Intensity Frontier Workshop, Argonne National Lab, April 25 2013

The model-discriminating power of μ-to-e conversion

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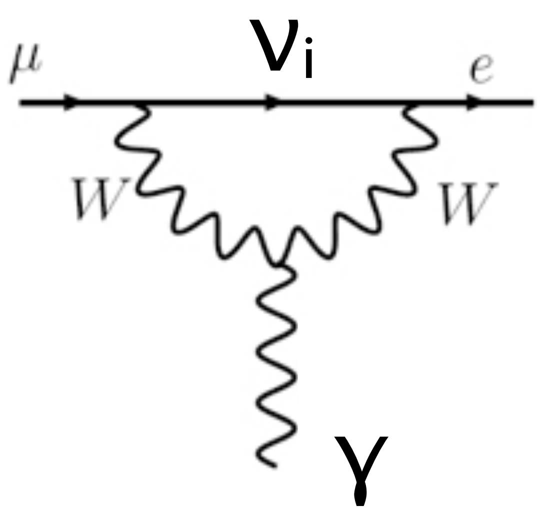
Charged LFV: general considerations

- ν oscillations imply that individual lepton family numbers are not conserved (after all $L_{e,\mu,\tau}$ are "accidental" symmetries of SM)
- In SM + massive "active" ν, CLFV rates are tiny (GIM-suppression)



$$Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi} \left| \sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M^2_W} \right|^2 < 10^{-54}$$

Petcov '77, Marciano-Sanda '77 ....

Extremely clean probe of BSM physics

Charged LFV: general considerations

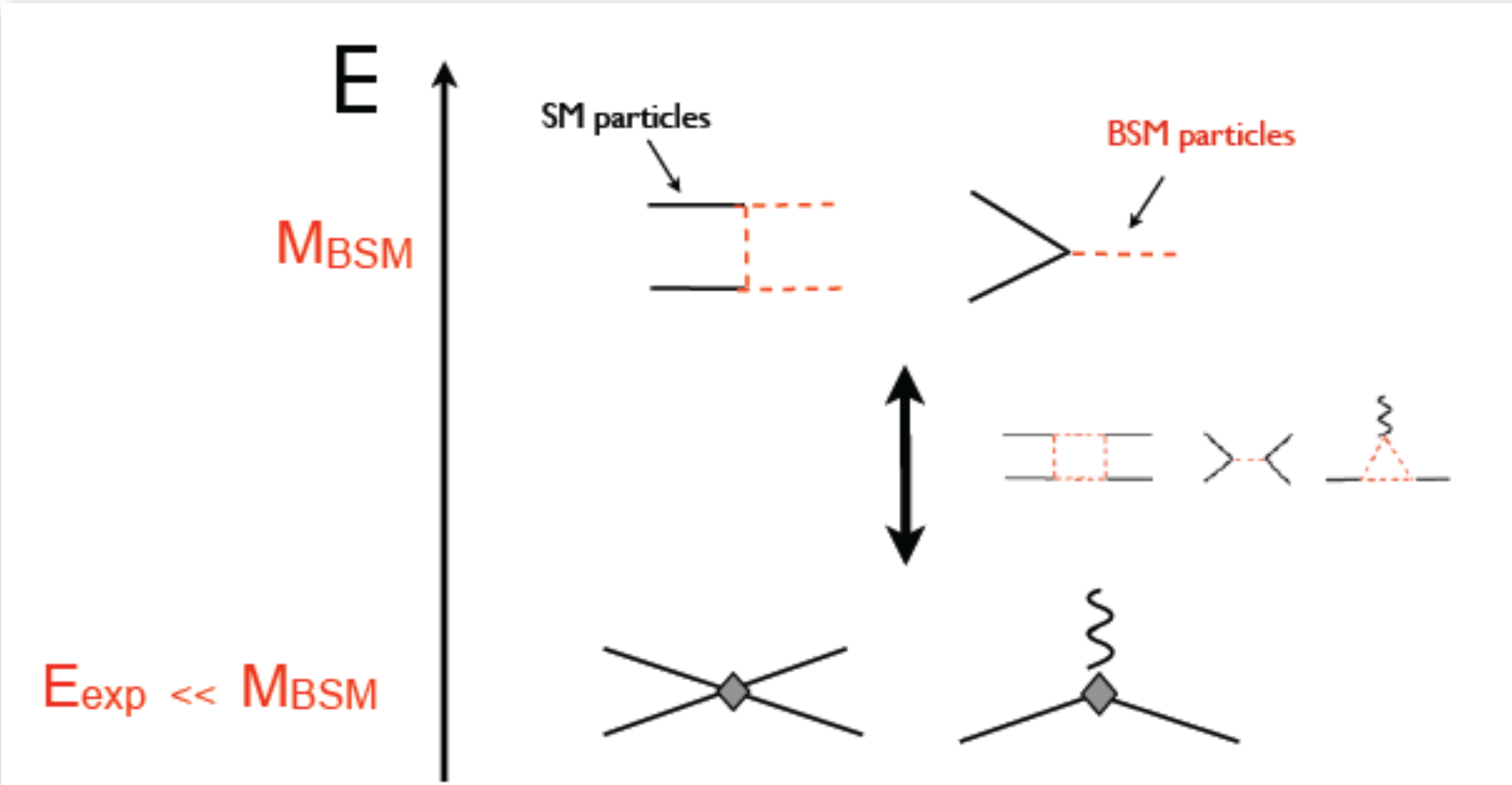
- Great "discovery" tools
  - Observation near current limits $\Rightarrow$ BSM physics
- Great "model-discriminating" tools
  - Comparing $\mu \to 3e$ vs $\mu \to e\gamma$ vs $\mu \to e$ conversion (Z) and $\mu \to e$ vs $\tau \to \mu$ vs $\tau \to e$ $\Rightarrow$

    learn about structure and flavor couplings of $\mathcal{L}_{BSM}$

In this talk I will discuss these points within an EFT framework

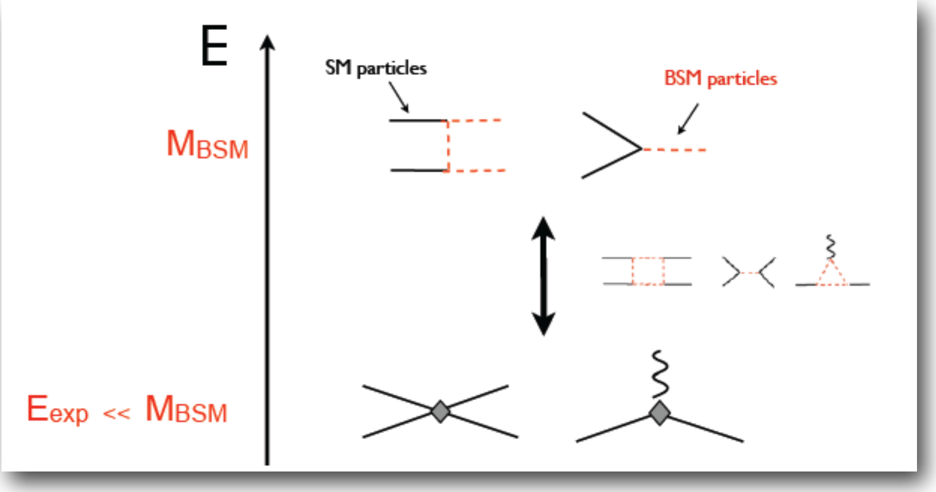
(assumption: new physics originates at a high scale)

Effective theory framework



At low energy, BSM physics is described by local operators

Effective theory framework



- Dynamics described by an effective Lagrangian

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

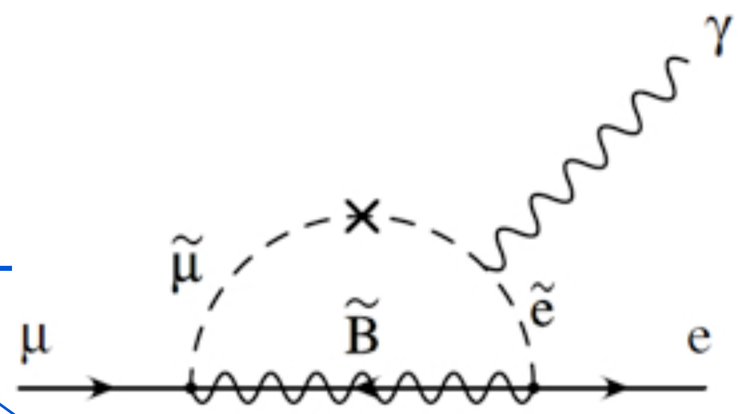
$\Lambda \leftrightarrow M_{\rm BSM}$ $\qquad$ $C_i\ [g_{\rm BSM},\ M_a/M_b]$

- Key point: each model generates its unique pattern of operators / couplings $\rightarrow$ distinctive signature in LE experiments

- LFV: probe strength of different operators and their flavor structure

- Several operators generated at dim6: rich phenomenology

Dominant in SUSY-GUT and SUSY see-saw scenarios



$$\mathcal{L}_{eff} \supset \frac{[\alpha_D]^{ij}}{\Lambda^2}\, \varphi^\dagger\, \bar{e}^i_R \sigma_{\mu\nu} \ell^j_L\, F^{\mu\nu}$$

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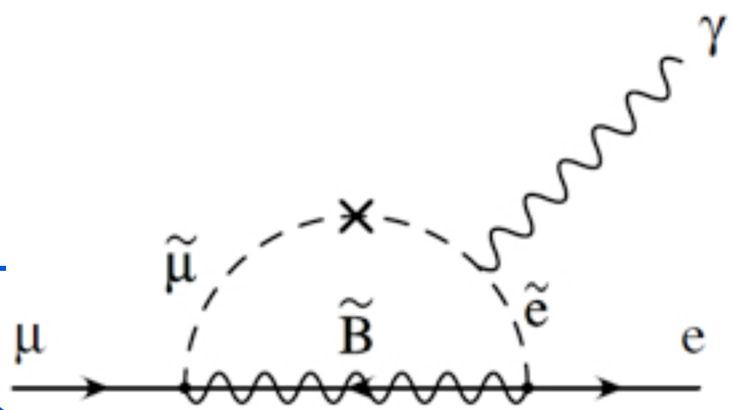
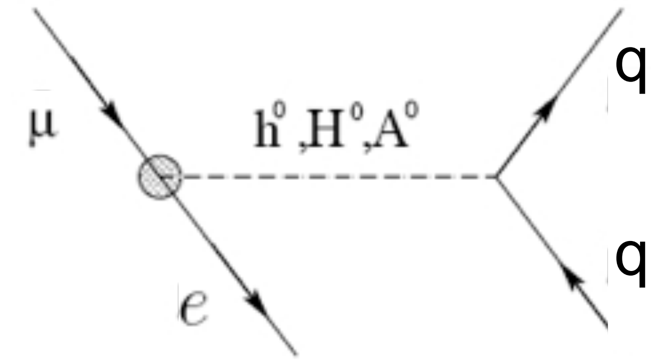


