Hadronic resonances from lattice QCD

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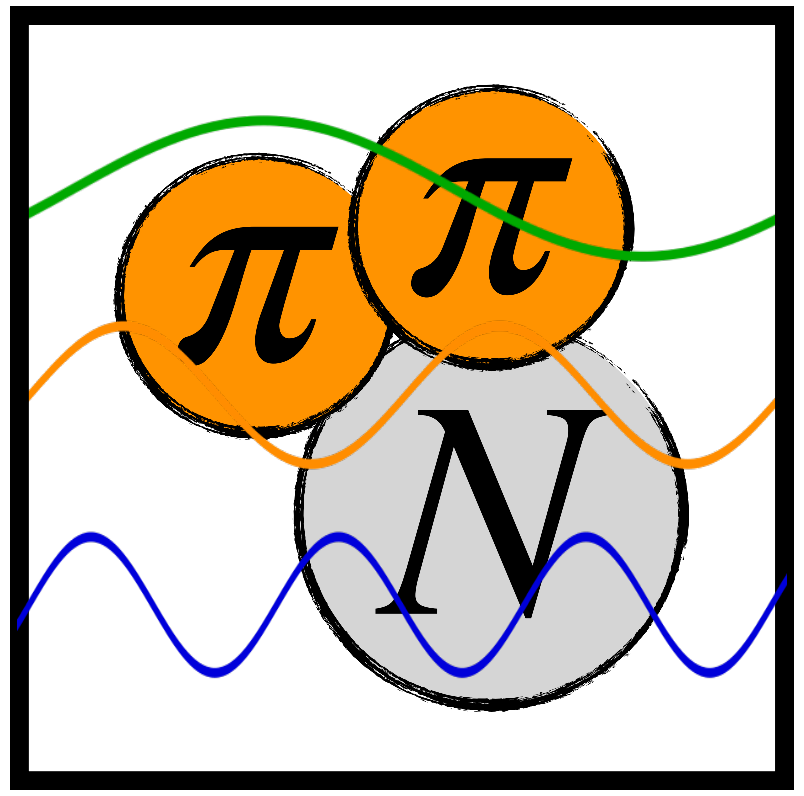
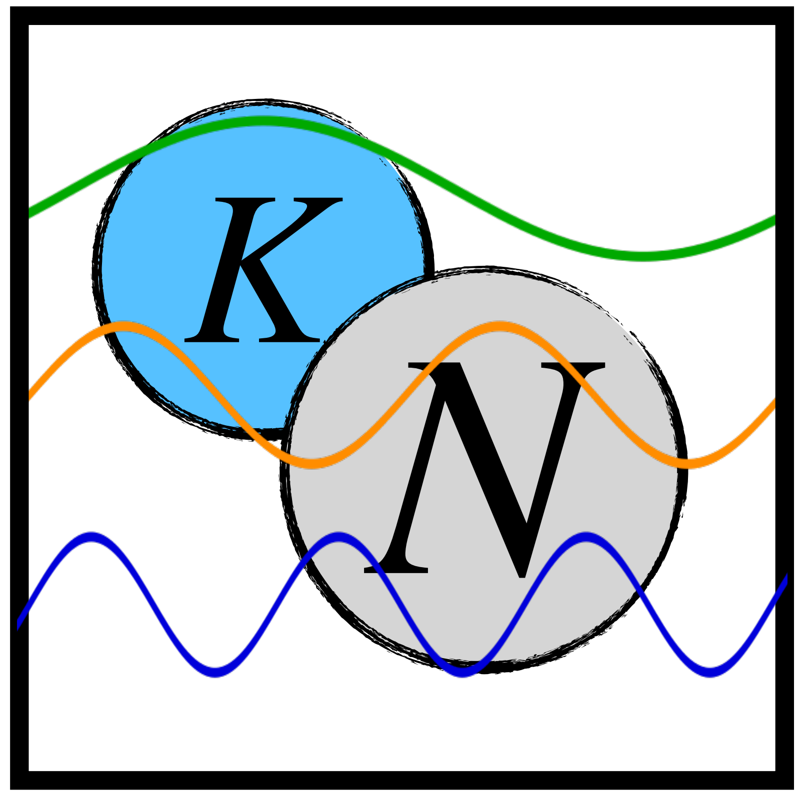
FNAL

March 14th





QCD and hadrons

$$\mathscr{L}_{QCD} = \sum_i^{N_f} \bar{q}_i \left(D_\mu \gamma^\mu + m_i \right) q_i + \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$

→ Confinement & Hadron spectrum

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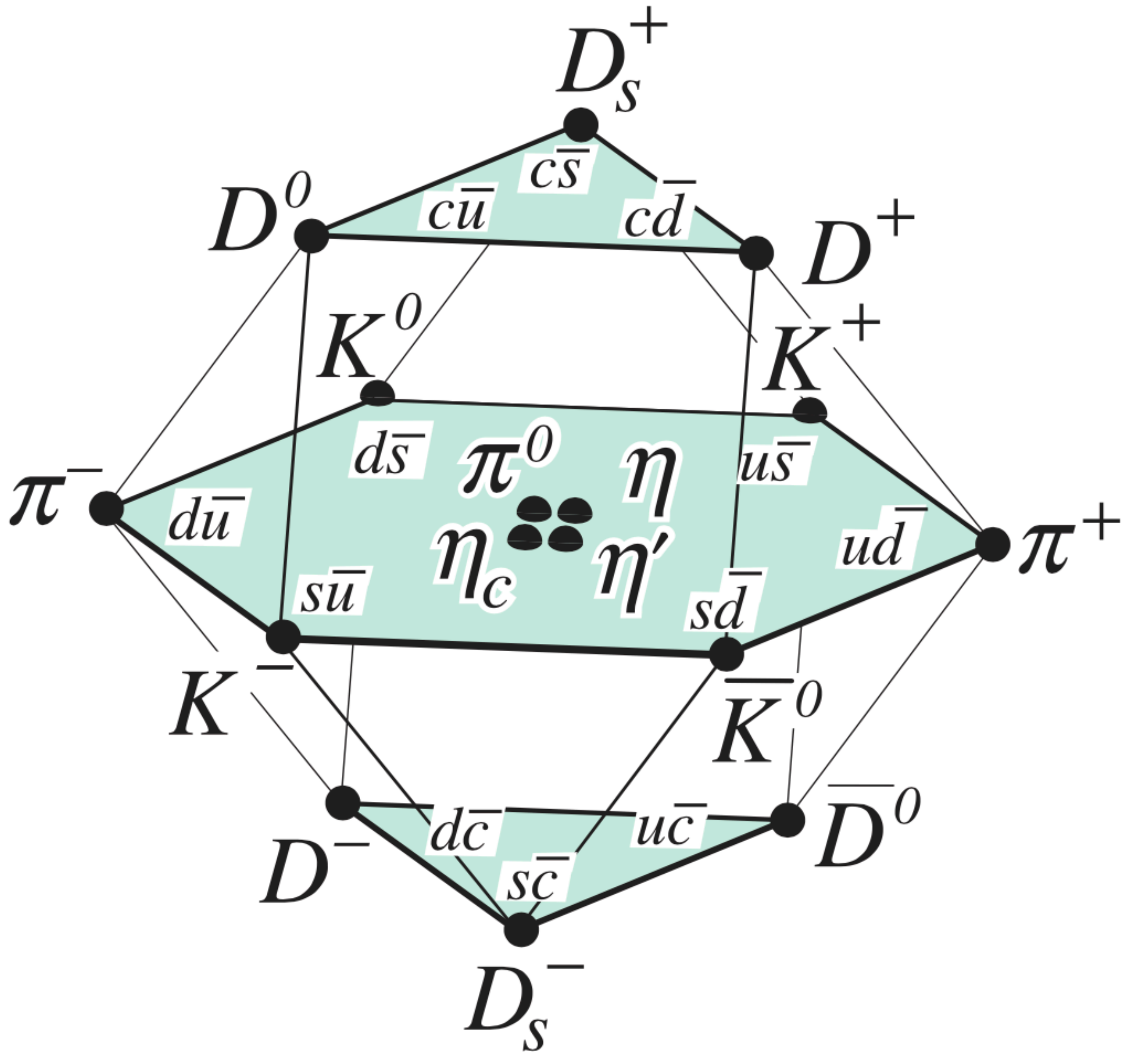
- **QCD stable hadrons, e.g. pseudo-Goldstone bosons**



$|\pi\rangle, |K\rangle \in \text{QCD Fock}$

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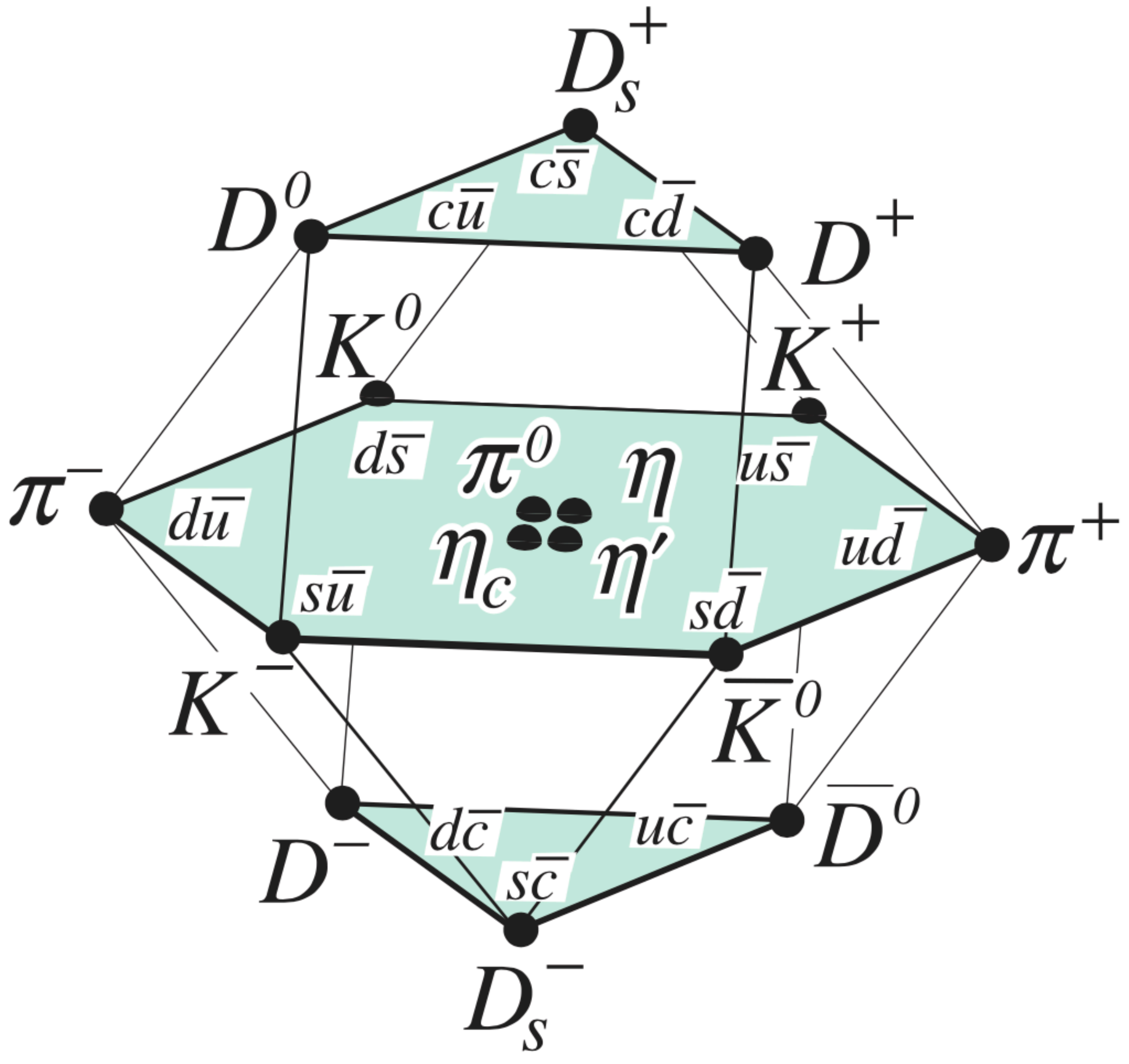

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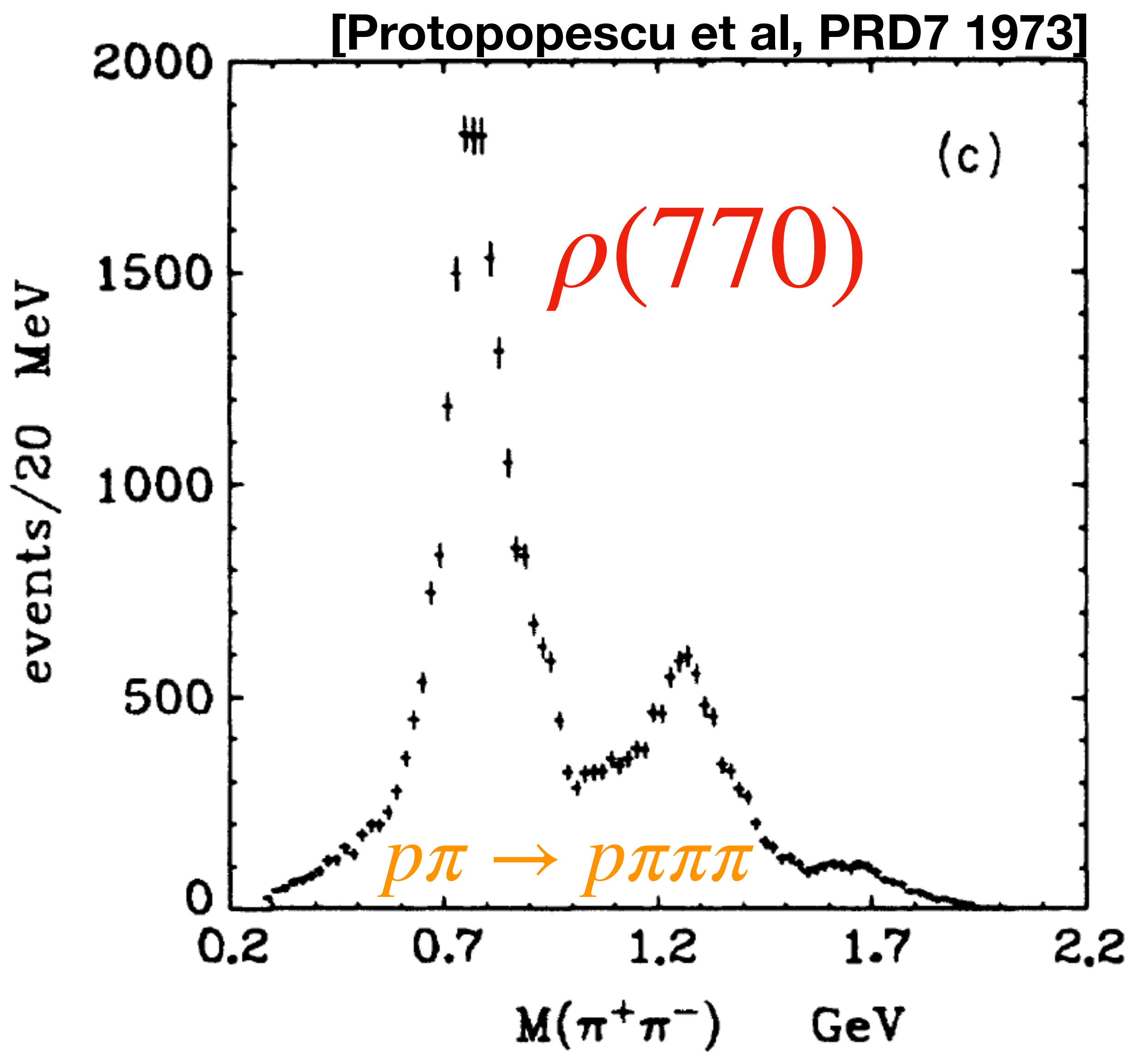
- Resonances show up in scattering processes

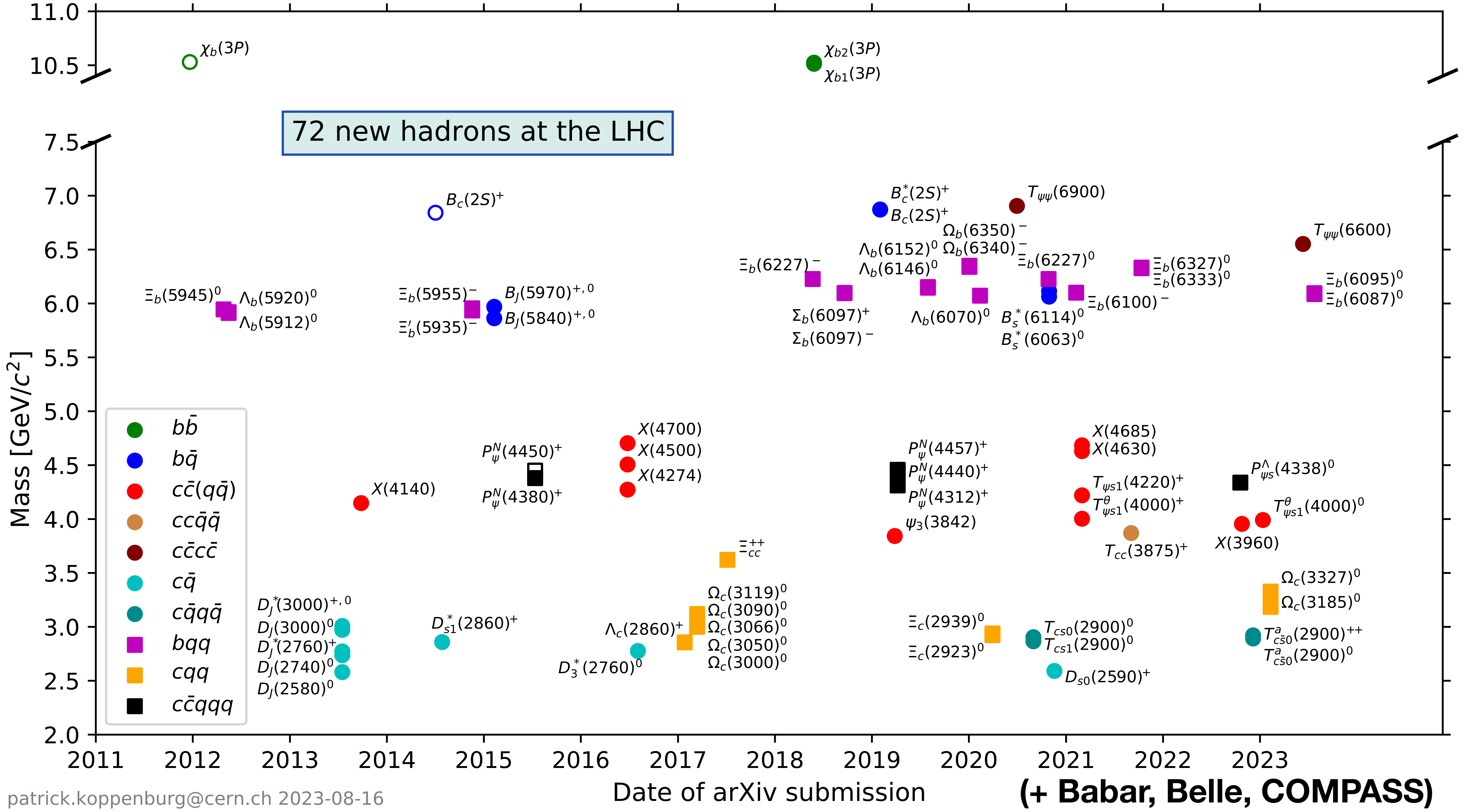


$|\rho\rangle \notin \mathrm{QCD\ Fock}$

The Hadron Spectrum

A growing hadron spectrum still requires first principles understanding

[https://www.nikhef.nl/~pkoppenb/particles.html]



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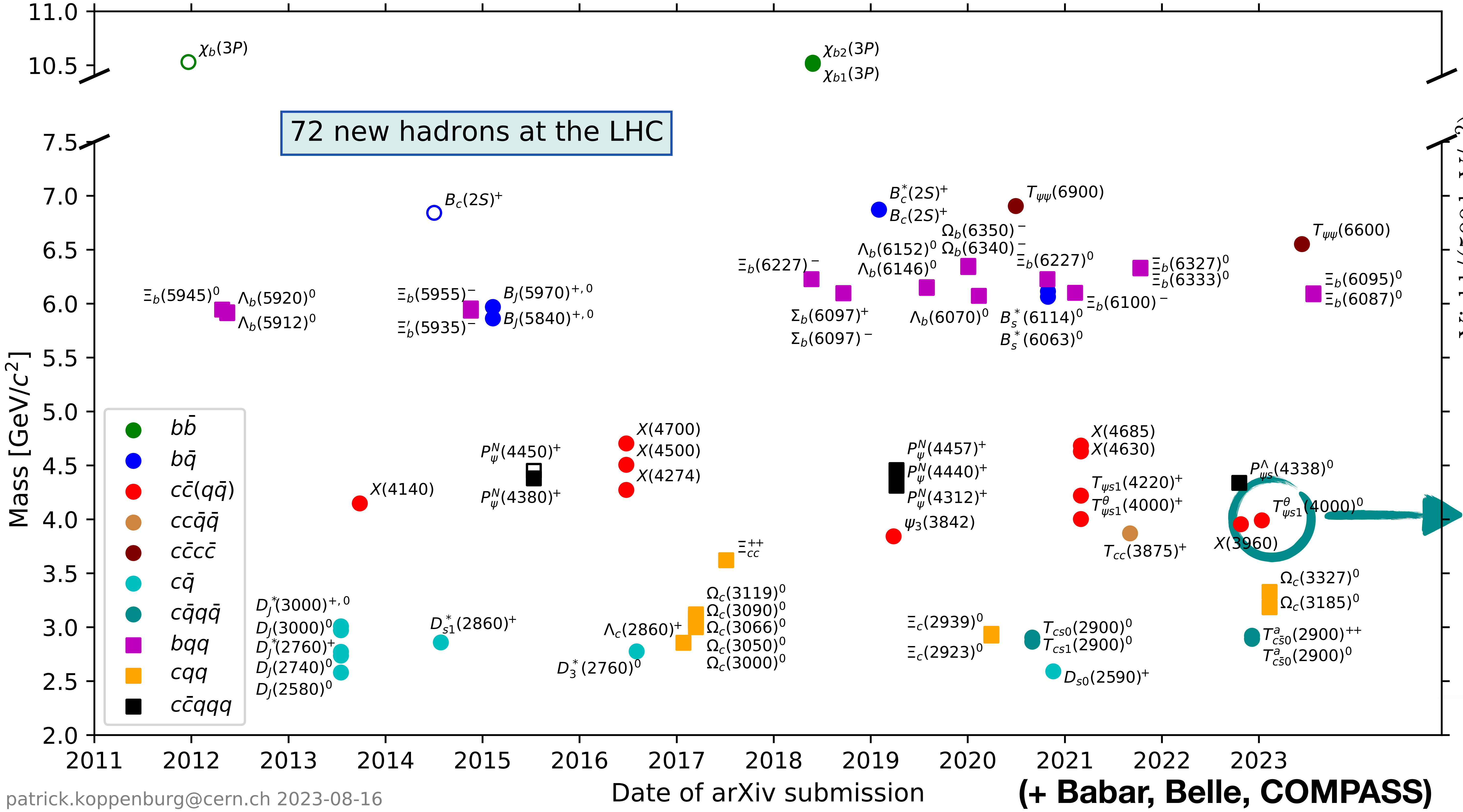
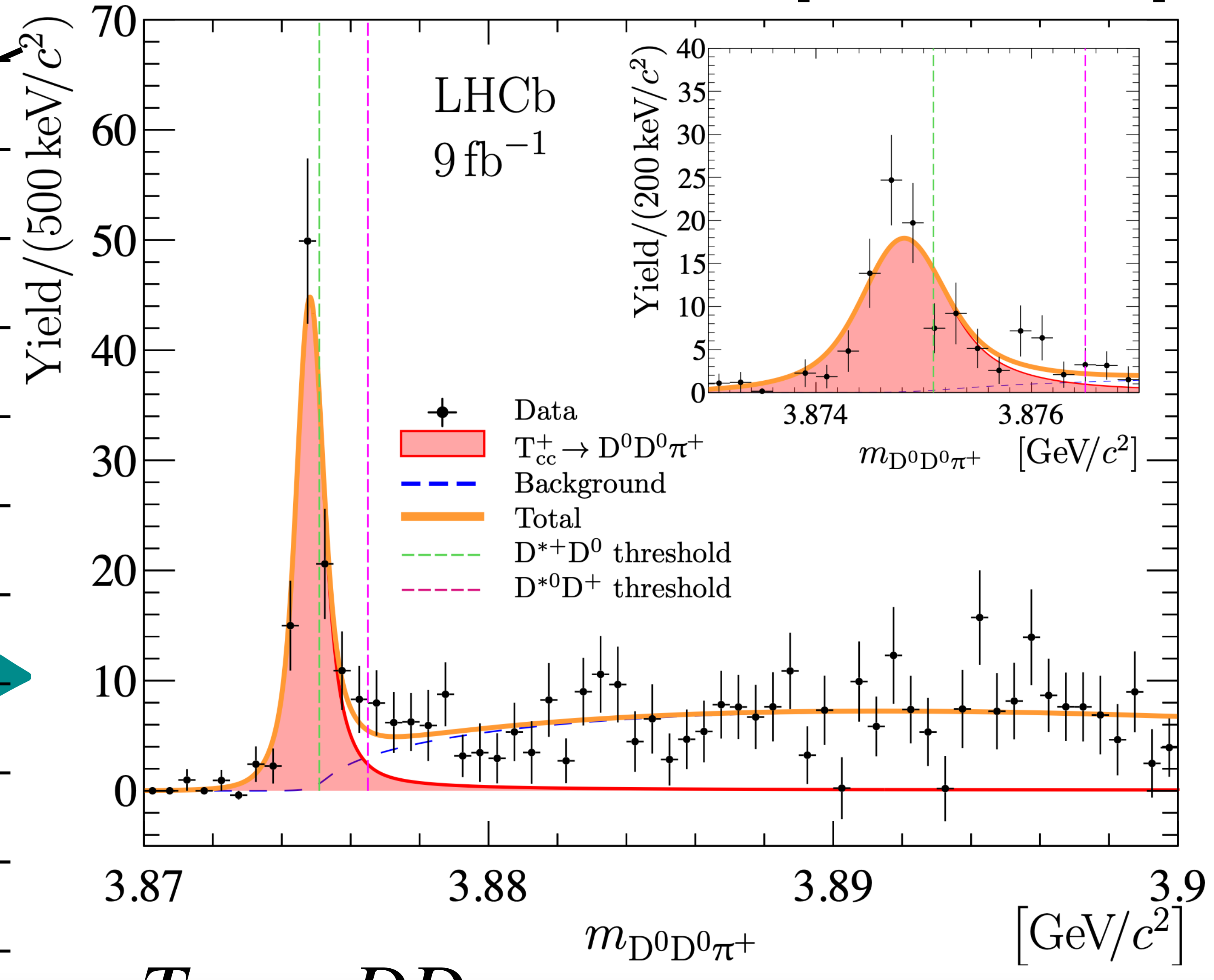


(+ Babar, Belle, COMPASS)

[arXiv:2109.01038]



$T_{cc} \to DD\pi$

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Multi-hadron interactions

Beyond QCD, understanding resonance properties is important for new physics searches

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- **Tests of the Standard Model in meson weak decays**

CP violation in $K \to \pi\pi$ weak decays

$$\left(\varepsilon'/\varepsilon\right)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}$$

[NA48 & KTeV, 2002 & 2009]

σ resonance

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$f_0(1710)$ **resonance**

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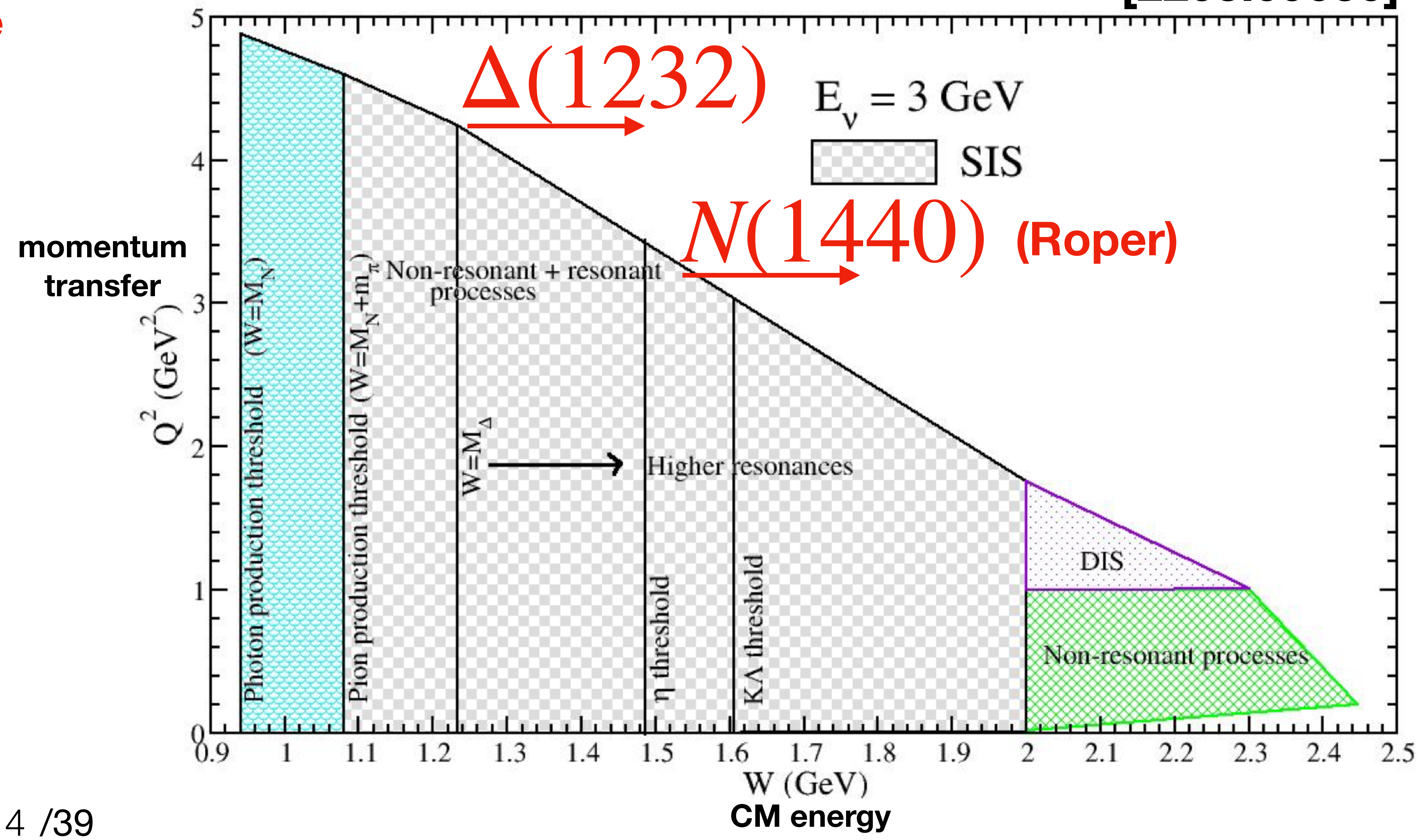
- **Neutrino-nucleus scattering (DUNE, Hyper-K)**

$$\nu N \to \ell N \pi$$

[2203.09030]



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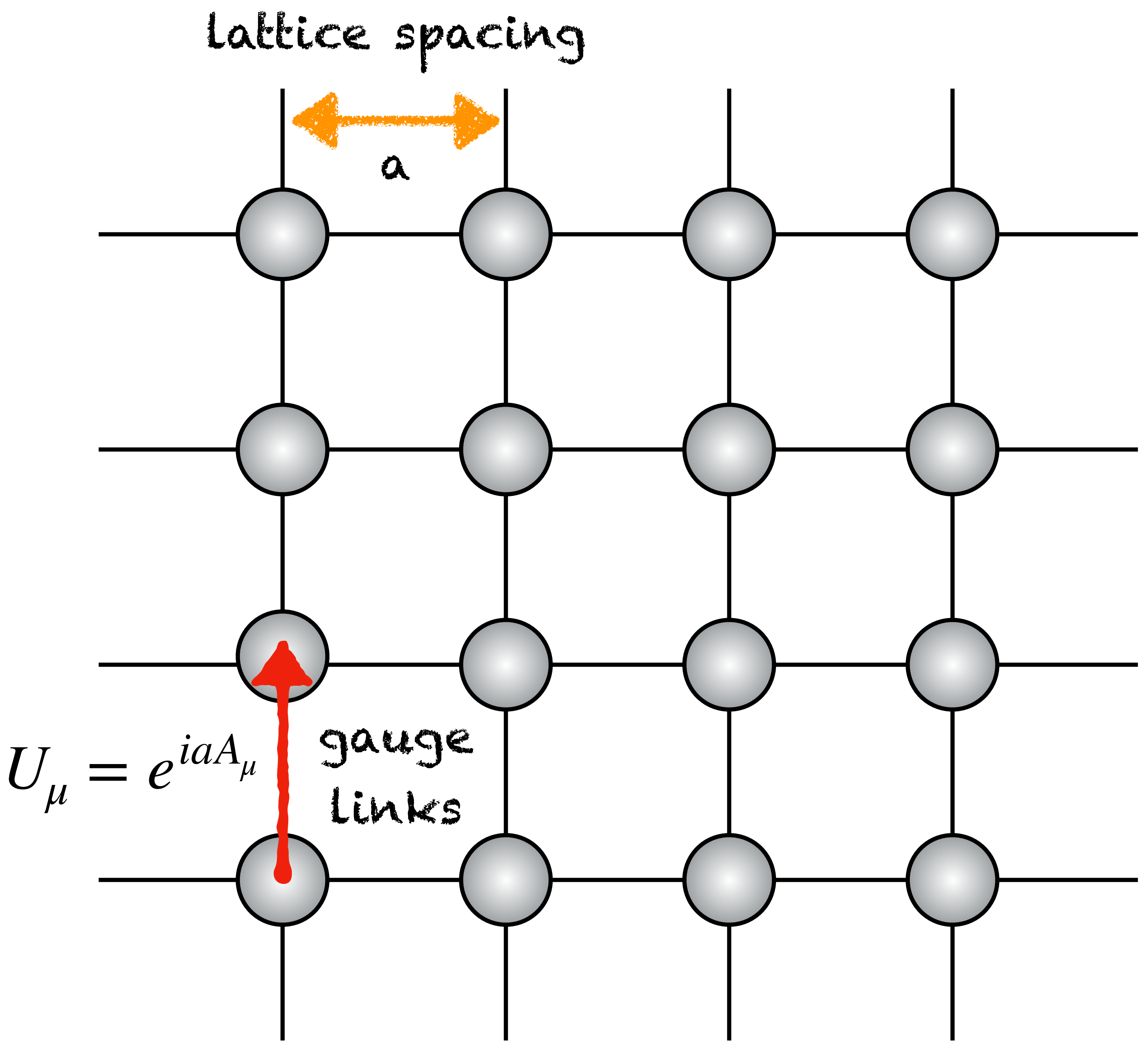
Lattice QCD

- Lattice QCD is a first-principles numerical approach to the strong interaction

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle = \frac{1}{\mathcal{Z}}\int D\psi D\bar{\psi} DA\, \mathcal{O}(t)\mathcal{O}(0) e^{-S_E(\psi,\bar{\psi},A_\mu)}$$

Euclidean action



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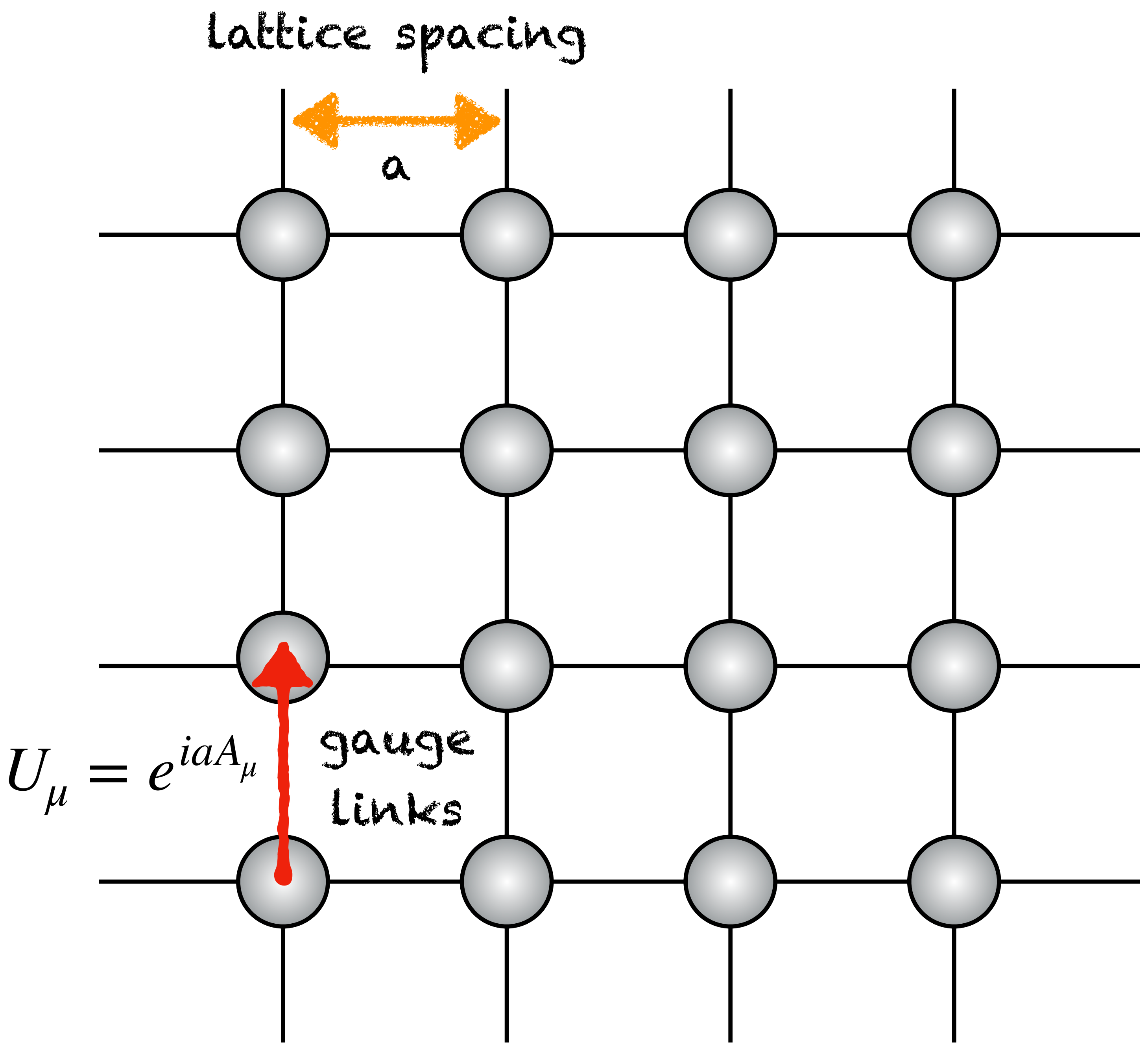

Can we obtain resonance properties from Euclidean correlation functions?

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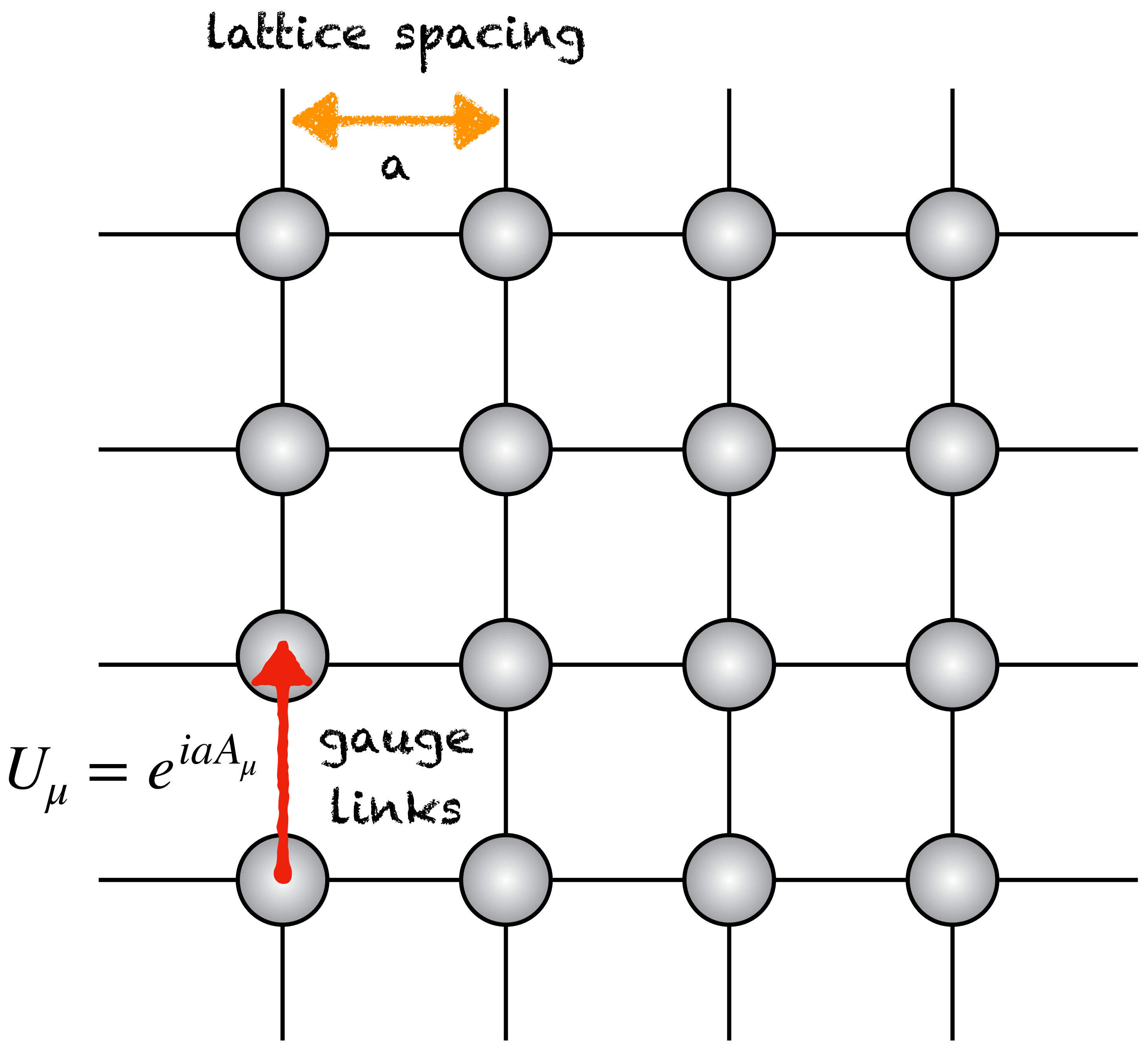


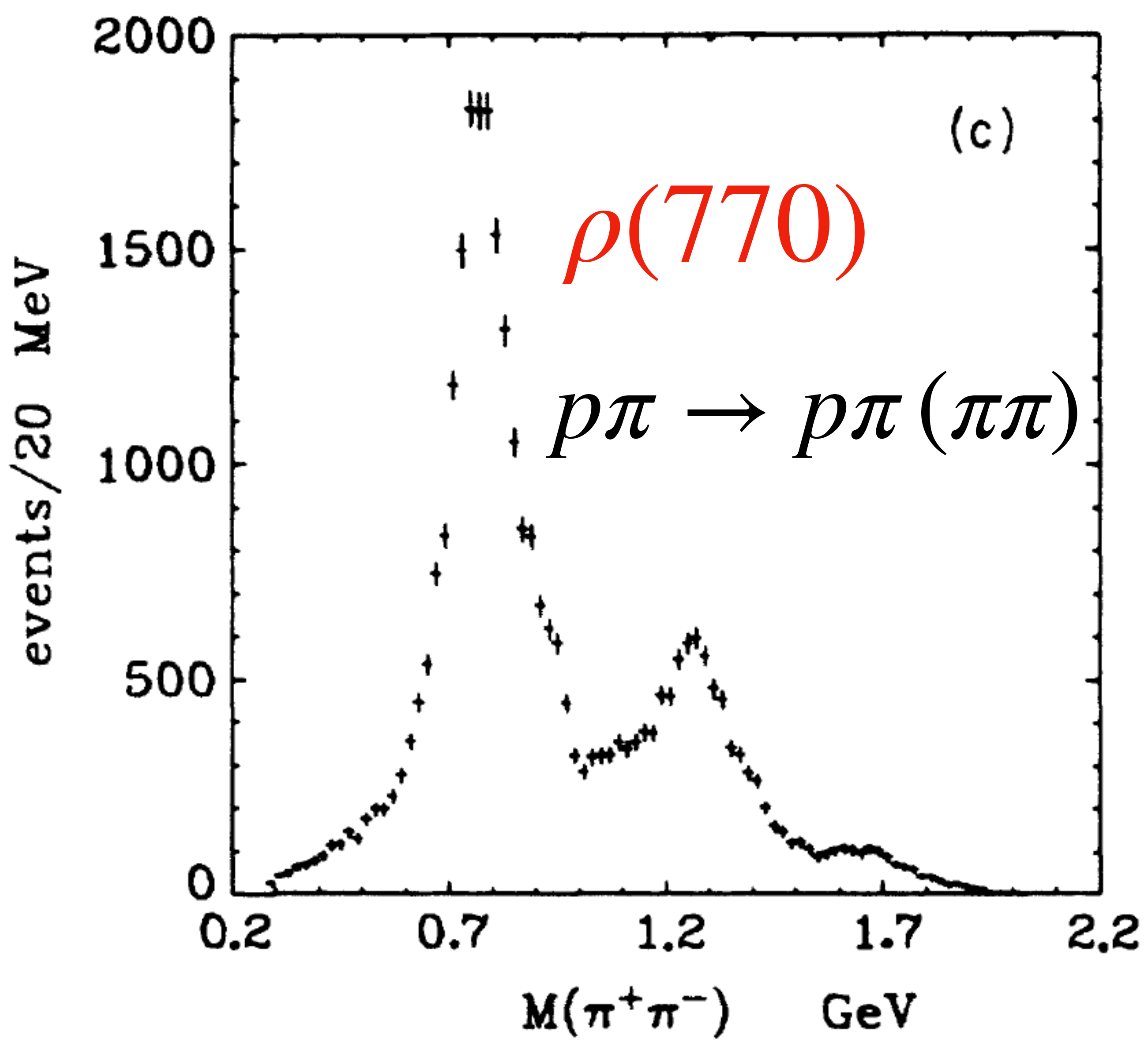
Can we obtain resonance properties from Euclidean correlation functions?

→ Yes, but not that simple!

