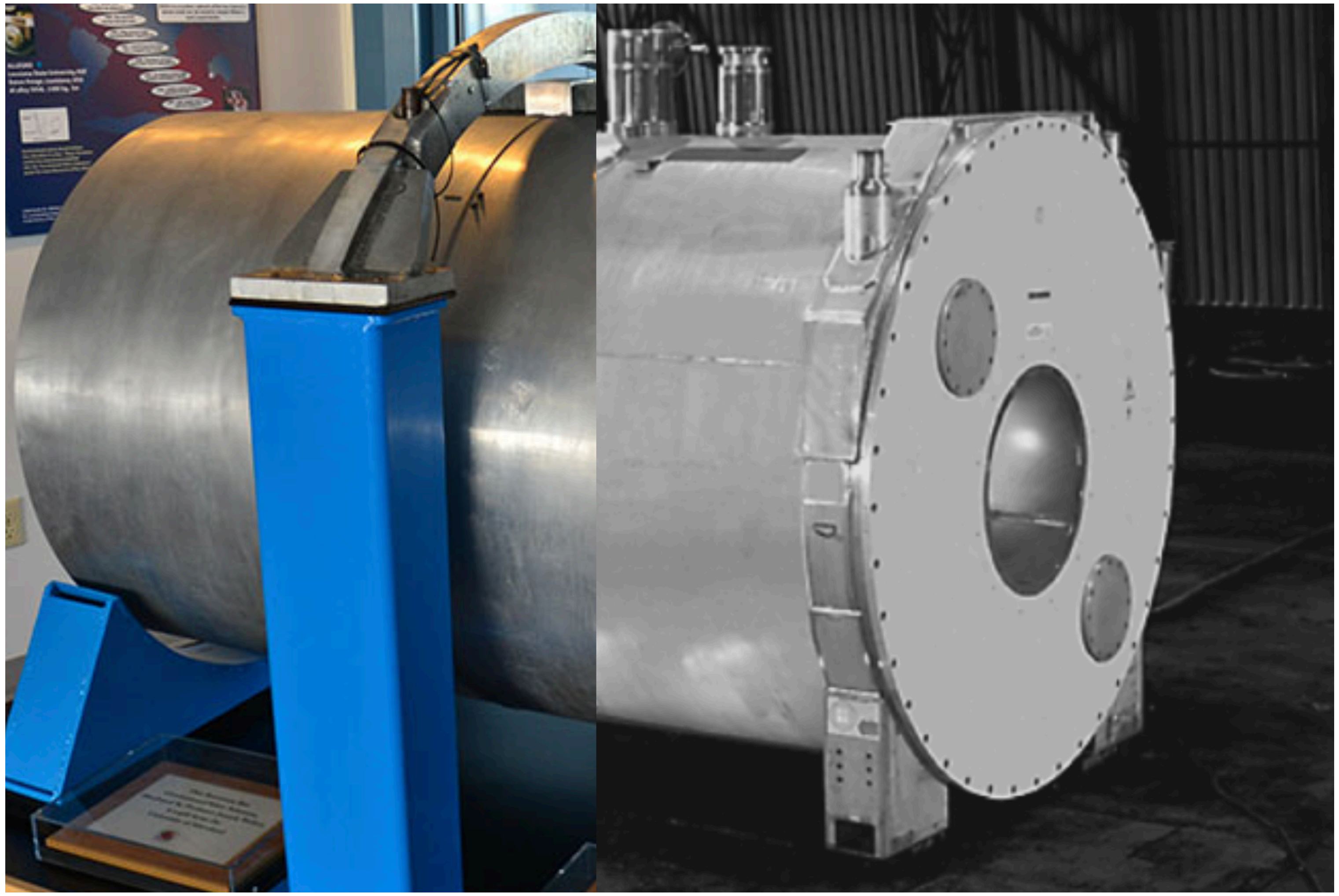


Static Magnets are Weber Bars

w/ Valerie Domcke & Sebastian Ellis

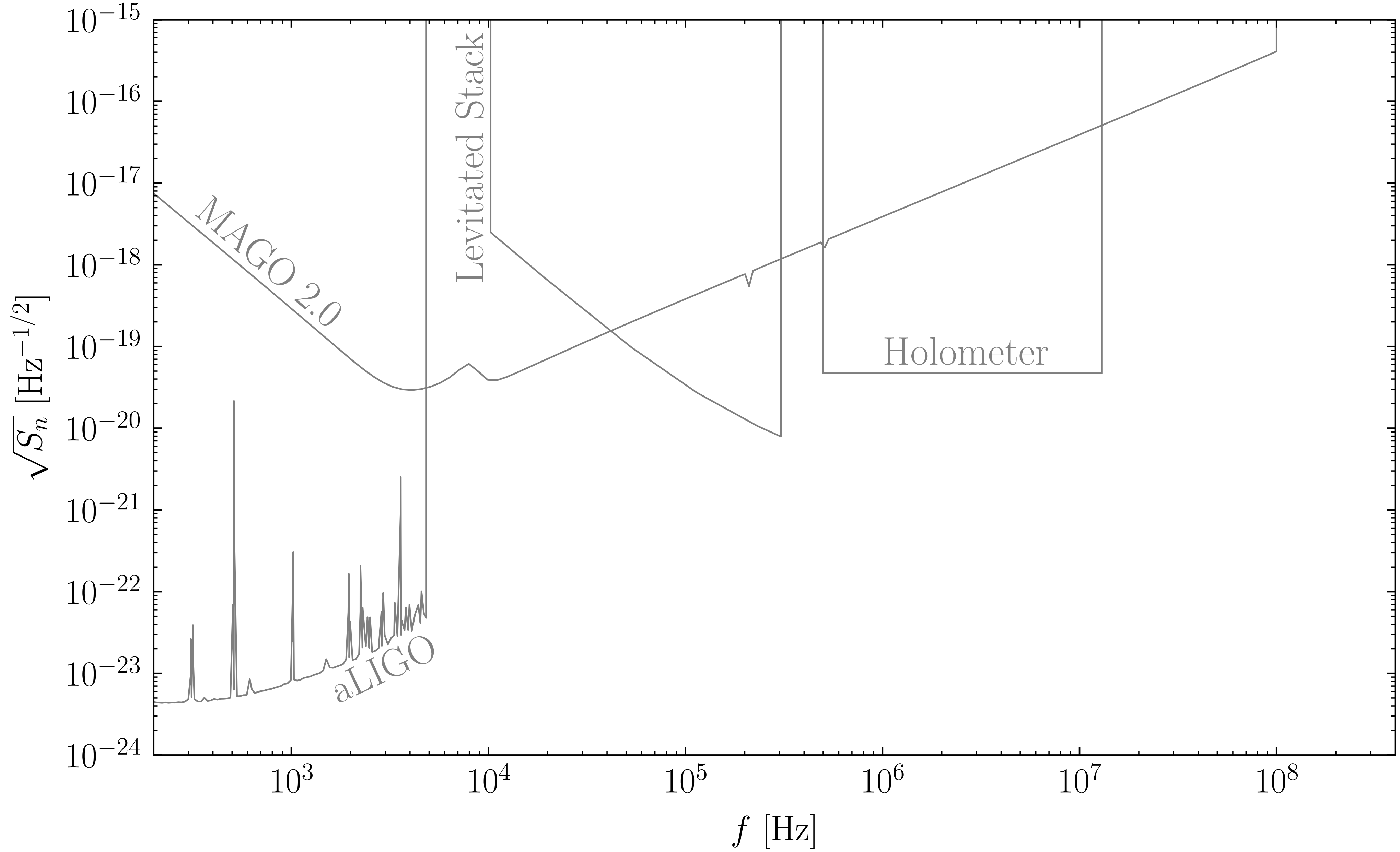
BERKELEY LAB

Motivation

What is the sensitivity of a large solenoidal magnet to gravitational waves?

Challenge

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$



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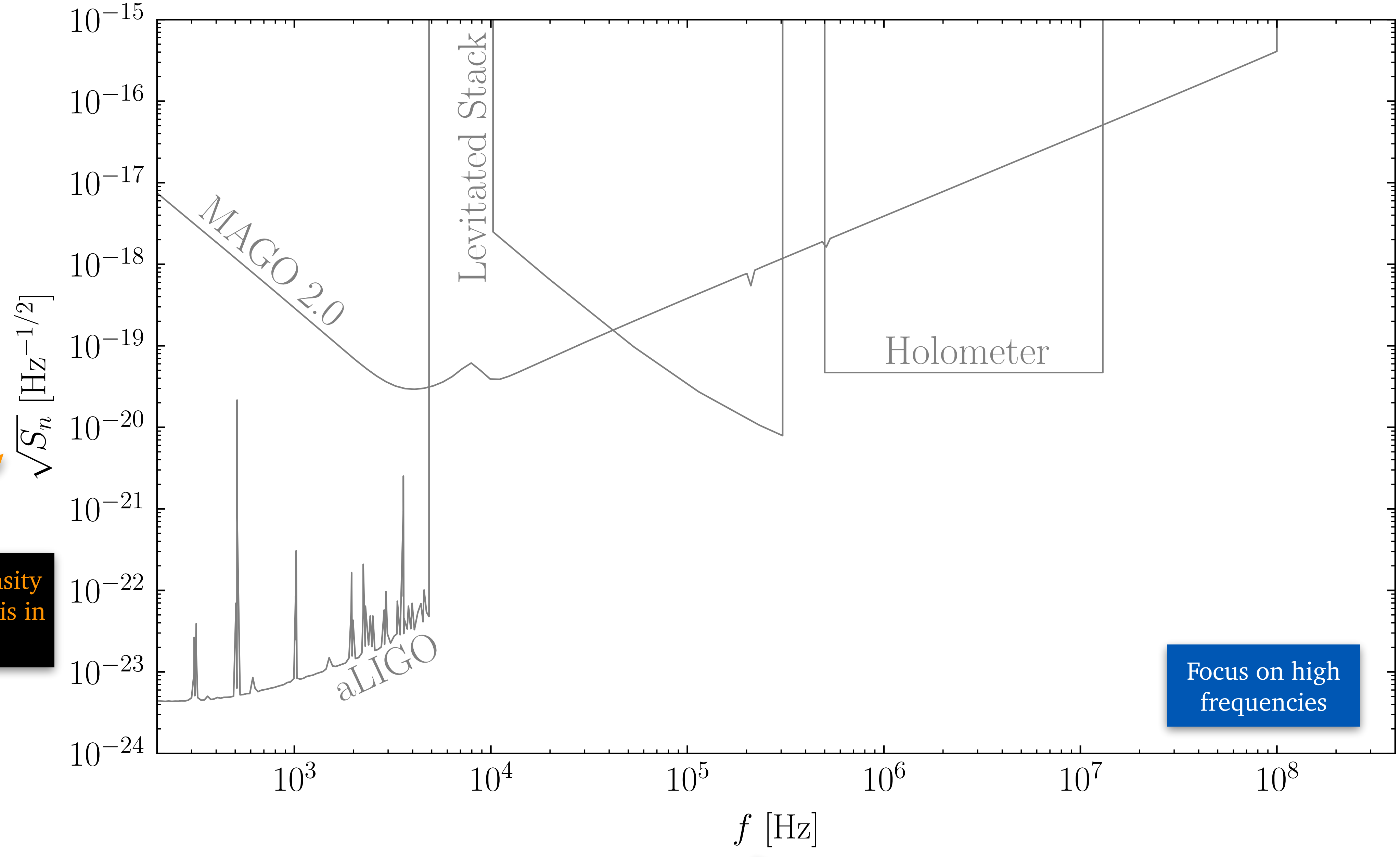


Noise spectral density (How much noise is in the detector?)

