Long Distance Physics Mediated by Continuous Spin Particles

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SLAC

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based on 2303.04816 (JHEP) with N. Toro, **K. Zhou,** 2308.16218 with N. Toro,
work in progress with **Kundu**, Toro, **Sundaresan**, **Zhou**,
(see also 1302.1198, 1302.1577, 1404.0675 with N. Toro)

Outline

- Helicity states can mix under Lorentz boosts – controlled by ***spin scale*** ρ [Wigner 1939]
- **Coupling to matter particles is predictive (new features in the IR), and closely connected to familiar theories** – we've had hints for a while [1302.1577], now have exact scheme to calculate both classical physics and amplitudes in putative theory
  - Part I — Top line summary of the *what, why, and how* of CSPs, and a few illustrative results.
  - Part II — Superspace-like formalism as a useful tool [1404.0675]. Coupling matter particles to fields with nonzero ρ, aka "Continuous spin fields".
  - Part III — Example results and correspondence with familiar theories (EM & Gravity)
- Speculation about CSPs in the Standard Model and future directions (as time permits)

Massless Spin, Covariantly

Spin state → σ; Internal charges → n

Physical states take the form $|p^\mu, \sigma, n\rangle$

Spin σ characterizes state's transformation under **little group** – subgroup of Lorentz that preserves p. Generators are 3 components of $W^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}J_{\nu\rho}p_\sigma$

E.g. for massive particle at rest, $W^\mu = (0, m\mathbf{J})$.

Spatial components generate SO(3).

Natural relativistic invariant is $W^2 = -m^2\mathbf{J}^2 = -m^2 s(s+1)$

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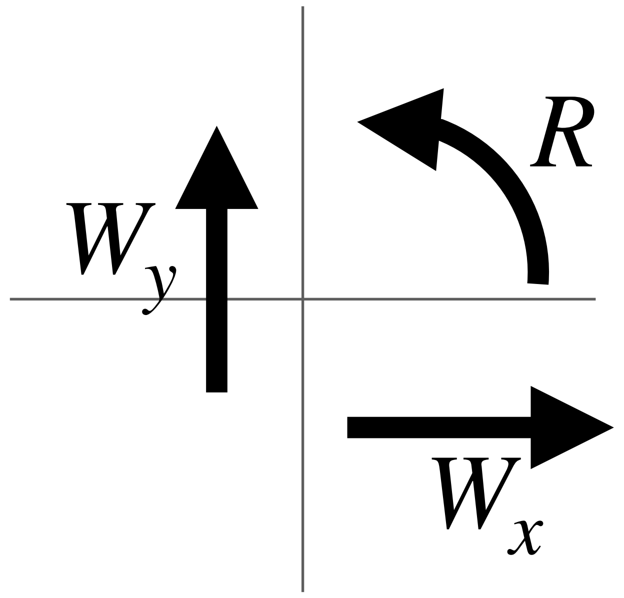
For massless particle:

$W^0, \mathbf{W} \cdot \hat{\mathbf{p}} \propto$ familiar helicity generator $R = \mathbf{J} \cdot \hat{\mathbf{p}}$.

Transverse spatial components are less familiar, involve rotation **and** boost

e.g. for $\mathbf{p} \propto \hat{\mathbf{z}}$, $W_x \propto J_x + K_y$ and $W_y \propto -J_y + K_x$



They commute! Group structure is ISO(2)

The natural relativistic invariant is $W^2 = -(W_x^2 + W_y^2)$ – independent of helicity R!

Massless Spin, Covariantly

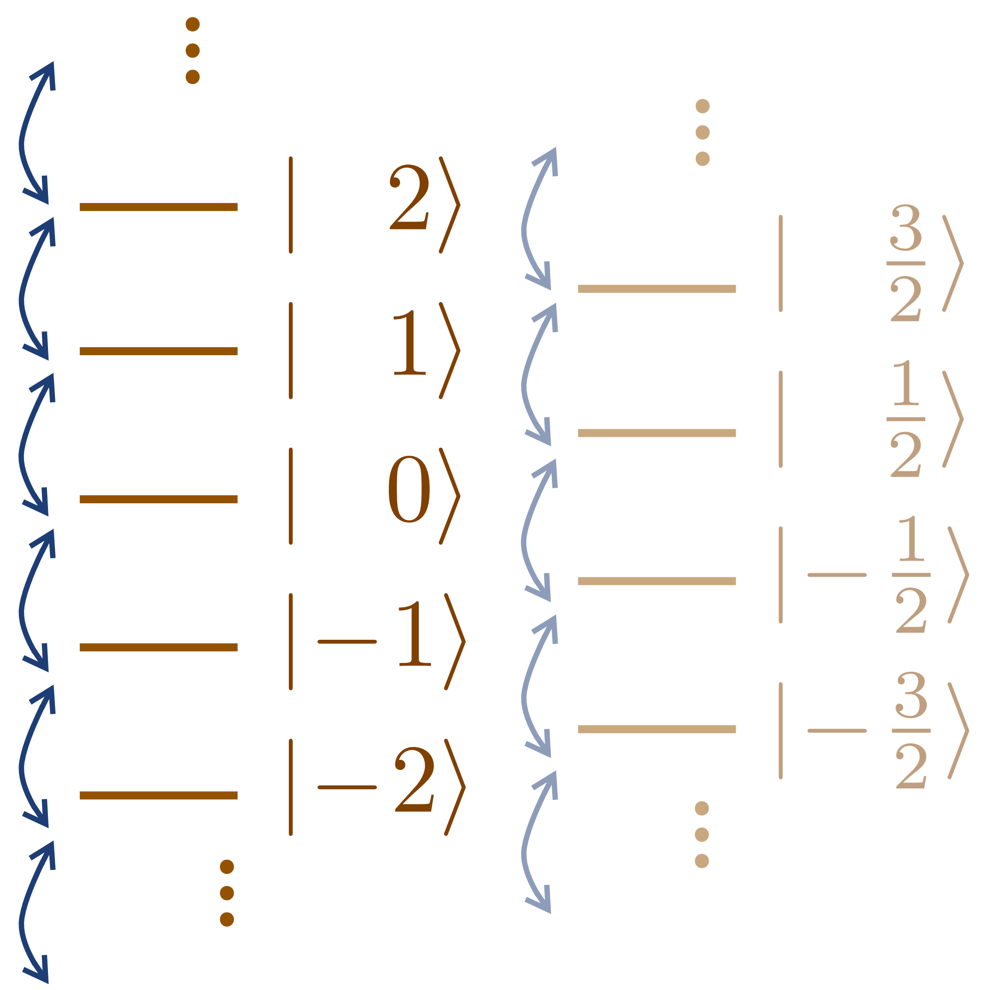
It's convenient to work in a helicity eigenstate basis: $\mathbf{J} \cdot \hat{\mathbf{p}} \, | p, \sigma \rangle = \sigma \, | p, \sigma \rangle$,

Eigenvalues σ must be (half-)integer so that 4π rotation returns state to itself, since Lorentz group is doubly connected.

Build raising/lowering operators from "translations": $W_\pm = W_x \pm i W_y$, with $[R, W_\pm] = \pm W_\pm$

$W_\pm \, | p, \sigma \rangle = \rho \, | p, \sigma \pm 1 \rangle$ where the invariant $W^2 = -\rho^2$ sets the **spin-scale** ρ.

σ-independent coefficient $\Rightarrow$ ***infinite*** ladder of states in one representation.



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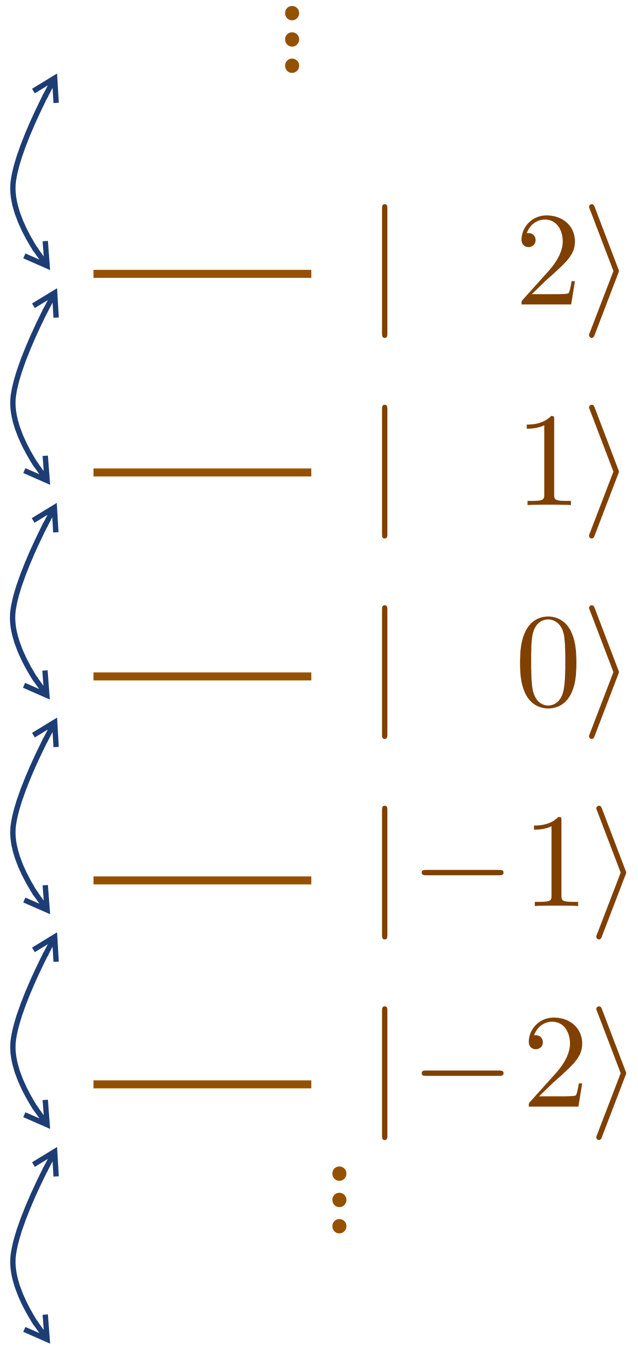

Exception: if $\rho = 0$ the states decouple. Each $|\sigma\rangle$ is a singlet representation, related only to $|-\sigma\rangle$ by CPT. **Familiar massless theories fall under this exception.**

The general case $\rho \neq 0$, where helicities mix under Lorentz boosts – just as they do for massive particles – is known as "infinite" or **"continuous" spin**.

Quick, *but too quick*, reasons to ignore $\rho \neq 0$

(to explain why this case has largely been ignored until recently)

Continuous spin includes high helicity states. Massless high spin is sick. Aren't these?

Robust constraints on high helicities (e.g. Weinberg soft theorems, Weinberg-Witten) all rely deeply on boost-invariance of helicity, so they don't directly apply when $\rho \neq 0$.

Massive high spin is a somewhat better analogy, and can be consistent – *e.g.* nuclei and string theory

Are infinitely many states at fixed energy a problem? (Cross-sections, thermodynamics)

Very interesting resolution follows from Lorentz symmetry (at least for best-controlled calculations)

At frequencies $\gg \rho$, **all but one helicity have parametrically suppressed interactions.**
The dominant interaction can be "scalar-like", "vector-like", or "tensor-like"

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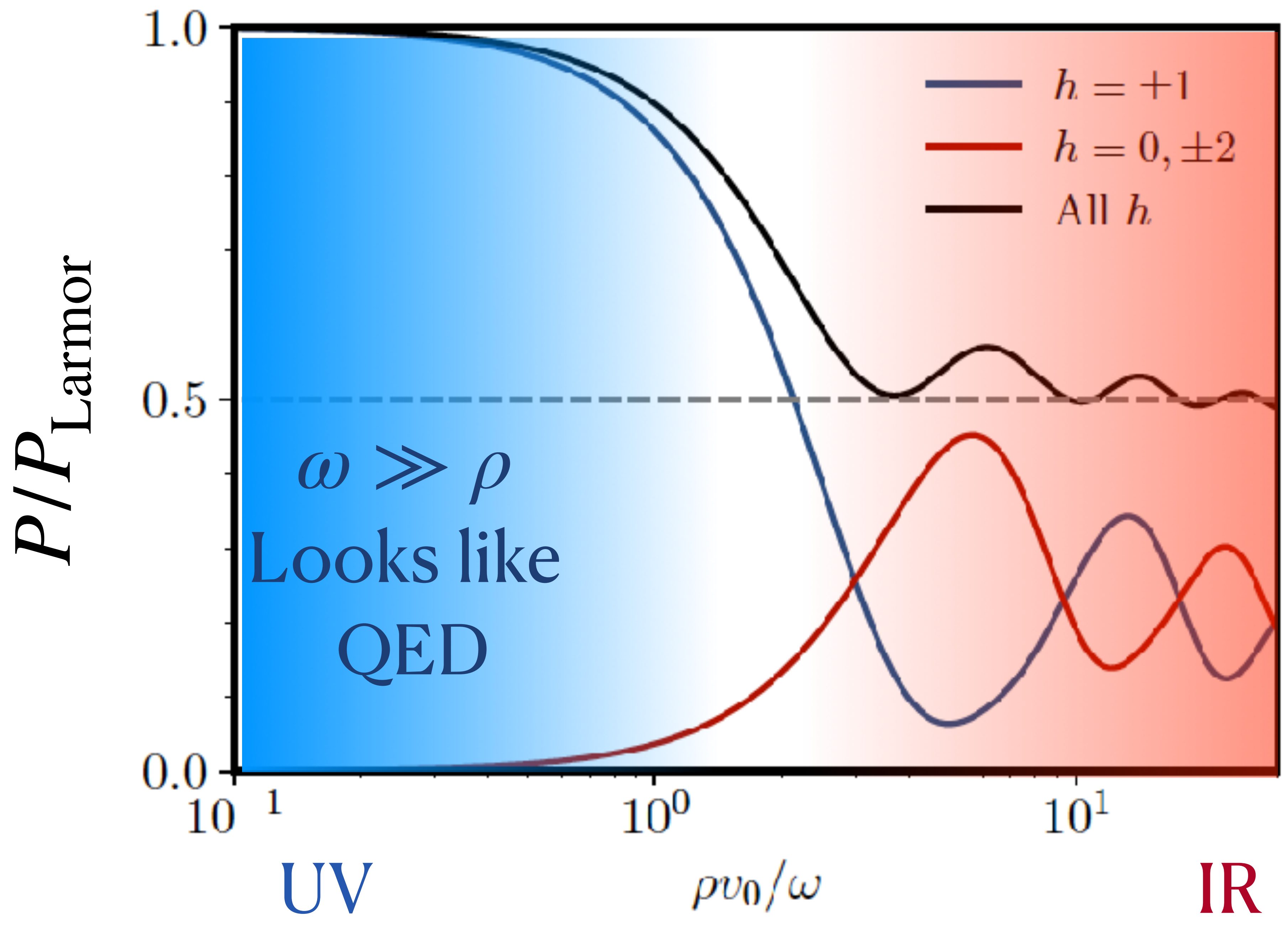
This is the same basic story that intrigued us a decade ago.
New: sharp predictive calculations that affirm the story!

Concrete Predictions: Vector-Like Coupling Class

Classical radiation from an oscillating particle:

Larmor power — Leading correction

$$P = \frac{e^2\omega^2 v_0^2}{12\pi}\left(1 - \frac{9}{80}\frac{\rho^2 v_0^2}{\omega^2} + \dots\right)$$

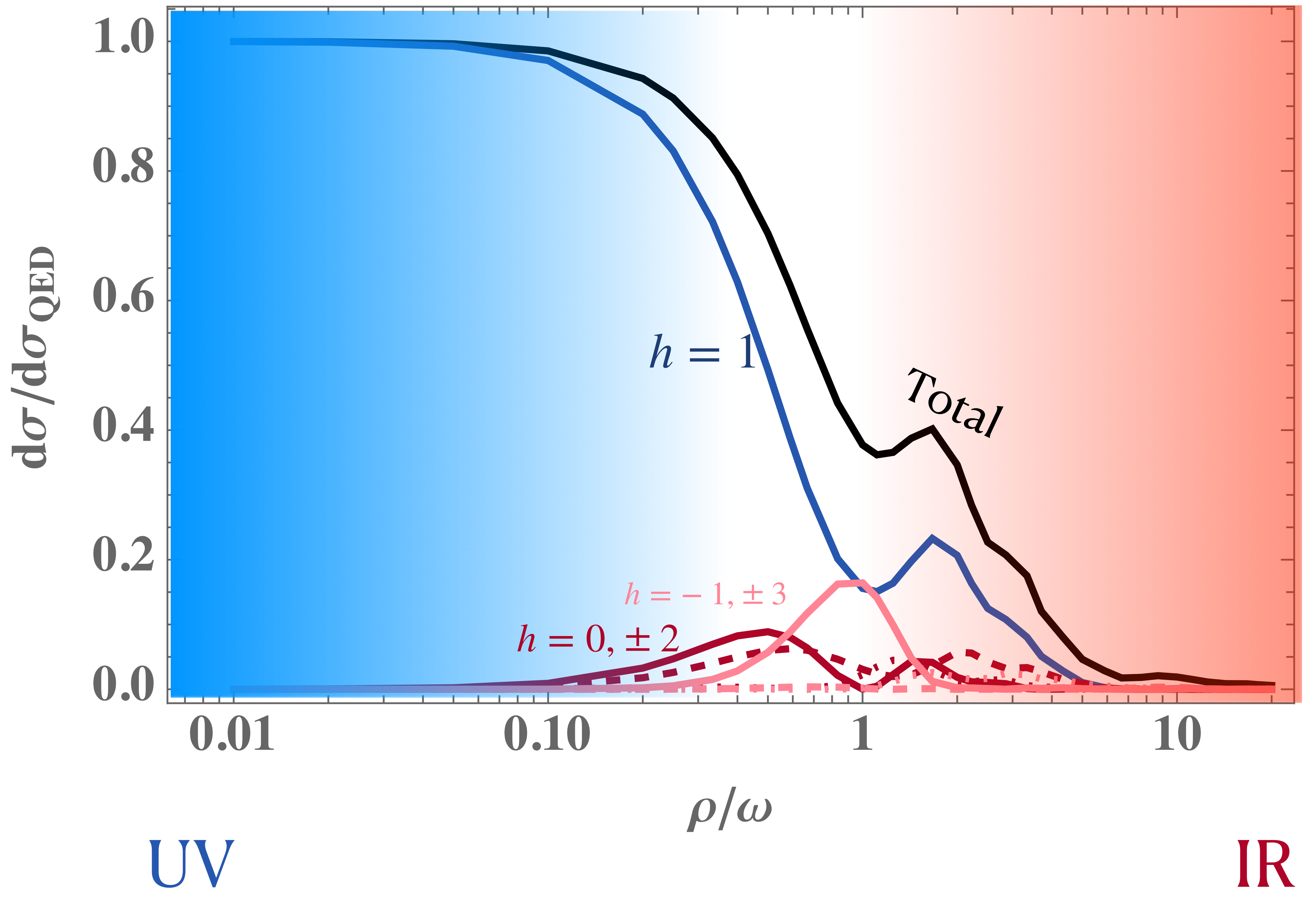


$\omega \ll \rho$

All $h \lesssim \rho/\omega$ relevant; finite limiting power

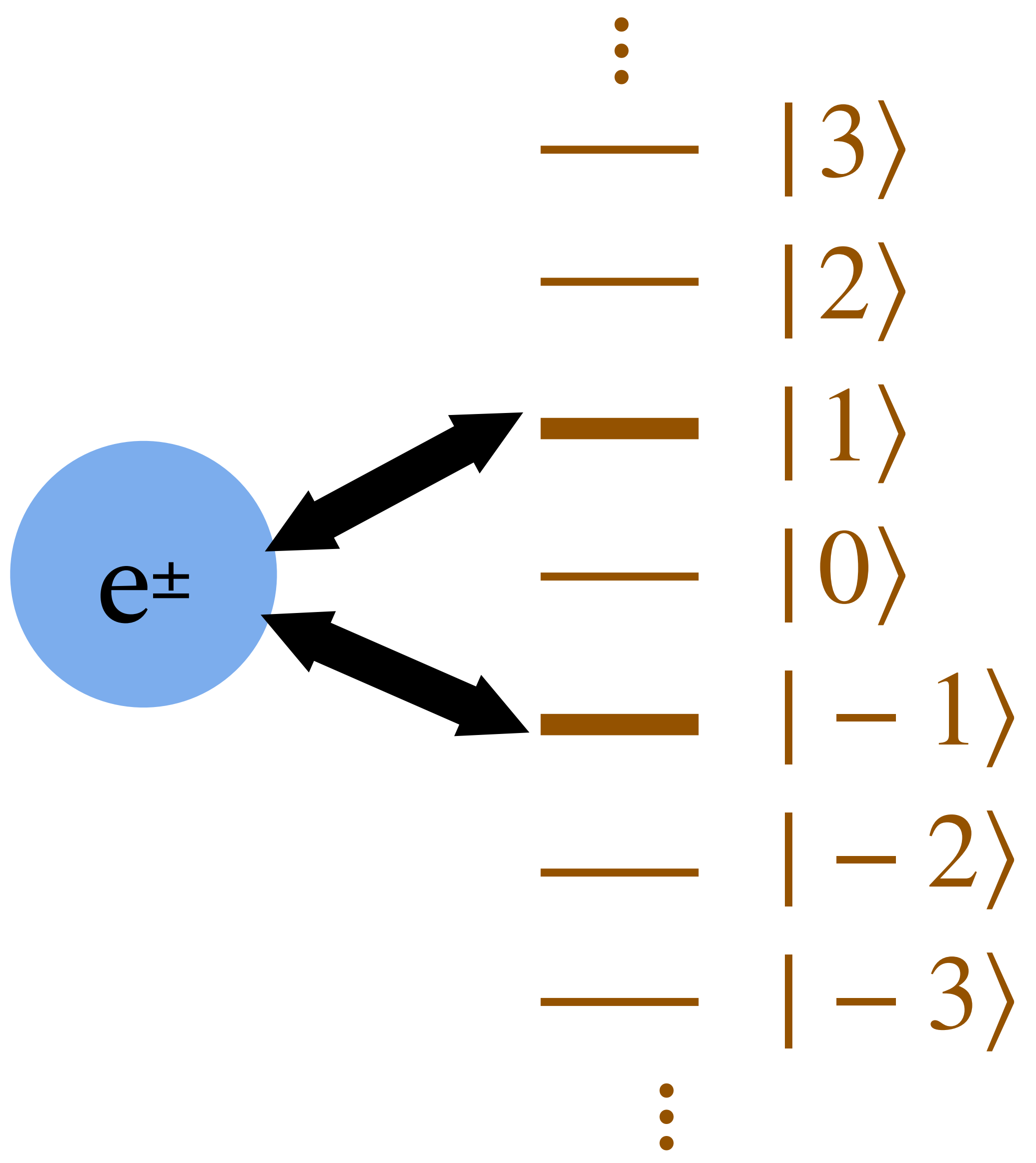
Scattering amplitudes computed using vertex operators

Massless scalar Compton (fixed kinematics)



Continuous Spin Particles are like familiar massless particles with an associated dark sector

Covariant interactions single out **one** helicity with unsuppressed coupling (e.g. |h|=1)