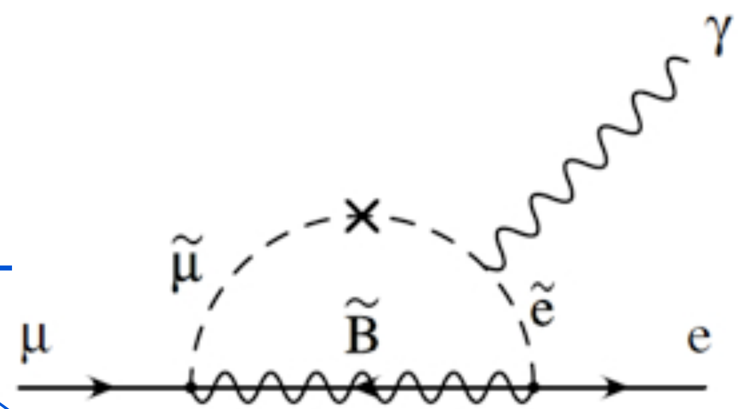
Dominant in RPV SUSY and RPC SUSY for large tan(β) and low m_A



$$\mathcal{L}_{eff} \supset \frac{[\alpha_D]^{ij}}{\Lambda^2}\, \varphi^\dagger\, \bar{e}^i_R \sigma_{\mu\nu} \ell^j_L\, F^{\mu\nu} + \frac{[\alpha_S]^{ij}}{\Lambda^2}\, \bar{e}^i_R \ell^j_L\, \bar{q}_L d_R$$

- Several operators generated at dim6: rich phenomenology

Dominant in SUSY-GUT and SUSY see-saw scenarios



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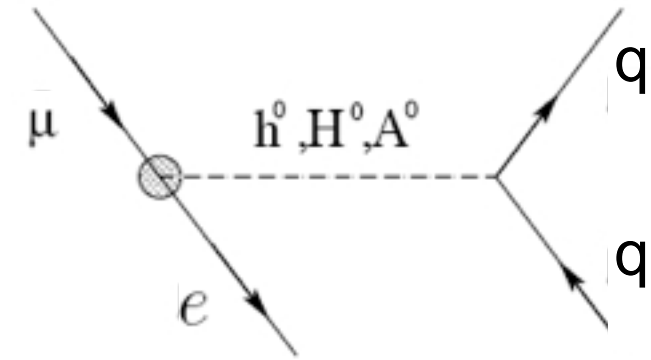

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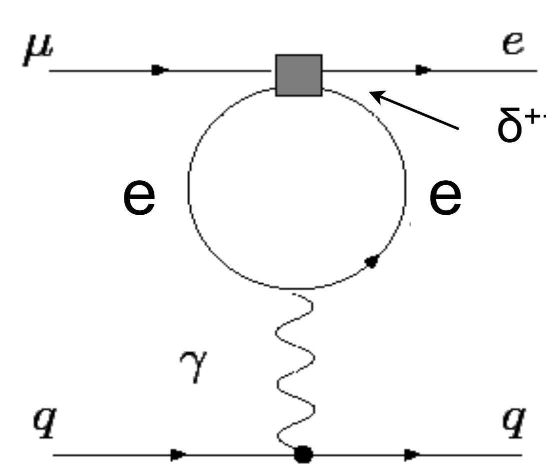
$$+ \frac{[\alpha_{V(z)}]^{ij}}{\Lambda^2} \bar{\ell}^i_L \gamma_\mu \ell^j_L \, \varphi^\dagger D^\mu \varphi + \frac{[\alpha_{V(\gamma)}]^{ij} e_q}{\Lambda^2} \bar{\ell}^i_L \gamma_\mu \ell^j_L \, \bar{q}_L \gamma^\mu q_L + \ldots$$

Z-penguin

Enhanced in triplet models, Left-Right symmetric models



... + 4-lepton operators

- EFT framework: ask questions on LFV dynamics without choosing a specific model (answers will help discriminating among models)
  - What is the sensitivity to the effective scale Λ? What is the relative sensitivity of various processes?
  - What is relative the strength of various operators (α_D vs α_S ...)? What experiments are needed to disentangle this?
  - What is the flavor structure of the couplings ($[\alpha_D]^{e\mu}$ vs $[\alpha_D]^{\tau\mu}$...)? How can we probe it? How does it relate to neutrino mixing?

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in this talk

Sensitivity to NP scale

- What combination of scale Λ + couplings produces observable rates?

$$BR_{\alpha\to\beta} \sim (v_{EW}/\Lambda)^4 * (\alpha_n)_{\alpha\beta}{}^2$$

Observable CLFV @ $10^{-1?}$ ⇔ new physics between weak and GUT scale

- Current limit from μ →eγ implies

$$\Lambda/\sqrt{[\alpha_D]^{e\mu}} > 2 \times 10^4\,\mathrm{TeV}$$

↓ even after taking into account loop factors

New physics at TeV scale (and reasonable mixing pattern) ⇒

LFV signals are within reach of planned searches

Sensitivity to NP scale

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Observable CLFV @ $10^{-1?}$ $\Leftrightarrow$ new physics between weak and GUT scale

- Current limit from $\mu \to e\gamma$ implies

$$\Lambda/\sqrt{[\alpha_D]^{e\mu}} > 2 \times 10^4\ \mathrm{TeV}$$

- What about other processes? Relative sensitivity depends on the model: each process probes a different combination of operators

(related to model-discriminating question)

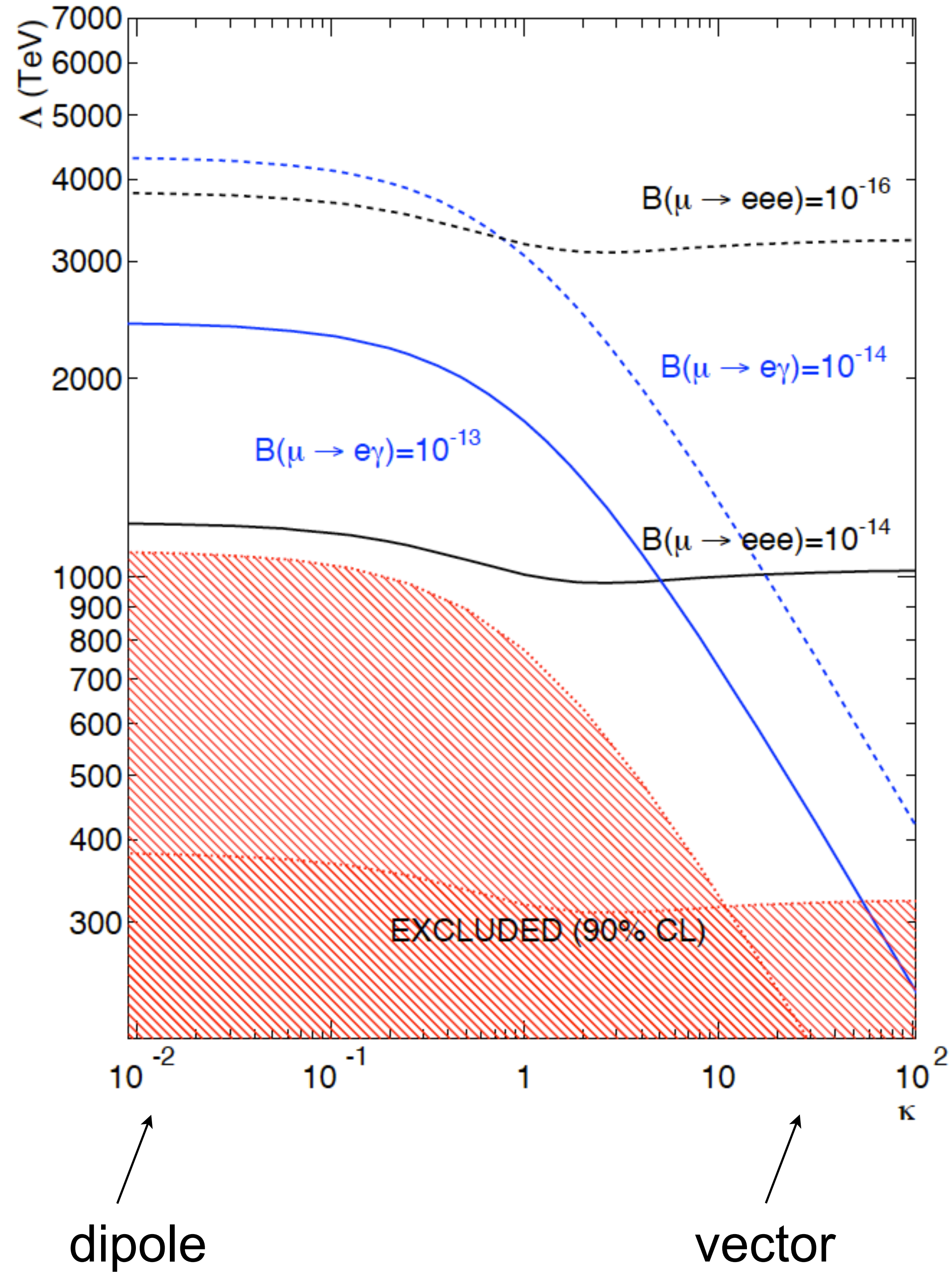
μ → eγ vs μ → 3e

- A simple example with two operators

De Gouvea, Vogel 1303.4097

$$\mathcal{L}_{\rm CLFV} = \frac{m_\mu}{(\kappa+1)\Lambda^2}\bar{\mu}_R\sigma_{\mu\nu}e_L F^{\mu\nu} + h.c.$$
$$\frac{\kappa}{(1+\kappa)\Lambda^2}\bar{\mu}_L\gamma_\mu e_L\left(\bar{e}\gamma^\mu e\right) + h.c.\,.$$

