Short-range Wake Fields in Plasma Accelerators

Valeri Lebedev
Fermilab

Outline

- Objective
- Longitudinal wakes
- Transverse wake
- Conclusions

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and Test Beams
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Objective

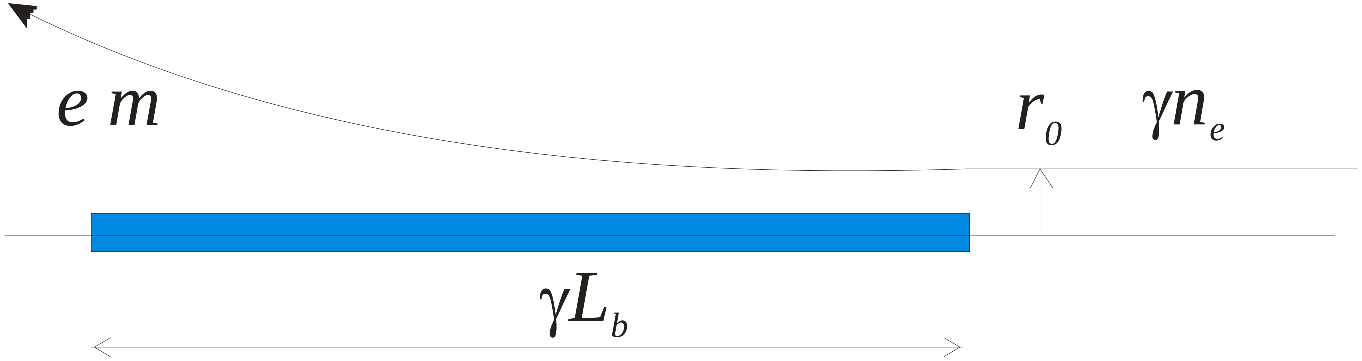
- Active discussion on the R&D for plasma accelerators in application to e+e- colliders in the TeV energy range
- Short wake range wake-fields look as serious show stoppers for collider
    - not for plasma acceleration
- Figure of merit
    - Luminosity $$L = \frac{fN^2}{4\pi\sigma_x\sigma_y} = \frac{P_{beam}}{4\pi E_b}\frac{N}{\sigma_x\sigma_y}$$
    - $N/\sigma_x\sigma_y$ is limited by disruption and beamstrahlung
    - Energy efficiency (P_{beam}/P_{total}) is the primary issue

Energy Loss and its Dependence along Bunch

- For beam sizes much smaller than the plasma wave length ($\lambda_p=2\pi c/\omega_p$) the average energy loss per particle is well-known

$$\frac{d\mathrm{E}}{ds}=eE=\frac{4\pi n_e e^4 N_e}{mc^2}L_c\ ,\quad L_c=\frac{\rho_{\max}}{\rho_{\min}}$$

L_c is the Coulomb logarithm. It is different for a finite size bunch in comparison to a point-like macro-particle

