key proton structure observables, such as its mechanical radius, mass radius, and D-term.

Gravitational structure of hadrons

Gravitational form factors (GFFs)



GFFs are EMT form factors

Schematically, for any hadron:

$$\langle \mathrm{hadron}(p') | T(\Delta) | \mathrm{hadron}(p) \rangle = \sum_i (\text{Lorentz structure})_i \ \mathrm{GFF}_i(t = \Delta^2)$$

Graviton scattering ~ symmetric EMT

$$T^{\{\mu\nu\}} = \frac{2}{\sqrt{-g}} \frac{\delta S_{QCD}}{\delta g_{\mu\nu}} = 2\,\mathrm{Tr}\left[-G^{\alpha\mu} G^{\nu}_{\alpha} + \frac{1}{4} g^{\mu\nu} G^{\alpha\beta} G_{\alpha\beta} \right] + \bar{q}\, \gamma^{\{\mu} i \overleftrightarrow{D}^{\nu\}}\, q$$

$$a^{\{\mu} b^{\nu\}} \equiv \frac{1}{2}(a^\mu b^\nu + \mathrm{a}^\nu \mathrm{b}^\mu)$$

Gravitational form factors

$$T^{\{\mu\nu\}} = 2\,\mathrm{Tr}\left[-G^{\alpha\mu}G^{\nu}_{\alpha} + \frac{1}{4}g^{\mu\nu}G^{\alpha\beta}G_{\alpha\beta}\right] + \bar{q}\,\gamma^{\{\mu}i\overleftrightarrow{D}^{\nu\}}\,q$$

$a^{\{\mu}b^{\nu\}} \equiv \frac{1}{2}(a^{\mu}b^{\nu} + a^{\nu}b^{\mu})$
$\overleftrightarrow{D} = (\overrightarrow{D} - \overleftarrow{D})/2$
$U, \bar{U}$ = Dirac spinors
$P = (p' + p)/2$
$\Delta = p' - p$
$t = \Delta^2$

Nucleon:

$$\langle N(p')|T^{\{\mu\nu\}}|N(p)\rangle = \bar{U}(p')\left[A(t)\frac{P^{\{\mu}P^{\nu\}}}{M} + J(t)\frac{i\,P^{\{\mu}\sigma^{\nu\}\rho}\Delta_{\rho}}{2M} + D(t)\frac{\Delta^{\{\mu}\Delta^{\nu\}} - g^{\mu\nu}\Delta^2}{4M}\right]U(p)$$

Pion:

$$\langle\pi(p')|T^{\{\mu\nu\}}|\pi(p)\rangle = A(t)\,2P^{\mu}P^{\nu} + D(t)\frac{1}{2}(\Delta^{\mu}\Delta^{\nu} - g^{\mu\nu}\Delta^2)$$

Why are these interesting?

Global properties

$$\langle N(p')|T^{\{\mu\nu\}}|N(p)\rangle = \bar{U}(p')\left[A(t)\frac{P^{\{\mu}P^{\nu\}}}{M} + J(t)\frac{i\,P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M} + D(t)\frac{\Delta^{\{\mu}\Delta^{\nu\}} - g^{\mu\nu}\Delta^2}{4M}\right]U(p)$$

$\partial_\mu T^{\mu\nu} = 0 \rightarrow$ GFFs are scale- and scheme-independent

Forward GFFs are fundamental, global properties:

$A(0) = 1 \Leftrightarrow \langle p|T^{tt}|p\rangle = M$

$J(0) = \frac{1}{2} =$ Total spin

$B(0) = 2J(0) - A(0) = 0$ "vanishing of the anomalous gravitomagnetic moment"

$D(0) =$??? (internal forces)

Similar for pion, except no J

$D(0)$: "the last global unknown"

[Polyakov Schweitzer 1805.06596]

em: $\partial_\mu J^\mu_{\text{em}} = 0$	$\langle N' \vert J^\mu_{\text{em}} \vert N \rangle$	$\longrightarrow$	Q	$= 1.602176487(40) \times 10^{-19}\text{C}$
			μ	$= 2.792847356(23)\mu_N$
weak: PCAC	$\langle N' \vert J^\mu_{\textbf{weak}} \vert N \rangle$	$\longrightarrow$	g_A	$= 1.2694(28)$
			g_p	$= 8.06(55)$
gravity: $\partial_\mu T^{\mu\nu}_{\textbf{grav}} = 0$	$\langle N' \vert T^{\mu\nu}_{\textbf{grav}} \vert N \rangle$	$\longrightarrow$	m	$= 938.272013(23)\,\text{MeV}/c^2$
			J	$= \frac{1}{2}$
			$\boldsymbol{D}$	$= \boldsymbol{?}$

Table I. The global properties of the proton defined in terms of matrix elements of the conserved currents associated with respectively electromagnetic, weak, and gravitational interaction. Notice the weak currents include the partially conserved axial current, and g_A or g_p are strictly speaking defined in terms of transition matrix elements in the neutron β-decay or muon-capture. The values of the properties are from the particle data book [107] and [108] (for g_p) except for the unknown D-term.

What is a nucleon made of?

Gluons $T_g^{\{\mu\nu\}} = 2\,\mathrm{Tr}[G^{\alpha\{\mu}G^{\nu\}\alpha}]$

Quarks $T_q^{\{\mu\nu\}} = \bar{q}\,\gamma^{\{\mu} i\overleftrightarrow{D}^{\nu\}}\,q$

$$\left\langle N(p') \middle| T_{g,q}^{\{\mu\nu\}} \middle| N(p) \right\rangle = \bar{u}(p') \left[A_{g,q}(t) \frac{P^{\{\mu}P^{\nu\}}}{M} + J_{g,q}(t) \frac{i\,P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M} + D_{g,q}(t) \frac{\Delta^{\{\mu}\Delta^{\nu\}} - g^{\mu\nu}\Delta^2}{4M} + \bar{c}_{g,q}(t) M g^{\mu\nu} \right] u(p)$$

(Similar for pion!)

What is a nucleon made of?

Gluons $T_g^{\{\mu\nu\}} = 2\,\mathrm{Tr}[G^{\alpha\{\mu}G^{\nu\}\alpha}]$ Quarks $T_q^{\{\mu\nu\}} = \bar{q}\,\gamma^{\{\mu}i\overleftrightarrow{D}^{\nu\}}\,q$

$$\left\langle N(p')\middle|T_{g,q}^{\{\mu\nu\}}\middle|N(p)\right\rangle = \bar{u}(p')\left[A_{g,q}(t)\frac{P^{\{\mu}P^{\nu\}}}{M} + J_{g,q}(t)\frac{i\,P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M} + D_{g,q}(t)\frac{\Delta^{\{\mu}\Delta^{\nu\}} - g^{\mu\nu}\Delta^2}{4M} + \bar{c}_{g,q}(t)Mg^{\mu\nu}\right]u(p)$$

Not conserved $\sum_q \bar{c}_q + \bar{c}_g = 0$

Power-divergent mixing

(Similar for pion!)

What is a nucleon made of?

Gluons $T_g^{\{\mu\nu\}} = 2\,\mathrm{Tr}[G^{\alpha\{\mu}G^{\nu\}\alpha}]$ Quarks $T_q^{\{\mu\nu\}} = \bar{q}\,\gamma^{\{\mu}i\overleftrightarrow{D}^{\nu\}}\,q$

Momentum fraction

$A_{q,g}(0) = \langle x\rangle_{q,g}$

$A_g(0) + \sum_q A_q(0) = 1$

Spin fraction

$J_g(0) + \sum_q J_q(0) = \frac{1}{2}$

$$\left\langle N(p')\middle|T_{g,q}^{\{\mu\nu\}}\middle|N(p)\right\rangle = \bar{u}(p')\left[A_{g,q}(t)\frac{P^{\{\mu}P^{\nu\}}}{M} + J_{g,q}(t)\frac{i\,P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M} + D_{g,q}(t)\frac{\Delta^{\{\mu}\Delta^{\nu\}} - g^{\mu\nu}\Delta^2}{4M} + \bar{c}_{g,q}(t)Mg^{\mu\nu}\right]u(p)$$

Internal forces

$D(0) = D_g(0) + \sum_q D_q(0)$

Not conserved $\sum_q \bar{c}_q + \bar{c}_g = 0$

Power-divergent mixing

(Similar for pion!)

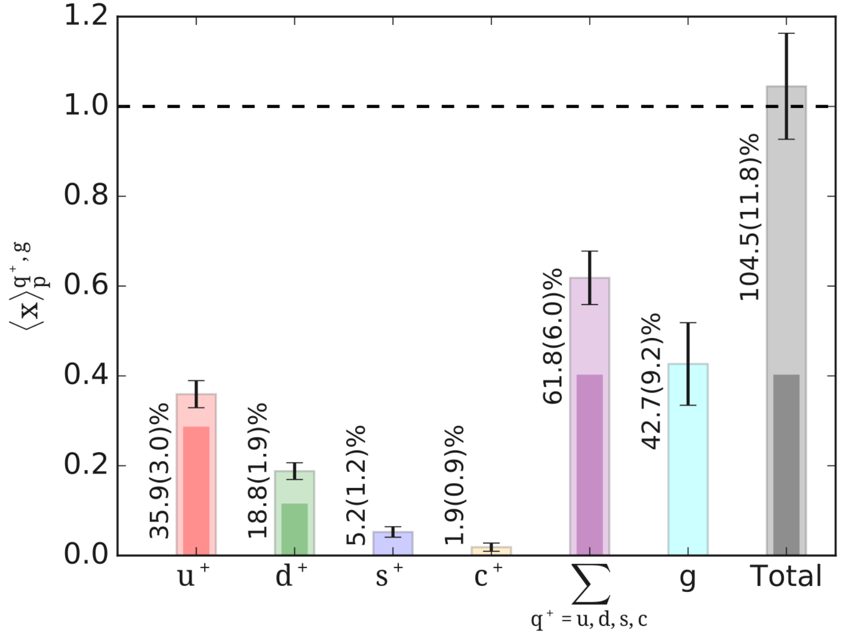
Forward GFFs & decompositions

$$\langle N(p')|T^{\{\mu\nu\}}_{g,q}|N(p)\rangle = \bar{U}(p')\left[A_{g,q}(t)\frac{P^{\{\mu}P^{\nu\}}}{M} + J_{g,q}(t)\frac{i\,P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M} + D_{g,q}(t)\frac{\Delta^{\{\mu}\Delta^{\nu\}} - g^{\mu\nu}\Delta^2}{4M} + \bar{c}_{g,q}(t)Mg^{\mu\nu}\right]U(p)$$

Momentum fraction

$A_{q,g}(0) = \langle x\rangle_{q,g}$

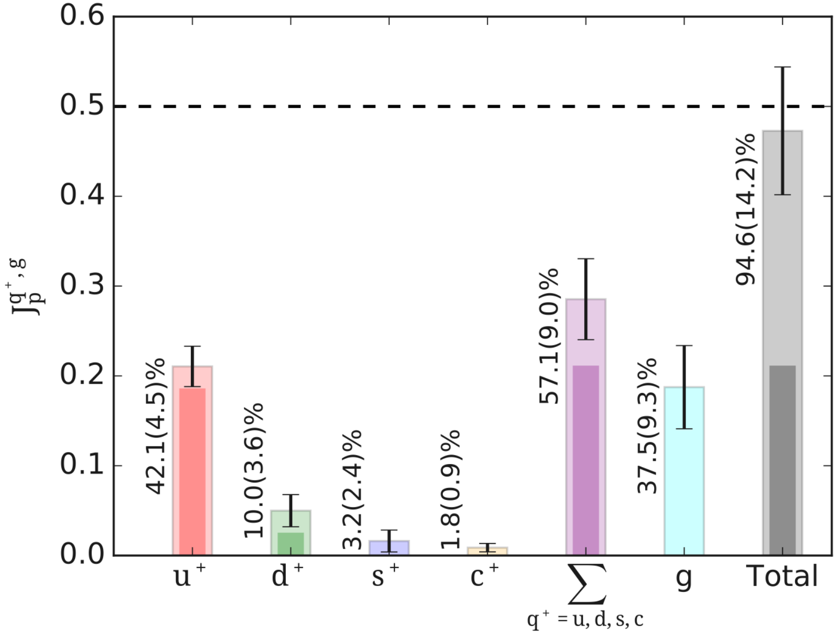
$\sum_q A_q(0) + A_g(0) = 1$



[ETMC 2003.08486]

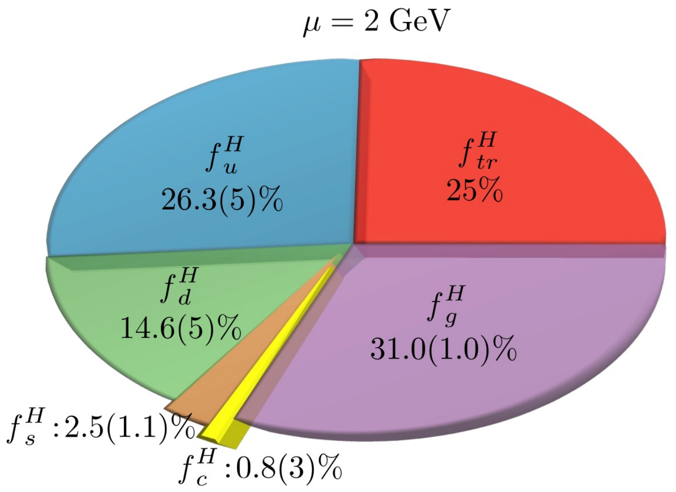
Spin fraction

$\sum_q J_q(0) + J_g(0) = 1/2$



[ETMC 2003.08486]

…and involved in others, e.g. Ji's rest energy decomp

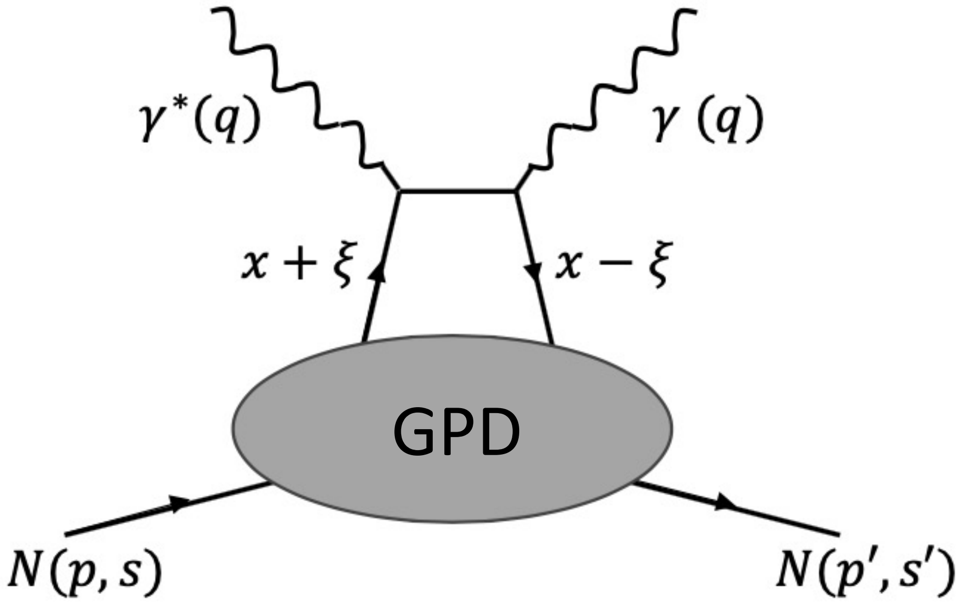


[Liu 2103.15768]

Experimental accessibility?