- κ controls relative strength of dipole vs vector operator



$\mu \to e\gamma$ vs $\mu \to e$ conversion

- A simple example with two operators

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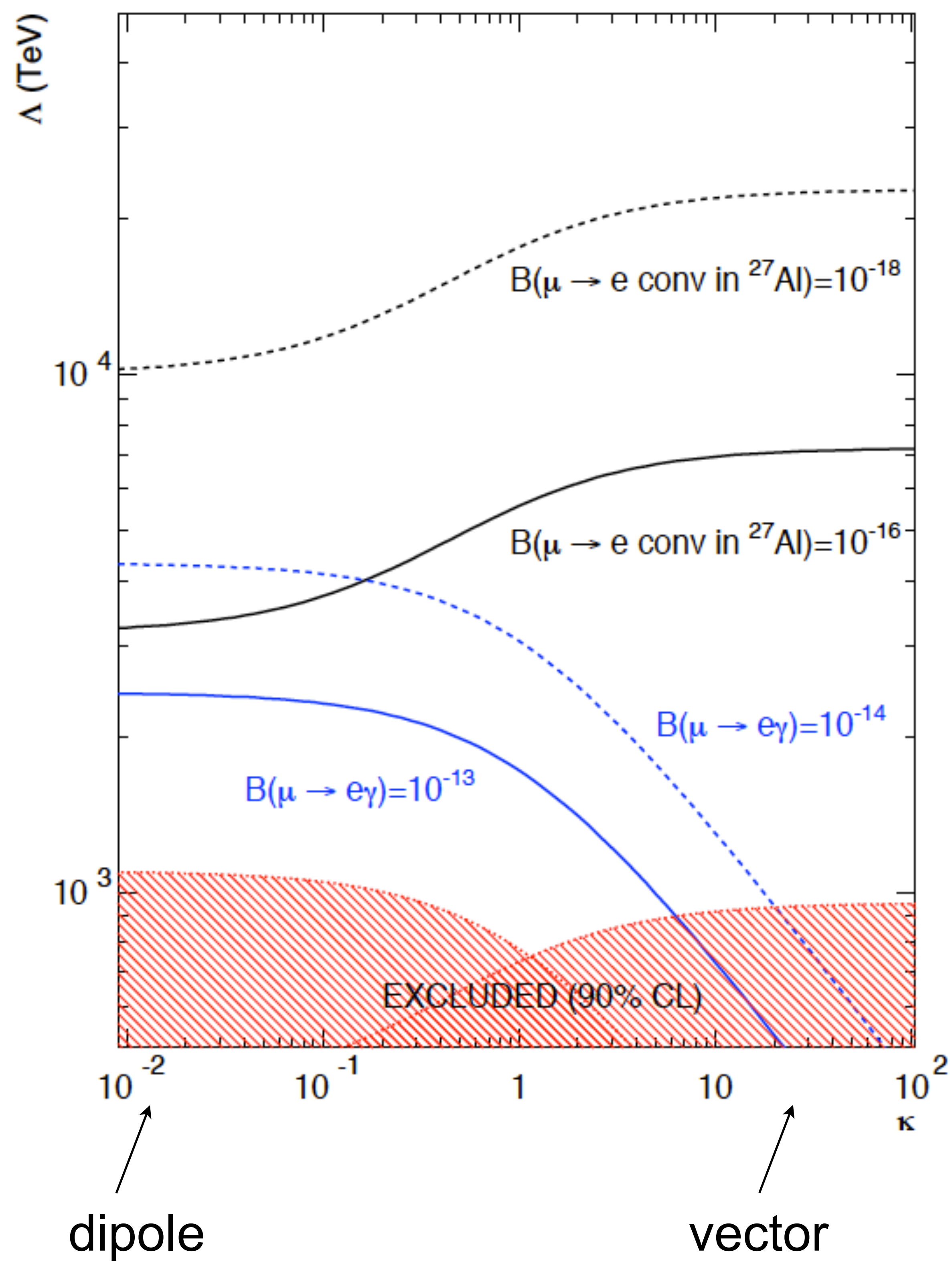
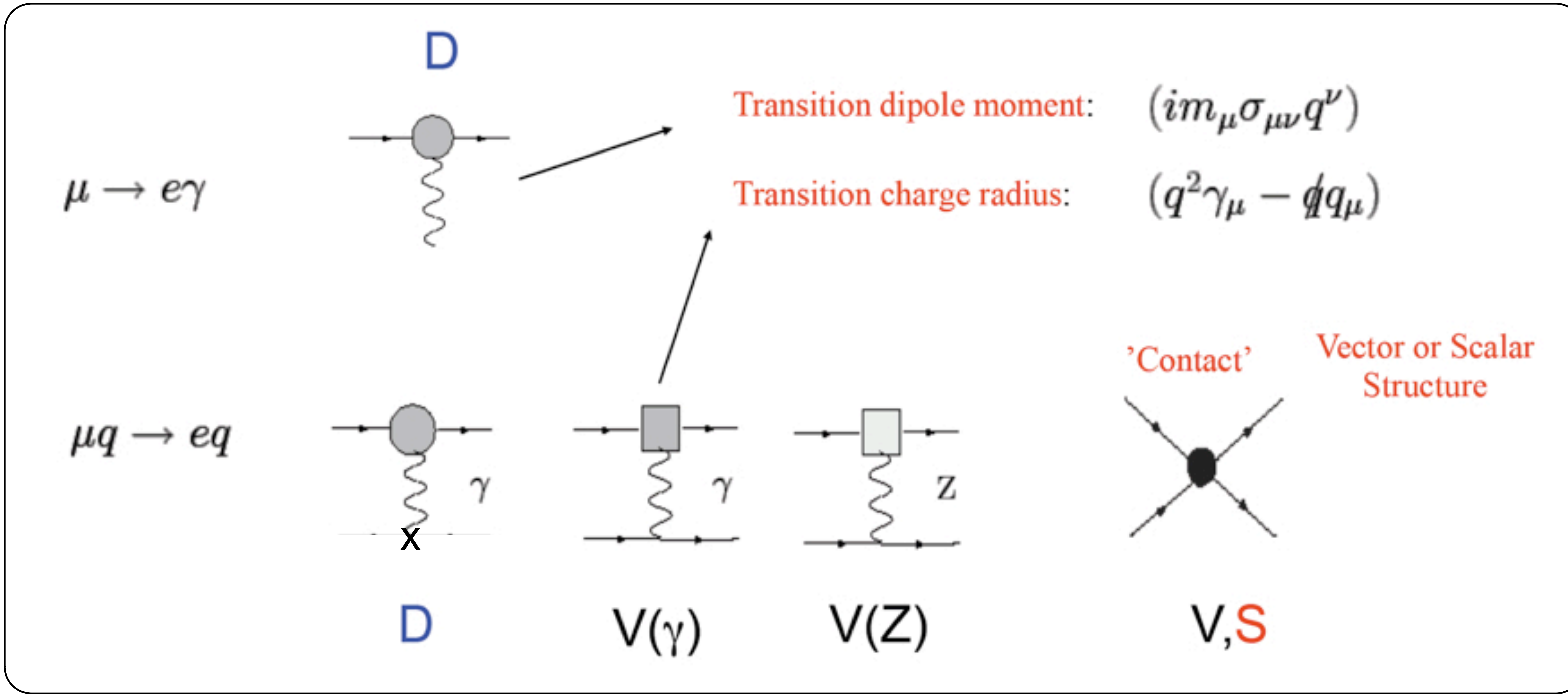


Model-discriminating power

- $\mu \to e\gamma$ and $\mu \to e$ conv. probe different combinations of operators



- By measuring the target dependence of $\mu \to e$ conversion (and ratio to $\mu \to e\gamma$ BR) we can infer the relative strength of effective operators

- How does this work? Conversion amplitude has non-trivial dependence on target nucleus, that distinguishes D,S,V underlying operators

$$M_{fi} \sim \langle e^-; A, Z| \int d^3x \, \hat{O}_\ell(x) \, \hat{O}_q(x) \, |\mu^-; A, Z\rangle$$

$$\sim \int d^3x \, \bar{\psi}_e O_\ell \psi_\mu \, \langle A, Z|\hat{O}_q|A, Z\rangle$$

Czarnecki-Marciano-Melnikov

Kitano-Koike-Okada

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Czarnecki-Marciano-Melnikov

Kitano-Koike-Okada

- Lepton wave-functions in EM field generated by nucleus
- Relativistic components of muon wave-function give different contributions to D,S,V overlap integrals. For example:

$$\bar{\psi}_e \gamma_0 \psi_\mu = \bar{\psi}_e \, \psi_\mu + O(v_\mu/c)$$

- Expect largest discrimination for heavy target nuclei

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- Sensitive to hadronic and nuclear properties

$$\langle A, Z| \, \bar{q}\Gamma q \, |A, Z\rangle$$

$$\downarrow$$

$$f^{(q)}_{\Gamma N} \langle A, Z| \, \bar{\psi}_N \Gamma \psi_N \, |A, Z\rangle$$

$$\downarrow$$

$$\langle A, Z| \, \bar{\psi}_p (\gamma_0) \psi_p \, |A, Z\rangle = Z \rho^{(p)}$$

$$\langle A, Z| \, \bar{\psi}_n (\gamma_0) \psi_n \, |A, Z\rangle = (A - Z) \rho^{(n)}$$

- Dominant sources of uncertainty:

  - Scalar matrix elements $\langle i|\, m_q q\bar{q}\, |i\rangle = \sigma_q^{(i)} \bar{\psi}_i \psi_i$

$$\sigma_{\pi N} \;=\; \frac{m_u + m_d}{2} \langle p|\bar{u}u + \bar{d}d|p\rangle \;\rightarrow\; 53\,^{+21}_{-10}\ \mathrm{MeV} \qquad (45 \pm 15)\ \mathrm{MeV}$$

ChPT JLQCD 2008 Lattice range 2012 (Kronfeld 1203.1204)

$$y = \frac{2\ \langle p|\bar{s}s|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle} \;\in\; [0, 0.4] \;\rightarrow\; [0, 0.05] \qquad [0.04, 0.12]$$

  - Neutron density (heavy nuclei)

Test hypothesis of single-operator dominance

- One unknown parameter ($[\alpha_{D,V,S}]^{e\mu}/\Lambda^2$) → predict ratios of LFV BRs

- If $\mu \to e\gamma$ and $\mu \to e$ conversion are observed, can test dipole model

D ⟷ $$\frac{B(\mu \to e, Z)}{B(\mu \to e\gamma)}$$

- In principle, any single-operator dominance model can be tested with two $\mu \to e$ conversion rates (even if $\mu \to e\gamma$ is not observed)

D,V,S ⟷ $$\frac{B(\mu \to e, Z_2)}{B(\mu \to e, Z_1)}$$

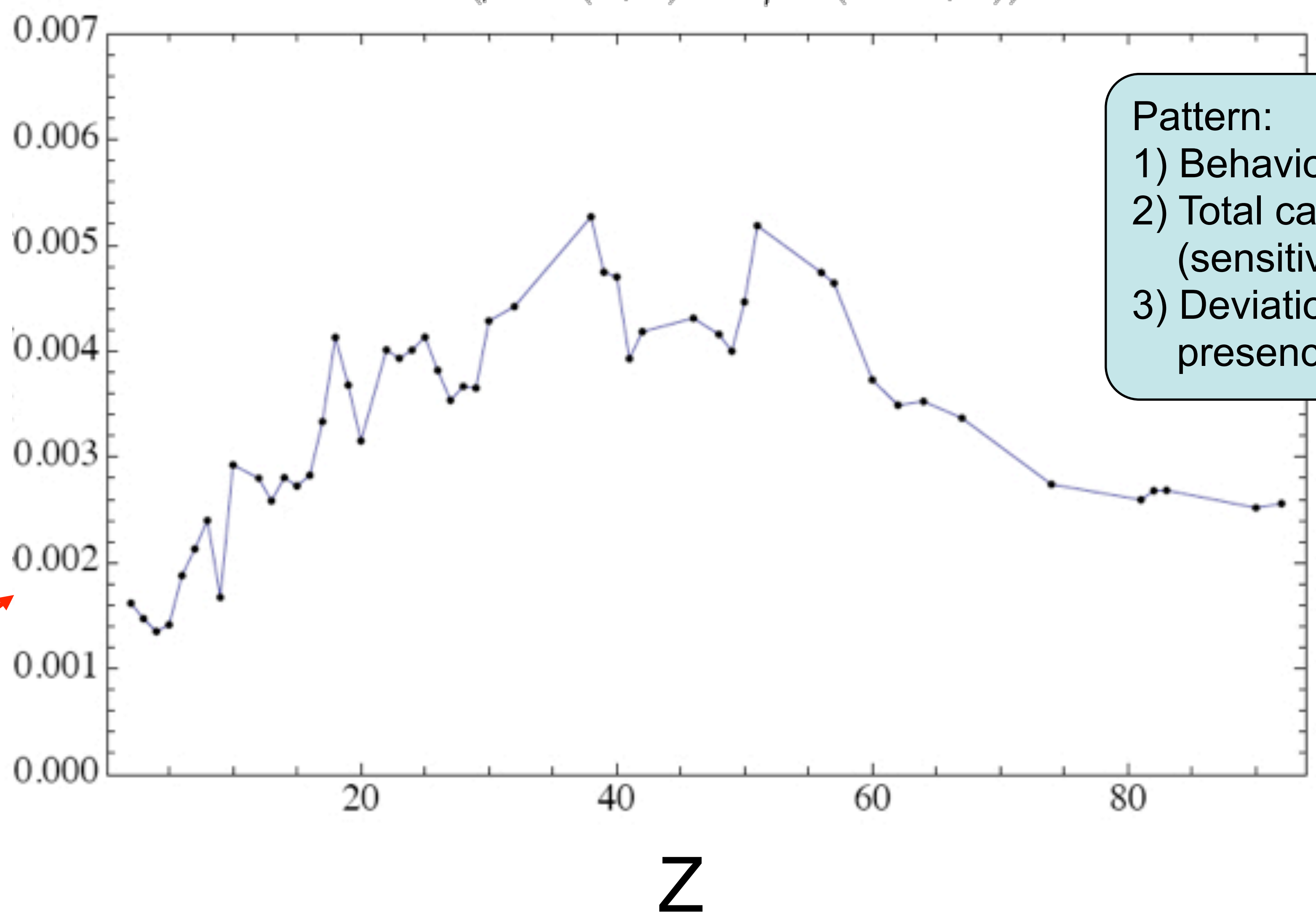
dipole vector scalar

- Test dipole-dominance model with μ→eγ and one μ→e rate

Kitano-Koike-Okada '02
VC-Kitano-Okada-Tuzon '09

$$B_{\mu\to e} = \frac{\Gamma(\mu^- + (Z,A) \to e^- + (Z,A))}{\Gamma(\mu^- + (Z,A) \to \nu_\mu + (Z-1,A))}$$

$$\frac{B(\mu \to e, Z)}{B(\mu \to e\gamma)}$$



O(α/π)

