Gravitational waves from Nnaturalness

arXiv: 2310.06905 [hep-ph]

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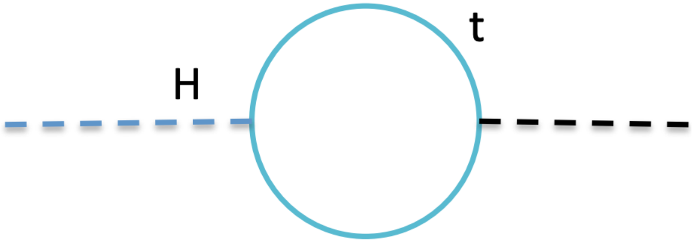
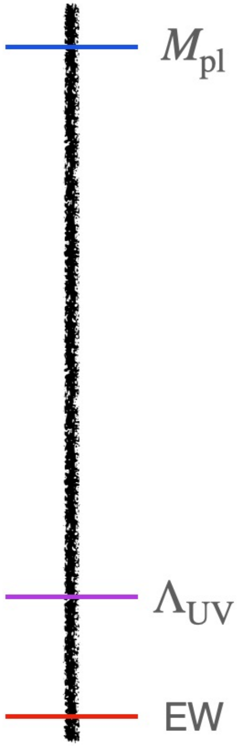
Motivation

- Naturalness : EFT should not be extremely sensitive to the structure of the underlying UV theory.
- 1-loop corrections from SM fields to the Higgs mass give quadratic corrections :

$$\Delta m_h^2 \propto \Lambda_{UV}^2$$



- No evidence of new physics at TeV scale.
- If new physics should enter at a new scale, Λ_{UV}, say at O(10) TeV then we want to explain this little hierarchy.



https://arxiv.org/pdf/1501.01035.pdf
https://arxiv.org/pdf/2205.05708.pdf

Nnaturalness(Arkani-Hamed et.al. 2016)

- Nnaturalness is a model which consists of a large N copies of the Standard Model with varying values of the Higgs mass parameter.

- Some sectors will accidentally have the Higgs mass parameters that are parametrically smaller than the cutoff.

- Our SM is to be identified with the sector having the smallest (negative) squared Higgs mass.

- Finally, a light `reheaton' can naturally transfer most of its energy to our sector and only slight fractional energy densities to the other sectors.

- We choose $N = 10^4$ and $\Lambda_H = 10\, TeV$ for our study.

Nnaturalness : Model

- Consider N copies of SM, with the Higgs squared mass parameters assumed to vary uniformly from one sector to another, (i=0 corresponds to our SM)

$$-\Lambda_{\rm UV}^2 < m_{H,i}^2 < \Lambda_{\rm UV}^2$$

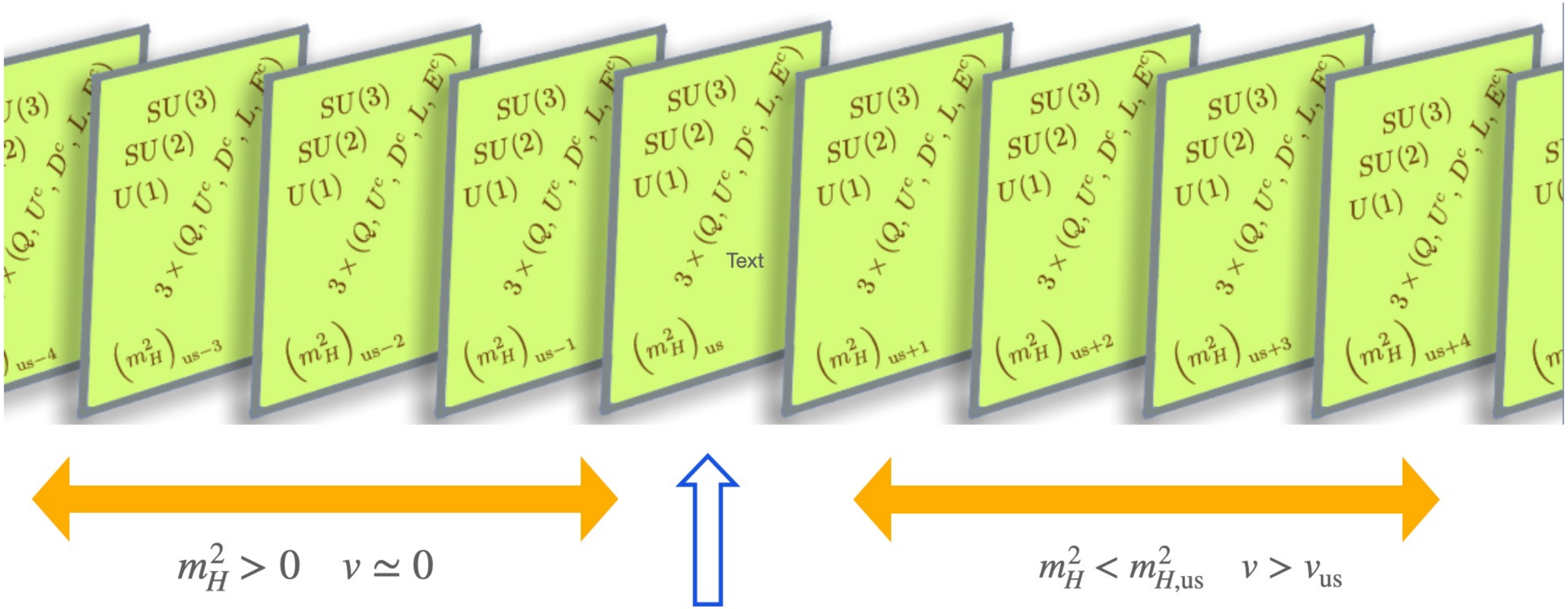
$$m_{H,i}^2 = -\frac{\Lambda_H^2}{N}(2i + r) = -(88\ {\rm GeV})^2\ (1 + \frac{2i}{r})$$

- We consider a scalar reheaton, which dominate the energy density of the universe at some time following inflation,

$$\mathcal{L}_\phi \supset -a\phi \sum_i |H_i|^2 - \frac{1}{2} m_\phi^2 \phi^2$$

arXiv:1607.06821 [hep-ph]

Nnaturalness



$m_H^2 > 0 \quad v \simeq 0$

$m_H^2 < m_{H,\mathrm{us}}^2 \quad v > v_{\mathrm{us}}$

$m_{H,\mathrm{us}}^2 = -(88\ \mathrm{GeV})^2 \quad v_{\mathrm{us}} = 246\ \mathrm{GeV}$