Infinite vs finite volume

Experiments

- Asymptotic states
- Direct access to scattering amplitudes



[Protopopescu et al, PRD7 1973]

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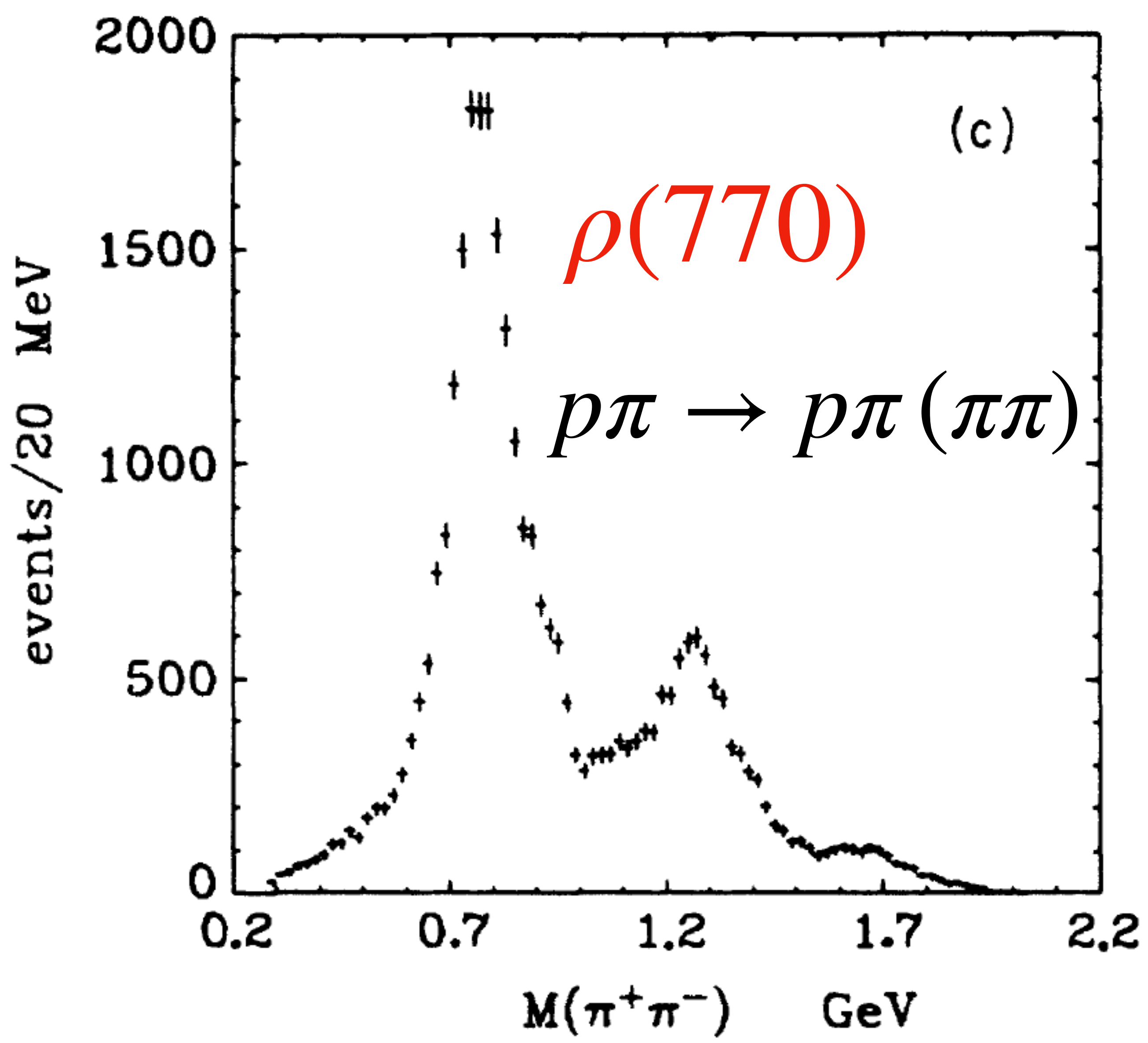

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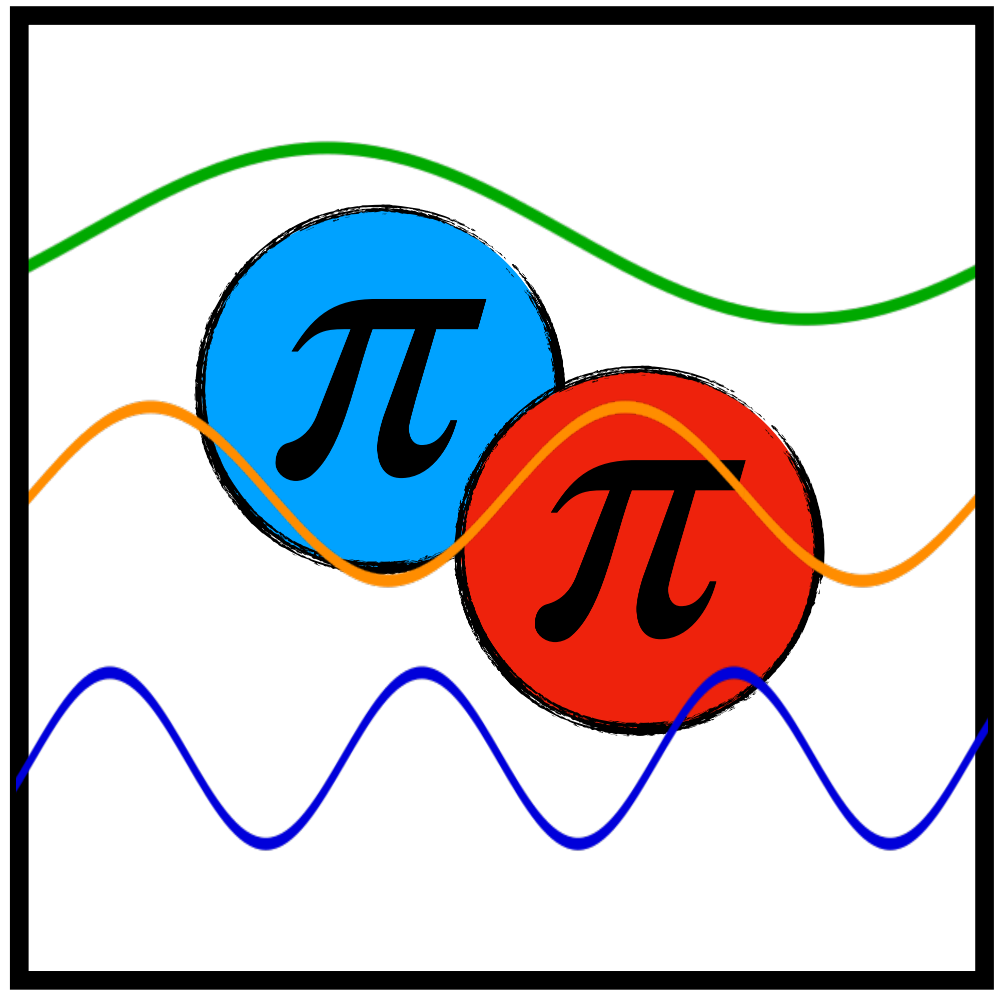
Lattice QCD

- Euclidean time
- Stationary states in a box



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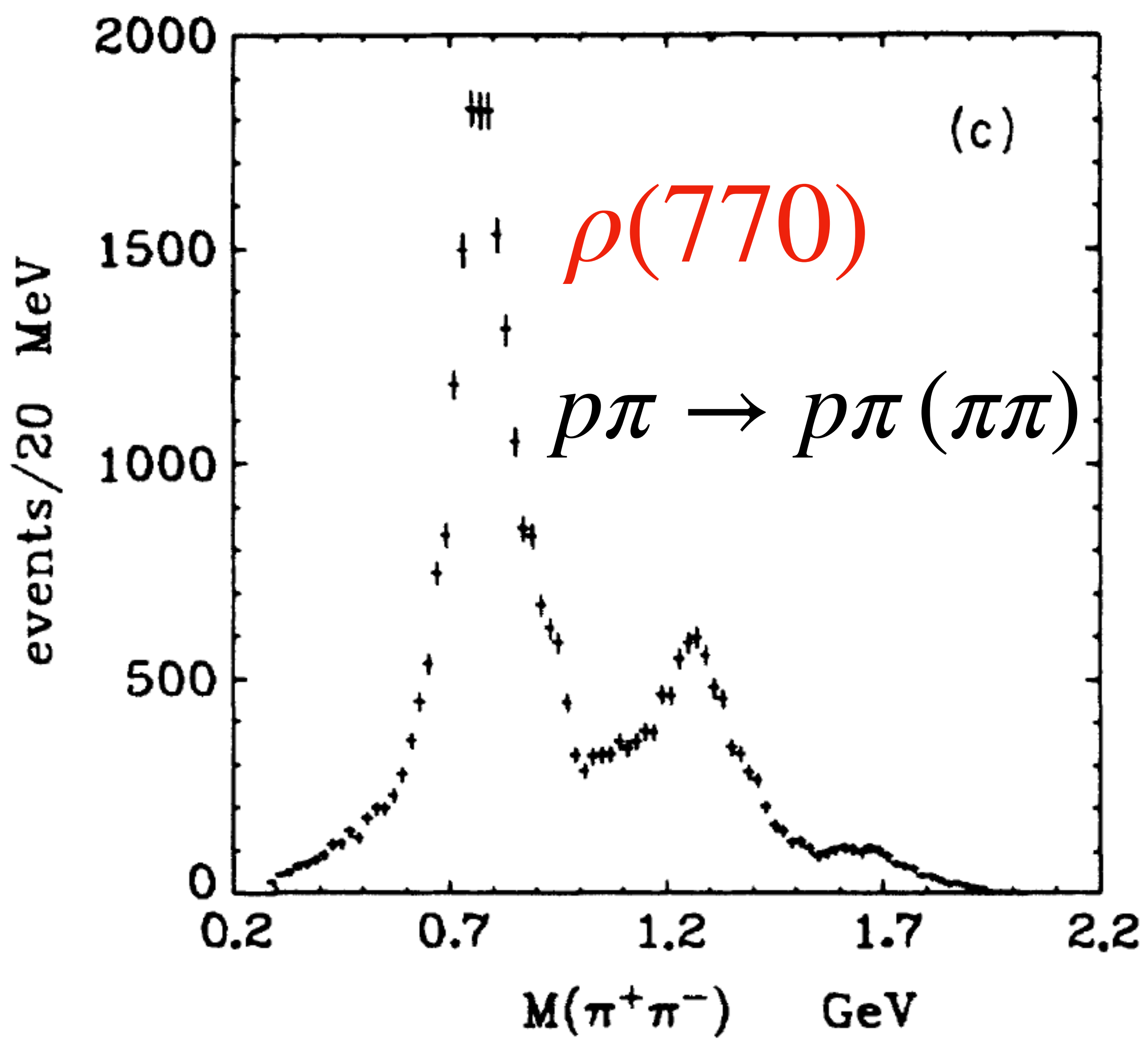

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Finite-volume formalism

[Lüscher, 89']

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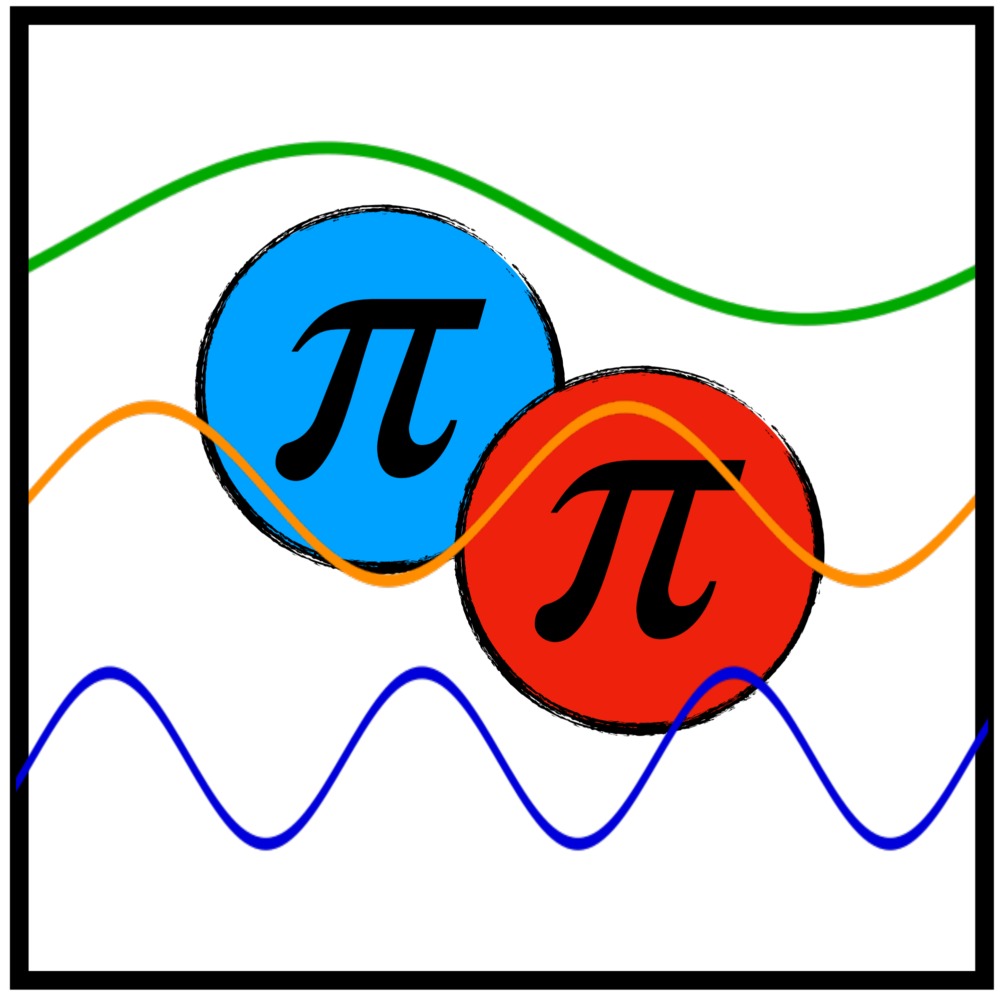

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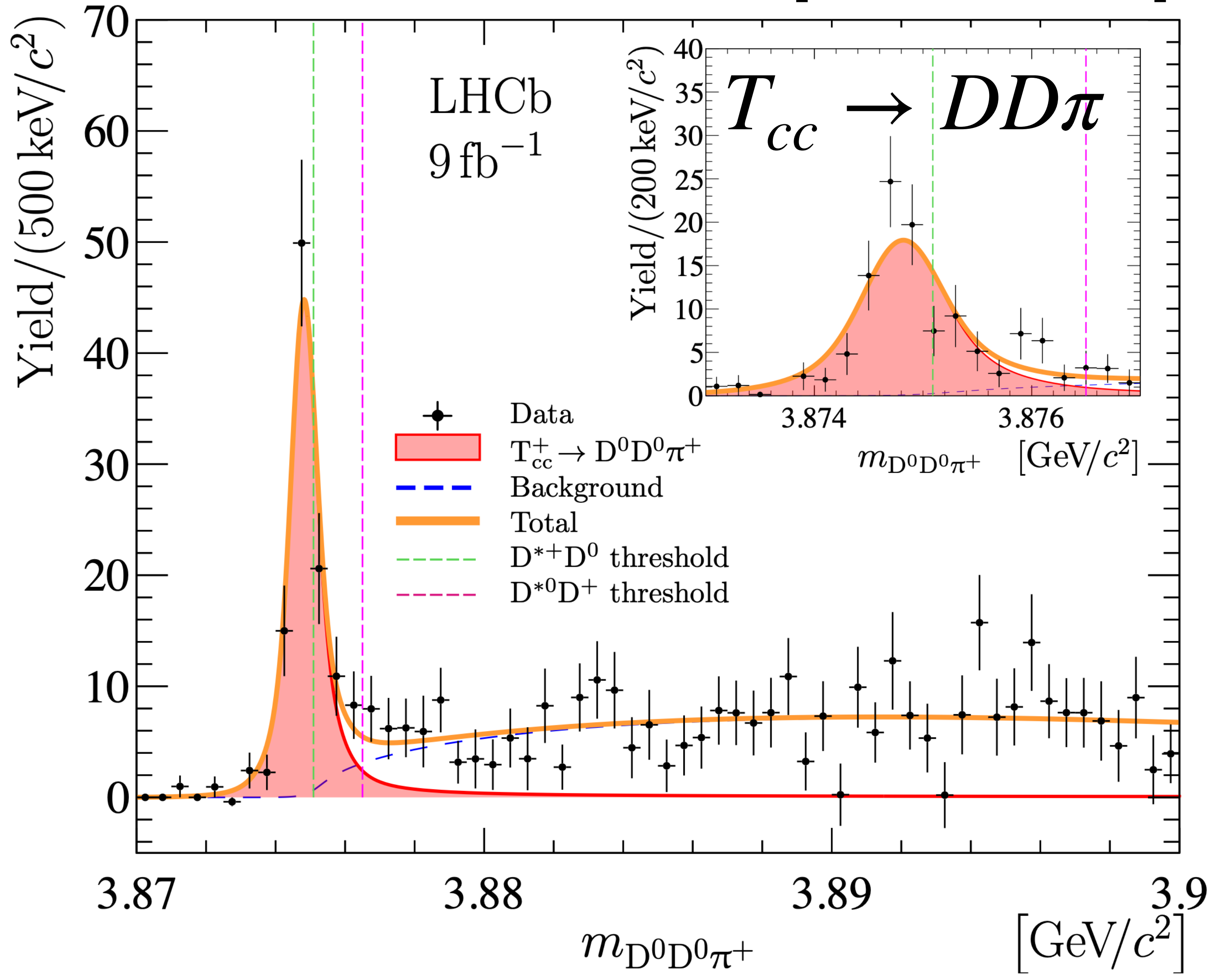

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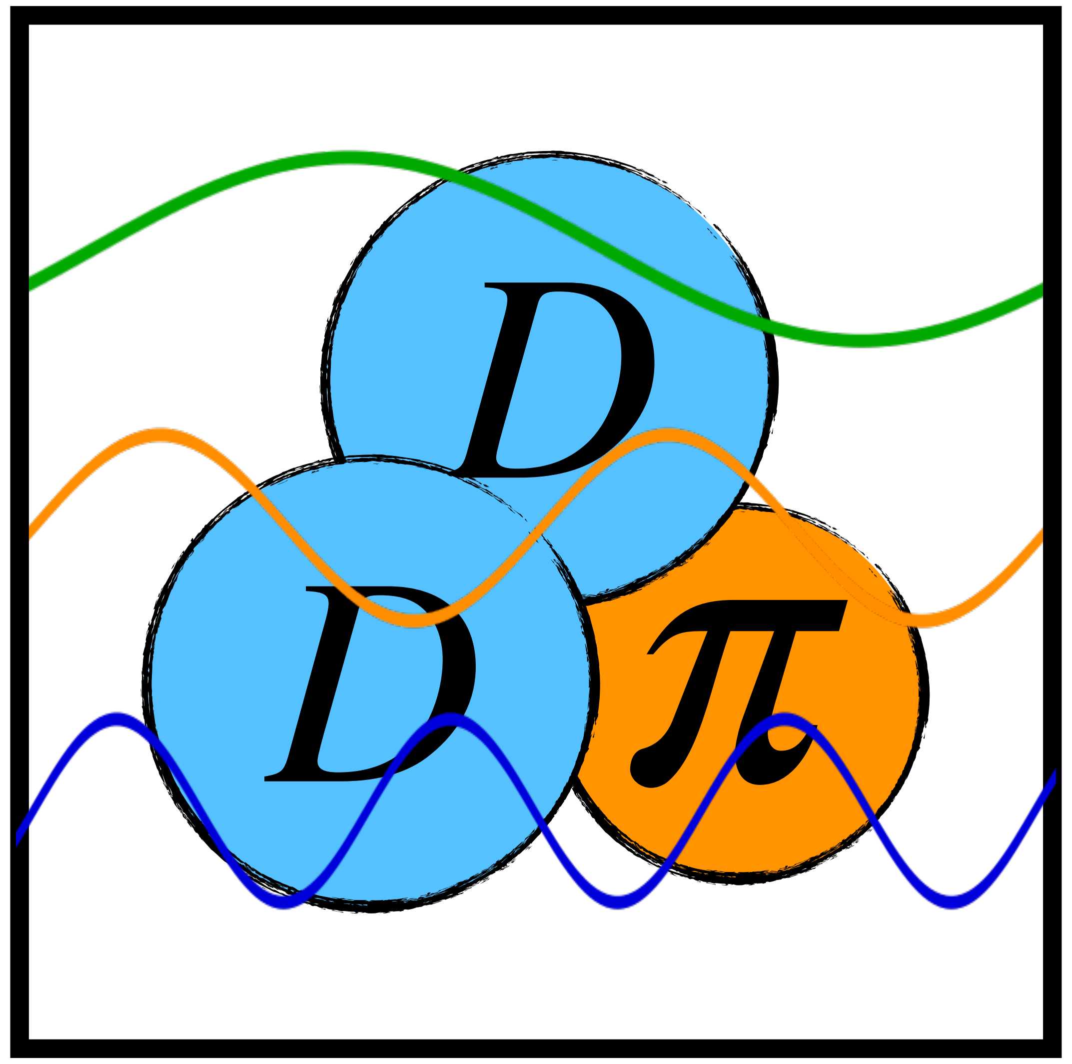

Finite-volume formalism ←

Need to include 3-body effects!

Towards the QCD S-Matrix

The S-Matrix contains the physical information of the theory:

$$S_{ab}(E) \equiv \langle \text{out}|\mathcal{S}|\text{in}\rangle$$

Lattice QCD → QCD S-matrix

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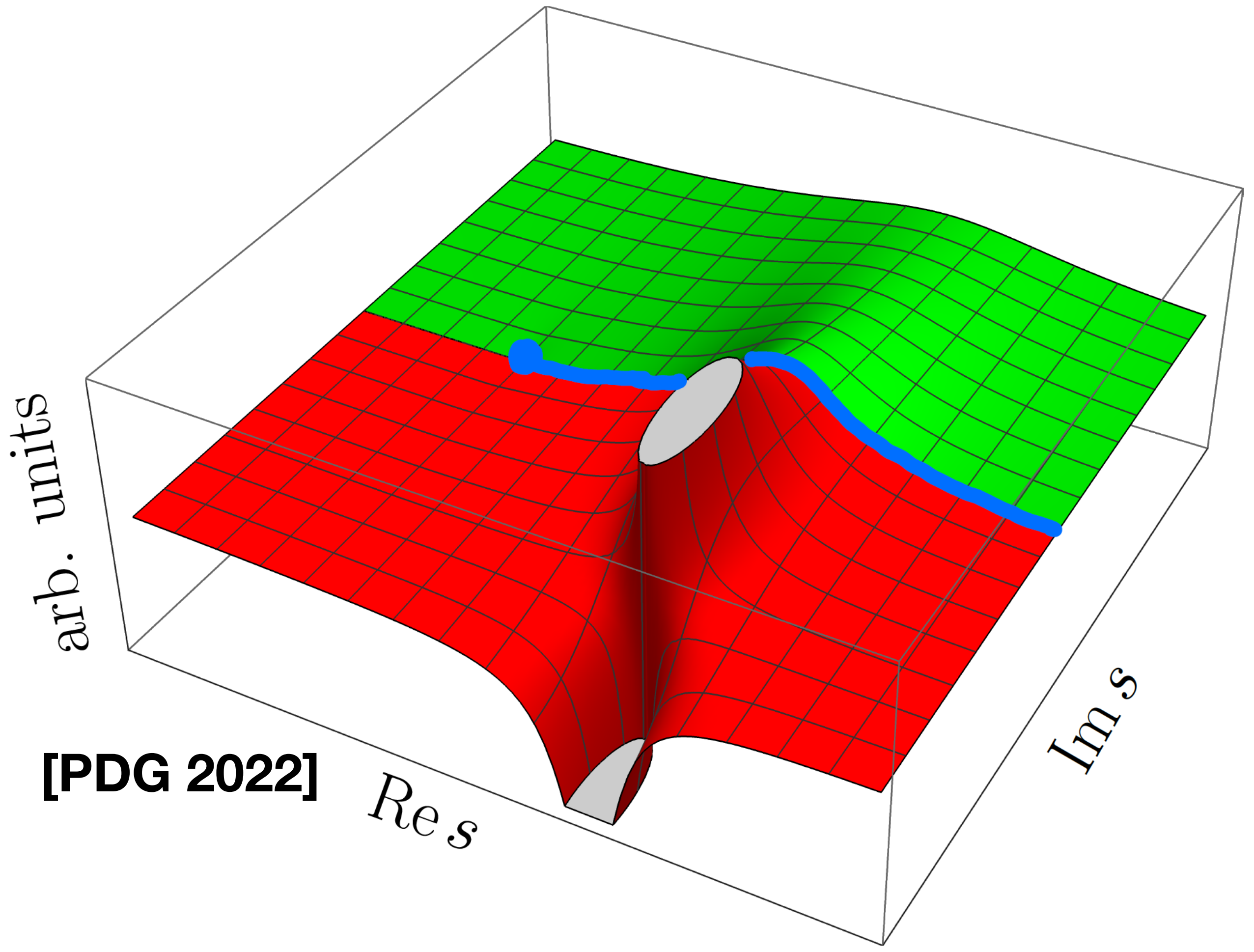
Lattice QCD ⟶ QCD S-matrix

- Resonances as poles in the S-matrix (or scattering amplitude)



[PDG 2022]

$$\sim \frac{g}{E^2 - E_R^2}$$

$$E_R = M_R - i\,\Gamma/2$$

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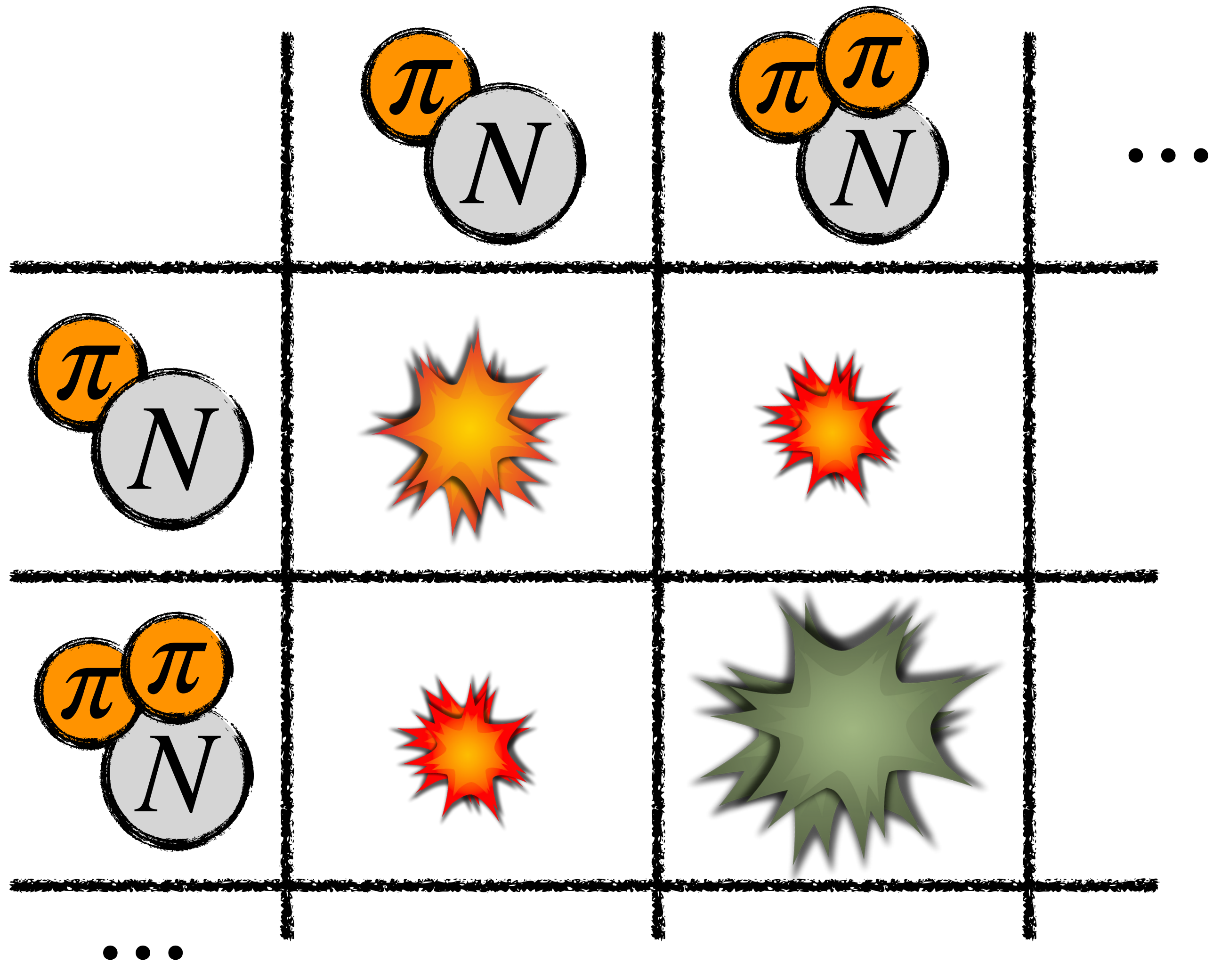

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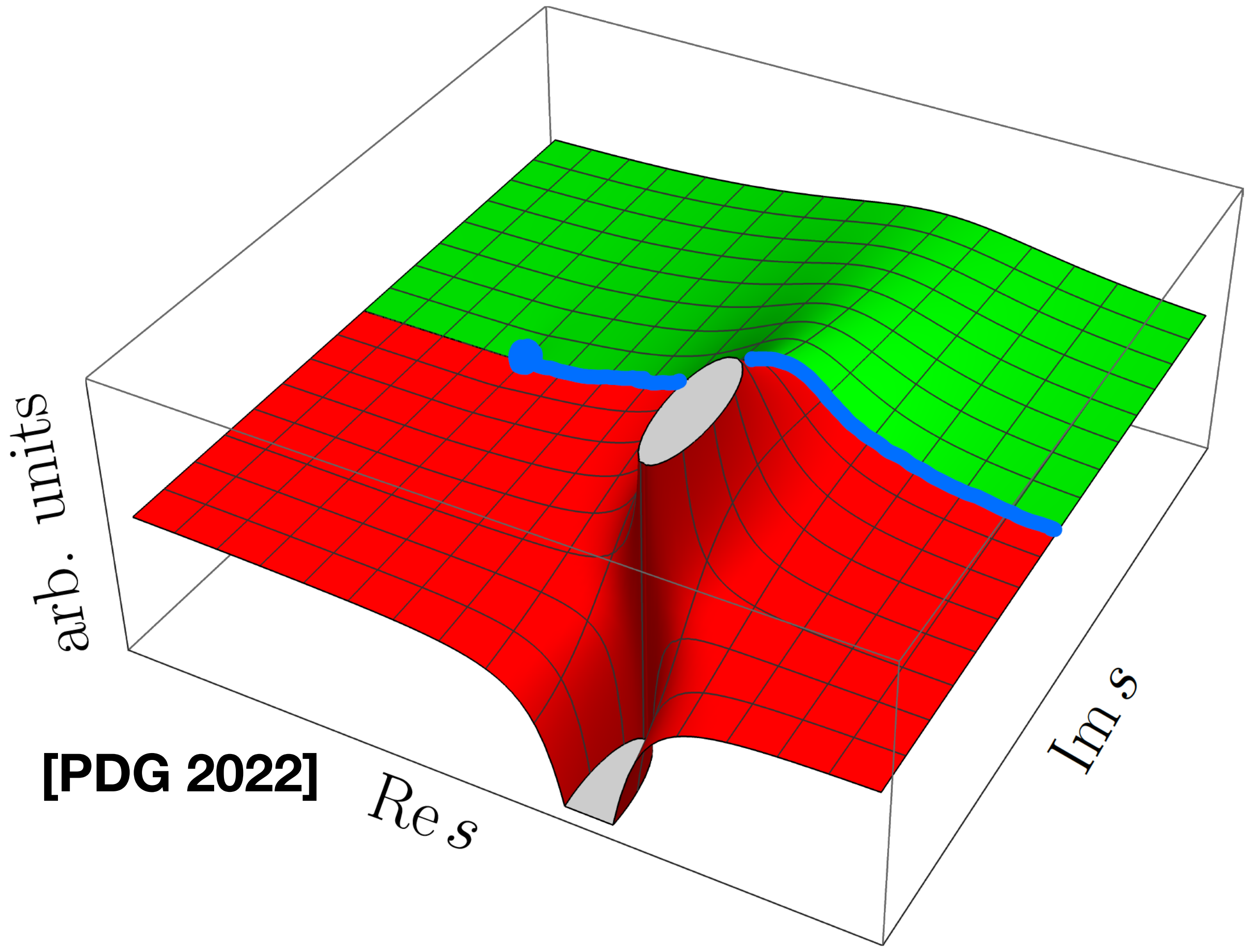

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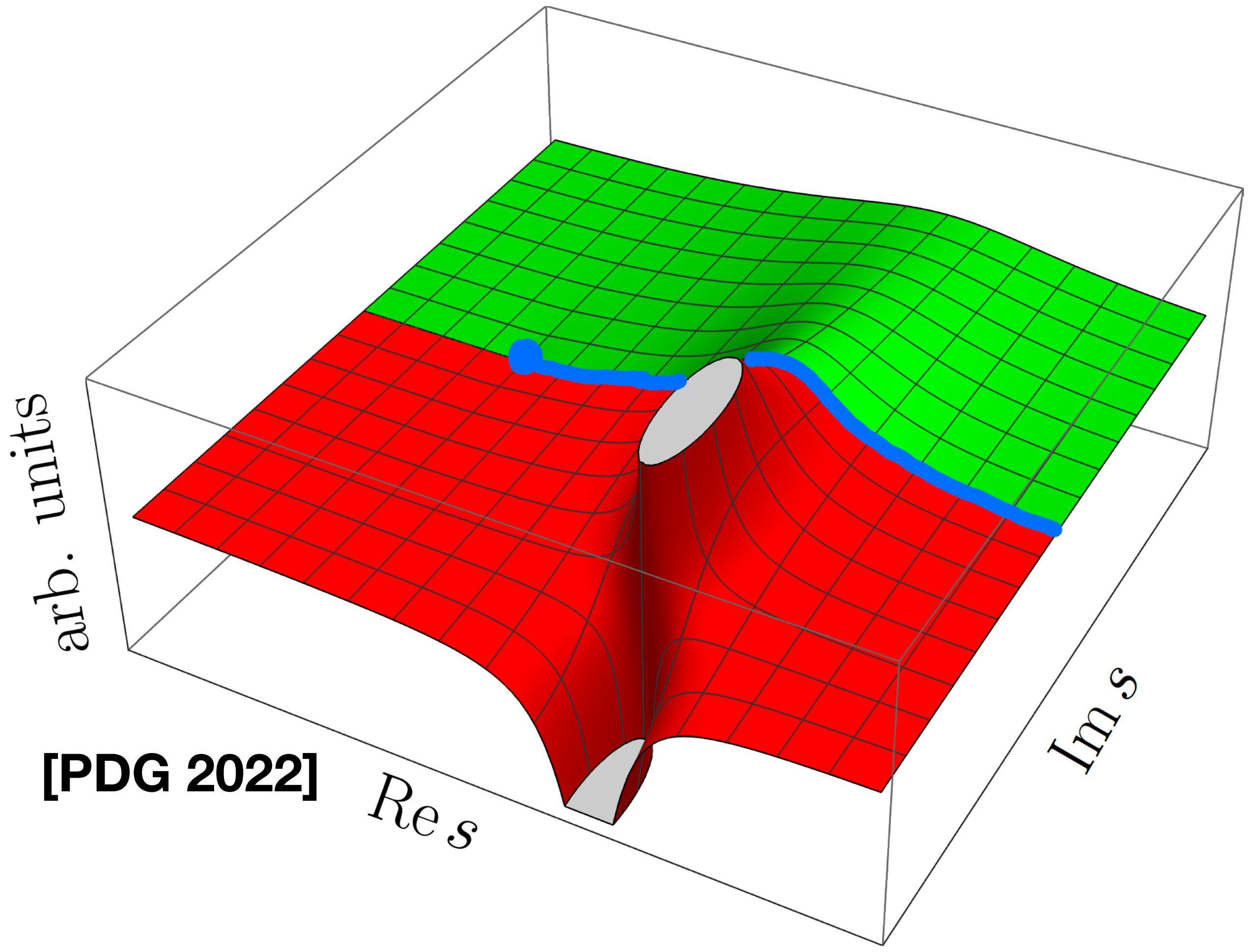

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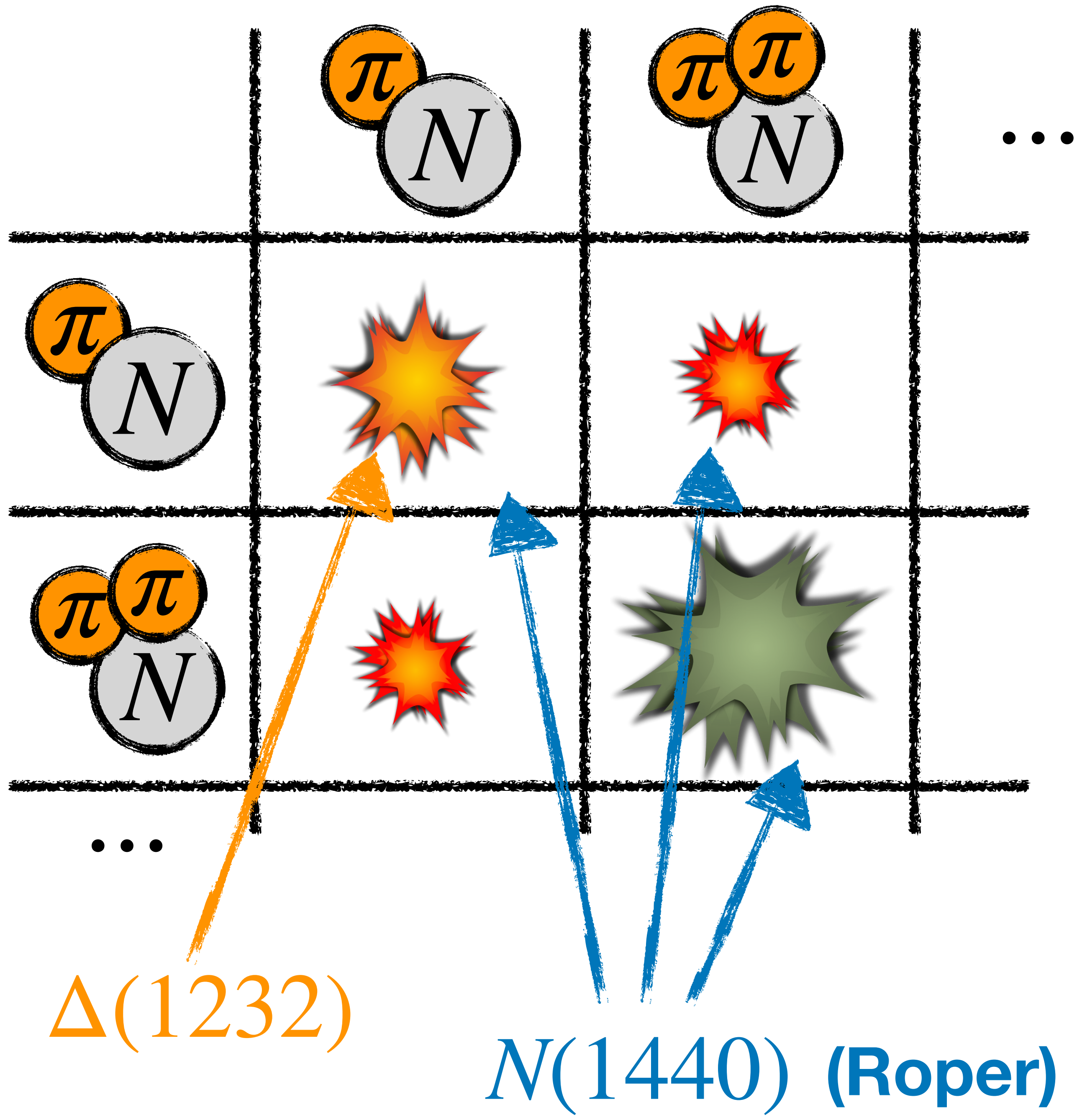

Outline

1. Scattering amplitudes from lattice QCD

2. Meson-Baryon scattering: $\Delta(1232)$ and $\Lambda(1405)$

3. Towards three-body systems: 3π, T_{cc} and more

Scattering amplitudes from lattice QCD

The general strategy

Spectrum

E_4
E_3
E_2
E_1
E_0

From Euclidean Correlation functions

Quantization condition

$$\det_{\ell m}\left[F^{-1} + \mathcal{K}\right] = 0$$

A bridge between:

finite volume ⟶ infinite volume

Infinite-volume scattering quantities

K matrix	Phase shift	Amplitude
$\mathcal{K}$	δ_ℓ	$\mathcal{M}$

Resonance poles

$$\mathcal{M}^{-1}(E_{\text{pole}}) = 0$$

Energy levels from lattice QCD

- The energy levels of the theory are measured from Euclidean correlation functions

$$C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle = \sum_n \left| \langle 0 | \mathcal{O}(0) | n \rangle \right|^2 e^{-E_n t}$$

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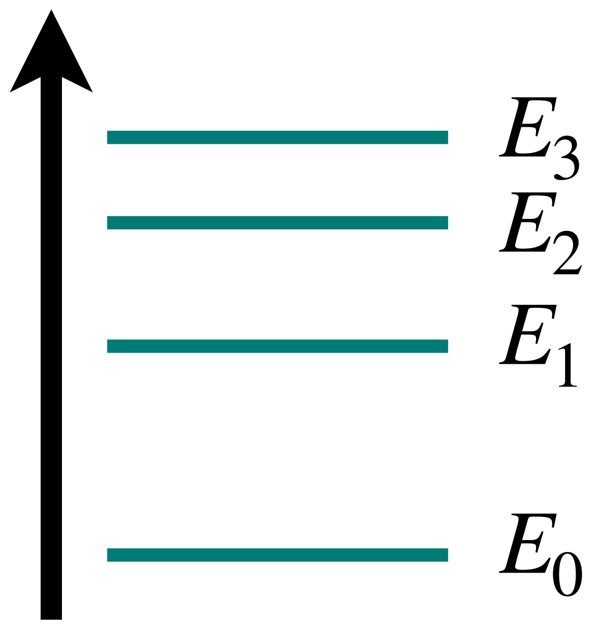
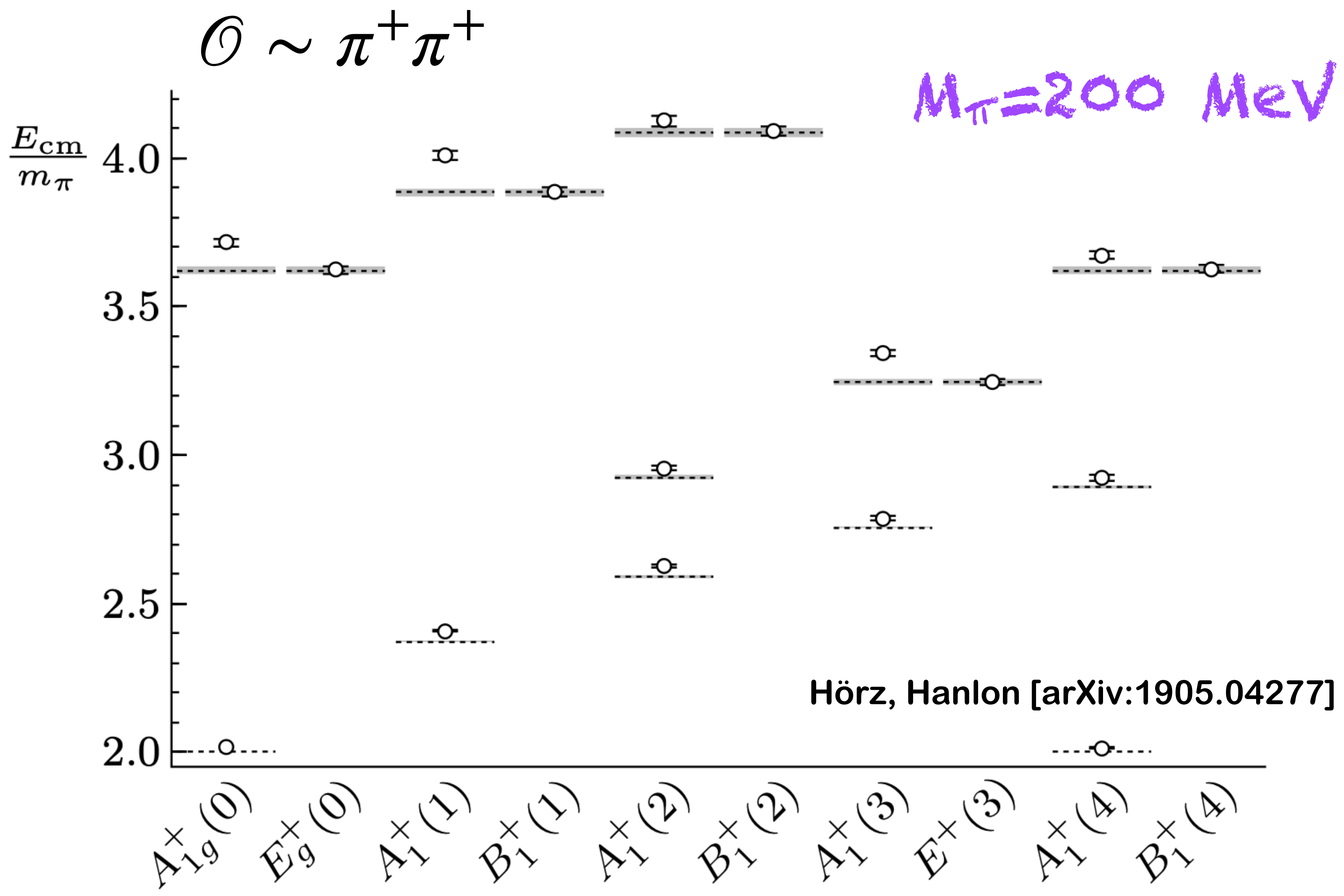
- Multiple operators with the same quantum names to obtain several energy levels

▶ Variational techniques
(Generalized EigenValue Problem, GEVP)

