

Fermilab Theory Seminar

μ - e scattering at 10ppm

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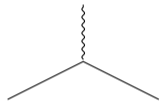
4 APRIL 2024

- what is up with $g - 2$?
- how is the theory value determined?
- what is MUonE?
- reaching 10^{-5} relative accuracy with McMULE

- magnetic moment of a charged lepton: $\vec{\mu} = g \frac{e}{2m} \vec{S}$

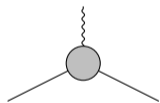
- Dirac: $g_{\mu}^{\text{Dirac}} = 2$

$$(-ie)\bar{u}\gamma^{\mu}u =$$

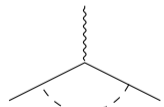


- SM quantum corrections: $g_{\mu}^{\text{SM}} = 2 \times (1 + a_{\mu}) = 2 \times (1 + F_2(0))$

$$(-ie)\bar{u}\left[F_1(Q^2)\gamma^{\mu} + F_2(Q^2)\frac{i\sigma^{\mu\nu}Q_{\nu}}{2m}\right]u =$$

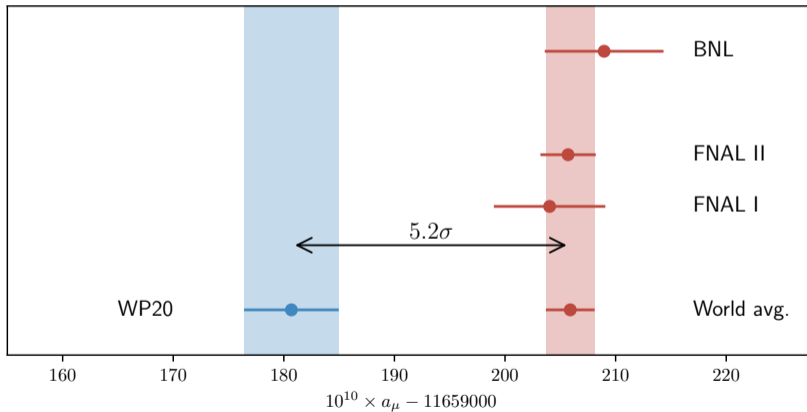


- BSM quantum corrections: $g_{\mu}^{\text{BSM}} \sim g_{\mu}^{\text{exp}} - g_{\mu}^{\text{SM}}$

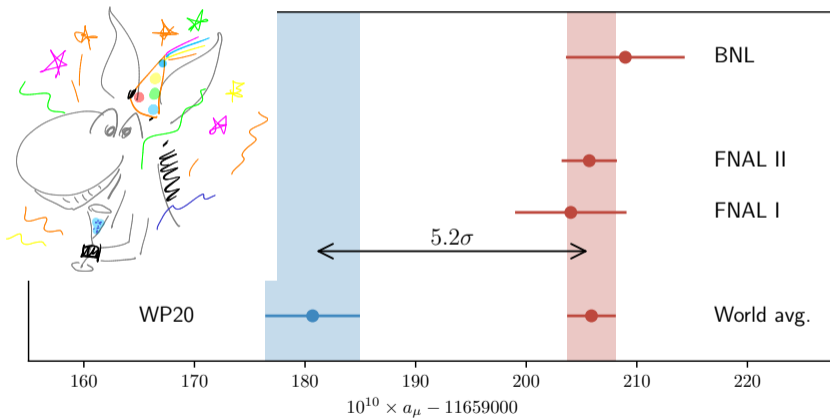












(insert favourite BSM)

most precise measurement of $g - 2$

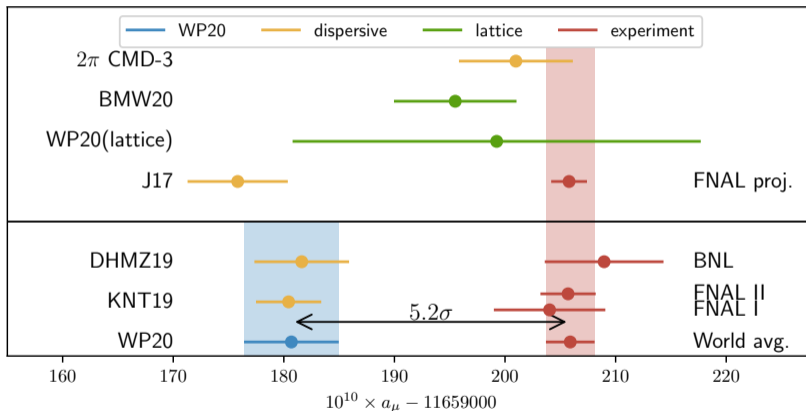


most precise measurement of $g - 2$



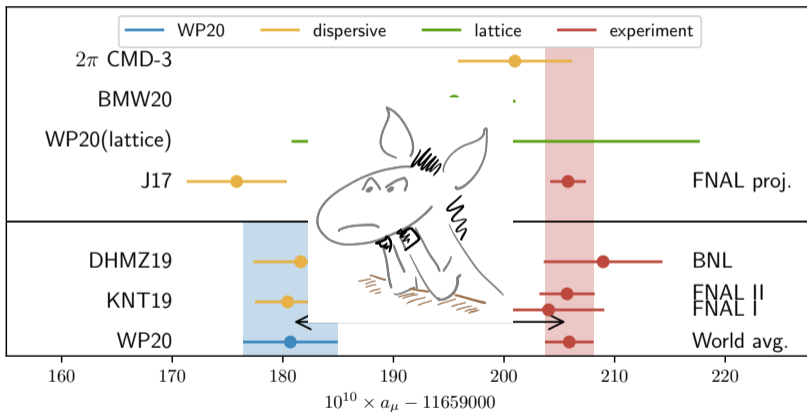
	value	diagrams
QED 1-loop	$\alpha/2\pi = 116\,140\,973$	
QED 2-loop	-177 231	 
QED 3-loop	1 480	 
more QED	-5	+ 3 others + 1 conspiracy theory + 70 others
EW	153	 
HVP	6 845(40)	+ others
HLbL	92(17)	  
total	116 591 810(43)	[g - 2 white paper 20]
FNAL+BNL	116 592 062(40)	

largest source of uncertainty & non-perturbative



this problem is bigger than $g - 2$! [CMD-3 23] [BMW 20]

largest source of uncertainty & non-perturbative



this problem is bigger than $g - 2$! [CMD-3 23] [BMW 20]

