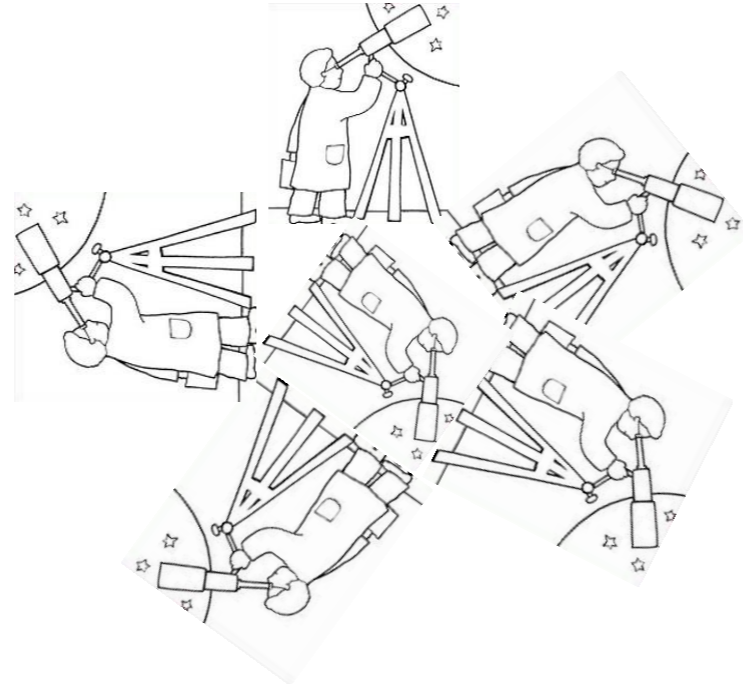


# Searching for (Ultra-)light Dark Matter Using Mössbauer Spectroscopy



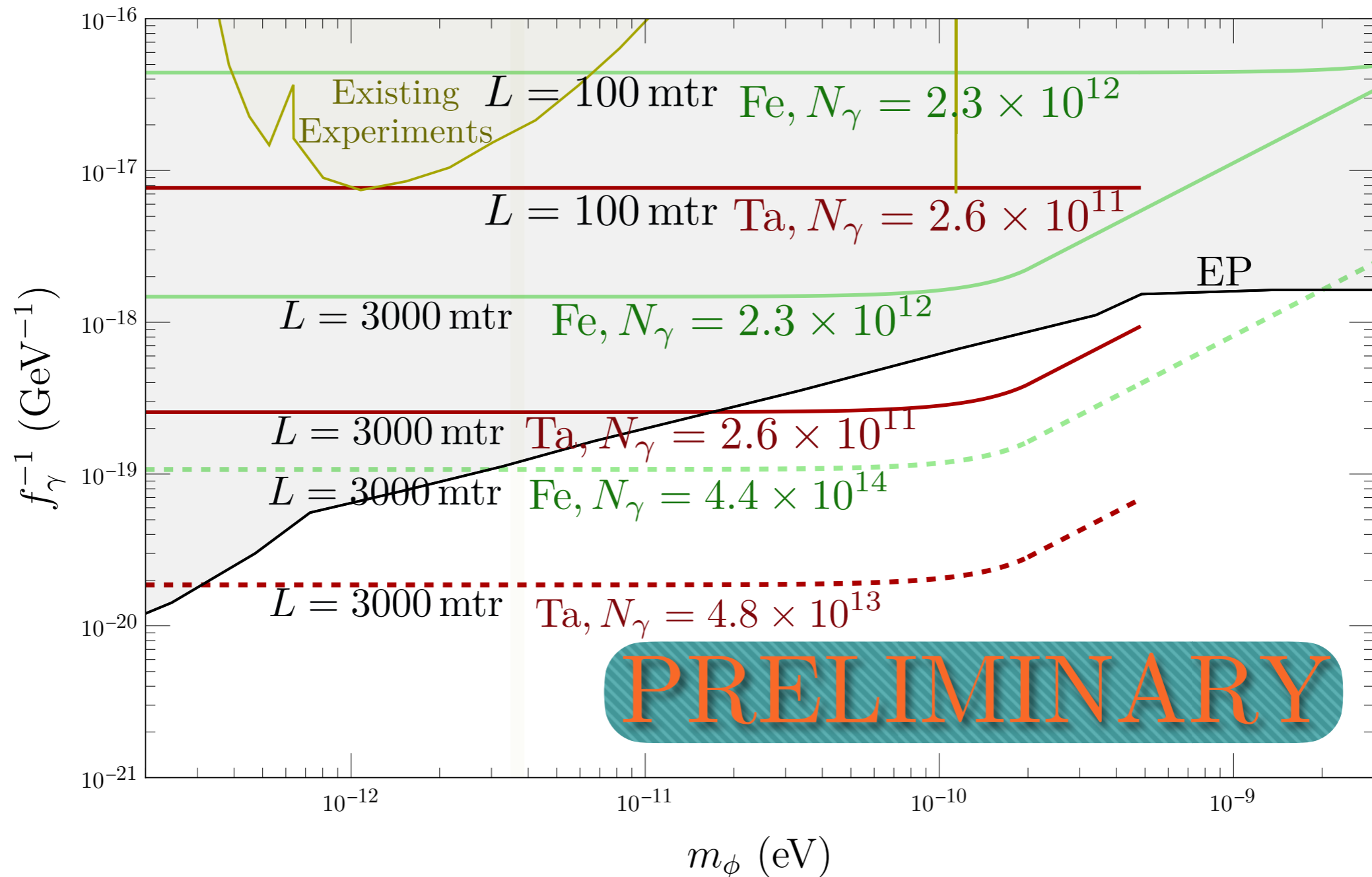
Abhishek Banerjee  
University of Maryland

Fermilab, 2024

[AB, and Rajendran (to appear), '24 ]

# Result

With the existing set-up, we are already probing interesting part of the parameter space, and a slight improvement would enable us to probe uncharted territory!



# Plan

---

- ▶ What is Mössbauer Spectroscopy?
- ▶ How to use it for probing Dark Matter (DM)?
- ▶ Experimental considerations
- ▶ Difference between our method and existing searches
- ▶ Results

# Introduction

---

We are interested in (sub-eV) dark matter (DM) which interacts with the SM CP even operators

$$\phi(t, \vec{x}) \simeq \phi_0 \cos \left[ m_\phi (t + \vec{\beta} \cdot \vec{x}) \right] \quad \phi_0 \simeq \sqrt{2\rho_{\text{DM}}/m_\phi}$$

Examples include

QCD axion DM  $\mathcal{L} \supset \frac{g_s^2}{32\pi^2} \frac{a}{f_a} G^{\mu\nu} \tilde{G}_{\mu\nu}$

For a dilaton like scalar DM  $\longrightarrow$  through scale anomaly

For a Higgs portal type models  $\longrightarrow$  through Higgs mixing

# Introduction

---

We are interested in (sub-eV) dark matter (DM) which interacts with the SM CP even operators

$$\phi(t, \vec{x}) \simeq \phi_0 \cos \left[ m_\phi (t + \vec{\beta} \cdot \vec{x}) \right] \quad \phi_0 \simeq \sqrt{2\rho_{\text{DM}}/m_\phi}$$

Examples include

QCD axion DM  $\mathcal{L} \supset \frac{g_s^2}{32\pi^2} \frac{a}{f_a} G^{\mu\nu} \tilde{G}_{\mu\nu}$  (Quadratic order)

(Linear order) For a dilaton like scalar DM  $\longrightarrow$  through scale anomaly

(Linear order) For a Higgs portal type models  $\longrightarrow$  through Higgs mixing

Leads to time dependence of (nuclear-) energy levels!

# Probing oscillating DM using Precision Spectroscopy

---

General: find 2 systems with different dependence on the fundamental constants

$$\overline{\Delta E_1} \uparrow f_1 \qquad \Delta E_{1,2} \equiv f_{1,2} = f_{1,2} \left( \alpha^{\xi_\alpha^{1,2}}, m_e^{\xi_{m_e}^{1,2}}, m_N^{\xi_{m_N}^{1,2}} \right) \qquad \overline{\Delta E_2} \uparrow f_2$$

[Safronova, Budker, DeMille, Kimball, Derevianko, Clark 18] [Antypas, Budker, Flambaum, Kozlov, Perez, Ye 20]

Fractional change of the frequency ratio:  $\frac{\Delta(f_1/f_2)}{(f_1/f_2)} = \frac{\Delta f_1}{f_1} - \frac{\Delta f_2}{f_2} = (\xi_\alpha^1 - \xi_\alpha^2) \frac{\Delta\alpha}{\alpha}$

Classical ex: various clock comparisons, comparison clocks with cavity,...