$v_i \sim \frac{y_t \Lambda_{QCD,i}^3}{m_{H,i}^2} \ll \Lambda_{QCD,i} \sim O(100\,MeV)$

$v_{i\geq 0}^2 = -(m_{H_i}^2)/\lambda = v^2(2i/r+1)$

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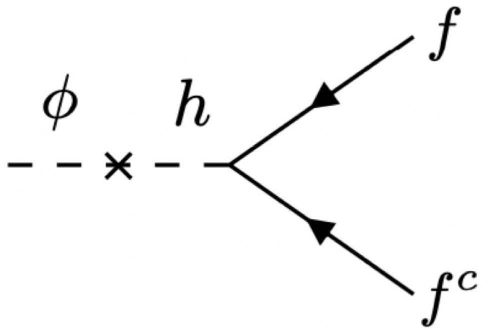
Model : Salient features

- Reheaton decays preferentially to lightest Higgs i.e. $\rho_i/\rho_{\rm SM} \approx \Gamma_i/\Gamma_{\rm SM}. < 1$ and this ratio is independent of coupling a (<Λ_H/N).
- Coupling strength only controls reheating temperature.
- Nnaturalness cosmology is completely determined by $\{m_\phi, r\}$ with a weak dependence on N.
- SM has a very tiny coupling to other sectors, making colliders searches futile.

Reheaton decays : SM-like Sectors



- Reheaton decays to all kinematically available final states via it’s mixing with the Higgs h_i (mixing angle $\theta_i \sim a/m_{H_i}$)

$$\Gamma_{\phi\to\{f_i\}} = \theta_i^2 \, \Gamma_{h_i\to\{f_i\}}(m_\phi) \propto 1/m_{h_i}^2 \sim \frac{1}{i}$$

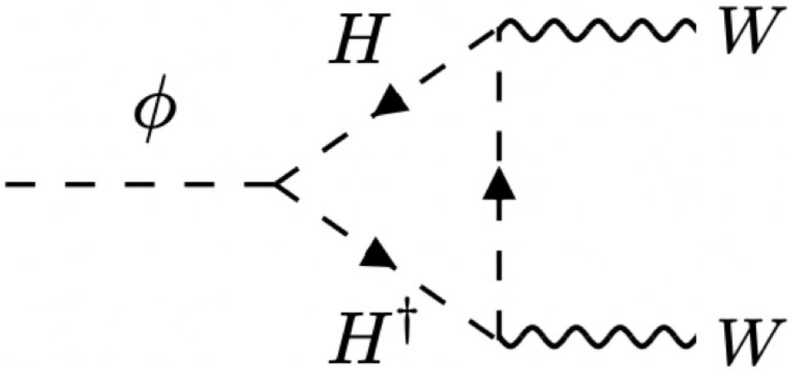
- Pre-dominant decay is to our SM (lightest higgs)

$$\rho_i/\rho_{\rm SM} \approx \Gamma_i/\Gamma_{\rm SM}.$$

- Energy density in other sectors contributes to relativistic d.o.f

$$\Delta N_{\rm eff} \sim \sum_{i\neq 0} \frac{\Gamma_i}{\Gamma_{i=0}} \sim \log(N)$$

Reheaton decays : Exotic sector



- There's no higgs-scalar mass mixing in the absence of vev, and decays generally involve doublet Higgs

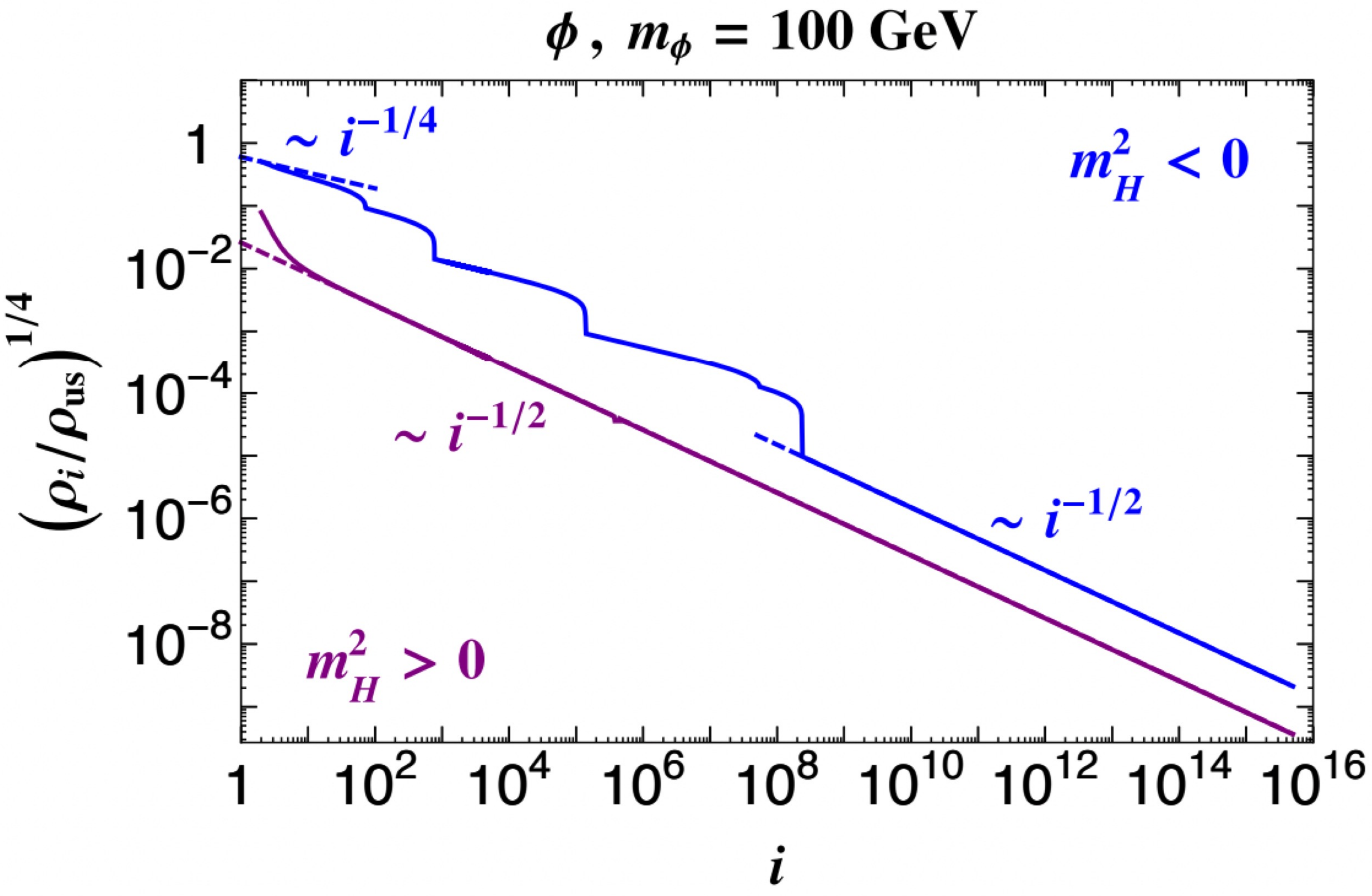
$$\Gamma_i \sim \frac{1}{m_{H,i}^4} \sim \frac{1}{i^2}$$