GFFs ⊂ GFFs

Gravitational Form Factors — Generalized Form Factors



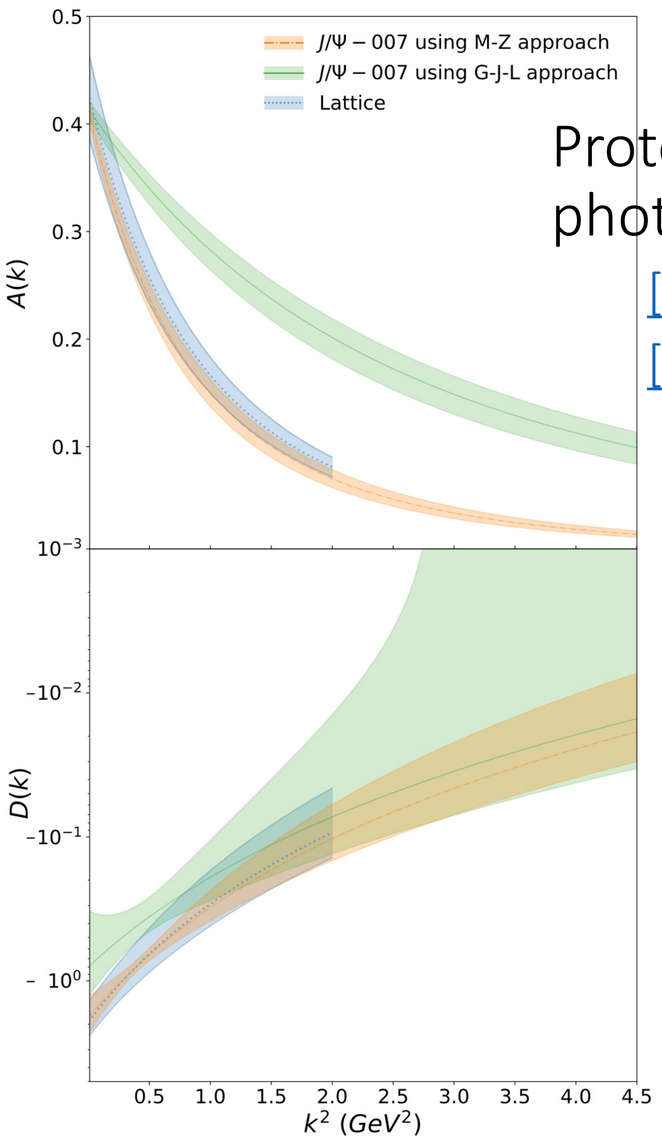
GFFs are Mellin moments of GPDs, e.g.

$$\int dx\, x\, H_q(x,\xi,t) = A_q(t) + \xi^2 D_q(t) \qquad \int dx\, H_g(x,\xi,t) = A_g(t) + \xi^2 D_g(t)$$

$$\int dx\, x\, E_q(x,\xi,t) = B_q(t) - \xi^2 D_q(t) \qquad \int dx\, E_g(x,\xi,t) = B_g(t) - \xi^2 D_g(t)$$

→ relate to experiment via factorization

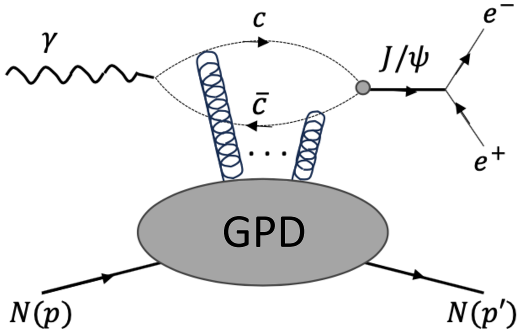
Experimental results



Proton: glue from J/Ψ photoproduction

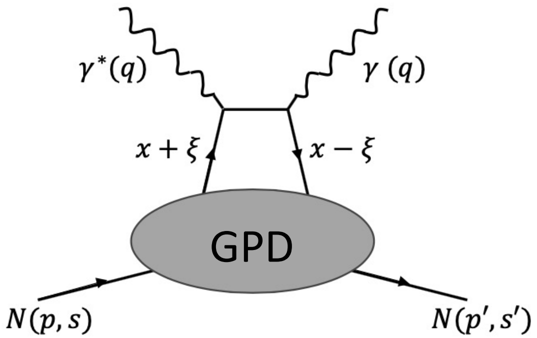
[Duran et al. 2207.05212]

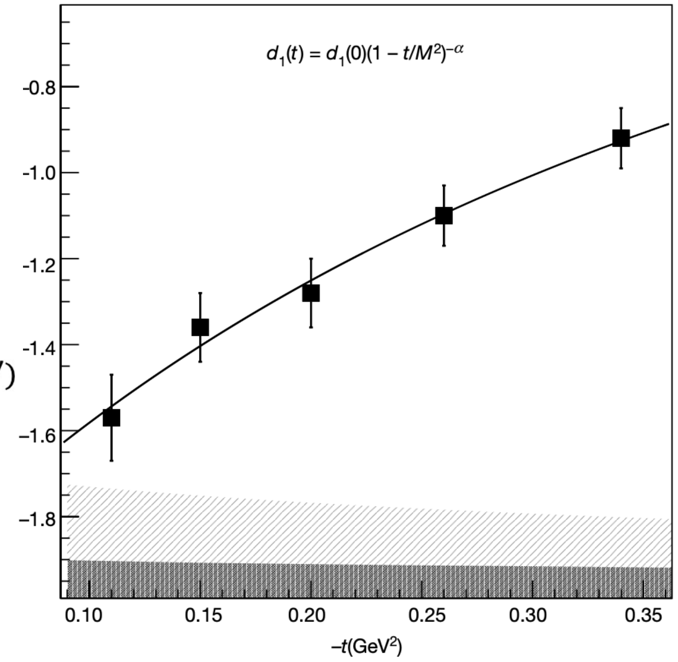
[Guo et al. 2305.06992]

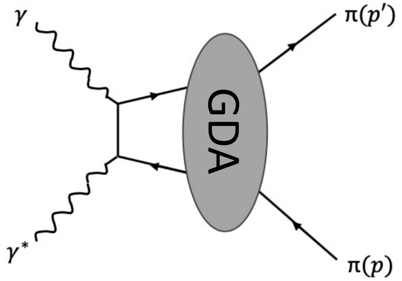


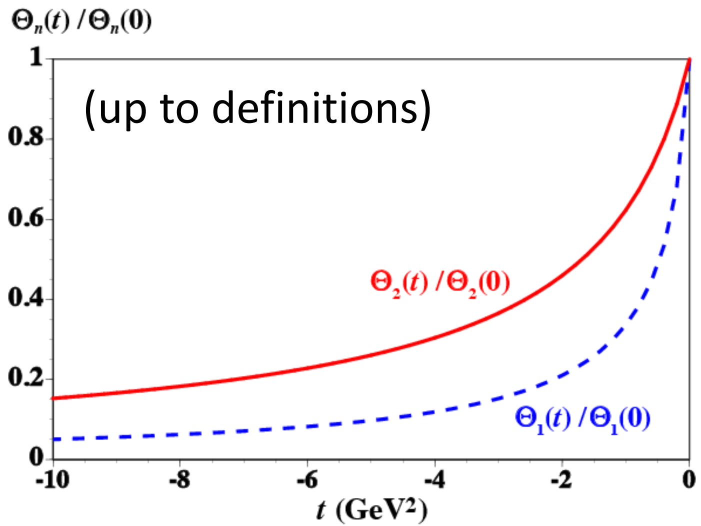
Proton: quark D from DVCS

[Burkert Elouadrhiri Girod 2018]









Pion: quark from $\gamma\gamma \to \pi^0\pi^0$

[Kumano Song Teryaev 1711.08088]

GFFs on the lattice

Overview of calculation

Need to compute:

Bare matrix elements for $f \in \{g, u, d, s\}$ to constrain bare GFFs

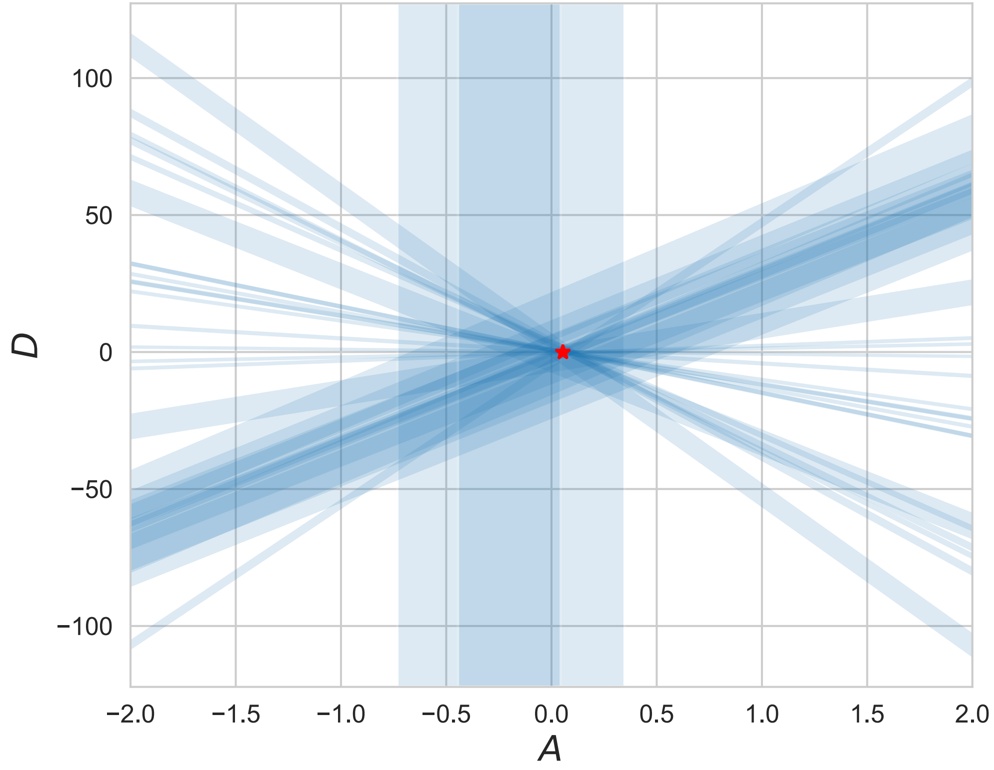
$$\langle p' | T_f^{\mathrm{b}}(\Delta) | p \rangle = c_A\, A_f^b(t) + c_J\, J_f^{\mathrm{b}}(t) + c_D\, D_f^{\mathrm{b}}(t)$$

Isosinglet mixing matrix (+ non-singlet Z_{u+d-2s})

$$\begin{bmatrix} T_q^{\overline{MS}} \\ T_g^{\overline{MS}} \end{bmatrix} = \begin{bmatrix} Z_{qq}^{\overline{MS}} & Z_{qg}^{\overline{MS}} \\ Z_{gq}^{\overline{MS}} & Z_{gg}^{\overline{MS}} \end{bmatrix} \begin{bmatrix} T_q^{\mathrm{bare}} \\ T_g^{\mathrm{bare}} \end{bmatrix}$$

→ Renormalized linear constraints on GFFs at different values of $t = \Delta^2 = (p' - p)^2$

Fit to extract GFFs(t)



Ensembles

Gauge action: tadpole-improved Luscher-Weisz

Fermion action: $2 + 1$ flavors, stout-smeared clover

	L/a	T/a	β	am_l	am_s	a [fm]	m_π [MeV]
A	48	96	6.3	-0.2416	-0.2050	0.091(1)	169(1)
B	12	24	6.1	-0.2800	-0.2450	0.1167(16)	450(5)

Bare matrix elements

Glue: 2511 configs

Quarks: 1381 configs (subset)

[“a091m170” (JLab/W&M/MIT/LANL)]

Renormalization

Conn. quark: 240 configs

Disco./glue: 20000 configs

Ensembles

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Renormalization

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Disco./glue: 20000 configs

“Single”-ensemble calculation: can’t quantify remaining artifacts due to discretization, unphysical quark masses, finite volume

Lattice EMT operators

Quark: $T_q^{\{\mu\nu\}} = \bar{q}\gamma^{\{\mu} i\overleftrightarrow{D}^{\nu\}} q$

Discretized covariant derivative

$\overleftrightarrow{D} = (\overrightarrow{D} - \overleftarrow{D})/2$

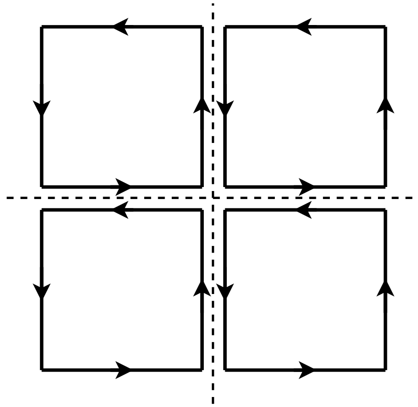
$(\overrightarrow{D}_\mu \psi)(x) = \frac{1}{2}\left[U_\mu(x)\psi(x+\mu) - U_\mu^\dagger(x-\mu)\psi(x-\mu)\right]$

$(\bar{\psi}\overleftarrow{D}_\mu)(x) = \frac{1}{2}\left[\bar{\psi}(x+\mu)U_\mu^\dagger(x) - \bar{\psi}(x-\mu)U_\mu(x-\mu)\right]$

Glue: $T_g^{\{\mu\nu\}} = \frac{2}{g^2}\,\mathrm{Tr}[G^{\alpha\{\mu}G^{\nu\}\alpha}]$

Clovers flowed to $t/a^2 = 2$

$G_{\mu\nu} \sim \left(Q_{\mu\nu} - Q_{\mu\nu}^\dagger\right)$



Project to irreps of hypercubic group

$$\tau_1^{(3)}: \quad \frac{1}{2}(T^{xx} + T^{yy} - T^{zz} + T^{tt}),\ \frac{1}{\sqrt{2}}(T^{zz} + T^{tt}),\ \frac{1}{\sqrt{2}}(T^{xx} - T^{yy})$$

$$\tau_3^{(6)}: \quad \left\{\frac{i^{\delta_{\mu 0}}}{\sqrt{2}}(T^{\mu\nu} + T^{\nu\mu}),\ 0 \le \mu \le \nu \le 3\right\}$$

Bare matrix elements from three-point functions

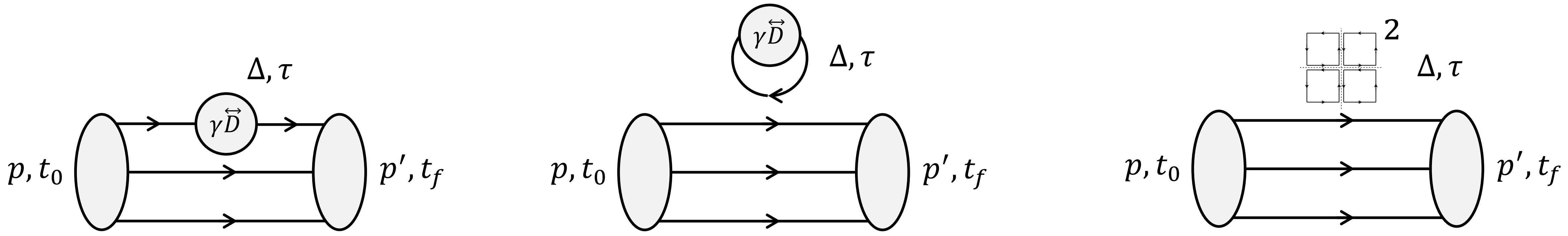
Can't compute matrix elements directly, must extract from

$$\langle \chi(p', t_f)\, T^{\mathrm{b}}(\Delta, \tau)\, \bar{\chi}(p, 0) \rangle \sim Z_{p'}\, Z_p \boxed{\langle p' | T^{\mathrm{b}}(\Delta) | p \rangle} e^{-E'(t_f - \tau) - E\tau} + \text{(excited states)}$$

