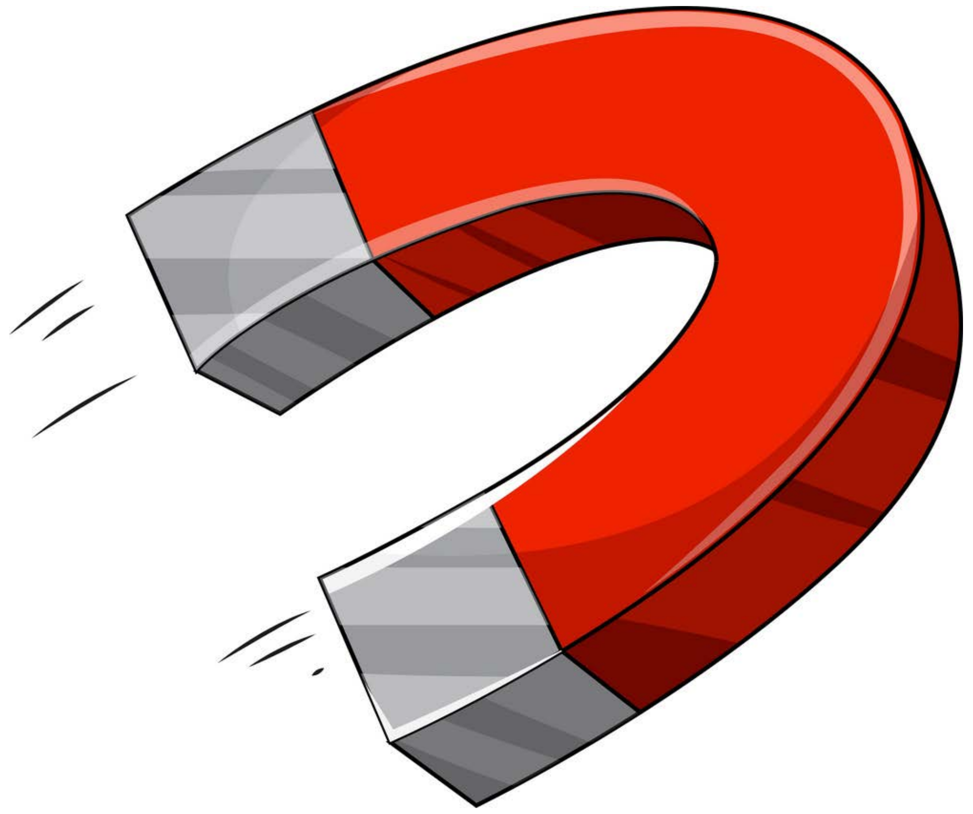
Recent Advancements in BELFEM

Christian Messe, Gregory Giard, Frédérik Sirois
April 30th, 2023

BERKELEY LAB

Outline



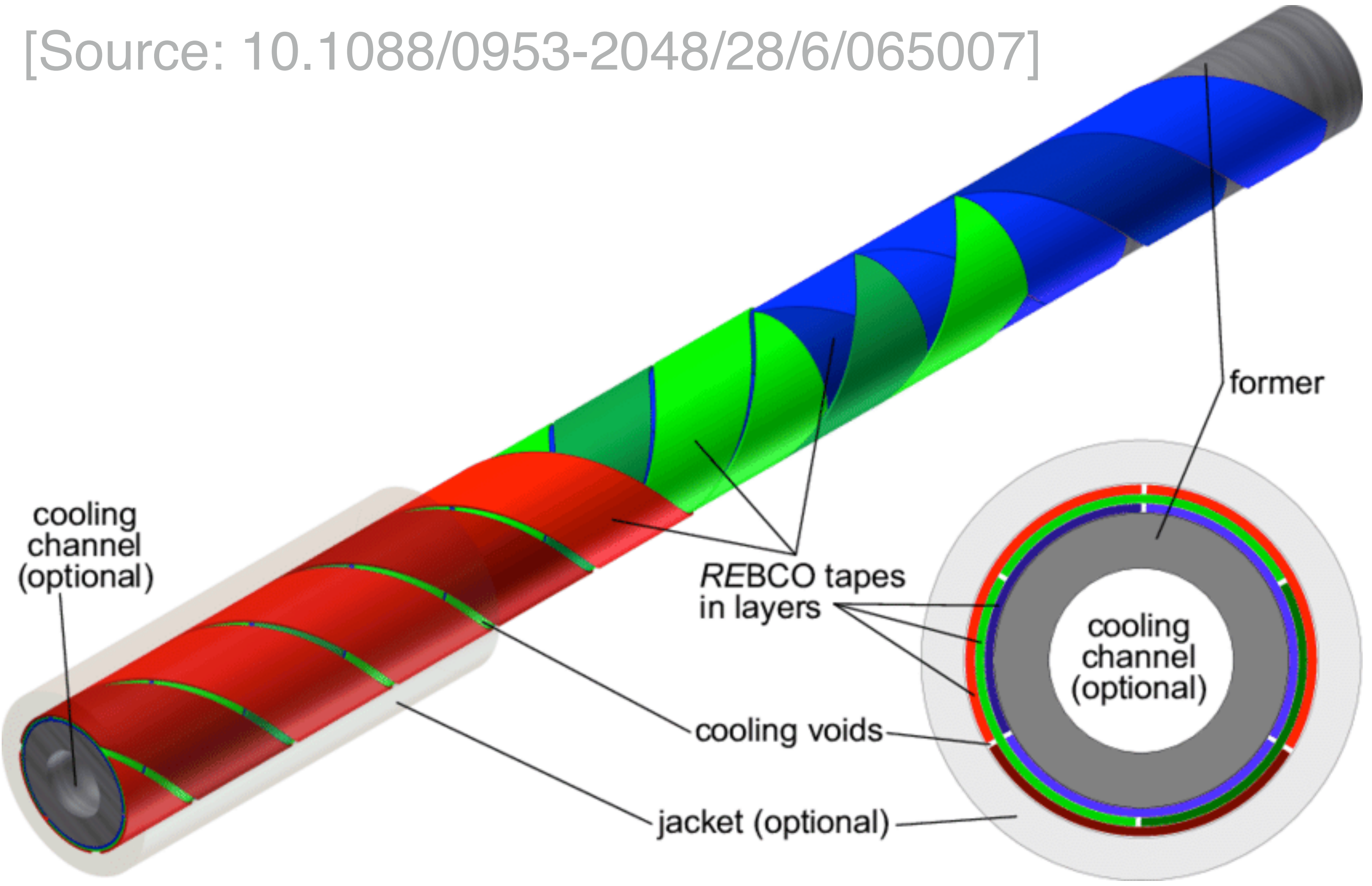
- Motivation
- h-ϕ fundamentals
- Why a custom codebase?
- Last Year’s progress
- Work in Progress: Thermal Coupling
- Work in Progress: Current Sharing
- Summary and Outlook

BELFEM: Motivation and Project Goals

want ability to model:

- **quasi-magnetodynamic modeling**
  - understand electromagnetic behavior of cables
- **coupled thermal modeling**
  - thermal behavior and physical coupling with EM
  - quench behavior
- **other phenomena**
  - current sharing
  - mechanical behavior
  - …

[Source: 10.1088/0953-2048/28/6/065007]



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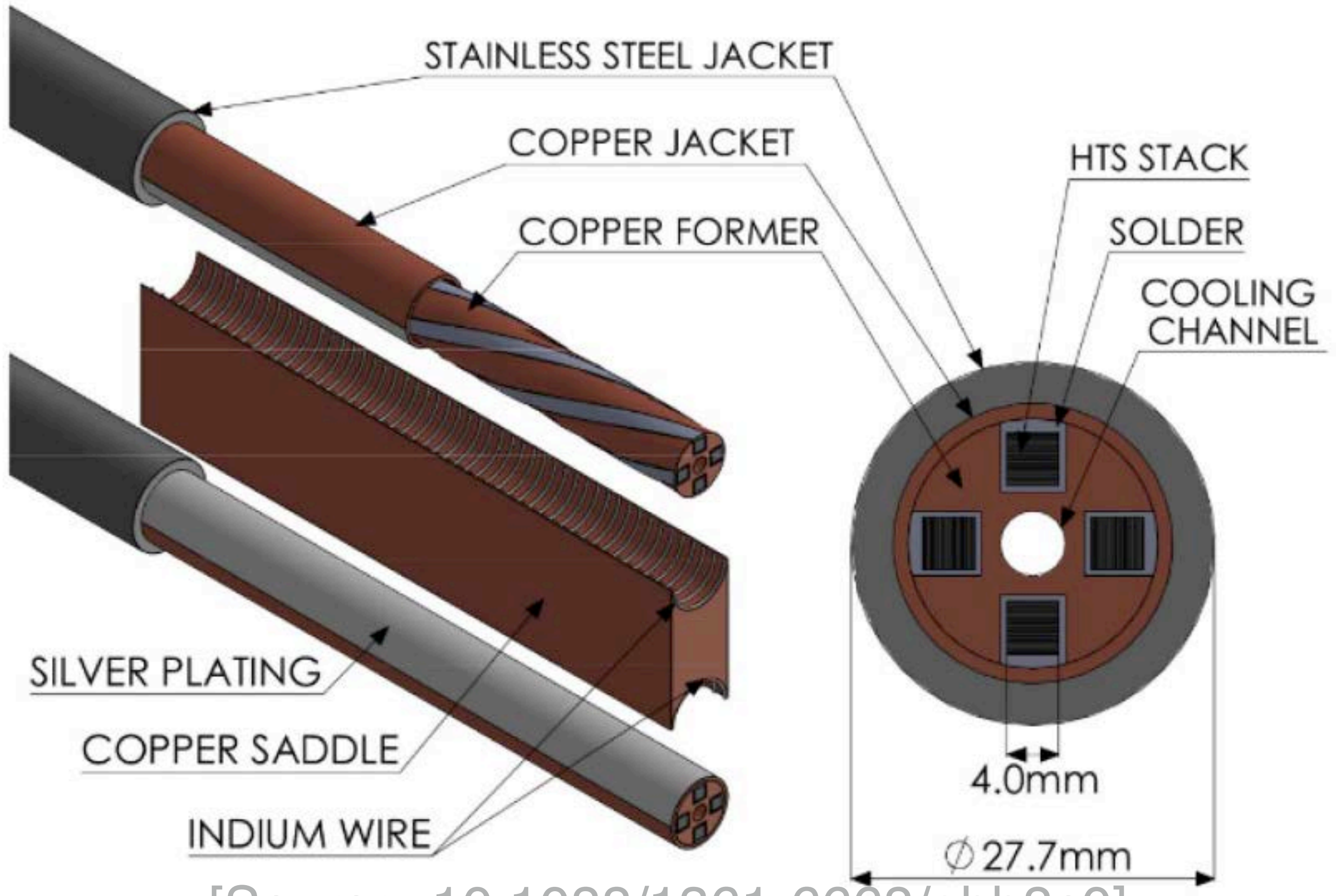


[Source: 10.1088/1361-6668/abb8c0]

H-ϕ formulation: Fundamentals

Solid Model

- got momentum in late 2010s to early 2020s
- very robust formulation
- significantly reduced degrees of freedom in non-conducting domains

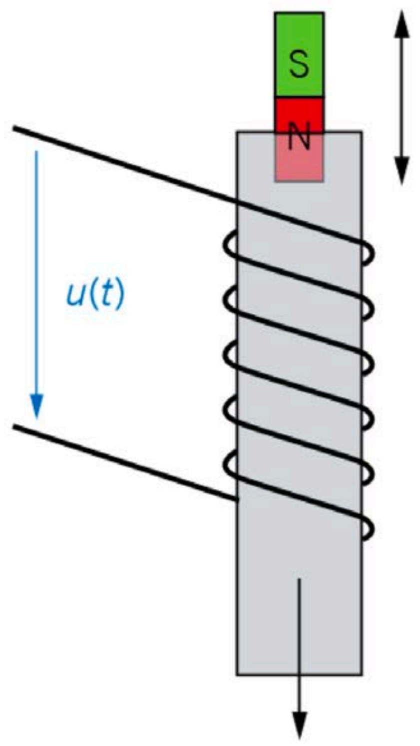
➡ very high performance gain

➡ ideal for large 3D models!

Thin-Shell Model

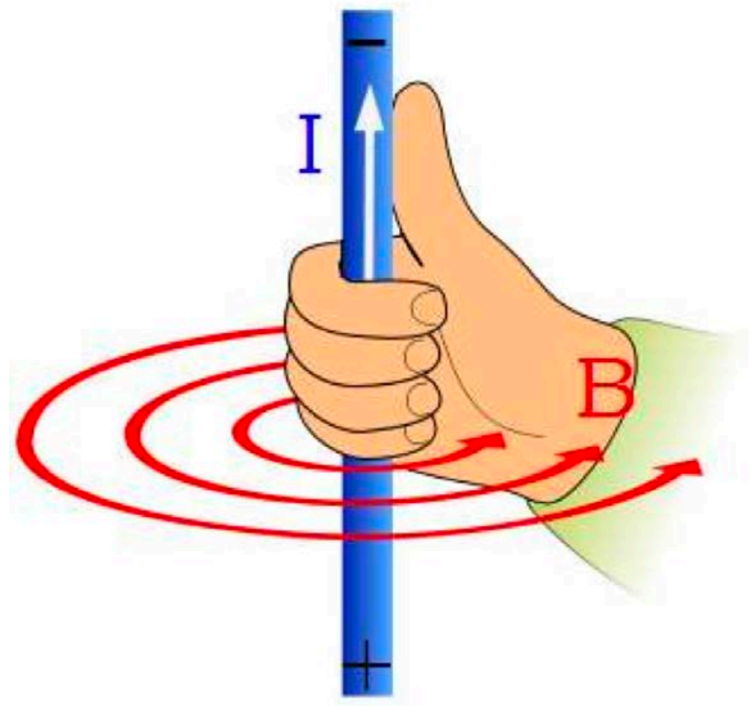
- first published in 2022
- can resolve individual layers of HTS tapes

➡ideal for current sharing and quench investigations



Faraday's law

$$\nabla \times \boldsymbol{e} = -\dot{\boldsymbol{b}}$$



Ampére's law

$$\nabla \times \boldsymbol{h} = -\boldsymbol{j}$$

Recipe:

- develop weak form around Faraday's law
- use $\boldsymbol{\delta h}$ as test function
- substitute **e** with Ohm's and Ampére

$$\boldsymbol{e} = -\rho \nabla \times \boldsymbol{h}$$

- for non-conducting region, define

$$\boldsymbol{h} = -\nabla \phi$$

H-ϕ formulation: Weak Forms

Conducting Region

$$\int_{\Omega} \delta\boldsymbol{h}^{\mathbf{T}} \frac{\partial(\mu\boldsymbol{h})}{\partial t}\,\mathrm{d}V + \int_{\Omega} \delta\boldsymbol{h}^{\mathbf{T}} \times \nabla^{\mathbf{T}} \rho \nabla \times \boldsymbol{h}\,\mathrm{d}V + \int_{\Gamma} \delta\boldsymbol{h}^{\mathbf{T}} \boldsymbol{n} \times \boldsymbol{e}\,\mathrm{d}S = 0$$

damping stiffness boundary

Non-Conducting Regions

substitute $-\nabla\phi = \boldsymbol{h}$

$$\int_{\Omega} \delta\phi^{\mathbf{T}} \nabla^{\mathbf{T}} \frac{\partial(\mu\nabla\phi)}{\partial t}\,\mathrm{d}V + \cancelto{\mathbf{0}}{\int_{\Omega} \delta\phi^{\mathbf{T}} \nabla^{\mathbf{T}} \times \nabla^{\mathbf{T}} \rho \nabla \times \nabla\phi\,\mathrm{d}V} - \int_{\Gamma} \delta\phi^{\mathbf{T}} \nabla^{\mathbf{T}} \boldsymbol{n} \times \boldsymbol{e}\,\mathrm{d}S = 0$$

damping boundary

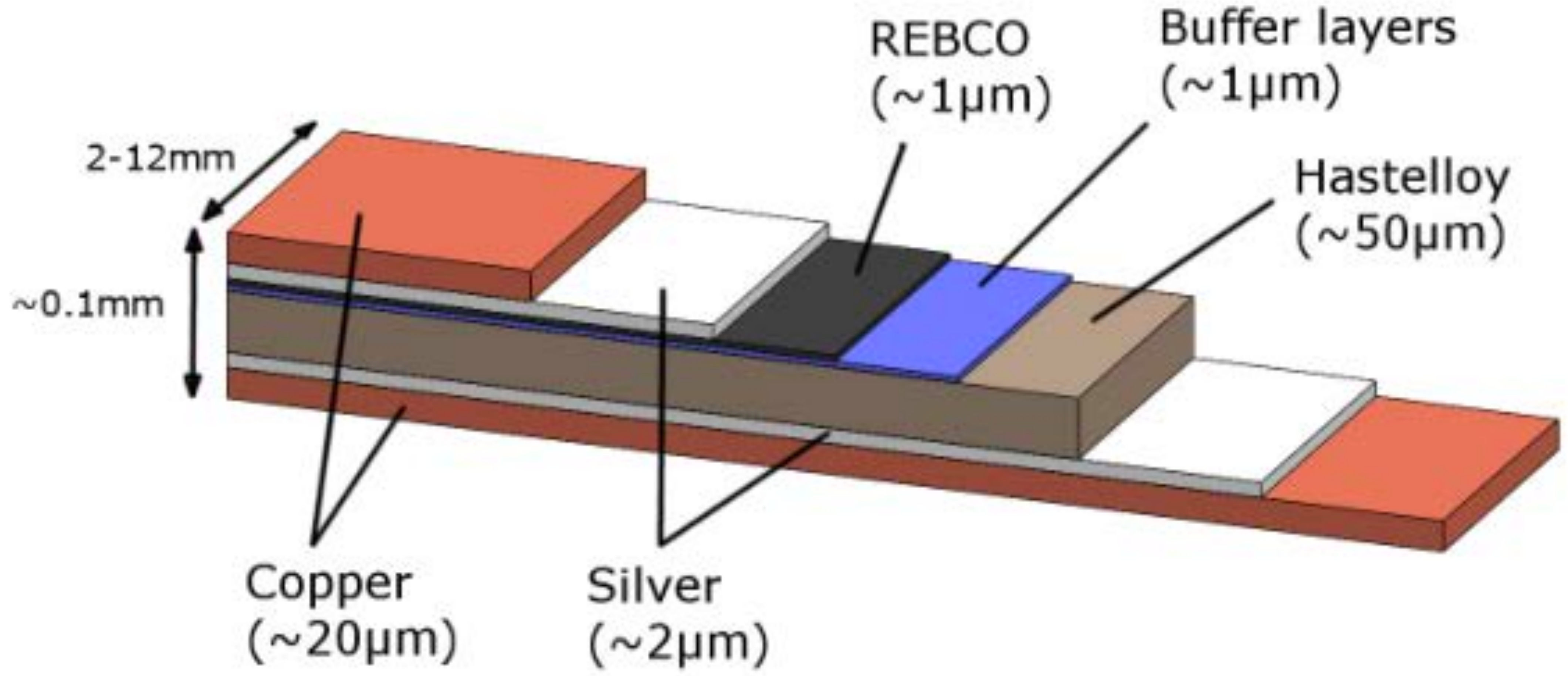
- traditionally, the ϕ-formulation is based on the Gauß law (∇B=0), we, however, use Faraday 's law
  → better convergence since same physical equation for all domains!

