

Our Evolving Understanding of Tokamak Confinement Physics:

Thoughts for the 80th Birthday of Roald Z. Sagdeev

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Outline

A.) 1) Foundations: Sagdeev's Greatest Hits

→ The Building Blocks of Confinement Theory

2) Where Things Stand Today:

→ Current Status of Our Understanding: An OV

B.) A Recent Perspective:

→ Heat Flux 'Jamitons' and the Formation of
the $E \times B$ staircase

- with Yusuke Kosuga and Ozgur Gurcan

C.) Looking Ahead

AI.) Sagdeev's Greatest Hits in MFE

i.) Microstability Theory

- basic physics of drift and ITG modes

 - early - mid 60's

 - with: Rosenbluth, Rudakov, Moiseev, Coppi, ...

(Coppi, MNR, RZS '67)

achieved:

- 'inverse dissipation' mechanism for electron drift waves

- 'negative compressibility' ITG modes

- first effects of magnetic geometry

➔ Defined fundamental agents of core anomalous transport: ITG, CTEM

ii.) Quasilinear (Mean Field) Theory

→ early - mid 60's

→ with: Vedenov, Velikov, ...

achieved:

– mean field theory for Vlasov system

– basic wave-particle energetics

– time scale hierarchy and link to chaos

➔ Formulated basis of all transport models currently in use in MFE community

iii.) Weak Turbulence Theory, especially nonlinear wave-particle interaction

→ early - mid 60's => Trieste lectures

→ with: Galeev, Rudakov, Zakharov, Shapiro, ...

achieved:

- complete perturbative theory of non-linear dynamics, with wave-wave and wave-particle interaction on an equal footing
- initial ideas on quasi-particle models
- set foundations for subsequent work on granulations, strong turbulence (T. H. Dupree, et al.)
- ➔ defined the structure. Potential use for G.K. verification not yet exploited.

iv.) Hamiltonian Chaos

→ early 60's – 80's

→ with: Zaslavsky, Rosenbluth, Taylor (RSTZ paper, Trieste) ...

achieved:

→ formulation of Hamiltonian chaos theory for plasma-relevant problems, especially magnetic field lines (c.f. Rechester-Rosenbluth '78)

→ linked Kolmogorov, Arnold, Chirikov to MFE-plasma community

→ initiated further work by Zaslavsky school

➔ defined and established standard techniques in everyday use – re: RMP

v.) Neoclassical Theory

→ 60's

→ with: Galeev, Rosenbluth, ...

achieved:

– basic structure and concepts of the theory

– fundamental ideas on heat transport, flow damping, electric fields, ...

➔ conceptual foundations of an industry initiated, followed by many others

vi.) Quasi-Particle Picture and Methodology

→ 60's – late 70's

→ with: Zakharov, Rudakov, Galeev, Shapiro, Shevchenko, ...

achieved:

– development of modulational interaction paradigm

→ Langmuir turbulence

→ convective cells (includes zonal flows)

