

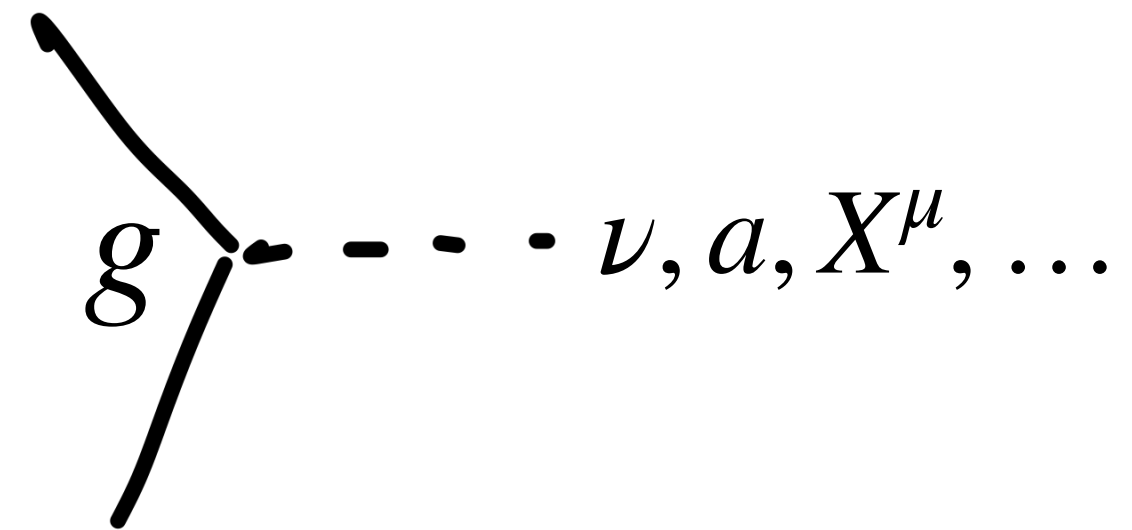
Optomechanical searches for invisible particles

Giacomo Marocco

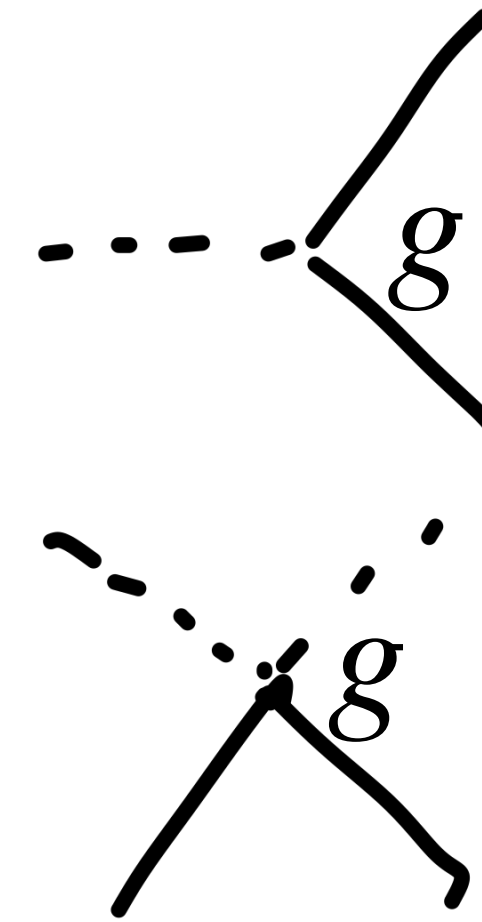
gmarocco@lbl.gov



Production



Detection



$$P(\text{production \& detection}) \sim g^4$$

- Can we infer properties of particle production without scattering/absorption?

Über die β -Strahlen des aktiven Niederschlags
des Thoriums.

Von Otto v. Baeyer, Otto Hahn und
Lise Meitner. (1911)

- Check conservation laws!
- Pauli (1931): “a desperate remedy to save ... the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles”
- Neutrino direct detection (1956)

$$\sigma_{\nu e} \sim G_F^2 E^2$$
$$\approx 10^{-43} \text{ cm}^2$$

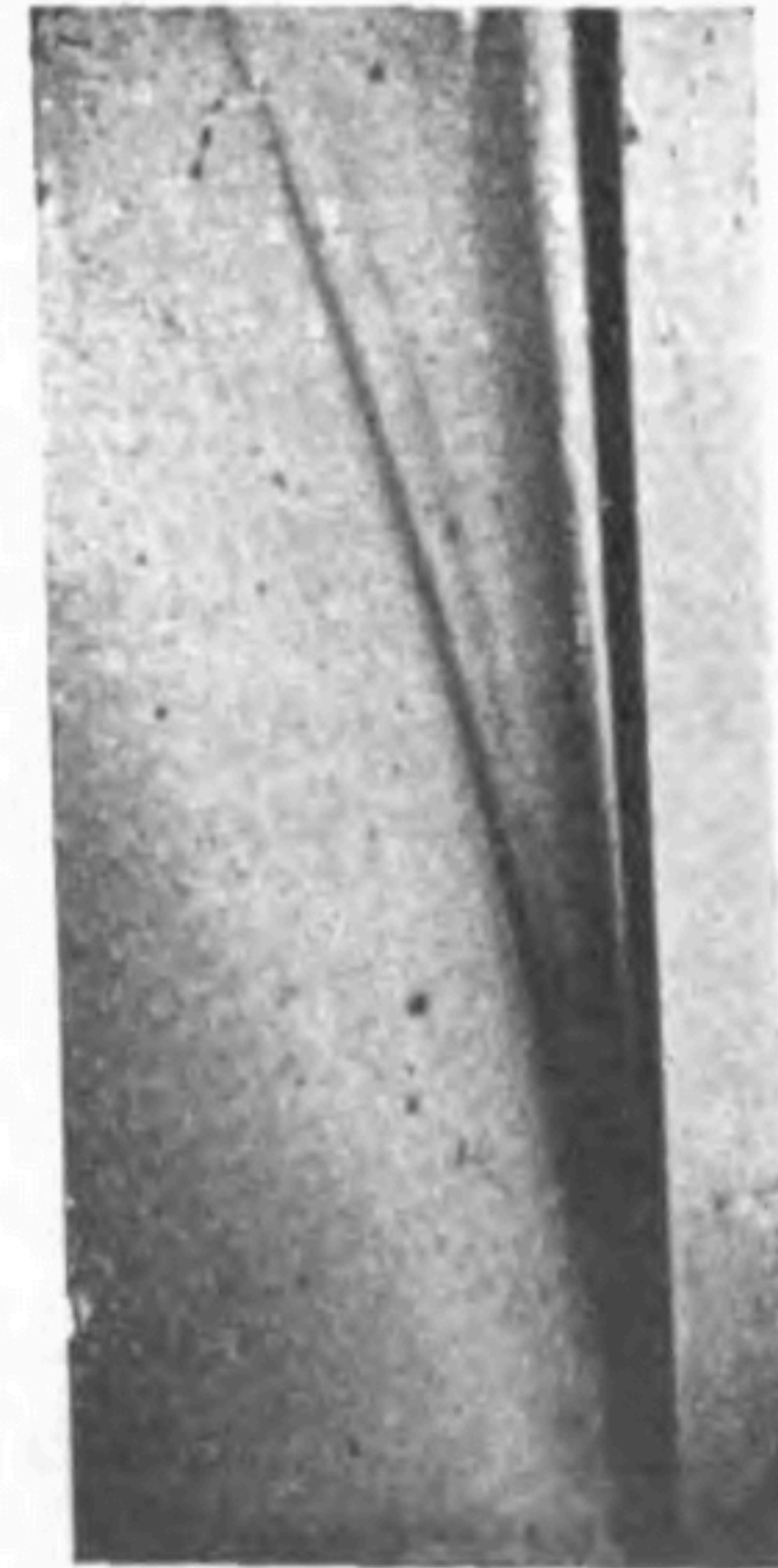
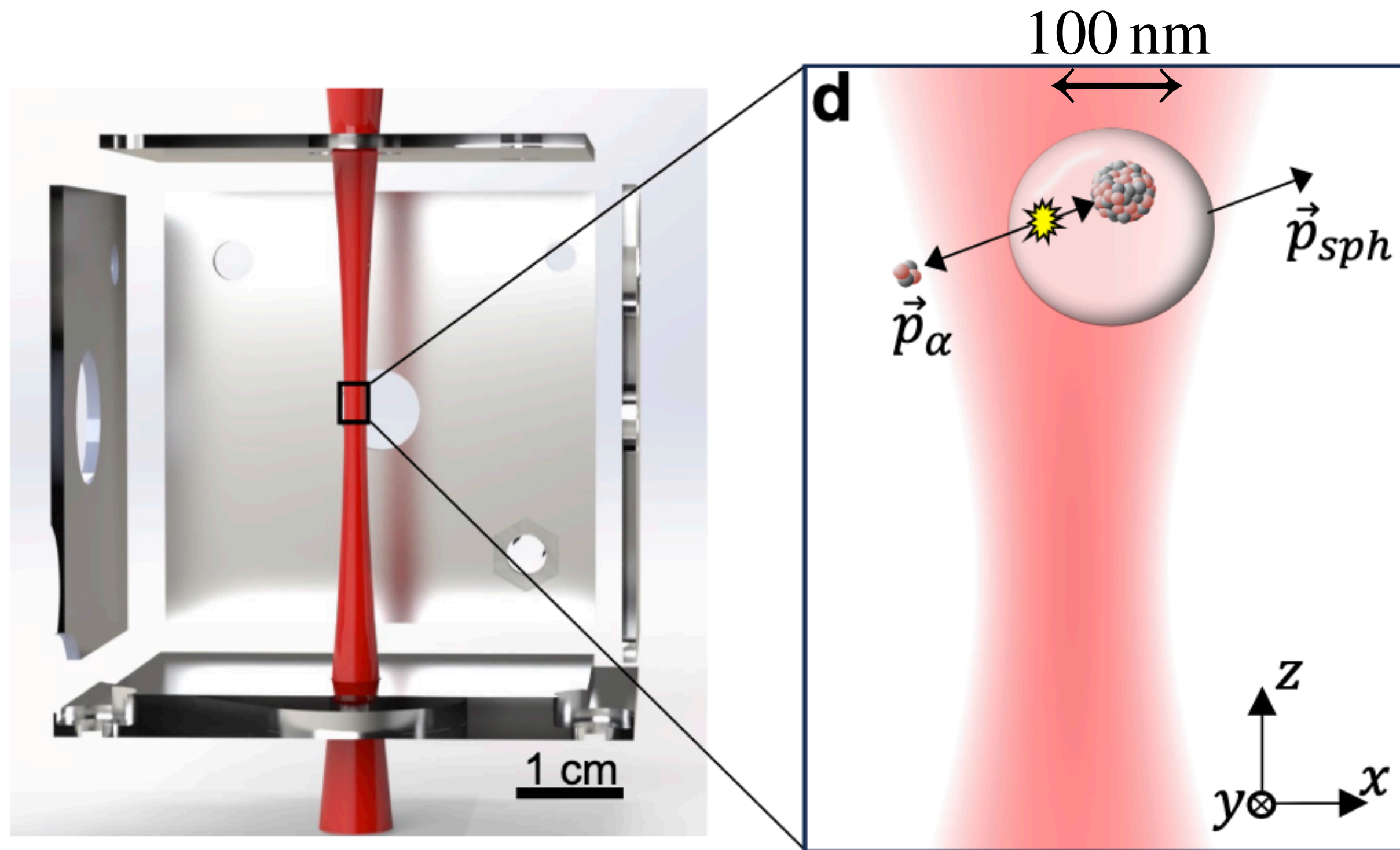


Fig. 2.

Quantum invisible particle sensor (QuIPS)



- Detect weakly coupled particles emitted in nuclear decays
- How can we minimize the quantum noise in missing momentum measurements?

$$\mathbf{p}_\alpha = -\mathbf{p}_{sph}$$

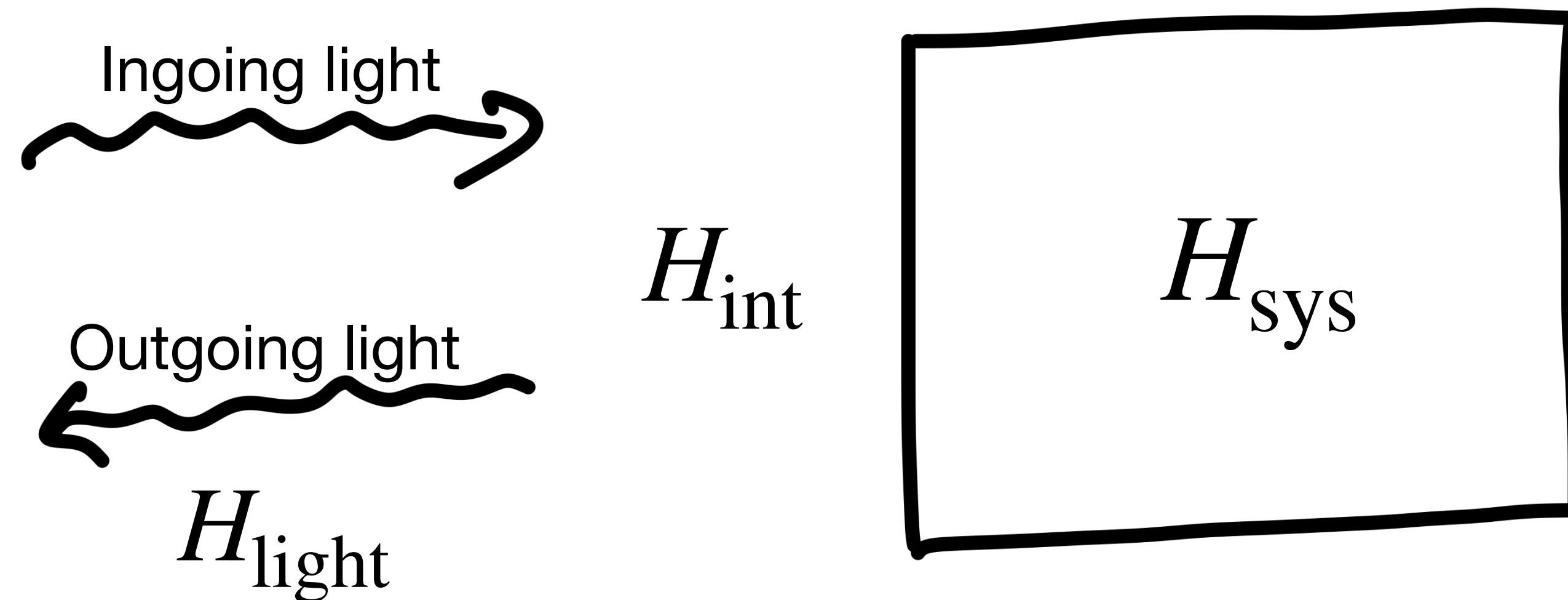
LBL: Sorensen, Garcia, Denes, Rofors, R. Carney, **GM**, Beckey, Knepper, Kodroff, D. Carney
Yale: Moore, Tseng, Penny

Plan

1. Quantum impulse measurements
2. Testing SM β decay
3. Testing SM γ decay
4. Quantum enhanced measurements

Quantum sensing

- Weak, continuous, measurement may be viewed as a scattering problem:
- How does the outgoing light depend on the ingoing light and the system/interaction dynamics?