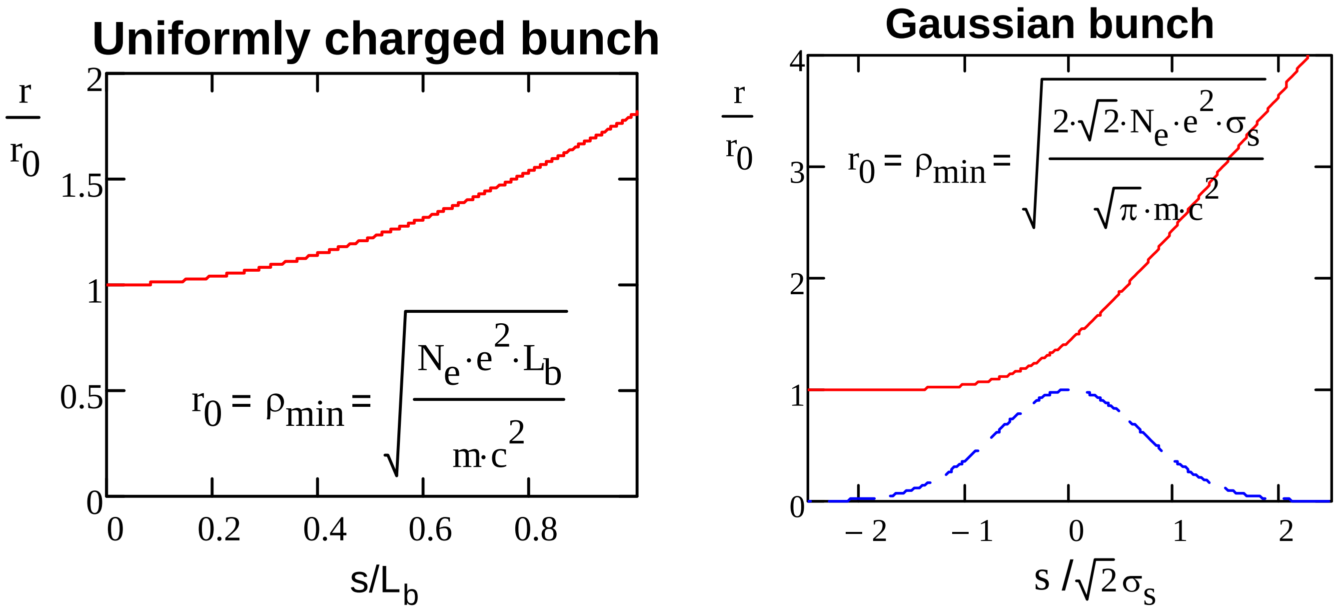


- Bunch length in the beam frame is much larger than any transverse size (λ_p, ⊥ beam size)

$$\Rightarrow\quad \delta\varphi(s)=2\delta Q\ln\frac{r(s,r_0)}{r_0}\quad\Rightarrow\quad eE=-e\frac{d\varphi}{ds}=4\pi\gamma n_e e^4\frac{d}{ds}\left(\int_{r_b}^{c/\omega_p}\rho\ln\left(\frac{r(s,\rho)}{\rho}\right)d\rho\right)$$

Minimum & Maximum Impact Parameters

- Longitudinal wake function is obtained in logarithmic approximation
- Minimum impact parameter, ρ_{min}, is determined by scattering where particle displacement is ~ρ_{min} to bunch end



Plasma electron scattering on the bunch with impact parameter ρ_{min}

- For all discussed parameter sets the beam radius is « ρ_{min} and can be neglected
- Particle scattering is described with perturbation theory
  - It is sufficiently accurate for the logarithmic approximation
- Maximum impact parameter $\rho_{max} = c/\omega_p$ the same as for point-like particle

