

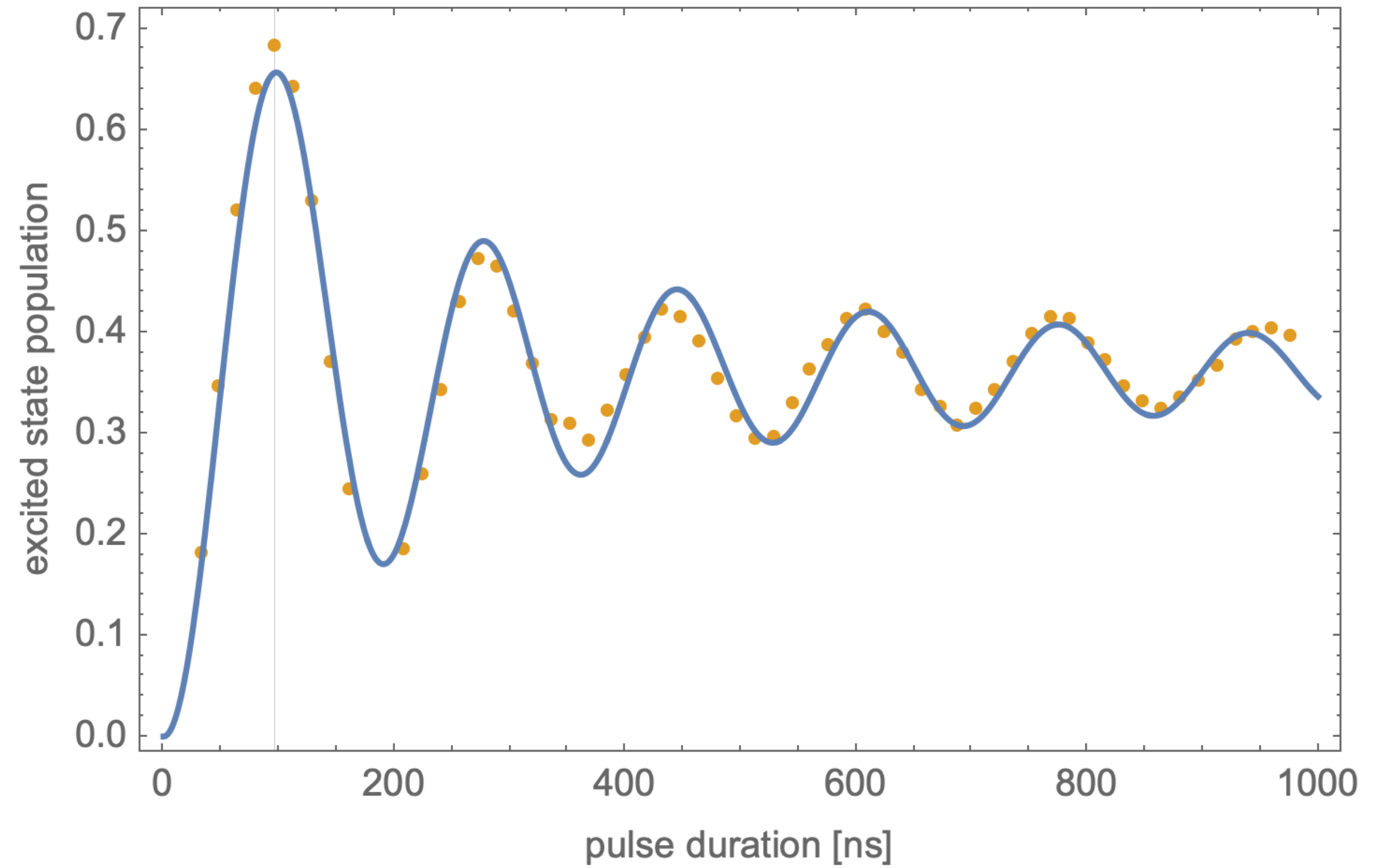
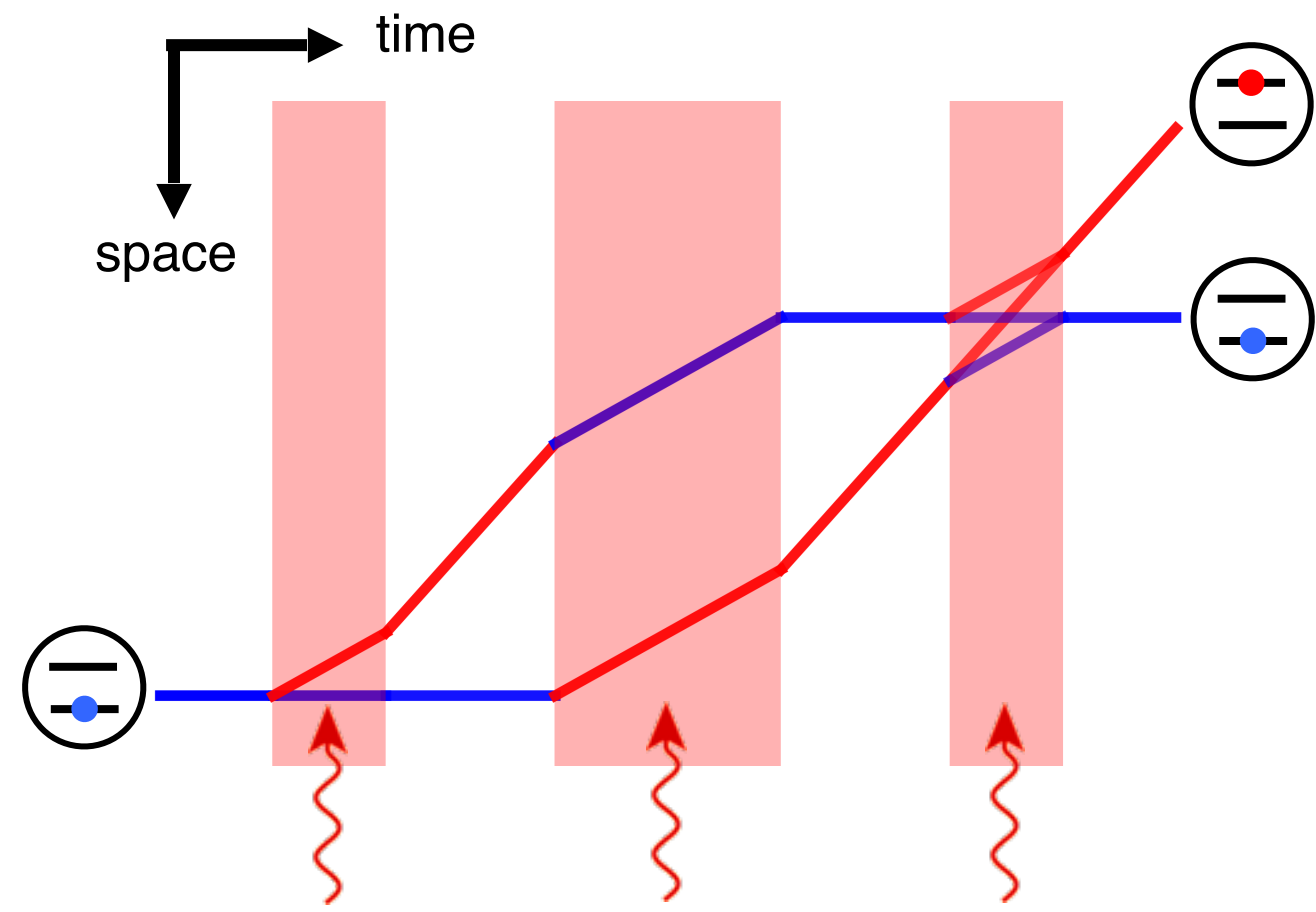
Two Simulation Tools For Improving Interferometer Response Under Imperfect Atom-Optics Laser Pulses

Classical trajectory stray path calculator & optical Bloch equation 'A matrix' solver

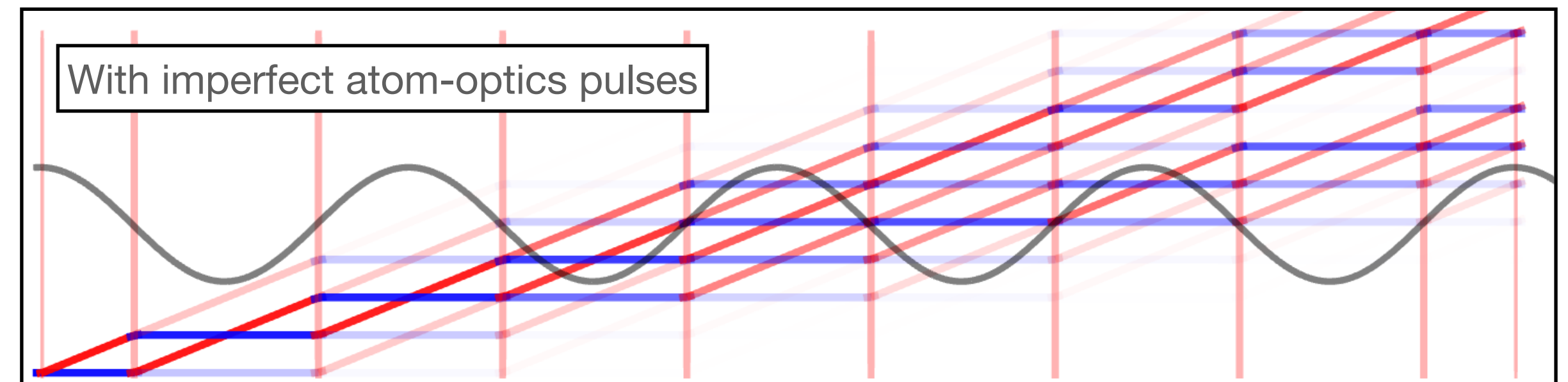
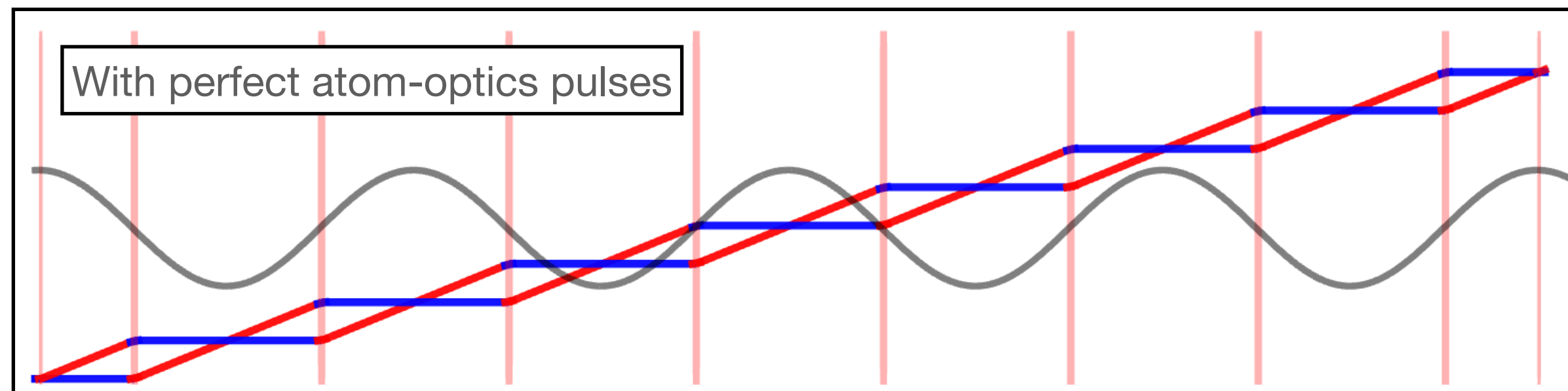
The Problem — Imperfect Pulses