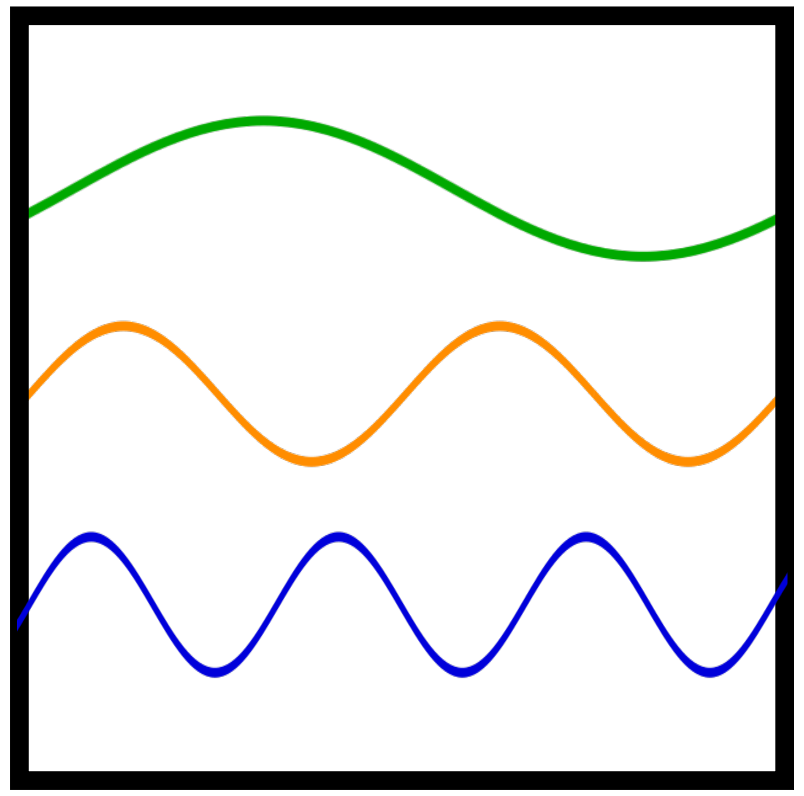
The Spectrum





Finite-Volume Spectrum

Free scalar particles in finite volume with periodic boundaries



$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

Two particles: $E = 2\sqrt{m^2 + \frac{4\pi^2}{L^2}\vec{n}^2}$

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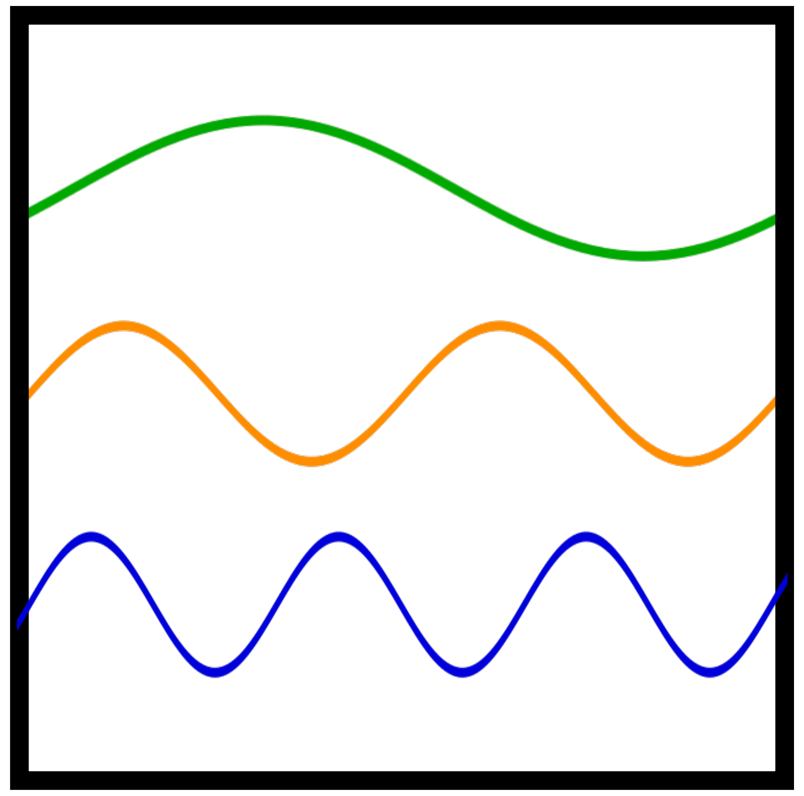
Free scalar particles in finite volume with periodic boundaries

→

Interactions change the spectrum: it can be treated as a perturbation



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Ground state to leading order

$$E_2 - 2m = \langle \phi(\vec{0})\phi(\vec{0}) \,|\, \mathbf{H_I} \,|\, \phi(\vec{0})\phi(\vec{0}) \rangle$$

$$\Delta E_2 = \frac{\mathcal{M}_2(E = 2m)}{8m^2L^3} + O(L^{-4})$$

[Huang, Yang, 1958]

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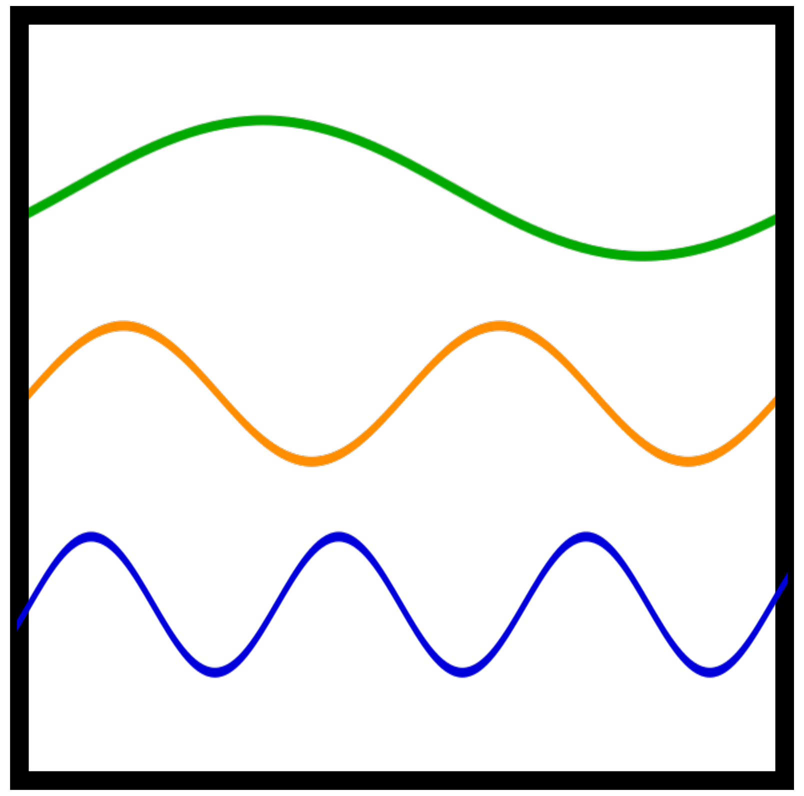

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The energy shift of the two-particle ground state
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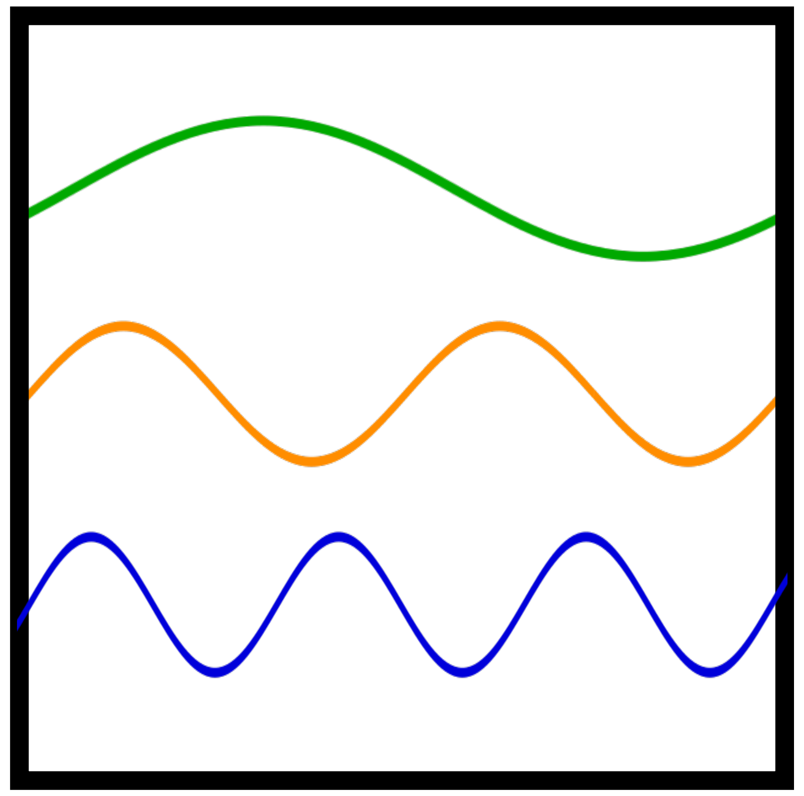

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In general a problem of
Quantum Field Theory
in finite volume

Interactions change the spectrum:
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Finite-Volume formalism

- Indirect connection between the spectrum and the two-particle scattering amplitude [Lüscher 89']

Two-particle Quantization Condition

$$\det_{\ell m}\left[\mathcal{K}_2(E) + F^{-1}(E, \vec{P}, L)\right]\Bigg|_{E=E_n} = 0$$

Scattering K-Matrix — Known kinematic function

"QC2"

! Note: only valid for two particles below inelastic thresholds.

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Finite-volume information

$$F_{00}(q^2) \sim \left[\frac{1}{L^3}\sum_{\vec{k}} - \int\frac{d^3k}{(2\pi)^3}\right]\frac{1}{k^2-q^2}$$

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Example I=2 ππ scattering

Two pions in s-wave

$$\mathcal{K}_2^{s-wave}(E_n) = \frac{-1}{F_{00}(E_n, \overrightarrow{P}, L)}$$

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one energy level → a phase shift point

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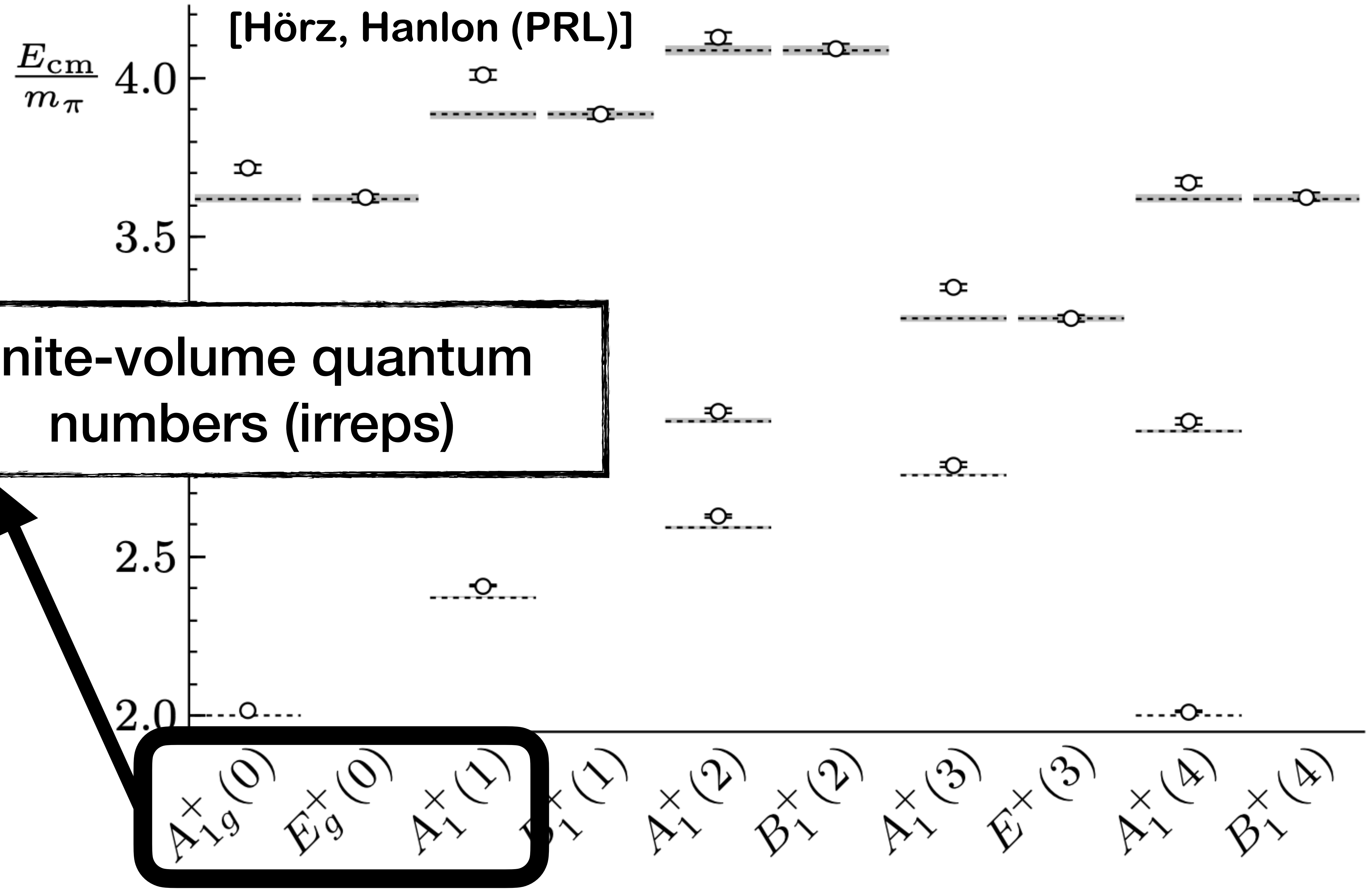
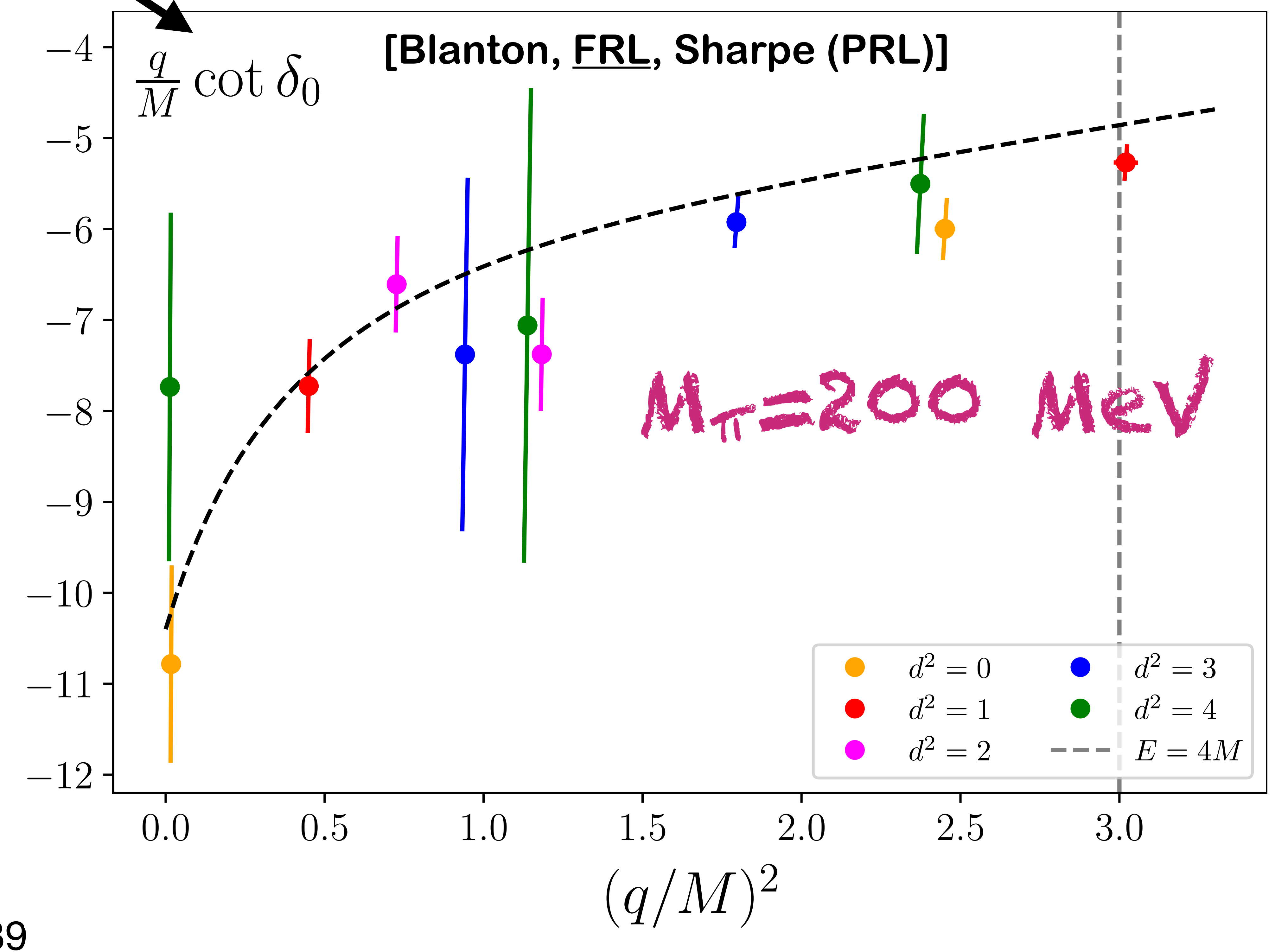


finite-volume quantum numbers (irreps)



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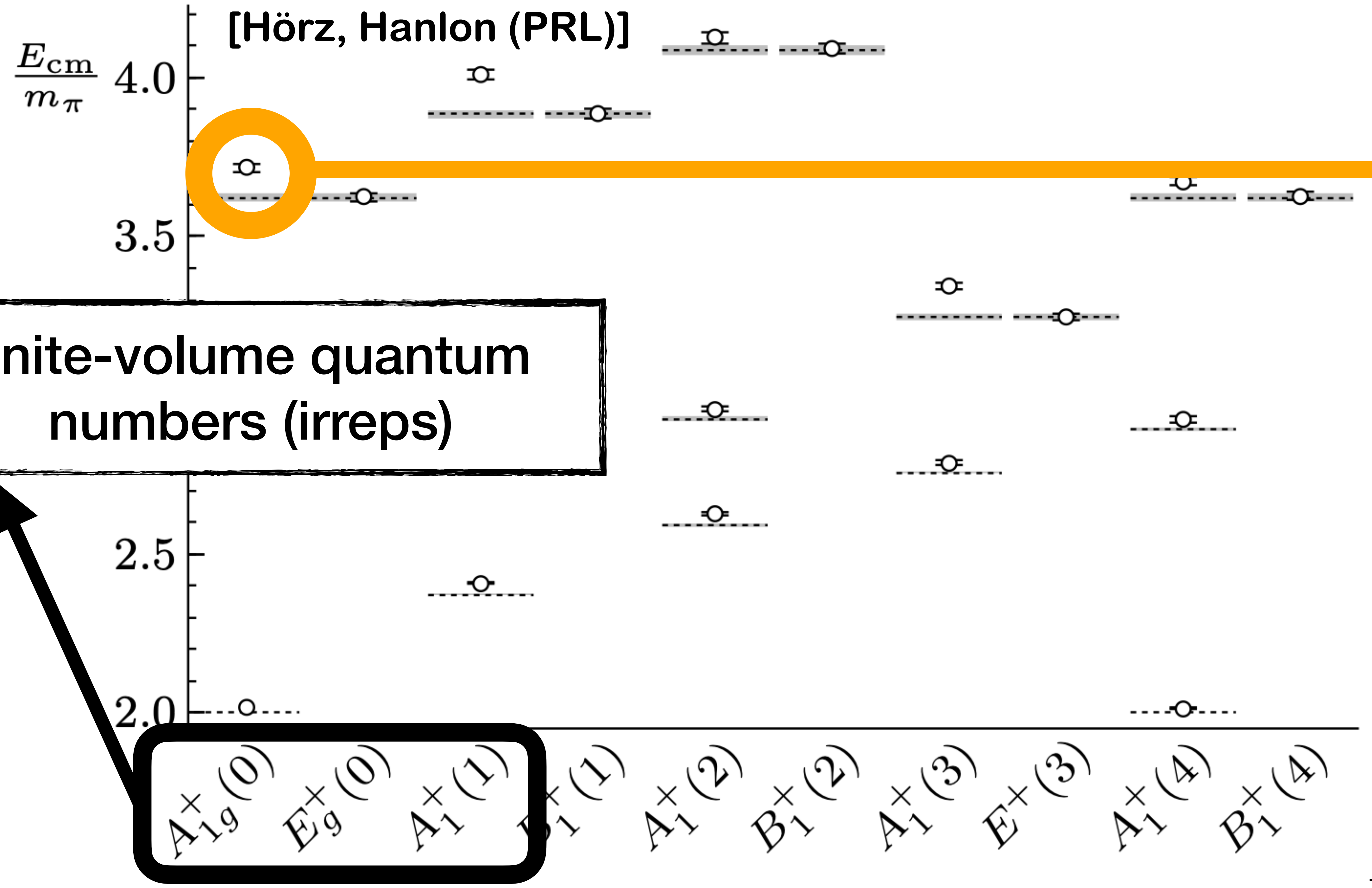

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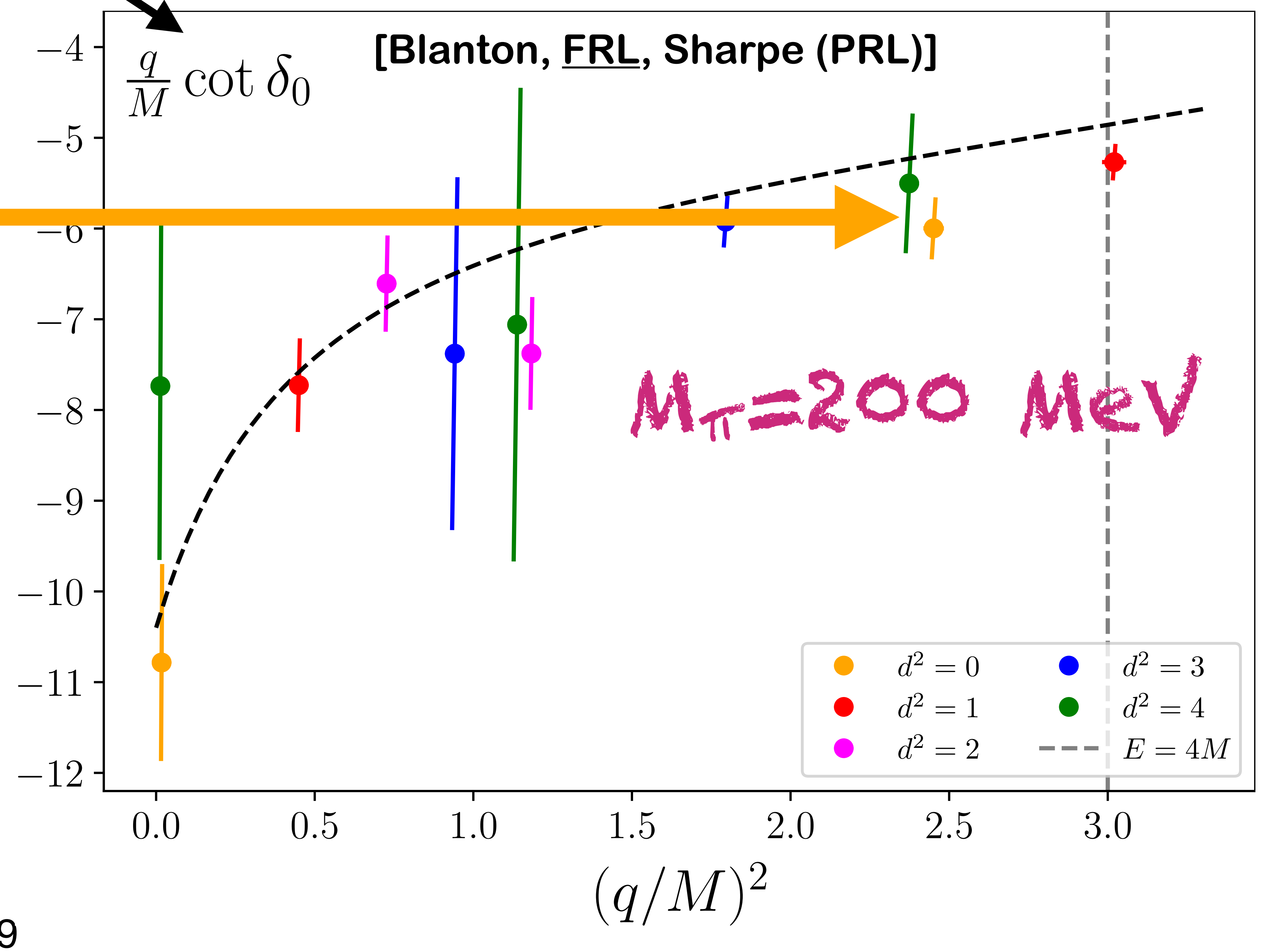

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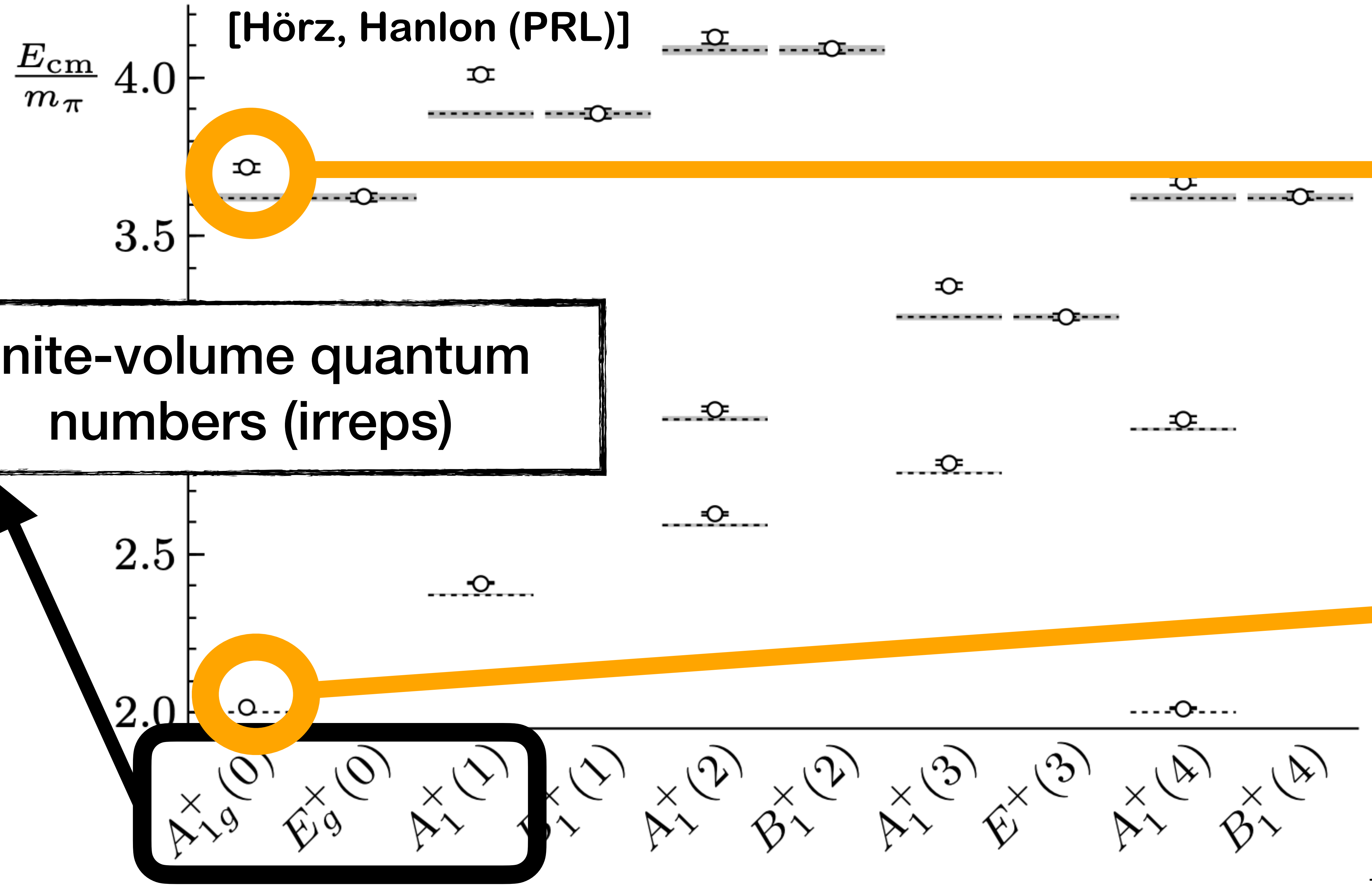

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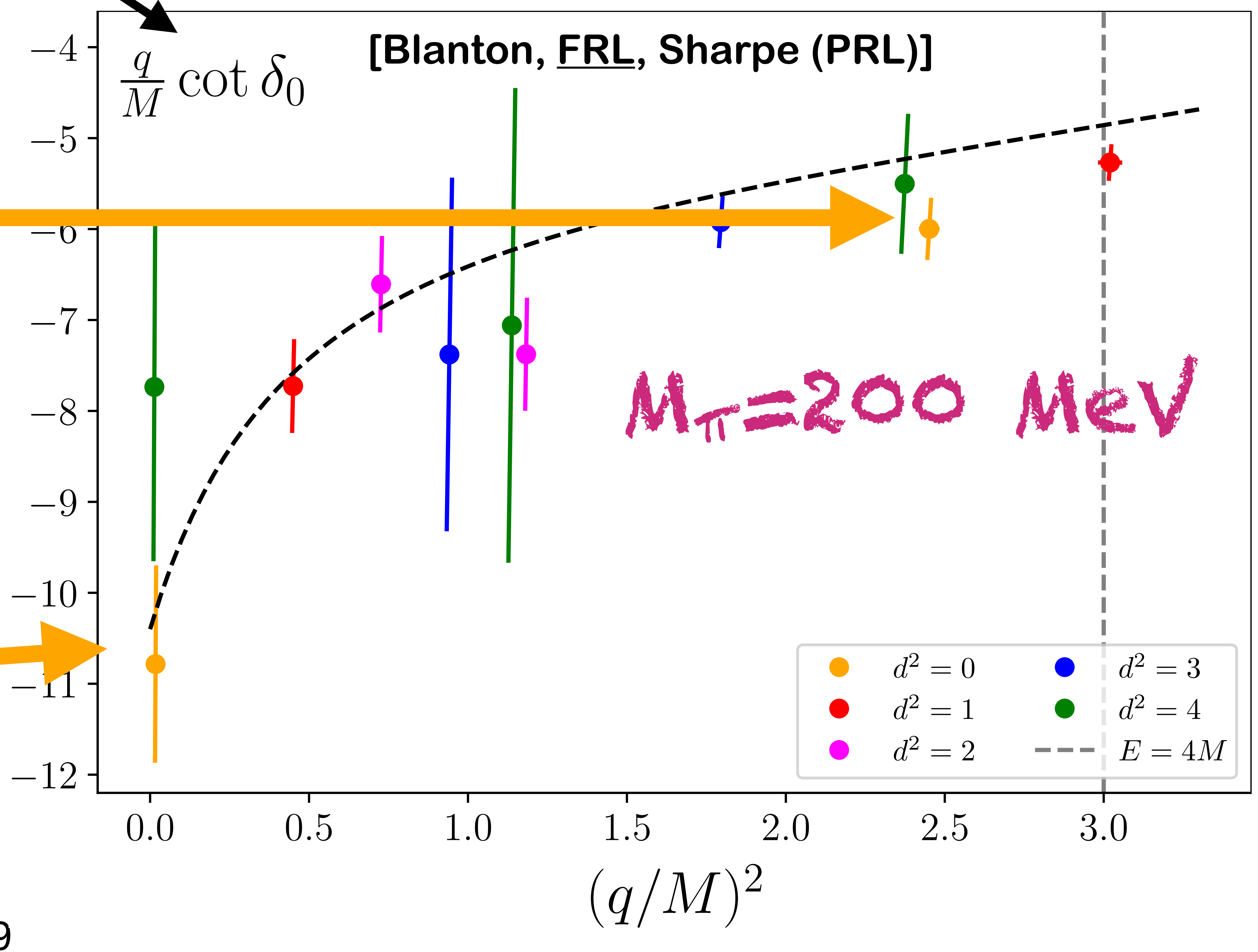

M_π=200 MeV

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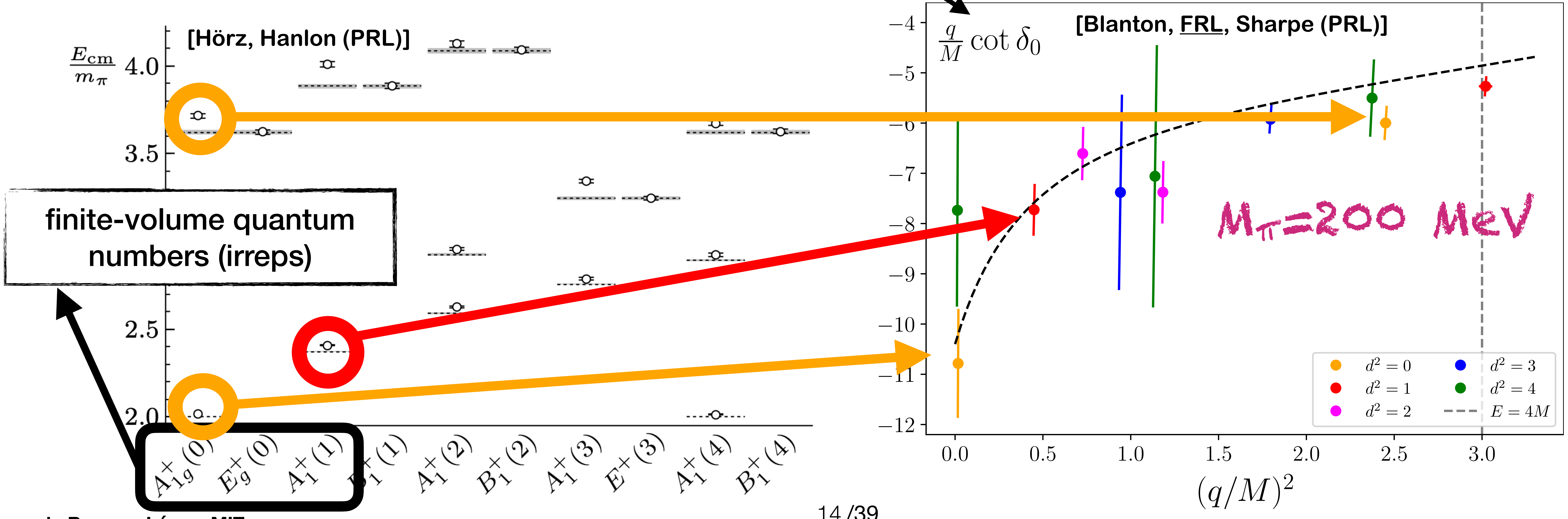
one energy level → a phase shift point



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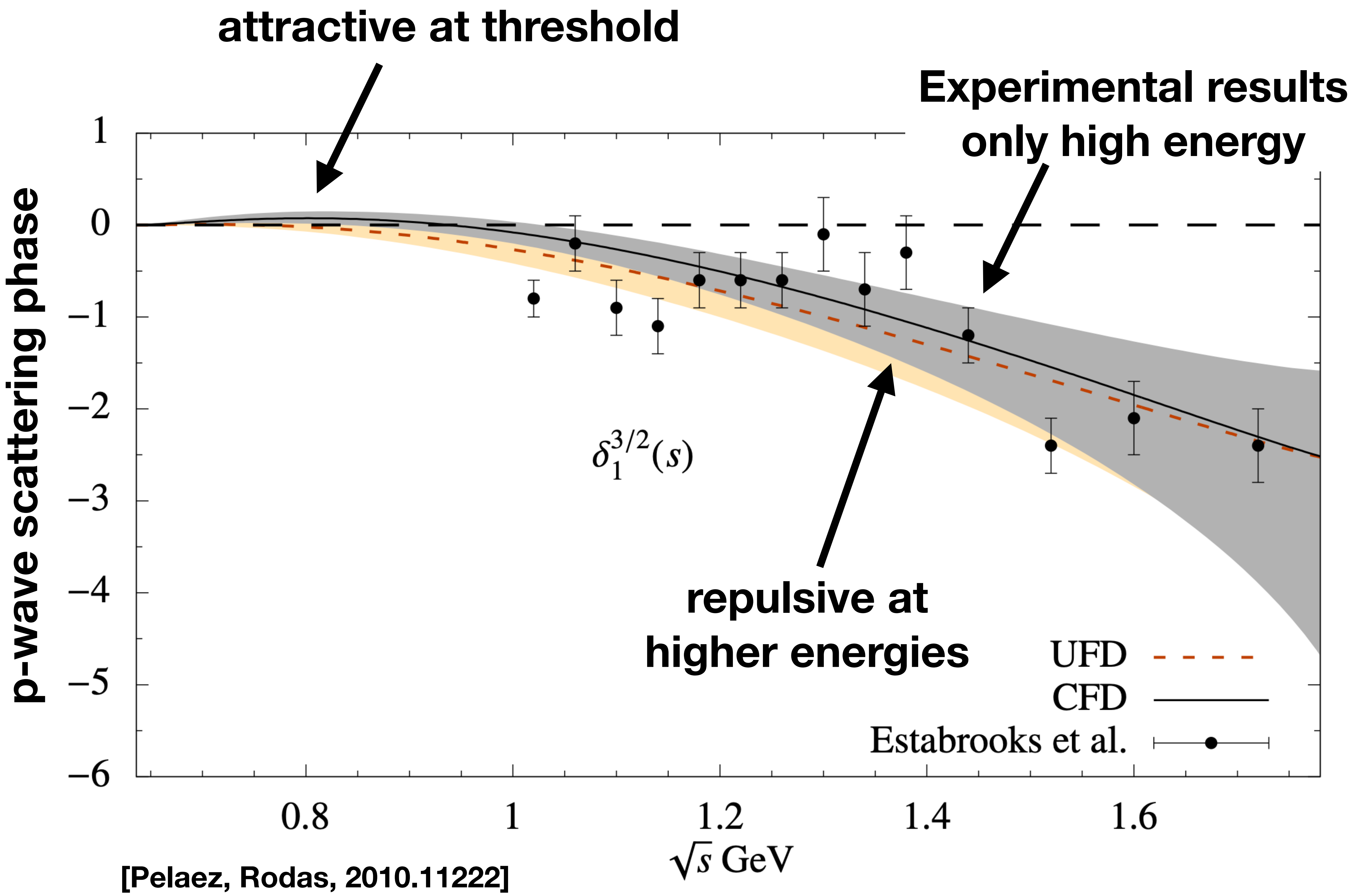
p-wave π⁺K⁺ scattering

- Lattice QCD results are complementary to experiment!



[Pelaez, Rodas, 2010.11222]

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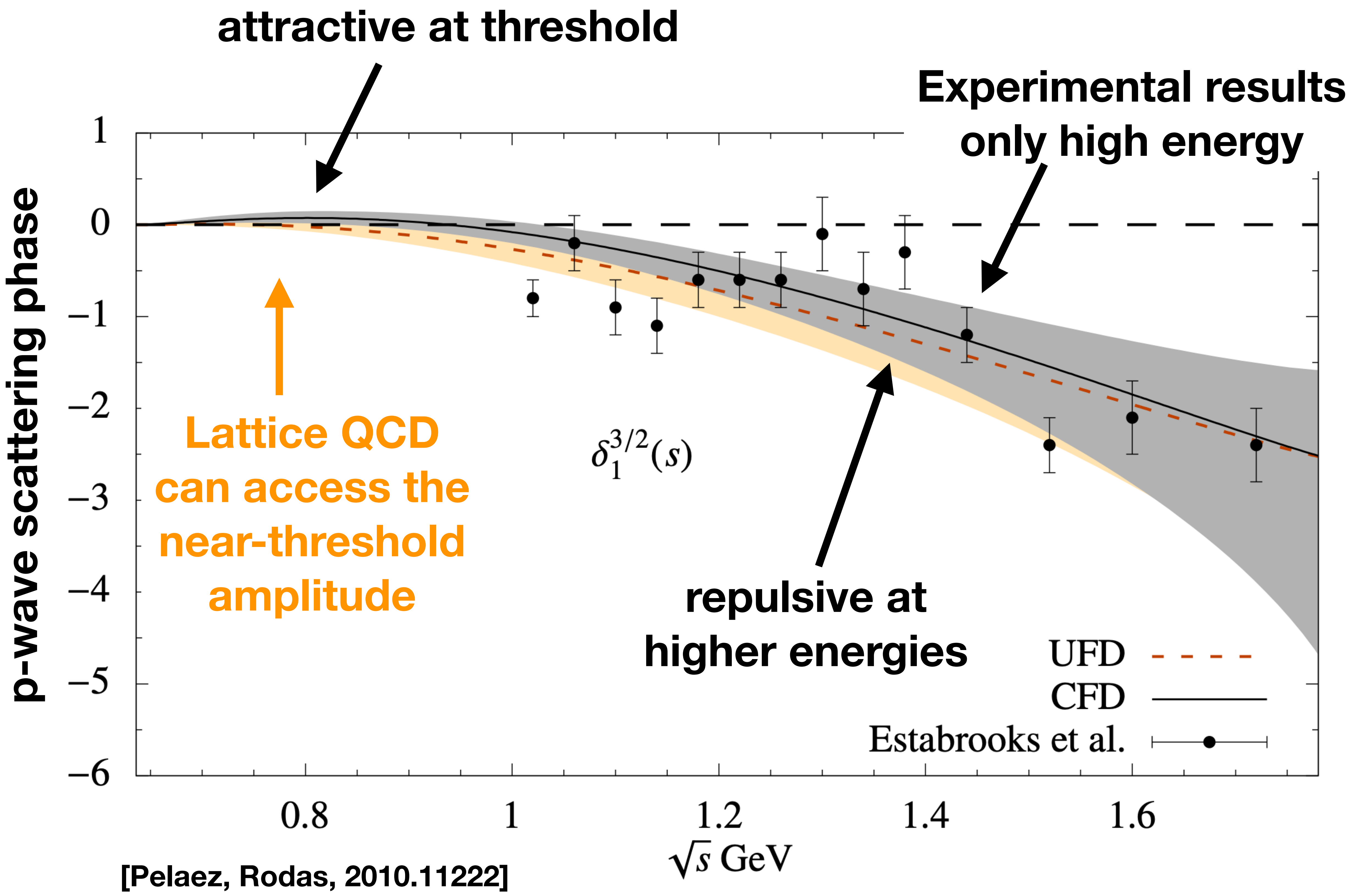
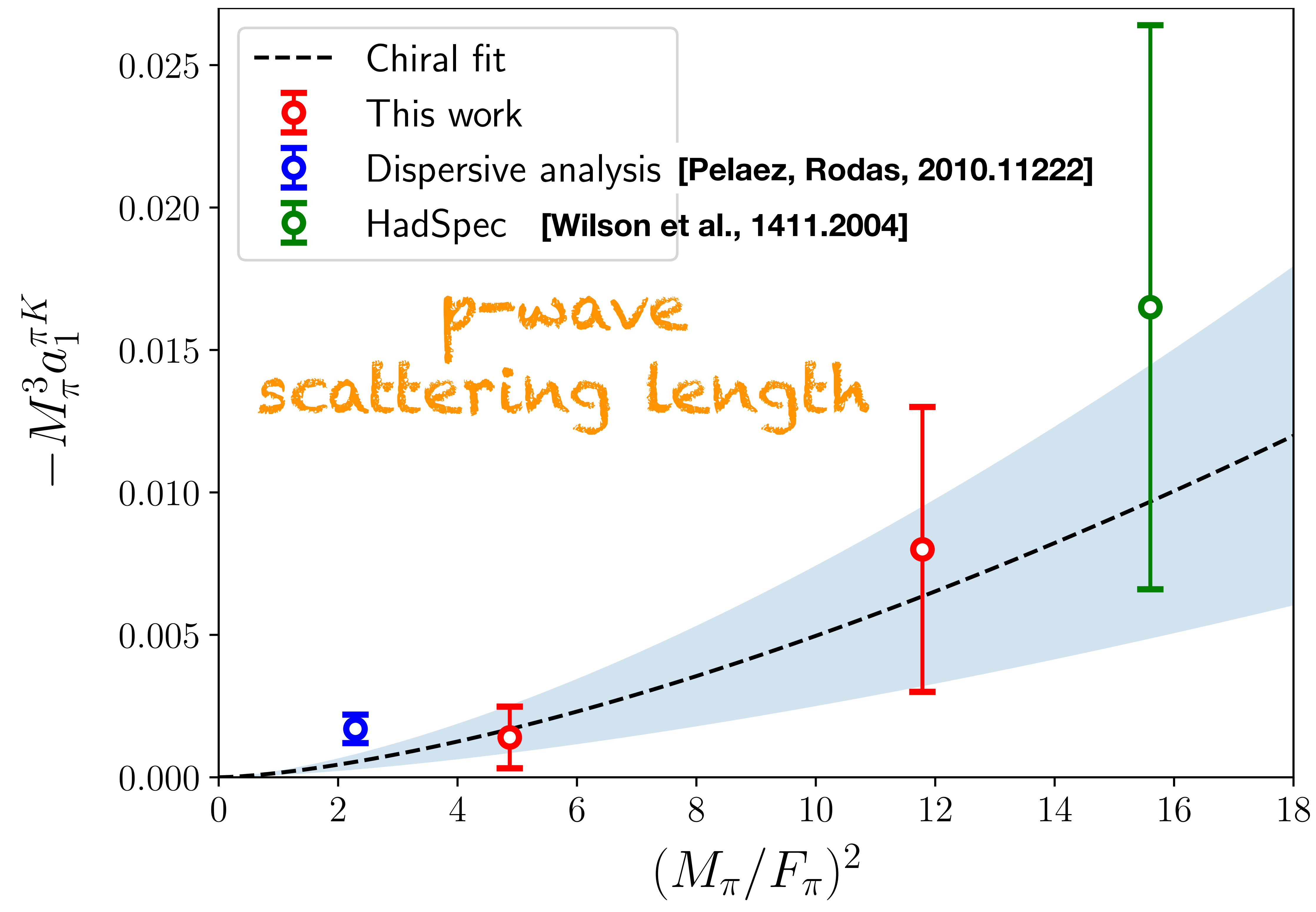


[Pelaez, Rodas, 2010.11222]

[Draper, Hanlon, Hörz, Morningstar, FRL, Sharpe, JHEP 2023]