Pattern:
1) Behavior of overlap integrals**
2) Total capture rate
(sensitive to nuclear structure)
3) Deviations would indicate
presence of scalar / vector terms

** Qualitative behavior of overlap integrals

$\phi_e(x)$ → free outgoing electron wf

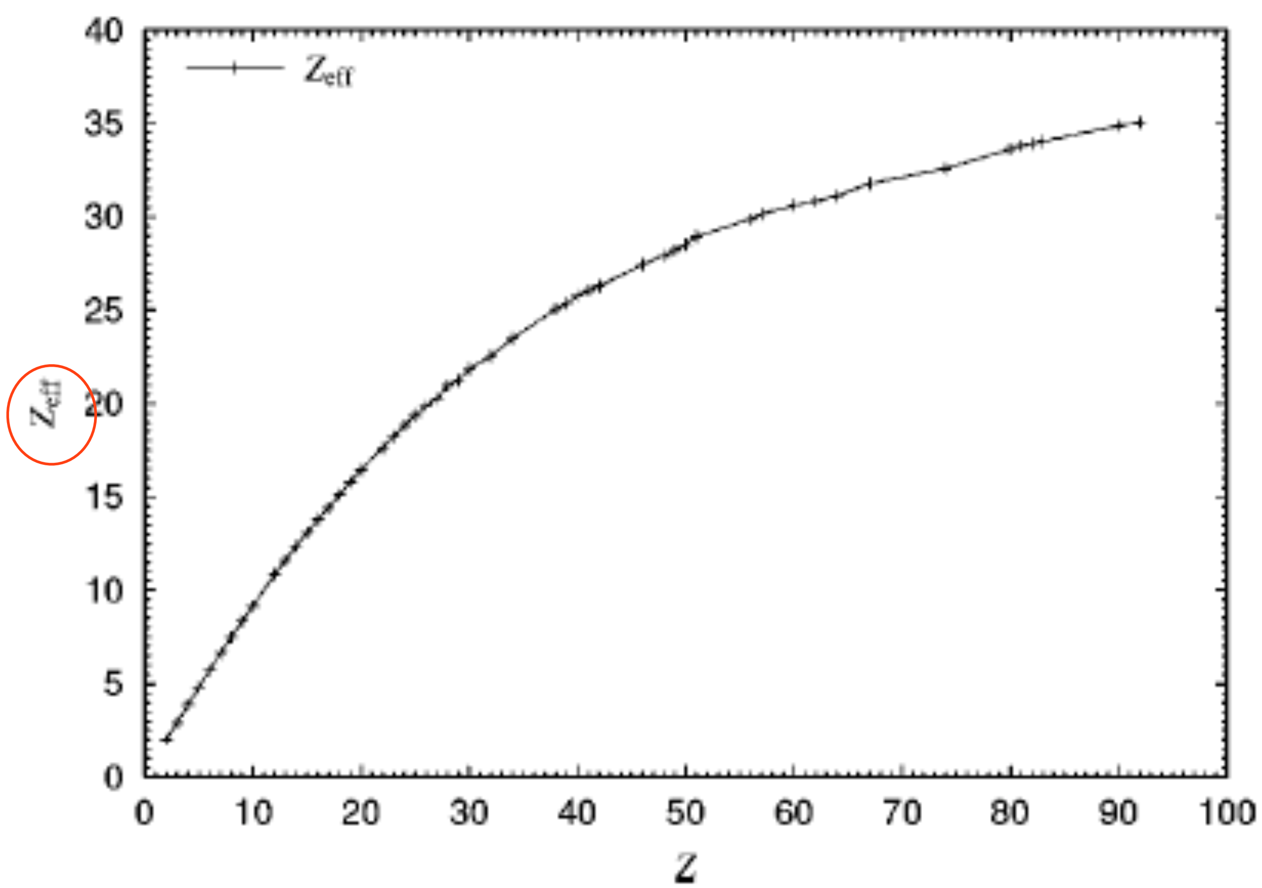
$\phi_\mu(x) \to \langle \phi_\mu(x) \rangle$ (average value)

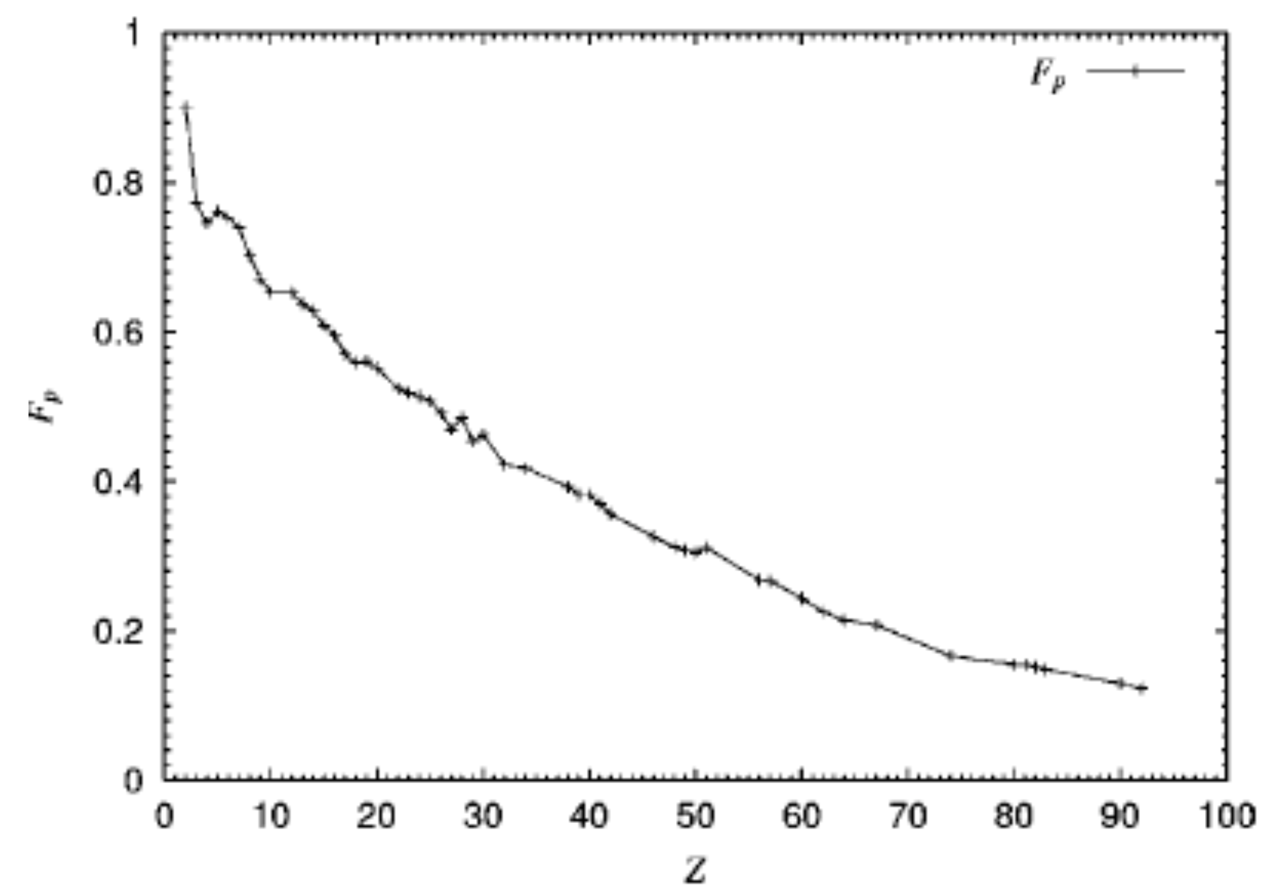
$$I \sim \int d^3x \, \phi_e^*(x) \phi_\mu(x) \, \rho_p(x) \quad \to \quad \langle \phi_\mu \rangle \, F_p$$

$p \sim m_\mu$

$$\langle \phi_\mu \rangle^2 = \int_0^\infty dr 4\pi r^2 (g_\mu^2 + f_\mu^2) \rho^{(p)} = \frac{4 m_\mu^3 \alpha^3 Z_{\text{eff}}^4}{Z}$$

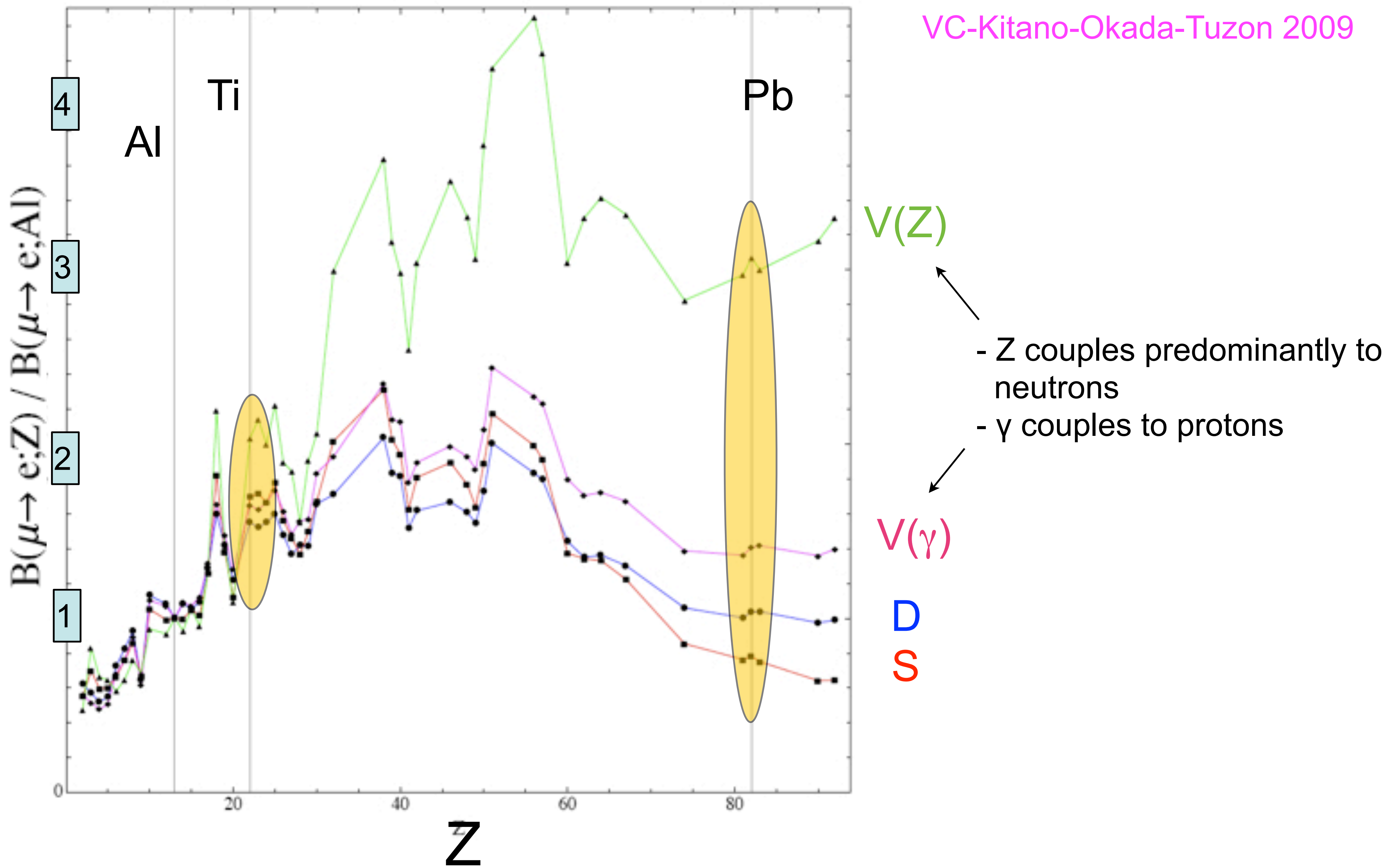
$$F_p = \int_0^\infty dr 4\pi r^2 \rho^{(p)} \frac{\sin m_\mu r}{m_\mu r}$$





