# Challenges \& prospects for kaon and D physics from lattice QCD 

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## Challenges \& prospects for kaon and $D$ physics from lattice QCD

Focus will be on "non-standard" (i.e. new) quantities for which standard methods don't apply

Little overlap with calculations discussed in preceding (Bernard) \& following (El-Khadra) talks

# Based partly on 2013 white paper <br> <br> LATTICE QCD AT THE INTENSITY FRONTIER 

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Thomas Blum, Michael Buchoff, Norman Christ, Andreas Kronfeld, Paul Mackenzie, Stephen Sharpe, Robert Sugar and Ruth Van de Water

(USQCD Collaboration)

## SUMMARY

Lattice QCD calculations now play an essential role in the search for new physics at the intensity frontier. They provide accurate results for many of the hadronic matrix elements needed to realize the potential of present experiments probing the physics of flavor. The methodology has been validated by comparison with a broad array of measured quantities, several of which had not been well measured in experiment when the first good lattice calculation became available. In the US, this effort has been supported in an essential way by hardware and software support provided to the USQCD Collaboration.

This document has laid out an ambitious five year vision for future LQCD calculations, explaining how they can provide essential and timely information for upcoming experiments at the intensity frontier, by undertaking calculations of new, more computationally challenging, quantities. In addition, steady improvements in lattice results for matrix elements which are already well calculated will ensure that existing experimental results are fully utilized in the search for new physics. Our plans rely on continuing hardware and software support at similar levels to those of the last decade.

> www.usqcd.org/documents/ I 3flavor.pdf

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Input from experimentalists and phenomenologists
We gratefully acknowledge suggestions and comments from Marina Artuso, Brendan Casey, Tim Gershon, Enrico Lunghi, Bob Tschirhart and Jure Zupan.

## Outline

- Overall aims
- Standard vs non-standard (i.e. new) quantities
- Prospects for new quantities
- $K \rightarrow \pi \pi$ decays
- $\Delta M_{K}$ (long distance)
- $K \rightarrow \pi v V$ and other rare decays
- $D \rightarrow \pi \pi, D \rightarrow K K$ and $D-D b a r$ mixing
- Summary


## Experimental vista (partial \& optimistic)



Adapted from Ruth Van de Water

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ANCIENT


Quantities discussed in this talk


2014
$2014 \quad 2016$

$\sin (2 \beta), B \rightarrow \tau v, B \rightarrow \pi\left|v, B \rightarrow D^{(*)}\right| v$, rare $b \rightarrow c y \& b \rightarrow c l l$ decays, CPV in D-decays, D-mixing, ...

## Aims

- Determine electroweak (\& dark matter) matrix elements sufficiently accurately that searches for new physics in CKM fits, in rare decays, in extremely precise measurements ( $g$ - 2 , dipole moments, ...), and in dark matter experiments are limited by experimental rather than theory errors
- Determine fundamental parameters of standard model with every increasing accuracy (quark masses and $\Lambda_{\mathrm{QCD}}$ )
- As precision improves, continue to cross-check methods by comparisons of spectrum with experiment \& comparisons between different discretizations

$$
\begin{aligned}
\text { Expt }= & (C K M)(p Q C D)(\text { non-pert QCD) } \\
& + \text { BSM(non-pert QCD) }
\end{aligned}
$$

LQCD provides first-principles method to calculate (some)
non-perturbative QCD matrix elements

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## Standard vs non-standard quantities

- Standard means matrix elements involving single particles

$\Rightarrow \mathrm{K} \rightarrow \mathrm{\pi}$ form factor
(similarly $\mathrm{B} \rightarrow \mathrm{D}$, etc)


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$\Rightarrow B_{K}\left(\right.$ similarly $\left.B_{D}, B_{B}\right)$
- Nearly 20 standard matrix elements are fully controlled with small errors
- Decay constants: $f_{T}, f_{\mathrm{K}}, \mathrm{f}_{\mathrm{D}}, \mathrm{f}_{\mathrm{Ds}}, \mathrm{f}_{\mathrm{B}}, \mathrm{f}_{\mathrm{B}}$
- Form factors: $K \rightarrow \pi, D \rightarrow K, D \rightarrow \pi, B \rightarrow D, B \rightarrow D^{*}, B_{s} \rightarrow D_{s} \& B \rightarrow \pi$
- Mixing matrix elements: $B_{k}, B_{B}, B_{B s}$


## Standard vs non-standard quantities

- Non-standard: matrix elements involving two or more particles and/or quark-disconnected contributions



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## Standard quantities well controlled

2013 white paper
Forecasts
$\begin{array}{ccccccc}\hline \hline \text { Quantity } & \text { CKM } & \text { Present } & 2007 \text { forecast } \\ \text { element expt. error }\end{array}$ lattice error $\left.\begin{array}{c}\text { Present } \\ \text { lattice error }\end{array}\right)$

## Expanding portfolio of standard q'ties

- Contributions of BSM physics to K, D \& B-meson mixing
- $\left.\mathrm{B} \rightarrow \mathrm{K} \mathrm{I}^{+}\right|^{-},\left.\Lambda_{b} \rightarrow \mathrm{I}^{+}\right|^{-}$and related form factors
- Nucleon beta-decay BSM form factors
- Nucleon-decay matrix elements
- Neutron-antineutron mixing
- Also, nucleon EDM matrix elements (from SM and BSM theories)
- ...