- stiffness vanishes for ϕ since ∇ x ∇ ϕ = 0

H-ϕ formulation: Thin Shell Approach

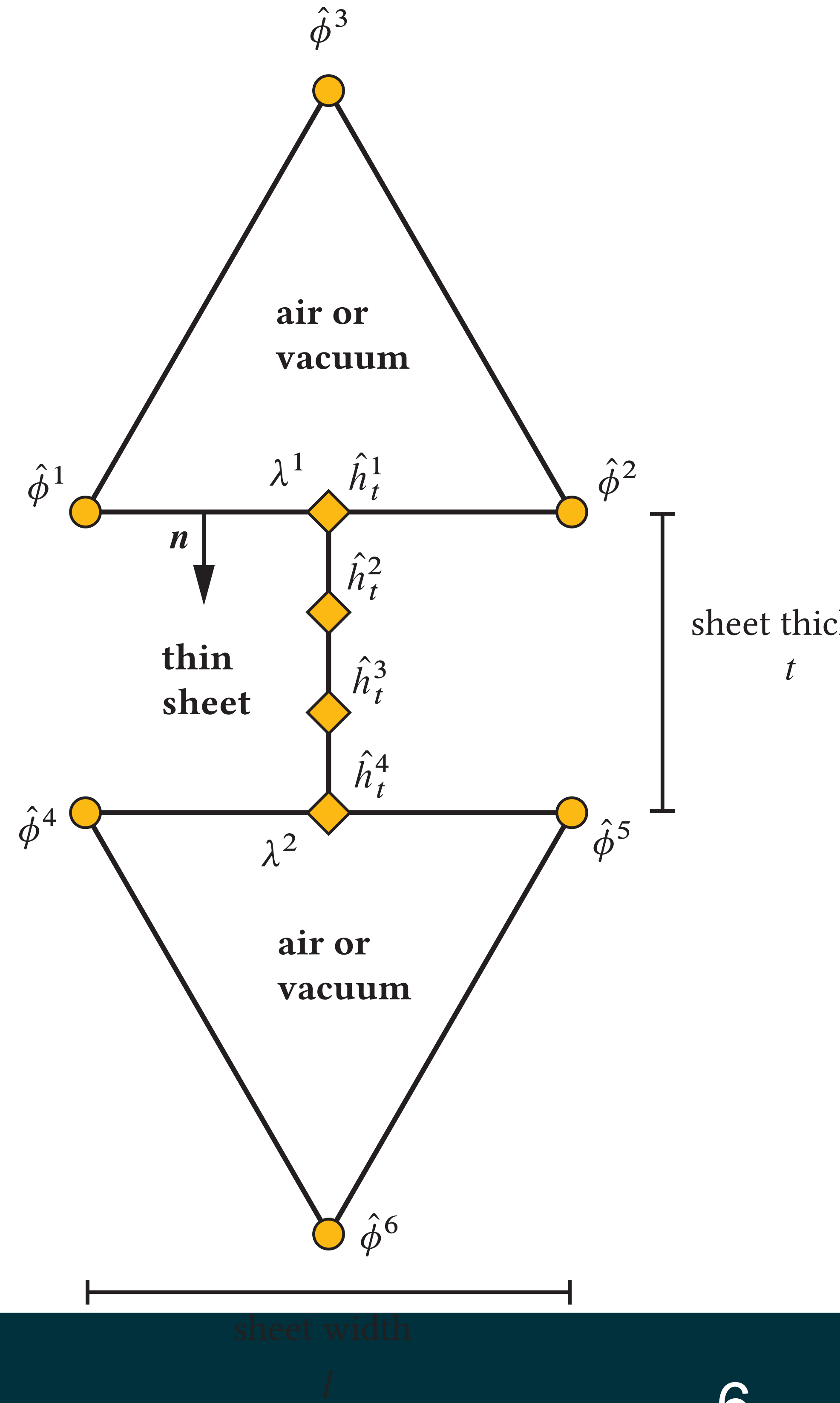
Challenges

- high aspect ratio
- strongly nonlinear material behavior
- electromagnetic-thermal interaction
- boundary conditions
- current sharing



Mixed h-ϕ Formulation for thin shells

- ongoing development in corporation with Polytechnique Montreal



H-ϕ formulation: Homologies

Boundary Conditions

current is applied over Ampere’s circuital law:

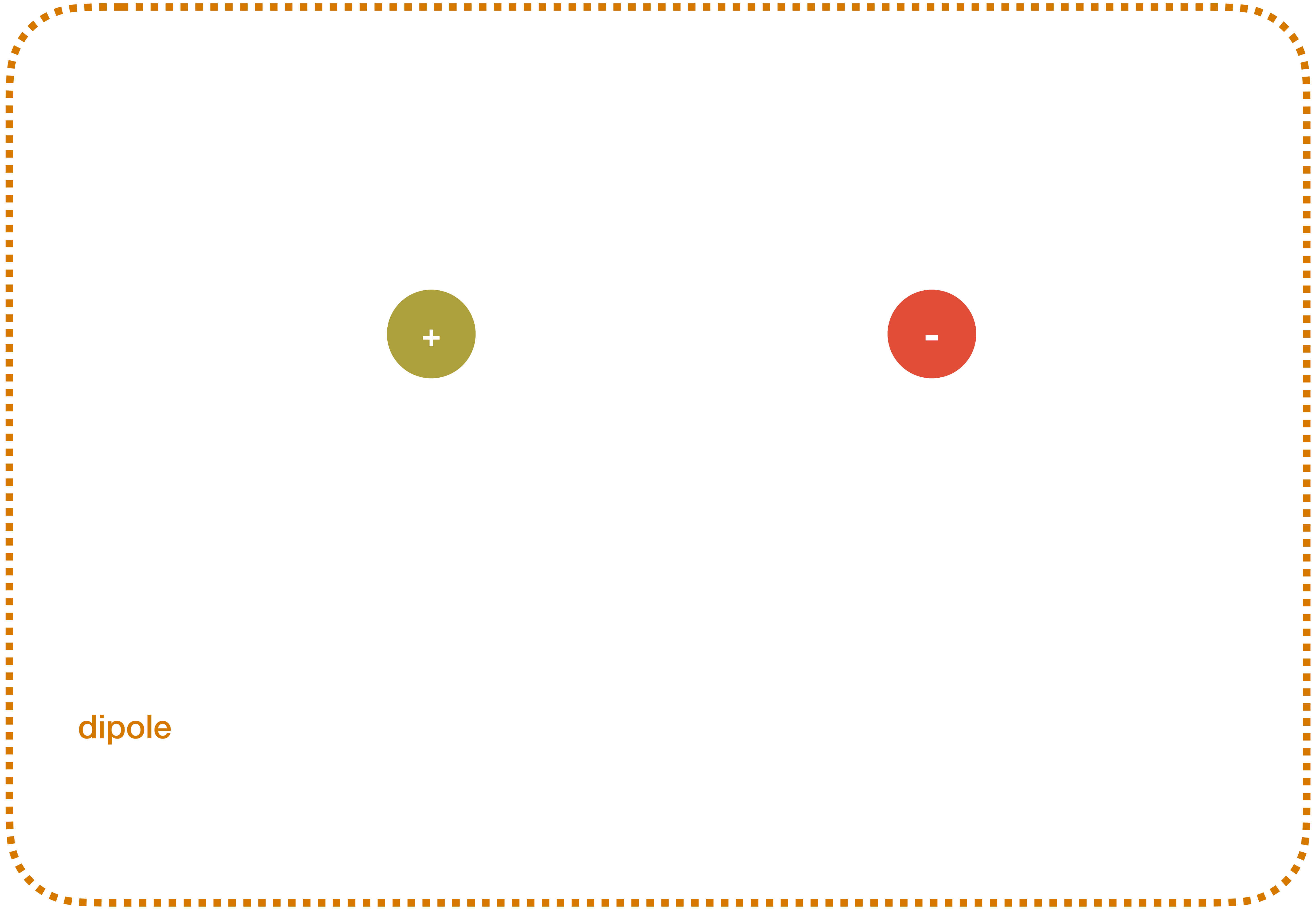
- homologies represent the loops that can be drawn around the conducting regions that fulfill Ampere’s law

- only integral current I needs to be known

- cohomologies are cuts in the domain over which jumps in the magnetic potential ϕ are imposed so that Δϕ=I.

➡ very elegant mathematics!
➡ homology definition not user friendly
➡ difficult to implement in commercial codes

Ampére’s circuital law $\oint \boldsymbol{h}\, \mathrm{d}l = I$



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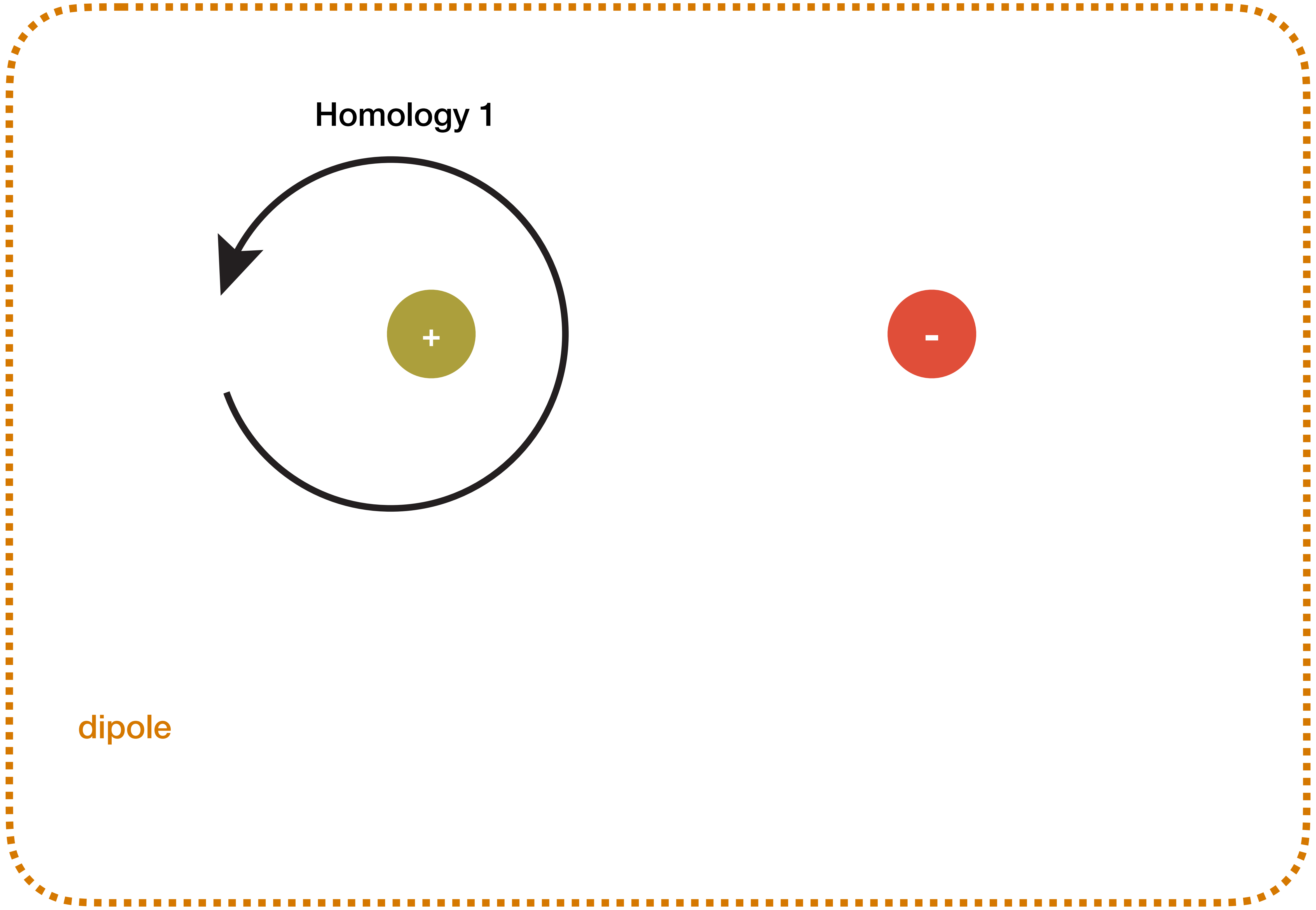

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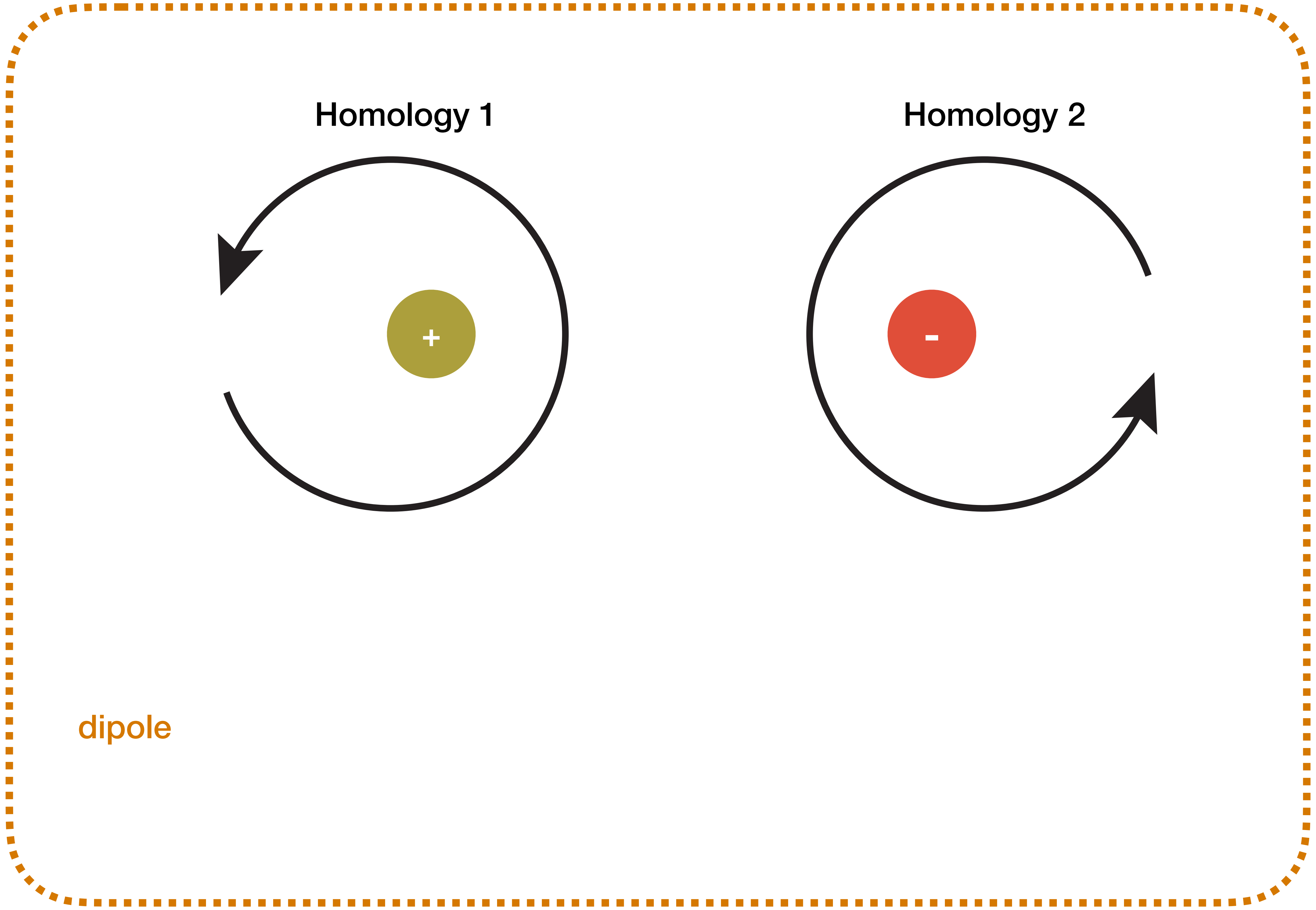

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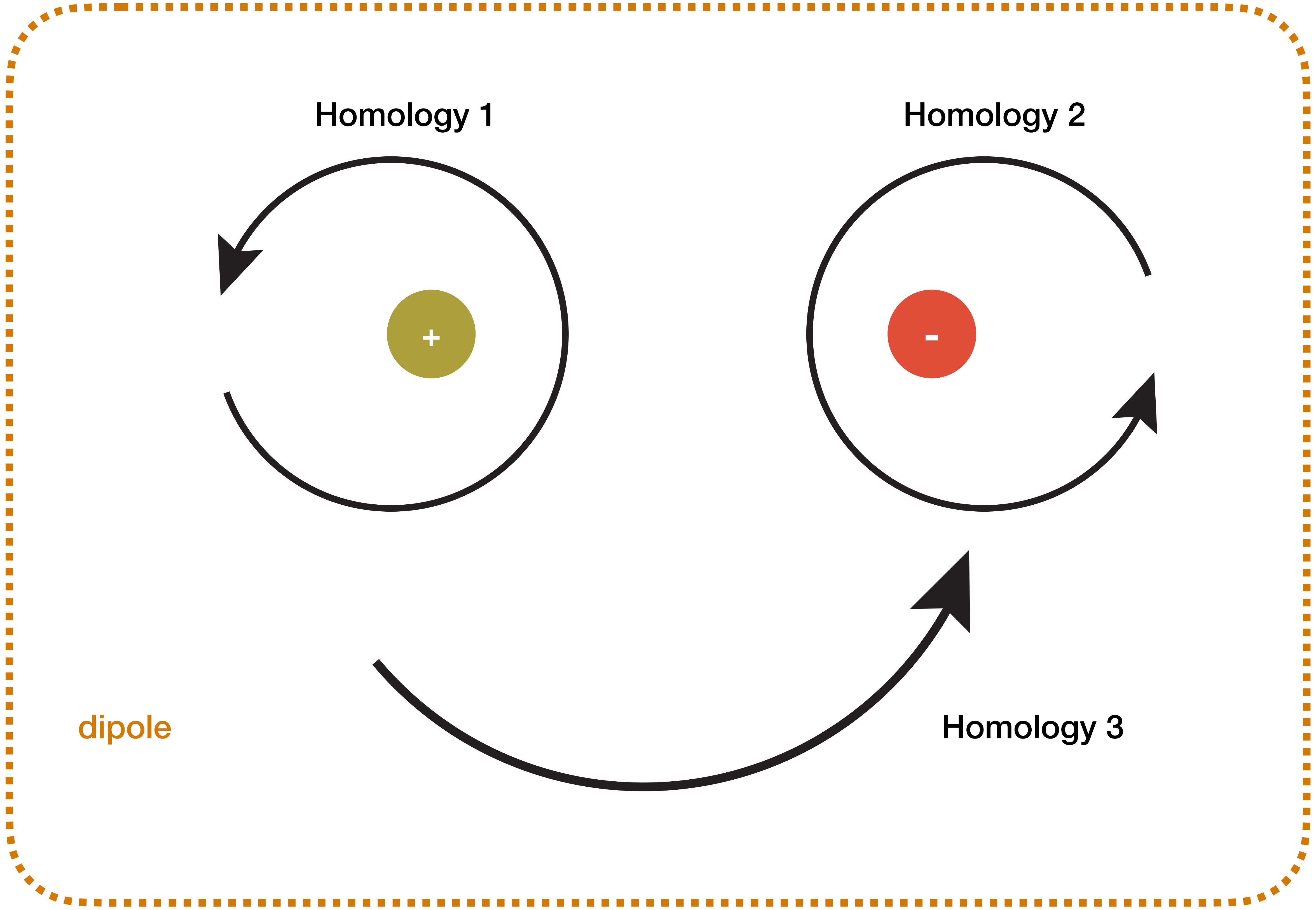

