



## Non-linear Integrable Optics, Landau Damping

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IOTA Collaboration Meeting

12 March 2024



THE UNIVERSITY OF  
**CHICAGO**

# Outline

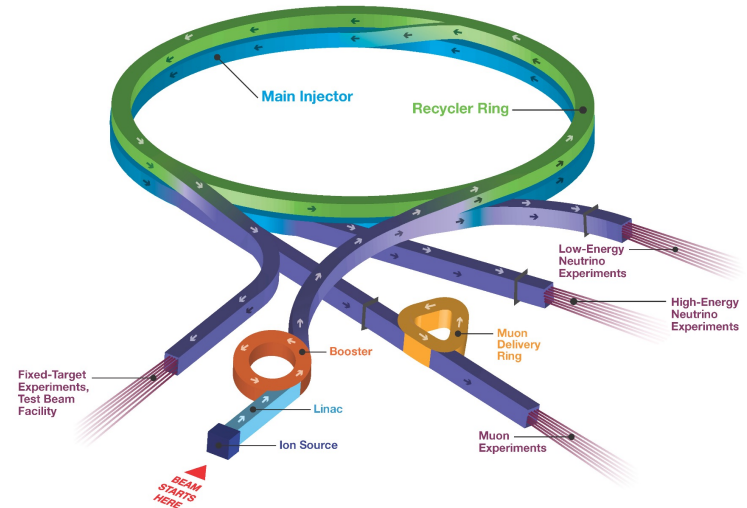
- Introduction
  - Motivation
  - Introduction to Landau Damping
  - Stability Diagram Theory
  - Method to Measure Stability Diagram
  - Measurement Predictions
- NIOLD
  - Experimental Setup
  - Data Collection methods
  - Analysis
- Conclusions
  - Successes
  - Next Steps

# Introduction

# Motivation

- In most Particle Accelerators, we want to:
  - Maximize beam intensity
- An accelerator's beam intensity is limited by collective instabilities
- If beam is unstable:
  - Produce losses, irradiate components, etc.
- We approach the grand challenge of beam intensity through studying beam stability

Fermilab Accelerator Complex



<https://www.fnal.gov/pub/science/particle-accelerators/accelerator-complex.html>

# Background/Motivation – Landau Damping

- Landau Damping (LD) Historical Context:
  - Derived originally for plasmas with purely mathematical approach by Landau (1946)
  - First experimental evidence for LD observed in 1964 by Malmberg and Wharton:

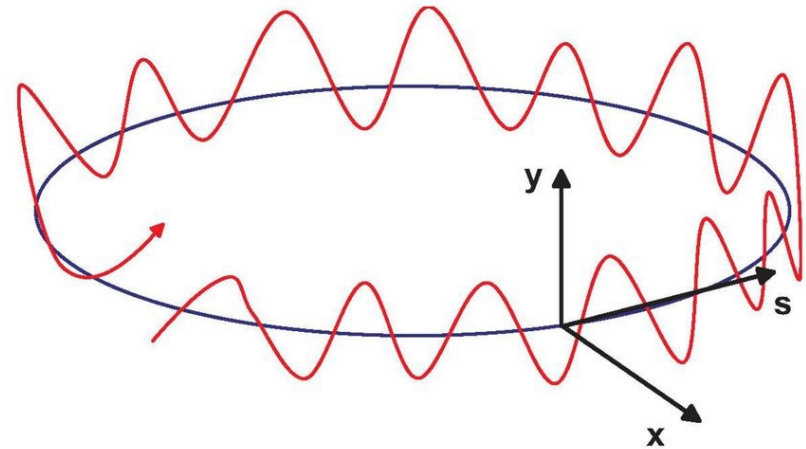
COLLISIONLESS DAMPING OF ELECTROSTATIC PLASMA WAVES\*  
J. H. Malmberg and C. B. Wharton  
John Jay Hopkins Laboratory for Pure and Applied Science,  
General Atomic Division of General Dynamics Corporation, San Diego, California  
(Received 6 July 1964)

- In collisionless plasmas, Landau damping (LD) is the effect of damping of collective oscillation modes.
- Landau Damping acts a defense against collective instabilities

# Background/Motivation – Landau Damping

## Landau Damping in Accelerators:

- In our case, LD depends on spread of betatron frequencies
- Works through energy transfer between coherent & incoherent oscillation modes
- ‘damps’ collective oscillation modes in energy transfer, stabilizing intense beams



Detailed knowledge of LD strength is essential to predictions on beam stability

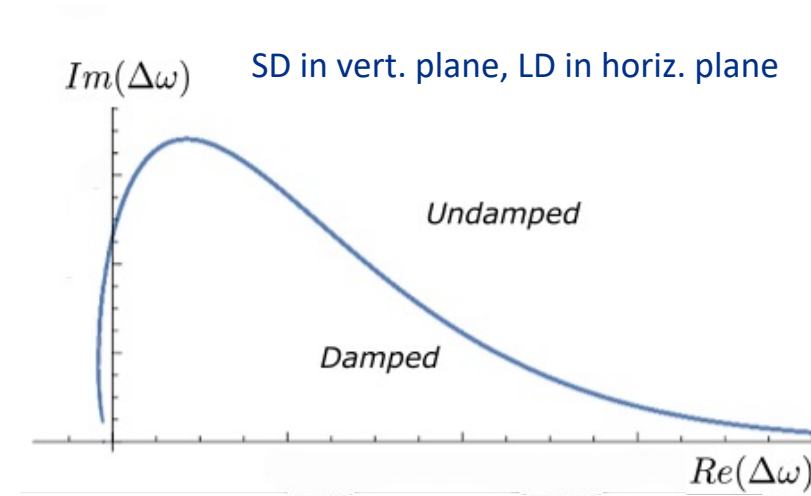
LD studies approached via Stability Diagram Theory

# Stability Diagram Theory

- Given Collective mode tune shift (from external source in vert. plane), find resulting frequency in presence of LD using dispersion relation:

$$\Delta\omega = -1/ \int \frac{J_y \partial F / \partial J_y}{\Omega + \delta\omega(J_x, J_y) + i0} dJ_x dJ_y$$

Collective mode tune shift  $\rightarrow \Delta\omega$   
 Action in y  $\rightarrow J_y$   
 Bunch distribution function  $\rightarrow F$   
 Freq. in presence of LD  $\rightarrow \Omega$   
 Freq. shift of oscillating particles  $\rightarrow \delta\omega$   
 Vanishingly small term (avoid singularities)  $\rightarrow i0$



Alexey Burov. "Inverse stability problem in beam dynamics." In: *Phys. Rev. Accel. Beams* 26 (8 Aug. 2023),

- Often to find  $\Omega$ : solve dispersion equation for all  $\Delta\omega$ .
- If map  $\text{Im}[\Omega] = 0$  onto the complex plane of  $\Delta\omega$ , obtain threshold separating stable and unstable states of bunch: **Stability Diagram**

# SD Measurement Methods

- Current techniques measuring LD have limitations: Don't directly measure LD strength
- We investigate alternative method to measure LD strength:
  - Revert polarity of transverse feedback to excite collective mode: **antidamper**
  - Produces coupling impedance ( $G$  - gain,  $\phi$  - phase):

$$Z(\omega) \propto Ge^{i\phi} \delta(\omega)$$

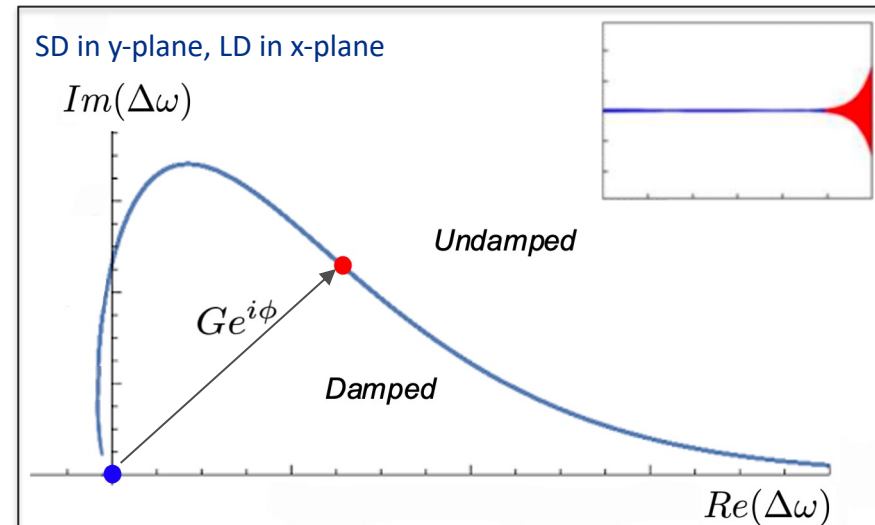
Delta function: antidamper  
kicks bunch as whole