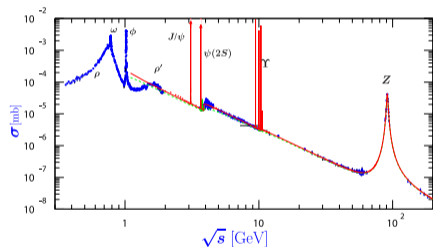
using optical theorem $s > 0$

- measure $ee \rightarrow \text{hadrons}$
- remove radiative corrections
- extrapolate to $s \rightarrow \infty$ using pQCD
- integrate over s

$$\int ds \left(K(s) \left[\text{Diagram: } ee \rightarrow \text{hadrons} \right] \right)$$

The diagram shows an electron-positron annihilation process into hadrons, represented by a wavy line connecting two vertices. The right vertex is a semi-circle representing a hadronic final state.

- 72% (78%) of value (uncertainty) from the $ee \rightarrow \pi\pi$ channel $s \lesssim 1 \text{ GeV}$

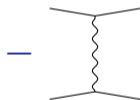
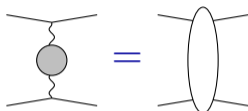


measure low Q^2 regions

- instead measure in t -channel, i.e. space-like
- no resonances \rightarrow much cleaner signal
- HVP is loop-induced \rightarrow much smaller signal ($10^{-3} \times \text{LO}$)
- competitive extraction @ 10^{-2}

\Rightarrow goal for MUonE: measure $e\mu \rightarrow e\mu$ @ 10^{-5}

$$\int dt \left(K'(t) \text{ [diagram of loop-induced photon exchange]} \right) \quad [\text{MUonE 19}]$$

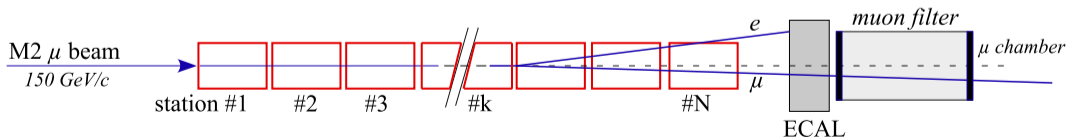


textbook QED

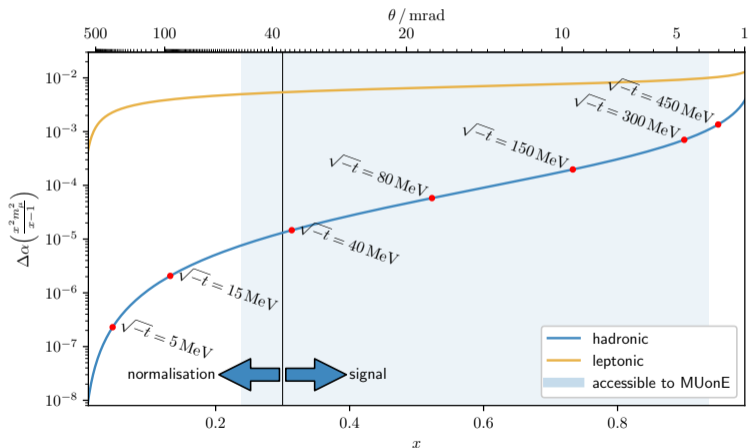
– QED

5+ years,
4+ workshops,
34+ authors

- scattering μ of low- Z material (${}_4\text{Be}$)
- pure t -channel $-s \simeq Q^2 \simeq 0$
- \Rightarrow high $s \leftrightarrow$ measure more of the curve
- beam energy needs to be quite high $E_\mu \simeq 160 \text{ GeV}$
- \Rightarrow M2 muon beam at CERN North Area
- main measurement: θ_e, θ_μ
 - + E_{beam} for calibration
 - + E_μ for particle ID



cancel systematic effects $\left(\frac{d\sigma}{d\theta}\right)_{\text{sig}} / \left(\frac{d\sigma}{d\theta}\right)_{\text{norm}}$



6 MUonE (adjacent) theory workshops over 7+ years



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	problem	solution	what?	doable up to?
	lots of masses	massification	expand in m_e^2/Q^2	LP, three-loop
①	numerical issues in real corrections	NTS stabilisation	expand in $E_\gamma/\sqrt{Q^2}$	NLP, all-orders
		jettification	expand in $\cos\theta \rightarrow 1$	LP, one-loop
	phase space	FKS ^ℓ	YFS-inspired subtraction scheme	all-orders
②	negative weights	gres	bubbles in event space	any order



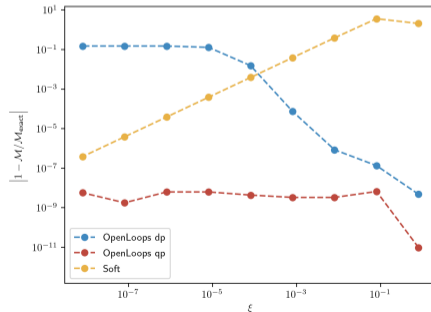
real-virtual corrections trivial in principle, extremely delicate numerically

$$\text{Diagram} = \frac{1}{E_\gamma^2} \underbrace{\varepsilon \text{ Diagram}}_{\text{eikonal}} + \mathcal{O}(E_\gamma^{-1})$$

example $ee \rightarrow ee\gamma$

[Engel, Signer, YU 21; Kollatzsch, YU 22; Engel 23]

- soft limit $E_\gamma = \xi \sqrt{s}/2$
- arbitrary prec. calculation vs **dp**, **qp**, **eikonal**
- stability problem