Reach of the experiments is limited by the stability of the clocks  $t_{\text{avg}} \gtrsim \mathcal{O}(\text{sec})$

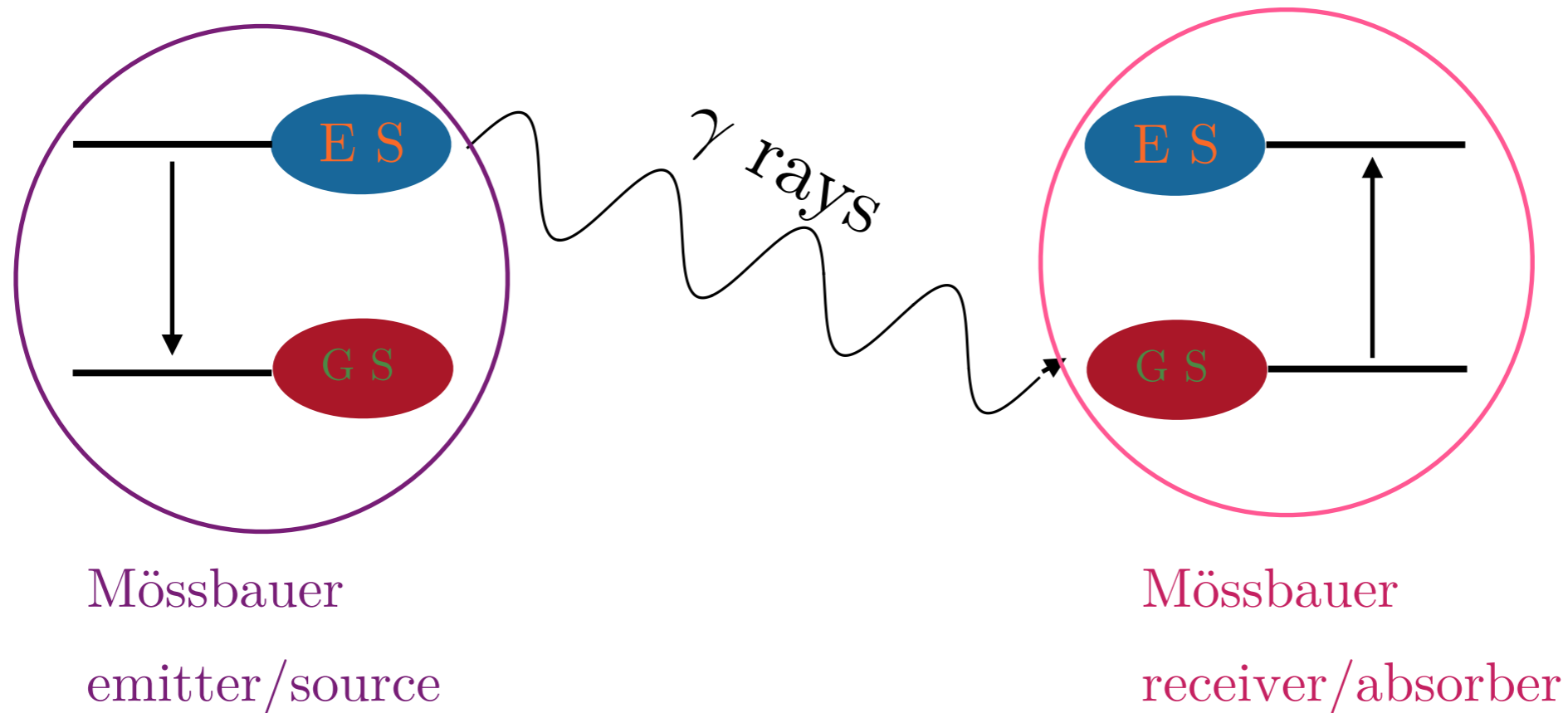
Efficiently can only probe DM parameter space in the low mass region

We are proposing to use Mössbauer Spectroscopy to probe oscillating DM through the time dependence of nuclear energy levels

# Mössbauer Spectroscopy

---

Mössbauer Spectroscopy relies on recoil free resonant re-absorption of gamma photons



These are nuclear transitions, and only possible to see in a lattice



# Mössbauer Spectroscopy

---

Absorption cross-section

$$\sigma_{\text{res}}(E_0, E_\gamma, \Gamma) \propto \frac{2\pi}{E_\gamma^2} \frac{(\Gamma/2)^2}{(E_0 - E_\gamma)^2 + (\Gamma/2)^2}$$

Cross section changes drastically while going from the “on resonance” to “off resonance” region, sometimes by several orders of magnitude

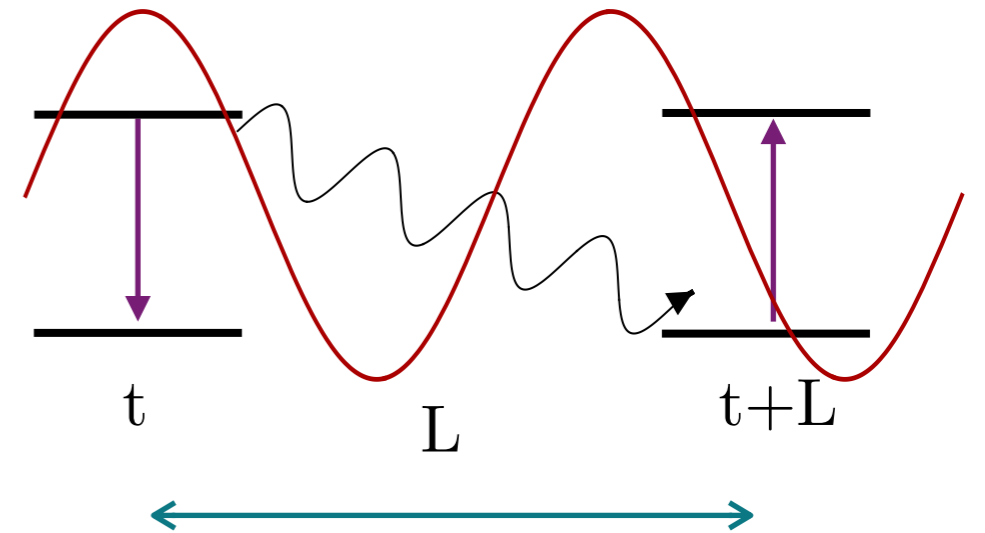
For a typical Mössbauer transition  $E_\gamma \sim (6 - 100) \text{ keV}$ ,  $\Gamma \sim (0.1 - 1000) \text{ neV}$

Amazing probe of new physics!

# Probing DM

---

Oscillating DM background inducing time dependence to the energy levels



Change in the DM field

$$\Delta\phi \simeq \phi(t) - \phi(t + L) \simeq \frac{\sqrt{2\rho_{\text{DM}}}}{m_\phi/2} \sin\left(\frac{m_\phi L}{2}\right) \simeq \sqrt{2\rho_{\text{DM}}} L \times \text{sinc}(m_\phi L/2)$$

To obtain the change in the energy levels, we need to specify DM-SM interaction!

For  $N_\gamma$  number of photons, the sensitivity of DM search

$$\Delta E = \frac{\Gamma}{\sqrt{N_\gamma}} \quad \sigma_{\text{res}}(E_0, E_\gamma, \Gamma) \propto \frac{2\pi}{E_\gamma^2} \frac{(\Gamma/2)^2}{(E_0 - E_\gamma)^2 + (\Gamma/2)^2}$$

# Probing DM

---

For a linearly coupled DM, change in the energy levels

$$\Delta E = E_{\text{tran}}(t) - E_{\text{tran}}(t + L) \propto \Delta\phi$$

$$\Delta\phi \simeq \phi(t) - \phi(t + L) \simeq \frac{\sqrt{2\rho_{\text{DM}}}}{m_\phi/2} \sin\left(\frac{m_\phi L}{2}\right) \simeq \sqrt{2\rho_{\text{DM}}}L \times \text{sinc}(m_\phi L/2)$$

$$\text{sinc}(x) = \begin{cases} 1 & \text{for } x \lesssim 1 \\ 1/x & \text{for } x \gtrsim 1 \end{cases}$$

# Probing DM

---

For a linearly coupled DM, change in the energy levels

$$\Delta E = E_{\text{tran}}(t) - E_{\text{tran}}(t + L) \propto \Delta\phi$$

$$\Delta\phi \simeq \phi(t) - \phi(t + L) \simeq \frac{\sqrt{2\rho_{\text{DM}}}}{m_\phi/2} \sin\left(\frac{m_\phi L}{2}\right) \simeq \sqrt{2\rho_{\text{DM}}} L \times \text{sinc}(m_\phi L/2)$$

$$\text{sinc}(x) = \begin{cases} 1 & \text{for } x \lesssim 1 \\ 1/x & \text{for } x \gtrsim 1 \end{cases}$$

Maximum sensitivity is obtained for  $m_\phi \sim 1/L \sim \text{neV} \left(\frac{100 \text{ mtr}}{L}\right)$

Enables us to probe higher(!) masses

# Constraints on DM mass

---

DM better be coherent between the Mössbauer emitter and the absorber

$$m_\phi \lesssim 4 \times 10^{-4} \text{eV} \left( \frac{3 \text{ km}}{L} \right) \quad \tau_{\text{coh}} = \frac{2\pi}{m_\phi \beta^2}$$

The spread of both the emission, and the absorption lines in the time-domain should follow the oscillation

$$t_{\text{osc}} = \frac{2\pi}{m_\phi} \gtrsim \tau_{\text{dec}} = \frac{1}{\Gamma_{\text{nat}}} \Rightarrow m_\phi \lesssim 2\pi \Gamma_{\text{nat}}$$

For a given transition  
allowed mass range

$$m_\phi \lesssim 8.3 \times 10^{-5} \text{eV} \times \text{Min} \left[ \frac{15 \text{ km}}{L}, \frac{50 \text{ ps}}{\tau_{\text{dec}}} \right]$$

# Challenges?

---

Effective reach of the experiment

$$\frac{\Gamma}{\sqrt{N_\gamma}} = \Delta E \propto \Delta\phi \propto \sqrt{\rho_{\text{DM}}} L$$

For a Gaussian beam

$$N_\gamma \propto 1/(L\theta)^2$$

Mitigating the apparent gain obtained by separating the emitter and the absorber

# Challenges?

---

Effective reach of the experiment  $\frac{\Gamma}{\sqrt{N_\gamma}} = \Delta E \propto \Delta\phi \propto \sqrt{\rho_{\text{DM}}} L$

For a Gaussian beam  $N_\gamma \propto 1/(L\theta)^2$

Mitigating the apparent gain obtained by separating the emitter and the absorber

Flux at a distance L  $N_\gamma(L) \propto \frac{z_R^2}{L^2 + z_R^2}$   $L \gg z_R$

the Rayleigh range — qualitatively, a beam remains “parallel” over this distance

$$z_R = \frac{\pi w_0^2}{\lambda} \sim \frac{\lambda}{\Omega}$$

We want a lot of well collimated emitted photons!