- Shifts frequencies of collective modes by ( $g$  - growth rate):

$$\Delta\omega \propto ge^{i\phi}$$

- Change gain and phase: controlled impedance
- Observe gain where beam goes unstable: get SD

- We measure SD in the vert. plane, LD provided in horiz. plane



Alexey Burov. "Inverse stability problem in beam dynamics." In: *Phys. Rev. Accel. Beams* 26 (8 Aug. 2023),

I. Fadelli, "A procedure to directly measure the strength of landau damping," *Phys. Org*, 2021. <https://phys.org/news/>



# SD Measurement Method

- Proof-of-principle study of this method has been performed at the LHC:

## Proof-of-Principle Direct Measurement of Landau Damping Strength at the Large Hadron Collider with an Antidamper

S. A. Antipov<sup>1,2,\*</sup>, D. Amorim<sup>1,3</sup>, N. Biancacci<sup>1</sup>, X. Buffat<sup>1</sup>, E. Métral<sup>1</sup>,  
N. Mounet<sup>1</sup>, A. Oeftiger<sup>1,4</sup> and D. Válich<sup>1</sup>

<sup>1</sup>European Organization for Nuclear Research (CERN), Geneva 1211, Switzerland

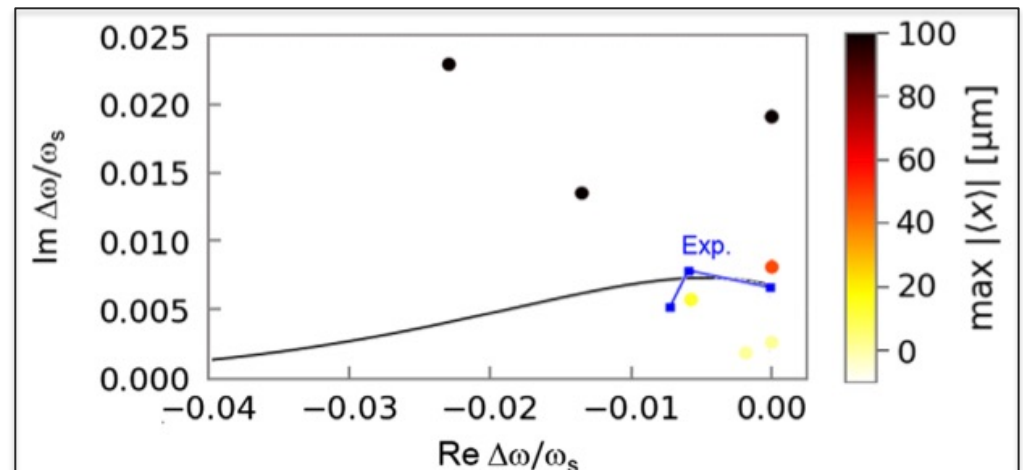
<sup>2</sup>Deutsches Elektronen-Synchrotron (DESY), Hamburg 20459, Germany

<sup>3</sup>Synchrotron SOLEIL, Gif-sur-Yvette 91192, France

<sup>4</sup>Helmholtzzentrum für Schwerionenforschung (GSI), Darmstadt 64291, Germany

(Received 7 August 2020; revised 13 January 2021; accepted 12 March 2021; published 20 April 2021)

- Why expand on this research?
  - More phase measurements
  - investigate impact of machine's impedance
  - obtain beam distribution function



‘The color of the dots depicts the maximum beam excursion. The blue squares and line show the measured limit of stability. The solid black line shows a SD prediction for a gaussian beam.’

S. Antipov et. al., Phys. Rev. Lett.126, 164801, 2021.

# SD Measurement Predictions

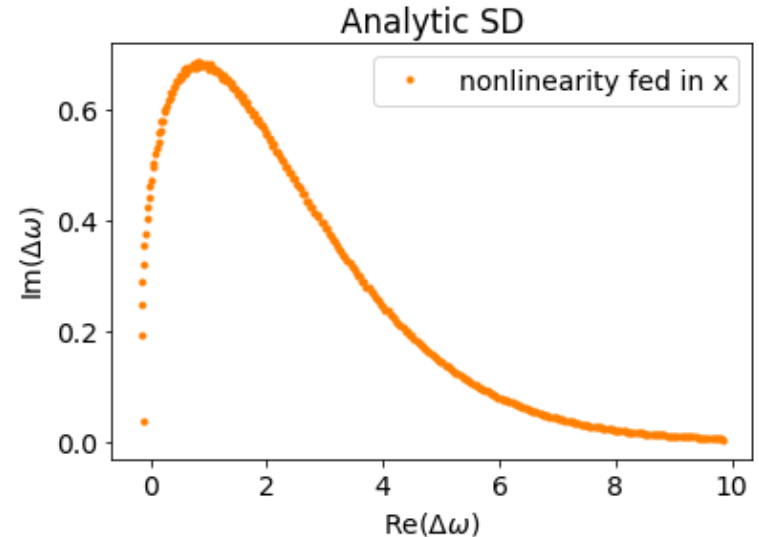
- Normalized analytical stability diagram:
  - Octupoles as LD source in horizontal plane (x)
  - Excitation fed through vertical plane (y)

$$\Delta\omega = \left[ \int dJ_x \frac{F_x(J_x)}{\Omega - J_x + i0} \right]^{-1}$$

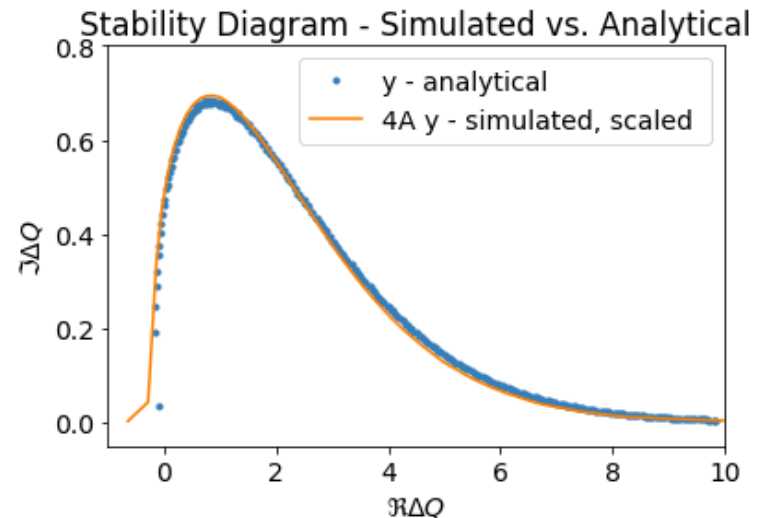
Collective mode tune shift  $\Delta\omega$   
 Action in x  $F_x(J_x)$   
 Frequency in presence of LD  $\Omega - J_x + i0$

## Simulated SD in IOTA

- IOTA madx lattice, Xsuite for simulation
- Simulate at high LD: octupoles dominate
- Left: scaled simulation with analytical



Alexey Burov. "Inverse stability problem in beam dynamics." In: *Phys. Rev. Accel. Beams* 26 (8 Aug. 2023),



# NIOLD Experiment

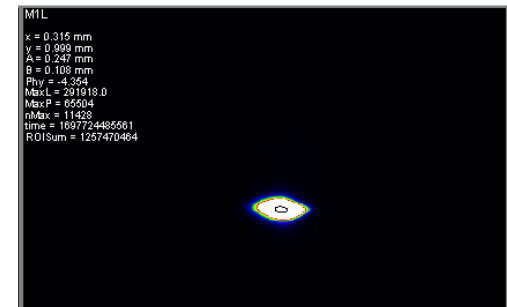
Current experiment does not involve Non-linear Integrable Optics

# Experimental Shifts (IOTA Run 4)

- NIOld Run 4 Experimental Shifts:
  - Difficulties collecting necessary data
    - did not know was necessary until post data-collection
  - Took many data sets with too small octupole strengths

| Date            | Notes   |
|-----------------|---|
| 09/26/23        | Commissioning                                 |
| 09/28/23        | Commissioning                                 |
| 09/29/23        | Data Collection – missing necessary data      |
| 10/03/23        | Data Collection – missing necessary data      |
| 10/05/23        | Data Collection – missing necessary data      |
| 10/14/23        | Data Collection – low octupole currents       |
| 10/17/23        | Data Collection – low octupole currents       |
| <b>10/19/23</b> | <b>Data Collection – Have the right data!</b> |