Mechanisms for pulse inefficiency

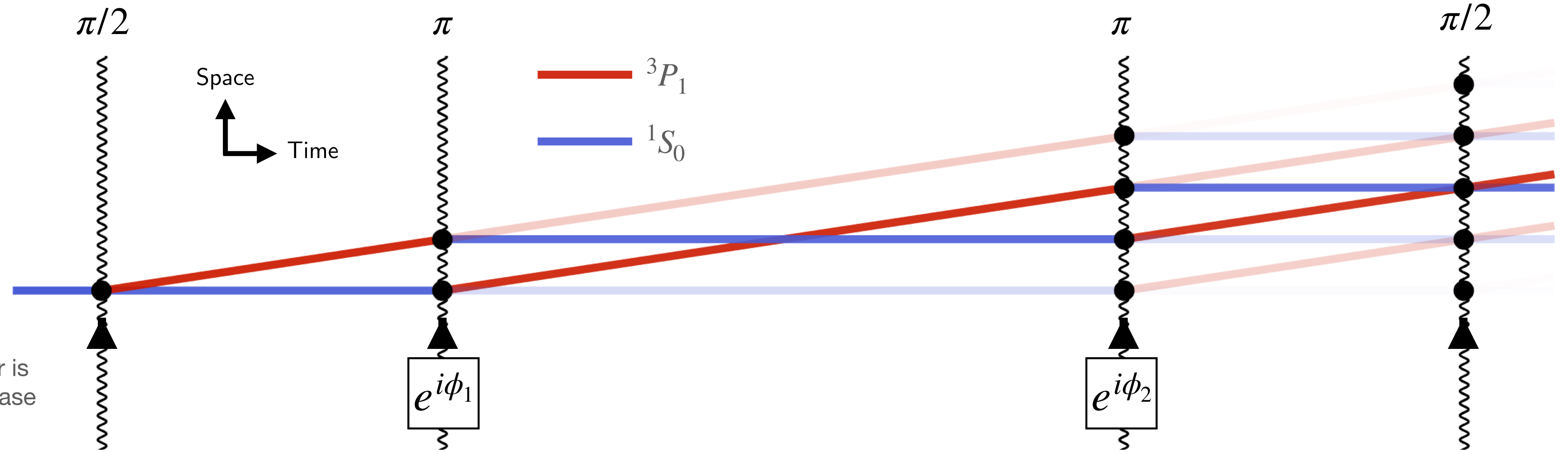
- Detuning spread — the atom cloud is too hot relative to the Rabi frequency
- Rabi frequency inhomogeneity across the atom cloud — interferometer beam too small relative to the atom cloud



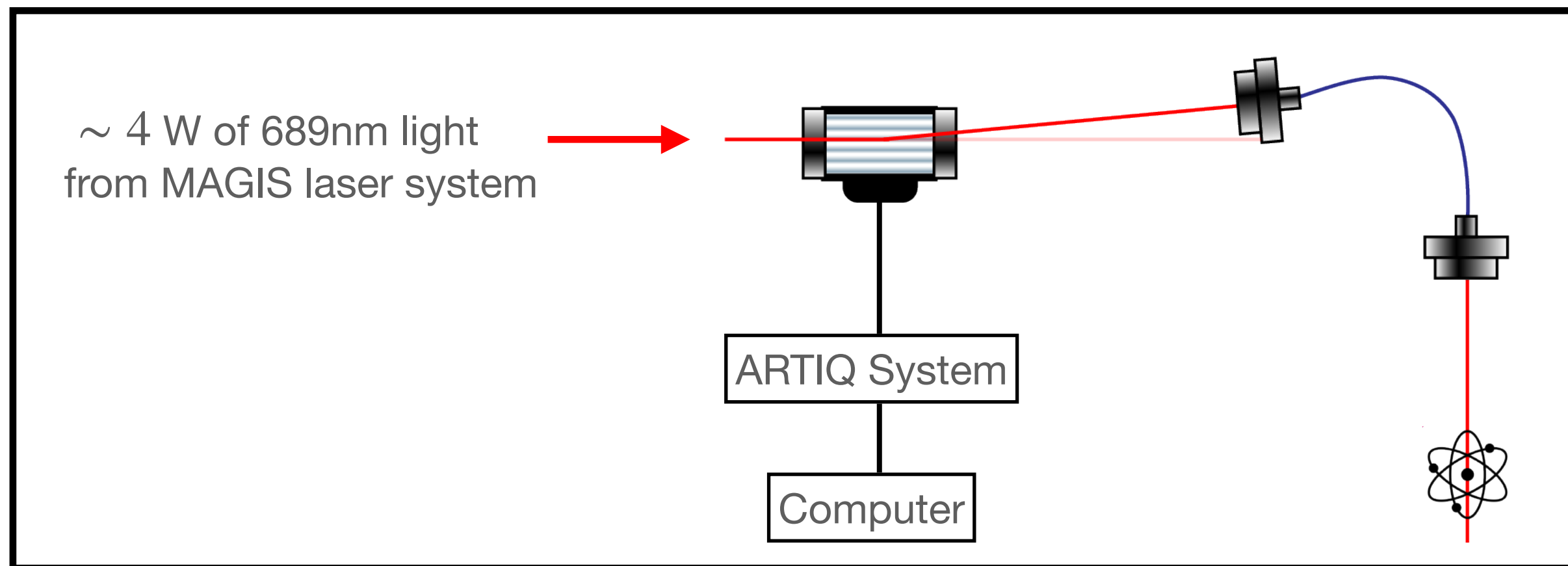
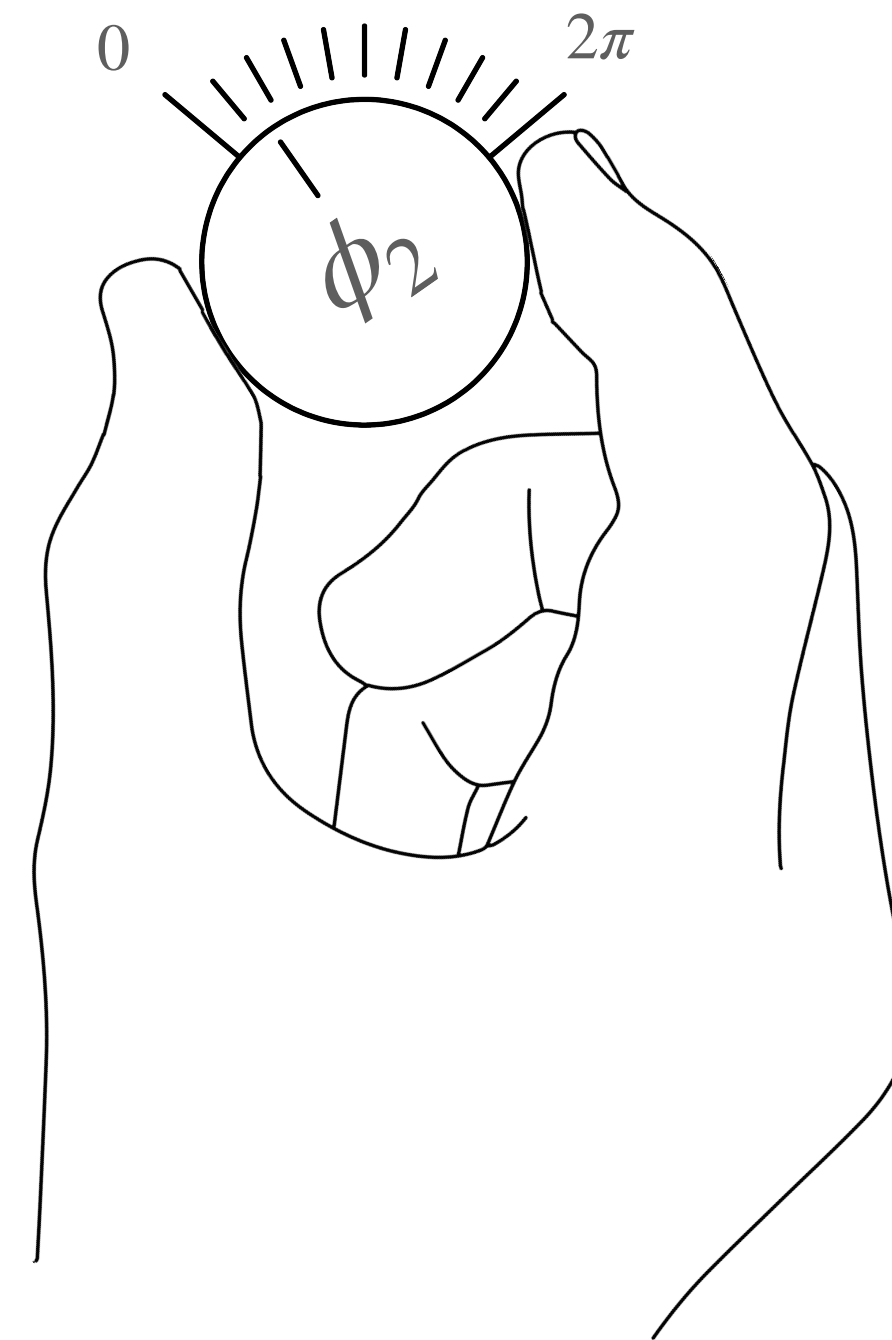
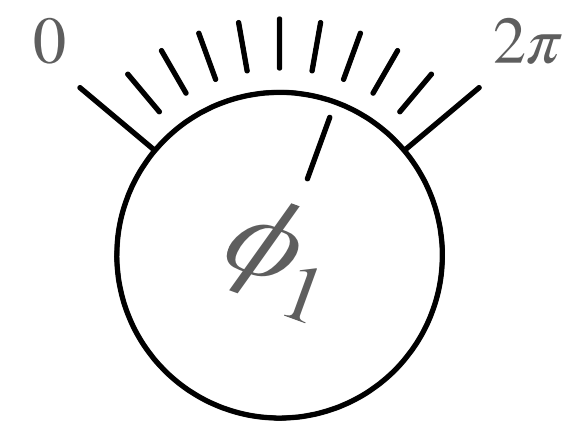
In general, the response of an atom interferometer to an oscillating signal is amplified by multi-loop interferometry, but imperfect pi pulses can limit the total number of loops that can be performed