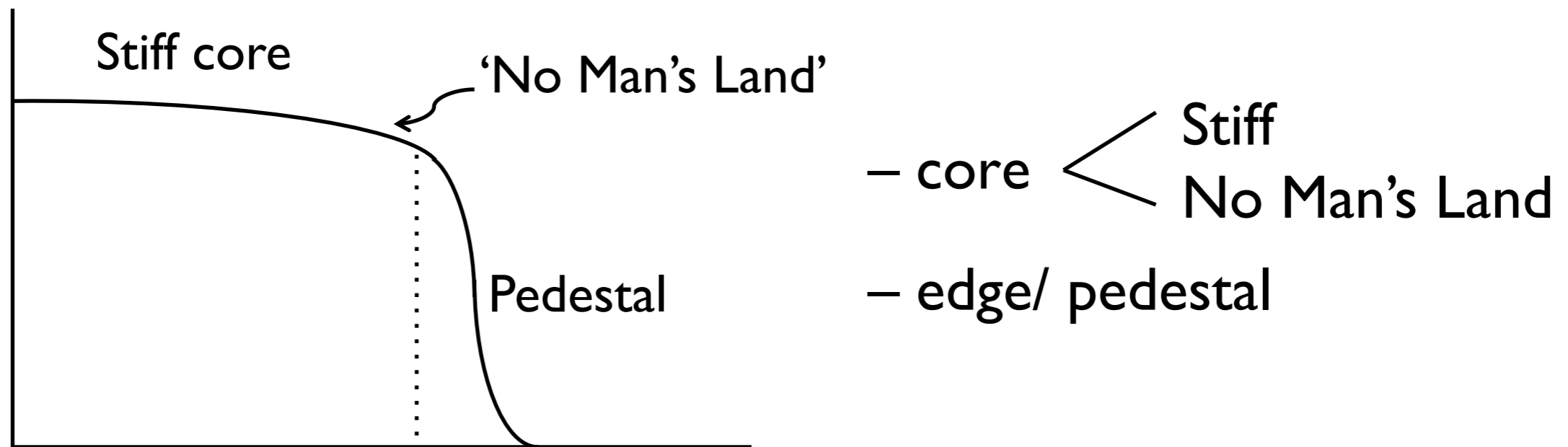
cf: Sagdeev, et al. '78

– concept of pattern selection in secondary structures

➔ mapped out a useful and elegant approach to structure formation in turbulence

A2.) Where Things Stand Today

→ The Multi-Zone Picture



Core:

– The Good News – Understanding

→ Core Turbulence : ITG, TEM modes (“standard model”)

→ $D \sim D_B \rho_*^\alpha$, $\alpha \leq 1$ → close to GyroBohm

→ Zonal Modes critical to saturated state

→ ∇n peaking → TEP, thermoelectric pinch

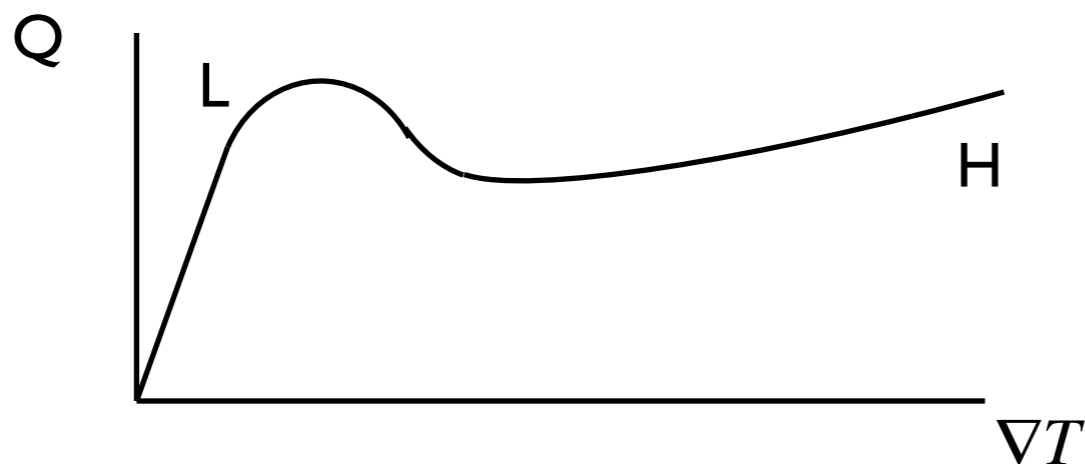
→ intrinsic rotation → turbulent (residual) stress

- The Bad News – Performance
- Stiffness → Core profiles difficult to move off criticality
- No Man’s Land:
 - standard GK codes under-predict layer linking pedestal to stiff core
 - “standard model” fails
 - spreading, non-locality?
 - Significant impact on ITER predictions
- ‘The Fix’ – of questionable reactor relevance : ITBs
 - ITB’s require externally driven V_{ϕ} shear → how ?
 - MHD stability of ITB plasmas dubious
- We understand a great deal about the core, but will probably need to live with its stiffness...

Edge

→ Good News → H-Mode

- L→H transition, first observed in 1982, by F. Wagner on ASDEX, is THE high point of MFE since Trieste.
- L→H transition enables confinement with plausible extrapolation to ignition, and is 'robust' state.
- physics : H-mode is due spontaneous ETB formation, very likely due to enhanced ExB shear suppression induced transport bifurcation.



→ Current Status :

- L→H transition likely triggered by synergy between Reynolds stress driven (zonal) flows and ∇P -driven diamagnetic flow
 - world wide 'hot topic', especially 2009 - present
- Transition is 1st order → hysteresis

Major unknowns :

- $P_{\text{Thresh}}(r)$ scalings, especially low density regime
- strength of hysteresis → important near P_{Th}
- width of H-mode pedestal
- pedestal transport physics → especially electron thermal

→ The Bad News → H-Mode

- H-mode is fundamentally state with strong suppression of particle transport

- ∇n and ∇P steepen to (MHD) stability limits

→ ELMs happen!

- ELM : Edge Localized Modes

- edge relaxation oscillation

- kink-ballooning, etc.

- unacceptable transient power load on PFC

- Basic Problem: H-mode (Particle) Confinement is

TOO GOOD.