- The lightest $i = -1$ is the only relevant sector and gets reheated substantially

$$\Delta N_{\text{eff}} \sim \sum_{i=-1} \frac{\Gamma_{i=-1}}{\Gamma_{i=0}} \propto \frac{1}{m_{H,-1}^4}$$

- Decay's parametric dependence can be evaded by having H_{-1} light, $\mathrm{m_{H_{-1}}} \leq \frac{\mathrm{m}_\phi}{2}$ which opens on-shell or nearly on-shell two body decays.

Reheaton decay : recap



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First exotic sector (i=-1) : mass spectrum

- Formation of quark condensates lead to spontaneous breaking of the electroweak (SU2) symmetry at around $T_{-1,c} \sim 85\ MeV$,

$$< q_L q_R > \simeq 4\pi f_{\pi,-1}^3 \ \rightarrow \ v_{-1} \simeq \frac{f_\pi^3}{m_{H,-1}^2}$$

- 3 goldstones of $SU(6)_L \times SU(6)_R \rightarrow SU(6)_V$ gets eaten by W and Z

$$m_{W,Z} \simeq f_{\pi,-1} \simeq \mathcal{O}(10\mathrm{MeV})$$

- The rest 32 gets their masses from the explicit symmetry breaking via Yukawas

$$m_{\pi,-1} \simeq \mathcal{O}(\mathrm{keV} - \mathrm{MeV})$$

- Leptons are the lightest

$$m_e \simeq \mathcal{O}(10^{-4}\mathrm{eV}),\ m_\mu \simeq \mathcal{O}(10^{-2}\mathrm{eV}),\ m_\tau \simeq \mathcal{O}(10^{-1}\mathrm{eV})$$

First order phase transition (Nf=6)

- We follow the conventional arguments by [Pisarski,Wilczek 1984] which suggests that the QCD with 3 or more massless quarks undergoes FOPT.
- First exotic sector with 6 massless quarks cools and as its temperature reaches $\Lambda_{QCD,-1} \sim 100\ MeV$, there's a FOPT due to quark condensates.
- $T_{-1,c} \sim 85\ MeV$ and $50 < T_{-1,perc} < 85\ MeV$, where the latter is the temperature by which 1/3 of the universe is in true vacuum.
- Near recombination($T \sim 0.3\ eV$), the exotic sector relativistic species comprise of neutrinos as well as photons and electrons still in interacting bath.

Phase transition : Strength

- Strength of the phase transition is roughly given by

$$\alpha_{-1} = \frac{\Delta V}{\rho_{\mathrm{rad},-1}} \quad \alpha_{\mathrm{tot}} = \frac{\Delta V}{\rho_{\mathrm{rad,tot}}}$$

- First principles calculation of α_{-1} for QCD is complicated due to strong coupling regime, instead we consider distinct phase transition scenarios.

- Entropy ratio after and before the QCD FOPT is related via,

$$D_{s,-1} = \frac{s_{-1}^{rh}}{s_{-1}^{perc}} \propto (1+\alpha_{-1})^{3/4}$$

ΔN_{eff} in Nnaturalness

- Phenomenologically, Nnaturalness can be probed via its effects on ΔN_{eff} during recombination.
- For our benchmarks, we use interacting radiation bound from CMB,

$$\Delta N_{\rm eff}^{\rm CMB} \leq 0.7$$

(Planck + SH0ES)

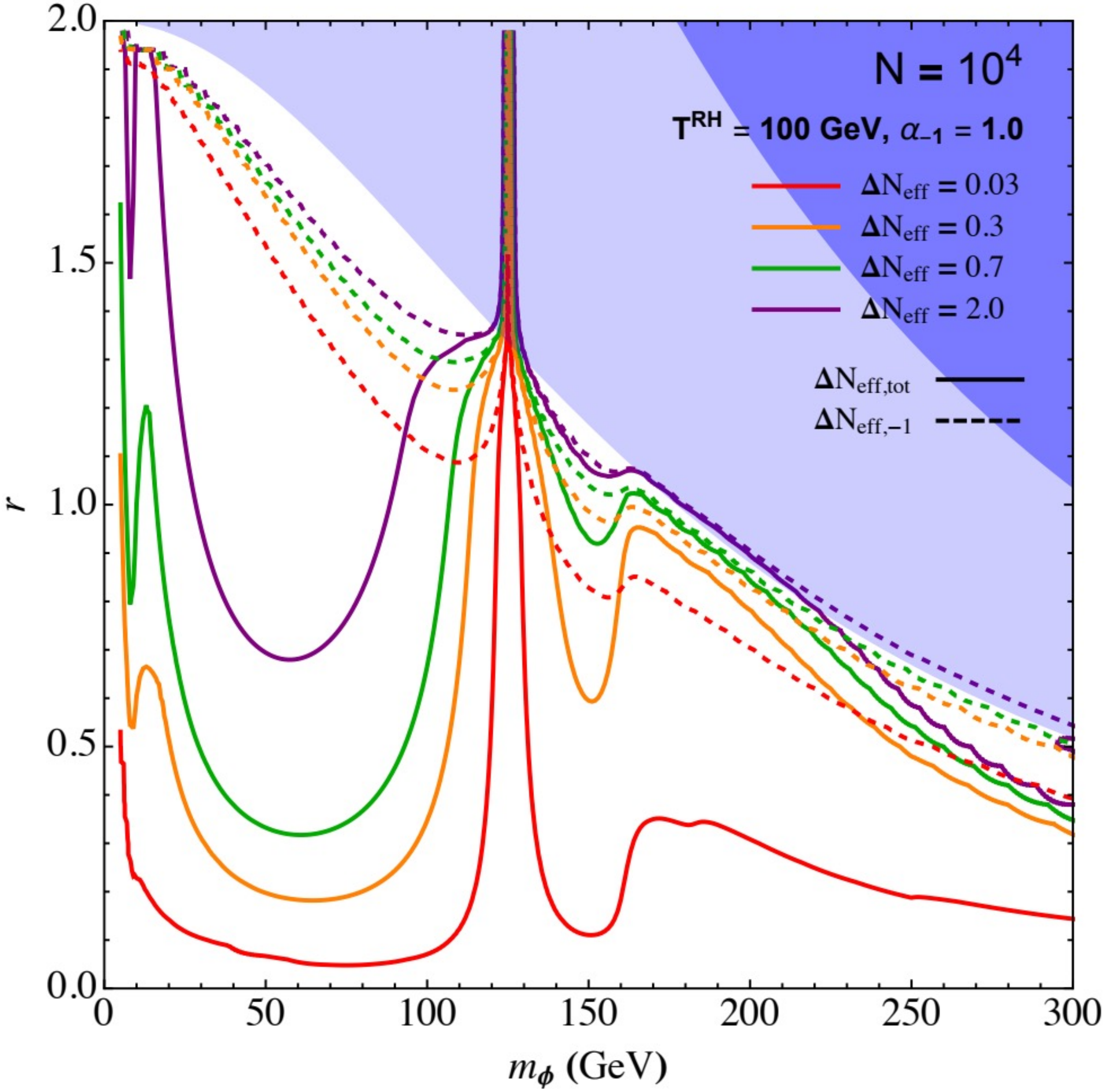
- SM like sectors have contributions via free streaming neutrinos

