Electro-Thermal Feedback Physics in Transition Edge Sensor

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Radiation Impact on Superconducting Qubits @ Fermilab
G4CMP Satellite Workshop

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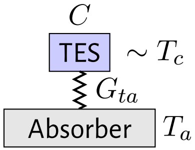
Outline

TES thermal connection

- TES is thermally connected to the absorber by a thermal conductance G_{ta}.



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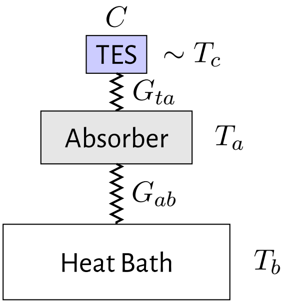
- Absorber is thermally connected to the heat bath (fridge) by a thermal conductance G_{ab}.



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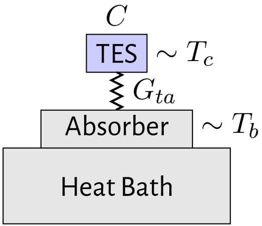
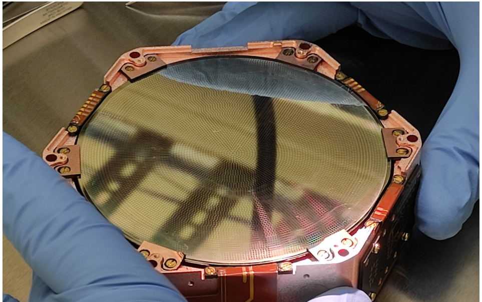
- TESSim assumes that the absorber and the heat bath are identical. G_{ab} is infinite.





TES electro-thermal connection

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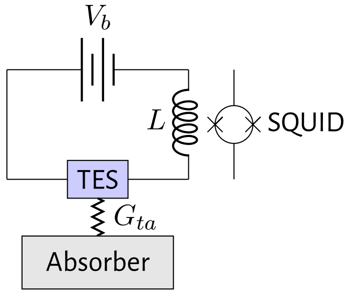
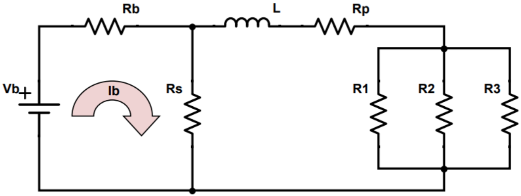
- TESSim assumes that the absorber and the heat bath are identical. G_{ab} is infinite.
- TES in SuperCDMS devices is kept under a constant voltage.
- Change in current changes the magnetic flux of the inductor L
- Magnetic flux changes are sensed and amplified by the SQUID



Current equation

Kirchoff's loop rule:

$$0 = -V_b + I_b R_b + (I_b - I_L)\, R_s$$



TES Channel

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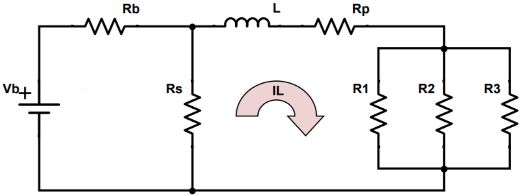
$$0 = (I_L - I_b)\, R_s + L\frac{dI_L}{dt} + I_L R_p + I_L \frac{1}{\sum_i \frac{1}{R_i}}$$



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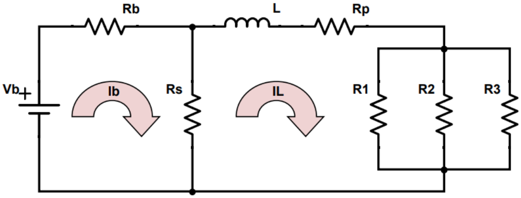
$$\dot{I}_L = \frac{V_{b0} - I_L \left(R_0 + \frac{1}{\sum_k G_k(A_k)} \right)}{L}$$