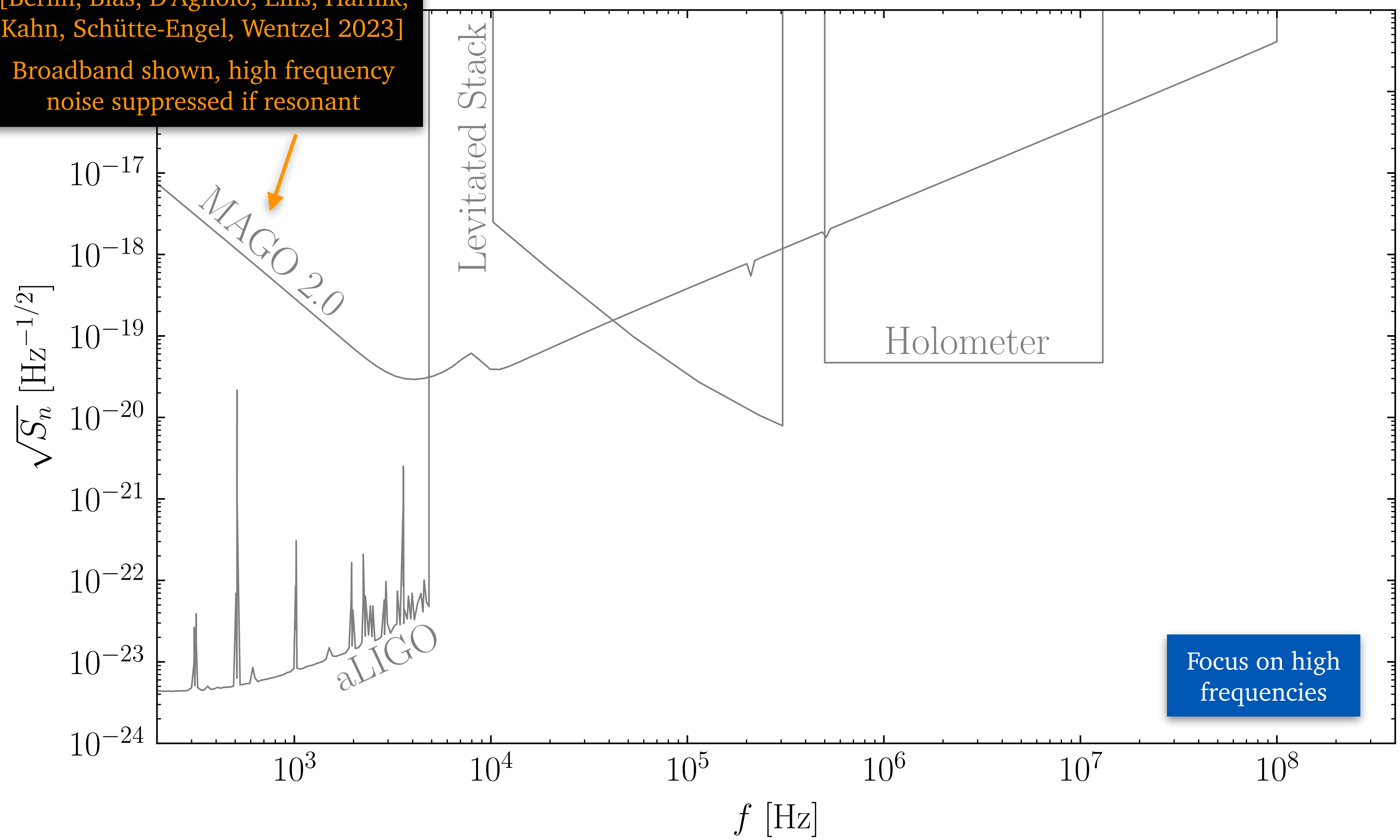
$h(t) \sim h e^{-2\pi i f t}$

Challenge

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

[Berlin, Blas, D'Agnolo, Ellis, Harnik, Kahn, Schütte-Engel, Wentzel 2023]

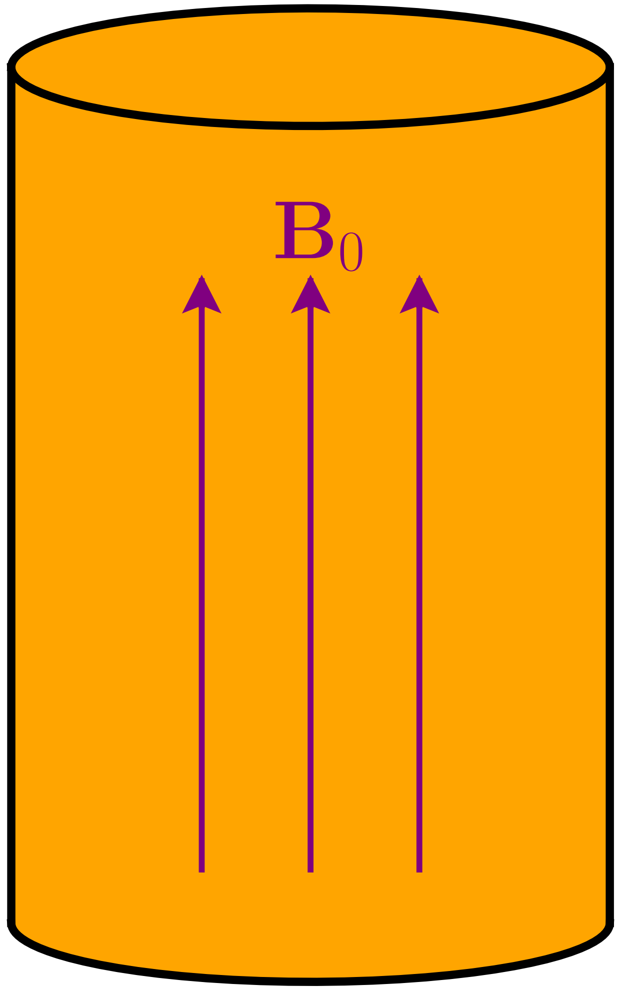
Broadband shown, high frequency noise suppressed if resonant



Focus on high frequencies

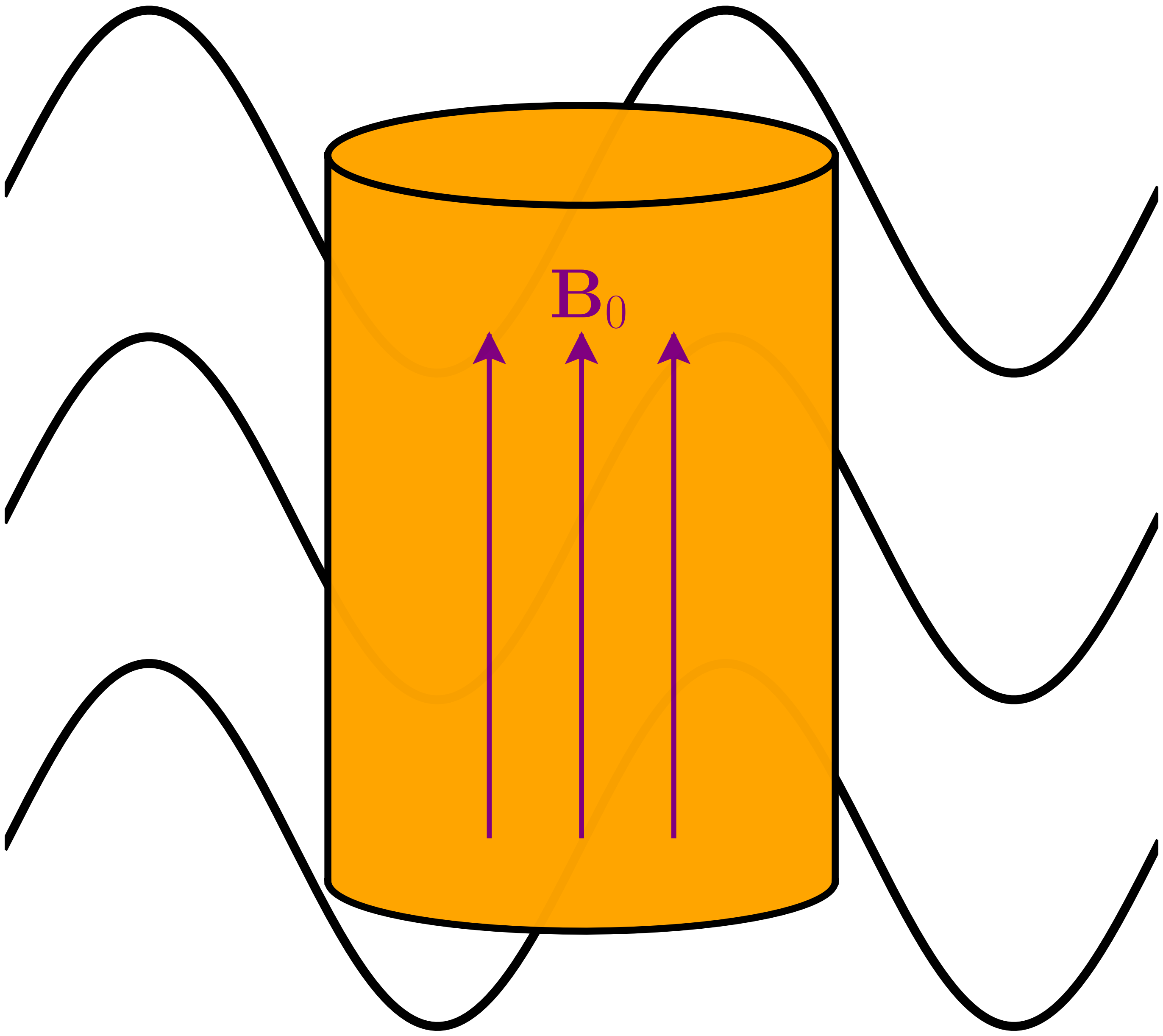
Small subset of ideas, see [Aggarwal+ 2020]