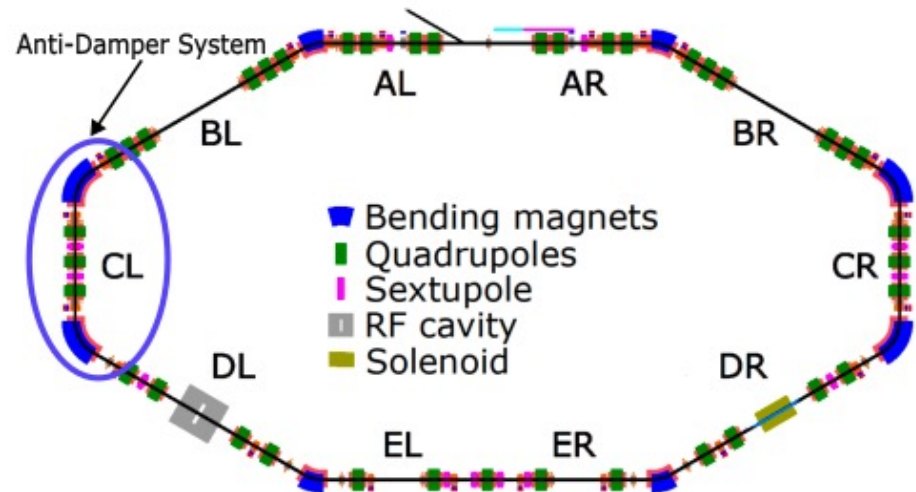
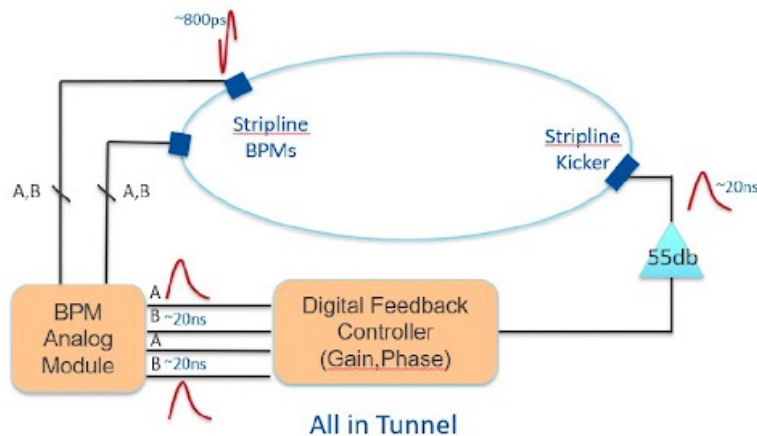
- Obtained beam size data from M1L camera:



- Obtained bunch length data from wall current monitor

# Experimental Setup

- Reverse polarity of transverse feedback to excite collective mode in the beam
- Main experimental elements:
  - Stripline Kicker
  - Two stripline BPMs: approx.  $90^\circ$  apart in phase
  - Octupoles



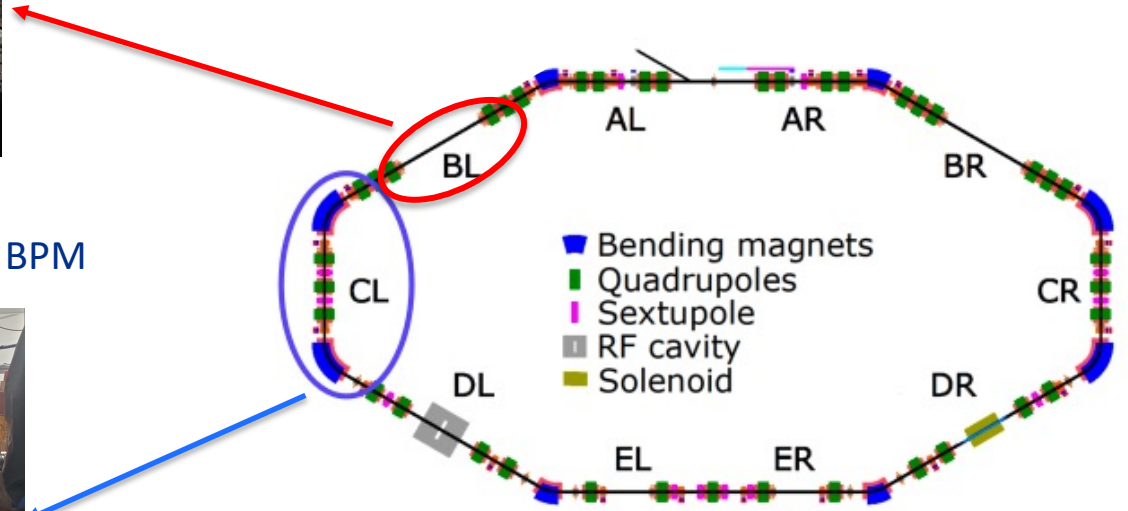
*N. Eddy et al, 'IOTA Experiment Nonlinear Optics: Landau Damping (NIOLD)', Experiment Proposal, April 2022*

# Experimental Setup

Octupole line (9 in total)



- Octupoles as strong source of nonlinear focusing
- Strength of Landau Damping changes depending on octupole strength



Stripline BPM    Stripline kicker    Stripline BPM



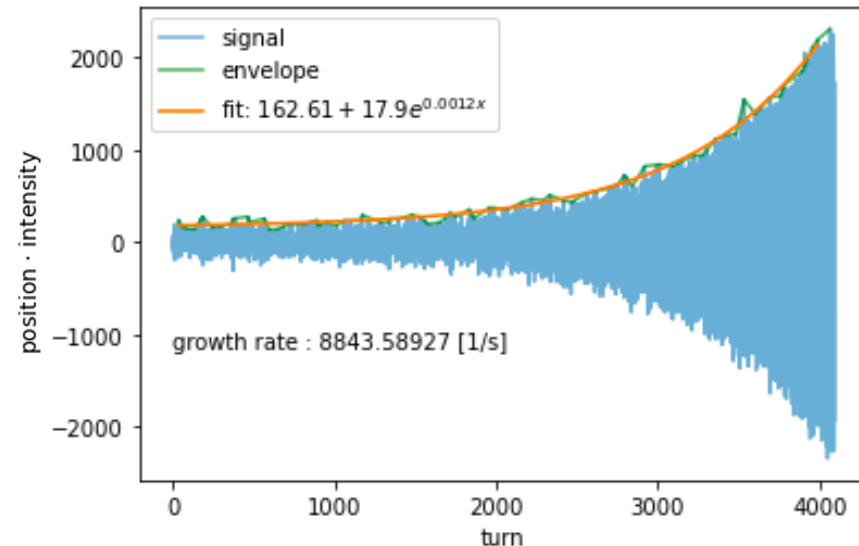
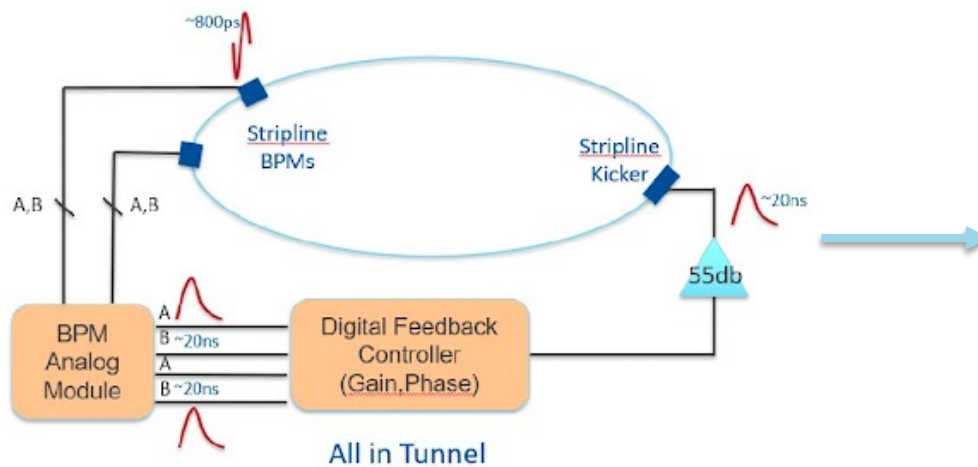
Note: striplines are located within the quadrupoles

# Data Collection

- Phase set by the BPM coefficients, gain set by the kicker

$$Z(\omega) \propto Ge^{i\phi}\delta(\omega)$$

- Stripline BPMs measure beam position,  $\sim 110^\circ$  apart
- Experiment Process:
  - Sweep through  $\phi$  of the antidamper
  - At each  $\phi$ , observe what kicker gain beam becomes unstable
  - Obtain growth rate of this instability -> experimentally obtain  $g, \phi$