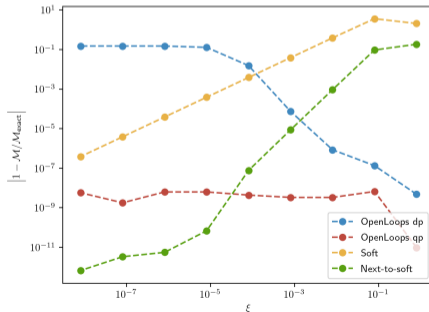
real-virtual corrections trivial in principle, extremely delicate numerically

$$\text{Diagram} = \frac{1}{E_\gamma^2} \underbrace{\mathcal{E} \text{ Diagram}}_{\text{eikonal}} + \frac{1}{E_\gamma} \left\{ \underbrace{D \left[\text{Diagram} \right]}_{\text{LBK}} + \underbrace{\mathcal{S} \text{ Diagram}}_{\text{soft function}} \right\} + \mathcal{O}(E_\gamma^0)$$

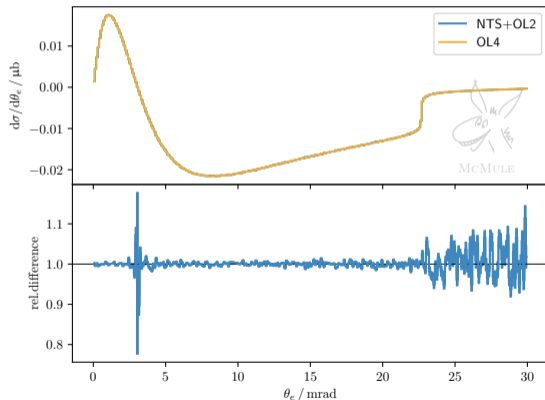
example $ee \rightarrow ee\gamma$

[Engel, Signer, YU 21; Kollatzsch, YU 22; Engel 23]

- soft limit $E_\gamma = \xi \sqrt{s}/2$
- arbitrary prec. calculation vs **dp**, **qp**, **eikonal**, **NTS**
- stability problem solved & speed-up



test next-to-soft stabilisation vs OL4 (OpenLoops quad) for $\mu e \rightarrow \mu e$ real-virtual



- same statistics, same result
 - 70 days vs 4 days
 - integrated results for different cuts
- ⇒ this is **not** an approximation but a numerical tool

NTS	OL4
-0.29268(4)	-0.29267(4)
-0.44789(6)	-0.44778(6)
-0.64662(9)	-0.64649(9)

... at NLO for simplicity

$$\sigma_{\text{NLO}} = \int \text{[tree]} + \frac{\alpha}{4\pi} \int \text{[loop]} + \frac{\alpha}{4\pi} \int \text{[loop]}'$$

- slicing: fairly few negative weights **but** numerically construct $\log \omega_c$

$$= \int \underbrace{\left(\text{[tree]} + \frac{\alpha}{4\pi} \text{[loop]} + \frac{\alpha}{4\pi} \int_1 \text{[loop]}' \right)}_{\text{mostly } > 0} + \frac{\alpha}{4\pi} \int_{\omega > \omega_c} \underbrace{\text{[loop]}'}_{> 0}$$

- subtraction: easier integration **but** lots and lots of negative weights ($\mathcal{O}(5\%)$ at NLO, more at NNLO)

$$= \int \underbrace{\left(\text{[tree]} + \frac{\alpha}{4\pi} \text{[loop]} + \frac{\alpha}{4\pi} \int_1 \text{[loop]}' \right)}_{\text{mostly } > 0} + \frac{\alpha}{4\pi} \int \underbrace{\left(\text{[loop]}' - \text{[loop]}' \right)}_{\text{whatever}}$$

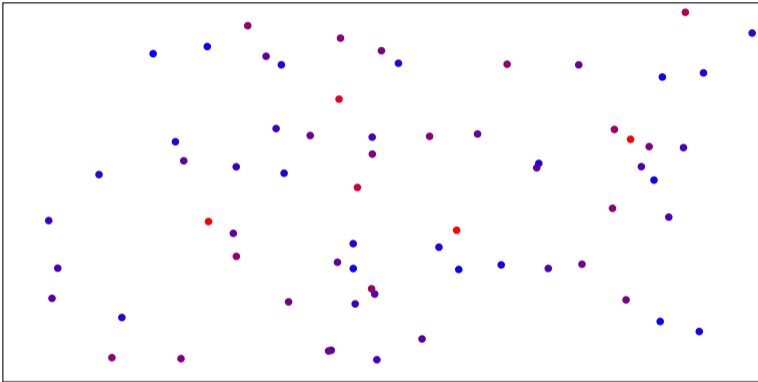
two observations

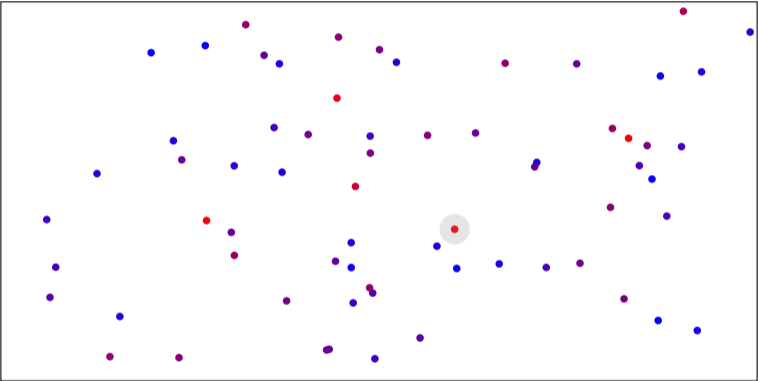
- ① cross section $\sigma = \int_{\mathcal{C}} d\sigma > 0$, irregardless of the size of integration region \mathcal{C}
- ② experiments have a finite resolution
(we already knew that because we can't see soft photons)