First approach: hF^2



Goal: exploit large stored energy in a magnetic field

First approach: hF^2

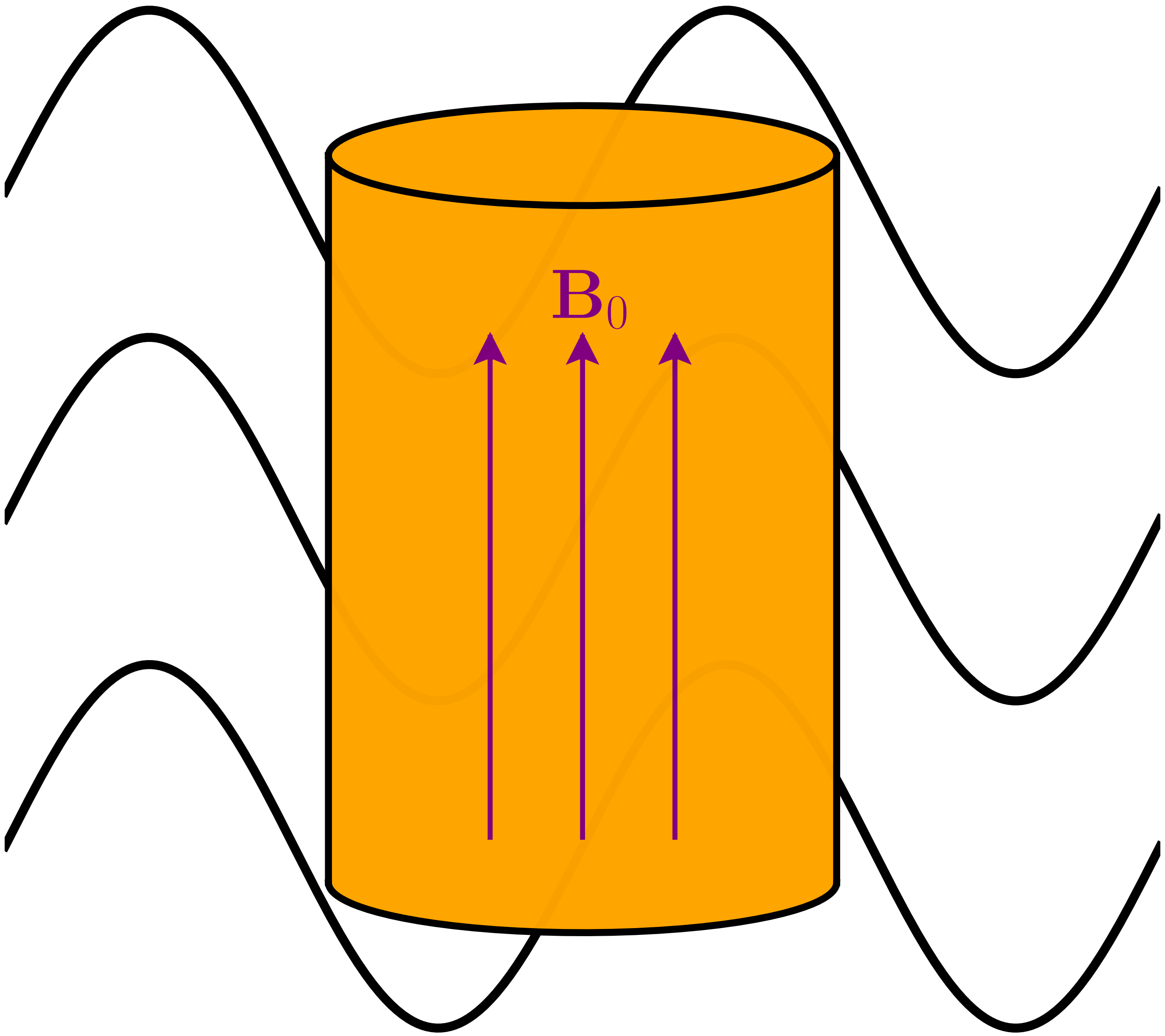


Familiar use

$g_{a\gamma\gamma} a F\tilde{F}$

Axion detection with solenoidal geometry [Sikivie, Sullivan, Tanner 2014]
Earlier suggestion by Romalis & Thomas, see [Svrcek, Witten 2006]

First approach: hF^2



Familiar use

$$g_{a\gamma\gamma} a F \tilde{F}$$

Exploit direct analogy for GW

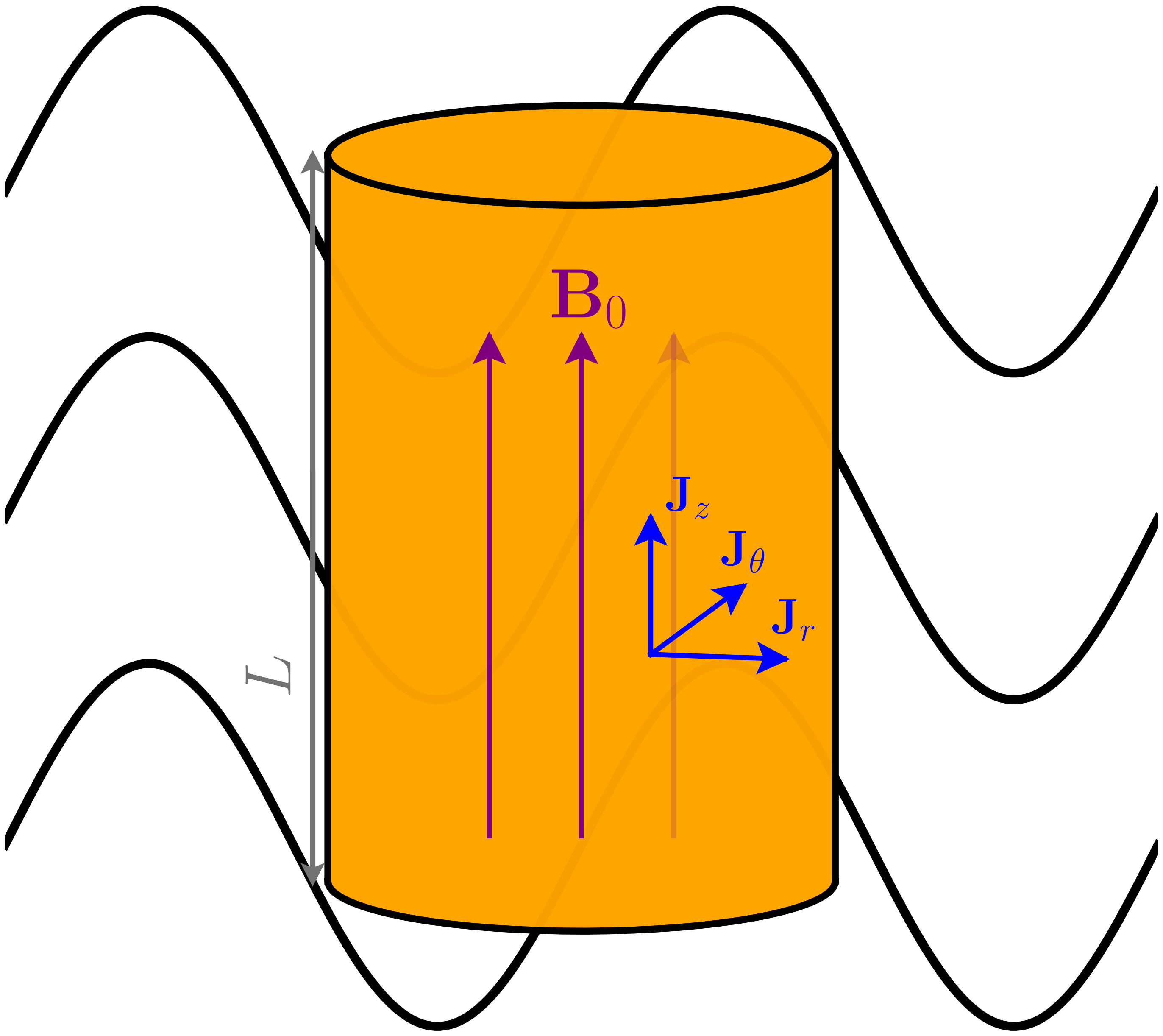
$$hF^2$$

Axion haloscopes are GW telescopes

Expand $S \supset \int d^4x \sqrt{-g} \left(-\frac{1}{4} F^2 \right)$ for $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

[Berlin, Blas, Tito D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel 2021]
[Domcke, Garcia-Cely, NLR 2022]

First approach: hF^2



Familiar use

$$g_{a\gamma\gamma} a F \tilde{F}$$

Exploit direct analogy for GW

$$hF^2$$

Generates AC magnetic field

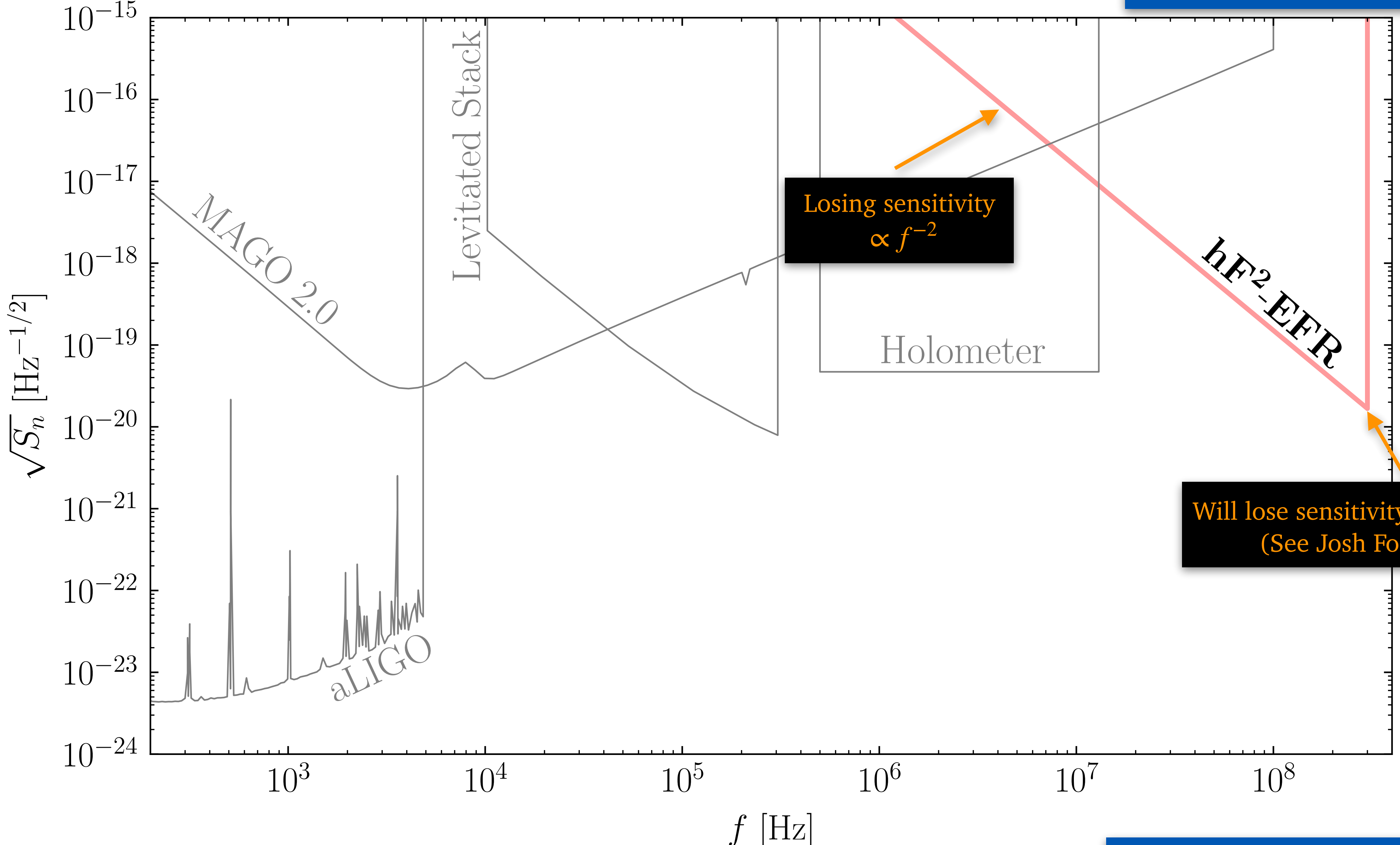
$$B_h^{\rm EM} \sim h B_0 (\omega L)^2$$