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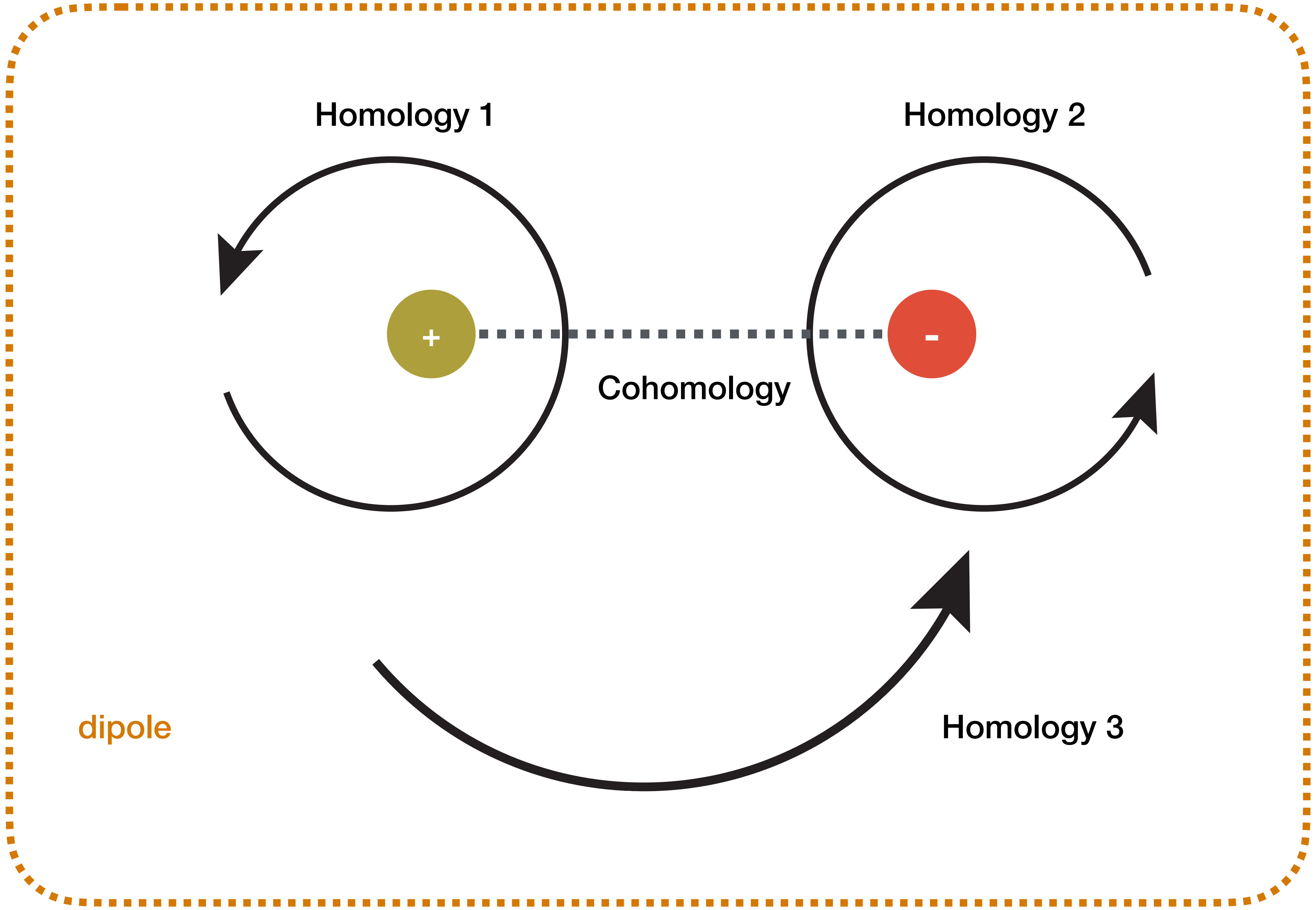
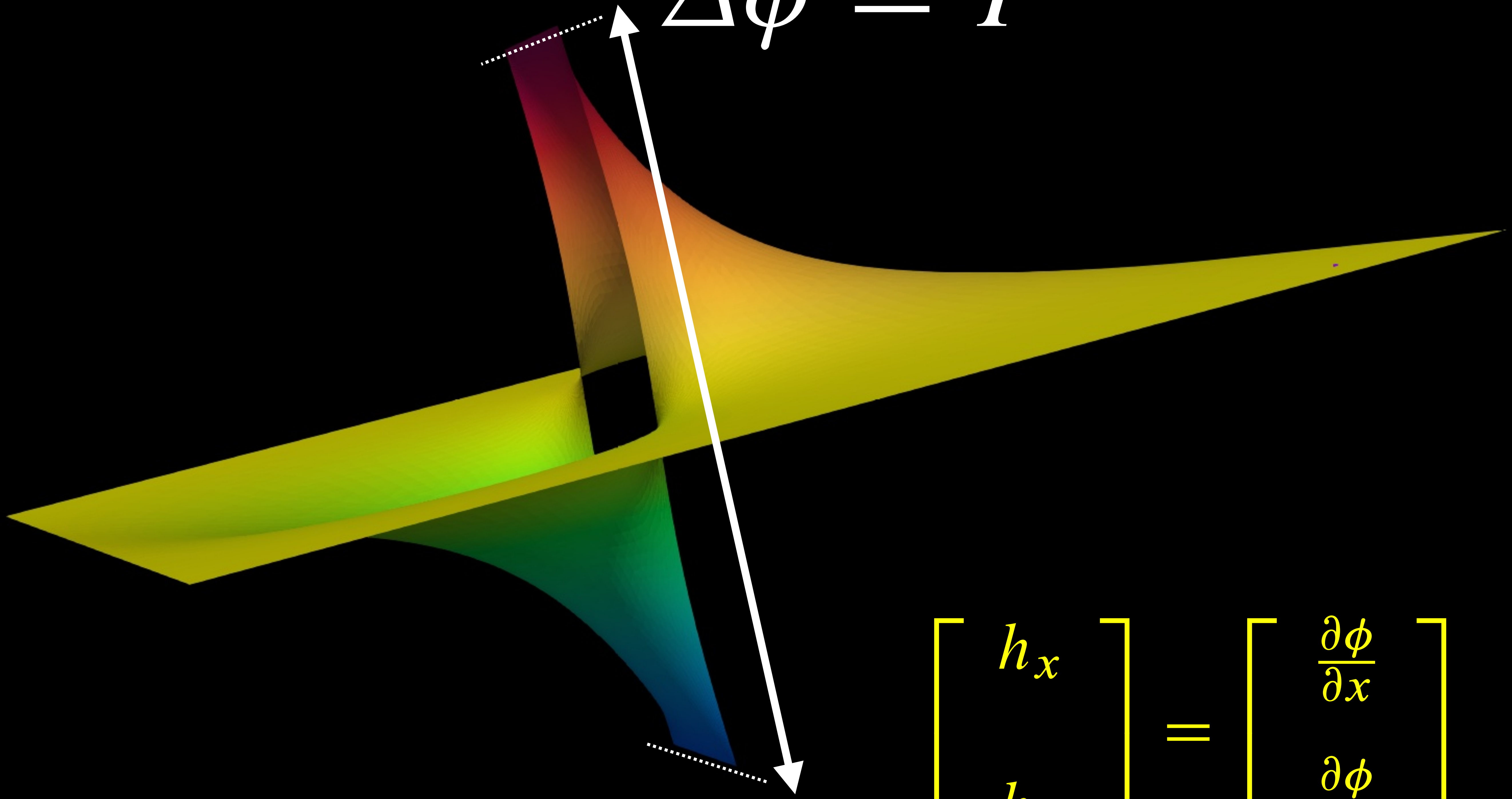
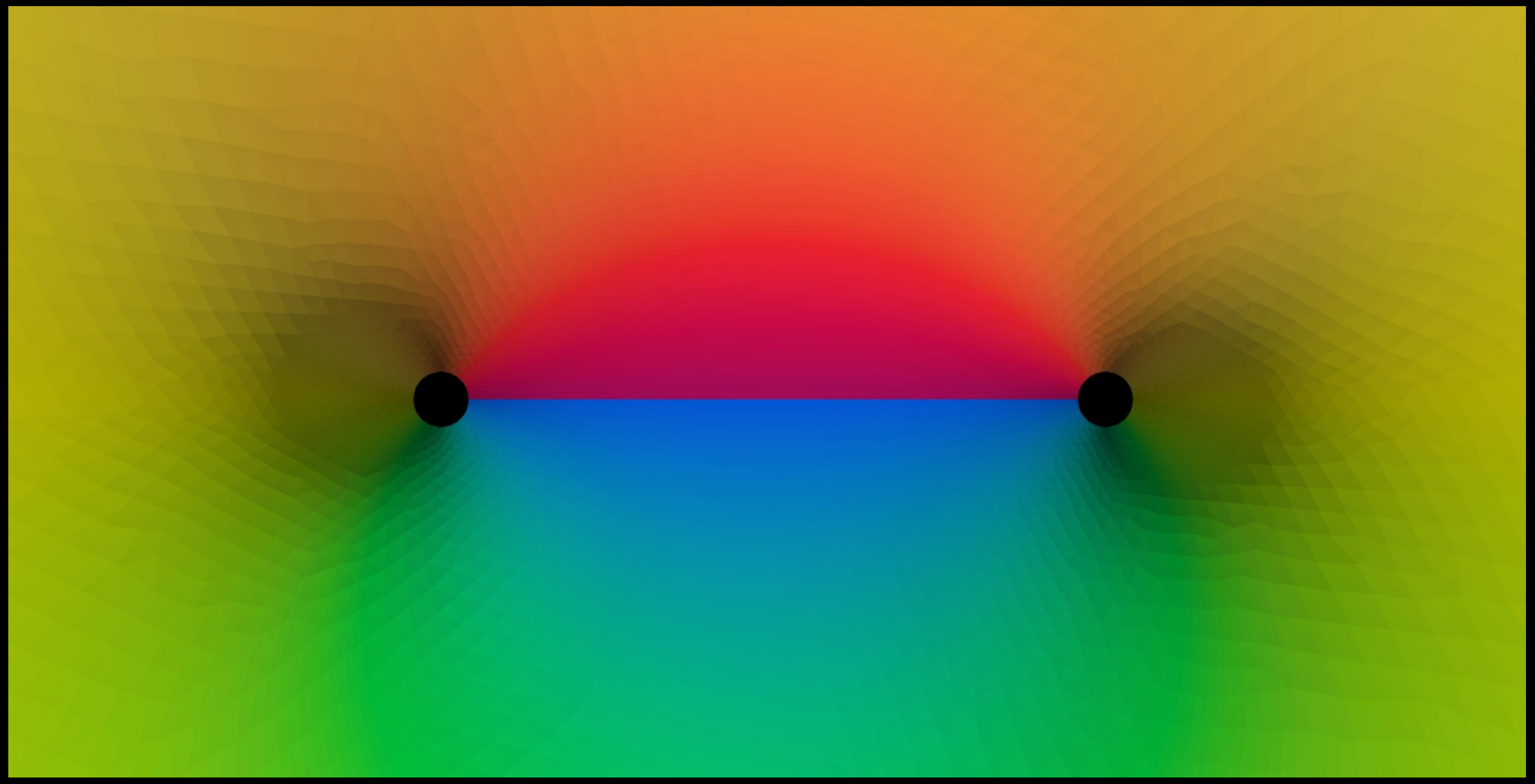


H-ϕ formulation: Homologies



$$\Delta\phi = I$$

$$\begin{bmatrix} h_x \\ h_y \end{bmatrix} = \begin{bmatrix} \frac{\partial\phi}{\partial x} \\ \frac{\partial\phi}{\partial y} \end{bmatrix}$$



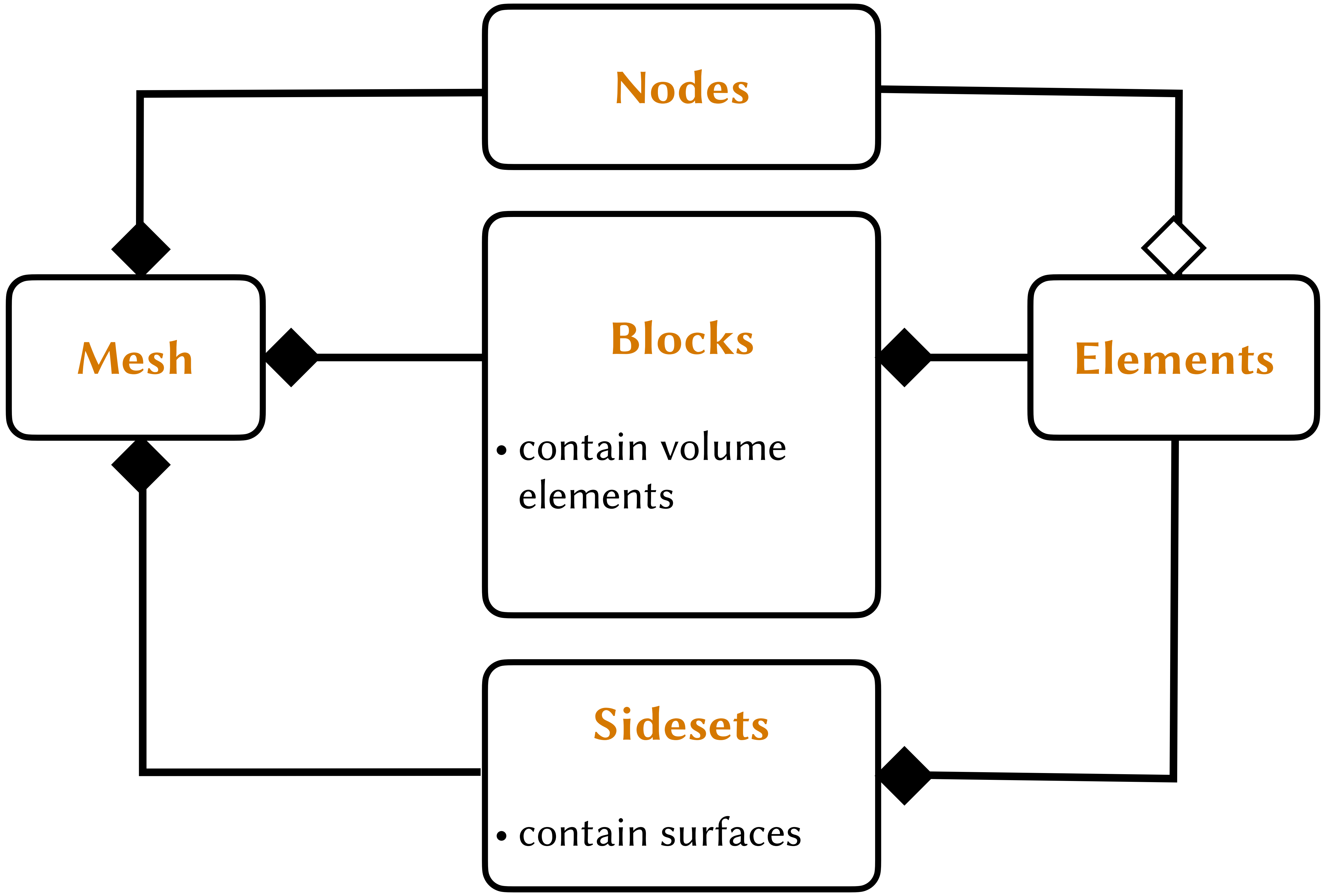
Why a custom codebase?

- need to predict quench behavior of HTS
  ⟶ coupled electromagnetic-thermal simulation

- needs to be sufficiently fast
  ⟶ uses state of the art h-ϕ formulation
  ⟶ uses state of the art solver libraries (STRUMPACK)
  ⟶ ability to run on a HPC node

- need complex geometries and current sharing
  ⟶ uses state of the art thin shell models
  ⟶ **need low level access to data structure**

- need to handle highly nonlinear material properties
  ⟶ custom database
  ⟶ using 3D-B-splines to allow smooth derivatives


Nodes

Mesh

Blocks

- contain volume elements

Elements

Sidesets

- contain surfaces

Why a custom codebase?

