

Recent Advancements in BELFEM

**Christian Messe, Gregory Giard, Frédéric Sirois
April 30th, 2023**





- **Motivation**
- **h - ϕ fundamentals**
- **Why a custom codebase?**
- **Last Year's progress**
- **Work in Progress: Thermal Coupling**
- **Work in Progress: Current Sharing**
- **Summary and Outlook**

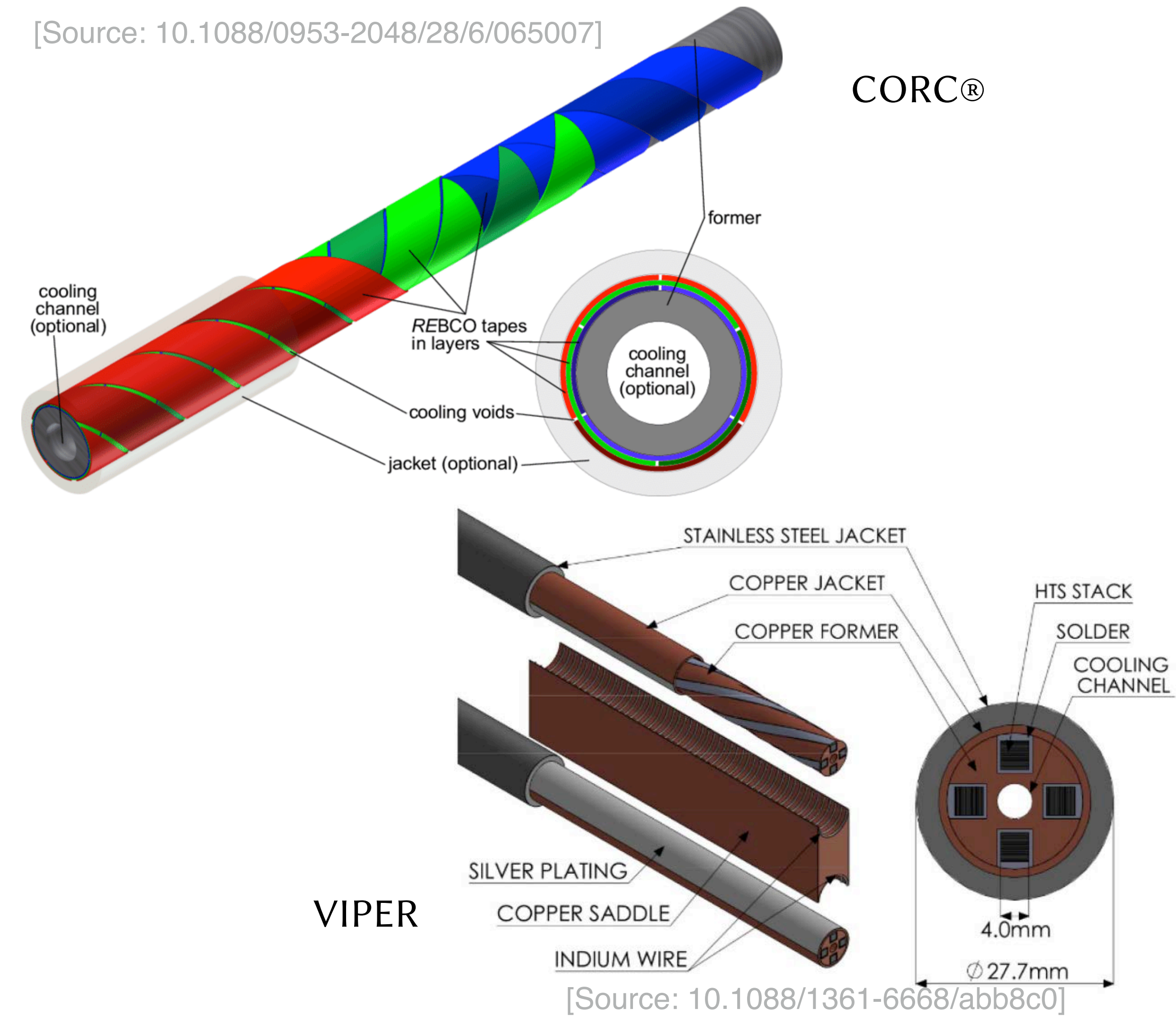
want ability to model:

- **quasi-magnetodynamic modeling**
 - understand electromagnetic behavior of cables

- **coupled thermal modeling**
 - thermal behavior and physical coupling with EM
 - quench behavior

- **other phenomena**
 - current sharing
 - mechanical behavior
 - ...

[Source: 10.1088/0953-2048/28/6/065007]



[Source: 10.1088/1361-6668/abb8c0]

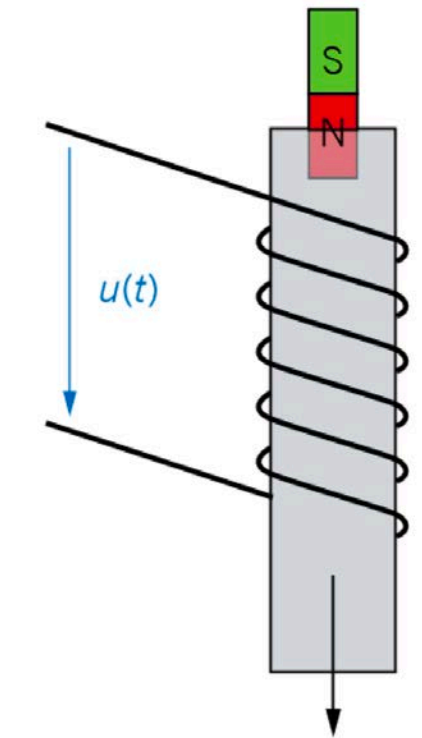
Solid Model

- got momentum in late 2010s to early 2020s
- very robust formulation
- significantly reduced degrees of freedom in non-conducting domains

- ➔ very high performance gain
- ➔ ideal for large 3D models!

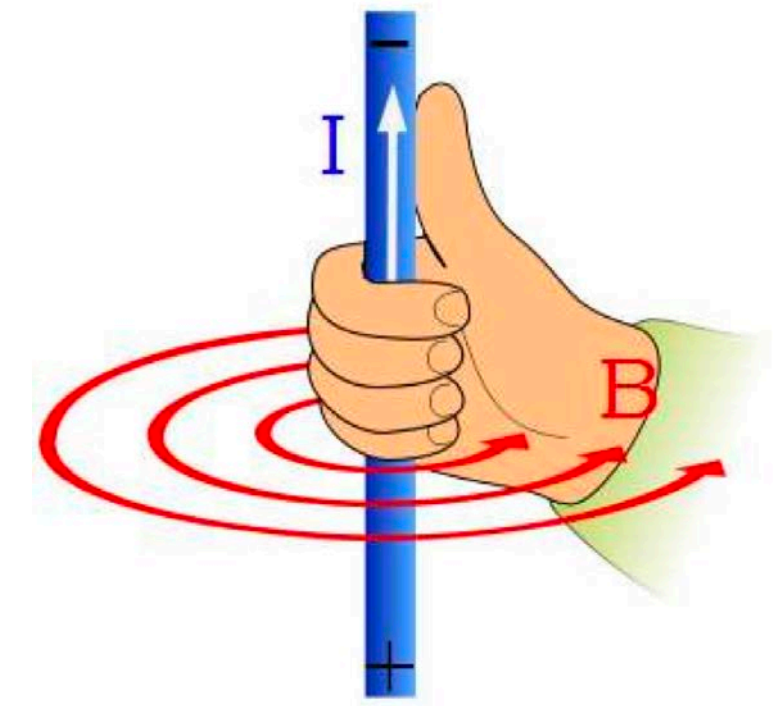
Thin-Shell Model

- first published in 2022
 - can resolve individual layers of HTS tapes
- ➔ ideal for current sharing and quench investigations



Faraday's law

$$\nabla \times \mathbf{e} = -\dot{\mathbf{b}}$$



Ampère's law

$$\nabla \times \mathbf{h} = -\mathbf{j}$$

Recipe:

- develop weak form around Faraday's law
- use $\delta \mathbf{h}$ as test function
- substitute \mathbf{e} with Ohm's and Ampère

$$\mathbf{e} = -\rho \nabla \times \mathbf{h}$$

- for non-conducting region, define

$$\mathbf{h} = -\nabla \phi$$

H- ϕ formulation: Weak Forms

Conducting Region

$$\int_{\Omega} \delta \mathbf{h}^T \frac{\partial (\mu \mathbf{h})}{\partial t} dV + \int_{\Omega} \delta \mathbf{h}^T \times \nabla^T \rho \nabla \times \mathbf{h} dV + \int_{\Gamma} \delta \mathbf{h}^T \mathbf{n} \times \mathbf{e} dS = 0$$

damping
stiffness
boundary

Non-Conducting Regions

substitute $-\nabla \phi = \mathbf{h}$

$$\int_{\Omega} \delta \phi^T \nabla^T \frac{\partial (\mu \nabla \phi)}{\partial t} dV + \int_{\Omega} \delta \phi^T \nabla^T \times \nabla^T \rho \nabla \times \nabla \phi dV - \int_{\Gamma} \delta \phi^T \nabla^T \mathbf{n} \times \mathbf{e} dS = 0$$

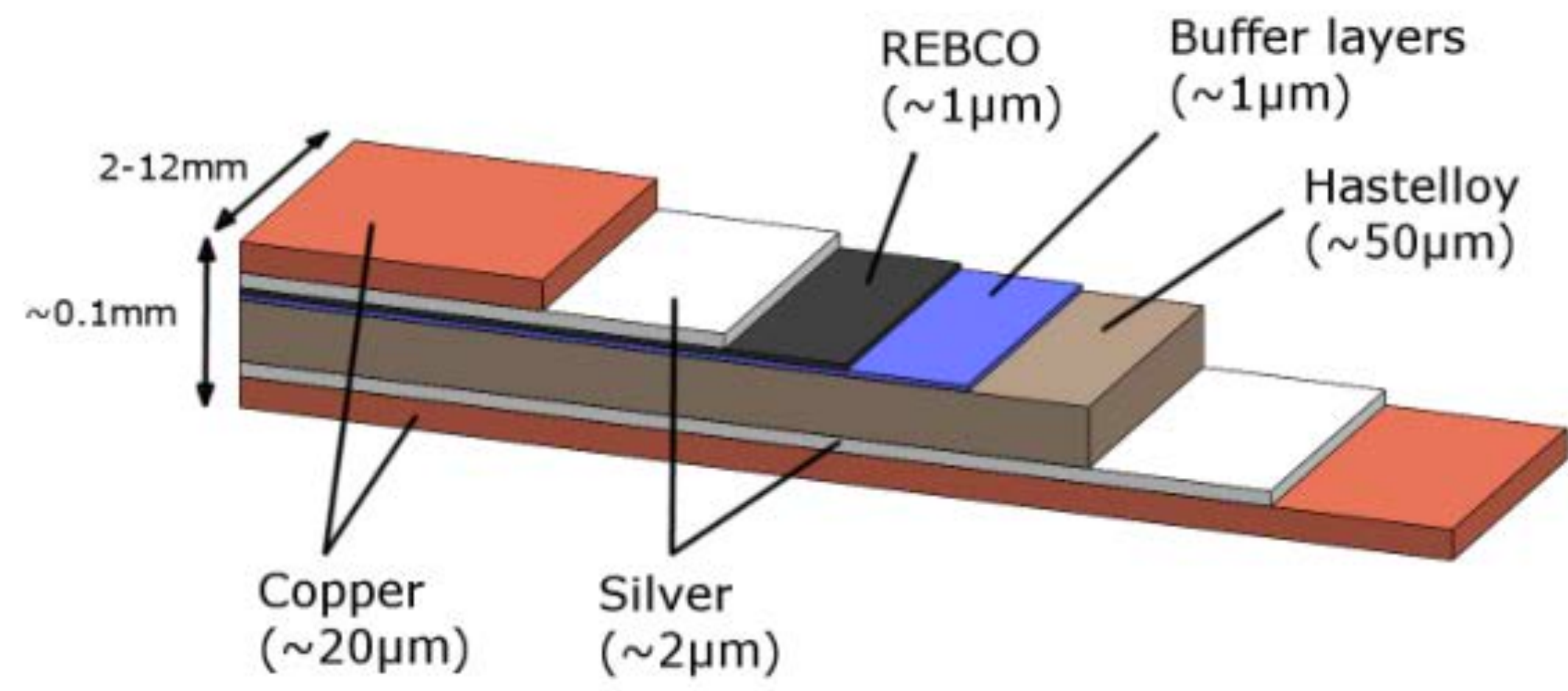
damping
~~stiffness~~
boundary

0

- traditionally, the ϕ -formulation is based on the Gauß law ($\nabla B=0$), we, however, use Faraday 's law
→ better convergence since same physical equation for all domains!
- stiffness vanishes for ϕ since $\nabla \times \nabla \phi = 0$