# Challenges?

---

Effective reach of the experiment  $\frac{\Gamma}{\sqrt{N_\gamma}} = \Delta E \propto \Delta\phi \propto \sqrt{\rho_{\text{DM}}}L$

For a Gaussian beam  $N_\gamma \propto 1/(L\theta)^2$

Mitigating the apparent gain obtained by separating the emitter and the absorber

We want a lot of well collimated emitted photons

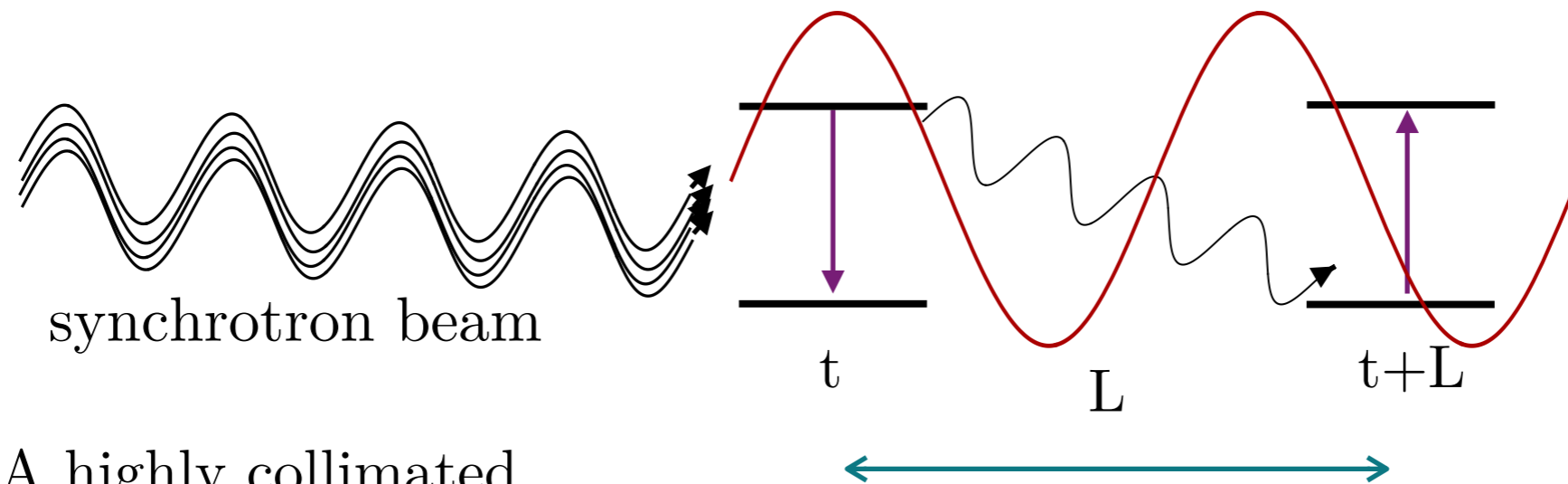
Synchrotron based Mössbauer Spectroscopy (SMS)



# Synchrotron based Mössbauer Spectroscopy (SMS)

---

A sample is directly excited through the absorption of an x-ray photon



A highly collimated  
high intensity beam

In the case of SMS, the emitted radiation after the synchrotron light excitation is coherent

Leads to an enhanced collimated x-ray photons, which happens from the whole source

The flux is equal to the number of source excited source nuclei

# Synchrotron based Mössbauer Spectroscopy (SMS)

---

Number of excited source nuclei?

The resolution of the synchrotron source  $(\Delta E)_{\text{SR}} \gg \Gamma$

Only a fraction of the nuclei can be excited

Effective flux at the Mössbauer source  $(N_\gamma)_{\text{MS}} = (N_\gamma)_{\text{SR}} \frac{\Gamma}{(\Delta E)_{\text{SR}}}$

DM search translates to find a suitable synchrotron source!

$$\Delta E = \frac{\Gamma}{\sqrt{N_\gamma}} = \sqrt{\frac{\Gamma (\Delta E)_{\text{SR}}}{(N_\gamma)_{\text{SR}}}}$$

How to even improve the set-up?

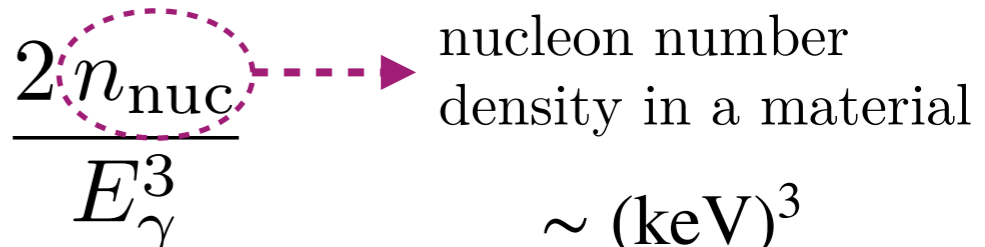
# Synchrotron based Mössbauer Spectroscopy (SMS)

---

How to achieve a higher Rayleigh range?  $z_R = \frac{\pi w_0^2}{\lambda}$

For that purpose, we want to refocus the radiation by using a lens

The effective radius of curvature  $R = (n - 1)f$  (Lensmaker's equation)

For x-ray energies, the resonant nuclear contribution to the refractive index  $n - 1 \simeq \frac{2n_{\text{nuc}}}{E_\gamma^3}$   nucleon number density in a material  $\sim (\text{keV})^3$

Rayleigh range  $z_R \sim 3.2 \text{ km} \left( \frac{20 \text{ keV}}{E_\gamma} \right)^5 \left( \frac{f}{1 \text{ m}} \right)^2$

Go to larger distances without losing flux!

# Synchrotron Sources

---

SMS can be done with various transitions with different widths

Available synchrotron sources: APS in USA, ESRF in Europe, and SPring-8 in Japan...

Different transitions  $\Rightarrow$  various DM mass  $m_\phi \lesssim 8.3 \times 10^{-5} \text{ eV} \times \text{Min} \left[ \frac{15 \text{ km}}{L}, \frac{50 \text{ ps}}{\tau_{\text{dec}}} \right]$

Different transitions  $\Rightarrow$  different sensitivity  $\Delta E = \frac{\Gamma}{\sqrt{N_\gamma}} = \sqrt{\frac{\Gamma (\Delta E)_{\text{SR}}}{(N_\gamma)_{\text{SR}}}}$

# Synchrotron Sources

SMS can be done with various transitions with different widths

Available synchrotron sources: APS in USA, ESRF in Europe, and SPring-8 in Japan...

Different transitions  $\Rightarrow$  various DM mass  $m_\phi \lesssim 8.3 \times 10^{-5} \text{ eV} \times \text{Min} \left[ \frac{15 \text{ km}}{L}, \frac{50 \text{ ps}}{\tau_{\text{dec}}} \right]$

Different transitions  $\Rightarrow$  different sensitivity  $\Delta E = \frac{\Gamma}{\sqrt{N_\gamma}} = \sqrt{\frac{\Gamma (\Delta E)_{\text{SR}}}{(N_\gamma)_{\text{SR}}}}$

Transition List					Beam	
Transition	$E_0$ (keV)	$\Gamma_{\text{exp}}$ (eV)	$\tau_{\text{dec}}$	$N_\gamma$ ( $\gamma$ /month)	$(\Delta E)_{\text{SR}}$	$(N_\gamma)_{\text{SR}}$ $\gamma$ /s
$^{181}_{73}\text{Ta}$ [58]	6.2	$5.5 \times 10^{-10}$	8.7 $\mu\text{s}$	$2.6 \times 10^{11}$	35.7 meV	$6.3 \times 10^{12}$
$^{57}_{26}\text{Fe}$ [59]	14.4	$5 \times 10^{-9}$	141.8 ns	$2.3 \times 10^{12}$	35.7 meV	$6.3 \times 10^{12}$
$^{187}_{76}\text{Os}$ [60, 61]	9.8	$1.9 \times 10^{-7}$	3.4 ns	$8.8 \times 10^{13}$	35.7 meV	$6.3 \times 10^{12}$
$^{187}_{76}\text{Os}$ [61, 62]	74.4	$1.2 \times 10^{-5}$	0.0534 ns	$4.6 \times 10^{13}$	70 eV	$1 \times 10^{14}$

BLXU09 at  
SPring-8

20-ID-D at  
APS

Improvement: effective flux can be improved by a factor of 100

3-ID beam line  
at the APS

---

Reach on DM parameter space

# DM-SM Interaction

$$\mathcal{L} \supset \sum_{\psi=u,d} \left( -\frac{m_\psi}{f_\psi} \phi \bar{\psi} \psi \right) - \frac{\alpha \phi}{4 f_\gamma} F^2 - \frac{\beta(g_s) \phi}{2 g_s f_g} G^2$$

Alpha module:

$$\alpha \rightarrow \alpha - \frac{\alpha^2 \phi}{f_\gamma} \Rightarrow \frac{\Delta \alpha}{\alpha(0)} = \frac{\alpha(\phi) - \alpha(0)}{\alpha(0)} \simeq -\frac{\alpha \phi}{f_\gamma}$$

Yukawa module:

$$\hat{m} \rightarrow \hat{m} - \frac{\hat{m} \phi}{f_{\hat{m}}} \Rightarrow \frac{\Delta \hat{m}}{\hat{m}} = \frac{\hat{m}(\phi) - \hat{m}(0)}{\hat{m}(0)} = \frac{\phi}{f_{\hat{m}}} \quad \hat{m} = \frac{m_u + m_d}{2} \text{ and } \frac{2\hat{m}}{f_{\hat{m}}} = \frac{m_u}{f_u} + \frac{m_d}{f_d}$$

Gluon module:

$$\frac{\Delta \alpha_s}{\alpha_s(0)} = \frac{\alpha_s(\phi) - \alpha_s(0)}{\alpha_s(0)} \simeq -\frac{2\beta(g_s) \phi}{g_s f_g} \Rightarrow \frac{\partial \ln \Lambda_{\text{IR}}}{\partial \phi} \simeq -\frac{\alpha_s}{\beta(\alpha_s)} = \frac{1}{f_g}$$

# Energy Level change

---

$$\mathcal{L} \supset \sum_{\psi=u,d} -\frac{m_\psi}{f_\psi} \phi \bar{\psi} \psi - \frac{\alpha \phi}{4 f_\gamma} F^2 - \frac{\beta(g_s)}{2g_s} \frac{\phi}{f_g} G^2$$

Alpha module:

$$\alpha \rightarrow \alpha - \frac{\alpha^2 \phi}{f_\gamma} \Rightarrow \frac{\Delta \alpha}{\alpha(0)} = \frac{\alpha(\phi) - \alpha(0)}{\alpha(0)} \simeq -\frac{\alpha \phi}{f_\gamma}$$