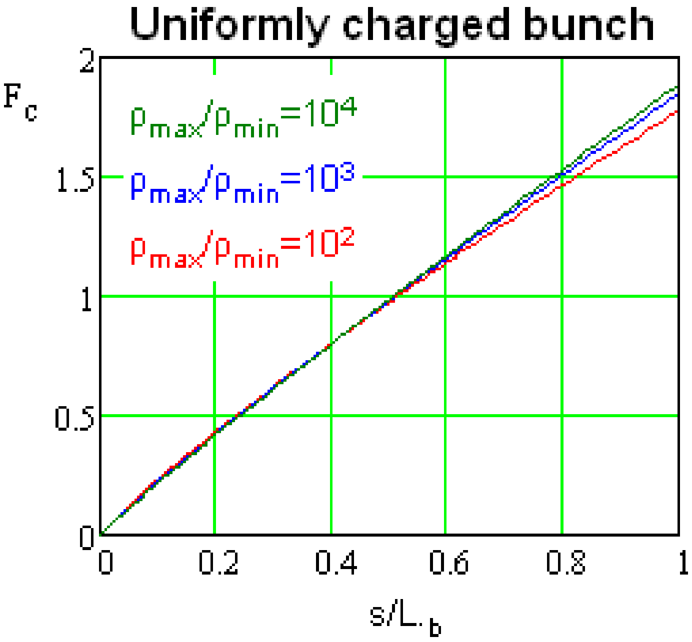
Energy Loss along Bunch

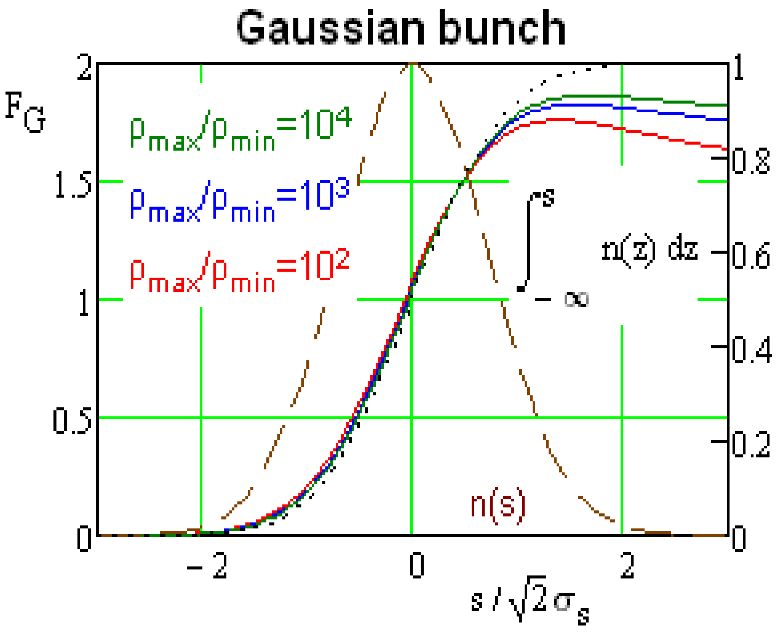
- For uniformly charged bunch

$$\frac{dE}{ds}=\frac{4\pi\gamma n_e N_e e^4}{mc^2}\ln\left(\frac{1.3\rho_{\max}}{\rho_{\min}}\right)F_c\left(\frac{\rho_{\max}}{\rho_{\min}},\frac{s}{L_b}\right),\quad F_c\left(X,s\right)\approx\frac{1}{\ln\left(1.3X\right)}\int_0^X\frac{\left(2\rho+0.23s\right)s\rho\,d\rho}{\left(\rho^2+s\left(1+0.23\rho\right)\right)\left(\rho+0.23s\right)}$$

- For Gaussian bunch

$$\frac{dE}{ds}=\frac{4\pi\gamma n_e N_e e^4}{mc^2}\ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right)F_G\left(\frac{\rho_{\max}}{\rho_{\min}},\frac{s}{\sqrt{2}\sigma_s}\right),\quad F_G\left(X,s\right)\approx\frac{2-\mathrm{erfc}(s)}{2\ln X}\ln\left(\frac{2X^2}{\sqrt{\pi}s\left(2-\mathrm{erfc}(s)\right)+\exp\left(-s^2\right)}\right)$$