Optomechanics

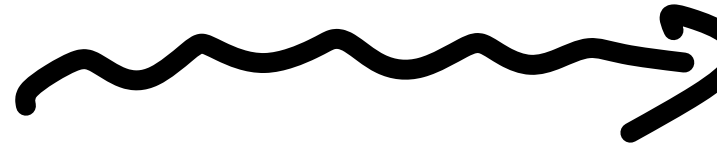
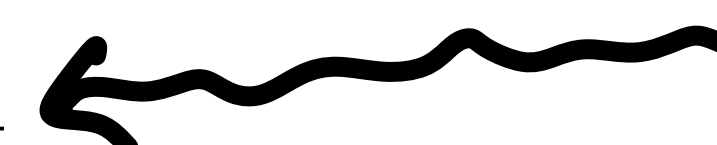
- Dielectric coupling:

$$H_{\text{int}} = \int_{\text{sph}} d^3\mathbf{x} \mathbf{P}(\mathbf{x}) \cdot \mathbf{E}(\mathbf{x})$$

- Strong background laser field – linearized Hamiltonian:

$$H_{\text{int}} = gxX$$

$$H_{\text{int}} = gxX$$

$X_{\text{in}}, Y_{\text{in}}$ 
 $X_{\text{out}}, Y_{\text{out}}$ 

$$H_{\text{sys}} = \frac{p^2}{2m} + \frac{m\omega_m^2 x^2}{2}$$

$$H_{\text{light}} = \omega_0 (X^2 + Y^2)$$

Amplitude quadrature

$$X = \frac{1}{2}(a + a^\dagger)$$

Phase quadrature

$$Y = \frac{i}{2}(a^\dagger - a)$$

Force estimation

- Force estimated from output phase: $F_E = \chi_{YF}^{-1} Y_{\text{out}}$
- Uncertainty in estimator given by power spectral density PSD S_{FF} :

$$2\pi S_{FF}(\nu) \delta(\nu - \nu') = \langle F_E(\nu) F_E^\dagger(\nu') \rangle_{\text{sym}}$$

- PSD has **back action** and **shot noise** contributions:

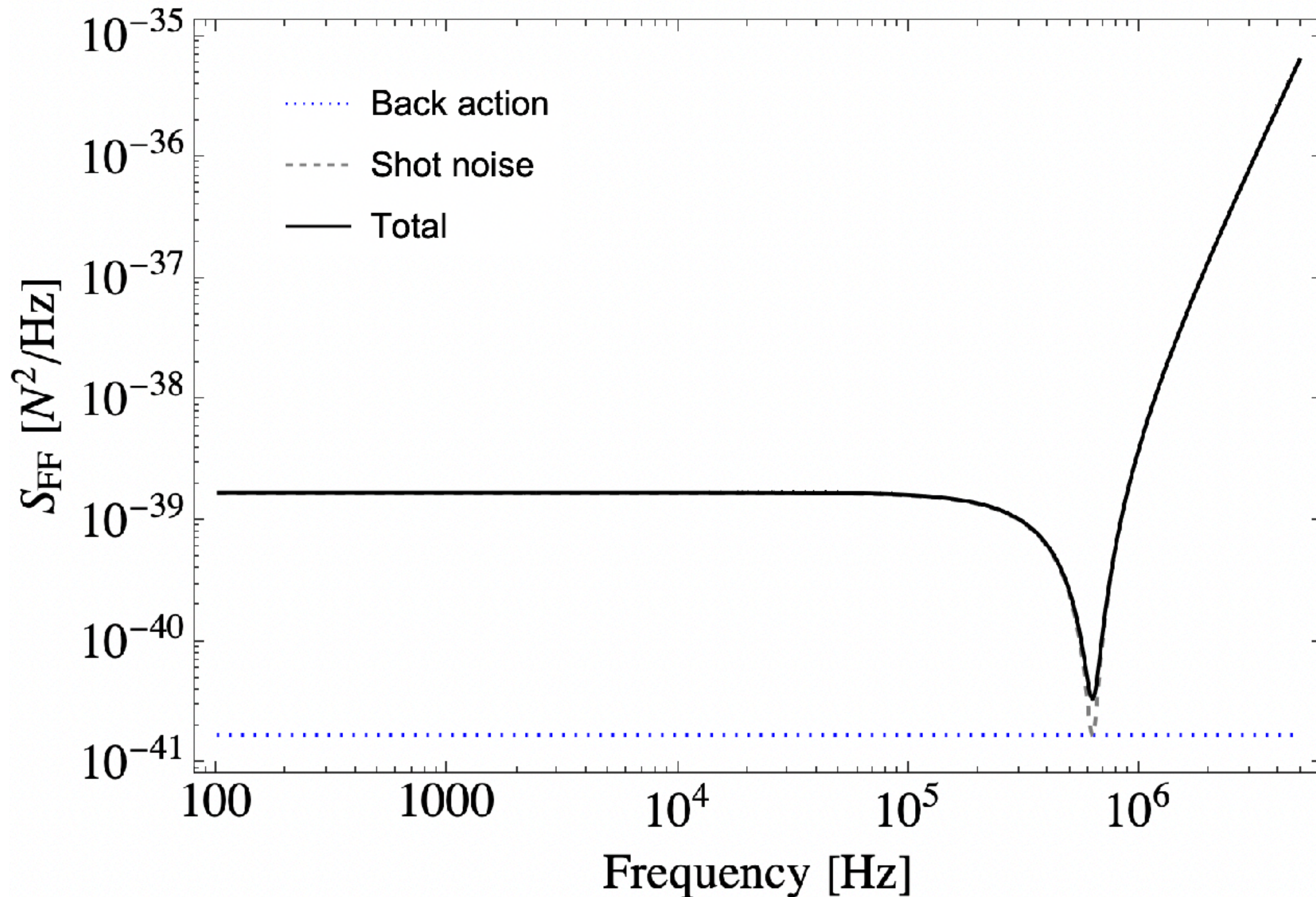
$$S_{FF}(\nu) = \frac{2P\pi}{\lambda_0} + \frac{m^2 \lambda_0 (\gamma^2 \nu^2 + (\nu^2 - \omega_m^2)^2)}{4\pi P}$$

Scattered power P

Laser wavelength λ_0

Mechanical damping γ

Uncertainty



Signal force

$$F(t) = \Delta p \delta(t - t_0)$$

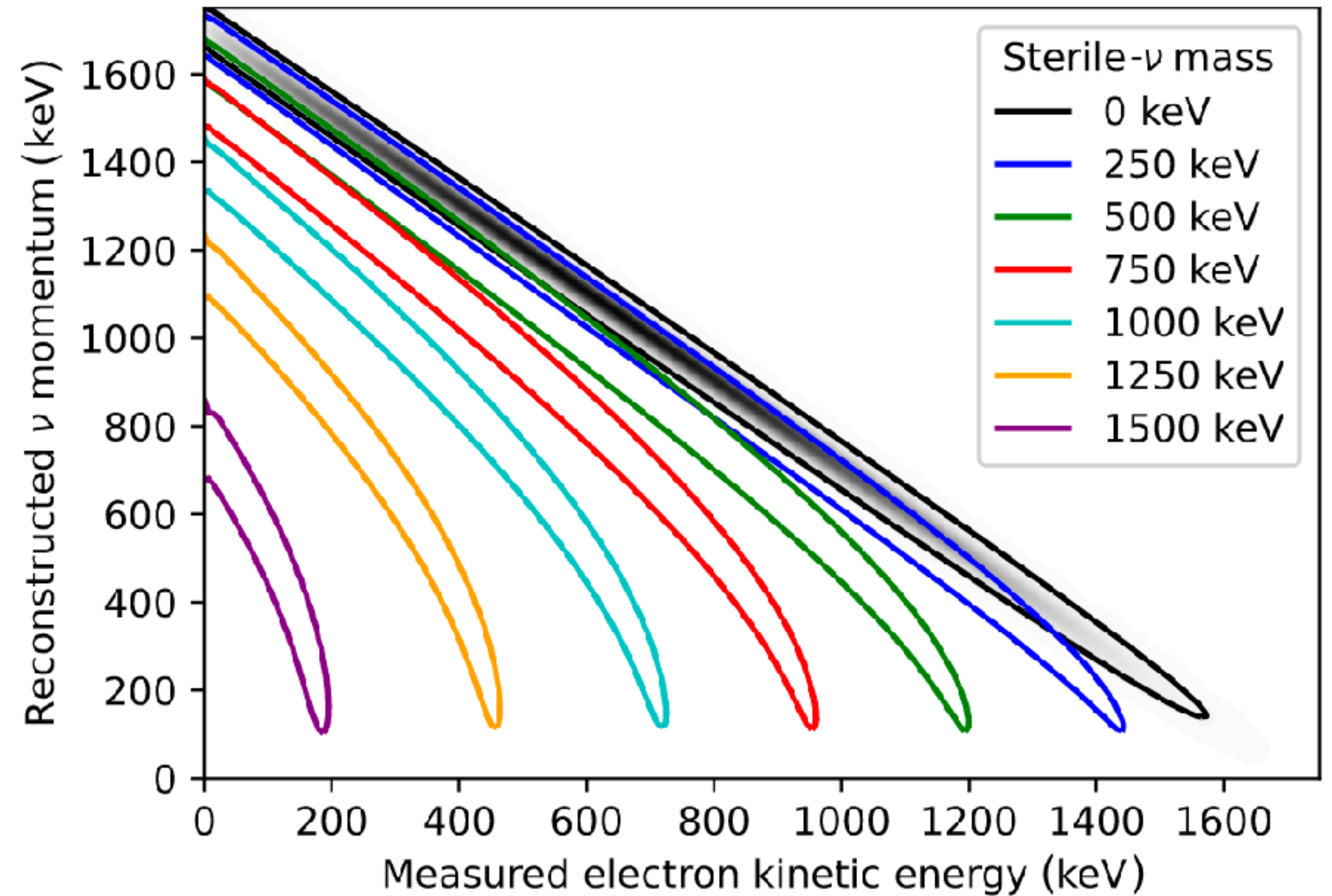
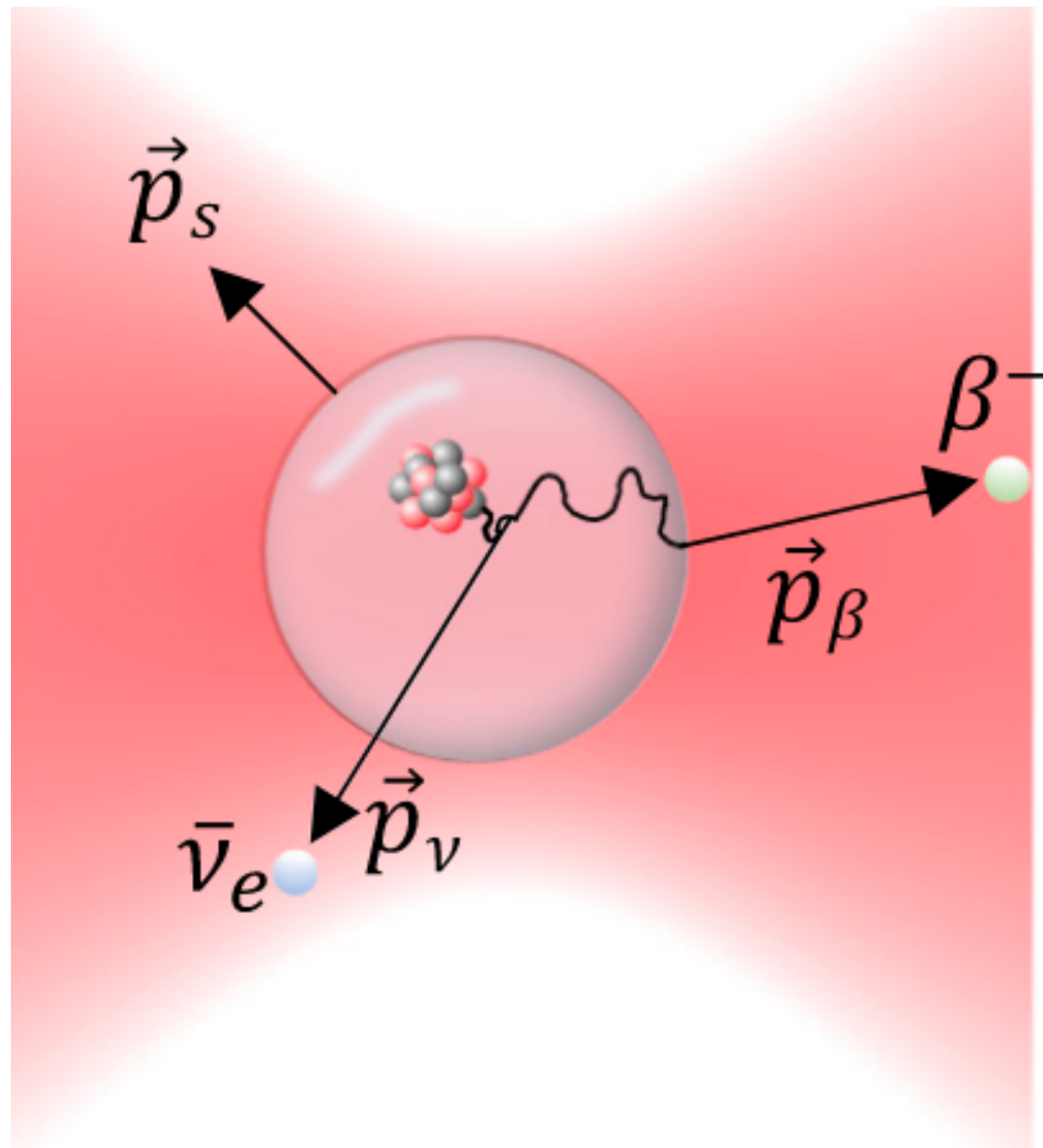
Impulse threshold:

$$\begin{aligned} \Delta p_{\text{th}} &= \left(\int \frac{d\nu}{S_{FF}(\nu)} \right)^{-1/2} \\ &\approx \sqrt{m\omega_m} \\ &\approx 15\text{keV} \left(\frac{m}{\text{fg}} \frac{\omega_m}{10\text{kHz}} \right)^{1/2} \end{aligned}$$