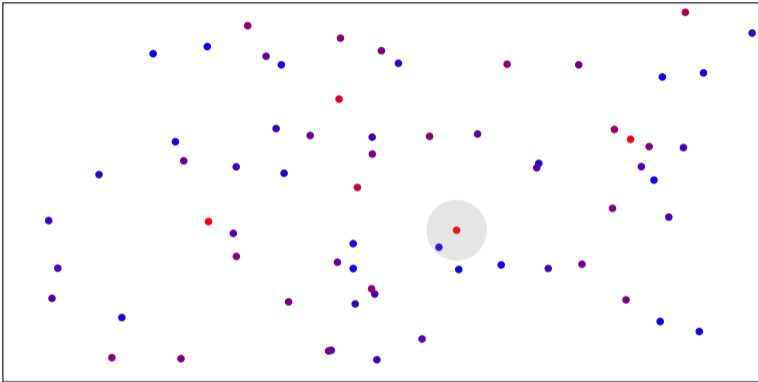
algorithm to remove negative weights [Andersen, Maier 21]

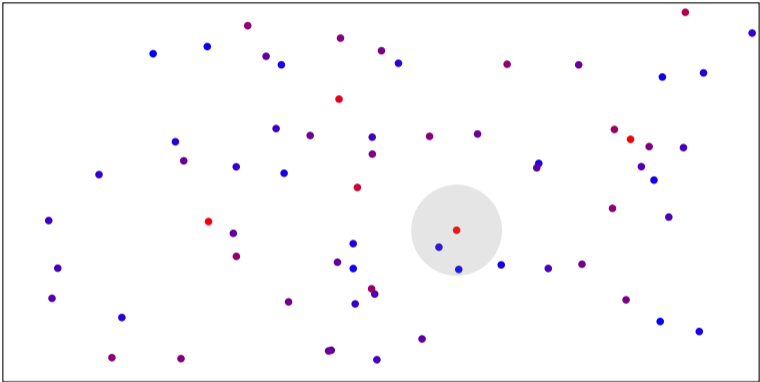
- pick an event with $w_i < 0$
- find nearby events until $\sum_{i \in \mathcal{C}} w_i > 0$
- if \mathcal{C} gets too big (events become resolvable), abort (or add more events)
- else $w_i \rightarrow \frac{\sum_{j \in \mathcal{C}} w_j}{\sum_{j \in \mathcal{C}} |w_j|} w_i$

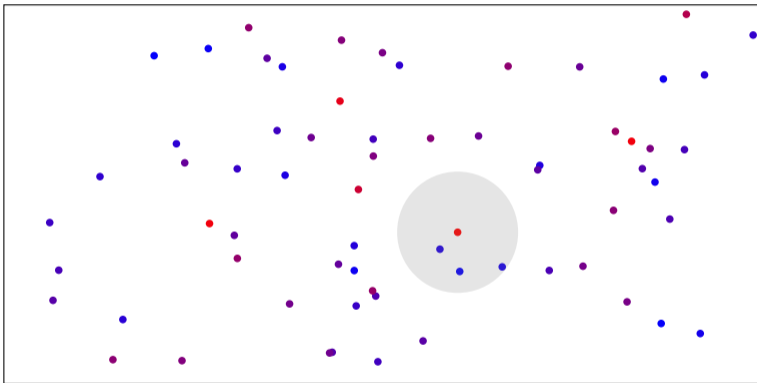
we can remove negative weights without biasing physical observables!

