Basic Kaon Physics from Lattice QCD

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Intensity Frontier Workshop
Argonne National Lab
April 25-27, 2013

Take-Home Message

- ✦ Lattice QCD has been highly successful for basic kaon quantities: f_K, $f_+(0)$, B_K.
  - Calculations of these quantities have progressed so far that lattice errors are no longer (or will relatively soon no longer be) the limiting precision for phenomenological applications.
    - Note that lattice errors *still dominate* for almost all heavy-quark quantities, or for more complicated light quark quantities (*e.g.*, ε'/ε).
- ✦ New approaches need to be developed, both on and off the lattice, in order to reduce errors that heretofore have been subdominant (*e.g.*, EM errors, perturbative errors).

Lattice QCD in a Nutshell

1. Make QCD well-defined nonperturbatively. Replace infinite number of degrees of freedom of continuum field by finite number of degrees of freedom.
   - Introduce lattice with spacing of points a ⇒ discretization or continuum-extrapolation errors.
   - Consider only a finite volume of space-time ⇒ finite-volume errors.
   - Go to Euclidean space ($t \to it$) so that path integral is absolutely convergent and amenable to numerical methods. (No associated errors *per se*, but calculation of standard Minkowski quantities like phase shifts becomes rather involved.)
2. Do path integral by Monte Carlo methods.
   - Choose gluon fields on lattice ("configurations") according to probability given by $\exp(-S_{eff})$, where S_{eff} includes gluon action + back reaction from sea-quark loops (the fermion determinant).
   - Set of configurations is an "ensemble."
   - Calculate some physical quantity on each configuration in an ensemble.

Lattice QCD in a Nutshell (cont.)

2. Do path integral by Monte-Carlo Methods.

- Calculate some physical quantity on each configuration in an ensemble.

  ⋆ Usually will involve finding the propagators of valence quarks in that gluon background. (E.g. for a pion propagator, from which you can get m_π, find propagators of **d**-quark and $\bar{\mathbf{u}}$-quark.)

- Average the quantity over the configurations in an ensemble. Number of configurations in an ensemble is finite ⇒ <u>statistical errors</u>.

- Computer time increases sharply as $m_{quark} \to 0$. May make light (u,d) quarks heavier than in real world and extrapolate. ⇒ <u>chiral extrapolation error</u> (or χ PT error).

  ⋆ Computer + algorithmic advances have recently made it possible to do simulations at physical light quark masses and avoid the chiral extrap. error, but trade-offs still required.

Caveat on my Lattice Averages

- I made "rough and ready" averages of results from various groups.
  - Included recent results: posted, but not yet published.
  - No attempt to take into account correlations of statistical or systematic errors between groups.
    - correlations not very strong, overall, but there are some.
  - The resulting lattice errors are usually somewhat underestimated, but qualitatively similar to those from a more careful job.
  - Point is to convey a sense of where field is, not to produce numbers to be used in phenomenological CKM analyses.
- New official averages from FLAG will be out this summer.
  - FLAG = *Flavor Averaging Lattice Group*
    - Goal is to be the PDG for lattice results.
    - Merger of original Europe-based group, US-based *LatticeAverages.org*, plus additional lattice physicists from around the world.
    - Now includes heavy-quark quantities as well as light-quark ones.

$K_{\ell 2}$ decay: f_{K^+}

- Since V_{ud} has small errors, can get V_{us} in the SM from (photon inclusive) rates:

$$\frac{\Gamma_{K_{\ell 2}}}{\Gamma_{\pi_{\ell 2}}} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{f_K^2}{f_\pi^2} \frac{m_K(1 - m_\ell^2/m_K^2)^2}{m_\pi(1 - m_\ell^2/m_\pi^2)^2} (1 + \delta_{EM})$$

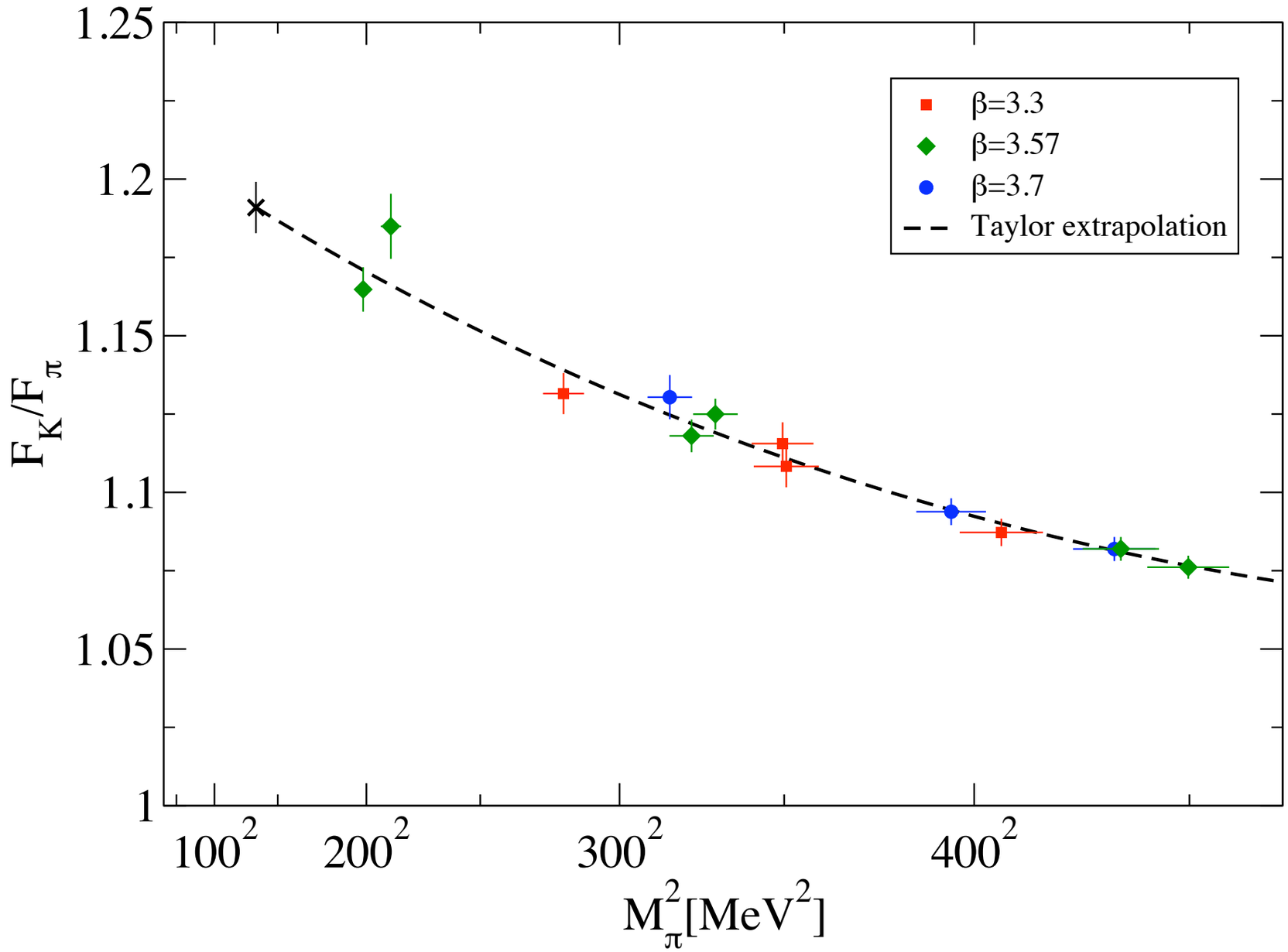
  - δ_{EM} is effect of $O(\alpha)$ electromagnetic corrections.

- Lattice needs to calculate f_K/f_π.
  - Need to evaluate matrix elements like:

$$\langle 0| \, \bar{s} \, \gamma_\mu \gamma_5 \, u \, |K^+(p)\rangle_{\rm QCD} = i p_\mu f_{K^+}$$

  - "Easy" quantity; requires only a two-point function.
  - Some statistical and systematic errors cancel in the ratio.
  - Calculations in a mature phase; done by many groups; fraction of percent errors.

f_K/f_π

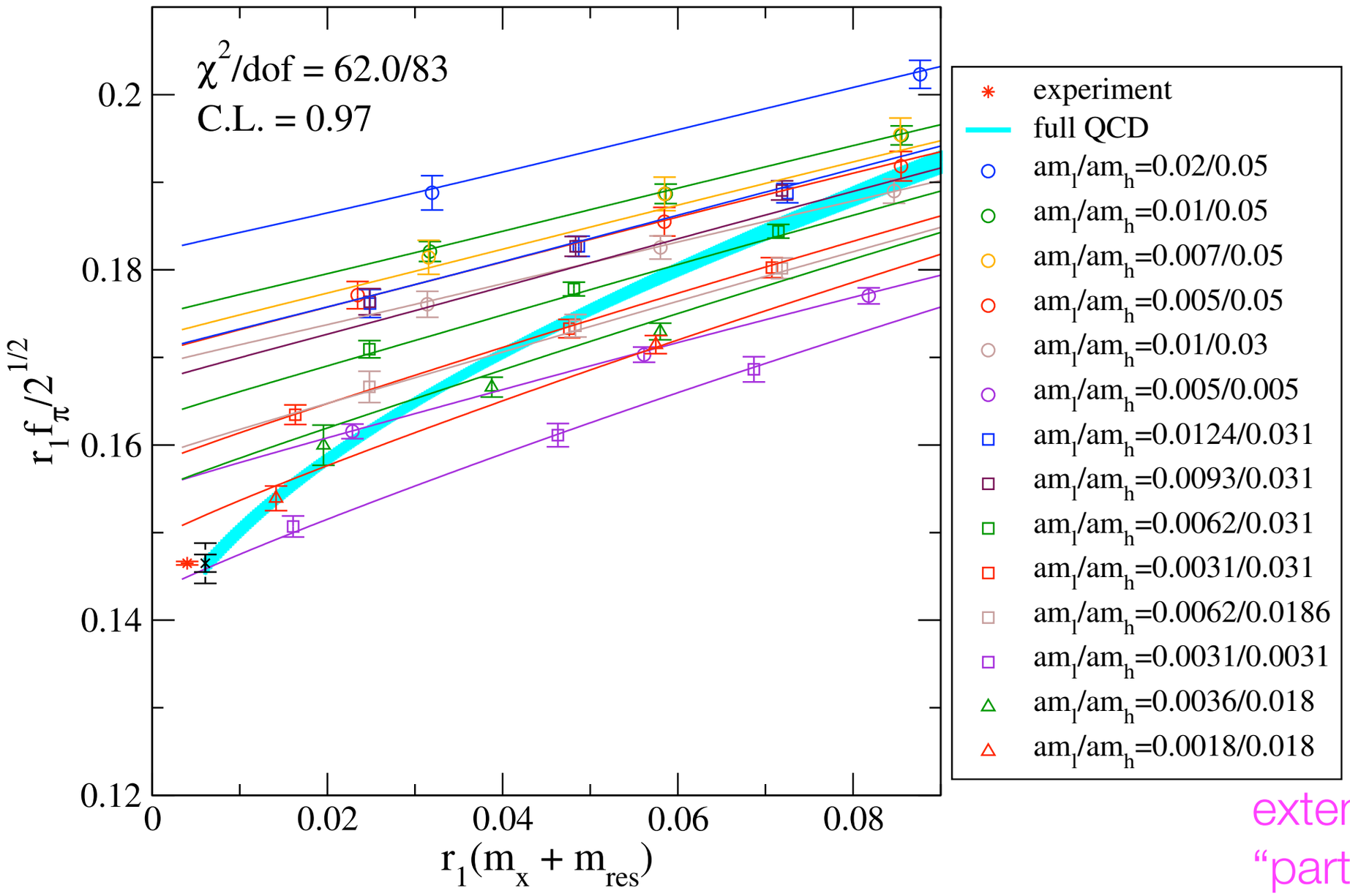


"HEX" smeared Wilson quarks;

u, d, s sea quarks, with $m_u=m_d$: "2+1" sea quark flavors.