The Central Idea — Modulate the Phase of the Interferometer Beam to Destructively Interfere Stray Paths

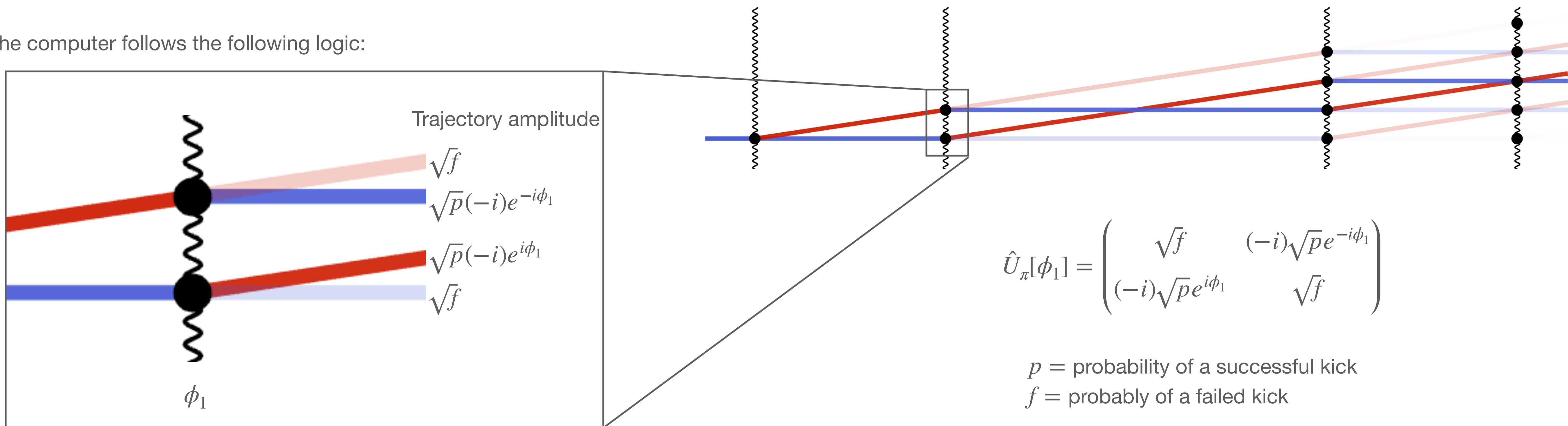


The phase of the laser is imprinted onto the phase of the kicked atoms



Simulation Tool – Classical Trajectory Stray Path Calculator

The computer follows the following logic:



For a sequence for which there are N paths which end up in the ground state

$$P_{g, \text{tot}} = \sum_{i=1}^N \sum_{j=1}^N \sqrt{P_i P_j} e^{-\frac{(x_i - x_j)^2}{2w_0^2}} e^{-\frac{m^2 w_0^2 (v_{x_i} - v_{x_j})^2}{8\hbar^2}} \text{Cos} \left[\phi_i^{\text{LF}} - \phi_j^{\text{LF}} - \frac{m (v_{x_i} + v_{x_j}) (x_i - x_j)}{2\hbar} \right]$$

Total population in the ground state

Increments over all N trajectories

Trajectories 'far away' from one another don't interfere

Trajectories with disparate velocities have imperceptible interference under spatial averaging

'Laser' and 'Free' propagation phase associated with a trajectory

Separation phase between trajectories

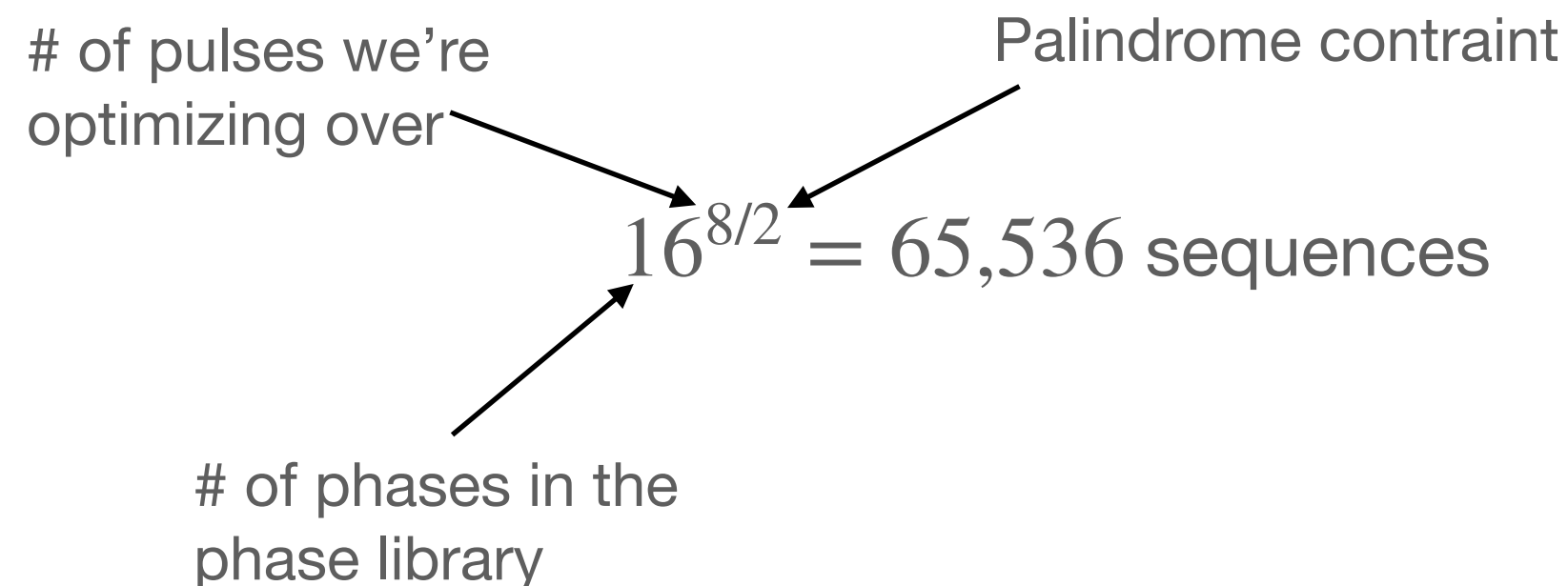
Initial idea – Computing a Sequence Which Maximizes the Interferometer Contrast

The initial idea was hunt for a new pulse sequence by

- Only considering optimizations over 8 pulses
- Only consider palindromic sequences
- Only consider phases that belonged a pre-defined phase library

Then brute force compute the interferometer contrast using the stray path calculator and sort the sequences by the best resulting contrast

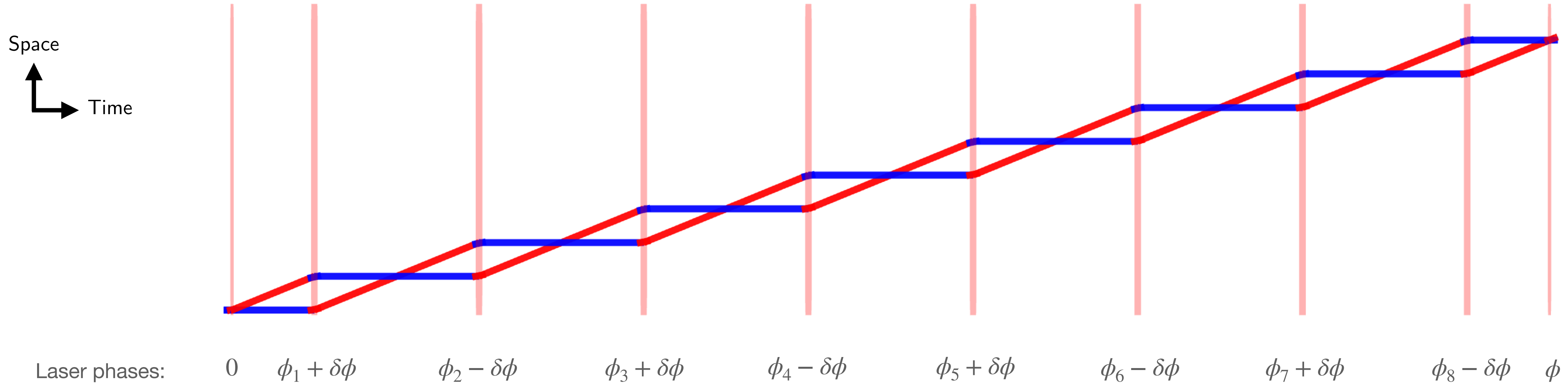
Phase library								
0	$\frac{\pi}{8}$	$\frac{2\pi}{8}$	$\frac{3\pi}{8}$	$\frac{4\pi}{8}$	$\frac{5\pi}{8}$	$\frac{6\pi}{8}$	$\frac{7\pi}{8}$	$\frac{8\pi}{8}$
	$\frac{9\pi}{8}$	$\frac{10\pi}{8}$	$\frac{11\pi}{8}$	$\frac{12\pi}{8}$	$\frac{13\pi}{8}$	$\frac{14\pi}{8}$	$\frac{15\pi}{8}$	



Trying these sequences out on the actual apparatus, most gave us quiet a high contrast at 504 loops, but very few were sensitive to a small oscillating laser phase

