

# Diamagnetic EDMs and Nuclear Structure

J. Engel

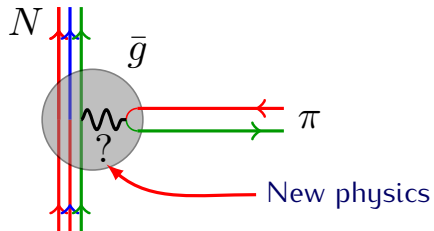
University of North Carolina

February 15, 2013

# One Way Things Get EDMs

Starting at fundamental level and working up:

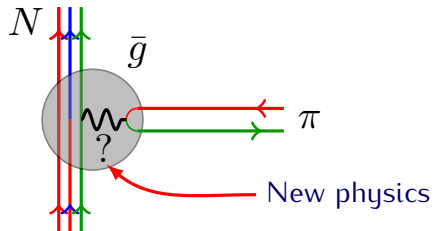
- ▶ Underlying fundamental theory generates three  $T$ -violating  $\pi NN$  vertices:



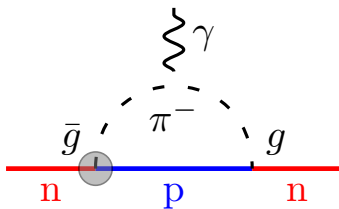
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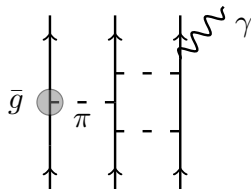


- ▶ Then neutron gets EDM, e.g., from chiral-PT diagrams like this:



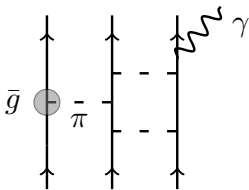
# How Diamagnetic Atoms Get EDMs

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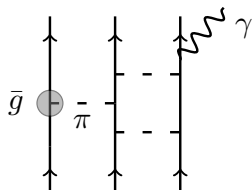
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$$V_{PT} \propto \left\{ \left[ \bar{g}_0 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 - \frac{\bar{g}_1}{2} (\tau_1^z + \tau_2^z) + \bar{g}_2 (3\tau_1^z \tau_2^z - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \right] (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) - \frac{\bar{g}_1}{2} (\tau_1^z - \tau_2^z) (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \right\} \cdot (\nabla_1 - \nabla_2) \frac{\exp(-m_\pi |\mathbf{r}_1 - \mathbf{r}_2|)}{m_\pi |\mathbf{r}_1 - \mathbf{r}_2|}$$

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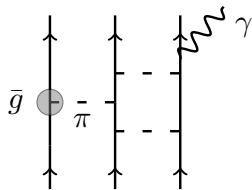


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- ▶ Finally, atom gets one from nucleus. Electronic shielding makes the relevant nuclear object the “Schiff moment”  $\langle S \rangle \approx \langle \sum_p r_p^2 z_p + \dots \rangle$  rather than the dipole moment  $\langle D_z \rangle$ .

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Job of nuclear theory: calculate dependence of  $\langle S \rangle$  on the  $\bar{g}$ 's.

# How Does Shielding Work?

## Theorem (Schiff)

*The nuclear dipole moment causes the atomic electrons to rearrange themselves so that they develop a dipole moment opposite that of the nucleus. In the limit of nonrelativistic electrons and a point nucleus the electrons' dipole moment exactly cancels the nuclear moment, so that the net atomic dipole moment vanishes.*



# How Does Shielding Work?

## Proof

Consider atom with nonrelativistic constituents (with dipole moments  $\vec{d}_k$ ) held together by electrostatic forces. The atom has a “bare” edm

$\vec{d} \equiv \sum_k \vec{d}_k$  and a Hamiltonian

$$H = \sum_k \frac{p_k^2}{2m_k} + \sum_k V(\vec{r}_k) - \sum_k \vec{d}_k \cdot \vec{E}_k$$

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K.E. + Coulomb

dipole perturbation

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$$+ i \sum_k (1/e_k) [\vec{d}_k \cdot \vec{p}_k, H_0]$$

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## How Does Shielding Work?

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$$\begin{aligned} |\tilde{0}\rangle &= |0\rangle + \sum_m \frac{|m\rangle \langle m| H_d |0\rangle}{E_0 - E_m} \\ &= |0\rangle + \sum_m \frac{|m\rangle \langle m| i \sum_k (1/e_k) \vec{d}_k \cdot \vec{p}_k |0\rangle (E_0 - E_m)}{E_0 - E_m} \\ &= \left( 1 + i \sum_k (1/e_k) \vec{d}_k \cdot \vec{p}_k \right) |0\rangle \end{aligned}$$

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So the net EDM is zero!