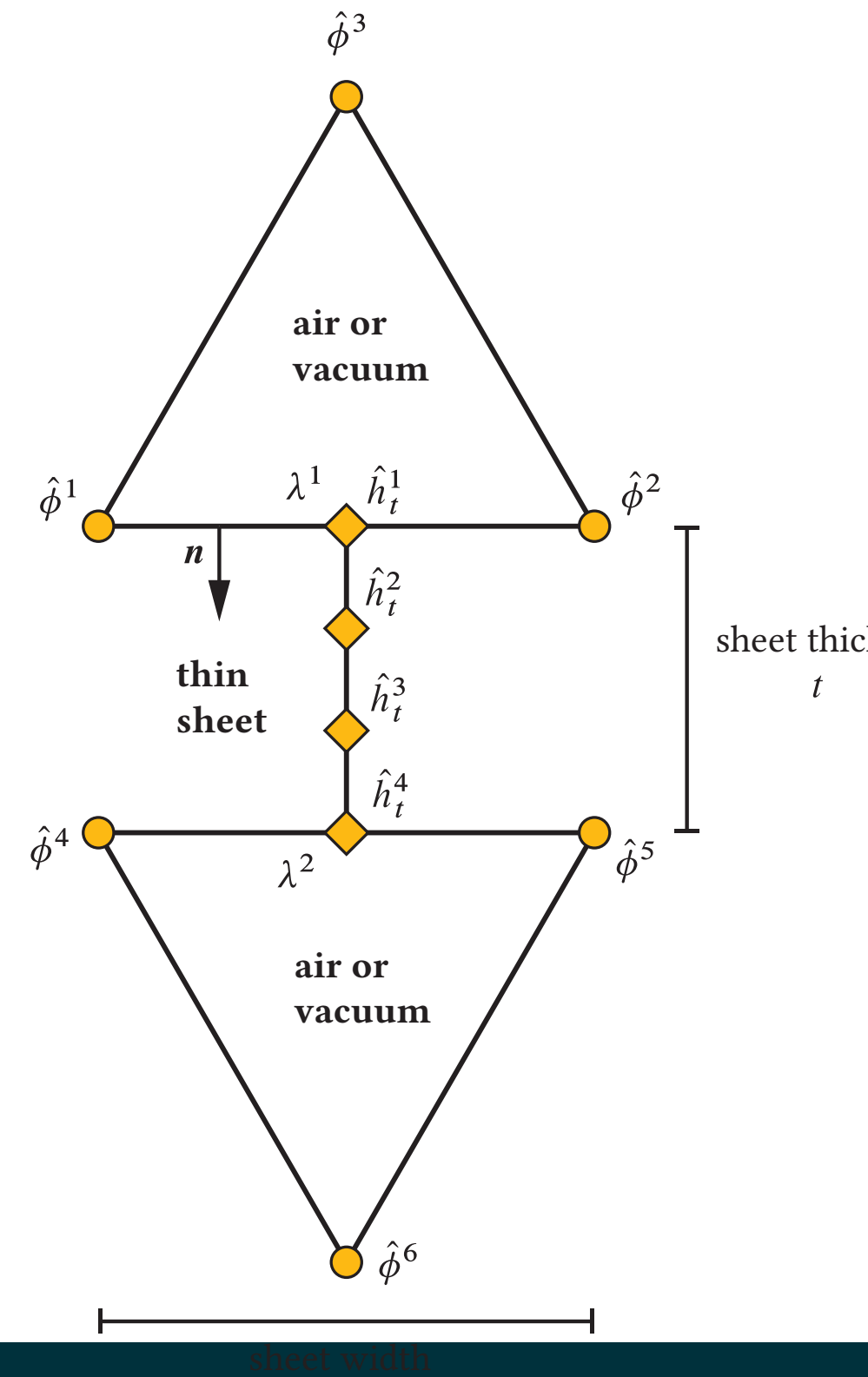
Challenges

- high aspect ratio
- strongly nonlinear material behavior
- electromagnetic-thermal interaction
- boundary conditions
- current sharing



Mixed h- ϕ Formulation for thin shells

- ongoing development in corporation with Polytechnique Montreal



Boundary Conditions

current is applied over Ampere's circuital law:

- homologies represent the loops that can be drawn around the conducting regions that fulfill Ampere's law
- only integral current I needs to be known

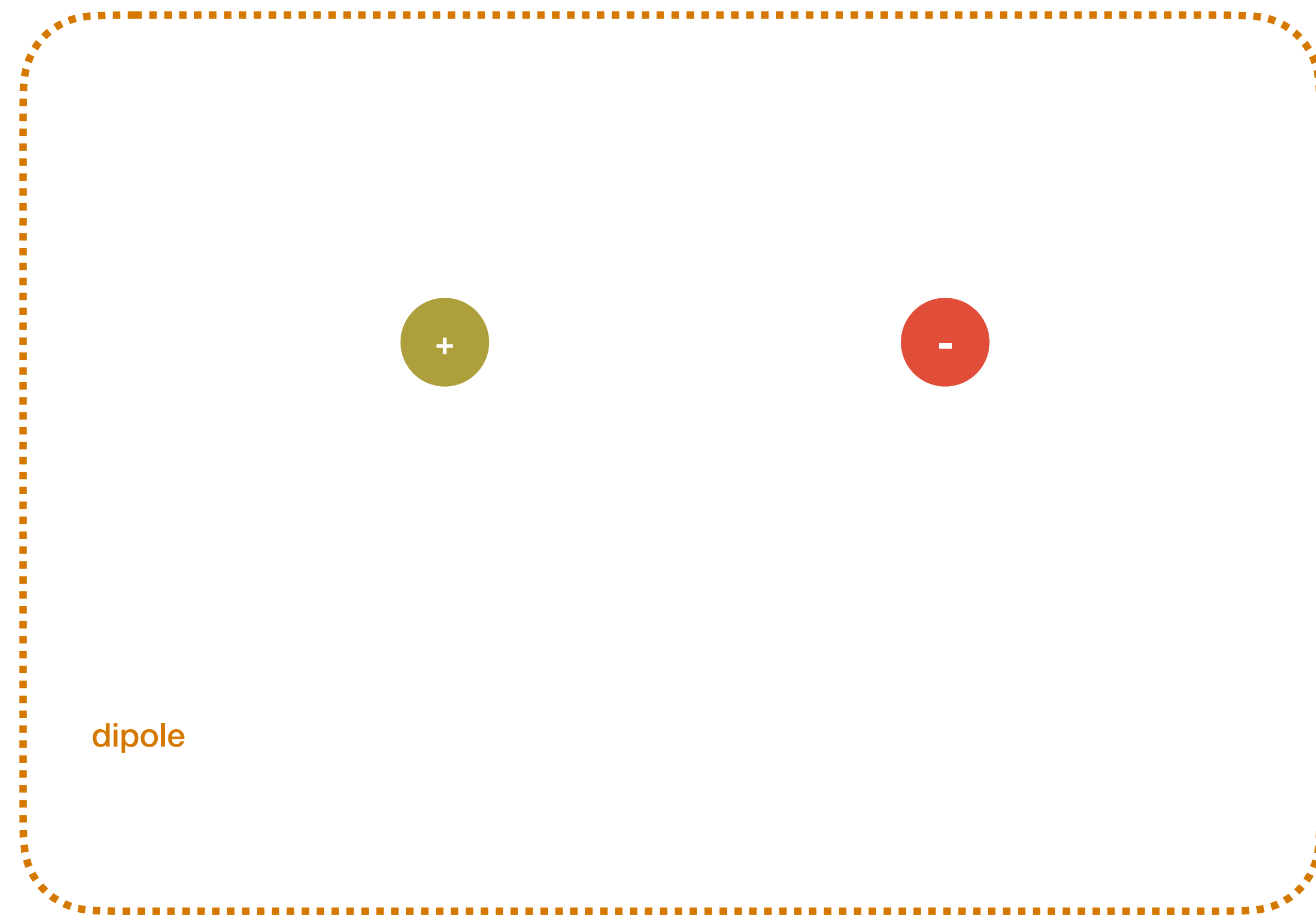


- cohomologies are cuts in the domain over which jumps in the magnetic potential ϕ are imposed so that $\Delta\phi=I$.

- ➔ very elegant mathematics!
- ➔ homology definition not user friendly
- ➔ difficult to implement in commercial codes

Ampère's circuital law

$$\oint h \, dl = I$$



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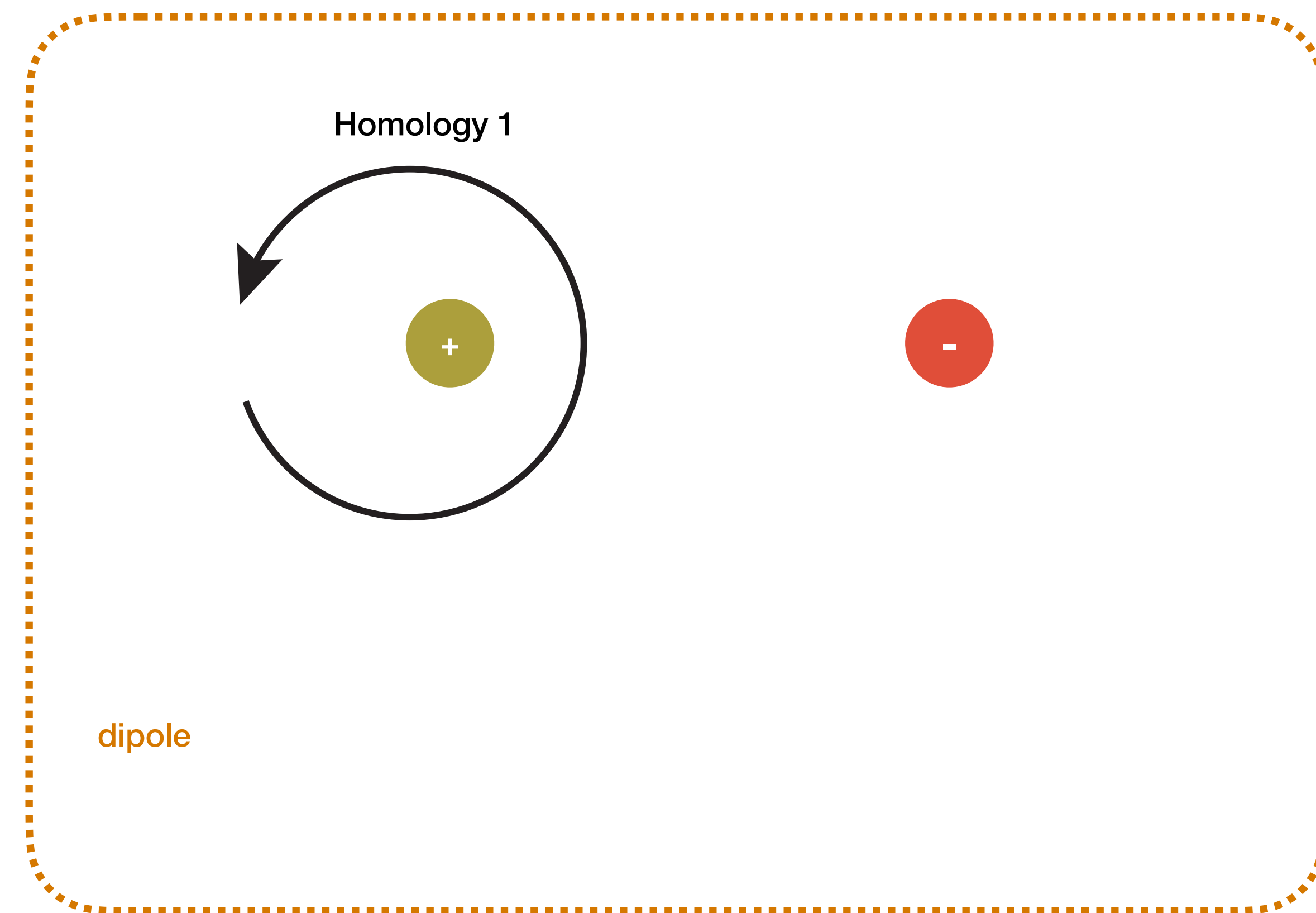