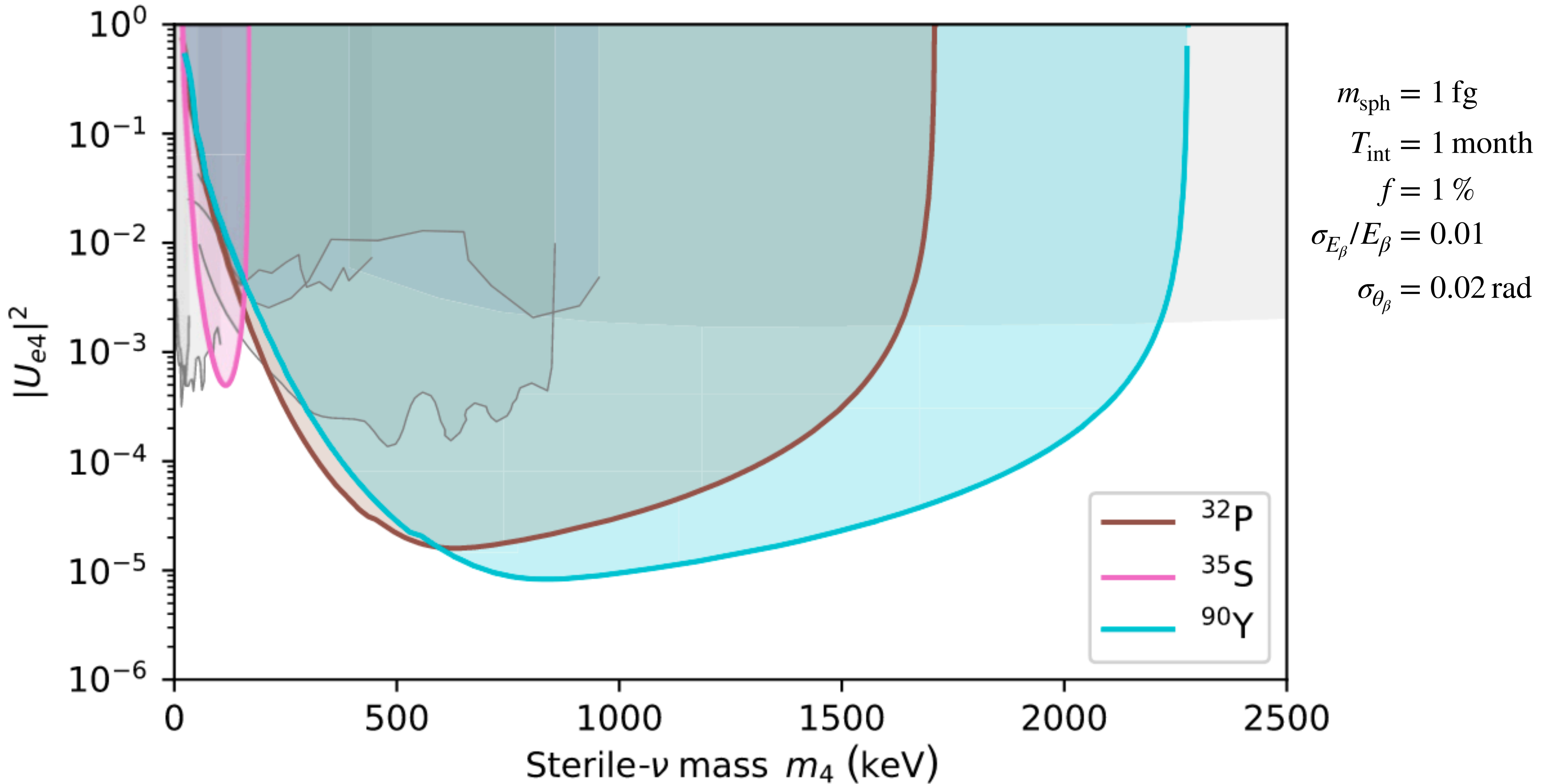
Physics goals

Sterile neutrinos

- Radioisotopes undergoing beta decay can emit heavy sterile neutrinos



$$E_\nu \approx Q - E_\beta$$

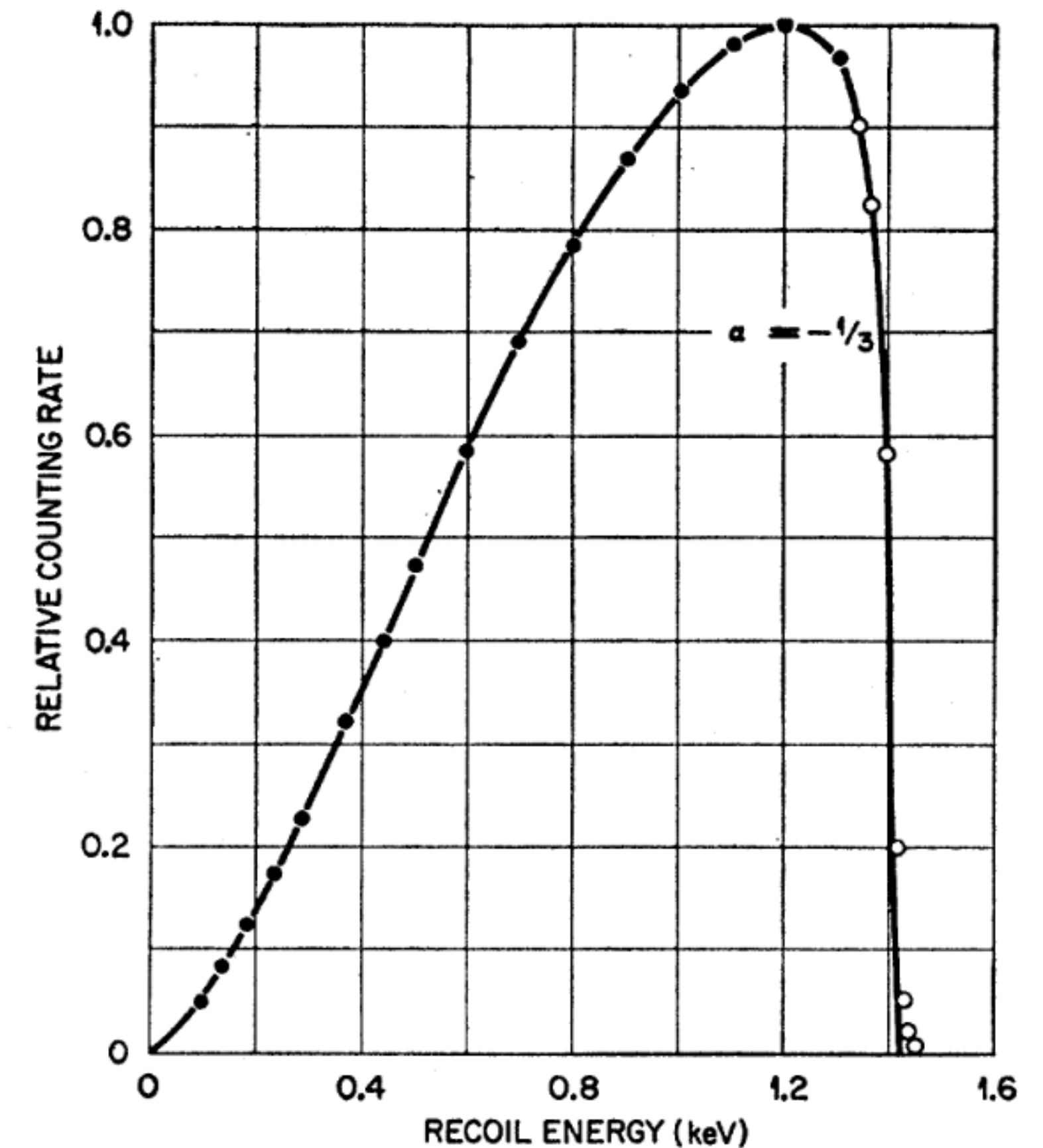


Precision electroweak physics

- Beta decay differential distribution determined by electroweak sector of Standard Model (+ nuclear corrections)

$$\frac{d^5\Gamma}{d^2\Omega_\nu d^2\Omega_\beta dE_\beta}$$

- What can we learn by measuring more than E_β ?



[Johnson et al Phys Rev (1963)]

Angular correlations

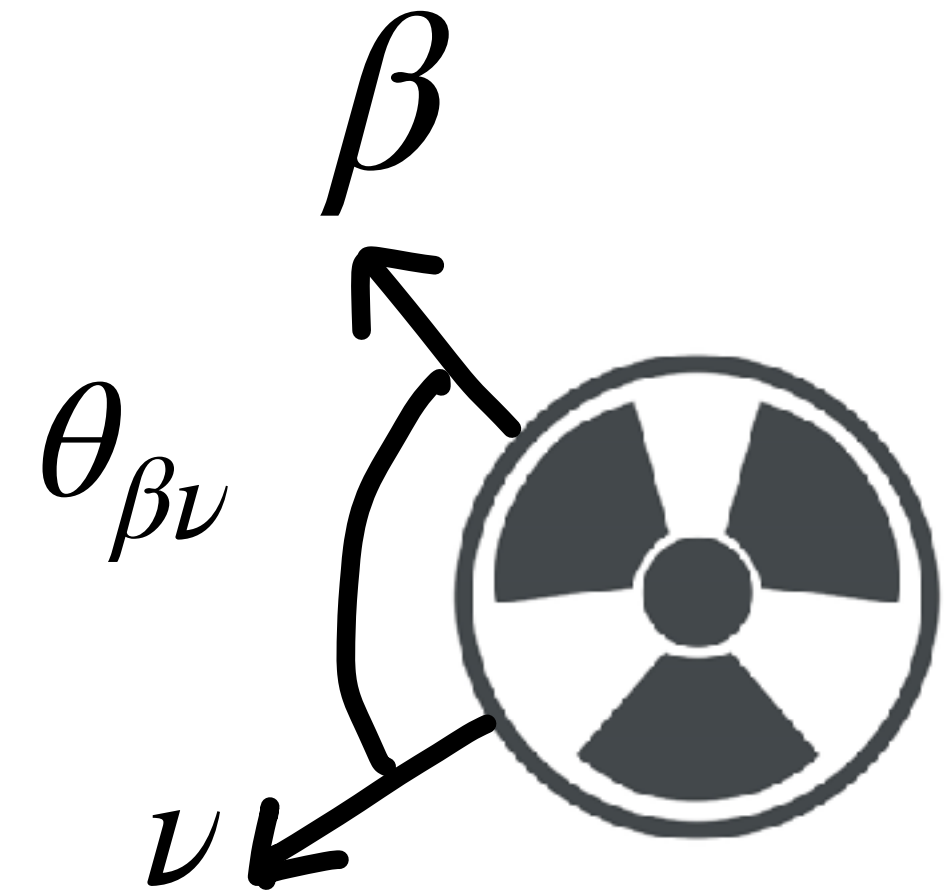
- For unpolarised nuclei,

$$\frac{d^2\Gamma}{dE_\beta d\cos\theta_{\beta\nu}} = F_0(Z, E_\beta, Q)\xi \left[1 + a_{\beta\nu} \frac{\mathbf{p}_\beta \cdot \mathbf{p}_\nu}{E_\beta E_\nu} + b \frac{m_e}{E_e} \right]$$

- Simple observable: correlation of $\beta - \nu$ angle

$$\frac{d^2\Gamma}{dE_\beta d\cos\theta_{\beta\nu}} = A \cos\theta_{\beta\nu} + B$$

- In SM for Gamow-Teller decays, $a_{\beta\nu} = \frac{A}{B} \cdot \frac{p_\beta}{E_\beta} = -\frac{1}{3}$



New interactions?

- The beta spectrum may be modified by additional tensor interactions

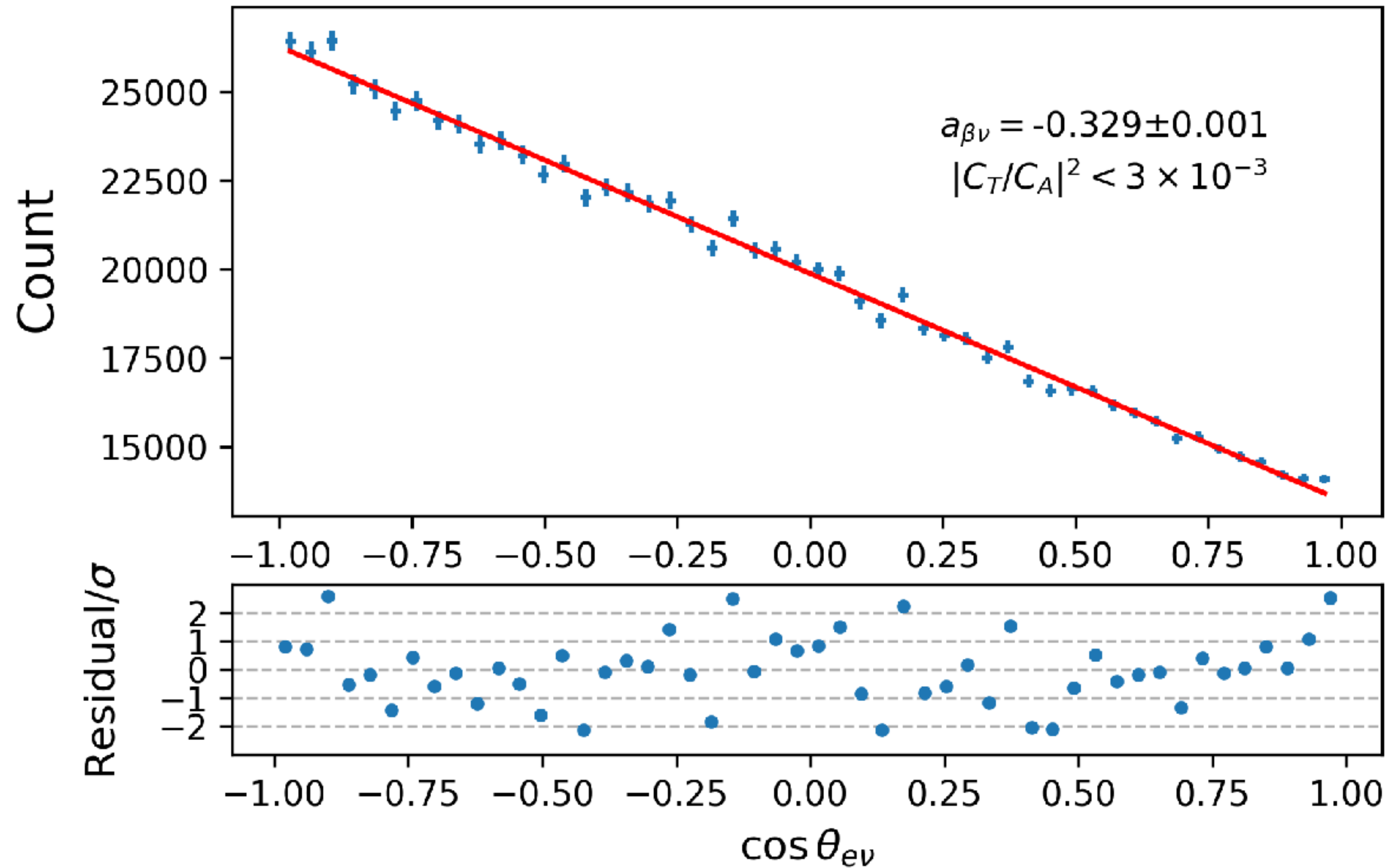
$$a_{\beta\nu} = -\frac{1}{3} \left(1 - \left| \frac{C_T}{C_A} \right|^2 \right) \quad \mathcal{L} \supset C_T G_F \bar{n} \sigma^{\mu\nu} p \bar{\nu} \sigma_{\mu\nu} e$$

- β -decay measurements constrain $\left| \frac{C_T}{C_A} \right|^2 \lesssim 7.6 \times 10^{-3}$ – neutron decay

$$\text{tension } \left| \frac{C_T}{C_A} \right|^2 \approx 4.7 \times 10^{-3}?$$

- Complementary to $p p \rightarrow e^+ e^- + X$ at LHC

Sensitivity projection

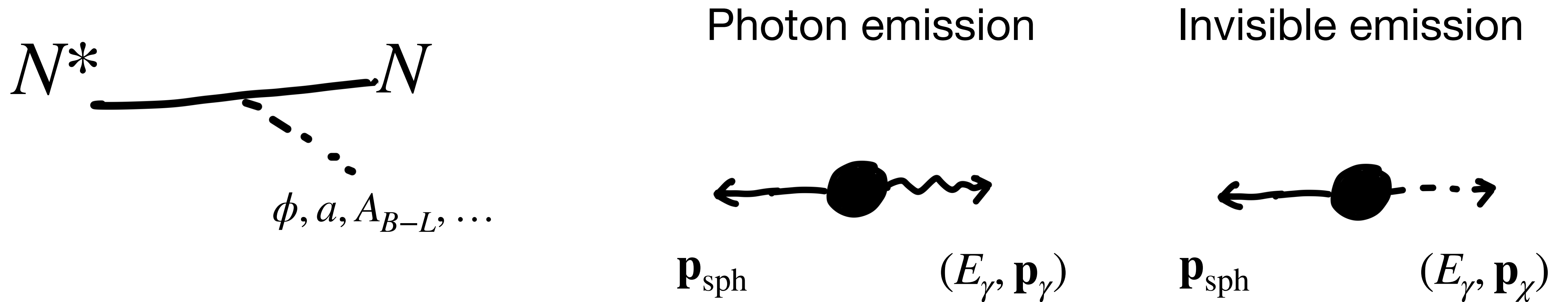


^{66}Ni
 $m_{\text{sph}} = 1 \text{ fg}$
 $N_{\text{decays}} = 10^7$
 $\sigma_{E_\beta}/E_\beta = 0.01$
 $\sigma_{\theta_\beta} = 0.02 \text{ rad}$

- Sensitive to $\left| \frac{C_T}{C_A} \right|^2 \approx 5 \times 10^{-3}$

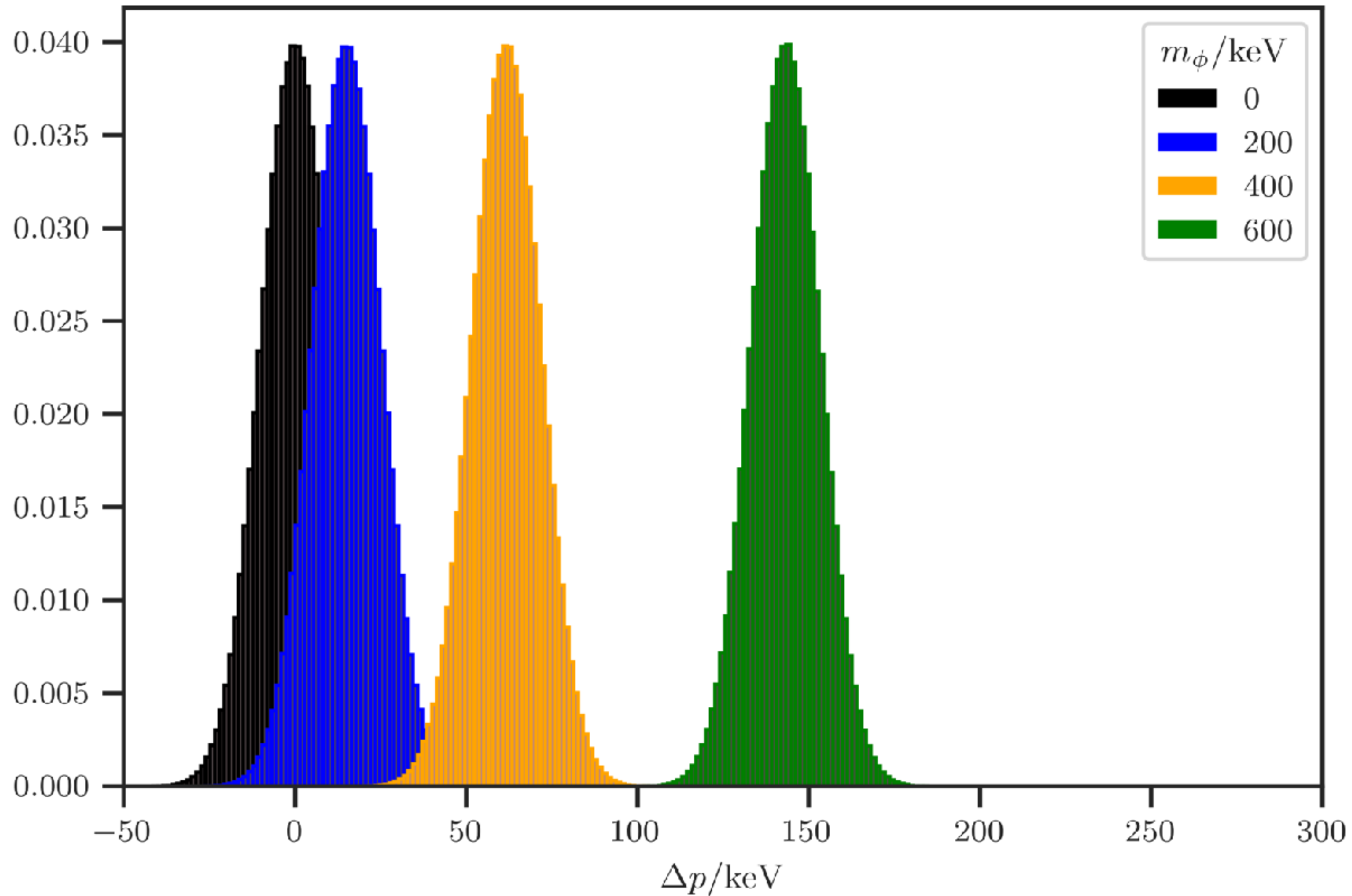
Gamma decays

- Excited nuclear states can de-excite by emitting a photon
- Or, may emit a **massive**, invisible state