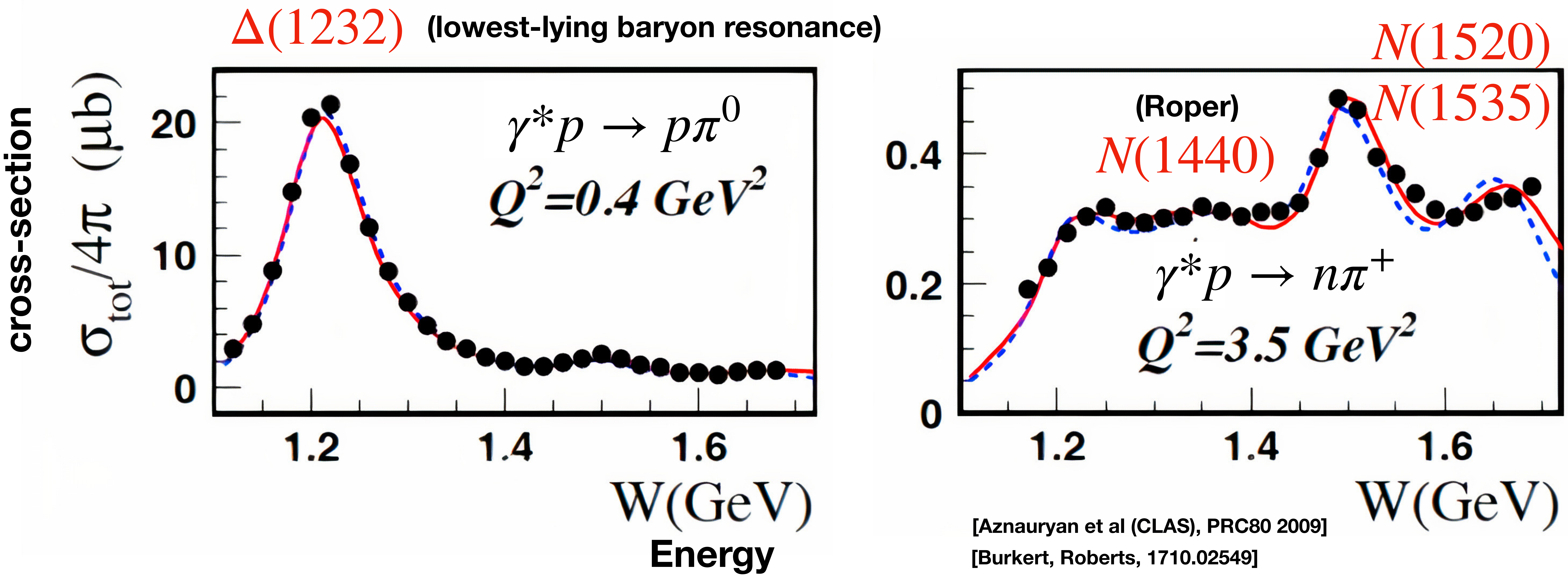
Fernando Romero-López, MIT

πN scattering & the $\Delta(1232)$ resonance

[Bulava, Hanlon, Hörz, Morningstar, Nicholson, FRL, Skinner, Vranas, Walker-Loud, 2208.03867]

πN scattering

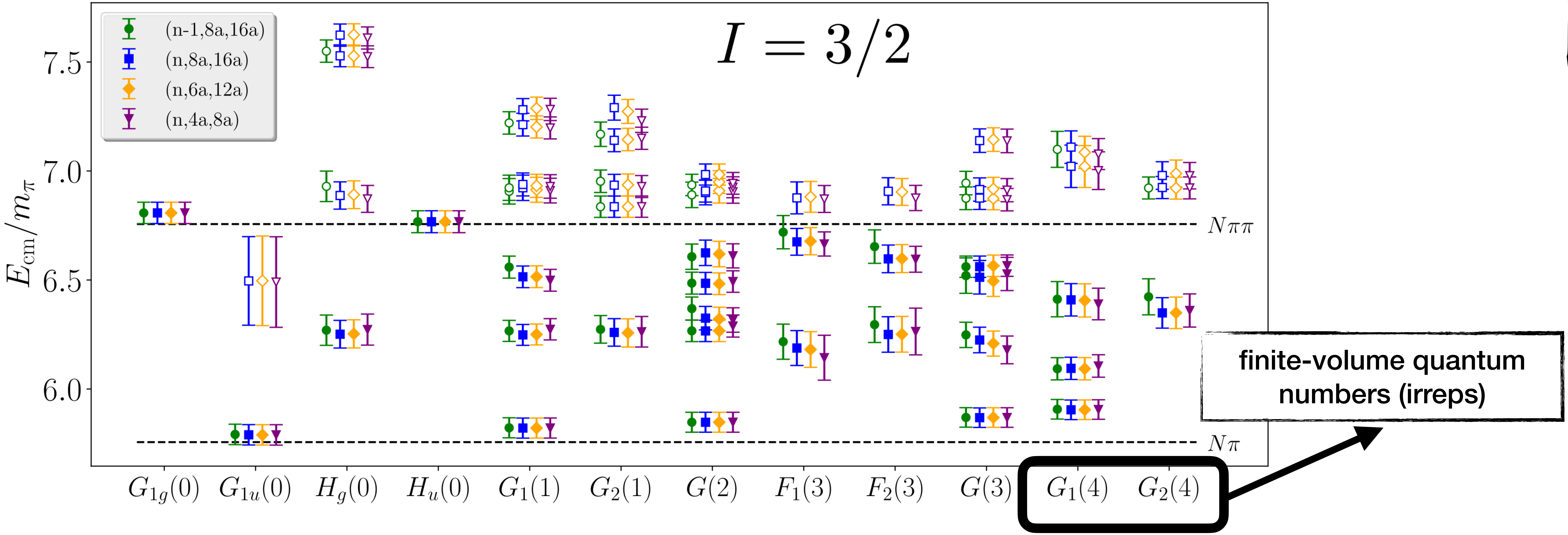
- Pion-nucleon scattering is an important process in QCD



[Aznauryan et al (CLAS), PRC80 2009]
[Burkert, Roberts, 1710.02549]

Lattice QCD πN spectrum

- Key ingredient: reliable variational extractions of the lattice QCD energy levels: GEVP + stability



Fernando Romero-López, MIT

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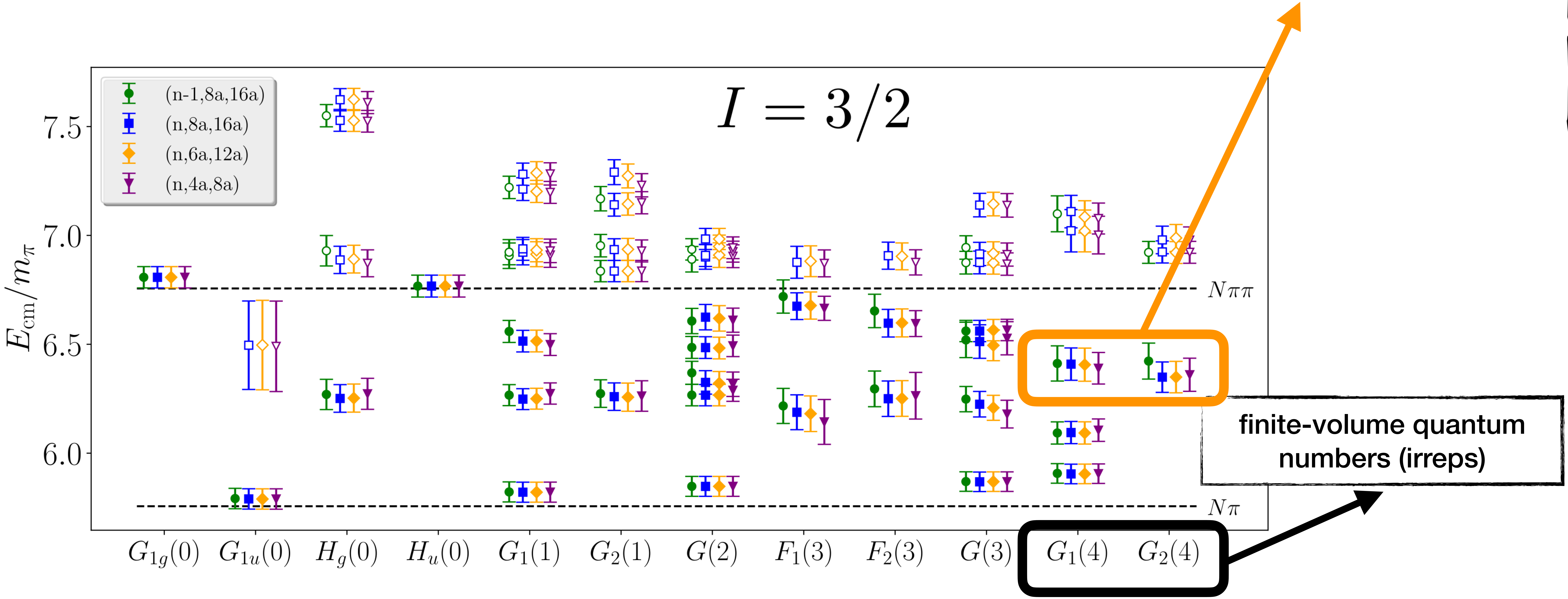
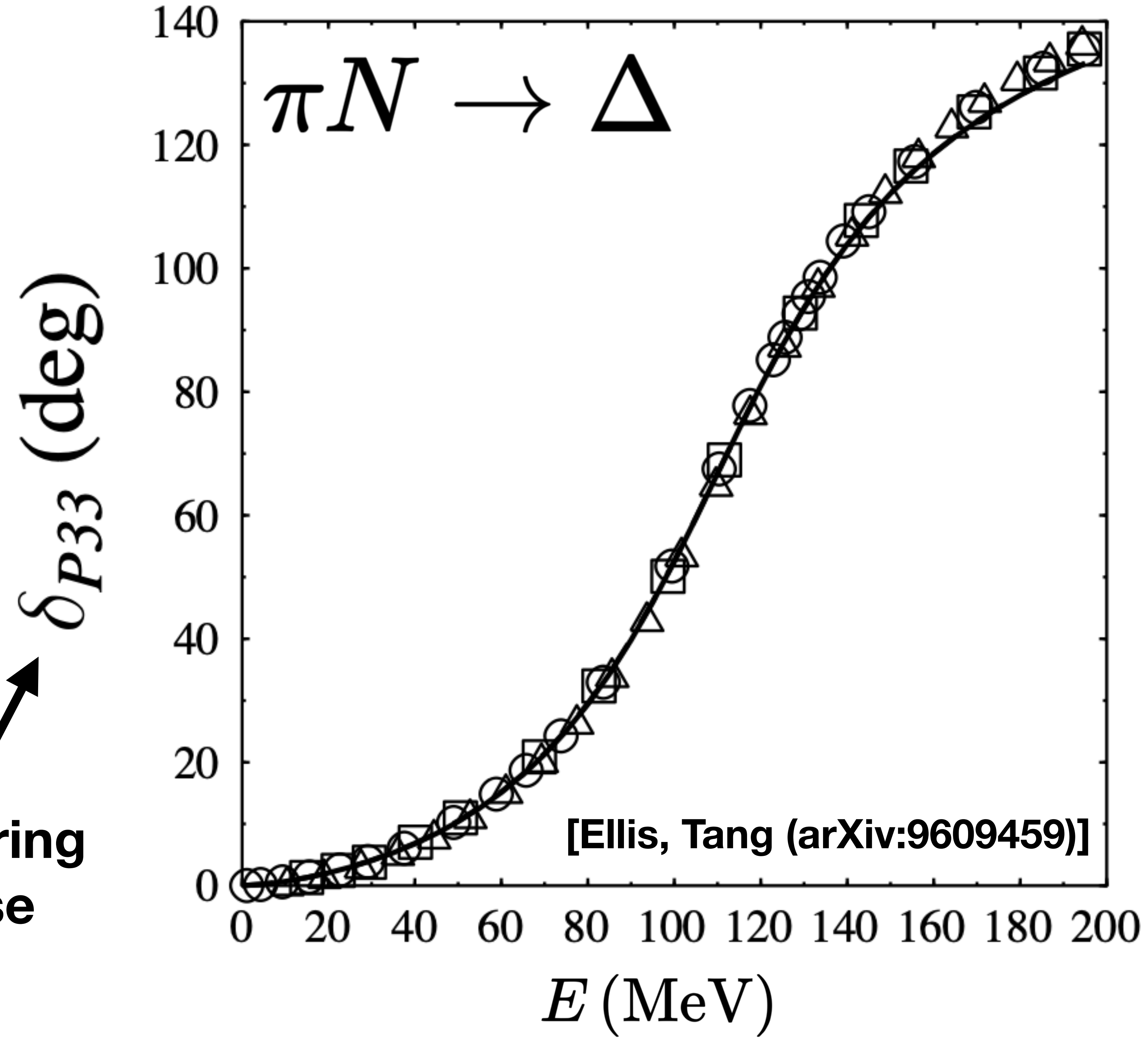


The Δ(1232) resonance

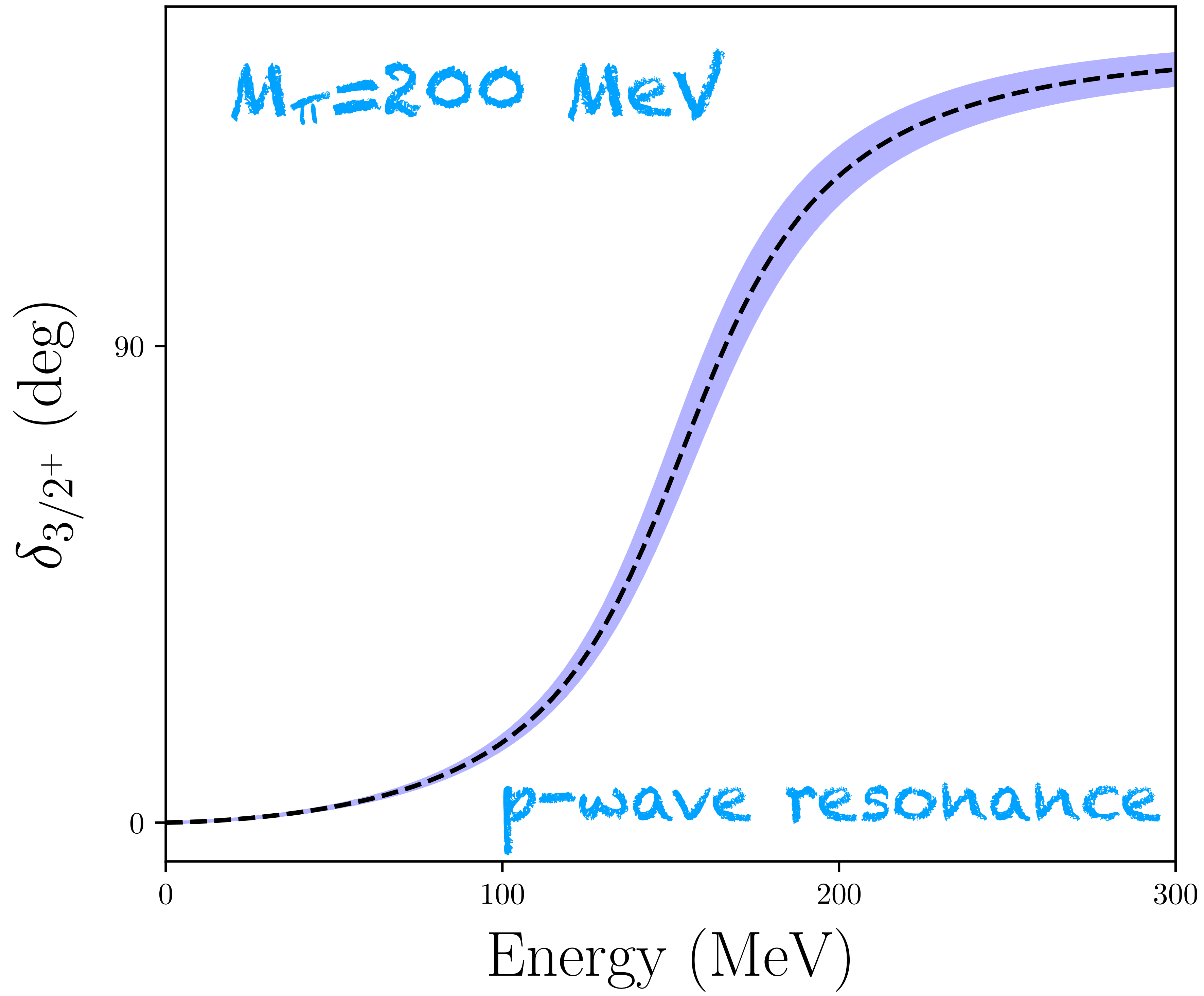
Experiment



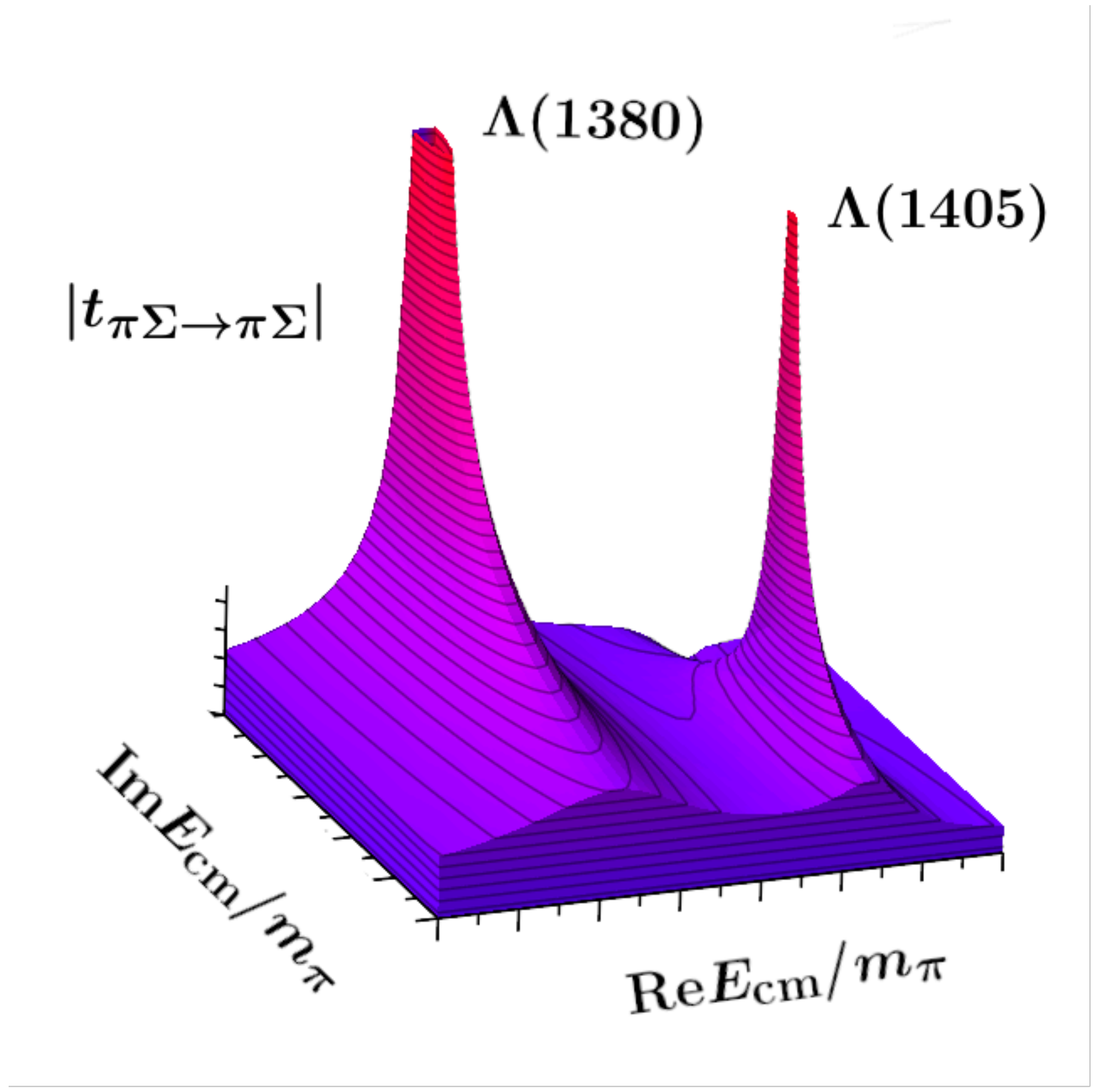
scattering phase

Lattice QCD

[Bulava, Hanlon, Hörz, Morningstar, Nicholson, FRL, Skinner, Vranas, Walker-Loud, 2208.03867]



Λ(1405)

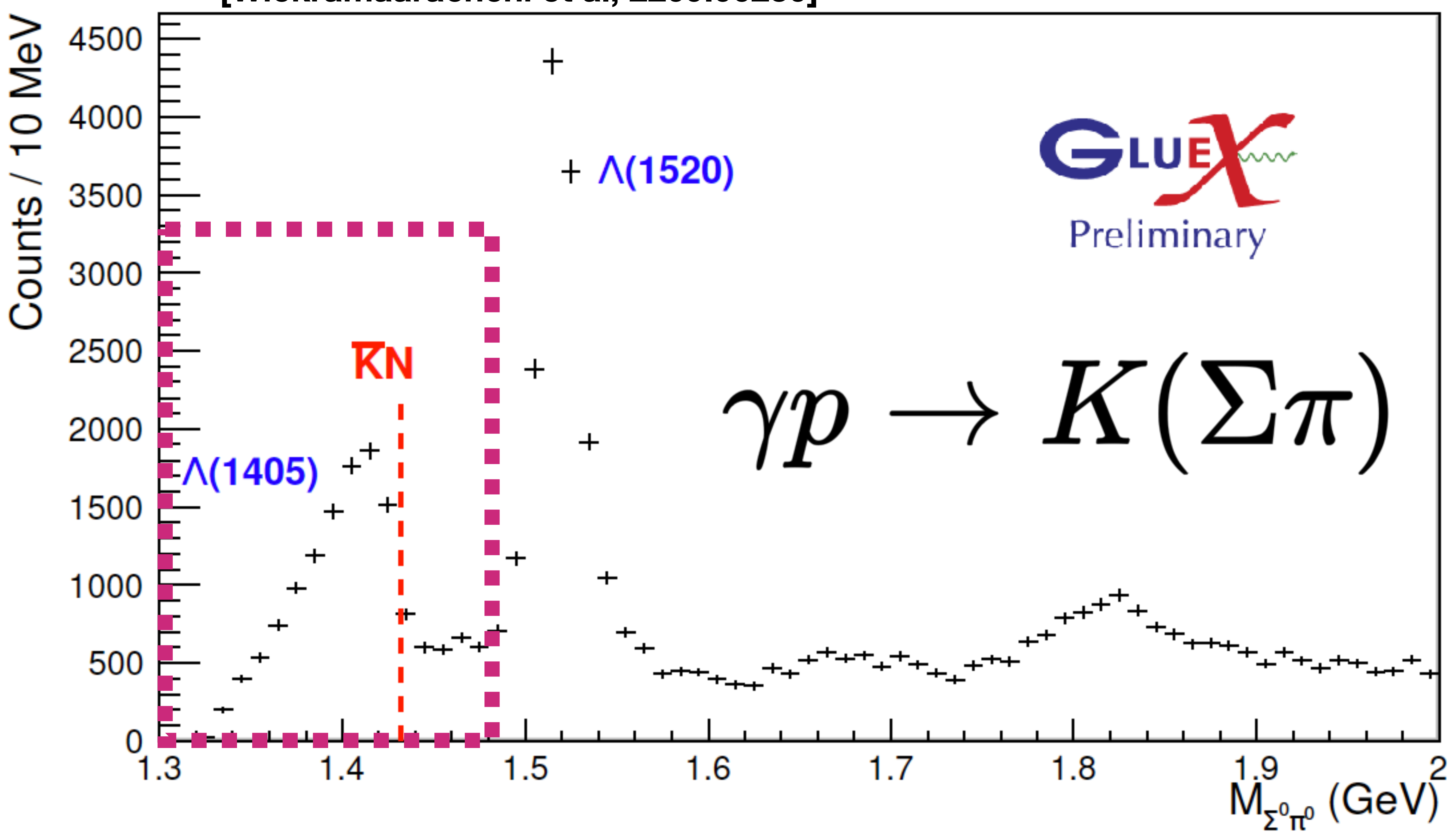


[Bulava, Cid Mora, Hanlon, Hörz, Mohler, Morningstar, Moscoso, Nicholson, FRL, Skinner, Walker-Loud, 2307.13471 & 2307.10413 (PRL Editor's suggestion)]

The Λ(1405) resonance

- **Known since 1950s, still under investigation**
  [Dalitz, Tuan PRL 1959]

[Wickramaarachchi et al, 2209.06230]



- **Appears in coupled-channel scattering**

$$\begin{pmatrix} \pi\Sigma \to \pi\Sigma & \pi\Sigma \to Kp \\ Kp \to \pi\Sigma & Kp \to Kp \end{pmatrix}$$

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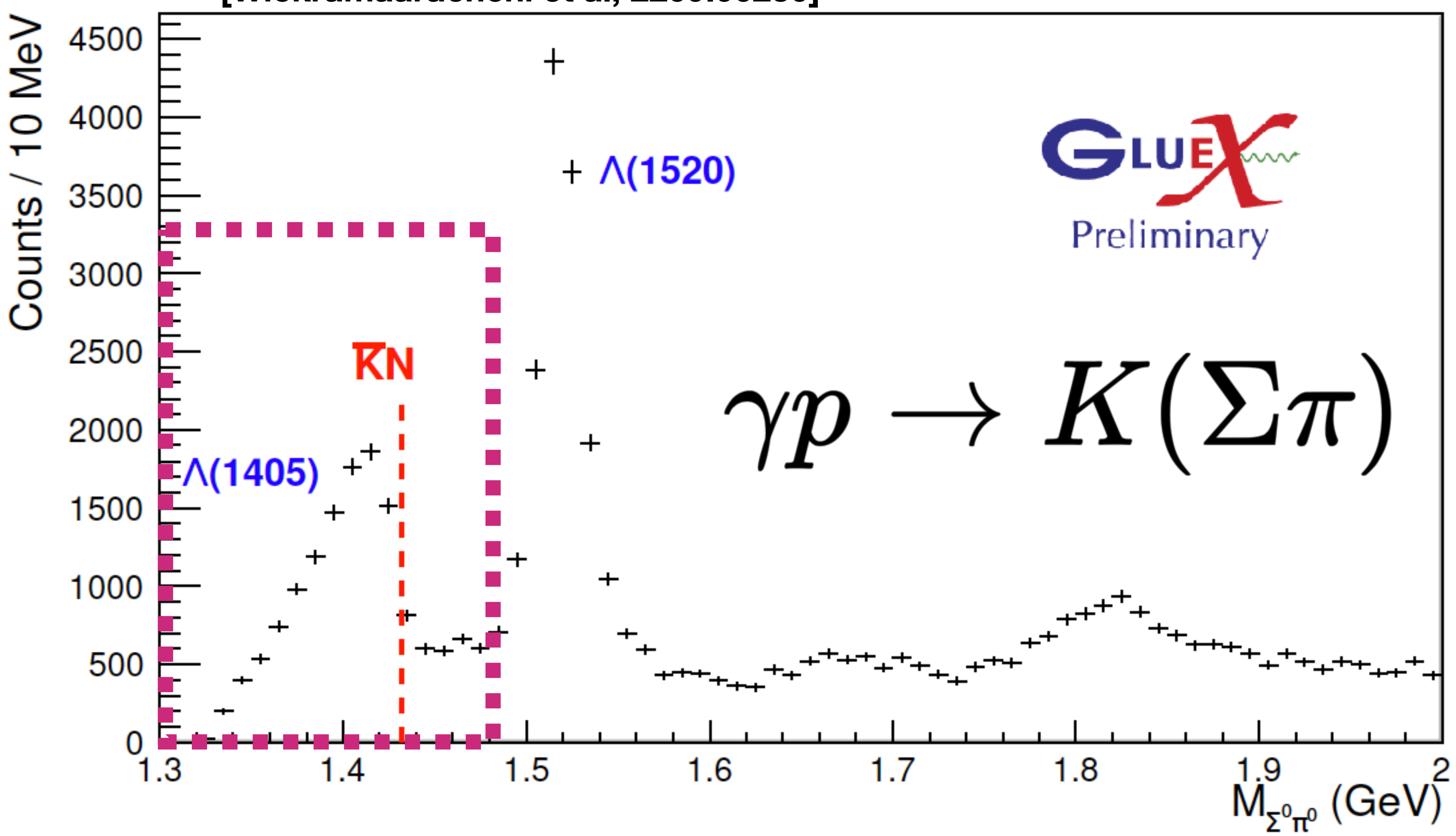

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- **Latest PDG lists two resonances in the energy region**

$\Lambda(1405)\ 1/2^-$	$I(J^P) = 0(\frac{1}{2}^-)$	Status: ∗∗∗∗
$\Lambda(1380)\ 1/2^-$	$J^P = \frac{1}{2}^-$	Status: ∗∗

∗∗∗∗	Existence is certain.
∗∗∗	Existence is very likely.
∗∗	Evidence of existence is fair.
∗	Evidence of existence is poor.

One or two resonances?

The nature of the $\Lambda(1405)$ is a theoretical and experimental challenge

One or two resonances?

The nature of the $\Lambda(1405)$ is a theoretical and experimental challenge

Experiment

- Quantum numbers $J^P = 1/2^-$ @ CLAS
  [CLAS Collaboration, arXiv:1402.22967]
- Different CLAS analysis favor two poles:
  [Mai, Meißner, EPJA 2014] [Roca, Oset, PRC 2013]
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  [BGOOD, arXiv:2108.12235] [ALICE, arXiv:2205.15176]
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Theory

- Simple quark models predict one state
  [Isgur, Karl PRD 1987]
- Chiral Unitarity approach predicts two poles
  [Oller, Meißner, hep-ph/0402261] See review: [Mai, arXiv:2010.00056]
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  [Gubler et al PRD 2016], [Menadue et al PRL 2012], [Engel et al PRD 2013], [Hall et al, PRL 2015] , [Takahashi, Oka, PRD 2010]

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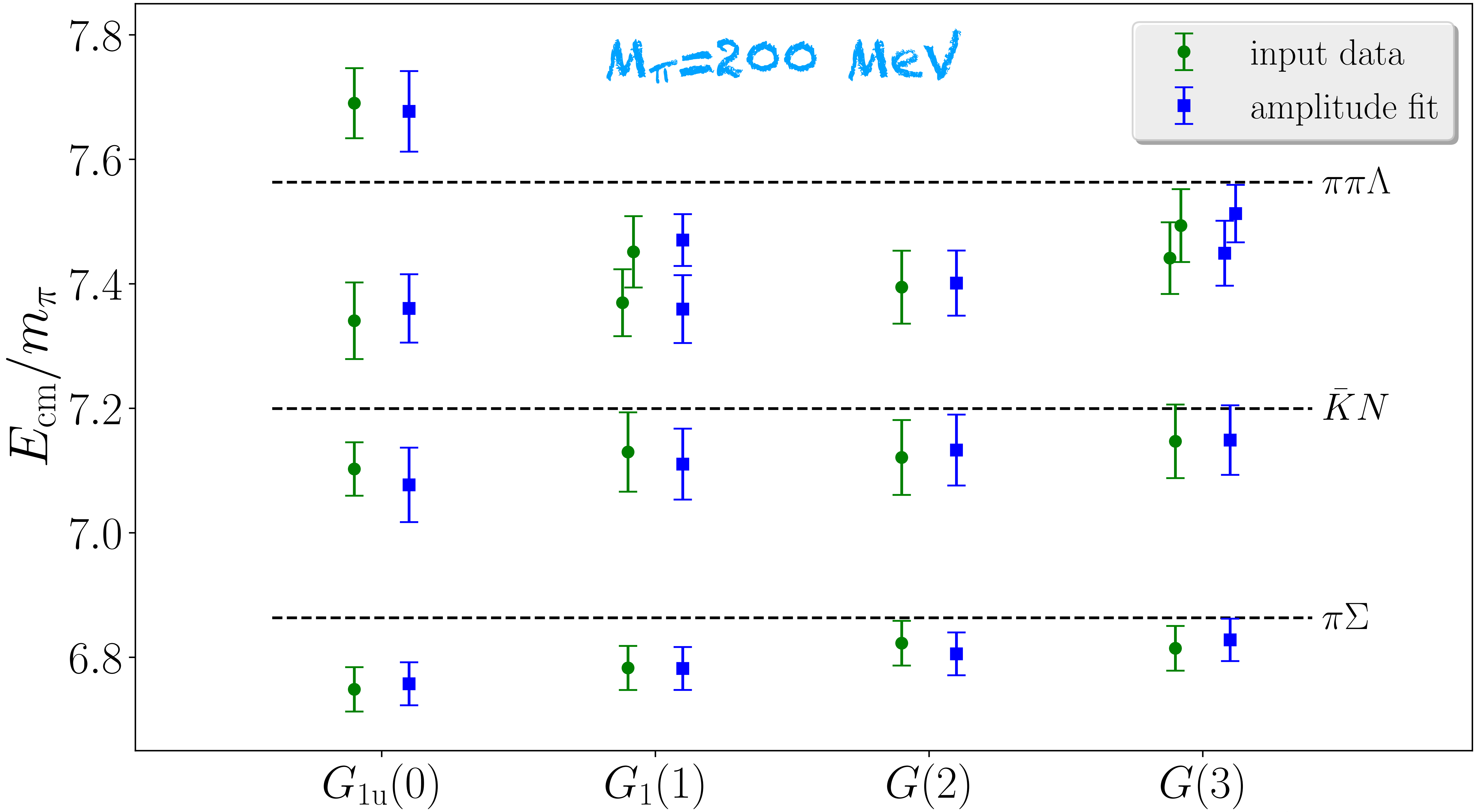
This talk!

The Λ(1405) in Lattice QCD



[Bulava, Cid Mora, Hanlon, Hörz, Mohler, Morningstar, Moscoso, Nicholson, FRL, Skinner, Walker-Loud, 2307.13471 & 2307.10413]

- **Fit with ERE in 2x2 K matrix**

$$(K_2)_{ij} = A_{ij} + B_{ij}E_{\rm cm}^2 + \ldots$$

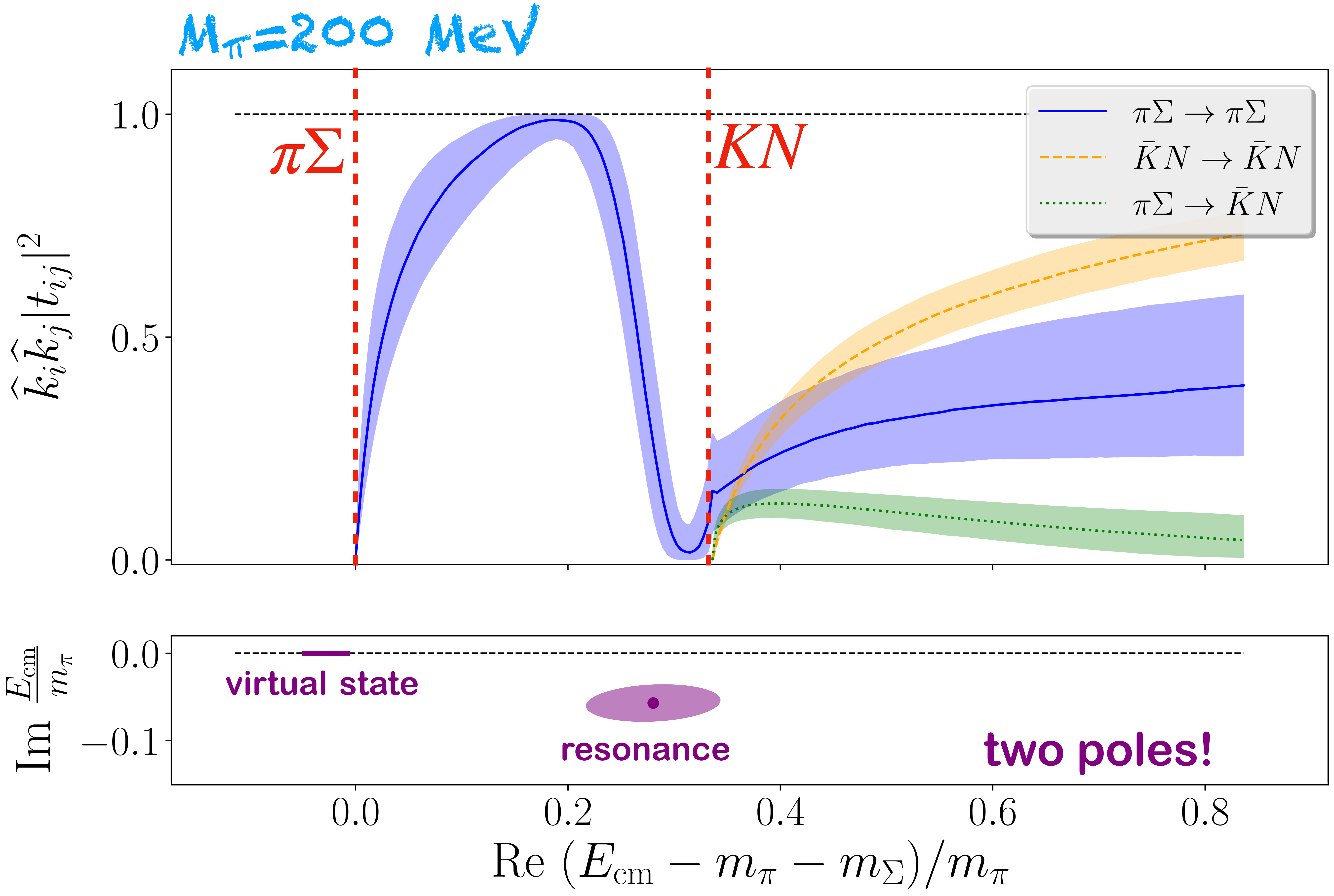
4 parameters
15 energies

$$\chi^2/\mathrm{dof} = 0.96$$

finite-volume quantum numbers (irreps)

Fernando Romero-López, MIT

Amplitudes and poles



- Scattering amplitudes for "preferred" fit

  i.e. with lowest $\mathrm{AIC} = \chi^2 - 2\,\mathrm{dof}$

- Pole positions for "preferred" fit
- All other parametrization find two poles!

[Bulava, Cid Mora, Hanlon, Hörz, Mohler, Morningstar, Moscoso, Nicholson, FRL, Skinner, Walker-Loud, 2307.13471 & 2307.10413]

Double-pole picture

Two poles with $(\text{sign Im}\, k_{\pi\Sigma}, \text{sign Im}\, k_{KN}) = (-,+)$

Virtual bound state

$$E_1 = 1392(9)_{\text{stat}}(2)_{\text{model}}(16)_a \text{ MeV}$$

$$\left|\frac{c^{(1)}_{\pi\Sigma}}{c^{(1)}_{\bar{K}N}}\right| = 1.9(4)_{\text{stat}}(6)_{\text{model}}$$

Stronger coupling to $\pi\Sigma$

ratio of residues of the pole

Resonance pole

$$E_2 = [1455(13)_{\text{stat}}(2)_{\text{model}}(17)_a - i \times 11.5(4.4)_{\text{stat}}(4.0)_{\text{model}}(0.1)_a] \text{ MeV}$$

$$\left|\frac{c^{(2)}_{\pi\Sigma}}{c^{(2)}_{\bar{K}N}}\right| = 0.53(9)_{\text{stat}}(10)_{\text{model}}$$

Stronger coupling to KN

- ☑ Qualitative agreement with chiral approaches
  [See PDG, section 83]

$$\text{Re } E_1 = 1325 - 1380 \text{ MeV}$$

$$\text{Re } E_2 = 1421 - 1434 \text{ MeV}$$

- Poles are at slightly larger energies
- Lower pole on the real axis
  - Unphysical pion mass effect?

$M_\pi = 200$ MeV

Three-particle processes

Why three particles?

- The two-body formalism is restricted to few interesting resonances
  - **Exotics:** $T_{cc} \to DD^*, DD\pi$
  - **Roper:** $N(1440) \to \Delta\pi \to N\pi\pi$

Resonance	$I_{\pi\pi\pi}$	J^P
$\omega(782)$	0	1^-
$h_1(1170)$	0	1^+
$\omega_3(1670)$	0	3^-
$\pi(1300)$	1	0^-
$a_1(1260)$	1	1^+
$\pi_1(1400)$	1	1^-
$\pi_2(1670)$	1	2^-
$a_2(1320)$	1	2^+
$a_4(1970)$	1	4^+

(with $\geq 3\pi$ decay modes)

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- ☑ Major developments in the three-particle finite-volume formalism

[Hansen, Sharpe, PRD 2014 & 2015], [Hammer, Pang, Rusetsky, JHEP 2017] x 2
[Mai, Döring, EPJA 2017]
[...]
[Blanton, FRL, Sharpe, JHEP 2019], [Hansen, FRL, Sharpe, JHEP 2020]
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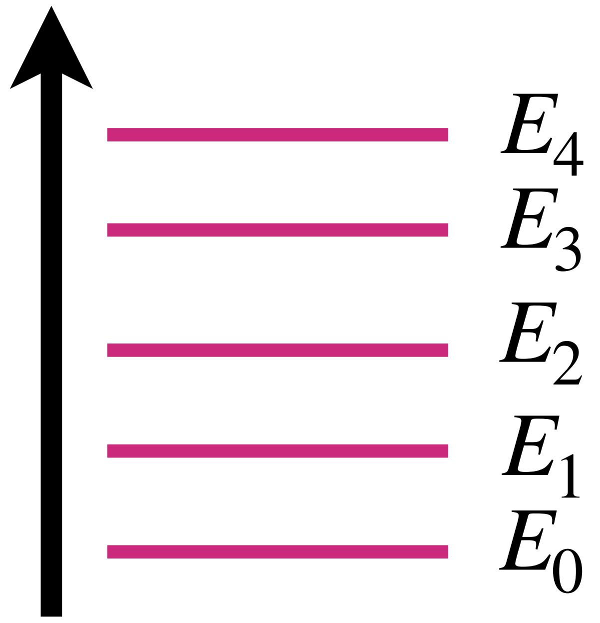
Relativistic three-particle formalism for identical particles

[Hansen, Sharpe, PRD 2014 & 2015]

2π Spectrum



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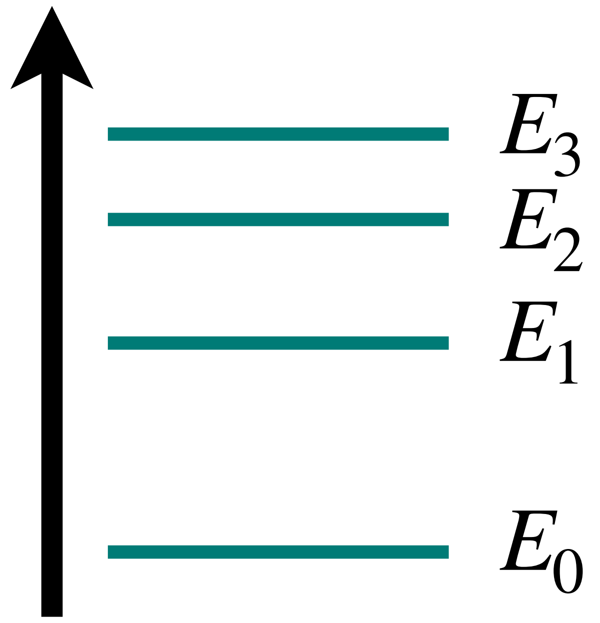

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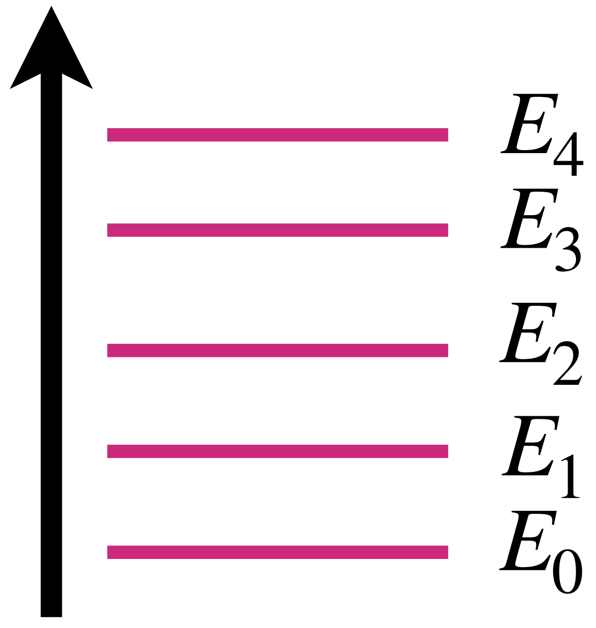

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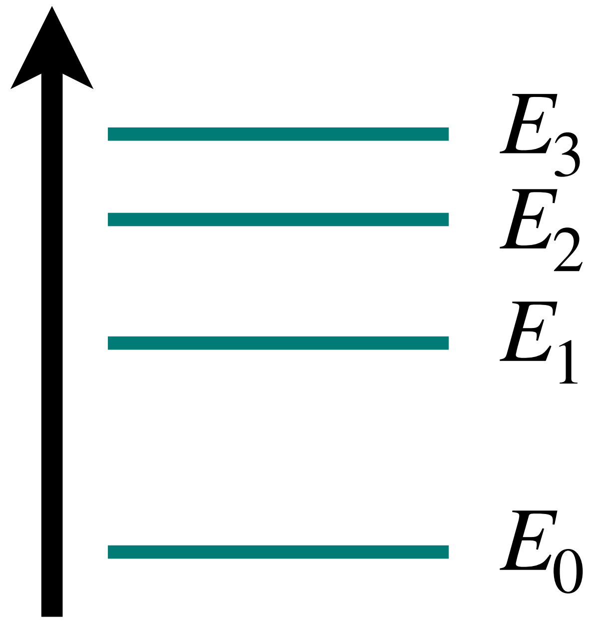

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Matrix indices describe three on-shell particles

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2π Spectrum

E_4, E_3, E_2, E_1, E_0

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Fit →

K-matrices

$\mathcal{K}_2$

$\mathcal{K}_{df,3}$

Parametrize:

$$\mathcal{K}_2 = c_0 + c_1 k^2 + \ldots$$

$$\mathcal{K}_{\mathrm{df},3} = \mathcal{K}_{\mathrm{df},3}^{\mathrm{iso},0} + \mathcal{K}_{\mathrm{df},3}^{\mathrm{iso},1}\left(\frac{s-9m^2}{9m^2}\right) + \ldots$$

[Blanton, FRL, Sharpe, JHEP 2019]