Bare matrix elements from three-point functions

Can't compute matrix elements directly, must extract from

$$\langle \chi(p', t_f)\, T^{\mathrm{b}}(\Delta, \tau)\, \bar{\chi}(p, 0) \rangle \sim Z_{p'}\, Z_p \boxed{\langle p' | T^{\mathrm{b}}(\Delta) | p \rangle} e^{-E'(t_f - \tau) - E\tau} + (\text{excited states})$$



Connected Quark (u, d)

Sequential source (thru sink)

- 3 sink momenta
- 1 spin channel
- Sources / cfg varies w/ t_f

Disconnected Quark $(u = d, s)$

- 1024 sources / cfg
- 4 spin channels
- Hierarchical probing w/ 512 Hadamard vectors
- 2 Z_4 noise shots / cfg

Glue (disconnected)

- 1024 sources / cfg
- 4 spin channels

Extracting bare matrix elements

Isolate matrix element by constructing ratios

$$R(p,p';\tau,t_f) = \frac{C^{3\mathrm{pt}}(p,p';t_f,\tau)}{C^{2\mathrm{pt}}(p';t_f)} \sqrt{\frac{C^{2\mathrm{pt}}(p;t_f-\tau)}{C^{2\mathrm{pt}}(p';t_f-\tau)} \frac{C^{2\mathrm{pt}}(p';\ t_f)}{C^{2\mathrm{pt}}(p;t_f)} \frac{C^{2\mathrm{pt}}(p';\ \tau)}{C^{2\mathrm{pt}}(p;\tau)}}$$

$$= \#\, \boxed{\langle p'|T^b(\Delta)|p\rangle} + O\left(e^{-\Delta E\,\tau - \Delta E'(t_f-\tau)}\right)$$

Extracting bare matrix elements

Isolate matrix element by constructing ratios

$$R(p,p';\tau,t_f) = \frac{C^{3\text{pt}}(p,p';t_f,\tau)}{C^{2\text{pt}}(p';t_f)} \sqrt{\frac{C^{2\text{pt}}(p;t_f-\tau)}{C^{2\text{pt}}(p';t_f-\tau)} \frac{C^{2\text{pt}}(p';\,t_f)}{C^{2\text{pt}}(p;t_f)} \frac{C^{2\text{pt}}(p';\,\tau)}{C^{2\text{pt}}(p;\tau)}}$$

$$= \# \langle p'|T^b(\Delta)|p\rangle + O\left(e^{-\Delta E\,\tau - \Delta E'(t_f-\tau)}\right)$$

Noisy → use all data available → too much data

→ Bin all ratios with same kinematic coefficients

$$R \sim k_A\, A(t) + k_J\, J(t) + k_D\, D(t)$$

where k depend on momenta, spin channel, irrep basis element, …

Channel	Multiplicities		
Conn. u	6982		3081
Conn. d	6982		3081
Disco. $u = d$	1200296	Bin →	11452
Disco s	1200296		11452
Glue	1200296		11452
TOTAL	**3614852**		**40518**

Extracting bare matrix elements

Have ~ 40k ratios

$$R_{\text{binned}}(\tau, t_f) \sim \# \boxed{\langle p' | T^b(\Delta) | p \rangle} + O\left(e^{-\Delta E\, \tau - \Delta E' (t_f - \tau)}\right)$$

Need to analyze each to extract matrix element → Large-scale automated analysis

Extracting bare matrix elements

Have ~ 40k ratios

$$R_{\text{binned}}(\tau, t_f) \sim \# \boxed{\langle p' | T^b(\Delta) | p \rangle} + O\left(e^{-\Delta E\,\tau - \Delta E'(t_f - \tau)}\right)$$

Need to analyze each to extract matrix element → Large-scale automated analysis

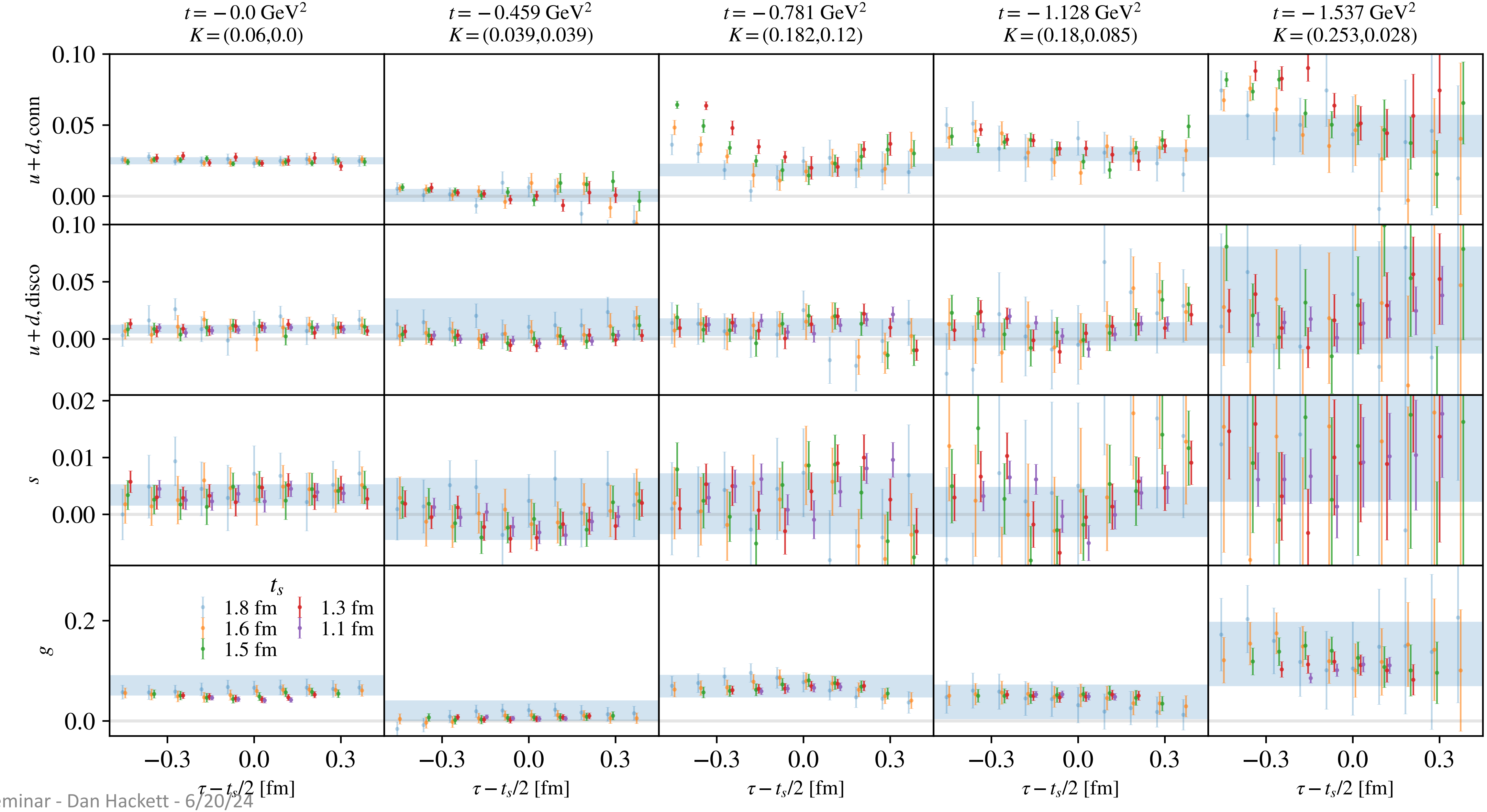
Approach: use "summation method"

$$\Sigma(t_f) = \sum_{\tau=\tau_{\text{cut}}}^{t_f - \tau_{\text{cut}}} R(\tau, t_f) = (\text{const}) + \# \langle p' | T^b(\Delta) | p \rangle\, t_f + O\left(e^{-\delta E\, t_f}\right)$$

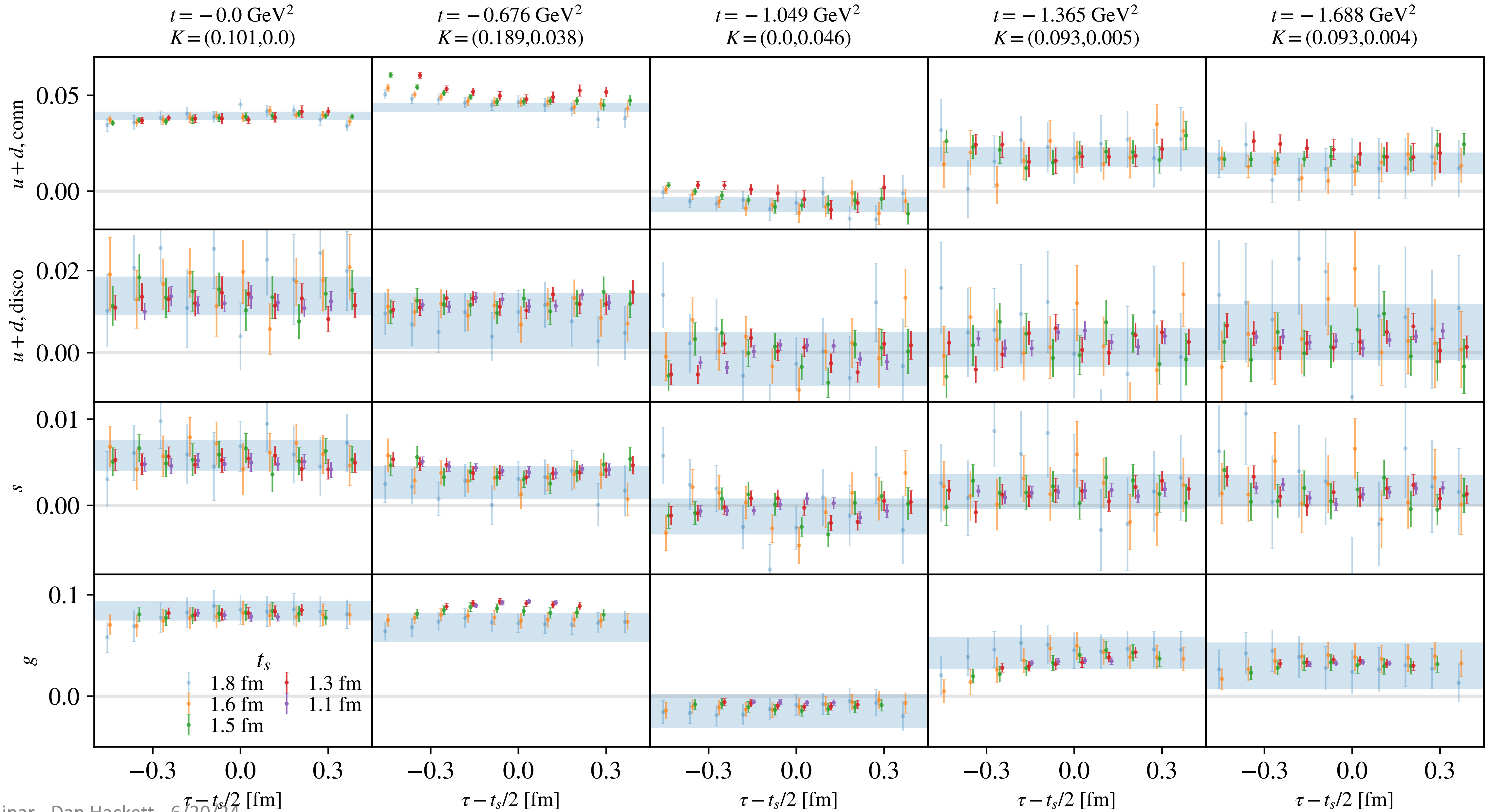
… w/ Bayesian model averaging over fit ranges, τ_{cut} [Jay Neil 2008.01069]