- BMW, arXiv:1001.4692.
- chiral/continuum extrapolation of f_K/f_π
- $f_K/f_\pi = 1.192(6)_{\mathrm{stat}}\,(7)_{\mathrm{systematic}} \Rightarrow 0.8\%$ error

f_K/f_π



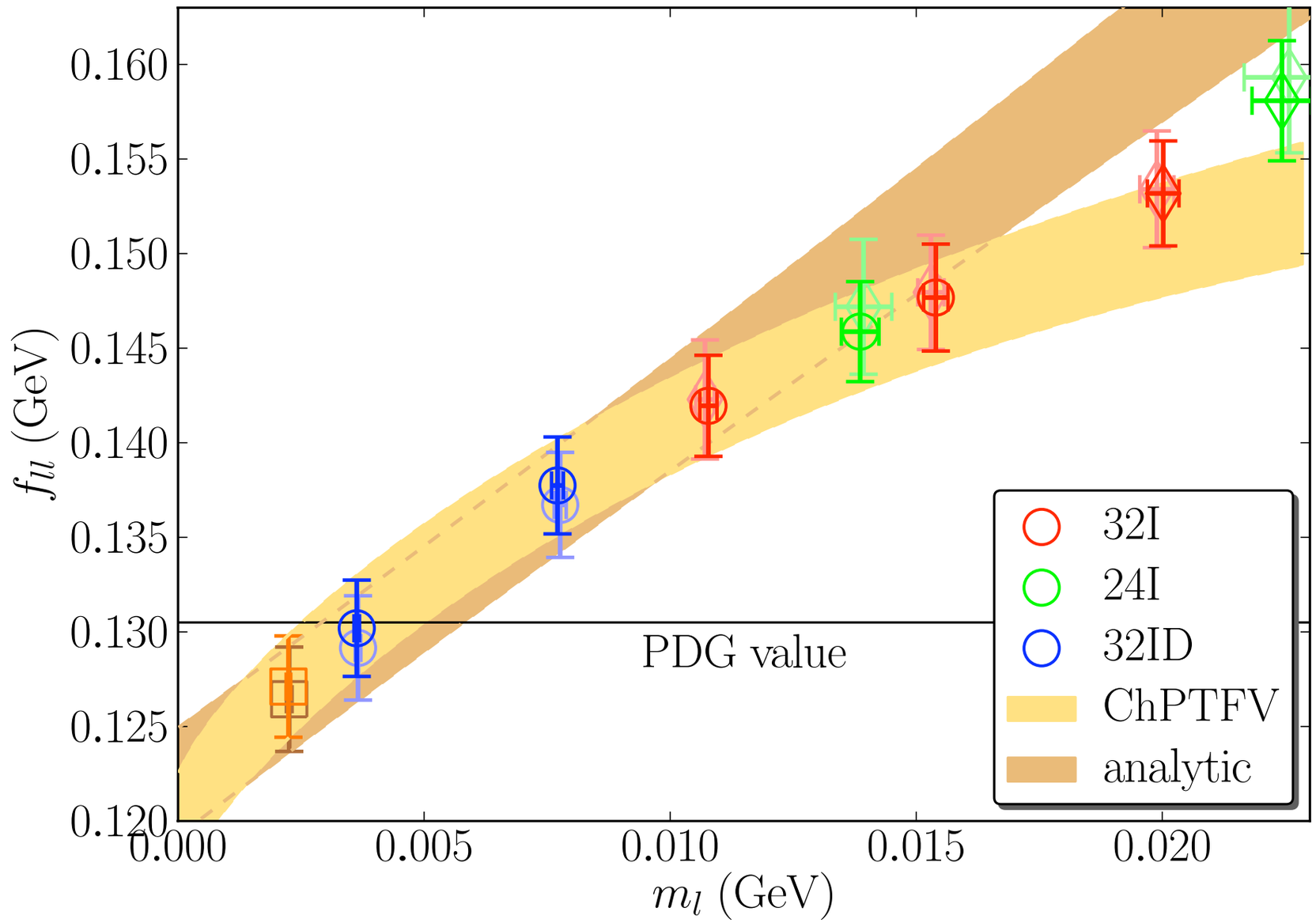
Domain wall valence quarks on MILC "asqtad" staggered sea quarks: "Mixed Action."

2+1 sea quark flavors.

extensive use of "partial quenching": sea and valence masses can be chosen to be different.

- Laiho & Van de Water, arXiv:1112.4861.
- chiral & continuum extrapolation of f_π.
- $f_{K+}/f_{\pi+} = 1.202(11)_{stat}\,(9)_{\chi PT}\,(8)_{FV}\,(2)_{r1}\,(5)_{mq} \Rightarrow$ 1.4% error

f_K/f_π

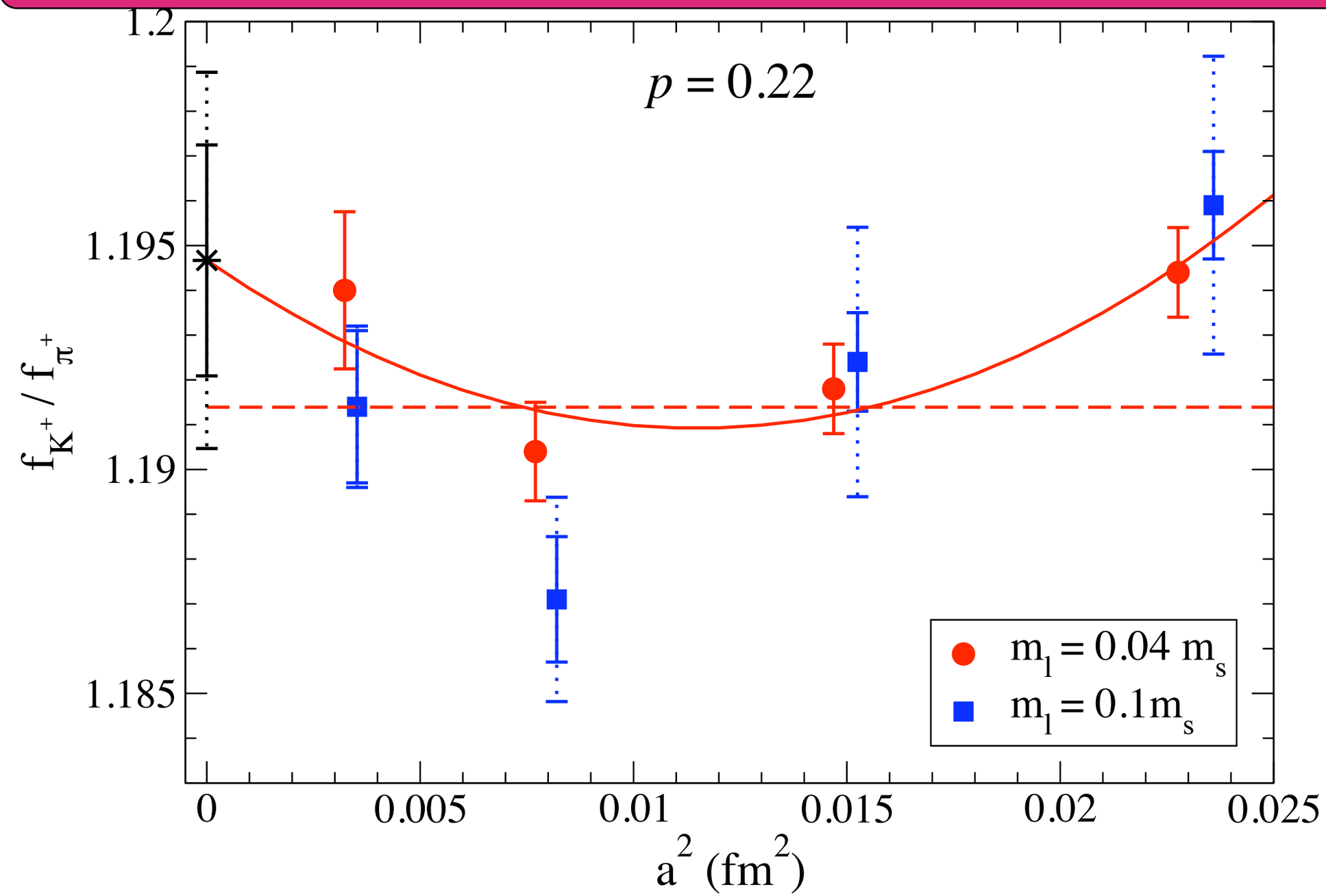


Domain wall quarks;

2+1 sea quark flavors.

- RBC/UKQCD, arXiv:1208.4412.
- chiral extrapolation of f_π.
- $f_K/f_\pi = 1.1991(116)_{\text{stat}}\,(69)_{\chi\text{PT}}\,(116)_{\text{FV}} \Rightarrow$ 1.5% error

f_K/f_π

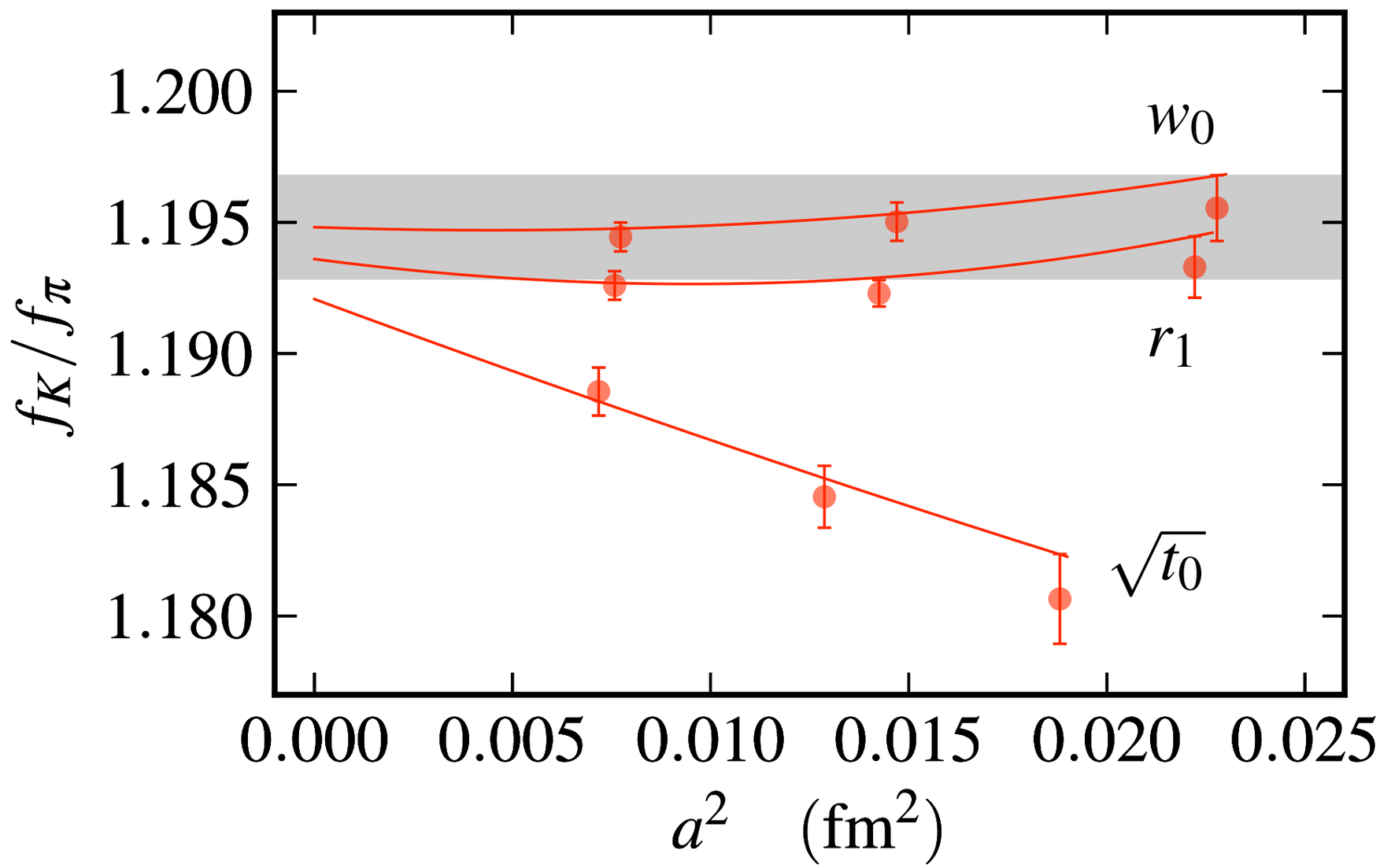


Staggered "HISQ" quarks by MILC.

2+1+1 sea quark flavors.

- MILC, arXiv:1301.5855.
- continuum extrapolation of $f_{K+}/f_{\pi+}$
- red points have physical quark masses; no chiral extrap. necessary!
- $f_{K+}/f_{\pi+}$ = 1.1947(26)$_{\text{stat}}$ (33)$_{\text{continuum-extrap}}$ (17)$_{\text{FV}}$ (2)$_{\text{EM}}$ ⇒ 0.4% error