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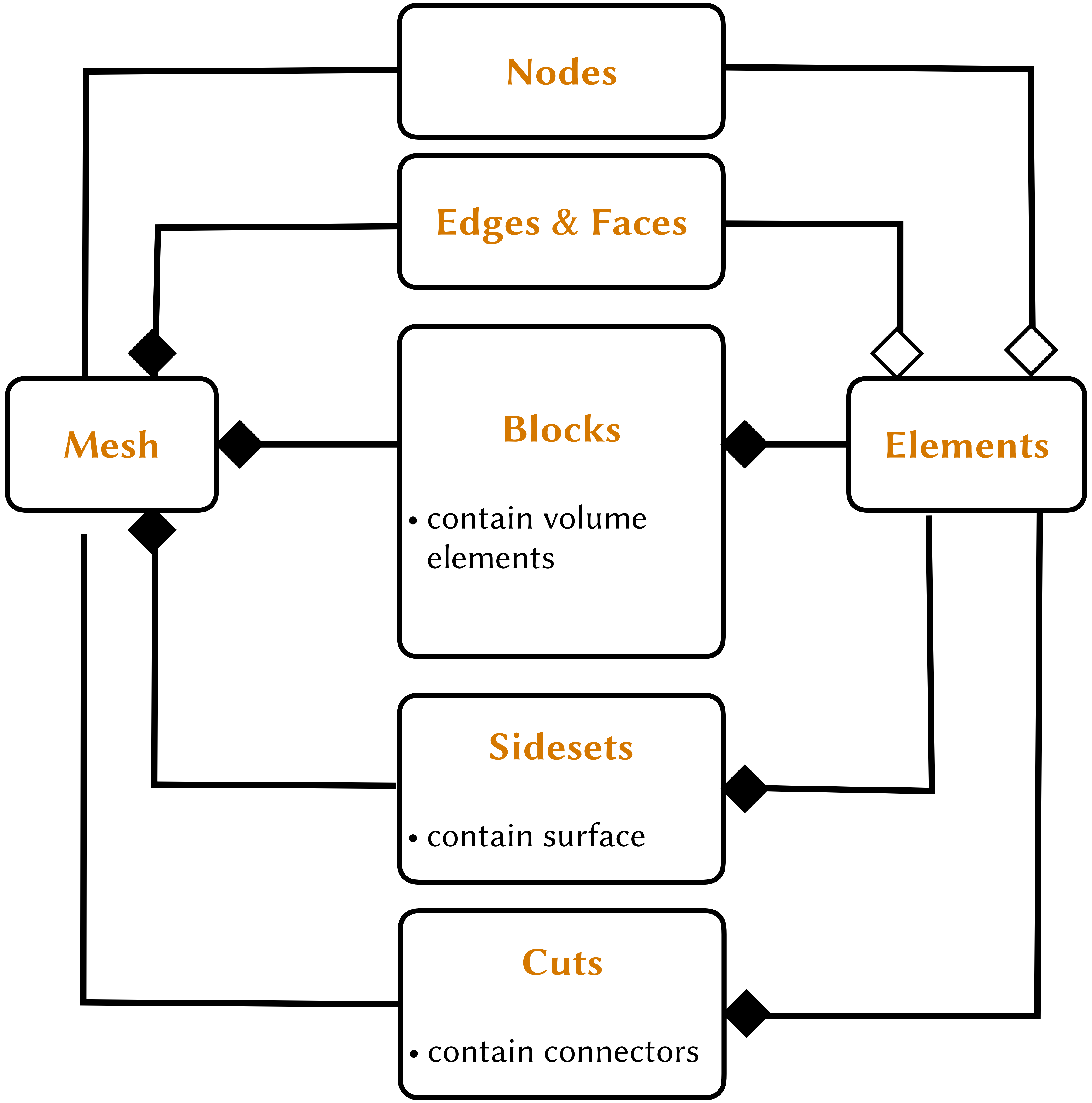
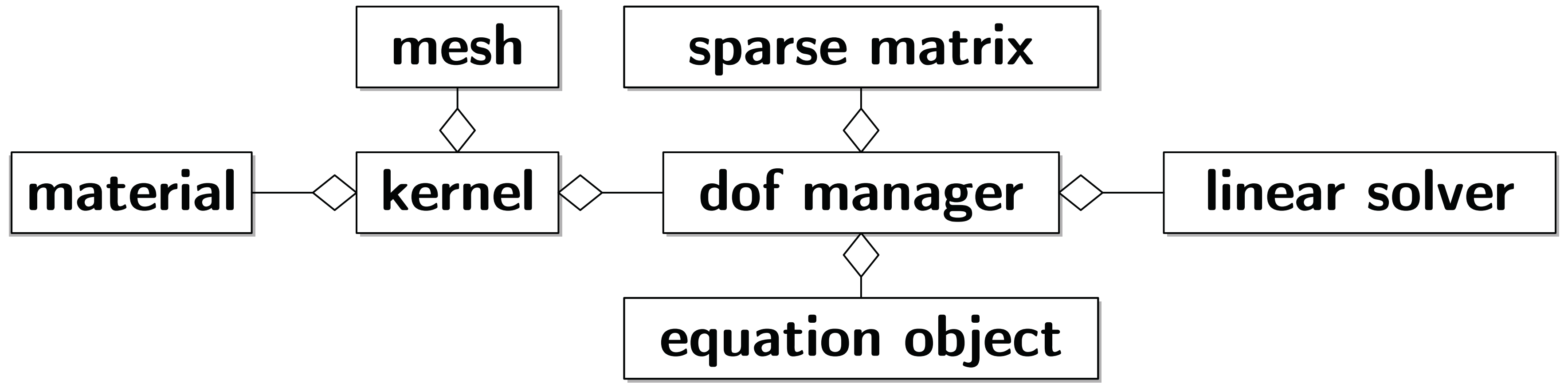
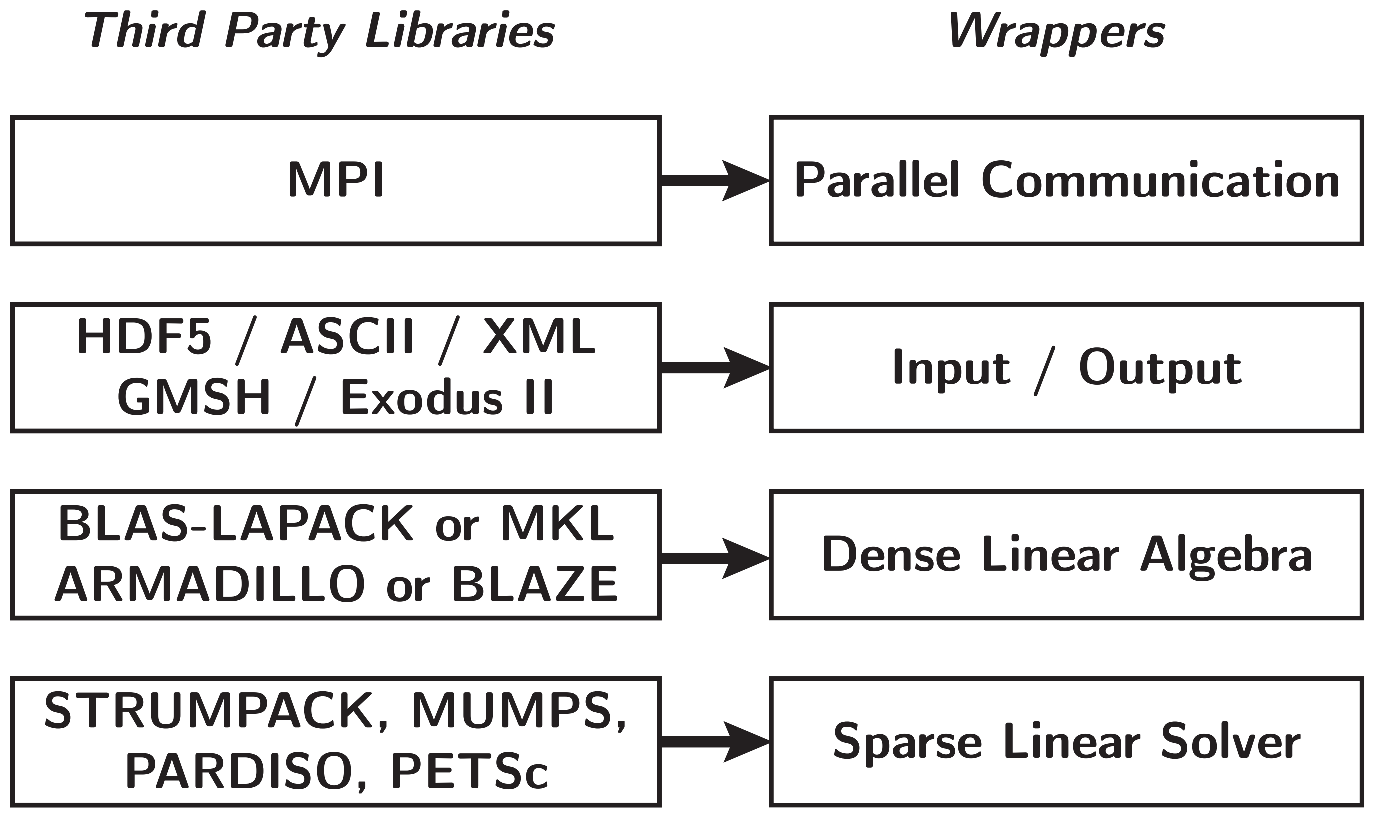


Codebase Philosophy


mesh
sparse matrix
material
kernel
dof manager
linear solver
equation object



Third Party Libraries	*Wrappers*
MPI	Parallel Communication
HDF5 / ASCII / XML GMSH / Exodus II	Input / Output
BLAS-LAPACK or MKL ARMADILLO or BLAZE	Dense Linear Algebra
STRUMPACK, MUMPS, PARDISO, PETSc	Sparse Linear Solver

- **Flexibility & Maintainability**
  - consistent naming scheme of functions and classes
  - lots of comments in code!
  - modular structure

- **simple MATLAB-like dense linear algebra**
  - through ARMADILLO or BLAZE

- **text based interface tailored to magnet development**
  - ability to write scripts in BASH and Python

- **utilization of community software**
  - use open source data formats (GMSH, HDF5, Exodus II)
  - link against community libraries: MUMPS, PETSc, STRUMPACK, ...