Vary analysis hyperparameters to ensure stability in final results

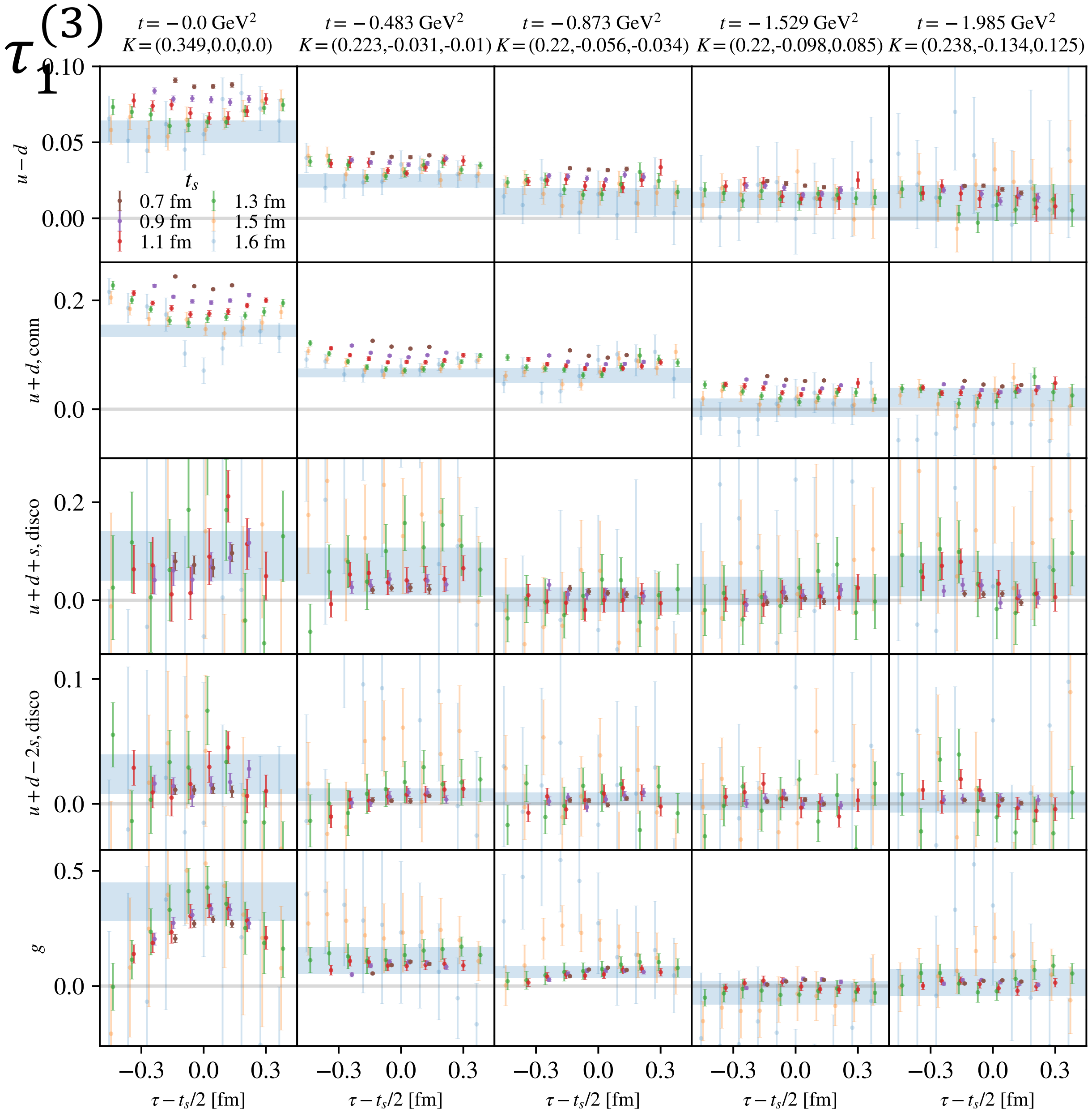
Example pion ratios: $\tau_1^{(3)}$

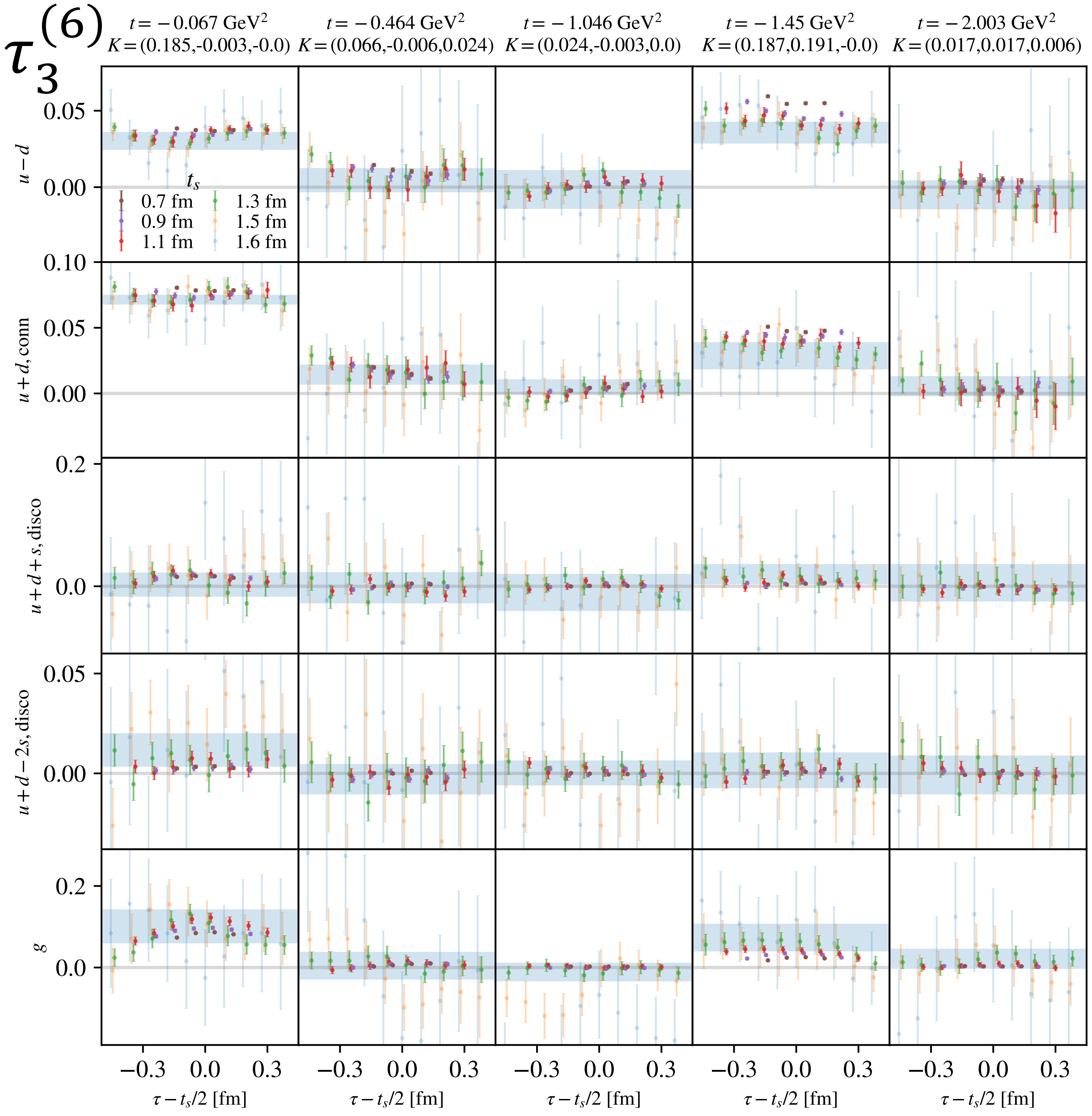


Example pion ratios: $\tau_3^{(6)}$



Example nucleon ratios





Renormalization

Assert RI-MOM conditions at scale $\mu^2 = p^2$

$$\langle q(p)\, T_f(0)\, \bar{q}(p) \rangle_{\text{lattice}} = Z_q\, R^{\text{RI}}_{fq}\, \langle q(p)\, T_f(0)\, \bar{q}(p) \rangle_{\text{tree}}$$

$$\langle A(p)\, T_f(0)\, A(p) \rangle_{\text{lattice}} = Z_g\, R^{\text{RI}}_{fg}\, \langle A(p)\, T_f(0)\, A(p) \rangle_{\text{tree}}$$

…in Landau gauge

…flow T_g to $t/a^2 = 1.2$ to match glue operator in bare matrix elements

Renormalization

Assert RI-MOM conditions at scale $\mu^2 = p^2$

$$\langle q(p)\, T_f(0)\, \bar{q}(p) \rangle_{\text{lattice}} = Z_q\, R^{\text{RI}}_{fq}\, \langle q(p)\, T_f(0)\, \bar{q}(p) \rangle_{\text{tree}}$$

$$\langle A(p)\, T_f(0)\, A(p) \rangle_{\text{lattice}} = Z_g\, R^{\text{RI}}_{fg}\, \langle A(p)\, T_f(0)\, A(p) \rangle_{\text{tree}}$$

...in Landau gauge

...flow T_g to $t/a^2 = 1.2$ to match glue operator in bare matrix elements

Apply perturbative matching to $\overline{\text{MS}}$ and run to $\mu = 2$ GeV

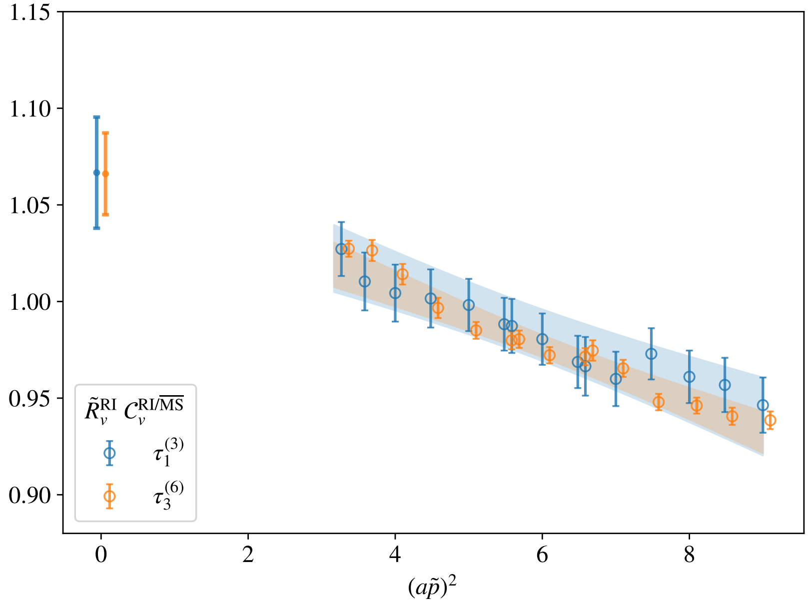
$$\left(Z_v^{\overline{MS}}\right)^{-1}(\mu^2) = C_v^{\text{RI}/\overline{MS}}(\mu^2, \mu_R^2)\, R_v^{\text{RI}}(\mu_R^2)$$

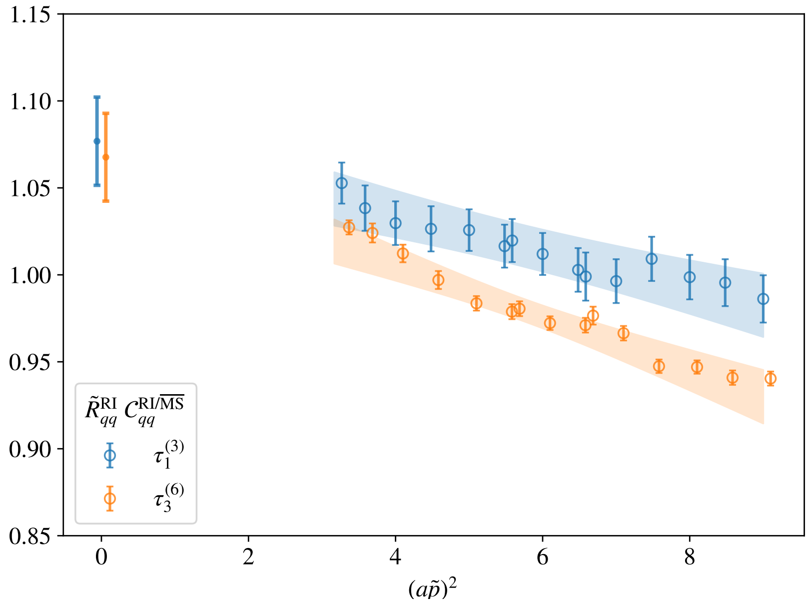
$$\begin{bmatrix} Z_{qq}^{\overline{MS}} & Z_{qg}^{\overline{MS}} \\ Z_{gq}^{\overline{MS}} & Z_{gg}^{\overline{MS}} \end{bmatrix}^{-1}(\mu^2) = \begin{bmatrix} R_{qq}^{\text{RI}} & R_{qg}^{\text{RI}} \\ R_{gq}^{\text{RI}} & R_{gg}^{\text{RI}} \end{bmatrix}(\mu_R^2) \begin{bmatrix} C_{qq}^{\text{RI}/\overline{MS}} & C_{qg}^{\text{RI}/\overline{MS}} \\ C_{gq}^{\text{RI}/\overline{MS}} & C_{gg}^{\text{RI}/\overline{MS}} \end{bmatrix}(\mu^2, \mu_R^2)$$

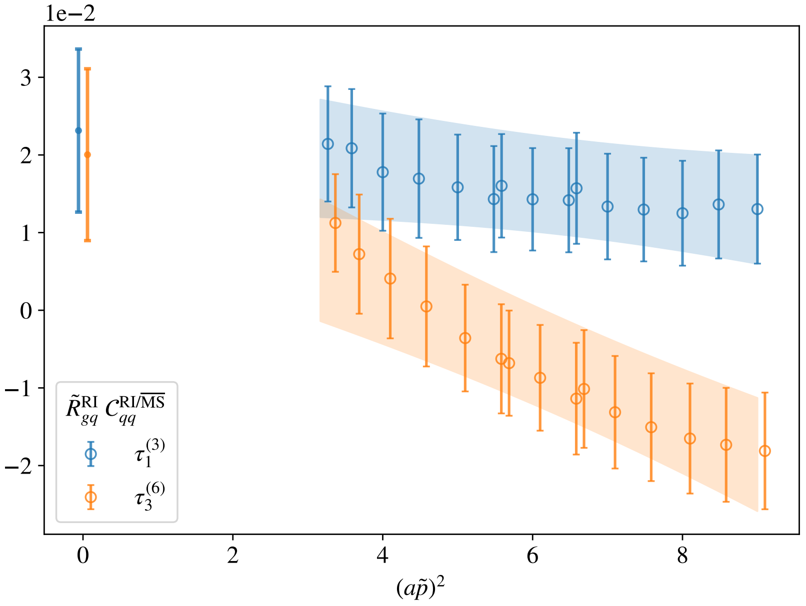
Model and fit residual $(ap)^2$ dependence in each of product $R^{\text{RI}}\, C^{\text{RI}/\overline{MS}}$

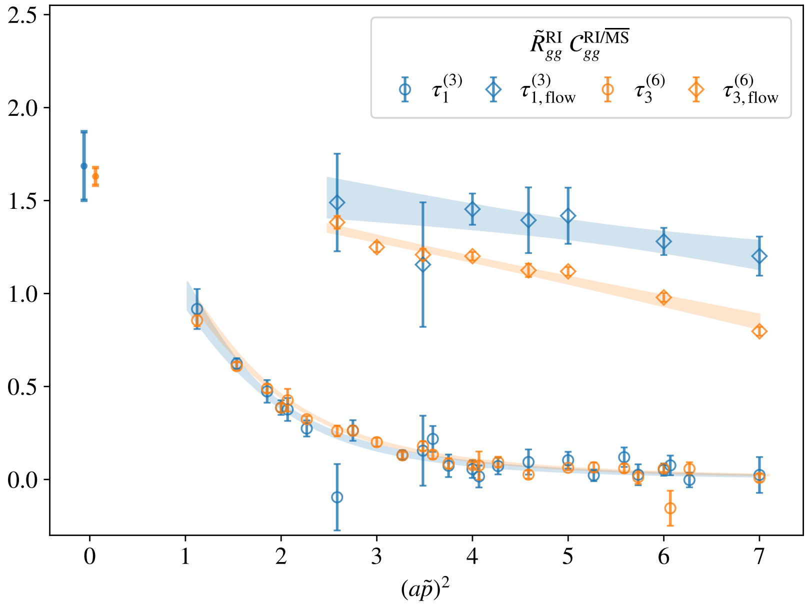
Renormalization: removing discretization artifacts

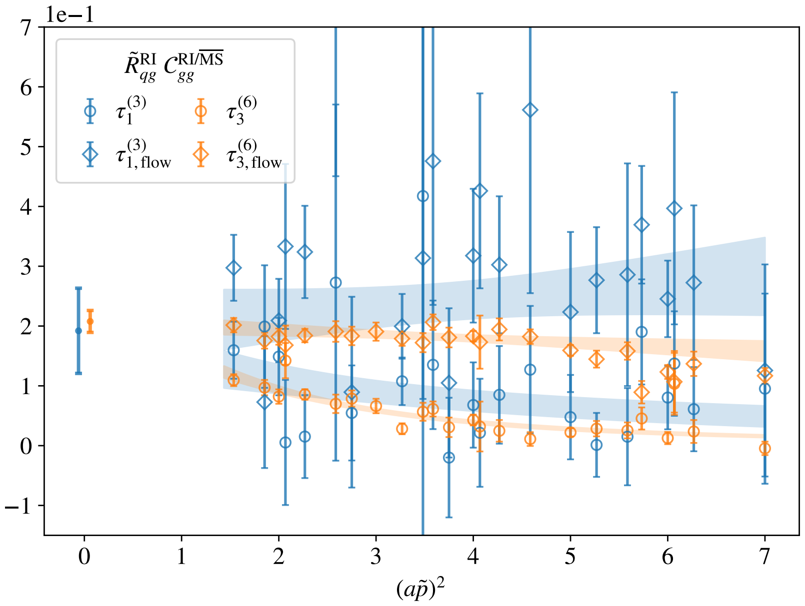
Model discretization artifacts as polynomials, inverse polynomials