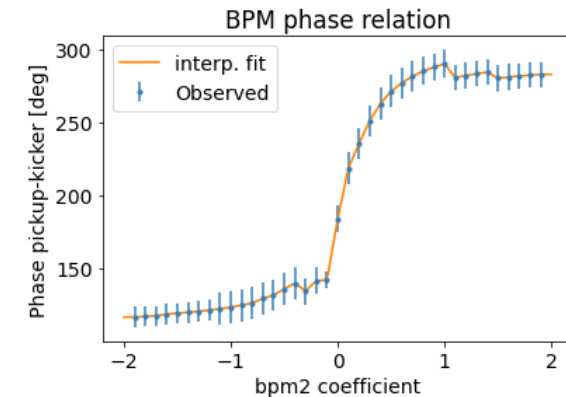
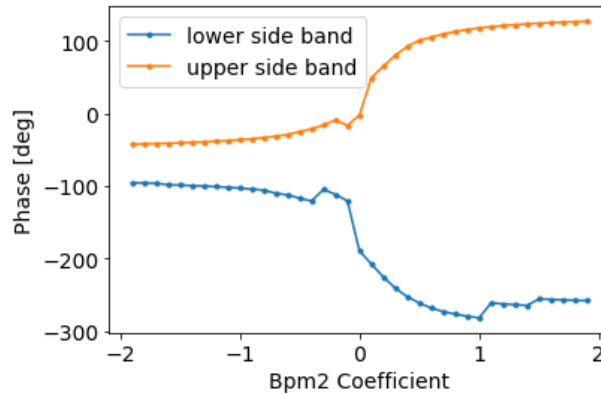
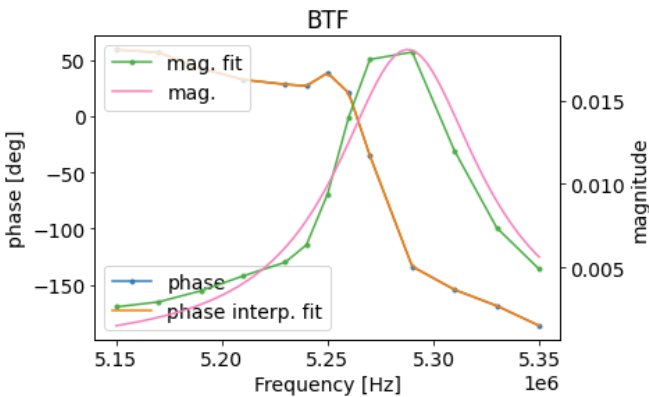
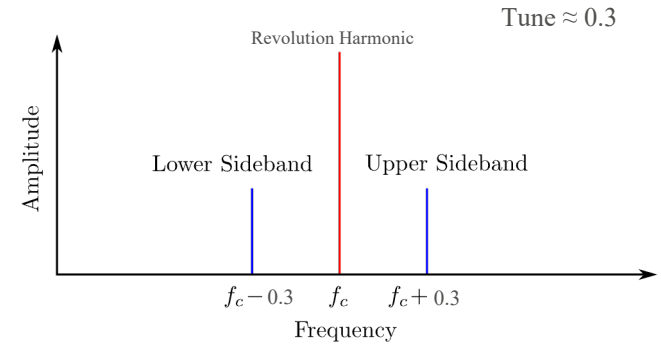


N. Eddy et al, 'IOTA Experiment Nonlinear Optics: Landau Damping (NIOLD)', Experiment Proposal, April 2022

# Analysis: BTFs

To obtain phase advance  $\phi$ :

- Sweep through BPM coef.
- Perform a Beam Transfer Function (BTF): frequency dependence of the response to forced beam oscillations
  - Excite beam at frequencies, get upper and lower sidebands



Perform BTFs: upper and lower sidebands

$$((\theta_{usb} - \theta_{lsb}) + 180)/2 = \phi$$

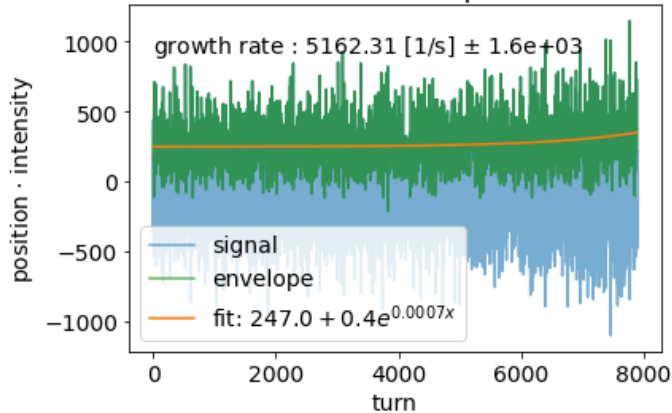
- BTF does not cover full phase map, took BTF data at different turn delays (turns before collected phase), each turn advanced phase by  $\sim 60^\circ$ . This allowed us to cover more phases.



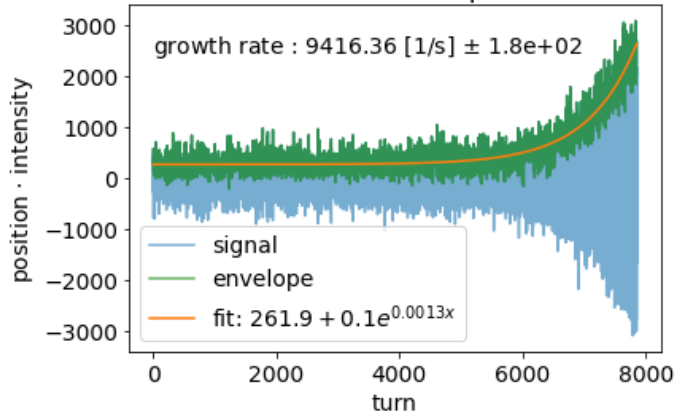
# Analysis: Growth Rate

- Get growth rate for each phase: which kicker gain has first growth
- Linear fit between kicker gain and growth rate.
- Take linear fit value where see first growth
- Perform for each phase (bpm coefficient)

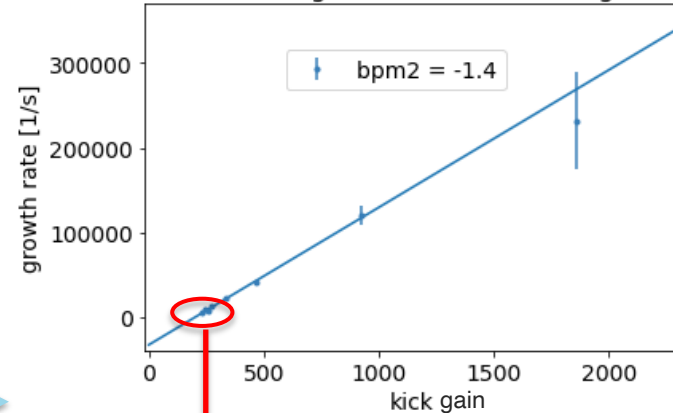
total kick = 229.34, bpm2 = -1.4



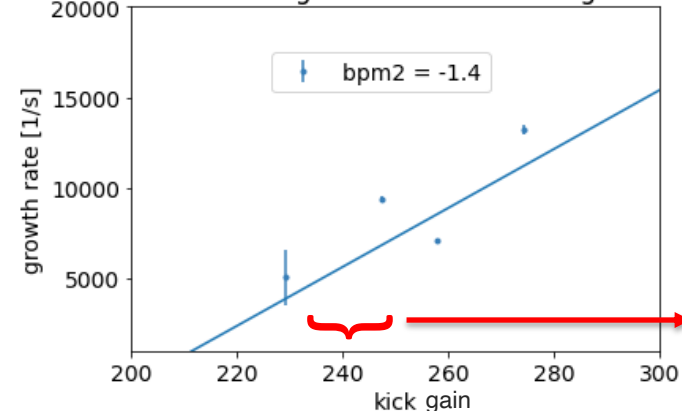
total kick = 247.46, bpm2 = -1.4



linear fits: growth rate to kicker gain



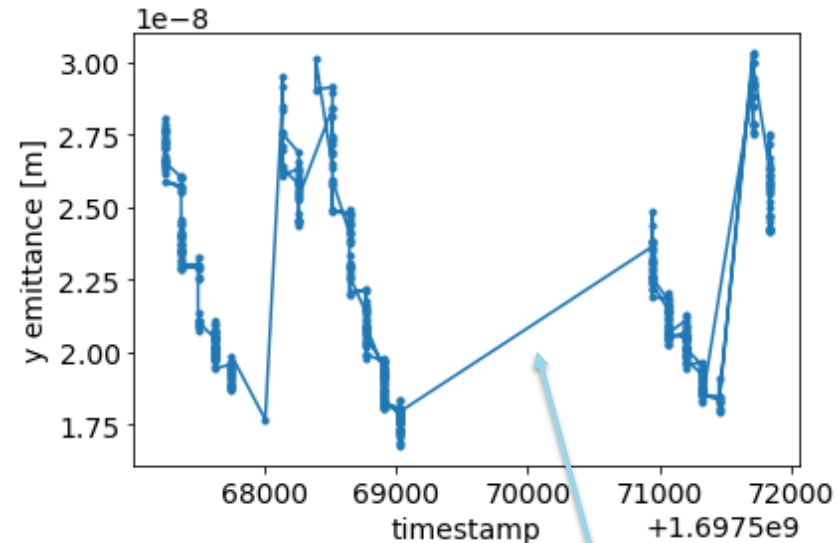
linear fits: growth rate to kicker gain



Growth rate here is used for SD

# Analysis: Growth Rate

- During measurements: beam emittance and intensity changed over time
  - SD size relies on detuning from octupoles: octupole strength, emittance, as well as intensity
  - Normalize each growth by octupole detuning matrix and beam intensity



$$\Delta Q_y = k_{yx}\epsilon_x + k_{yy}\epsilon_y$$

Where:

$$k_{xy}, k_{yx} = -\frac{1}{8\pi} K_3 L \beta_x \beta_y$$

$$k_{yy} = \frac{1}{16\pi} K_3 L \beta_y^2$$

$$\text{where } K_3 = \frac{1}{B\rho} 3! \frac{B_y}{x^3}$$

Normalize each Vert. GR by this to account for emittance variations. Normalize also by intensity.