Pulse sequence	Interferometer Contrast @ 504 loops w/ no ac phase offset	Phase sensitivity?	Rank (listed in mathematica file)
$\frac{3\pi}{8}, \frac{15\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{11\pi}{8}, \frac{7\pi}{8}, \frac{15\pi}{8}, \frac{3\pi}{8}$	0.0440828	Yes (1 pi)	7
$\frac{\pi}{4}, \frac{7\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{4}$	0.0377814	Didn't check	8
$\frac{3\pi}{8}, \frac{\pi}{8}, \frac{5\pi}{8}, \frac{15\pi}{8}, \frac{15\pi}{8}, \frac{5\pi}{8}, \frac{\pi}{8}, \frac{3\pi}{8}$	0.0617513	No (3 pi)	15
$\frac{3\pi}{8}, \frac{\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{7\pi}{8}, \frac{5\pi}{8}, \frac{\pi}{8}, \frac{3\pi}{8}$	0.0329247	No (2 pi)	17
$\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{7\pi}{8}, \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{3\pi}{8}$	0.053459	Sorta?(4 pi)	18
$0, \frac{5\pi}{8}, \frac{13\pi}{8}, \frac{9\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}, \frac{5\pi}{8}, 0$	0.0490297	No (1 pi)	26
$0, \frac{3\pi}{8}, \frac{11\pi}{8}, \frac{7\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{3\pi}{8}, 0$	0.0515638	No (1 pi)	28
$\frac{3\pi}{8}, \frac{\pi}{4}, \frac{7\pi}{8}, \frac{7\pi}{4}, \frac{7\pi}{4}, \frac{7\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}$	0.0646001	No (1 pi)	32
$\frac{3\pi}{8}, 0, 0, \frac{3\pi}{4}, \frac{3\pi}{4}, 0, 0, \frac{3\pi}{8}$	0.0637466	No (1 pi)	35
$\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{15\pi}{8}, \frac{11\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}, \frac{7\pi}{8}, \frac{3\pi}{8}$	0.0404615	Yes (1 pi)	3
$\frac{3\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}, \frac{15\pi}{8}, \frac{9\pi}{8}, \frac{3\pi}{8}, \frac{3\pi}{8}$	0.0611053	No (1 pi)	36
$\frac{3\pi}{8}, \frac{11\pi}{8}, \frac{7\pi}{8}, \frac{13\pi}{8}, \frac{13\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{3\pi}{8}$	0.0557966	No (1 pi)	37
$\frac{3\pi}{8}, 0, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{7\pi}{8}, \frac{5\pi}{8}, 0, \frac{3\pi}{8}$	0.0545032	No (1 pi)	40
$\frac{\pi}{8}, \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{5\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{8}$	0.0490887	No (2 pi)	43
$\frac{3\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{13\pi}{8}, \frac{13\pi}{8}, \frac{3\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{8}$	0.0665145	No (1 pi)	44
$\frac{3\pi}{8}, \frac{15\pi}{8}, \frac{3\pi}{8}, \frac{13\pi}{8}, \frac{13\pi}{8}, \frac{3\pi}{8}, \frac{15\pi}{8}, \frac{3\pi}{8}$	0.0581659	No (1 pi)	45
$\frac{\pi}{8}, \frac{7\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{8}$	0.0523706	No (1 pi)	46
$\frac{3\pi}{8}, \frac{\pi}{4}, \frac{5\pi}{8}, \frac{5\pi}{4}, \frac{5\pi}{4}, \frac{5\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}$	0.0795381	No (1 pi)	47
$\frac{3\pi}{8}, \frac{13\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}, \frac{15\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}, \frac{3\pi}{8}$	0.0569317	No (1 pi)	50
$\frac{3\pi}{8}, \frac{\pi}{4}, \frac{\pi}{8}, \frac{15\pi}{8}, \frac{15\pi}{8}, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}$	0.0626831	No (1 pi)	52

Phase Sensitivity



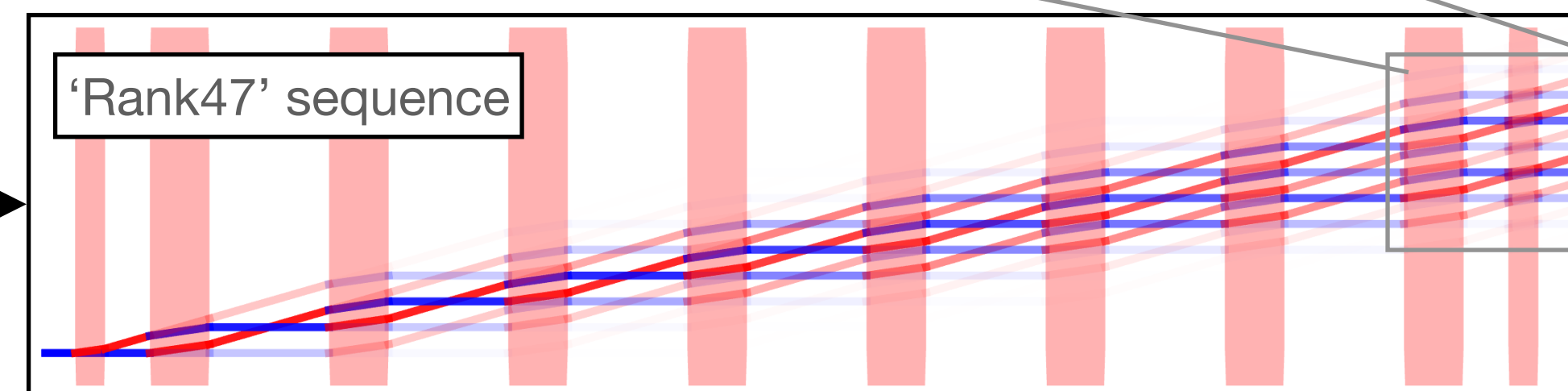
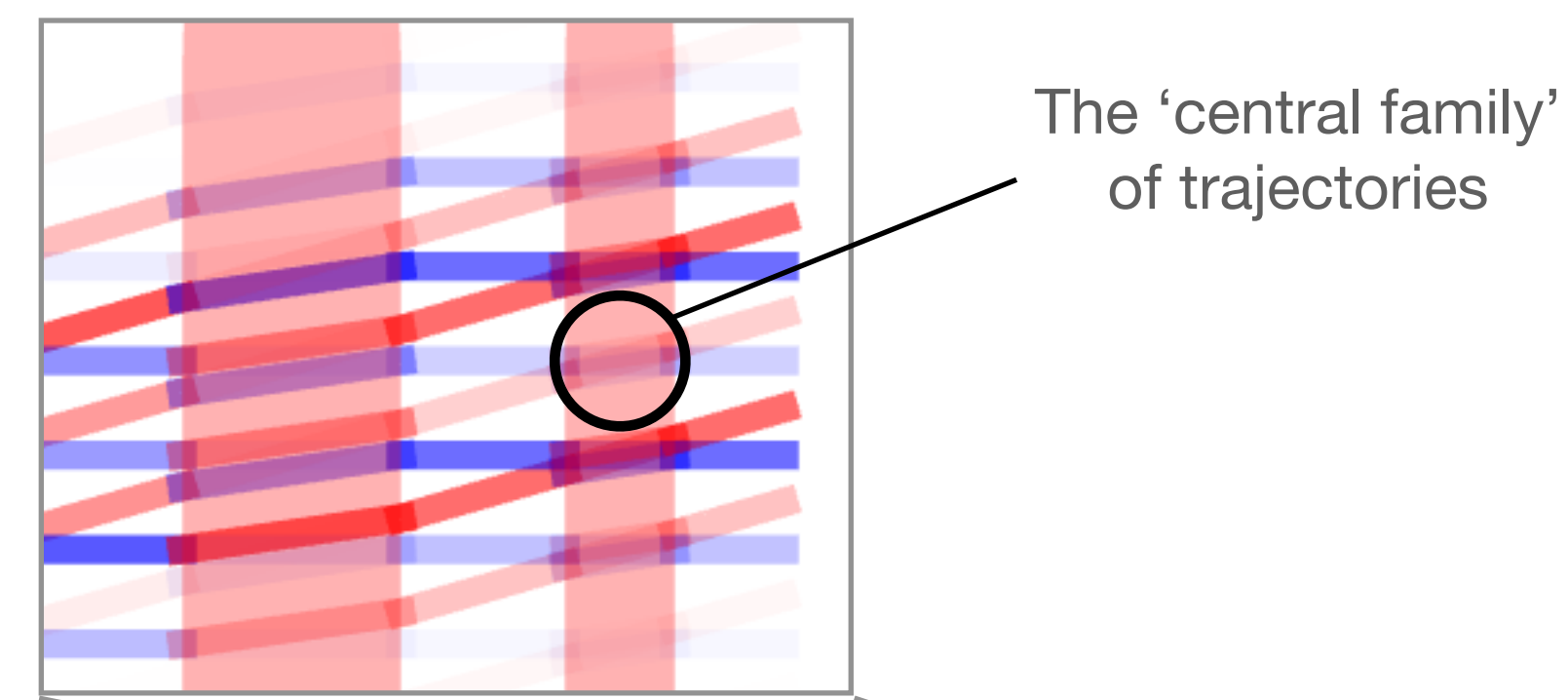
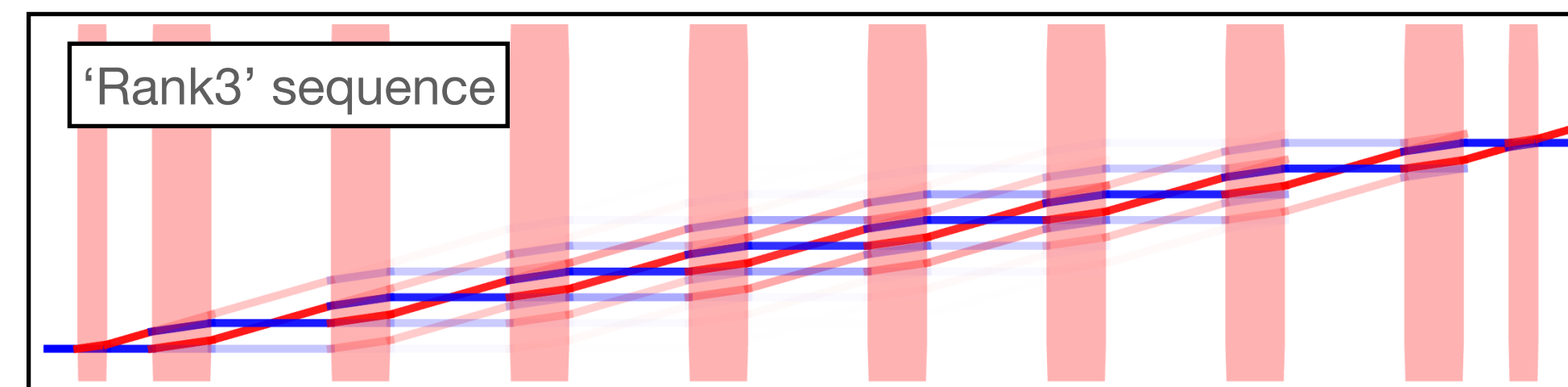
The interferometer phase should be $\Delta\phi = 2 \times 8 \times \delta\phi$ for an 8 loop interferometer

In general, for an ' L ' loop interferometer, the interferometer phase shift should be $\Delta\phi = 2 \times L \times \delta\phi$