$$\Delta p = \mathbf{p}_\gamma - \mathbf{p}_\chi = E_\gamma(1 - v_\chi)$$

Missing momentum

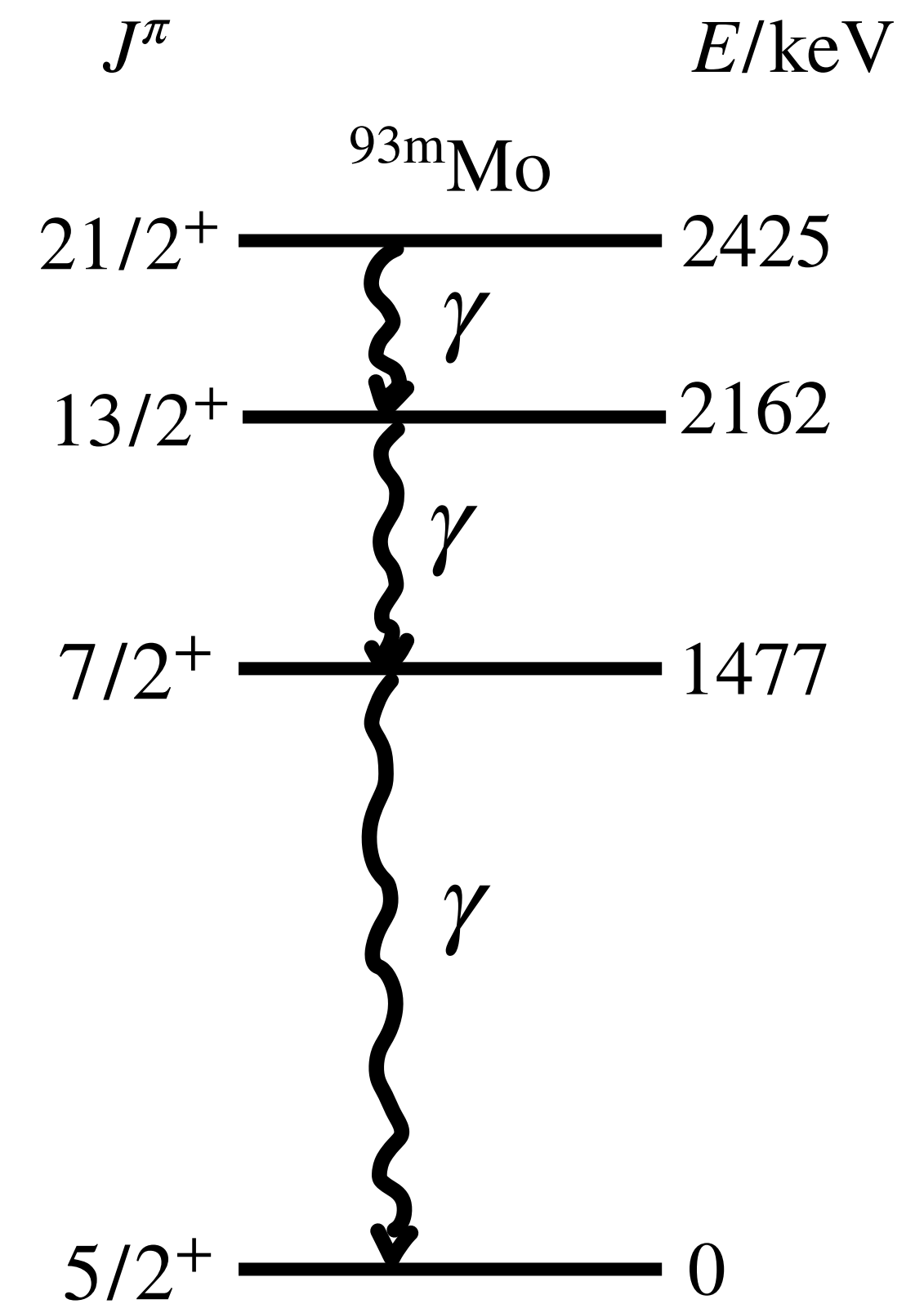


Nuclear batteries

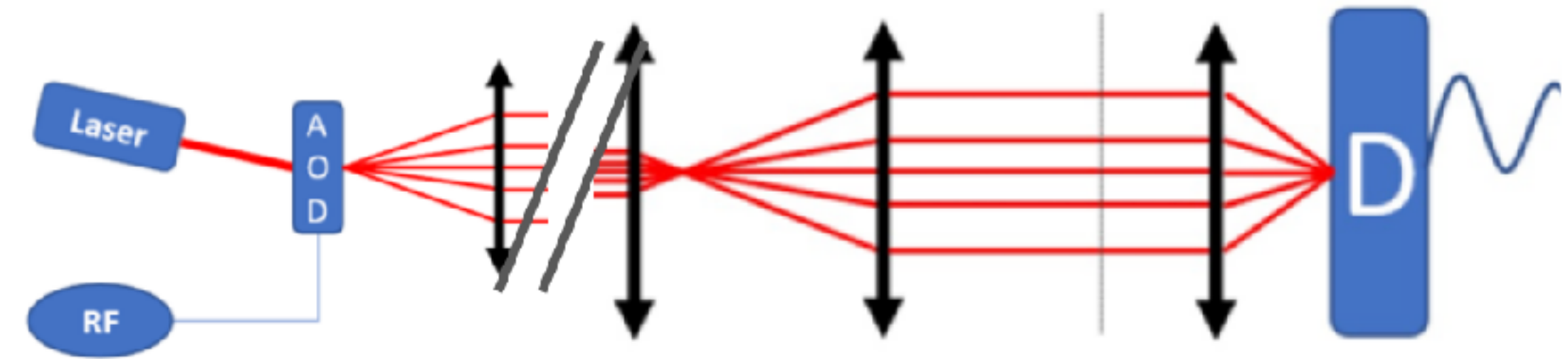
- Metastable excited nuclear isomers: long lifetimes (seconds - years)
- Isomer transition initiates prompt gamma cascade
- Aim to constrain branching ratio to invisible states, e.g.

$$\mathcal{L}_{\text{int}} = g_{\phi} \phi \bar{N} N$$

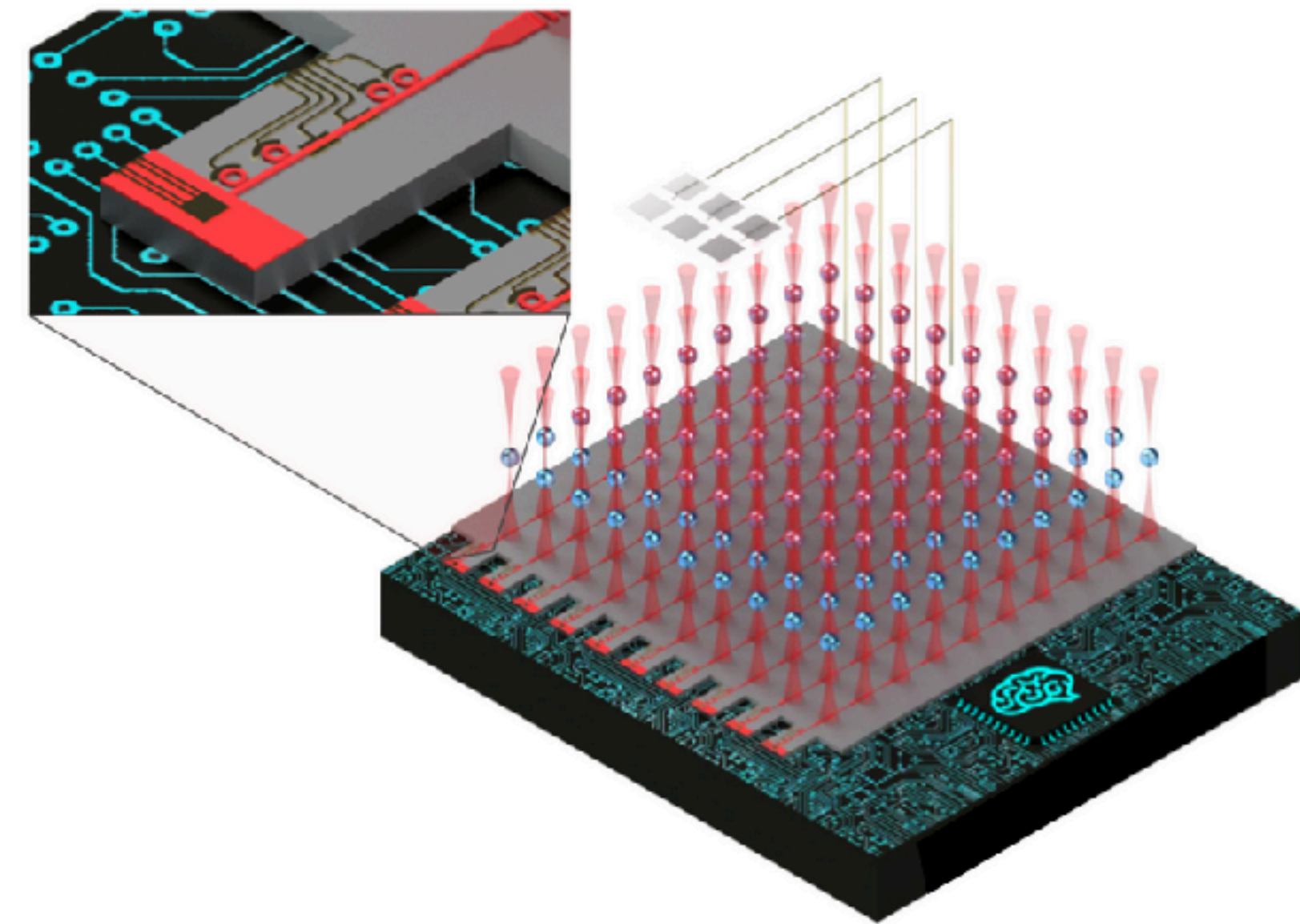
$$\text{BR}_{\phi}(E\ell) \approx \frac{1}{2} \left(\frac{g_{\phi}}{e} \right)^2 v_{\phi}^{2\ell+1}, \quad l > 0$$



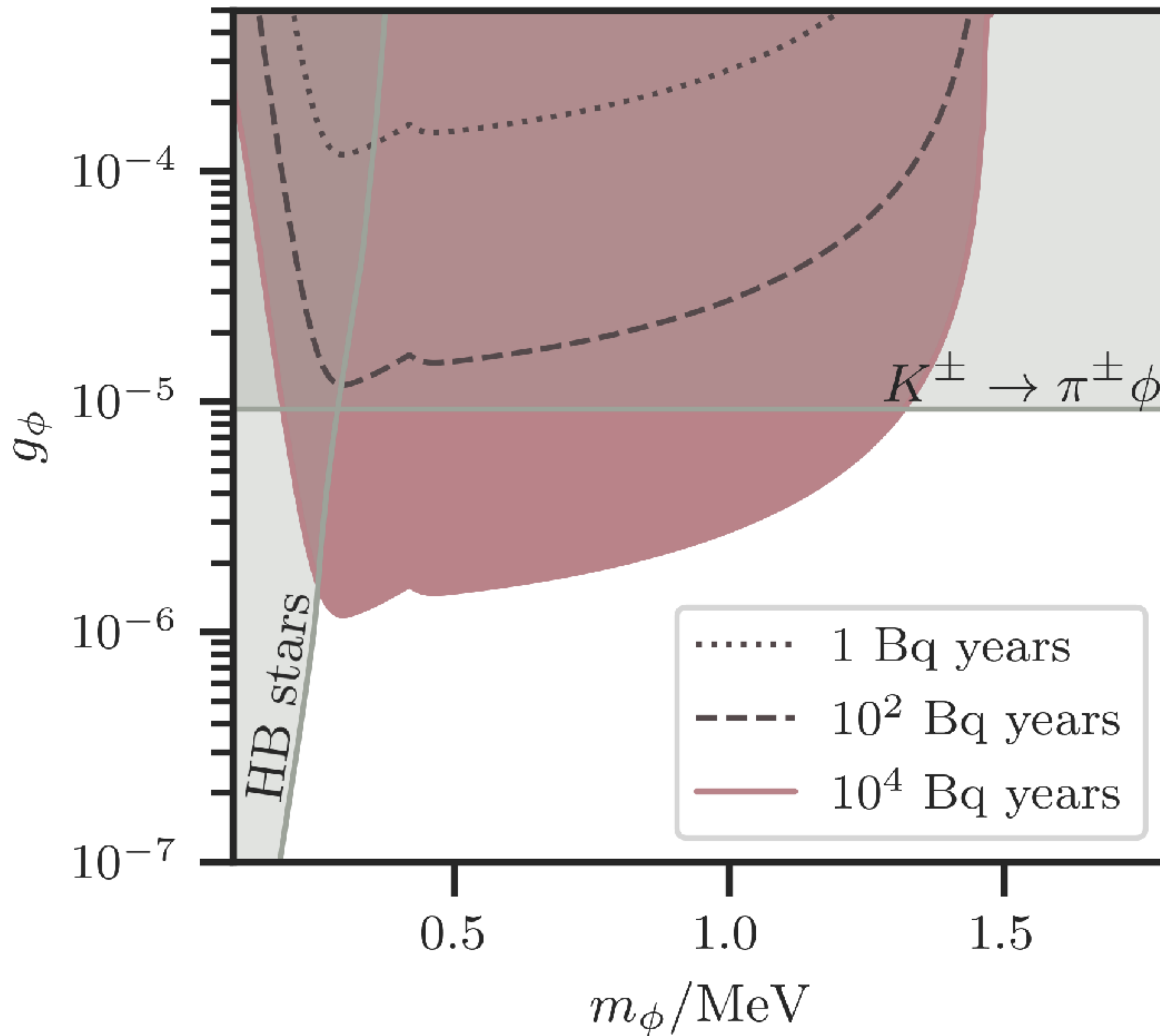
Sphere arrays



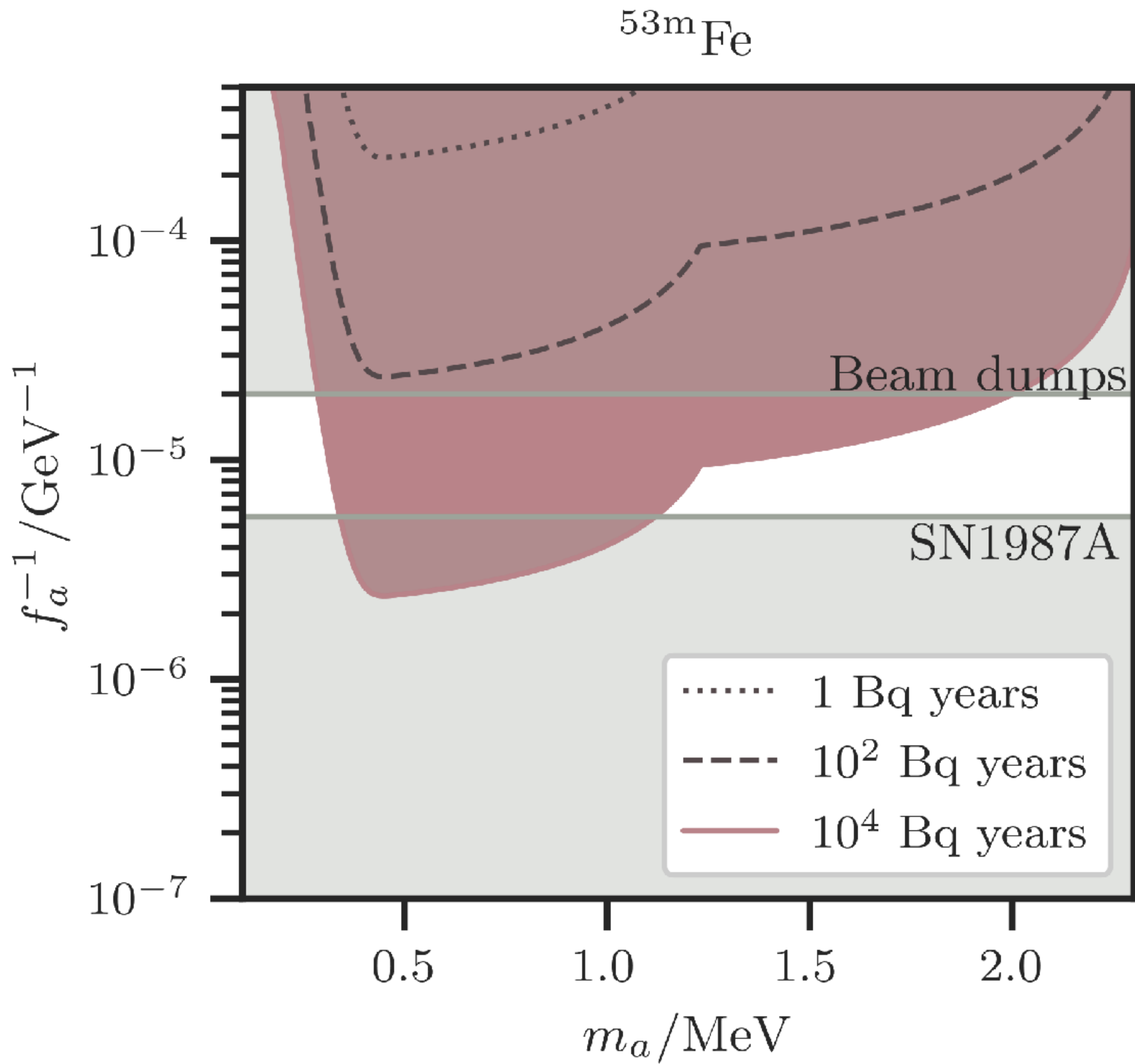
- Using beam splitters, can currently levitate + readout 100s of spheres
- Aim to scale this up to 100x100 sphere array