Fernando Romero-López, MIT

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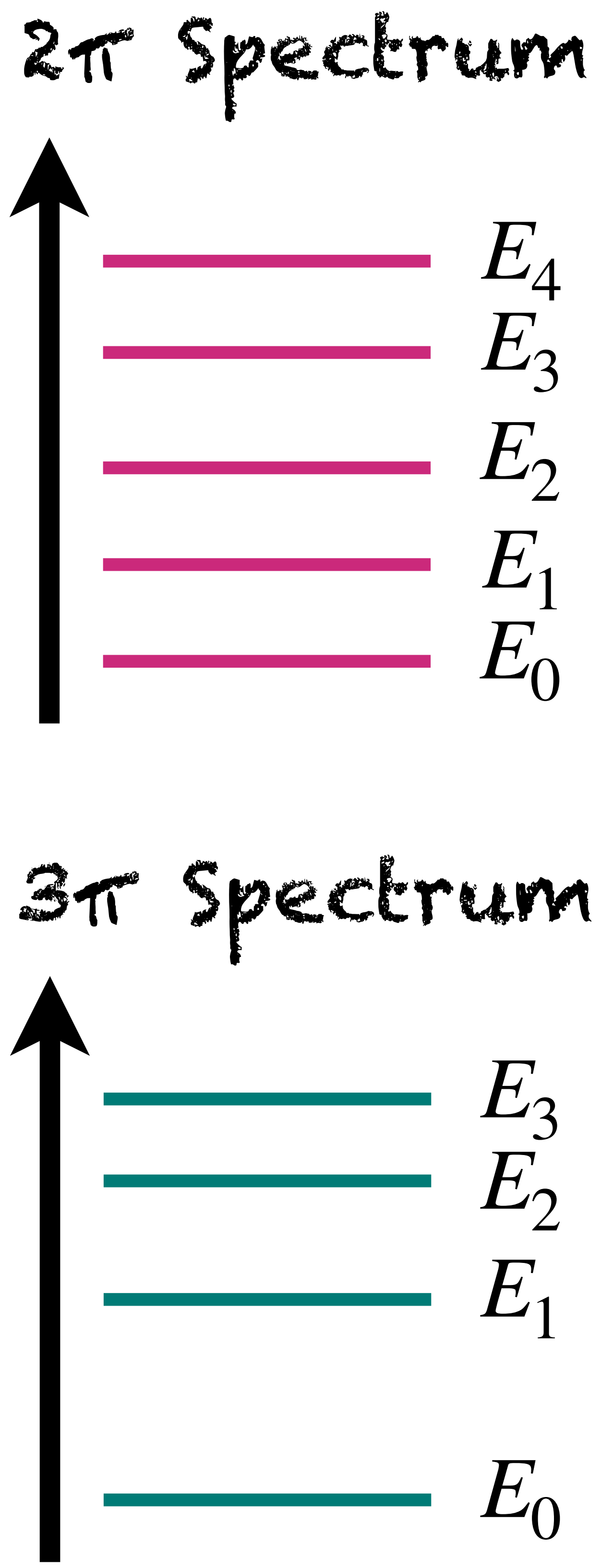

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Unitarity relations

Integral equations

[Briceño et al., PRD 2018]
[Hansen et al., PRL 2021]
[Jackura et al., PRD 2021]
[Dawid et al., 2303.04394]

Scattering amplitudes

$\mathcal{M}_2$

$\mathcal{M}_3$

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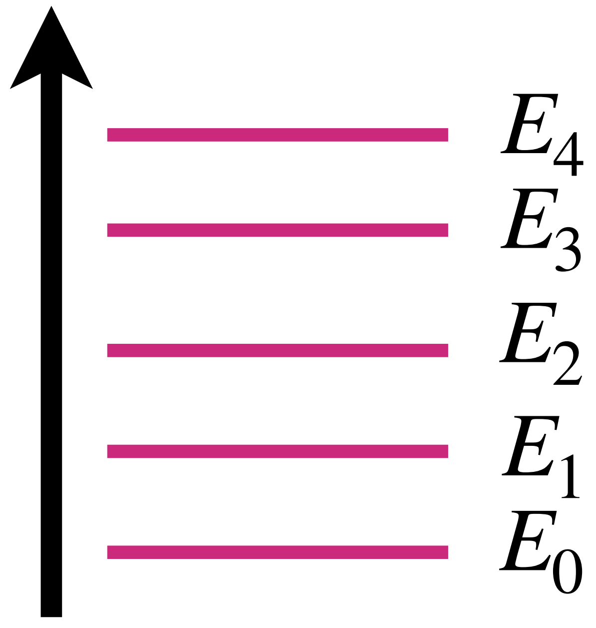

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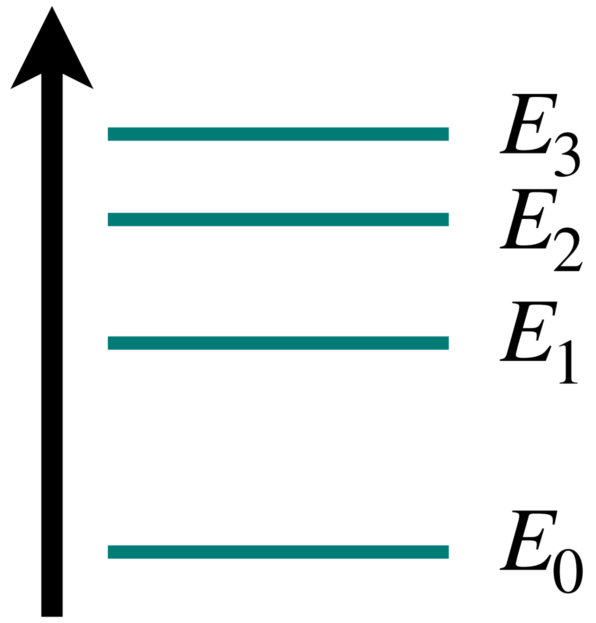

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↕

Experiments & resonance properties

Three-meson systems

- Three-particle formalism applied to weakly-interacting (non-resonant) systems: $\pi^+\pi^+\pi^+,\ \pi^+\pi^+K^+$

  [Blanton … FRL … et al., PRL 2020 & JHEP 2021], [Draper … FRL … et al., JHEP 2023], [Fischer … FRL … et al, EPJC 2021]

  [Alexandrou et al, Brett et al, Culver et al, Hansen et al, Mai et al]

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Example: $\pi^+\pi^+\pi^+$ scattering

Lattice data: [Blanton, Hanlon, Hörz, Morningstar, FRL, Sharpe, JHEP 2021]

NLO ChPT: [Baeza-Ballesteros, Bijnens, Husek ,FRL, Sharpe, Sjö, JHEP 2023]

parametrized by the three-particle K-matrix

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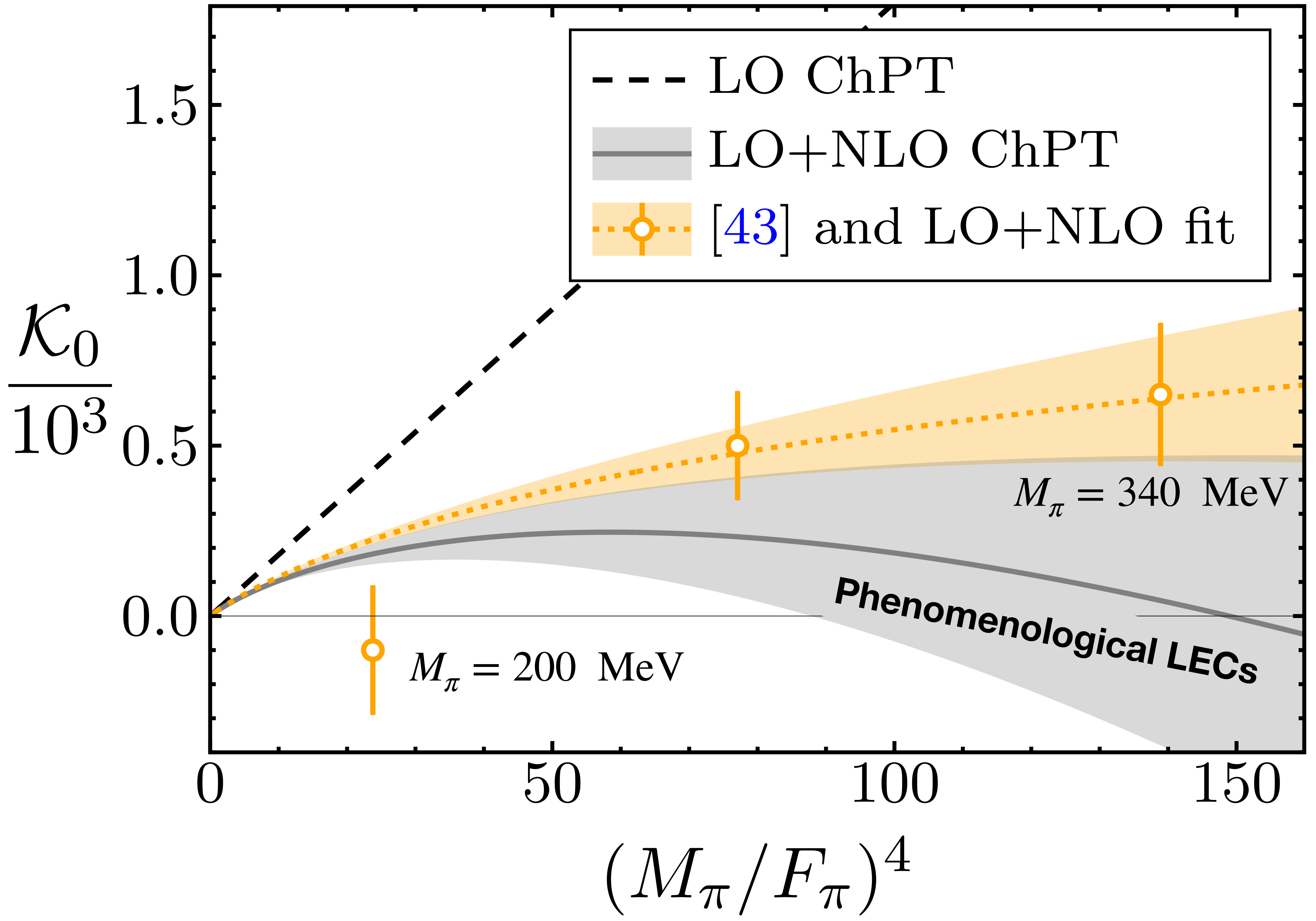

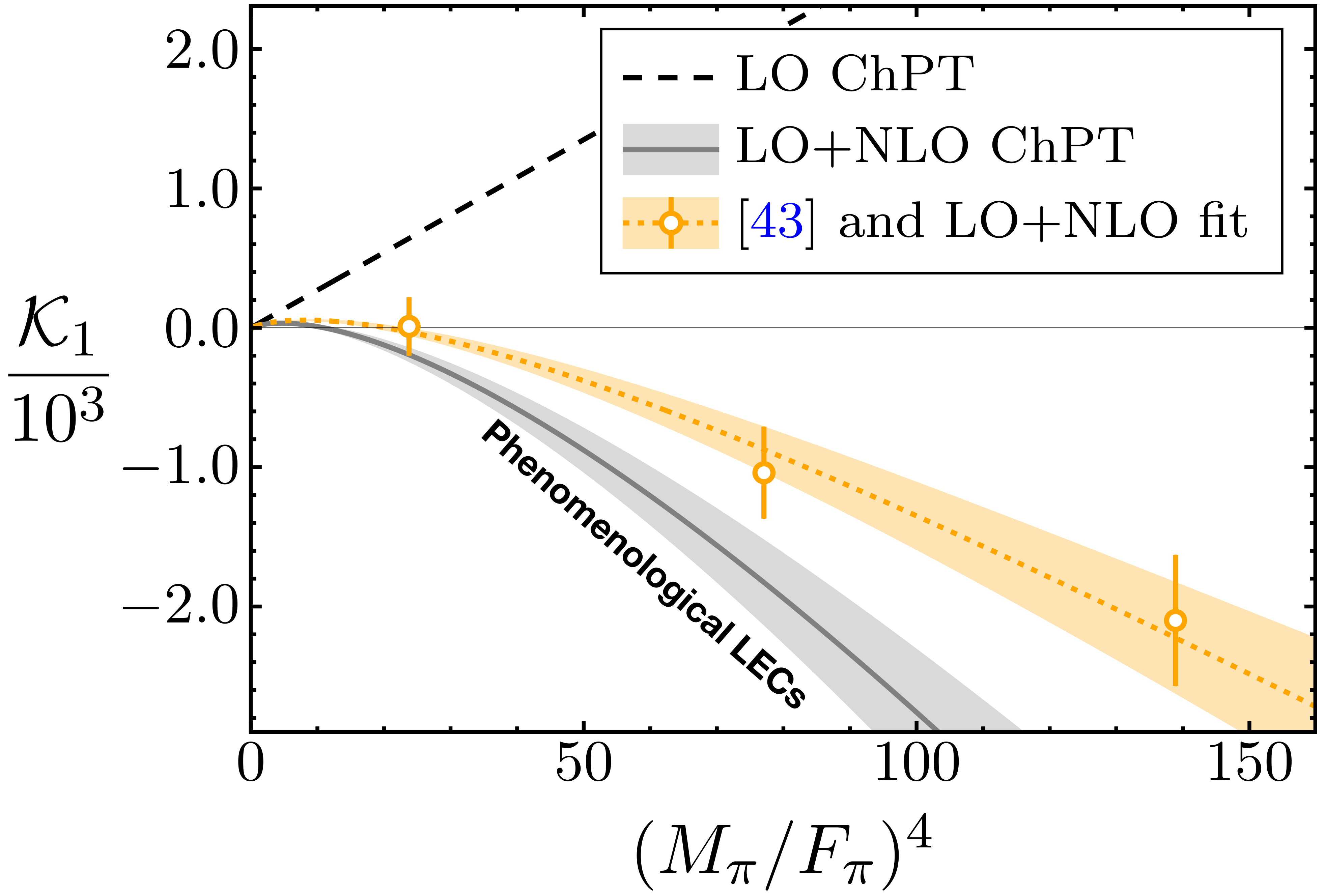

Three-body resonances

- Formalism to study relevant three-pion resonances is available
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- Extract resonance properties on a toy model [Garofalo, Mai, FRL, Rusetsky, Urbach (2211.05605)]

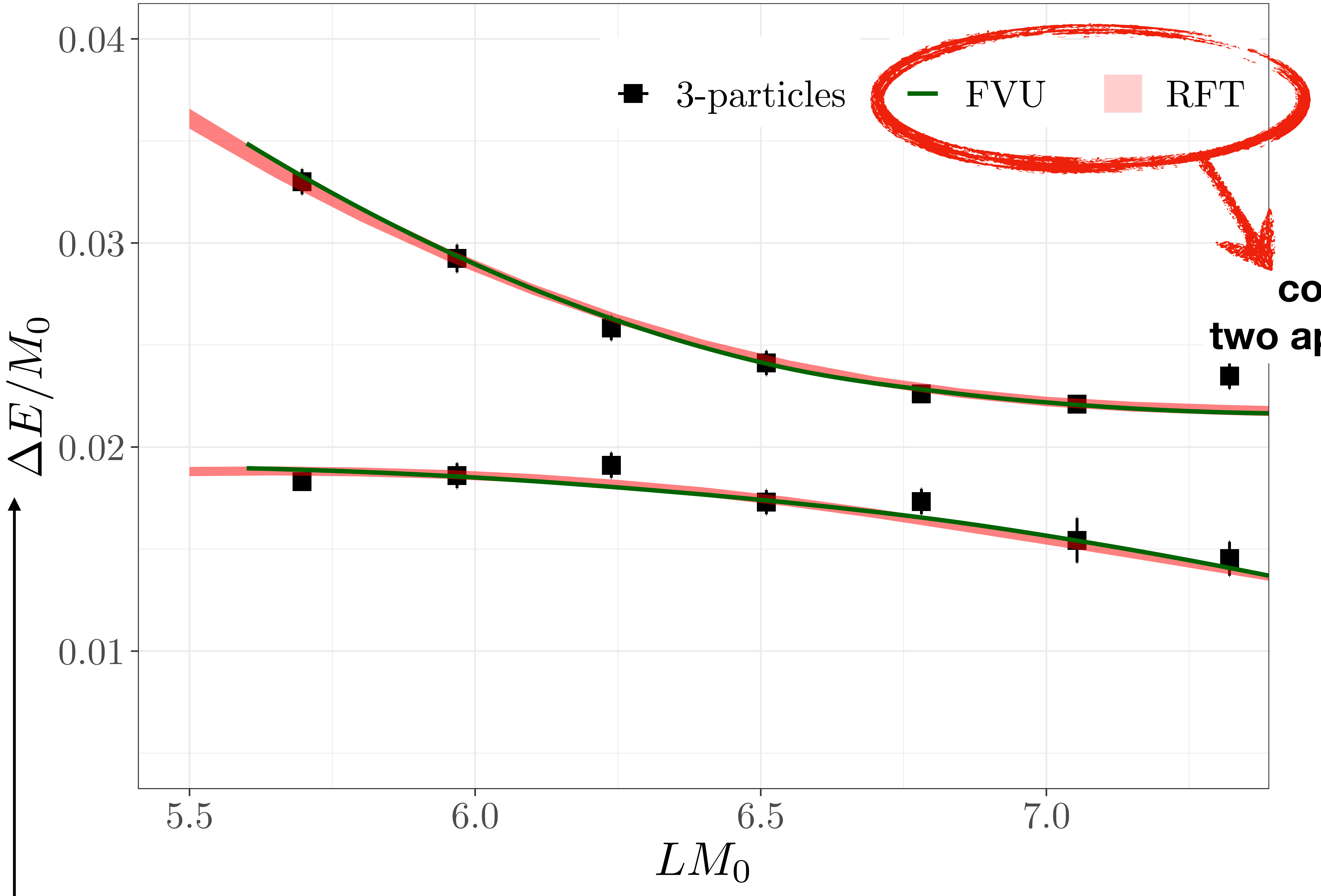
$$\mathcal{L} = \sum_{i=0,1} \left(\partial_\mu \phi_i^\dagger \partial_\mu \phi_i + m_i \phi_i^\dagger \phi_i + \lambda_i \phi_i^4 \right) + \frac{g}{2} \phi_1^\dagger \phi_0^3 + h.\,c.$$

Induces transitions:

$$\phi_1 \to 3\phi_0$$
$$(M_1 > 3M_0)$$

  - Test formalism in a controlled setup
  - Computationally cheaper

Three-body spectrum

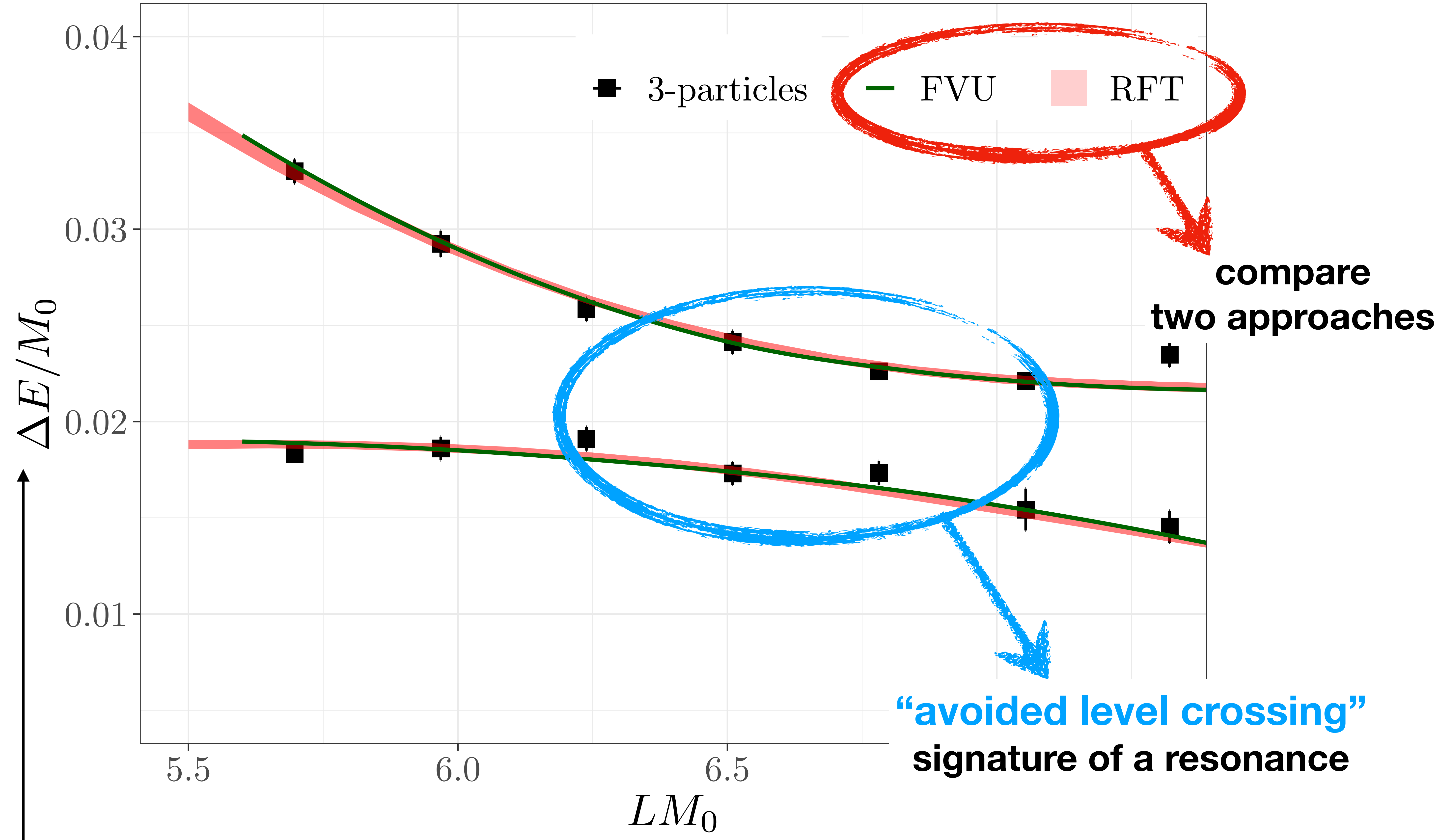


compare two approaches

$\Delta E = E - 3M_0$

[Garofalo, Mai, FRL, Rusetsky, Urbach (2211.05605)]

Three-body spectrum



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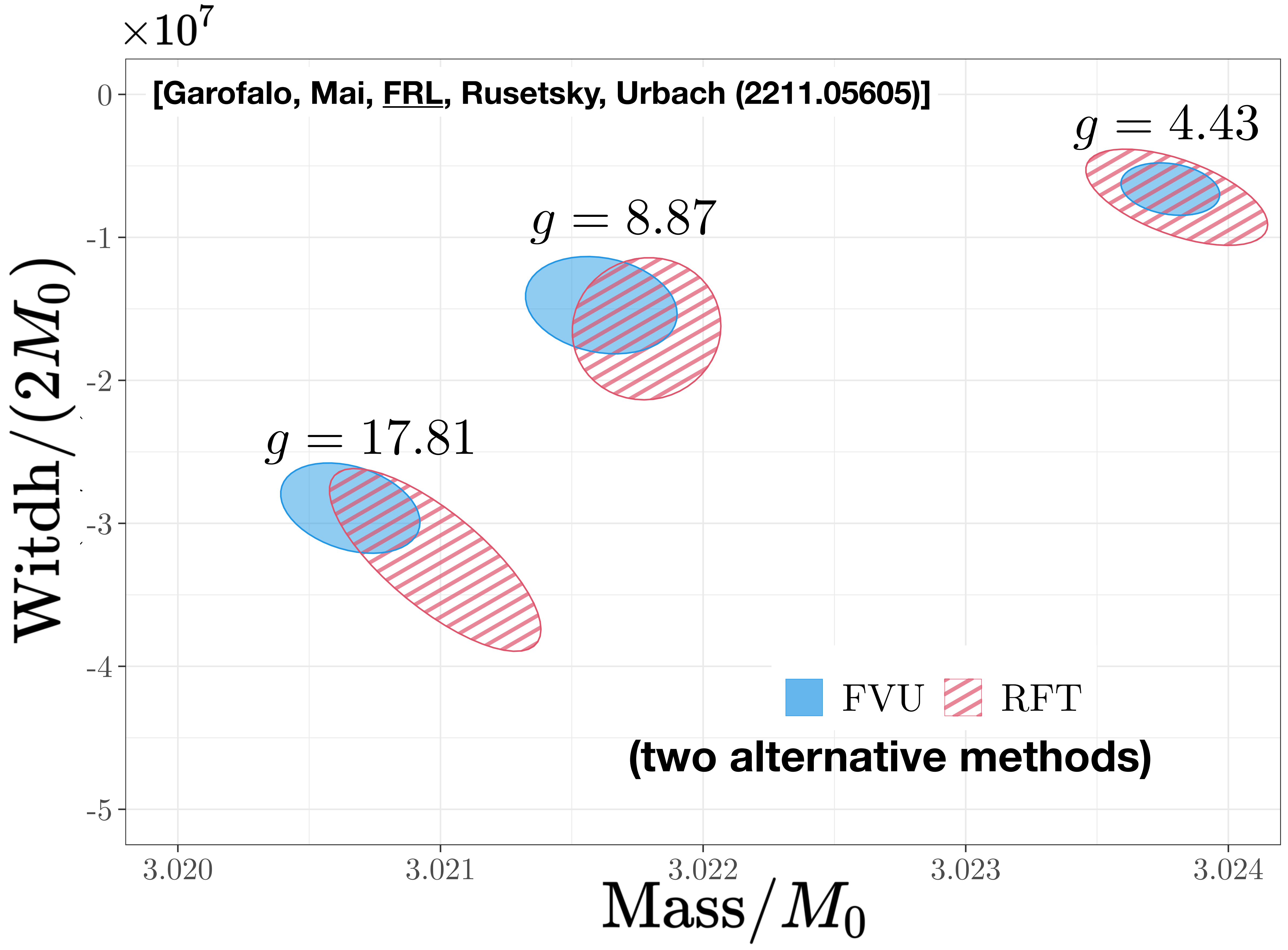
A toy three-body resonance



$$3\phi_0 \to \phi_1 \to 3\phi_0$$

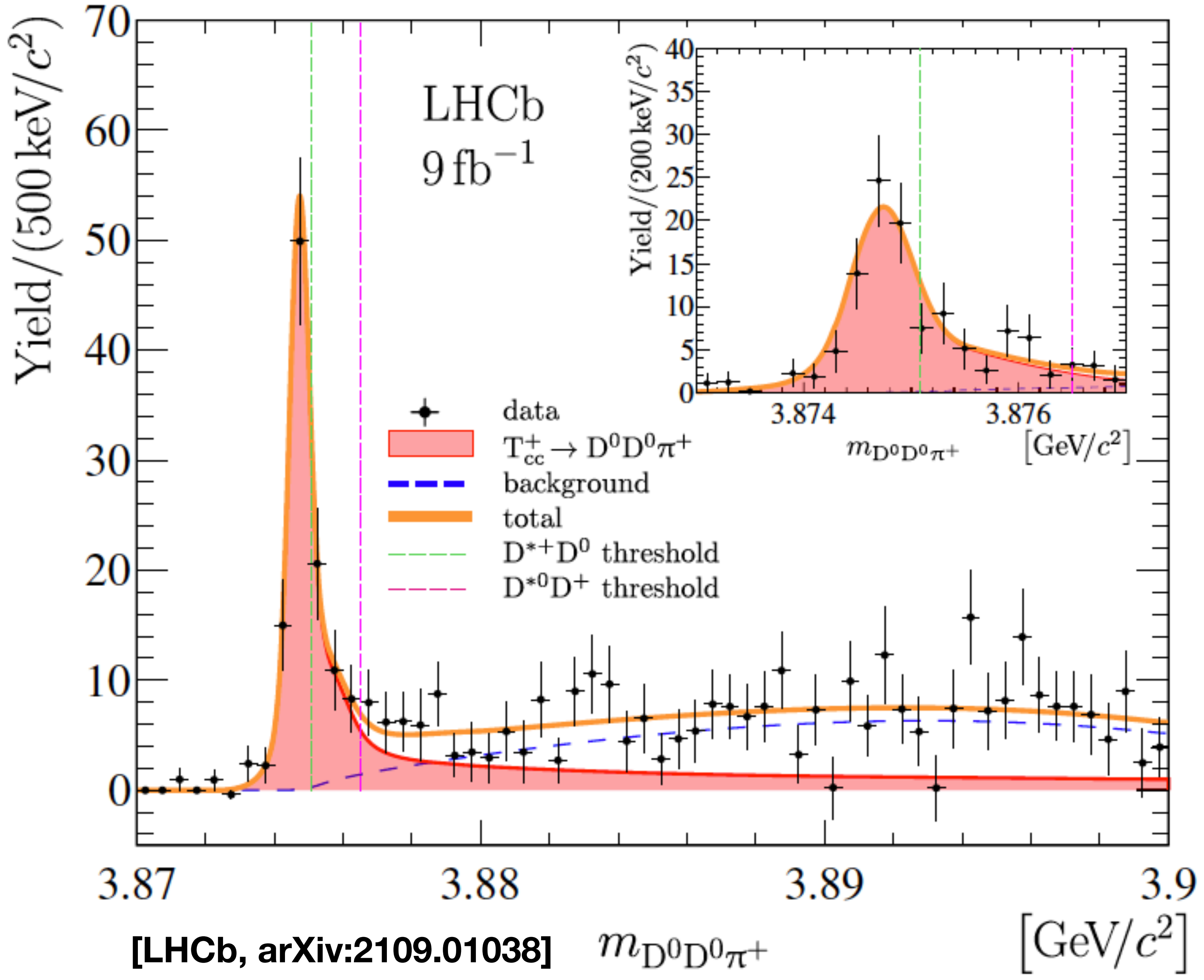
Parametrize three-body K-matrix:

$$\mathcal{K}_{\mathrm{df},3} = \frac{c_0}{E_{\mathrm{CM}}^2 - m_R^2} + c_1$$

+ solve integral equations

- ☑ **Successfully determined properties of three-particle resonance for the first time!**
- ☑ **Good agreement between methods**

The formalism for the doubly-charm tetraquark

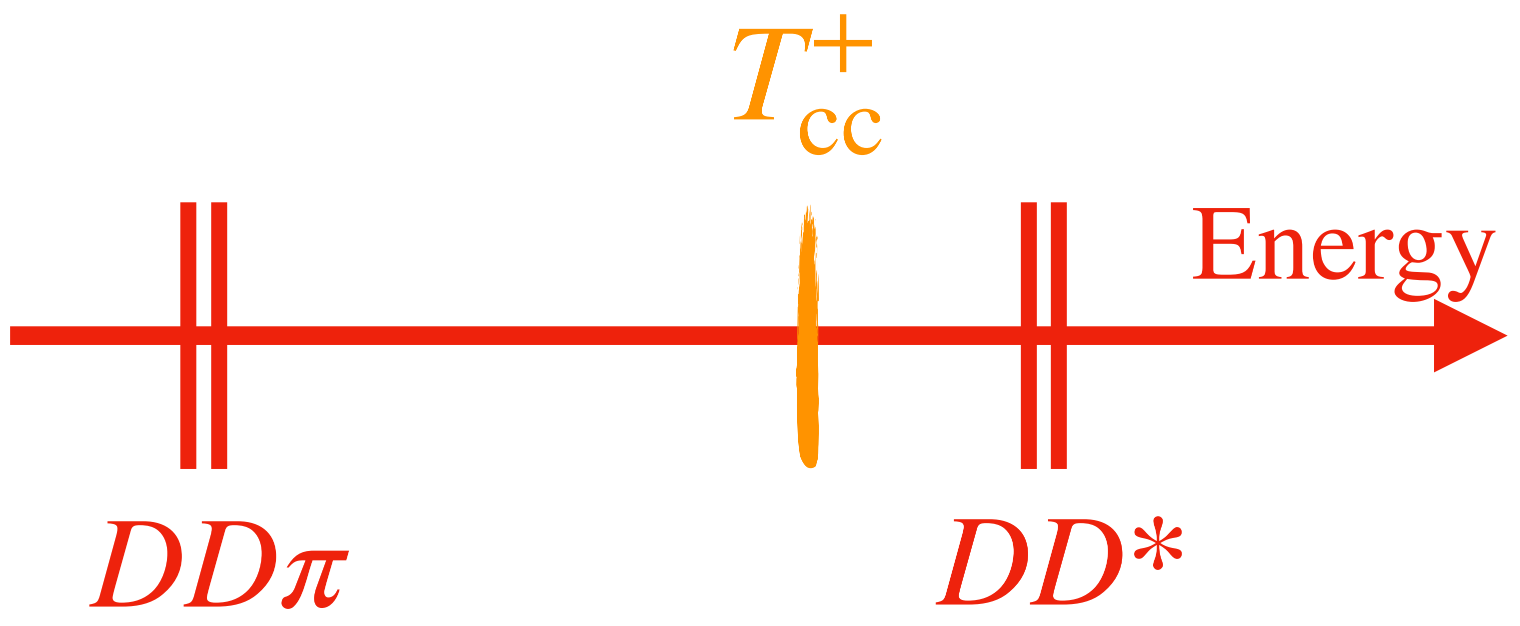


[LHCb, arXiv:2109.01038]

Fernando Romero-López, MIT

Doubly-charmed tetraquark

Experiment



- **For physical quark masses is a three-body resonance**
  $T_{cc} \to DD\pi$

  → need three-body formalism!

Doubly-charmed tetraquark

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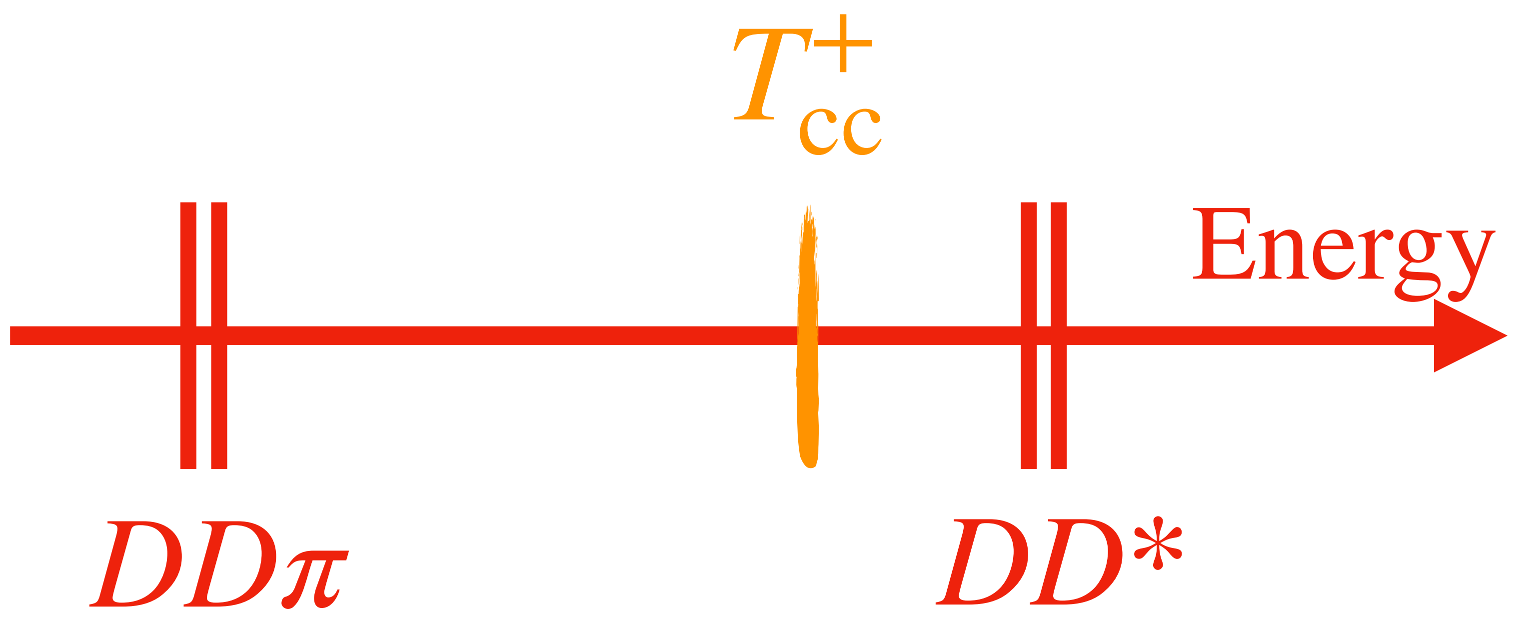
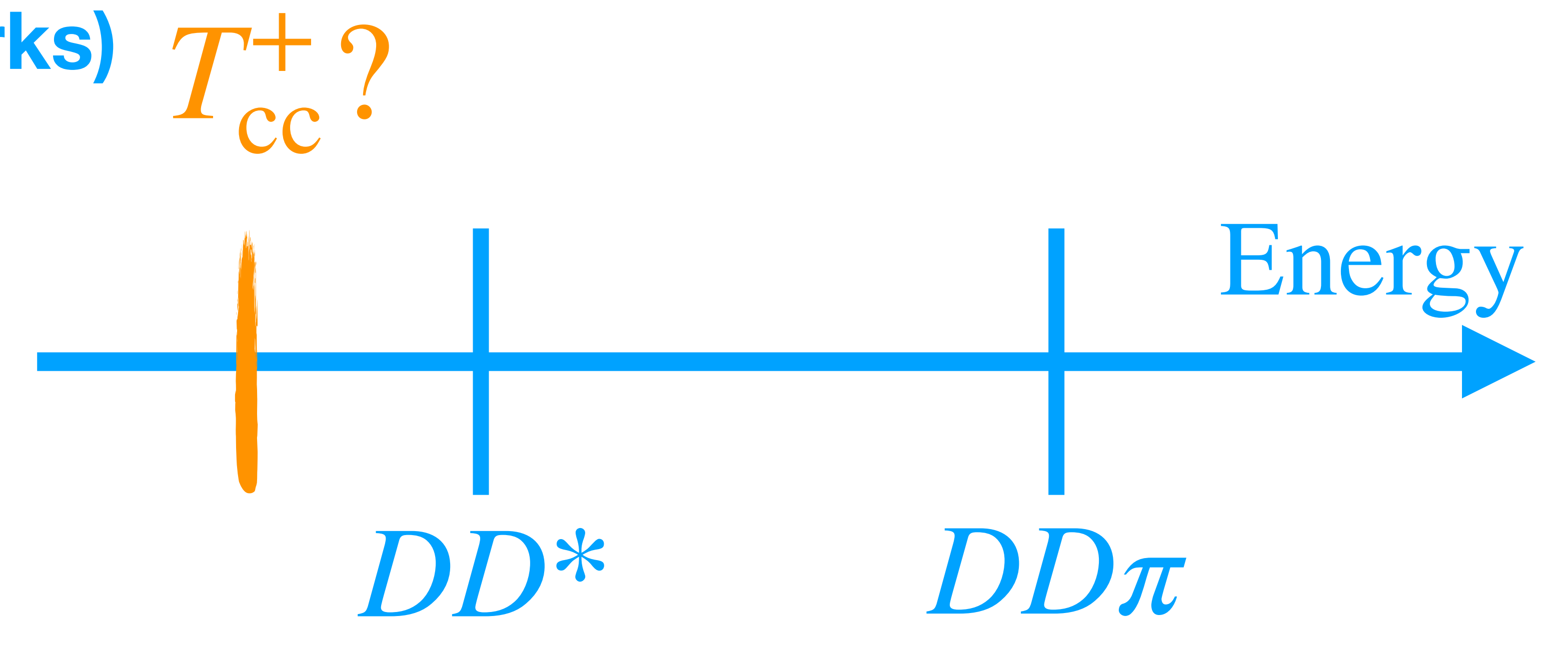


▸ **For physical quark masses is a three-body resonance**

$T_{cc} \to DD\pi$

need three-body formalism!

N_f=2+1+1 QCD (heavier quarks)



▸ **Stable D* at slightly heavier-than-physical quark mases**

$T_{cc} \to DD^*$?

suitable for the two-body Lüscher formalism?

D-D* scattering

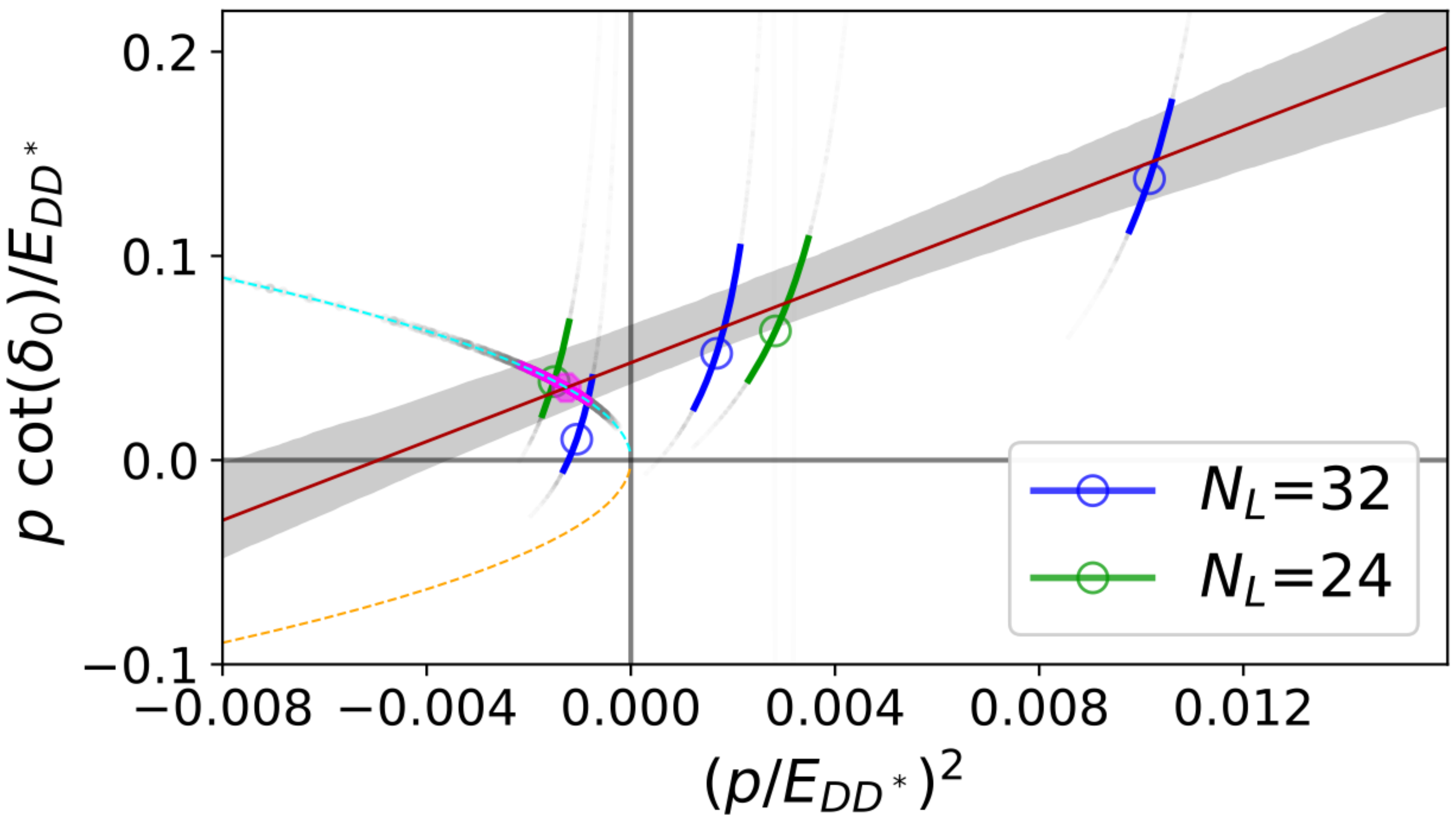
- Several work study the T_{cc} channel in this setup

  [Chen et al., 2206.06185] [Lyu et al. (HALQCD), 2302.04505]

  [Padmanath & Prelovsek, 2202.10110]

  - Signature of virtual bound state?