→ ITER Crisis Du Jour – ELM Mitigation

- Seek reduce/eliminate ELM by relieving ∇P using stochastic field induced by additional applied coil
- RMP – edge layer becomes fundamentally 3D
 - a step toward stellarator !?
- Counter-intuitively, RMPs degrade particle confinement more than thermal
- RMP's have achieved some success in ELM suppression, but windows of operation limited. Understanding is poor.
- Alternatives : QH-mode, pellets, SMBI, ...
- New Problem : SOL widths too narrow

→ Current Research Foci on Confinement Physics

- ‘Non-Locality’ and Transport in No Man’s Land

 - (microstability)

- Flow Physics : Zonal and Intrinsic Rotation

 - (quasi particles)

- L→H Transition : Dynamics, Thresholds

 - (quasi particles, neoclassical)

- Pedestal Modelling

- RMP, 3D Physics

 - (Hamiltonian chaos)

➔ Continuing impact of R.Z. Sagdeev in MFE world of 2013 indisputable.

B.) A Recent Perspective :The Heat Flux 'Jamiton'

“On how the propagation of heat flux modulations triggers ExB flow pattern formation”

- Y. Kosuga, P.H. Diamond, O.D. Gurcan
(PRL '13, in press)

“Transport of Radial Heat Flux and Second Sound in Fusion Plasmas”

- O.D. Gurcan, P.H. Diamond, X. Garbet, et al.
(PoP '13, in press)

Outline

- Motivation
- Extension of heat avalanche concept
- Relation to traffic flow problem
- Analysis
 - Instability calculation: suggestive of flux jam formation
 - maximal growth: comparison to staircase structure
- Implications for staircase and transport
- Summary

Motivation

- ExB flows are important for tokamak confinement
 - Shear suppression of turbulent transport
- ExB flows are self-organized in tokamak plasmas
 - hierarchy of flow patterns:

Type of ExB flows	generation mechanisms
Mean ExB	transport bifurcation, strong mean gradients, mean shear flows
radially quasi-periodic zonal flows	drift wave turbulence, non-local energy transfer in k-space
ExB staircase	?????

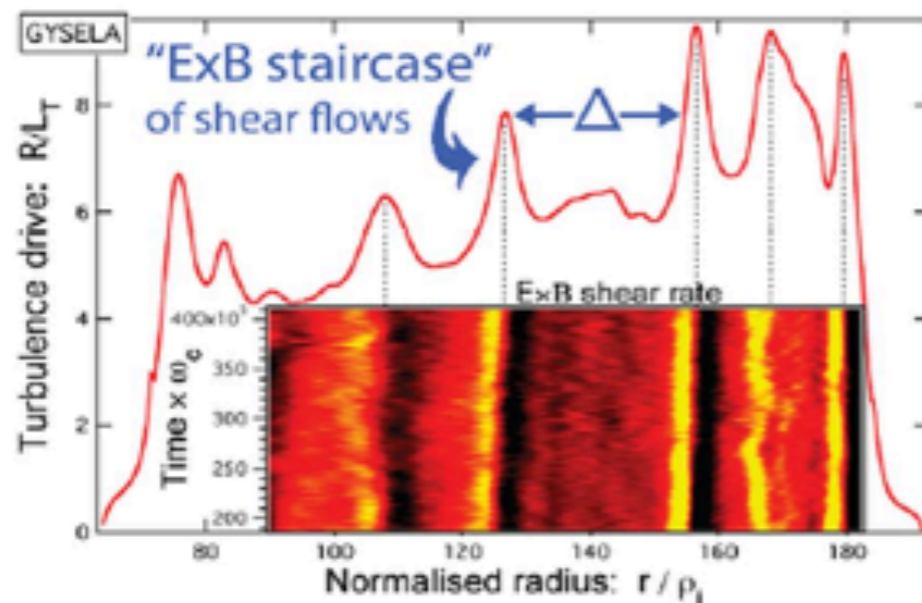
see
Sagdeev, et. al.
'78

What is the staircase??

→ The Most Interesting Thing in GK Simulation in 10 years!

With G. Dif-Pradalier et. al.

Analogy with geophysics: the ' $\mathbf{E} \times \mathbf{B}$ staircase'



- Quasi-regular pattern of shear layer and profile corrugations

$$Q = -n\chi(r)\nabla T \Rightarrow Q = -\int \kappa(r, r')\nabla T(r') dr'$$

- ' $\mathbf{E} \times \mathbf{B}$ staircase' width \equiv kernel width Δ
- coherent, persistent, jet-like pattern
⇒ the ' $\mathbf{E} \times \mathbf{B}$ staircase'
- staircase NOT related to low order rationals!



Dif-Pradalier, P.D. et. al., Phys Rev E. 2010

What is the staircase??