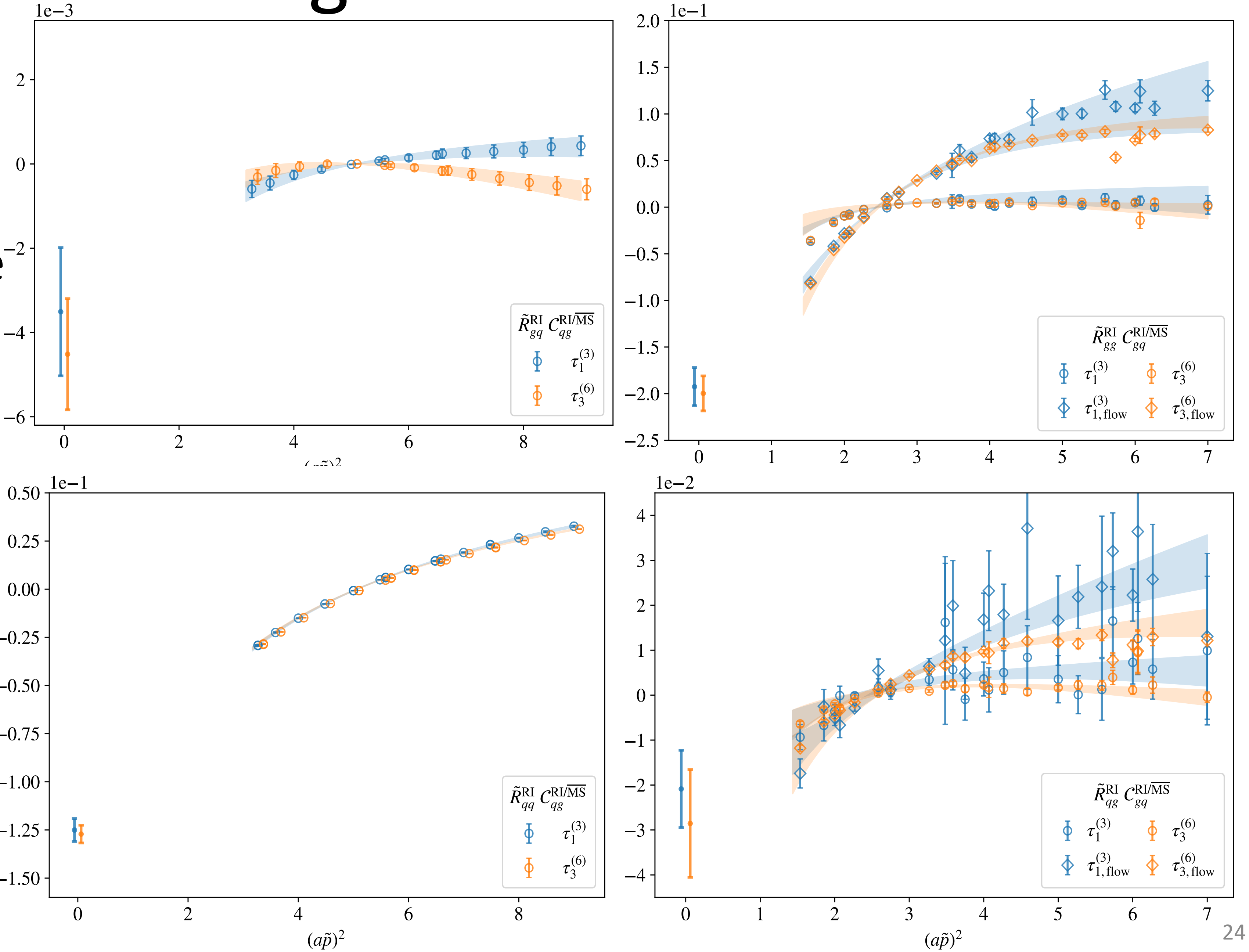




Renormalization: removing discretization artifacts

Model discretization artifacts as polynomials, inverse polynomials

+ logs for nonperturbative effects



Overview of calculation

Need to compute:

Bare matrix elements for $f \in \{g, u, d, s\}$ to constrain bare GFFs

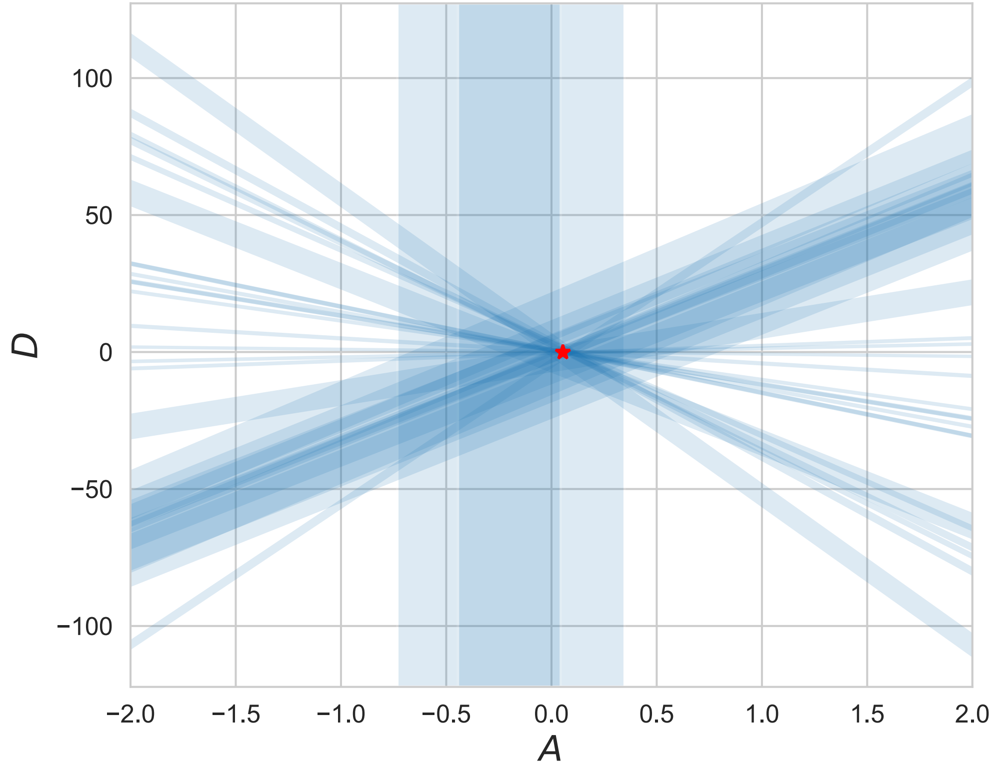
$$\langle p' | T_f^{\mathrm{b}}(\Delta) | p \rangle = c_A\, A_f^b(t) + c_J\, J_f^{\mathrm{b}}(t) + c_D\, D_f^{\mathrm{b}}(t)$$

Isosinglet mixing matrix (+ non-singlet Z_{u+d-2s})

$$\begin{bmatrix} T_q^{\overline{MS}} \\ T_g^{\overline{MS}} \end{bmatrix} = \begin{bmatrix} Z_{qq}^{\overline{MS}} & Z_{qg}^{\overline{MS}} \\ Z_{gq}^{\overline{MS}} & Z_{gg}^{\overline{MS}} \end{bmatrix} \begin{bmatrix} T_q^{\mathrm{bare}} \\ T_g^{\mathrm{bare}} \end{bmatrix}$$

→ Renormalized linear constraints on GFFs at different values of $t = \Delta^2 = (p' - p)^2$

Fit to extract GFFs(t)



Results

Pion GFFs (flavor decomp)

$$G(t) \sim \sum_{k=0}^{k_{\max}=2} \alpha_k [z(t)]^k$$

Hatched bands: monopole

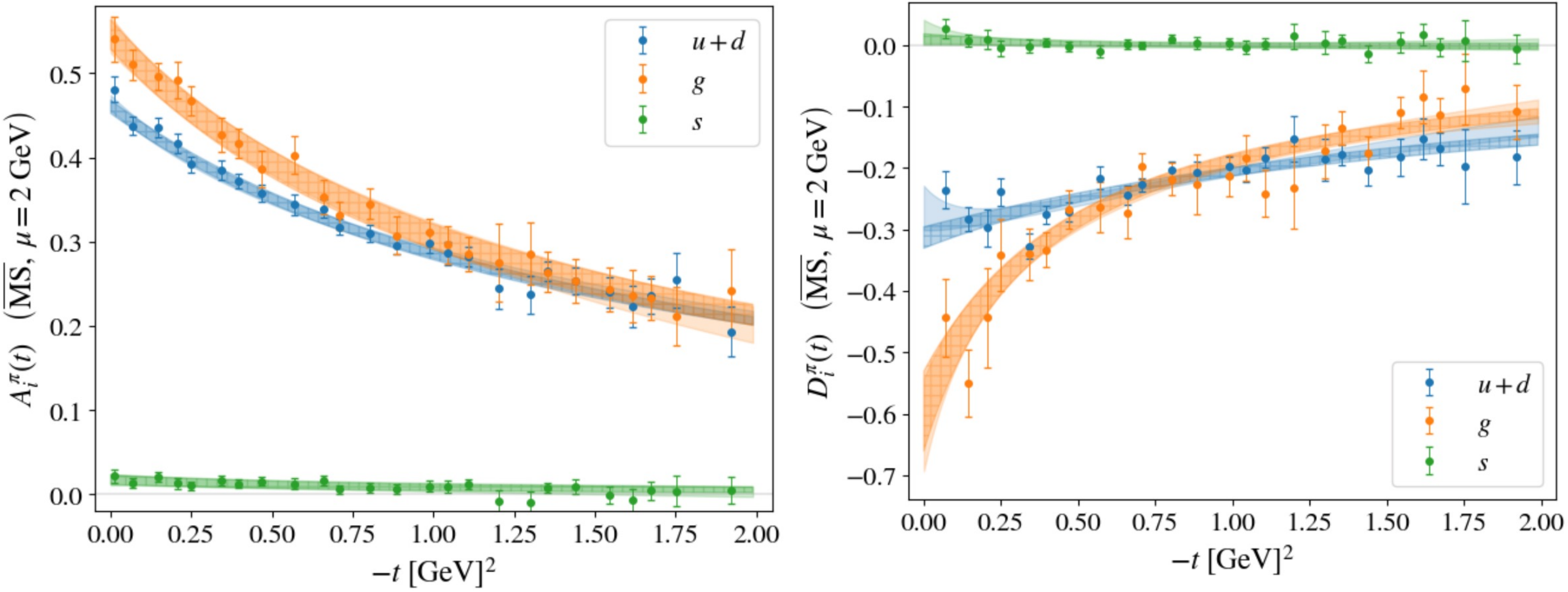
$$G(t) \sim \frac{\alpha}{(1-t/\Lambda^2)}$$

Solid bands: z-expansion

$$z(t) = \frac{\sqrt{t_{\mathrm{cut}}-t} - \sqrt{t_{\mathrm{cut}}-t_0}}{\sqrt{t_{\mathrm{cut}}-t} + \sqrt{t_{\mathrm{cut}}-t_0}}$$

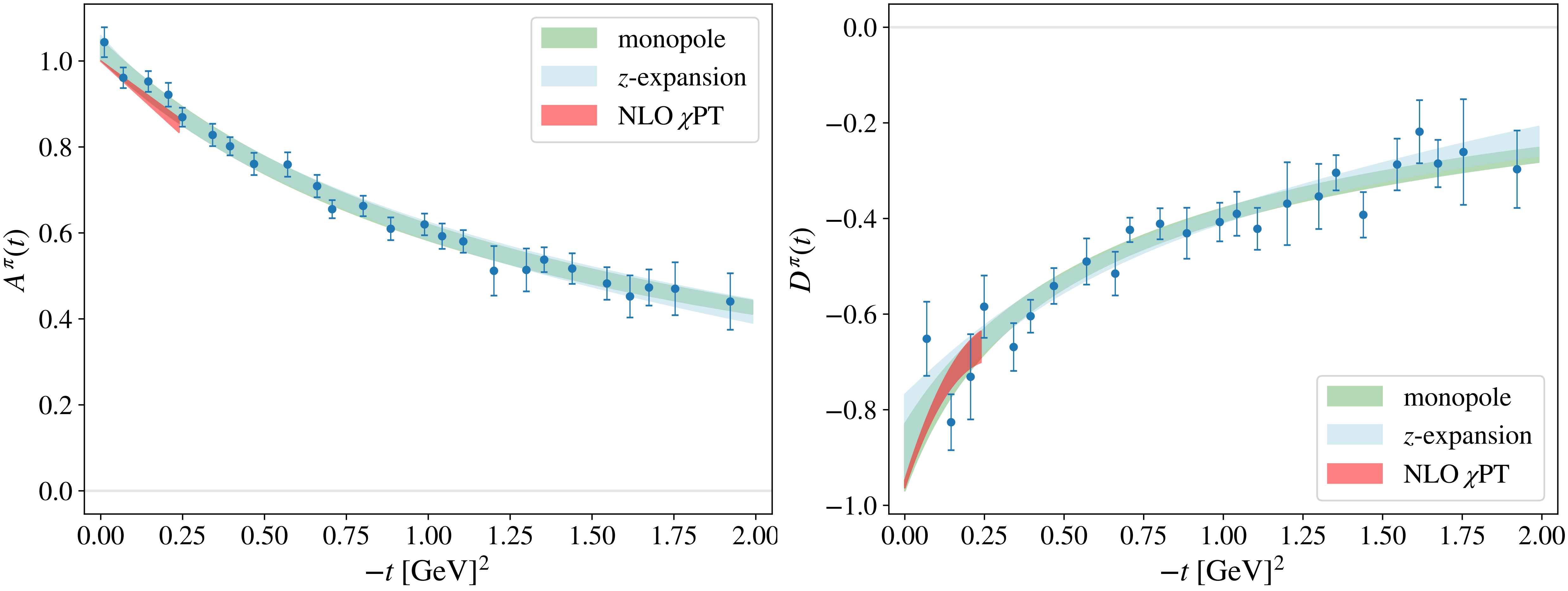
$$t_{\mathrm{cut}} = 4M_\pi^2$$

$$t_0 = t_{\mathrm{cut}}\left(1 - \sqrt{1 + (2\ \mathrm{GeV}^2)/t_{\mathrm{cut}}}\right)$$



Pion GFFs (total)

Error on χPT estimate due to different estimates for LECs [Donaghue Leutwyler 1991]