## All is Not Lost, Though...

The nucleus has finite size. Shielding is not complete, and nuclear  $T$  violation can still induce atomic EDM  $\vec{d}$ .

Post-screening nucleus-electron interaction proportional to Schiff moment:

$$\vec{S} \equiv \sum_p e_p \left( r_p^2 - \frac{5}{3} \langle R_{\text{ch}}^2 \rangle \right) \vec{r}_p + \dots$$

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If, as you'd expect,  $\langle \vec{S} \rangle \approx R_N^2 \langle \vec{D} \rangle$ , then  $\vec{d}$  is down from  $\langle \vec{D} \rangle$  by

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Overall suppression of  $\langle \vec{D} \rangle$  is only about  $10^{-3}$ .

## Theory for Heavy Nuclei

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Nuclear version: Mean-field theory with density-dependent interactions (called Skyrme interactions) built from delta functions and derivatives of delta functions plus whatever corrections one can manage, e.g.

- ▶ projection of deformed wave functions onto states with good angular momentum
- ▶ mixing of several mean fields
- ▶ ...

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Density functional still obtained largely through phenomenology.



# Nuclear Deformation

$\lambda = 0$   
**Sphere**



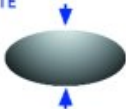
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$\lambda = 2$   
Quadrupoles

OBLATE



PROLATE



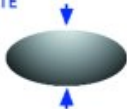
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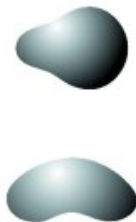
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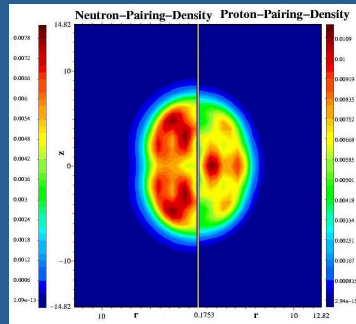
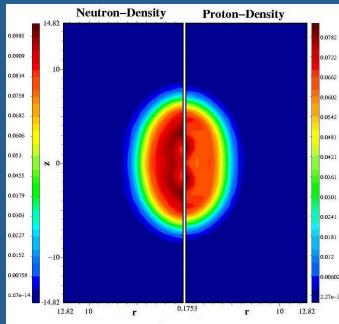
$\lambda = 3$   
Octupoles



# Deformed Skyrme Mean-Field Theory

Zr-102: normal density and pairing density  
HFB, 2-D lattice, SLy4 + volume pairing

Ref: Artur Blazkiewicz, Vanderbilt, Ph.D. thesis (2005)

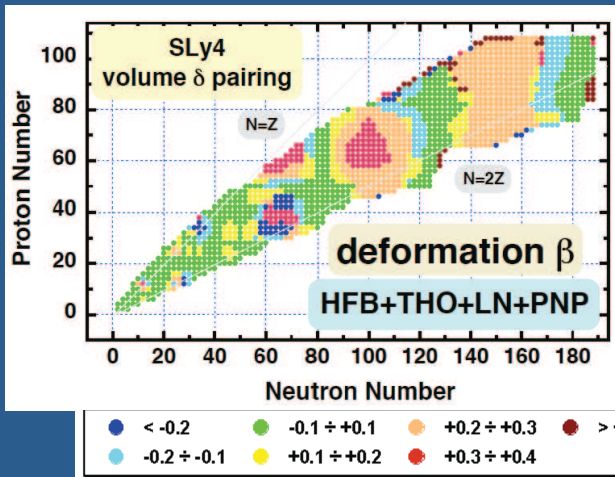


HFB:  $\beta_2^{(p)}=0.43$

exp:  $\beta_2^{(p)}=0.42(5)$ , J.K. Hwang et al., Phys. Rev. C (2006)

## Nuclear ground state deformations (2-D HFB)

Ref: Dobaczewski, Stoitsov & Nazarewicz (2004) arXiv:nucl-th/0404077



# Varieties of Recent Schiff-Moment Calculations

Need to calculate

$$S = \langle S_z \rangle = \sum_m \frac{\langle 0 | V_{PT} | m \rangle \langle m | S_z | 0 \rangle}{E_0 - E_i} + c.c.$$

where  $H = H_{strong} + V_{PT}$ .

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where  $H = H_{strong} + V_{PT}$ .

- ▶  $H_{strong}$  represented either by Skyrme density functional or by simpler effective interaction, treated non-self-consistently.
- ▶  $V_{PT}$  either included nonperturbatively or via explicit sum over intermediate states.
- ▶ Nucleus either forced artificially to be spherical or allowed to deform.