f_K/f_π

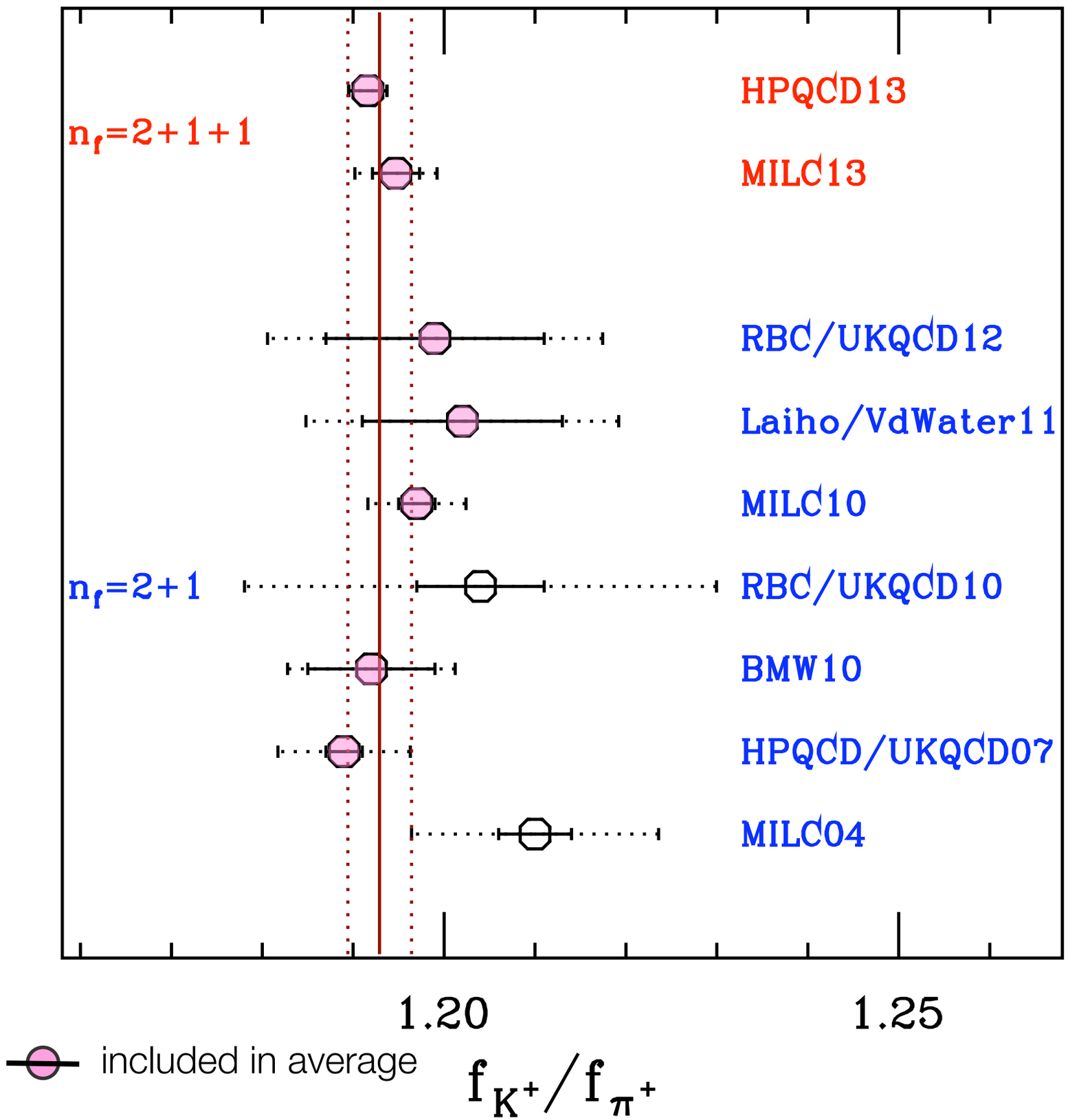


Valence "HISQ" staggered quarks by HPQCD on HISQ sea-quark ensembles by MILC.

2+1+1 sea quark flavors.

- HPQCD, arXiv:1303.1670.
- continuum extrapolation of $f_{K+}/f_{\pi+}$ with various scale-setting procedures.
- have physical quark masses; also include unphysical masses + chiral extrap.
- $f_{K+}/f_{\pi+}$ = 1.1916(21)$_{\text{stat+systematic}}$ ⇒ 0.2% error!

f_K/f_π



- my average:

$$f_{K^+}/f_{\pi^+} = 1.193(3)$$

- 0.25% error

V_{us} from f_K/f_π

- With my average plus experimental rates:

$$|V_{us}| = 0.2254\,(3)_{\rm expt}\,(2)_{\rm EM}\,(6)_{\rm lattice}\,(1)_{V_{ud}}$$

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- If take HPQCD errors, get (blue).

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- Note that EM error comes from power counting estimate of higher order terms in ChPT by Cirigliano and Rosell, JHEP 10 (2007) 005.

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- If take more conservative estimate of Marciano, PRL 93 (2004) 231803, which is what is used in PDG2012, get (red).

V_{us} from f_K/f_π

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V_{us} from f_K/f_π

- ✦ If lattice errors are not yet comparable in size to the EM errors, they will be fairly soon.
- ✦ Will then become much more difficult to reduce errors in V_{us} in this approach.
  - For some quantities, *e.g.*, hadron masses, it is straightforward to include EM effects into lattice simulation.
    - Especially easy for corrections to Dashen's theorem for π, K masses: quenched EM is sufficient for a controlled calculation.
    - This has been/is being/ done by several groups.
  - But for decay constants of charged mesons, a simple change of QCD → QCD+EM is not possible.
    - Charged current $\bar{u}\gamma_\mu s$ is not EM gauge invariant!
    - Would need to compute decay, *e.g.*, $K \to \mu\nu$, directly, say in a 4-fermi theory with μ and ν fields, rather than the simple matrix element of a current between meson and vacuum.

$K \to \pi \ell \nu$

✦ Standard model (photon inclusive) rate:

$$\Gamma_{K_{\ell 3}} = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{\rm EW} \left[|V_{us}|^2 f_+(0)\right]^2 I_{K\ell} \times \left(1 + \delta_{\rm EM}^{K\ell} + \delta_{\rm iso}^{K\pi}\right)$$

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Clebsch for $K^{+/-}$ decays compared to K^0

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Phase-space

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Form factor at q^2=0 to be determined by lattice; normalized to $K^0 \to \pi^-$ channel

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- Form factors defined by

$$\langle \pi^-(p_\pi)| \, \bar{s}\, \gamma^\mu \, u \, |K^0(p_K)\rangle = f_+(q^2) \left[p_K^\mu + p_\pi^\mu - \frac{m_K^2 - m_\pi^2}{q^2} q^\mu \right] + + f_0^{K\pi}(q^2) \frac{m_K^2 - m_\pi^2}{q^2} q^\mu \,,$$

$$q^\mu \equiv p_K^\mu - p_\pi^\mu \,.$$

- with kinematic constraint:

$$f_0(0) = f_+(0)$$

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long distance, structure dependent EM corrections; error ~0.2%

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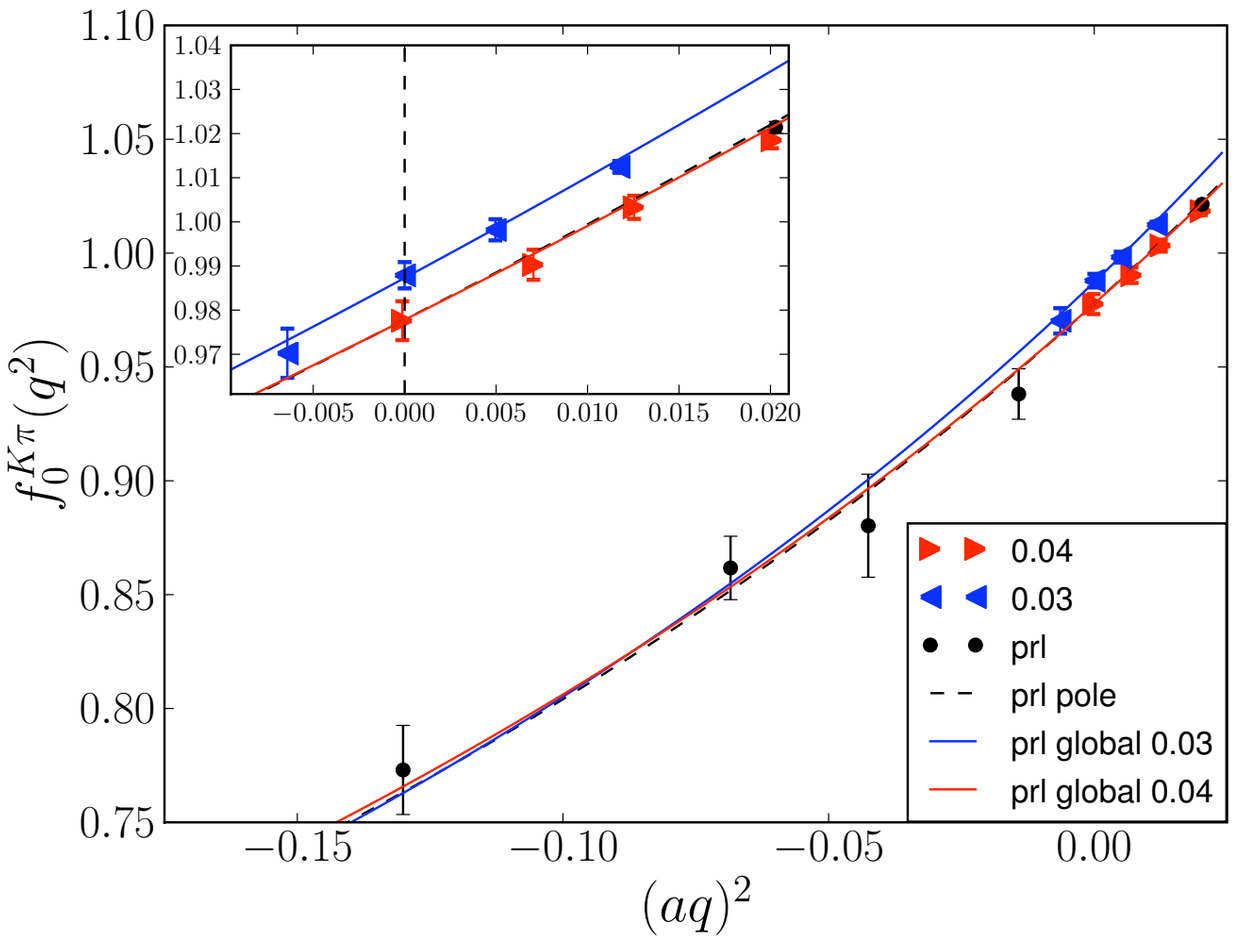
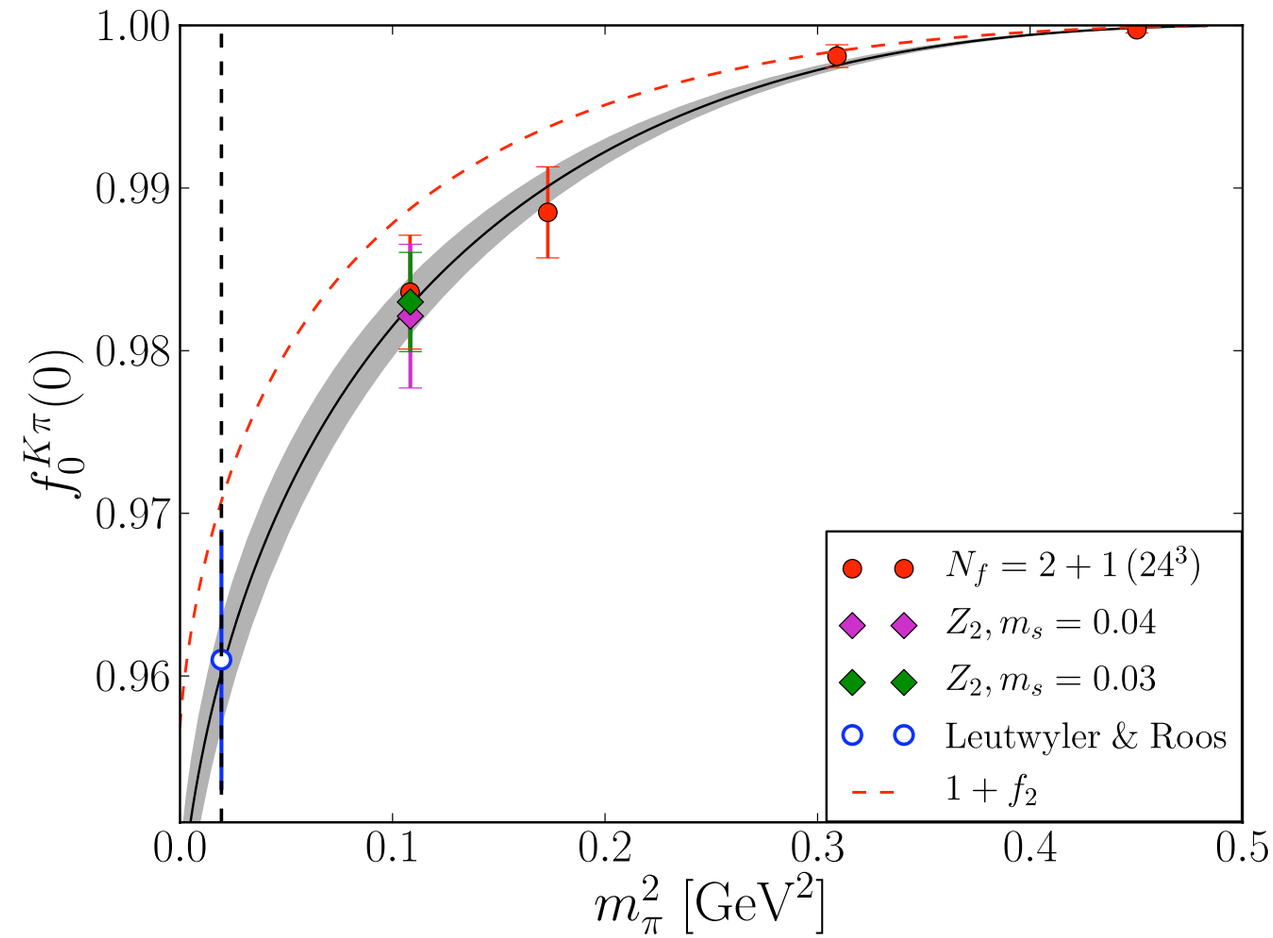
long distance, structure dependent EM corrections; error ~0.2%

Corrections to $K^{+/-}$ modes relative to K^0 from isospin violation. Errors comparable to experimental errors in those modes.

$K \to \pi \ell \nu$

Domain wall quarks; 2+1 sea quark flavors.