\* Note:  $\epsilon_x \gg \epsilon_y$  for 4A data, so  $k_{yx} \epsilon_x$  term dominates

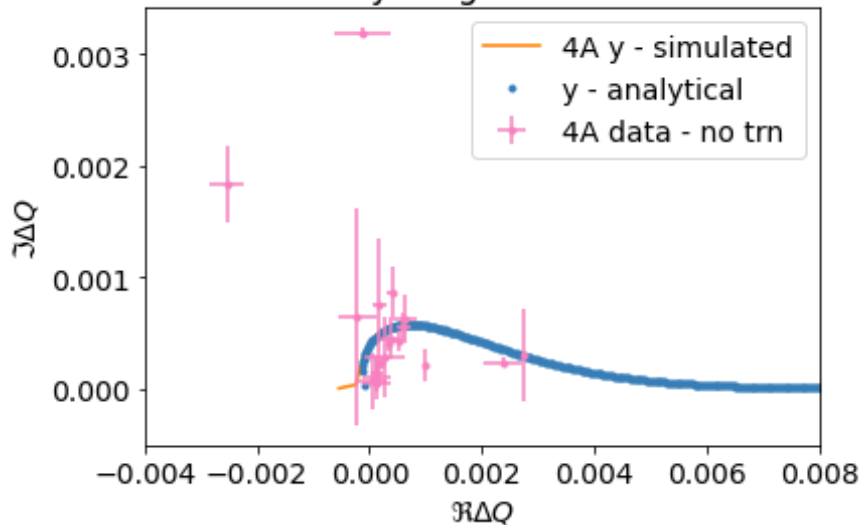
Re-injection

# Analysis: Stability Diagram

- Data taken with all octupoles at 4A
  - Two sub-sets of data: where the beam has no turn delays, where beam has two turns delays before data collection

No turn delay

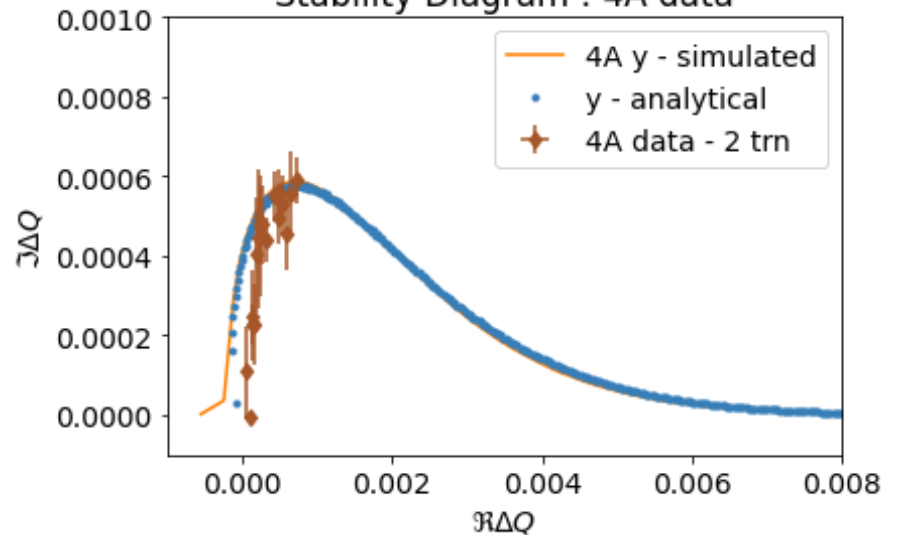
Stability Diagram : 4A data



Large error bars, large outliers,  
unexpected results with phases

two turn delay

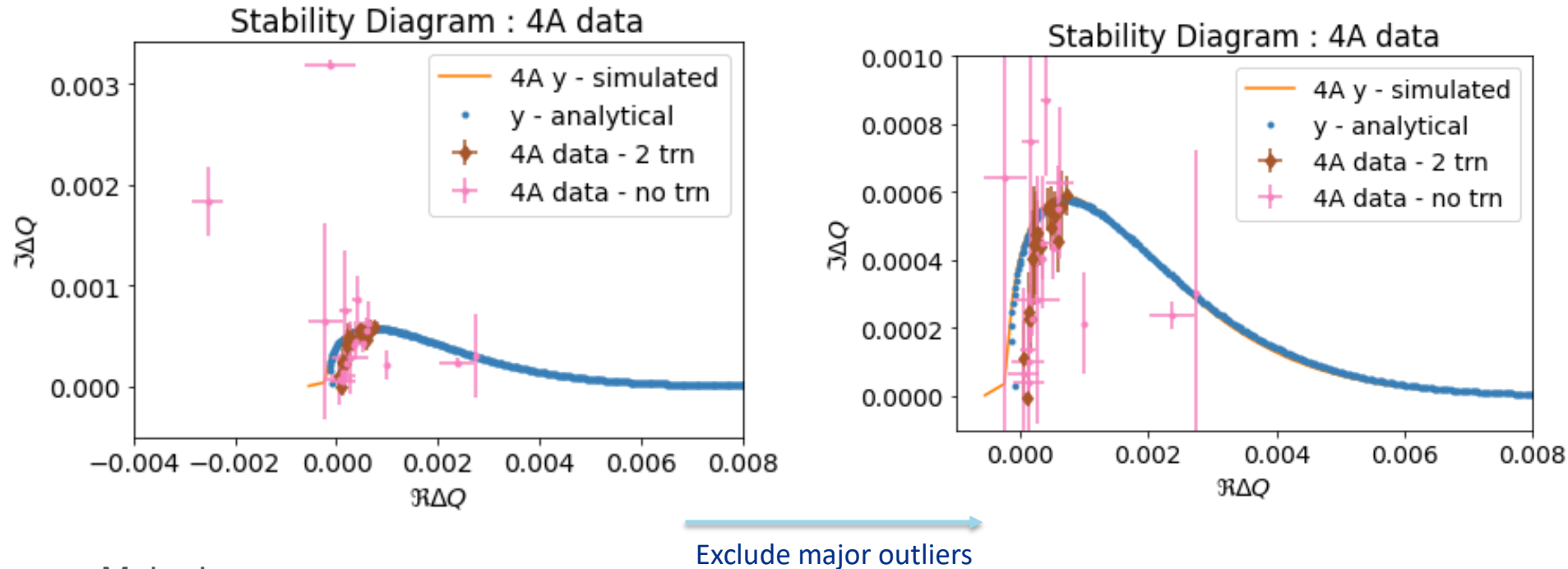
Stability Diagram : 4A data



systematic shift rightward

# Analysis: Stability Diagram

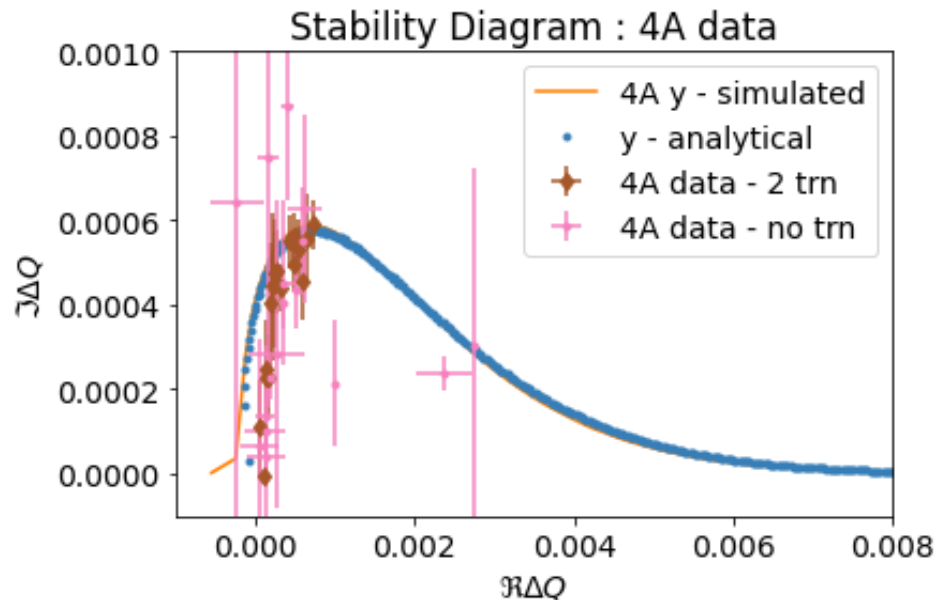
- All together: analytical, simulated, data