$$\Delta N_{eff,i>0}^{CMB} \propto \sum_{i>0} \frac{\Gamma_i}{\Gamma_{SM}}$$

- Exotic sectors contribute via free streaming neutrinos and interacting radiation

$$\Delta N_{eff,-1}^{CMB} \propto (1 + \alpha_{-1}) \frac{\Gamma_{-1}}{\Gamma_{SM}}$$

Cosmological Probes of NNaturalness



Gravitational waves parameters

- The spectra of GW from FOPT depends on :
- α: Strength of phase transition
- β: Inverse time scale of phase transition
- T_N: Nucleation Temperature at which first true vacuum bubble arise
- v_w : Bubble wall velocity, the expansion speed of true vacuum bubbles.

Strength of GW signal

- Amplitude of GW signal is controlled by strength parameter, α_{tot} which is given in terms of product of α_{-1} and fraction of energy density in exotic sector.

$$\alpha_{\rm tot} \simeq \alpha_{-1} \left[\frac{g_{*\rho,-1}^{\rm perc}}{g_{*\rho,{\rm SM}}^{\rm perc}} (\xi_{-1}^{\rm perc})^4 \right]$$

$$\simeq \begin{cases} 0.02 \times \left(\dfrac{\Delta N_{{\rm eff},-1}^{\rm CMB}}{0.7} \right) & (\alpha_{-1} = 0.3, \quad {\rm Non-runaway\ scenario}), \\ 0.1 \times \left(\dfrac{\Delta N_{{\rm eff},-1}^{\rm CMB}}{0.7} \right) & (\alpha_{-1} = 10, \quad {\rm Runaway\ scenario}). \end{cases}$$

Duration of PT

- This parameter gives a measure of the duration of PT
- Defined in terms of the Euclidean bounce action as,

$$\frac{\beta}{H} = T_{-1} \frac{d}{dT_{-1}} \frac{S_3}{T_{-1}} \bigg|_{T_{-1,\mathrm{nuc}}}$$

- Unfortunately, we cannot calculate it from first principles as QCD becomes strong.
- There has been studies which predict large values for beta, but none can be mapped 1-1 to our case.
- We thus consider a large range, $\beta/H \in [3, 10^4]$.

General features of Hidden sector PT

- Strong GW signal and safety from Neff constraints impose opposite conditions on hidden sectors.
- ΔN_{eff} constraints impose $\xi_H = \frac{T_H}{T_{SM}} < 1$, whereas $\alpha \propto \xi_H^4$ means that strong GW signals require it isn't much smaller than 1.
- Large β means faster transitions, which results in signals that are weaker and peaking at higher frequencies vs slower transitions.
- Duration parameter β/H is generally expected to be inversely correlated with the strength parameter α.

GW spectrum

- Cosmological FOPT leads to production of stochastic GW via :
  1. **Collisions of bubble walls**: This contribution depends only on the dynamics of scalar field, not on the effects of the background plasma.
  2. **Collision of sound waves** : Bubble expansion leads to production of sound waves from plasma surrounding the bubbles. Lasts longer, enhanced by factor of β/H.
  3. **Hydrodynamic turbulence** in the plasma[1] : Happens after collision of sounds waves, generally suppressed by O(5-10 %) compared to sounds waves.

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[1] neglected in this study

Gravitational Waves signal

- Differential GW density parameter characterizes them :

$$\Omega_{GW} = \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\log f}, \qquad \rho_c = 3M_{pl}^2 H^2$$

- Semi-analytical parametrizations can be used to describe them,

$$\Omega_{\rm GW}^{\rm em}(f_{\rm em}) = \sum_{I={\rm BW,SW}} N_I\,\Delta_I(v_{\rm w})\left(\frac{\kappa_I(\alpha_{-1})\,\alpha_{\rm tot}}{1+\alpha_{\rm tot}}\right)^{p_I}\left(\frac{H}{\beta}\right)^{q_I} s_I(f_{\rm em}/f_{{\rm p},I});$$

$$\Longrightarrow \quad h^2\,\Omega_{\rm GW}^0(f) = h^2\mathcal{R}\,\Omega_{\rm GW}^{\rm em}\left(\frac{a_0}{a_{\rm perc}}f\right).$$

GW signal parametrization

- Normalization factors and exponents :

$$(N_{\mathrm{BW}}, N_{\mathrm{SW}}) = (1, 0.159) \qquad (p_{\mathrm{BW}}, p_{\mathrm{SW}}) = (2, 2) \qquad (q_{\mathrm{BW}}, q_{\mathrm{SW}}) = (2, 1)$$