## Spherical Calc.: $^{198}\text{Hg}$ + Polarization by Last Neutron

---

1. Skyrme HFB (mean-field treatment of pairing) in  $^{198}\text{Hg}$ .
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$$\langle S_z \rangle_{\text{Hg}} \equiv a_0 g\bar{g}_0 + a_1 g\bar{g}_1 + a_2 g\bar{g}_2 \quad (\text{e fm}^3)$$

	$a_0$	$a_1$	$a_2$
SkM*	0.009	0.070	0.022
SkP	0.002	0.065	0.011
SIII	0.010	0.057	0.025
SLy4	0.003	0.090	0.013
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Range of variation here doesn't look too bad. But these calculations are not the end of the story.

## Deformation and Angular-Momentum Restoration

If deformed state has good intr.  $J_z = K$ , averaging over angles gives:

$$|J, M\rangle = \frac{2J+1}{8\pi^2} \int D_{MK}^{J*}(\Omega) \hat{R}(\Omega) |\Psi_K\rangle d\Omega$$

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Matrix elements;

$$\begin{aligned} \langle J, M | \hat{S}_i | J', M' \rangle &\propto \int \int \sum_j d\Omega d\Omega' \times (\text{some D-functions}) \\ &\times \langle \Psi_K | \hat{R}^{-1}(\Omega') \hat{S}_j \hat{R}(\Omega) | \Psi_K \rangle \end{aligned}$$

$$\xrightarrow[\Omega \approx \Omega']{\text{rigid defm.}} (\text{Geometric factor}) \times \underbrace{\langle \Psi_K | \hat{S}_z | \Psi_K \rangle}_{\langle \hat{S} \rangle_{\text{intr.}}}$$

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For expectation value in  $J = \frac{1}{2}$  state:

$$S = \langle \hat{S}_z \rangle_{J=\frac{1}{2}, M=\frac{1}{2}} \implies \begin{cases} \langle \hat{S} \rangle_{\text{intr.}} & \text{spherical nucleus} \\ \frac{1}{3} \langle \hat{S} \rangle_{\text{intr.}} & \text{rigidly deformed nucleus} \end{cases}$$

Exact answer somewhere in between.

## Deformed Calculation Directly in $^{199}\text{Hg}$

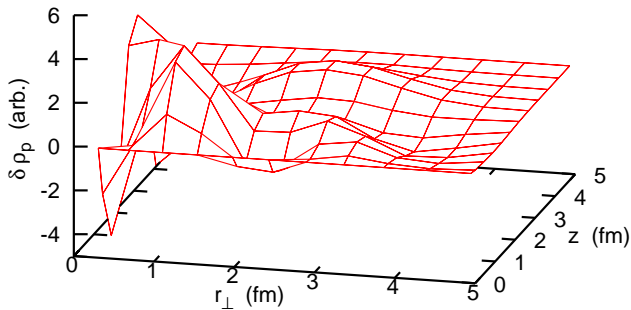
Deformation actually small and soft — perhaps worst case scenario for mean-field. But in odd nuclei, that's the limit of current technology<sup>1</sup>.  $V_{PT}$  included nonperturbatively and calculation done in one step. Includes more physics (deformation) than RPA calculations, plus an economy of approach. Otherwise more or less equivalent.

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Induced change in density distribution indicates delicate Schiff moment.

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## Results of "Direct" Calculation

Like before, use a number of Skyrme functionals:

		$E_{\text{gs}}$	$\beta$	$E_{\text{exc.}}$	$a_0$	$a_1$	$a_2$
SLy4	HF	-1561.42	-0.13	0.97	0.013	-0.006	0.022
SIII	HF	-1562.63	-0.11	0	0.012	0.005	0.016
SV	HF	-1556.43	-0.11	0.68	0.009	-0.0001	0.016



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SkM*	HFB	-1564.03	0	0.82	0.041	-0.027	0.069

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Fav. RPA	QRPA	—	—	—	0.010	0.074	0.018

Hmm...

# What to Do About Discrepancy

- ▶ Authors of these papers need to revisit/recheck their results.
- ▶ Improve treatment further:
  - ▶ Variation after projection
  - ▶ Triaxial deformation

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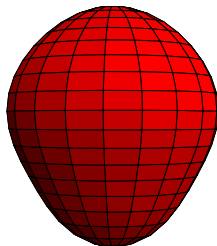
Ultimate goal: mixing of many mean fields (aka “generator coordinates”)

# Schiff Moment with Octupole Deformation

Here we treat always  $V_{PT}$  as explicit perturbation:

$$S = \sum_m \frac{\langle 0 | S_z | m \rangle \langle m | V_{PT} | 0 \rangle}{E_0 - E_m} + c.c.$$

where  $|0\rangle$  is unperturbed ground state.