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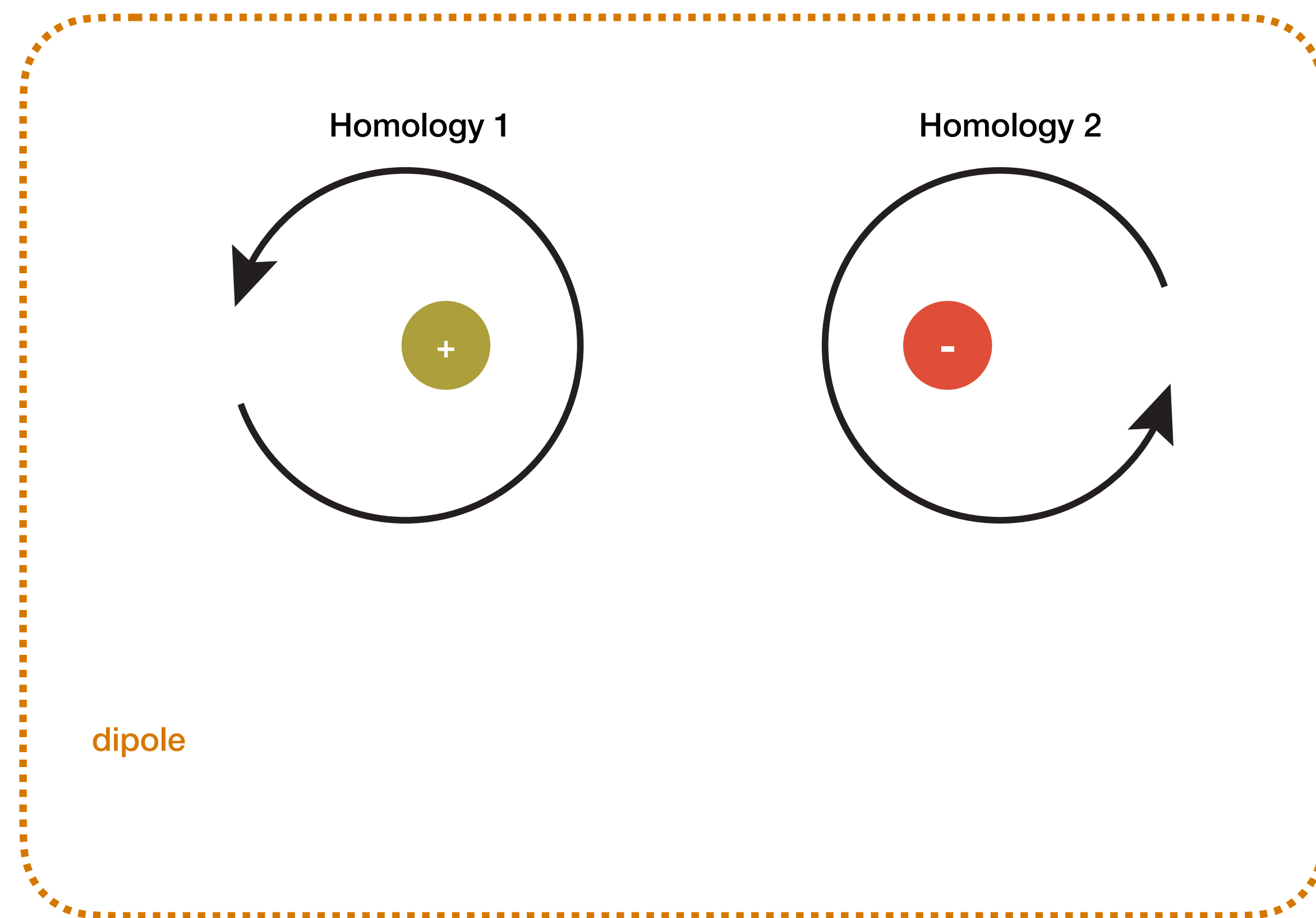


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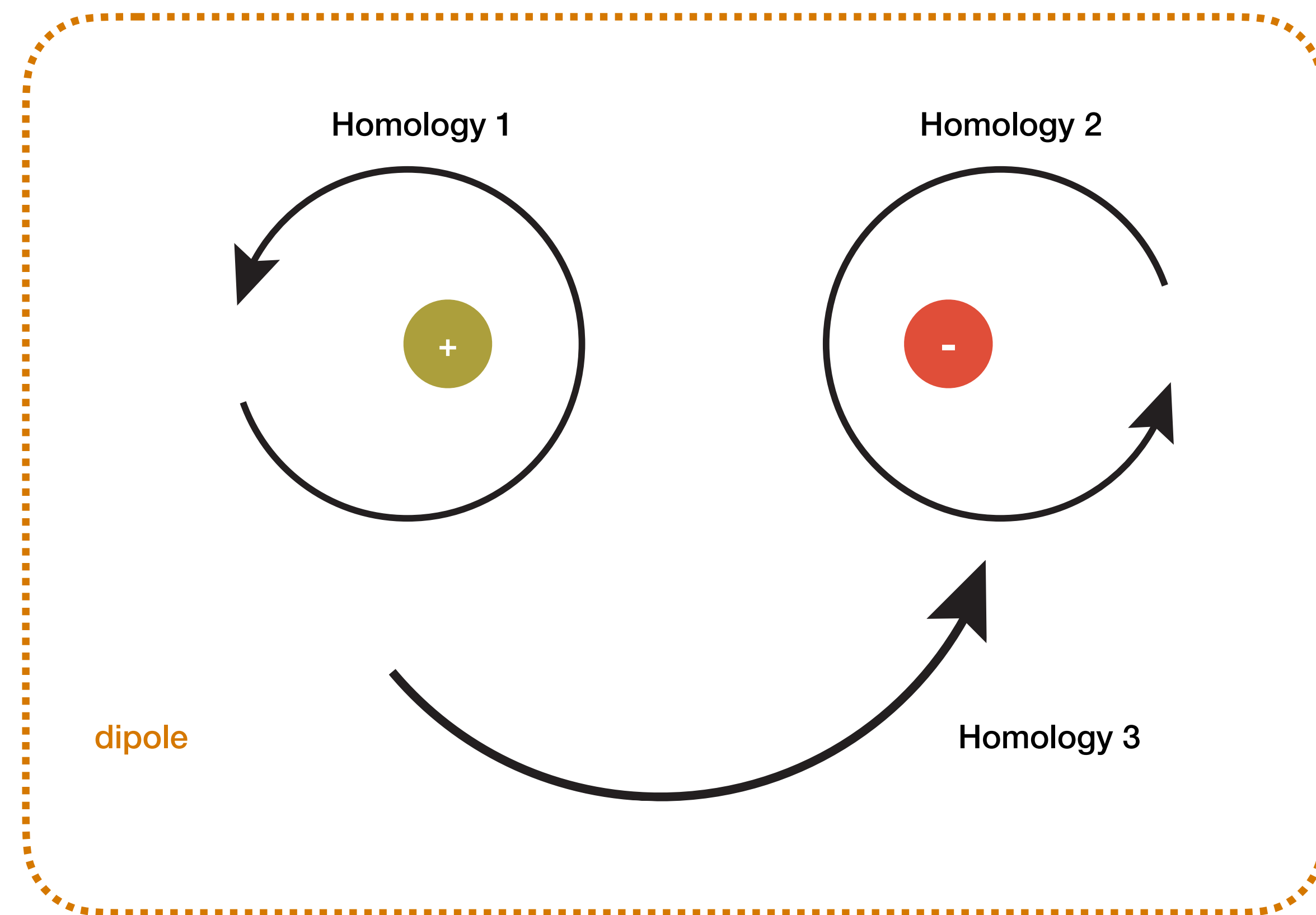


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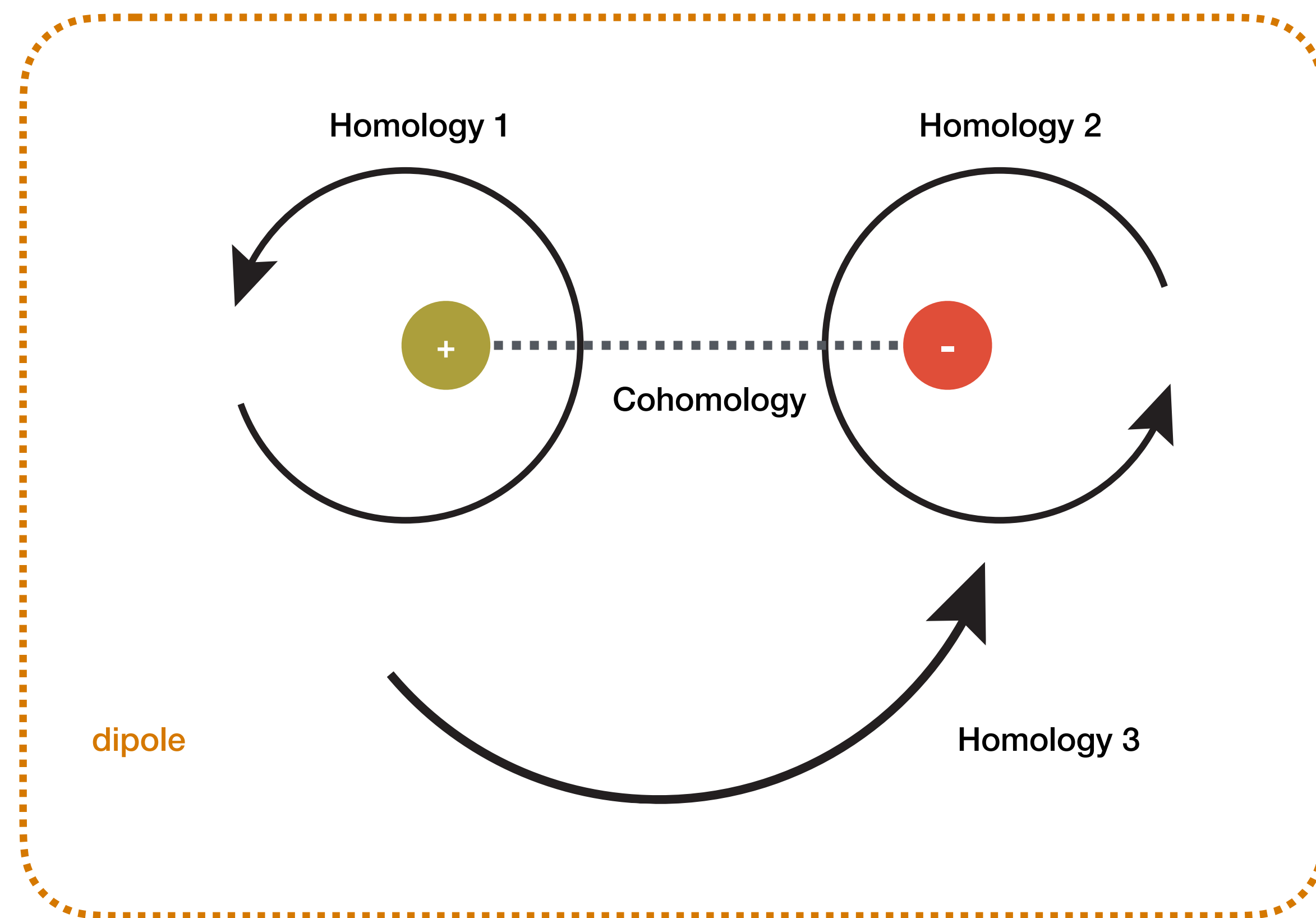


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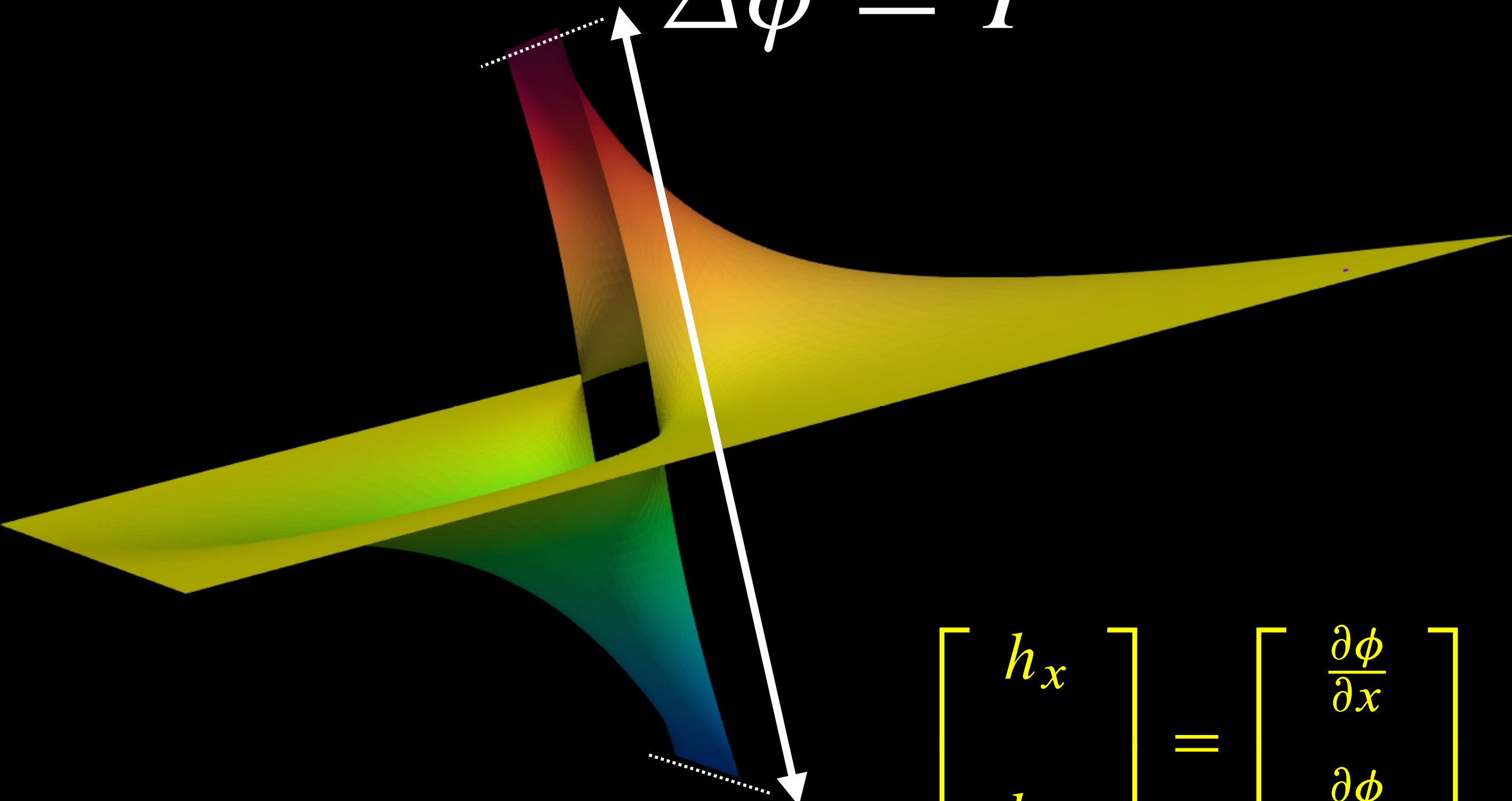
Ampère's circuital law