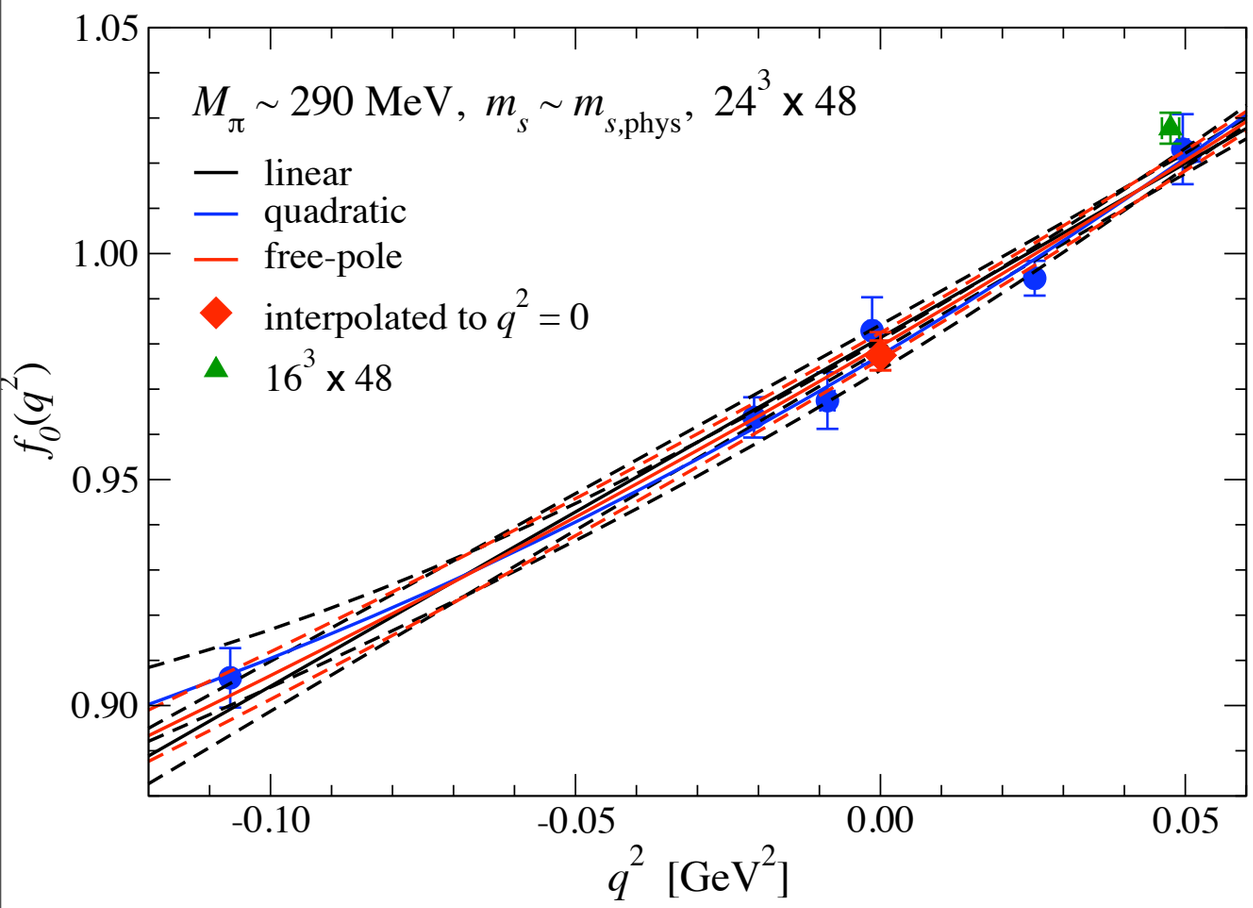


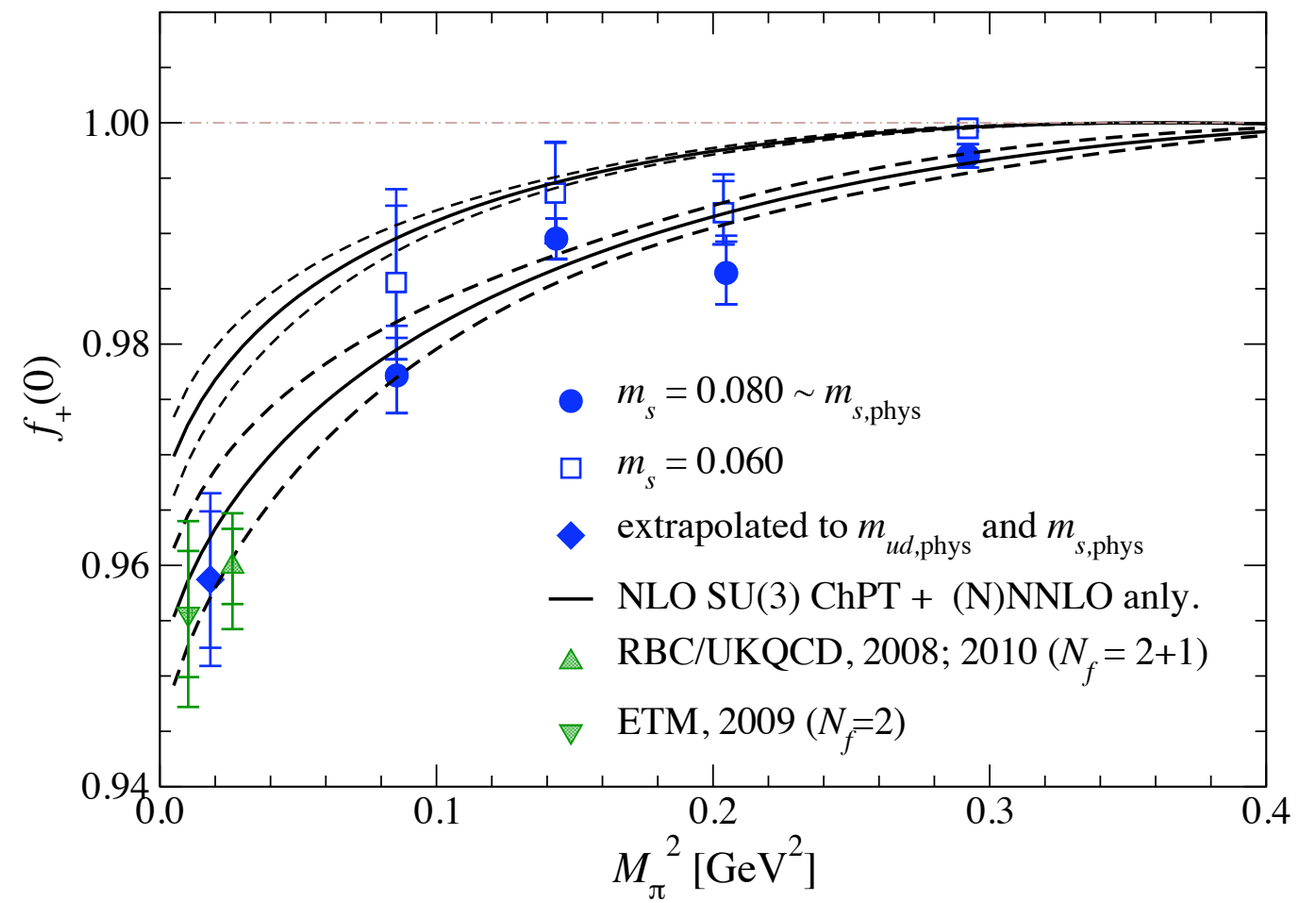


- RBC/UKQCD, arXiv:1004.0866.
- Single lattice spacing, a=0.11 fm.
- left: interpolation of $f_0(q^2)$ to $q^2=0$ for two different strange quark masses.
- right: chiral extrapolation of $f_0(0)=f_+(0)$.
- $f_+(0)=0.9699\,(34)_{stat}\,(^{+34}_{-43})_{\chi PT}\,(14)_{discretization}$ ⇒ 0.6% error

$K \to \pi \ell \nu$

Overlap quarks by JLQCD; 2+1 sea quark flavors.

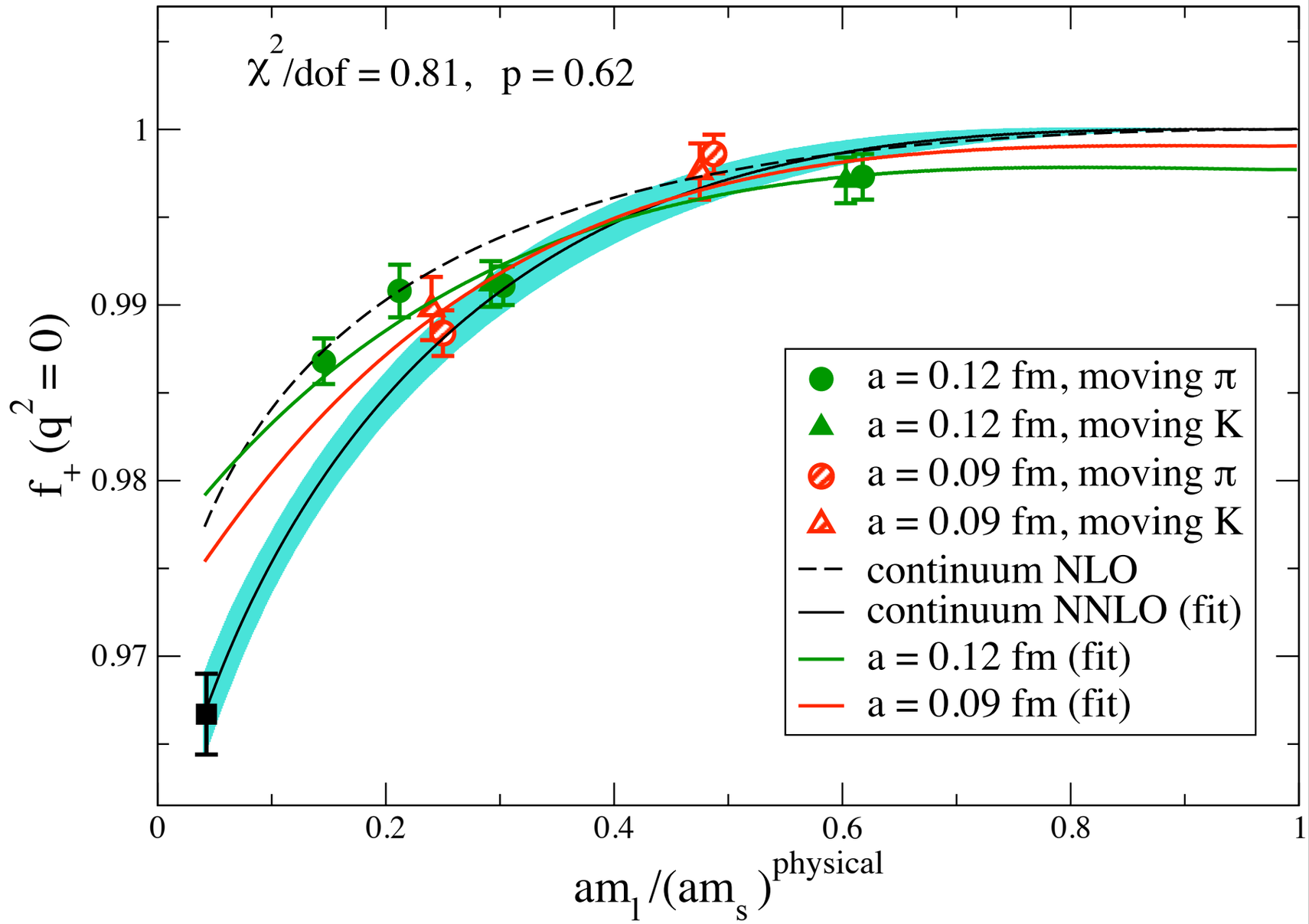




- JLQCD, arXiv:1211.6180 [Single lattice spacing, a=0.11 fm].
- left: interpolation of $f_0(q^2)$ to q^2=0 for lightest pion mass.
- right: chiral extrapolation of $f_+(0)$.
- $f_+(0)$=0.959 (6)$_{stat}$ (4)$_{\chi PT}$ (3)$_{discretization}$ ⇒ 0.8% error

$K \to \pi \ell \nu$

- Fermilab Lattice/MILC, arXiv:1212.4993.
- Chiral extrapolation of $f_+(0)$ for 2 lattice spacings.