- **be open source once mature**
  - Berkeley Lab specific BSD-3 like license

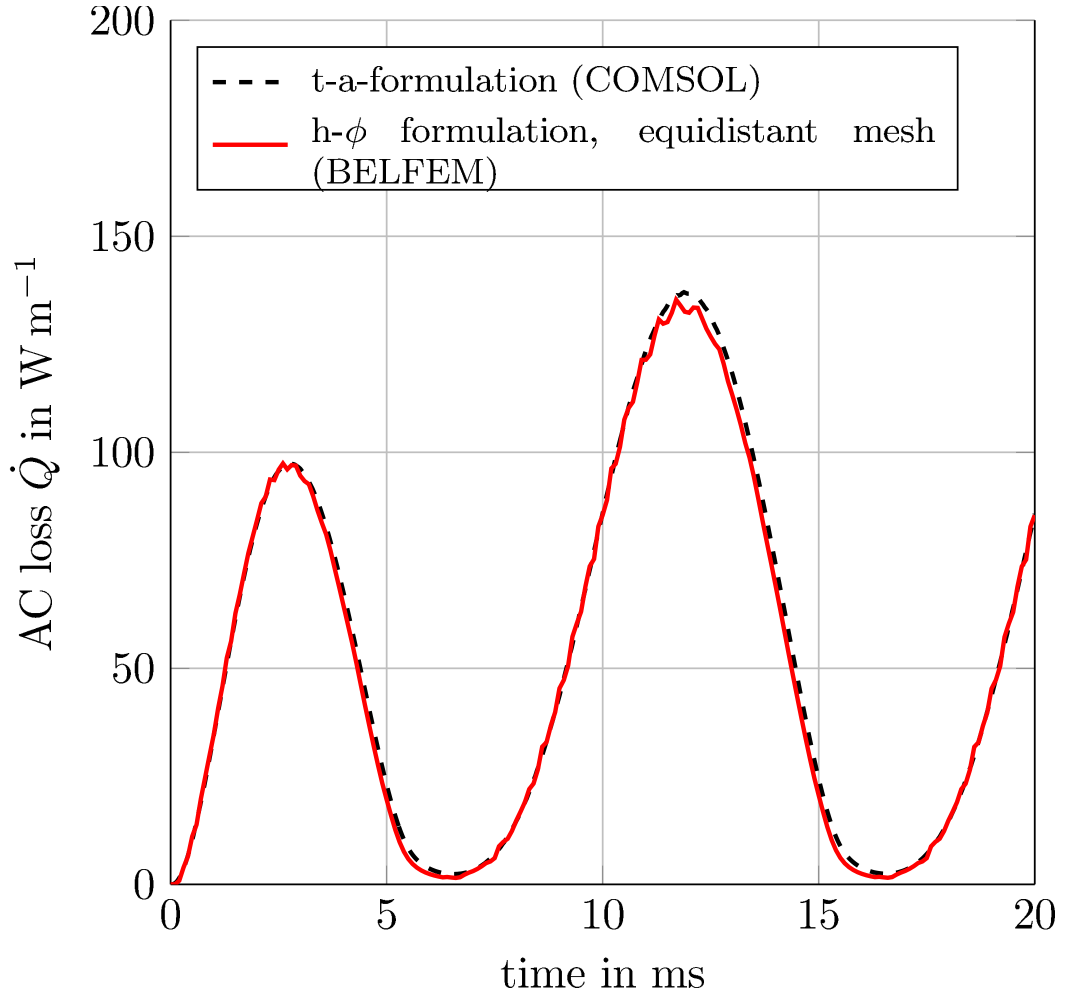
Current Status

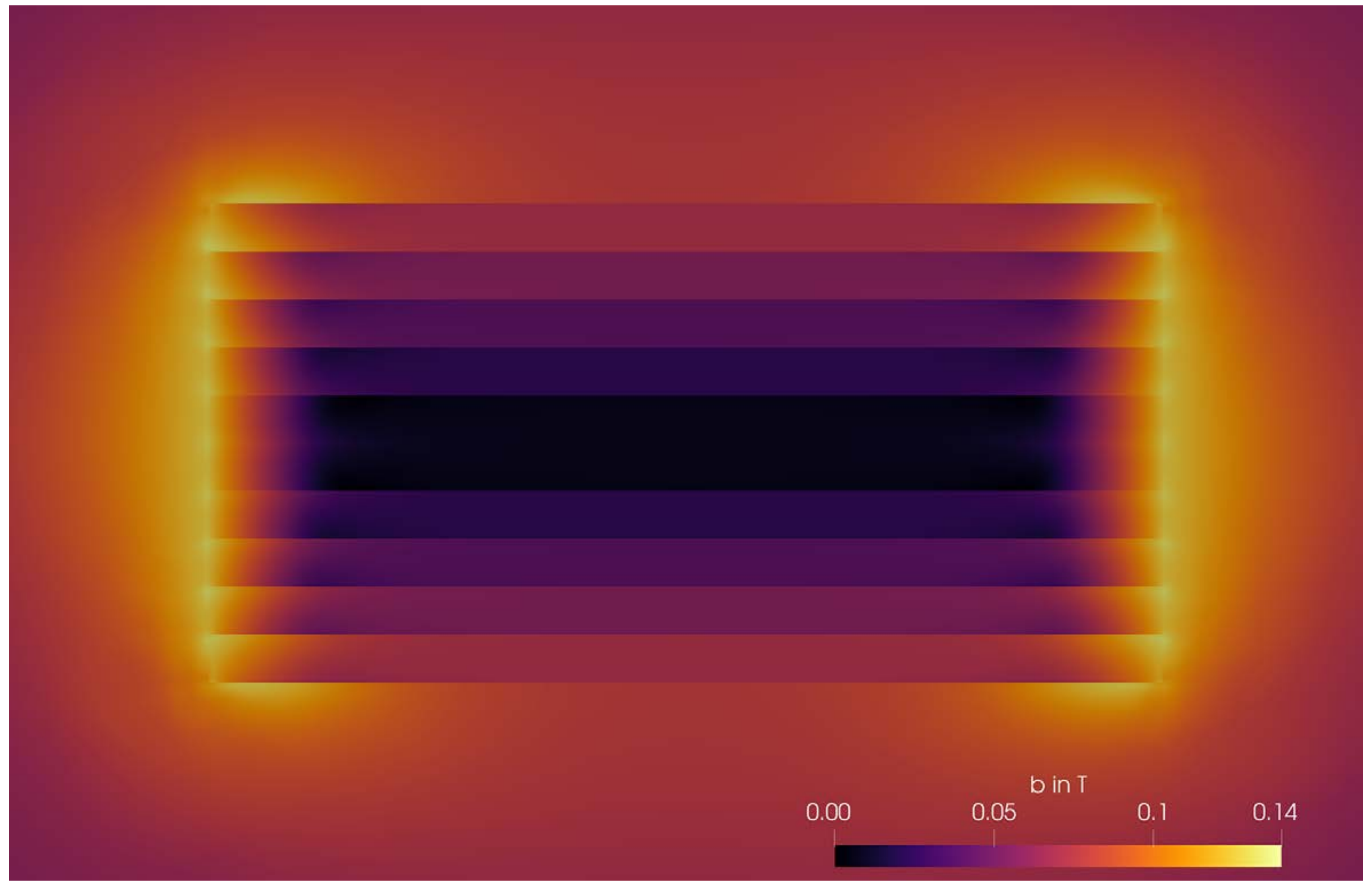
Thin Shell Formulation: 2D Results 2023

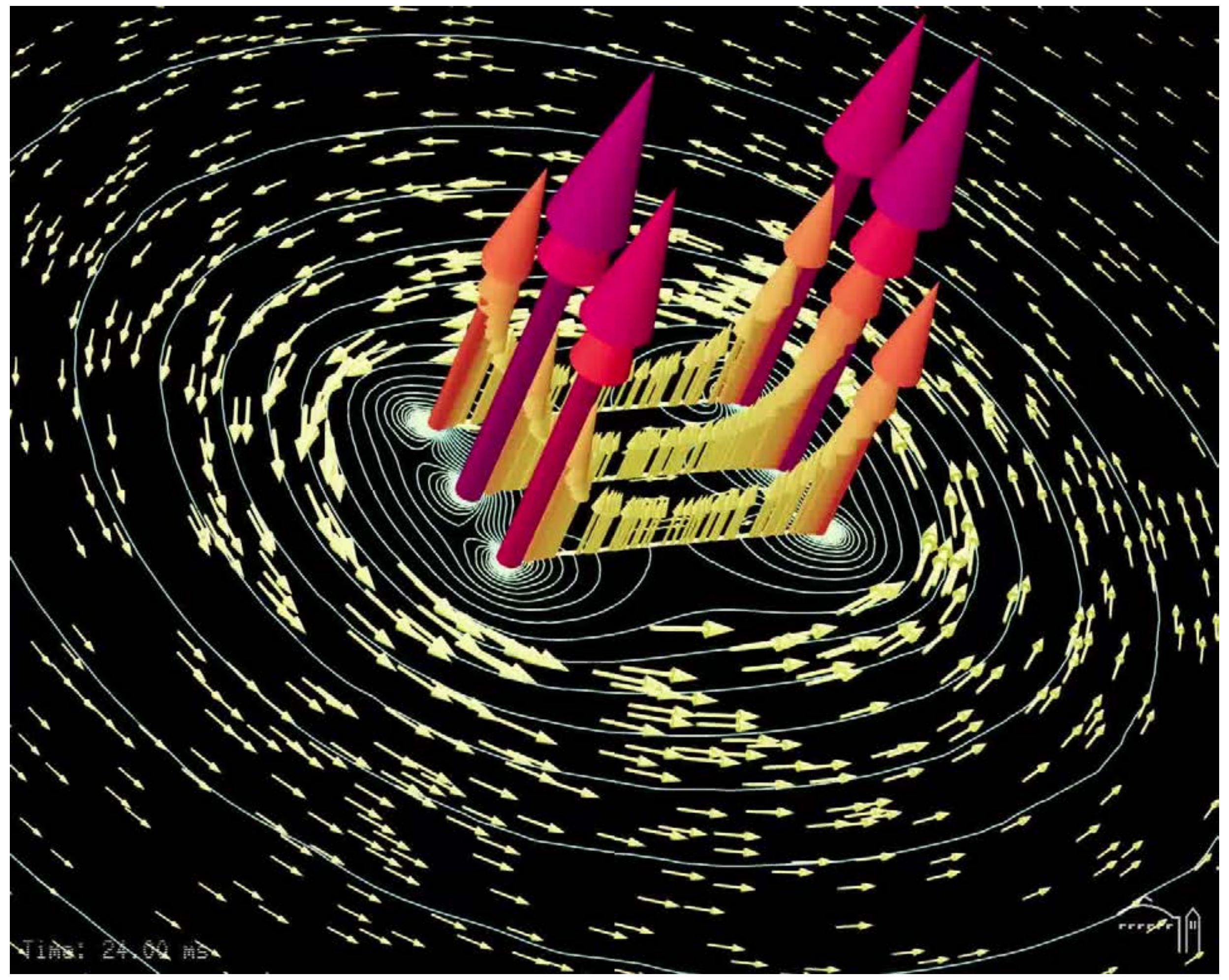
Published Paper in SuST 2023

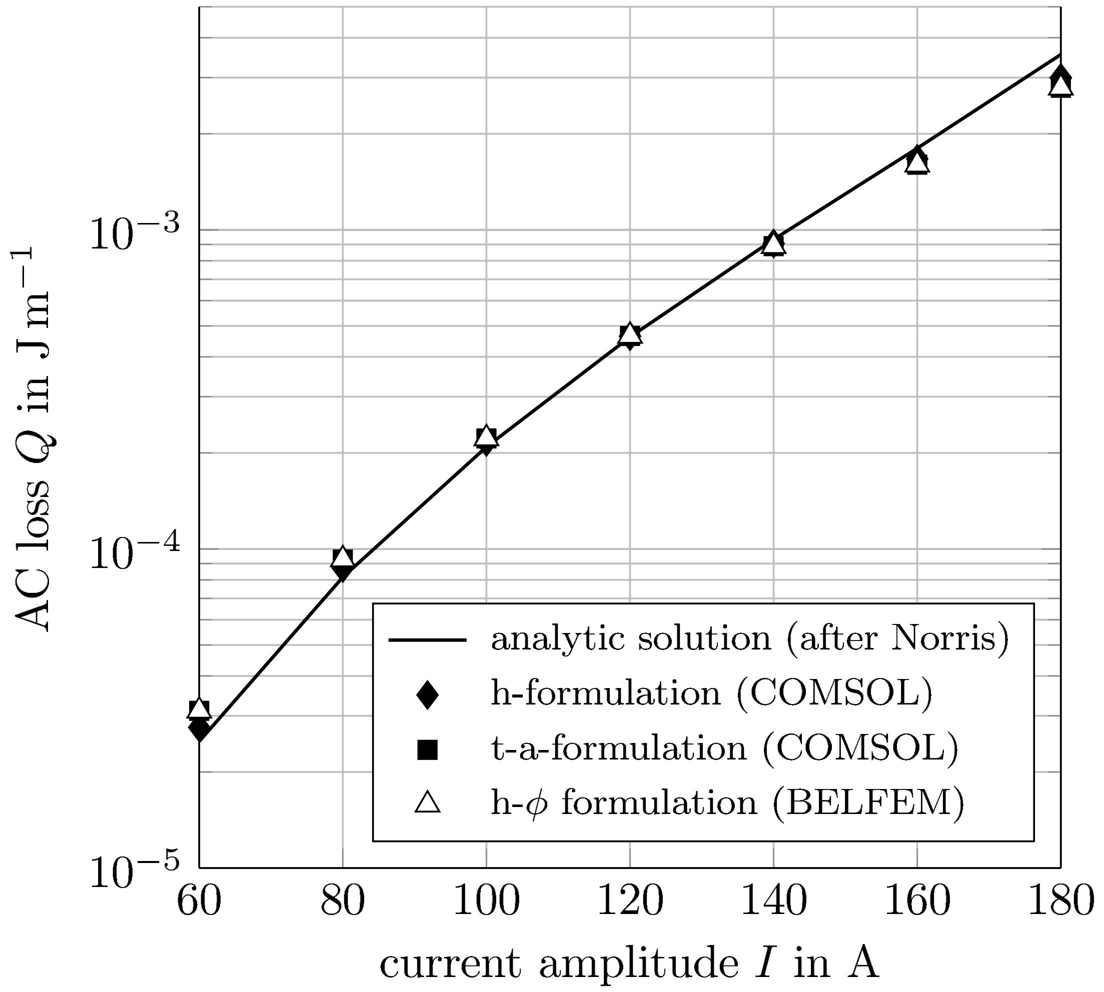
- Christian Messe, Berkeley Lab
- Nico Riva, MIT
- Sofia Viarengo, Politechnico di Torino
- Gregory Giard & Frédéric Sirois, Polytechnique Montreal

- validated against analytical methods + COMSOL / GetDP
- first research promises faster and more detailed results than other established methods such as t-a

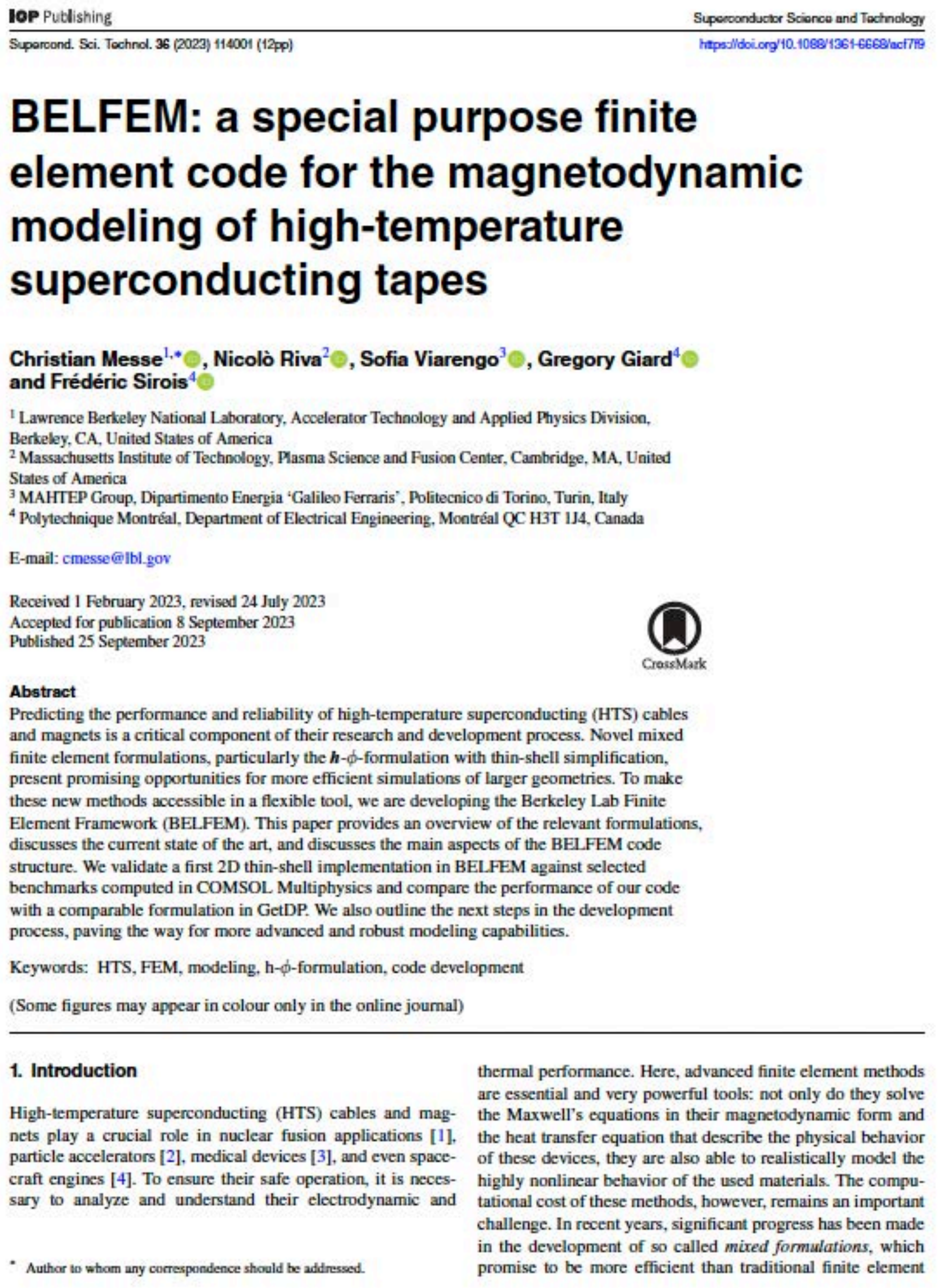








Thin Shell Formulation: 2D Results 2023



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BELFEM: a special purpose finite element code for the magnetodynamic modeling of high-temperature superconducting tapes

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[2] Massachusetts Institute of Technology, Plasma Science and Fusion Center, Cambridge, MA, United States of America
[3] MAHTEP Group, Dipartimento Energia 'Galileo Ferraris', Politecnico di Torino, Turin, Italy
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Abstract
Predicting the performance and reliability of high-temperature superconducting (HTS) cables and magnets is a critical component of their research and development process. Novel mixed finite element formulations, particularly the $\boldsymbol{h}$-ϕ-formulation with thin-shell simplification, present promising opportunities for more efficient simulations of larger geometries. To make these new methods accessible in a flexible tool, we are developing the Berkeley Lab Finite Element Framework (BELFEM). This paper provides an overview of the relevant formulations, discusses the current state of the art, and discusses the main aspects of the BELFEM code structure. We validate a first 2D thin-shell implementation in BELFEM against selected benchmarks computed in COMSOL Multiphysics and compare the performance of our code with a comparable formulation in GetDP. We also outline the next steps in the development process, paving the way for more advanced and robust modeling capabilities.

Keywords: HTS, FEM, modeling, h-ϕ-formulation, code development

(Some figures may appear in colour only in the online journal)

1. Introduction

High-temperature superconducting (HTS) cables and magnets play a crucial role in nuclear fusion applications [1], particle accelerators [2], medical devices [3], and even spacecraft engines [4]. To ensure their safe operation, it is necessary to analyze and understand their electrodynamic and thermal performance. Here, advanced finite element methods are essential and very powerful tools: not only do they solve the Maxwell's equations in their magnetodynamic form and the heat transfer equation that describe the physical behavior of these devices, they are also able to realistically model the highly nonlinear behavior of the used materials. The computational cost of these methods, however, remains an important challenge. In recent years, significant progress has been made in the development of so called *mixed formulations*, which promise to be more efficient than traditional finite element

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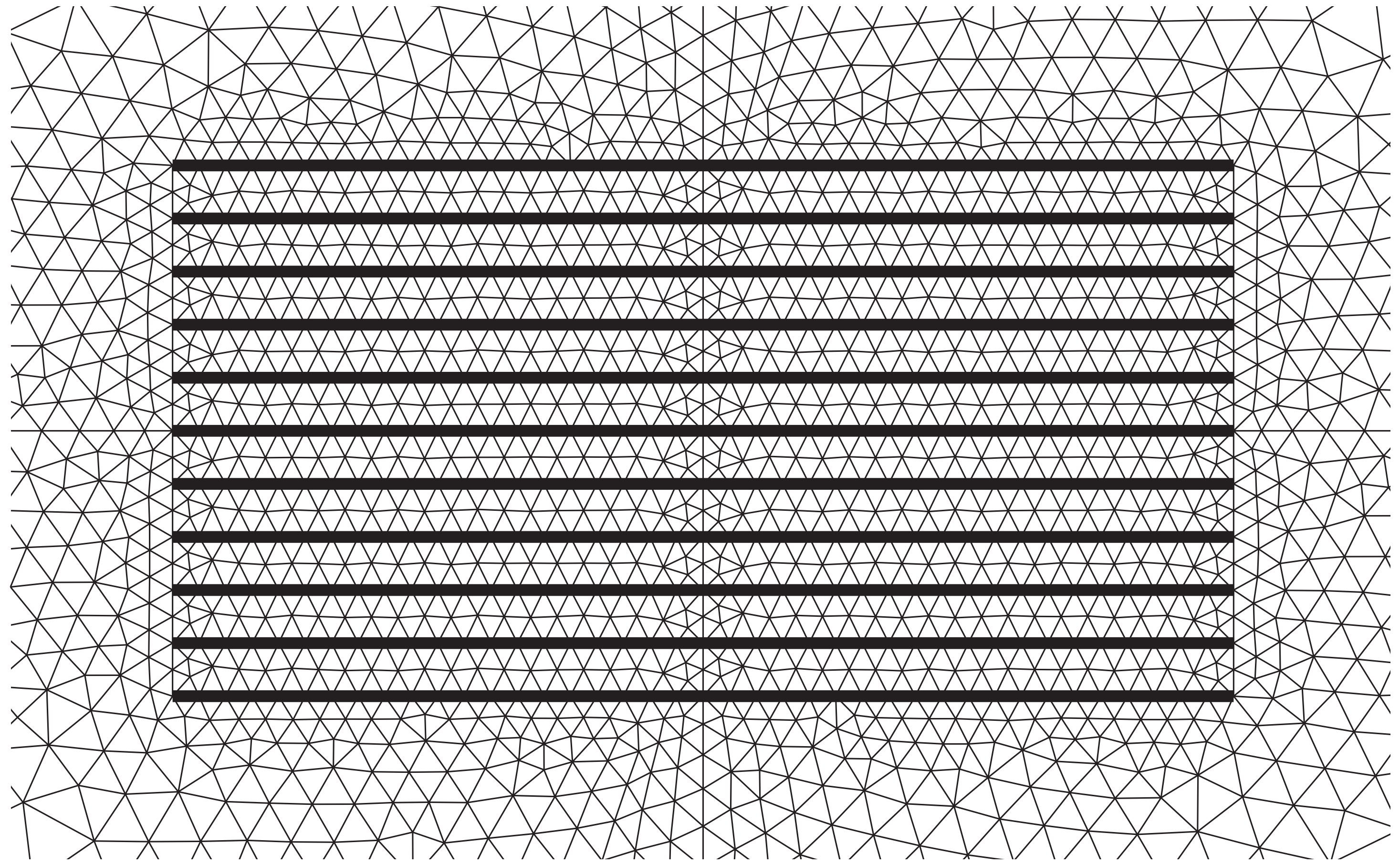
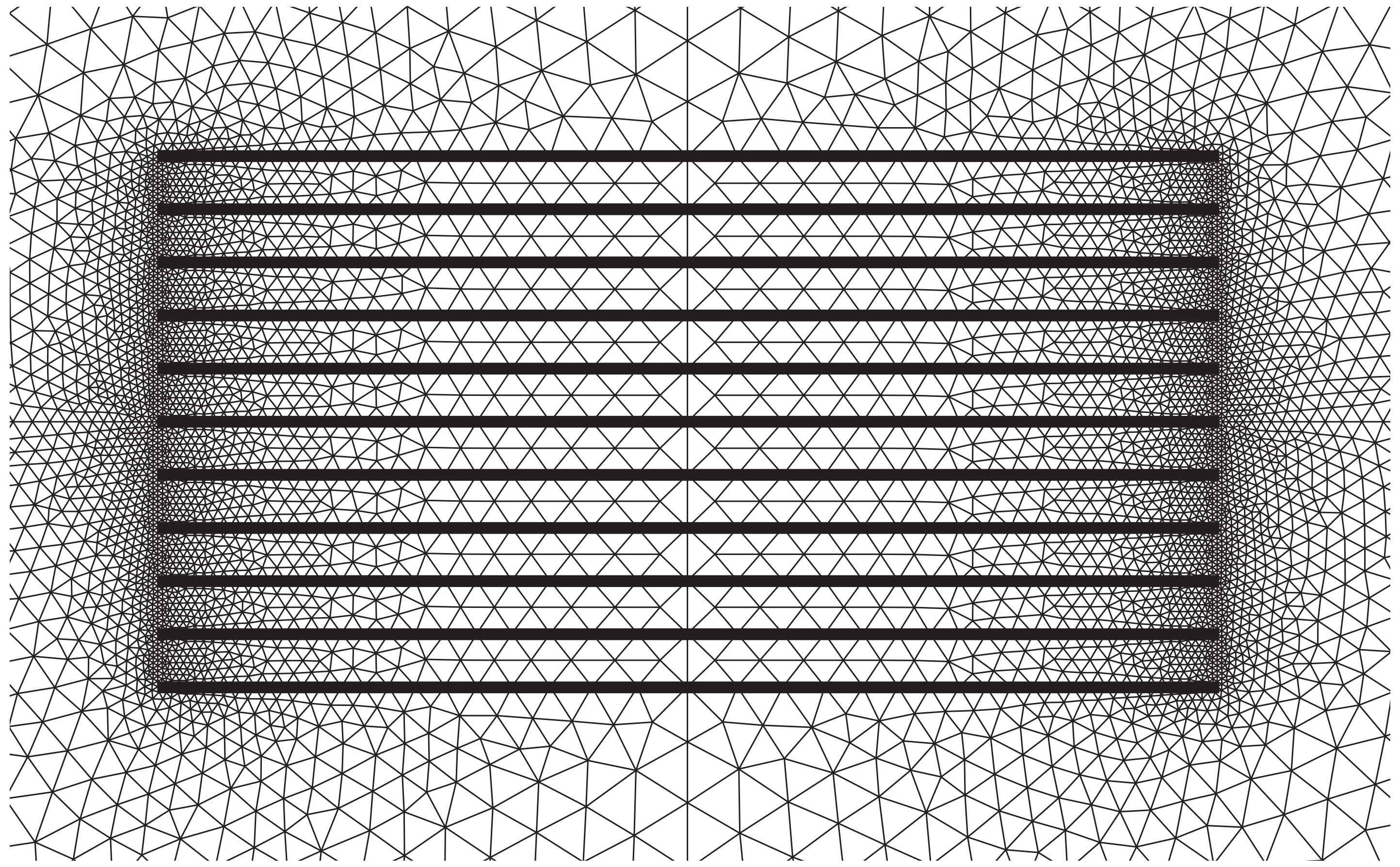
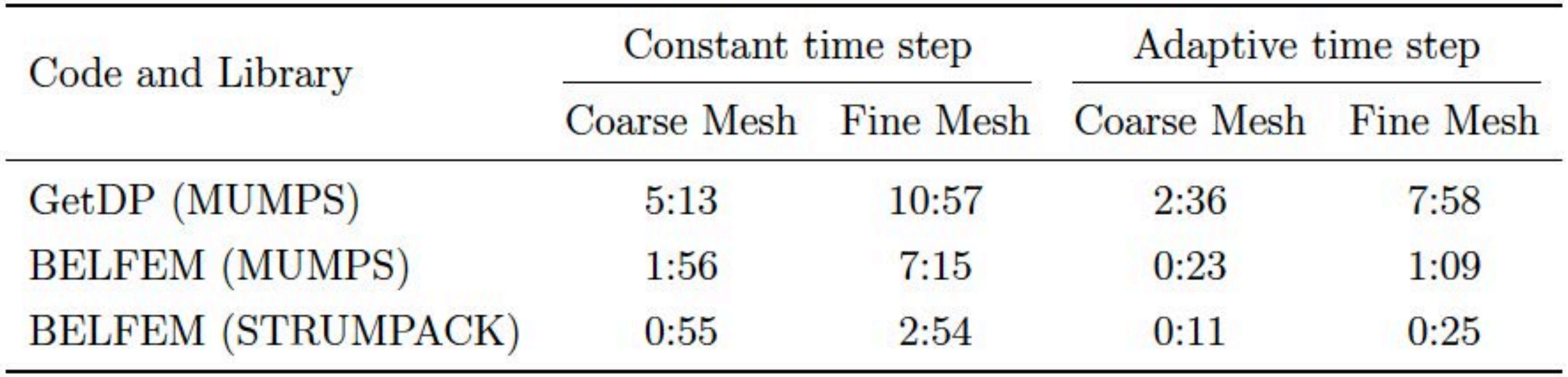
[*] Author to whom any correspondence should be addressed.