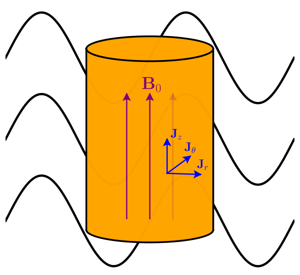
Leading gauge invariant contribution, full solenoidal calculation in [Domcke, Garcia-Cely, Lee, NLR 2024]

First approach: hF^2

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Compute full response for EFR, SQUID noise limited, broadband readout (ideal for transients - see Yoni Kahn's talk)





As for axion, improves with resonant readout (also true for MAGO 2.0)

Why $B_h \sim (\omega L)^2$?

TT gauge: $h \sim e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} \sim \omega^0$, but $B_0 \sim ?$

Proper detector frame: $B_0 \sim \omega^0$, $h \sim \omega^2$

PDF advocated in [Berlin+ 2021], see also [Fortini and Gualdi 1982], [Marzlin 1994], [Rakhmanov 2014]

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PDF: locally inertial frame along a geodesic x_0

ω^0 contribution is $\eta_{\mu\nu}$

Locally flat coordinates $\Rightarrow \mathcal{O}(\omega)$ must vanish

$$0 = \Gamma^{\mu}_{\nu\rho}(x_0) \sim \partial g(x_0)$$

See [Domcke, Garcia-Cely, Lee, NLR 2024]

PDF advocated in [Berlin+ 2021], see also [Fortini and Gualdi 1982], [Marzlin 1994], [Rakhmanov 2014]

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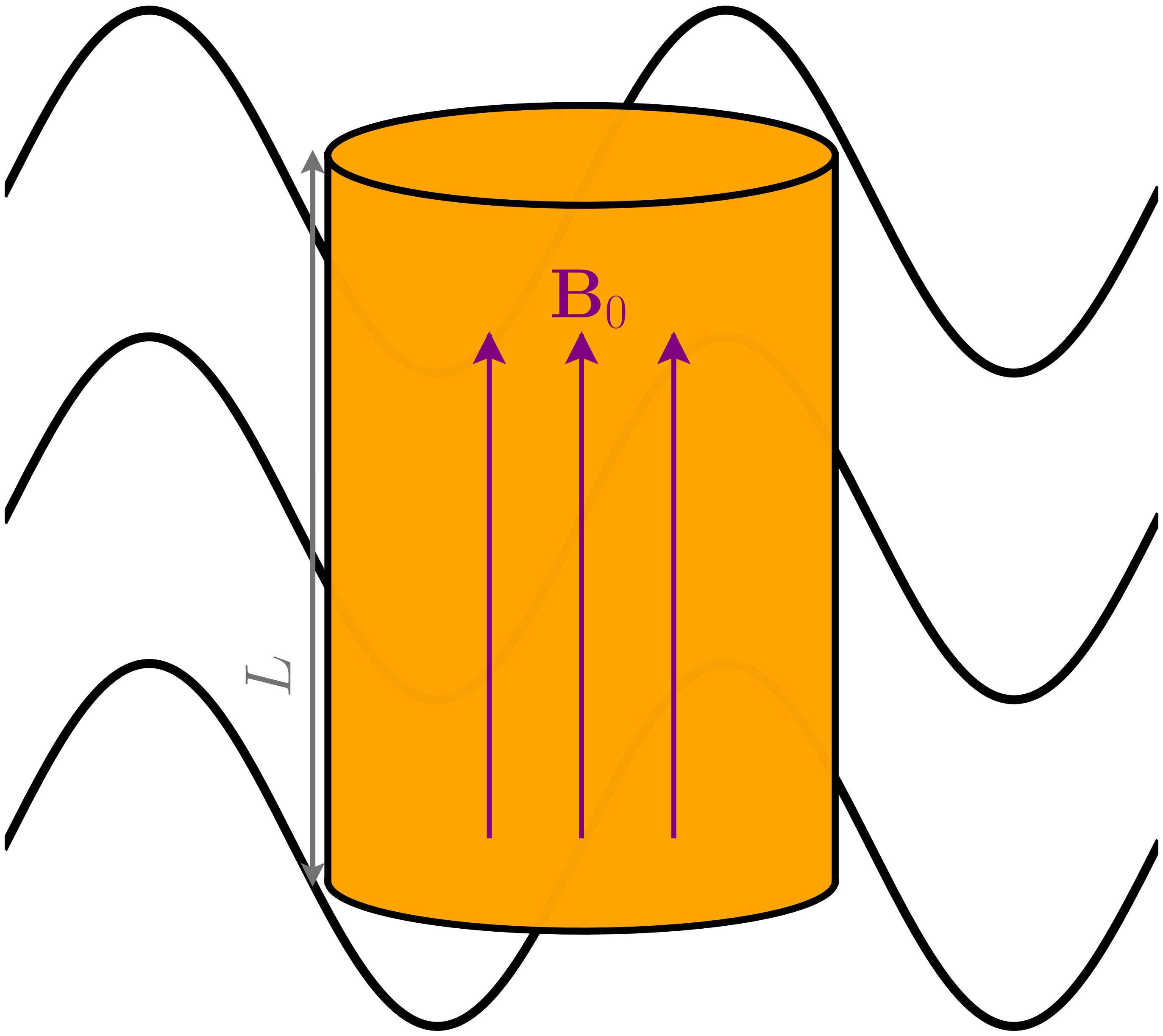
Proper detector frame: $B_0 \sim \omega^0$, $h \sim \omega^2$

Couples to an EM modes

$$B_h \sim \frac{\omega^2 h}{\omega^2 - \omega_{\rm EM}^2 + i(\omega\omega_{\rm EM}/Q_{\rm EM})^2} \sim \omega^2 h$$

$\omega \ll \omega_{\rm EM} \sim 500$ MHz

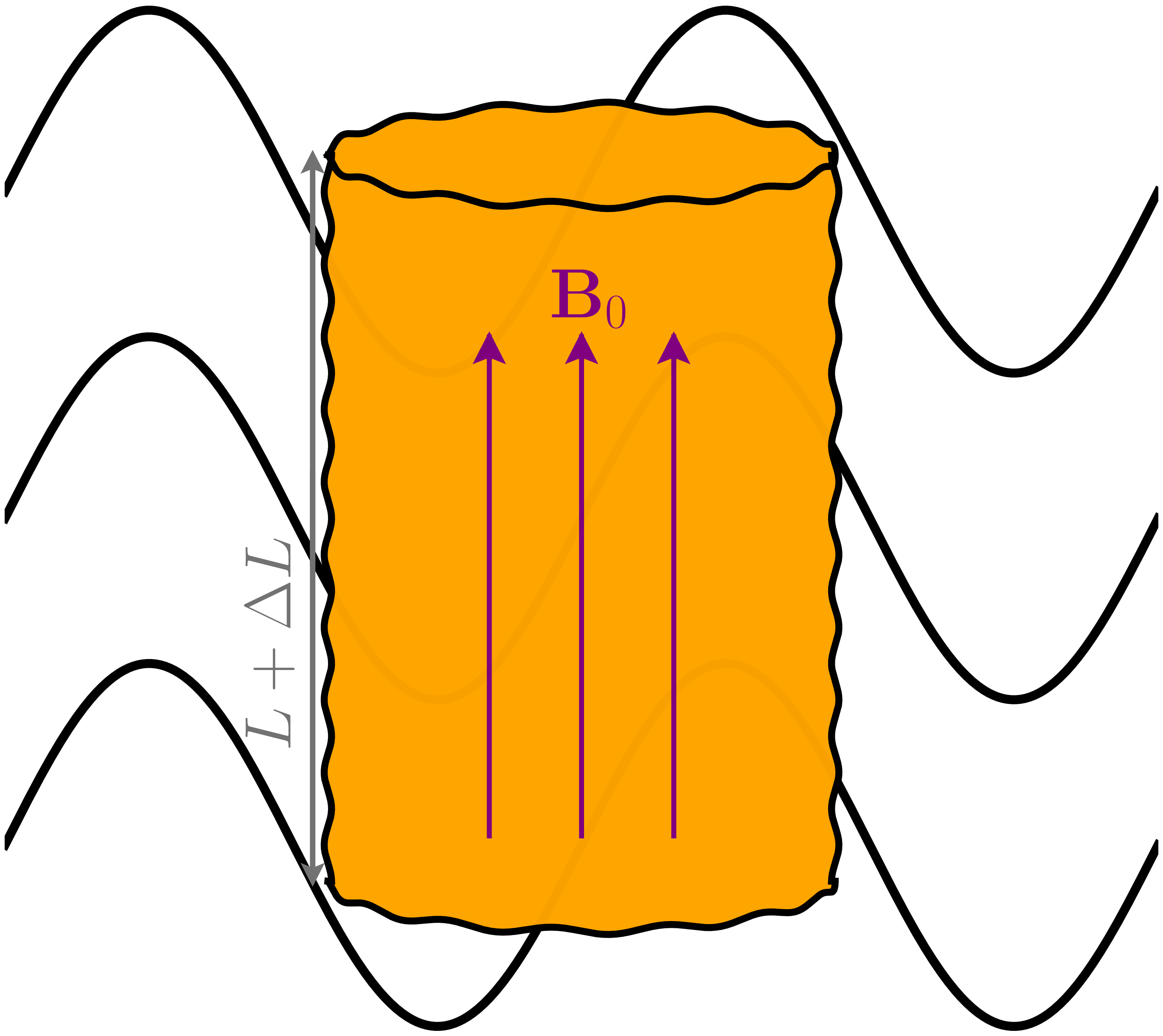
Mechanical Coupling



Solenoidal Magnet:

$$B_0 = \frac{NI}{L}$$

Mechanical Coupling



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GW oscillates the magnet length

$$L \to L + hL$$

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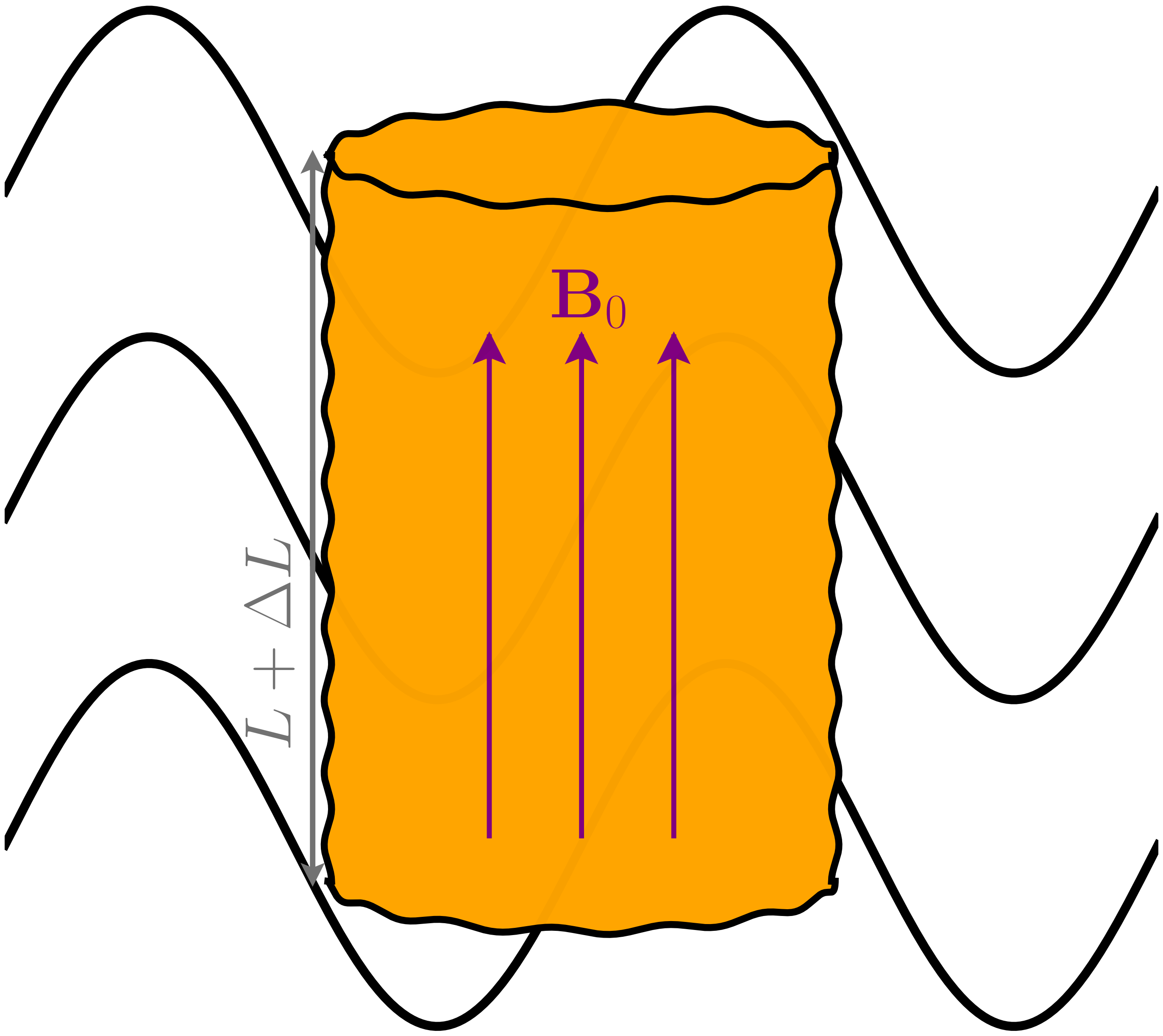

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$$B_h^{\rm mech} \sim hB_0 \gg B_h^{\rm EM}$$

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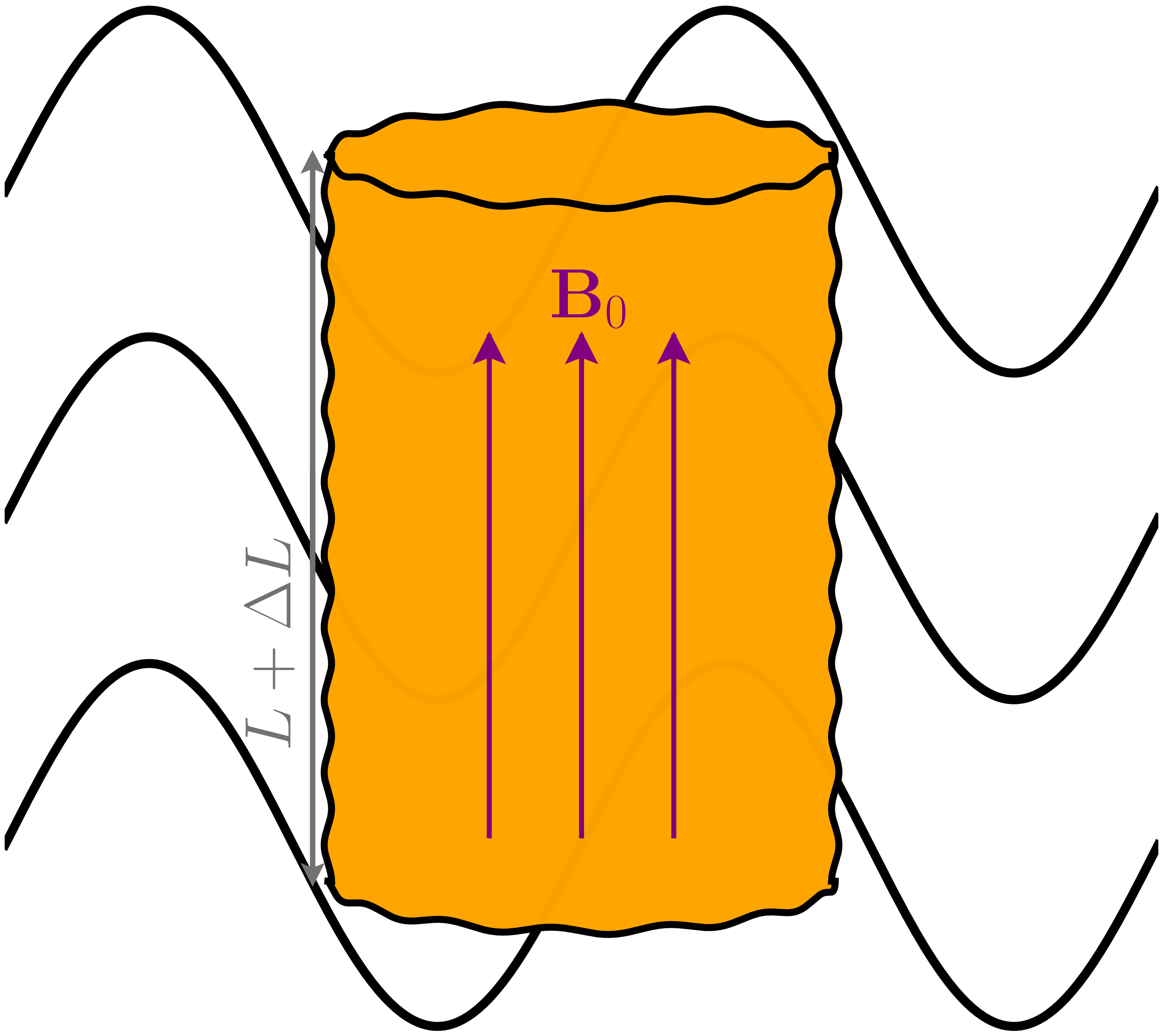

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A static magnet is a Weber bar!

Where is the ω^2

PDF: GW generates a force density

$$[\mathbf{f}_g]_i = \frac{1}{2}\rho \ddot{h}_{ij}^{\rm TT} x^j$$

$\ddot{h}^{\rm TT} \sim \omega^2 h$

Where is the ω^2

PDF: GW generates a force density

$$[\mathbf{f}_g]_i = \frac{1}{2}\rho \ddot{h}^{\rm TT}_{ij} x^j$$

Force density $\Rightarrow$ displacements in solenoid,

find $\mathbf{U}(\mathbf{r}) = \mathbf{r}' - \mathbf{r}$ by Navier-Cauchy equation

$$\rho \ddot{\mathbf{U}} = (\lambda + \mu)\nabla(\nabla \cdot \mathbf{U}) + \mu \nabla^2 \mathbf{U} + \mathbf{f}_g$$

Where is the ω^2

PDF: GW generates a force density

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Force density $\Rightarrow$ displacements in solenoid,

find $\mathbf{U}(\mathbf{r}) = \mathbf{r}' - \mathbf{r}$ by Navier-Cauchy equation