Nucleon GFFs

Dark bands: dipole

$$G(t) \sim \frac{\alpha}{(1-t/\Lambda^2)^2}$$

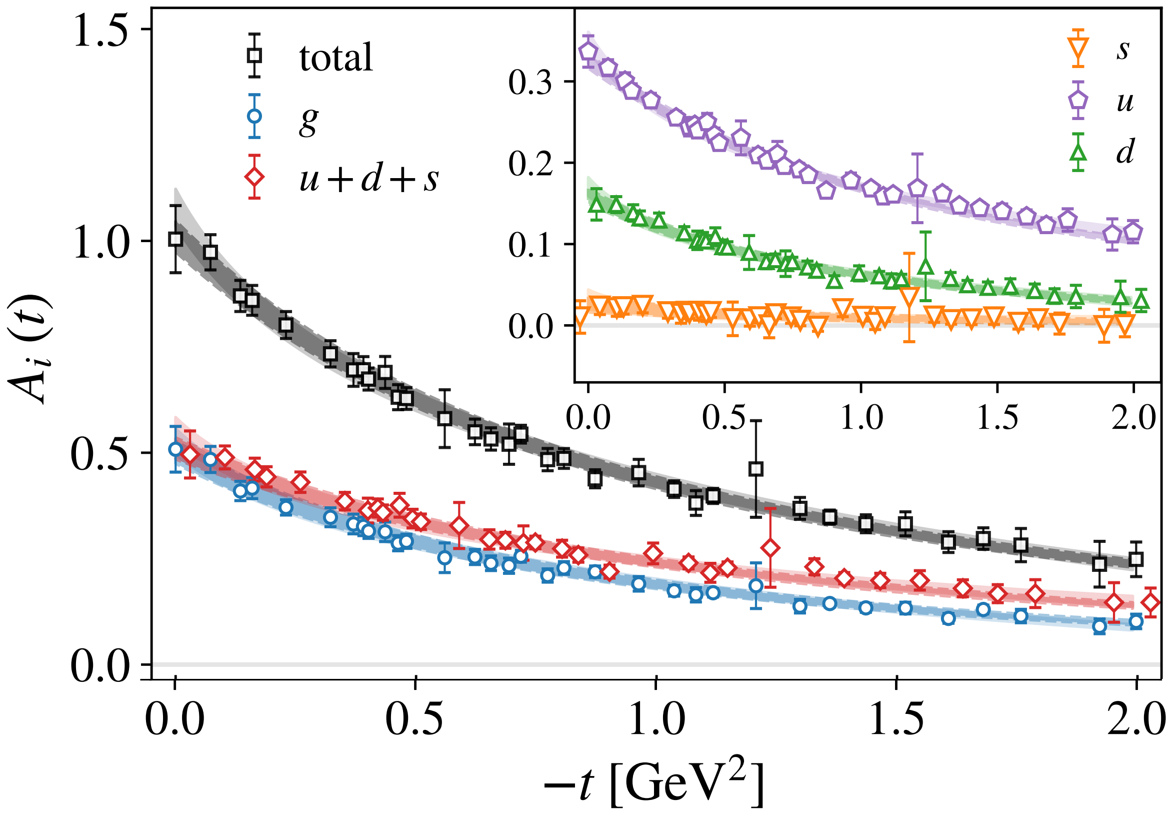
Light bands: z-expansion

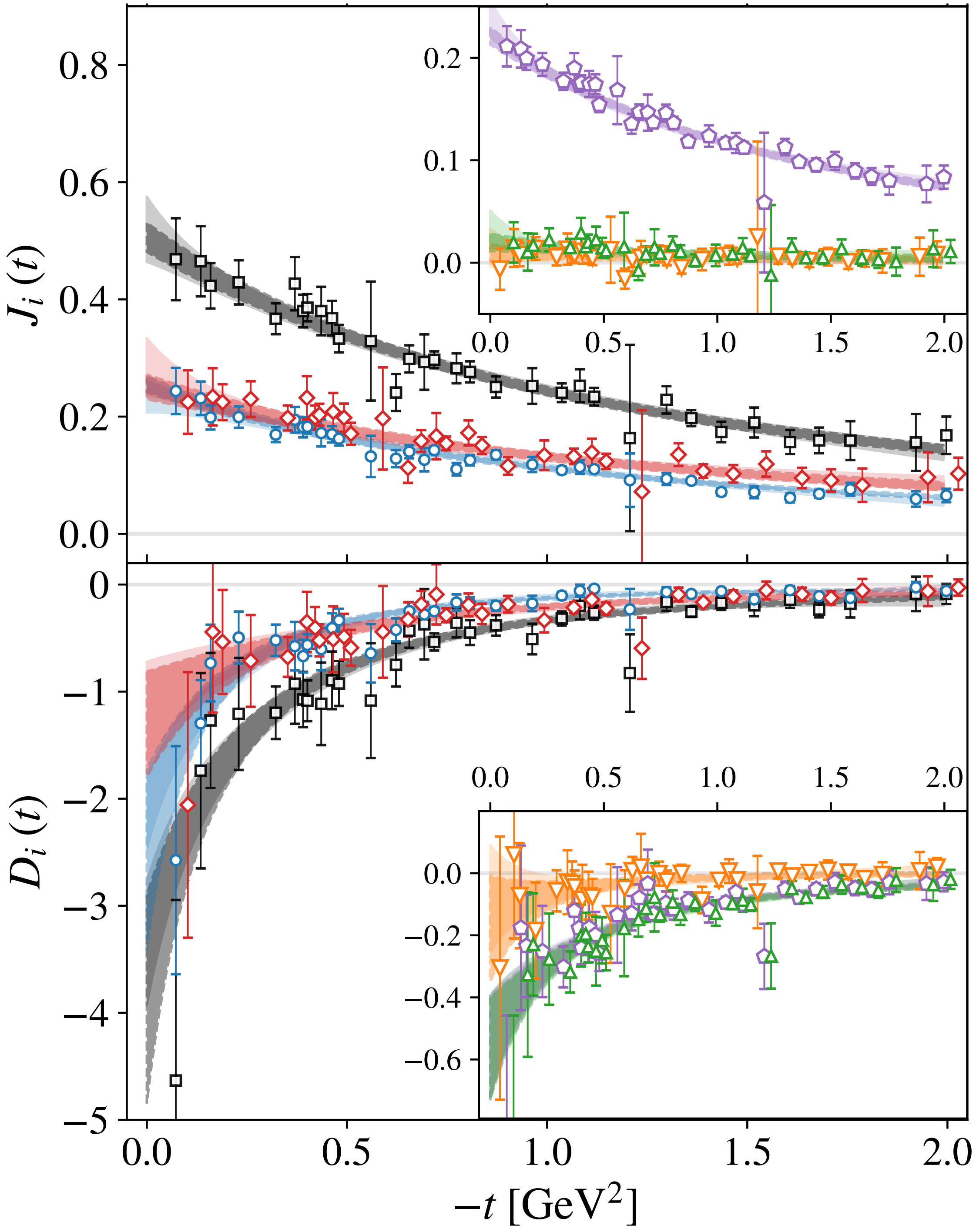
$$G(t) \sim \sum_{k=0}^{k_{\max}=2} \alpha_k [z(t)]^k$$

$$z(t) = \frac{\sqrt{t_{\text{cut}}-t} - \sqrt{t_{\text{cut}}-t_0}}{\sqrt{t_{\text{cut}}-t} + \sqrt{t_{\text{cut}}-t_0}}$$

$$t_{\text{cut}} = 4M_\pi^2$$

$$t_0 = t_{\text{cut}}\left(1 - \sqrt{1 + (2\ \text{GeV}^2)/t_{\text{cut}}}\right)$$





Forward limits

Nucleon

	Dipole			z-expansion		
	A_i	J_i	D_i	A_i	J_i	D_i
u	0.3255(92)	0.2213(85)	−0.56(17)	0.349(11)	0.238(18)	−0.56(17)
d	0.1590(92)	0.0197(85)	−0.57(17)	0.171(11)	0.033(18)	−0.56(17)
s	0.0257(95)	0.0097(82)	−0.18(17)	0.032(12)	0.014(19)	−0.08(17)
$u+d+s$	0.510(25)	0.251(21)	−1.30(49)	0.552(31)	0.286(48)	−1.20(48)
g	0.501(27)	0.255(13)	−2.57(84)	0.526(31)	0.234(27)	−2.15(32)
Total	1.011(37)	0.506(25)	−3.87(97)	1.079(44)	0.520(55)	−3.35(58)

Pion	monopole	z-expansion
$A_g^\pi(0)$	0.546(18)	0.546(22)
$A_q^\pi(0)$	0.481(15)	0.485(18)
$A_{u+d}^\pi(0)$	0.463(11)	0.468(12)
$A_s^\pi(0)$	0.0176(57)	0.0174(66)
$A^\pi(0)$	1.026(23)	1.031(28)
$D_g^\pi(0)$	−0.596(65)	−0.618(75)
$D_q^\pi(0)$	−0.304(26)	−0.242(53)
$D_{u+d}^\pi(0)$	−0.313(17)	−0.265(36)
$D_s^\pi(0)$	0.0092(94)	0.023(19)
$D^\pi(0)$	−0.900(70)	−0.860(92)

Forward limits

Nucleon	Dipole			z-expansion		
	A_i	J_i	D_i	A_i	J_i	D_i
u	0.3255(92)	0.2213(85)	−0.56(17)	0.349(11)	0.238(18)	−0.56(17)
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Sum rules (consistency check)

Forward limits

Nucleon

	Dipole A_i	Dipole J_i	Dipole D_i	z-expansion A_i	z-expansion J_i	z-expansion D_i
u	0.3255(92)	0.2213(85)	−0.56(17)	0.349(11)	0.238(18)	−0.56(17)
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Pion

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$A^\pi_g(0)$	0.546(18)	0.546(22)
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cf. global fit result

$$A_g(0) = 0.414(8)$$

[Hou et al. 1912.10053]

Sum rules (consistency check)

Forward limits

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	A_i	J_i	D_i	A_i	J_i	D_i
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cf. global fit result

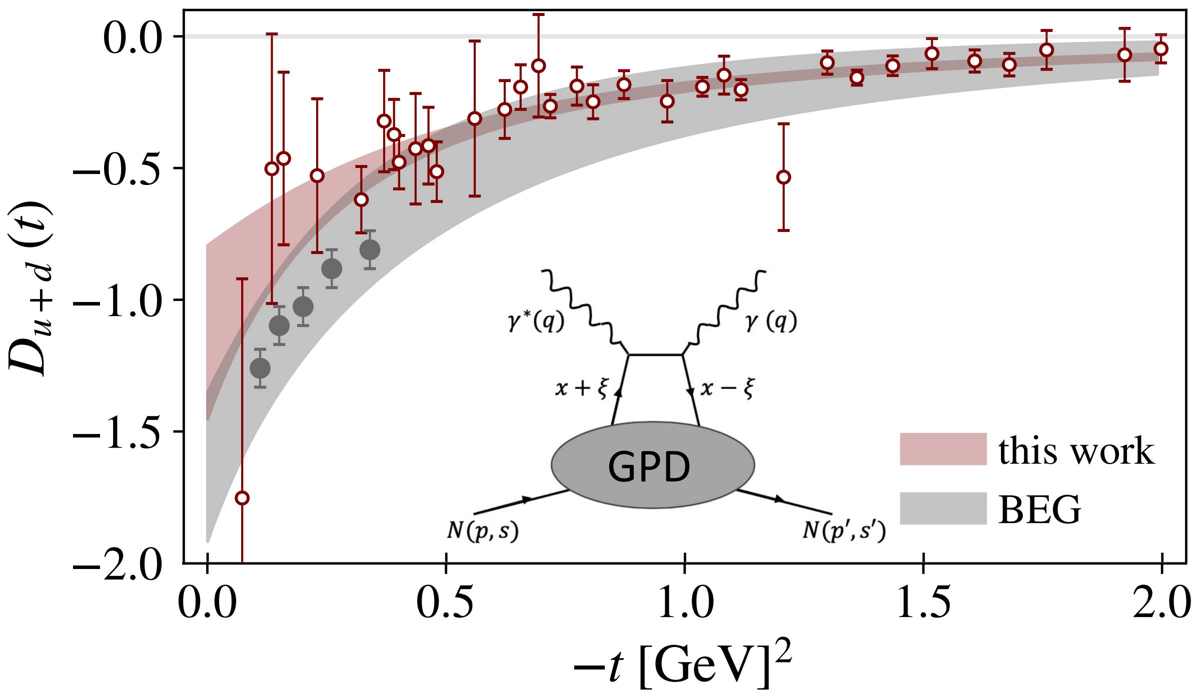
$$A_g(0) = 0.414(8)$$

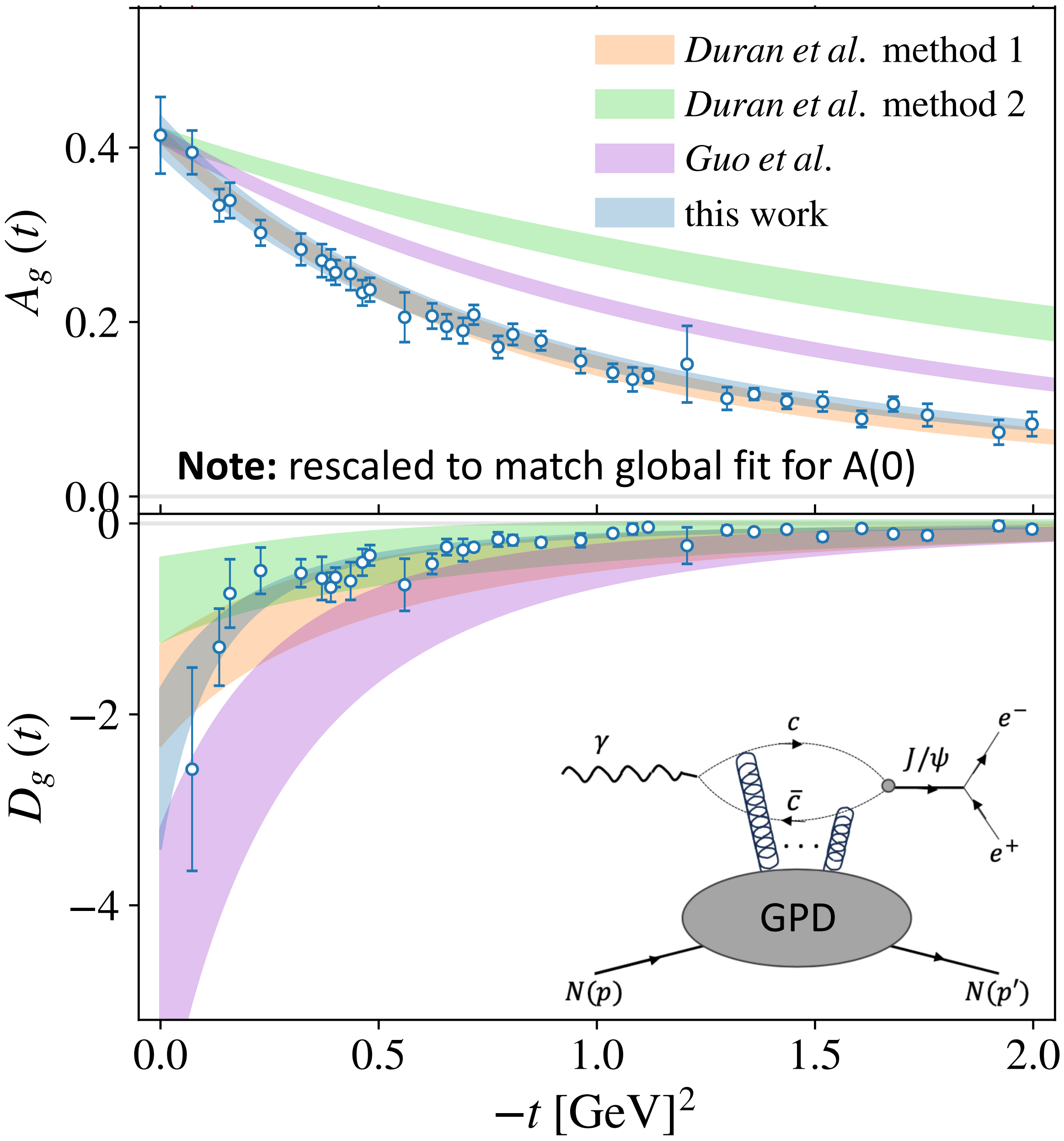
[Hou et al. 1912.10053]