$^{93\text{m}}\text{Mo}$



$m_{\text{sph}} = 1 \text{ fg}$
 $\sigma_{E_\gamma}/E_\gamma = 0.01$
 $\sigma_{\theta_\gamma} = 0.01 \text{ rad}$



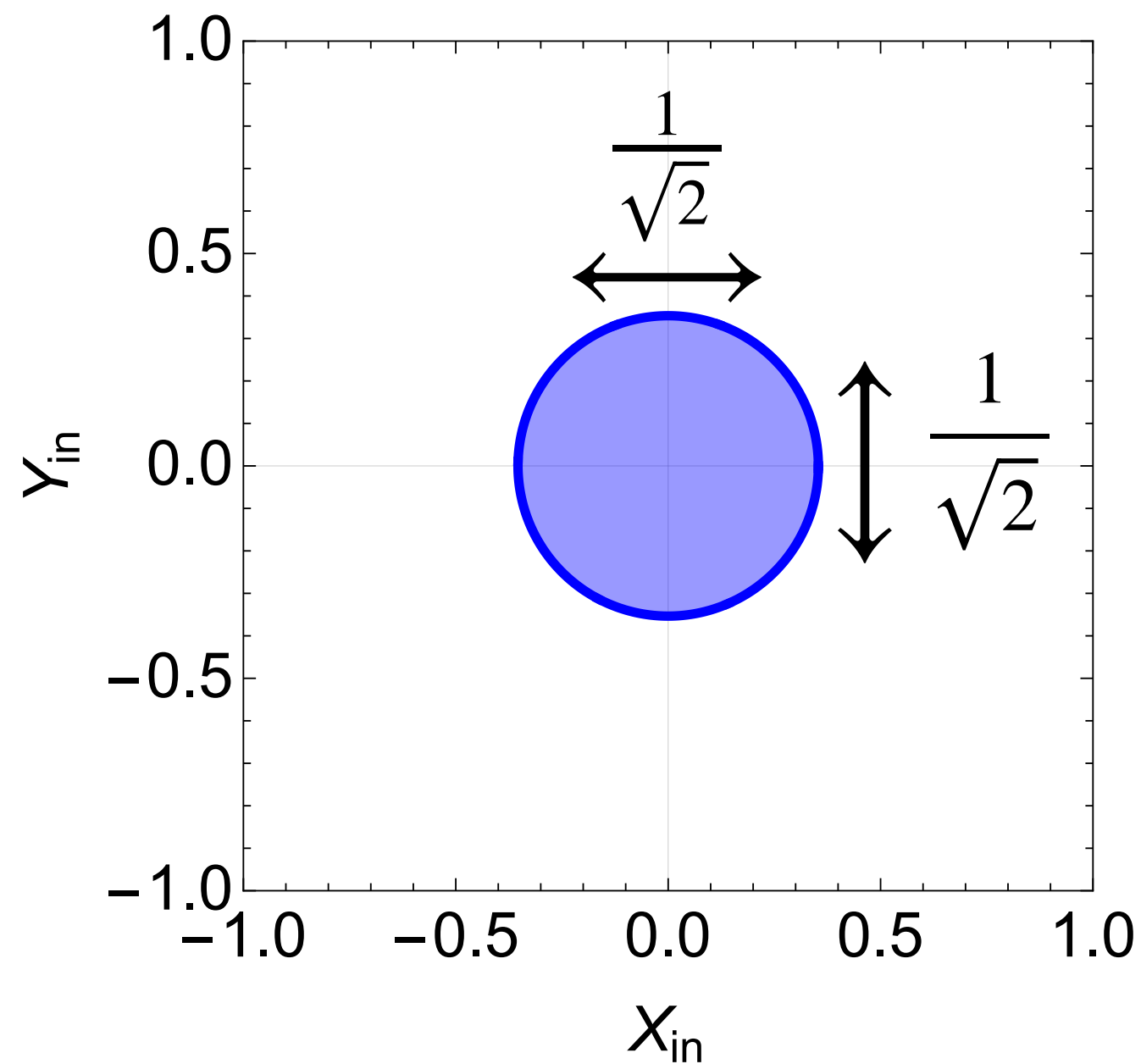
$$\mathcal{L}_{\text{int}} = \frac{1}{f_a} \partial_\mu a \bar{N} \gamma^\mu \gamma^5 N$$

Summary of BSM searches

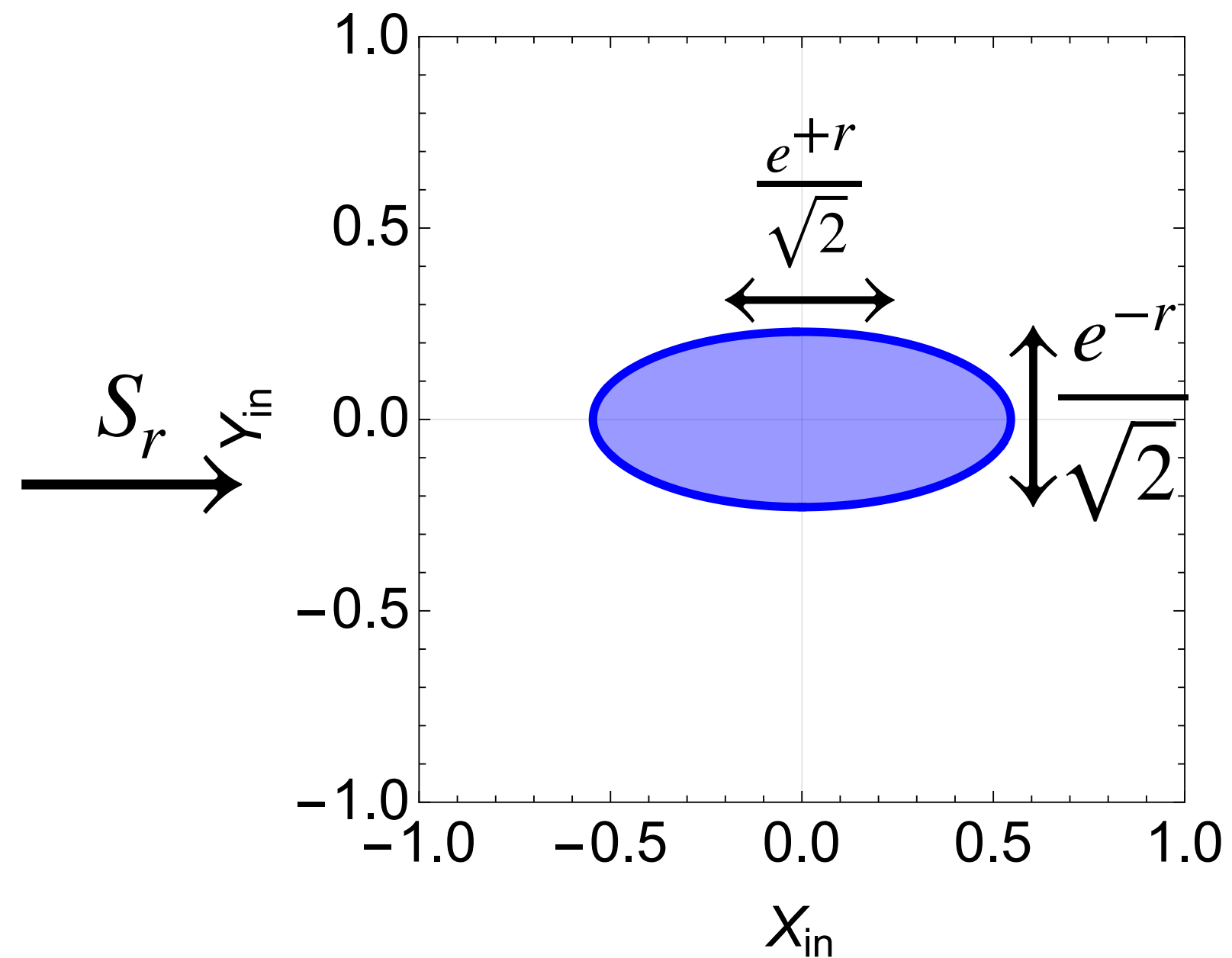
- Reconstruction of \mathbf{p}_ν by measuring \mathbf{p}_{sph} and \mathbf{p}_β allows:
 1. Searches for heavy neutrino ν_4 mixing with ν_e ;
 2. Searches for modifications to $a_{\beta\nu}$ parameter of β -spectrum.
- Infer \mathbf{p}_γ to search for heavy states emitted in γ -decay

Squeezing

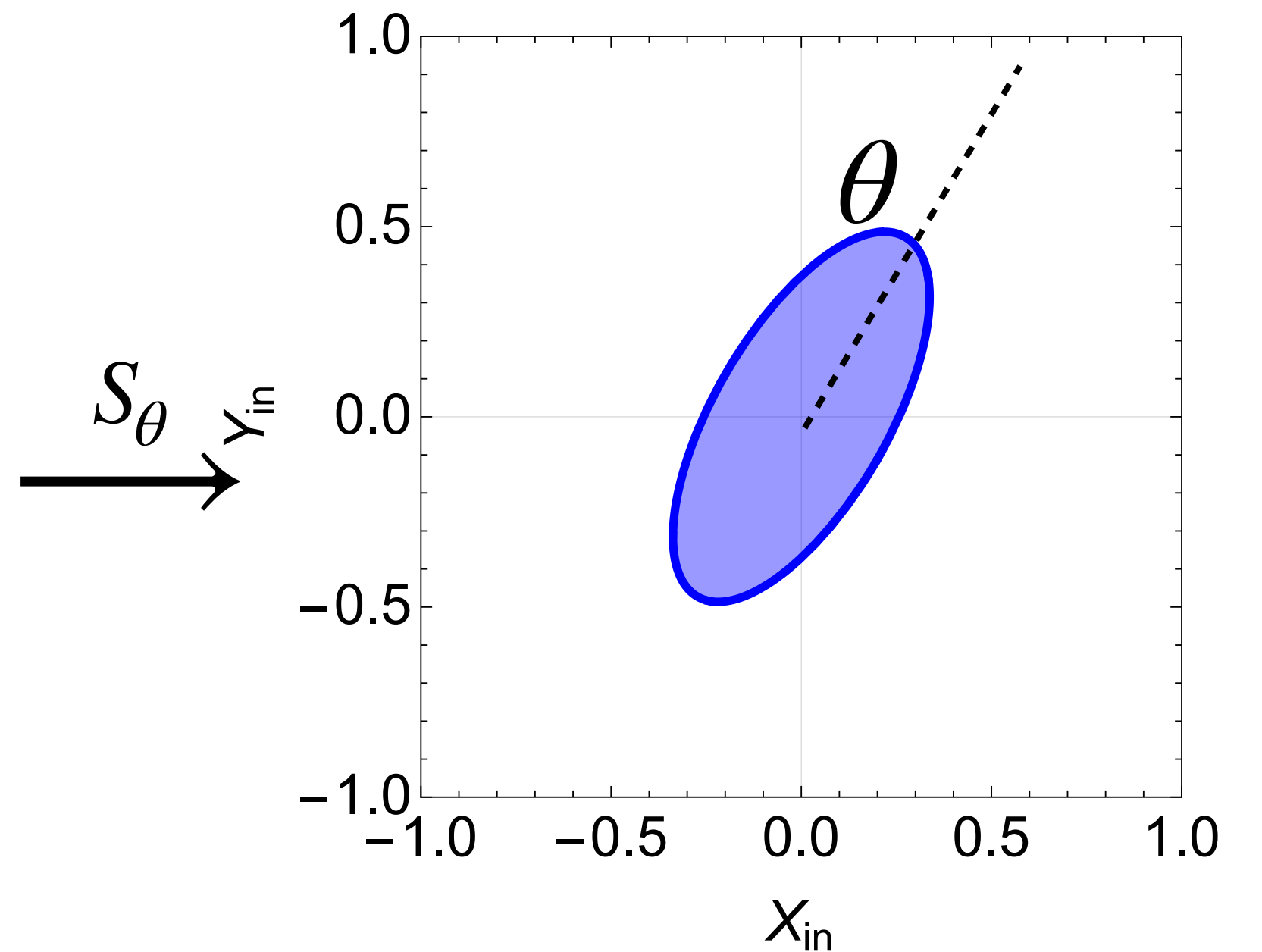
- Transform incoming light: $\begin{pmatrix} a_{\text{in}} \\ a_{\text{in}}^\dagger \end{pmatrix} \rightarrow U \begin{pmatrix} a_{\text{in}} \\ a_{\text{in}}^\dagger \end{pmatrix} U^\dagger$, with $U = S_\theta \overset{\text{SU}(1,1)}{\mathbb{M}} S_r$.