- The point:

- fit: $Q = -\int dr' \kappa(r, r') \nabla T(r')$ $\kappa(r, r') \sim \frac{S^2}{(r - r')^2 + \Delta^2}$ → some range in exponent

- $\Delta \gg \Delta_c$ i.e. $\Delta \sim$ Avalanche scale $\gg \Delta_c \sim$ correlation scale

- Staircase 'steps' separated by Δ ! → **stochastic avalanches produce quasi-regular flow pattern!?** → How?

N.B.

- The notion of a staircase is not new – especially in systems with natural periodicity (i.e. NL wave breaking...)

- What IS new is the connection to stochastic avalanches, independent of geometry

- What is process of self-organization linking avalanche scale to zonal pattern step?

i.e. How does system resolve pattern selection competition between zonal flow and streamer/ avalanche?

→ *spatial, domain decomposition, ala' spinodal decomposition?*

Towards a model

- How do we understand quasi-regular pattern of ExB staircase, generated from stochastic heat avalanche???

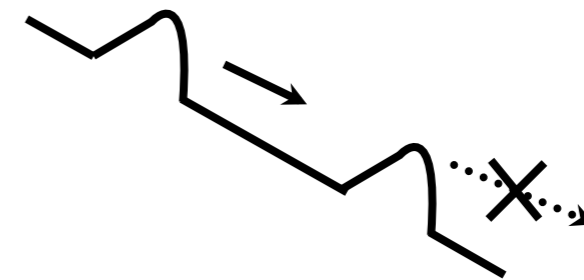
→ ‘eigenfunction’ idea dubious, as no identifiable growth phase for staircase

- An idea: **jam of heat avalanche**

ExB staircase \leftrightarrow corrugated profile

→ corrugation of profile occurs by ‘jam’ of heat avalanche flux

* → time delay between $Q[\delta T]$ and δT is crucial element



→ accumulation of heat increment
→ stationary corrugated profile

- How do we actually model heat avalanche ‘jam’ ??? → what is dynamical origin?

- $\delta T(x) \leftrightarrow v'_E$ via radial force balance \Rightarrow onset

Heat avalanche dynamics

→ Hwa+Kardar '92, P.D. + Hahm '95, Carreras, et al. '96, ... GK simulation, ... Dif-Pradalier '10

- δT : deviation from marginal profile → conserved order parameter

- Heat Balance Eq.: $\partial_t \delta T + \partial_x Q[\delta T] = 0$ → up to source and noise

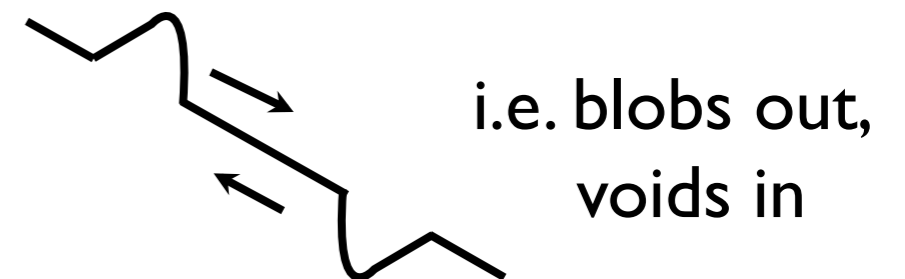
- Heat Flux $Q[\delta T]$? – utilize symmetry argument, ala' Ginzberg-Landau

- **Usual**: Diamond+Hahm '95

→ ala **joint reflectional symmetry** (Hwa+Kardar)

$$Q = Q_0(\delta T) \\ = \frac{\lambda}{2} \delta T^2 - \chi_2 \partial_x \delta T + \chi_4 \partial_x^3 \delta T$$

↘ hyperdiffusion



lowest order → Burger's equation

$$\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$$

An extension of the heat avalanche dynamics model

- **An extension:** allow a finite time relaxation of Q toward SOC flux state

$$\partial_t Q = -\frac{1}{\tau} (Q - Q_0(\delta T)) \qquad Q_0[\delta T] = \frac{\lambda}{2} \delta T^2 - \chi_2 \partial_x \delta T + \chi_4 \partial_x^3 \delta T$$


 Relaxation time: $\tau(\delta T, Q_0)$ lag between mean and instantaneous flux

i.e. τ enforces **time delay** between δT and heat flux,
large near criticality

- Dynamics of heat avalanche:

$$\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T - \chi_4 \partial_x^4 \delta T - \tau \partial_t^2 \delta T$$

→ Burger's
(P.D. + T.S.H. '95)

↓
Hyper-diffusion

New: due finite response time

→ Instability, 'Flux jamiton'

→ heat equation

→ Burgers Eqn.

→ Wave Eqn.