First determination!
Satisfies χPT bound
$D(0)/M \leq -1.1(1)\ \mathrm{GeV}^{-1}$

Sum rules (consistency check)

Nucleon vs. experiment





BEG = [Burkert Elouadrhiri Girod 2018] (DVCS)

[Duran et al. 2207.05212] (J/ψ)

Method 1: holographic QCD (Mamo Jahed, PRD 21,22)

Method 2: GPD (Guo Ji Liu, PRD 2021)

[Guo et al. 2305.06992]

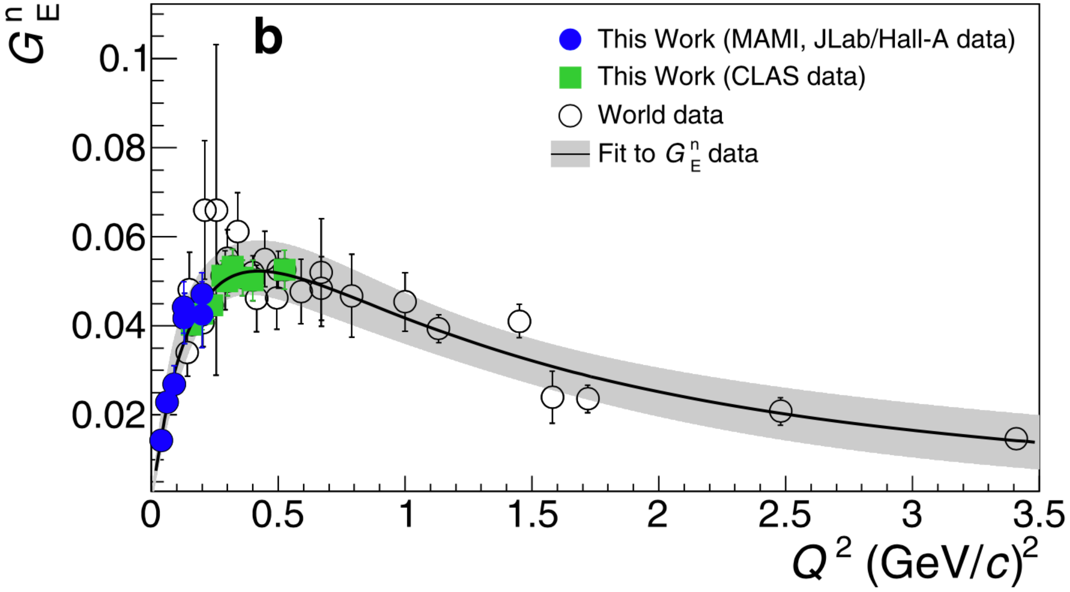
Updated GPD analysis + GlueX data

(G)FFs and Tomography

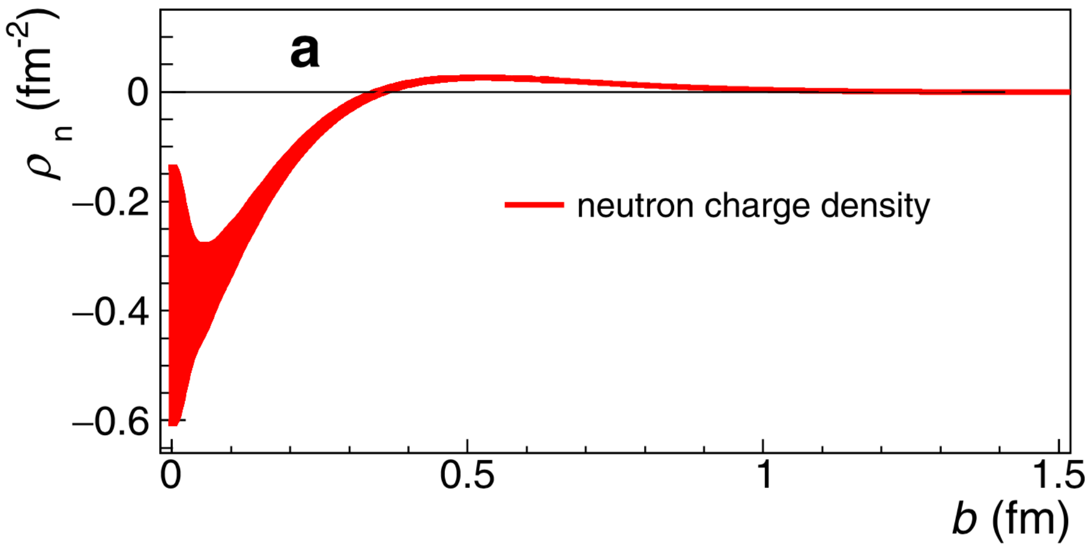
Fourier-transformed form factors provide information about spatial densities

Example: electric charge density in the neutron from G_E^n

[Atac, Constantinou, Meziani, Paolone, Sparveris 2103.10840]



Fourier transform →



Applies also for GFFs → mechanical densities

Mechanical densities from GFFs

1. Parametrize $T_{\mu\nu}(t)$ with GFFs
2. Fourier transform $T_{\mu\nu}(t) \rightarrow T_{\mu\nu}(r)$
3. Identify

$$[f(t)]_{\mathrm{FT}} = \int \frac{d^3\mathbf{\Delta}}{(2\pi)^3} e^{-i\mathbf{\Delta}\cdot\mathbf{r}} f(t)$$

$$T_{\mu\nu}(r) = \begin{bmatrix} T_{tt}(r) & \\ & T_{ij}(r) \end{bmatrix} = \begin{bmatrix} \epsilon(r) & \\ & \left(\frac{r_i r_j}{r^2} - \frac{1}{d}\delta_{ij}\right) s(r) + \delta_{ij} p(r) \end{bmatrix}$$

Mechanical densities from GFFs

1. Parametrize $T_{\mu\nu}(t)$ with GFFs
2. Fourier transform $T_{\mu\nu}(t) \to T_{\mu\nu}(r)$
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→ Spatial densities (Breit frame)

energy $\epsilon(r) = M\left[A(t) - \frac{t}{4M^2}\left(D(t) + A(t) - 2J(t)\right)\right]_{FT}$

pressure $p(r) = \frac{1}{6M}\frac{1}{r^2}\frac{d}{dr} r^2 \frac{d}{dr} [D(t)]_{FT}$

shear forces $s(r) = -\frac{1}{4M} r \frac{d}{dr}\frac{1}{r}\frac{d}{dr} [D(t)]_{FT}$

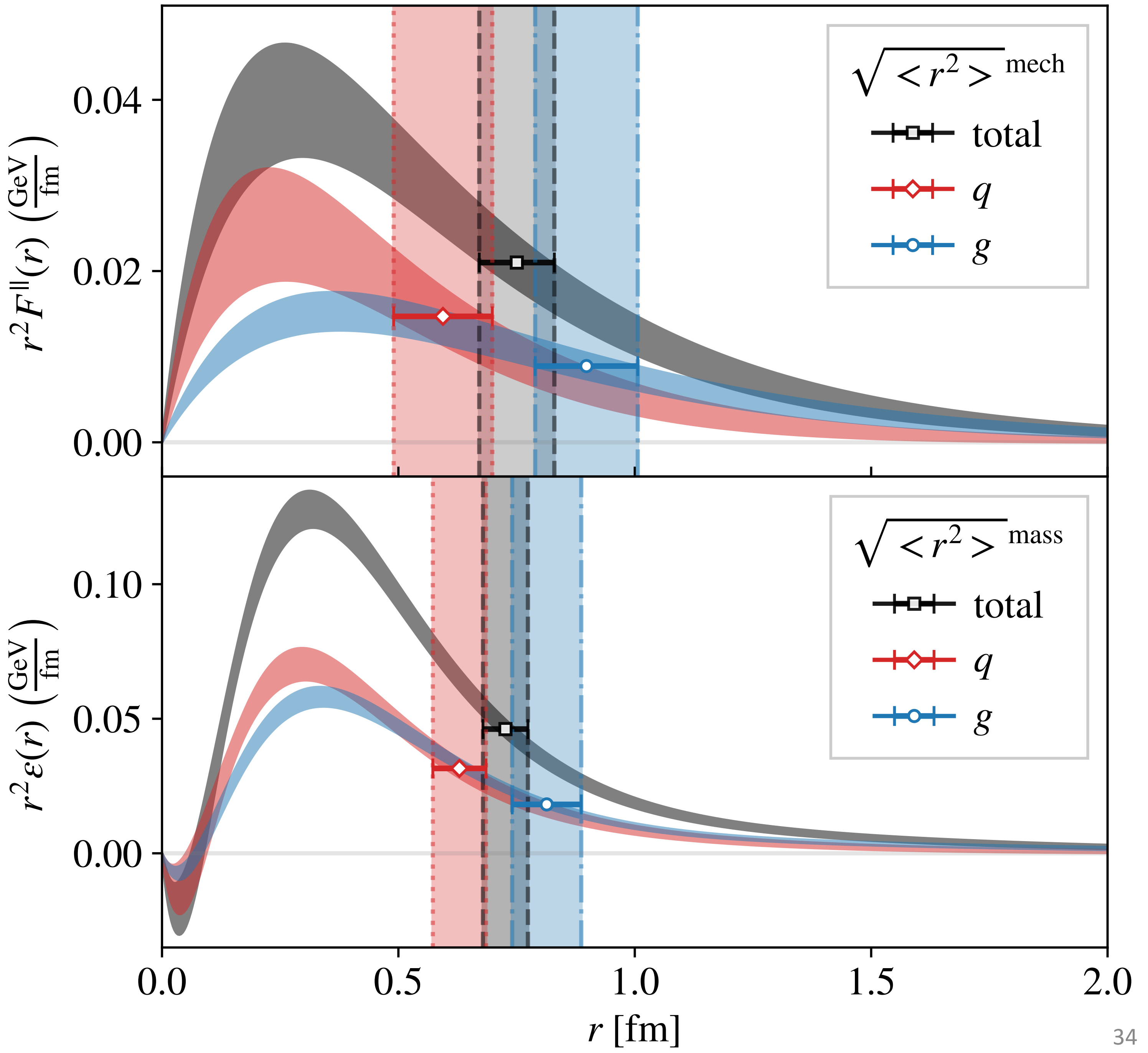
longitudinal force $F^{\parallel}(r) = p(r) + 2s(r)/3$

Caveat: physical significance of these analogies is under debate

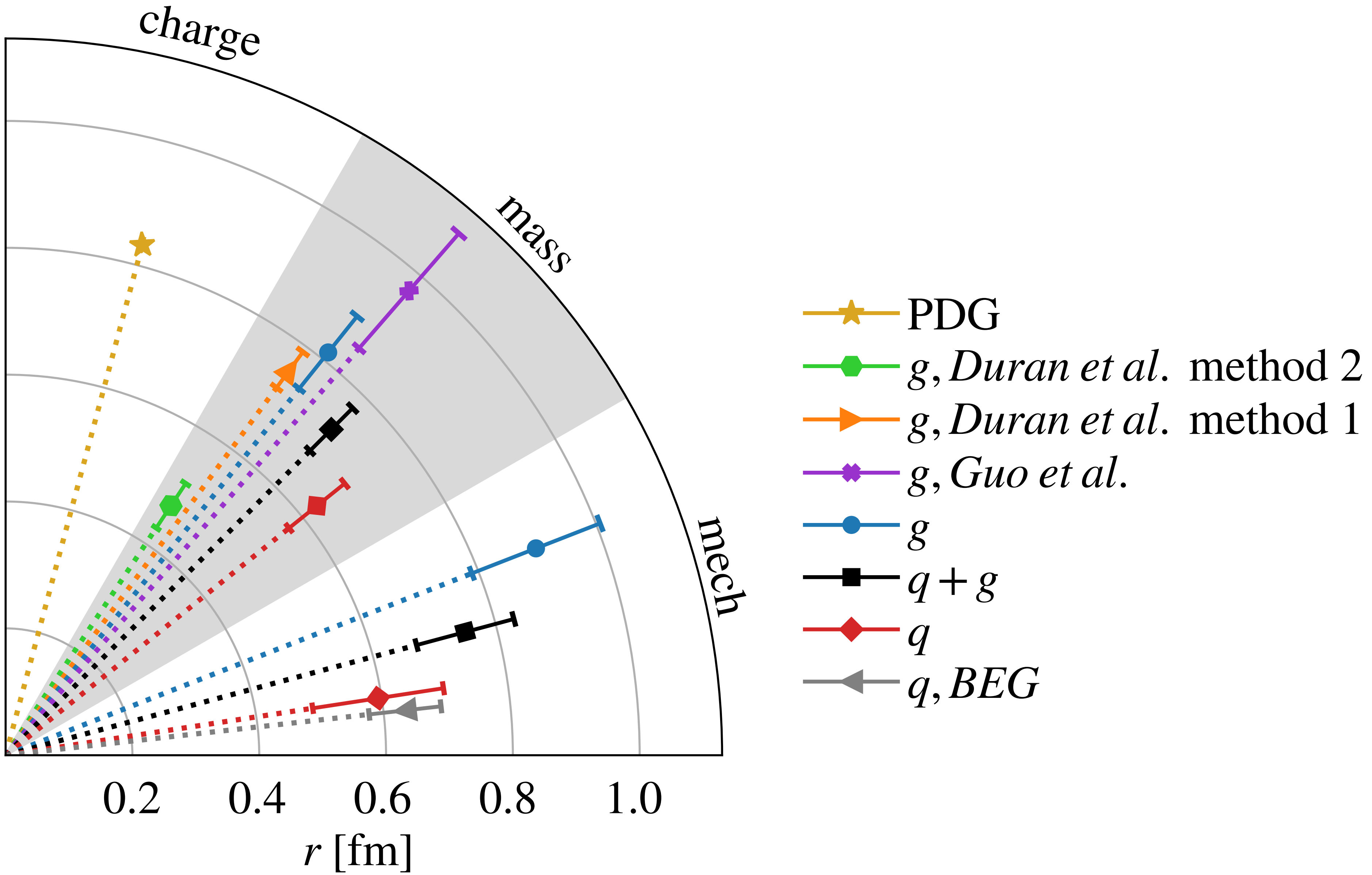
Densities & radii

$$\langle r_i^2 \rangle^{\text{mass}} = \frac{\int d^3\mathbf{r}\ r^2\ \epsilon_i(r)}{\int d^3\mathbf{r}\ \epsilon_i(r)}$$

$$\langle r_i^2 \rangle^{\text{mech}} = \frac{\int d^3\mathbf{r}\ r^2 F_i^{\parallel}(r)}{\int d^3\mathbf{r}\ F_i^{\parallel}(r)}$$



How big is a proton?



Glueball GFFs

Glueball GFFs: Overview

Idea: are "exotic" states glueballs? Can structure observables discriminate?