$$\oint \mathbf{h} \, d\mathbf{l} = I$$

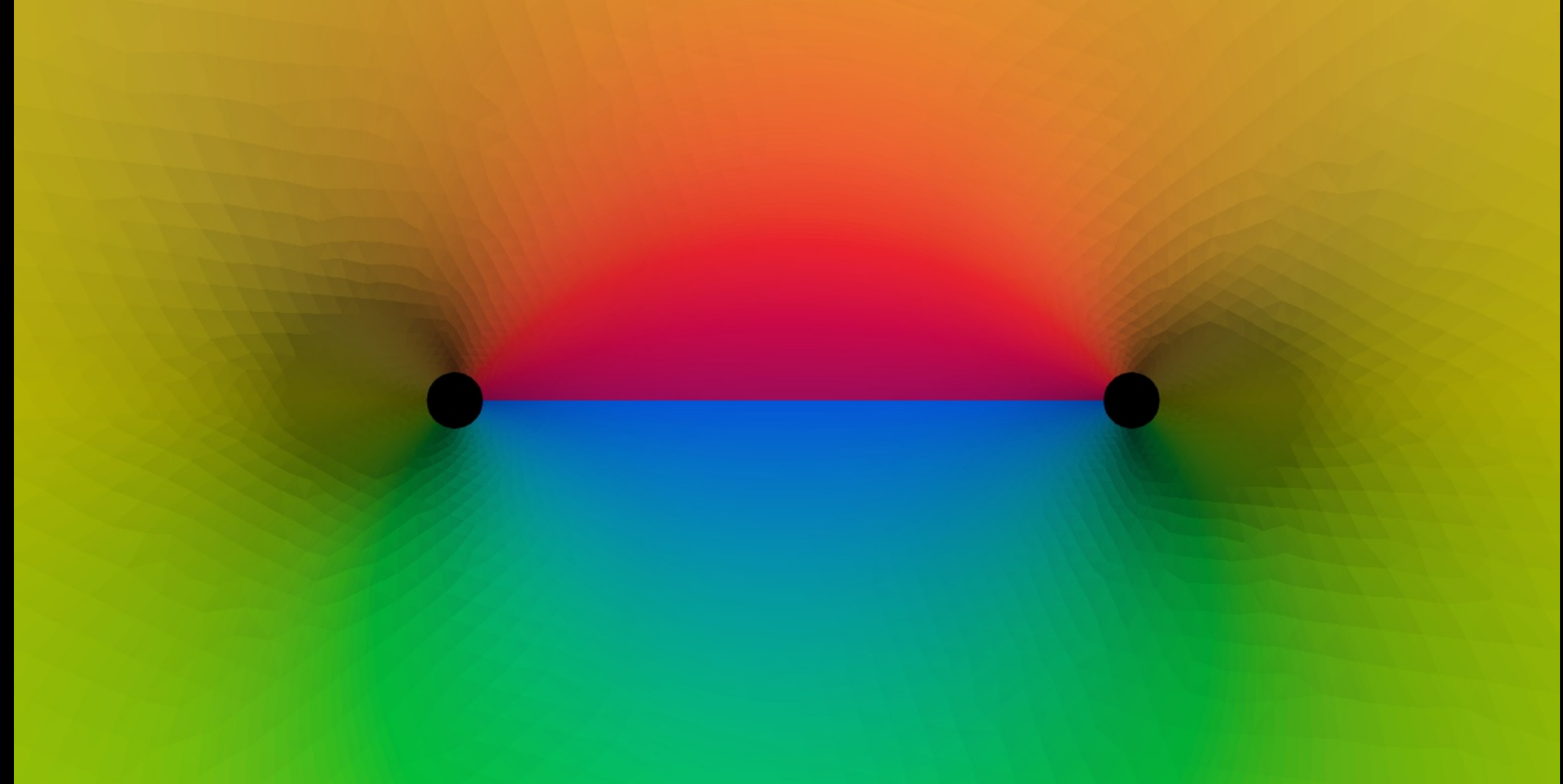


H- ϕ formulation: Homologies

$$\Delta\phi = I$$

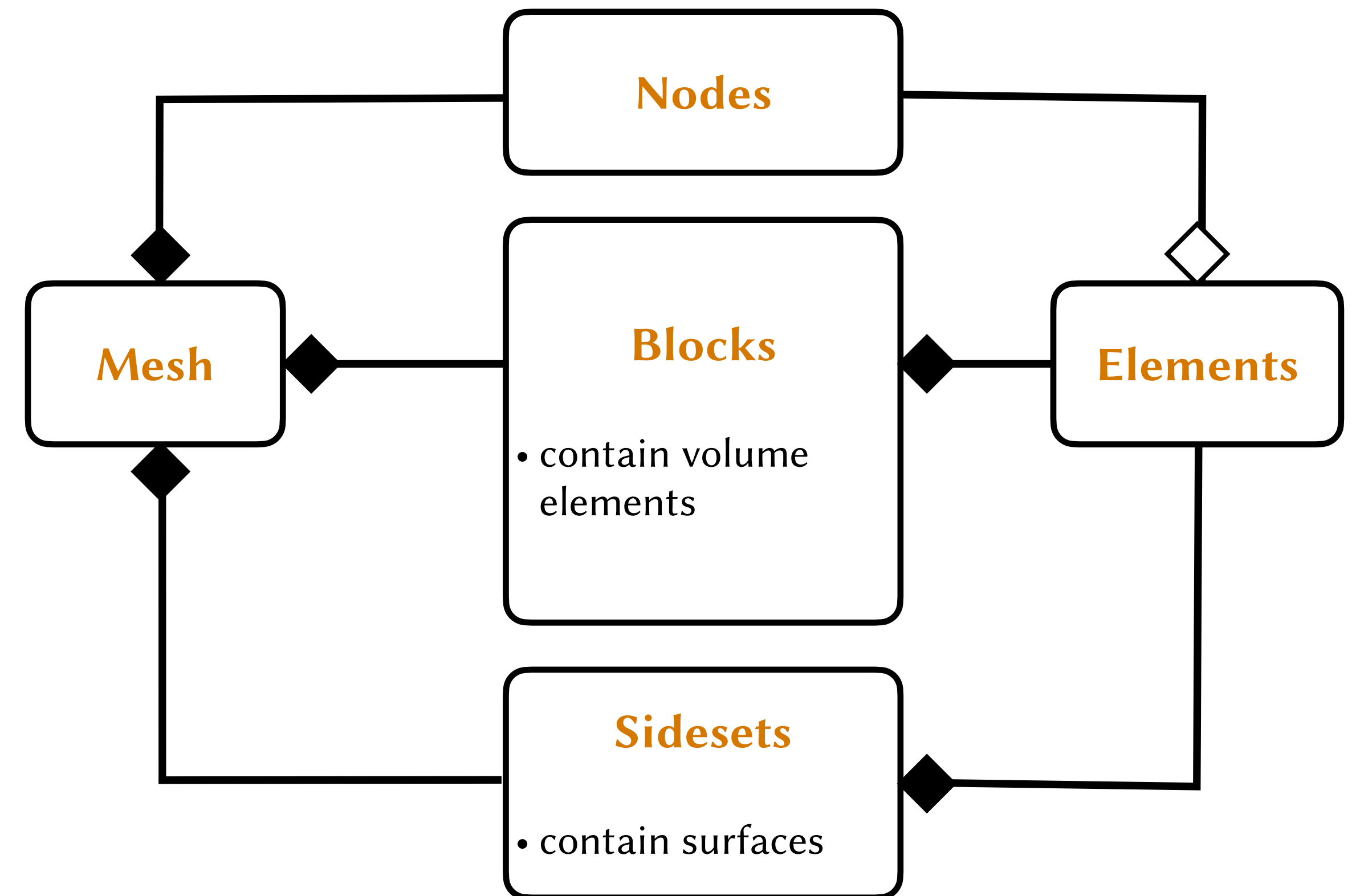


$$\begin{bmatrix} h_x \\ h_y \end{bmatrix} = \begin{bmatrix} \frac{\partial\phi}{\partial x} \\ \frac{\partial\phi}{\partial y} \end{bmatrix}$$



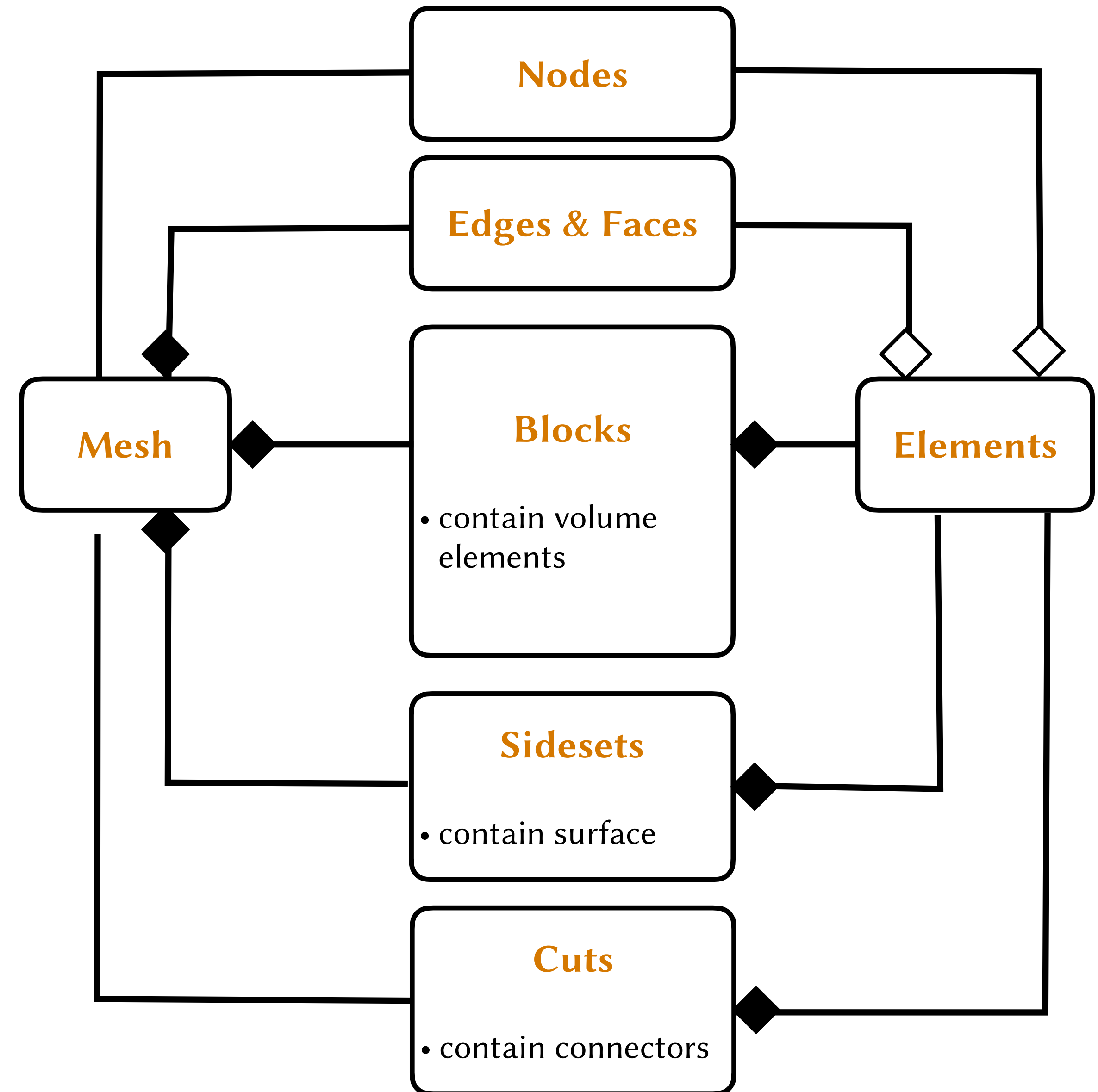
Why a custom codebase?