Can achieve few- I0\% accuracy on few year timescale, which is commensurate with experimental program, and significantly enhances search for BSM physics

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## $K \rightarrow \pi \pi(I=2)$ decay amplitudes

[RBC/UKQCD arXiv:IIII.1699, I206.5|42]

- First controlled result for an amplitude involving two particles
- Isospin $2 \Rightarrow$ no quark-disconnected contributions
- Uses physical kinematics (physical quark masses, moving pions so $M_{K}=2 E_{\pi}$ ) which requires box with $L \approx 6 \mathrm{fm}$
- Finite volume effects small (6\%) but corrected for by calculating phase shift
- Error is $\sim 15 \%$, dominated by discretization errors since $a \approx 0.14 \mathrm{fm}$



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## Results

$$
\begin{aligned}
& \operatorname{Re~} A_{2}=1.38(5)_{\text {stat }}(26)_{\text {syst }} 10^{-8} \mathrm{GeV} \longleftrightarrow \begin{array}{c}
\text { expt. I.479(4) } 10^{-8}\left[\mathrm{~K}^{+}\right] \\
\mathrm{I} .57(6) 10^{-8}[\mathrm{Ks}]
\end{array} \\
& \operatorname{Im~A}=-6.5(5)_{\text {stat }}(12)_{\text {syst }} 10^{-13} \mathrm{GeV} \longleftrightarrow \begin{array}{l}
\text { New information! } \\
\text { Can use with expt result for } \varepsilon^{\prime} \\
\text { to determine Im A } \mathrm{A}_{0}
\end{array}
\end{aligned}
$$

Expect errors of $\sim 5 \%$ in 2-3 years (by adding a smaller lattice spacing)

## $K \rightarrow \pi \pi(I=0)$ amplitude $\& \Delta I=1 / 2$ rule

[RBC/UKQCD arXiv:I2|2.1474]

- $\mathrm{I}=0$ involves disconnected contractions $\Rightarrow$ numerics much more challenging
- Several other technical challenges too. Fermions with chiral symmetry essential.
- Pilot calculation completed: decay at threshold for $M_{K} \sim 660,880 \mathrm{MeV}$
- Demonstrates that technology (\& related theory) exists. Statistical errors ~ $15 \%$.
- "Emerging understanding of $\Delta I=I / 2$ rule"
- Re $A_{0} \sim$ experiment
- $\operatorname{Re} A_{2} / \operatorname{Re} A_{0}$ suppressed due to cancellation between color contractions
- Penguins unimportant at $\mu \approx 2 \mathrm{GeV}$


Contraction (1).


Contraction (2).

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Errors of $\sim 15 \%$ (also for $\operatorname{Im} A_{0} \Rightarrow \varepsilon^{\prime}$ ) expected in $\sim 2$ years

- Beyond this precision, likely require dynamical charm quark


## Long-distance part of $\Delta M_{K}$

[RBC/UKQCD arXiv:I2|2.593I]

- Very challenging since two insertions of $\mathrm{H}_{\mathrm{w}}$ \& quark-disconnected diagrams
- Also requires new theoretical developments [Christ]
- Dynamical charm enforces GIM cancellations
- Pilot calculation keeping non-disconnected contractions at unphysical masses and with valence (but not dynamical) charm
- Proof of principle giving results with correct order of magnitude

Kept


Dropped


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## Prospects

- First complete calculation (all contractions) is underway

■ Extension to long-distance part of $\varepsilon_{\kappa}$ next step (needs dynamical charm)

- Can make detailed comparisons with PT to test convergence at $\mu \sim m_{c}$


## Other applications?

- Correlators with four operators can now be calculated. Other applications?
- Example: can LQCD help solidify SM predictions for $\mathrm{K} \rightarrow \pi \mathrm{e}^{+} \mathrm{e}^{-}$?
- Is calculating the sign of $K_{s} \rightarrow \pi \gamma^{*} \rightarrow \pi \mathrm{e}^{+} \mathrm{e}^{-}$useful (since that seems to be the main uncertainty in predicting $\mathrm{K}_{\mathrm{L}} \rightarrow \pi \mathrm{e}^{+} \mathrm{e}^{-}$) ?
- Are present estimates of charm and long-distance contributions to $K \rightarrow \pi V V$ accurate enough?

Examples of diagrams for $\mathrm{K}_{\mathrm{s}} \rightarrow \pi \gamma^{*}$


## $C P$ violation in $D \rightarrow \pi \pi, K K$

- Evidence for CP-violation puts us in the same situation as we've been in with $\varepsilon$ ' for decades: can we reliably predict the SM contribution?
- Many challenges, both computational and theoretical
- Hardest (still unsolved) is that, at energy $M_{D}, 2 \pi \& 2 K$ states mix in a finite box with $4 \pi, 6 \pi$, etc. and need to disentangle



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- Some progress with $3 \pi$ case [Hansen \& SS]
- I expect progress on $\mathrm{D} \rightarrow \pi \pi$, KK on a 5 year timescale
- D-Dbar mixing is more challenging


## Summary \& Outlook

- LQCD provides a quantitative method for determining non-perturbative matrix elements
- ~20 standard matrix elements now well controlled
- Several puzzles to be understood, e.g. $\mathrm{V}_{\mathrm{ub}} \& \mathrm{~V}_{\mathrm{cb}}$ exclusive vs. inclusive
- Next 5 years will bring a mix of steady improvements in standard quantities, extensions to several additional standard quantities, and calculations of new types of matrix element requiring qualitatively different (and more challenging) methods
- Steady improvements eventually require isospin breaking \& EM
- Are we missing quantities that these methods can be used for?
- Method is powerful but limited
- No known method for non-leptonic $B$ decays, e.g. $B \rightarrow D \pi$