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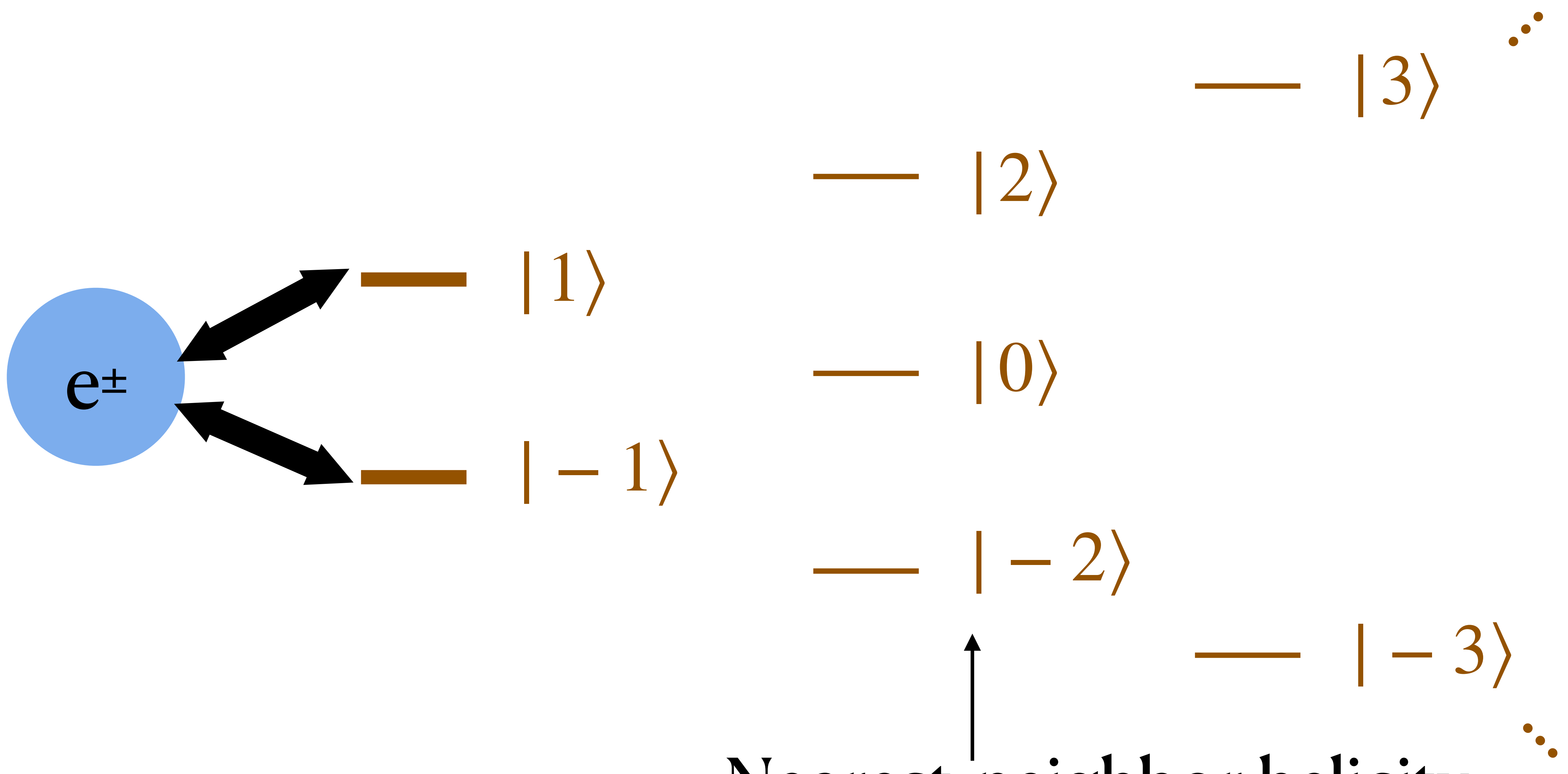

Nearest-neighbor helicity states' interactions suppressed by ρ/ω

Higher helicities' interactions suppressed by more powers of ρ/ω

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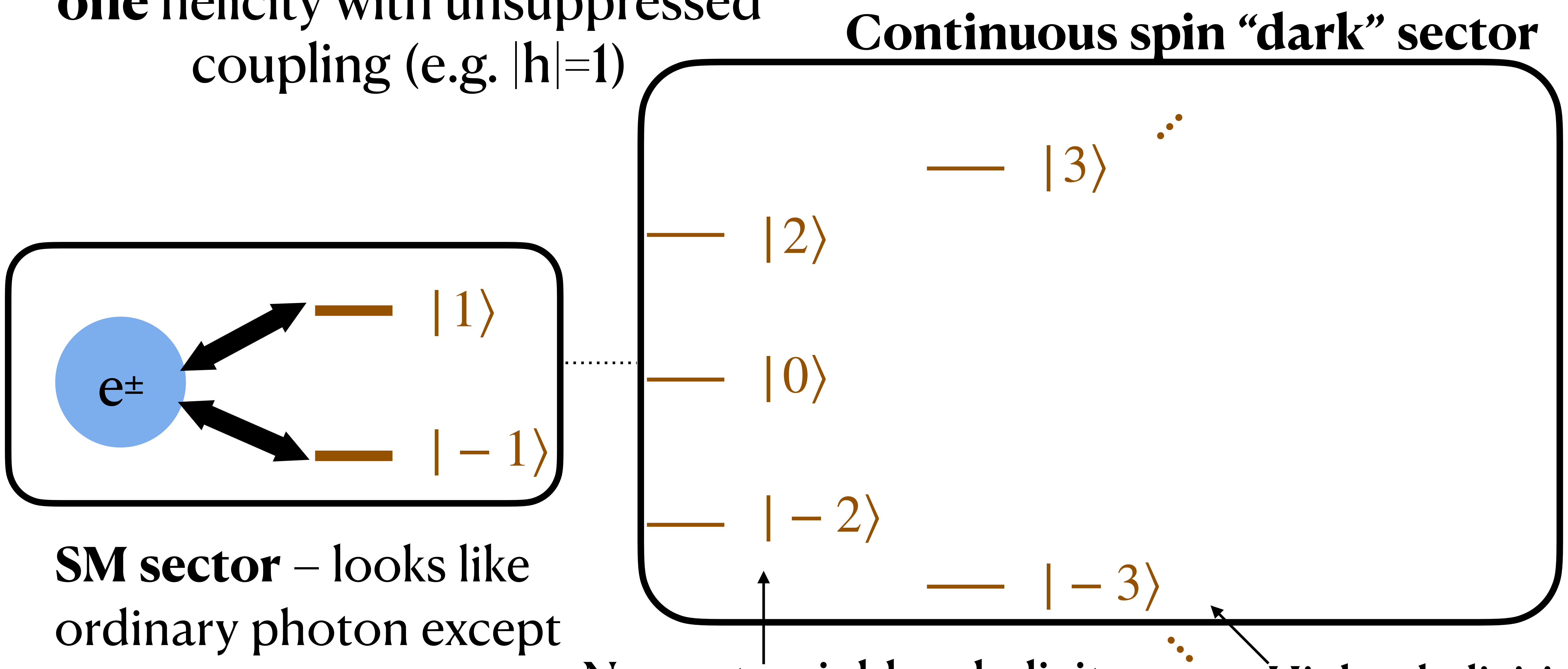


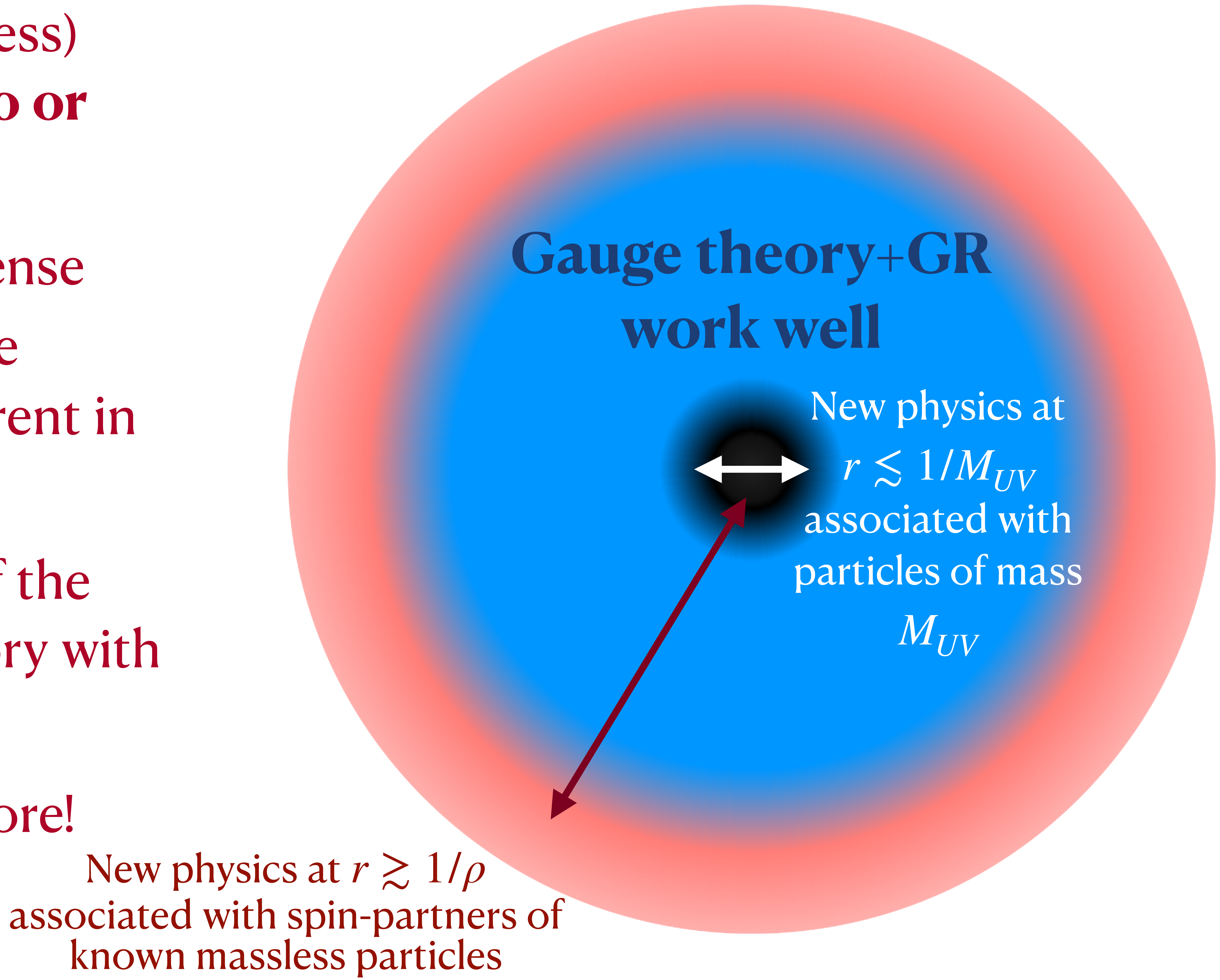
SM sector – looks like ordinary photon except in deep IR $\omega \lesssim \rho$

Nearest-neighbor helicity states' interactions suppressed by ρ/ω

Higher helicities' interactions suppressed by more powers of ρ/ω

Top Line Summary

- Lorentz invariance → known (massless) particles have a spin-scale. **Is it zero or non-zero?**
- The non-zero option makes more sense than previously thought → looks like familiar theories in the UV, but different in the IR.
- If viable, perhaps we should think of the Standard Model as an effective theory with both UV and IR completions.
- Opens up many possibilities to explore!



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- Continuous spin particle has modes of every helicity, that separate into singleton representations as $\rho \to 0$
- Helicity $\pm h$ modes typically described by gauge theory of rank-h tensor fields
  - In the free (linear) case, expect Abelian gauge redundancy *(focus of this talk!)*
- Continuous spin field should, in $\rho \to 0$ limit, decompose into similar modes.
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$$\Psi(\eta, x) \equiv \phi^{(0)}(x) + \eta^\mu \phi^{(1)}_\mu(x) + \eta^\mu \eta^\nu \phi^{(2)}_{\mu\nu}(x) + \dots$$

Lorentz acts as $x \to \Lambda x, \eta \to \Lambda \eta$

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- Action:

$$S = \frac{1}{2}\int_{\eta,x} \delta'(\eta^2+1)(\partial_x\Psi)^2 + \frac{1}{2}\delta(\eta^2+1)(\Delta\Psi)^2 \quad \text{with } \Delta\Psi \equiv \partial_\eta \cdot \partial_x + \rho$$

(Free) Abelian Field Theory

When $\rho = 0$, components encode familiar actions, e.g. decomposing

$$\Psi(\eta, x) = \phi(x) + \eta^\mu A_\mu(x) + \left(\eta^\mu \eta^\nu - \frac{1}{2}(\eta^2 + 1) g^{\mu\nu}\right) h_{\mu\nu}(x) + \ldots$$

and plugging into action yields

$$\mathscr{L}[\Psi] = \frac{1}{2}(\partial_x \phi)^2 + \frac{1}{2}\left((\partial_\mu A_\nu)^2 + \partial_\mu A^\mu)^2\right) + \textbf{Fierz-Pauli} + \sum \textbf{Fronsdal}$$

Working in η-space directly is much more compact – and **vastly** simpler for $\rho \neq 0$.

(Free) Abelian Field Theory

Strong Analogy with Maxwell Action!

Action

$$\int_x -\frac{1}{2}(\partial_\mu A_\nu)^2 + \frac{1}{2}(\partial \cdot A)^2$$

$$\frac{1}{2}\int_{x,\eta} \delta'(\eta^2+1)(\partial_x \Psi)^2 + \frac{1}{2}\,\delta(\eta^2+1)(\Delta\Psi)^2$$

Equation of Motion and gauge invariance

$$\Box A_\mu - \partial_\mu \partial \cdot A = 0$$

$$A_\mu \simeq A_\mu + \partial_\mu \epsilon(x)$$

$$\delta'(\eta^2+1)\,\Box\,\Psi(\eta,x) - \frac{1}{2}\Delta(\delta(\eta^2+1)\Delta\Psi) = 0$$

$$\Psi(\eta,x) \simeq \Psi(\eta,x) + \left(\eta\cdot\partial_x - \frac{1}{2}(\eta^2+1)\Delta\right)\epsilon(\eta,x) + (\eta^2+1)^2\chi(\eta,x)$$

(Free) Abelian Field Theory

Strong Analogy with Maxwell Action!

Covariant Gauge Fixing	$\partial \cdot A = 0$	$\delta(\eta^2+1)\Delta\Psi(\eta,x) = 0$
Gauge-Fixed EOM and	$\Box A_\mu = 0$	$\delta'(\eta^2+1)\Box\Psi = 0$
Basis of physical states	$\psi_{\pm,k}(x) = e^{-ik\cdot x}\,\epsilon^\mu_\pm$	$\Psi_{k,h} = e^{-ik\cdot x}\,(\eta\cdot\epsilon_\pm)^{\lvert h\rvert}\,e^{-i\rho\eta\cdot q}$

Abelian Field Theory

Coupling to currents

Current Term in Action	$\delta S = -\int_x A^\mu(x) J_\mu(x)$	$\delta S = \int_{x,\eta} \delta'(\eta^2+1)\Psi(\eta,x)J(\eta,x)$
Continuity condition from gauge-invariance	$\partial_\mu J^\mu = 0$	$\delta(\eta^2+1)\Delta J(\eta,x) = 0$
EOM in suitable gauge	$\Box A^\mu = J^\mu$	$\Box \Psi(\eta,x) = J(\eta,x)$

Once we have found a suitable current, can use familiar machinery to compute physical quantities, e.g.

- Classical radiation and CSP-exchange forces [2303.04816]
- Scattering amplitudes [2308.16218]

Currents from Worldlines

Ordinary EM Example

For technical reasons, we've worked with matter **particles** and their **worldlines** rather than matter **fields**.

Well-established if less familiar, e.g. EM current for scalar matter described by $z^\mu(\tau)$

$$\delta S = -\int_x A^\mu(x) J_\mu(x)$$

$$J^\mu(x) = \int d\tau\, q\, \dot{z}^\mu(\tau)\, \delta^{(4)}(x - z(\tau))$$

$$\partial \cdot J(x) = -\int d\tau\, \partial_\tau \left[q\, \delta^{(4)}(x - z(\tau)) \right]$$

Conserved as long as worldlines only begin and end at charge-conserving vertices.

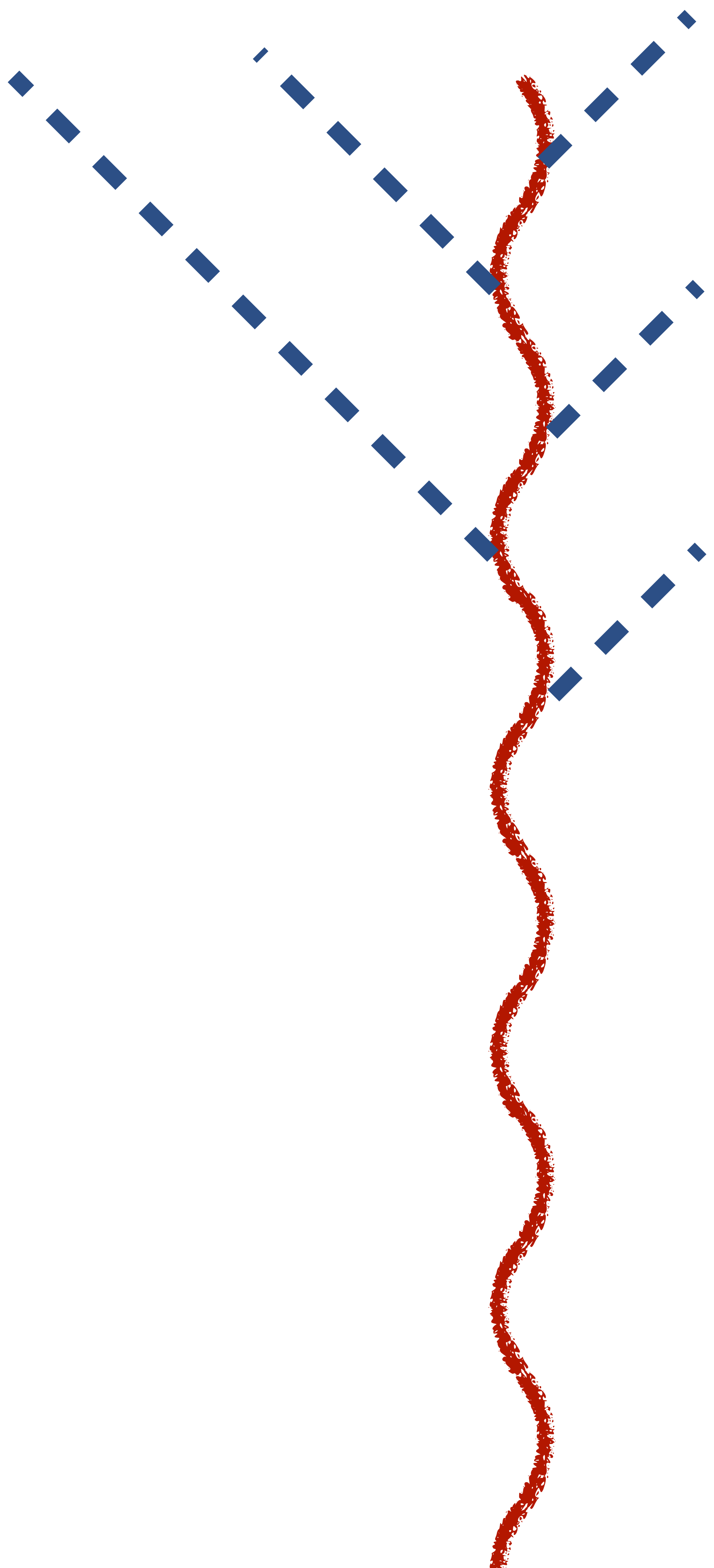
Currents from Worldlines

Maxwell EM: Classical Radiation from a Moving Particle

$$\frac{dP_h}{d\omega d\hat{\mathbf{r}}} = \frac{\omega^2}{8\pi^2} |\epsilon^{*\ \mu}_{h,k} J_\mu(k)|^2 \quad \text{with k} = (\omega, \omega\hat{\mathbf{r}})$$

For simple harmonic motion, power

$$P_{Larmor} = \frac{e^2\omega^2 v_0^2}{12\pi}$$



Currents from Worldlines

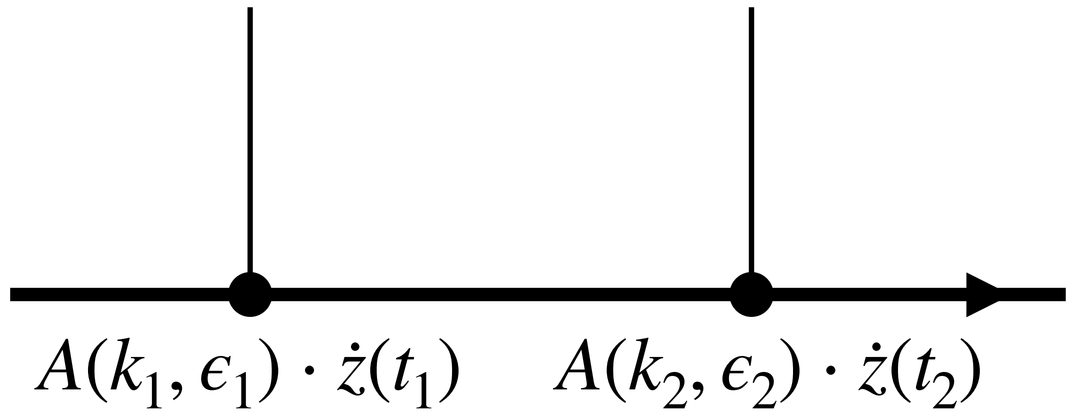
Familiar QED: Amplitudes

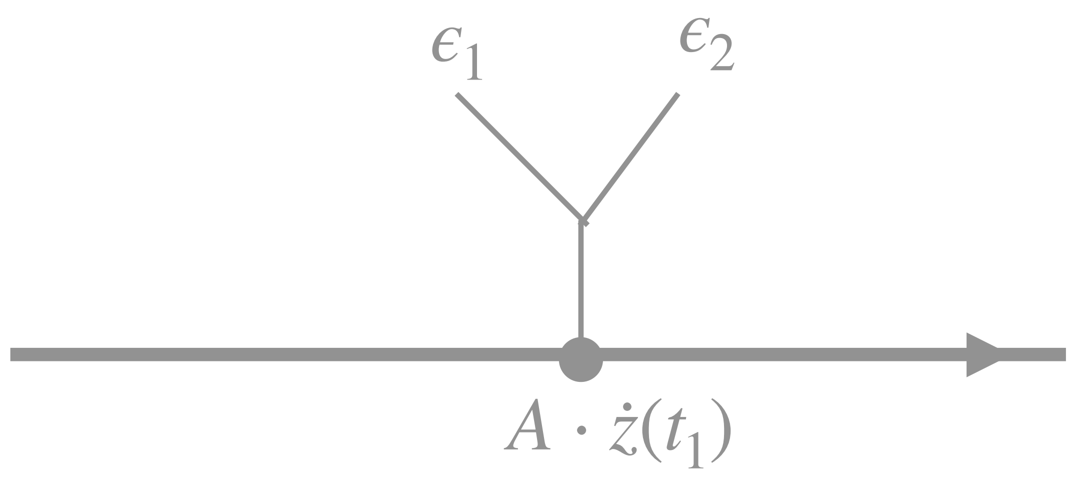
Compute amplitudes from path integral for worldline in EM field (Feynman 1950)

$$A(p,p',k_i,\epsilon_i) = \int_{\mathscr{P}[x,x']} Dz(\tau)\, e^{-S_{free}[z]}\, e^{-ip\cdot x}\, e^{ip'\cdot x'} \prod \int \underbrace{dt_i \left(\epsilon_i \cdot \dot{z}(t_i) e^{-ik_i\cdot z(t_i)}\right)}_{A^\mu_{ext} J_\mu} \Bigg|_{LSZ}$$

Current and Maxwell field theory are all you need to know to build amplitudes!