- Collective Dynamics of Turbulent Eddy
'Aether' I – First Quasi-Particle Model of Transport?!

– Kelvin, 1887

*XLV. On the Propagation of Laminar Motion through a turbulently moving Inviscid Liquid. By Sir WILLIAM THOMSON, LL.D., F.R.S.**

1. **I**N endeavouring to investigate turbulent motion of water between two fixed planes, for a promised communication to Section A of the British Association at its coming Meeting in Manchester, I have found something seemingly towards a solution (many times tried for within the last twenty years) of the problem to construct, by giving vortex motion to an incompressible inviscid fluid, a medium which shall transmit waves of laminar motion as the luminiferous æther transmits waves of light.

2. Let the fluid be unbounded on all sides, and let u, v, w be the velocity-components, and p the pressure at (x, y, z, t) . We have

$$\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0 \quad . \quad . \quad . \quad . \quad (1),$$

* Communicated by the Author, having been read before Section A of the British Association at its recent Meeting in Manchester.

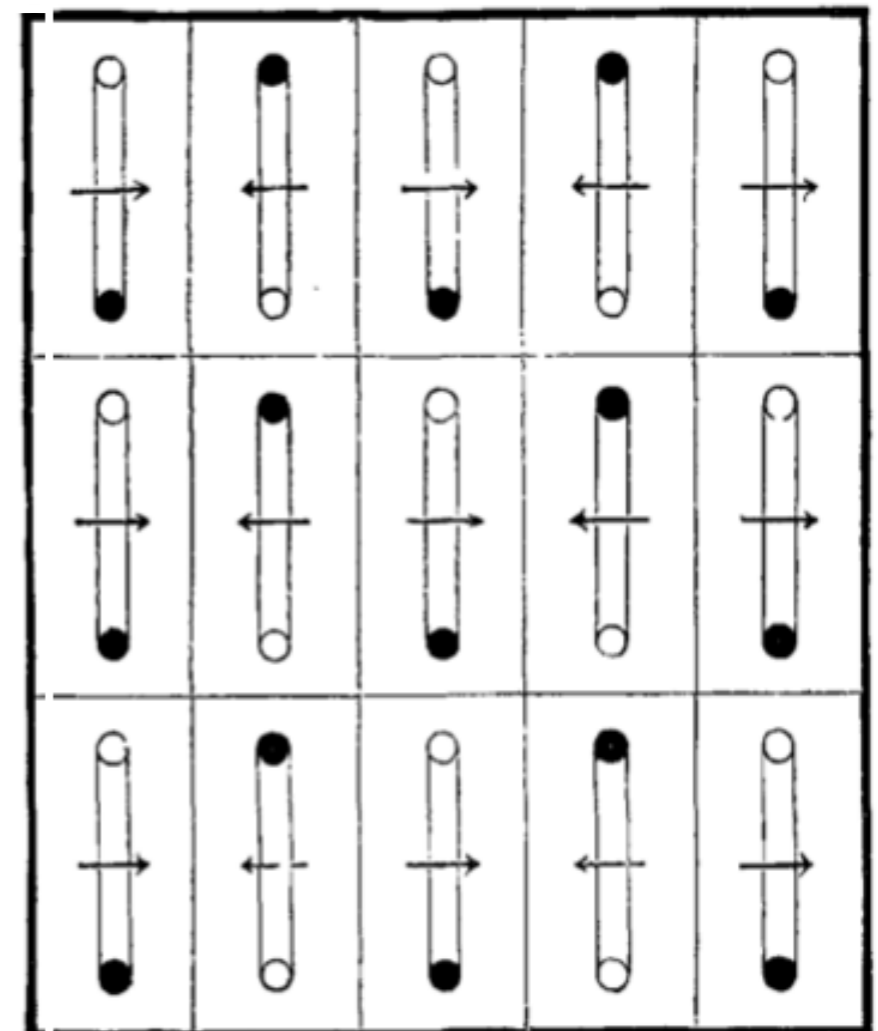
21. Eliminating the first member from this equation, by (34), we find

$$\frac{d^2 f}{dt^2} = \frac{2}{9} R^2 \frac{d^2 f}{dy^2} \quad \cdot \cdot \cdot \cdot \cdot \cdot \quad (51).$$

Thus we have the very remarkable result that laminar disturbance is propagated according to the well-known mode of waves of distortion in a homogeneous elastic solid ; and that the velocity of propagation is $\frac{\sqrt{2}}{3} R$, or about .47 of the average velocity of the turbulent motion of the fluid.

- time delay between Reynolds stress and wave shear introduced
- converts diffusion equation to wave equation
- describes wave in ensemble of vortex quasi-particles
- c.f. “Worlds of Flow”, O. Darrigol

Fig. 1.