Kitano-Koike-Okada

- Test any single-operator model via target-dependence of μ→e rate



- Essentially free of theory uncertainty (largely cancels in ratios)
- Discrimination: need ~5% measure of Ti/Al or ~20% measure of Pb/Al
- Ideal world: use Al and a large Z-target (D,V,S have largest separation): challenge for experiments

Test "two-operator" models

- If "single-operator" dominance hypothesis fails, consider next simplest case: two-operator dominance (DV, DS, SV)

- Unknown parameters: $[\alpha_1]^{e\mu}/\Lambda^2$, $[\alpha_2]^{e\mu}/\Lambda^2$

- Hypothesis can be tested with two double ratios (three LFV measurements!!). For example:

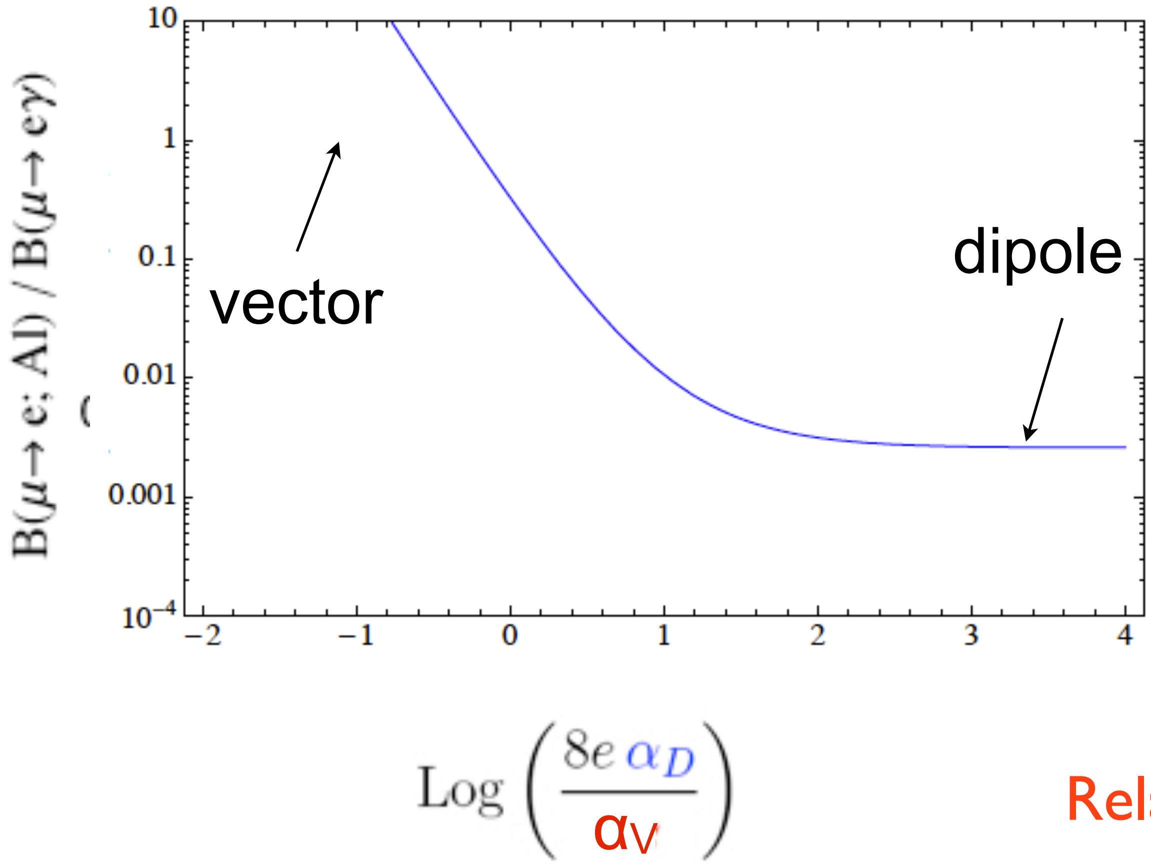
DV, DS $\longleftrightarrow$ $\dfrac{B(\mu \to e, Al)}{B(\mu \to e\gamma)}$ $\dfrac{B(\mu \to e, Pb)}{B(\mu \to e, Al)}$

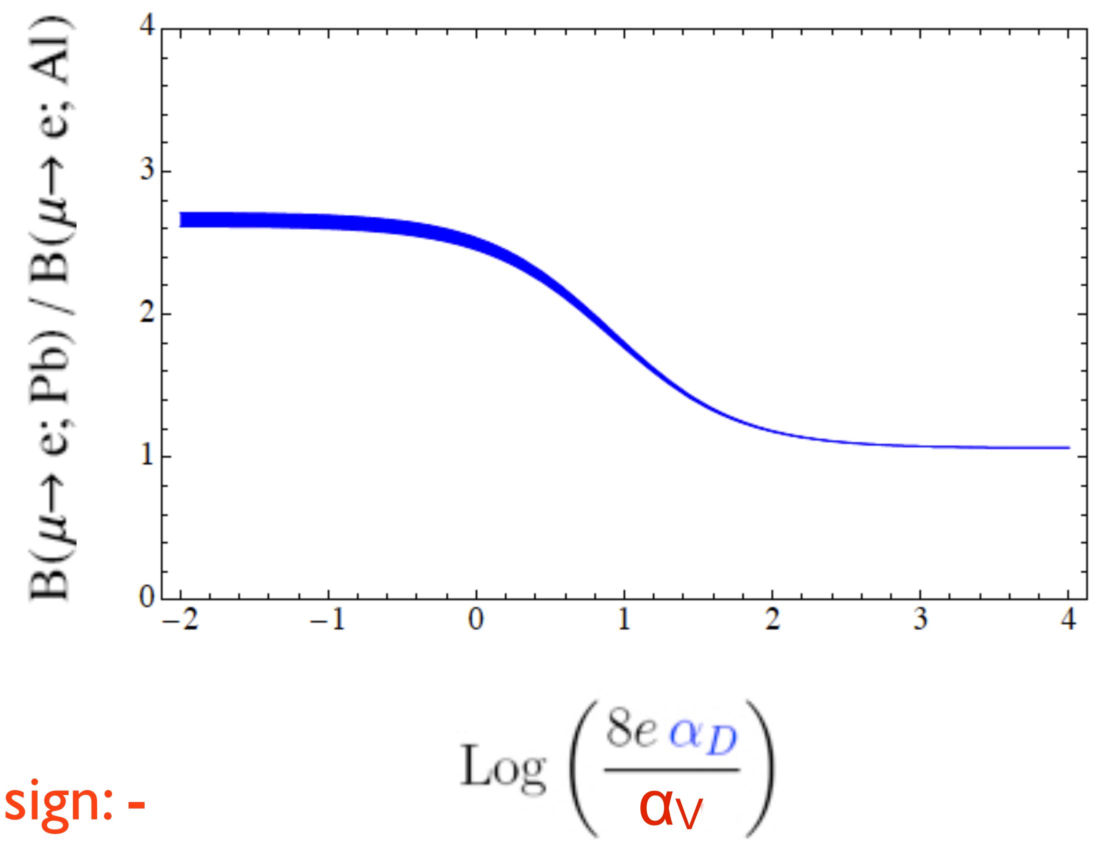
SV $\longleftrightarrow$ $\dfrac{B(\mu \to e, Ti)}{B(\mu \to e, Al)}$ $\dfrac{B(\mu \to e, Pb)}{B(\mu \to e, Al)}$