(More pieces needed for YM or GR theories with self-interacting fields)





Matter Currents Appropriate for CSP Field Interaction

To couple a particle's worldline to CSP field, need to find current from worldline data satisfying continuity condition.

$$J(\eta, x) = \int d\tau\, f(x - z(\tau), \dot{z}(\tau), \eta)$$ <— Worldline-local ansatz

$$= \int d\tau\, d^4k\ e^{ik\cdot(z(\tau)-x)} f(k, \dot{z}, \eta)$$

continuity condition $(-ik \cdot \partial_\eta + \rho) f = 0$

Matter Currents — General Solution

Most general solution to continuity condition (up to total derivative terms) can be written as

$$f(k, \dot{z}, \eta) = e^{-i\rho \frac{\eta \cdot \dot{z}}{k \cdot \dot{z}}} \hat{g}(k \cdot \dot{z}) + \mathcal{O} X(k, \dot{z}, \eta)$$

Free EOM operator

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Worldline interactions with on-shell radiation **fully** determined by $\hat{g}$.

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Free EOM operator

Worldline interactions with on-shell radiation **fully** determined by $\hat{g}$.

Expanding $\hat{g}$ in Taylor series gives "universality classes" of currents:

$$\hat{g} = \begin{cases} g & \text{scalar-like current} \\ \frac{e}{\rho}k\cdot\dot{z} & \text{vector-like current} \\ (k\cdot\dot{z})^n/\Lambda^n & \text{Tensor-like \& non-minimal currents*} \end{cases}$$

scalar-like current, vector-like current: Classical results in these cases are main focus of 2303.04816

Tensor-like & non-minimal currents*: GR-like is a special case

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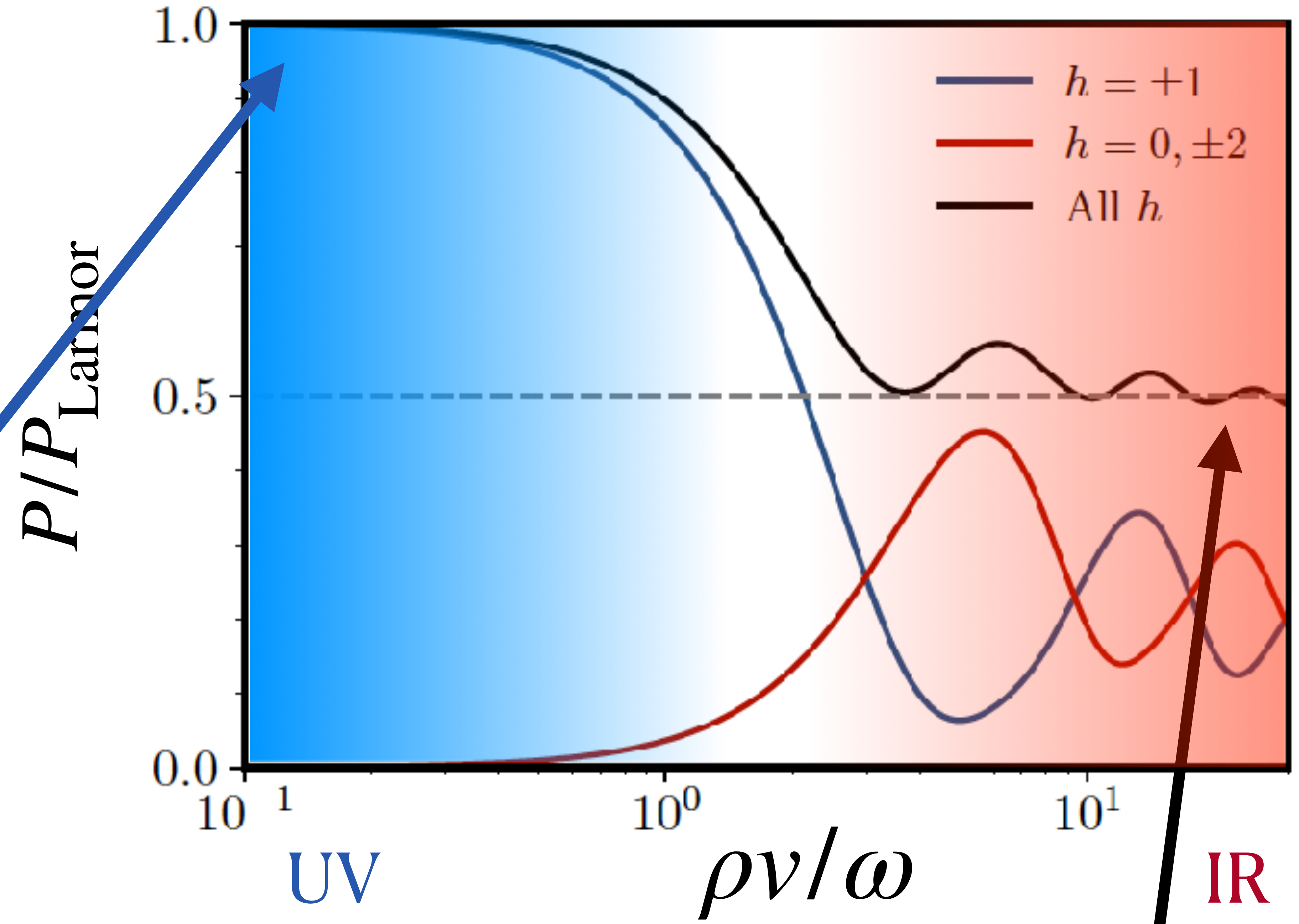
Radiation from a Moving (Oscillating) Particle

$$f(k,\eta,\dot{z}) = -\frac{e}{\rho} k\cdot\dot{z}(\tau)\, e^{-i\rho\frac{\eta\cdot\dot{z}}{k\cdot\dot{z}}}$$

For "vector-like" currents:

$$P = \frac{e^2\omega^2 v_0^2}{12\pi}\left(1 - \frac{9}{80}\frac{\rho^2 v_0^2}{\omega^2} + \dots\right)$$

Standard Larmor power



For small $\rho v/\omega$, power **matches Larmor** and **dominated by h=±1 modes**

(Physical manifestation of formal correspondence noted earlier)

At large $\rho v/\omega$, power spread among many modes, harmonics **but total power emitted has finite limit.**

Classical Electromagnetism with $\rho \neq 0$

= CSP with vector-like interaction!

$$f(k,\eta,\dot{z}) = -\frac{e}{\rho} k \cdot \dot{z}(\tau)\, e^{-i\rho\frac{\eta\cdot\dot{z}}{k\cdot\dot{z}}}$$

(Vector-like current)

$$= -\frac{e}{\rho} k \cdot \dot{z}(\tau) + ie\,\eta \cdot \dot{z}(t) + \mathcal{O}(\rho)$$

(Small-ρ behavior of current)

Physically irrelevant (changes J by total τ-derivative)

η-space form of usual vector current

⇒ Leading physical effects should be QED-like!

$$J(\eta, x) = \int d\tau\, d^4k\ e^{ik\cdot(z(\tau)-x)} f(k,\dot{z},\eta)$$

Scalar QED with $\rho \neq 0$

(Compton-like scattering amplitudes)

Structure of the calculation is identical to QED – η-dependent vertex operator yields matrix elements which can be contracted with basis wave-functions to get polarization amplitudes.

$$M(p_0, p_3, \{k_1, \eta_1\}, \{k_2, \eta_2\}) = 2\int_{-1}^{1} dx \left(\eta_1 - \frac{\eta_1 \cdot P_1(x)}{k_1 \cdot P_1(x)} k_1\right) \cdot \left(\eta_2 - \frac{\eta_2 \cdot P_2(x)}{k_2 \cdot P_2(x)} k_2\right) e^{-i\rho \frac{\eta_1 \cdot P_1(x)}{k_1 \cdot P_1(x)} - i\rho \frac{\eta_2 \cdot P_2(x)}{k_2 \cdot P_2(x).}}$$

$P_{1,2}(x) = p_3 - p_0 \pm x\, k_{2,1}$ → at endpoints $x = \pm 1$, these are momenta appearing in $s(u)$-channel photon vertex

(1) no unphysical singularities, (2) factorization at physical singularities, (3) finite angle-differential cross-section at all energies.

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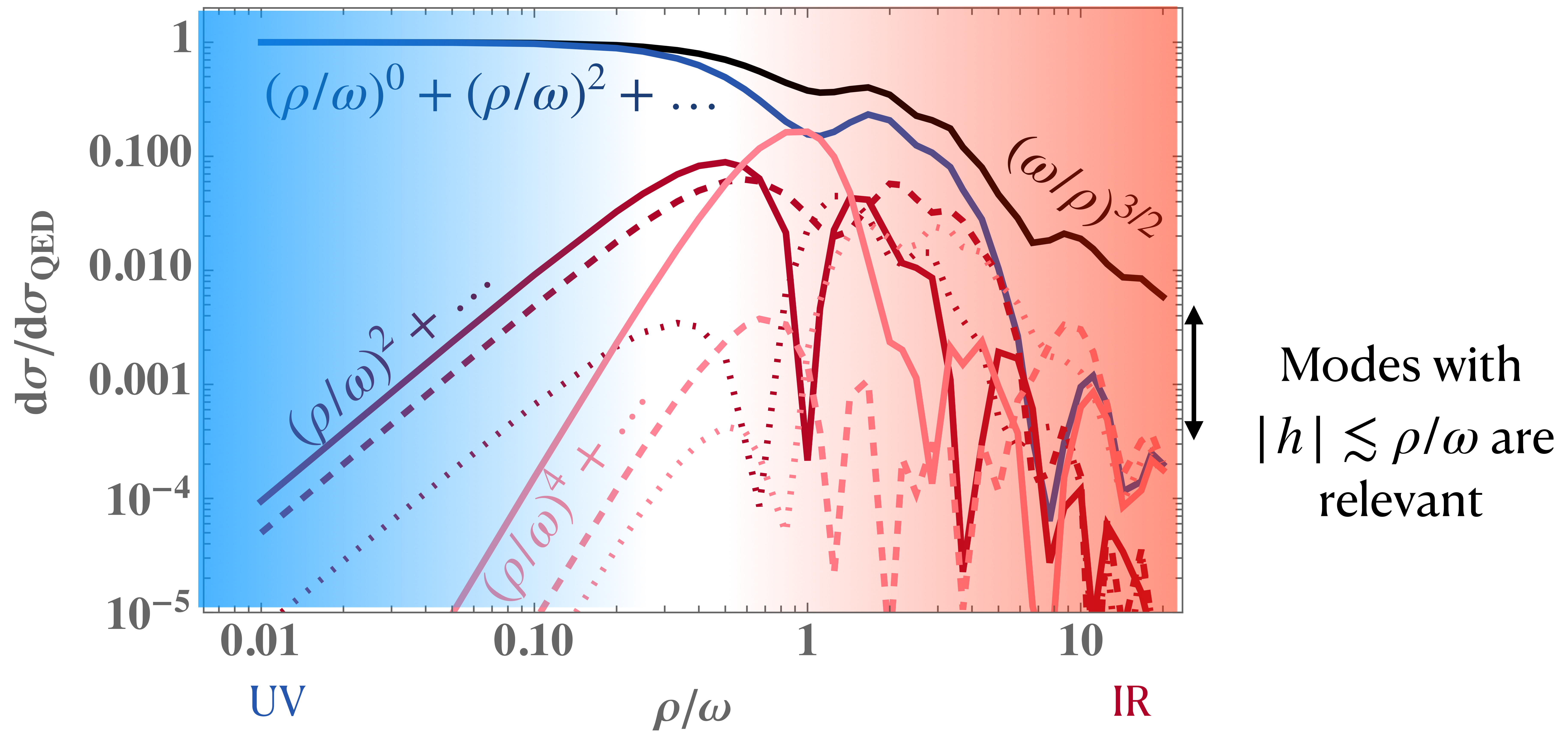
1

This clearly has smooth $\rho \to 0$ limit (just drop phase).

Linear in η_1 and η_2 implies only modes $h = \pm 1$ survive. [In this case, $\eta_i \sim \epsilon_i$]

Compton Cross-Section at $\rho \neq 0$: UV to IR

(Similar for CSP pair production)



Scalar-like current → qualitatively similar behavior.

Gravity (GR) at non-zero ρ

= CSP with tensor-like interaction current!

$$f(k, \eta, \dot{z}) = \kappa \left(\frac{k \cdot \dot{z}}{\rho} \right)^2 \left(e^{-i\rho \frac{\eta \cdot \dot{z}}{k \cdot \dot{z}}} + \dots \right)$$

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$$= \text{total derivatives } + \boxed{\kappa(\eta\cdot\dot{z})^2} + O(\rho)$$ (Small-ρ behavior of current)

η-space form of usual stress energy current (massless in this case)

$\Rightarrow$ Leading physical effects should be GR-like!

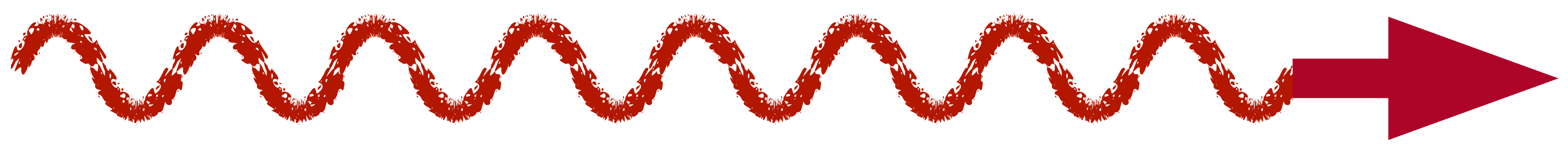
Note: As in familiar GR, this treatment only works to leading order in κ.
Graviton self-interactions and acceleration dependent terms enter at $O(\kappa^2)$

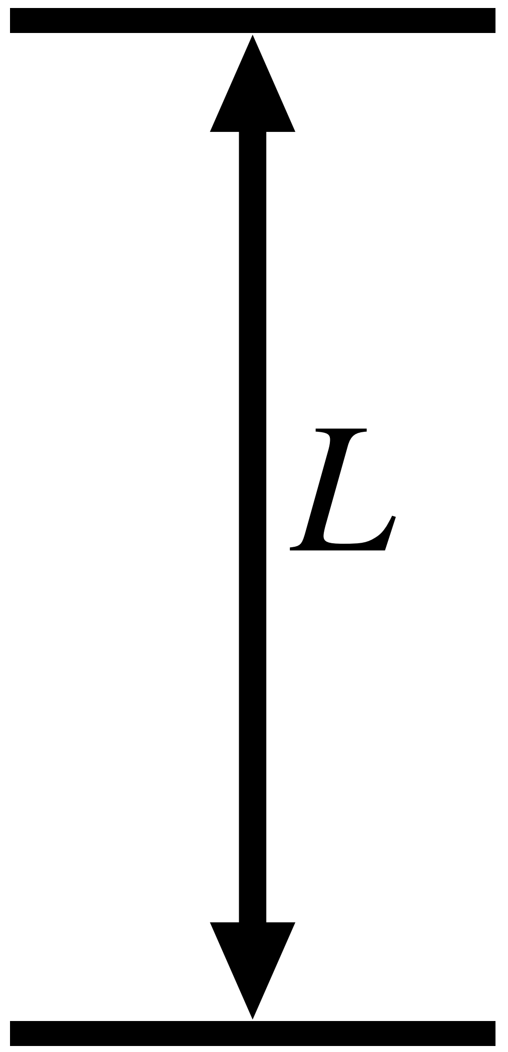
Gravitational Time-Delay

(CSP with tensor-like interaction current)

Simplest (most natural) case: h=2 mode of gravitational CSP on-shell wave:

$$\Psi_{h=2} = h_+ \left((i\eta \cdot \epsilon_+)^2 + (i\eta \cdot \epsilon_-)^2\right) \left(e^{-i\rho\eta\cdot q}e^{-ik_0\cdot x} + e^{i\rho\eta\cdot q}e^{ik_0\cdot x}\right)$$





Consider time delay of massless particle traversing two mirrors in the presence of gravitational wave

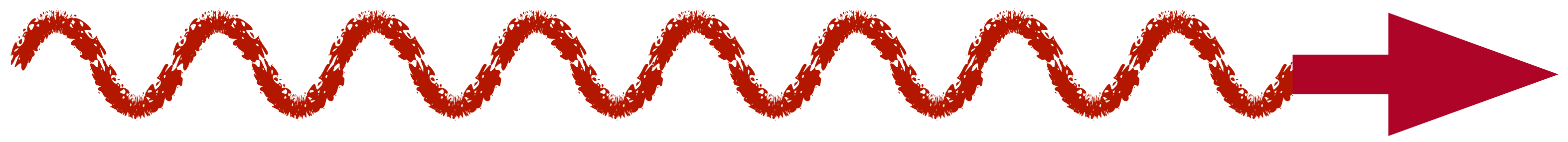
[Work in “TT” gauge where the mirrors remain at rest]

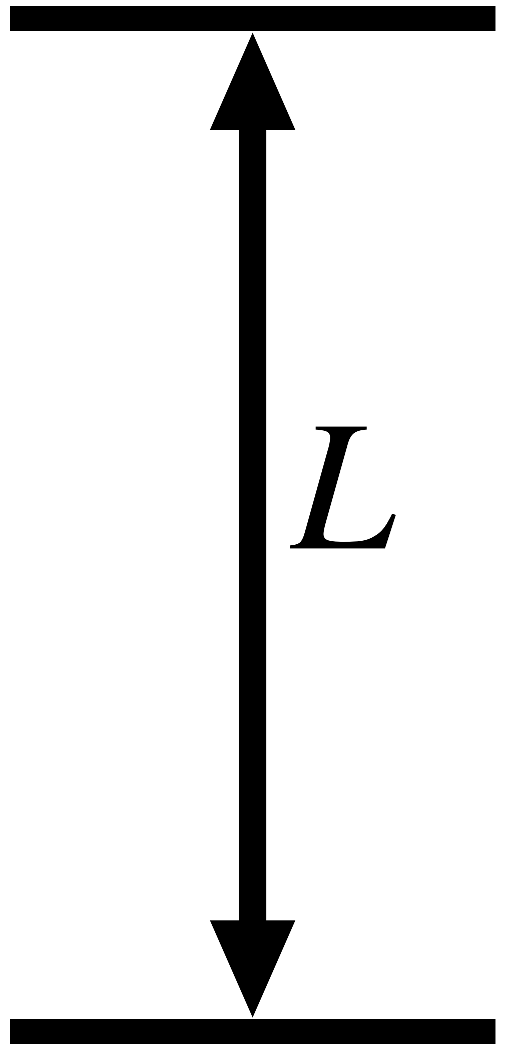
Gravitational Time-Delay

(CSP with tensor-like interaction current)

Simplest (most natural) case: h=2 mode of gravitational CSP on-shell wave:

$$\Psi_{h=2} = h_+ \left((i\eta \cdot \epsilon_+)^2 + (i\eta \cdot \epsilon_-)^2 \right) \left(e^{-i\rho\eta\cdot q} e^{-ik_0\cdot x} + e^{i\rho\eta\cdot q} e^{ik_0\cdot x} \right)$$





Consider time delay of massless particle traversing two mirrors in the presence of gravitational wave

[Work in “TT” gauge where the mirrors remain at rest]

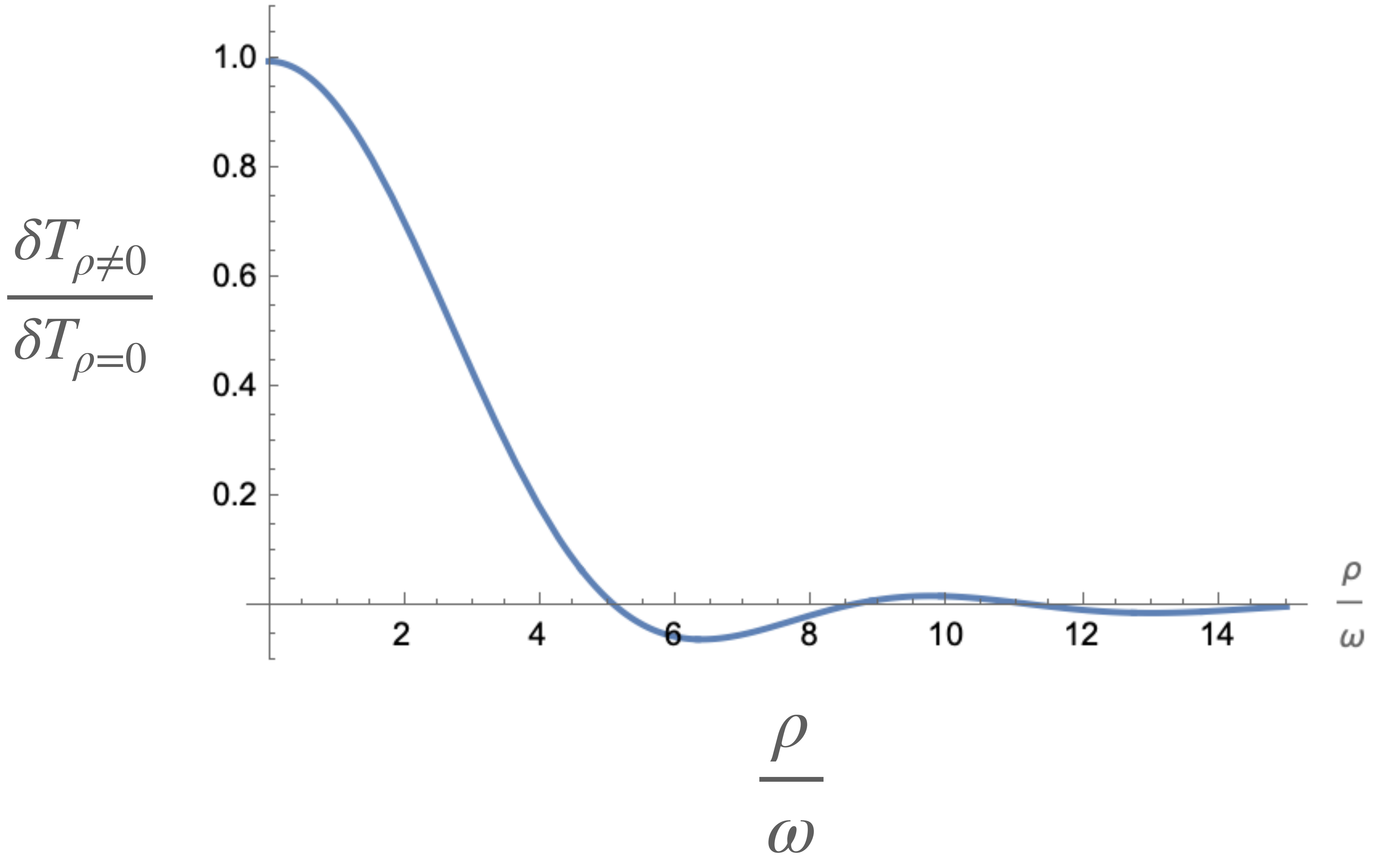
$$\frac{\delta T_{\rho \ll \omega}}{T} = h_+ \frac{\sin(2L\omega)}{2L\omega} \left(1 - \frac{1}{12} \left(\frac{\rho}{\omega} \right)^2 + O(\rho^4) \right)$$

Standard GR result

Corrections when $\rho \neq 0$

Gravitational Time-Delay

(CSP with tensor-like interaction current)



$$\frac{\delta T_{\rho\neq 0}}{\delta T_{\rho=0}} = \left[8\left(\frac{\omega}{\rho}\right)^2 J_2\left(\frac{\rho}{\omega}\right)\right]$$

Simple analytic form describes full result:

- UV correspondence with $\rho = 0$ GR
- Screening behavior in the IR

To appear in work by S. Kundu, P.S., N. Toro

Matter Currents — General Solution

Most general solution to continuity condition (up to total derivative terms) can be written as

$$f(k, \dot{z}, \eta) = e^{-i\rho\frac{\eta \cdot \dot{z}}{k \cdot \dot{z}}}\hat{g}(k \cdot \dot{z}) + \underline{\mathcal{O}X(k, \dot{z}, \eta)}$$

"Shape" terms

where free eom is $\delta'(\eta^2 + 1)\mathcal{O}\Psi = 0$

Analogous to charge radius etc. operators in E&M

$\Rightarrow$ As in E&M, shape terms do not couple to continuous spin radiation

$\Rightarrow$ Worldline interactions with radiation **fully** determined by $\hat{g}$.