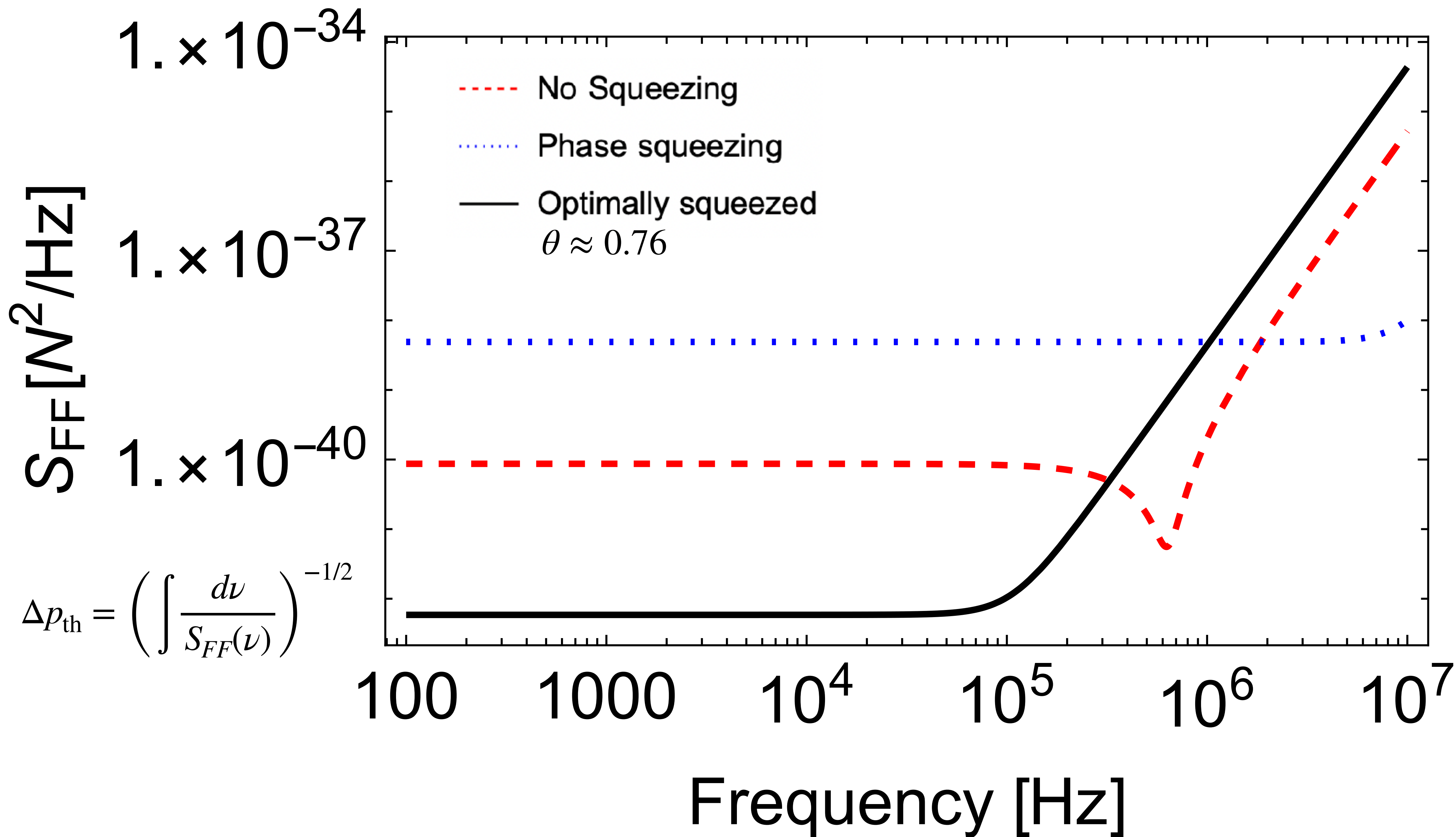
Coherent state



Phase squeezed



General squeezed

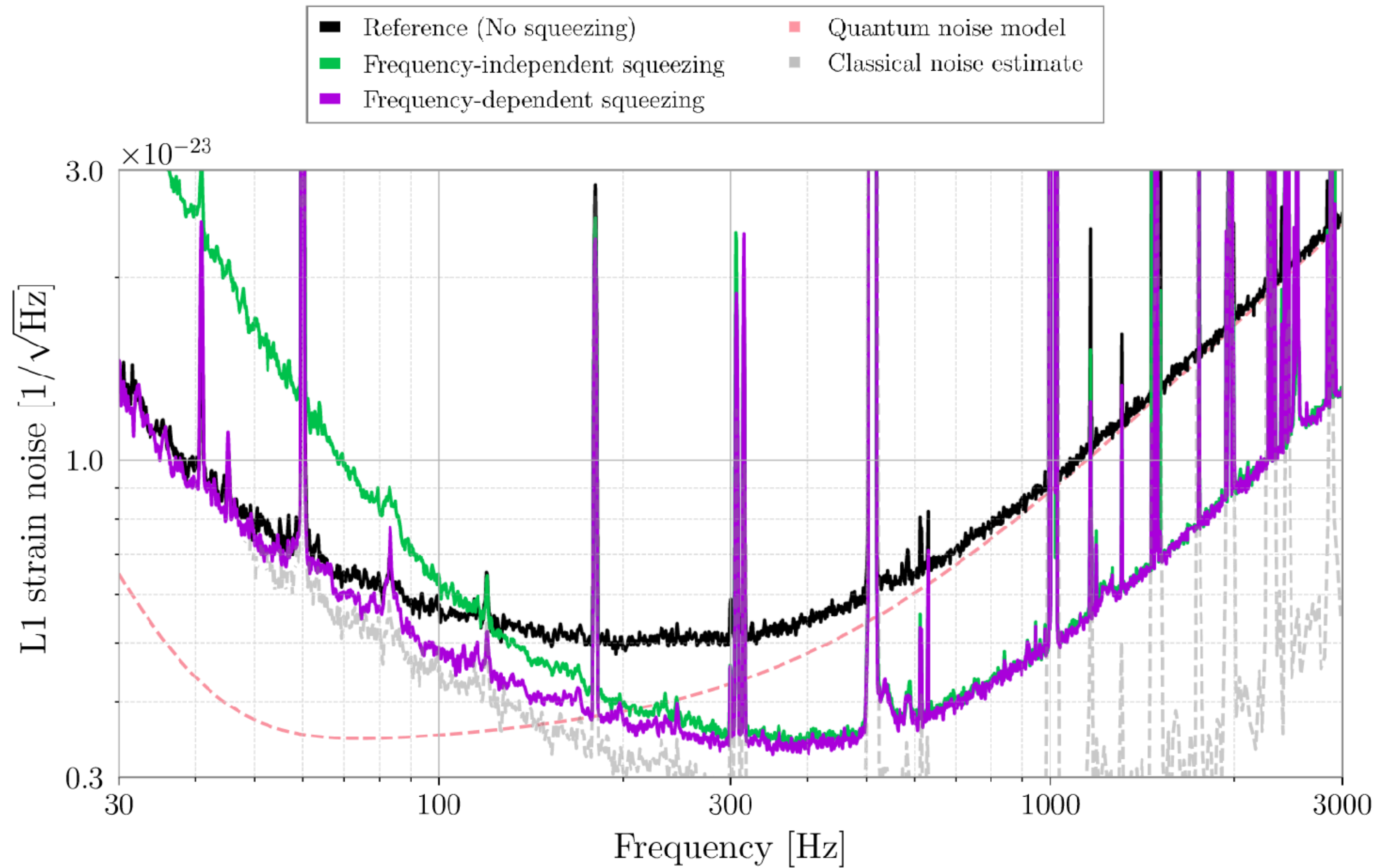


Frequency-dependent squeezing

- Squeezing angle may be frequency-dependent $S(r, \theta(\nu))$
- Cavities introduce phase shifts $\alpha_i(\nu) = \arctan\left(\frac{\nu - \omega_i}{\omega_i} + \nu/\gamma_i\right)$
- Use a sequence of cavities to tune $\theta(\nu)$ to minimise noise:

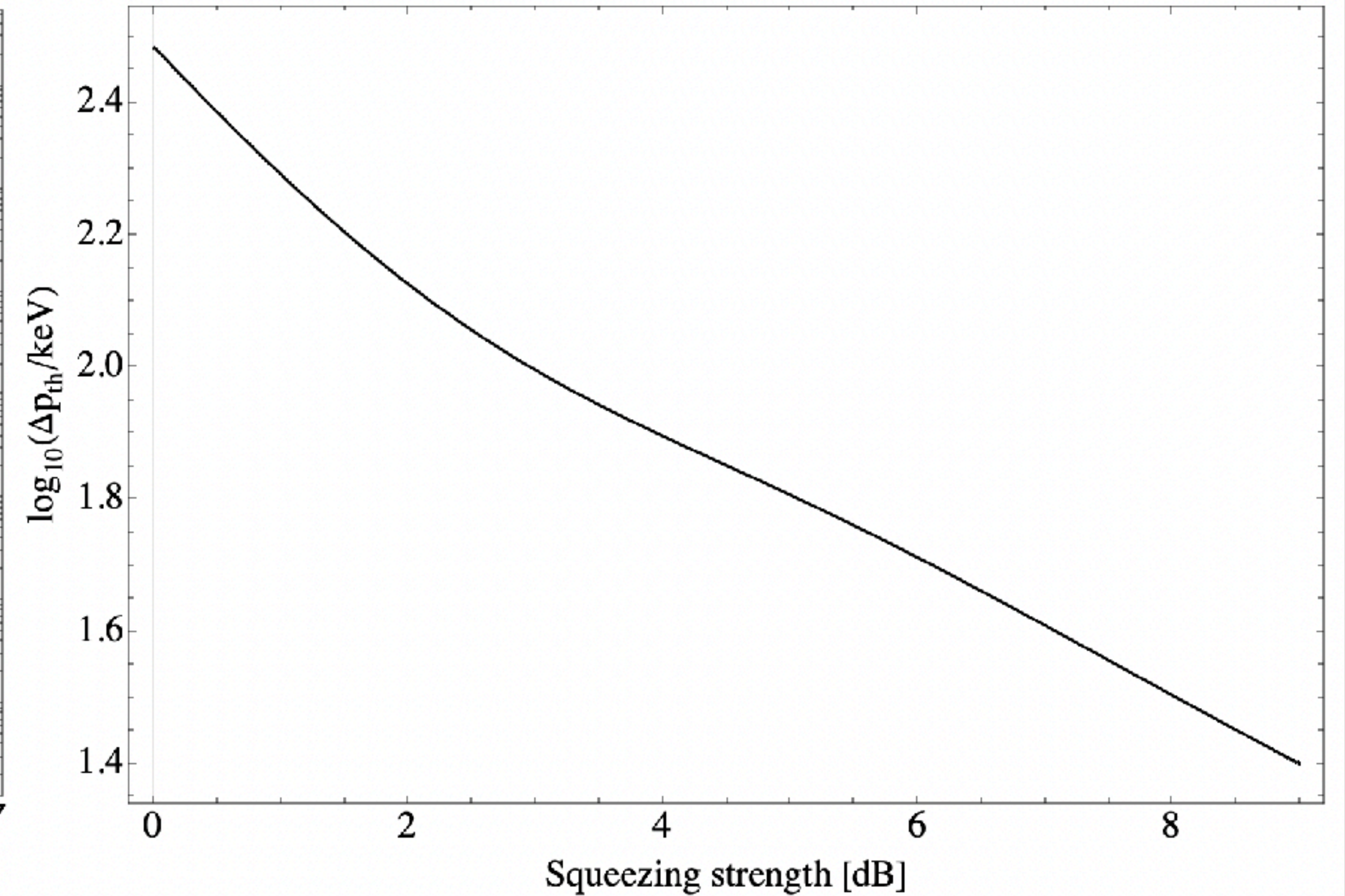
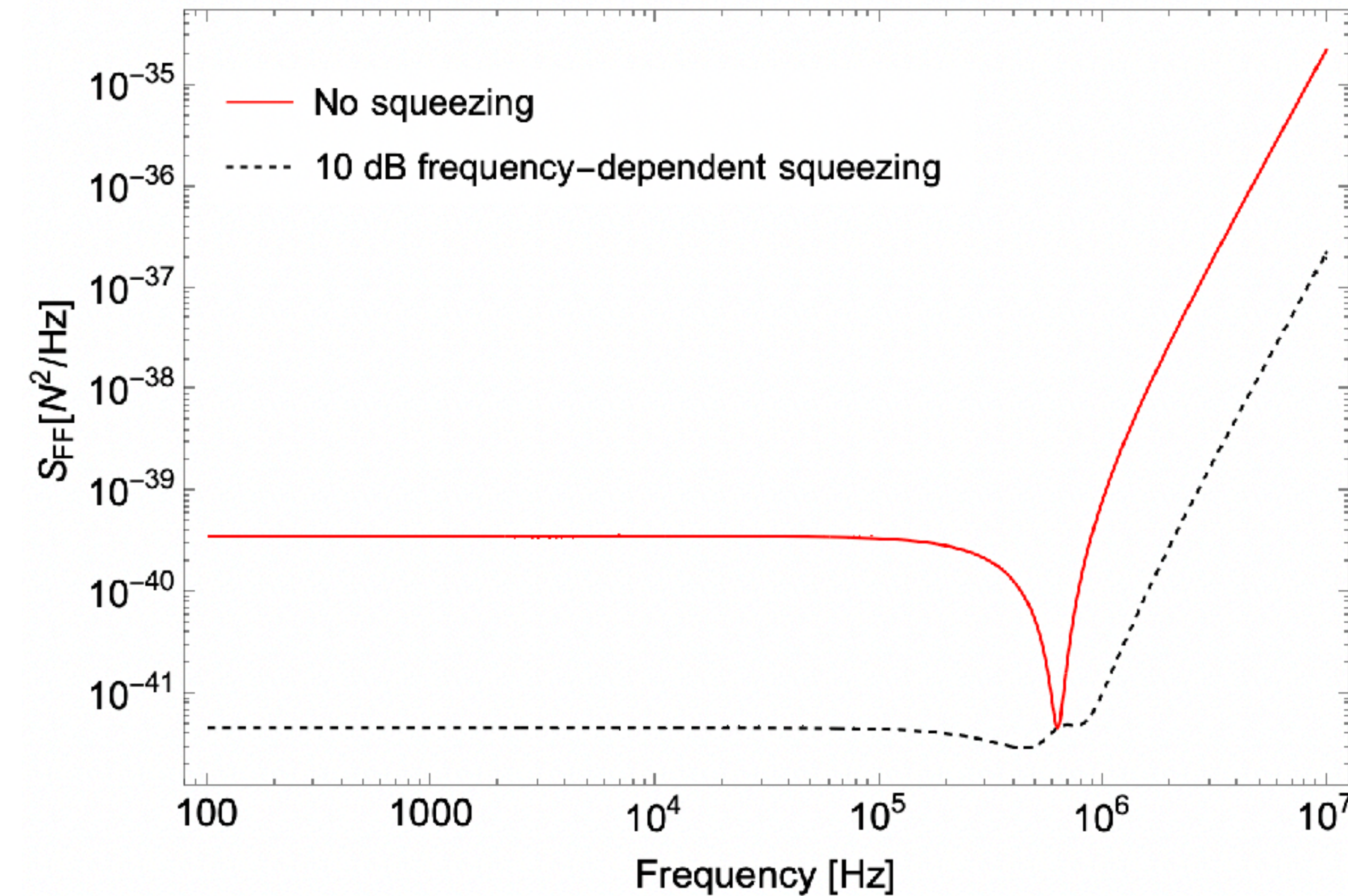
$$\tan \theta_*(\nu) = \frac{2\text{Re } \chi_{YX}\chi_{YY}^*}{\chi_{YX}^2 - \chi_{YY}^2}$$

$$S_{FF}(r, \theta_*) \approx e^{-2r} S_{FF}(r = 0)$$

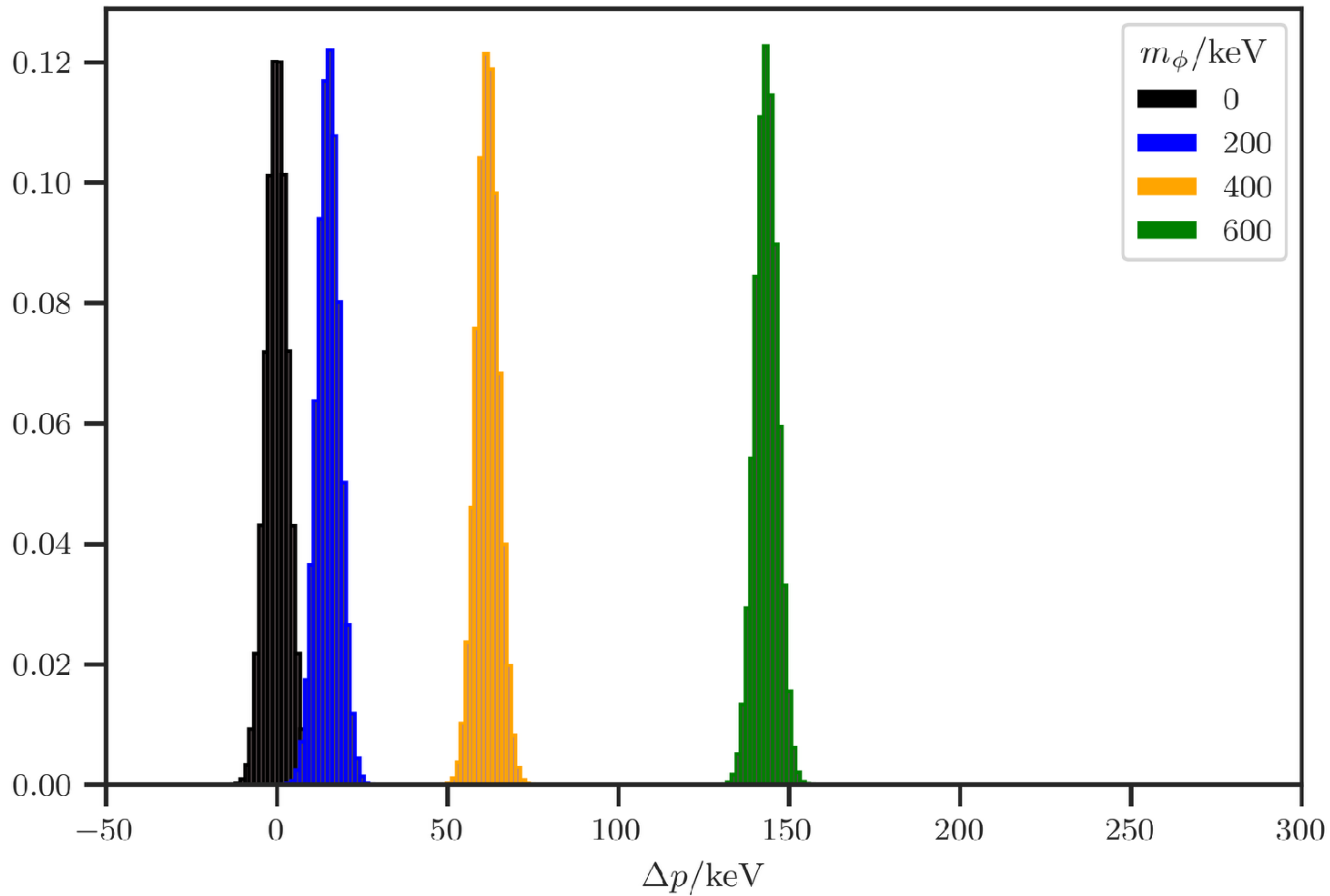


[Ganapathy et al., PRX Quantum (2023)]