Calculated  $^{225}\text{Ra}$  density

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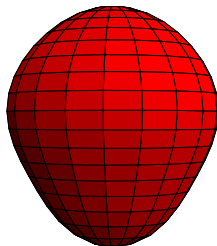
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Ground state has nearly-degenerate partner  $|\bar{0}\rangle$  with same opposite parity and same intrinsic structure, so:

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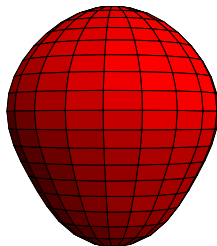
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$S$  is large because  $\langle S \rangle_{\text{intr.}}$  is collective and  $E_0 - E_{\bar{0}}$  is small.



Calculated  $^{225}\text{Ra}$  density

## A Little on Parity Doublets

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These are nearly degenerate if deformation is rigid. So with  $|0\rangle = |+\rangle$  and  $|\bar{0}\rangle = |-\rangle$ , we get

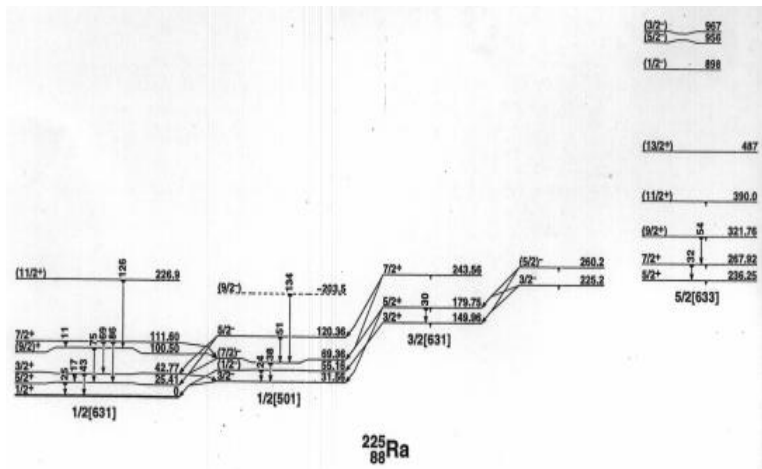
$$S \approx \frac{\langle 0 | S_z | \bar{0} \rangle \langle \bar{0} | V_{PT} | 0 \rangle}{E_0 - E_{\bar{0}}} + \text{c.c.}$$

And in the rigid-deformation limit

$$\langle 0 | \hat{O} | \bar{0} \rangle \propto \langle \bullet | \hat{O} | \bullet \rangle = \langle \hat{O} \rangle_{\text{intr.}}$$

again like angular momentum.

# Spectrum of $^{225}\text{Ra}$



Hartree-Fock calculation with our favorite interaction SkO' gives

$$S_{\text{Ra}} = -1.5 g\bar{g}_0 + 6.0 g\bar{g}_1 - 4.0 g\bar{g}_2 \text{ (e fm}^3\text{)}$$

Larger by over 100 than in <sup>199</sup>Hg!

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Variation a factor of 2 or 3.



## Current “Assessment” of Uncertainties

Judgment in upcoming review article (based on spread in reasonable calculations):

Nucl.	Best value			Range		
	$a_0$	$a_1$	$a_2$	$a_0$	$a_1$	$a_2$
$^{199}\text{Hg}$	0.01	0.01	0.02	0.005 – 0.02	-0.03 – 0.09	0.01 – 0.03
$^{129}\text{Xe}$	-0.008	-0.006	-0.009	-0.005 – -0.05	-0.003 – -0.05	-0.005 – -0.1
$^{225}\text{Ra}$	-1.5	6.0	-4.0	-1 – -6	4 – 20	-2 – -15

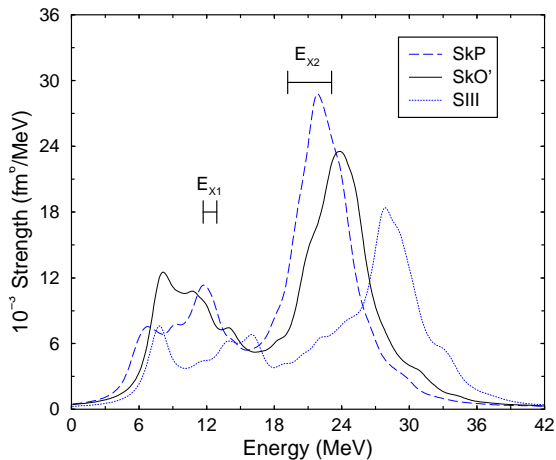
Uncertainties pretty large, particularly for  $g_1$  in  $^{199}\text{Hg}$  (range includes zero). How can we reduce them?