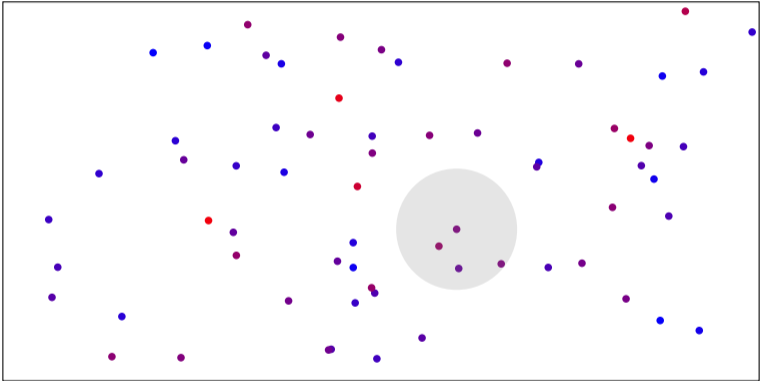


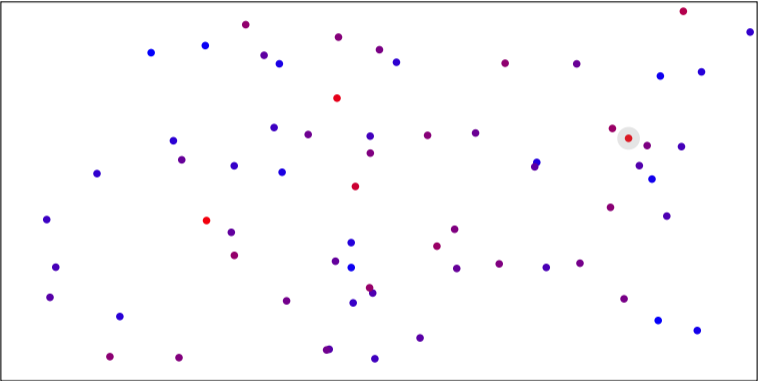


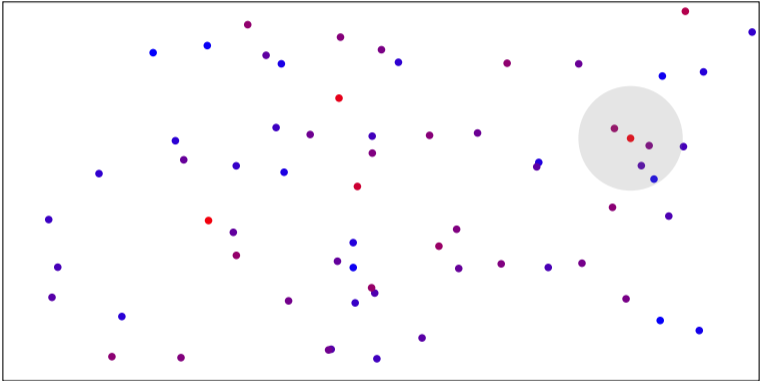


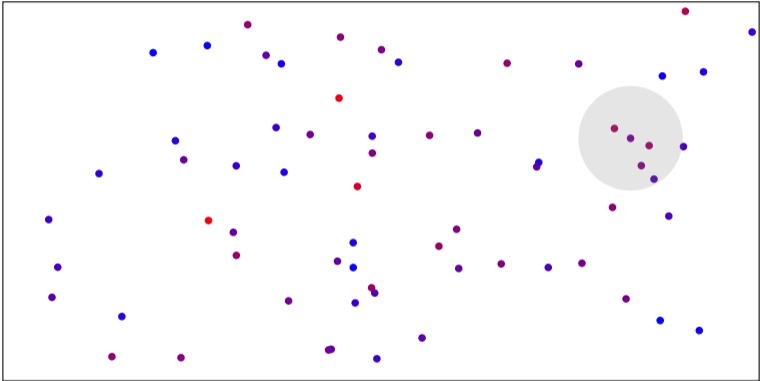


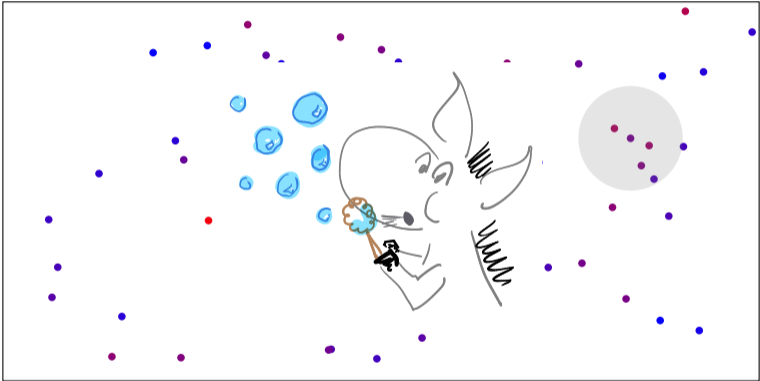






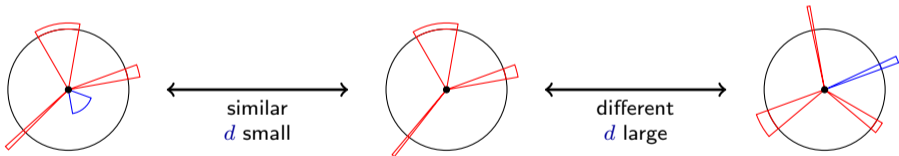






we need to define a metric in event space $d(e_1, e_2) \geq 0$

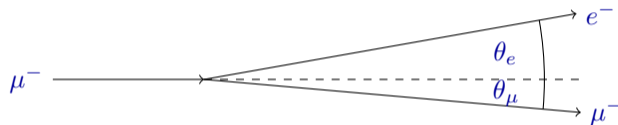
- doesn't really matter how we do this as long as IR safe
(events with soft photons are near each other)
- ideally: events that look similar are closer to each other than those that don't