The dominant electromagnetic effect is encoded in the  $\alpha$  dependence of the nuclear binding energy

$$\Delta E_{\text{bind}} \simeq \frac{Z^2 \Delta \alpha}{A^{1/3} r} = \frac{Z^2 \alpha}{A^{1/3} r} \frac{\Delta \alpha}{\alpha}$$

The energy shift of a highly deformed nucleus due to one nucleon transition

$$\Delta E \simeq \frac{Z \Delta \alpha}{r_{\text{nuc}}} \simeq \frac{Z \alpha}{A^{1/3} r_0} \frac{\Delta \alpha}{\alpha} \simeq -\frac{Z \alpha^2}{A^{1/3} r_0} \frac{\Delta \phi}{f_\gamma}$$



# Reach of the Proposed Method

SMS can be done with various transitions with different widths

Transition List					Beam	
Transition	$E_0$ (keV)	$\Gamma_{\text{exp}}$ (eV)	$\tau_{\text{dec}}$	$N_\gamma$ ( $\gamma$ /month)	$(\Delta E)_{\text{SR}}$	$(N_\gamma)_{\text{SR}}$ $\gamma$ /s
$^{181}_{73}\text{Ta}$ [58]	6.2	$5.5 \times 10^{-10}$	8.7 $\mu\text{s}$	$2.6 \times 10^{11}$	35.7 meV	$6.3 \times 10^{12}$
$^{57}_{26}\text{Fe}$ [59]	14.4	$5 \times 10^{-9}$	141.8 ns	$2.3 \times 10^{12}$	35.7 meV	$6.3 \times 10^{12}$
$^{187}_{76}\text{Os}$ [60, 61]	9.8	$1.9 \times 10^{-7}$	3.4 ns	$8.8 \times 10^{13}$	35.7 meV	$6.3 \times 10^{12}$
$^{187}_{76}\text{Os}$ [61, 62]	74.4	$1.2 \times 10^{-5}$	0.0534 ns	$4.6 \times 10^{13}$	70 eV	$1 \times 10^{14}$

Different transitions  $\Rightarrow$  various DM mass

$$m_\phi \lesssim 8.3 \times 10^{-5} \text{ eV} \times \text{Min} \left[ \frac{15 \text{ km}}{L}, \frac{50 \text{ ps}}{\tau_{\text{dec}}} \right]$$

Different transitions  $\Rightarrow$  different sensitivity

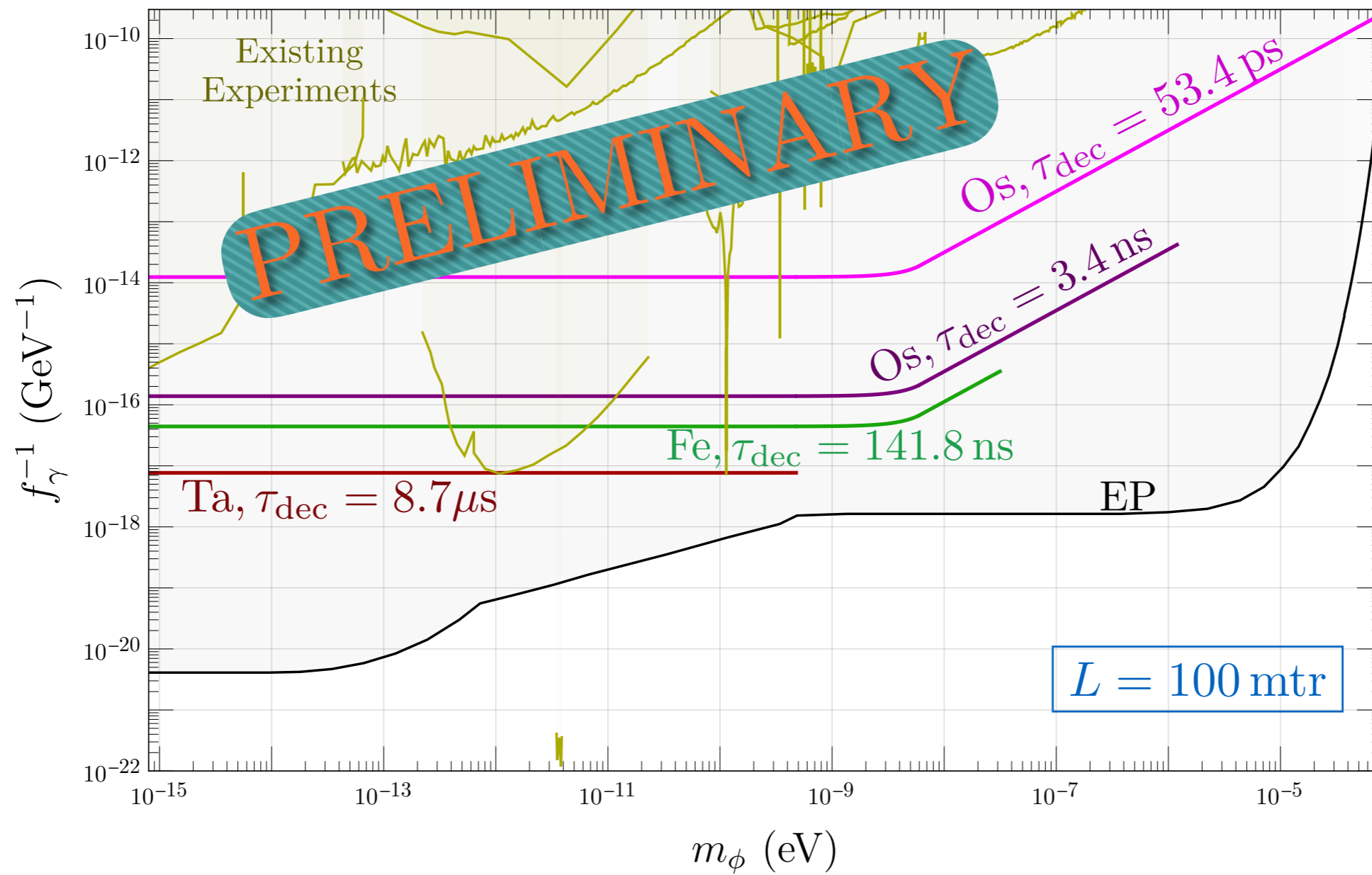
$$\Delta E = \frac{\Gamma}{\sqrt{N_\gamma}} = \sqrt{\frac{\Gamma (\Delta E)_{\text{SR}}}{(N_\gamma)_{\text{SR}}}}$$

DM  $\Rightarrow$  Changing alpha  $\Rightarrow$  changing binding energy

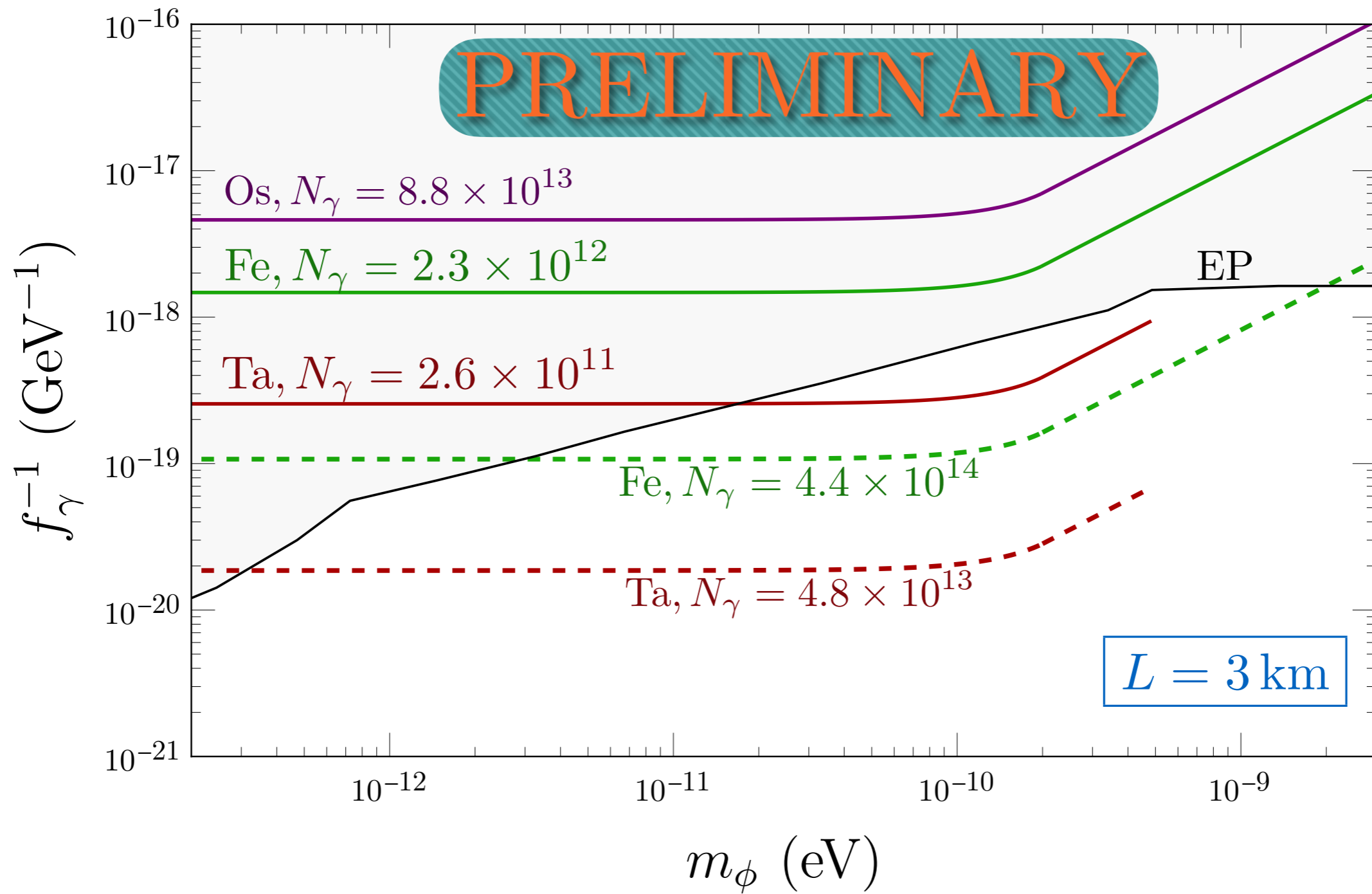
$$\Delta E \simeq \frac{Z \Delta \alpha}{r_{\text{nuc}}} \simeq -\frac{Z \alpha^2}{A^{1/3} r_0} \frac{\Delta \phi}{f_\gamma}$$

$$\Delta \phi \simeq \sqrt{2 \rho_{\text{DM}} L} \times \text{sinc}(m_\phi L/2)$$