- Potential suppression due to wall velocity :

$$(\Delta_{\mathrm{BW}}, \Delta_{\mathrm{SW}}) = \left(\tfrac{0.11 v_{\mathrm{w}}^3}{0.42 + v_{\mathrm{w}}^3}, 1\right)$$

- Spectral shape function and peak frequencies :

$$s_{\mathrm{BW}}(x) = \frac{3.8\, x^{2.8}}{1 + 2.8\, x^{3.8}}, \qquad s_{\mathrm{SW}}(x) = x^3 \left(\frac{7}{4 + 3\, x^2}\right)^{7/2},$$

$$f_{\mathrm{p,BW}} = 0.23\, \beta, \qquad f_{\mathrm{p,SW}} = 0.53\, \beta / v_{\mathrm{w}}.$$

Scenario I : Runaway phase transition

- Bubble walls overcome friction from plasma, accelerating to become ultra-relativistic.
- Gravitational waves are sourced by bubble wall collisions.
- α_{-1} should be large to create sufficient pressure resulting in accelerating bubble.
- Benchmark scenario :

$$v_{\mathrm{w}} = 1, \quad \kappa_{\mathrm{BW}} = 1, \quad \kappa_{\mathrm{SW}} = 0,$$

$$(\alpha_{-1}, \beta/H) = (10, 3),\ (5, 10),\ (1, 10^3).$$

Scenario II : Non-runaway phase transition

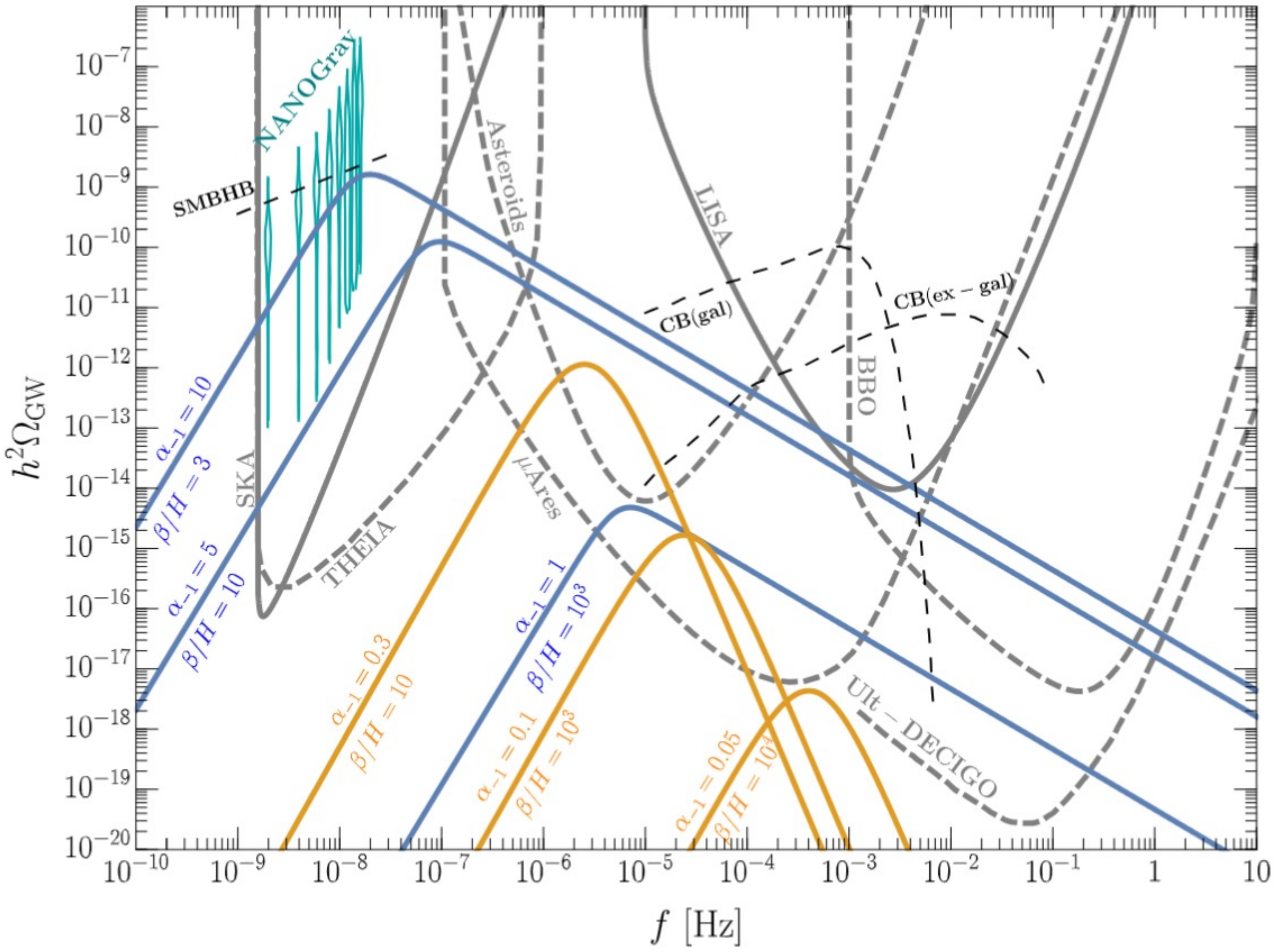
- Bubble walls cannot overcome friction due to plasma and reaches terminal velocity.
- Expanding walls pushes plasma creating a coherent motion of plasma.
- Gravitational waves are sourced by sounds waves.
- Benchmark scenario :

$$v_{\rm w} = \frac{1}{\sqrt{3}}, \quad \kappa_{\rm BW} = 0, \quad \kappa_{\rm SW} = \frac{\alpha_{-1}^{2/5}}{0.017 + (0.997 + \alpha_{-1})^{2/5}},$$