implemented in McMULE v0.4.2

<https://mule-tools.gitlab.io>

- $\mu^- e^- \rightarrow \mu^- e^-$

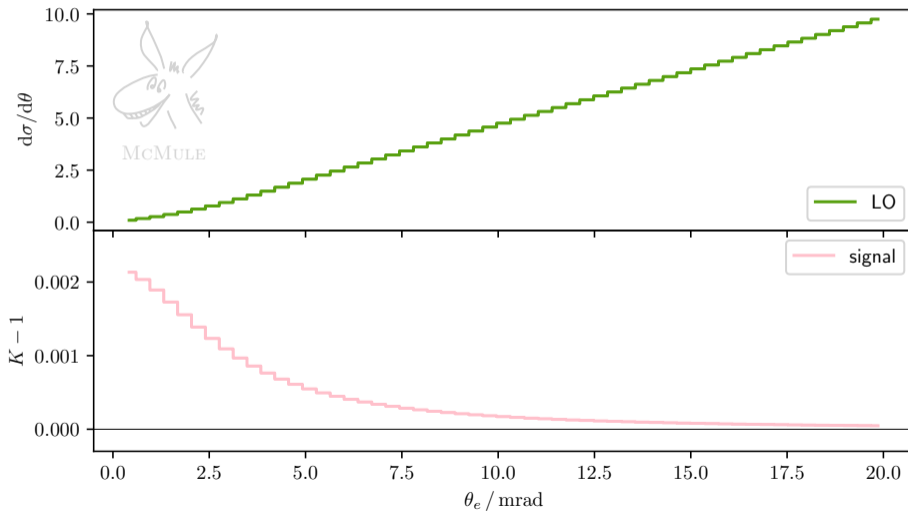


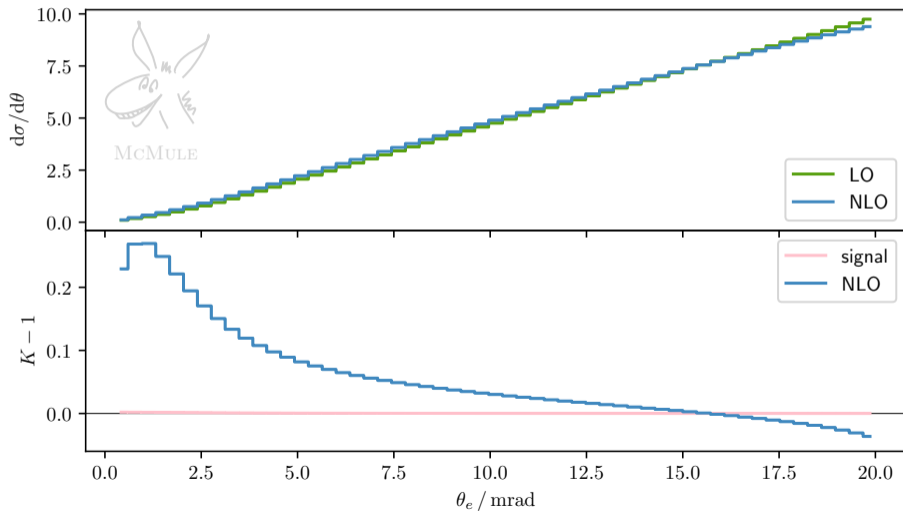
- S1: $E_e > 1 \text{ GeV}$, $\theta_\mu > 0.3 \text{ mrad}$
- run for 2.5 CPU yr
(290 kWh energy / 3.5 kgCO₂e)

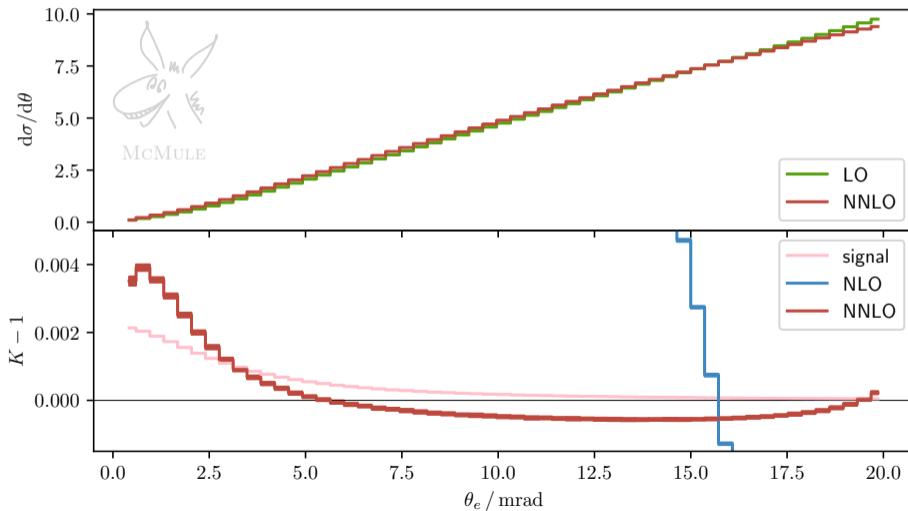


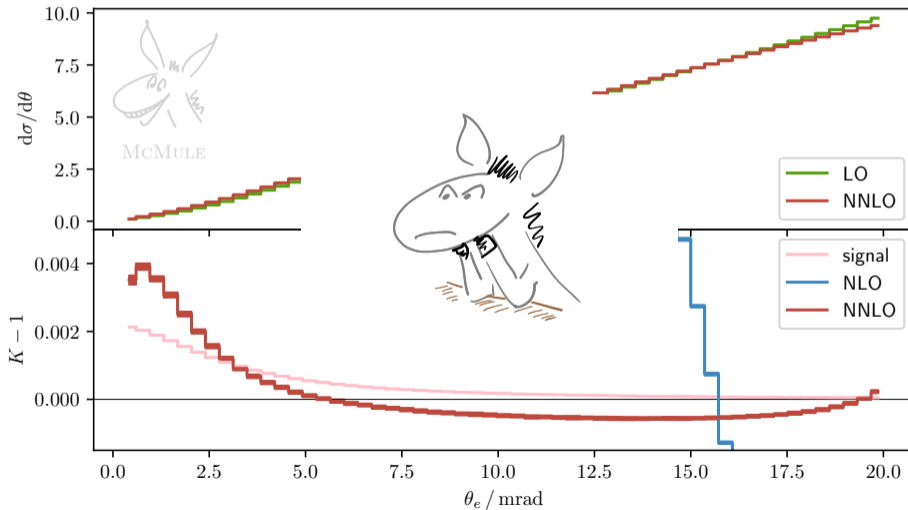
[Broggio, Engel, Ferrogli, Mandal, Mastrolia, Rocco, Ronca, Signer, Torres Bobadilla, Zoller, YU 22]

all results and data: <https://mule-tools.gitlab.io/user-library/mu-e-scattering/muone-full-legacy/>