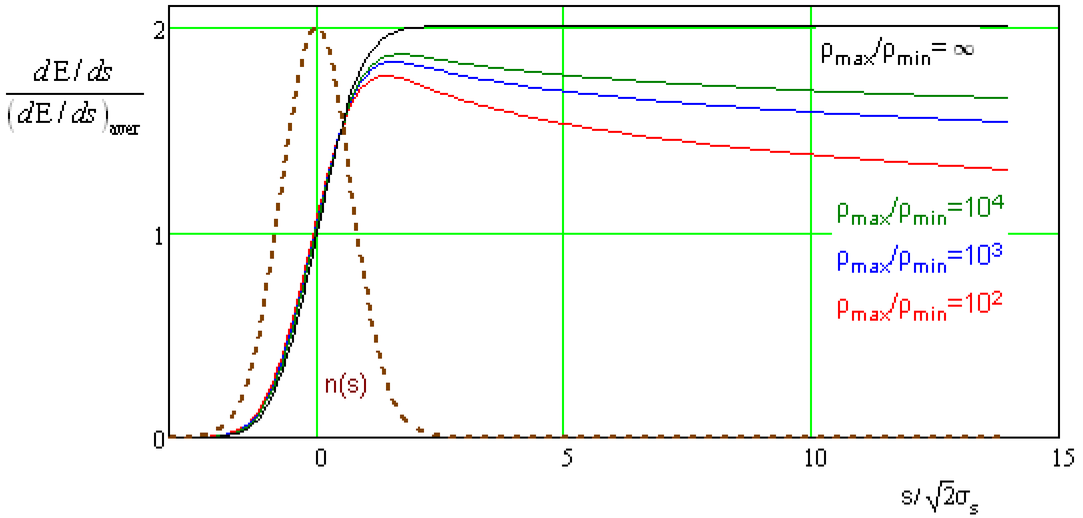


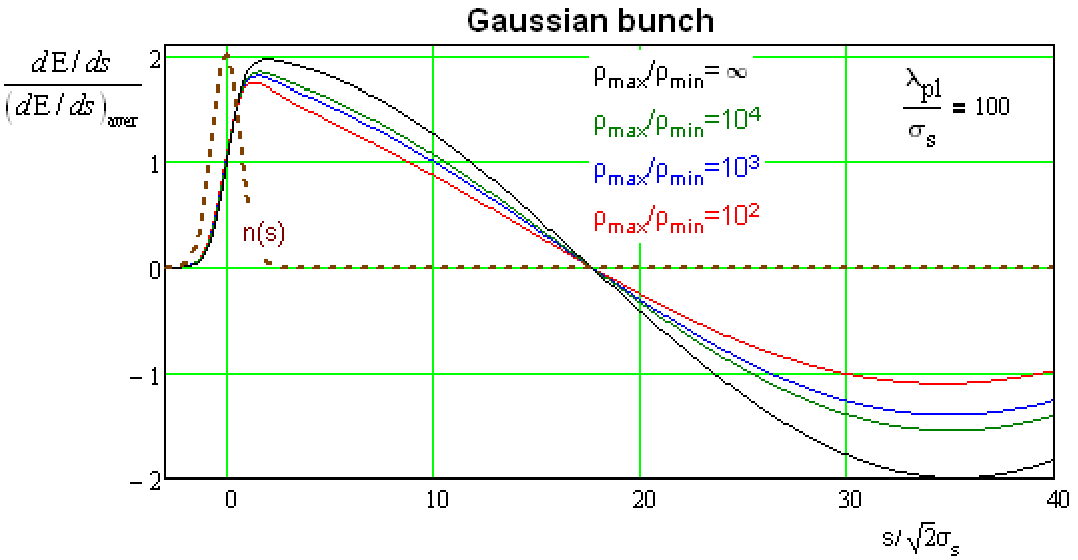
$$\mathrm{erfc}(s)=\frac{2}{\sqrt{\pi}}\int_s^\infty e^{-x^2}dx$$

- The longitudinal wake is close to a step function
    - strictly speaking it is dependent on longitudinal density distribution but it has only logarithmic correction

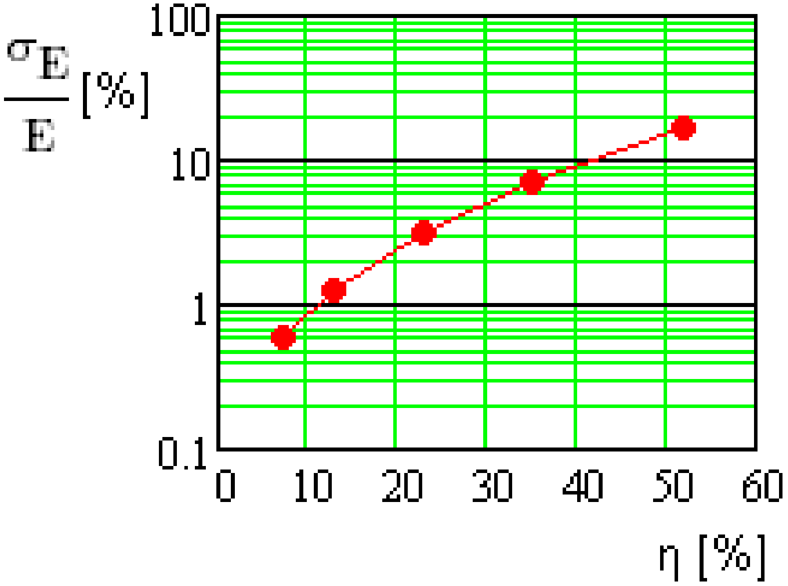
Efficiency of Plasma-to-Beam Energy Transfer

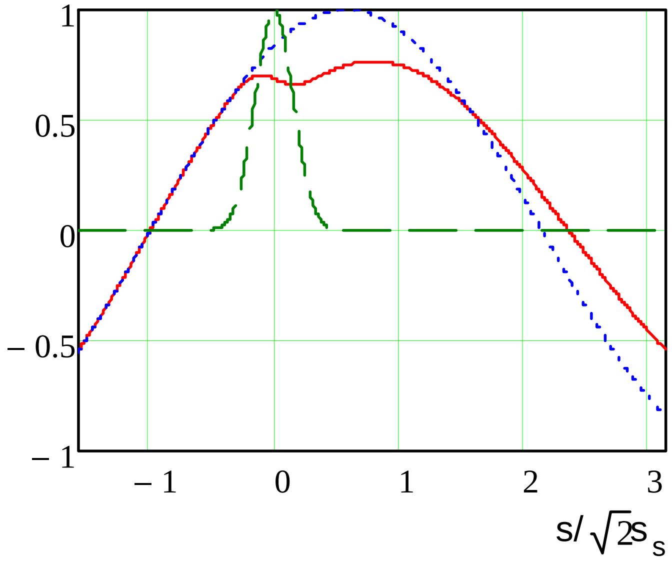
- Deceleration of the tail particles is almost twice stronger than the deceleration of bunch center (or average deceleration)
- For finite values of ρ_{max}/ρ_{min} there is also some wake reduction after the bunch
- All this limits the total amount of bunch energy transferred to plasma to less than 50%