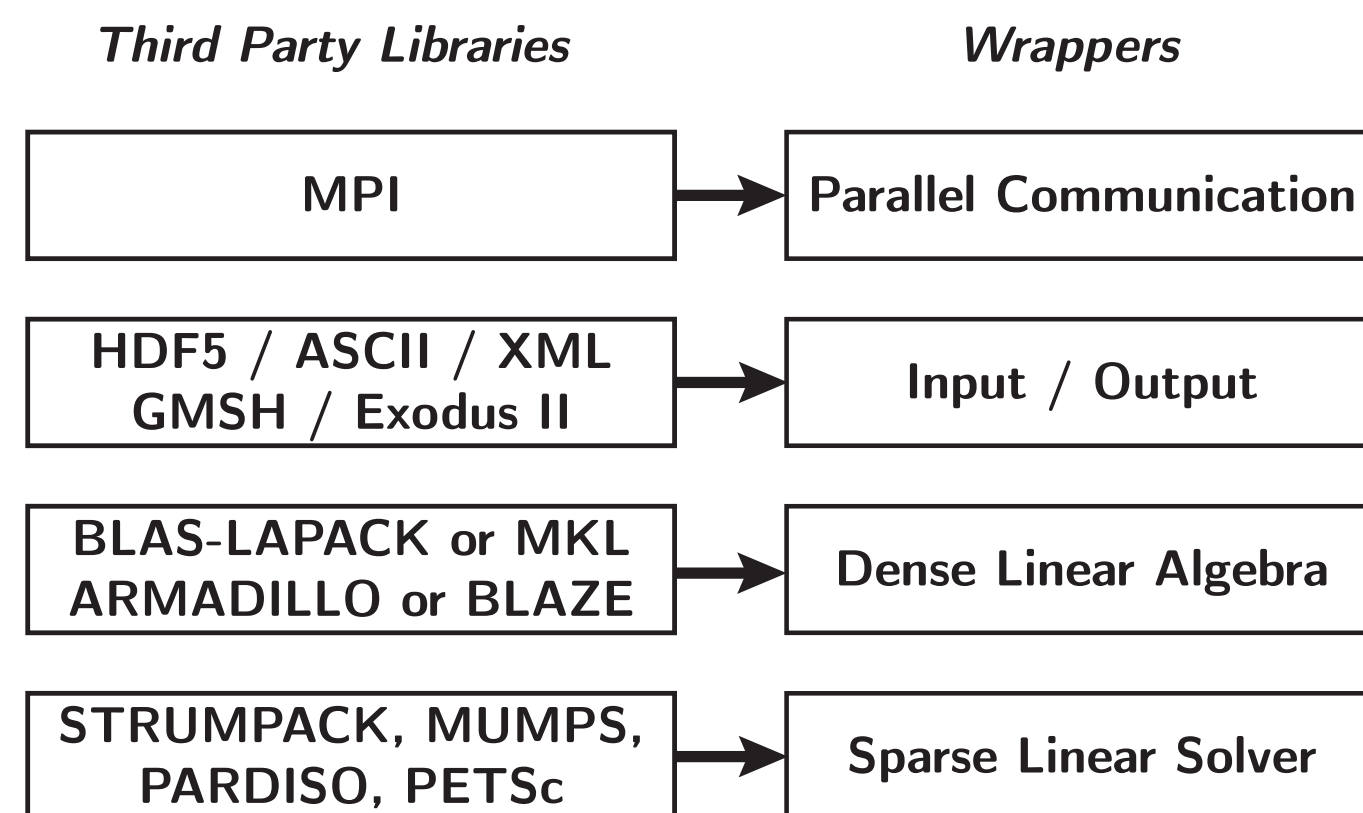
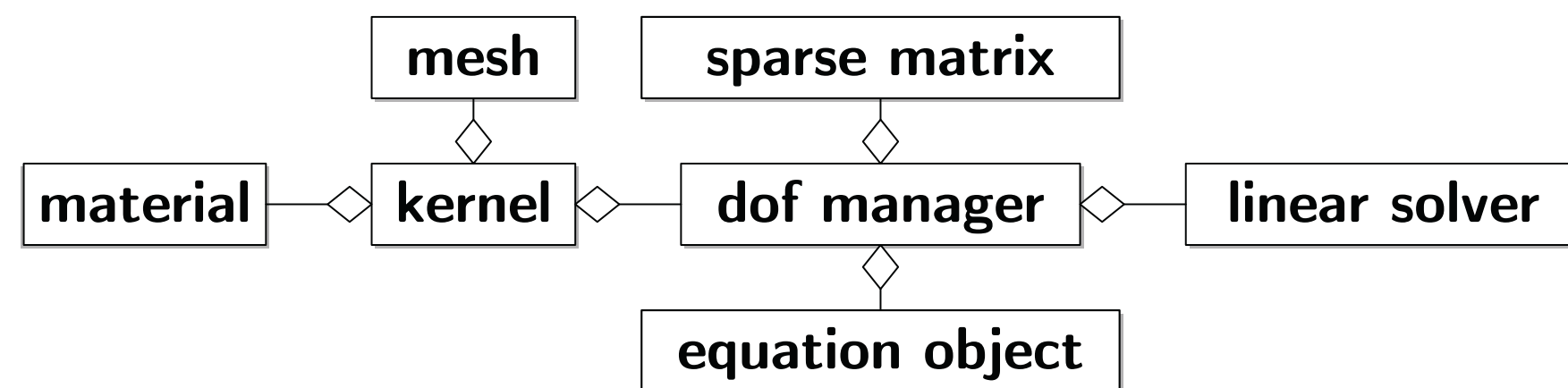
- need to predict quench behavior of HTS
 - coupled electromagnetic-thermal simulation
- needs to be sufficiently fast
 - uses state of the art $h-\phi$ formulation
 - uses state of the art solver libraries (STRUMPACK)
 - ability to run on a HPC node
- need complex geometries and current sharing
 - uses state of the art thin shell models
 - **need low level access to data structure**
- need to handle highly nonlinear material properties
 - custom database
 - using 3D-B-splines to allow smooth derivatives



Why a custom codebase?

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- **Flexibility & Maintainability**

- consistent naming scheme of functions and classes
- lots of comments in code!
- modular structure

- **simple MATLAB-like dense linear algebra**

- through ARMADILLO or BLAZE

- **text based interface tailored to magnet development**

- ability to write scripts in BASH and Python

- **utilization of community software**

- use open source data formats (GMSH, HDF5, Exodus II)
- link against community libraries: MUMPS, PETSc, STRUMPACK, ...

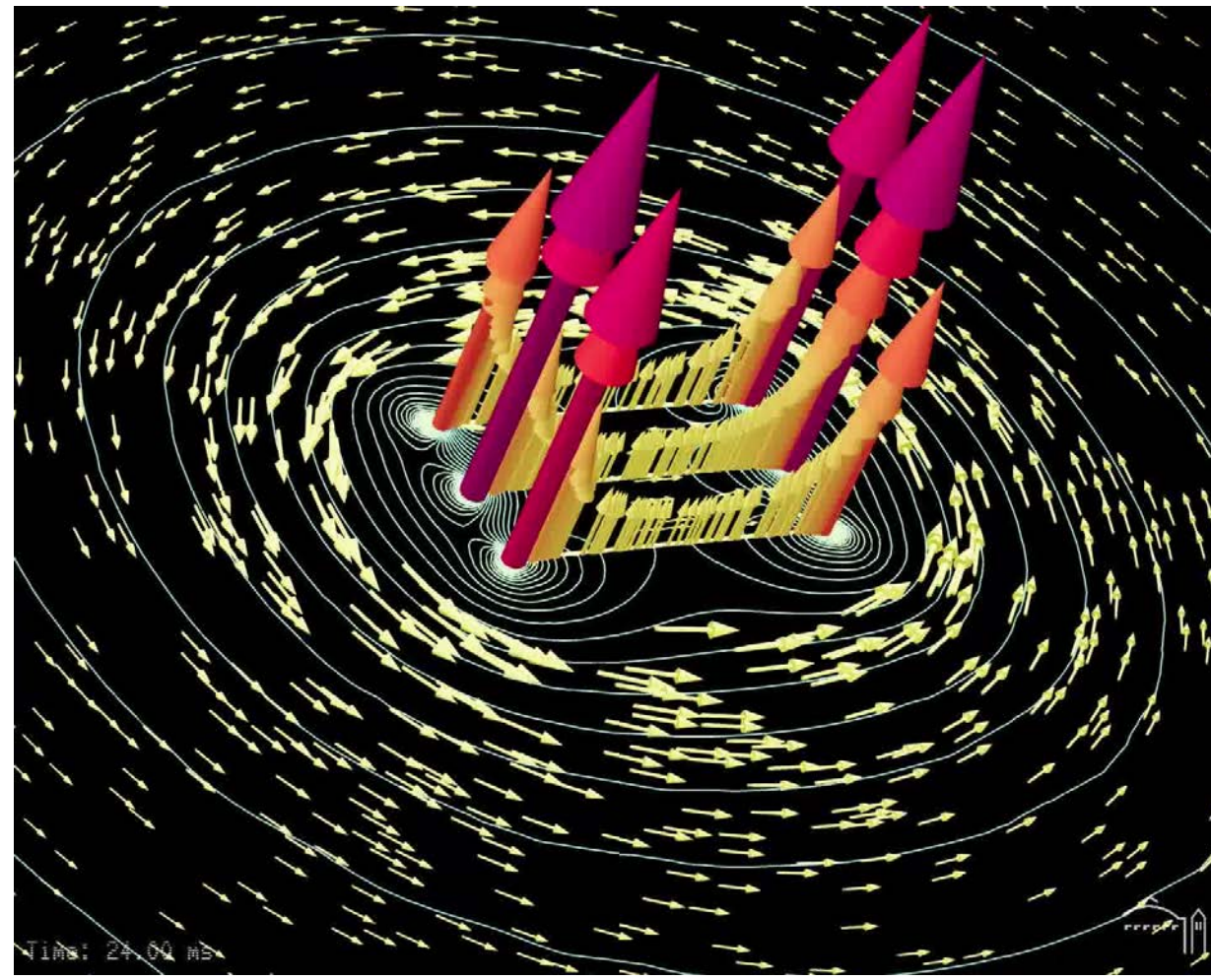
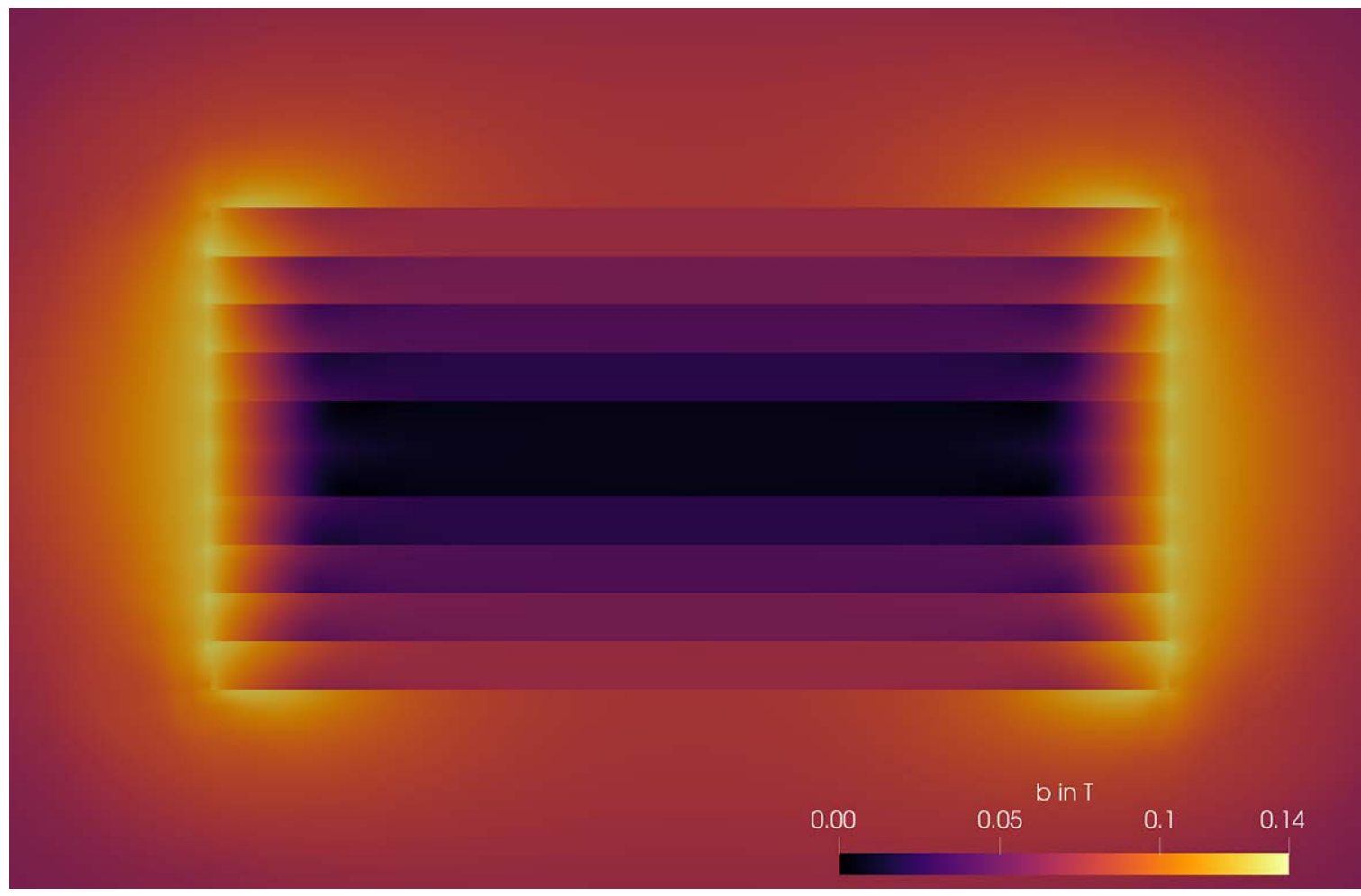
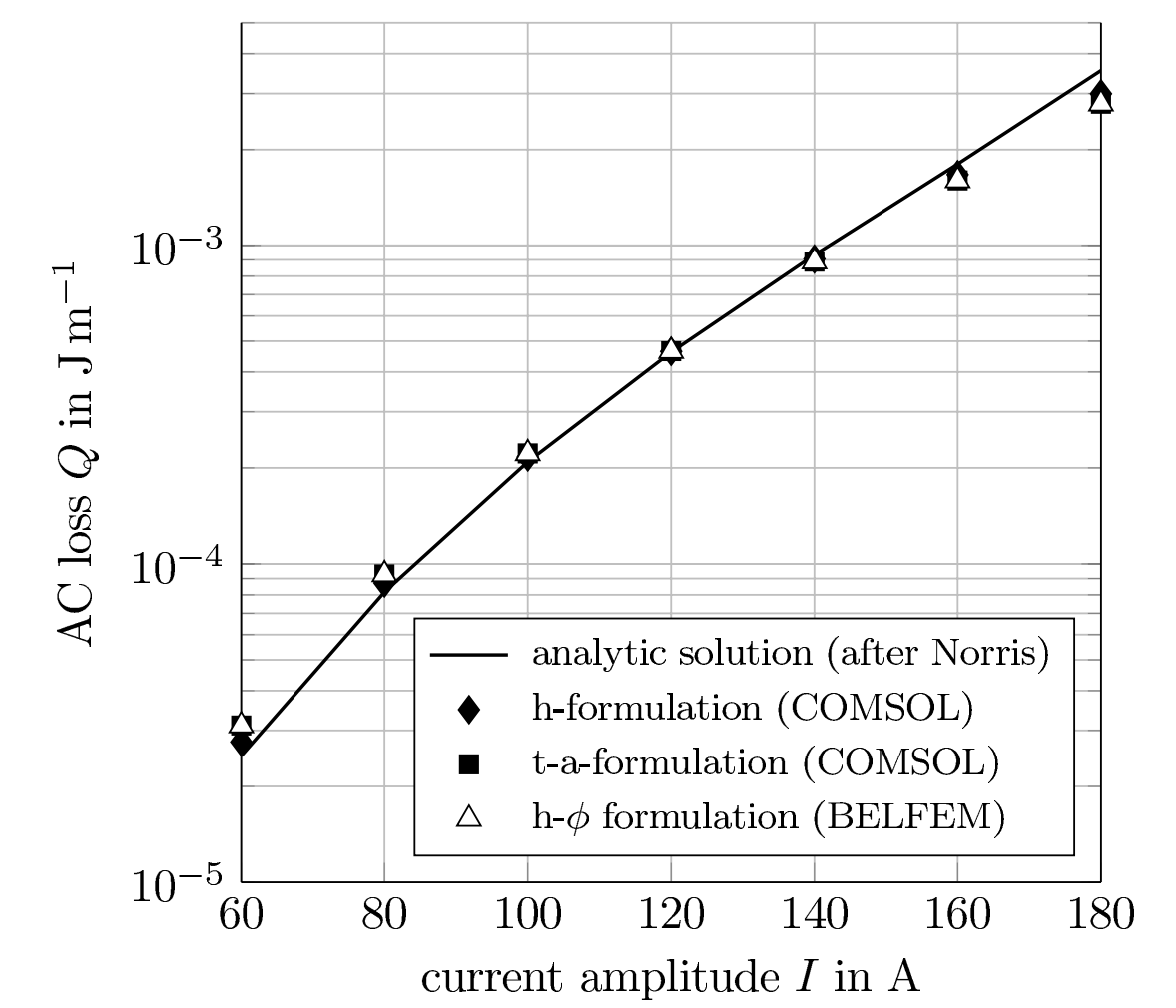
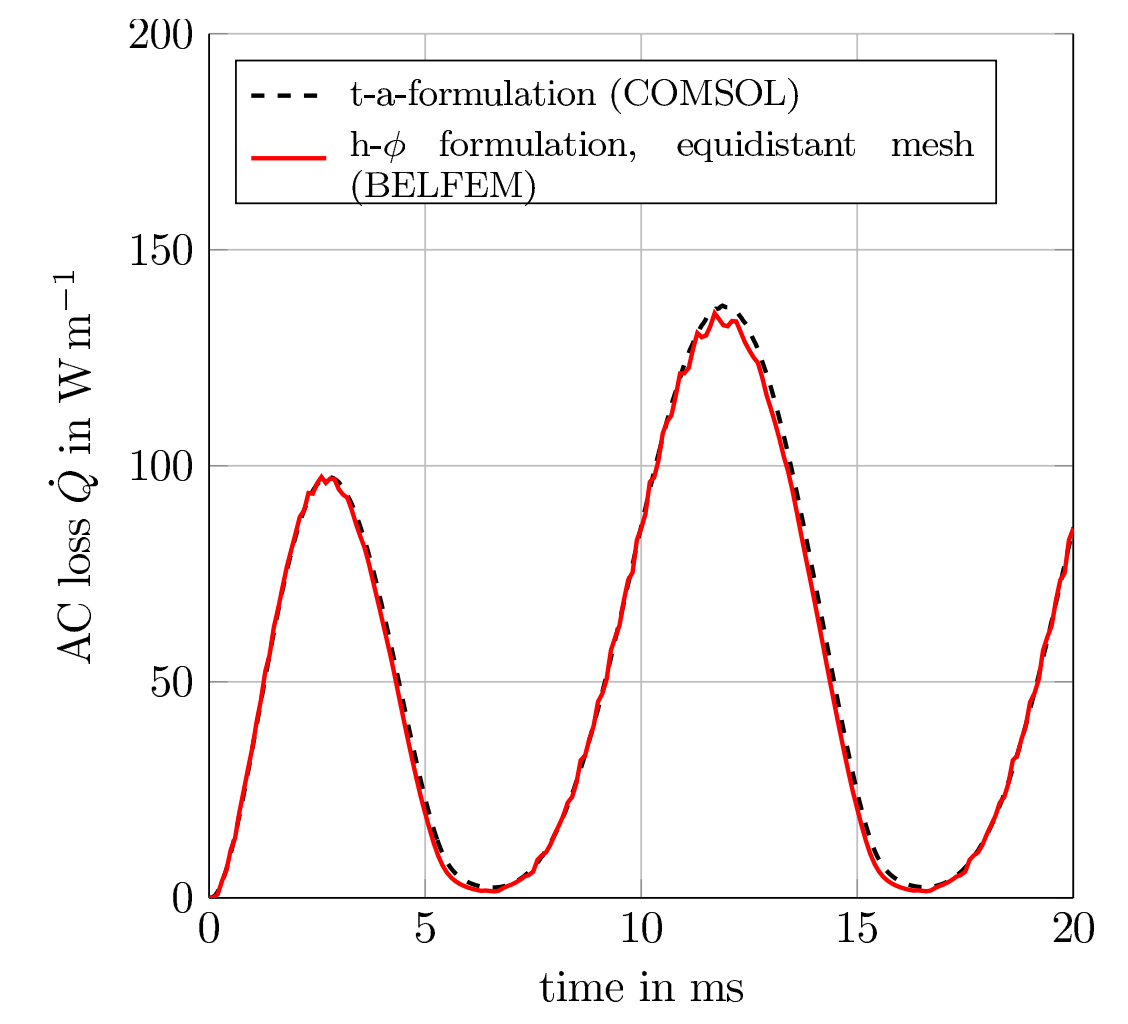
- **be open source once mature**