Relaxation time - the idea

- What is ' τ '? → Analogy from **traffic jam dynamics**
→ c.f. Whitham!
- A useful **analogy**:

avalanche dynamics	traffic flow dynamics
temp. deviation from marginal profile	local car density
heat flux	traffic flow
mean SOC flux (ala joint reflection symmetry)	equilibrium, steady traffic flow $V(\rho)$
heat flux relaxation time τ	driver's reaction time τ



- driver's response can induce **jam**
- jam in avalanche → profile corrugation → staircase?!?
- key distinction: instantaneous flux vs. mean flux

Excitons in Traffic: The 'jamiton'

- A model for Traffic jam dynamics → Whitham

$$\rho_t + (\rho v)_x = 0$$

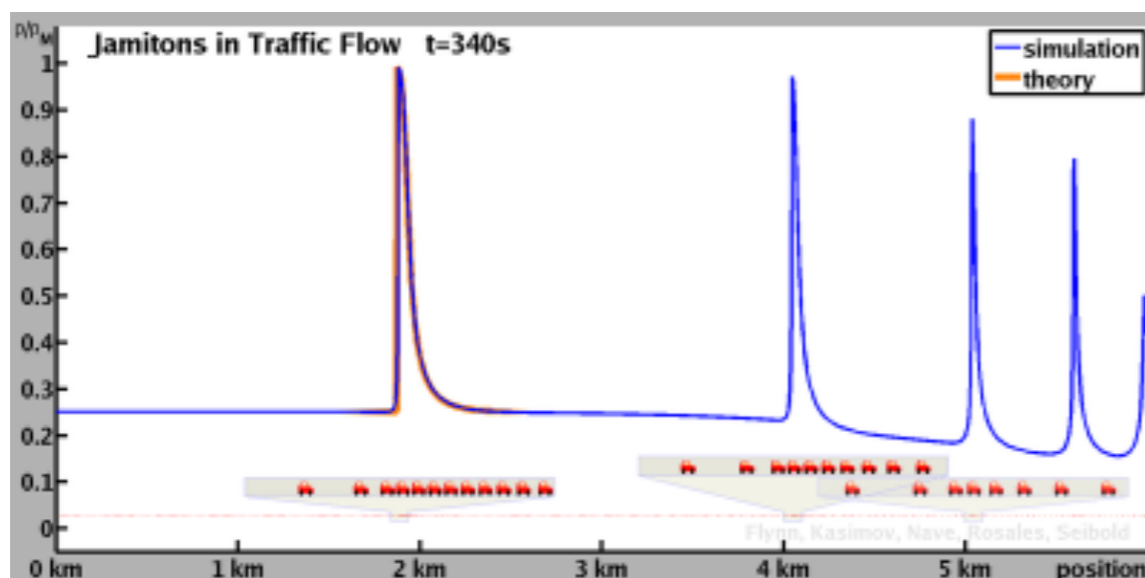
$$v_t + vv_x = -\frac{1}{\tau} \left\{ v - V(\rho) + \frac{\nu}{\rho} \rho_x \right\}$$

→ **Instability** occurs when $\tau > \nu / (\rho_0^2 V_0'^2)$

$$D_{eff} = \nu - \tau \rho_0^2 V_0'^2 < 0 \rightarrow \text{clustering instability}$$

→ Indicative of jam formation

- Simulation of traffic **jam formation**



ρ → car density

v → traffic flow velocity

$V(\rho) - \frac{\nu}{\rho} \rho_x$ → an equilibrium traffic flow

τ → driver's response time

<http://math.mit.edu/projects/traffic/>

→ **Jamitons** (Flynn, et.al., '08)

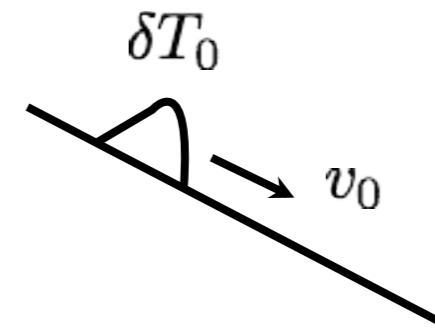
n.b. : I.V.P. → decay study

Analysis of heat avalanche dynamics: 1

- How do heat avalanches jam?