# Results



# Results



# Probing QCD axion DM

---

$$\mathcal{L} \supset \frac{g_s^2}{32\pi^2} \frac{a}{f_a} G^{\mu\nu} \tilde{G}_{\mu\nu}$$

$$m_\pi^2(\theta_{\text{eff}}) = B \sqrt{m_u^2 + m_d^2 + 2m_u m_d \cos(\theta_{\text{eff}})} \quad \text{where } \theta_{\text{eff}} = \bar{\theta} + a/f_a, \quad B = -\langle \bar{q}q \rangle_0 / f_\pi^2$$

The oscillating QCD axion DM induces an oscillating component to pion mass

$$\frac{\Delta m_\pi^2}{m_\pi^2} = \frac{m_\pi^2(\theta_{\text{eff}}) - m_\pi^2(0)}{m_\pi^2(0)} \simeq -\frac{m_u m_d \theta_{\text{eff}}^2(t)}{2(m_u + m_d)^2} \quad \theta_{\text{eff}}(t) = \frac{(a - \langle a \rangle)}{f_a} = \frac{\sqrt{2\rho_{\text{DM}}}}{m_a f_a} \cos(m_a t)$$

The nucleon mass also depends on the pion mass

$$m_N(\theta_{\text{eff}}) = m_0 - 4c_1 m_\pi^2(\theta_{\text{eff}}) - \frac{3g_A^2 m_\pi^3(\theta_{\text{eff}})}{32\pi f_\pi^2} \quad \frac{\partial \ln m_N}{\partial \ln m_\pi^2} \simeq 0.06$$

The QCD axion DM => oscillating pion mass and nucleon mass

# Probing QCD axion DM

---

How does pion mass and nucleon mass affect nuclear transition energy?

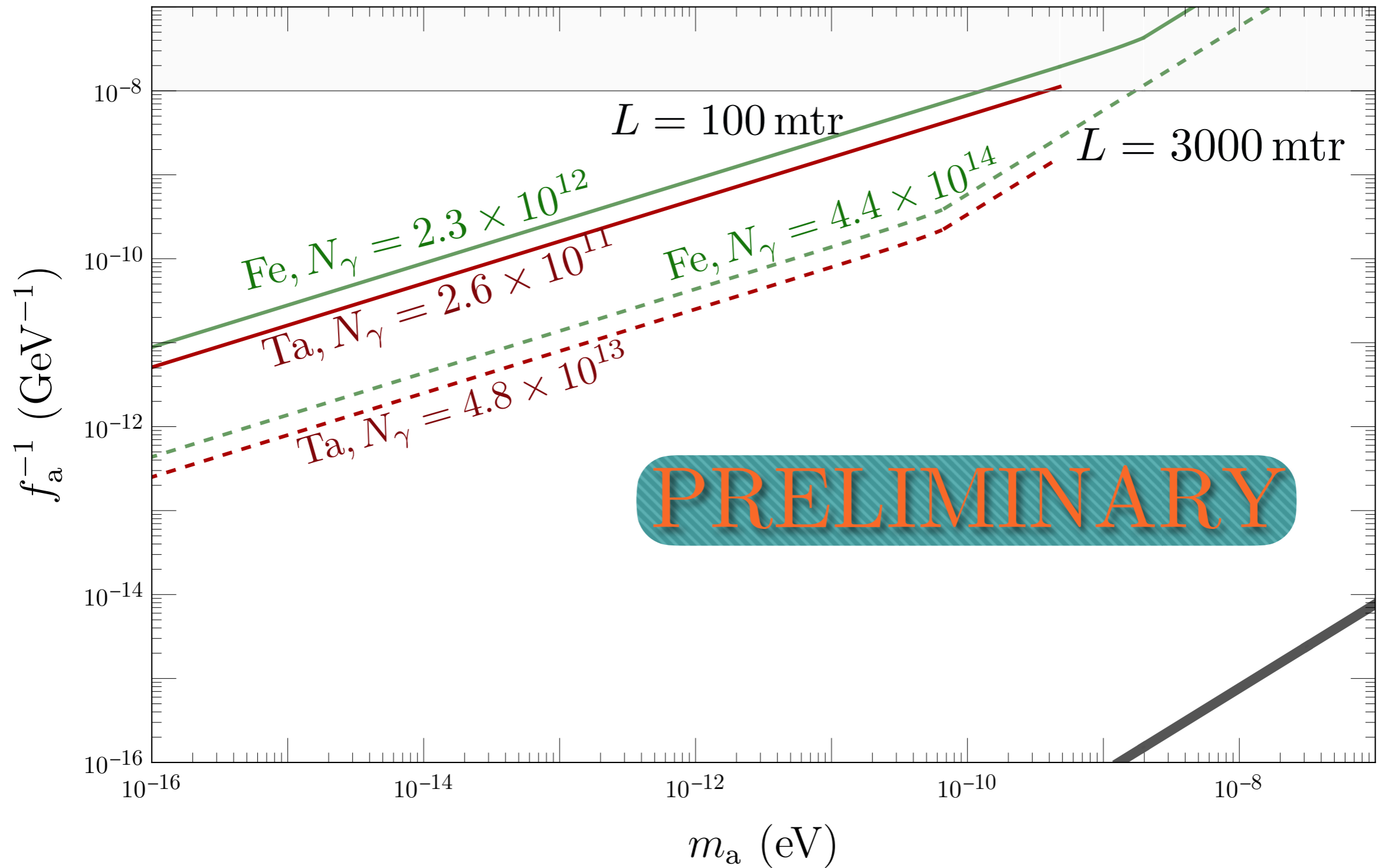
We estimate it using one pion exchange potential

$$V_{\text{OPE}}(r) = -\frac{g_{\pi NN}^2}{4\pi} \frac{m_\pi^2}{4m_N^2} \frac{e^{-m_\pi r}}{r}$$

The energy shift due to changing pion and nucleon mass

$$\Delta E \simeq \frac{g_{\pi NN}^2}{(4\pi) 4r} \left[ \frac{\partial(m_\pi^2/m_N^2)}{\partial \ln m_\pi^2} \right] \frac{\Delta m_\pi^2}{m_\pi^2} \quad \frac{\Delta m_\pi^2}{m_\pi^2} = \frac{m_\pi^2(\theta_{\text{eff}}) - m_\pi^2(0)}{m_\pi^2(0)} \simeq -\frac{m_u m_d \theta_{\text{eff}}^2(t)}{2(m_u + m_d)^2}$$
$$\theta_{\text{eff}}(t) = \frac{(a - \langle a \rangle)}{f_a} = \frac{\sqrt{2\rho_{\text{DM}}}}{m_a f_a} \cos(m_a t)$$

# Results QCD axion plot



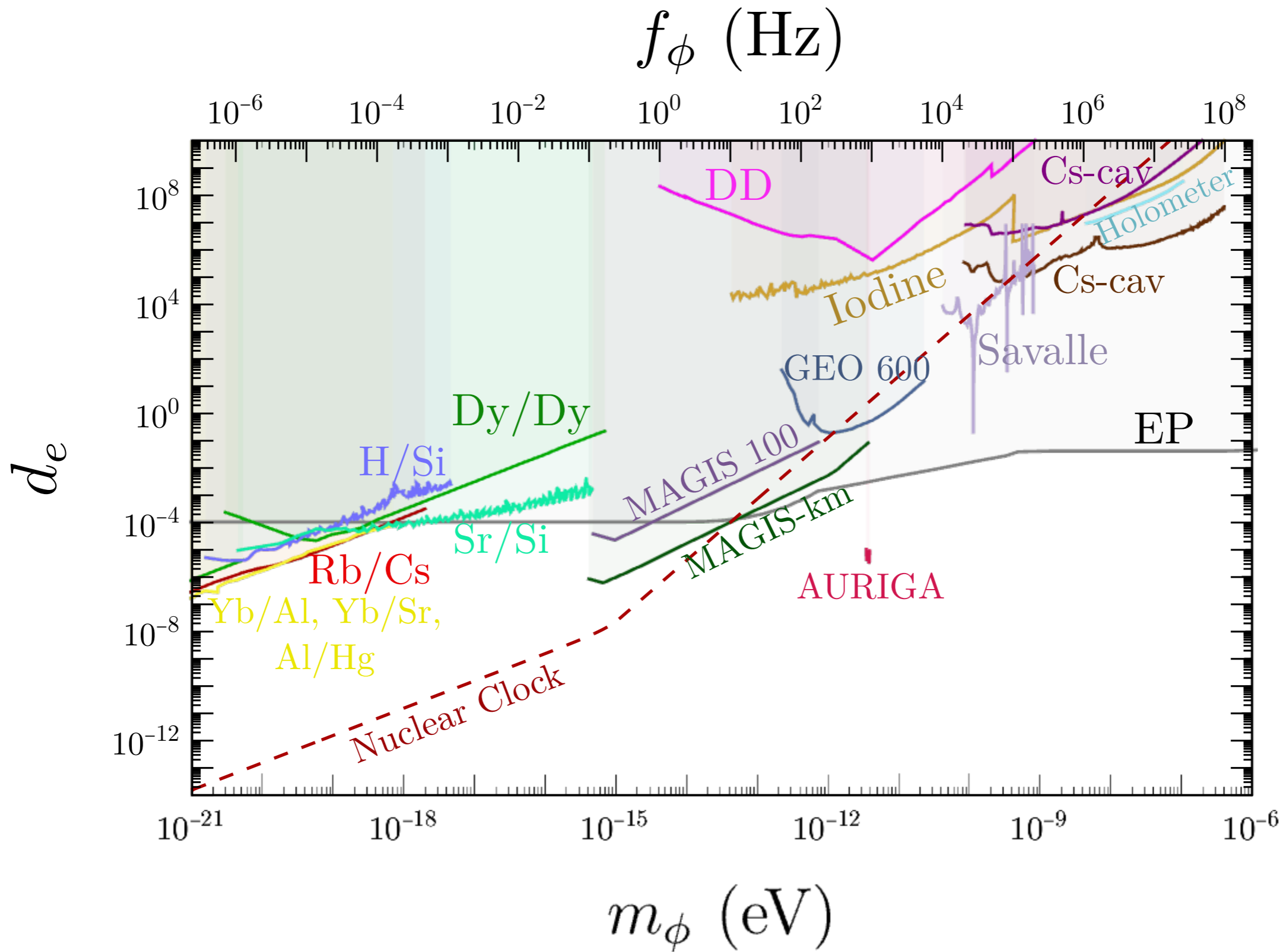
# CONCLUSION

---

Mössbauer Spectroscopy can be used to look for New Physics and especially DM

We show the bounds from our suggested method are at a similar level compared to the most stringent current limits, and future investigations promise significant improvements