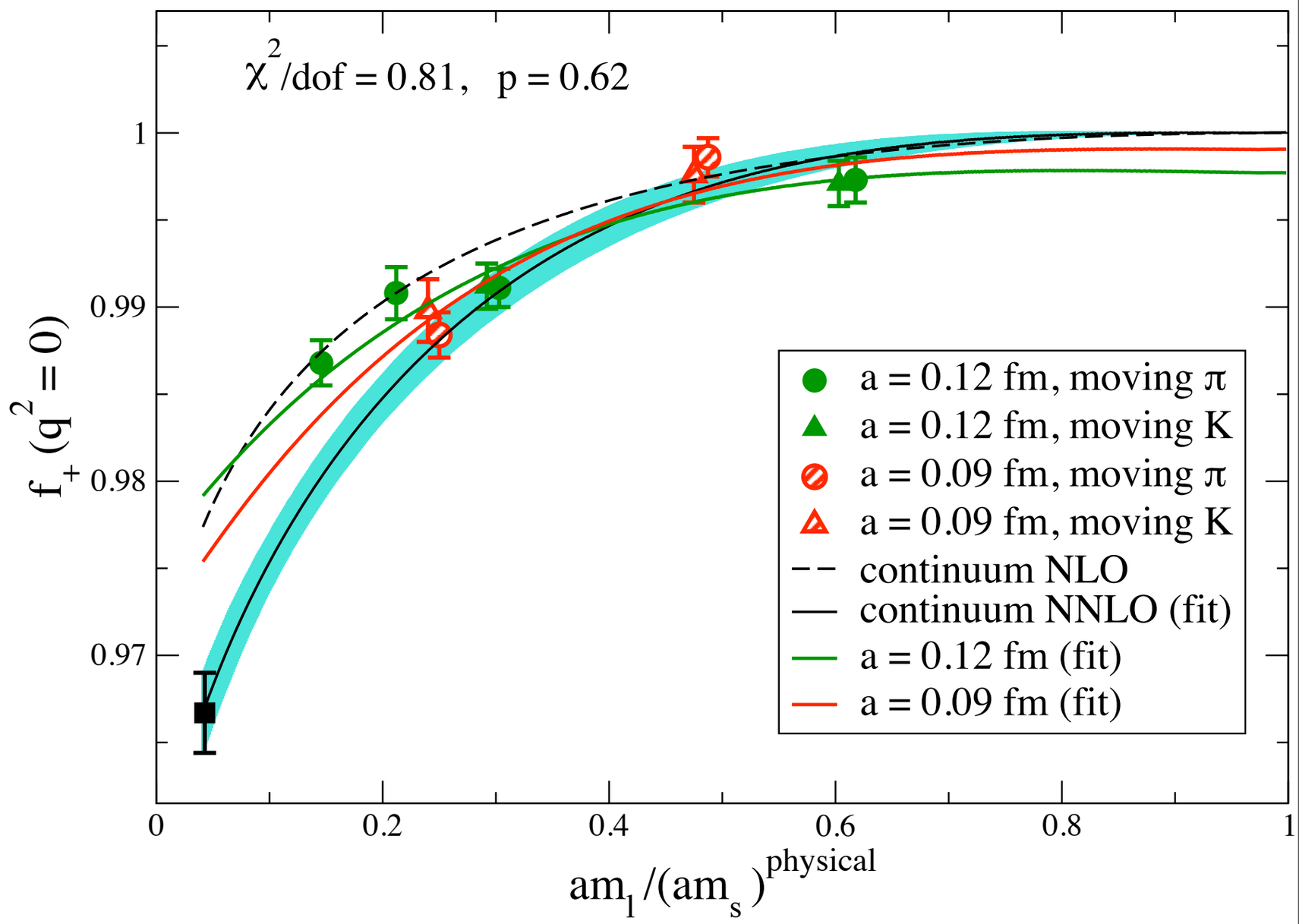


HISQ valence quarks on asqtad staggered sea quarks by MILC.

2+1 sea quark flavors.

$K \to \pi \ell \nu$

- Fermilab Lattice/MILC, arXiv:1212.4993.
- Chiral extrapolation of $f_+(0)$ for 2 lattice spacings.
- Fit includes NNLO chiral logs so sensitivity to form of NLO terms is eliminated.



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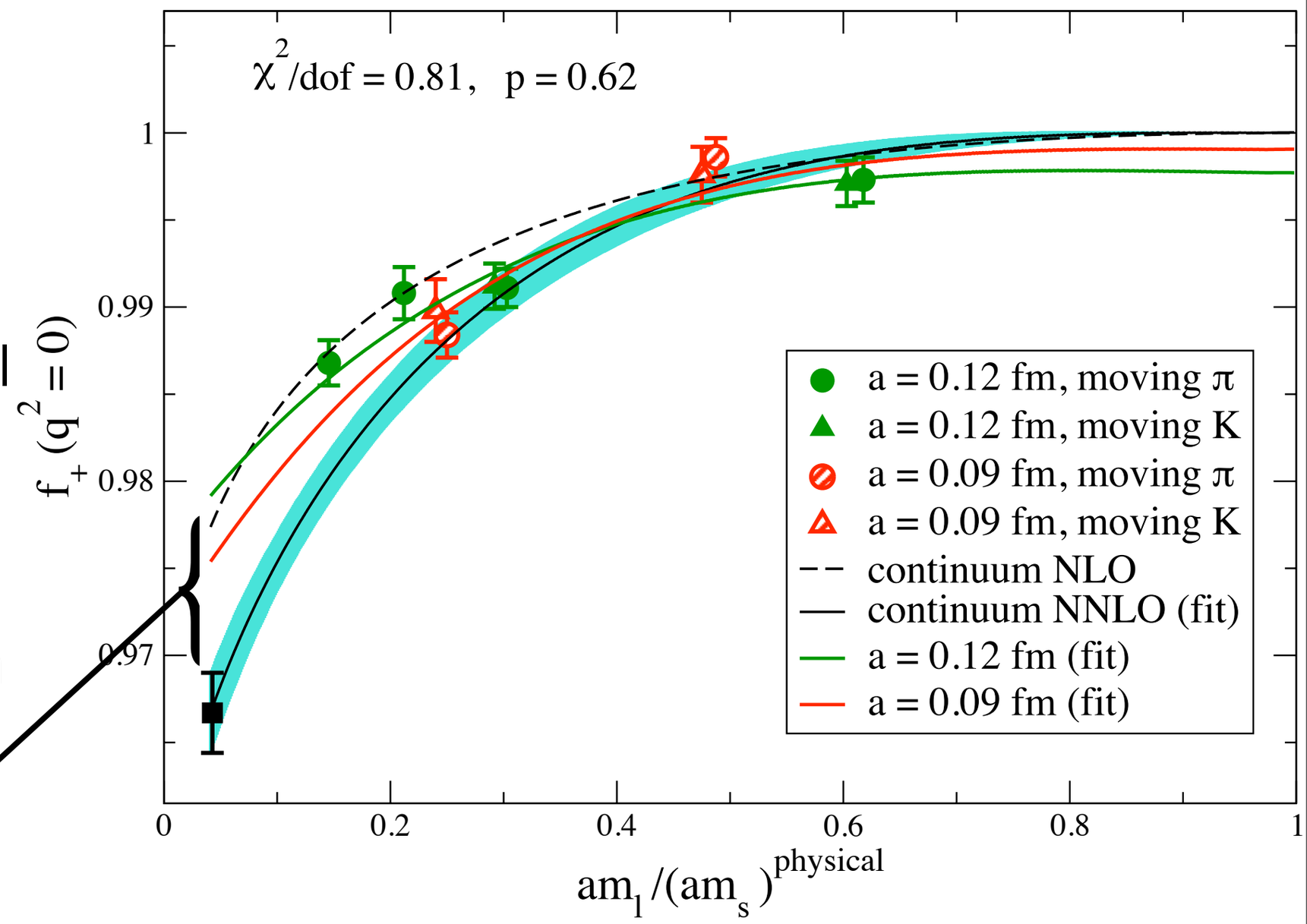
- staggered discretization effects relatively more impt. at low mass, so must understand them well.



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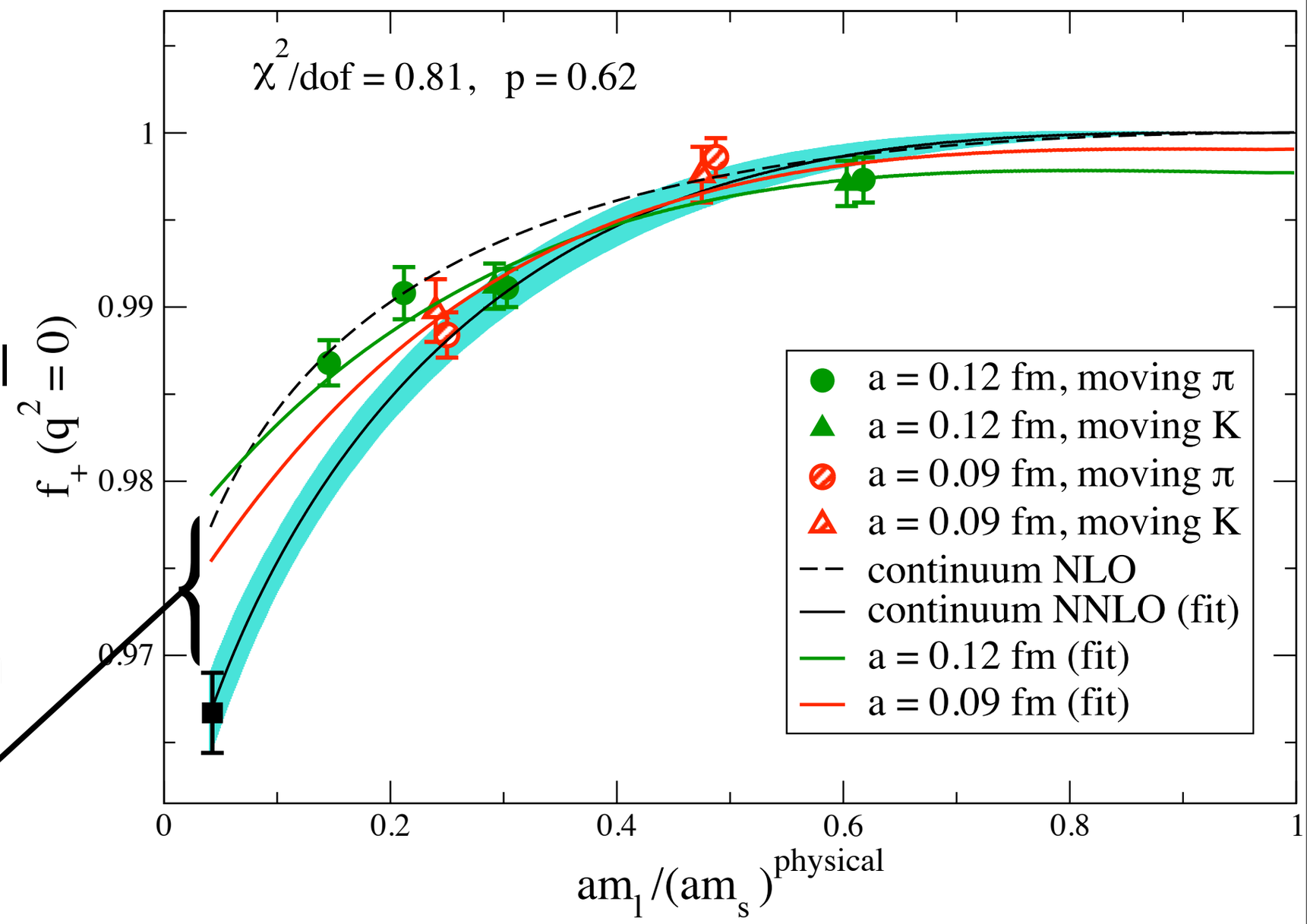
- $f_+(0)=0.9667(23)_{stat}(33)_{sys}$ $\Rightarrow$ 0.4% error



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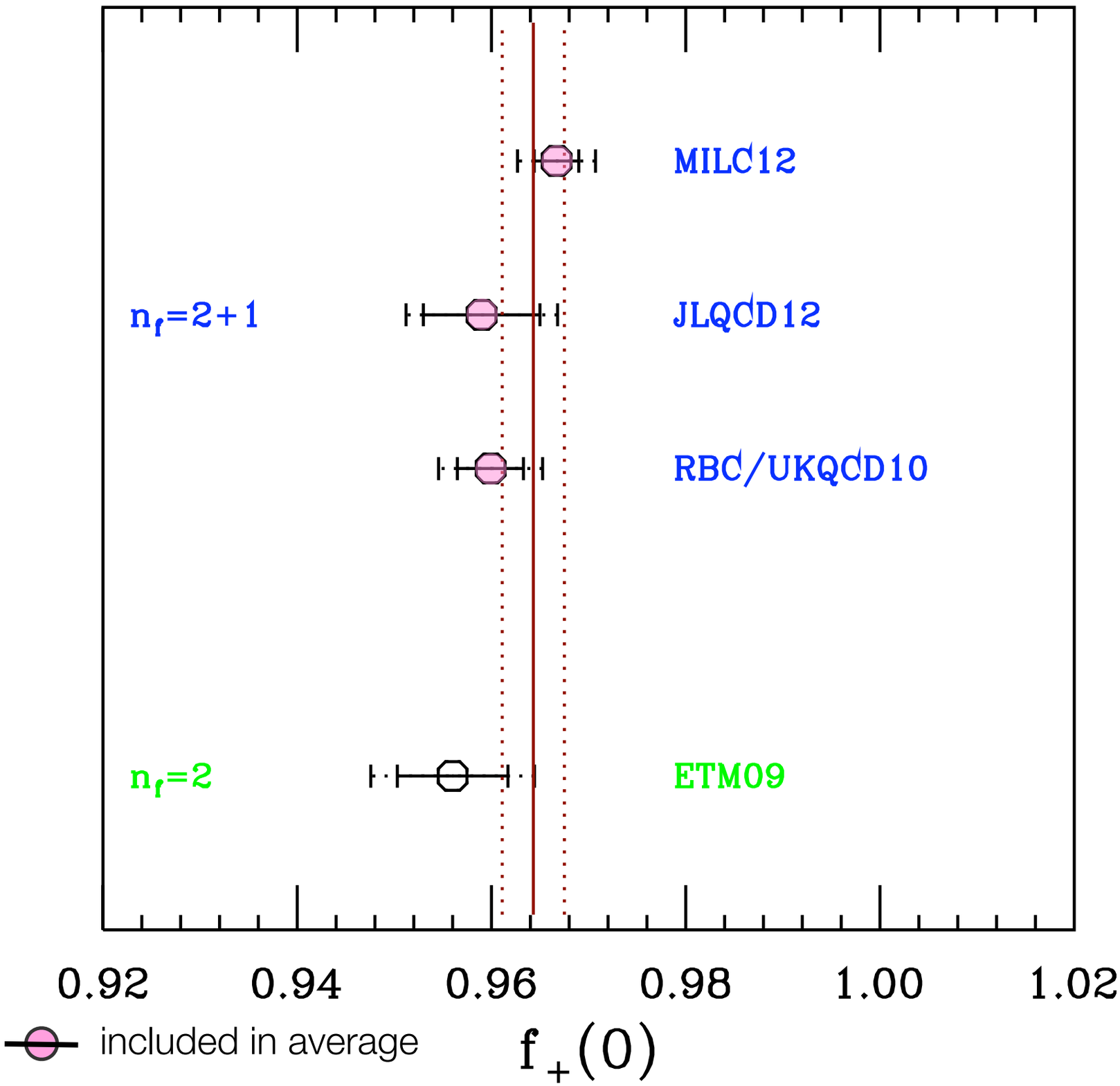
$K \to \pi \ell \nu$