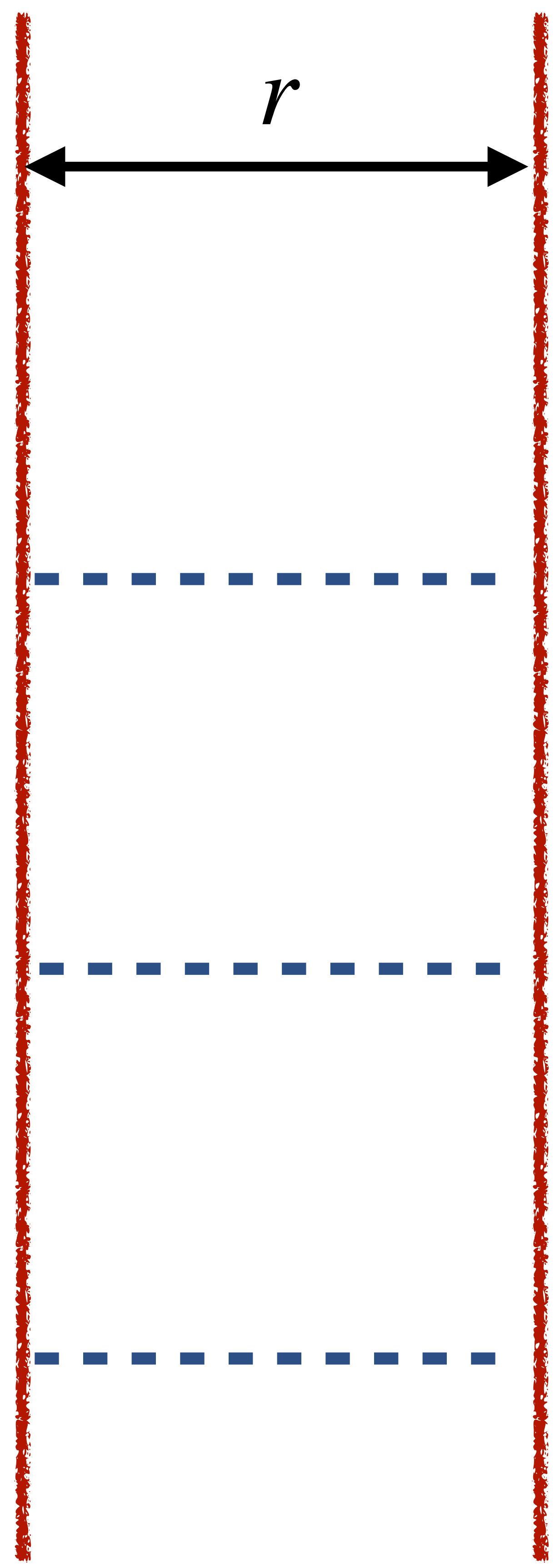
But shape terms **do** qualitatively change the impact **of** off-shell CSPs in scattering interactions and long-range force calculations

What is the "minimal" current?

Most general solution to continuity condition (up to total derivative terms) can be written as

$$f(k, \dot{z}, \eta) = e^{-i\rho\frac{\eta\cdot\dot{z}}{k\cdot\dot{z}}}\hat{g}(k\cdot\dot{z}) + \underline{\mathcal{O}X(k, \dot{z}, \eta)}$$

"Shape" terms



Need additional constraints (from field theory?) to fix the "minimal" interaction current!

Examples calculations of interaction potential for different "shape" terms [from arXiv:2303.0481]

$$V(r) = \frac{g^2}{4\pi r} \times \begin{cases} 1 & \text{spatial current} \\ 1 & \text{temporal current} \\ 1 - c_1\sqrt{\rho\beta}r + c_2\rho\beta r^2 + \ldots & \text{inhomogeneous current} \end{cases}$$

Always unchanged at small r, varies at large r

Outline — CSP's in Nature?

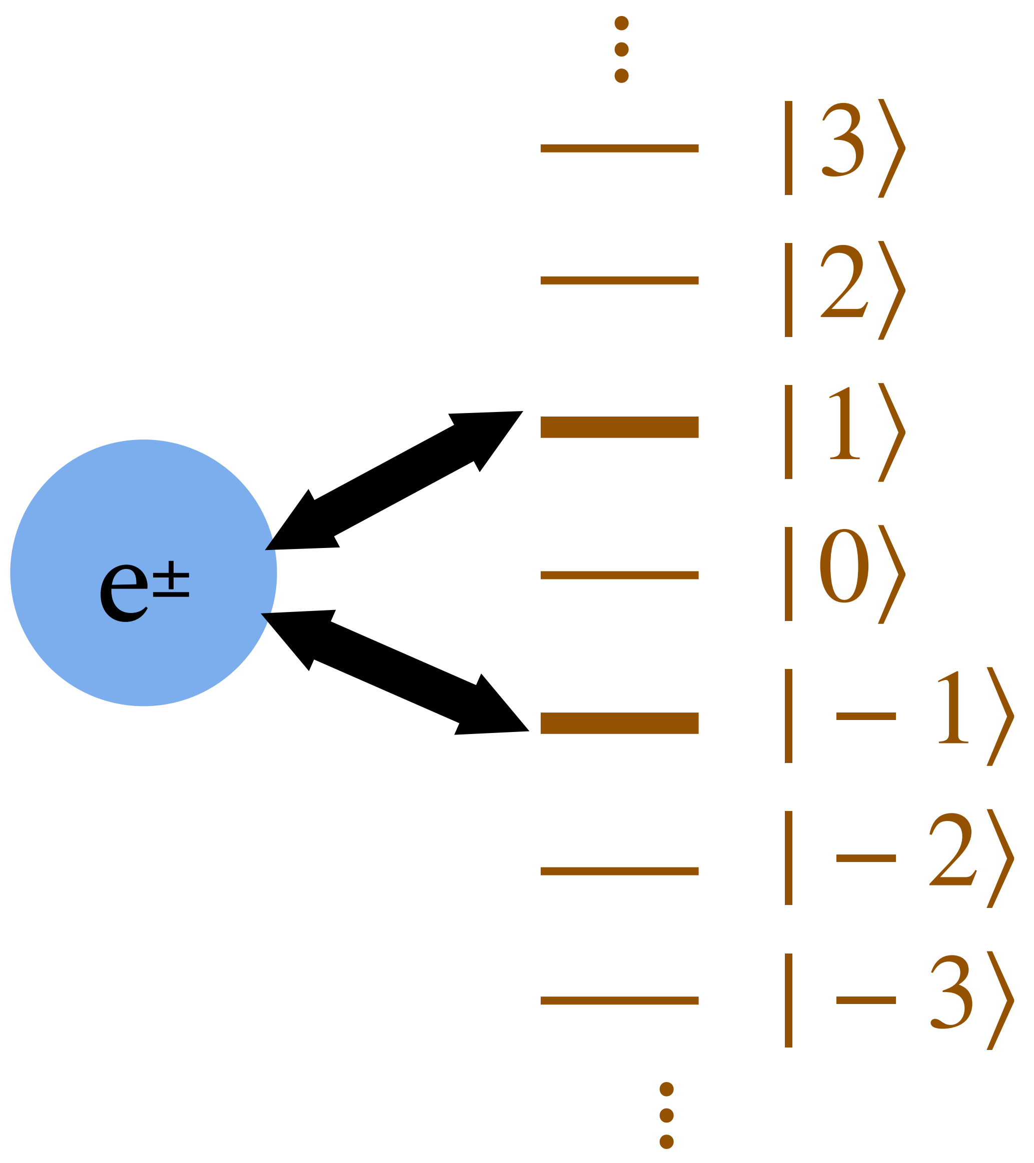
- Helicity states can mix under Lorentz boosts – controlled by ***spin scale*** ρ [Wigner 1939]
- **Coupling to matter particles is predictive (new features in the IR), and closely connected to familiar theories** – we've had hints for a while [1302.1577], now have exact scheme to calculate both classical physics and amplitudes in putative theory
  - Part I — Top line summary of the *what, why, and how* of CSPs, and a few illustrative results.
  - Part II — Superspace-like formalism as a useful tool [1404.0675]. Coupling matter particles to fields with nonzero ρ, aka "Continuous spin fields".
  - Part III — Example results and correspondence with familiar theories (EM & Gravity)
- Speculation about CSPs in the Standard Model and future directions (as time permits)

Are Known SM Particles CSPs?

- Small ρ recovers familiar theories, so phenomenologically viable! SM built out of (naturally) massless or unnaturally light states, so natural to consider.
- Recent advances can address many theory and phenomenological aspects of QED and GR for $\rho \neq 0$. *Self-interactions and non-abelian generalizations still needed*.
- At first glance, many ways in which CSP physics might touch on BSM and puzzles of SM. I will flag only three categories:
  - Interesting early Universe physics, thermodynamic signatures, new signatures in the IR
  - New dark matter candidates
  - Radiative/renormalization structure is distinctive (mass terms protected)

Any $\rho \neq 0$ part of the SM has a "dark sector"

Covariant interactions single out **one** helicity with unsuppressed coupling (e.g. |h|=1)



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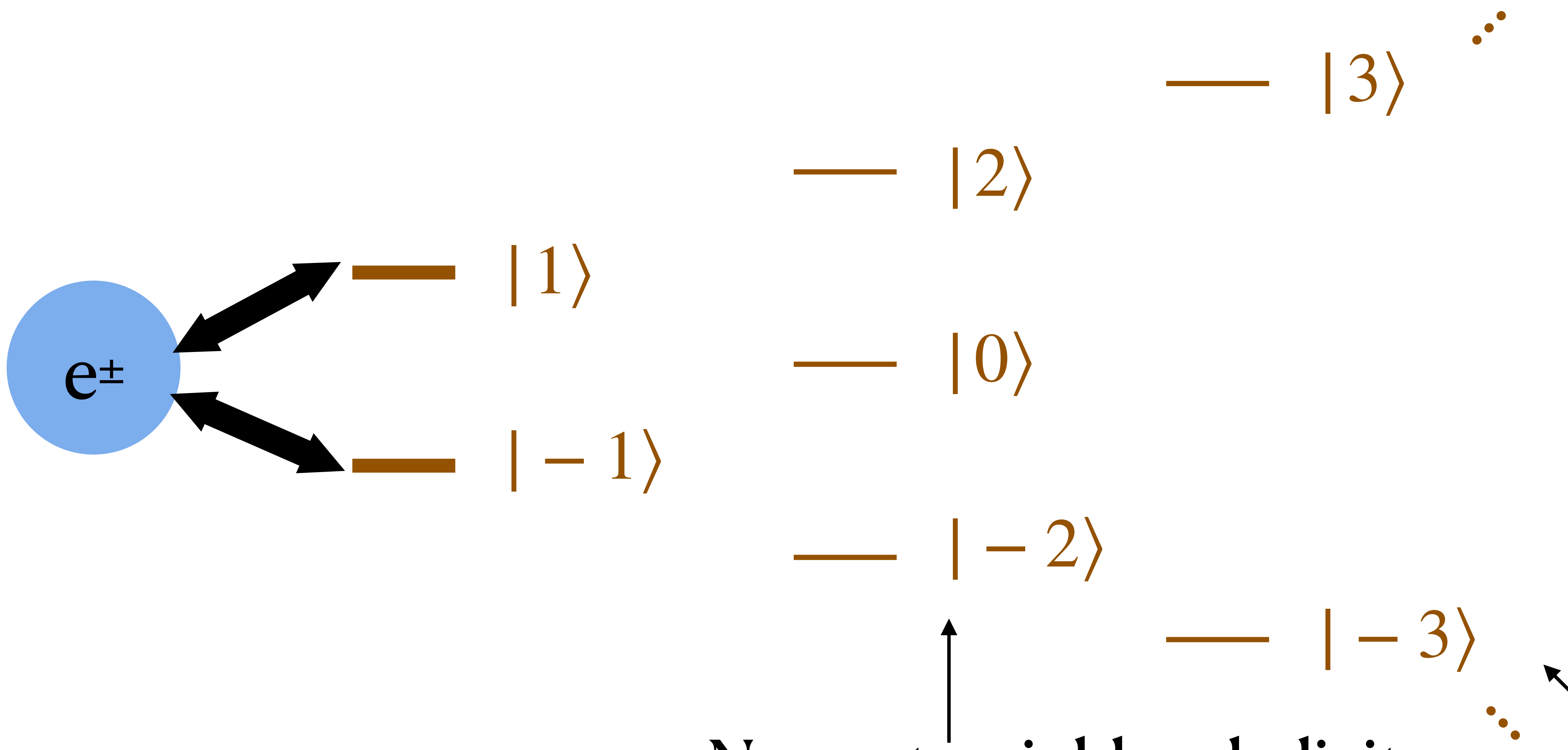

Nearest-neighbor helicity states' interactions suppressed by ρ/ω

Higher helicities' interactions suppressed by more powers of ρ/ω

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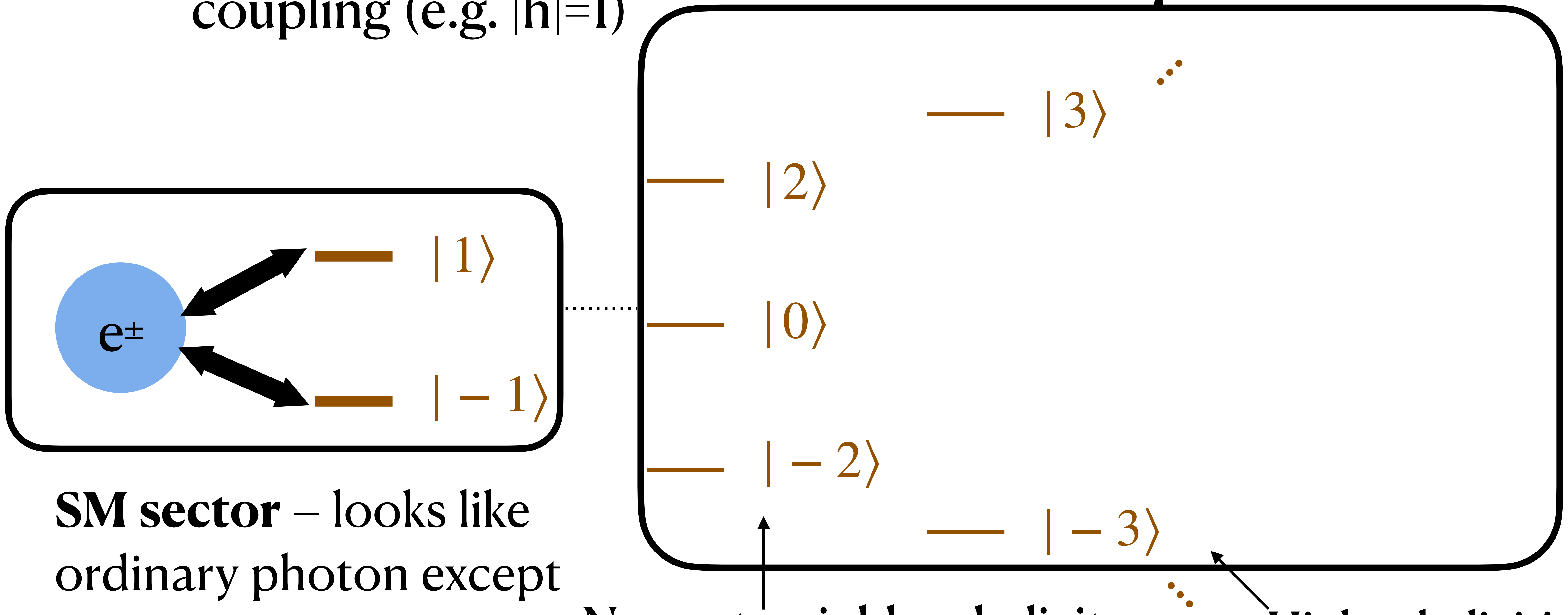
Covariant interactions single out **one** helicity with unsuppressed coupling (e.g. |h|=1)

Continuous spin "dark" sector



SM sector – looks like ordinary photon except in deep IR $\omega \lesssim \rho$

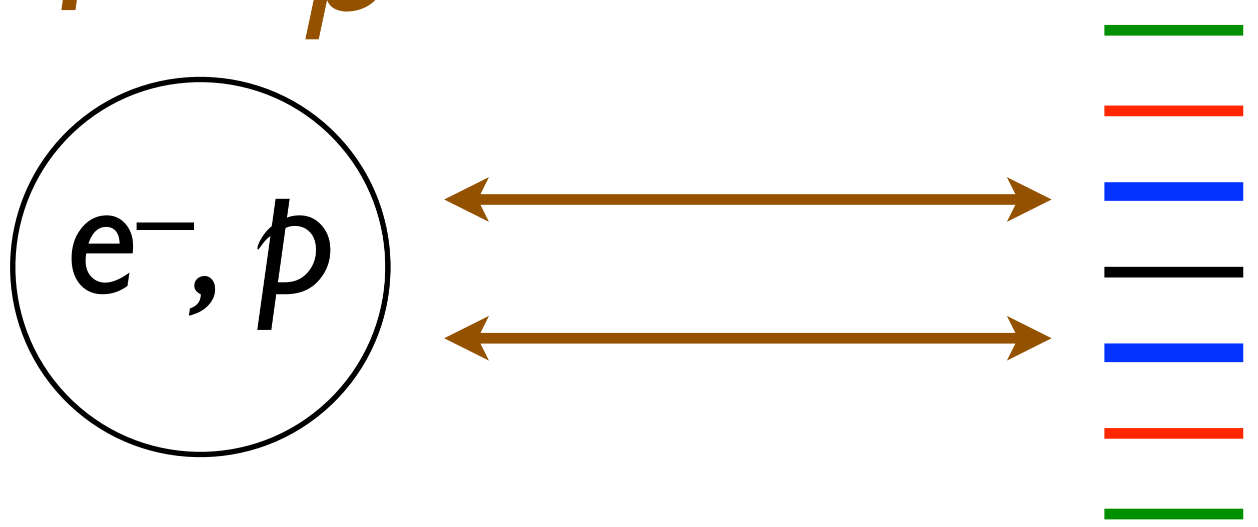
Nearest-neighbor helicity states' interactions suppressed by ρ/ω

Higher helicities' interactions suppressed by more powers of ρ/ω

Early Universe Production

$T \gg \rho$

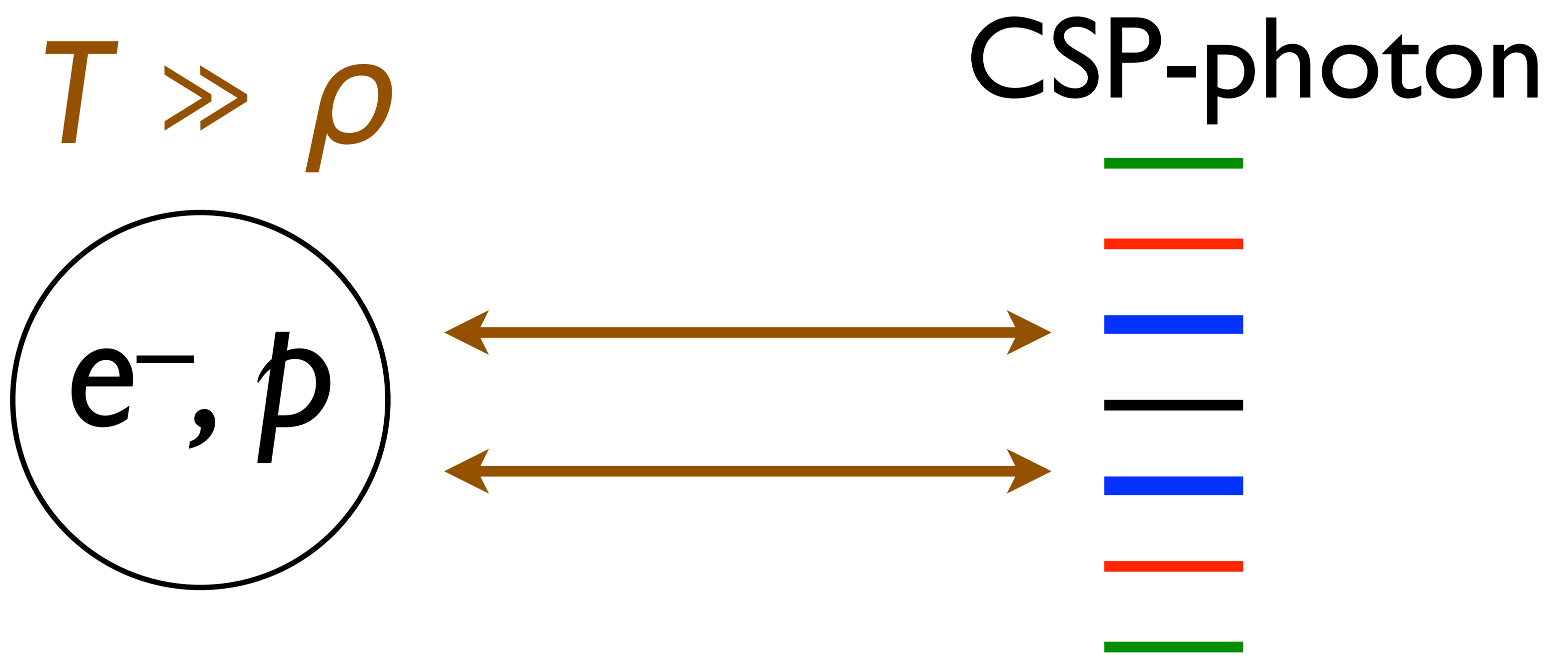
CSP-photon



h=±1 modes
equilibrate rapidly

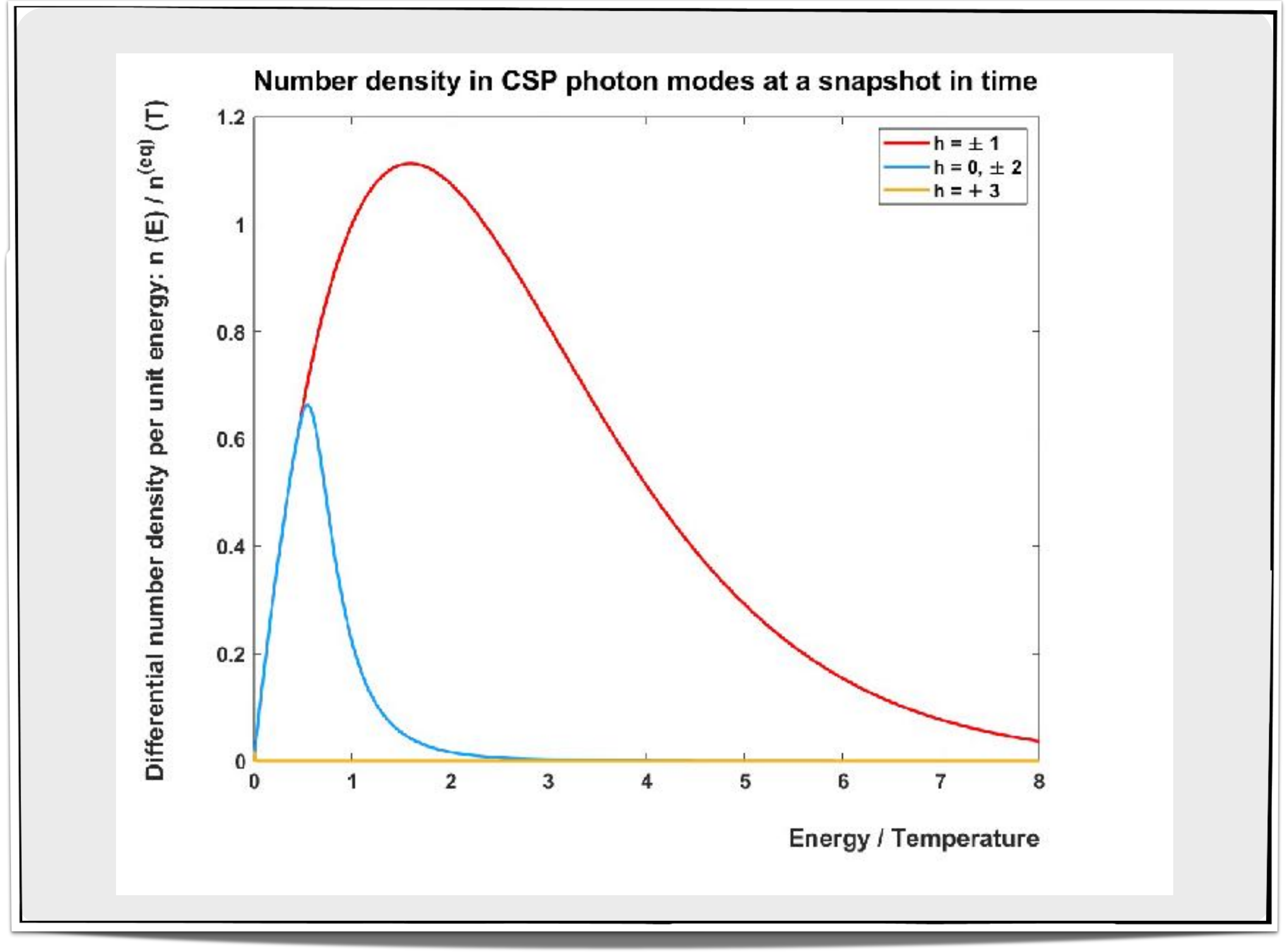
other modes'
equilibration time
$\sim\tau\, (T/\rho)^2$ or longer