# Existing Experiments plot





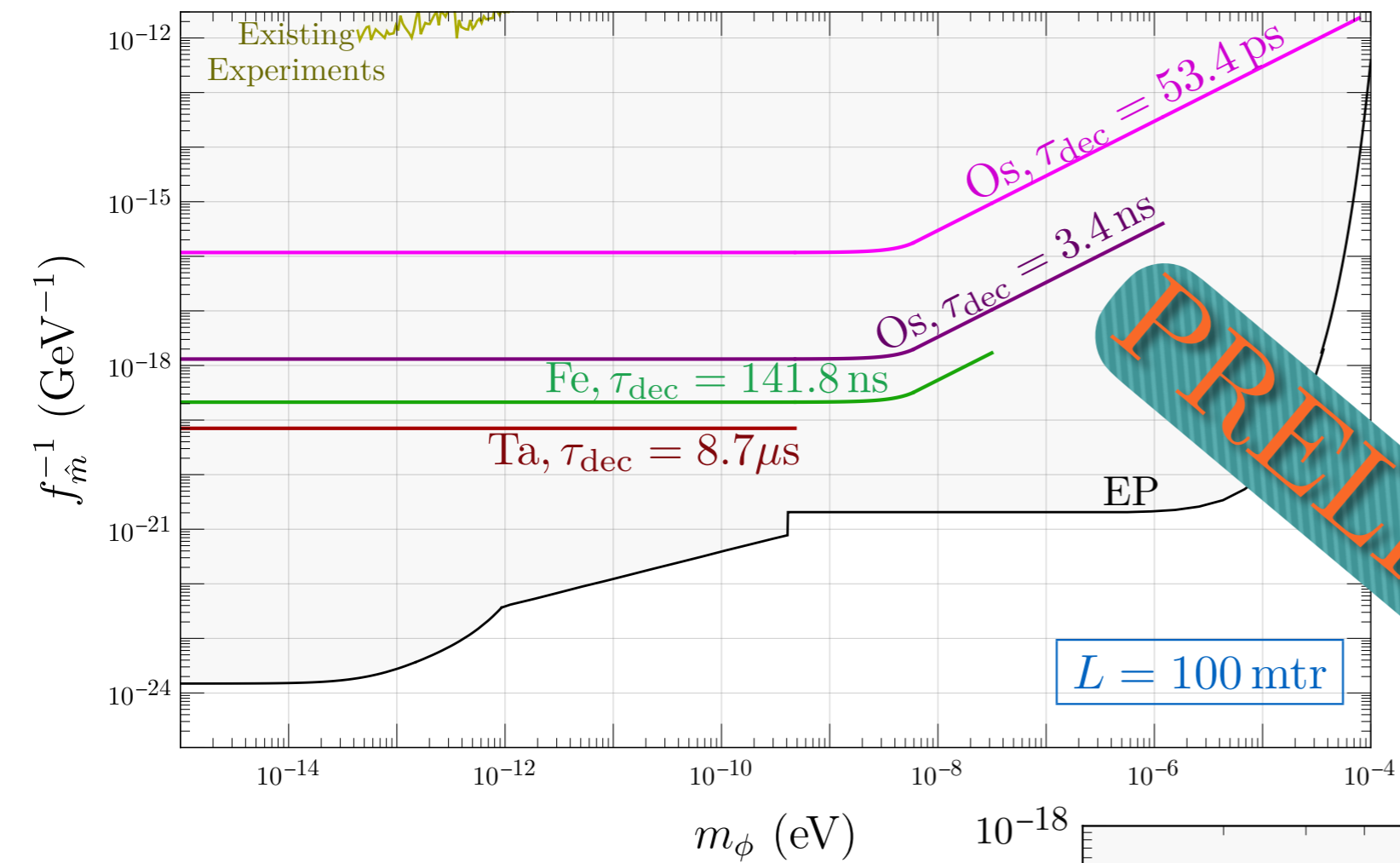
# Ultralight DM

---

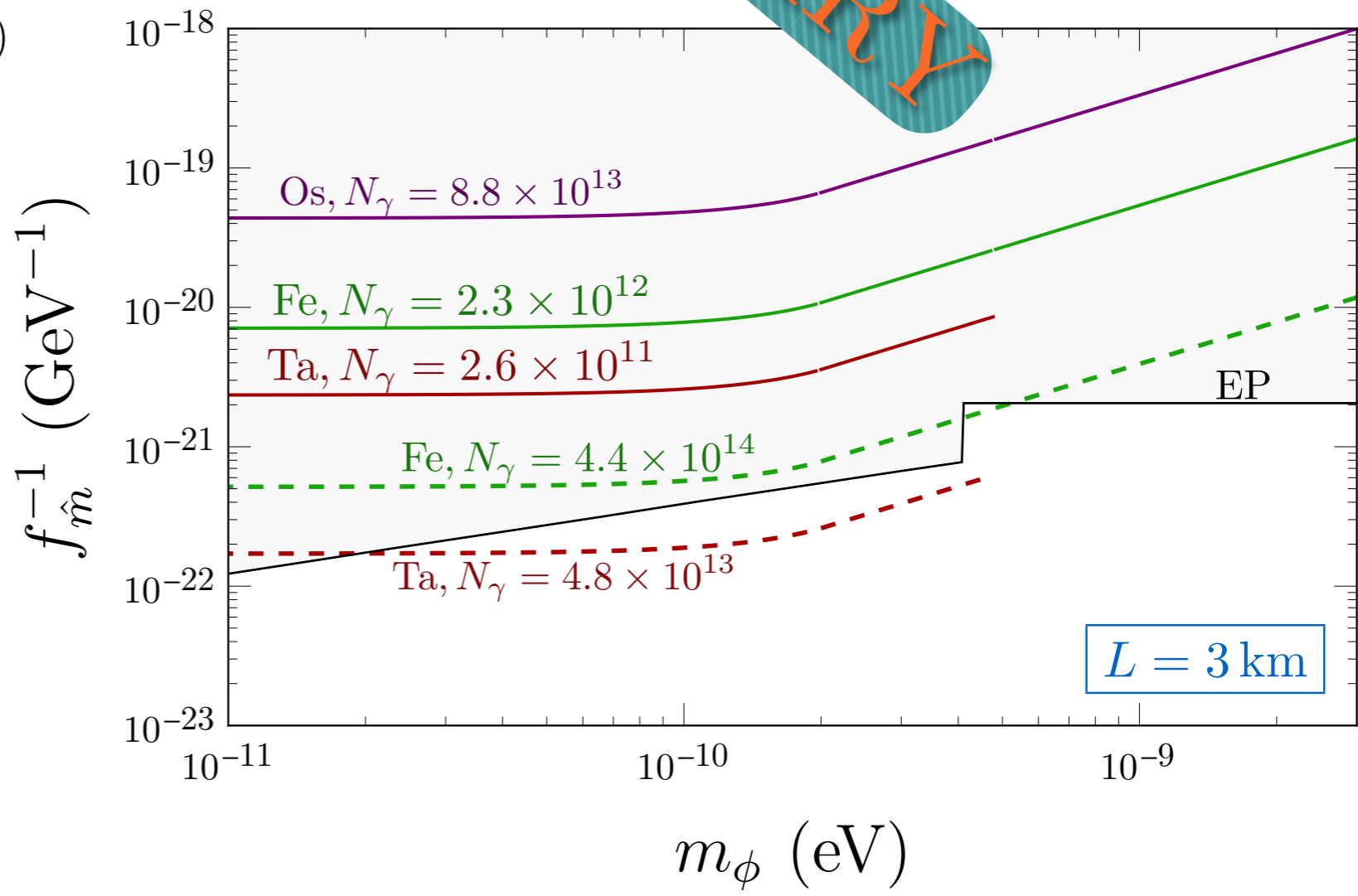
The number of particle in a coherent volume