## Grounding the Calculations: Hg

Improving the many-body theory to handle soft deformation, though probably necessary, is tough. But can also try to optimize density functional.

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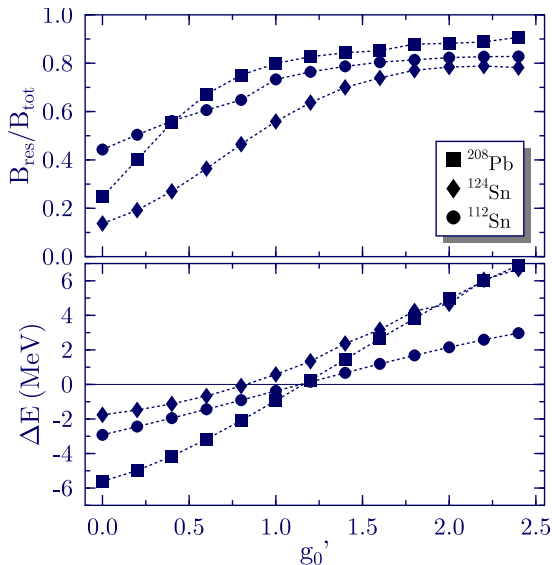


Isoscalar dipole operator contains  $r^2z$  just like Schiff operator. Can see how well functionals reproduce measured distributions, e.g. in  $^{208}\text{Pb}$ .



# More on Grounding Hg Calculation

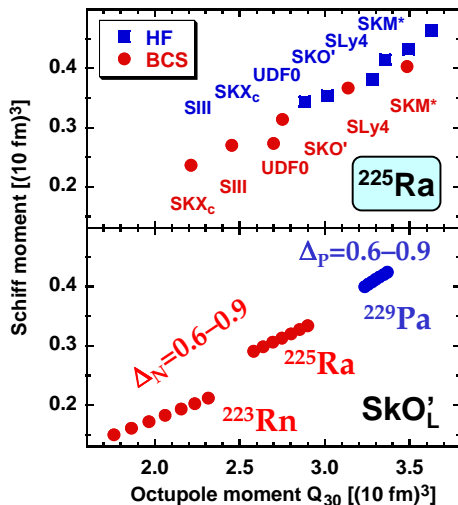
$V_{PT}$  probes spin density;  
functional should have good  
spin response. Can adjust  
relevant terms in, e.g. SkO',  
to Gamow-Teller resonance  
energies and strengths.



# Grounding the Calculations: Ra

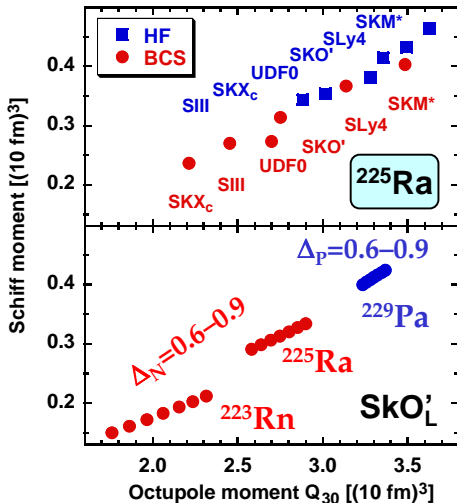
Here there have been important recent developments.

$\langle S \rangle_{\text{intr.}}$  correlated with octupole moment, which will be extracted from measurements of E3 transitions.

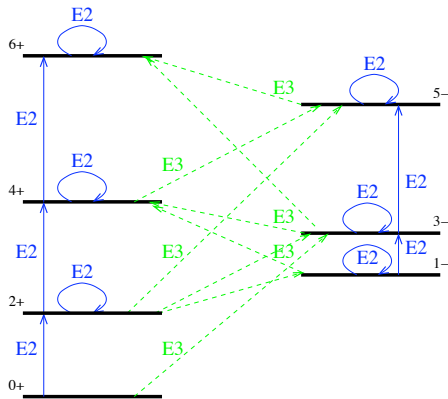


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This is  $^{224}\text{Ra}$ ; transitions in  $^{225}\text{Ra}$  will be measured soon.

# THE END

Thanks for your kind attention.