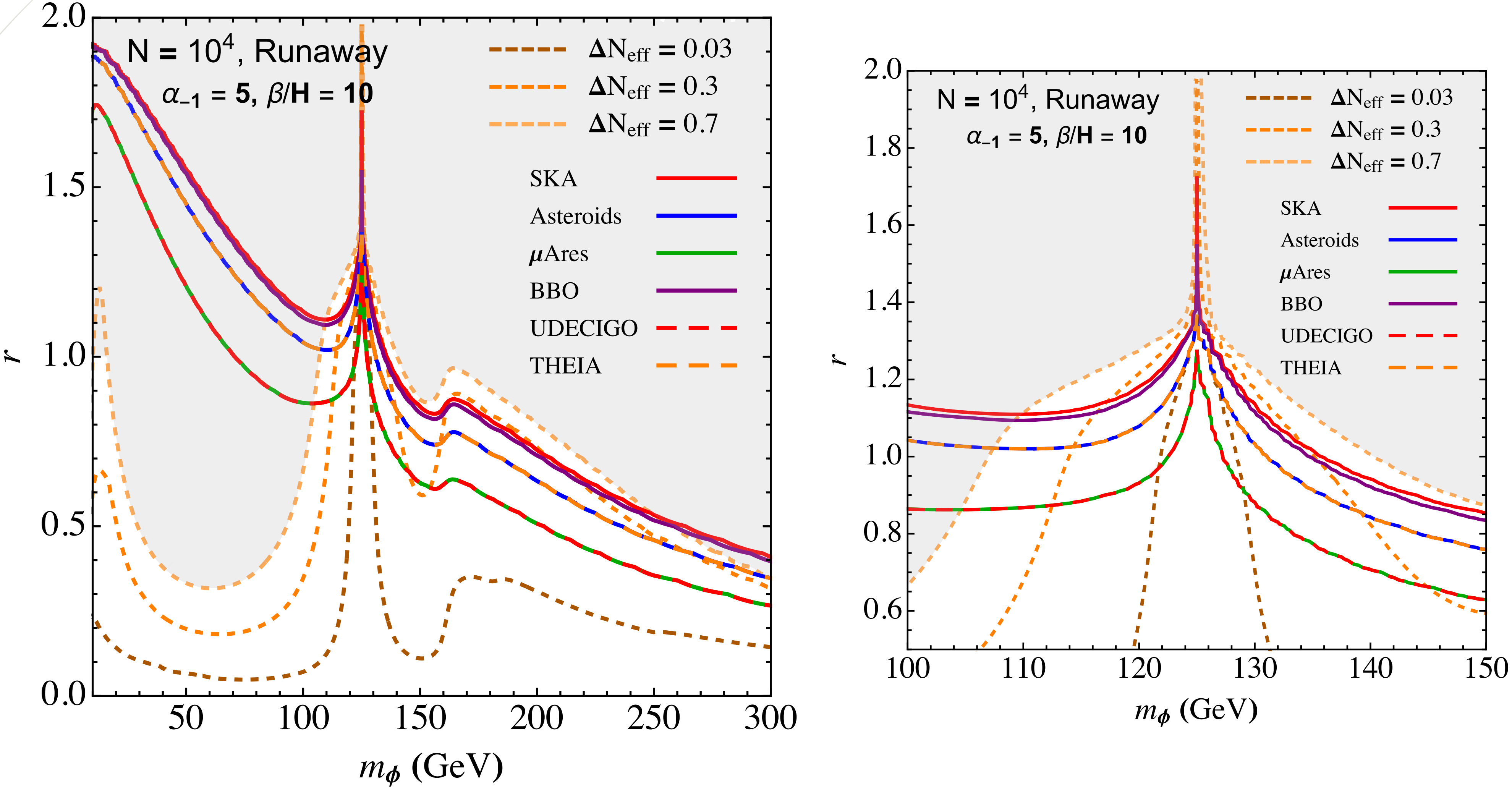
$$(\alpha_{-1}, \beta/H) = (0.3, 10^2), \ (0.1, 10^3), \ (0.05, 10^4).$$