- Major issues:
  - Large error bars:
    - Error bar sources mainly from BTF step size and kick gain step size
  - Systematic shift rightward in curve

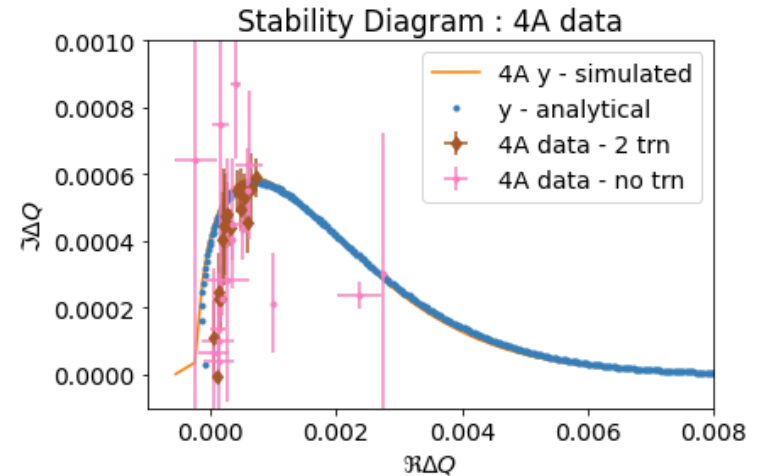
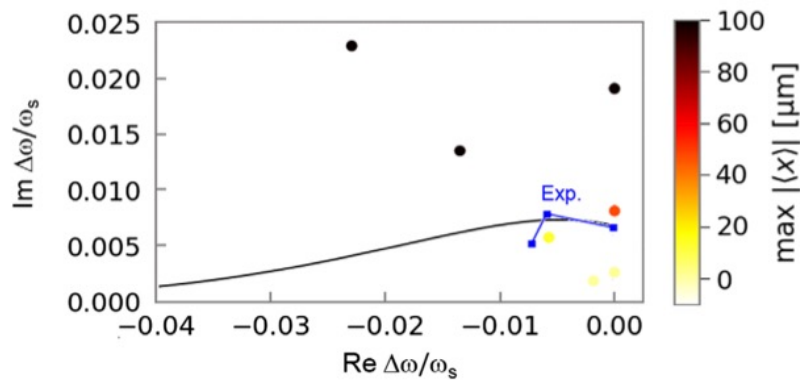
# Analysis: Stability Diagram – other factors

- Landau Damping is not only source of damping:
  - Synchrotron Radiation damping (SRD)
  - SRD would supply vertical shift to SD
- Stability Diagram Shift from SRD:
  - $\Delta Q$  (SRD) =  $4.4E-07+3.4E-07j$
- Impact of Synchrotron radiation damping is negligible compared to LD



# Successes

- Significantly increased quantity of phase measurements for a stability diagram:

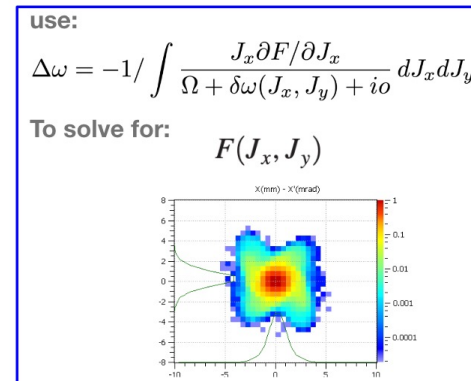
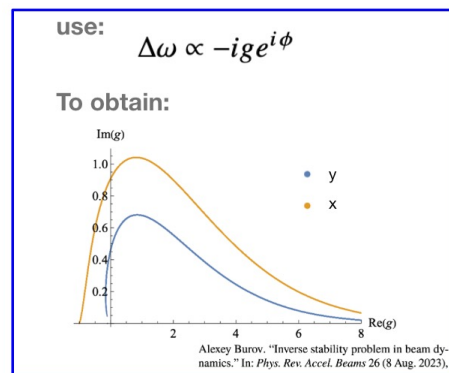
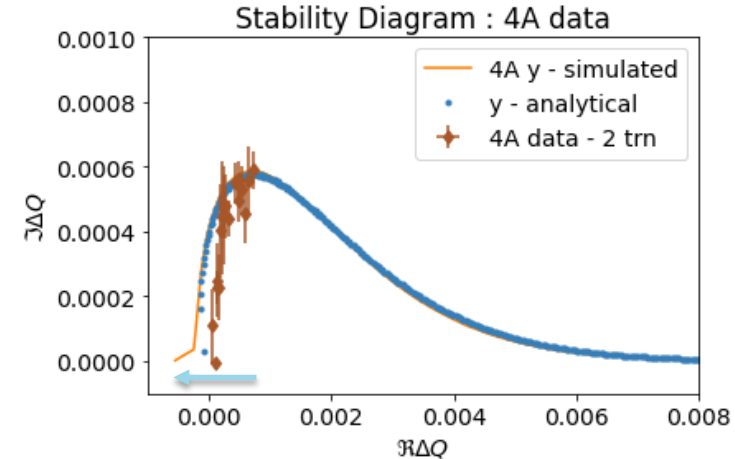


- Experimental data, simulation, and analytical predictions qualitatively agree.
- This first run was critical for understanding what we need to know and what we need in data collection

# Next Steps – Current Data

Next Steps with current data:

- Investigate the systematic rightward shift in data
- Further investigate issues in phase-wrapping
- Further analyze other sets of data - First data sets did not include beam size, we later understood the importance of it for analysis.
- Try to obtain beam distribution function from SD
- Investigate machine impedance impact on SD



# Next Steps – Future Runs

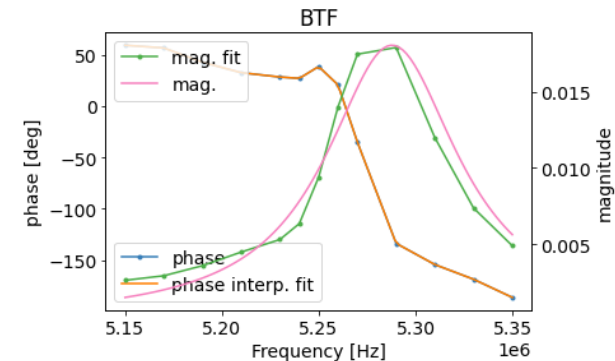
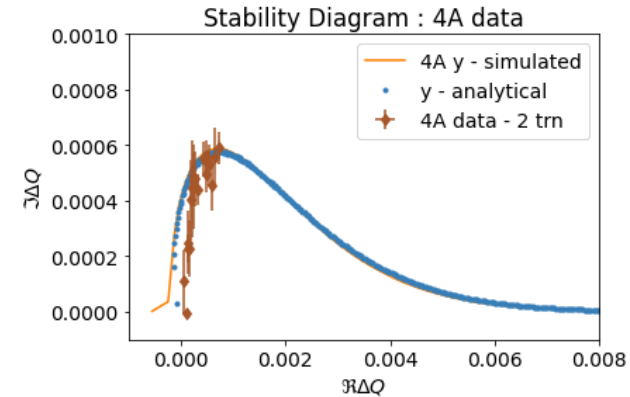
Future runs:

## Electrons:

- Measure beam size for all the data collected - normalize by emittance
- Obtain more data for tails of the SDs: Tails are essential to obtain distribution function
  - More measurements at small phases
  - Better understand phase wrapping
- Take tighter measurements for the BTFs, so to minimize phase error
- Directly measure the amplitude detuning matrix
- Currently limited with the antidamper acting and measuring in one plane at a time: add capability to measure both planes at the same time

## Protons:

- Analyze SDs with Impact of Electron Lens (protons)
- Investigate LD with bunched beams and space charge (protons)