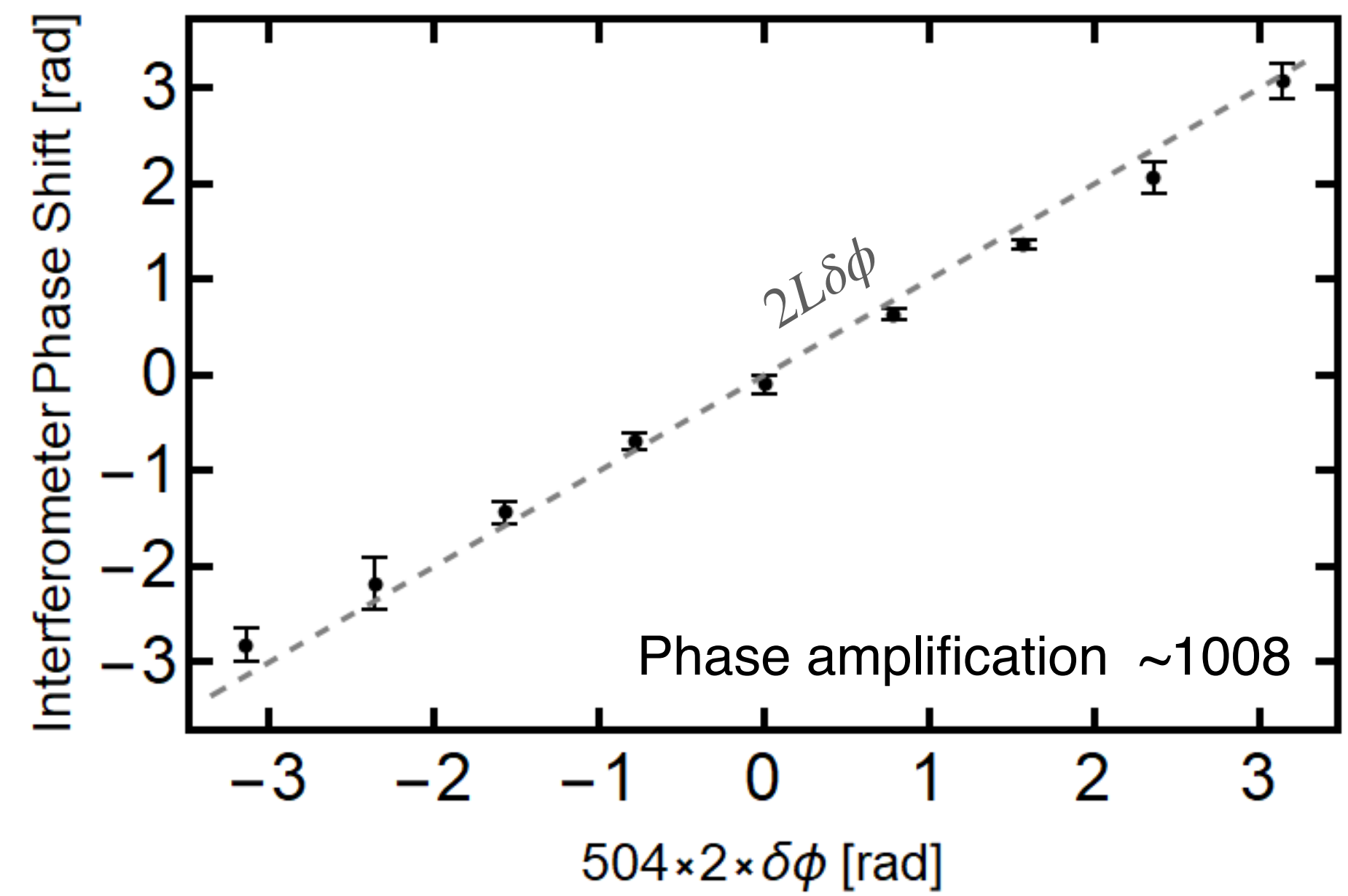
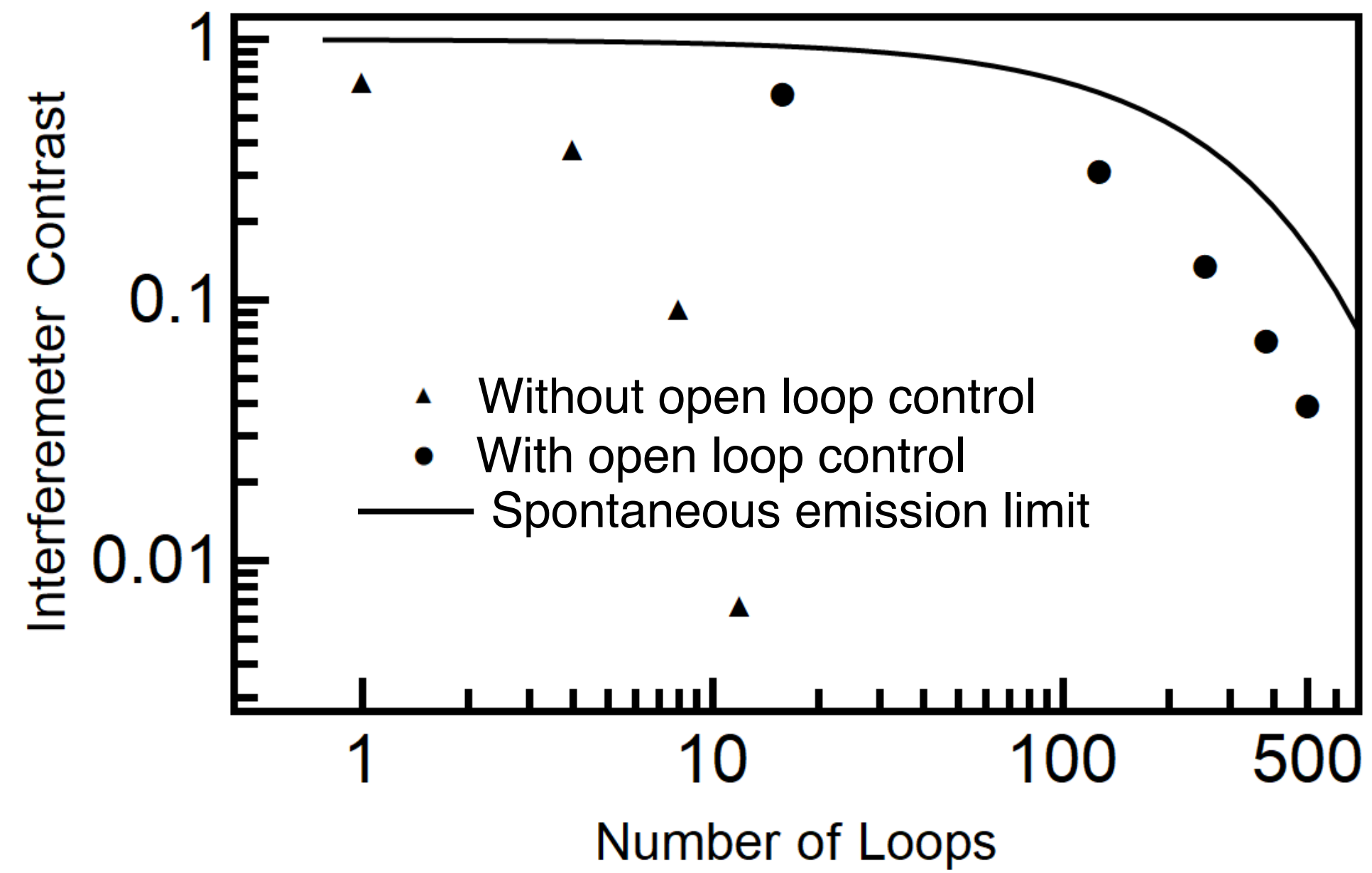
Increasing the number of loops enhances the sensitivity to oscillating signals

One Common Denominator — Sequences with Phase Sensitivity Also Have a Large ‘Central Family’ Population

Pulse sequence	Interferometer Contrast @ 504 loops w/ no ac phase offset	Phase sensitivity?	Rank (listed in mathematica file)	Central family pop after 8 pulses for f=10%
$\frac{3\pi}{8}, \frac{15\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{11\pi}{8}, \frac{7\pi}{8}, \frac{15\pi}{8}, \frac{3\pi}{8}$	0.0440828	Yes (1 pi)	7	0.9998
$\frac{\pi}{4}, \frac{7\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{4}$	0.0377814	Didn't check	8	0.9998
$\frac{3\pi}{8}, \frac{\pi}{8}, \frac{5\pi}{8}, \frac{15\pi}{8}, \frac{15\pi}{8}, \frac{5\pi}{8}, \frac{\pi}{8}, \frac{3\pi}{8}$	0.0617513	No (3 pi)	15	0.783584
$\frac{3\pi}{8}, \frac{\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{7\pi}{8}, \frac{5\pi}{8}, \frac{\pi}{8}, \frac{3\pi}{8}$	0.0329247	No (2 pi)	17	0.641068
$\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{7\pi}{8}, \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{3\pi}{8}$	0.053459	Sorta?(4 pi)	18	0.992614
$0, \frac{5\pi}{8}, \frac{13\pi}{8}, \frac{9\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}, \frac{5\pi}{8}, 0$	0.0490297	No (1 pi)	26	0.972587
$0, \frac{3\pi}{8}, \frac{11\pi}{8}, \frac{7\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{3\pi}{8}, 0$	0.0515638	No (1 pi)	28	0.972587
$\frac{3\pi}{8}, \frac{\pi}{4}, \frac{7\pi}{8}, \frac{7\pi}{4}, \frac{7\pi}{4}, \frac{7\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}$	0.0646001	No (1 pi)	32	0.335838
$\frac{3\pi}{8}, 0, 0, \frac{3\pi}{4}, \frac{3\pi}{4}, 0, 0, \frac{3\pi}{8}$	0.0637466	No (1 pi)	35	0.26379
$\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{15\pi}{8}, \frac{11\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}, \frac{7\pi}{8}, \frac{3\pi}{8}$	0.0404615	Yes (1 pi)	3	0.9998
$\frac{3\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}, \frac{15\pi}{8}, \frac{9\pi}{8}, \frac{3\pi}{8}, \frac{3\pi}{8}$	0.0611053	No (1 pi)	36	0.283067
$\frac{3\pi}{8}, \frac{11\pi}{8}, \frac{7\pi}{8}, \frac{13\pi}{8}, \frac{13\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{3\pi}{8}$	0.0557966	No (1 pi)	37	0.863374
$\frac{3\pi}{8}, 0, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{7\pi}{8}, \frac{5\pi}{8}, 0, \frac{3\pi}{8}$	0.0545032	No (1 pi)	40	0.675956
$\frac{\pi}{8}, \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{5\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{8}$	0.0490887	No (2 pi)	43	0.972587
$\frac{3\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{13\pi}{8}, \frac{13\pi}{8}, \frac{3\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{8}$	0.0665145	No (1 pi)	44	0.294655
$\frac{3\pi}{8}, \frac{15\pi}{8}, \frac{3\pi}{8}, \frac{13\pi}{8}, \frac{13\pi}{8}, \frac{3\pi}{8}, \frac{15\pi}{8}, \frac{3\pi}{8}$	0.0581659	No (1 pi)	45	0.881099
$\frac{\pi}{8}, \frac{7\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{8}$	0.0523706	No (1 pi)	46	0.972587
$\frac{3\pi}{8}, \frac{\pi}{4}, \frac{5\pi}{8}, \frac{5\pi}{4}, \frac{5\pi}{4}, \frac{5\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}$	0.0795381	No (1 pi)	47	0.0809828
$\frac{3\pi}{8}, \frac{13\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}, \frac{15\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}, \frac{3\pi}{8}$	0.0569317	No (1 pi)	50	0.760004
$\frac{3\pi}{8}, \frac{\pi}{4}, \frac{15\pi}{8}, \frac{15\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}$	0.0626831	No (1 pi)	52	0.392031
$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}$		Sorta	NMR	0.992614