GW detection

$\Delta N^{CMB}_{eff,-1} = 0.7$



Mapping to Nnaturalness : Runaway



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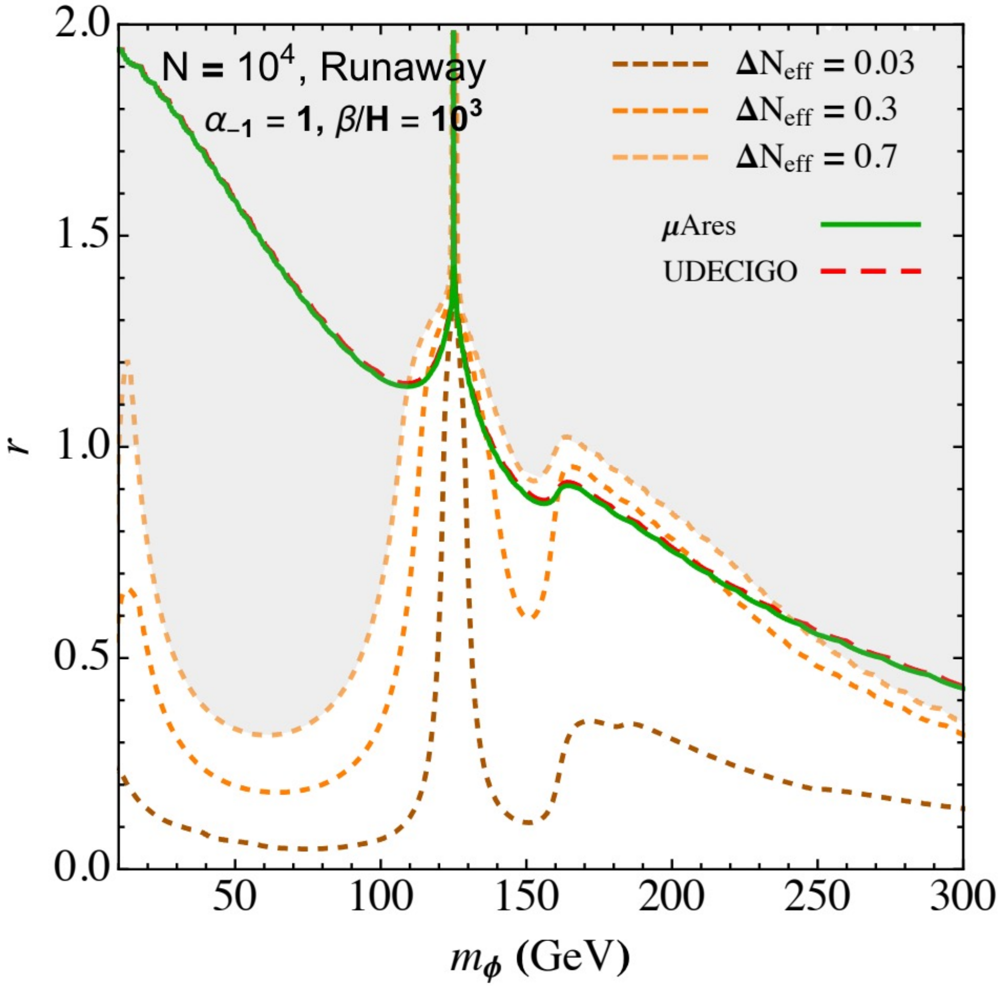
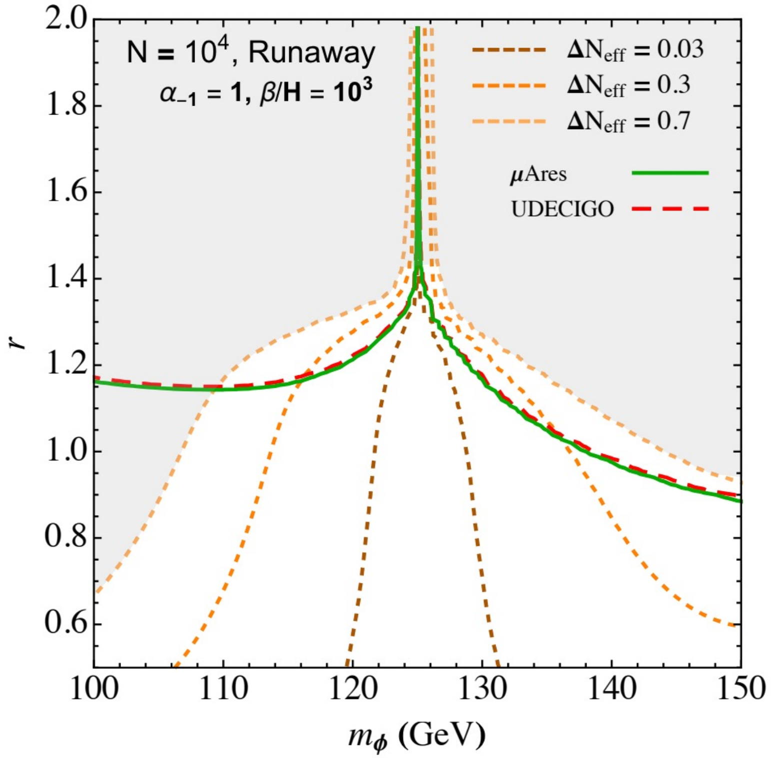
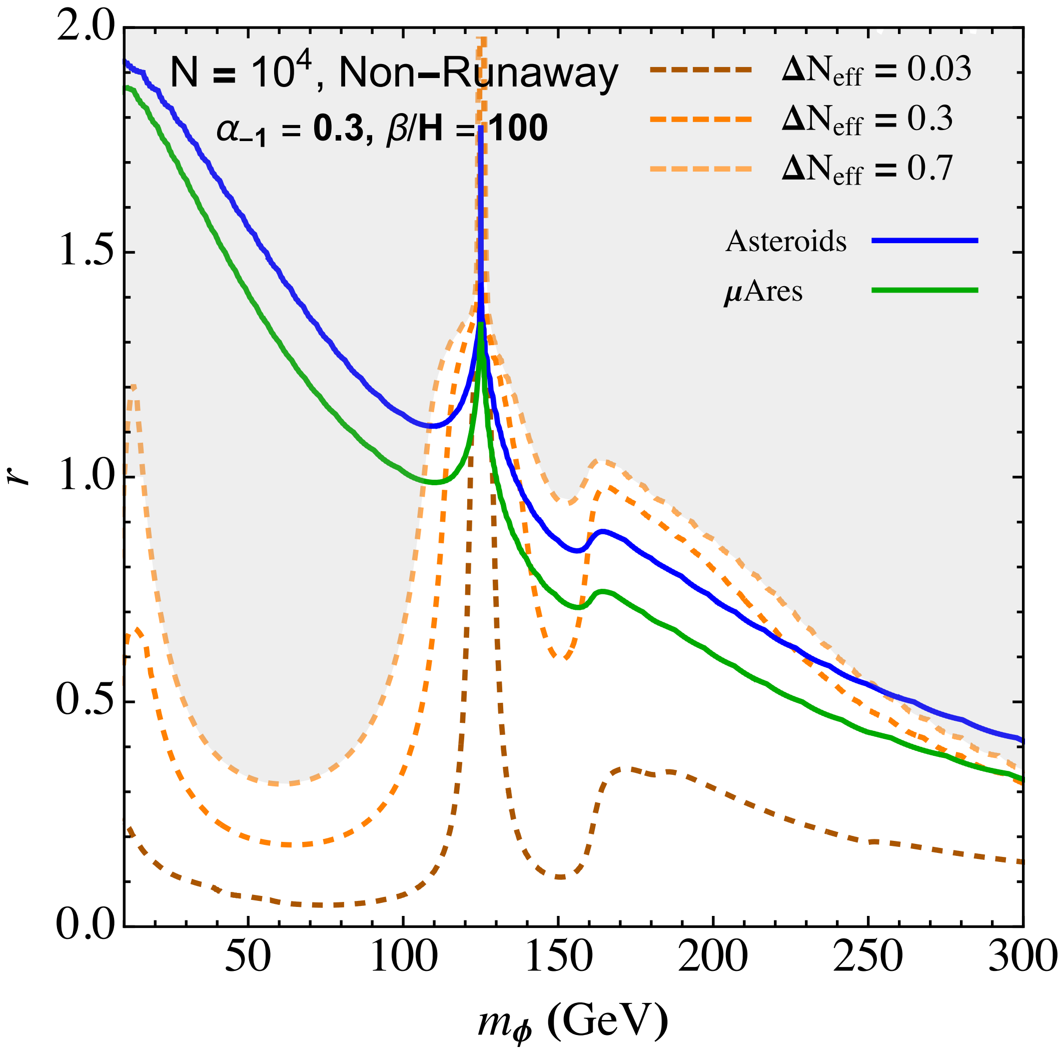
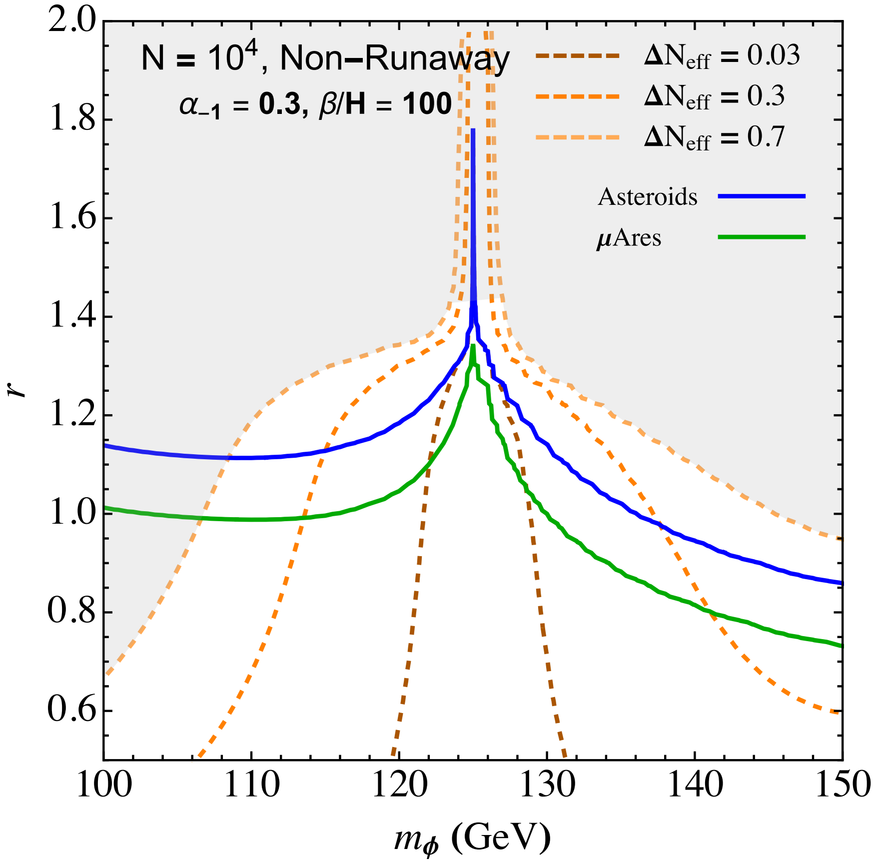