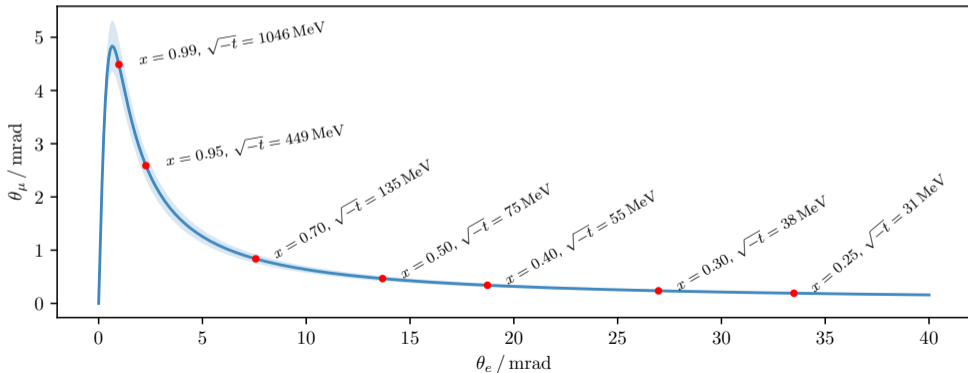


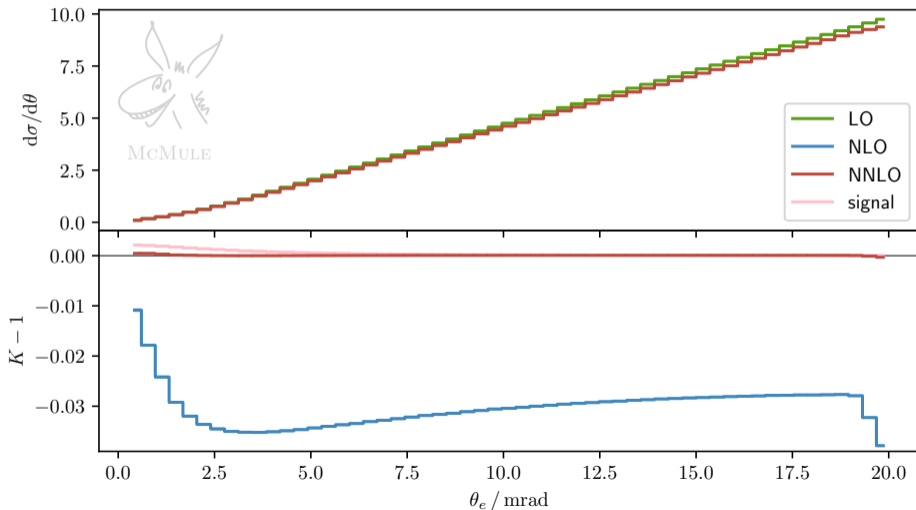


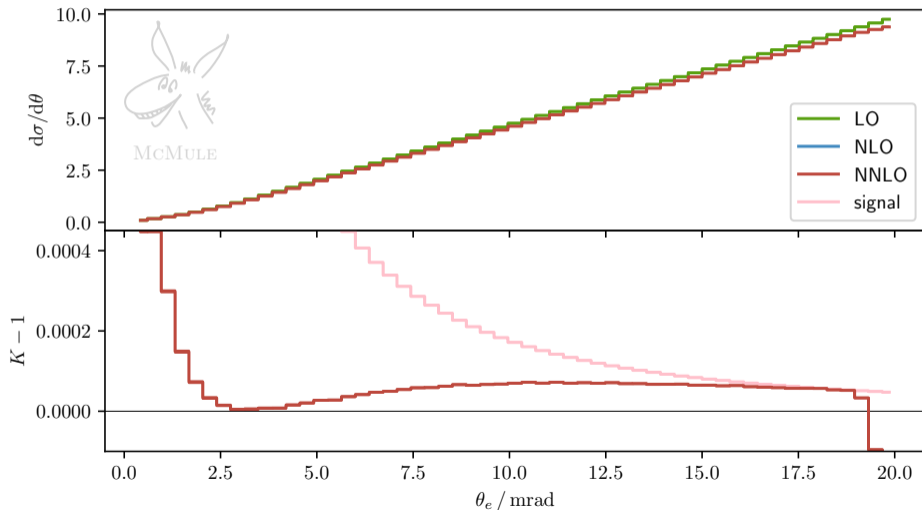


this clearly isn't working

- at this rate ($\sim 10\%$ NLO, $\sim 0.1\%$ NNLO), we would need N⁴LO to reach 10^{-5}
- most of this is due to hard radiation
- S2: same as S1 + needs to be in the band







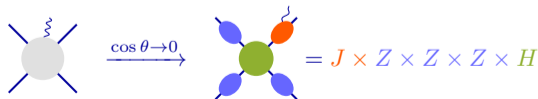
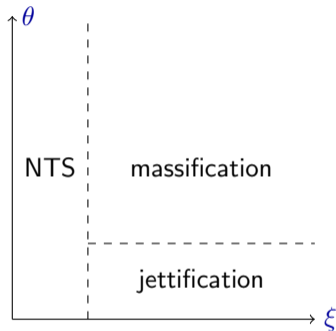
$ee \rightarrow \gamma^*$ can be taken to N^3 LO

- VVV: known
[Fael, Lange, Schönwald, Steinhauser 22]
- RRR: “trivial”
- RRV: OpenLoops + NTS stabilisation
- RVV: massless known (three-jet @ NNLO), massive (DiffExp?)

\Rightarrow LBK + jettification at two-loop

jettification

- expand for small emission angles



- ✓ first NNLO with multiple external masses
[Broggio, Engel, Ferroglia, Mandal, Mastrolia, Rocco, Ronca, Signer, Torres Bobadilla, Zoller, YU 22]
- ✓ event generation
- ✓ iterative HVP extraction procedure
[Fael 18]
- ✓ precision now: $\mathcal{O}(10^{\{-3,-4\}})$, goal: $\mathcal{O}(10^{-5})$
 - lots of optimisation still possible
(observable, μ^+ vs. μ^- beam, polarisation etc)
 - resummation (analytic & parton shower)
 - partial N³LO ($Q_e^8 Q_\mu^2$)





f.l.t.r.: F.Hagelstein (Mainz), A.Coutinho (IFIC), N.Schalch (Bern), L.Naterop (Zurich & PSI),
S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), M.Rocco (PSI), T.Engel (Freiburg),
V.Sharkovska (Zurich & PSI), Y.Ulrich (Bern), A.Gurgone (Pavia)
not pictured: P.Banerjee (IIT Guwahati), D.Moreno (PSI), D.Radic (PSI)