[Padmanath, Prelovsek, arXiv:2202.10110]



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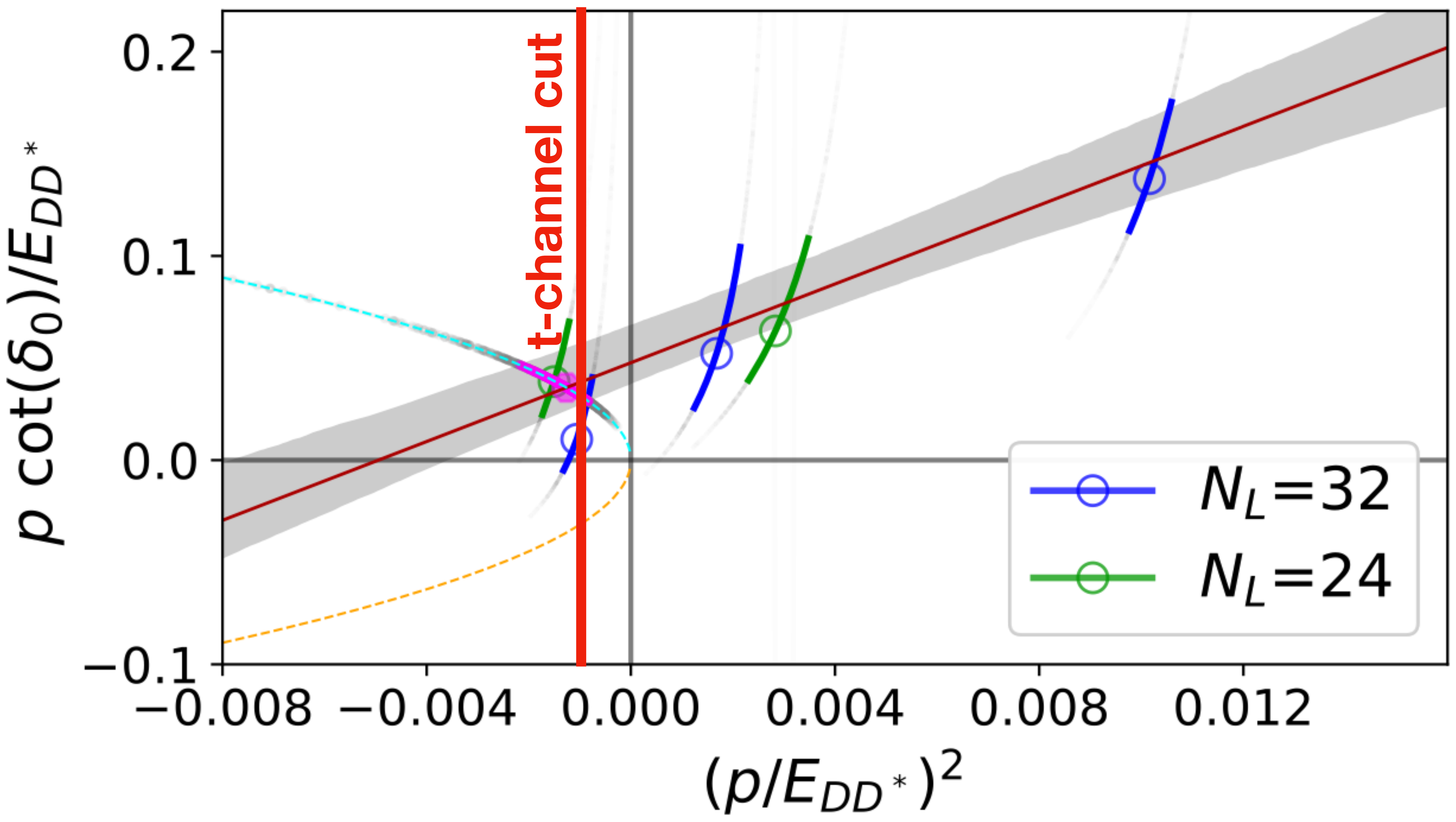
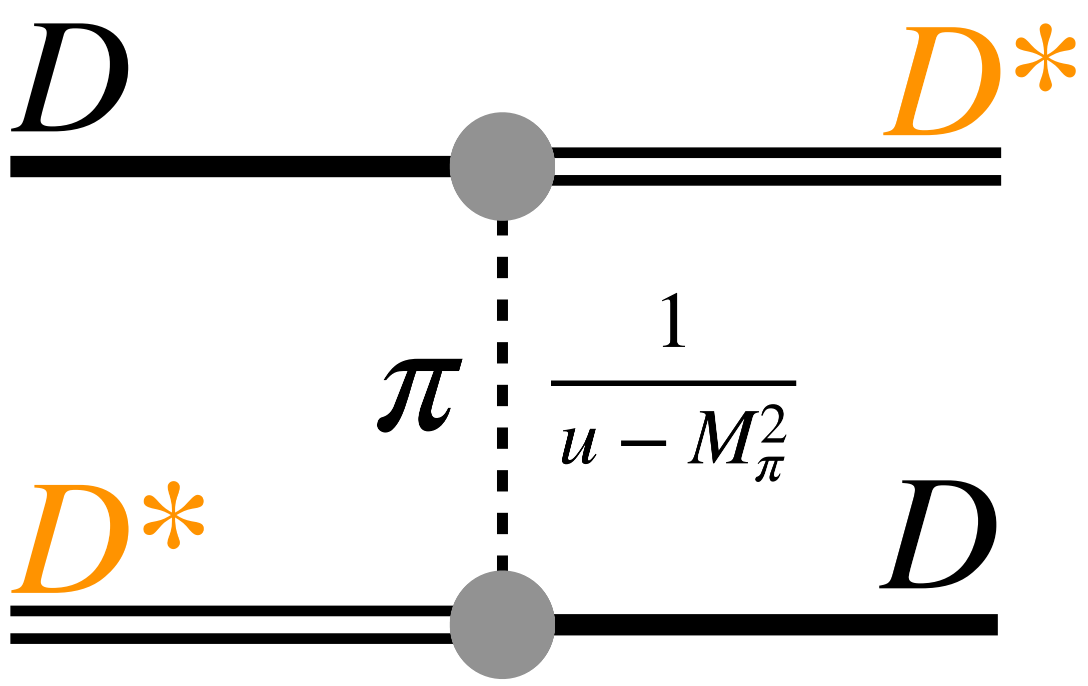
  - Signature of virtual bound state?
  - But two-particle formalism breaks down
    i.e. complex phase shift

[Padmanath, Prelovsek, arXiv:2202.10110]



! one-pion exchange creates non analytic behavior:



$$u = M_\pi^2, \quad t = 0, \quad s - s_{th} = -M_\pi^2 + (M_D - M_{D^*})^2$$

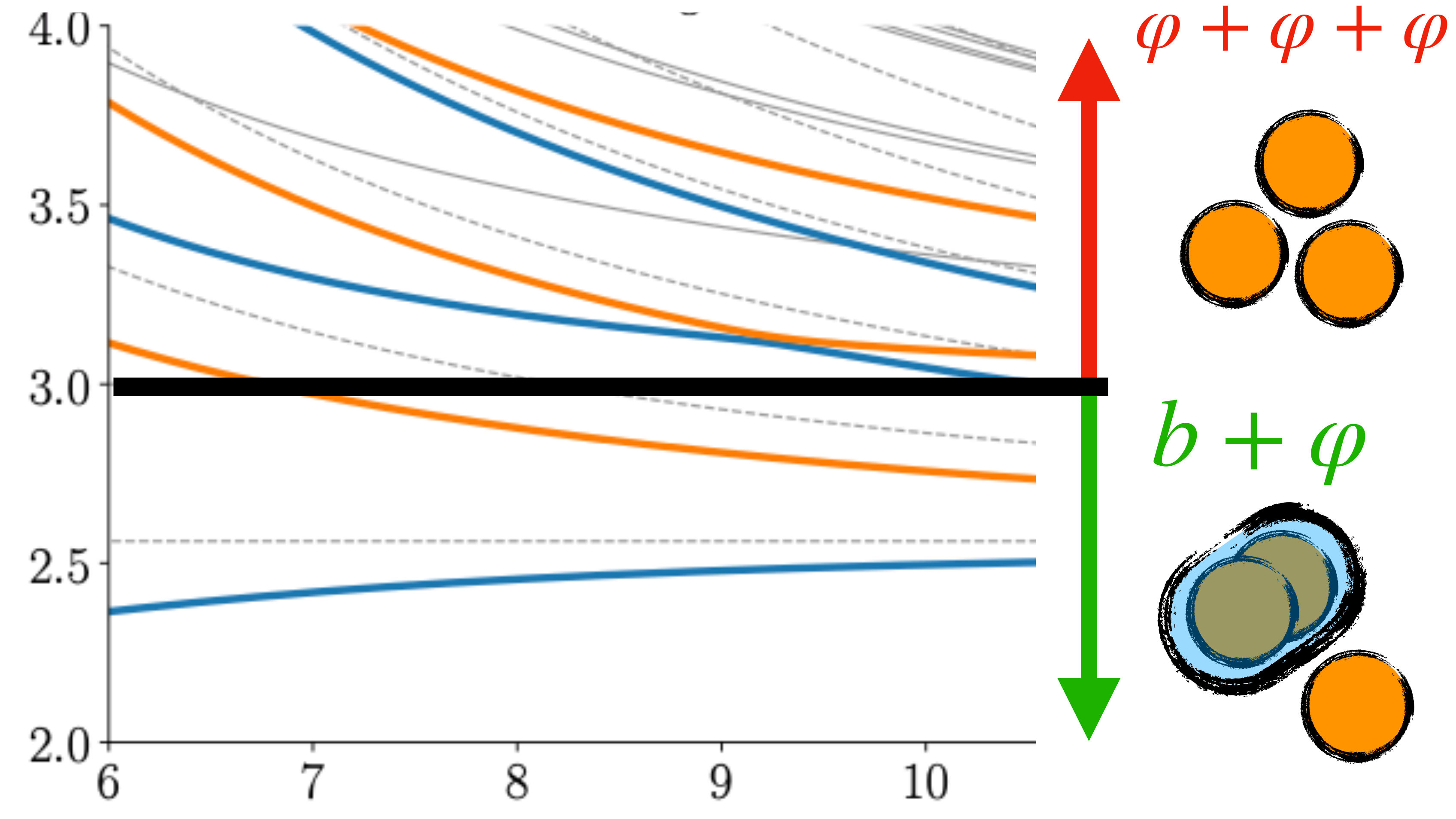
just 8 MeV below threshold!

A three-body solution

- In the presence of a two-body bound state:
  - Below the three-particle threshold, effective "particle-dimer"

    [FRL et al 2302.04505] [Jackura et al 2010.09820]
    [Dawid, Islam, Briceño, 2303.04394] [Pefkou et al (in prep)]

[FRL, Sharpe, Blanton, Briceño, Hansen 2302.04505]



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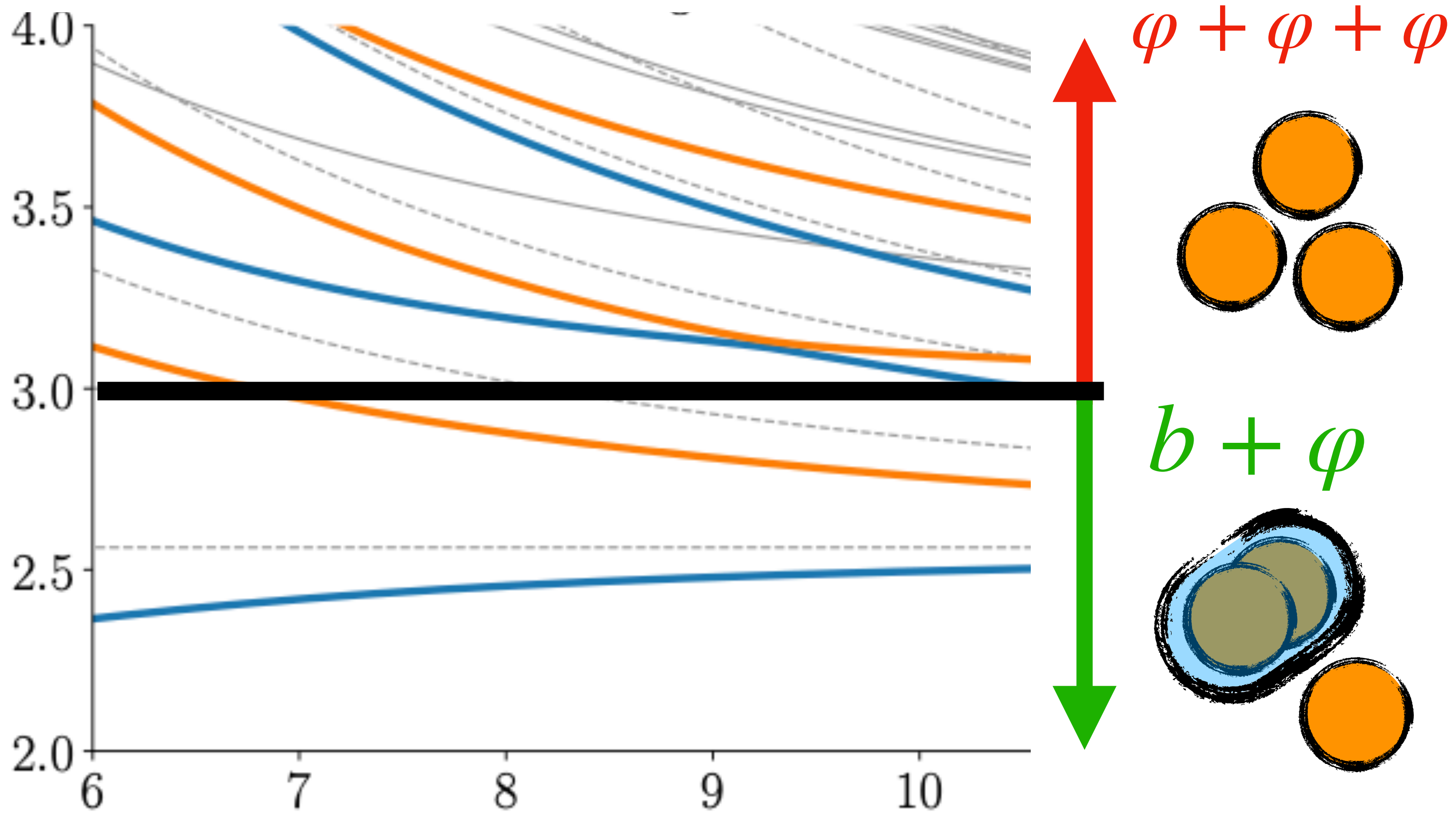
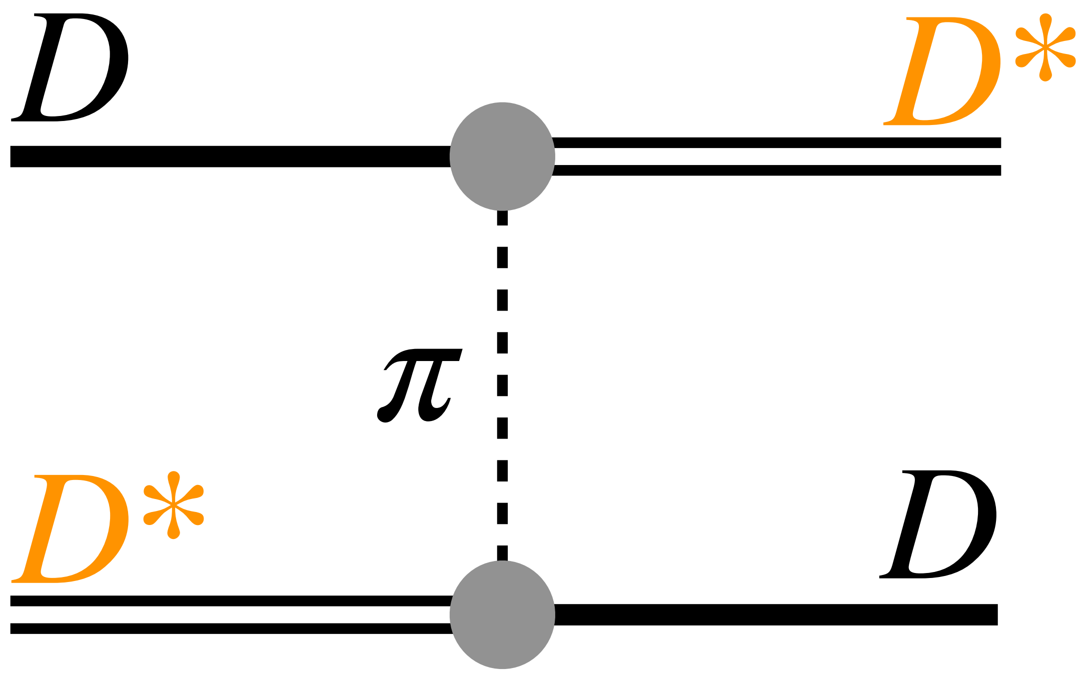
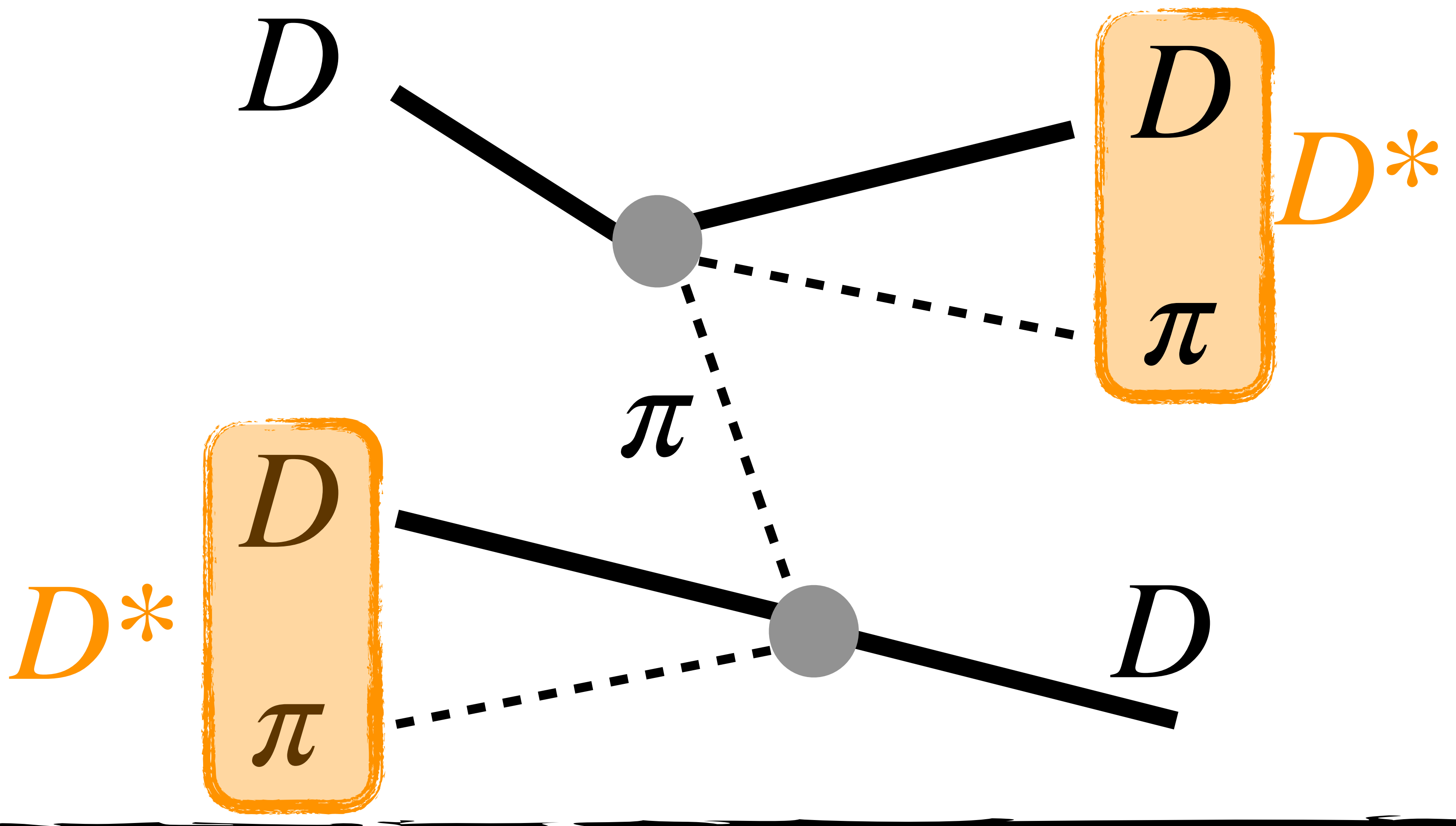


- This solves the left-hand cut problem:
  - Finite-volume effects from one-pion exchange naturally incorporated



[Hansen, FRL, Sharpe, arXiv:2401.06609]

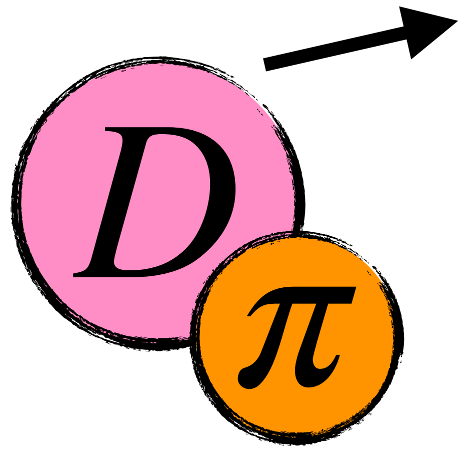


The strategy for the T_{cc}

[Hansen, FRL, Sharpe, arXiv:2401.06609]

Two-meson spectra

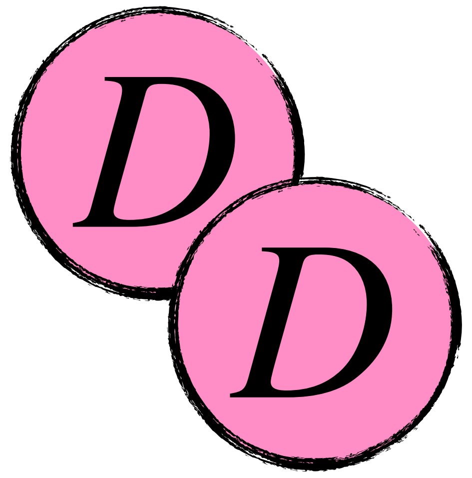
$I_{D\pi} = 1/2$



D^* as a bound state or resonance

s and p waves

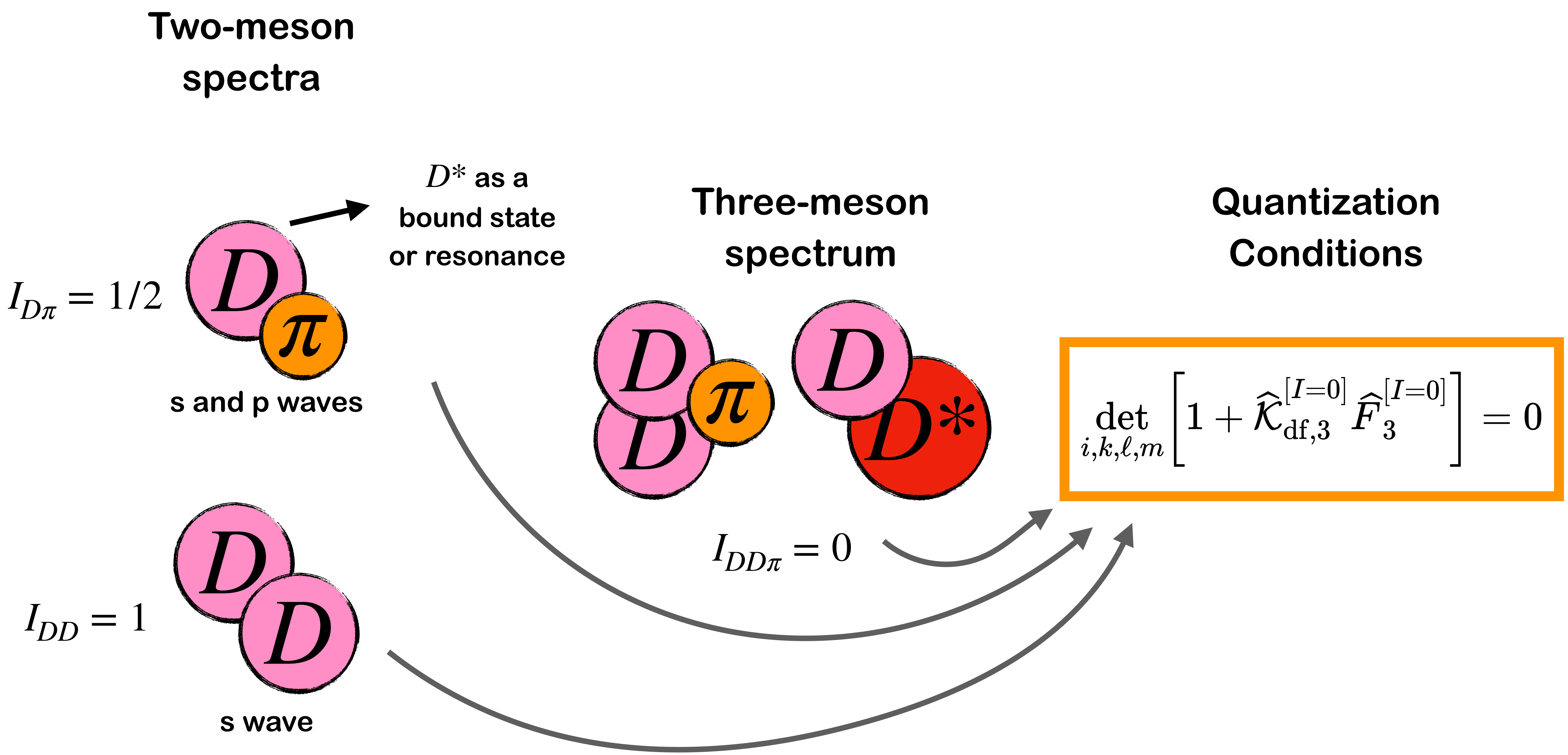
$I_{DD} = 1$



s wave

The strategy for the T_{cc}

[Hansen, FRL, Sharpe, arXiv:2401.06609]



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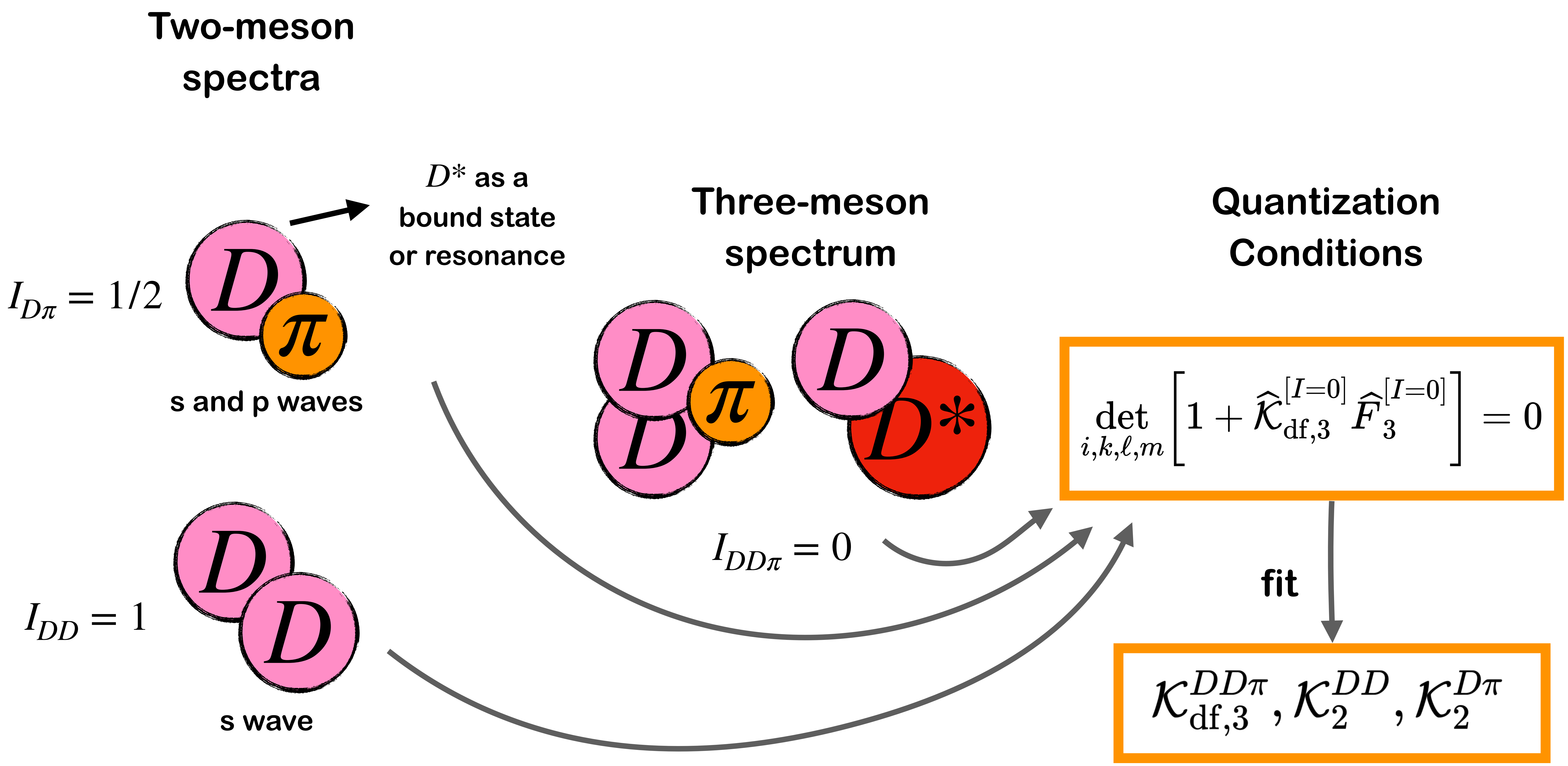

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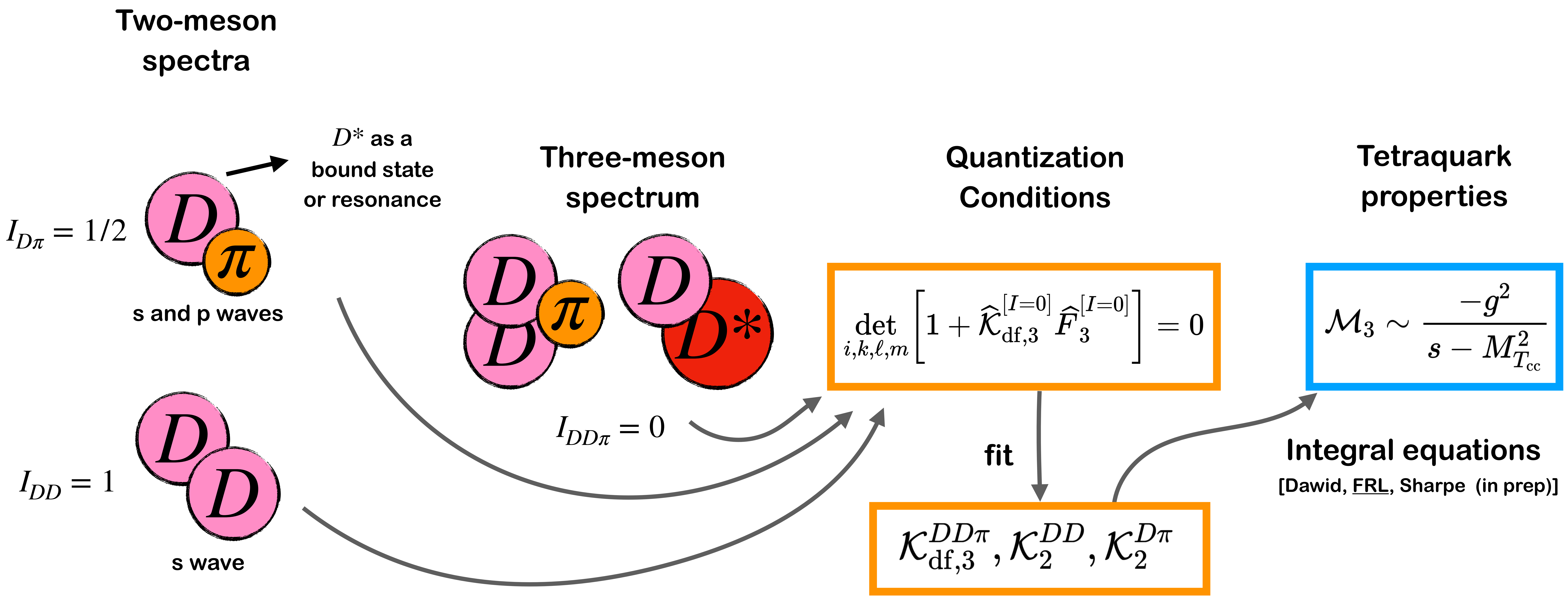

Summary & Outlook

Summary and outlook

- ☑ Lattice QCD provides a first-principle tool to investigate the hadron spectrum
- ☑ Several studies of scattering lattice QCD: $\Delta(1232), \Lambda(1405)$
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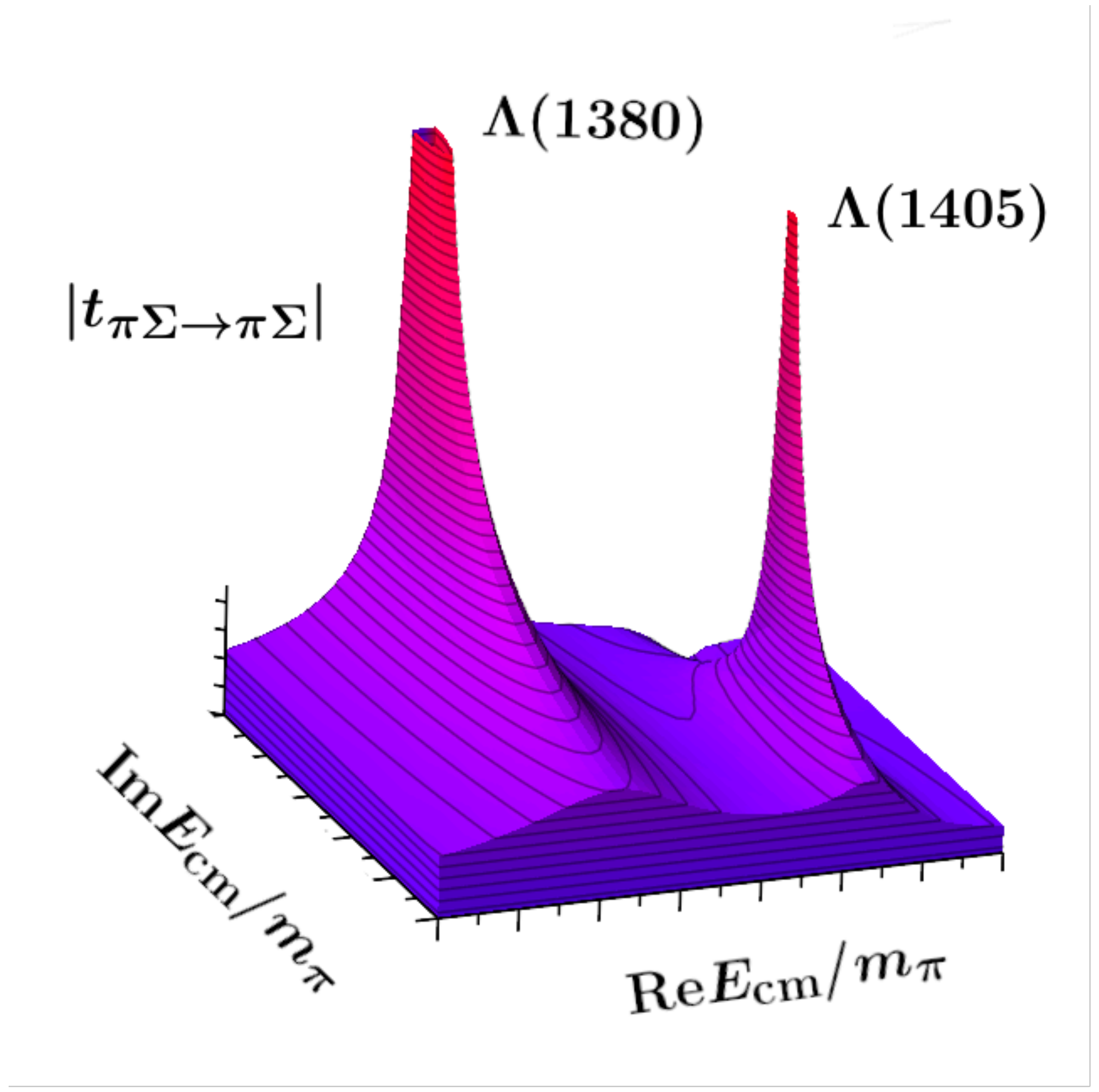
- ☑ The formalism for DDπ systems, allowing the study of the T_{cc}



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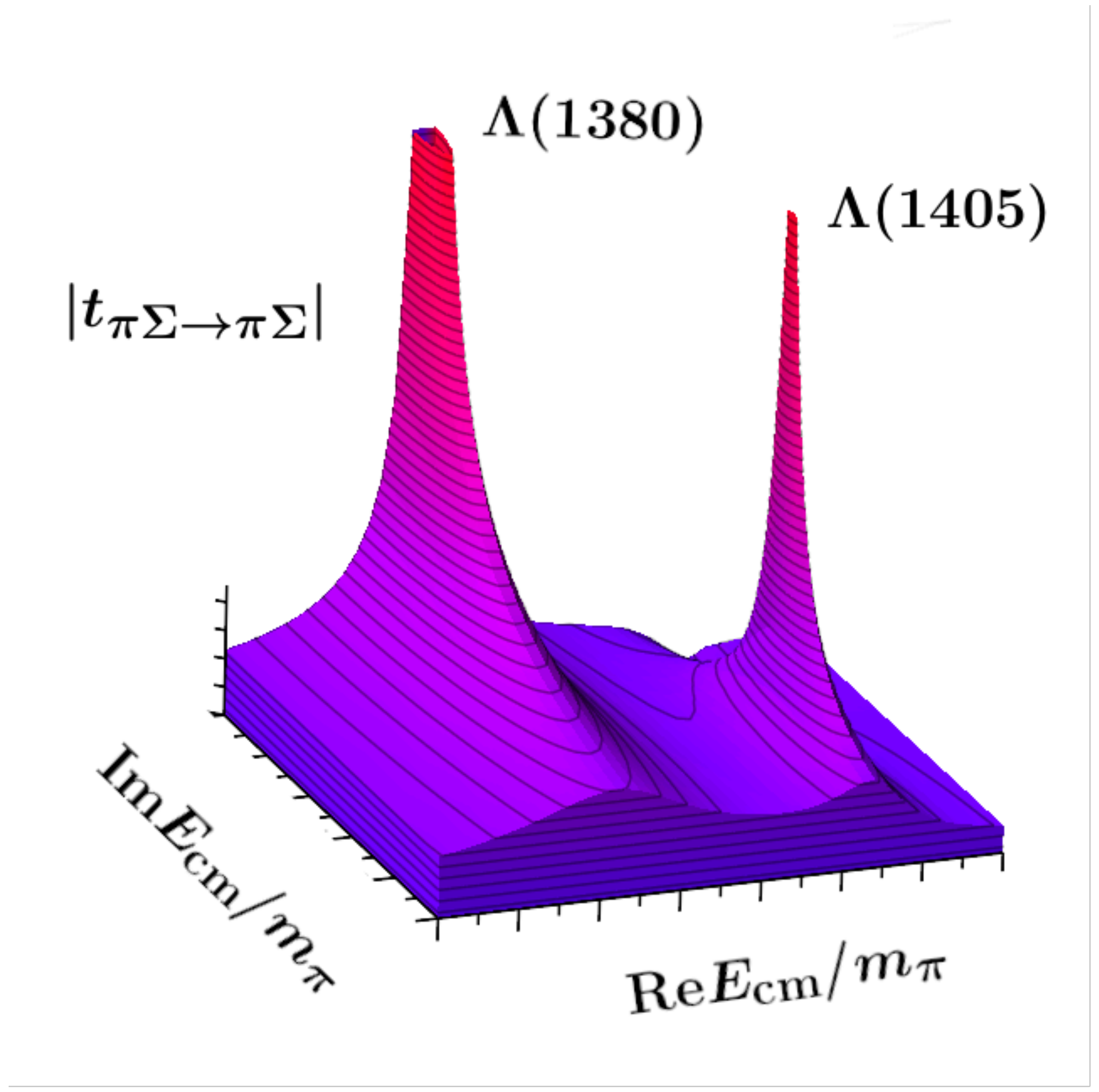

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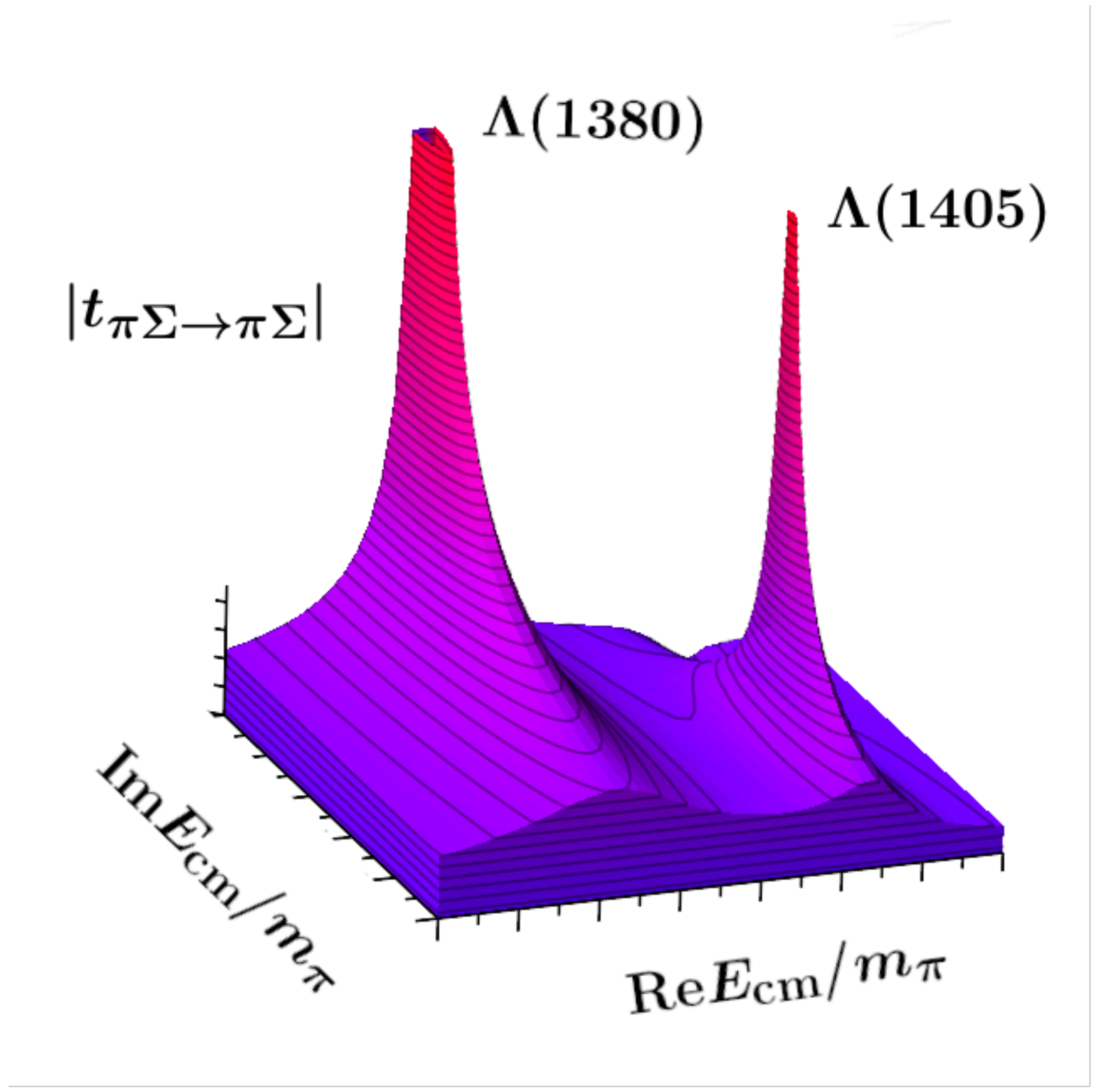


Thanks!

Back-up

Poles

$$\mathcal{M}_2(s) = \frac{16\pi\sqrt{s}}{k\cot\delta(k) - ik},$$

$$k = \pm\sqrt{k^2}$$

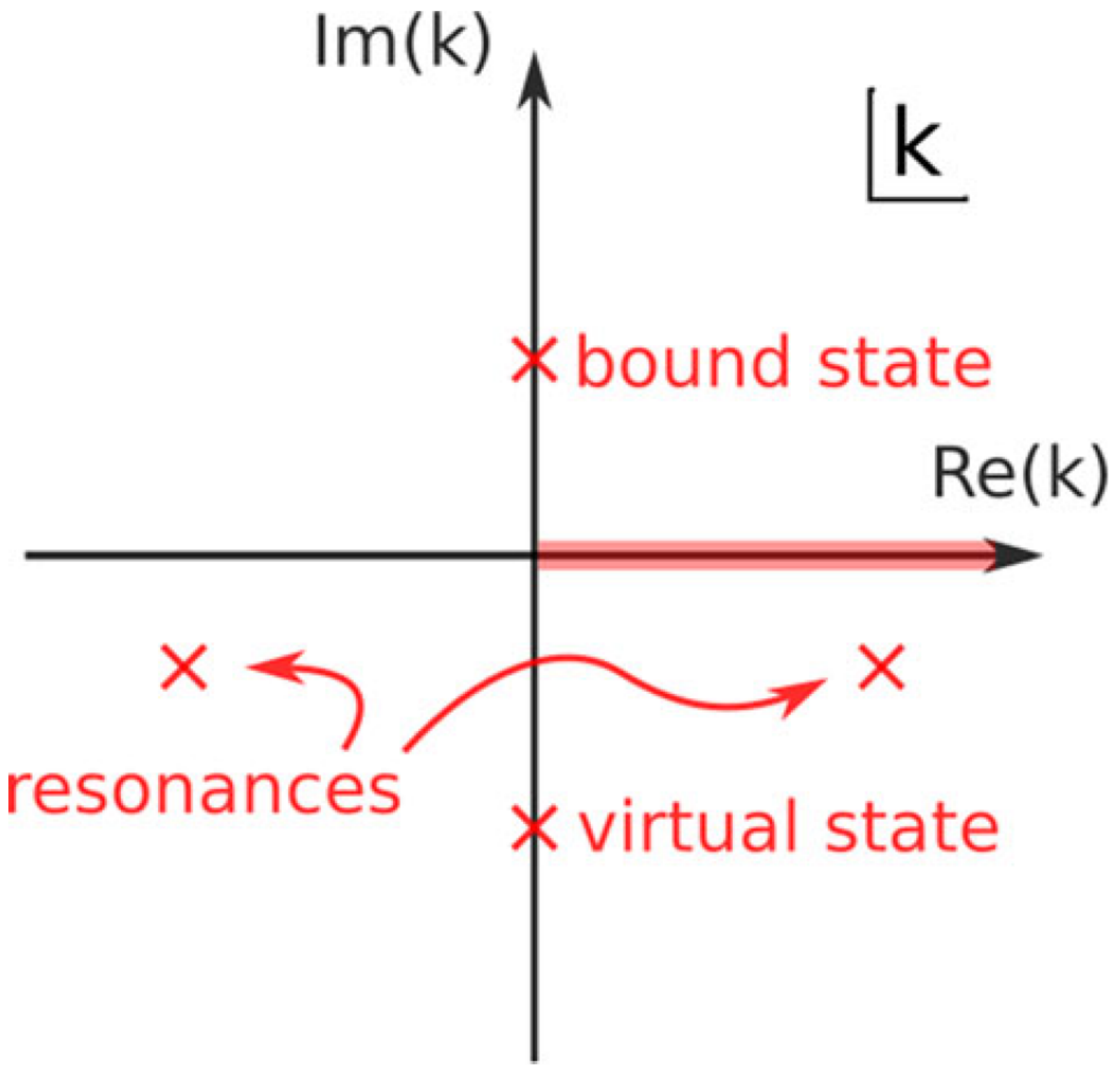


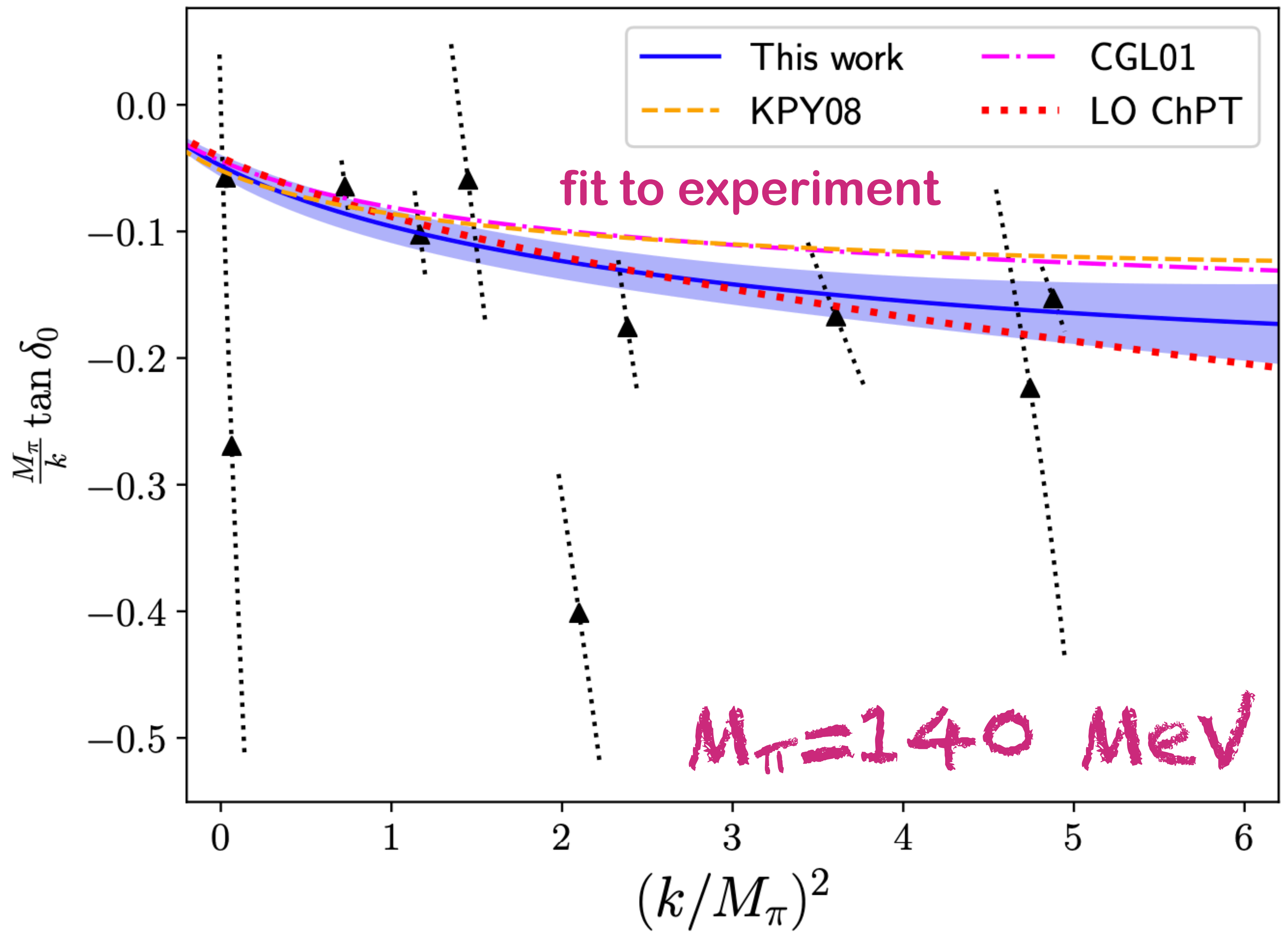
Fig. 1 Naming convention for the poles in the k-plane. The thick red line for positive real valued k marks the physical momenta in the scattering regime

[Matuschek et al, EPJA 2021]

Towards the physical point

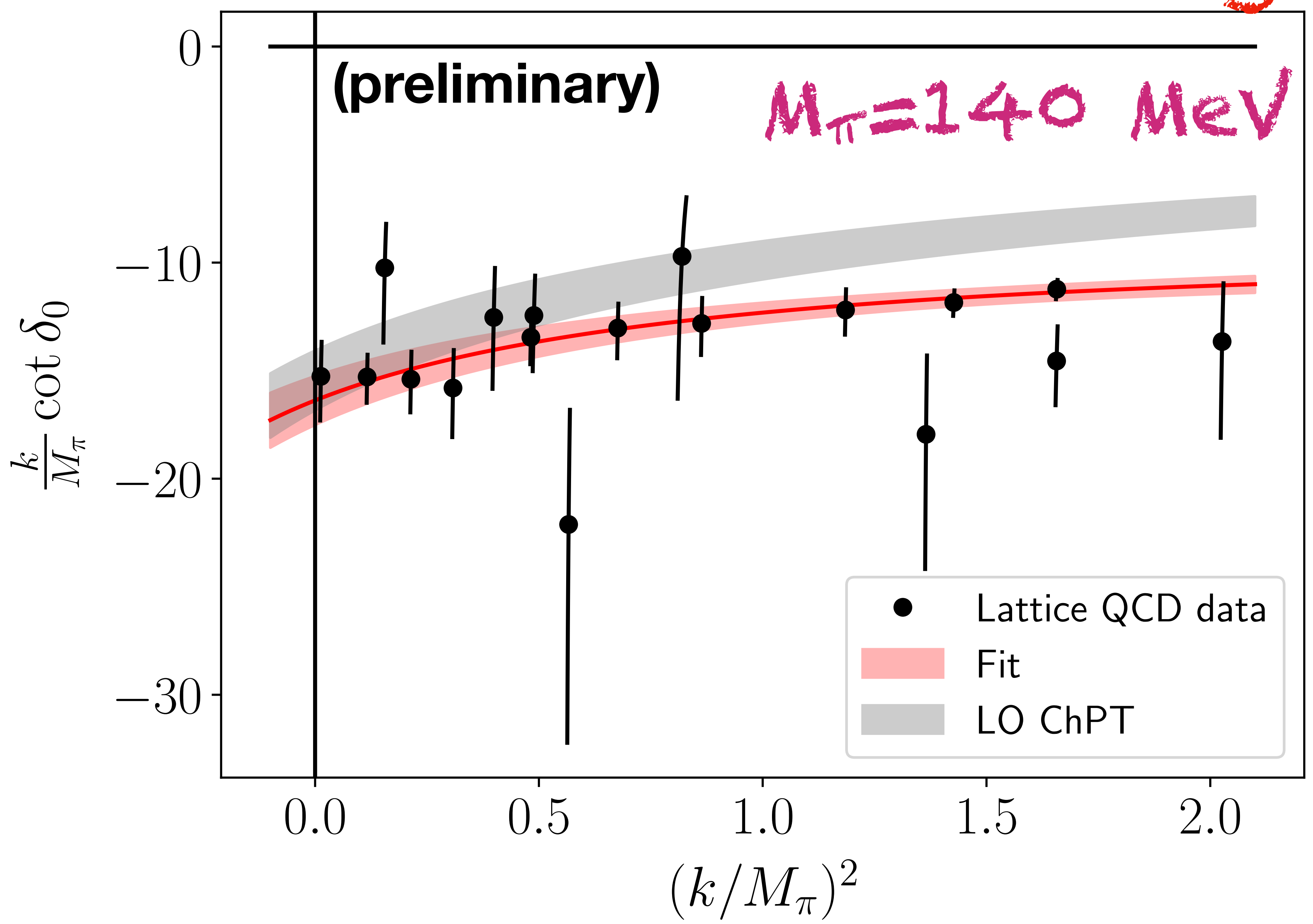
- Some systems already being studied at the **physical point**!