- Consider an initial avalanche, with amplitude δT_0 ,

propagating at the speed $v_0 = \lambda \delta T_0$



- negative heat conduction instability occurs if $v_0^2 \tau$ (related to streaming) beats $\chi_2 \Rightarrow$ overshooting \Rightarrow clustering, as Q can depart $Q_0(\delta T)$

$$\begin{aligned} \partial_t \widetilde{\delta T} + v_0 \partial_x \widetilde{\delta T} &= \chi_2 \partial_x^2 \widetilde{\delta T} - \chi_4 \partial_x^4 \widetilde{\delta T} - \tau \partial_t^2 \widetilde{\delta T} \\ &\rightarrow \underline{(\chi_2 - v_0^2 \tau) \partial_x^2 \widetilde{\delta T} - \chi_4 \partial_x^4 \widetilde{\delta T}} \text{ ; in moving frame of avalanche} \end{aligned}$$

n.b. akin to negative viscosity instability

→ Recall ZF as secondary mode in the gas of primary DW

→ Heat flux ‘**jamiton**’ as secondary mode in the gas of primary avalanches

→ More generally, both are examples of ‘second sound’, as in phonon gas

Analysis of heat avalanche dynamics: 2

- Growth rate of the pulse: \rightarrow negative heat conduction instability

$$\gamma = -\frac{1}{2\tau} + \frac{1}{2\tau} \sqrt{\frac{r+1}{2} - 2\tau\chi_2 k^2 \left(1 + \frac{\chi_4 k^2}{\chi_2}\right)} \quad r = \sqrt{\left\{4\tau\chi_2 k^2 \left(1 + \frac{\chi_4 k^2}{\chi_2}\right) - 1\right\}^2 + 16v_0^2 k^2 \tau^2}$$

- Threshold for instability: $\tau > \frac{\chi_2}{v_0^2} \left(1 + \frac{\chi_4 k^2}{\chi_2}\right)$ but $1/\tau = 1/\tau[\mathcal{E}]$
clustering instability strongest near criticality

- Scale for maximum growth (assuming $k^2 < \frac{\chi_2}{\chi_4}$, scale not so small that χ_4 dominates)

$$k^2 \simeq \frac{\chi_2}{\chi_4} \sqrt{\frac{\chi_4 v_0^2}{4\chi_2^3}} \text{ from } \frac{\partial \gamma}{\partial k^2} = 0 \Rightarrow 8\tau \frac{\chi_4^2}{\chi_2} k^6 + 4\tau \chi_4 k^4 + 2\frac{\chi_4}{\chi_2} k^2 + 1 - \frac{v_0^2 \tau}{\chi_2} = 0$$

\rightarrow staircase size $\Delta_{stair}^2(\delta T)$, δT from saturation \leftrightarrow consider shearing!

Analysis of heat avalanche dynamics: 3

- Saturation: shearing strength to kill clustering instability

avalanche growth \rightarrow profile corrugation \rightarrow ExB staircase $\rightarrow v'_{E \times B}$

$$v'_{E \times B} \sim \gamma_{max} \quad \begin{array}{l} \uparrow \\ \text{-----} \\ \downarrow \end{array}$$

\rightarrow guesstimate, only \rightarrow onset, only, of step formation

- Saturated amplitude: $\frac{\delta T}{T_i} \sim \frac{1}{v_{thi} \rho_i} \sqrt{\frac{\chi_4}{\tau}}$

from $\gamma_{max} \cong \frac{1}{2\tau} \sqrt{\frac{v_0^2 \tau}{\chi_2}} \sim \frac{\lambda T_i}{2\sqrt{\chi_2 \tau}} \frac{\delta T}{T_i}$ and $v'_{E \times B} \cong \frac{c}{eB} \delta T'' \sim \omega_{ci} \rho_i^2 k_{max}^2 \frac{\delta T}{T_i} \sim \omega_{ci} \frac{\rho_i^2 \lambda T_i}{2\sqrt{\chi_2 \chi_4}} \left(\frac{\delta T}{T_i}\right)^2$

\rightarrow Radial Force Balance

- Staircase width: $\Delta_{stair}^2 \sim k_{max}^{-2} (\delta T) \sim \frac{2v_{thi}}{\lambda T_i} \rho_i \sqrt{\chi_2 \tau}$ $\chi_2 \sim \chi_{neo}$

\rightarrow Geometric mean of ρ_i and $\sqrt{\chi_2 \tau}$

\rightarrow neoclassical diffusion length in 1 relaxation time

\rightarrow 'standard' numbers: $\Delta_{stair} \sim 10\Delta_c$

Toward Systematics (1):

- to evolve \mathcal{T} self-consistently, need $\partial_t \mathcal{E}$ eqn. for $1/\tau[\mathcal{E}]$

→ Turbulence intensity evolution

- Feedback system for δT and \mathcal{E} , with Q as control parameter

- Key idea: \mathcal{T} long, near marginal

- $Q = -\chi \nabla T$ replaced by

$$\tau \partial_t Q + Q + \chi_T \nabla T + \text{heat flux scattering} = S_0$$

akin intensity equation for Q \longleftrightarrow c.f. turbulence spreading

Toward Systematics (2):

- can 'derive' $\partial_t Q$ eqn.