TES Channel

Current equation

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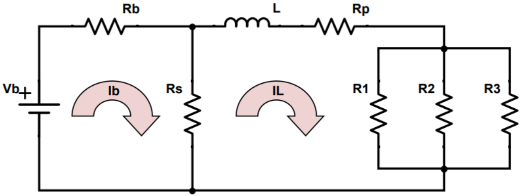
$$I_i = I_L \frac{G_i}{\sum_k G_k},$$

$$G_i = \frac{1}{R_i} = \frac{2}{R_n}\cosh A_i e^{-A_i},$$

$$V_{b0} = V_b \frac{R_s}{R_b + R_s}$$

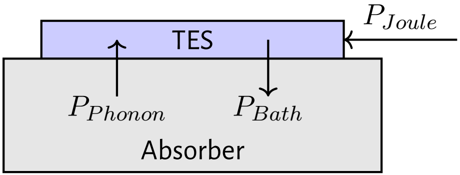
$$R_0 = R_p + \frac{R_b R_s}{R_b + R_s}$$



TES Channel

Heat equation

$$\frac{dT}{dt}C = P_J + P_B + P_P$$

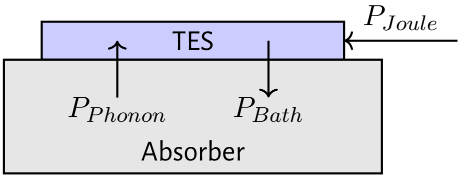


Heat equation

$$\frac{dT_s}{dt}C = P_J + P_B + P_P$$

$$P_J = IR^2(T) = \frac{I}{G^2(T)}$$

$$P_B = K\left(T_b^n - T^n\right) = \Sigma\mathcal{V}_{eff}\left(T_b^n - T^n\right)$$



- Σ is the electron-phonon coupling in the TES
- $n = 5$ for SuperCDMS detectors
- Phonon power comes from CrystalSim

Coupled ODEs

$$\dot{I}_L = \frac{V_{b0} - I_L\left(R_0 + \frac{1}{\sum_k G_k}\right)}{L}$$

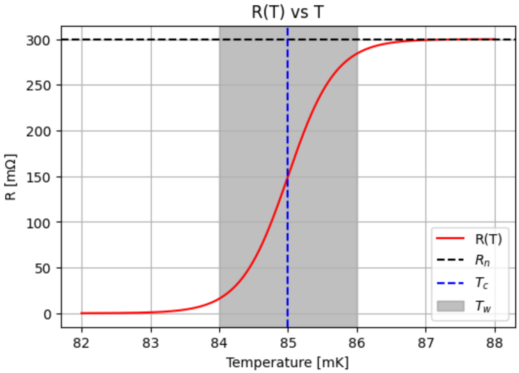
$$\dot{A}_i = \frac{K\left(T_b^5 - T_i^5\left(A_i\right)\right) + I_L^2 \frac{G_i(A_i)}{\left(\sum_k G_k(A_k)\right)^2} + P_{P,i}}{T_w C}.$$

- Not the whole story!
- Current effects have to be taken into account

TES resistance model

$$R(T) = \frac{1}{G(T)} = \frac{R_n}{2}\left[1 + \tanh\left(\frac{T - T_c}{T_w}\right)\right]$$