Approach:

Glueballs are noisy → need high stats

→ Study in $\mathrm{SU}(3)$ Yang-Mills (no quarks; cheaper)

→ Simplest glueball structure observable: GFFs

Calculation:

$\beta = 5.97$ on $24^3 \times 48$, $a \approx 0.1\ \mathrm{fm}$, $M_{0^{++}} \approx 1.6\ \mathrm{GeV}$

2×10^7 configs w/ heatbath, overrelaxation

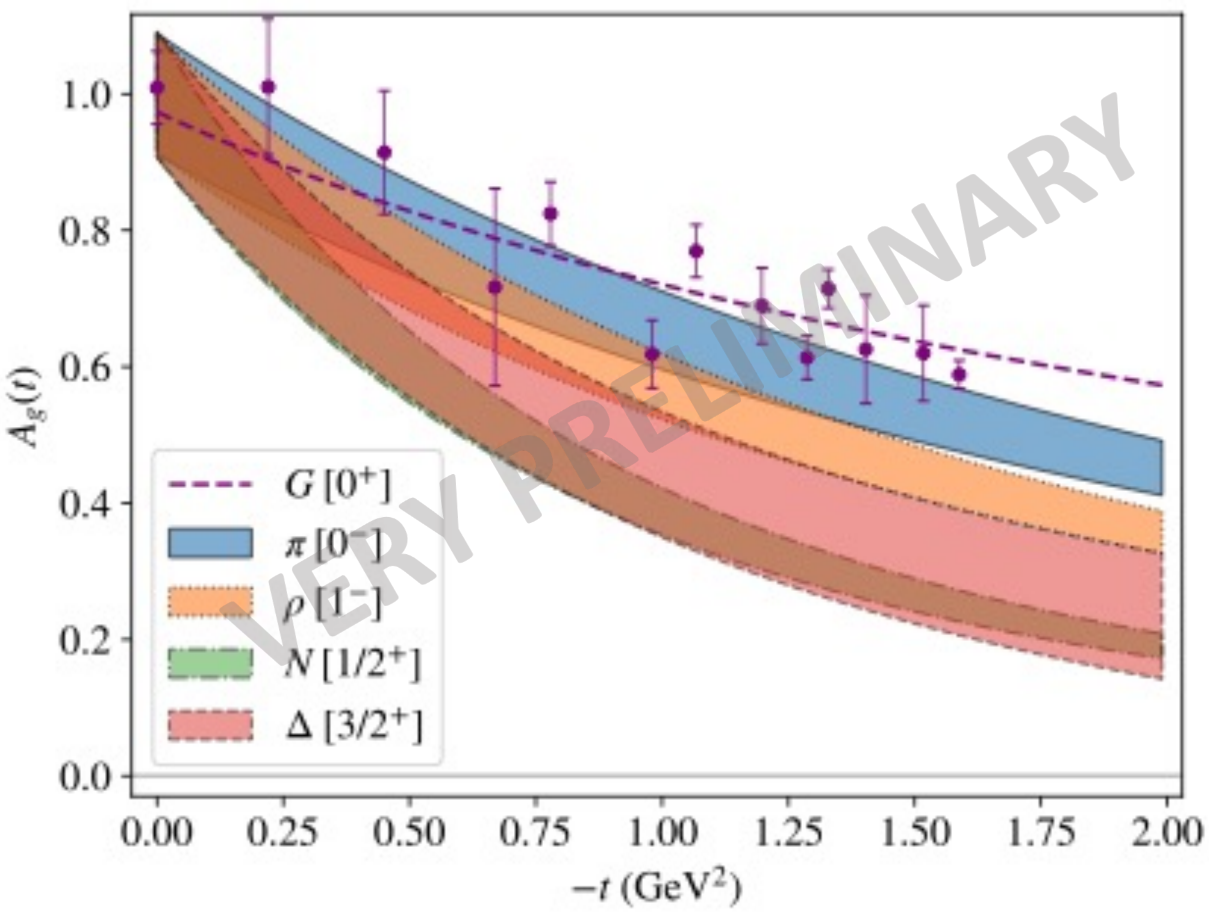
Variational method (GEVP) to control excited states

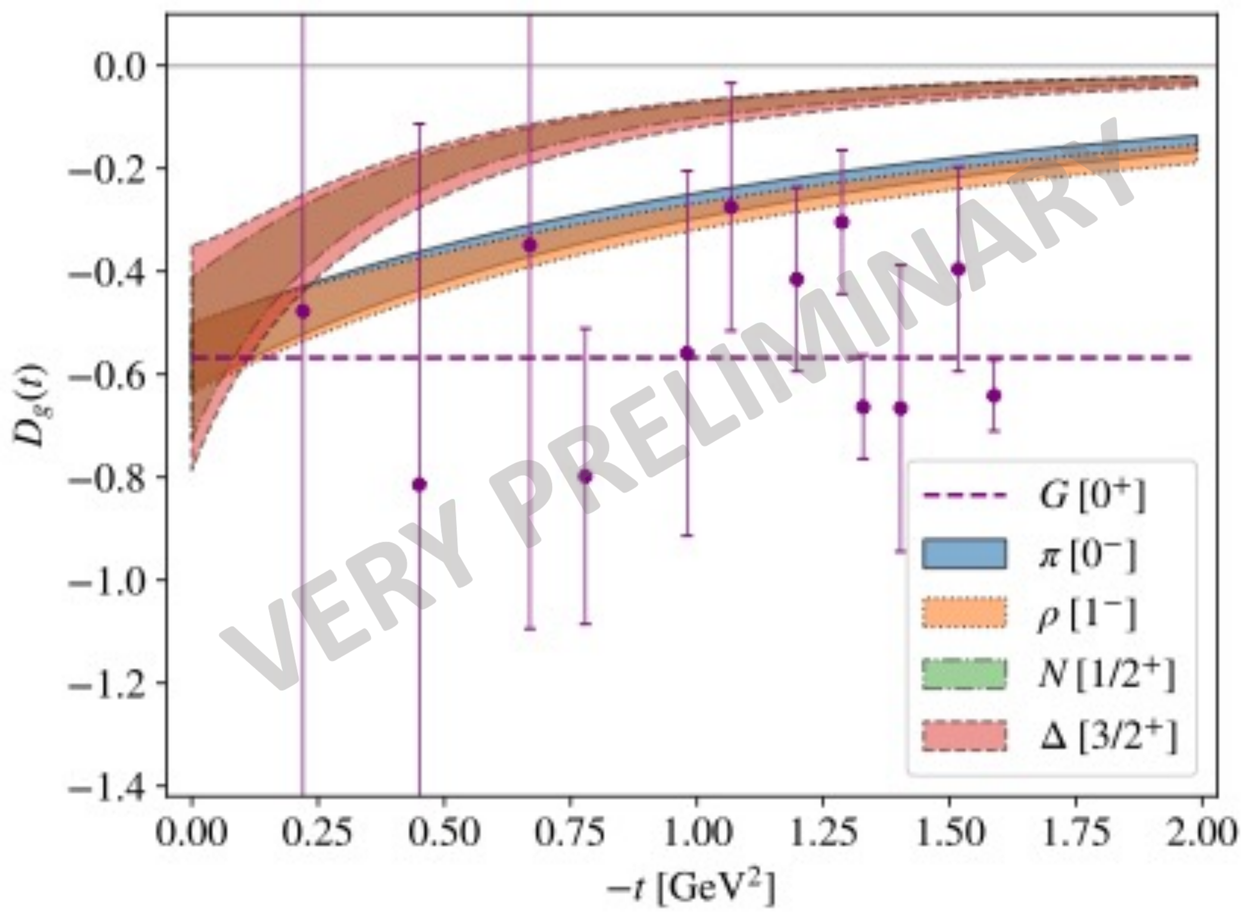
+ construct optimal interpolators for 3pts

Look at lightest glueball 0^{++} (for now)

Preliminary Results: Gluon GFFs

Other hadrons from [Pefkou DH Shanahan 2107.10368]: $a \approx 0.11$ fm, $M_\pi \approx 450$ MeV

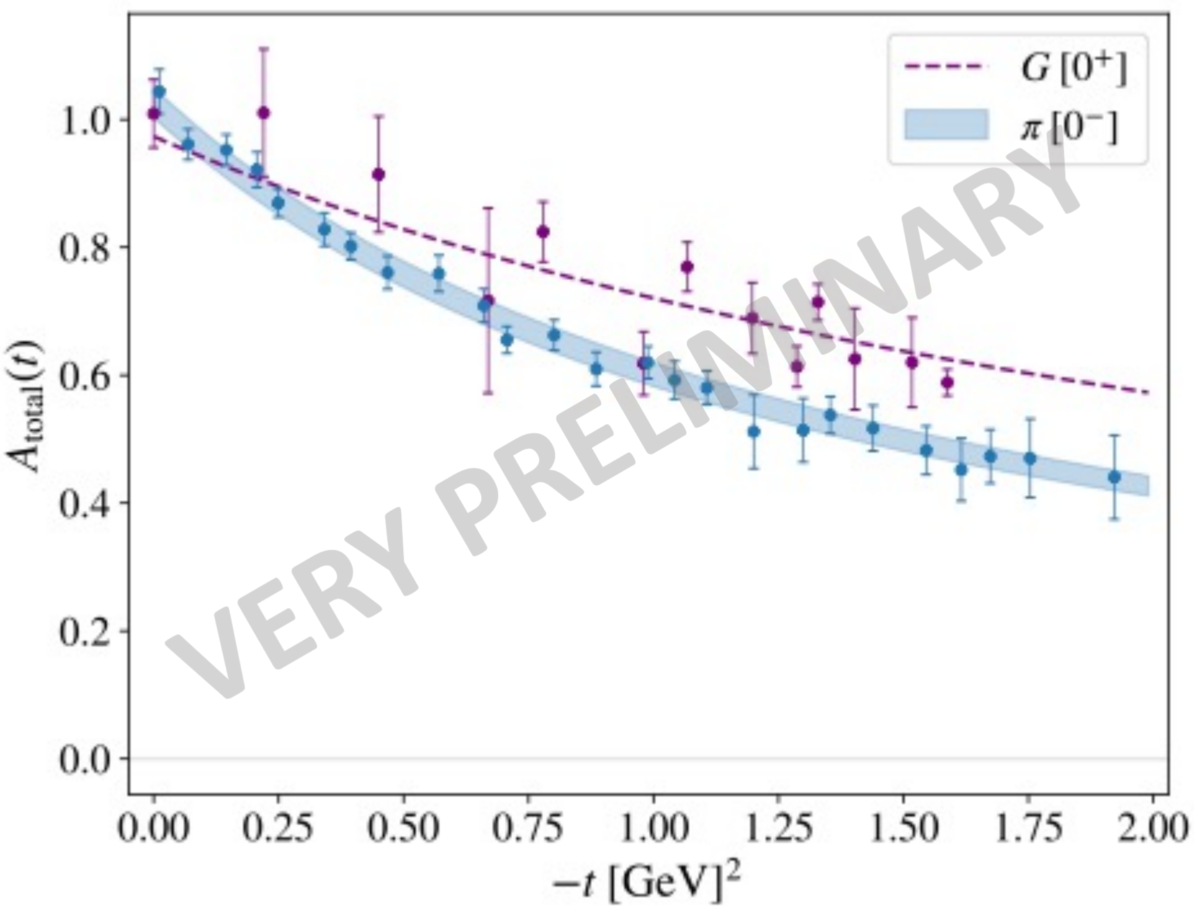


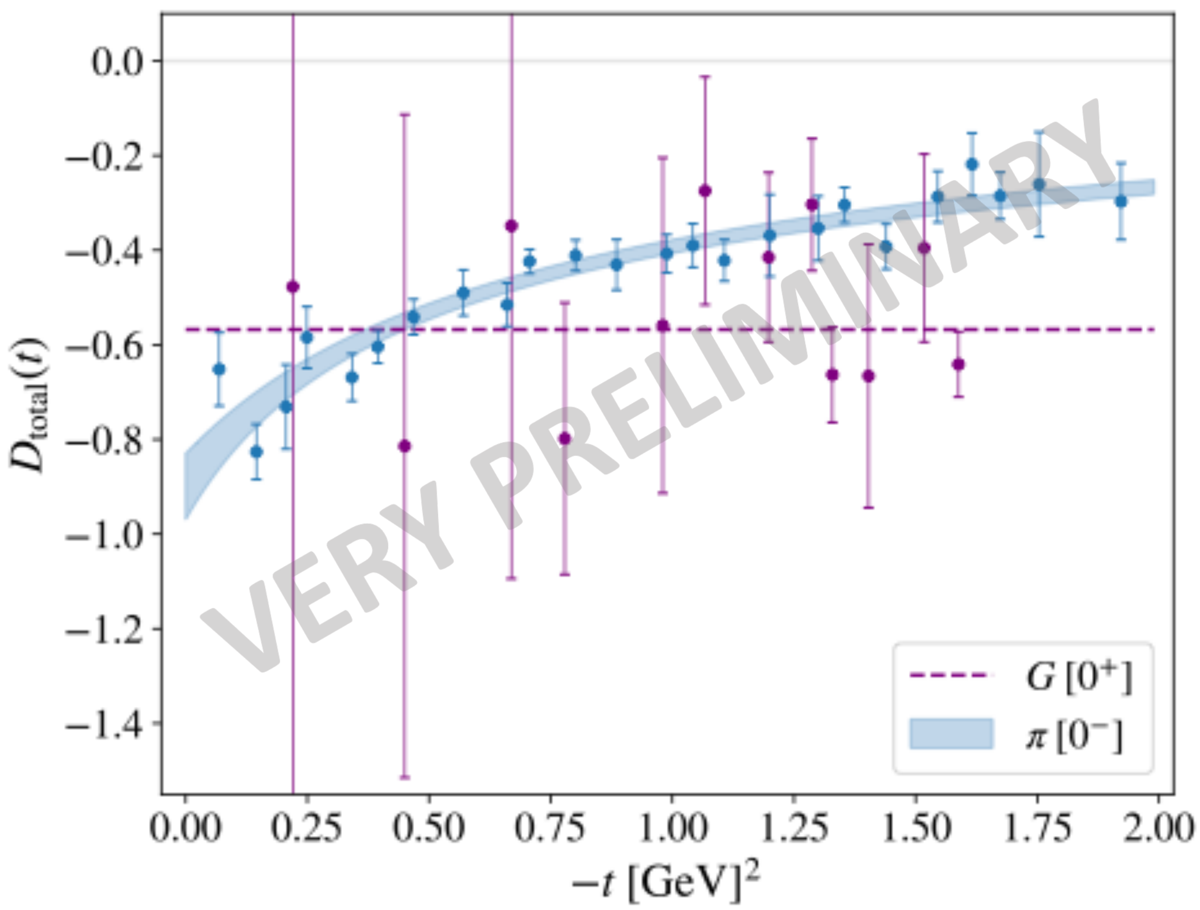


Preliminary Results: Total GFFs

Glueball: Gluon GFFs = total GFFs in Yang-Mills

Pion: total GFFs in QCD from earlier this talk





Conclusion

First lattice calculation of:

- complete flavor decomposition of nucleon GFFs
- *total* GFFs → *physical* (i.e. RGI) densities, radii
- $D(0)$

New first-principles descriptions of size and shape of nucleon

Results can help discriminate between different experimental extractions

Towards a precision calculation, need:

- Multiple ensembles to take continuum, physical-mass limits
- Improved renormalization (GIRS? Flow? Sum rules?)
- Better methods to control excited state effects

Glueballs WIP