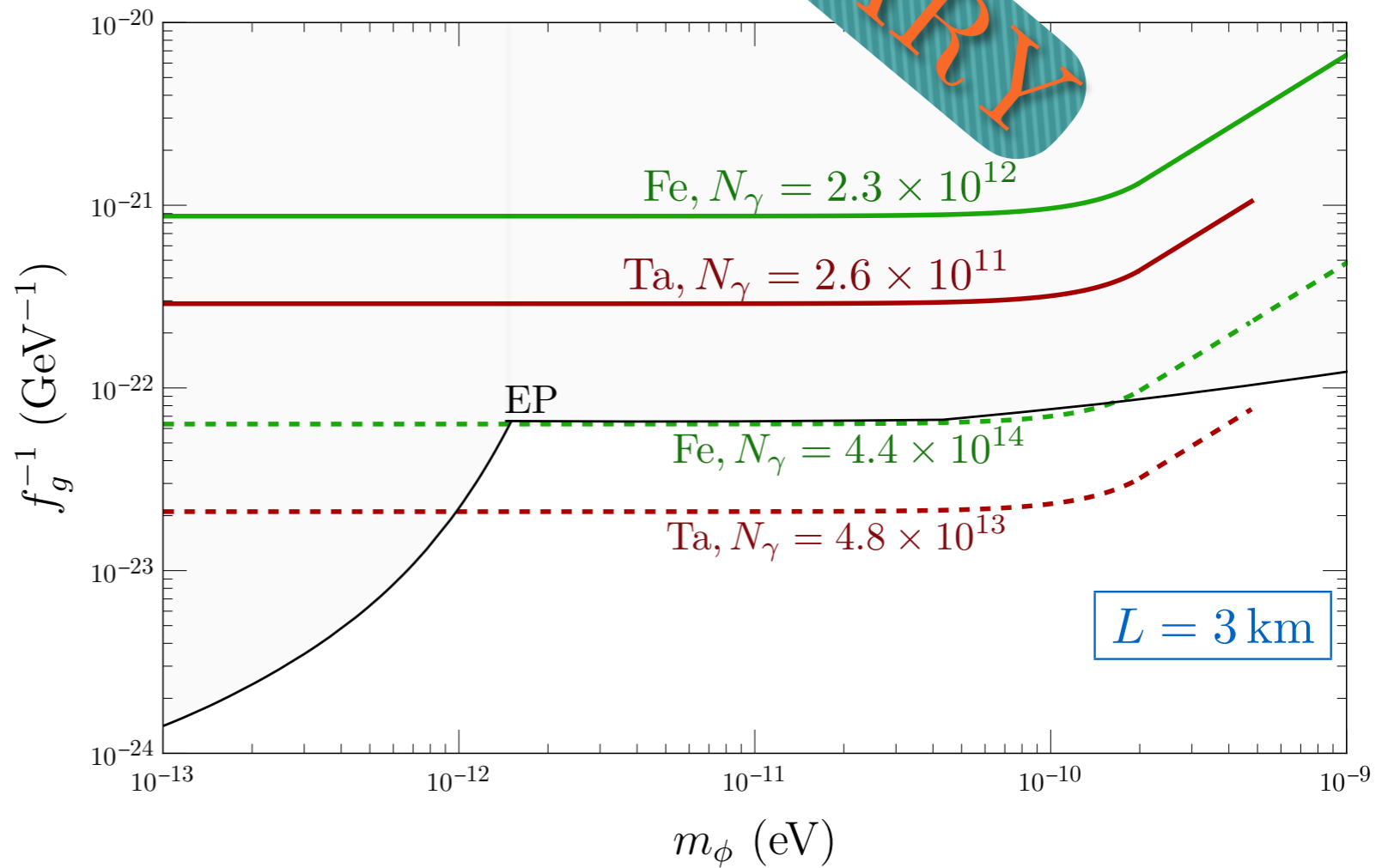
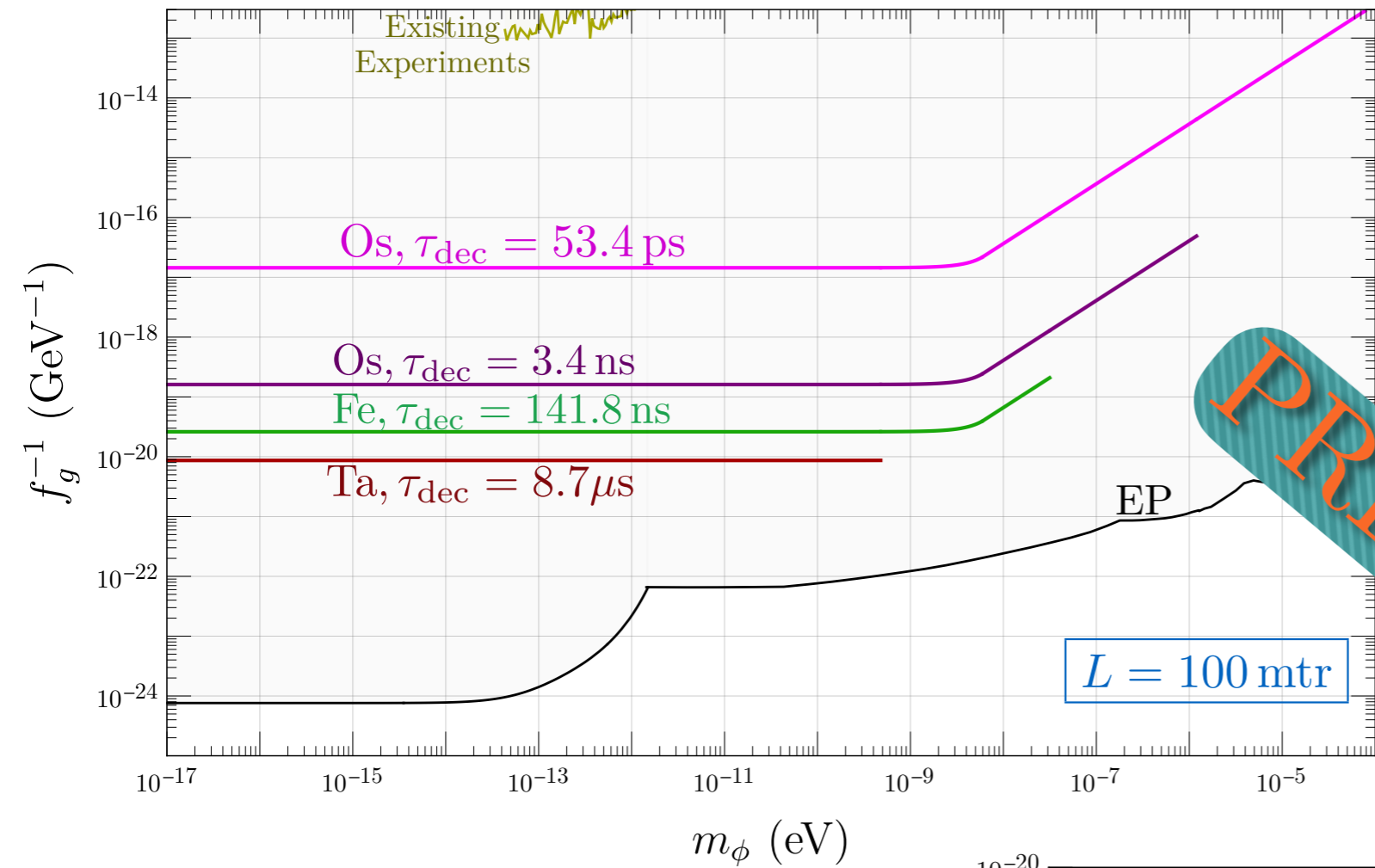
$$N = n \times V_{\text{coh}} = \frac{\rho_{\text{DM}}}{m_{\phi}} \frac{1}{(m_{\phi}\beta)^3}$$