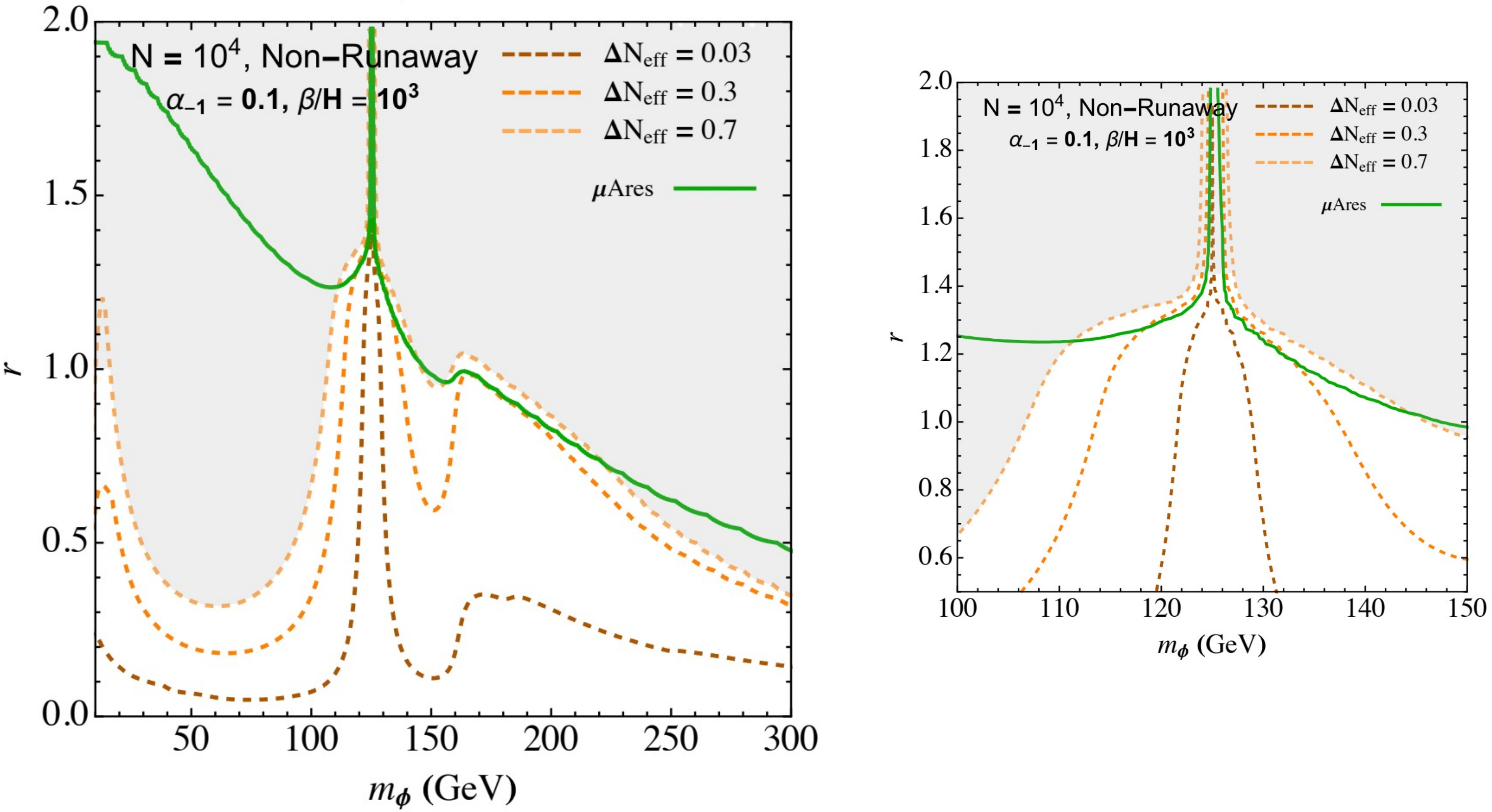


Mapping to Nnaturalness : non-runaway





Mapping to Nnaturalness : non-runaway



Conclusion

- Nnaturalness addresses the small hierarchy puzzle associated with the Higgs mass via N copies of SM with a range of Higgs mass squared values.
- The first exotic sector undergoes a QCD FOPT, with the GW peaked in the $\mathrm{n}Hz - \mathrm{m}Hz$ frequency.
- Nnaturalness by itself cannot explain the recent observation of stochastic GW background observed by NANOGrav.
- We find however that a large parameter of Nnaturalness parameter space can be probed by future PTA experiments as well as planned (LISA, BBO) and proposed (μ-Ares) space based gravitational wave observatories.

THANK YOU!