- Consider V and D

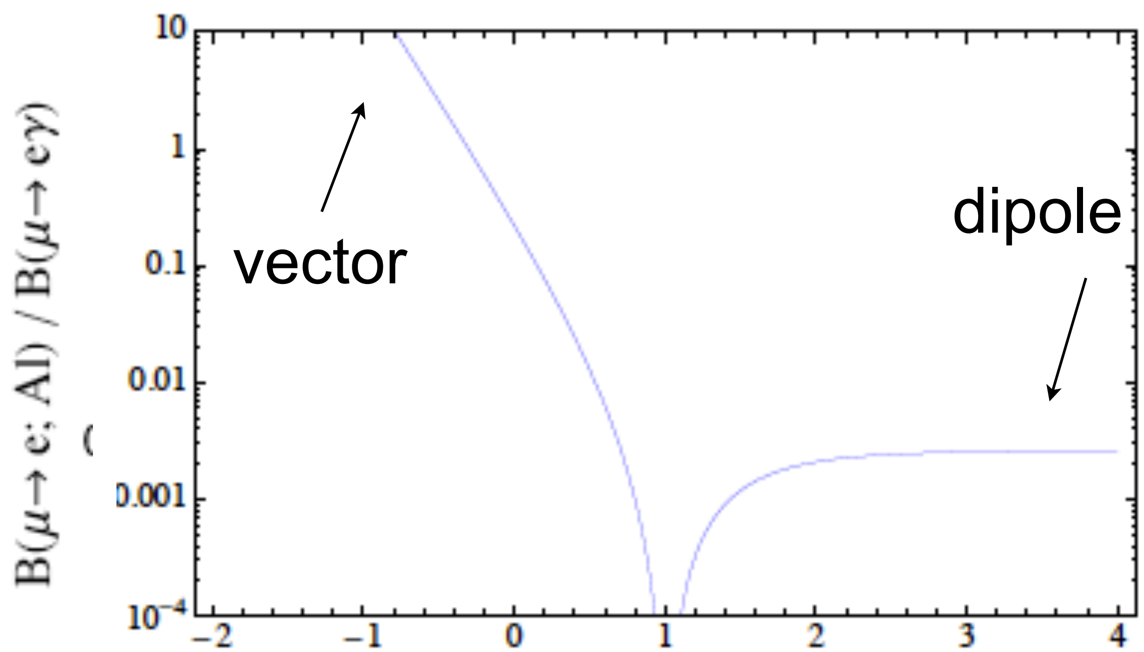
VC-Kitano-Okada-Tuzon 2009

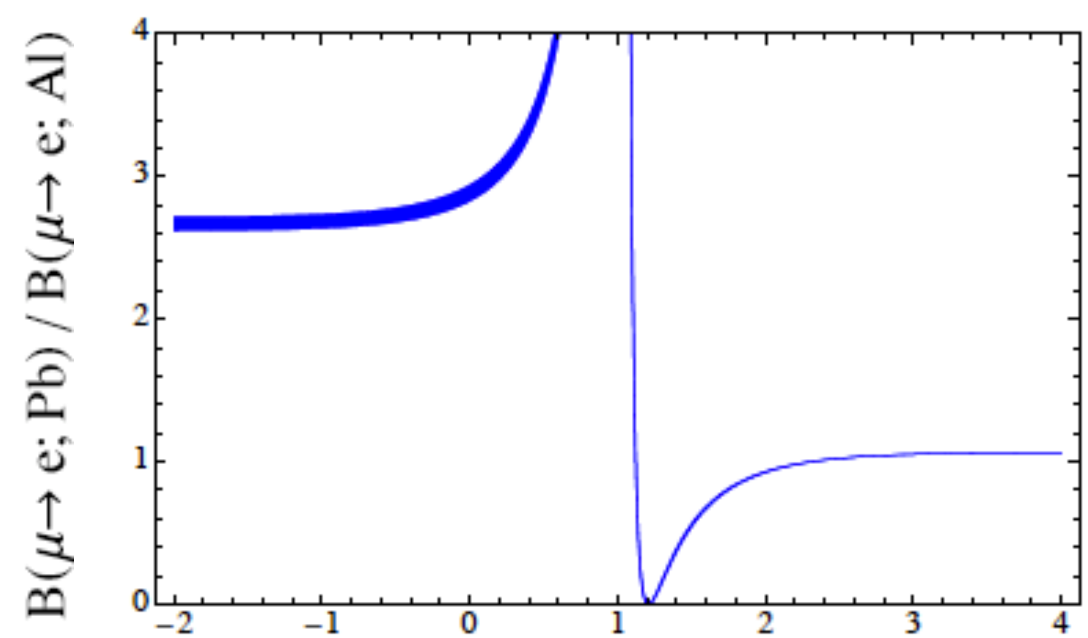
Relative sign: +





Relative sign: -

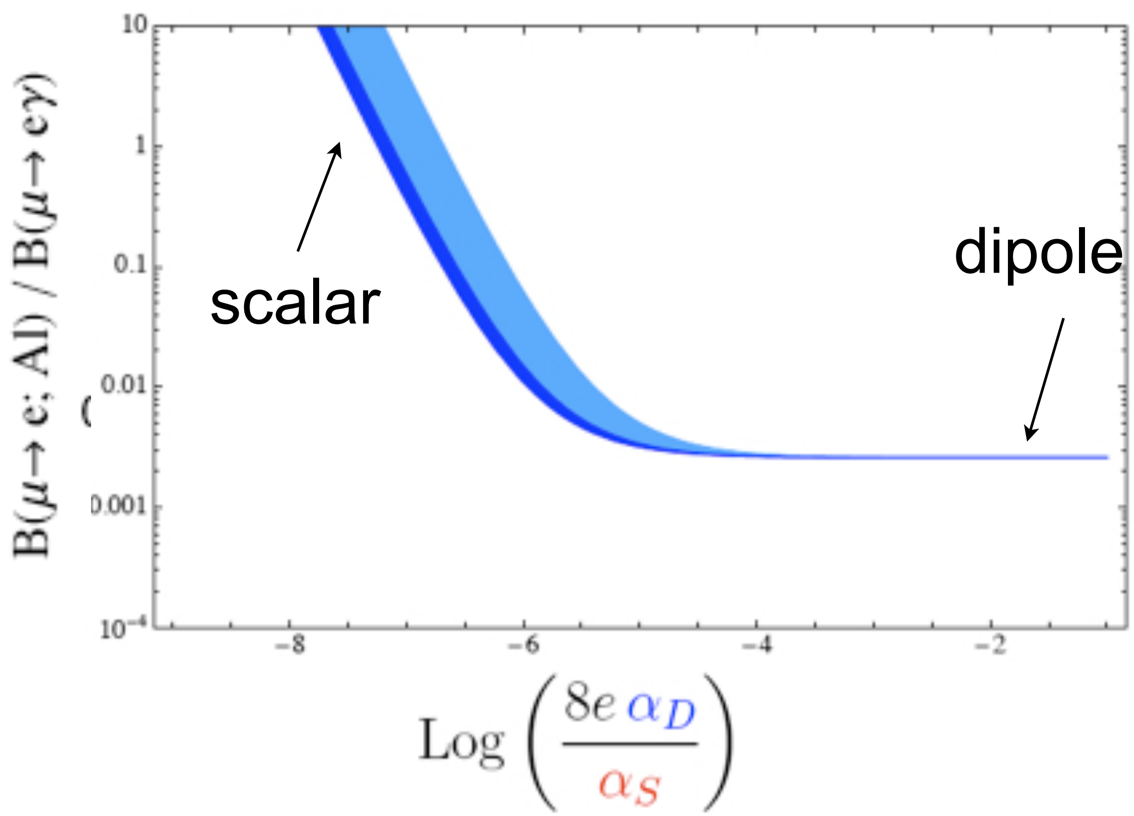


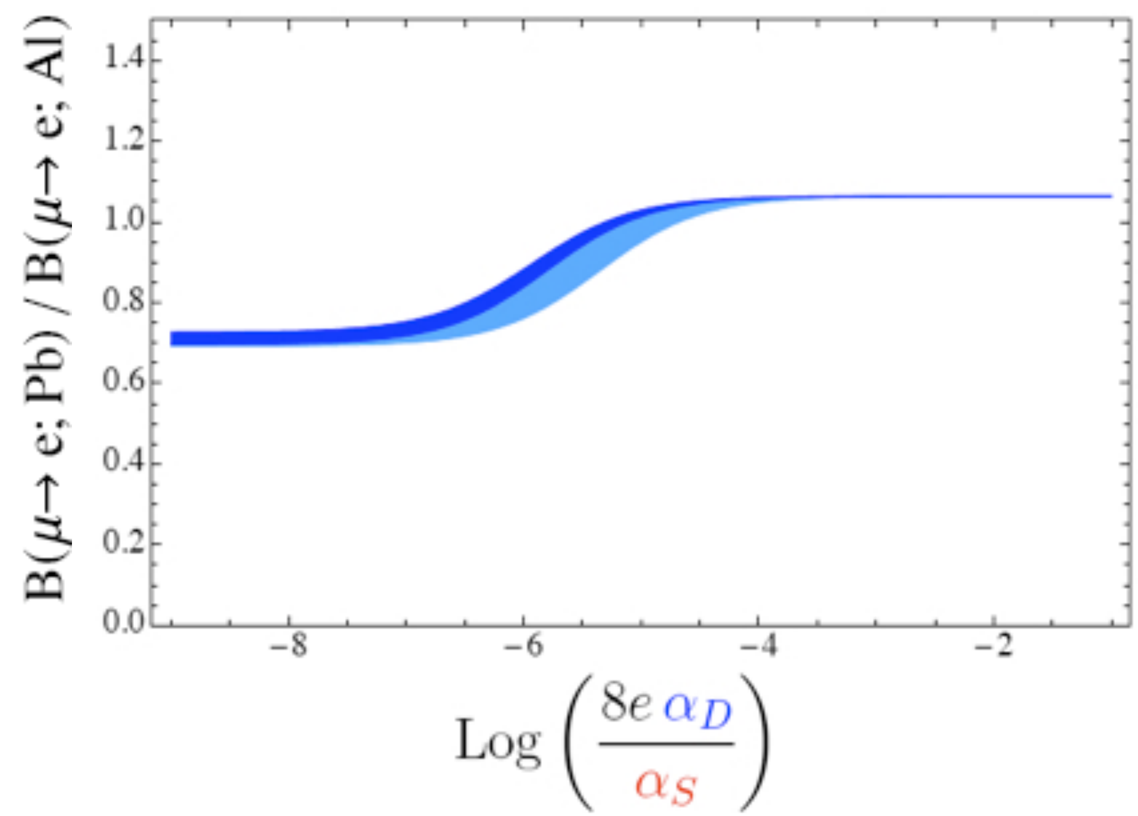


• Consider S and D: realized in SUSY via competition between dipole and scalar operator (mediated by Higgs exchange)

Relative sign: +

VC-Kitano-Okada-Tuzon 2009





- Uncertainty from strange form factor largely reduced by lattice QCD

$$y = \frac{2 \langle p|\bar{s}s|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle} \in [0, 0.4] \rightarrow [0, 0.05]$$