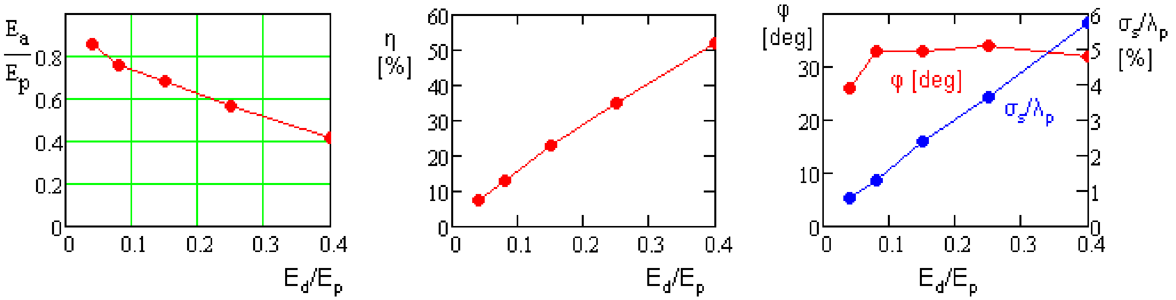


Energy Spread and Efficiency of Acceleration





Longitudinal electric field with and without bunch field; $\Delta E/E_p$=0.15, ϕ=33^0,, σ_s/lp=0.024 => E_{acc}/E_p=0.68.



- Small energy spread is required to transfer the beam from one accelerating section to another and to focus the beam in IP
  - For 1% rms energy spread only ~9% of plasma energy can be transferred to the beam
    - ±2.5% total spread is a huge number

E_p – amplitude of plasma accelerating electric field

E_d – decelerating electric field in the bunch center

η - percentage of energy transferred from plasma to beam

σ_E/E – rms energy spread in accelerated beam

E_a - average accelerating field

Transverse Wake

- There is no transverse wake in uniform plasma
  - However focusing of trailing particles do exist (detuning wake)
- Beam acceleration perturbs plasma density and creates accelerating channel and, consequently, transverse wake
- For small beam size ($\sigma_{b\perp} \ll c/\omega_p$) the wake field is nearly uniform in transverse plane
  - The wake-function grows almost linearly
  - In logarithmic approximation it is

$$W_\perp = 2\left(\frac{\omega_p}{c\sigma_\perp}\right)^2 \left(\frac{\Delta n}{n}\right)_e (s-s')\ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right) \xrightarrow{c/\omega_p=\sigma_\perp} 2\left(\frac{\Delta n}{n}\right)_e \frac{s-s'}{\sigma_\perp{}^4}\ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right)$$

- Comparing it with focusing strength of plasma channel we obtain at the bunch end

$$\frac{E_{wake}(s=L_b)}{E_{plasma\_foc}} \approx \frac{2L_b}{\sigma_\perp}\frac{(dE/ds)_{loss}}{(dE/ds)_{\max}}\ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right)$$

where $(dE/ds)_{loss}$ – average energy loss in plasma

$(dE/ds)_{max}$ – maximum accelerating field for given plasma density

Conclusions

- Beam interaction with plasma puts severe limitations on efficiency of energy transfer from plasma to beam
  - Requirement to have "a collider quality" beam limits this efficiency to well below 10%
- If plasma is excited by an electron bunch the energy transfer efficiency from beam to plasma cannot exceed 50%
- The transverse wake does not represent a fundamental problem but needs to be accounted in optimization of plasma acceleration
- Plasma acceleration presents
  - interesting scientific subject
  - can find good application in a number of fields
  - But it hardly can be a valuable tool for future e^+e^- colliders of TeV energy scale
    - Its energy efficiency is well below of ILC or other possible choices based on traditional acceleration