Experimental Characterization of the 'rank3' Sequence



Using the Classical Stray Path Calculator to Find a General Analytic Expression for Laser Phases That Maximize the Central Family Population — The ‘NL’ sequence

Studying the sequences that maximize the central family population at allow loop number, looking for a pattern, and extrapolating to higher loop #

$\phi_k^N = \text{optimal } (k + 1)^{\text{th}} \text{ phase in an } N\text{-group sequence}$

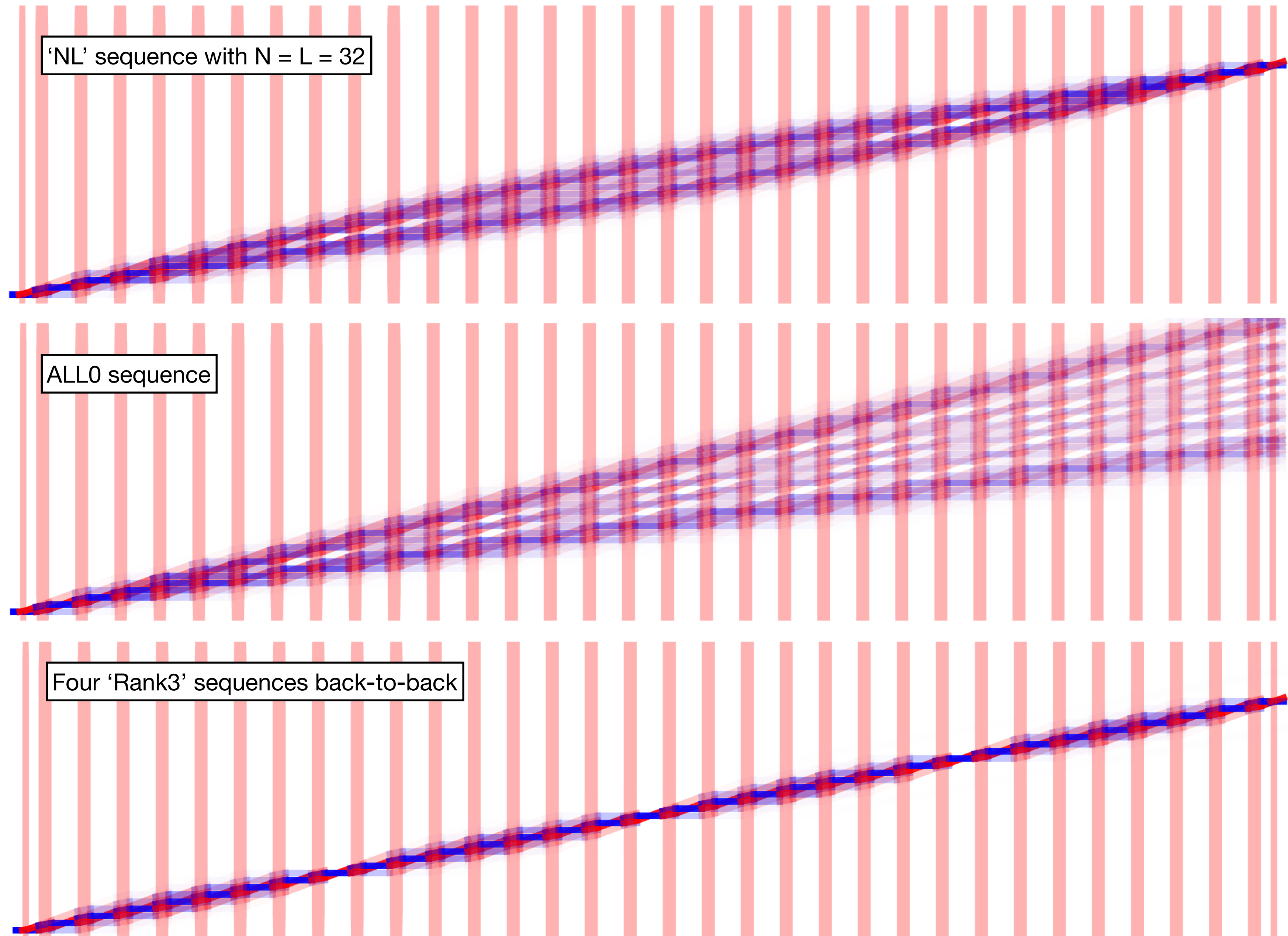
$$\phi_k^N = \frac{2\pi}{N}k(N - (k + 1))$$

But in the lab, this sequence doesn't perform very well

So likely we would need to maximize some cost function that includes both the CFP and the spread in trajectories

Motivation for making an optical Bloch equation solver:

- Initial tests of sequence hunting with a cost which was dependent on central family population & trajectory ‘spread’ didn't result in productive sequences
- It is possible that the coupling between the spread in detunings and spontaneous emission is relevant physics to account for, and that's not something the classical trajectory calculator has a language for



Simulation Tool — Optical Bloch Equation ‘A Matrix’ Solver

$$\partial_t \hat{\rho}[t] = -\frac{i}{\hbar} [\hat{H}, \hat{\rho}[t]] - \frac{\gamma}{2} (\hat{a}^\dagger \hat{a} \hat{\rho}[t] - 2\hat{a} \hat{\rho}[t] \hat{a}^\dagger + \hat{\rho}[t] \hat{a}^\dagger \hat{a})$$

Schrödinger Equation

Spontaneous Emission

$$\hat{\rho} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}$$

Spontaneous emission can produce states which cannot be described by a single wave function — expressing our system with a density matrix whose time evolution follows the Optical Bloch equations can allow us to compute the effect of Spontaneous emission in our interferometer

$$\frac{1}{2} \left(1 + C \cos[\theta + \phi] \right) = \underbrace{\int_{-\infty}^{\infty} \frac{d\delta}{k_z} \left(\frac{1}{2\pi\beta} \right)^{1/2} e^{-\frac{1}{2\beta} \left(\frac{\delta}{k_z} \right)^2}}_{\text{Integrating over thermal distribution of detunings}} \underbrace{\int_0^{\infty} 2\pi r dr \left(\frac{1}{2\pi\sigma^2} \right)^{2/2} e^{-\frac{1}{2\sigma^2} r^2}}_{\text{Integrating over a radial Rabi frequency inhomogeneity}} \rho_{22} \left[\delta, \Omega_0 e^{-\frac{1}{w_0^2} r^2} \right]$$

Integrating over thermal distribution of detunings

Integrating over a radial Rabi frequency inhomogeneity

$\rho_{22}[\delta, \Omega]$ = the excited state population after the final beamsplitter pulse with detuning δ and Rabi frequency Ω

Simulation Tool — Optical Bloch Equation ‘A Matrix’ Solver

The problem is that solving the optical Bloch equations numerically in time for a range of different detunings and Rabi frequencies can be computationally expensive, especially at high loop #

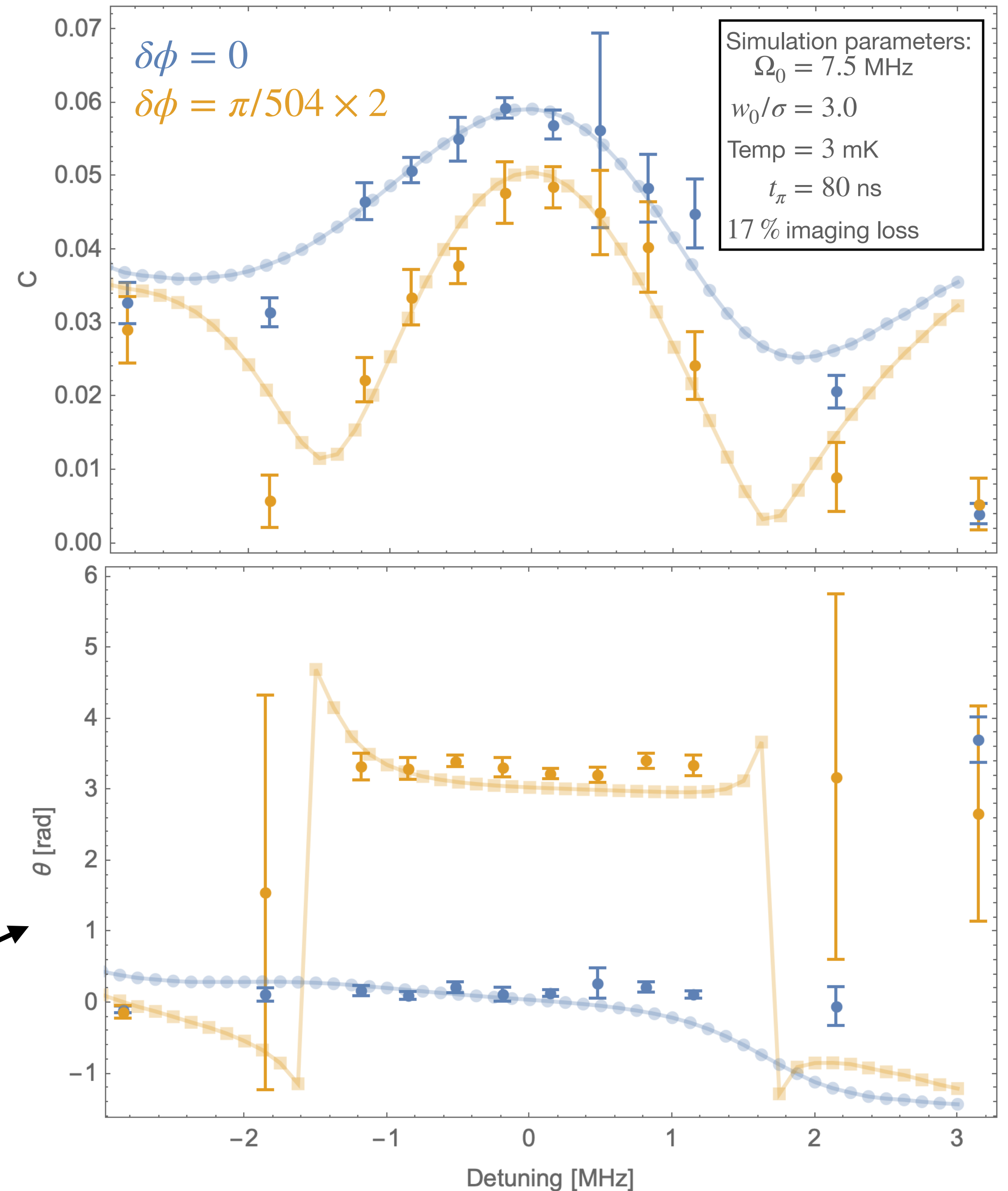
The idea is to save on computational cost by evaluating for the time evolution for a single π pulse, and applying that time evolution operator many times — we’re allowed to do this b/c the optical Bloch equations are just a linear system of equations

$$\partial_t \hat{A}_{ij}[t] = -\frac{i}{\hbar} [\hat{H}, \hat{A}_{ij}[t]] - \frac{\gamma}{2} (\hat{a}^\dagger \hat{a} \hat{A}_{ij}[t] - 2\hat{a} \hat{A}_{ij}[t] \hat{a}^\dagger + \hat{A}_{ij}[t] \hat{a}^\dagger \hat{a})$$

$$\hat{A}_{11}[0] = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \hat{A}_{12}[0] = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \hat{A}_{21}[0] = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad \hat{A}_{22}[0] = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\hat{\rho}[t] = \hat{A}_{11}[t]\rho_{11}[0] + \hat{A}_{12}[t]\rho_{12}[0] + \hat{A}_{21}[t]\rho_{21}[0] + \hat{A}_{22}[t]\rho_{22}[0]$$