Early Universe Production



h=±1 modes
equilibrate rapidly

other modes' equilibration time ~$\tau\,(T/\rho)^2$ or longer

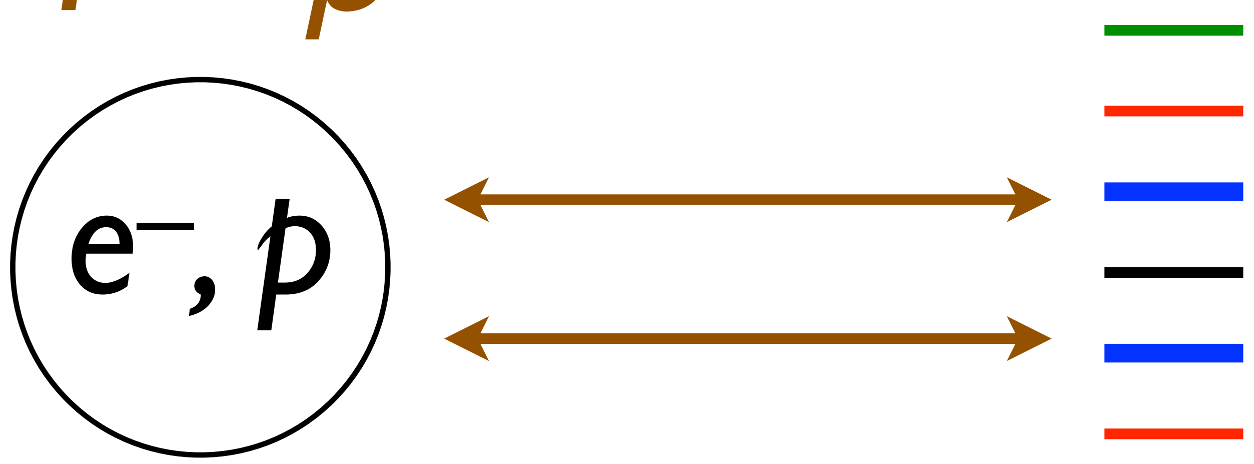


For ρ≲(meV), effectively ≪1 dof thermalizes in early universe

Early Universe Production

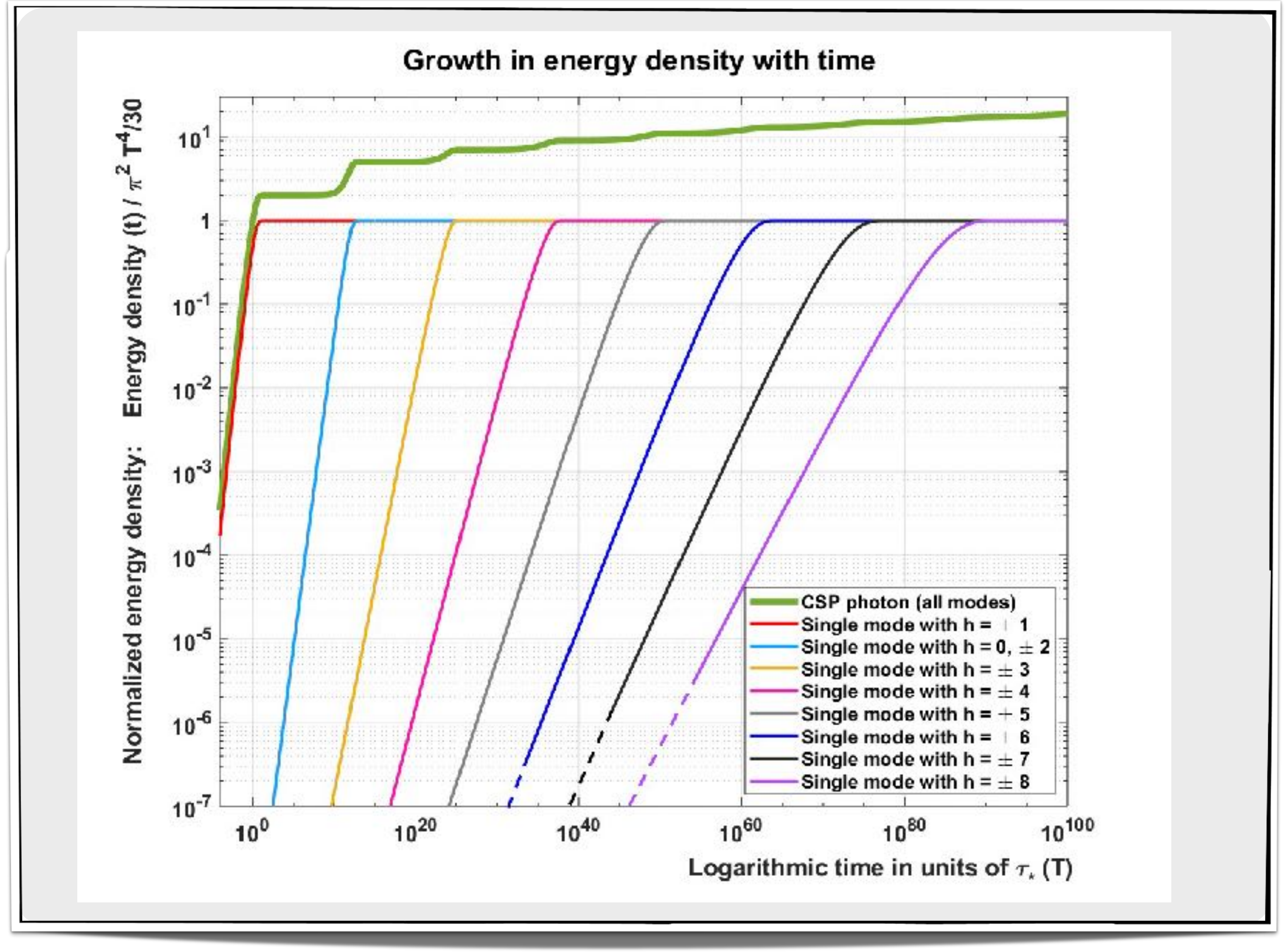
$T \gg \rho$

CSP-photon



h=±1 modes equilibrate rapidly

other modes' equilibration time $\sim\tau\,(T/\rho)^2$ or longer

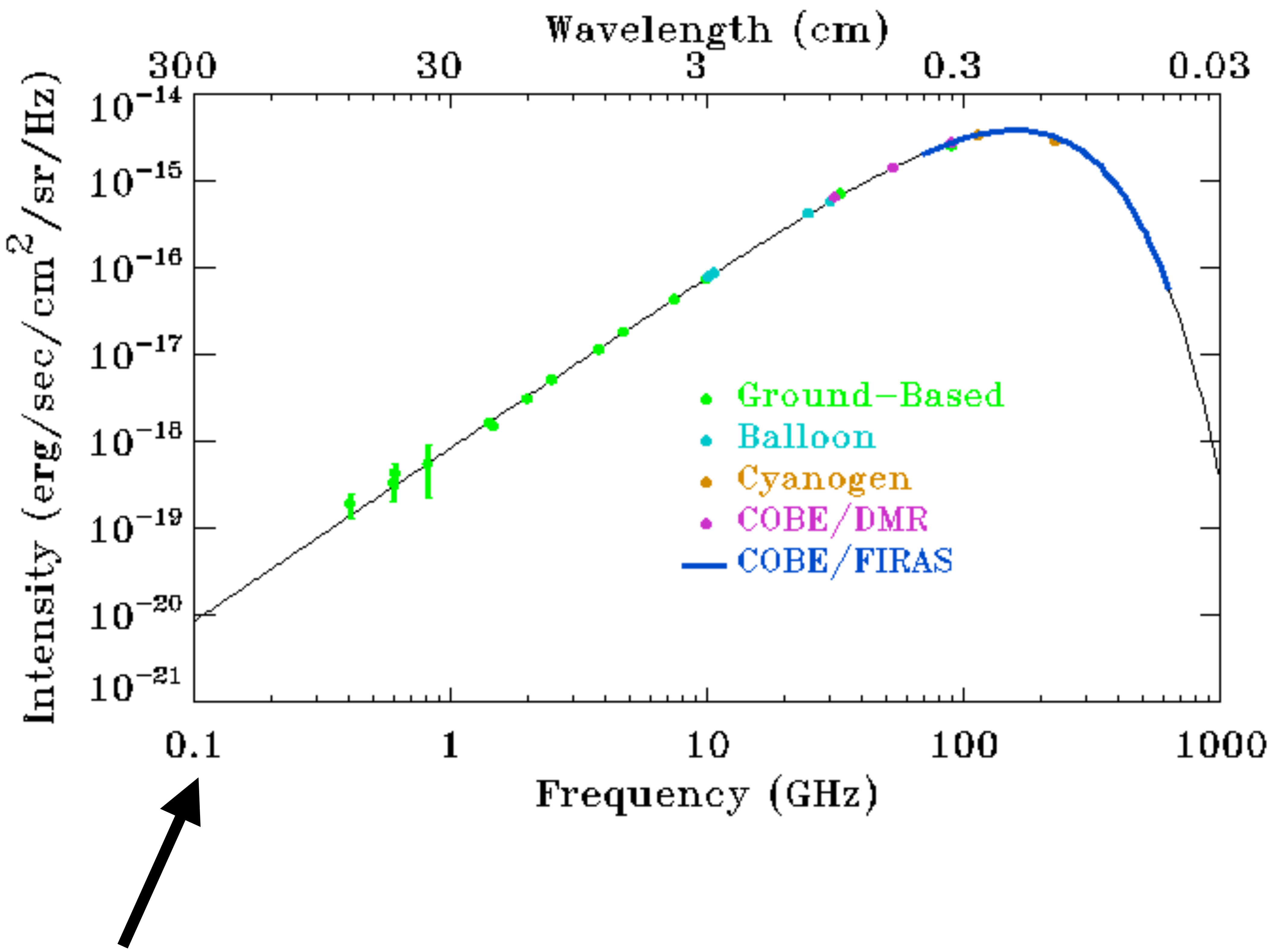


Only lowest-energy phase space of partner modes thermalizes, with finite (polynomial → logarithmically growing) $\sum_h \delta n(h)$

To appear in work by P.S., G. Sundaresan, N. Toro

Dark Radiation

For CSP E&M, evading N_eff constraints suggest meV scale or smaller for spin scale — dark radiation modes and primary mode distortion from black body spectrum at low frequency



After redshift, blackbody distortions at low sub-100 MHz frequency

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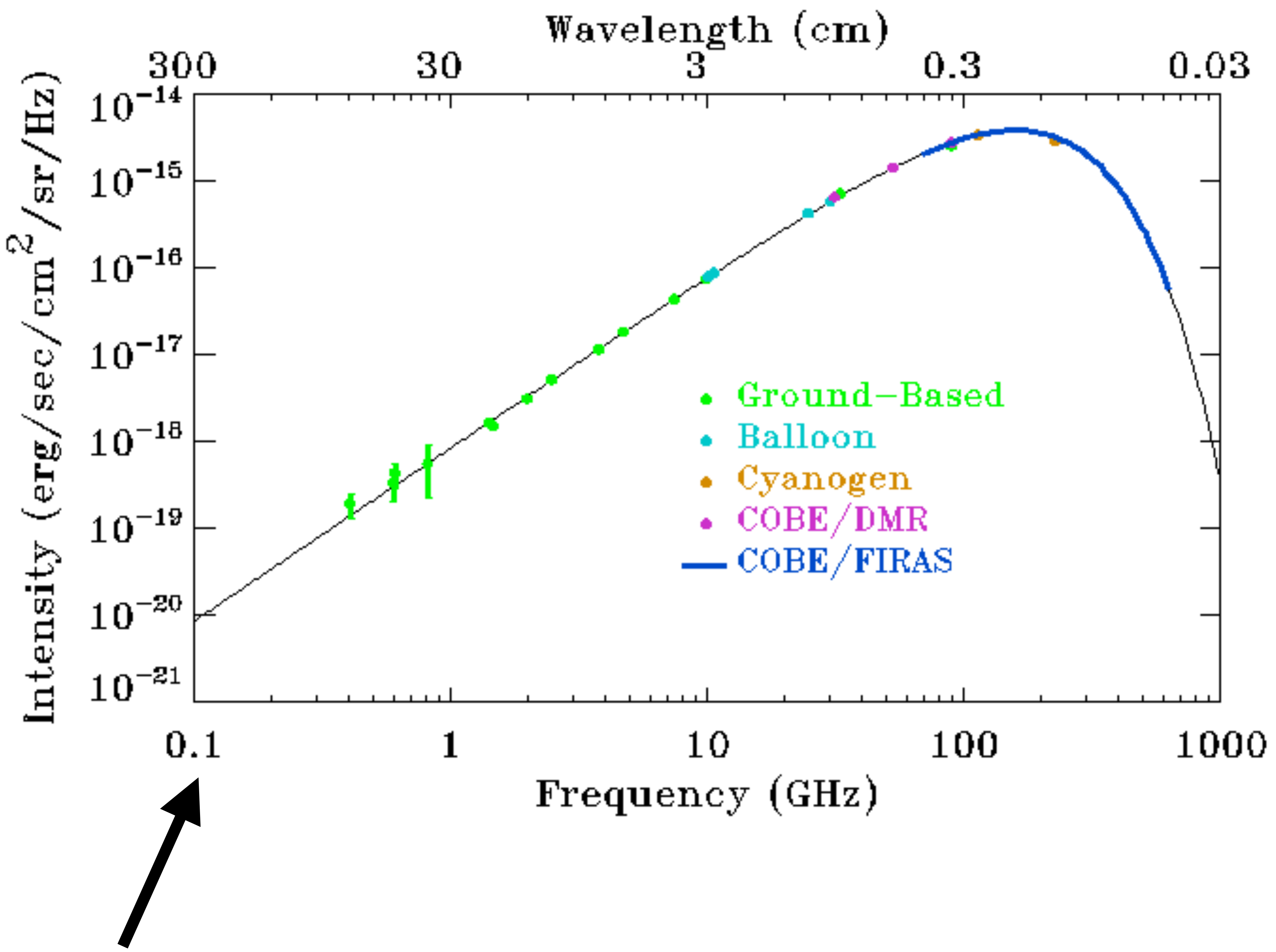


Possibly better to consider detection of the h=0, +/-2 partner modes of the CMB

… what kind of shielded antenna pickup would work for that?

After redshift, blackbody distortions at low sub-100 MHz frequency

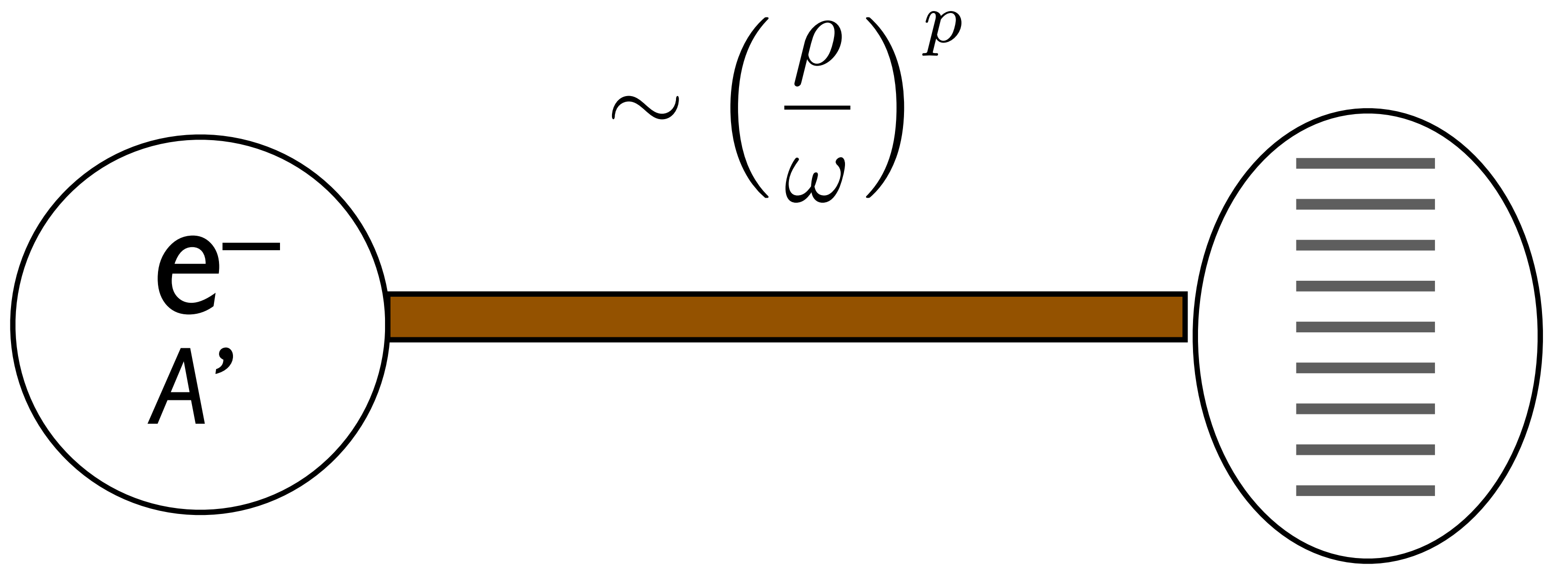
Dark Matter

Can also consider massive CSP phase:

Ongoing work by P.S., N. Toro, K. Zhou

$$S[\Psi] = \int d^4x d^4\eta \left(\delta'(\partial\Psi)^2 + \frac{1}{2}\delta(\Delta\Psi)^2 + \frac{1}{2}m^2\Psi^2 \right)$$

In Abelian theory, can study this with stueckelberg mass term



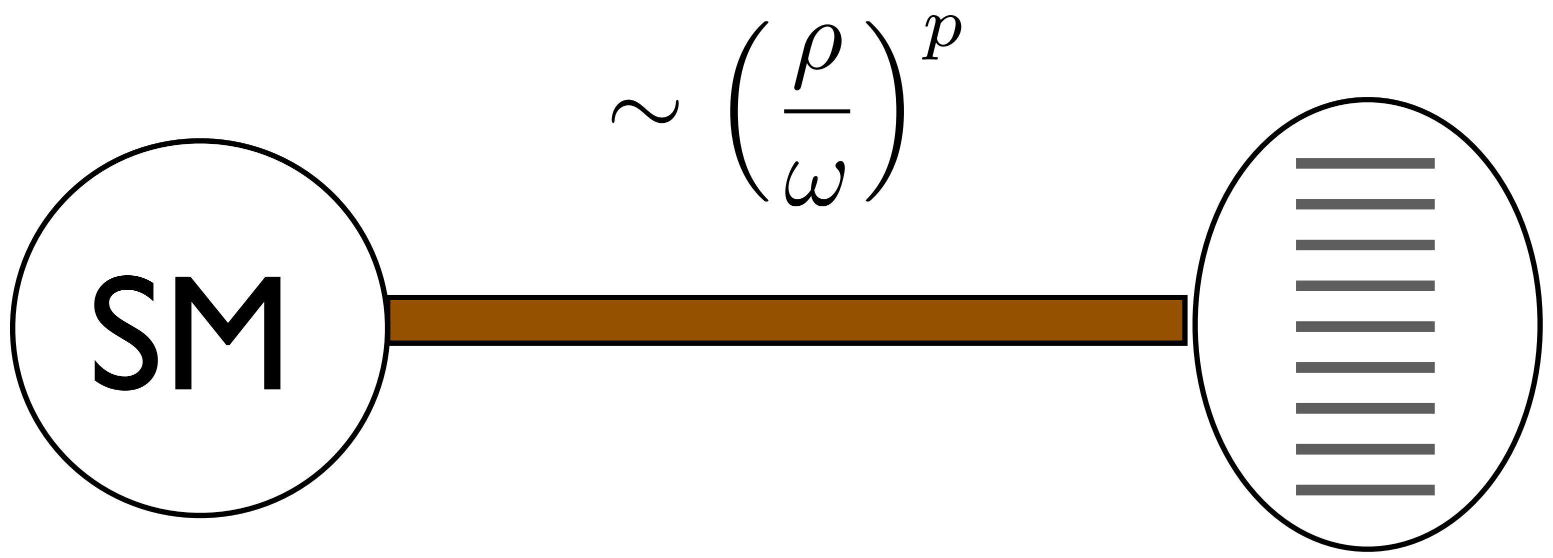
matter,
s=1 modes
of heavy boson

partner modes
of heavy boson;
tower of high spin
states

"Dark Sector" with
massive partner
polarizations

Abelian model not
realistic, but useful for
estimating interesting
values of spin scale

Dark Matter



Dark Sector = mass degenerate high spin partner modes of Standard Model particles

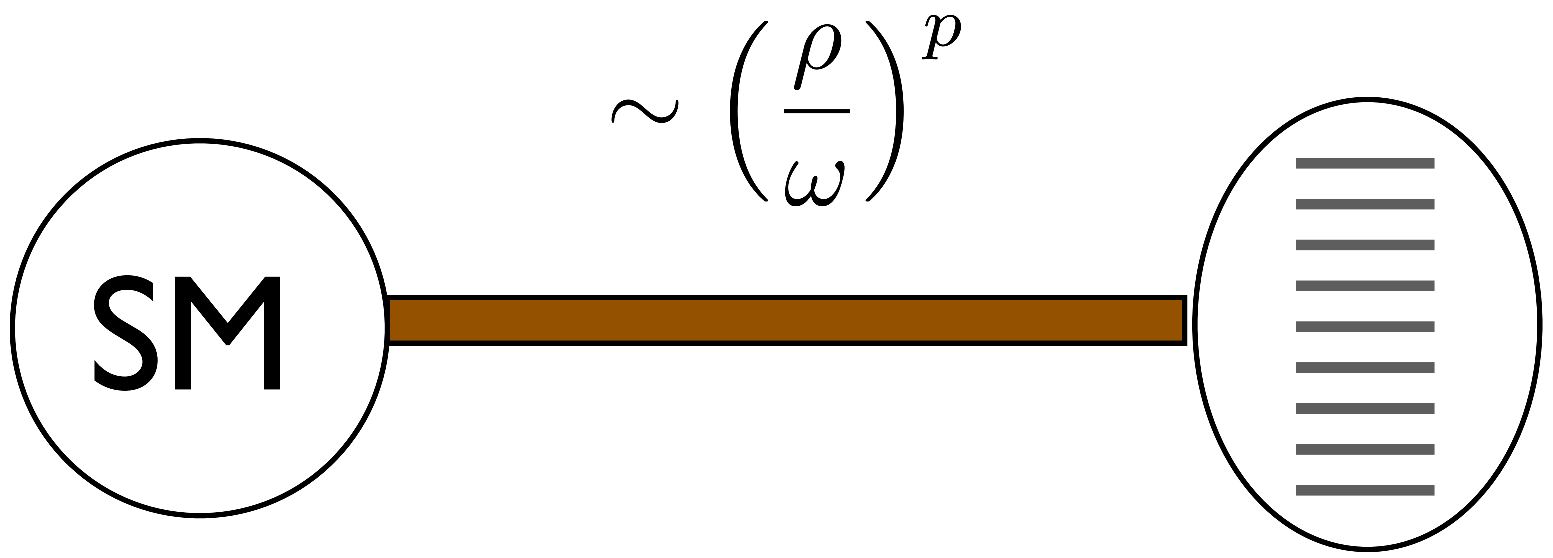
More realistic (<u>but speculative!</u>) possibilities:

If electroweak sector has nonzero rho, massive phase

Spin-3 partner modes (freeze-in), cosmologically long-lived

$$\rho_{W,Z} \lesssim 10^{-4} eV$$

Dark Matter



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If electroweak sector has nonzero rho, massive phase

Spin-3 partner modes (freeze-in), cosmologically long-lived $\rho_{W,Z} \lesssim 10^{-4} eV$

If matter (i.e. e-…etc) has nonzero rho, massive phase

Spin-3/2 partner of e- (likely) stable, freeze-in estimate $\rho_e \lesssim 10^{-6} eV$ smaller if co-produced

In any scenario, non-trivial departure from standard CDM possible due to (partial) freeze-in & decay of partner modes

CSPs in the Early Universe

Dark radiation is generic, though suppressed at small spin scale

Many DM candidates, and interesting departures from standard CDM thermodynamics; more theory work and realistic models needed

Estimates of spin scales motivated by dark matter/radiation becoming non-trivial in early Universe

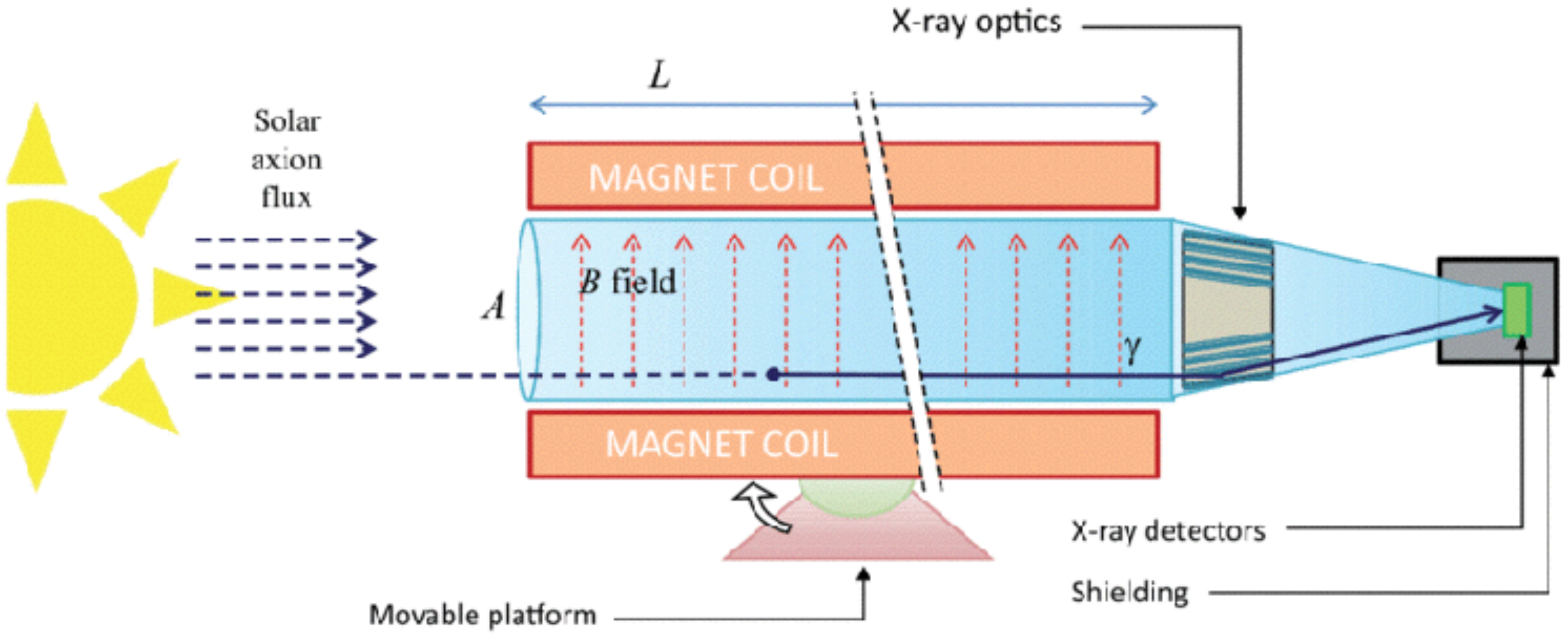
$$\rho_{W,Z,\gamma} \sim 10^{-10} eV - 10^{-4} eV$$

Stellar Production & Detection

If rho is large enough for CSP dynamics to play a role in the early Universe, or provide dark matter/radiation, then it can't be far from stellar cooling limits

$$\rho_\gamma \sim (10^{-8} - 10^{-6})eV$$

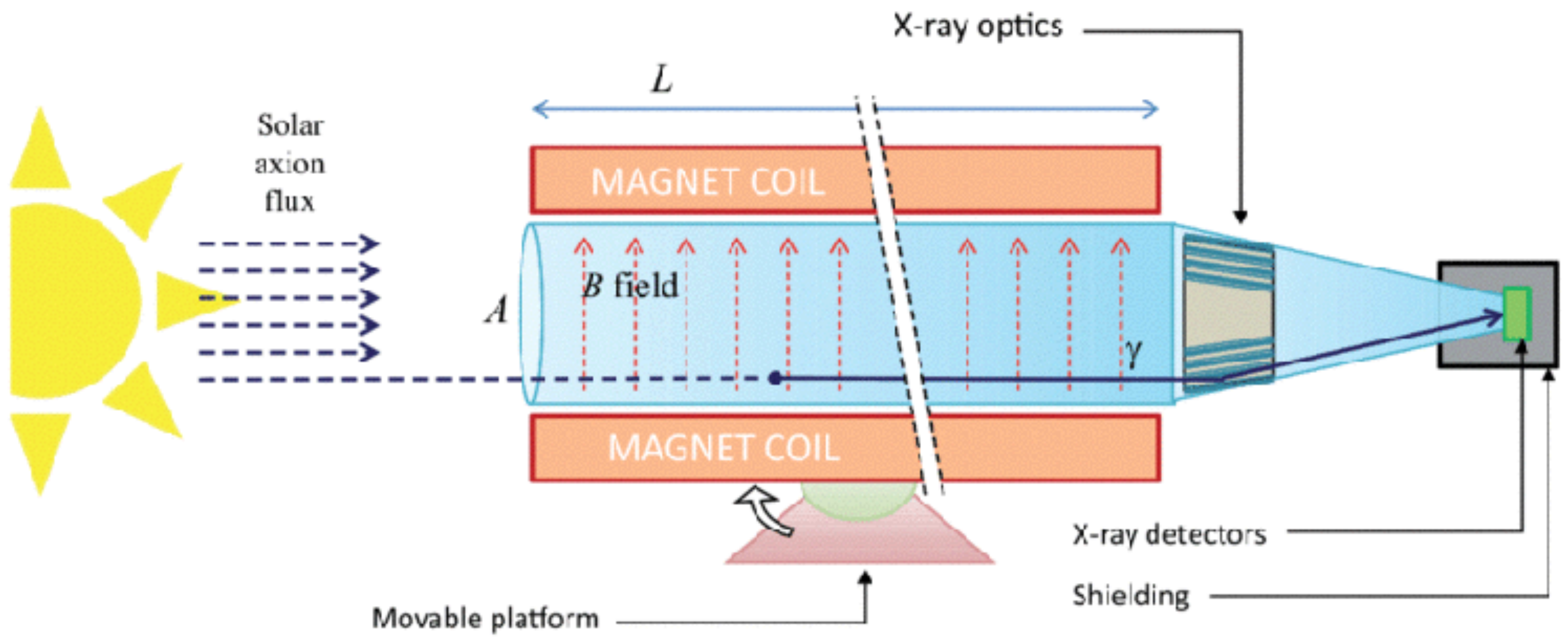
Should consider helioscope detection! ...borrow from ALP-searches



Journal of Physics Conference Series 460(1):012002

Do not need magnet for CSP photon partner mode detection

Stellar Production & Detection



Journal of Physics Conference Series 460(1):012002

Don't need magnet for CSP photon partner mode detection

$$F_{\text{X-ray modes}} \sim \frac{10^{18}}{m^2 s}\left(\frac{\rho}{10^{-8} eV}\right)^2$$

naive production estimate: very large flux of photon partner modes at X-ray freq.

$$S_{\text{X-ray modes}} \sim \frac{10^{-4}}{m^2 s}\left(\frac{\rho}{10^{-8} eV}\right)^4$$

Naive estimate of detection rate

Proper calculation needed, but stellar production seems like a viable way to probe photon CSP for DM inspired range of spin scale

Symmetry and Mass Terms

Symmetry and Mass Terms

Consider tensor fields used to describe (scalar interaction) CSP:

$$L = g\phi(x)O(x) + g\phi_\mu O^\mu(x) + \dots$$

Gives rise to 1/r potential

The theory is invariant under the transformations:

$$\delta\phi = \rho\epsilon(x) \qquad \delta\phi_\mu = \partial_\mu\epsilon(x) + \rho\epsilon_\mu$$

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This is not a shift symmetry

$$\cancel{m^2}\phi^2 + \ldots$$

but it's enough to forbid mass terms

Mass term naively protected by gauge "symmetry"

Symmetry and Mass Terms

Consider tensor fields used to describe (scalar interaction) CSP:

$$L = g\phi(x)O(x) + g\phi_\mu O^\mu(x) + \ldots$$

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The theory is invariant under the transformations:

$$\delta\phi = \rho\epsilon(x) \qquad\qquad \delta\phi_\mu = \partial_\mu\epsilon(x) + \rho\epsilon_\mu$$

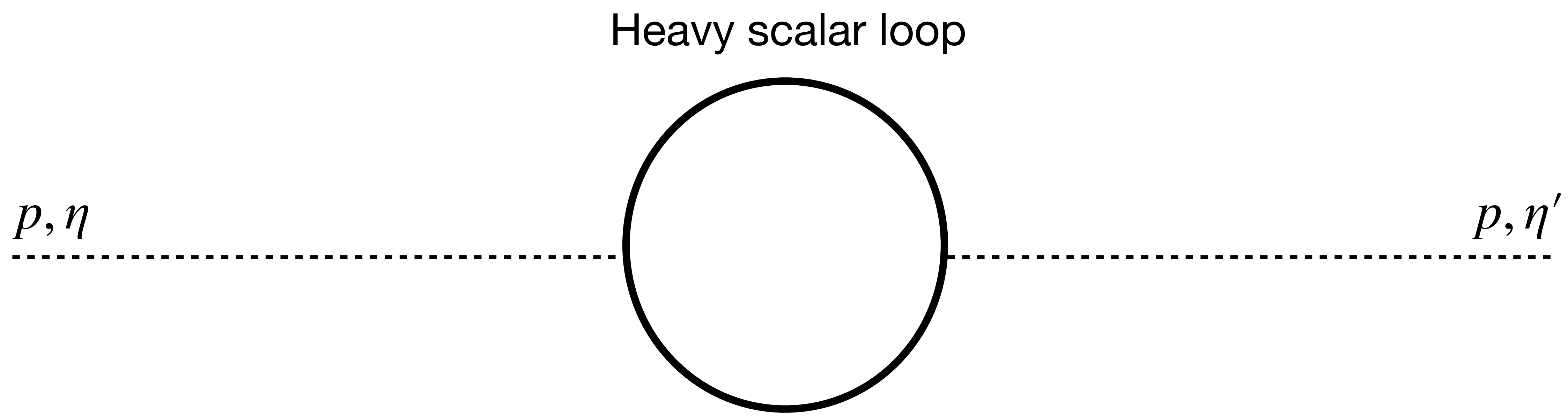
$$\delta L \propto (\rho O(x) - \partial_\mu O^\mu(x)) + \text{total derivate}$$

Vanishes by current continuity!

Partner mode contributions ensure gauge variation vanishes

and thus compatible with $\sim \dfrac{1}{r}$

Novel Radiative Structure



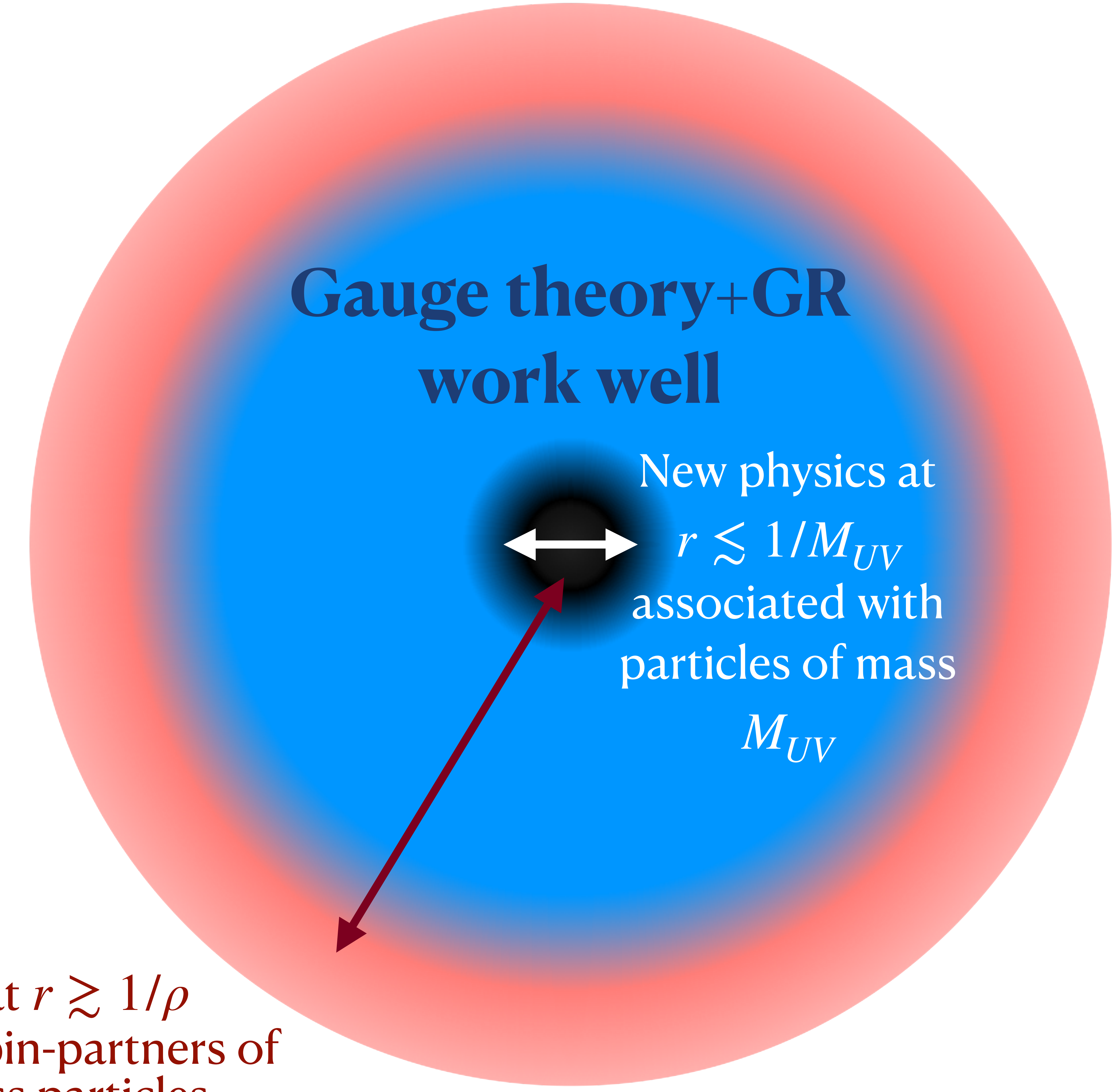
Use the path integral to calculate "renormalization" of 2-point function at 1-loop

At any $\rho \neq 0$, 1PI 2-point function vanishes as $p^2 \to 0$, so it looks like location of $m = 0$ physical singularity is less sensitive to heavy scalar mass than in $\rho = 0$ case.

Working on understanding this now…

Conclusion

- Lorentz invariance → massless particles have a spin-scale. **Is it zero or non-zero?**
- The non-zero option has testable consequences and potentially interesting applications
- If theoretically inconsistent, deserves a proper burial. Results thus far suggest otherwise.
- If viable, we should think of the Standard Model as an effective theory with both UV and IR completions.



New physics at $r \gtrsim 1/\rho$ associated with spin-partners of known massless particles

Backup

Self-interactions
Gravity
Spinor Helicity
Path Integral amplitudes
Thermodynamics
Stellar production of CSP
Vector superspace and intuition
Non-local currents, causality
Force law corrections
Long-range potential and cosmology

Self-Interactions

CSP self-interactions are required for consistent Yang-Mills or graviton-like CSPs. [Maybe even for photon/scalar-like CSPs at higher orders in ρ]

- Probably best approached in field theory
- 3-CSP Vertex = Permutation-invariant function $A(k_1, \eta_1;\ k_2, \eta_2;\ k_3, \eta_3)$.
- Gauge-invariance of physical amplitudes ⇒ when legs 1 and 2 are on-shell, must satisfy leg-3 continuity condition $(k_3 \cdot \partial_{\eta_3} + i\rho)\,A = 0$
- Other constraints from gauge invariance (e.g. when leg 1 is on-shell and leg 2 is contracted into a conserved matter current) are plausible, but can be avoided by adding explicit matter-matter-CSP-CSP couplings to theory
- Would be interesting to classify most general such functions.

Gravity

I think there are structural questions best understood in flat space before gravity.

1) In what sense are continuous spin theories gravitational?
   - Evidence from soft factors for graviton-like CSP interactions
   - Linearized-graviton-like worldline current (conserved when particles accelerate consistently)
   - Non-linear theory must involve self-interacting continuous spin field → warmup: non-Abelian theories (or even self-interacting scalar-like continuous spin).
2) Can continuous spin matter couple to helicity 2?
   - Minimal coupling to metric breaks continuous spin gauge invariance. Remedy?
   - Weinberg-Witten looks very different – writing down nonzero covariant matrix elements is easy, e.g. $$\langle p', n'|T^{\mu\nu}(k)|p, n\rangle = (p^\mu p'^\nu + p'^\mu p^\nu - p.p' g^{\mu\nu})\tilde{J}_{n'}\left(\rho\frac{\epsilon_+(p').p}{p.p'}\right)\tilde{J}^*_n\left(\rho\frac{\epsilon_+(p).p'}{p.p'}\right)$$
   - But limiting behavior still violates standard EP assumption (is this is ok?)
   - Warmup problem: Can continuous spin field carry ordinary electromagnetic charge?
   - If not, maybe "the only CSP theories with gravity are theories of CSP gravity"

Gravity

3) Puzzles independent of detailed theory: Infinite Hawking radiation?

- For $\rho = 0$ low-lying helicities, angular momentum dependent graybody factors seem to penalize higher spin modes. Studies of "minimally coupled" high spin massless fields suggest fast enough fall-off at high spin to get modest enhancement of Hawking radiation.

Spinor Helicity for CSPs

Spinor helicity formalism simplest in a different basis of states, labeled by **two** spinors λ^α and μ^α with $\lambda_\alpha \bar{\lambda}_{\dot{\alpha}} = k_\mu \sigma^\mu_{\alpha\dot{\alpha}}$ and $\langle \lambda\mu \rangle = 1$, with redundancy $|\lambda, \mu\rangle \sim |\lambda, e^{i\phi}\mu\rangle \sim e^{-i\rho\alpha}|\lambda, \mu + \alpha\langle\lambda\mu\rangle\lambda\rangle$.

Note:
$|e^{i\phi}\lambda, \mu\rangle \nsim e^{-ih\phi}|\lambda, \mu\rangle$ – they are distinct states.

For $\rho = 0$, redundancy $\Rightarrow \mu-$independent amplitudes

Little group acts on amplitudes (functions of λ, μ) as

$$W_{\alpha\dot{\alpha}} \sim \frac{\lambda_\alpha \bar{\lambda}_{\dot{\alpha}}}{\langle\lambda\mu\rangle[\bar{\lambda}\bar{\mu}]} \underbrace{(\langle\lambda\partial_\lambda\rangle - [\bar{\lambda}\partial_{\bar{\lambda}}] - \langle\mu\partial_\mu\rangle) + [\bar{\mu}\partial_{\bar{\mu}}])}_{\text{Helicity rotation } R} + \frac{\lambda^\alpha \bar{\mu}^{\dot{\alpha}}}{\langle\lambda\mu\rangle} \underbrace{\frac{[\bar{\lambda}\partial_{\bar{\mu}}]}{[\bar{\lambda}\bar{\mu}]}}_{W^+} + \frac{\mu^\alpha \bar{\lambda}^{\dot{\alpha}}}{[\bar{\lambda}\bar{\mu}]} \underbrace{\frac{\langle\lambda\partial_\mu\rangle}{\langle\lambda\mu\rangle}}_{W^-}$$

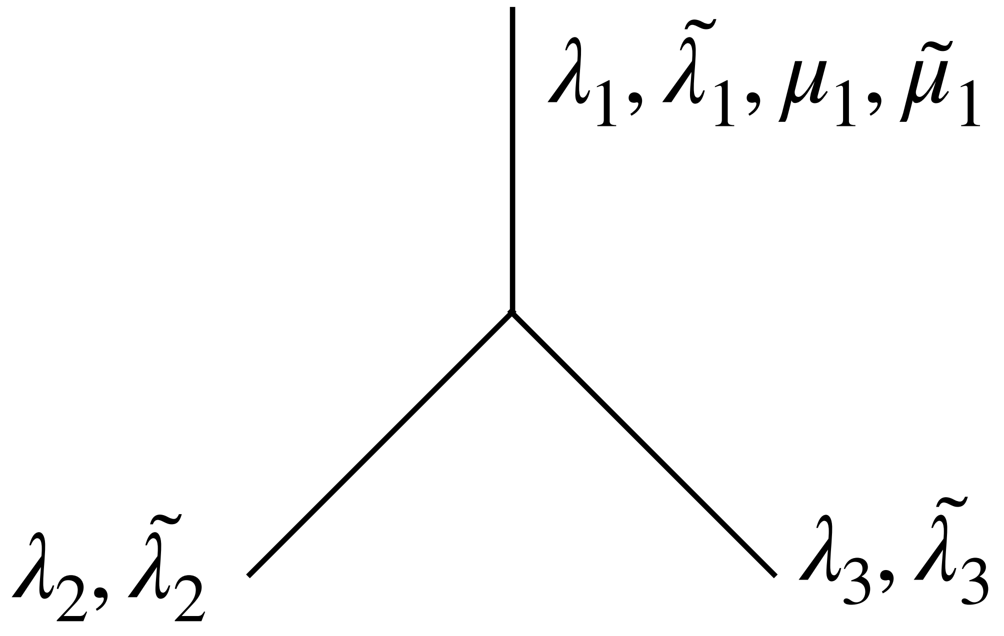
$T_+ A(\lambda, \mu) = \rho \dfrac{[\bar{\lambda}\bar{\mu}]}{\langle\lambda\mu\rangle} A(\lambda, \mu)$ ensures amplitudes respect the redundancy above

Helicity eigenstates are obtained by Fourier transform $\int d\phi\, e^{-ih\phi}|\lambda e^{i\phi}, \mu e^{-i\phi}\rangle$.