# Thank you!

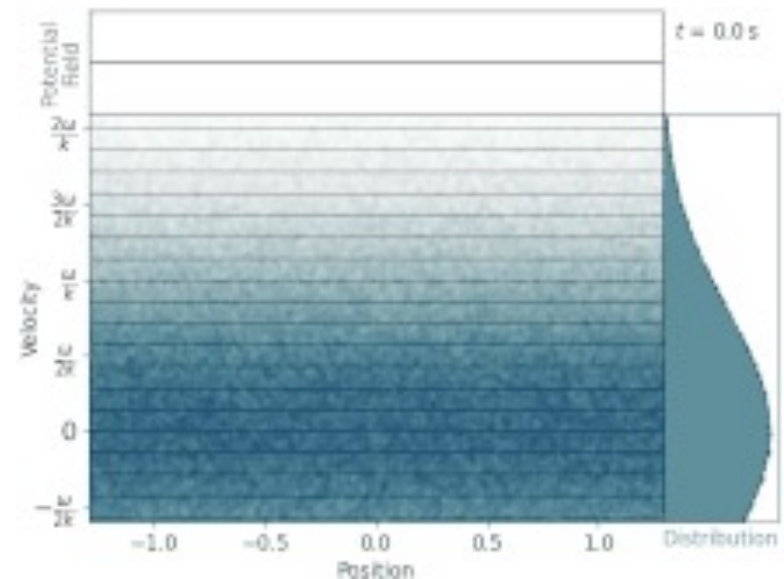
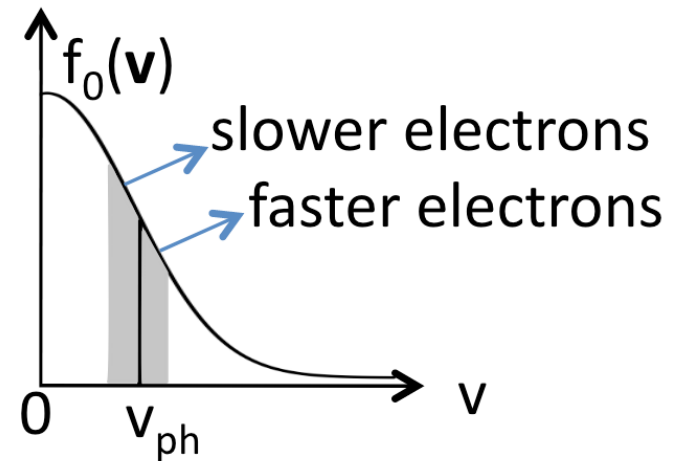


Photo Credit: Giulio Stancari

# Appendix

# Appendix: Background– Landau Damping

- Landau Damping (LD) Historical Context:
  - Derived originally for plasmas with purely mathematical approach (1946)
  - First experimental evidence for LD observed in 1964
- Physical Interpretation (plasmas)
  - Neutral plasma, Maxwellian vel. distribution
  - Perturbation in distribution: plasma wave propagates at  $V_{ph}$
  - Electrons near  $V_{ph}$  interact strongly.
  - Greater energy e's: decelerate, lose energy
  - Lesser energy e's: accelerate, gain energy
  - There are more e's with less energy: energy is transferred from perturbation to particles - damping the perturbation

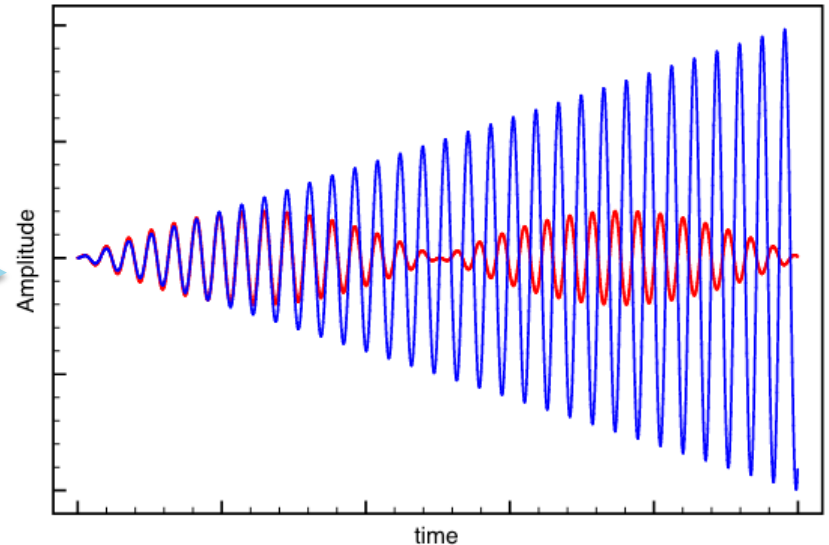
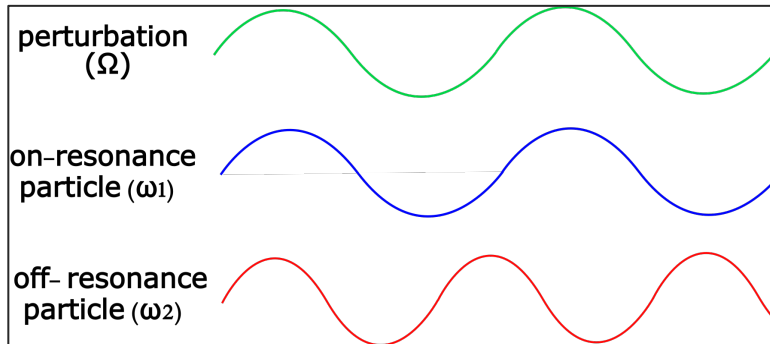


# Background/Motivation – Landau Damping

Accelerators suffer from collective instabilities:

- Transverse EM force as kick (freq.  $\Omega$ )
- Two particles:  $\Omega = \omega_1$ ,  $\Omega \neq \omega_2$ 
  - Amp. of  $\Omega = \omega_1$  particle grows w/time
  - $\omega_2$  particle reaches max, later loses phase synchronism with  $\Omega$ .
  - At time t, only oscillators  $\Omega - \omega \approx 0$  maintain phase relation with  $\Omega$ .
  - On-resonance particles absorb energy: bandwidth & number decrease w/time.

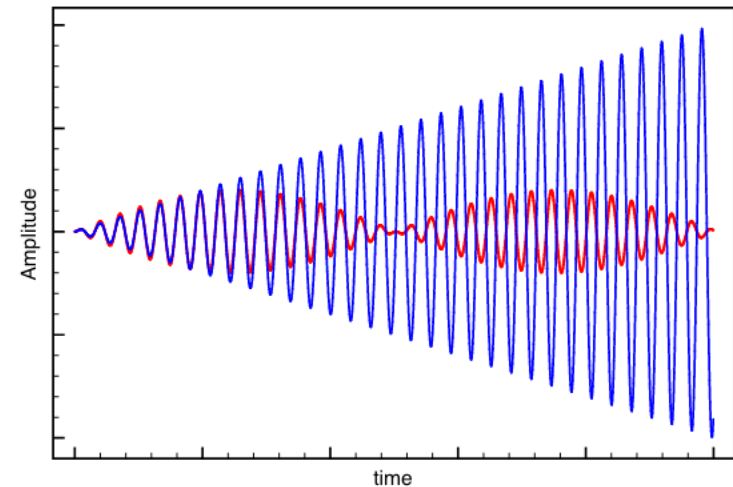
Energy absorption by particles while total energy of oscillations (coherent and incoherent) remains constant



# Appendix: Landau Damping Mechanical model

- Physical Interpretation (mechanical model):

- Model consisting of infinite set of harmonic oscillators: frequency distribution  $G(\omega)$
- System driven by external sinusoidal force at frequency  $\Omega$ .
- All oscillator frequencies sufficiently close to one another and  $\Omega$  lies within their spectrum.
- Consider two oscillators:  $\Omega = \omega$ ,  $\Omega \neq \omega$ .
  - Amplitude of on resonance oscillator grows linearly with time. Other reaches maximum and after time is out of resonance, loses phase synchronism with  $\Omega$ .
  - At time  $t$ , only oscillators with frequency  $\omega$ , where  $\Omega - \omega$  is so small maintain phase relation with the external force. The later on, the narrower the frequency bandwidth  $\Omega - \omega$  of synchronous oscillators.
  - The 'on resonance' oscillators are in phase with the external force and absorb energy from it, but their frequency bandwidth and number decreases with time.
  - Net effect is absorption of energy by the system while average amplitude of oscillation remains constant.



# NIOLD Experiment History at IOTA

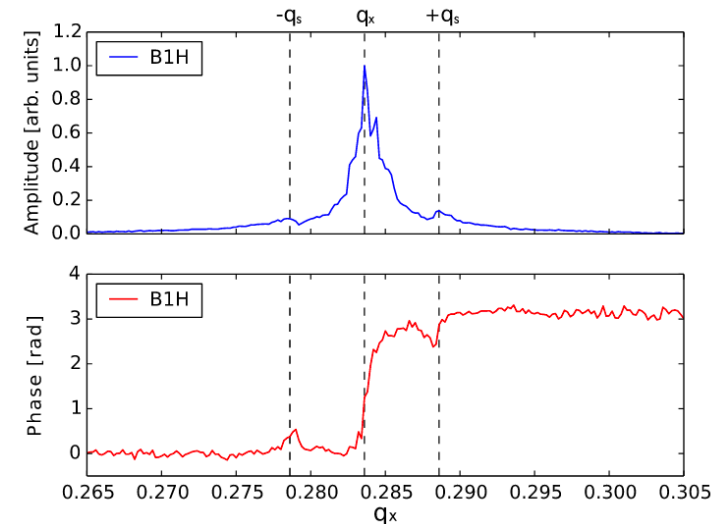
- Run 1:
  - simple analog system using a single button bpm and one of the horizontal kicker plates.
  - There was one experimental shift which showed improved stability threshold with octupoles powered.
- Run 2:
  - More extensive shifts were done
  - The same setup and more sophisticated data collection.
- Run 4:
  - Did major upgrades to install dedicated stripline pickups and kicker and implement the digital controller.
  - had major delays in stripline installation - final driving term was getting electricians to pull cables and install power to our crate.