$$A \equiv \frac{T - T_c}{T_w}$$

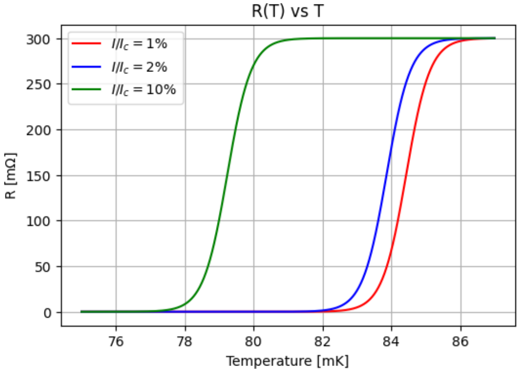


Current effects

- T_c is lowered due to a non-zero current.
- The effective T_c is modeled by

$$T_c^{\text{eff}} = T_c \left(1 - \frac{|I|}{I_c}\right)^n$$

- For SuperCDMS detectors $n = 2/3$

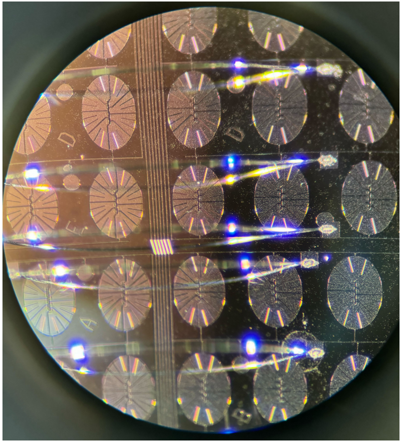


Current effects

- Current effects allow for the individual TESs to cross-talk through the matrix $\frac{\partial T_i}{\partial A_j}$

$$\sum_j \frac{\partial T_i}{\partial A_j} \dot{A}_j = \frac{P_{B,i} + P_{J,i} + P_{P,i}}{T_w C} - \frac{\partial T_i}{\partial I_L} \dot{I}_L$$

- Thermal conductivity between neighboring TESs through the substrate is not included.



Matrix inversion

- Applying chain rule the matrix $\frac{\partial T_i}{\partial A_j}$ can be written as:

$$\frac{\partial T_i}{\partial A_j} = \delta_{ij} a_i + u_i v_j$$

- Which can be inverted using the Sherman-Morrison formula:

$$\left(\frac{\partial T_i}{\partial A_j}\right)^{-1} = \frac{\delta_{ij}}{a_i} - \frac{(u_i/a_i)(v_j/a_j)}{1 + \sum_k u_k v_k / a_k}$$

$$a_i = T_w + B_i I_L \frac{2}{R_n} e^{-2A_j}$$

$$B_i = \frac{2}{3} \frac{T_c}{I_c} \frac{1}{\sum_k G_k} \operatorname{sgn} I_i \left(1 - \frac{|I_i|}{I_c}\right)^{-1/3}$$

$$u_i = B_i I_i, \qquad v_j = -\frac{2}{R_n} e^{-2A_j}$$

ODEs final form

- Putting all together

$$\dot{I}_L = \frac{V_{b0} - I_L\left(R_0 + \frac{1}{\sum_k G_k}\right)}{L}$$

$$\dot{A}_i = \frac{D_i}{a_i} - \frac{1}{1 + \sum_k u_k v_k / a_k} \frac{u_i}{a_i} \sum_j \frac{v_j}{a_j} D_j$$

Where

$$D_i = \frac{K\left(T_b^5 - T_i^5\right) + \left(\frac{I_L}{\sum_k G_k}\right)^2 G_i + P_i^P}{T_w C_n} + B_i G_i \frac{V_{b0} - I_L\left(R_0 + 1/\sum_k G_k\right)}{L}$$

- We have n+1 coupled ODEs with no numerical matrix calculations.

ODE solver and limitations

- The ODEs are easily solved using CVODE solver in $C++$.
- TESSim reports the pulse amplitude, temperature A, T_c, T_c^{eff}, R, G and power.
- The ODE solver time step is smaller than the rise time to be able to capture all the changes.
- The number of TESs in a detector can be huge ~ 1000, TESSim uses a lumping algorithm
- It lumps TESs into ~ 35 pseudo-TESs that are bigger in size.
- Volumetric quantities (C, K, R_n) are scaled accordingly.