$$N = 10^3 \left( \frac{\text{eV}}{m} \right)^4 \gg 1$$

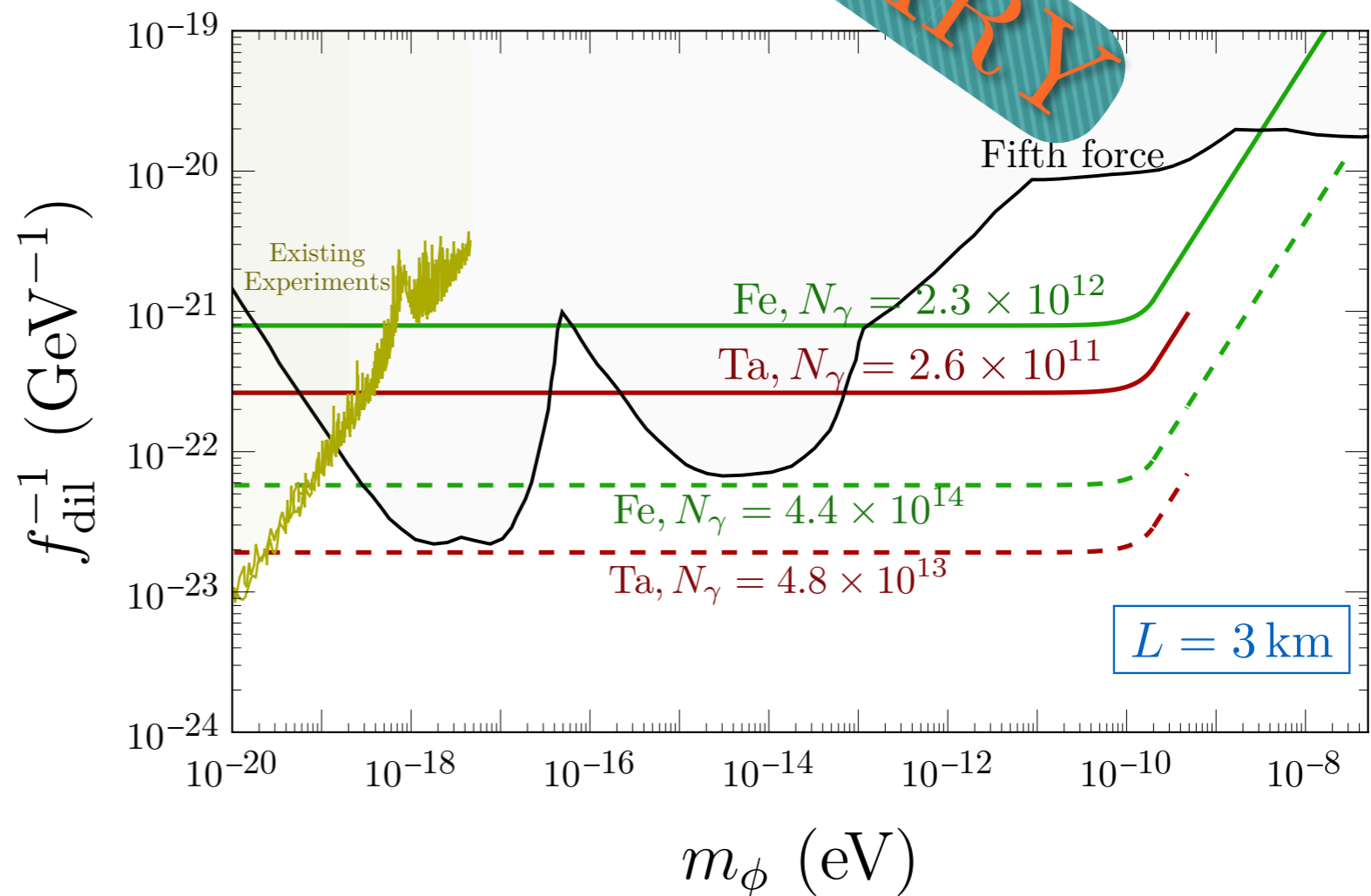
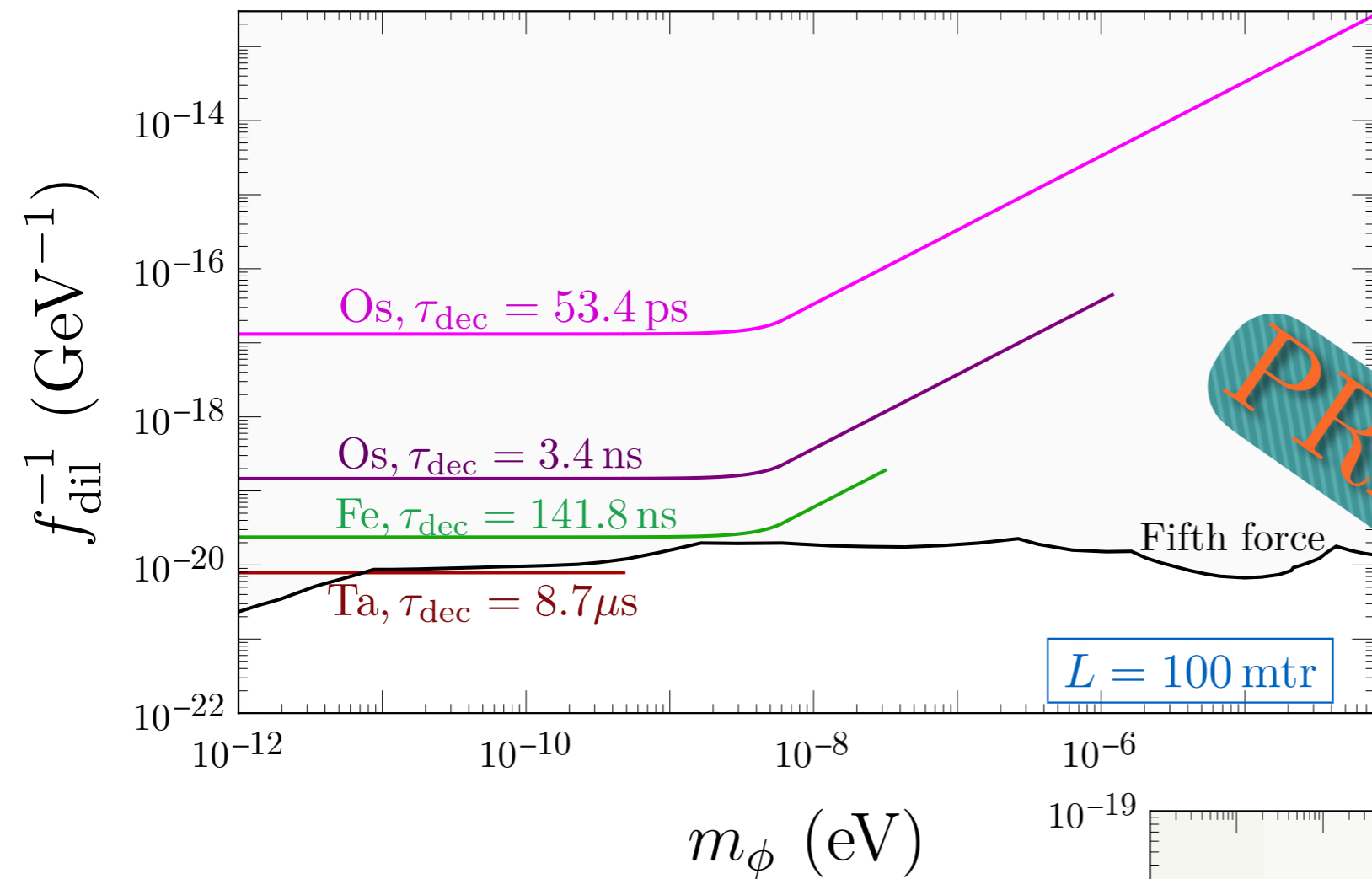


PRELIMINARY





PRELIMINARY



PRELIMINARY

# QCD axion DM at the quadratic order

---

$$\mathcal{L} \supset \frac{g_s^2}{32\pi^2} \frac{a}{f_a} G^{\mu\nu} \tilde{G}_{\mu\nu}$$

$$m_\pi^2(\theta_{\text{eff}}) = B \sqrt{m_u^2 + m_d^2 + 2m_u m_d \cos(\theta_{\text{eff}})} \quad \text{where } \theta_{\text{eff}} = \bar{\theta} + a/f_a, \quad B = -\langle \bar{q}q \rangle_0 / f_\pi^2$$

The oscillating QCD axion DM induces an oscillating component to pion mass

$$\frac{\Delta m_\pi^2}{m_\pi^2} = \frac{m_\pi^2(\theta_{\text{eff}}) - m_\pi^2(0)}{m_\pi^2(0)} \simeq -\frac{m_u m_d \theta_{\text{eff}}^2(t)}{2(m_u + m_d)^2} \quad \theta_{\text{eff}}(t) = \frac{(a - \langle a \rangle)}{f_a} = \frac{\sqrt{2\rho_{\text{DM}}}}{m_a f_a} \cos(m_a t)$$

The nucleon mass also depends on the pion mass

$$m_N(\theta_{\text{eff}}) = m_0 - 4c_1 m_\pi^2(\theta_{\text{eff}}) - \frac{3g_A^2 m_\pi^3(\theta_{\text{eff}})}{32\pi f_\pi^2} \quad \frac{\partial \ln m_N}{\partial \ln m_\pi^2} \simeq 0.06$$

The QCD axion DM  $\Rightarrow$  oscillating pion mass and nucleon mass

# Scalar DM interacting with the QCD Parameters

---

$$\mathcal{L} \supset -\frac{\phi}{\sqrt{2}M_{\text{pl}}} \left[ \sum_{q=u,d} d_{m_q} m_q \bar{q}q + \frac{d_g \beta(g_s)}{2g_s} G^{\mu\nu} G_{\mu\nu} \right]$$

Oscillating DM background  $\phi(t) \simeq \frac{\sqrt{2\rho_{\text{DM}}}}{m_\phi} \cos(m_\phi t)$

leads to time oscillation of the QCD parameters as

$$\alpha_s(t) = \alpha_s(0) \left( 1 - 2d_g \frac{\beta(g_s)\phi(t)}{g_s \sqrt{2}M_{\text{pl}}} \right), \quad \frac{\partial \ln \Lambda_{\text{QCD}}}{\partial \phi} = \frac{d_g}{\sqrt{2}M_{\text{pl}}}$$

$$\hat{m}(t) = \hat{m}(0) \left( 1 + d_{\hat{m}} \frac{\phi(t)}{\sqrt{2}M_{\text{pl}}} \right), \quad \frac{\partial \ln \hat{m}}{\partial \phi} = \frac{d_{\hat{m}}}{\sqrt{2}M_{\text{pl}}}$$

$$\hat{m} = (m_u + m_d)/2$$

$$d_{\hat{m}} = \frac{m_u d_{m_u} + m_d d_{m_d}}{m_u + m_d}$$

$$m_\pi^2 \propto \Lambda_{\text{QCD}} \hat{m} \Rightarrow \frac{\Delta m_\pi^2}{m_\pi^2} = (d_g + d_{\hat{m}}) \frac{\phi(t)}{\sqrt{2}M_{\text{pl}}}$$

scalar DM interacting with QCD parameter  $\Rightarrow$  oscillating pion

mass and nucleon mass

# How to probe time oscillation of hadron masses ?

---

Let us estimate the amplitude of such oscillations are for nucleon mass

For the QCD axion  $\frac{\Delta m_\pi^2}{m_\pi^2} = \frac{m_\pi^2(\theta_{\text{eff}}) - m_\pi^2(0)}{m_\pi^2(0)} \simeq -\frac{m_u m_d \theta_{\text{eff}}^2(t)}{2(m_u + m_d)^2}$   $\theta_{\text{eff}}(t) = \frac{(a - \langle a \rangle)}{f_a} = \frac{\sqrt{2\rho_{\text{DM}}}}{m_a f_a} \cos(m_a t)$

$$\frac{\Delta m_N}{m_N} \simeq 0.06 \frac{\Delta m_\pi^2}{m_\pi^2} \sim 10^{-16} \cos(2 m_a t) \left( \frac{10^{-15} \text{ eV}}{m_a} \right)^2 \left( \frac{10^{10} \text{ GeV}}{f_a} \right)^2$$

For the scalar DM  $\alpha_s(t) = \alpha_s(0) \left( 1 - 2d_g \frac{\beta(g_s)\phi(t)}{g_s \sqrt{2}M_{\text{pl}}} \right)$ ,  $\frac{\partial \ln \Lambda_{\text{QCD}}}{\partial \phi} = \frac{d_g}{\sqrt{2}M_{\text{pl}}}$   $\phi(t) \simeq \frac{\sqrt{2\rho_{\text{DM}}}}{m_\phi} \cos(m_\phi t)$

$$\frac{\Delta m_N}{m_N} \simeq \frac{\Delta \Lambda_{\text{QCD}}}{\Lambda_{\text{QCD}}} \sim 10^{-16} \cos(m_\phi t) \left( \frac{10^{-15} \text{ eV}}{m_\phi} \right) \left( \frac{d_g}{10^{-1}} \right)$$

Is it possible to probe such small oscillation? **Yes!**