# Appendix: Stability Diagram Measurement Method

## Measurement of Stability Diagrams

- Common technique to measure SDs: beam transfer function (BTF)
  - frequency dependence of response to forced beam oscillations
  - Stability diagram is inverse of BTF
- Technique has limitations:
  - Method does not test LD strength itself, but the BTF, relying on assumptions behind dispersion relation:
    - Synchrotron frequency is negligible
    - Betatron frequency spread is sufficiently small
    - Beam response to external excitation is linear
    - Coherent modes are uncoupled

Example of BTF measurement: amplitude and phase response for B1 at the LHC injection energy.



*'Beam transfer function measurements used to probe the transverse Landau damping at the LHC' C. Tambasco et al.*

Since other techniques have limitations, we investigate an alternative method to measure the strength of LD

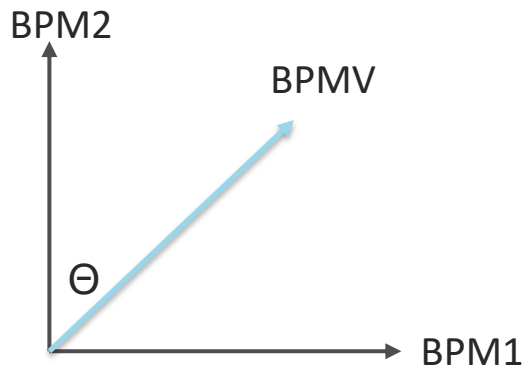
# Appendix: Landau Damping in Accelerators

- Landau Damping in Accelerators:
  - Accelerators suffer from collective instabilities limiting the current intensity
  - beam perturbation from interaction with surrounding walls -> em fields interacting back on the beam -> certain conditions lead to resonance in the beam motion excited with growing amplitude.
  - Here we focus on transverse beam dynamics of bunched beam.
    - Have transverse wake field -> introduce transverse equations of motion to be used in the Vlasov equation to obtain dispersion relation -> Apply dispersion relation to distribution function.
    - Consider transverse electromagnetic force produced by the charge as a transverse deflecting kick divided by the machine length
    - there is a contribution to the transverse Landau damping also from a possible spread on the synchrotron frequencies.
    - in order to get more efficient betatron frequency spread, the so-called Landau damping octupoles are sometimes used: introduce an amplitude frequency dependence which is thought to be more effective than the energy dependent betatron frequency spread produced by sextuples.



# BPM Coefficients

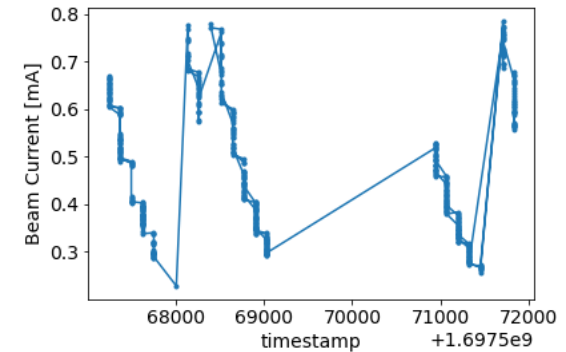
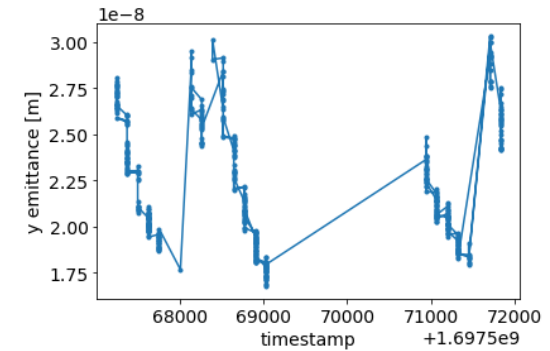
- BPMs are 90° apart in phase advance.
- Changing the BPM coefficients essentially changes the mapping of the virtual BPM's (BPMV) phase.
- C1, C2 represent the BPM coefficients, changing the phase advance according to their combination



$$\Theta = \tan^{-1}(C2 * BPM2 / C1 * BPM1)$$

# Analysis: Growth Rate

- During measurements: beam emittance and intensity changed over time
  - SD size relies on detuning from octupoles: octupole strength, emittance, as well as intensity
  - Normalize each growth by octupole detuning matrix and beam intensity



$$\Delta Q_y = k_{yx}\epsilon_x + k_{yy}\epsilon_y$$

$$\Delta Q_x = k_{xx}\epsilon_x + k_{xy}\epsilon_y$$

Where:

$$k_{xx} = \frac{1}{16\pi} K_3 L \beta_x^2$$

$$k_{xy}, k_{yx} = -\frac{1}{8\pi} K_3 L \beta_x \beta_y$$

$$k_{yy} = \frac{1}{16\pi} K_3 L \beta_y^2$$

$$\text{where } :K_3 = \frac{1}{B\rho} 3! \frac{B_y}{x^3}$$

Normalize each Vert. GR by this to account for emittance variations. Normalize also by intensity.

\* Note:  $\epsilon_x \gg \epsilon_y$  for 4A data, so  $k_{yx} \epsilon_x$  term dominates

# Appendix: IOTA Parameters

|   | Electrons                     | Protons               |
|---|-------------------------------|-----------------------|
| Circumference, $C$                                      | 39.96 m                       | 39.96 m               |
| Kinetic energy, $K_b$                                   | 100–150 MeV                   | 2.5 MeV               |
| Revolution period, $\tau_{\text{rev}}$                  | 133 ns                        | 1.83 $\mu\text{s}$    |
| Revolution frequency, $f_{\text{rev}}$                  | 7.50 MHz                      | 0.547 MHz             |
| Rf harmonic number, $h$                                 | 4                             | 4                     |
| Rf frequency, $f_{\text{rf}}$                           | 30.0 MHz                      | 2.19 MHz              |
| Max. rf voltage, $V_{\text{rf}}$                        | 1 kV                          | 1 kV                  |
| Number of bunches                                       | 1                             | 4 or coasting         |
| Bunch population, $N_b$                                 | $1 e^- - 3.3 \times 10^9 e^-$ | $< 5.7 \times 10^9 p$ |
| Beam current, $I_b$                                     | 1.2 pA – 4 mA                 | $< 2 \text{ mA}$      |
| Transverse emittances (rms, geom.), $\epsilon_{x,y}$    | 20–90 nm                      | 3–4 $\mu\text{m}$     |
| Momentum spread, $\delta_p = \Delta p/p$                | $1-4 \times 10^{-4}$          | $1-2 \times 10^{-3}$  |
| Radiation damping times, $\tau_{x,y,z}$                 | 0.2–2 s                       | –                     |
| Max. space-charge tune shift, $ \Delta\nu_{\text{sc}} $ | $< 10^{-3}$                   | 0.5                   |

# Appendix: beam distribution function

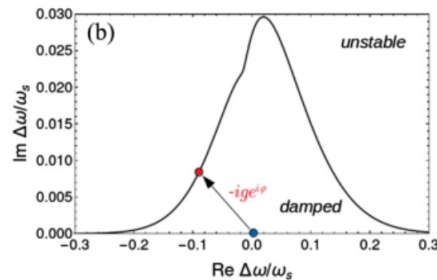
- Beam Distribution Function:
  - Although SD important alone: measurement can be also used as data about beam distribution function
  - SD is map of the real axes of eigenvalue plane onto complex gain plane, beam distribution function is transformation kernel
  - This mathematical problem has never been treated in the past, so its solution requires new methods

Experimentally  
obtain  $g, \phi$

use:

$$\Delta\omega \propto -ige^{i\phi}$$

To obtain:



use:

$$\Delta\omega = -1/\int \frac{J_x \partial F / \partial J_x}{\Omega + \delta\omega(J_x, J_y) + i0} dJ_x dJ_y$$

To solve for:

$$F(J_x, J_y)$$

For tails: take integral asymptotically to get simplified relations between  $\Delta\omega$  and  $F$

For core: assume distribution func. belongs to certain family of functions with few free parameters. Fit them to SD in core region