Recompute $\mathbf{B}$ from Biot-Savart, find

$$B_h \sim hB_0$$

Where is the ω^2

PDF: GW generates a force density

$$[\mathbf{f}_g]_i = \frac{1}{2}\rho \ddot{h}_{ij}^{\mathrm{TT}} x^j$$

Key: GW is coupling to mechanical not EM modes

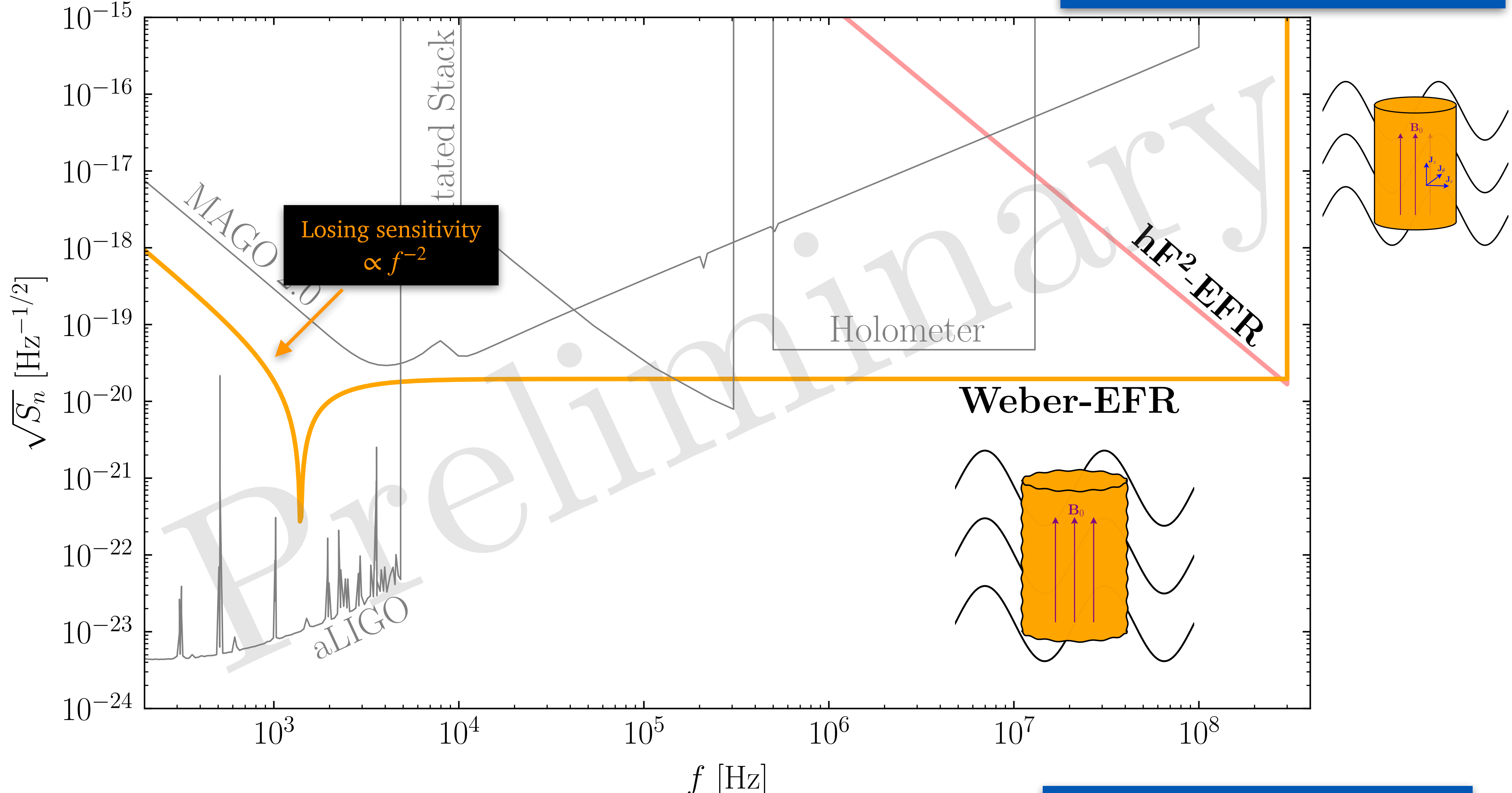
$$B_h \sim \frac{\omega^2 h}{\omega^2 - \omega_{\mathrm{Mech}}^2 + i(\omega\omega_{\mathrm{Mech}}/Q_{\mathrm{Mech}})^2} \sim h$$

$\omega \gg \omega_{\mathrm{Mech}} \sim 5\ \mathrm{kHz} \sim c_s \omega_{\mathrm{EM}}$

Mechanical Coupling

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Compute full response for EFR, SQUID noise limited, broadband readout (ideal for transients - see Yoni Kahn's talk)



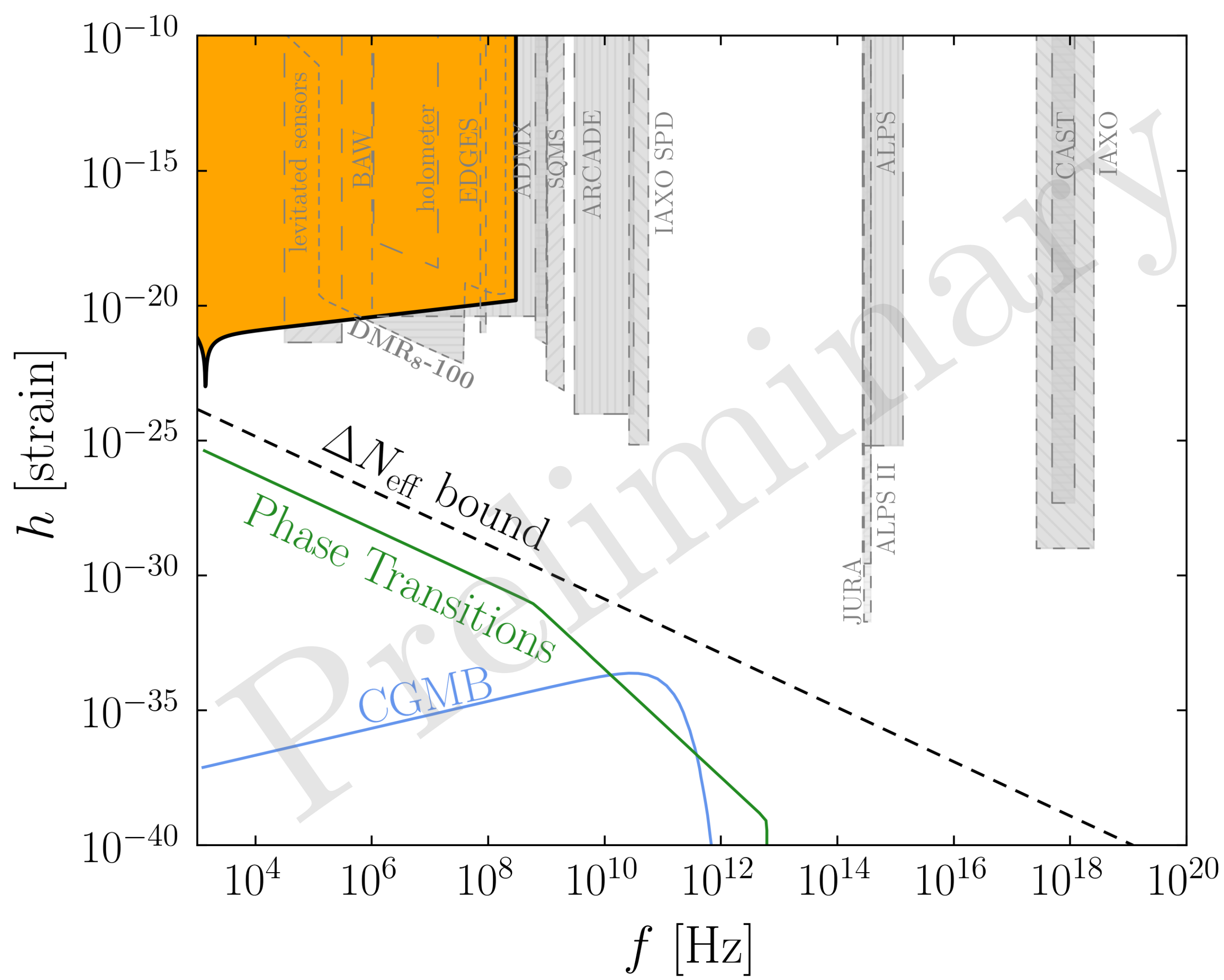
As for axion, improves with resonant readout (also true for MAGO 2.0)

Important Questions

1. Could we probe a signal from the early Universe?

Would have to exploit an EM and mechanical resonance

aLIGO also has sensitivity to a few kHz



Important Questions

1. Could we probe a signal from the early Universe?

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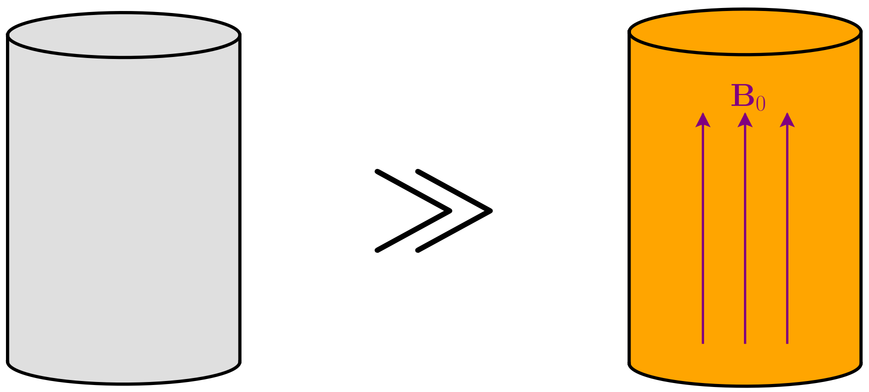
2. Is a static magnet the optimal Weber bar?