As part of characterizing the ‘rank3’ sequence, we studied the susceptibility to overall detuning errors of an interferometer operating at $L = 504$ loops with the rank3 sequence



Using the 'A matrix' Calculator to Find Optimal Sequences at Higher N

The brute force approach to finding new pulse sequences can be limited by the exponential scaling of the possible pulse sequences with increasing N , so constraints on the allowed pulse sequences need to be applied in order to limit the total number of sequences that are simulated

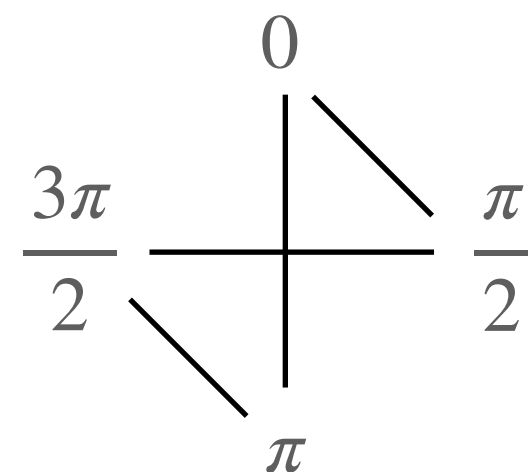
For hunting for $N = 16$ sequences, we can assume

- that the global phase doesn't matter — assume the first mirror phase is 0
- A phase library of factors of $\pi/2$
- The 16 pulse sequence is a palindrome

Which gives us

- $4^{16/2-1} = 16,384$ possible pulse sequences
- estimated compute time $2.5 \text{ s} \times 4^7 \times 2 \approx 23 \text{ hrs}$
- Actual compute time $\approx 18 \text{ hrs } 12 \text{ mins}$

For hunting for $N = 32$ sequences, if all we did was impose the constraints above, it would take $\approx 2.5 \text{ s} \times 4^{32/2-1} \times 2 \approx 170 \text{ yrs}$ to evaluate each pulse sequence, so we need to apply an additional constraint: Only allowing 'hourglass' sequences



Which reduces the total computation time to $2.5 \text{ s} \times 2^{32/2-1} \times 2 \approx 46 \text{ hrs}$

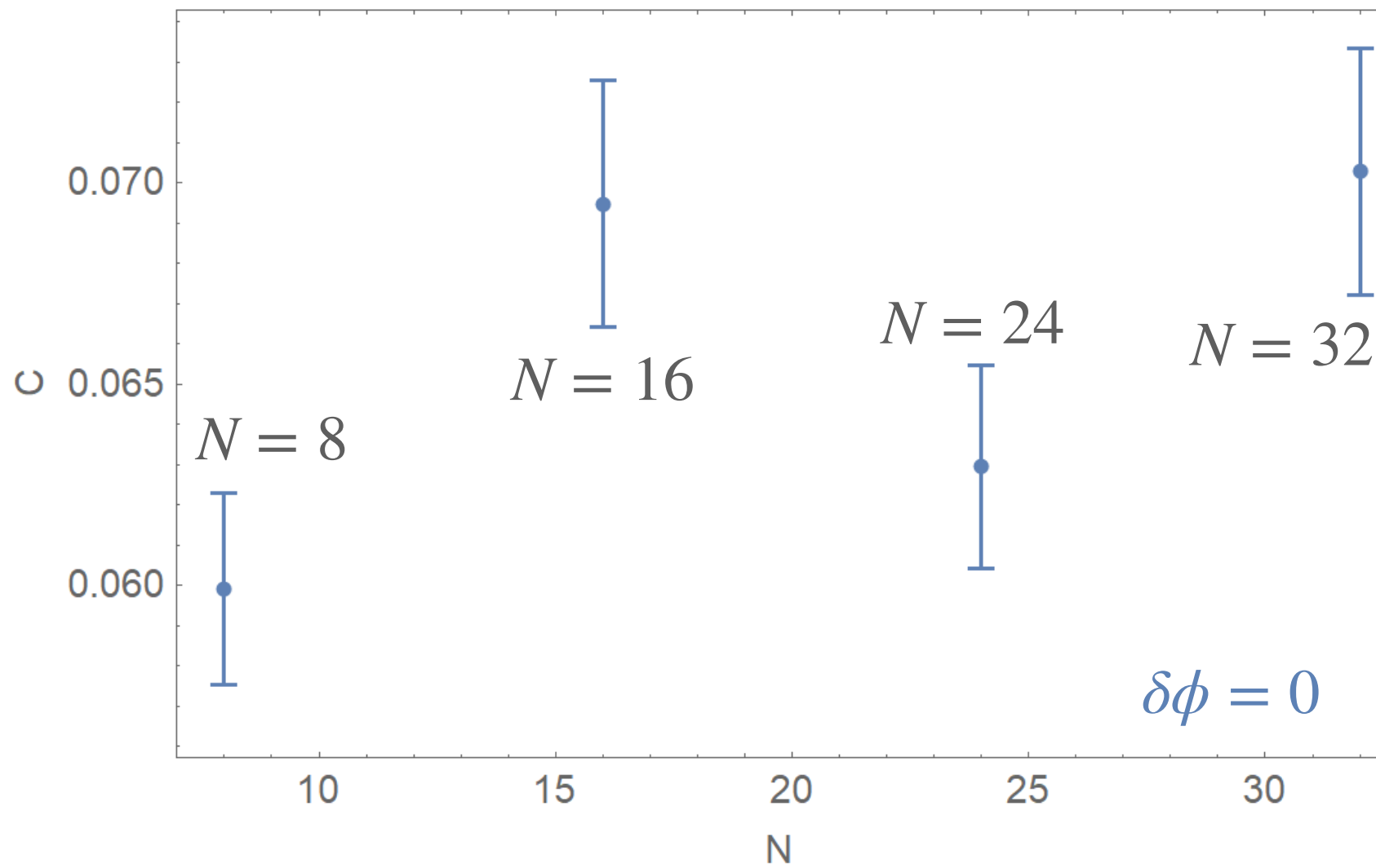
We don't have a good physical argument for why the best sequences should survive these constraints, but the best sequences at smaller N followed them, and we need to reduce the space of possible sequences we simulate

For each pulse sequence, we run the simulation twice : Once with no additional alternating phase and once with a small alternating phase $\delta\phi$ to check that with the sequence, the interferometer phase maintains a sensitivity to an alternating signal

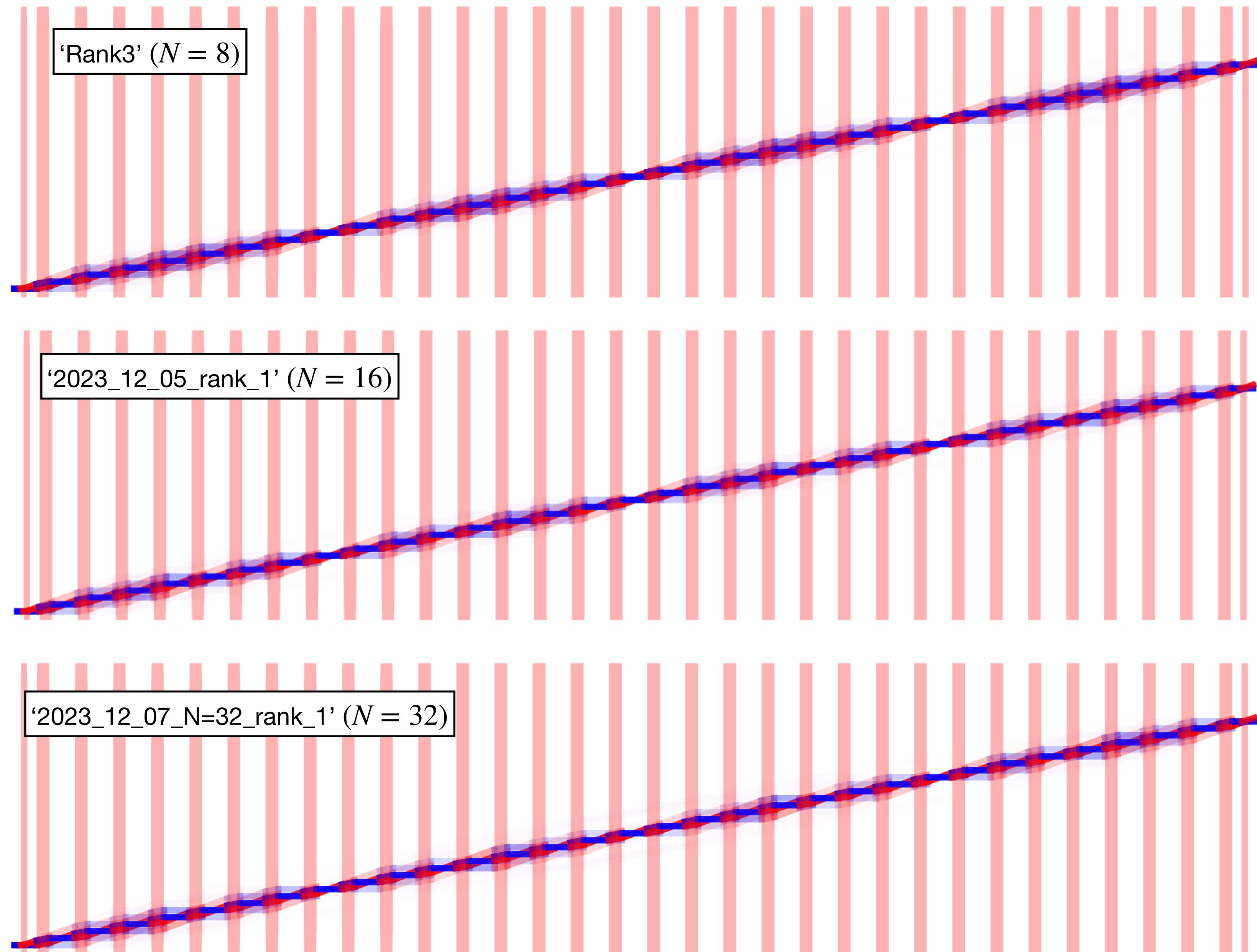
The output of the A-matrix calculator is a list of phase sensitive sequences, ranked in order of interferometer contrast at no alternating phase