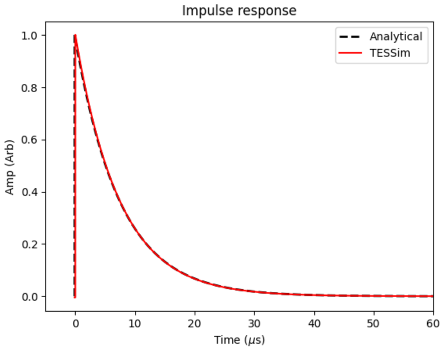
Pulse shape

- Impulse response for a single TES

$$\delta I(t) = A\left(E_{ph}\right)\left(e^{-\frac{t}{\tau_+}} - e^{-\frac{t}{\tau_-}}\right)$$

$$\frac{1}{\tau_\pm} = \frac{1}{2\tau_{LR}} + \frac{1}{2\tau_{TES}} \pm \frac{1}{2}\sqrt{\left(\frac{1}{\tau_{LR}} - \frac{1}{\tau_{TES}}\right)^2 - 4\frac{R_0}{L}\frac{\mathcal{L}(2+\beta)}{\tau}}$$

$$\tau_{TES} = \frac{C}{G\left(1-\mathscr{L}\right)}, \quad \tau_{LR} = \frac{L}{R_L + (1+\beta)R_0}$$

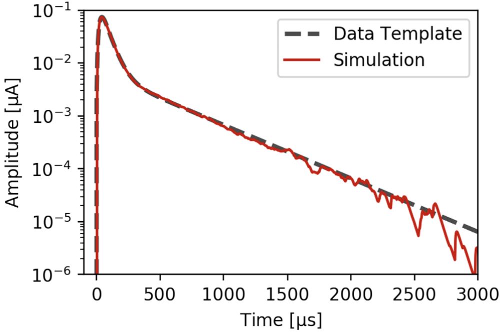


Important TES model parameters

Parameter	Description	Units
fin efficiency	Contribution of proximity effect to volumetric quantities	
L	Inductance	H
R_b	Bias resistance	Ω
R_s	Shunt resistance	Ω
R_p	Parasitic resistance	Ω
R_n	Normal-phase resistance of an individual TES	Ω
I_c	Critical current of an individual TES	A
T_c	TES critical temperature	K
T_w	Width of TES superconductivity transition curve	K
γ	Volumetric specific heat of the TES, $C = f_{sc}\mathcal{V}_{eff}\gamma T_c$	$J/(m^3K^2)$
Σ	Electron-phonon coupling constant, $K = \Sigma\mathcal{V}_{eff}$	$W/(m^3K^5)$

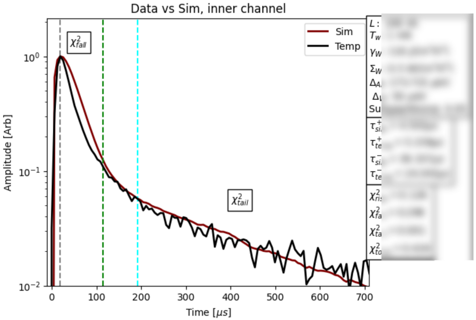
Pulse shape

- HVeV run 2



Credits: Warren Perry

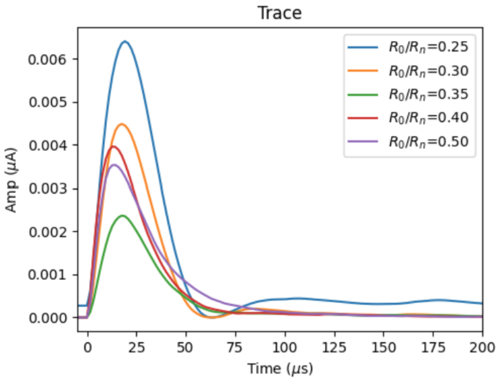
- HVeV run 4



Work in progress

Electro-Thermal Oscillations

- In HVeV and HV detectors, it was found that the baseline resolution worsens as the bias point is lowered
- ETO contributes a frequency component to the noise that degrades the resolution
- TESSim was used to check that the device in question lives in the ETO region.



Conclusion and Future Work

- TES coupled ODEs are solved numerically using BDF or Adams method from CVODE
- Sherman-Morrison formula is used to avoid matrix calculations
- TESSim is very successful in producing the correct pulse shape for single TESs and HVeV data
- Future code developments:
  - $3T$ model to include the thermal conductivity between the substrate and the heat bath
  - Thermal conductivity between neighboring TESs through the substrate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Qubit & Quark
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