I=2 ππ scattering



[Fischer, Kostrzewa, Liu, FRL, Ueding, Urbach (ETMC), EPJC 2021]

I=3/2 πK scattering

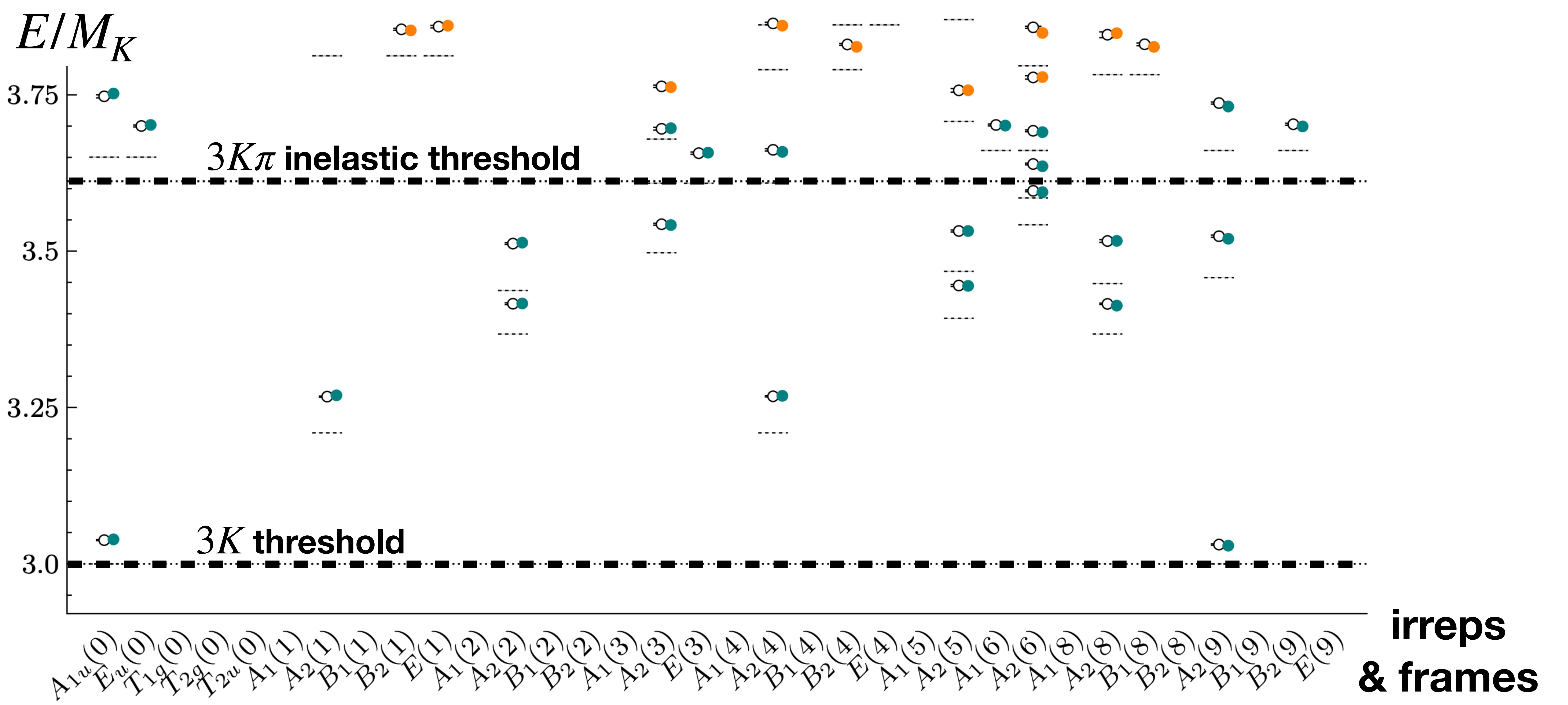


[Draper, Hanlon, Hörz, Morningstar, FRL, Sharpe (in preparation)]

Applying the three-body formalism

- Three pions and three kaons at maximal isospin have been explored by different groups
  [Alexandrou et al, PRD 2020], [Blanton et al., PRL 2020 & JHEP 2021], [Brett et al, PRD 2021], [Culver et al, PRD 2021], [Fischer et al, EPJC 2021], [Hansen et al, PRL 2021], [Mai et a PRL 2019 & 2021]
- Requires large sets of energy levels obtained using variational techniques
  Using stochastic LapH method [Morningstar et al, PRD 2011]

$3K^+$ energy levels



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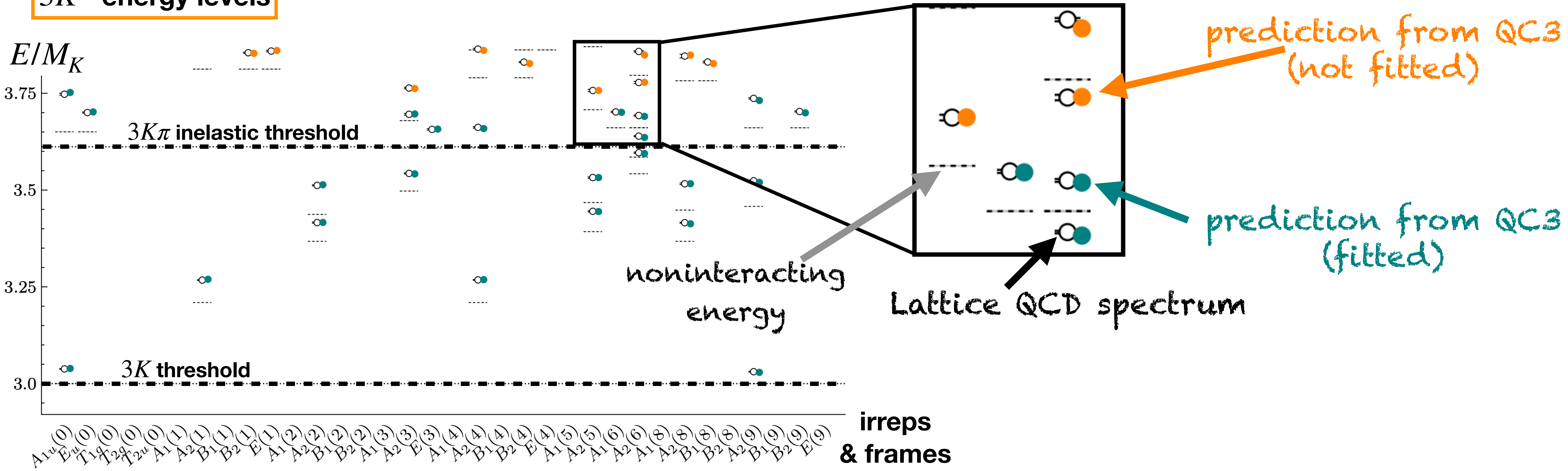

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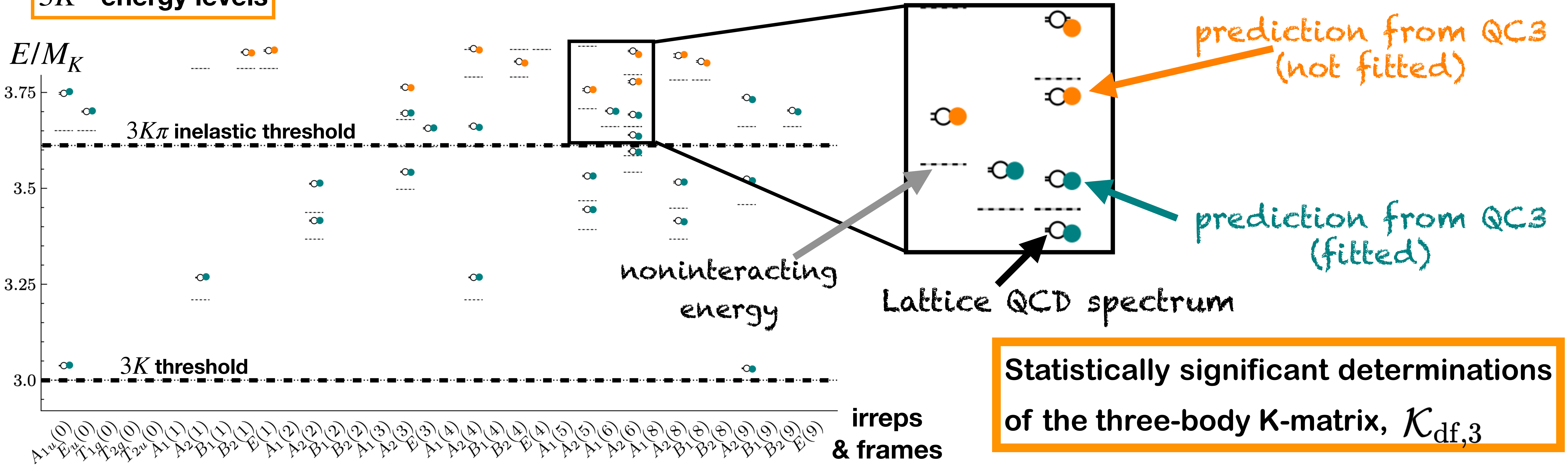

Statistically significant determinations of the three-body K-matrix, $\mathcal{K}_{\mathrm{df},3}$

[Blanton, Hanlon, Hörz, Morningstar, FRL, Sharpe, JHEP 2021]

$\mathcal{K}_{df,3}$ from lattice QCD

Depend of CM energy

Angular dependence

$$\mathcal{K}_{\text{df},3} = \mathcal{K}_{\text{df},3}^{\text{iso},0} + \mathcal{K}_{\text{df},3}^{\text{iso},1}\Delta + \mathcal{K}_{\text{df},3}^{\text{iso},2}\Delta^2 + \mathcal{K}_A\Delta_A + \mathcal{K}_B\Delta_B,$$

$$\Delta \equiv \frac{s-9m^2}{9m^2}$$

$$\Delta_B = \sum_{i,j=1}^{3} \tilde{t}_{ij}^2 - \Delta^2,$$

$\mathcal{K}_{\text{df},3}$ from Lattice QCD

Depend of CM energy | **Angular dependence**

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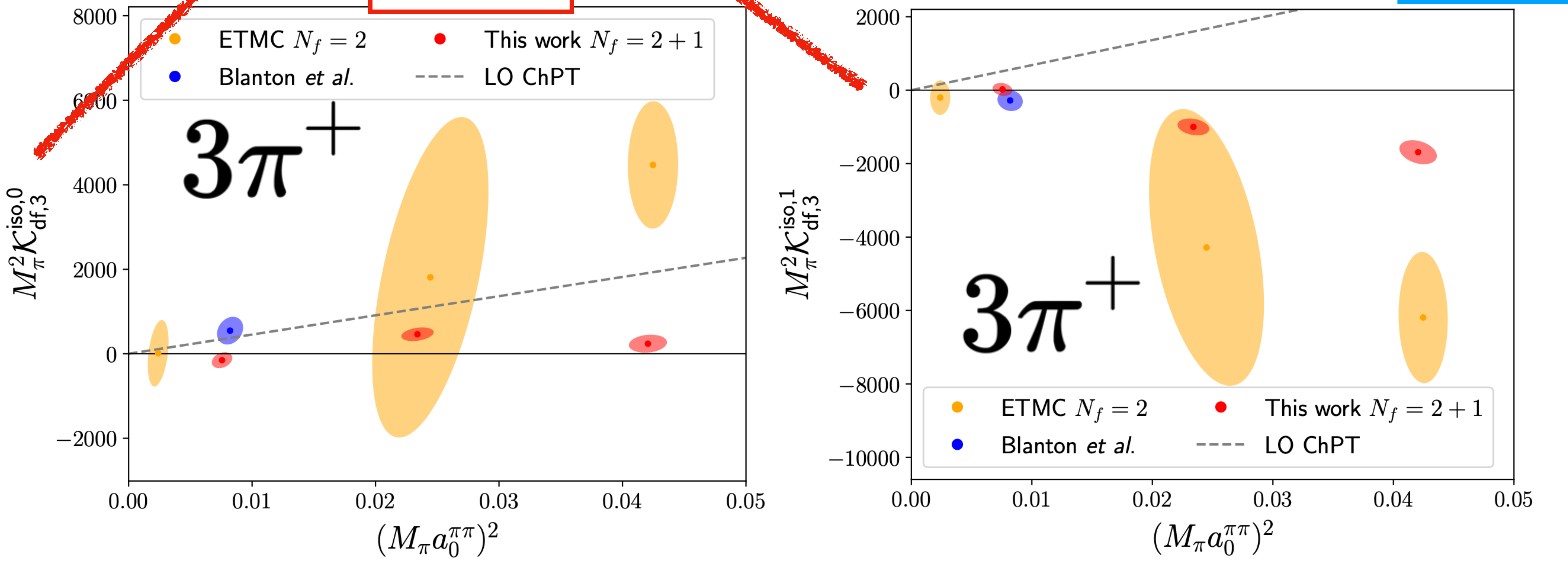

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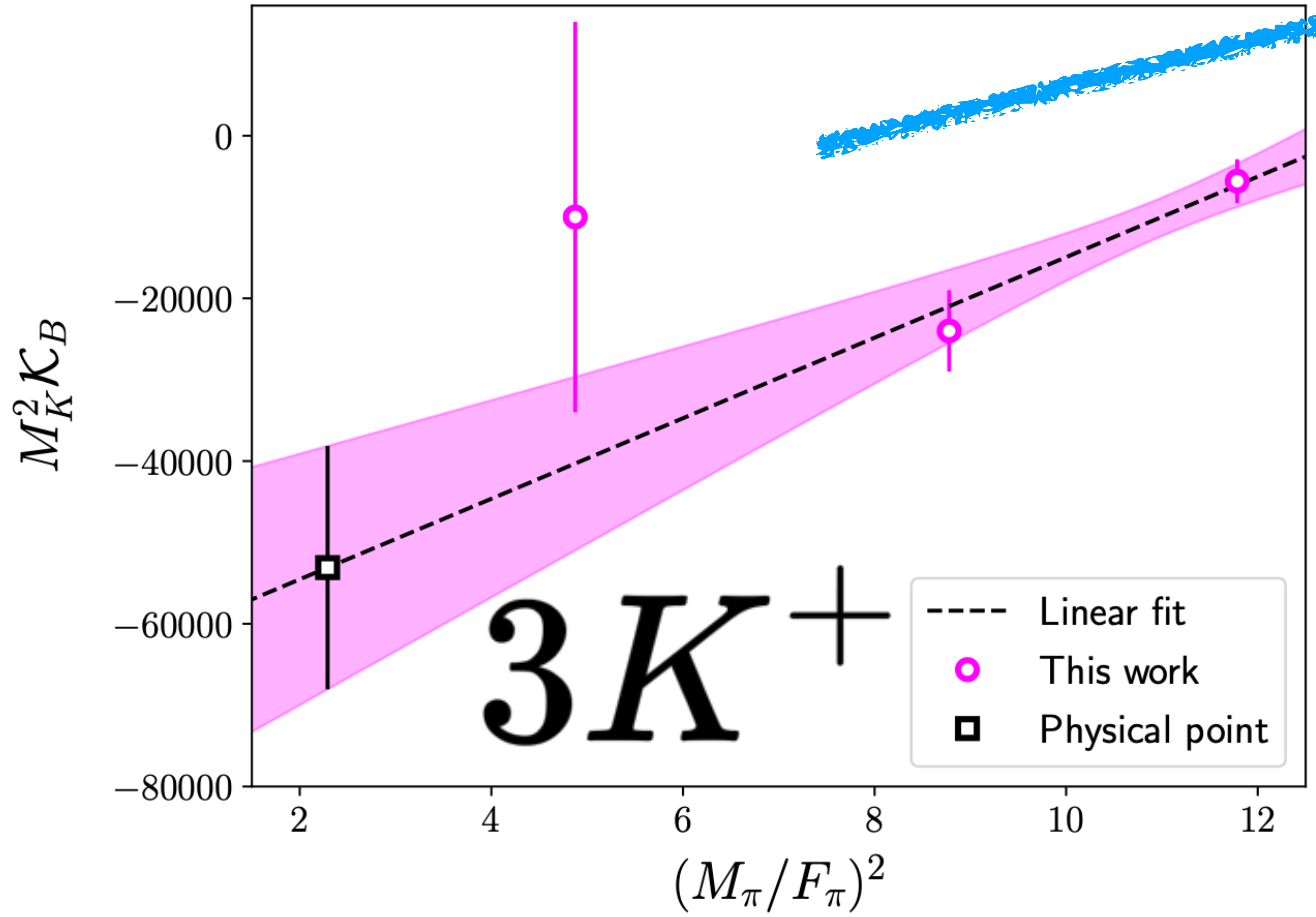

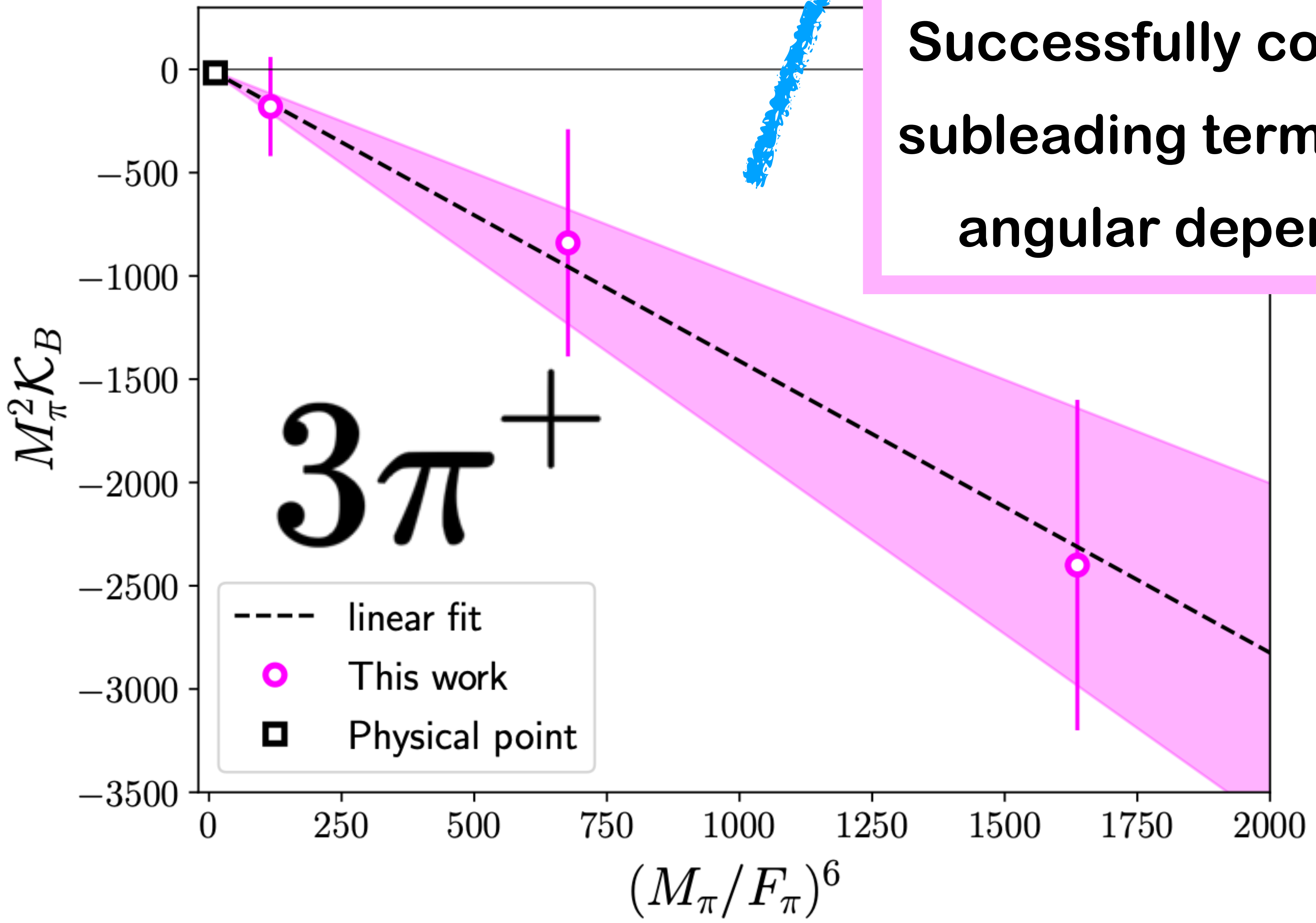

Successfully constrained subleading term including angular dependence!

[Blanton, Hanlon, Hörz, Morningstar, FRL, Sharpe, JHEP 2021]

Nondegenerate systems

- Relevant three-body systems involve nonidentical particles ($\pi\pi N$)
- First step: formalism for three nonidentical scalars

  e.g. $\pi^+\pi^0\pi^-$, $K^+K^+\pi^+$, $D_s^+D^0\pi^-$

  [Blanton, Sharpe, PRD 2021] x 2, [Hansen, FRL, Sharpe, JHEP 2021], [Mai et al (GWQCD), PRL 2021]

$$\det_{k,\ell,m,\mathbf{f}}[1 - \mathbf{K}_{\mathrm{df},3}(E^\star)\mathbf{F}_3(E,\boldsymbol{P},L)] = 0$$

determinant runs over an additional "flavor" index

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Example:
$\pi^+\pi^+K^+$ scattering

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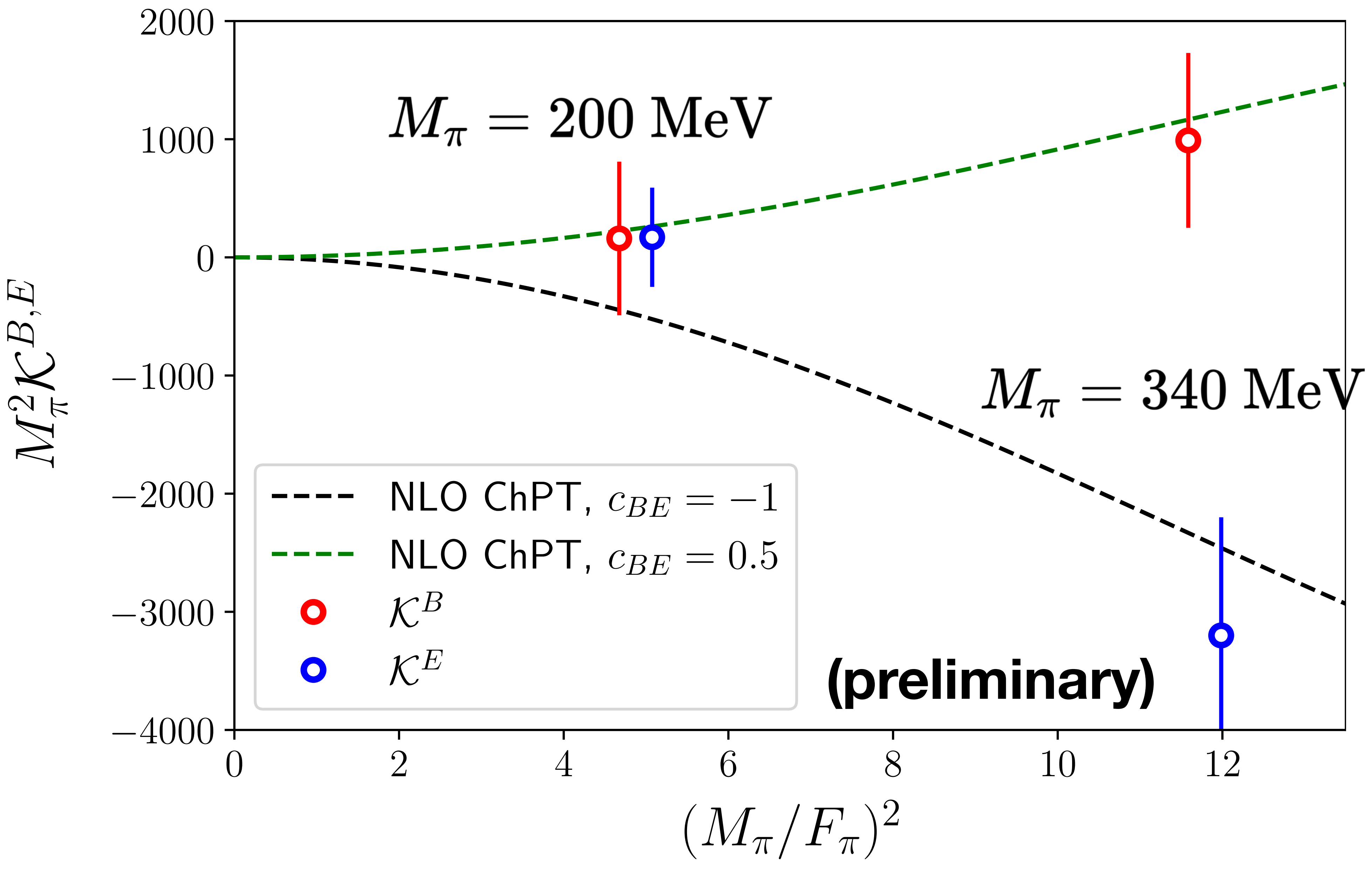
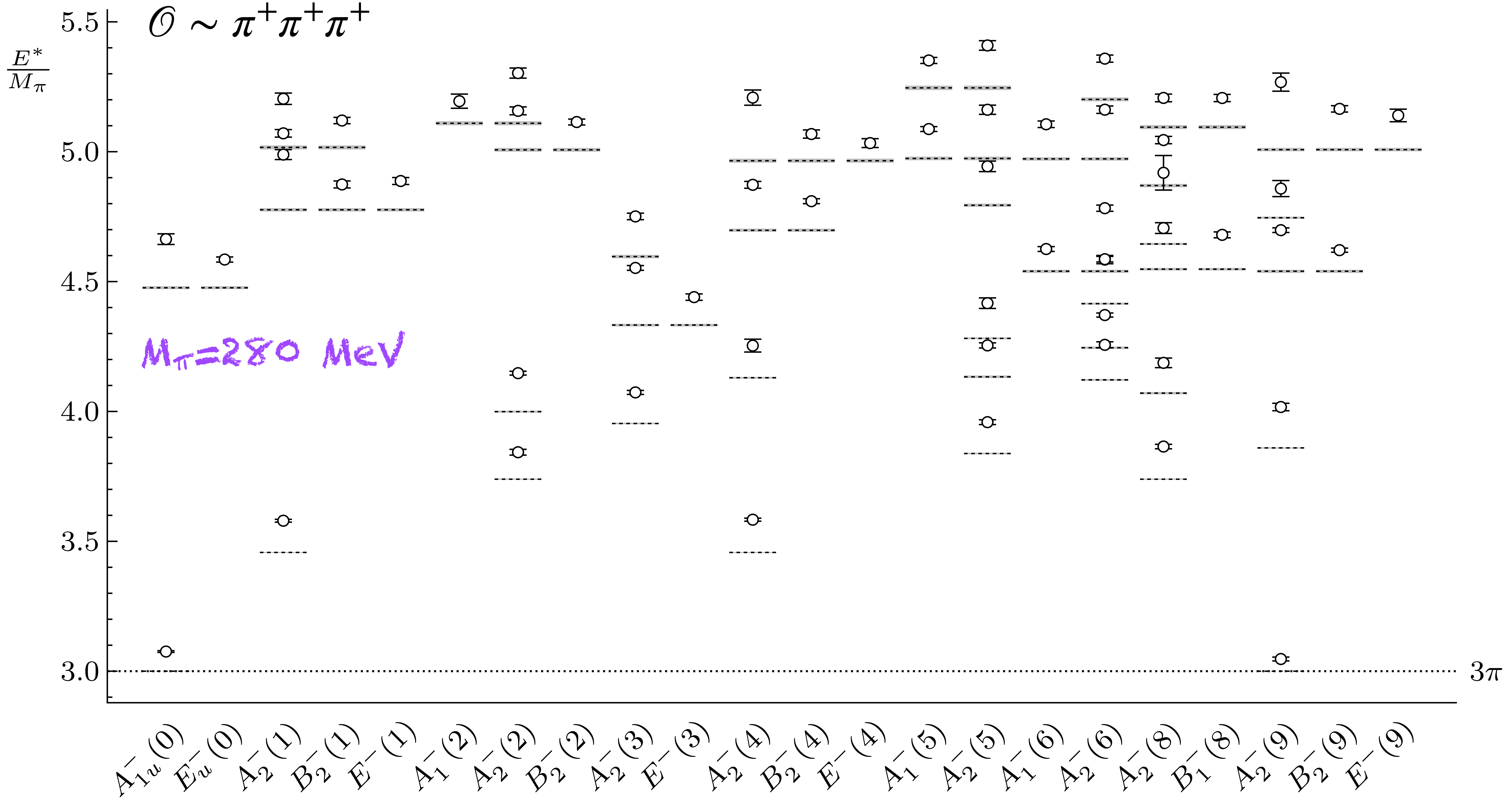
[Draper, Hanlon, Hörz, Morningstar, FRL, Sharpe (in preparation)]

[Talk by S. Sharpe]



3π+ energy levels



[Blanton, Hanlon, Hörz, Morningstar, FRL, Sharpe]

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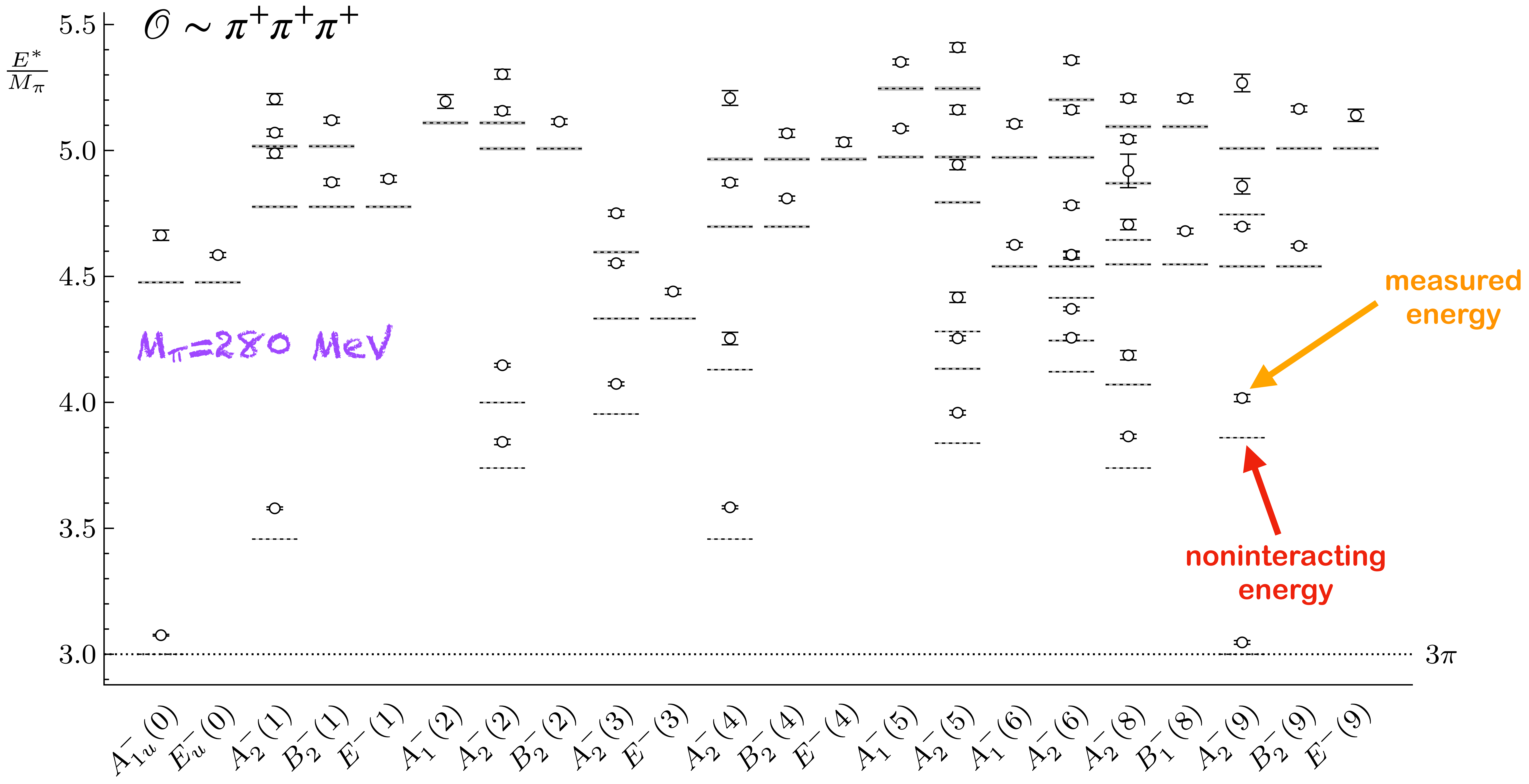

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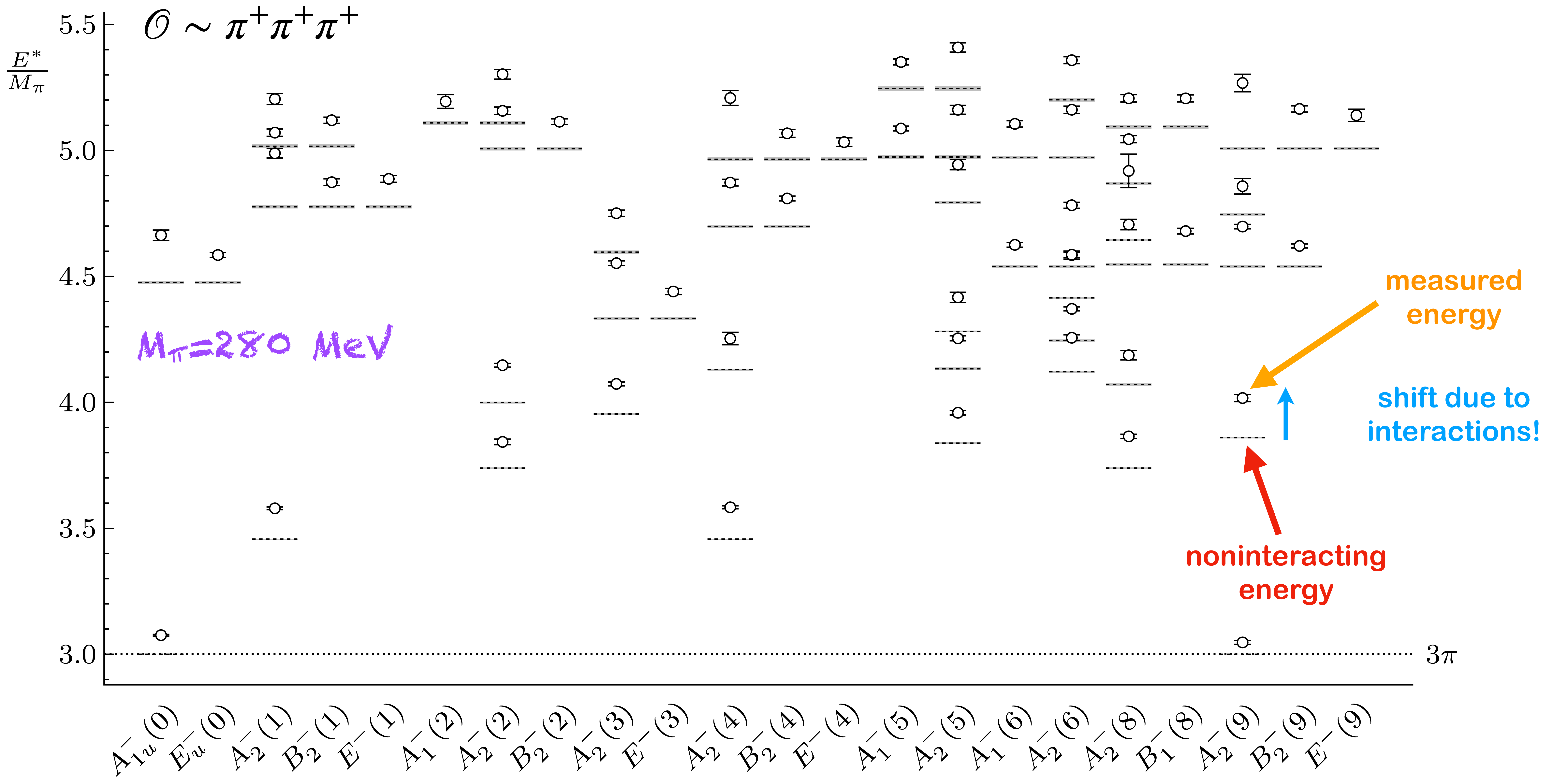

[Blanton, Hanlon, Hörz, Morningstar, FRL, Sharpe]

Integral equations (RFT)

Final step

Physical 3->3 amplitude

$$\mathcal{K}_2, \mathcal{K}_{df,3} \longrightarrow \mathcal{M}_3$$

Integral equations

Integral equations (RFT)

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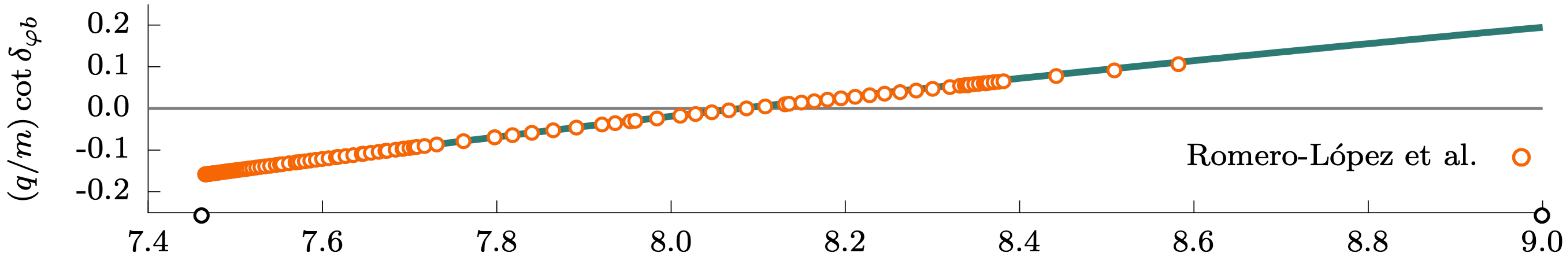
Physical 3->3 amplitude

$$\mathcal{K}_2, \mathcal{K}_{df,3} \xrightarrow{\text{Integral equations}} \mathcal{M}_3$$

Particle-Dimer phase shift **[Jackura et al.]**



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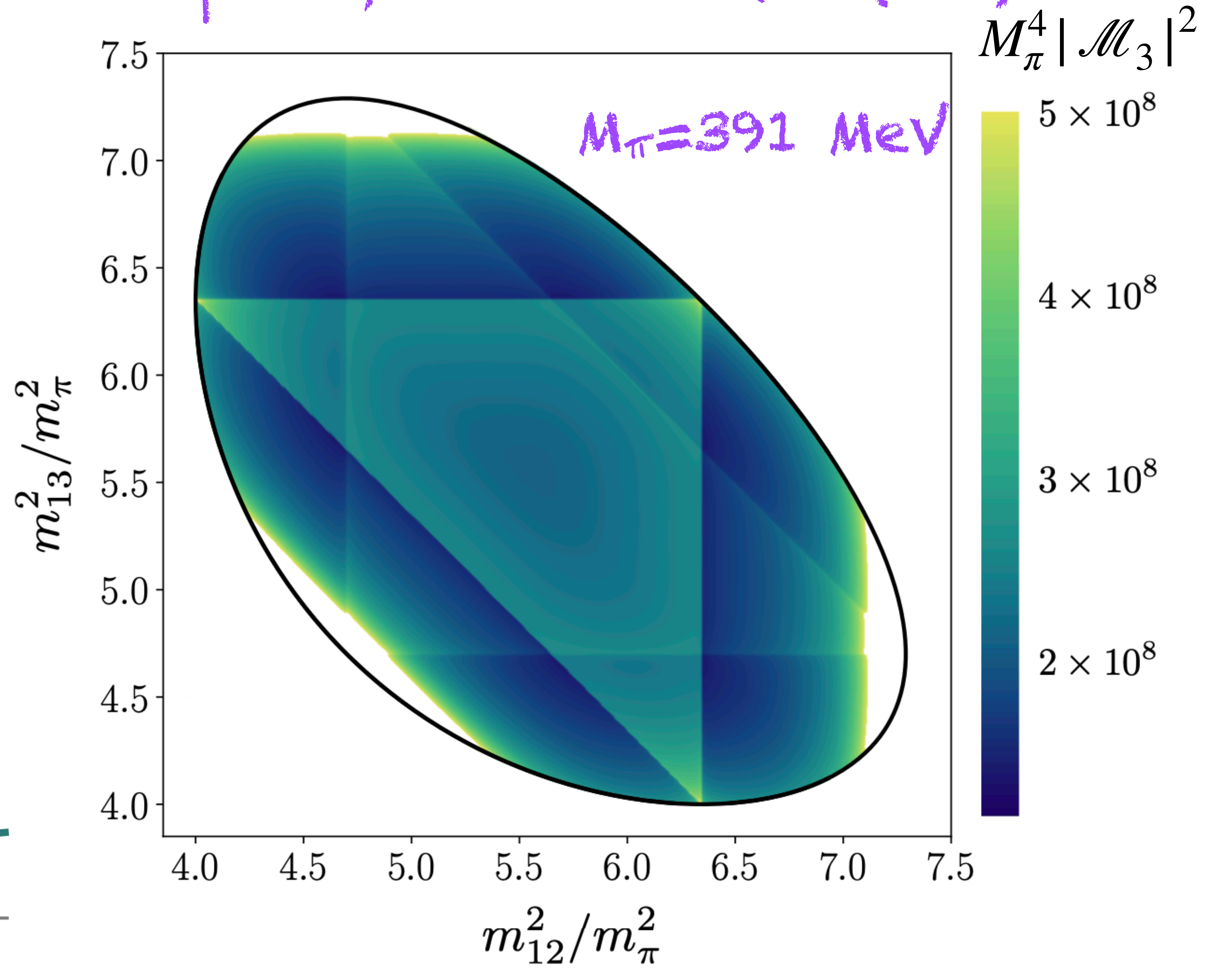
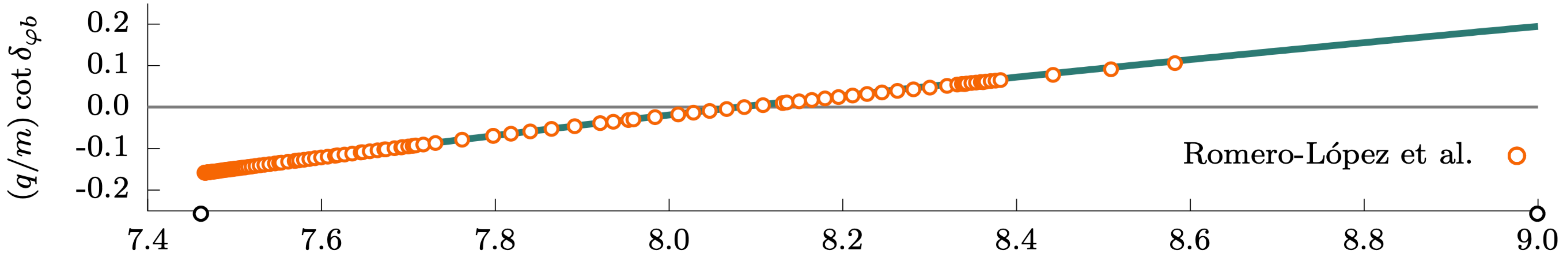
Dalitz plots from lattice QCD ($3\pi^+$)



[Hansen et al. (HadSpec)]

Particle-Dimer phase shift [Jackura et al.]