- Berkeley Lab specific BSD-3 like license

Current Status

Published Paper in SuST 2023

- Christian Messe, Berkeley Lab
- Nico Riva, MIT
- Sofia Viarengo, Politecnico di Torino
- Gregory Giard & Frédéric Sirois, Polytechnique Montreal
- validated against analytical methods + COMSOL / GetDP
- first research promises faster and more detailed results than other established methods such as t-a



IOP Publishing
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Superconductor Science and Technology
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BELFEM: a special purpose finite element code for the magnetodynamic modeling of high-temperature superconducting tapes

Christian Messe^{1,*}, Nicolò Riva², Sofia Viarengo³, Gregory Giard¹ and Frédéric Sirois⁴

¹ Lawrence Berkeley National Laboratory, Accelerator Technology and Applied Physics Division, Berkeley, CA, United States of America
² Massachusetts Institute of Technology, Plasma Science and Fusion Center, Cambridge, MA, United States of America
³ MAHTEP Group, Dipartimento Energia 'Galileo Ferraris', Politecnico di Torino, Turin, Italy
⁴ Polytechnique Montréal, Department of Electrical Engineering, Montréal QC H3T 1J4, Canada

E-mail: cmesse@lbl.gov

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Abstract
Predicting the performance and reliability of high-temperature superconducting (HTS) cables and magnets is a critical component of their research and development process. Novel mixed finite element formulations, particularly the h - ϕ -formulation with thin-shell simplification, present promising opportunities for more efficient simulations of larger geometries. To make these new methods accessible in a flexible tool, we are developing the Berkeley Lab Finite Element Framework (BELFEM). This paper provides an overview of the relevant formulations, discusses the current state of the art, and discusses the main aspects of the BELFEM code structure. We validate a first 2D thin-shell implementation in BELFEM against selected benchmarks computed in COMSOL Multiphysics and compare the performance of our code with a comparable formulation in GetDP. We also outline the next steps in the development process, paving the way for more advanced and robust modeling capabilities.

Keywords: HTS, FEM, modeling, h - ϕ -formulation, code development

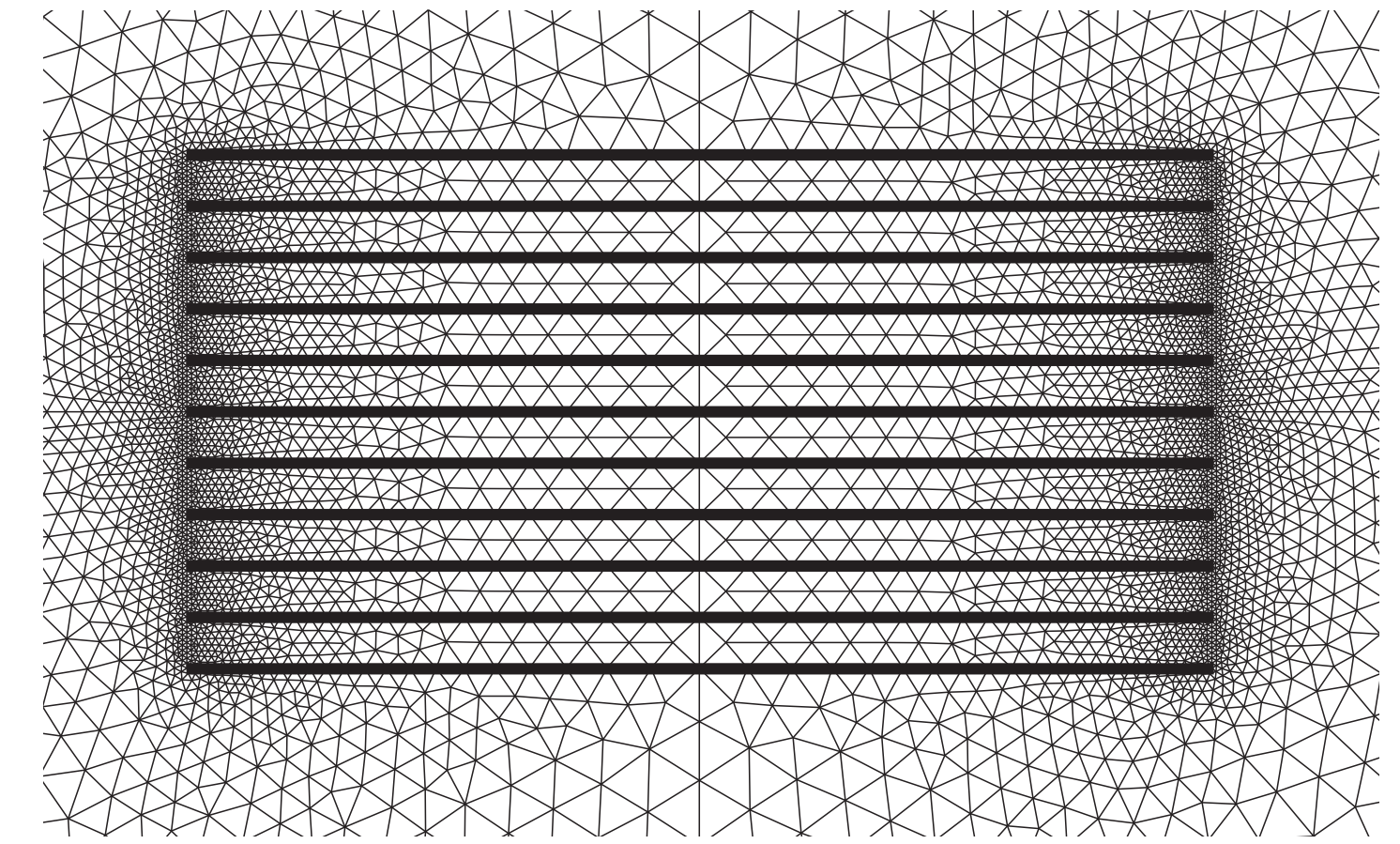
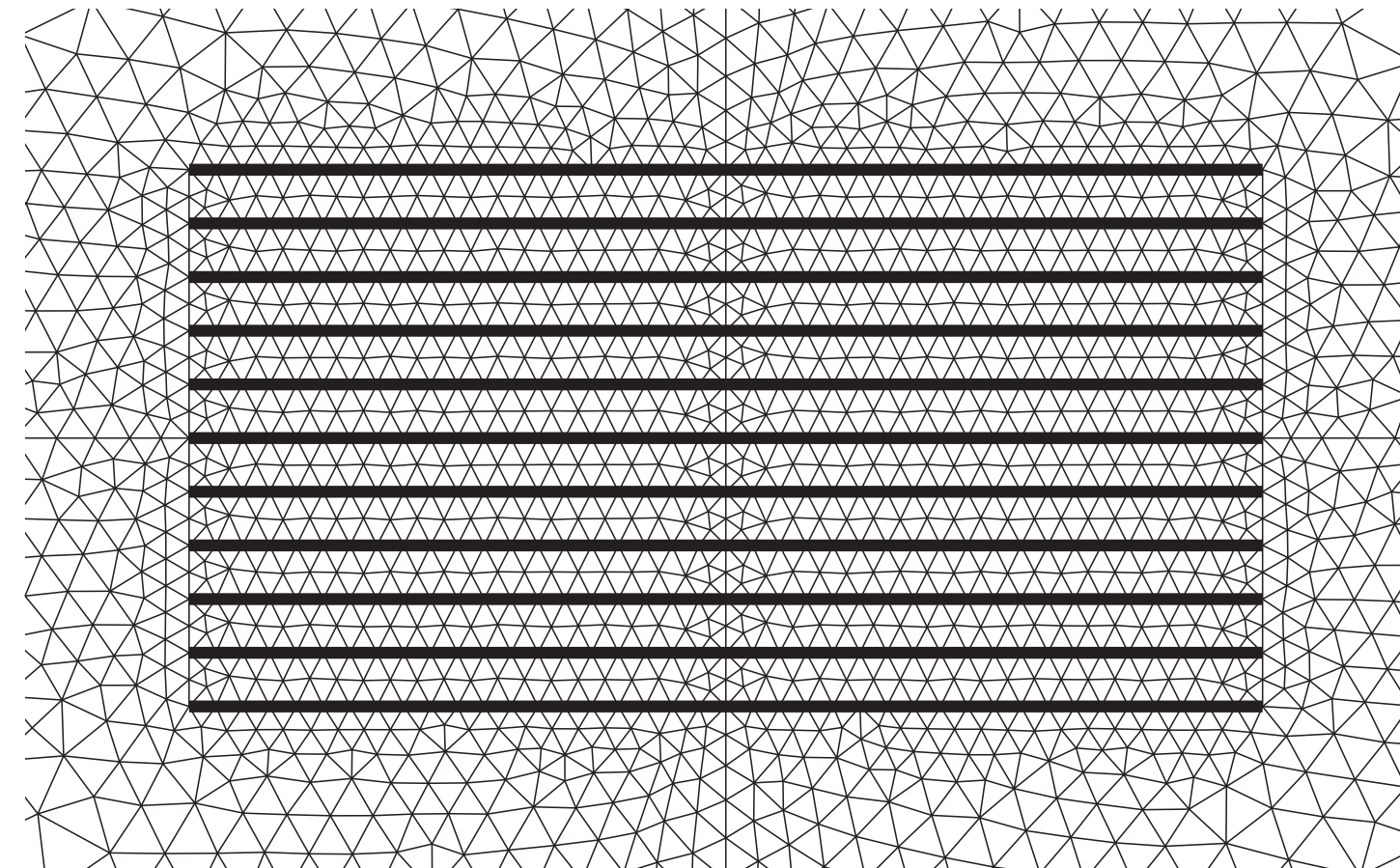
(Some figures may appear in colour only in the online journal)