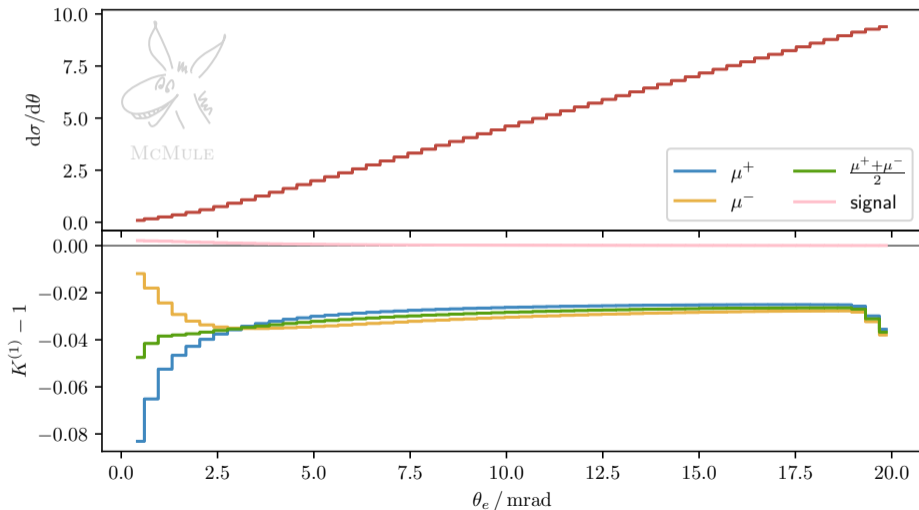
McMULE

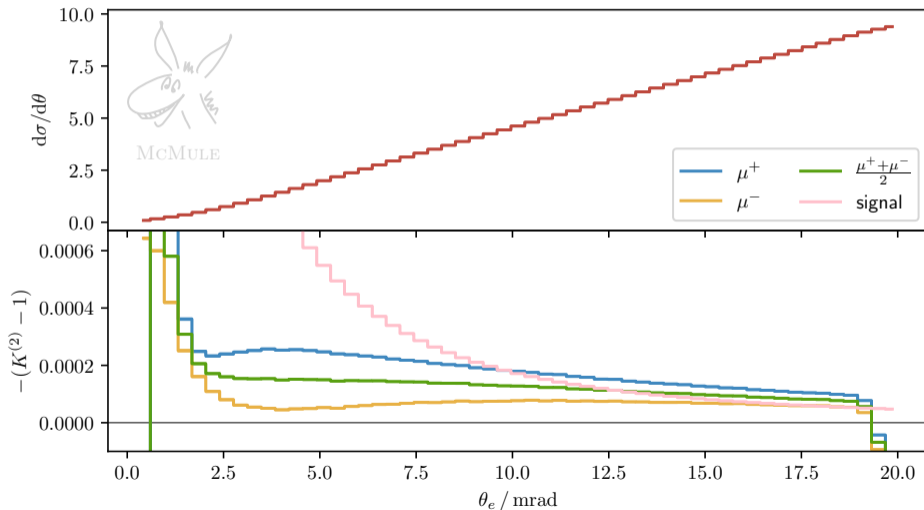
mule-tools.gitlab.io

the beam can do both μ^+ and μ^-

$$\begin{aligned}
 \sigma \sim Q_e Q_\mu & \left(Q_e^2 Q_\mu^1 \times \text{[diagram: photon exchange]} \right. \\
 & + \underbrace{Q_e^3 Q_\mu^1 \times \text{[diagram: photon exchange with electron loop]}}_{\text{easy}} + \underbrace{Q_e^2 Q_\mu^2 \times \text{[diagram: photon exchange with muon loop]}}_{\text{okay}} + \underbrace{Q_e^1 Q_\mu^3 \times \text{[diagram: photon exchange with electron loop]}}_{\text{easy}} \\
 & + \underbrace{Q_e^5 Q_\mu^1 \times \text{[diagram: photon exchange with electron loop]}}_{\text{easy}} + \underbrace{Q_e^4 Q_\mu^2 \times \text{[diagram: photon exchange with muon loop]}}_{\text{really difficult}} + \underbrace{Q_e^3 Q_\mu^3 \times \text{[diagram: photon exchange with muon loop]}}_{\text{really difficult}} + \underbrace{Q_e^2 Q_\mu^4 \times \text{[diagram: photon exchange with muon loop]}}_{\text{really difficult}} + \underbrace{Q_e^1 Q_\mu^5 \times \text{[diagram: photon exchange with electron loop]}}_{\text{easy}} \left. \right)
 \end{aligned}$$

- **proposal** $\sigma(\mu^+) + \sigma(\mu^-)$
- ⇒ some of the difficult stuff cancels





- universal soft limit $\mathcal{M}_{n+1}^{(\ell)} = \mathcal{E}\mathcal{M}_n^{(\ell)} + \mathcal{O}(E_\gamma^{-1})$
- universal pole structure $e^{\hat{\mathcal{E}}}\sum_{\ell=0}^{\infty}\mathcal{M}_n^{(\ell)} = \sum_{\ell=0}^{\infty}\mathcal{M}_n^{(\ell)f} = \text{finite}$

use this to construct an all-order subtraction scheme FKS^ℓ[Engel, Signer, YU 19]

- nothing complicated needed higher than $\mathcal{O}(\epsilon^0)$
- only one universal CT: $\hat{\mathcal{E}}$

$$\underbrace{\int d\Phi_\gamma}_{\text{divergent and complicated}} \left(\text{diagram with grey circle and wavy line} \right) = \underbrace{\int d\Phi_\gamma}_{\text{complicated but finite}} \left(\text{diagram with grey circle and wavy line} - \text{diagram with green circle} \right) + \underbrace{\int d\Phi_\gamma}_{\text{divergent but easy}} \left(\text{diagram with green circle} \right)$$

masses are physical in QED \Rightarrow keep masses

- drop polynomially suppressed terms at two-loop \rightarrow error $\sim \left(\frac{\alpha}{\pi}\right)^2 \log \frac{m^2}{Q^2} \times \frac{m^2}{Q^2}$
- based on factorisation, SCET, and method of regions
[Penin 06; Mitov, Moch 06; Becher, Melnikov 07; Engel, Gnendiger, Signer, YU 18]
- process e.g. $e\mu \rightarrow e\mu$ at two-loop:

$$\mathcal{A}(m) = \mathcal{S} \times \sqrt{Z} \times \sqrt{Z} \times \mathcal{A}(0) + \mathcal{O}(m) \supset \{1/\epsilon^2, L^2\}$$

- **soft**: process-dependent $\mathcal{S} = 1 + \text{fermion loops}$
 \rightarrow compute separately anyway to combine with hadron loops
- **collinear**: universal Z , converts $1/\epsilon \rightarrow \log(m^2/Q^2)$
- **hard**: massless calculation