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$$\det_{k\ell m}\left[\mathcal{K}_{\mathrm{df},3} + F_3^{-1}\right] = 0$$

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Finite-volume information & two-body interactions

$$F_3 = \frac{1}{L^3}\left[\frac{F}{3} - F\frac{1}{(\mathcal{K}_2)^{-1} + F + G}F\right]$$

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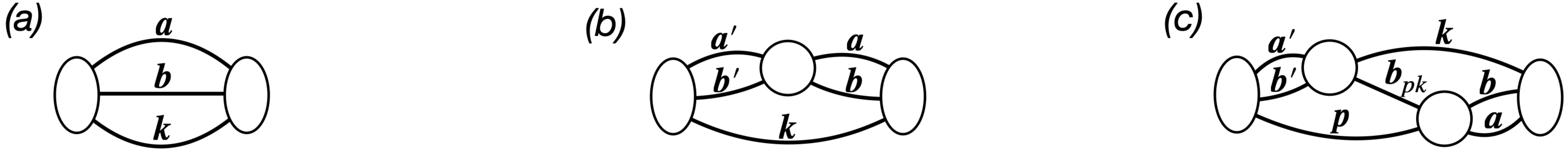


F $\qquad$ $\mathcal{K}_2$ $\qquad$ G

$$F_{00}(q^2) \sim \left[\frac{1}{L^3}\sum_{\vec{k}} - \int \frac{d^3k}{(2\pi)^3}\right]\frac{1}{k^2-q^2}$$

$$\mathcal{K}_2^{\ell} = \frac{16\pi\sqrt{s}}{q^{2\ell+1}\cot\delta_\ell}$$

$$G_{p00;k00} \equiv \frac{1}{L^3}\frac{1}{2\omega_p}\frac{H(\vec{p})H(\vec{k})}{b_{pk}^2 - m^2}\frac{1}{2\omega_k}$$

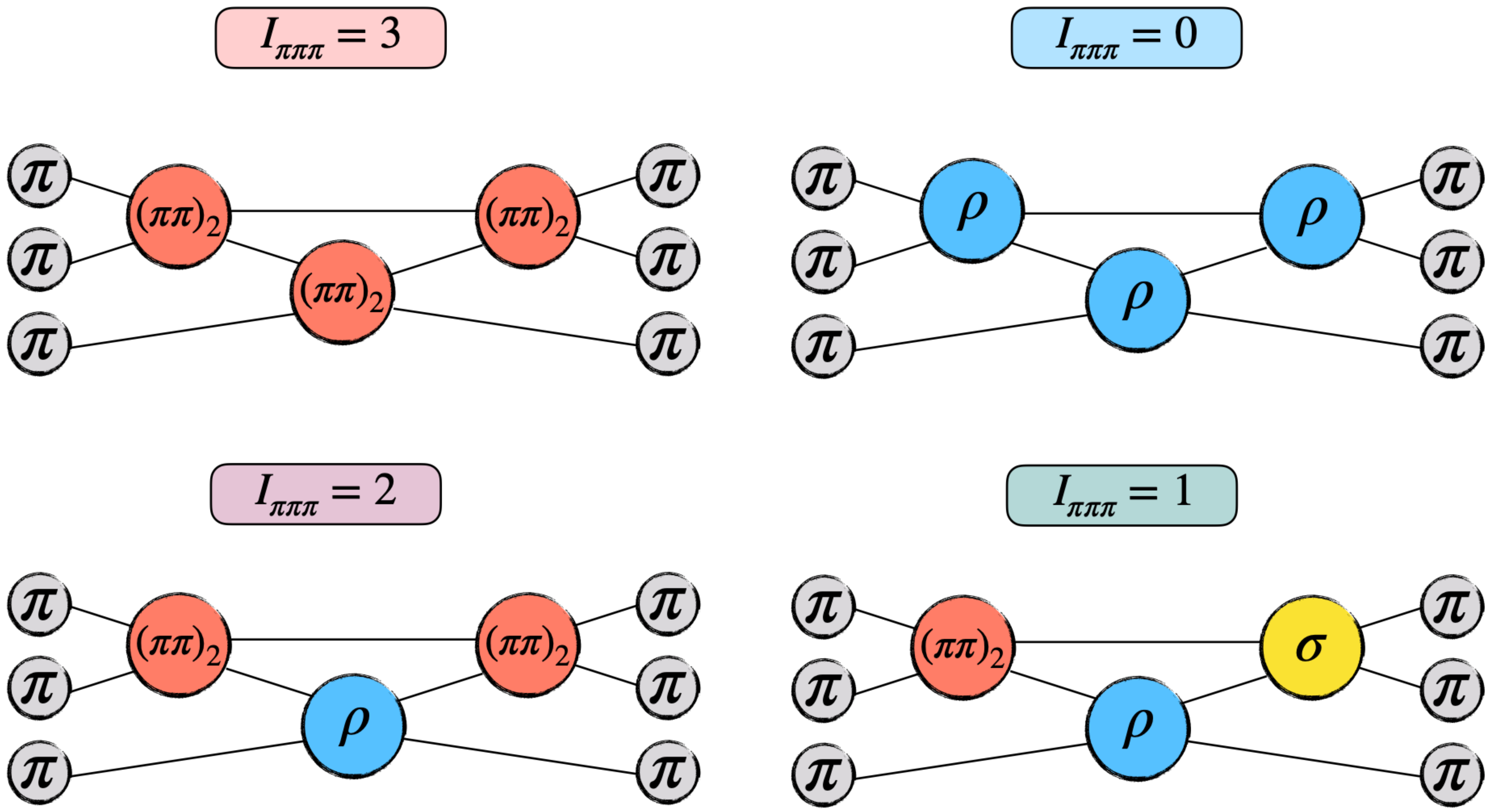
Fernando Romero-López, MIT

Including isospin

- Relevant three-body systems involve nonidentical particles ($\pi\pi N$)
- Let us consider mass-degenerate pions with different flavor [Hansen, FRL, Sharpe, JHEP 2020]

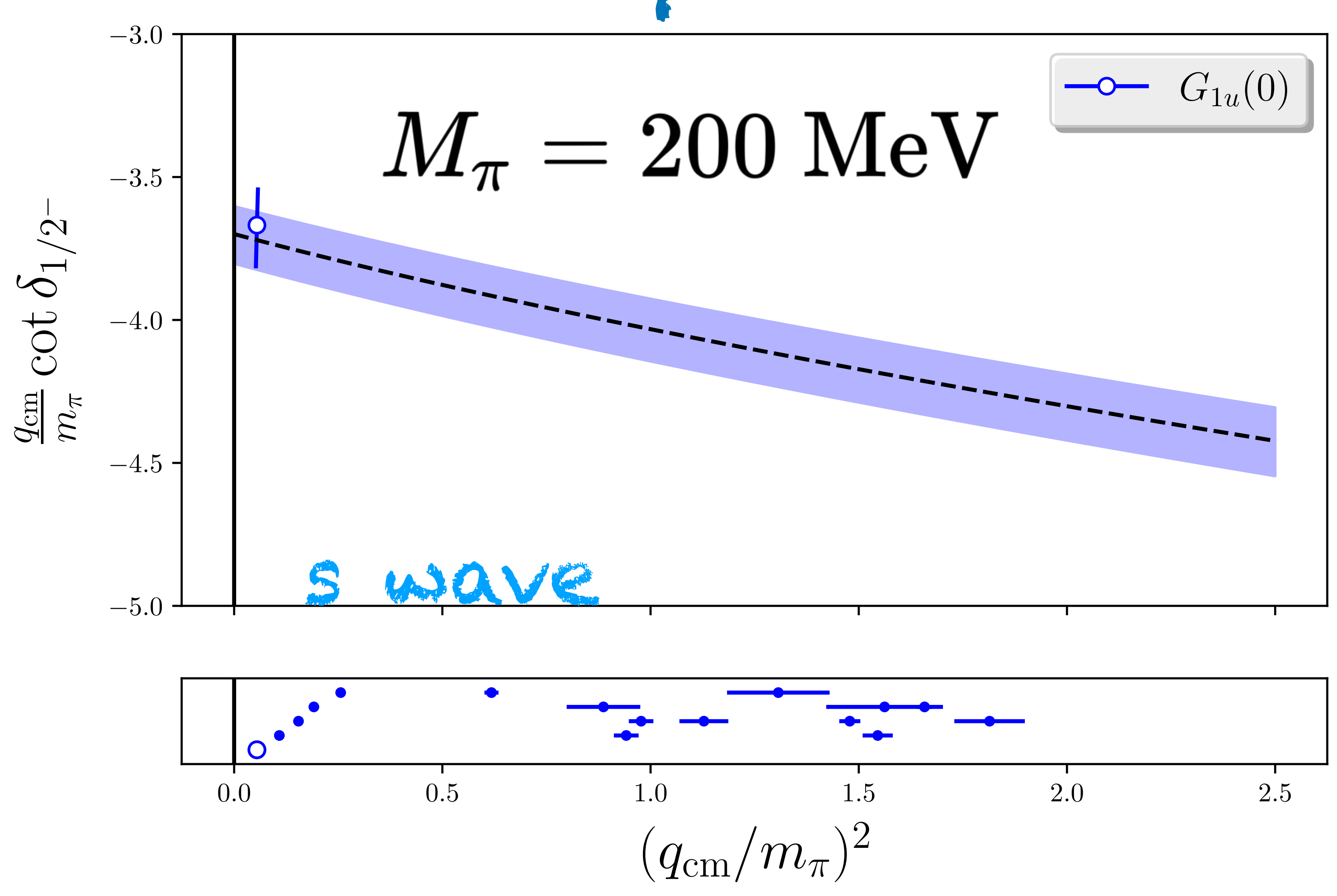
e.g. $\pi^+\pi^0\pi^-$

- All pions have the same mass
- Overall isospin is conserved
- Presence of resonances
- Example of multi-channel scattering



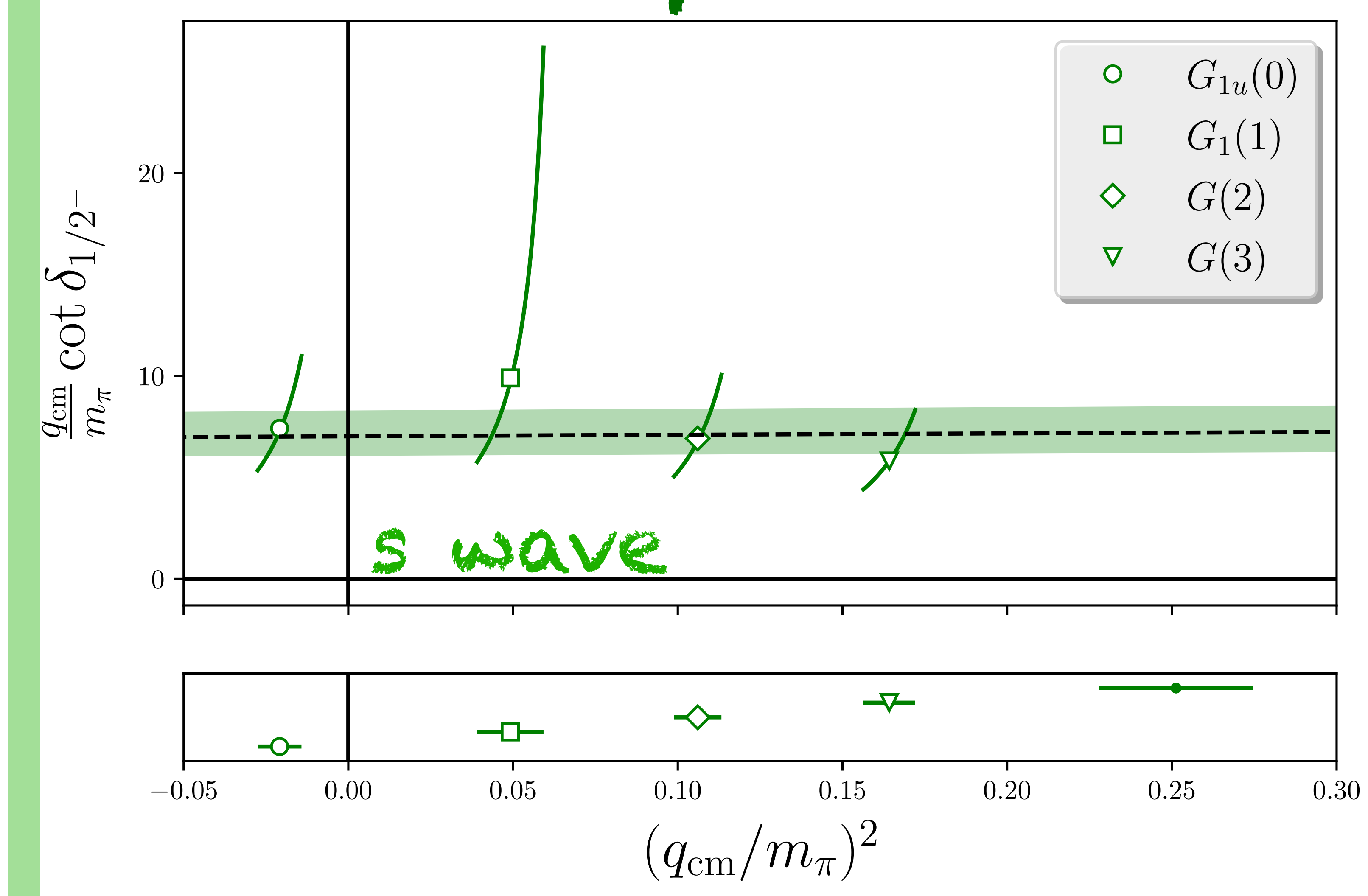
The πN scattering lengths

Isospin 3/2



$$m_\pi a_0^{3/2} = -0.2735(81)$$

Isospin 1/2

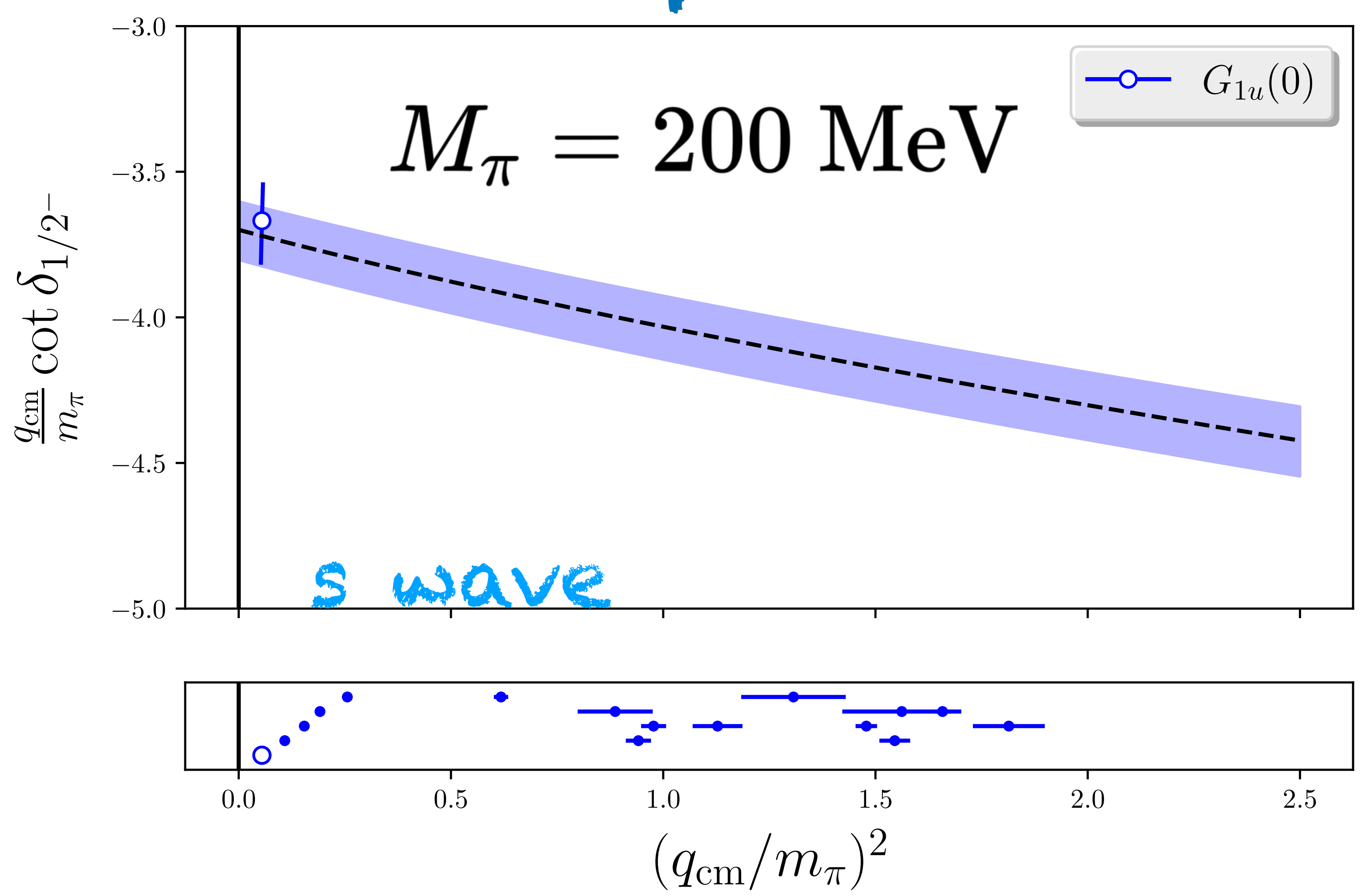


$$m_\pi a_0^{1/2} = 0.142(22)$$

[Bulava, Hanlon, Hörz, Morningstar, Nicholson, FRL, Skinner, Vranas, Walker-Loud, 2208.03867]

The πN scattering lengths

Isospin 3/2



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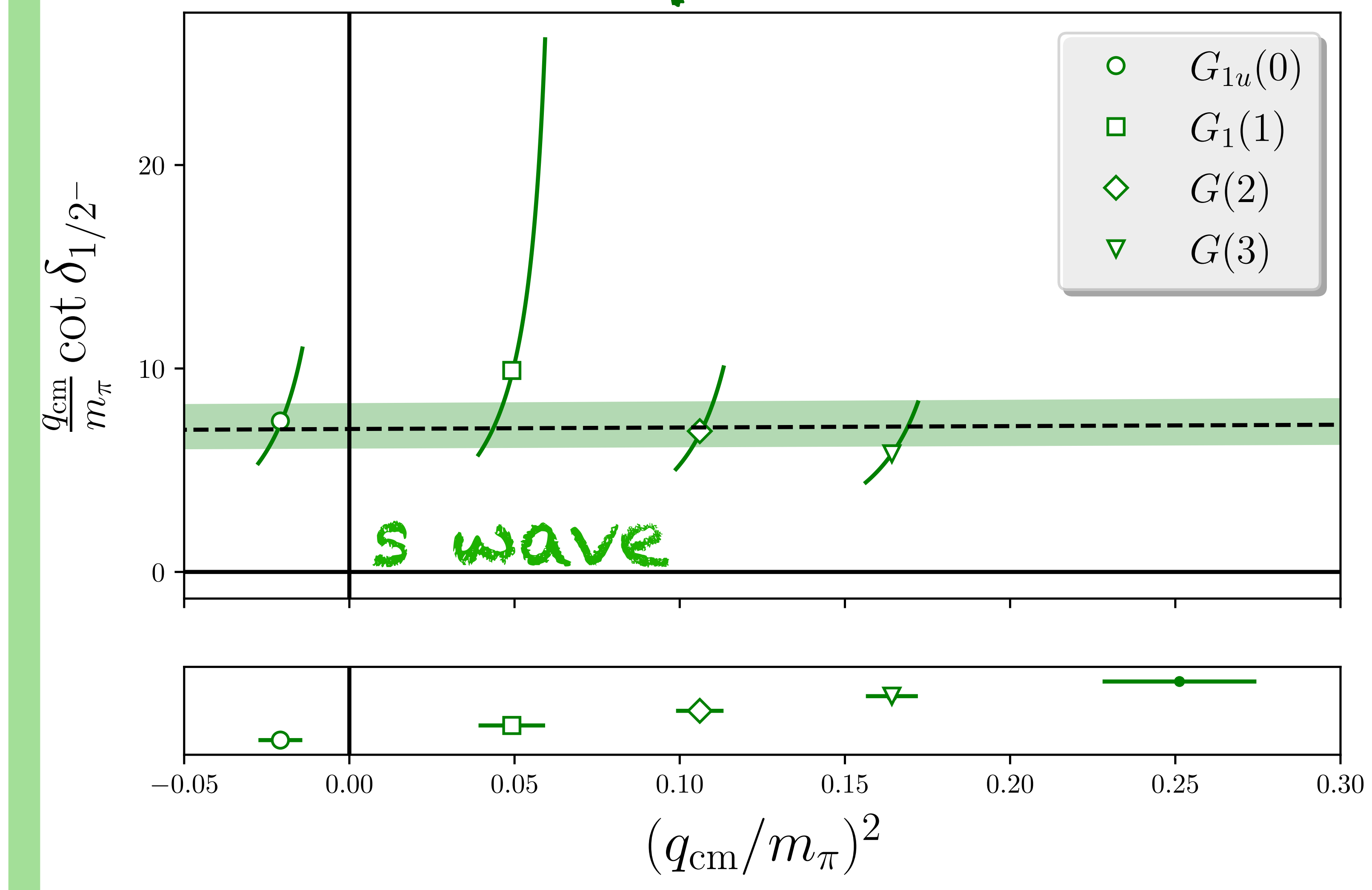
Isospin 1/2



$$m_\pi a_0^{1/2} = 0.142(22)$$

Determination of scattering lengths closest to the physical point!

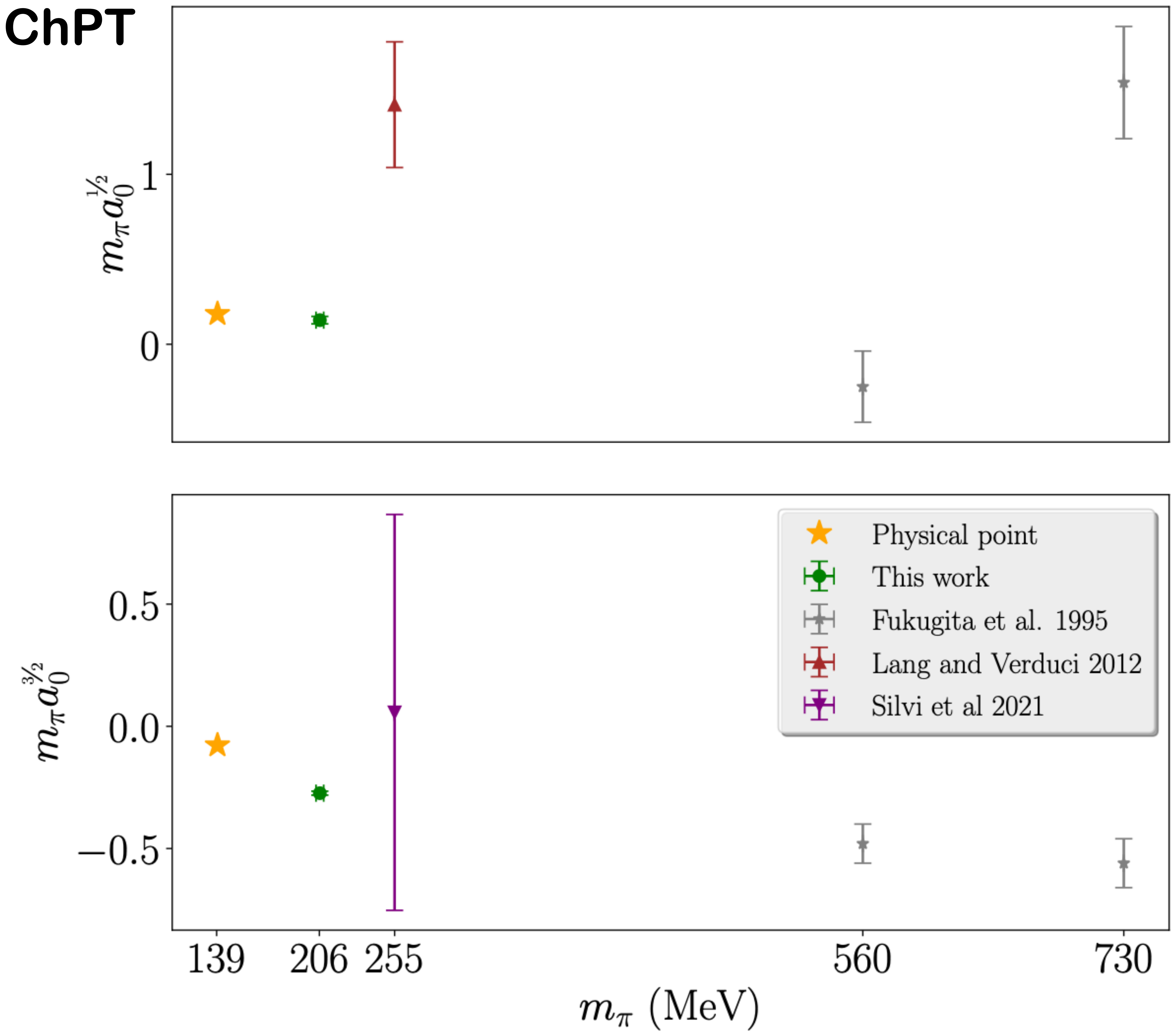
[Bulava, Hanlon, Hörz, Morningstar, Nicholson, FRL, Skinner, Vranas, Walker-Loud, 2208.03867]

The πN scattering lengths

[Bulava, Hanlon, Hörz, Morningstar, Nicholson, FRL, Skinner, Vranas, Walker-Loud, 2208.03867]

- **Our results can be used to test the convergence of baryon ChPT**

	m_π (MeV)	$m_\pi a_0^{1/2}$	$m_\pi a_0^{3/2}$
This work	200	0.142(22)	−0.2735(81)
LO χPT	200	0.321(04)(57)	−0.161(02)(28)



Fernando Romero-López, MIT

The πN scattering lengths

[Bulava, Hanlon, Hörz, Morningstar, Nicholson, FRL, Skinner, Vranas, Walker-Loud, 2208.03867]

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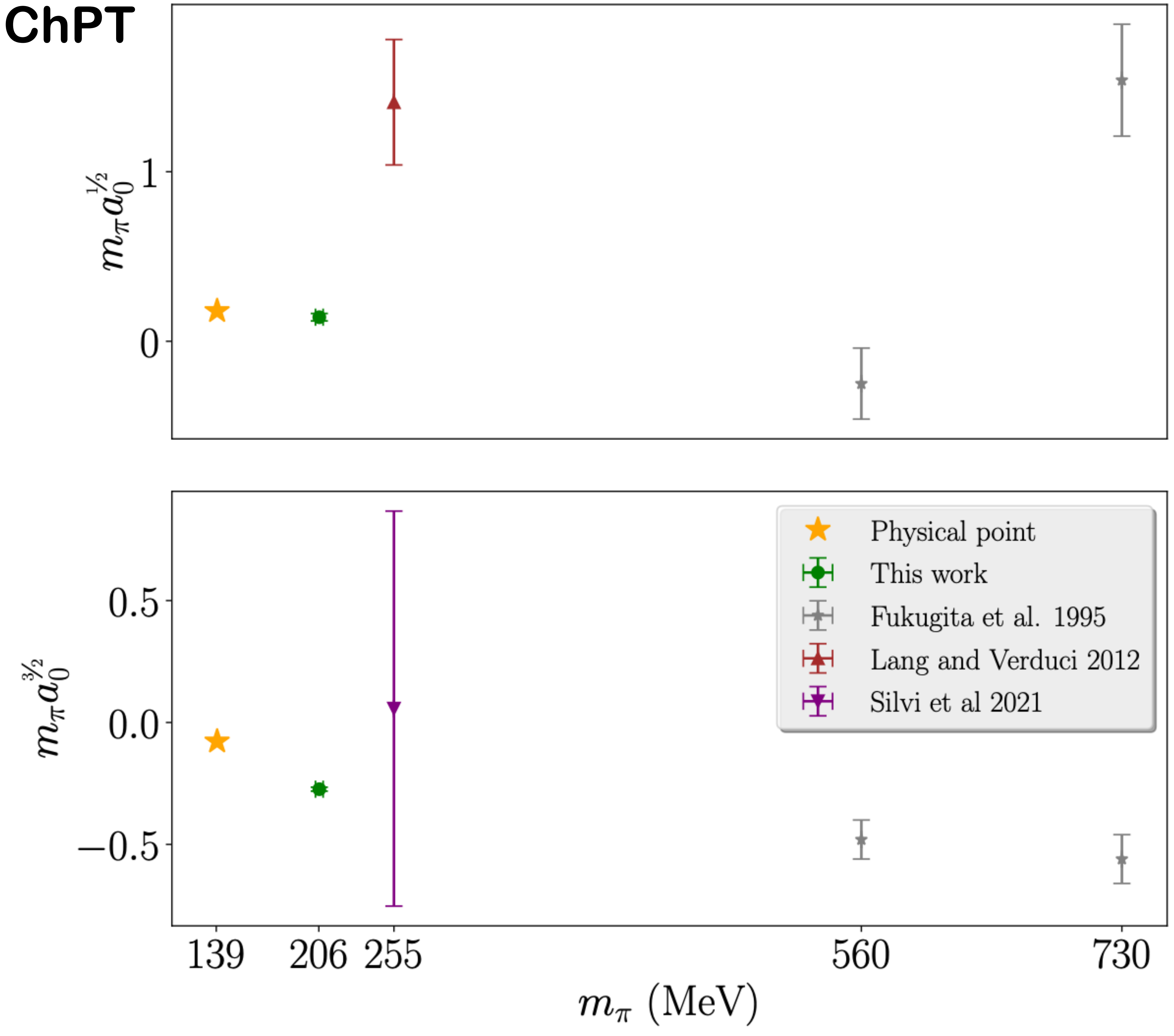
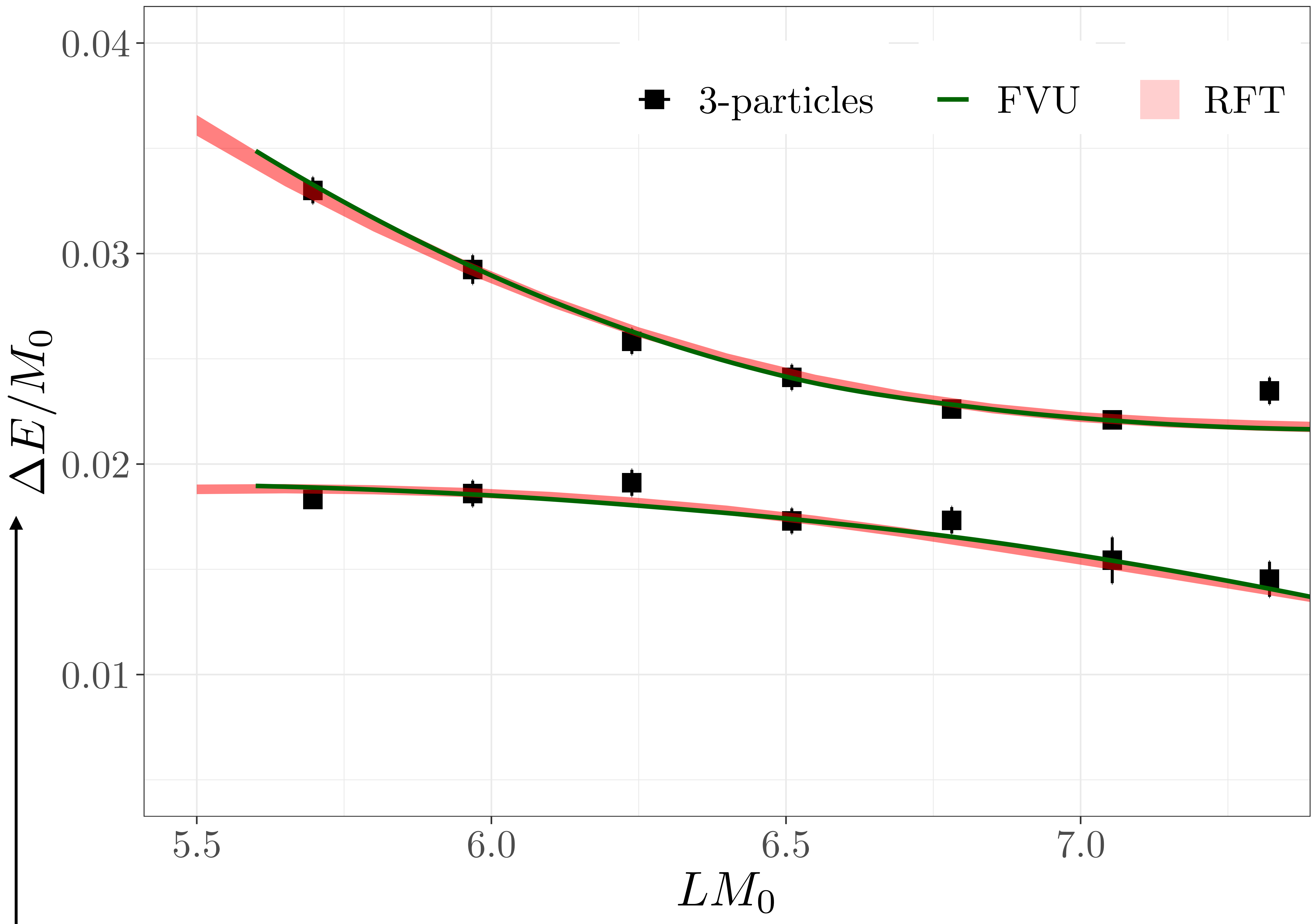
We find poor convergence at M_π = 200 MeV



- Additional values of the pion mass are needed!

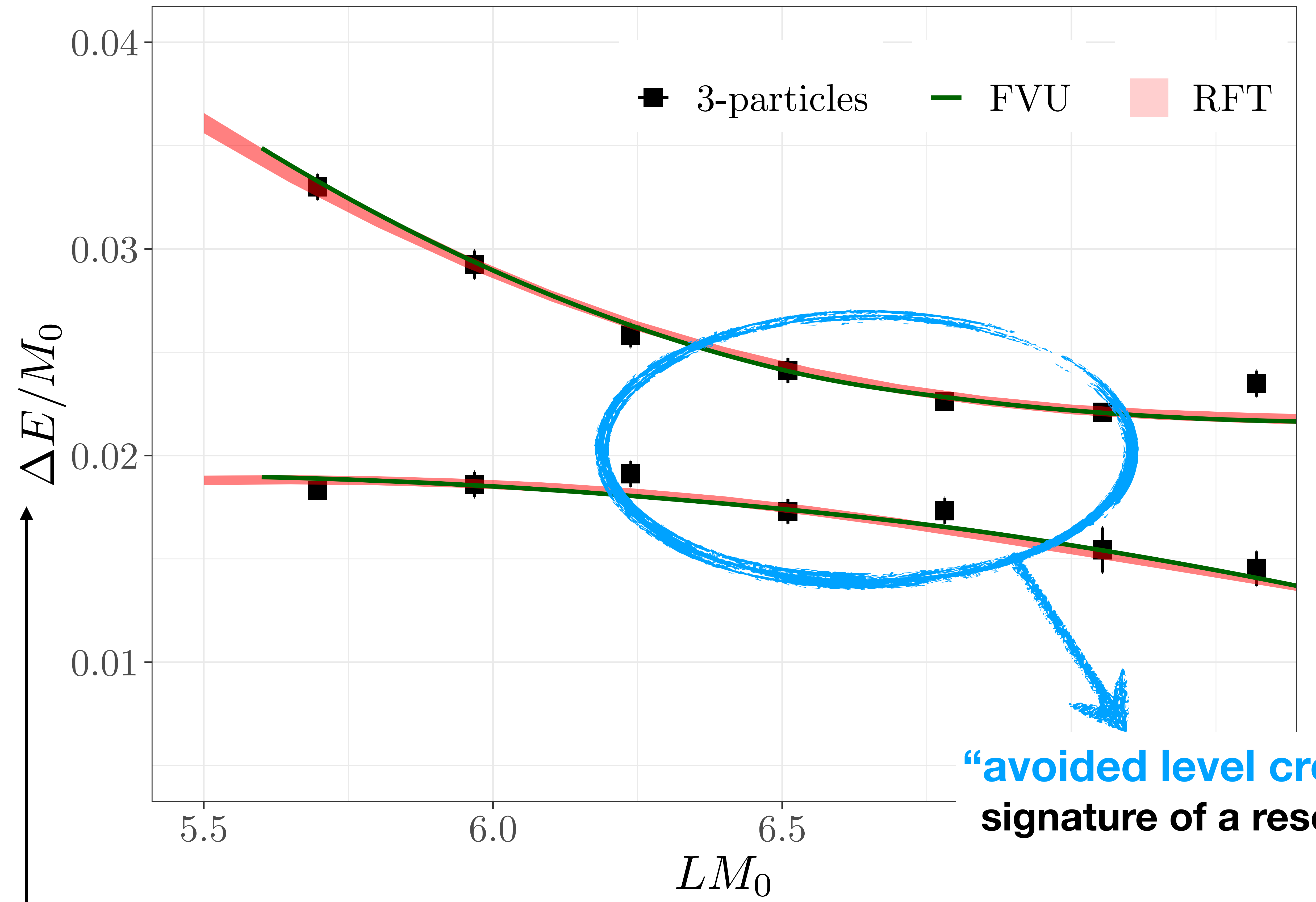
Three-body spectrum



[Garofalo, Mai, FRL, Rusetsky, Urbach (2211.05605)]

$\Delta E = E - 3M_0$

Three-body spectrum

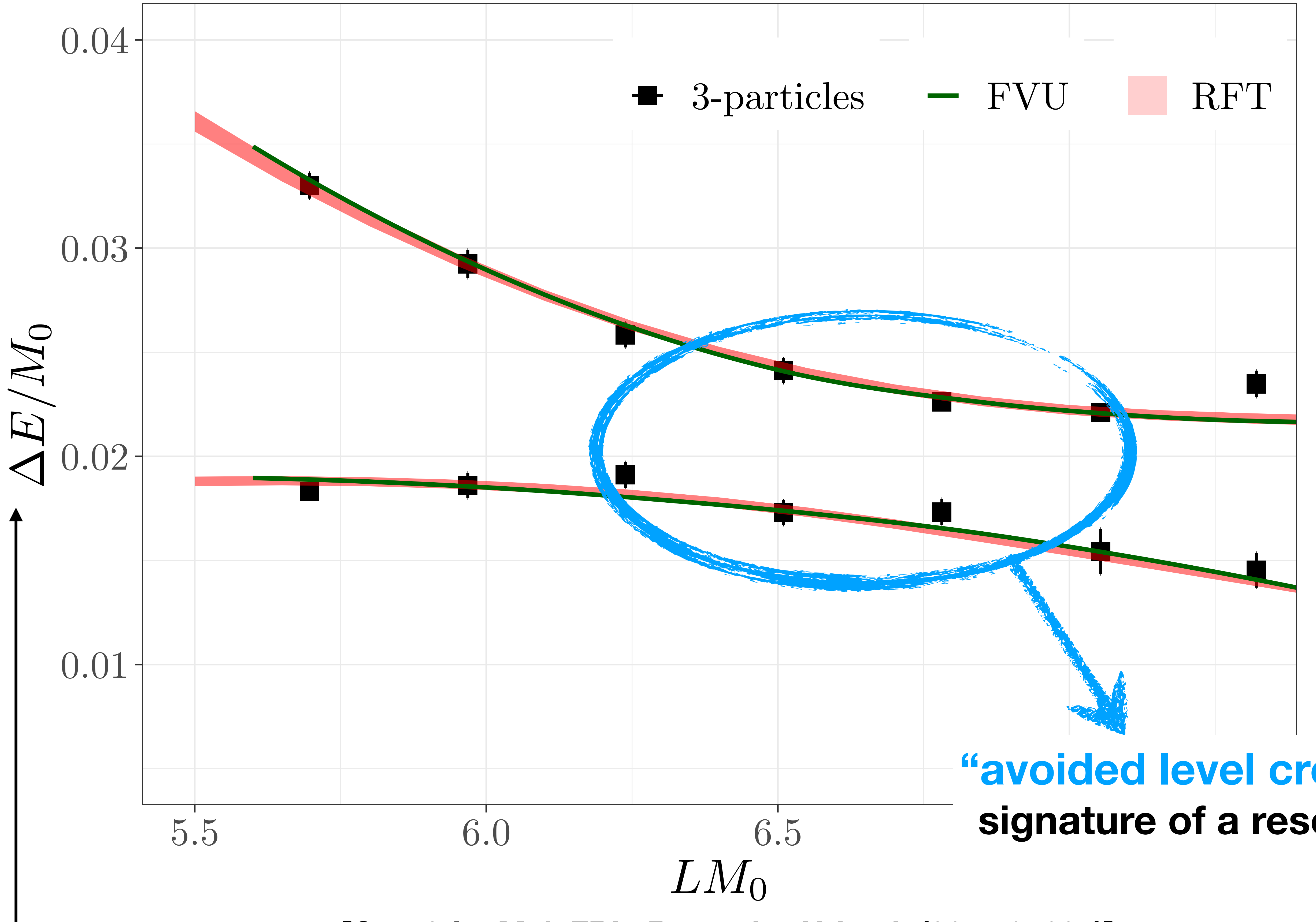


“avoided level crossing”
signature of a resonance

$\Delta E = E - 3M_0$

[Garofalo, Mai, FRL, Rusetsky, Urbach (2211.05605)]

Three-body spectrum



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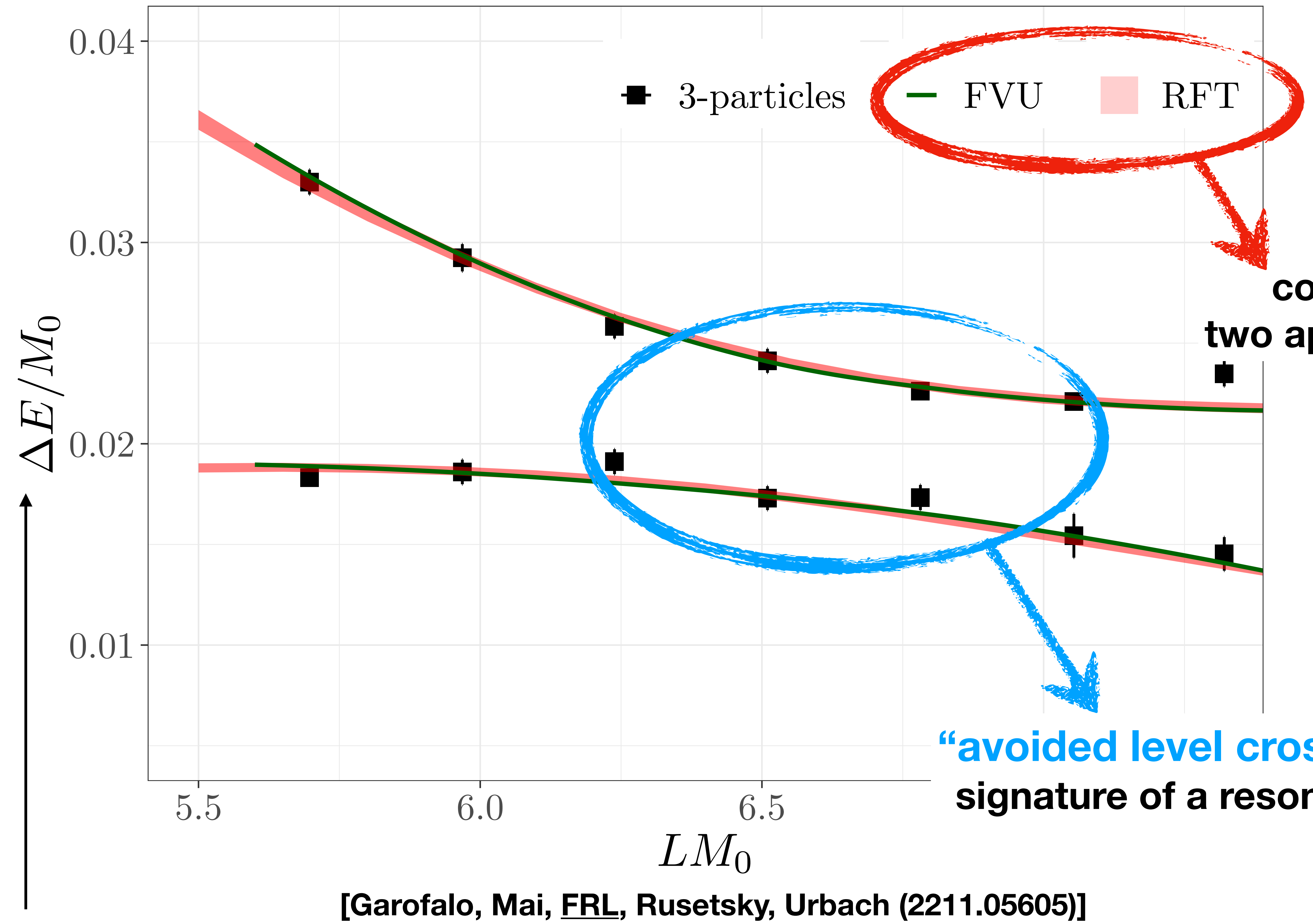
[Garofalo, Mai, FRL, Rusetsky, Urbach (2211.05605)]

Parametrize three-body K-matrix:

$$\mathcal{K}_{\mathrm{df},3} = \frac{c_0}{E_{\mathrm{CM}}^2 - m_R^2} + c_1$$

Three-body spectrum



$\Delta E = E - 3M_0$

[Garofalo, Mai, FRL, Rusetsky, Urbach (2211.05605)]

compare
two approaches

Similar results

$\chi^2/dof \sim 1.3$

"avoided level crossing"
signature of a resonance

Parametrize three-body K-matrix:

$$\mathcal{K}_{\mathrm{df},3} = \frac{c_0}{E_{\mathrm{CM}}^2 - m_R^2} + c_1$$