Hierarchy of energy densities: $U_M^{\text{Weber}} \gg U_B^{\text{Solenoid}}$



Important Questions

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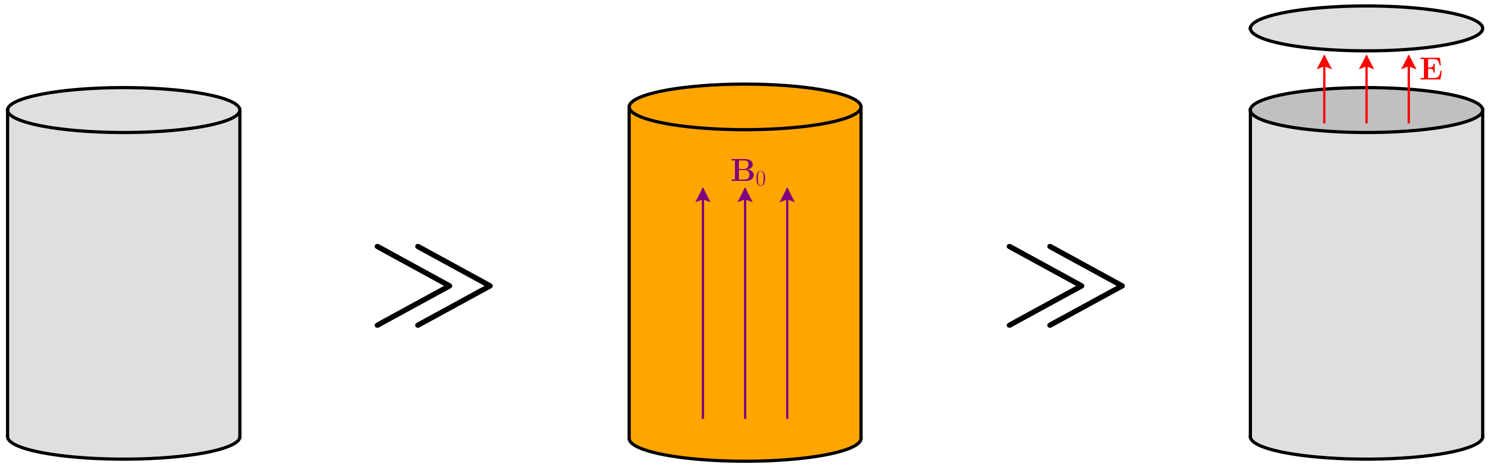
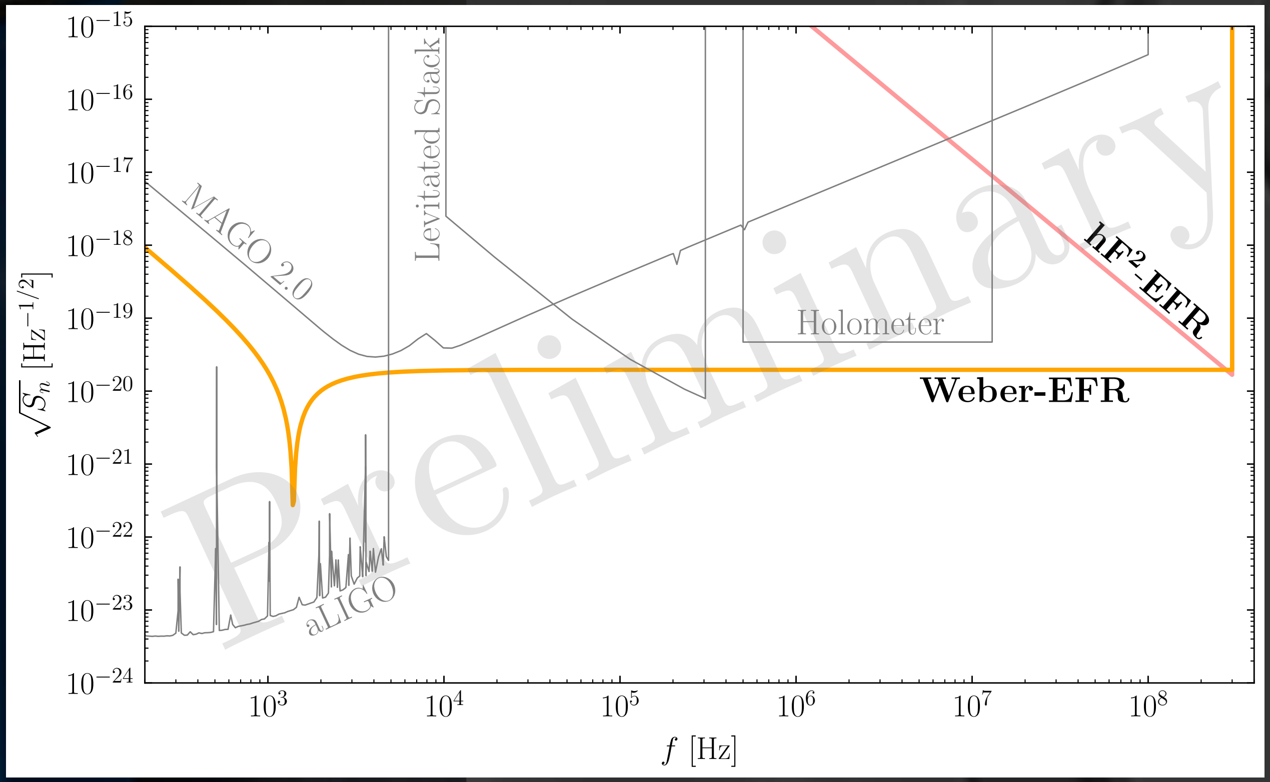
Hierarchy of energy densities: $U_M^{\rm Weber} \gg U_B^{\rm Solenoid} \gg U_E^{\rm Weber}$



Conclusion

Static magnets can have leading sensitivity to high-frequency gravitational waves



[Domcke, Ellis, NLR]

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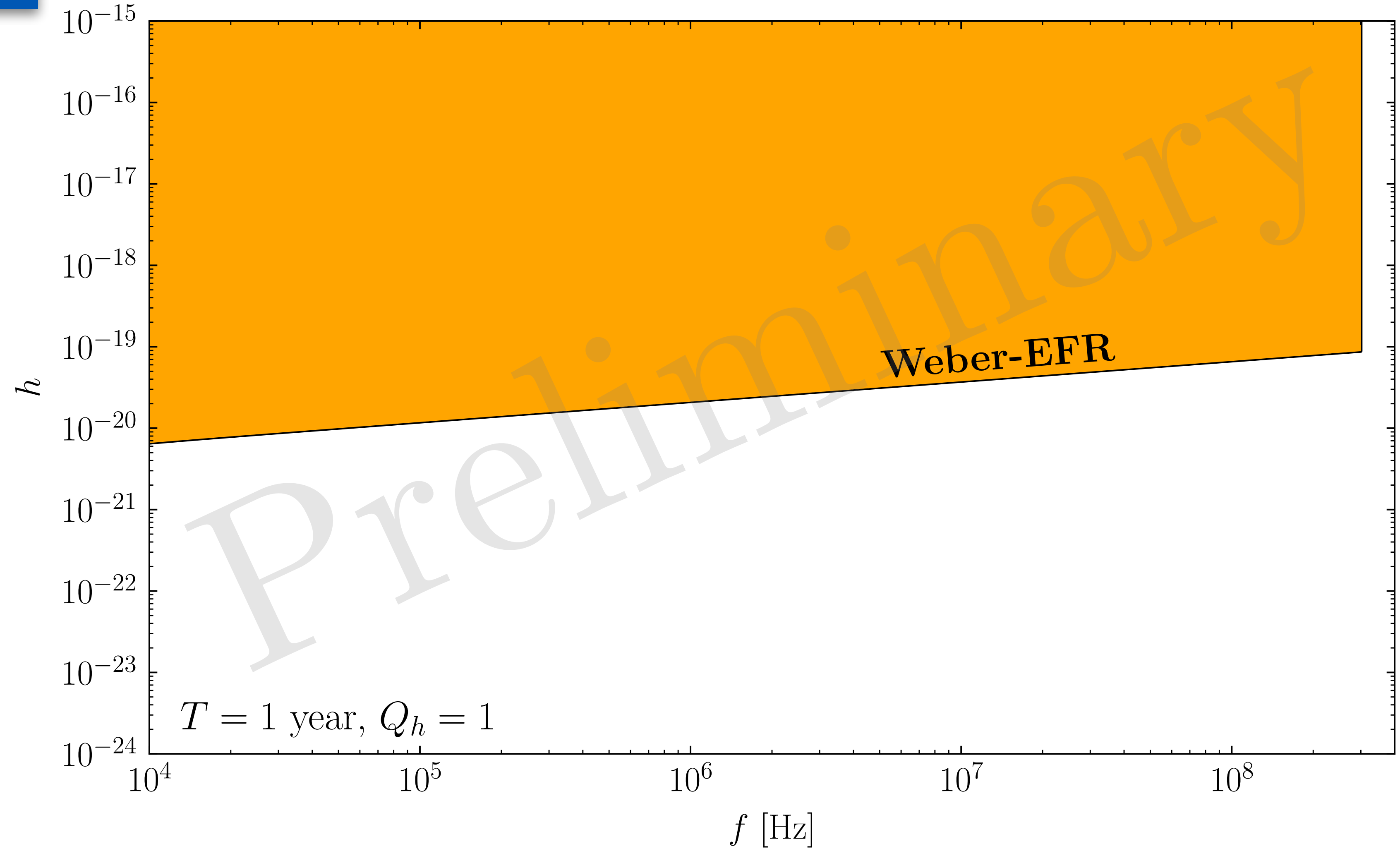
Backup Slides

Sensitivity to h

For a stochastic signal described by $(\bar{\omega}_h, Q_h)$, reach

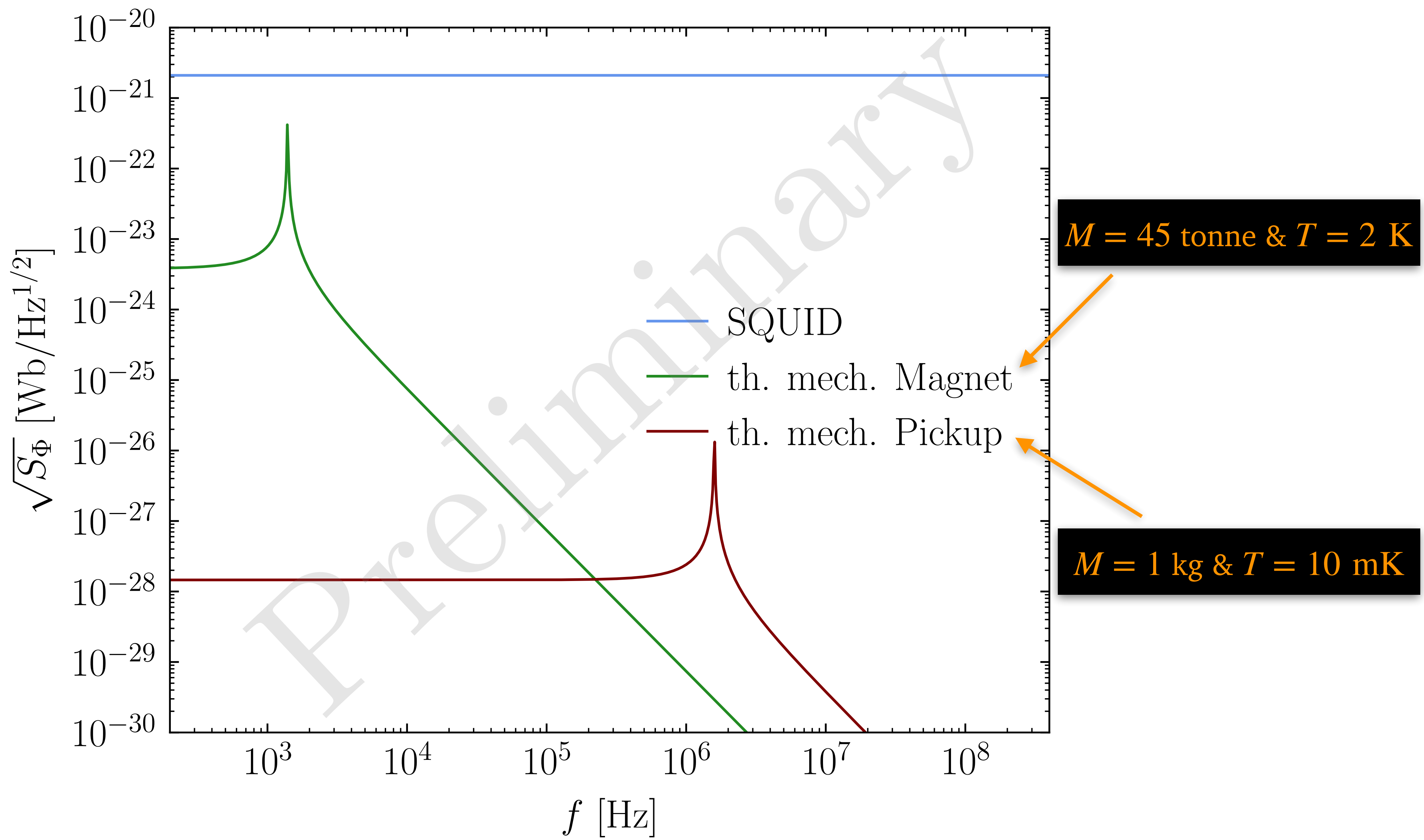
$$h \simeq \sqrt{S_n(\bar{\omega})} \left[\frac{\bar{\omega}_h}{TQ_h}\right]^{1/4} \simeq \sqrt{S_n(\bar{\omega})\ \text{Hz}} \left(\frac{\bar{f}}{1\ \text{Mhz}}\right)^{1/4} \left(\frac{T}{1\ \text{year}}\right)^{-1/4} \left(\frac{Q_h}{1}\right)^{-1/4}$$