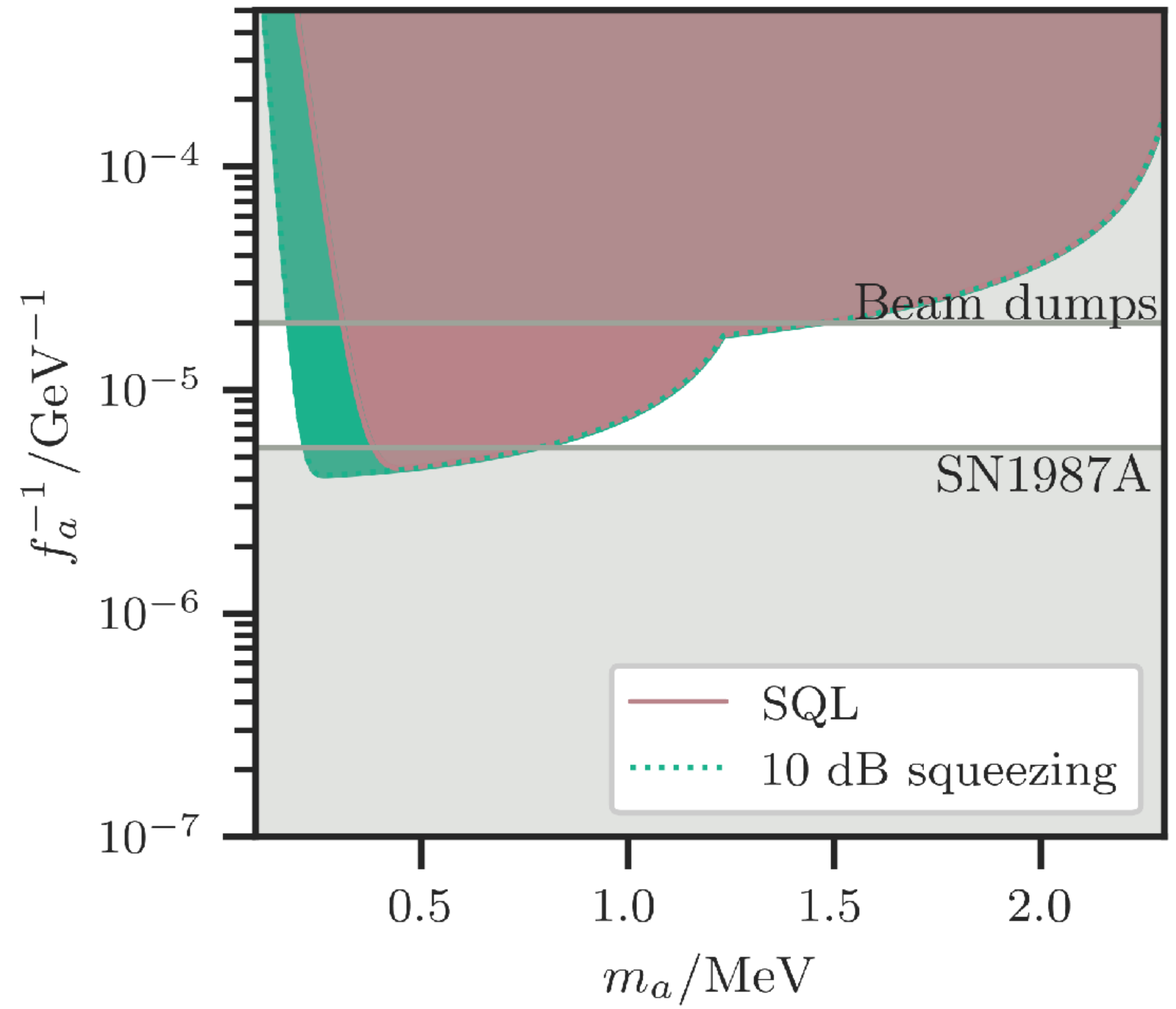
Impulse sensitivity



10 dB FD squeezing



$^{53\text{m}}\text{Fe}$



Conclusions

- Quantum-limited impulse measurements enable measurement of nuclear recoils
- Sensitive to sub-MeV dark state emissions in beta and gamma decay, as well as electroweak precision tests
- Quantum-enhanced techniques decrease noise, allowing access to lighter masses

Thank you

Backup slides

Quantum light

- Some notation:

- Hermitian photon operators:

$$\text{Amplitude quadrature } X_{\mathbf{p}} = \frac{1}{2}(a_{\mathbf{p}} + a_{\mathbf{p}}^{\dagger})$$

$$\text{Phase quadrature } Y_{\mathbf{p}} = \frac{i}{2}(a_{\mathbf{p}}^{\dagger} - a_{\mathbf{p}})$$

$$[X_{\mathbf{p}}, Y_{\mathbf{q}}] = i\delta_{\mathbf{p},\mathbf{q}}$$

$$\begin{aligned} A_{\mu}(\mathbf{x}) &= \sum_{\mathbf{p},r} \epsilon_{r,\mu}^*(\mathbf{p}) u_{\mathbf{p}}^*(\mathbf{x}) a_{\mathbf{p},r} + \text{h.c.} \\ &= \sum_{\mathbf{p},r} \text{Re} \left[\epsilon_{r,\mu}(\mathbf{p}) u_{\mathbf{p}}(\mathbf{x}) \right] X_{\mathbf{p}} + \text{Im} \left[\epsilon_{r,\mu}(\mathbf{p}) u_{\mathbf{p}}(\mathbf{x}) \right] Y_{\mathbf{p}} \end{aligned}$$

Input-output relations

- For quadratic dynamics, the input and output are linearly related:

$$Y_{\text{out}} = Y_{\text{in}} + \sqrt{\kappa}x$$

Measurement rate κ

- We may integrate out the mechanical d.o.f to find

$$Y_{\text{out}} = \chi_{YX}X_{\text{in}} + \chi_{YY}Y_{\text{in}} + \chi_{YF}F_{\text{in}}$$

Susceptibilities χ_{ij}

Input force F_{in}

- Construct an estimator for the force:

$$F = \chi_{YF}^{-1} Y_{\text{out}}$$

Unbiased:

$$\langle F \rangle = \langle F_{\text{in}} \rangle$$

Quantum uncertainties

- What about higher-point statistics? For Gaussian processes, all encoded in power spectral density (PSD) S_{FF} :

$$2\pi S_{FF}(\nu)\delta(\nu - \nu') = \langle F(\nu)F^\dagger(\nu') \rangle$$

$$S_{FF} = \chi_{YF}^{-2} \sum_{ij=X,Y} \chi_{Yi}^* \chi_{Yj} S_{ij} + S_{FF}^{\text{in}}$$

“Intrinsic uncertainty”
from input force fluctuations

“Measurement-related” uncertainty

- The uncertainty in the estimator of the force has additional contributions from the uncertainty in the input light

Standard quantum limit

- In coherent, classical state, $S_{XX} = S_{YY} = \frac{1}{2}$ & $S_{XY} = 0$

$$\chi_{YF}^2 S_{FF} = \underbrace{\frac{1}{2} \chi_{YX}^2}_{\text{Back action}} + \underbrace{\frac{1}{2} \chi_{YY}^2}_{\text{Shot noise}}$$

$$S_{FF}(\nu) = \frac{P\pi}{\lambda_0} + \frac{m^2 \lambda_0 (\gamma^2 \nu^2 + (\nu^2 - \omega_m^2)^2)}{4\pi P}$$

Scattered power P ,
Wavelength λ_0 ,
Mechanical damping γ

$$S_{FF}(0) \stackrel{\text{SQL}}{\approx} m\omega_m^2$$

- What does this mean for our impulse measurements?

Optimal filtering

- We are looking for an impulse kick

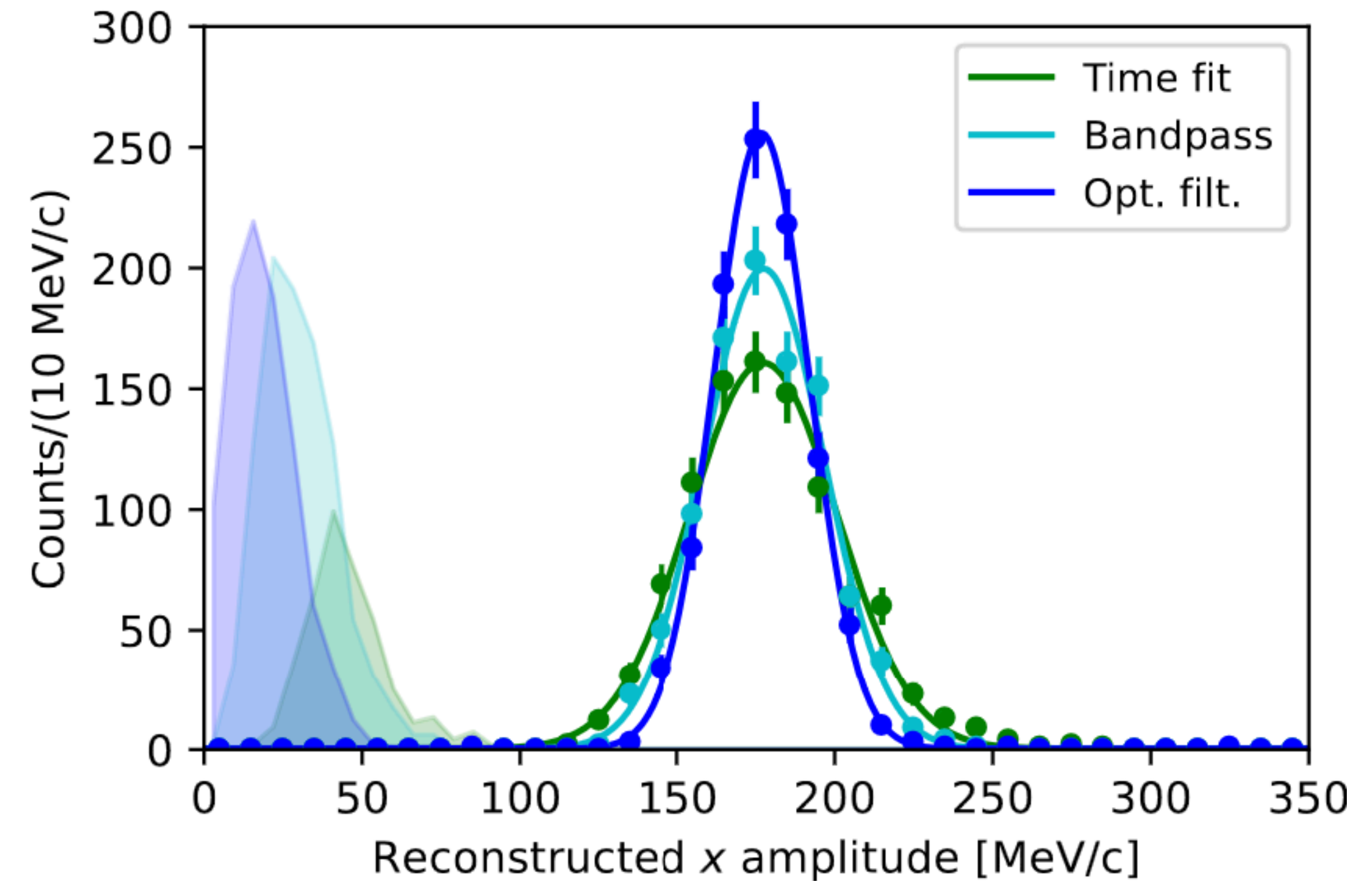
$$F_{\text{in}}(t, t_0) = \Delta p \delta(t - t_0)$$

- How do we maximize the SNR?

- Optimal filtering:

$$F_f(t) = \int d\nu e^{i\nu(t-t_0)} F(\nu) \cdot f(\nu)$$

- Filter $f(\nu) = 2\pi\Delta p \frac{1}{S_{FF}(\nu)}$



[2402.13257]

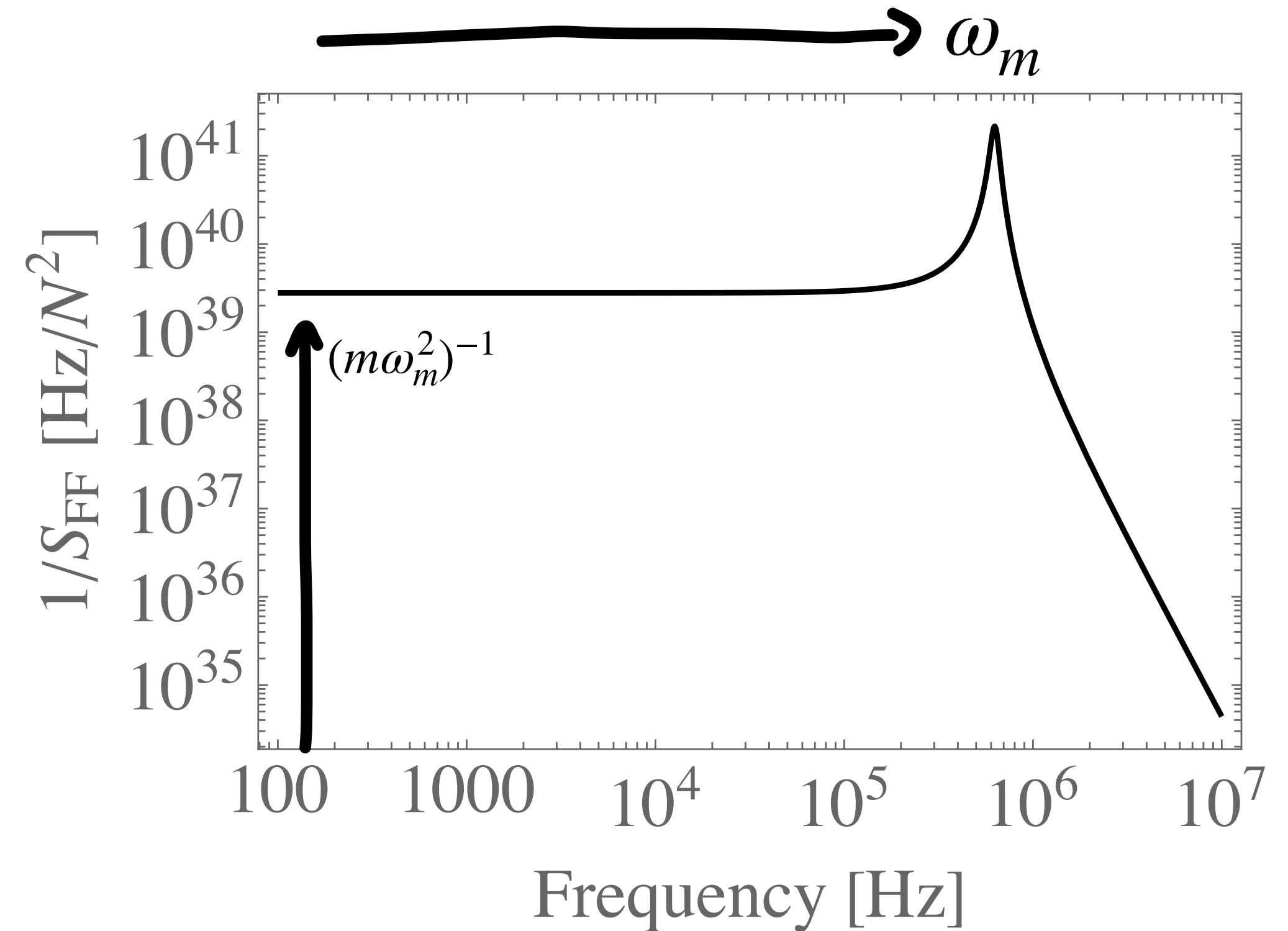
Momentum uncertainty

- Optimal SNR:

$$\text{SNR}_* = \Delta p \left(\int \frac{d\nu}{S_{FF}(\nu)} \right)^{1/2}$$

$$\text{SNR}_* > 1$$

$$\Rightarrow \Delta p > \left(\int \frac{d\nu}{S_{FF}(\nu)} \right)^{-1/2} \equiv \Delta p_{\text{th}}$$