Code and Library	Constant time step		Adaptive time step	
	Coarse Mesh	Fine Mesh	Coarse Mesh	Fine Mesh
GetDP (MUMPS)	5:13	10:57	2:36	7:58
BELFEM (MUMPS)	1:56	7:15	0:23	1:09
BELFEM (STRUMPACK)	0:55	2:54	0:11	0:25

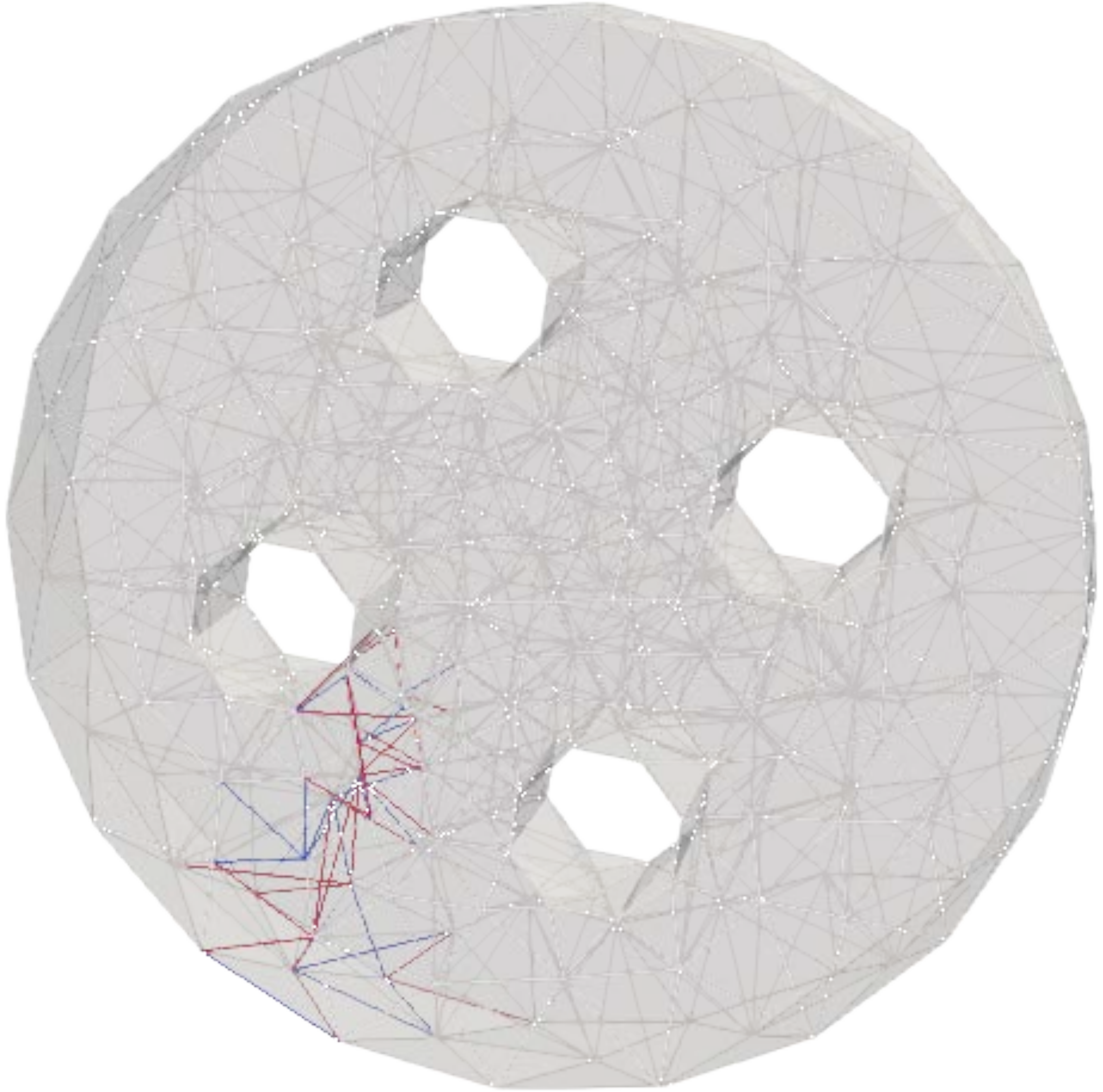
Current Efforts

PhD Student Gregory Giard (Polytechnique Montreal):

- visiting scholar at LBL from 01/23-06/23

- contribution to adaptive time stepping method
- development of 3D thermal conduction model

- implementing automated cohomology computation in 3D
- automated identification of weak BCs based on user provided currents ("the user shall not worry about cohomoligies")



POLYTECHNIQUE MONTRÉAL
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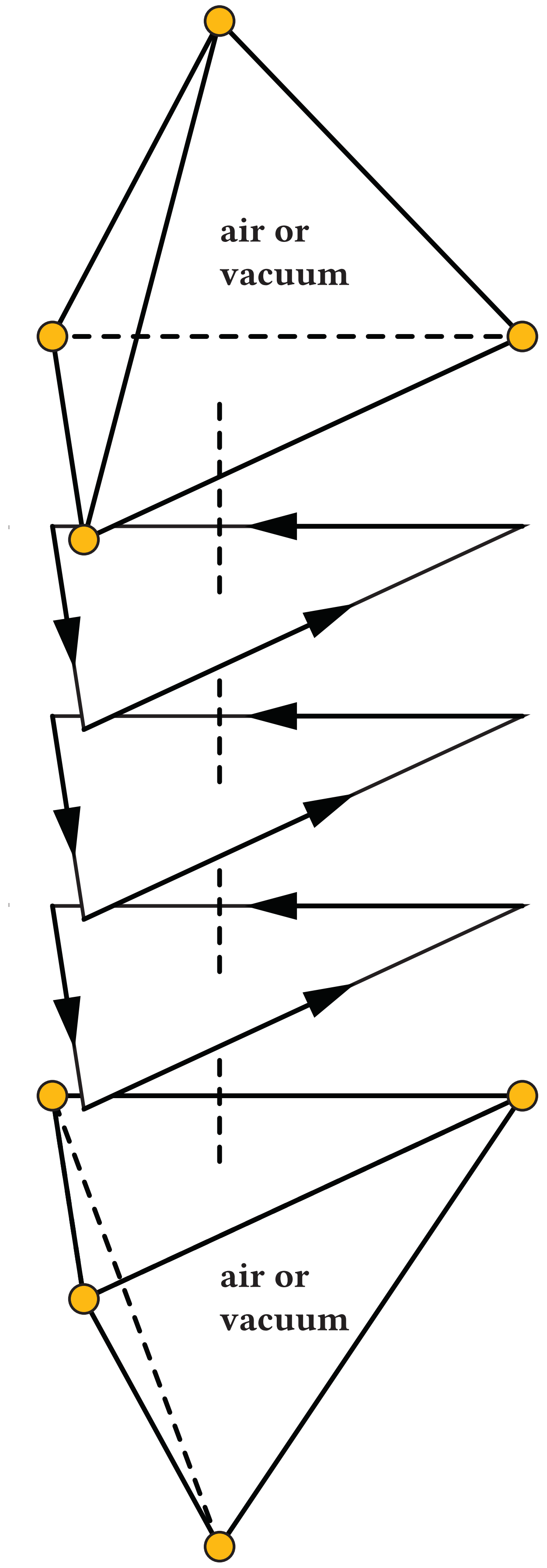
Current Efforts

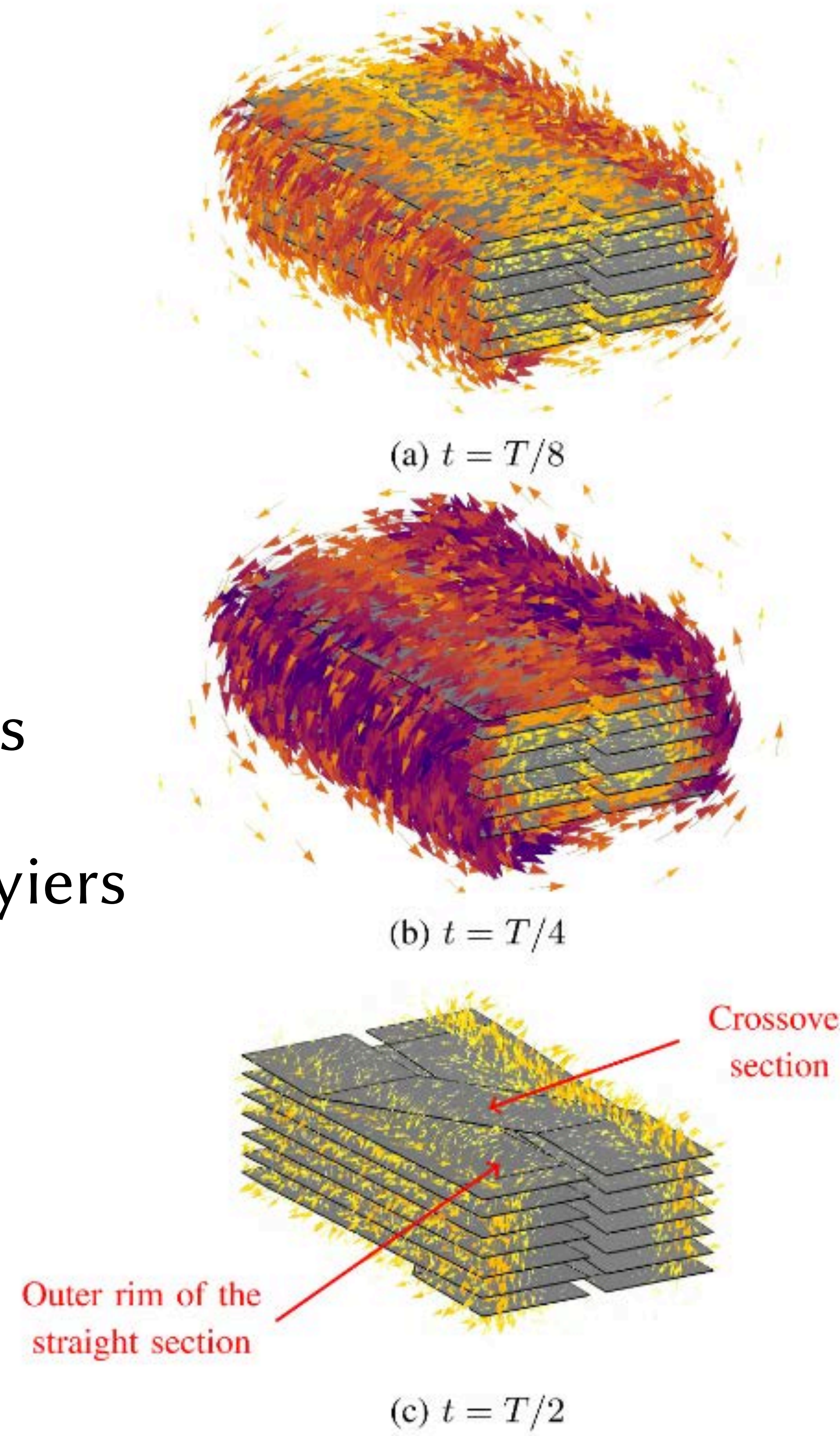


Goal:

- model a thin shell tapes tack in 3D after Alves et Al, 2022
- extend model to encompass solder and thermal model
- be able to do the coupled EM-Thermal quenching model by end of the year

Roadmap:

- overhaul data structure for simplified programming of weak governing equations
- implement "condensation" of degrees of freedom → get rid of Lagrange multiplyiers
- first benchmark with 3D tapestack
- implement solder and thermal model
- benchmark involving quench
- address contact sharing

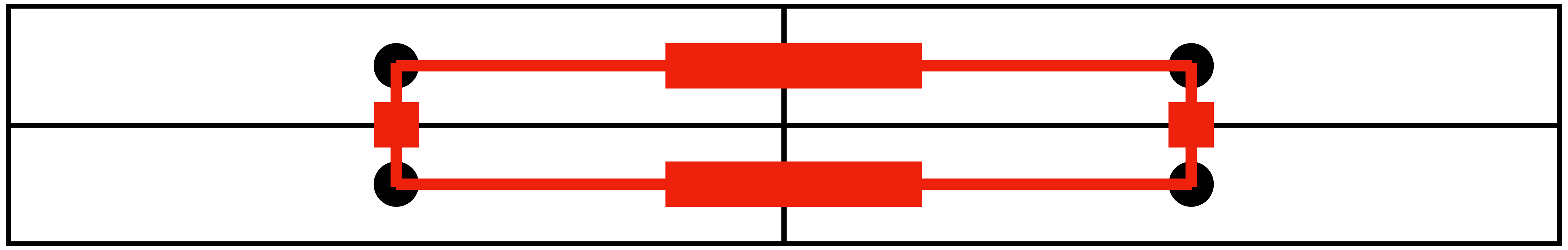

(a) $t = T/8$
(b) $t = T/4$
(c) $t = T/2$

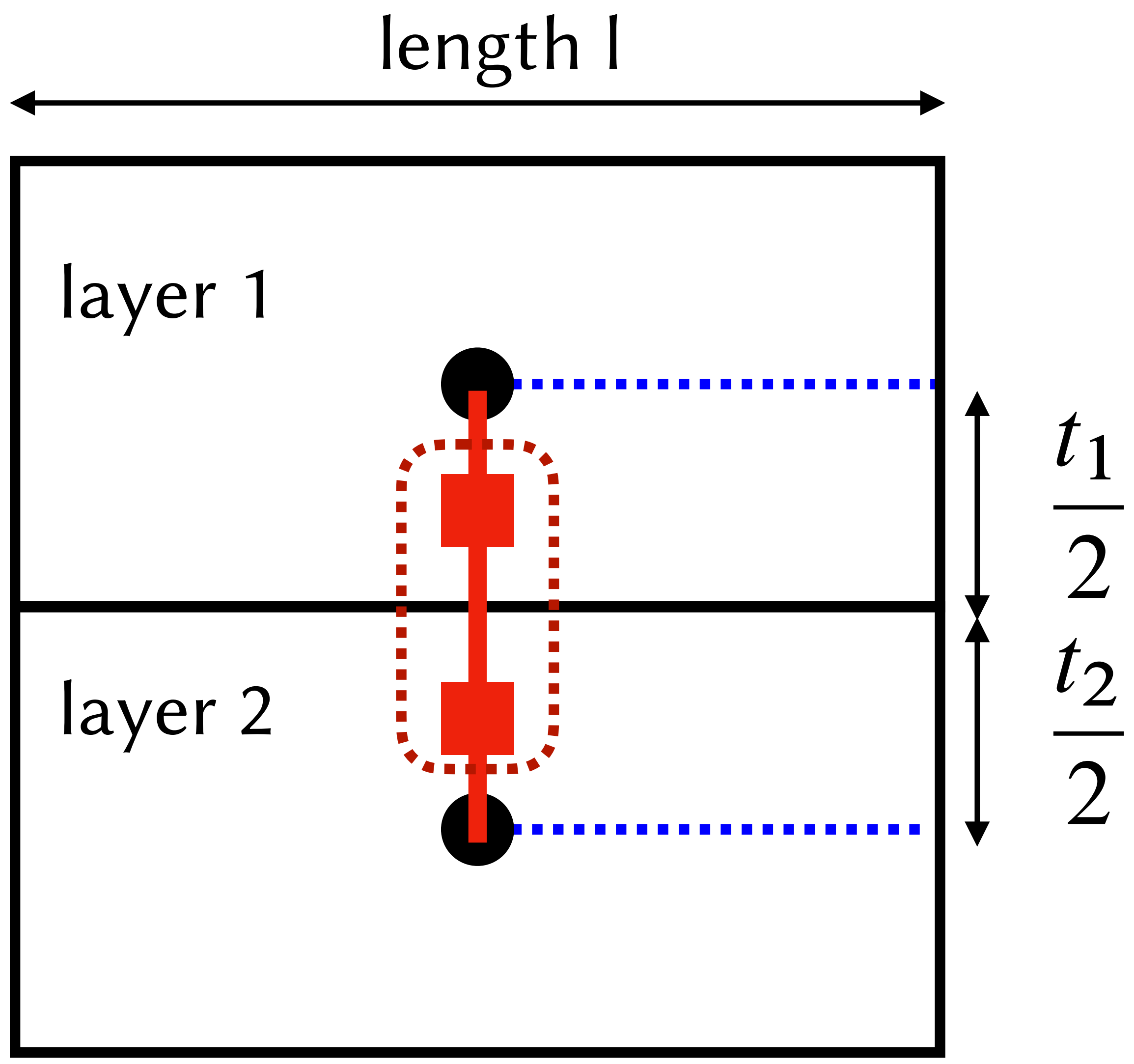
[Alves et al, 10.1109/TASC.2022.3143076]