(extended Guyer-Krumhansl relation, relevant to transport in phonon gas)

from moments of: $\partial_t \langle \delta f^2 \rangle \rightarrow$ 2pt correlation evolution

\rightarrow c.f. O.D.G, P.D., Garbet, et. al.

- re-enforces point that 2pt correlation evolution is **FUNDAMENTAL**. Those Space fluxes are production there.
- for experiment: need separate Q from $\nabla \langle T \rangle$, with high resolution!
i.e. severe challenge

A Glimpse at Some Results

- 2pt correlation evolution

$$\left(\frac{\partial}{\partial t} + \mathcal{L}_1 + \mathcal{L}_2\right) \langle \tilde{f}_1 \tilde{f}_2 \rangle + \mathcal{P}_1 \bar{f}_1 + \mathcal{P}_2 \bar{f}_2 = \nabla_1 \cdot \Gamma_{Q_1} + \nabla_2 \cdot \Gamma_{Q_2} + \partial_{v_{\parallel 1}} \mathcal{K}_1 + \partial_{v_{\parallel 2}} \mathcal{K}_2$$

$$\mathcal{L}_{1,2} = v_{\parallel 1,2} \nabla_{\parallel 1,2} + \frac{q}{m} \bar{E}_{\parallel 1,2} \frac{\partial}{\partial v_{\parallel 1,2}} \quad \mathcal{P}_{1,2} = \langle \tilde{f}_{1,2} \tilde{\mathbf{v}}_{E1,2} \rangle \cdot \nabla_{1,2} + \frac{q}{m} \langle \tilde{f}_{1,2} \tilde{E}_{\parallel 1,2} \rangle \frac{\partial}{\partial v_{\parallel 1,2}}$$

$$\Gamma_{Q_{1,2}} = \langle \tilde{\mathbf{v}}_{E1,2} \tilde{f}_1 \tilde{f}_2 \rangle \quad \mathcal{K}_{1,2} = \langle \tilde{E}_{\parallel 1,2} \tilde{f}_1 \tilde{f}_2 \rangle$$

- Energy moment \rightarrow Guyer-Krumhansl Structure

$$\partial_t Q - \partial_x (D_x(\mathcal{E}) \partial_x Q) = -D_y(\mathcal{E}) k_y^2 (Q - \chi(\mathcal{E}) \partial_x \langle T \rangle)$$

\downarrow
spreading

\downarrow
turbulent poloidal
decorrelation

\searrow
mean turbulent
heat flux

\rightarrow couples
relaxation to
heat flux
scattering

Summary

- A model for ExB staircase formation

Heat avalanche → Profile corrugation → ExB staircase

Profile corrugation due to ‘jam’ of heat flux avalanche!

Key:

- Analysis of heat flux jam dynamics

→ clustering instability when plasma response time is long

$$\gamma = -\frac{1}{2\tau} + \frac{1}{2\tau} \sqrt{\frac{r+1}{2} - 2\tau\chi_2 k^2 \left(1 + \frac{\chi_4 k^2}{\chi_2}\right)} \quad \tau > \frac{\chi_2}{v_0^2} \left(1 + \frac{\chi_4 k^2}{\chi_2}\right)$$

→ length scale of most jamming unstable modes → step size

$$\Delta_{stair}^2 \sim k_{max}^{-2} (\delta T) \sim \frac{2v_{thi}}{\lambda T_i} \rho_i \sqrt{\chi_2 \tau}$$

Summary

More importantly, new perspective on:

- SOC and relaxation
- Shear flow formation
- Formulation of heat transport and spreading

C.) Looking Ahead

- MFE confinement theory evolves and continues to be a dynamic research field
- a recent success study : intrinsic rotation
 - discovered in '95 (Ida), '97 (Rice)
 - extensively investigated by theory and simulation 2005 –
 - now entering maturity ...
- ➔ simulation, especially GK, increasingly central but need diversify simulation models, approaches

→ ITER issue

— need a change in concept:

→ '60 – now: design for optimal confinement

→ now – ITER, DEMO: design for optimal power handling

— corollary: Is the H-mode the way to go?

n.b. : really seek optimal energy, not particle, confinement

— a possibility: I-mode (c.f. AUG, Alcator C-Mod, ...)

— H-mode energy confinement with L-mode particle confinement

— sub-critical to transitions but strong zonal mode activity

— much to learn ...

→ R. Z. Sagdeev's impact on MFE physics, initiated in early 60's, is strong and visible in 2013