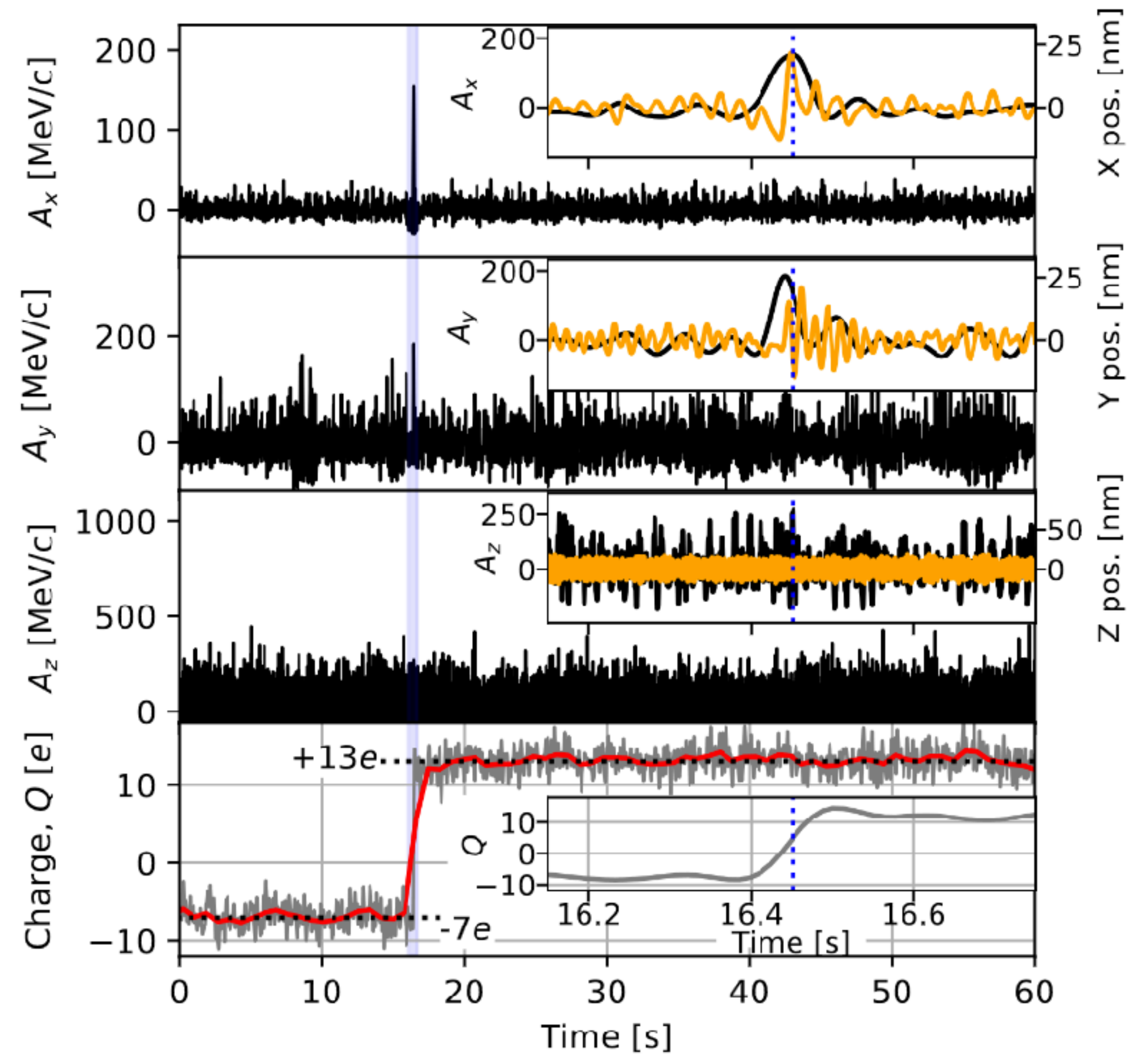


- For sensing at SQL, $\Delta p_{\text{th}} \approx \sqrt{m\omega_m}$

$$\approx 15\text{keV} \left(\frac{m\omega_m}{1\text{fg} \times 10\text{kHz}} \right)^{1/2}$$

Charge change

- Veto on charge state of sphere to remove backgrounds



[Wang et al. 2402.13257]

Squeezing

Squeezing operator: $S(r, \theta) = \exp\left(\frac{r}{2}(e^{-i\theta}a^2 - e^{i\theta}a^{\dagger 2})\right)$

How can we generate this dynamically?
Non-linear crystals

$$S_{ij} \rightarrow \begin{aligned} S_{XX} &= \frac{1}{2}(\cosh 2r - \cos \theta \sinh 2r) \\ S_{YY} &= \frac{1}{2}(\cosh 2r + \cos \theta \sinh 2r) \\ S_{XX} &= -\frac{1}{2} \sinh 2r \sin \theta \end{aligned}$$

$$H = \omega a^\dagger a + 2\omega b^\dagger b + i\chi^{(2)}(a^2 b^\dagger - a^{\dagger 2} b)$$

$$\text{Pump } b \rightarrow \beta e^{2i\omega t} + b:$$

$$\begin{aligned} U_I(t) &= \exp iH_I t \\ &= \exp\left(\chi^{(2)} t(\beta^* a^2 - \beta a^{\dagger 2})\right) \end{aligned}$$

Nuclear physics I

Nuclear EFT may be treated as an expansion in powers of nucleon momentum.

Recoil corrections goes as either

$$\frac{Q}{m_N} \approx 5 \times 10^{-4} \frac{Q}{0.5 \text{ MeV}},$$

$$\text{or as } (QR_{\text{nuc}})^2 \approx 10^{-4} \left(\frac{A}{100} \right)^{2/3} \left(\frac{Q}{0.5 \text{ MeV}} \right)^2 :$$

We may ignore higher-order derivative operators.

[Parity invariance of EM/strong force implies $\mathbf{q} \cdot \langle \mathbf{R}_N \rangle \approx 0$, where \mathbf{R}_N is nuclear centre-of-mass coordinate]

Nuclear physics II

The renormalisation of the vector and axial currents generically causes ambiguities even at leading order in recoil expansion:

$$L^\mu \langle f | V_\mu | i \rangle = a(q)^2 \frac{P \cdot \ell}{M + M'}$$

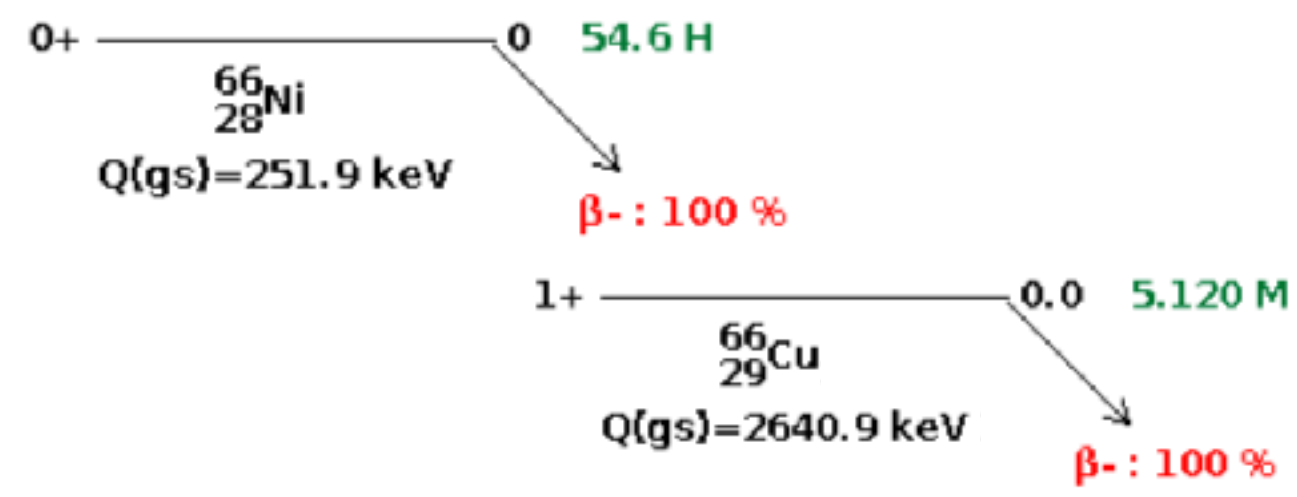
$$L^\mu \langle f | A_\mu | i \rangle = c(q)^2 \frac{P^\mu \ell^\nu C_{\mu\nu}}{M + M'}$$

The form factors $a(q^2)$ and $c(q^2)$ are incalculable for larger nuclei.

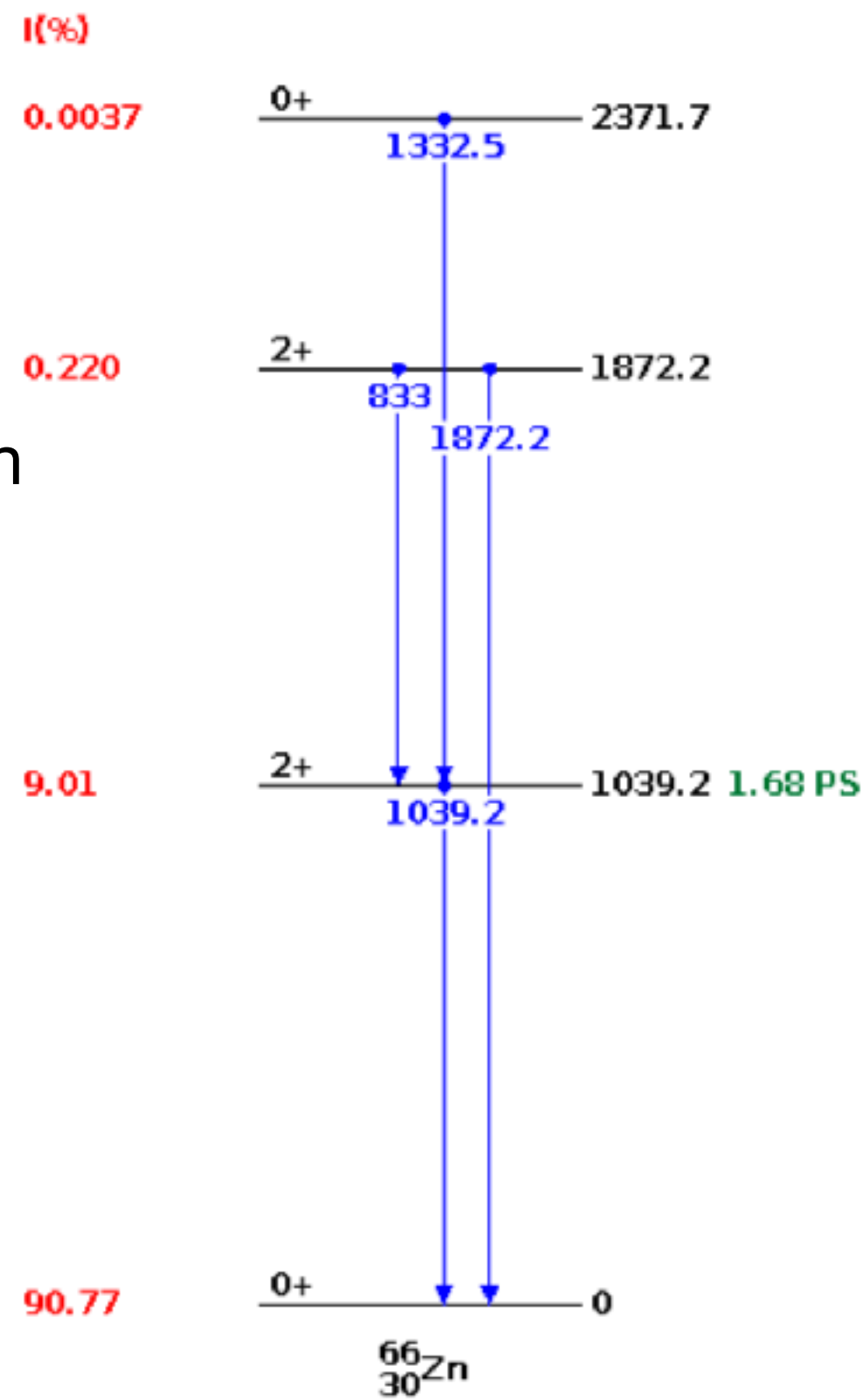
For pure Gamow-Teller transitions

$(J_i = 1, J_f = 0, \pi_i \cdot \pi_f = 1)$, $a(q^2) = 0$. Approximate $c(q^2) \approx c(0)$, which only affects total rate/normalisation – cancels in differential observable.

Nickel-66



- Ni-66 has desirable properties:
 - Reasonable half-life
 - Ground state to ground state transition
 - Small Q-value
 - Pure Gamow-Teller transition

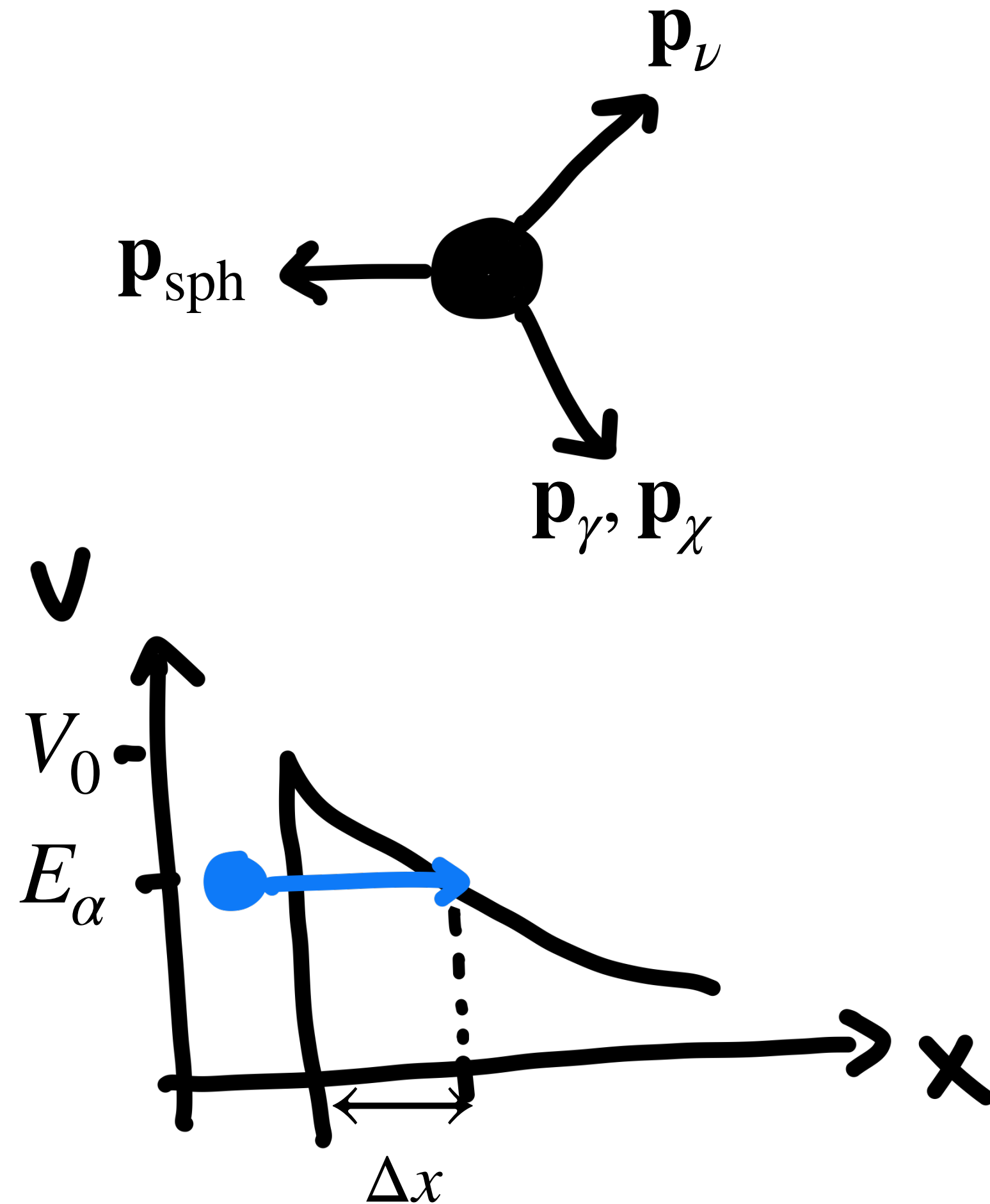


- BUT, child is unstable: need to differentiate decays
 - Transitions have very different Q -values

What gamma decays?

- Most gamma decays follow beta decay/ electron capture. The neutrino momentum is degenerate with invisible BSM momentum
- Alpha decay is predominantly to child ground state

$$P \sim \exp\left(-2\sqrt{m_\alpha(V_0 - E_\alpha)} \cdot \Delta x\right)$$



Dark state emission

e.g. Nucleon-coupled scalar

$$H_{\text{int}} = g_\phi \phi \bar{N}N$$



$$\mathcal{M} \sim \langle i; 0 | \mathcal{H}_{\text{int}} | f; \mathbf{k}_\phi \rangle$$

$$\sim g_\phi \langle i | e^{i\mathbf{k}_\phi \cdot \mathbf{x}} | f \rangle \cdot \langle 0 | \phi | \mathbf{k}_\phi \rangle$$

$$kr \approx 1 \text{ MeV} \times 5 \text{ fm} \\ \approx 0.03$$

Initial and final nuclear states are labelled by their angular momentum and parity J^π .

In multipole expansion, transitions come in two types:

$$E\ell : \Delta J = \ell, \pi_i \cdot \pi_f = (-1)^\ell$$

Scalars

$$M\ell : \Delta J = \ell, \pi_i \cdot \pi_f = (-1)^{\ell+1}$$

Pseudoscalars

Branching ratios

- Aim to constrain branching ratio to invisible states:

$$\text{BR}(J_i^{\pi_i} \rightarrow J_j^{\pi_f} + \chi) = \frac{\Gamma(J_i^{\pi_i} \rightarrow J_j^{\pi_f} + \chi)}{\Gamma_{\text{tot}}}$$

- Depends on selection rules of transition, as well as particle properties.
e.g. scalar emission:

$$\text{BR}_{\phi}(E\ell) \approx \frac{1}{2} \left(\frac{g_{\phi}}{e} \right)^2 v_{\phi}^{2\ell+1}, \quad l > 0$$