Using the 'A matrix' Calculator to Find Optimal Sequences at Higher N

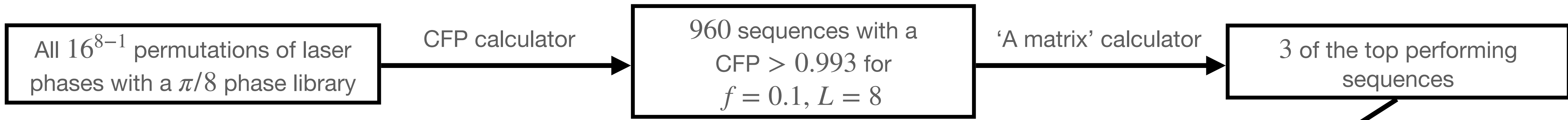
Running the best sequences in the lab for different N — it happens that some of the best sequences to come out of the 'A matrix' calculator, also have a large 'central family' in the classical stray trajectory calculator, and a narrow spread of populations in position space



- There is a pretty strong case here for improving interferometer contrast w/out a definitive drop-off in phase sensitivity as we go from $N = 8$ to $N = 16$, but it's not clear we see much of an additional improvement as we go from $N = 16$ to $N = 32$
- Indicates that the $N = 2^m, m \in \mathbb{Z}, m > 0$ sequences might be particularly well suited to a $\pi/2$ phase library



What Happens When we Relax the 'Palindrome' Constraint on the Pulse Sequences — Exploring for $N = 8$

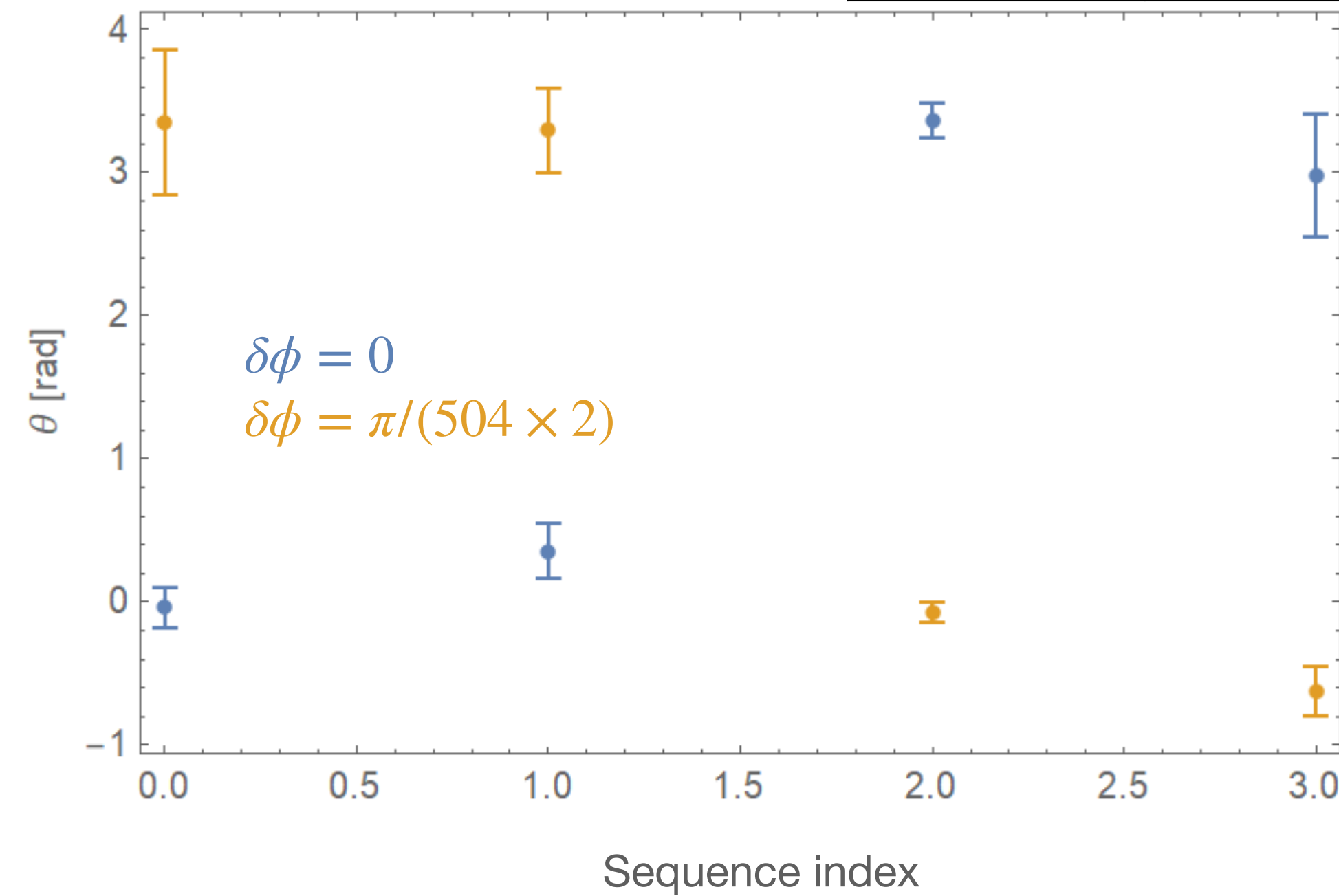
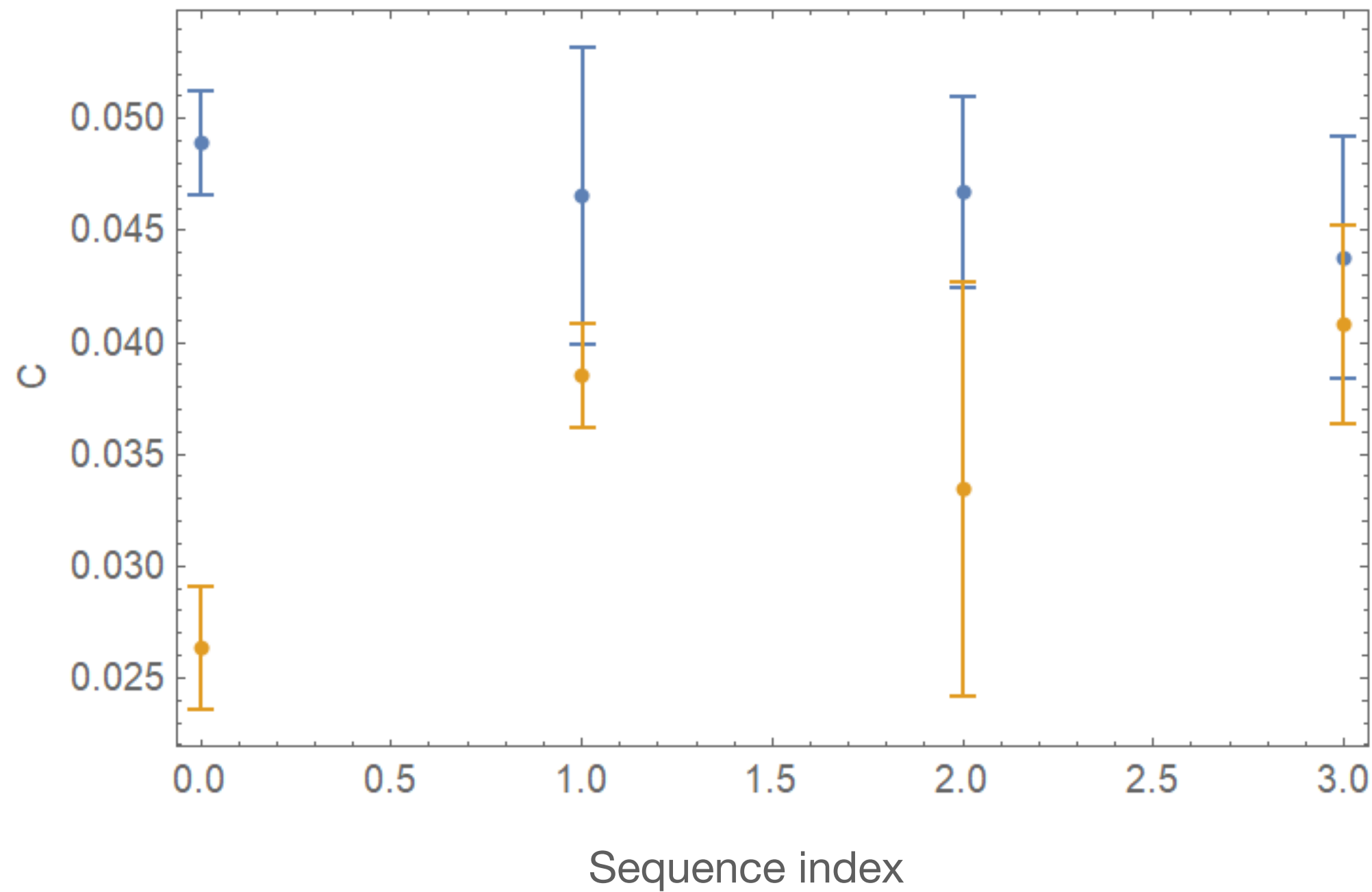


$16^{8-1} = 268,435,456$

- CFPs run in parallel (10 cores) with the python calculator
- Run in two sections so as not eat up all the RAM on that desktop
 - The first 100 million sequences took ≈ 8.5 hrs
 - The last ~ 160 million took ≈ 14.5 hrs
- Best CFP sequences written to a txt file

Sequence index	Sequence
0	$0, \frac{\pi}{2}, \frac{3\pi}{2}, \pi, \pi, \frac{3\pi}{2}, \frac{\pi}{2}, 0$
1	$0, \frac{3\pi}{4}, 0, \frac{7\pi}{4}, 0, \frac{3\pi}{4}, 0, \frac{7\pi}{4}$
2	$0, \frac{5\pi}{8}, \frac{3\pi}{4}, \frac{3\pi}{8}, \frac{3\pi}{2}, \frac{\pi}{8}, \frac{\pi}{4}, \frac{15\pi}{8}$
3	$0, \frac{11\pi}{8}, \frac{5\pi}{4}, \frac{13\pi}{8}, \frac{\pi}{2}, \frac{15\pi}{8}, \frac{7\pi}{4}, \frac{\pi}{8}$

— 'Rank3' for comparison



No sequence obviously outperformed 'rank3'

Conclusion

- We're continuing to play with and develop these tools in conjunction with experiment to better understand how we can improve resonant enhancement of atom interferometers under imperfect atom-optics pulses
- We're currently using optical procured for the MAGIS laser system for our interferometer beam, and the result of this work could likely improve the enhancement of MAGIS in a resonant, multi-loop operating mode, even though the experimental parameter space for MAGIS is different from the experiment we've built at northwestern

On the horizon:

- Exploring prospects for using these tools to design pulse sequences for LMT
- Exploring what additional power closed-loop control & trajectory optimization algorithms can provide