* Every equation here is probably off by some factors of 2, i, and ± 1

Complex-Momentum 3-particle Amplitudes?

Consider on-shell kinematics involving two scalar matter legs and one CSP.



$\langle \lambda_i \lambda_j \rangle = 0$

$T_+ A = i\lambda_1 \,.\, \partial_{\mu_1} A = \rho\, A$ can be satisfied by $A = e^{i\rho \frac{\mu_1 \cdot \lambda_2}{\lambda_1 \cdot \lambda_2}} f(\lambda_i, \tilde{\lambda}_i)$.

But $T\_A = i\tilde{\lambda}_1 \,.\, \partial_{\tilde{\mu}_1} A = \rho\, A$ **cannot** be satisfied in this kinematics – all Lorentz-scalars built from this kinematic data are annihilated by $\tilde{\lambda}_1 \cdot \partial_{\tilde{\mu}_1}$.

(But…note that in complexified LG, $T_\pm$ are not complex conjugates. Can try to recurse higher-point amplitudes for right- or left-CSP with $T_\pm A = \rho A$, $T_\mp A = 0$ from 3-point ansatz, and then perhaps combine the results and restrict to real momenta?)

Currents from Worldlines

EM: Amplitudes

Compute amplitudes from path integral for worldline in EM field (Feynman 1950)

$$A(p, p', k_i, \epsilon_i) = \int_{\mathscr{P}[x,x']} Dz(\tau)\, e^{-S_{free}[z]} \underbrace{e^{-ip\cdot x}}_{V_{in}} \underbrace{e^{ip'\cdot x'}}_{V_{out}} \prod \int dt_i \underbrace{\left(\epsilon_i \cdot \dot{z}(t_i) e^{-ik_i\cdot z(t_i)}\right)}_{V_\gamma^{k_i,\epsilon_i}(t_i)} \Bigg|_{LSZ}$$

Modern "string-inspired" approach to evaluation (Strassler, Schubert, ...): matter Fourier phases and photon-current couplings → vertex operators; solving Gaussian path-integral exactly leaves integral over the insertion points t_i.

Very different organization from Feynman diagrams but identical result.

Fully general treatment of loops, multiple worldlines, etc.

Scalar QED with $\rho \neq 0$

(Compton-like scattering amplitudes)

Structure of the calculation is identical to QED – η-dependent vertex operator yields matrix elements which can be contracted with basis wave-functions to get polarization amplitudes.

$$M(p_0, p_3, \{k_1, \eta_1\}, \{k_2, \eta_2\}) = 2\int_{-1}^{1} dx \left(\eta_1 - \frac{\eta_1 \cdot P_1(x)}{k_1 \cdot P_1(x)} k_1\right) \cdot \left(\eta_2 - \frac{\eta_2 \cdot P_2(x)}{k_2 \cdot P_2(x)} k_2\right) e^{-i\rho \frac{\eta_1 \cdot P_1(x)}{k_1 \cdot P_1(x)} - i\rho \frac{\eta_2 \cdot P_2(x)}{k_2 \cdot P_2(x)}.}$$

$P_{1,2}(x) = p_3 - p_0 \pm x\, k_{2,1}$ → at endpoints $x = \pm 1$, these are momenta appearing in $s(u)$-channel photon vertex

Polarization amplitudes are Fourier transforms of this expression,

$$A(p_0, p_3, \{k_i, h_i\}) = \int \frac{d\phi_i}{2\pi} e^{ih_i\phi_i} M(p_0, p_3, \{k_i, \eta_i(\phi_i)\})$$

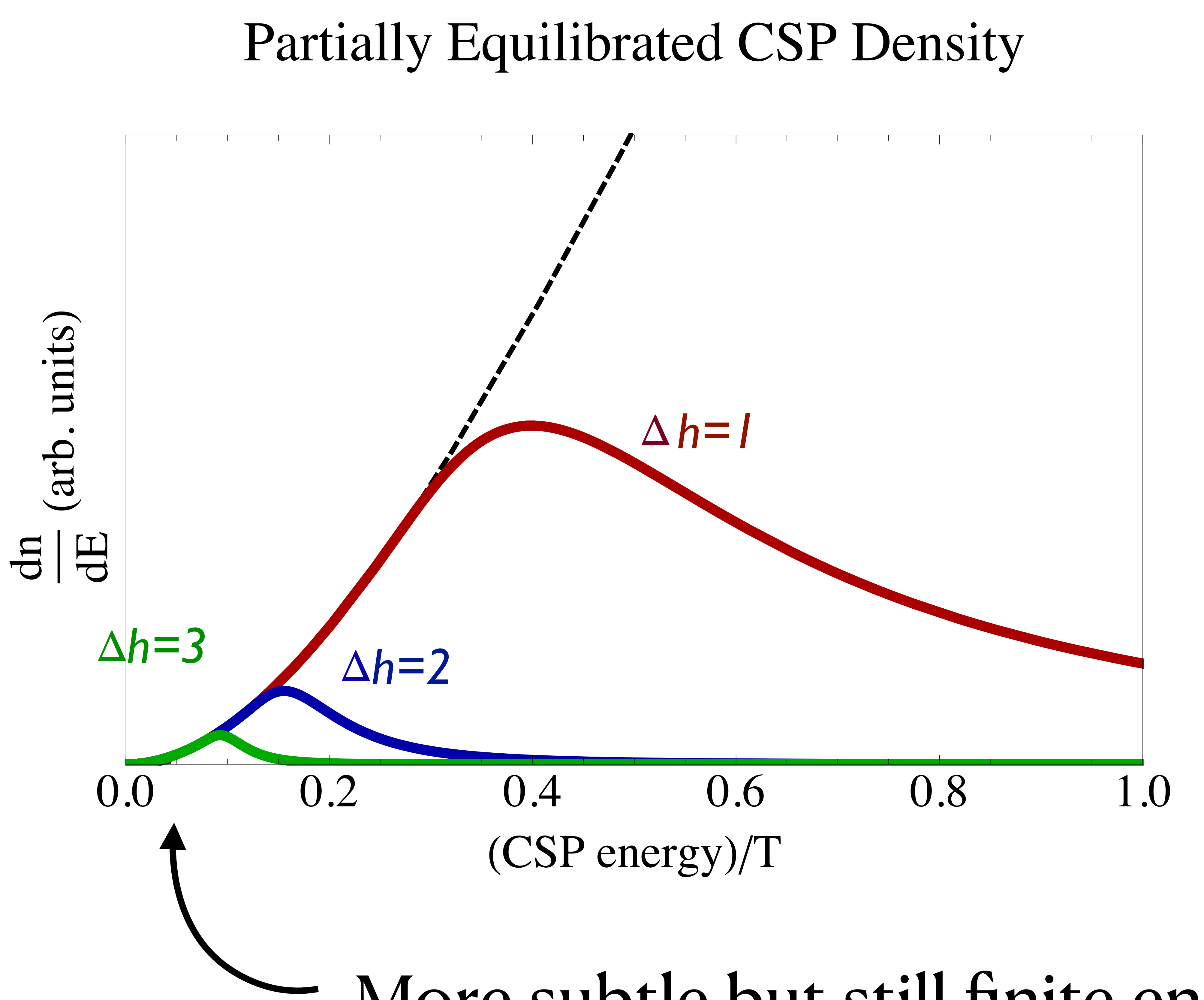
η(ϕ) lies on unit circle orthogonal to k, e.g. $(0, \cos\phi, i\sin\phi, 0)$ for $k = (k, 0, 0, k)$

(1) no unphysical singularities, (2) factorization at physical singularities, (3) finite angle-differential cross-section at all energies.

Thermodynamics

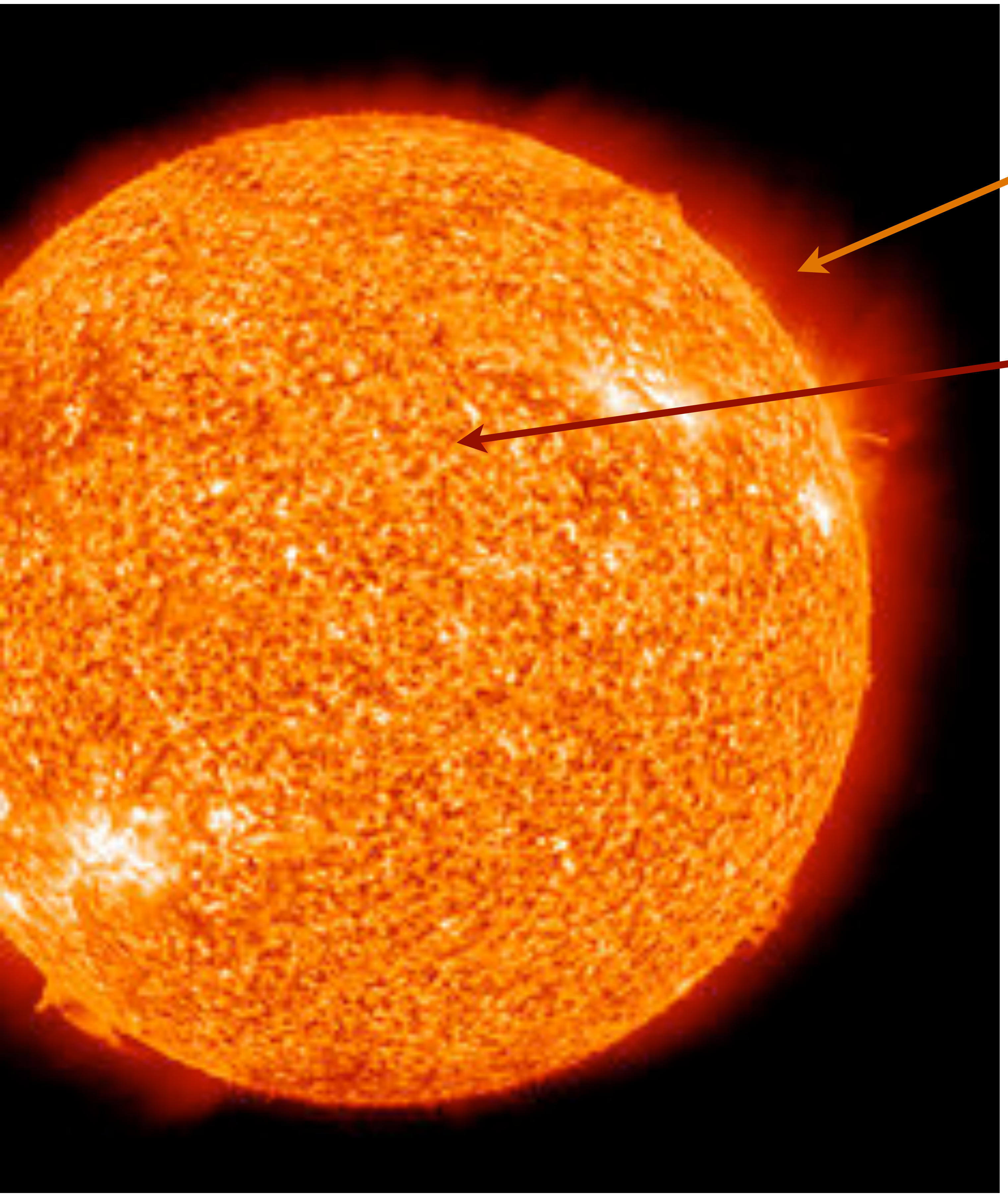
Qualitative picture is easy to understand for $T \gg \rho$ -- typical thermal modes are in UV helicity-correspondence regime.

- Primary $h = \pm 1$ modes thermalize as usual at timescale τ_0
- Adjacent modes' cross-sections suppressed by $(\rho/E)^2 \rightarrow$ slower thermalization except at low energies $\rightarrow \tau_1 \sim (T/\rho)^2 \tau_0$
- Successively slower equilibration of higher-helicity modes.



More subtle but still finite energy density: Contribution of very low energy modes $E \ll \rho$ with (Schuster, Sundaresan, NT)

Dark Radiation Production in Stars



Luminosity ~ 10^{34} erg/s

Power$_{(\text{brem})}$ ~ 10^{59} erg/s ≫ Lumi

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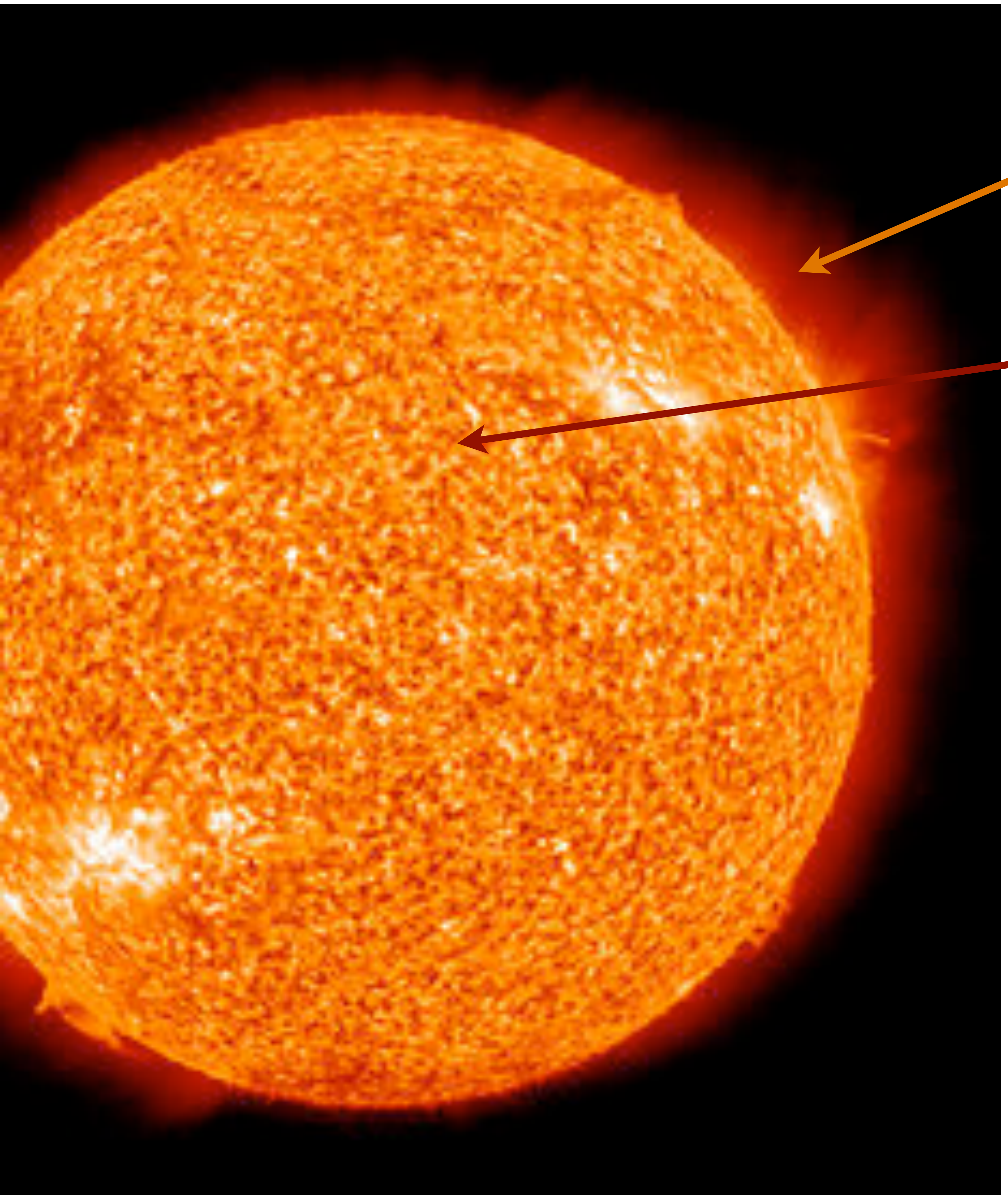

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[analogous to light-axion constraints]

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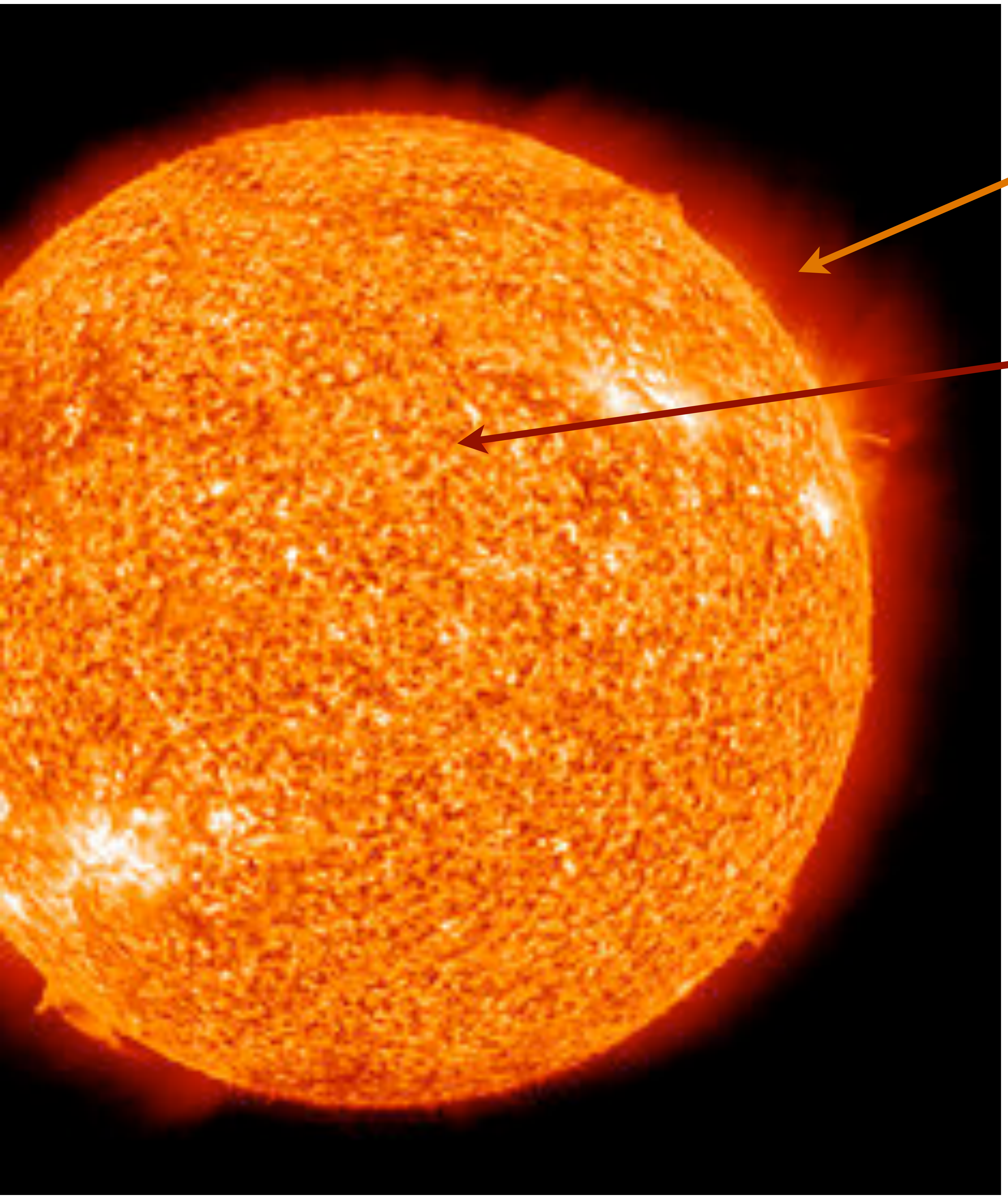

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If *one* h≠1 CSP was brem'd per 10^{26} γ's and escaped sun, luminosity and stellar evolution would change by O(10%).

$$\rho^2 \lesssim 10^{-26} m_e T \sim (10^{-8}\text{eV})^2$$

$$\rho^{-1} \gtrsim 10\text{m}$$

[analogous to light-axion constraints]

Cooler stars ⇒ few-10x stronger bound on ρ

What is Vector Superspace?

- Divergent – division by [infinite] volume $\int_\eta \delta(\eta^2+1)$ implicit. Regulate by $\eta^0 \to i\eta^0$ or use symmetry, e.g.

$$\int_\eta \delta(\eta^2+1)\eta^\mu\eta^\nu = \frac{1}{4}g^{\mu\nu}\int_\eta \delta(\eta^2+1)\eta^2 = -\frac{1}{4}g^{\mu\nu}$$

- Basic job: relating off-shell Lorentz transformation to little group action on-shell

Whenever $\delta(\eta^2+1)k\cdot\partial_\eta F(\eta) = 0$,

$$\int_\eta \delta'(\eta^2+1)F(\eta) = \int_C F(\eta)$$

C= unit circle of $\vec{\eta}$'s orthogonal to $\vec{k}$ ~ "$ISO(2)$ momentum-space"

Orthonormal basis, tree unitarity of CSP exchange, little group covariance of matrix elements follow from this identity

- Enables an enlarged spacetime symmetry that mixes spins

Free action is invariant under a "bosonic supertranslation" $\delta x^\mu = \omega^{\mu\nu}\eta_\nu$ [Rivelles '14].

A Field Theory for All Helicities: A Bit of Intuition

When $\rho = 0$, action encodes familiar actions for tensor components, e.g.

$$\mathscr{L}[\Psi \to \phi(x)] = \frac{1}{2}\int_\eta \underbrace{\delta'(\eta^2+1)}_{\text{gives } 1}\underbrace{(\partial_x\Psi)^2}_{\partial_x\phi} + \frac{1}{2}\,\delta(\eta^2+1)(\partial_x\cdot\underbrace{\partial_\eta\Psi)^2}_{=0} \quad = \frac{1}{2}\,(\partial_x\phi)^2$$

$$\mathscr{L}[\Psi \to \sqrt{2}\eta^\mu A_\mu] = \frac{1}{2}\int_\eta \delta'(\eta^2+1)\underbrace{(\partial_x\Psi)^2}_{2\,(\eta_\mu\partial_x A^\mu)^2} + \frac{1}{2}\,\delta(\eta^2+1)\underbrace{(\partial_x\cdot\partial_\eta\Psi)^2}_{2\,(\partial_\mu A^\mu)^2} = -\frac{1}{2}(\partial_\mu A_\nu)^2 + \frac{1}{2}(\partial_\mu A^\mu)^2$$

But working in η-space directly is compact, much simpler for non-zero ρ.