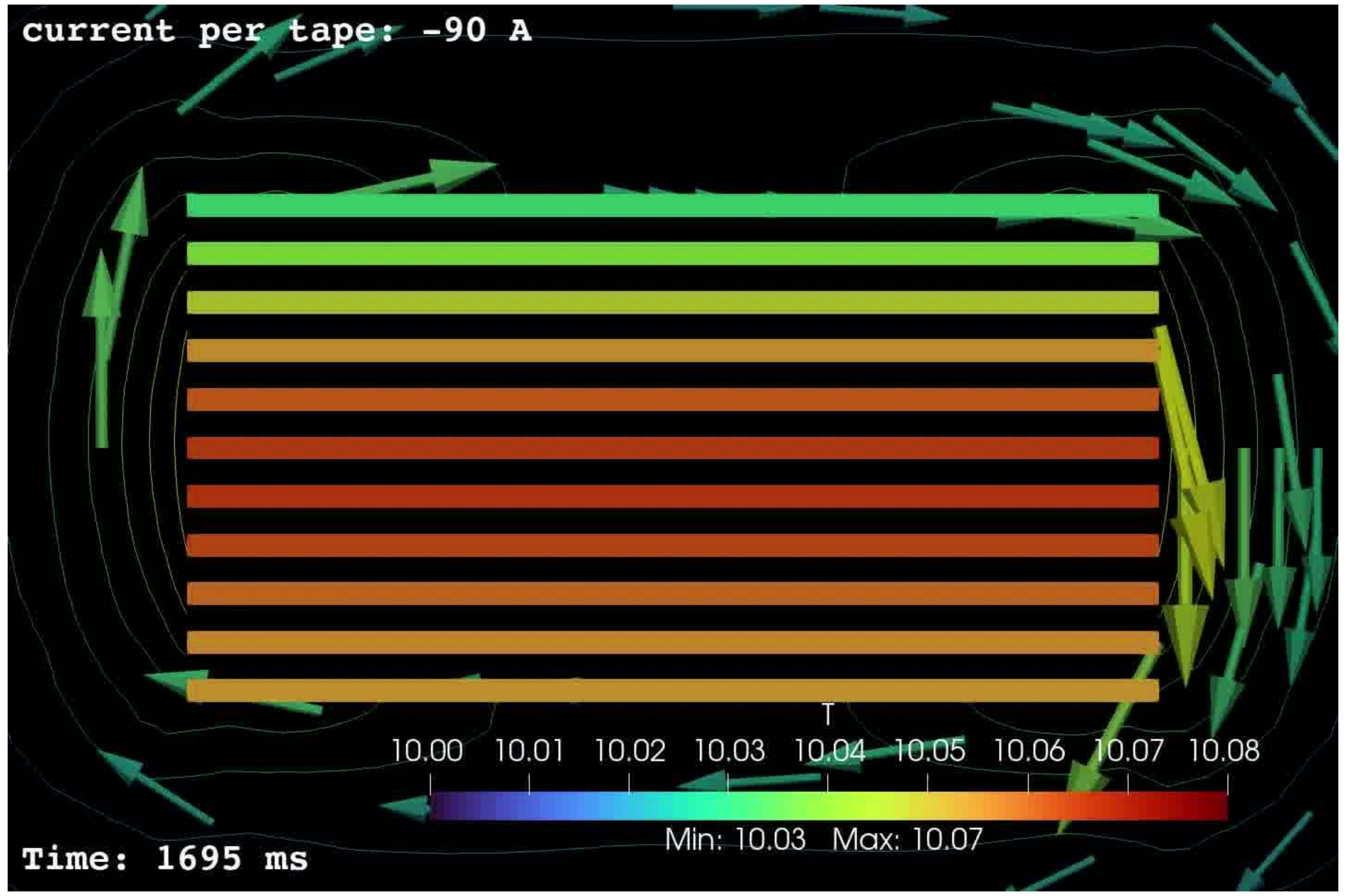
Work in Progress: Thermal Coupling

Work in progress: thermal coupling

- linear discretization + mass lumping collapses finite-element method to resistor grid
- can be first order even if Maxwell is second order
- degrees of freedom sit on the edges
  → assumes constant temperature per layer per element

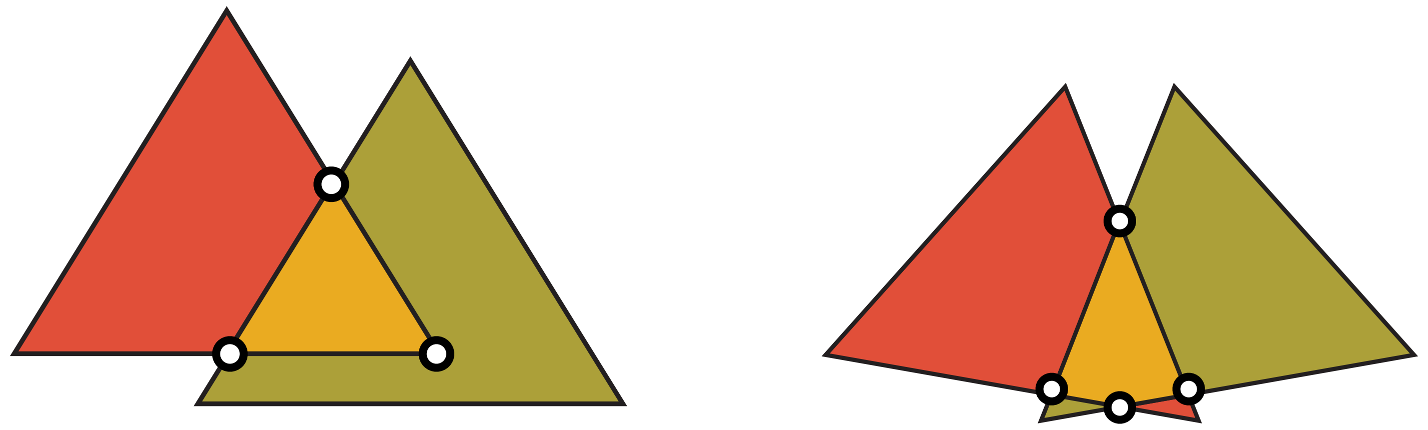


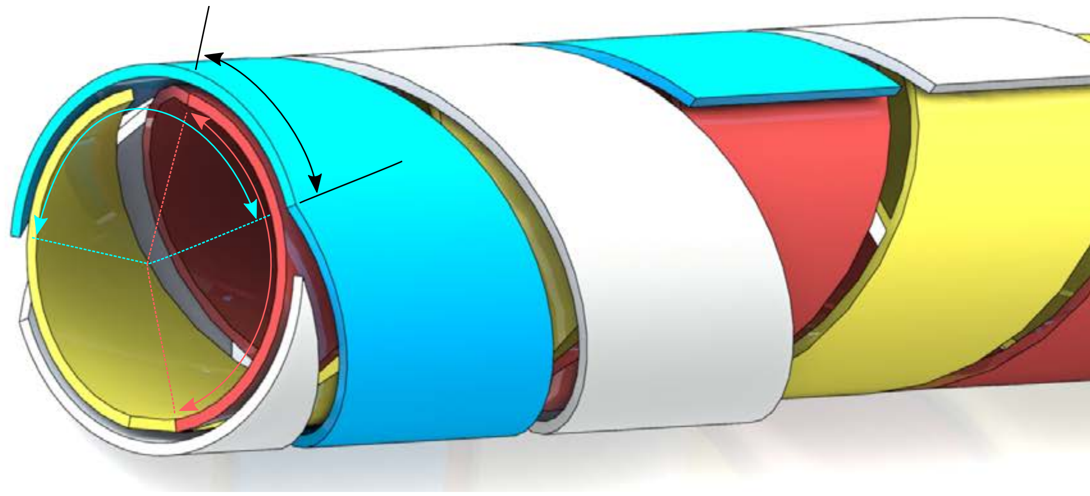




Work in Progress: Intertape current sharing

Toughts on Intertape Current Sharing





What we know:

- current sharing does not work in 2D!
- need a geometry preprocessor to compute overlapping surfaces
- coupling could work over the electric field

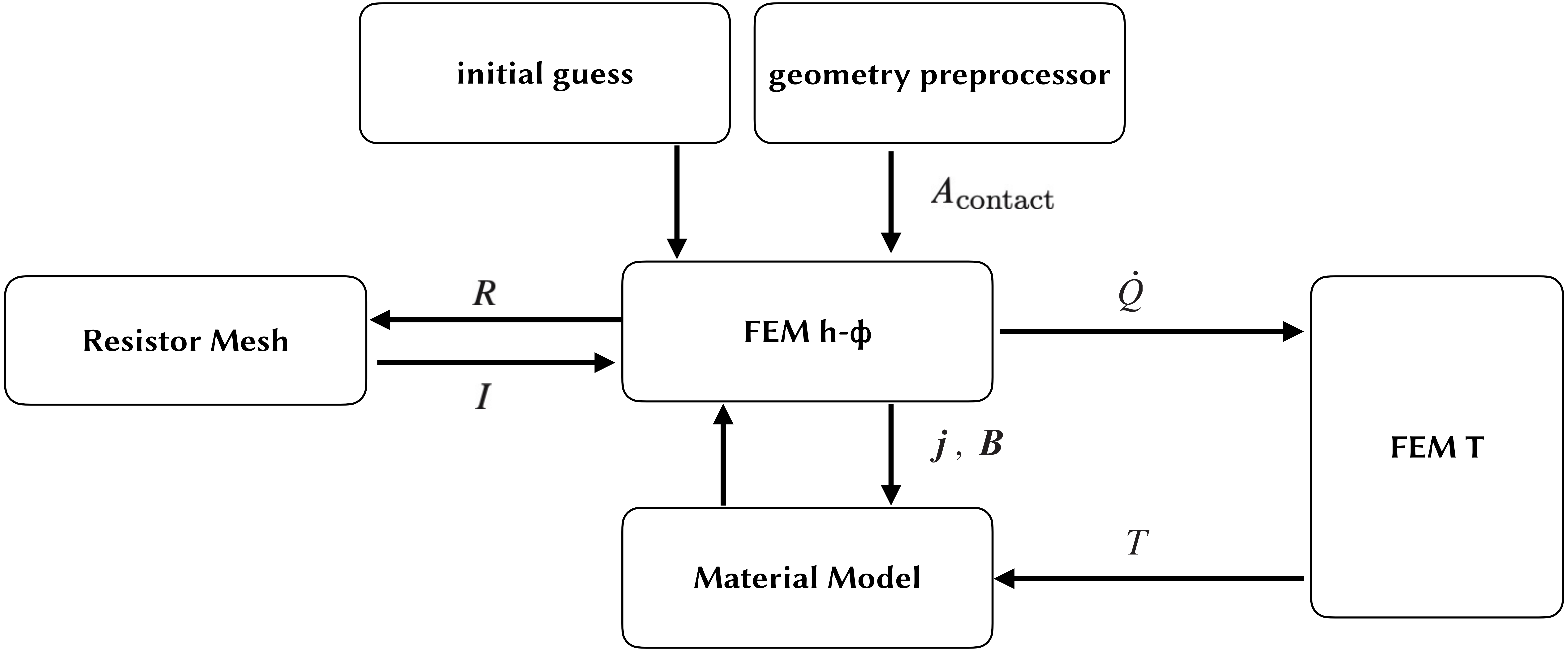
$$\int_\Omega \boldsymbol{\delta h}^{\mathbf{T}} \frac{\partial\left(\mu \boldsymbol{h}\right)}{\partial t}\,\mathrm{d}V + \int_\Omega \boldsymbol{\delta h}^{\mathbf{T}} \times \nabla^{\mathbf{T}} \rho\, \nabla \times \boldsymbol{h}\,\mathrm{d}V + \int_\Gamma \boldsymbol{\delta h}^{\mathbf{T}} \boldsymbol{n} \times \boxed{\boldsymbol{e}}\,\mathrm{d}S = 0$$

$$\begin{gathered}\Big\uparrow \\ \boldsymbol{e} = \rho\, \nabla \times \boldsymbol{h}\end{gathered}$$

Open Questions:

- Does contact resistivity affect the validity of cohomoligies?
- Is the model complete and free of contradiction?
- Does the model need to be extended with a resistor mesh ?
  - ⟶ modify current boundary conditions!
  - ⟶ use same resistor mesh as for thermal model!

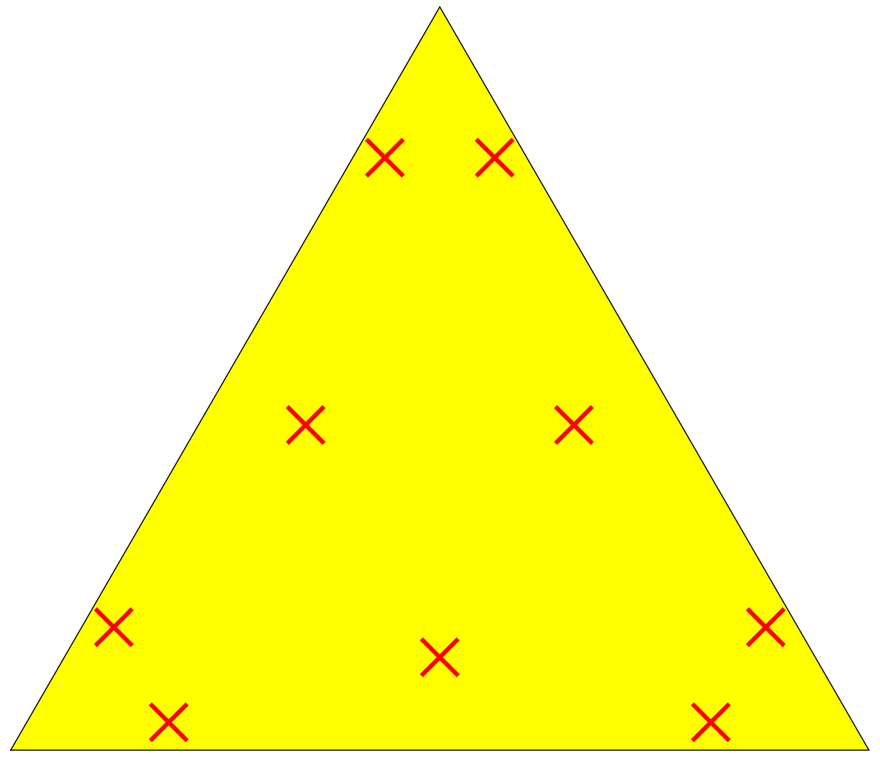
Conceptual Workflow



Material Modeling

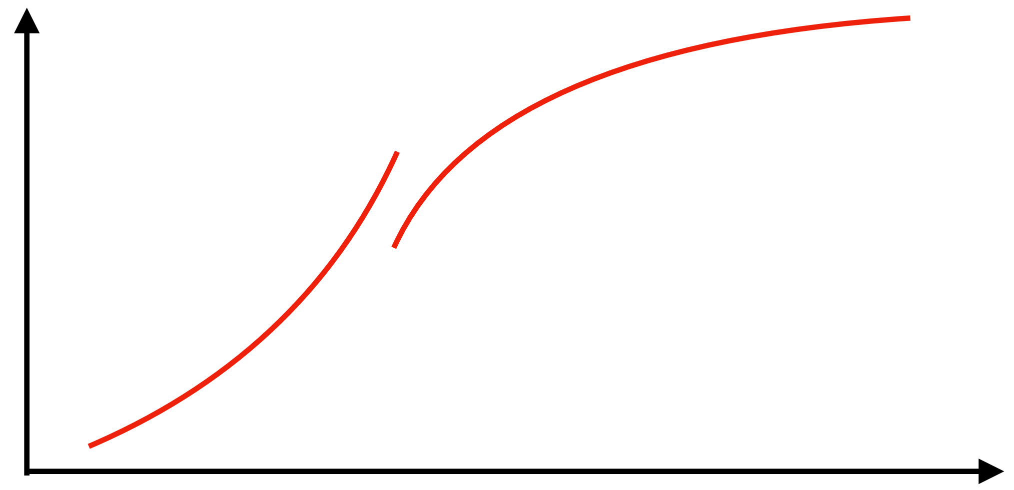
Material Modeling

- **High Nonlinearities for HTS Materials**
  - material properties must be evaluated at every integration point
  - nonlinearities require many iterations
  - material curves must be smooth (consistent derivatives, continuous)
  - avoid expensive functions such as exp or log



triangle integration points
(5th order)

- **Boobytraps in modeling and coding**
  - piecewise polynomials
  - wasteful implementations
  - validity range of functions



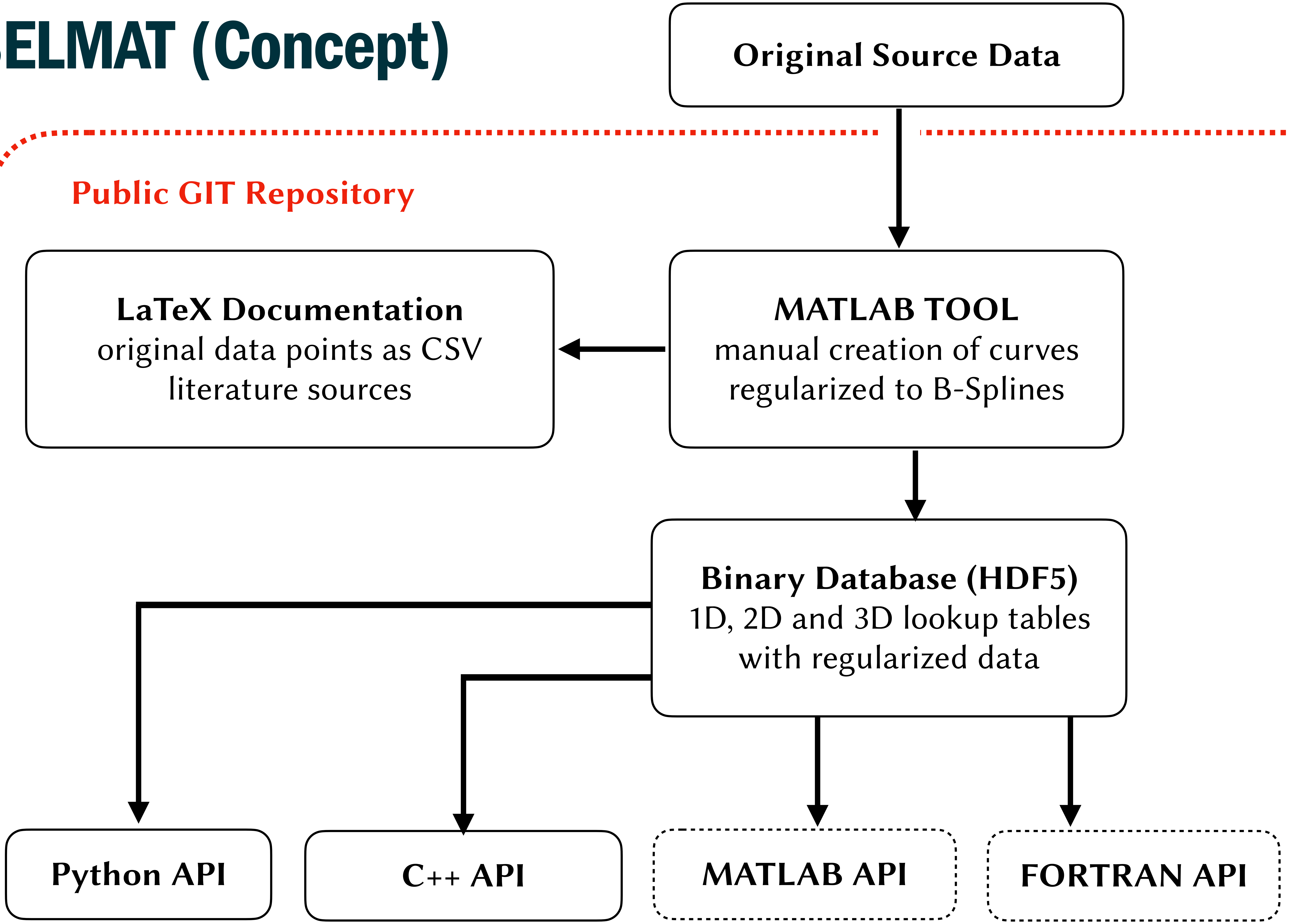
piecewise polynomials lead to convergence issues due to

- discontinuous material properties
- discontinuous derivatives

CSE can be up to ~3 x faster!!!

```
bad code:  wasting multiplications:

y = 10^( a + b * log(T) + c * log(T)^2 + d * log(T)^3 + ...  )

good code: Common Subexpression Elemination (CSE)

logT = log(T)

y = 10^( a + logT * ( b + logT * ( c + logT * ( d + ... ) ) ) ))
```

BELMAT (Concept)