- my average:

$$f_+^{K\pi}(0) = 0.9635(30)$$

- 0.3% error

V_{us} from $K \to \pi \ell \nu$

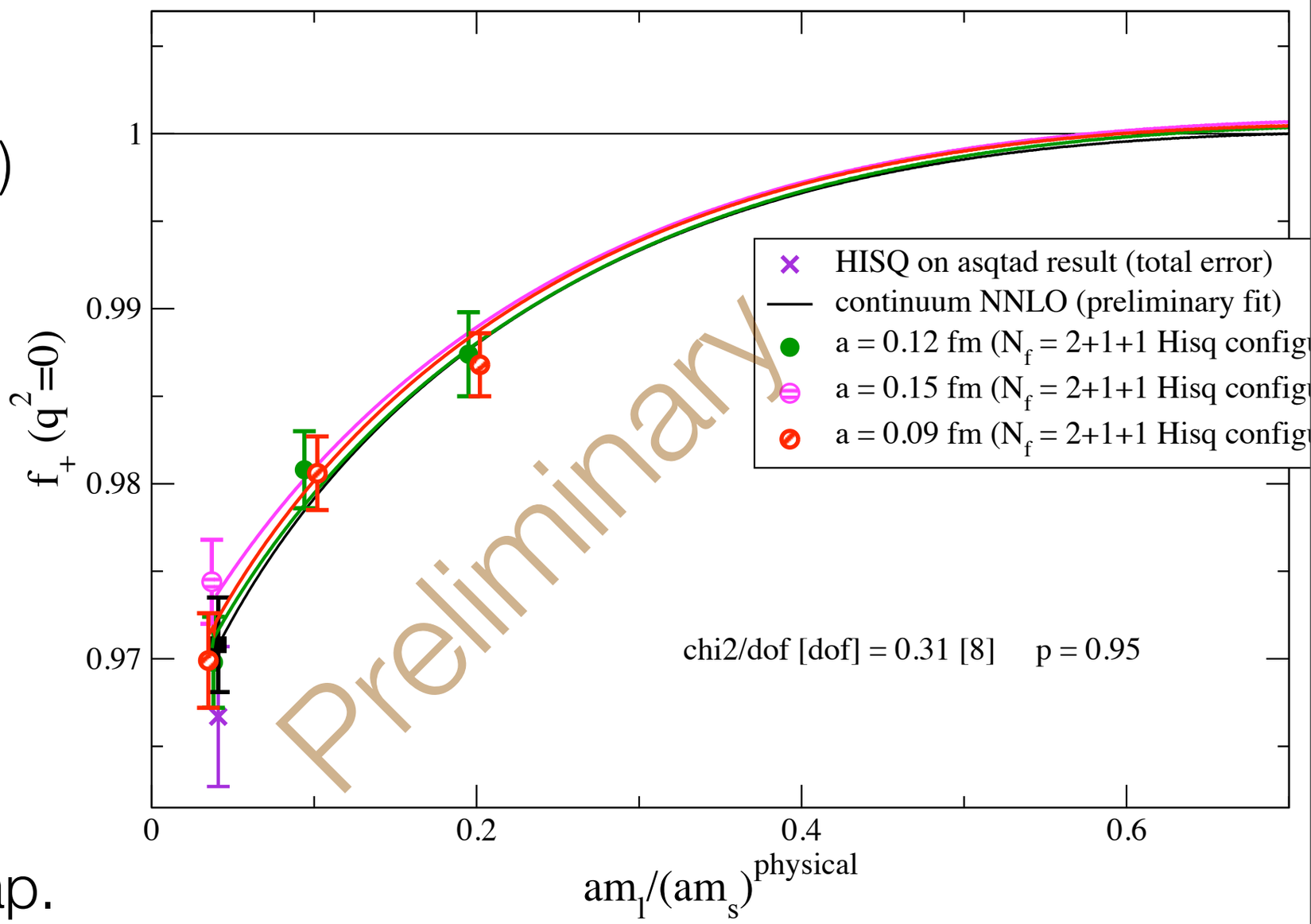
- From my lattice average plus experimental rate:

$$|V_{us}| = 0.2245(5)_{\rm expt}(7)_{\rm lat}(2)_{\rm EM}$$

- Consistent (<1 sigma) with $|V_{us}|$ from leptonic decay, with similar errors.
- EM error comes from power counting in ChPT [Cirigliano, Giannotti, Neufeld, JHEP 11 (2008) 006].
- But even if it is a bit optimistic, still some room for improvement of lattice and experiment.

$K \to \pi \ell \nu$

- Fermilab Lattice/MILC, preliminary.
- Chiral extrapolation of $f_+(0)$ for 4 lattice spacings with HISQ valence and sea quarks.
- Includes some physical-mass ensembles.
- Reduced discretization effects.
- ⇒ Reduced dependence on ChPT, continuum extrap.
- Good agreement with previous result.



HISQ valence quarks on HISQ sea quarks by MILC.

2+1+1 sea quark flavors.

$K \to \pi \ell \nu$

- ✦ Improvement in $f_+(0)$ anticipated with Fermilab/MILC HISQ on HISQ calculation (compared to previous HISQ on asqtad):
  - 0.33% systematic error (dominated by chiral extrap) → ~0.15%.
  - ~0.23% statistical error should remain.
  - overall 0.4% error → ~0.27%.
- ✦ I understand the RBC/UKQCD is making similar improvements, including points at physical light mass, to their calculations.
- ✦ This round won't push the EM errors, but the "EM wall" is waiting in the not-too-distant future.
  - Same issues as for leptonic decays make it difficult to deal with EM corrections directly on the lattice.

Neutral K Mixing: B_K

✦At lowest (2nd) order in weak interactions, K^0-$\bar{K}^0$ mixing comes from effective Hamiltonian proportional to 4-quark operator:

$$\begin{aligned}\langle \bar{K}^0 | \mathcal{H}_{\rm eff}^{\Delta S=2} | K^0 \rangle &= \frac{G_F^2 M_{\rm W}^2}{16\pi^2} \left[\lambda_c^2 S_0(x_c)\eta_{cc} + \lambda_t^2 S_0(x_t)\eta_{tt} + 2\lambda_c\lambda_t S_0(x_c,x_t)\eta_{ct} \right] \\ &\qquad \times c(\mu) \langle \bar{K}^0 | Q^{\Delta S=2}(\mu) | K^0 \rangle + \text{h.c.} \ , \\ Q^{\Delta S=2} &= \left[\bar{s}\gamma_\nu(1-\gamma_5)d \right] \left[\bar{s}\gamma_\nu(1-\gamma_5)d \right] \ .\end{aligned}$$

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Matrix element of 4-quark operator at scale **μ**; to be determined on lattice.

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$$Q^{\Delta S=2} = [\bar{s}\gamma_\nu(1-\gamma_5)d]\,[\bar{s}\gamma_\nu(1-\gamma_5)d] \; .$$

Matrix element of 4-quark operator at scale **μ**; to be determined on lattice.

scale dependence from OPE and RG; product with matrix element is **μ**-independent.

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coefficients from OPE, calculated in pert. theory; large source of error.

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- Getting from $\langle \bar{K}^0 | \mathcal{H}_{\rm eff}^{\Delta S=2} | K^0 \rangle$ to ϵ_K (CP violation in K^0-$\bar{K}^0$ mixing):

$$\epsilon_K \;=\; \exp(i\phi_\epsilon)\;\sin(\phi_\epsilon)\;\left[\frac{\mathrm{Im}[\langle \bar{K}^0 | \mathcal{H}_{\rm eff}^{\Delta S=2} | K^0 \rangle]}{\Delta M_K} \;+\; \rho\frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\right]$$

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Measured K_L-K_S mass difference

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Long distance contribution from absorptive part, $\mathrm{Im}(\Gamma_{12})$.
[Buras & Guadagnoli, arXiv:0805.3887]

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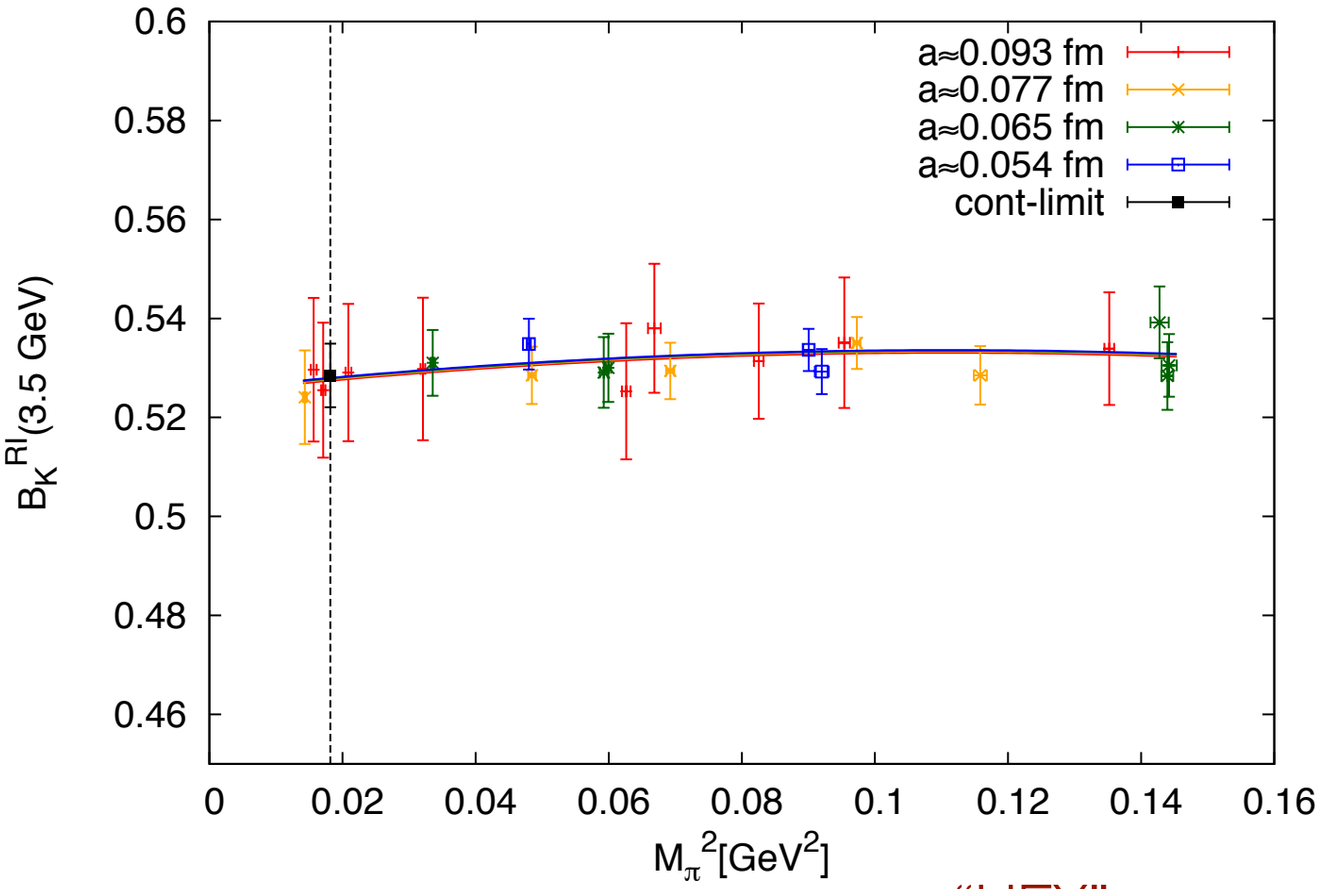
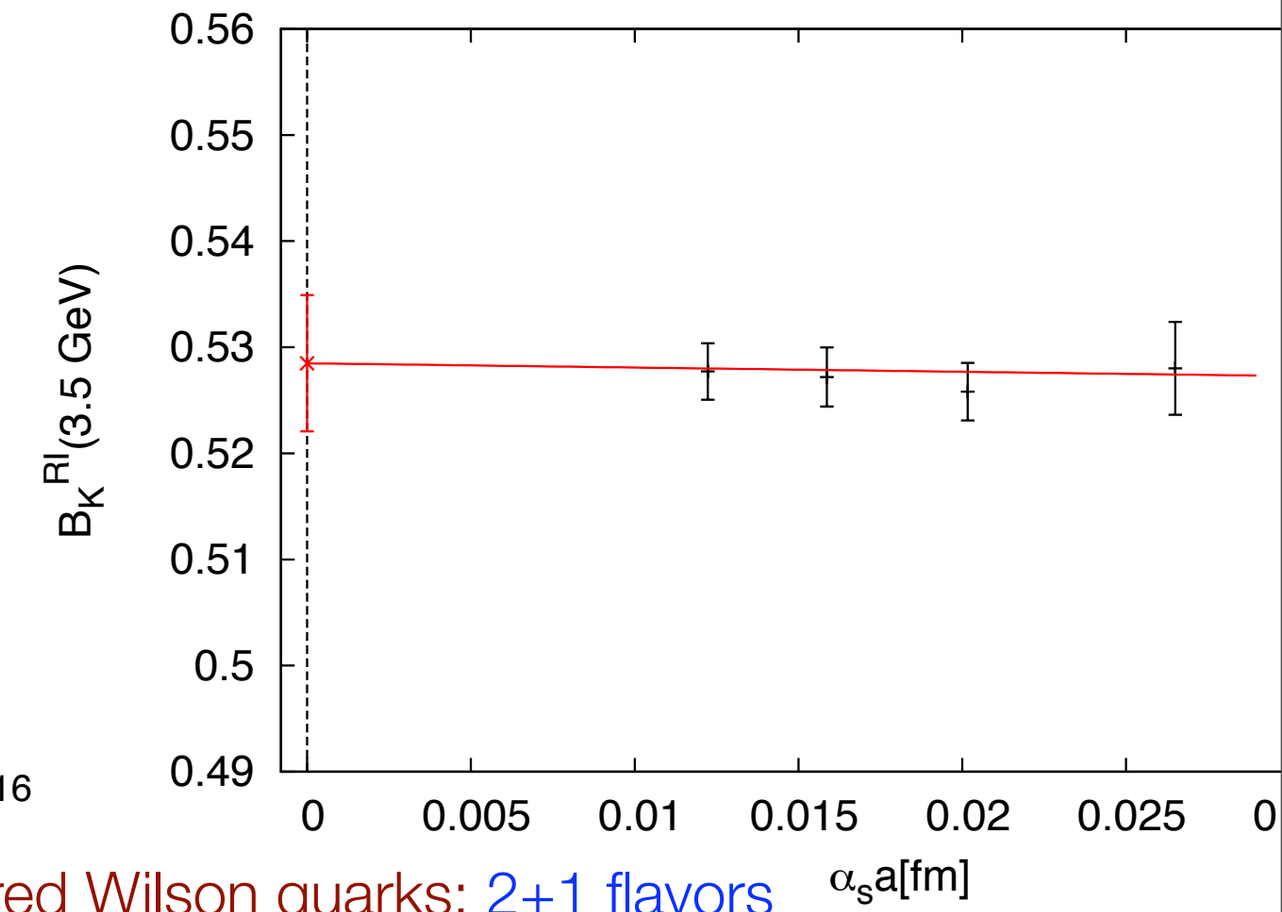
Correction to long distance contribution from dispersive part, Im(M_{12}).
[Buras, Guadagnoli, Isidori, arXiv:1002.3612]