A Field Theory for All Helicities

- Hyperboloid $\eta^2 + 1 = 0$ has infinite volume, so integrals must be regulated (e.g. by analytically continuing η°)

  But if we define $\int_\eta \delta(\eta^2+1) \equiv 1$, other integrals fixed by symmetry, e.g.

$$\int_\eta \delta(\eta^2+1)\eta^\mu\eta^\nu = \frac{1}{4}g^{\mu\nu}\int_\eta \delta(\eta^2+1)\eta^2 = -\frac{1}{4}g^{\mu\nu}.$$

A bit like Grassmann integration – can exchange $\int_\eta \delta'(\eta^2+1)$ for **differential** operator $J_0(\partial_\eta^2)$

- Action with $\rho = 0$ reduces to sum of familiar massless actions, e.g.

$$\mathscr{L}[\Psi \to \phi(x)] = \frac{1}{2}\int_\eta \underbrace{\delta'(\eta^2+1)}_{\text{gives } 1}\underbrace{(\partial_x\Psi)^2}_{\partial_x\phi} + \frac{1}{2}\delta(\eta^2+1)(\partial_x \cdot \underbrace{\partial_\eta\Psi)^2}_{=0} = \frac{1}{2}(\partial_x\phi)^2$$

$$\mathscr{L}[\Psi \to \sqrt{2}\eta^\mu A_\mu] = \frac{1}{2}\int_\eta \delta'(\eta^2+1)\underbrace{(\partial_x\Psi)^2}_{2\,(\eta_\mu\partial_x A^\mu)^2} + \frac{1}{2}\delta(\eta^2+1)\underbrace{(\partial_x \cdot \partial_\eta\Psi)^2}_{2\,(\partial_\mu A^\mu)^2} = -\frac{1}{2}(\partial_\mu A_\nu)^2 + \frac{1}{2}(\partial_\mu A^\mu)^2$$

Currents in Space-Time: Causality

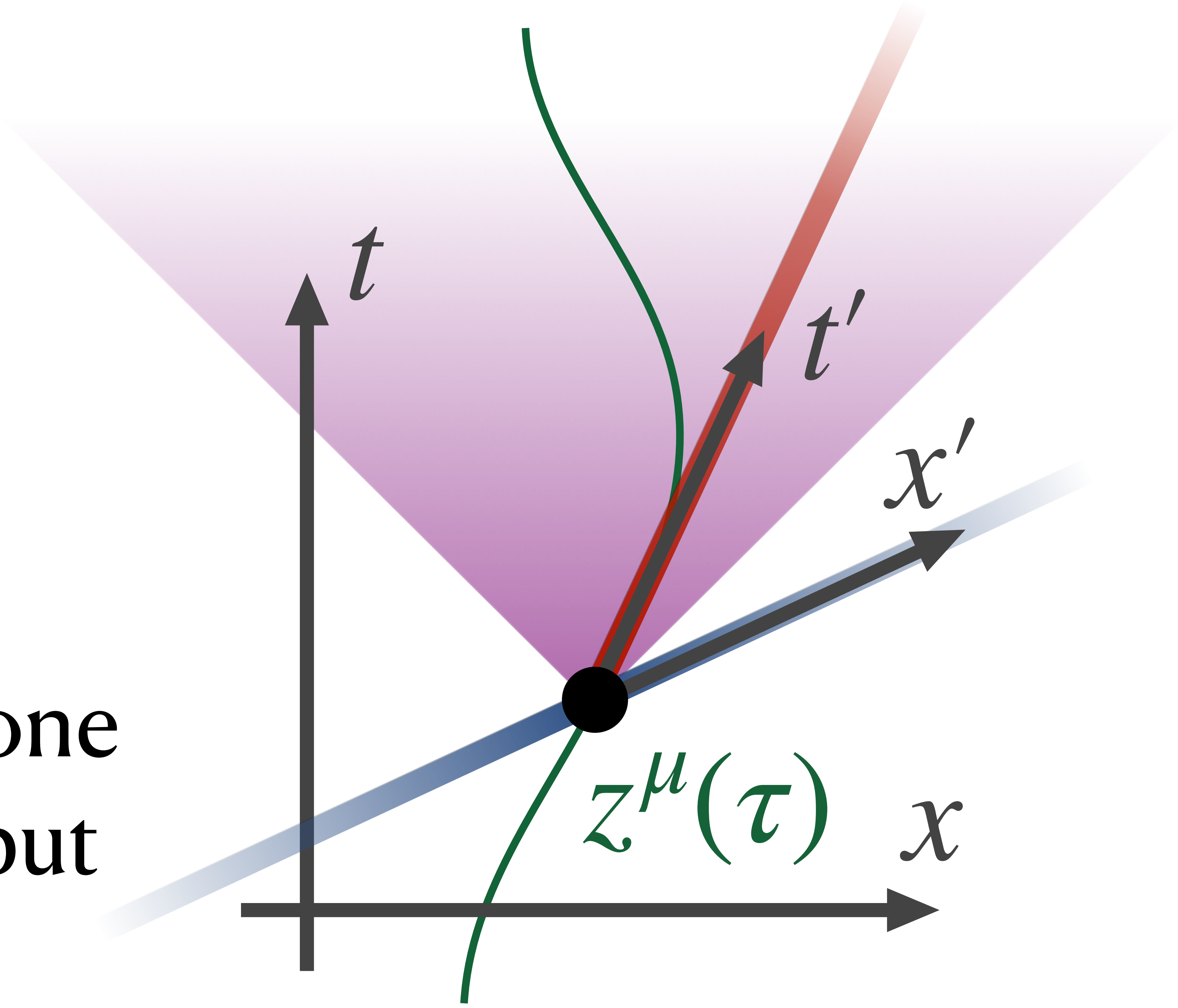
Some ansatz currents **admit retarded/advanced forms supported in source's forward/ backward lightcone → manifestly causal equations of motion**

$$\partial_x^2 \Psi(x) \propto \int d\tau\, j_R(x - z(\tau))$$

$$m\ddot{z}^\mu(\tau) \propto \int \Psi(\eta, x) j_A(x - z(\tau))$$

Field at point depends on particle trajectories in past causal cone

Particle's acceleration at point depends only on fields in its causal past



This feature, and detailed non-local structure, suggestive of integrating out intermediate fields. We suspect this can be done at Lagrangian level to yield local & manifestly causal action, but no concrete realization yet.

(Could Rivelles' supertranslation-like symmetry be a hint?)

Even equal-time interactions can yield causal dynamics (c.f. Coulomb-gauge QED) – so would be premature to rule out less causal-looking currents without further study

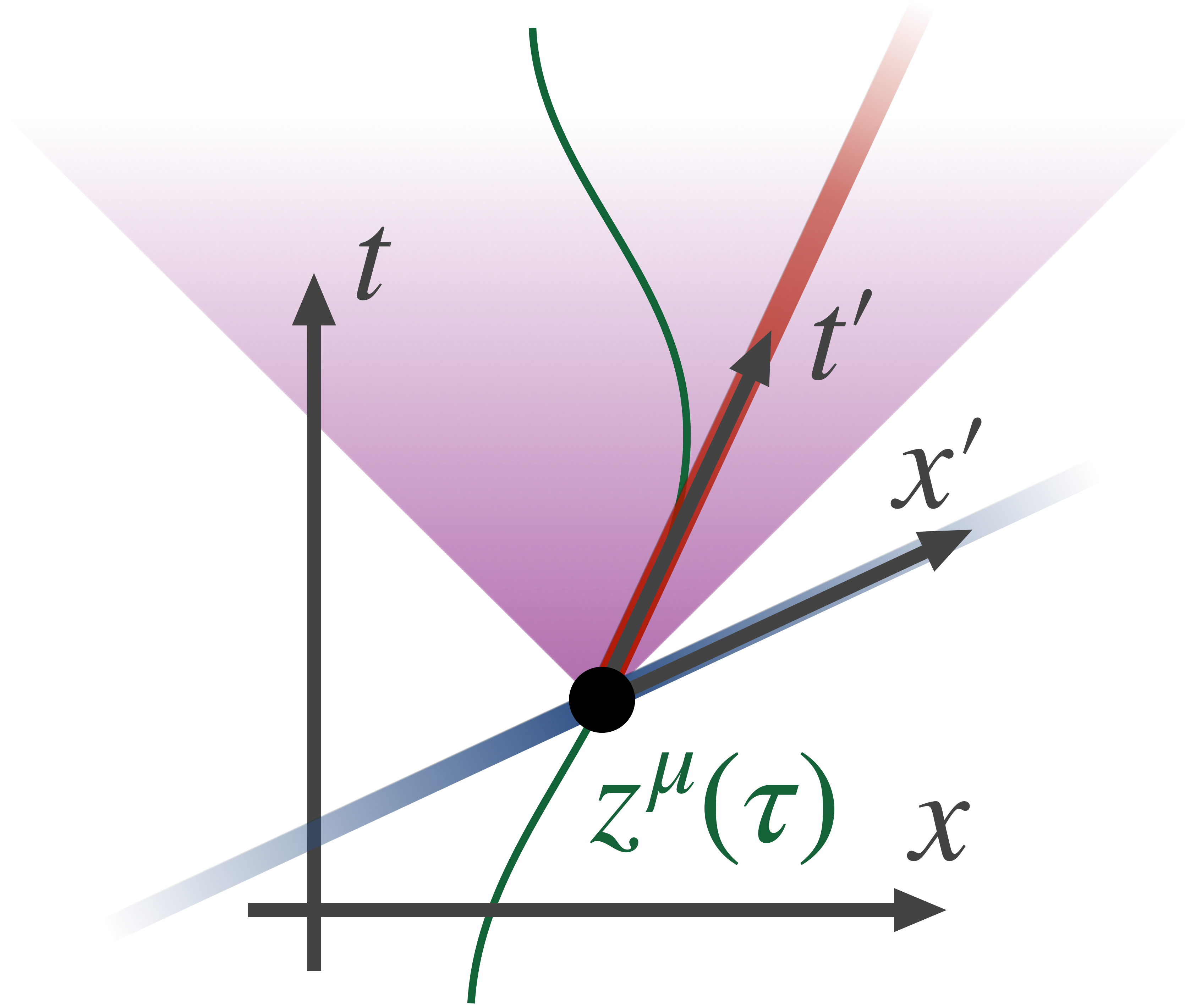
Shape Questions and Off-Shell Physics

Different "current shapes" do have different physics – whenever off shell continuous spin fields are involved, e.g.

— classical static force law (1/r for spatial and temporal, with ρ-corrections for inhomogeneous) and velocity-dependent corrections

— tree-level matter-matter scattering via CSP exchange

— **renormalized CSP and matter propagators**

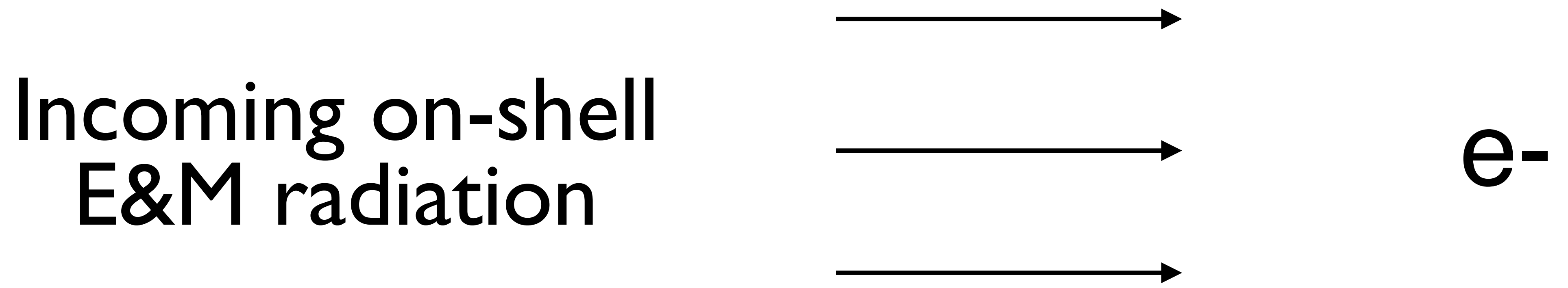


Exploring consistency properties of these less universal amplitudes will likely help to understand which current structures are consistent, physically "minimal".

Long Distance Force Measurements

Non-zero spin-scale for known fundamental forces motivates more precise (infrared) force-law tests

Most robustly predicted "force law" corrections are those from radiation backgrounds



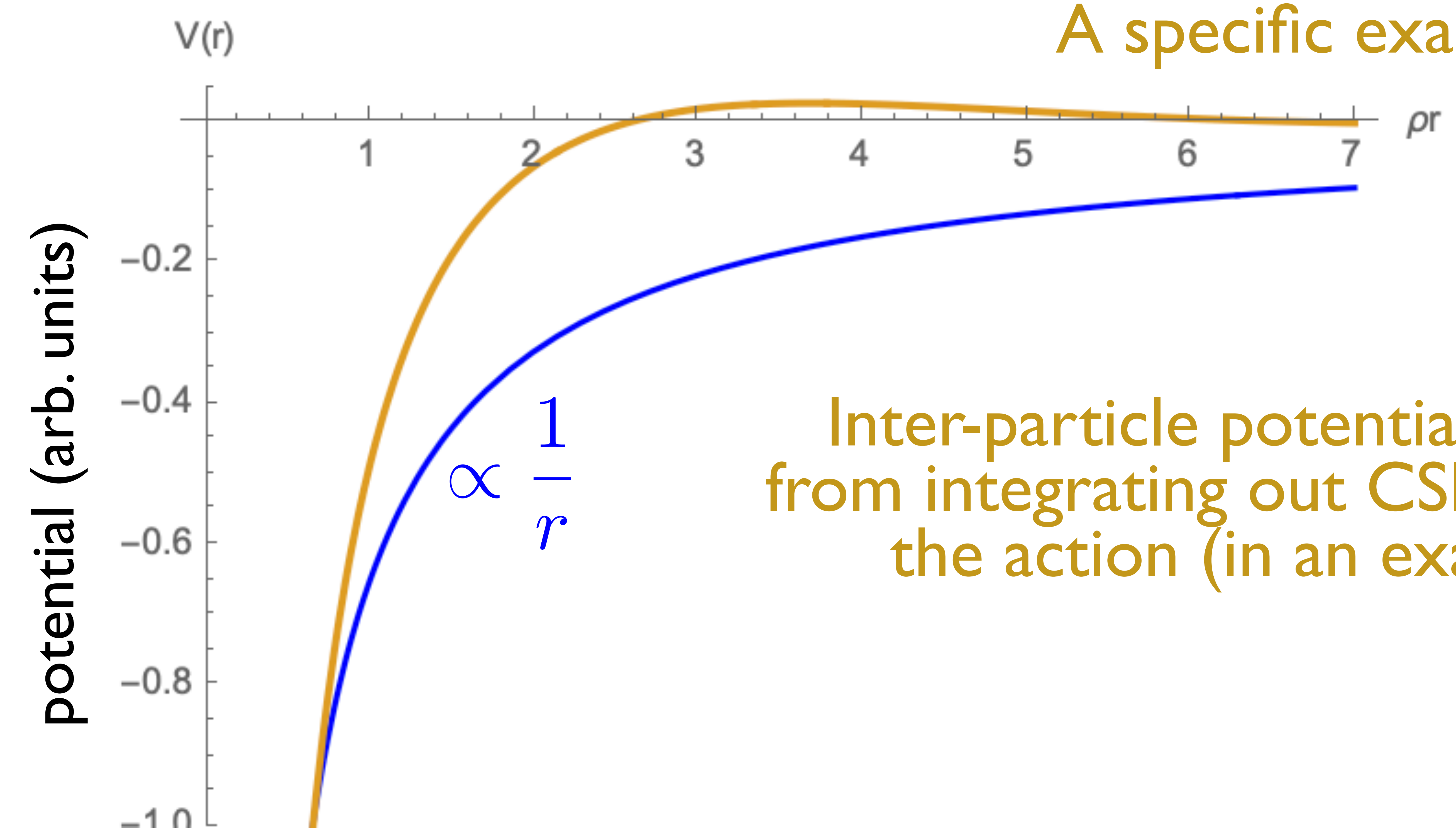
$$\frac{\mathbf{F}_{h=\pm 1}}{q} = \mathbf{E} + \mathbf{v} \times \mathbf{B} + \left(\frac{\rho}{\omega}\right)^2 \left(\frac{\mathbf{v}_\perp(\mathbf{v}_\perp \cdot \mathbf{E})}{4} - \frac{v_\perp^2 \mathbf{E}}{8}\right) + \ldots$$

We worked this out for E&M — would like to do leading order gravity case

Long Distance Potential Energy

Standard 1/r behavior at distance scales short compared to ρ^{-1}

A specific example

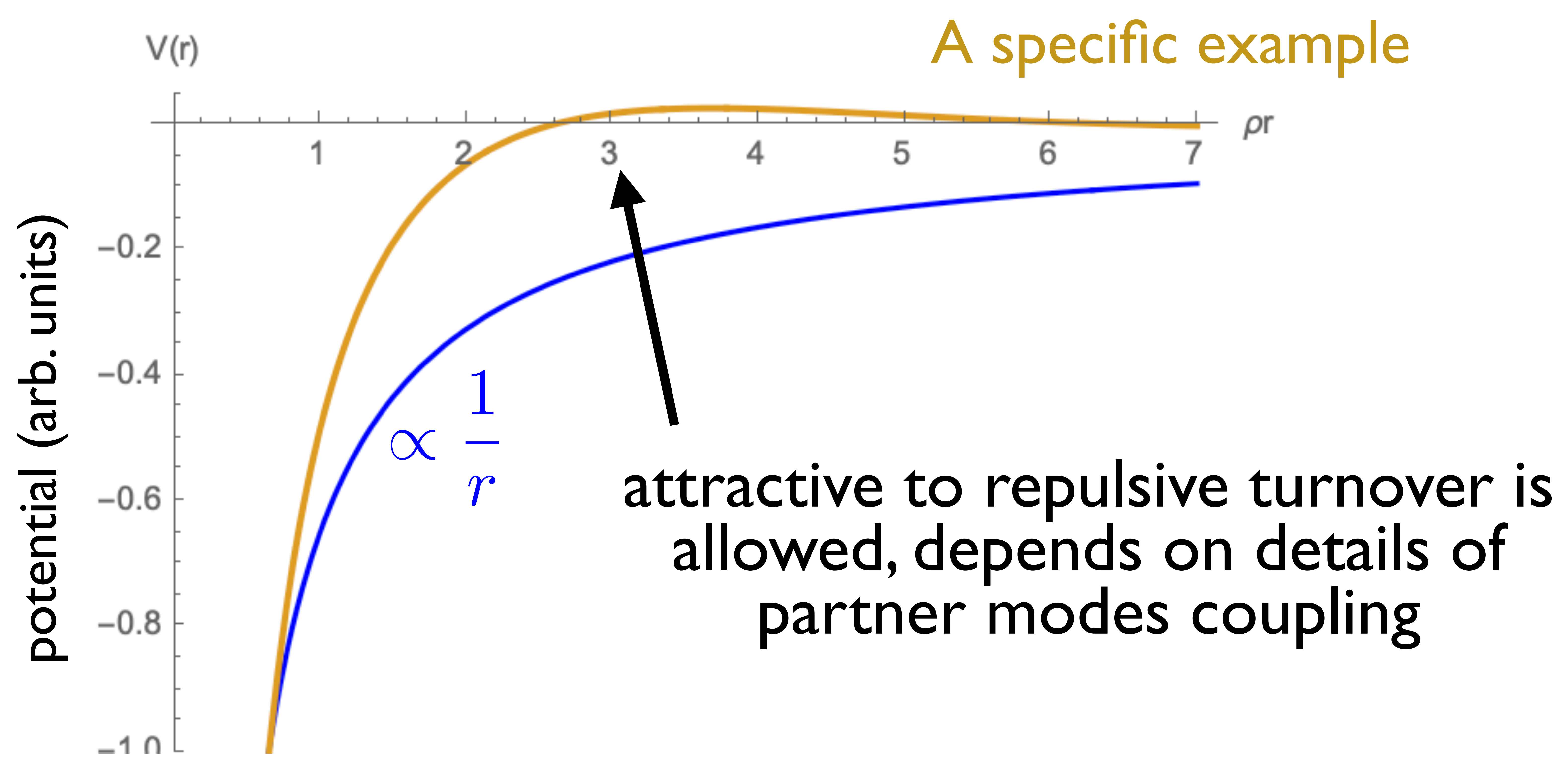


Inter-particle potential derived from integrating out CSP modes in the action (in an example)

Long distance departure from 1/r compatible with current conservation, but model-dependent

Long Distance Potential Energy

Consider gravity with non-zero spin scale



Long-distance acceleration behavior can arise from the potential, not just a dark energy term

In fact, the naive CC term is not gauge invariant for nonzero spin scale

Long Distance Potential Energy

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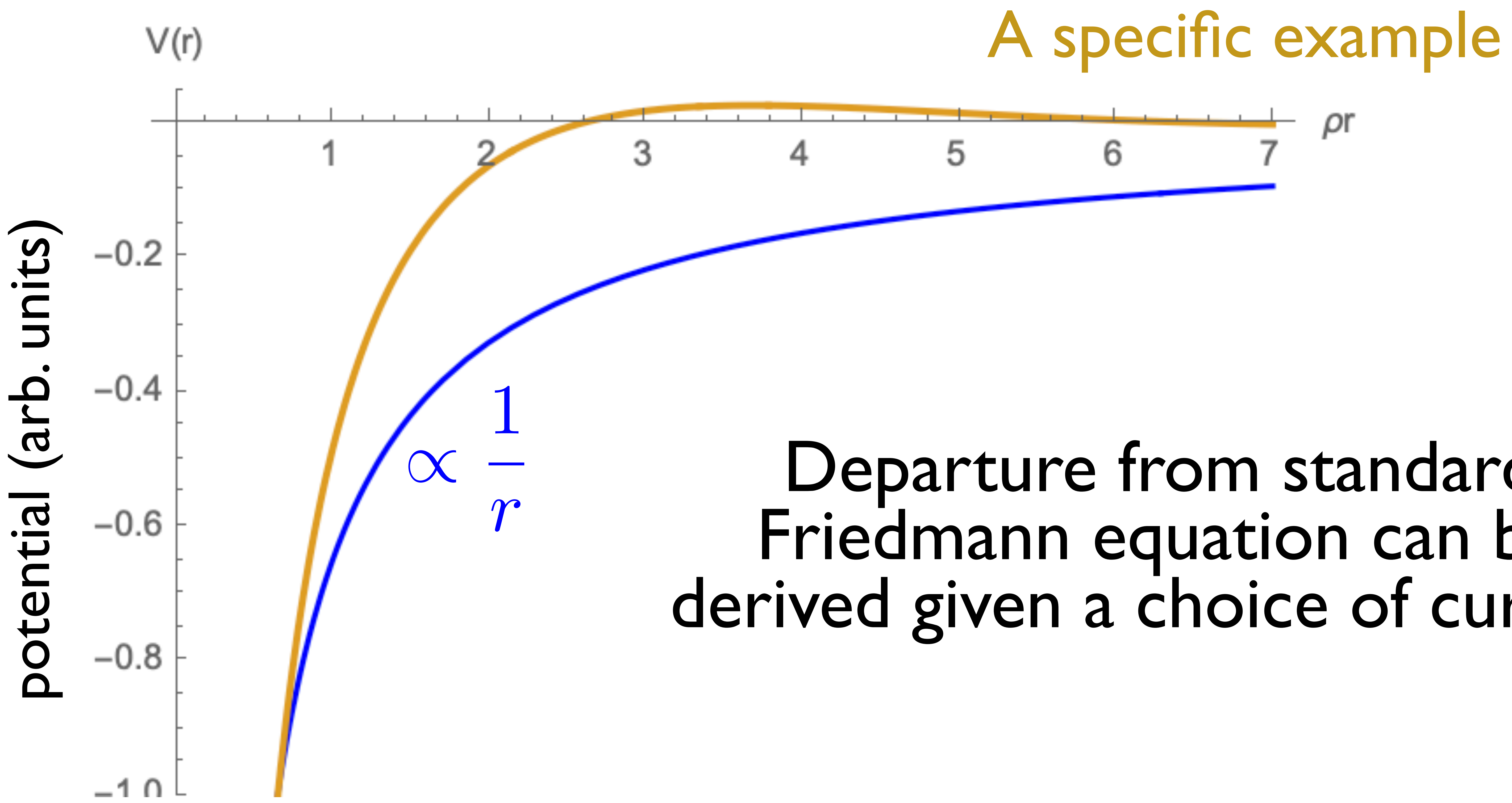

Departure from standard Friedmann equation can be derived given a choice of current

Very small spin scales for gravity natural to consider in the context of cosmology