Assumes $S_n(\omega)$ flat over the range signal has support



Backgrounds

For a broadband readout



Comparison of Stored Energies

$$\text{Elastic: } \frac{1}{2} m\omega^2 x^2$$

$$U_M \sim 2 \cdot 10^{12}\ \text{J} \times h^2 \left(\frac{f}{10\ \text{kHz}}\right)^2 \left(\frac{M}{1000\ \text{kg}}\right) \left(\frac{L}{1\ \text{m}}\right)^2$$

$$\text{Magnetic: } \frac{1}{2\mu_0} B^2 V$$

$$U_B \sim 4 \cdot 10^{7}\ \text{J} \times h^2 \left(\frac{B_0}{10\ \text{T}}\right)^2 \left(\frac{L}{1\ \text{m}}\right)^3$$

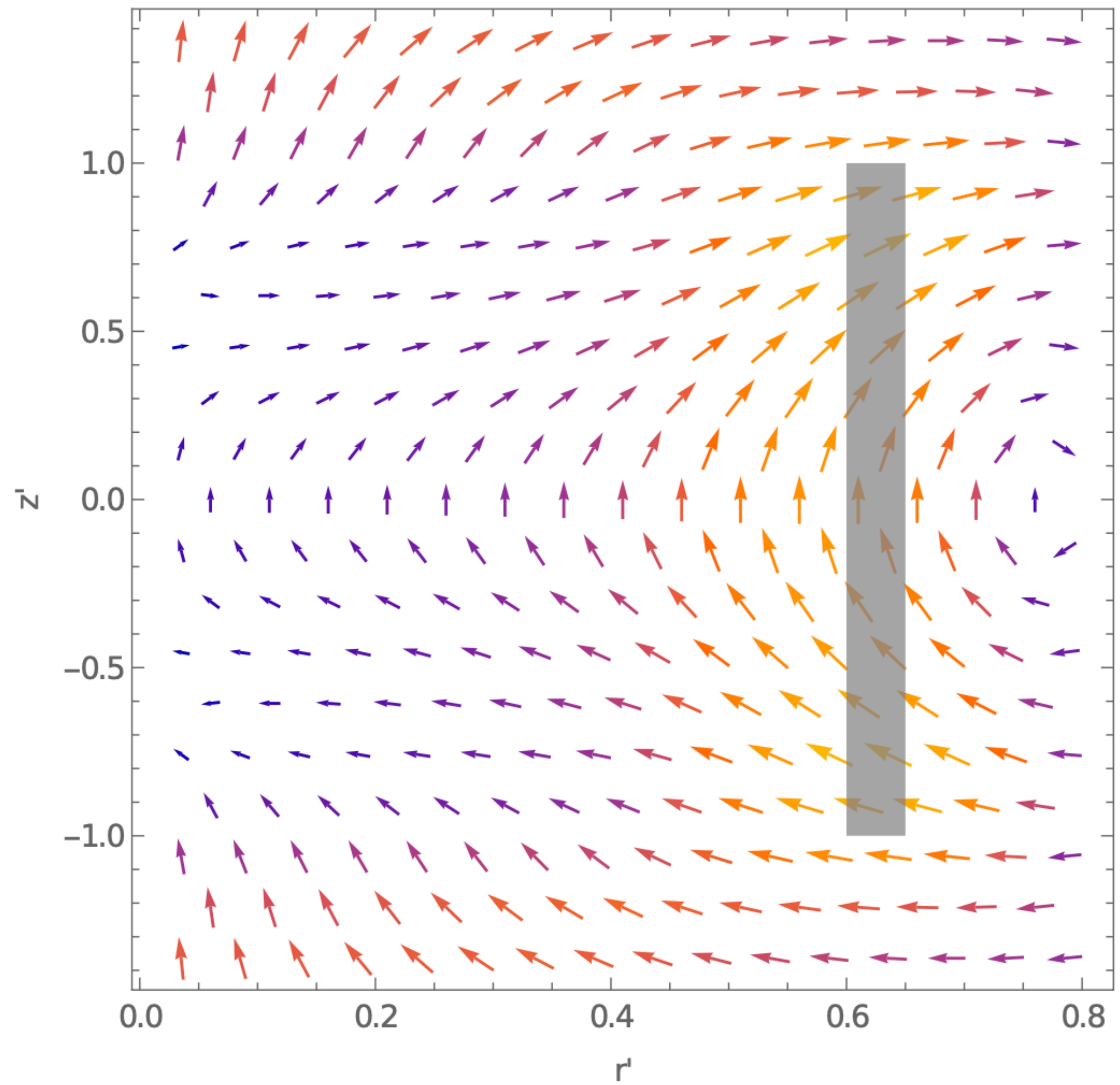
$$\text{Electric: } \frac{1}{2} C V^2$$

$$U_E \sim 5 \cdot 10^{5}\ \text{J} \times h^2 \left(\frac{C}{10^{-8}\ \text{F}}\right) \left(\frac{E}{10\ \text{MV/m}}\right)^2 \left(\frac{L}{1\ \text{m}}\right)^2$$

Taking typical values used by AURIGA

$\mathbf{B}_h$ profile

Response to h^+ & $\hat{\mathbf{k}} = \hat{\mathbf{x}}$ in the y-z plane



Proper Detector Frame

TT gauge: GW is a plane wave $\sim e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$

Proper Detector Frame: more involved

$$h_{00} = \omega^2 F(\mathbf{k}\cdot\mathbf{r})\,\mathbf{b}\cdot\mathbf{r}, \qquad b_j \equiv r_i h_{ij}^{\mathrm{TT}}\big|_{\mathbf{r}=0},$$
$$h_{0i} = \frac{1}{2}\omega^2 \left[F(\mathbf{k}\cdot\mathbf{r}) - iF'(\mathbf{k}\cdot\mathbf{r})\right]\left(\hat{\mathbf{k}}\cdot\mathbf{r}\; b_i - \mathbf{b}\cdot\mathbf{r}\;\hat{k}_i\right),$$
$$h_{ij} = -i\omega^2 F'(\mathbf{k}\cdot\mathbf{r})\left(|\mathbf{r}|^2\, h_{ij}^{\mathrm{TT}}\big|_{\mathbf{r}=0} + \mathbf{b}\cdot\mathbf{r}\,\delta_{ij} - b_i r_j - b_j r_i\right),$$
$$F(\xi) = (e^{i\xi} - 1 - i\xi)/\xi^2 = -1/2 + \mathcal{O}(\xi)$$

See [Berlin, Blas, Tito D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel 2021], [Domcke, Garcia-Cely, NLR 2022], [Domcke, Garcia-Cely, Lee, NLR 2024]

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$h_{00} = \omega^2 F(\mathbf{k}\cdot\mathbf{r})\,\mathbf{h}\cdot\mathbf{r}$ ... $h_i = x_j h^{\mathrm{TT}}$

$h_{0i} = \frac{1}{2}\omega^2 [F($ ...

$h_{ij} = -i\omega^2 F'($ ... $r_i),$

$F(\xi) = (e^{i\xi} - 1 - i\xi)/\xi^2 = -1/2 + \mathcal{O}(\xi)$

$$g_{\mu\nu}(x) = \underbrace{g_{\mu\nu}(x_0)}_{=\eta_{\mu\nu}} + \underbrace{(x-x_0)^\alpha \partial_\alpha g_{\mu\nu}(x_0)}_{=0\ (\because\ \Gamma^\mu_{\nu\rho}(x_0)=0)} + \underbrace{(x-x_0)^\alpha (x-x_0)^\beta \partial_\alpha \partial_\beta g_{\mu\nu}(x_0)}_{\mathcal{O}(\omega^2 R^2)} + \dots$$

See [Berlin, Blas, Tito D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel 2021], [Domcke, Garcia-Cely, NLR 2022], [Domcke, Garcia-Cely, Lee, NLR 2024]

Proper Detector Frame

Use Fermi normal coordinates

Locally inertial coordinates along a geodesic **[Fermi 1922]**

$$h_{ij} = -2 \sum_{n=0}^{\infty} \frac{n+1}{(n+3)!} \hat{R}_{ikjl,m_1 \ldots m_n} r_k r_l r_{m_1} \ldots r_{m_n} \,,$$

$$h_{0i} = -2 \sum_{n=0}^{\infty} \frac{n+2}{(n+3)!} \hat{R}_{0kil,m_1 \ldots m_n} r_k r_l r_{m_1} \ldots r_{m_n} \,,$$

$\hat{R}$ is evaluated at the coordinate origin

$$h_{00} = -2 \sum_{n=0}^{\infty} \frac{n+3}{(n+3)!} \hat{R}_{0k0l,m_1 \ldots m_n} r_k r_l r_{m_1} \ldots r_{m_n}$$

[Fortini and Gualdi 1982], [Marzlin 1994], [Rakhmanov 2014]

Proper Detector Frame

Proper detector frame:
Fermi normal coordinates transformed to the non-inertial reference frame of the detector

[Ni, Zimmermann 1978]

Non-inertial corrections (Earth's gravity, Coriolis effect, etc) are irrelevant at higher frequencies - effectively can just use Fermi normal coordinates