Long distance contribution from absorptive part, Im(Γ_{12}).
[Buras & Guadagnoli, arXiv:0805.3887]

Neutral K Mixing: B_K

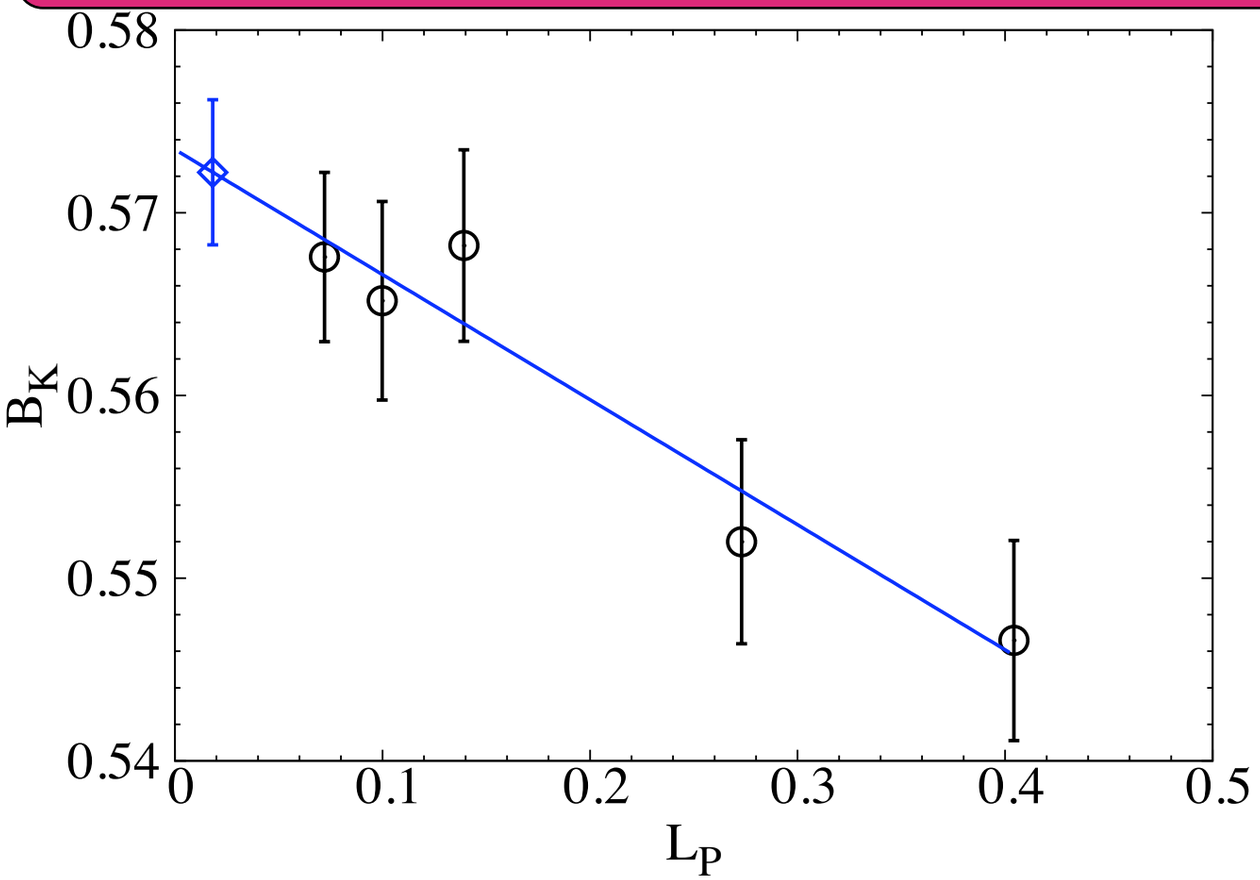


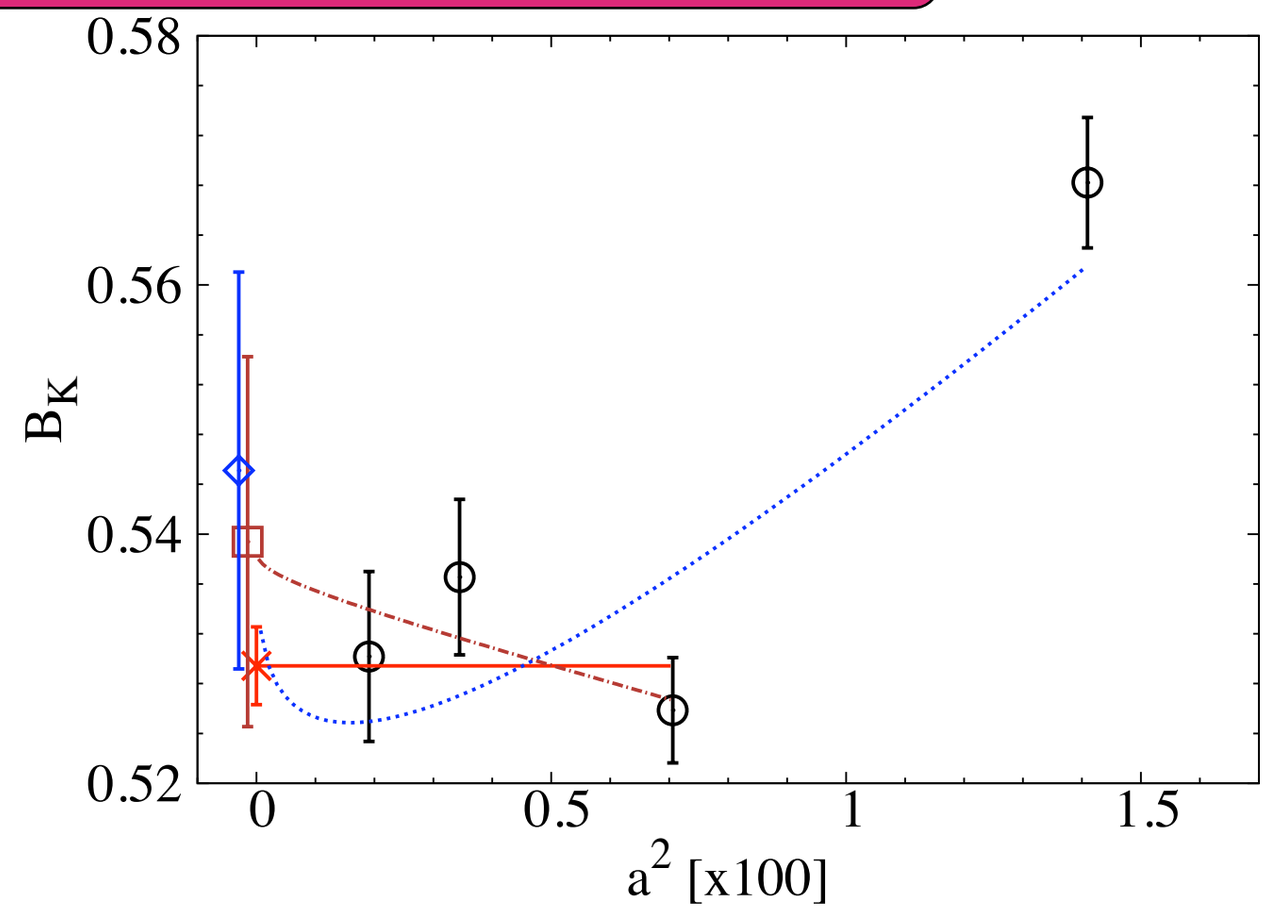


"HEX" smeared Wilson quarks; 2+1 flavors

- BMW, arXiv:1106.3230.
- Left: chiral extrapolation in light quark mass, *i.e.*, $(M_\pi)^2$.
  - mild mass dependence; includes physical mass ensembles.
- Right: continuum extrapolation.
  - very flat; well controlled.
- $\hat{B}_K = 0.7727\,(81)_{\rm stat}\,(34)_{\rm sys}\,(77)_{\rm PT} \Rightarrow 1.5\%$ error