JLQCD 2008

[0, 0.4] ↓ fat error band

[0, 0.05] ↓ thin error band → realistic discrimination

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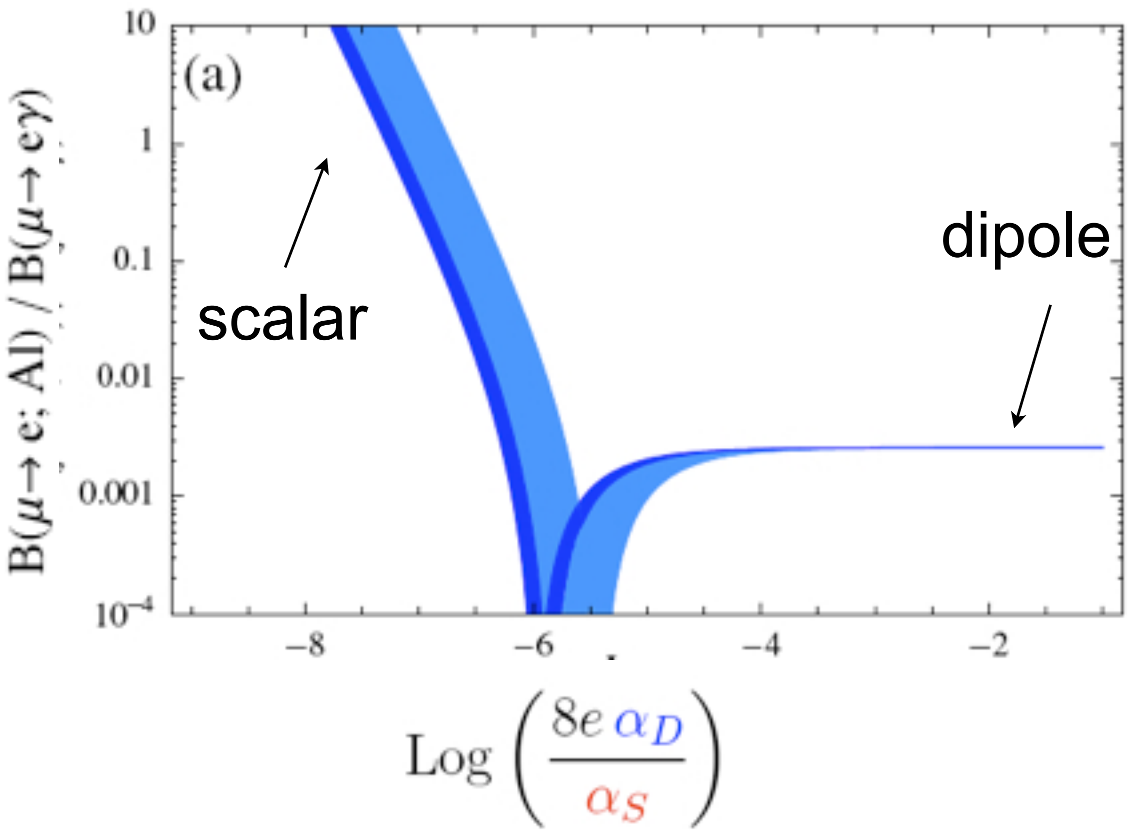
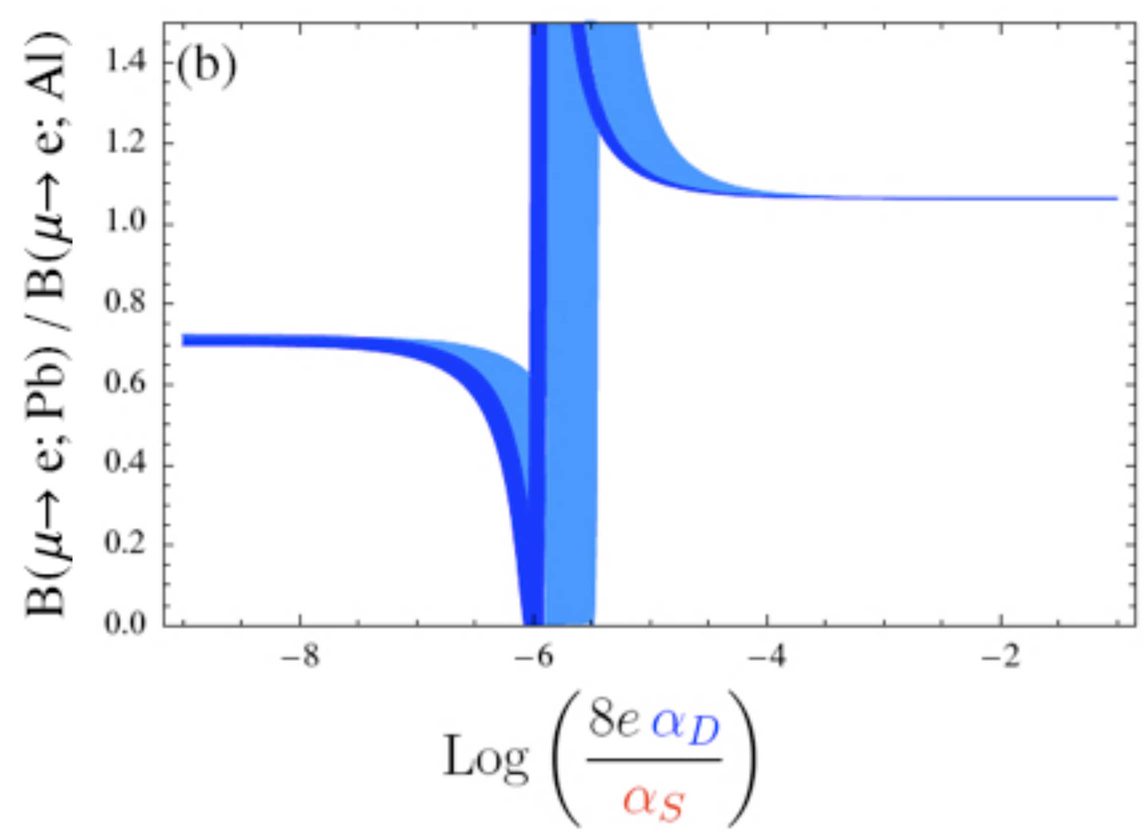
VC-Kitano-Okada-Tuzon 2009





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Relative sign: -

In summary:

- Theoretical hadronic uncertainties under control (OK for 1-operator dominance, need Lattice QCD for 2-operator models)

- Realistic model discrimination requires measuring Ti/Al at <5% or Pb/Al at <20%

- In principle, can perform similar analysis for hadronic vs radiative tau decays at next generation B factory

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JLQCD 2008

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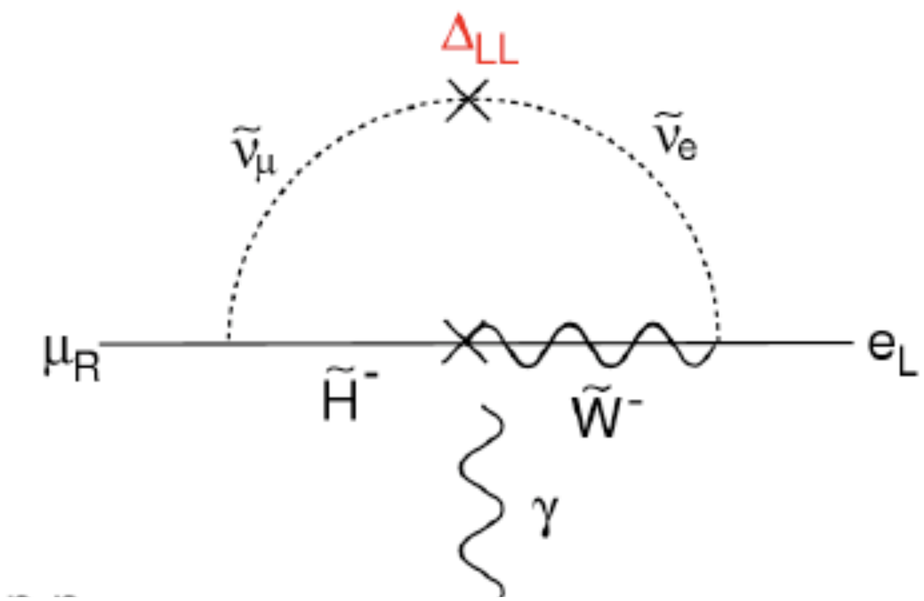
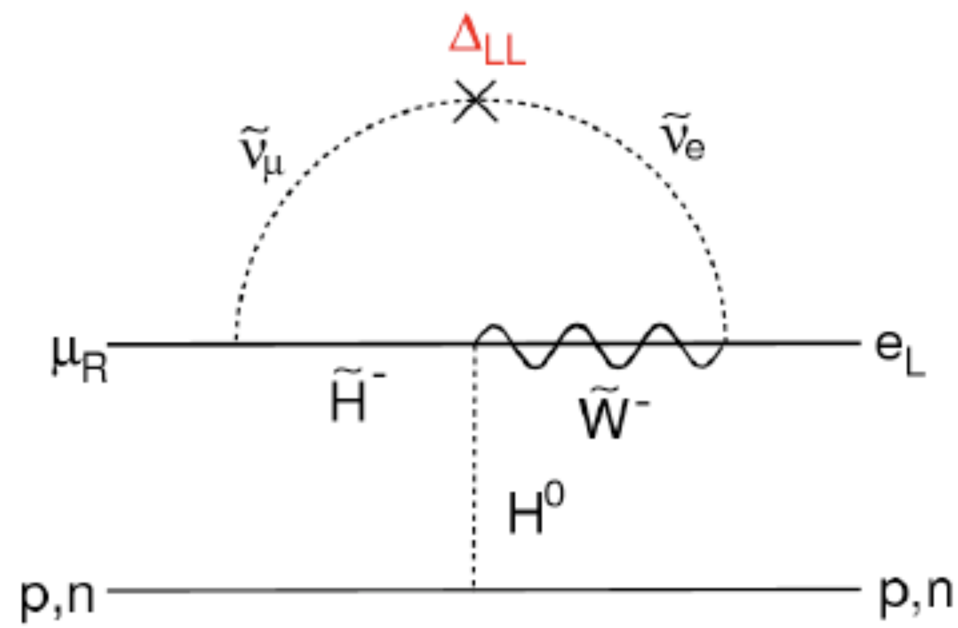
Explicit realization: SUSY see-saw scenario

- See-saw scenario: mixing in L-slepton mass matrices
- Dipole vs scalar operator, mediated by Higgs exchange

Kitano-Koike-Komine-Okada 2003



$\propto \tan\beta / m_{SL}^2$



$\propto \tan^3\beta / m_A^2$

Explicit realization: SUSY see-saw scenario

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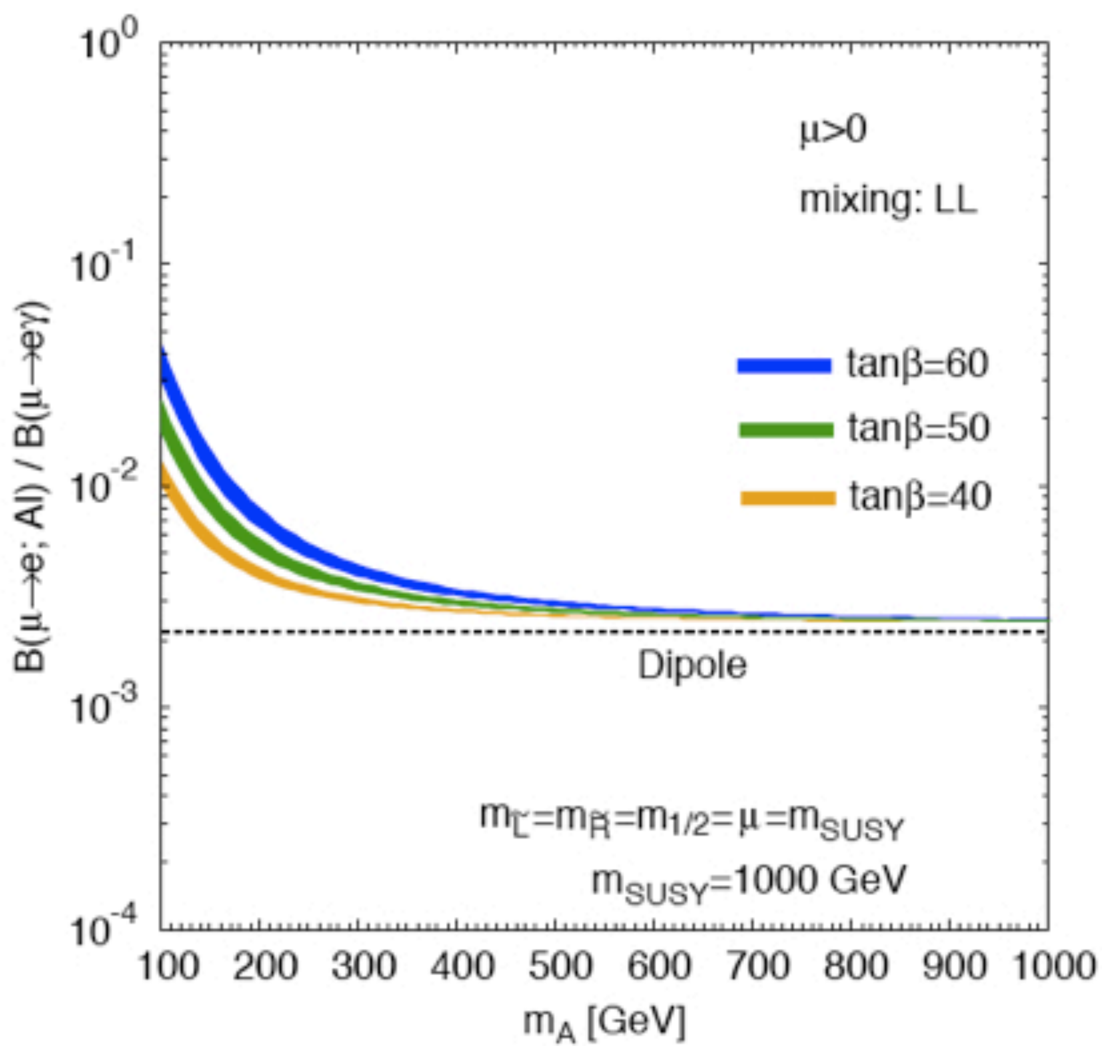
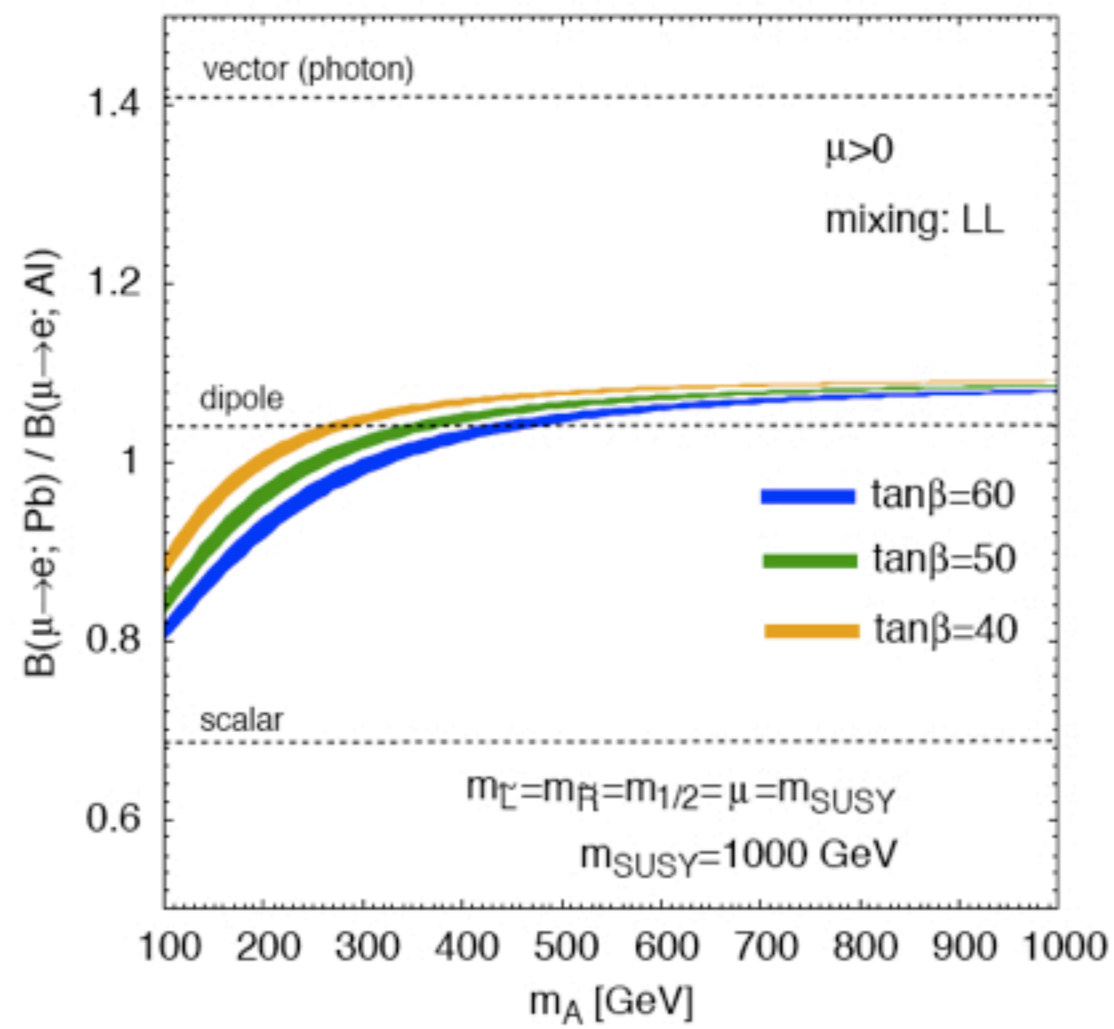