Original Source Data

Public GIT Repository

LaTeX Documentation
original data points as CSV
literature sources

MATLAB TOOL
manual creation of curves
regularized to B-Splines

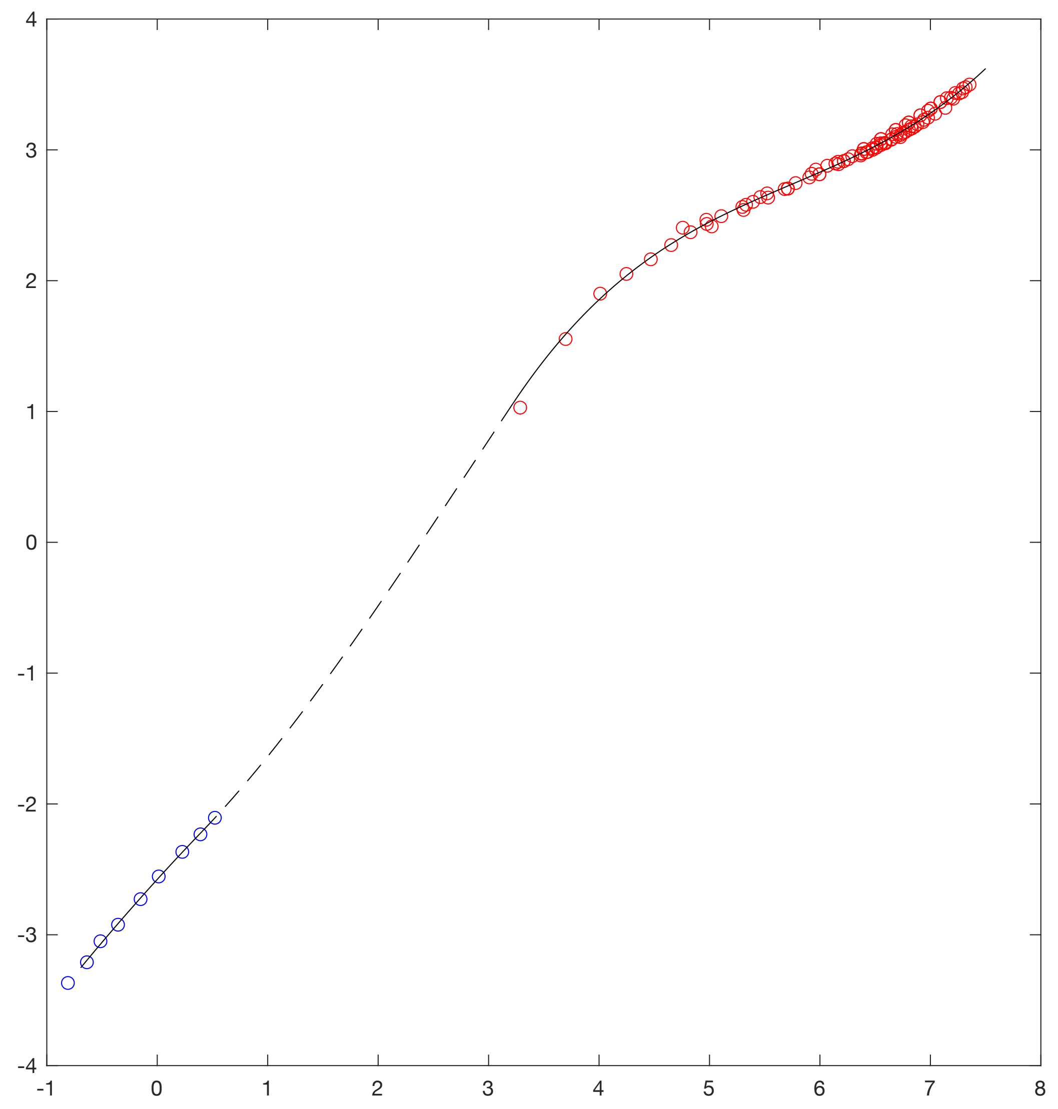
Binary Database (HDF5)
1D, 2D and 3D lookup tables
with regularized data

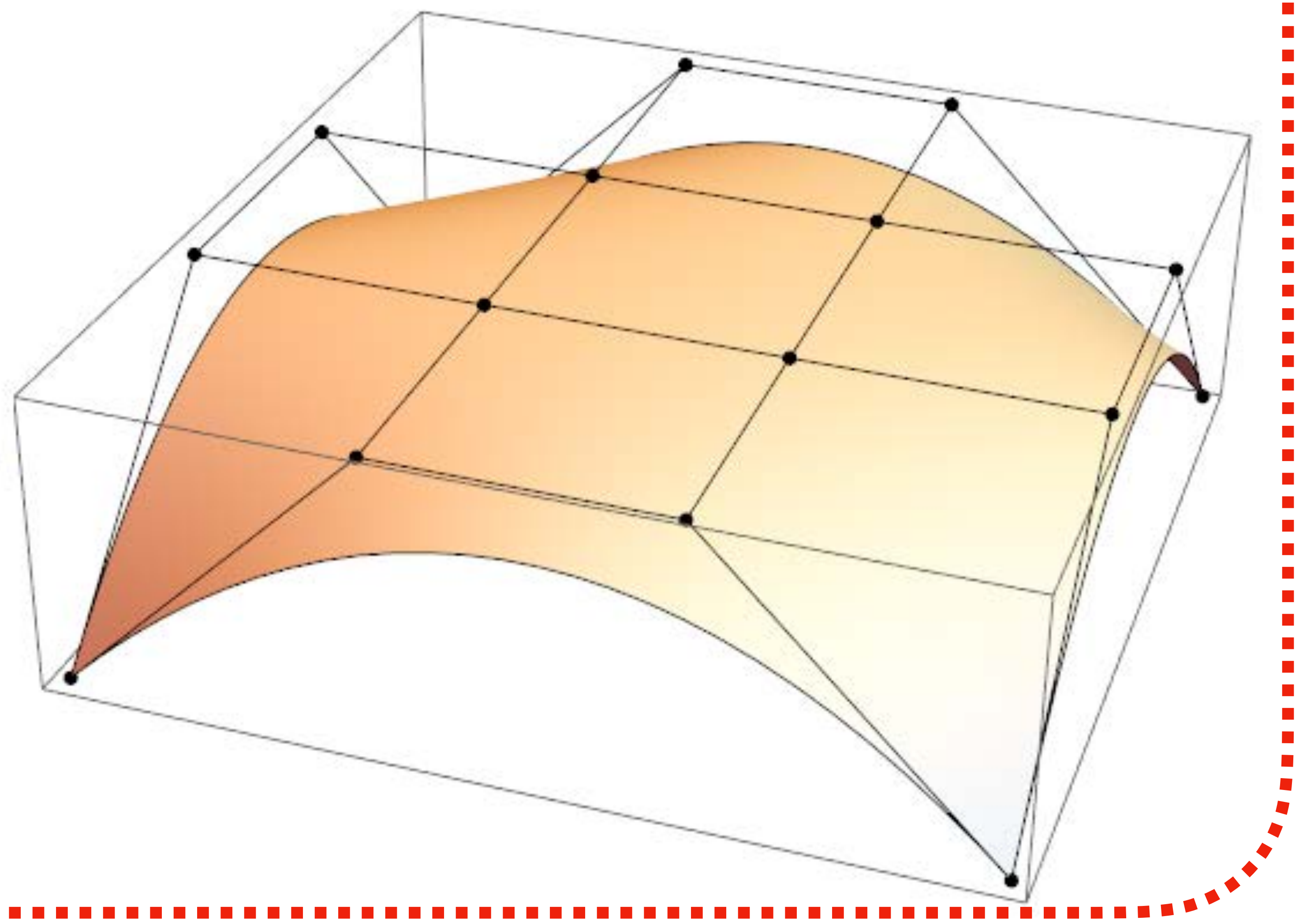
Python API

C++ API

MATLAB API

FORTRAN API





Usecase Example

- **implemented in SparseLizard: ASC 2022**
- **implemented in BELFEM: SUST 2023**

H-φ Formulation in Sparselizard Combined With Domain Decomposition Methods for Modeling Superconducting Tapes, Stacks, and Twisted Wires

N. Riva, A. Halbach, M. Lyly, C. Messe, J. Ruuskanen, and V. Lahtinen

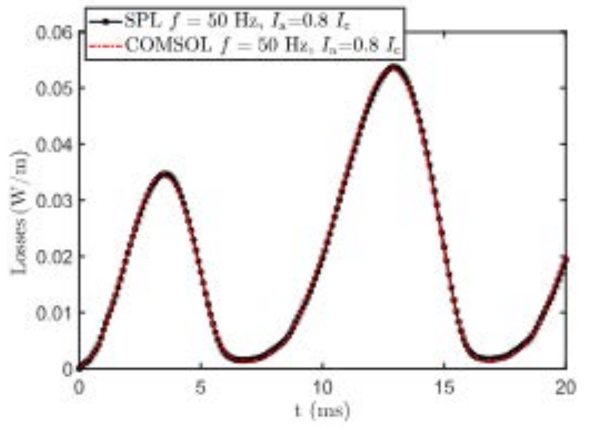


Fig. 5. Instantaneous losses computed on the entire assembly (NbTi wires + copper), with Sparselizard (continuous line + markers) and COMSOL (dashed line).

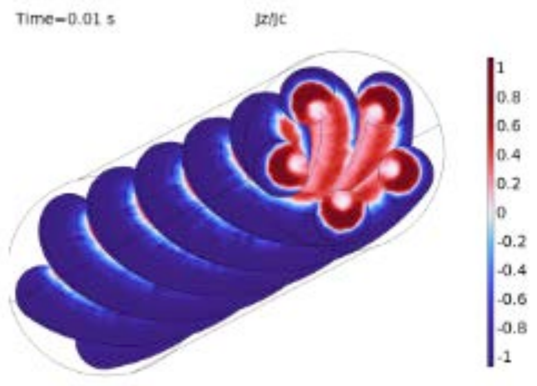
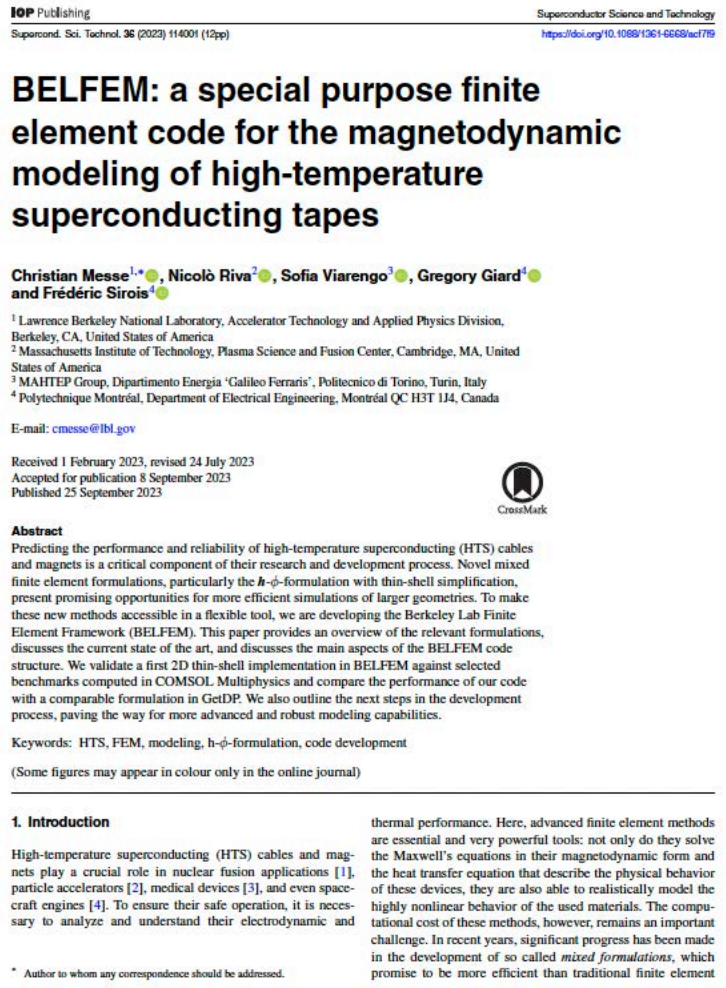


Fig. 6. Normalized current density at $t = 10$ ms.



BELFEM: a special purpose finite element code for the magnetodynamic modeling of high-temperature superconducting tapes

Christian Messe, Nicolò Riva, Sofia Viarengo, Gregory Giard and Frédéric Sirois

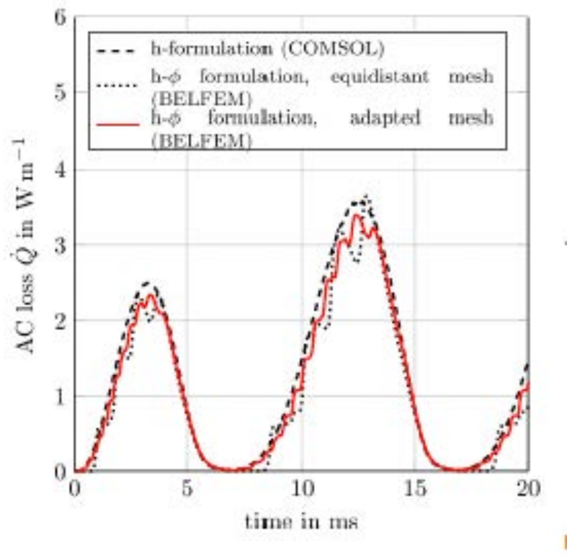


Figure 12. AC losses for the tape stack at $I = 90$ A per tape at $f = 50$ Hz.

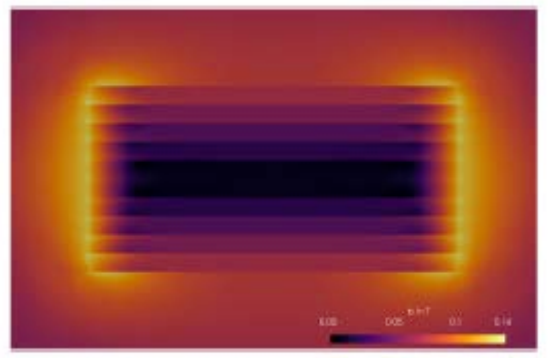
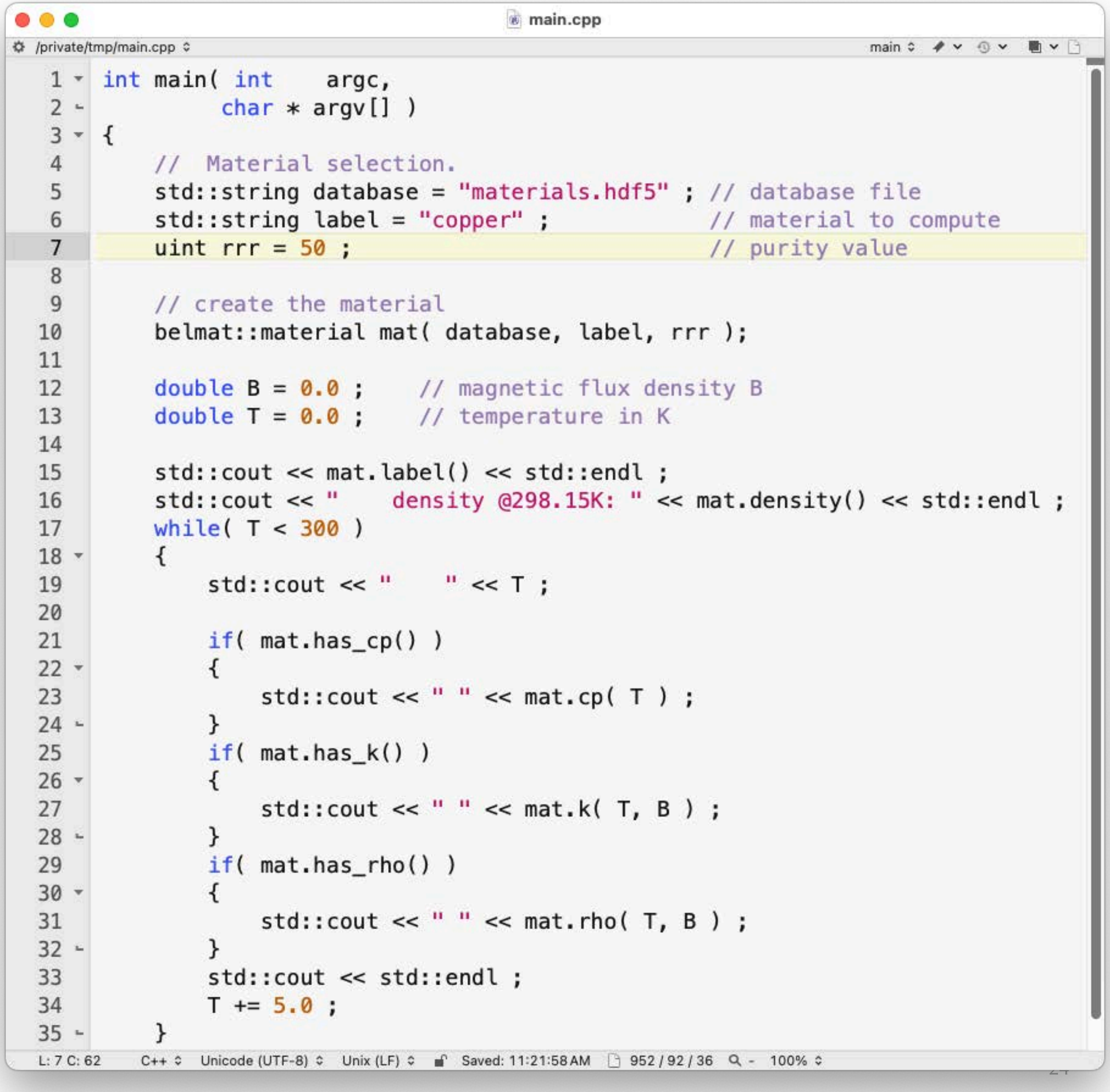


Figure 13. Magnetic flux density at 15 ms with $I = 90$ A per tape at $f = 50$ Hz.



```cpp
int main( int    argc,
          char * argv[] )
{
    //  Material selection.
    std::string database = "materials.hdf5" ; // database file
    std::string label = "copper" ;            // material to compute
    uint rrr = 50 ;                           // purity value

    // create the material
    belmat::material mat( database, label, rrr );

    double B = 0.0 ;    // magnetic flux density B
    double T = 0.0 ;    // temperature in K

    std::cout << mat.label() << std::endl ;
    std::cout << "     density @298.15K: " << mat.density() << std::endl ;
    while( T < 300 )
    {
        std::cout << "    " << T ;

        if( mat.has_cp() )
        {
            std::cout << " " << mat.cp( T ) ;
        }
        if( mat.has_k() )
        {
            std::cout << " " << mat.k( T, B ) ;
        }
        if( mat.has_rho() )
        {
            std::cout << " " << mat.rho( T, B ) ;
        }
        std::cout << std::endl ;
        T += 5.0 ;
    }
```

Summary

- developing a finite-element framework tailored to HTS cable & magnet development needs
- demonstrated proof of concept in 2D
- first performance tests very promising
- Work in progress: 3D tape model for quenching (goal: winter 2024)
- Work in progress: Intertape current sharing (goal: spring 2025)
- Work in progress: Material Database