Neutral K Mixing: B_K

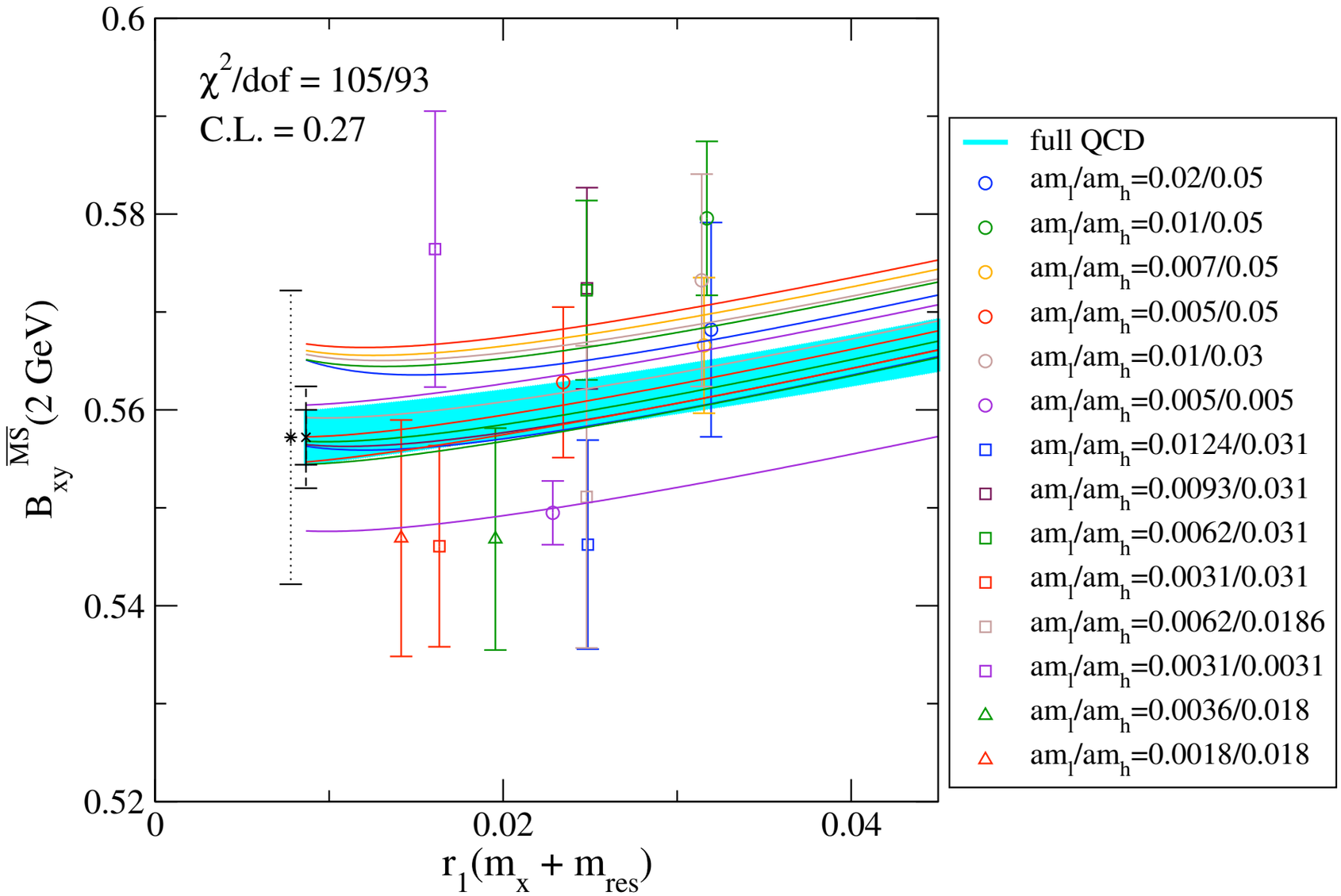




"HYP" smeared staggered valence quarks on MILC asqtad staggered sea; 2+1 flavors

- SWME, arXiv:1111.5698.
- Left: chiral extrapolation; L_P = (mass of sea pion)^2.
  - mild mass dependence.
- Right: continuum extrapolation, with various extrapolation methods.
- Error dominated by error of matching to continuum: use 1-loop PT at present.
- $\hat{B}_K = 0.727\,(4)_{\rm stat}\,(38)_{\rm sys}$ ⇒ 5.3% error

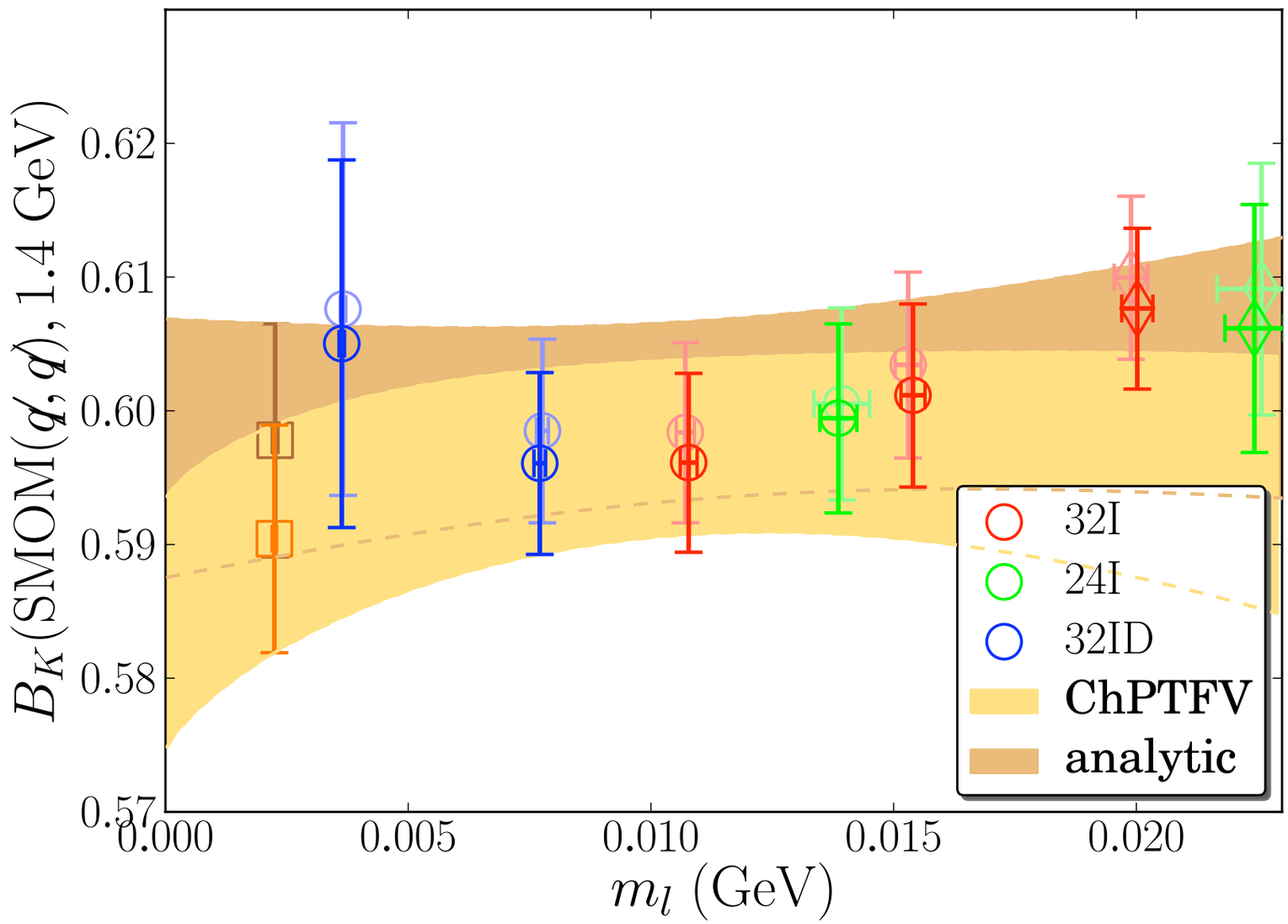
Neutral K Mixing: B_K



- Laiho/V.d.Water, arXiv:1112.4861.
  Domain wall valence quarks on MILC asqtad staggered sea; 2+1 flavors
- Chiral and continuum extrapolation for 3 lattice spacings (0.12, 0.09, 0.06 fm).
- Fit is to mixed action ChPT.

- Chiral and full errors shown.
- Dominant error is one-loop matching error (conversion of non-pert. renormalized lattice quantity to continuum scheme).
- $\hat{B}_K = 0.7628\,(38)_{\rm stat}\,(61)_{\chi\rm PT}\,(45)_{\rm FV}\,(53)_{\rm r1,mq}\,(8)_{\rm EM}\,(134)_{\rm match}$ ⇒ 2.8% error

Neutral K Mixing: B_K



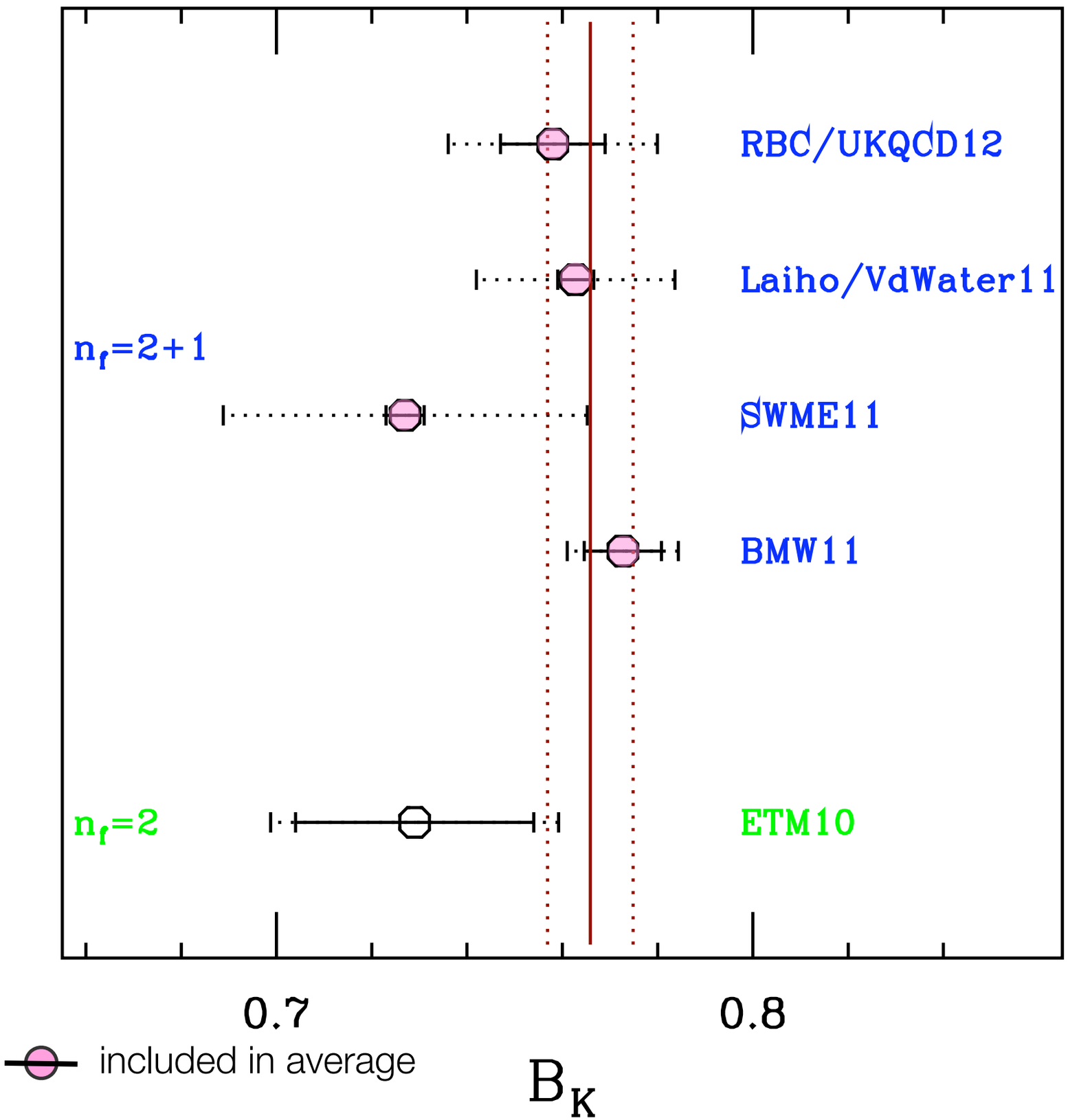
- RBC/UKQCD, arXiv: 1208.4412.

  Domain wall quarks;
  2+1 flavors

- chiral fit after correction of points to the continuum, using a global fit to 3 lattice spacings (0.14 , 0.11, 0.09 fm).

- Dominant error is one-loop matching error (conversion of non-pert. renormalized lattice quantity to continuum scheme).
- $\hat{B}_K = 0.758\,(11)_{\mathrm{stat}}\,(10)_{\chi\mathrm{PT}}\,(4)_{\mathrm{FV}}\,(16)_{\mathrm{match}} \Rightarrow 2.9\%$ error

Neutral K Mixing: B_K



- my average:

$$\hat{B}_K = 0.766(9)$$

- 1.2% error

Neutral K Mixing: B_K

- Errors in Standard Model calculation of ε_K:
  - $\hat{B}_K$ from lattice: 1.2%
  - Long distance contributions: 2.0%
    - from long distance absorptive part, $\mathrm{Im}(A_0)/\mathrm{Re}(A_0)$, only: 0.9%
    - from long distance dispersive part correction factor ρ: 1.8%
  - OPE coefficients [with NNLO perturbative calc., Brod & Gorbahn, arXiv:1108.2036]: 8.7%
    - from η_{cc} : 7.7%
    - from η_{tt} : 1.1%
    - from η_{ct} : 3.9%
  - Use of ε_K to constrain unitarity triangle is also limited by error on V_{cb} (normalizes sides): 6.2%.
    - But lattice evaluations there have lots of room for improvement.
- Experimental error on ε_K : 0.5%

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- The dispersive part is much more difficult, but RBC/UKQCD is working on it: arXiv:1212.5931. Requires active charm quark.

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- Symmetric (non-exceptional) momentum subtraction approach ("SMOM"), introduced in lattice calculations as an intermediate renormalization scheme to reduce lattice artifacts [RBC/UKQCD, arXiv:0712.1061 and Sturm *et al.*, arXiv:0901.2599] may also significantly improve perturbative coefficients [Gorbahn & Jager, arXiv: 1004.3997 and Almeida & Sturm, arXiv:1004.4613].

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- Active-charm approach [RBC/UKQCD, arXiv:1212.5931] should be better behaved in principle. But it is much harder.

Conclusions

- Standard kaon quantities have been calculated to high precision in lattice QCD:
  - f_K: ~0.25% ; $f_+^{K\pi}(0)$: ~0.3% ; $\hat{B}_K$: ~1.2%
- For leptonic decays, we are close to the "EM wall," where lattice errors in f_K are comparable to electromagnetic effects.
  - Going further will require new approaches/techniques on the lattice or in the continuum to calculate the EM effects.
- For semileptonic decays (f_+), the corresponding wall is further away, but is likely to be reached in ~3 years.
- For $\boldsymbol{\varepsilon}_K$, lattice errors from $\hat{B}_K$ already significantly smaller than those from perturbation theory and long distance effects.
  - Lattice beginning to deal with long distance effects, but it is difficult.
  - Approaches developed for the lattice calculations (SMOM, active charm) may also help reduce the perturbative errors.