VC-Kitano-Okada-Tuzon 2009

- Learn about SUSY parameters

Conclusions

- Charged LFV: deep probes of physics BSM
  - "Discovery" tools: clean, high scale reach (beyond LHC)
  - "Model-discriminating" tools
- Observation of more than one mode → diagnosing power:
  - Relative strength of operators through $\mu \to e\gamma$ vs $\mu \to e$ conversion in different nuclei [hadronic uncertainty OK]
  - (Structure of flavor breaking sources through μ vs τ LFV BRs)

Extra Slides

Definition of models: D, S, V$_{(Z)}$, V$_{(\gamma)}$

$$\mathcal{L}_{\rm eff}^{(q)} = -\frac{1}{\Lambda^2}\Big[(C_{DR}m_\mu\bar{e}\sigma^{\rho\nu}P_L\mu + C_{DL}m_\mu\bar{e}\sigma^{\rho\nu}P_R\mu)F_{\rho\nu}$$
$$+\sum_q (C_{VR}^{(q)}\bar{e}\gamma^\rho P_R\mu + C_{VL}^{(q)}\bar{e}\gamma^\rho P_L\mu)\bar{q}\gamma_\rho q$$
$$+\sum_q (C_{SR}^{(q)}m_\mu m_q G_F\bar{e}P_L\mu + C_{SL}^{(q)}m_\mu m_q G_F\bar{e}P_R\mu)\bar{q}q \ + \mathrm{H.c.}\Big]$$

Dipole model

$$C_D \equiv C_{DR} \neq 0, \qquad C_{\rm else} = 0.$$

Scalar model

$$C_S \equiv C_{SR}^{(d)} = C_{SR}^{(s)} = C_{SR}^{(b)} \neq 0,$$
$$C_{\rm else} = 0.$$

Vector model: V(γ)

$$C_V \equiv C_{VR}^{(u)} = -2C_{VR}^{(d)} \neq 0, \qquad C_{\rm else} = 0,$$

Vector model: V(Z)

$$C_V \equiv C_{VR}^{(u)} = \frac{C_{VR}^{(d)}}{a} \neq 0, \qquad C_{\rm else} = 0$$

$$a = \frac{T^3_{d_L} + T^3_{d_R} - (Q_{d_L} + Q_{d_R})\sin^2\theta_W}{T^3_{u_L} + T^3_{u_R} - (Q_{u_L} + Q_{u_R})\sin^2\theta_W} = -1.73. \qquad \tilde{C}_{VR}^{(n)}/\tilde{C}_{VR}^{(p)} = -9.26.$$

- Details on the uncertainties

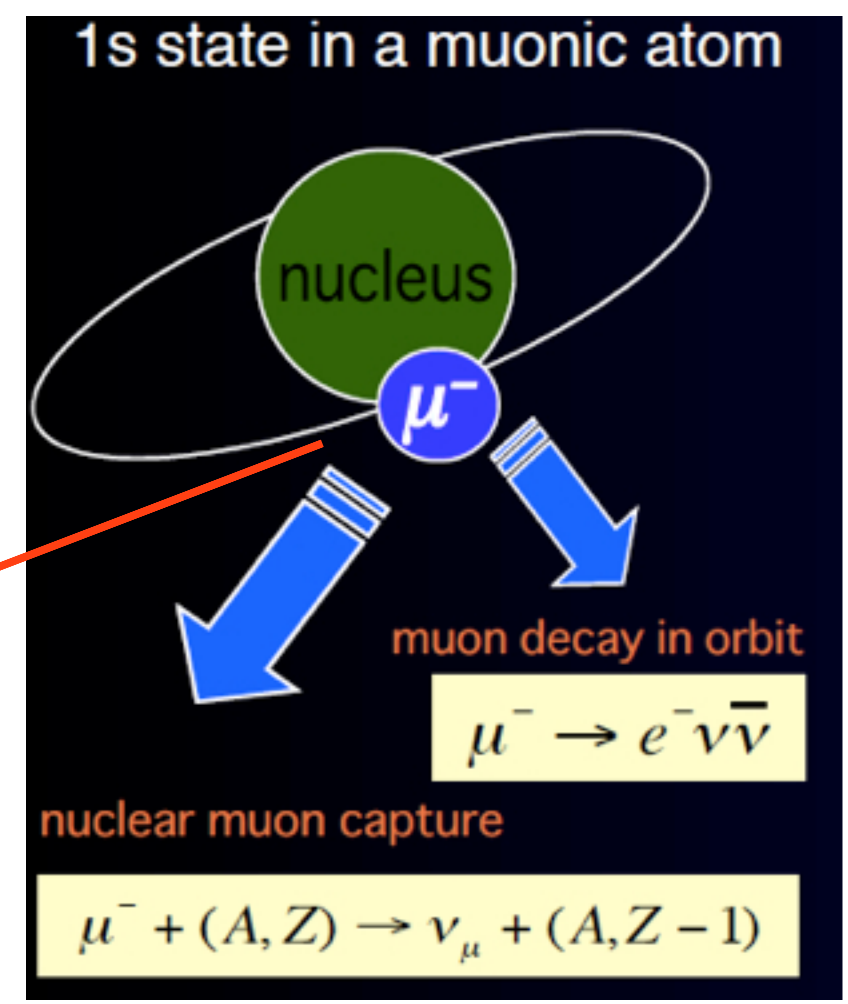
	S	D	$V^{(\gamma)}$	$V^{(Z)}$
$\frac{B(\mu\to e,\mathrm{Ti})}{B(\mu\to e,\mathrm{Al})}$	$1.70 \pm 0.005_y$	1.55	1.65	2.0
$\frac{B(\mu\to e,\mathrm{Pb})}{B(\mu\to e,\mathrm{Al})}$	$0.69 \pm 0.02_{\rho_n}$	1.04	1.41	$2.67 \pm 0.06_{\rho_n}$

- Experimental status (90% CL): muons

$B_{\mu\to e\gamma} < 5.7\times10^{-13}$	→	$10^{-/14}$ (MEG at PSI)
$B_{\mu\to 3e} < 1.0\times10^{-12}$	→	$10^{-14/16}$ (PSI *or* MuSIC?)
$B^{Ti}_{\mu-e} < 4.3\times10^{-12}$ $B^{Au}_{\mu-e} < 8\times10^{-13}$ $B^{Pb}_{\mu-e} < 4.6\times10^{-11}$	→	$10^{-16/17\to-18}$ (Mu2e, COMET)

- μ-to-e conversion rate (normalized to total muon capture rate)

$$B_{\mu\to e} = \frac{\Gamma(\mu^- + (Z,A) \to e^- + (Z,A))}{\Gamma(\mu^- + (Z,A) \to \nu_\mu + (Z-1,A))}$$



- Experimental status: taus (90% BR limits from PDG)

$e^- \gamma$	LF	$<$	1.1	$\times 10^{-7}$
$\mu^- \gamma$	LF	$<$	4.5	$\times 10^{-8}$
$e^- e^+ e^-$	LF	$<$	3.6	$\times 10^{-8}$
$e^- \mu^+ \mu^-$	LF	$<$	3.7	$\times 10^{-8}$
$e^+ \mu^- \mu^-$	LF	$<$	2.3	$\times 10^{-8}$
$\mu^- e^+ e^-$	LF	$<$	2.7	$\times 10^{-8}$
$\mu^+ e^- e^-$	LF	$<$	2.0	$\times 10^{-8}$
$\mu^- \mu^+ \mu^-$	LF	$<$	3.2	$\times 10^{-8}$
$e^- \pi^0$	LF	$<$	8.0	$\times 10^{-8}$
$\mu^- \pi^0$	LF	$<$	1.1	$\times 10^{-7}$
$e^- K^0_S$	LF	$<$	3.3	$\times 10^{-8}$
$\mu^- K^0_S$	LF	$<$	4.0	$\times 10^{-8}$
$e^- \eta$	LF	$<$	9.2	$\times 10^{-8}$
$\mu^- \eta$	LF	$<$	6.5	$\times 10^{-8}$

...

10^{-9} sensitivities at future super-B factory (KEK)