1. Introduction

High-temperature superconducting (HTS) cables and magnets play a crucial role in nuclear fusion applications [1], particle accelerators [2], medical devices [3], and even spacecraft engines [4]. To ensure their safe operation, it is necessary to analyze and understand their electrodynamic and

thermal performance. Here, advanced finite element methods are essential and very powerful tools: not only do they solve the Maxwell's equations in their magnetodynamic form and the heat transfer equation that describe the physical behavior of these devices, they are also able to realistically model the highly nonlinear behavior of the used materials. The computational cost of these methods, however, remains an important challenge. In recent years, significant progress has been made in the development of so called *mixed formulations*, which promise to be more efficient than traditional finite element

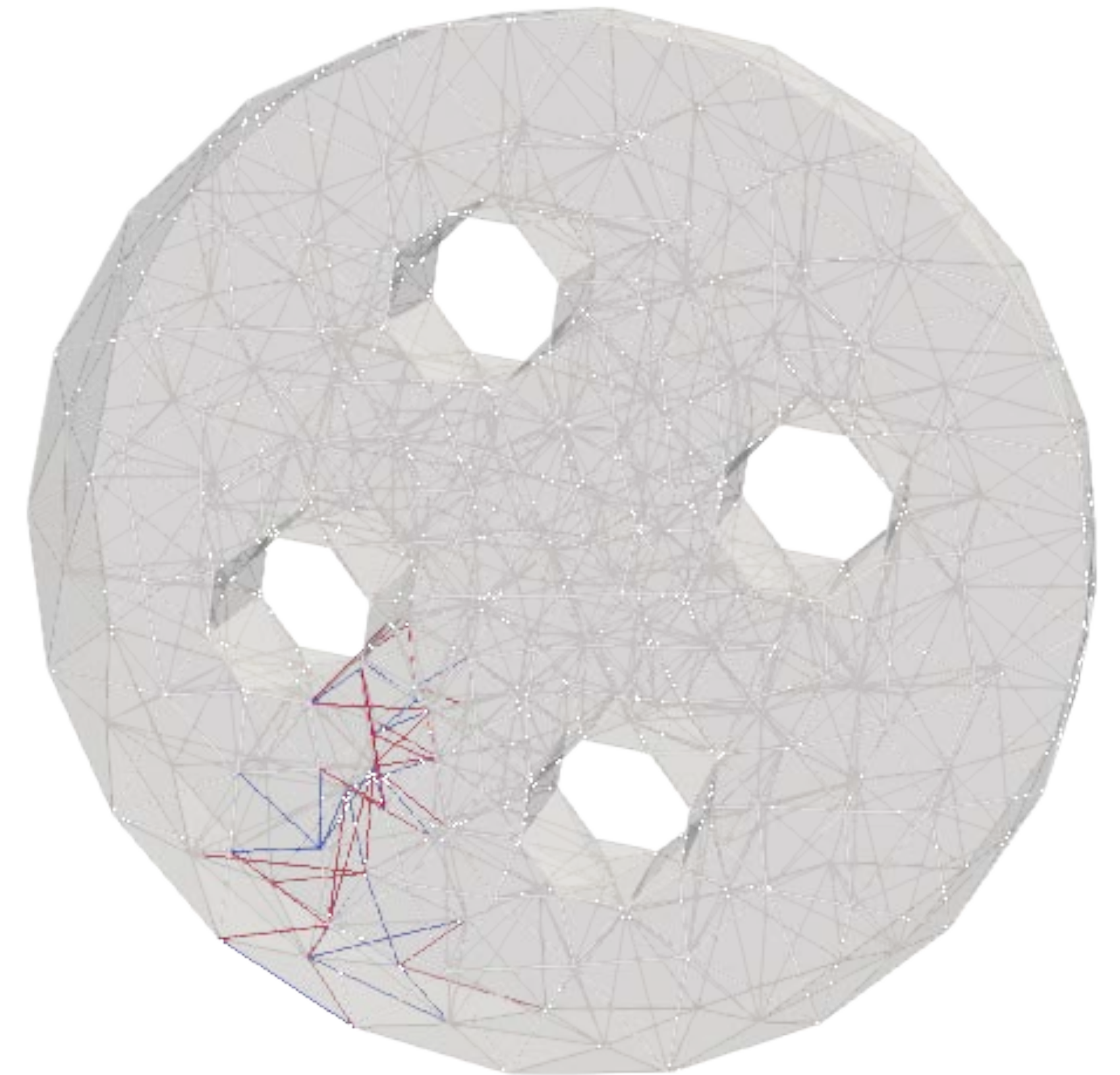
* Author to whom any correspondence should be addressed.

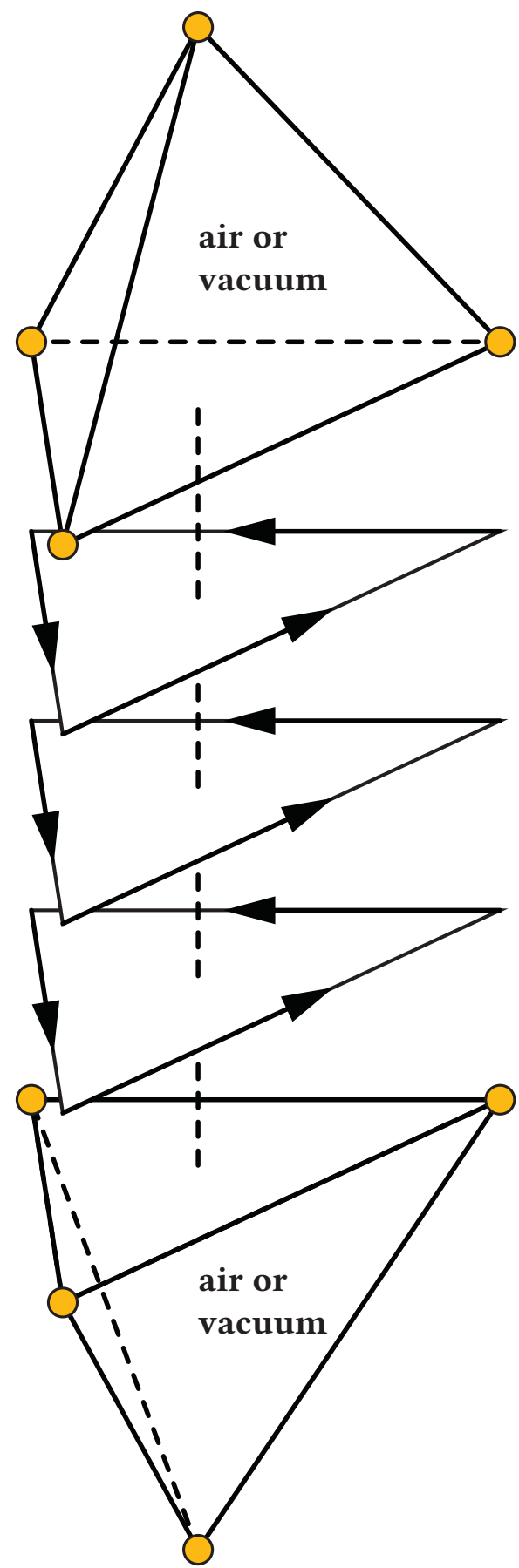


Code and Library	Constant time step		Adaptive time step	
	Coarse Mesh	Fine Mesh	Coarse Mesh	Fine Mesh
GetDP (MUMPS)	5:13	10:57	2:36	7:58
BELFEM (MUMPS)	1:56	7:15	0:23	1:09
BELFEM (STRUMPACK)	0:55	2:54	0:11	0:25

PhD Student Gregory Giard (Polytechnique Montreal):

- visiting scholar at LBL from 01/23-06/23
- contribution to adaptive time stepping method
- development of 3D thermal conduction model
- implementing automated cohomology computation in 3D
- automated identification of weak BCs based on user provided currents (“the user shall not worry about cohomologies”)



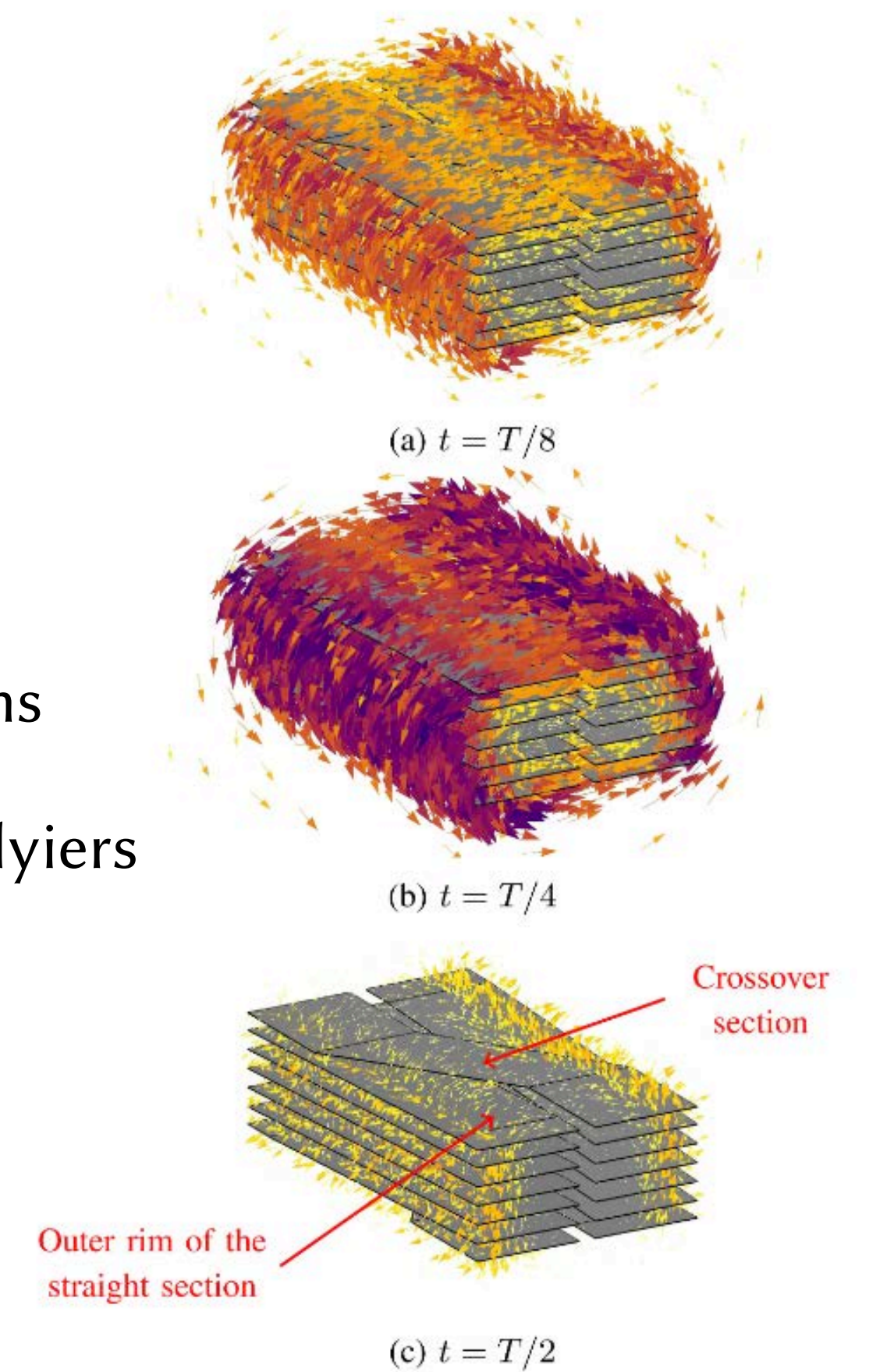


Goal:

- model a thin shell tapes tack in 3D after Alves et Al, 2022
- extend model to encompass solder and thermal model
- be able to do the coupled EM-Thermal quenching model by end of the year

Roadmap:

- overhaul data structure for simplified programming of weak governing equations
- implement “condensation” of degrees of freedom → get rid of Lagrange multipliers
- first benchmark with 3D tapestack
- implement solder and thermal model
- benchmark involving quench
- address contact sharing

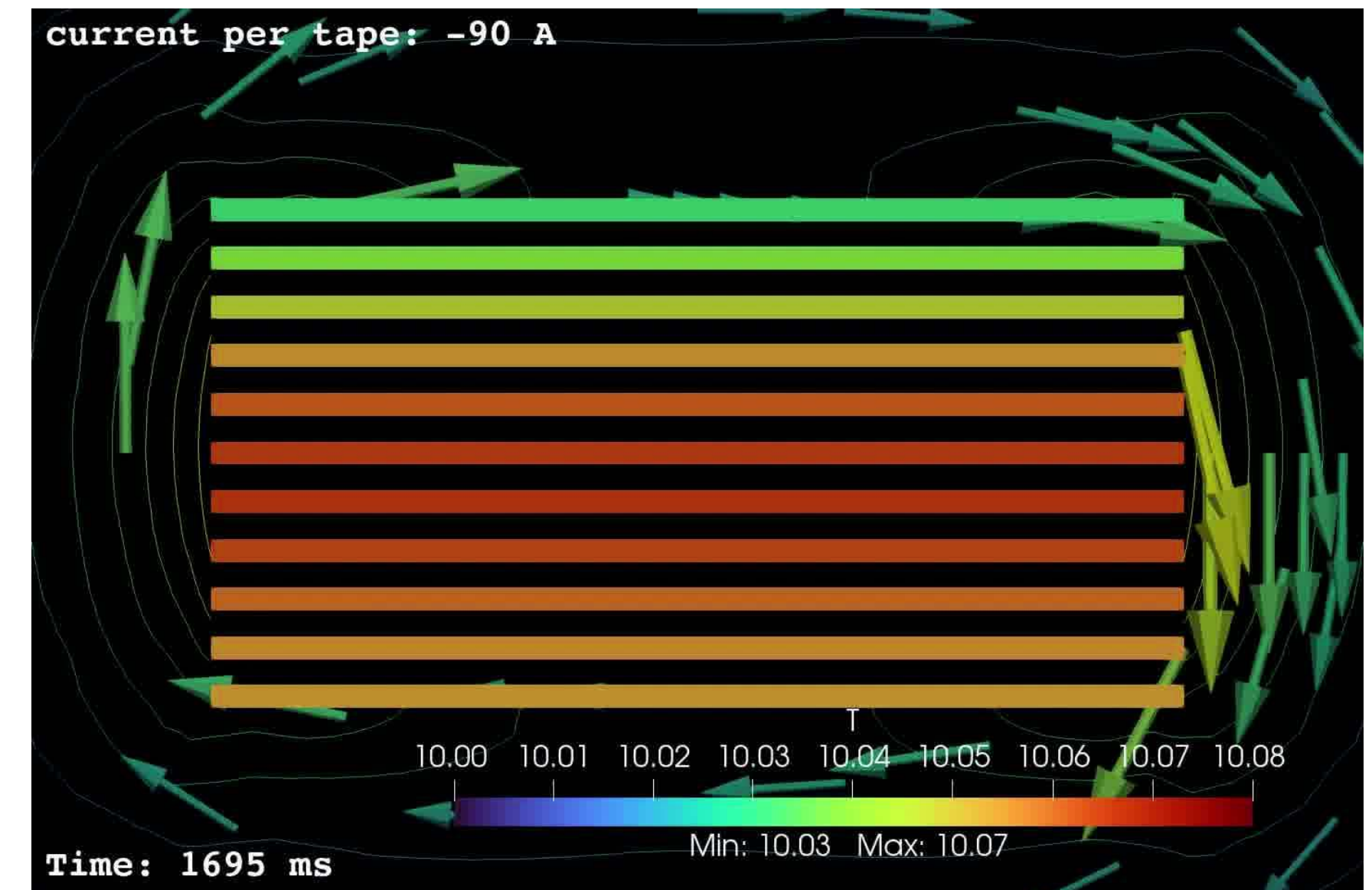
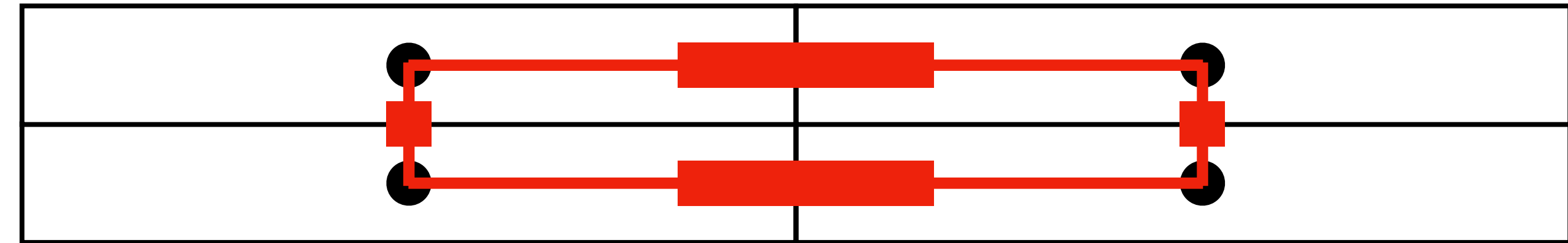
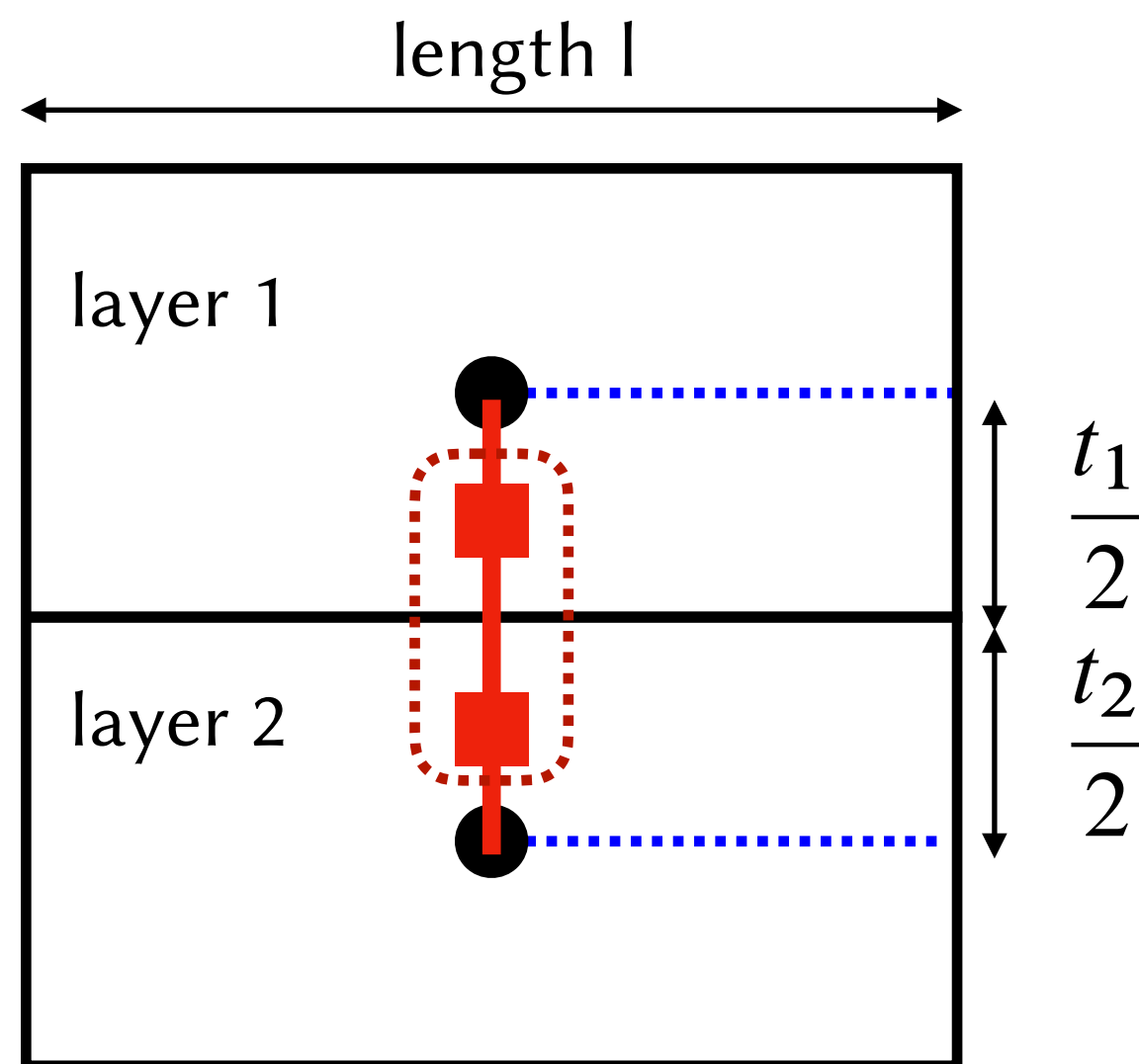


[Alves et al, 10.1109/TASC.2022.3143076]

Work in Progress: Thermal Coupling

Work in progress: thermal coupling

- linear discretization + mass lumping collapses finite-element method to resistor grid
- can be first order even if Maxwell is second order
- degrees of freedom sit on the edges
→ assumes constant temperature per layer per element



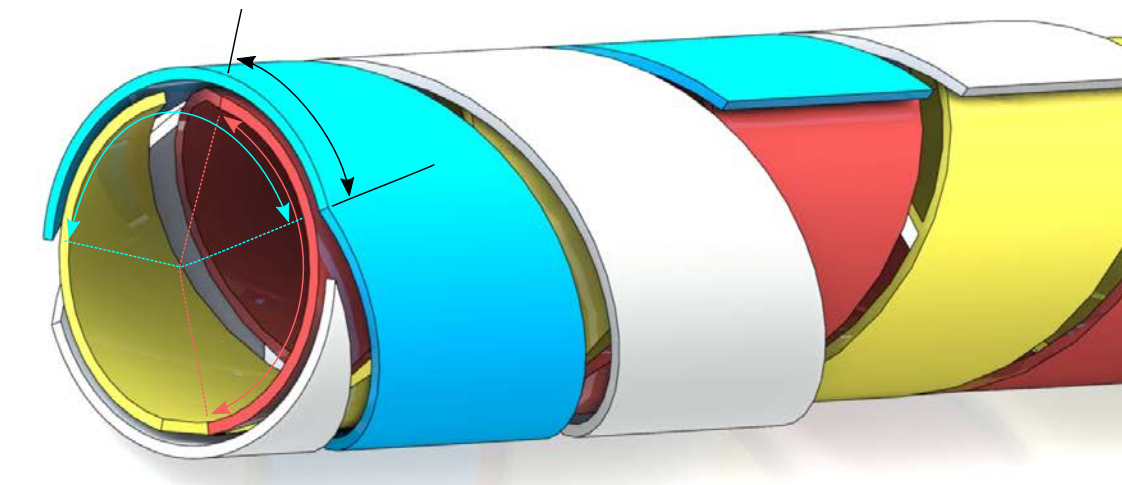
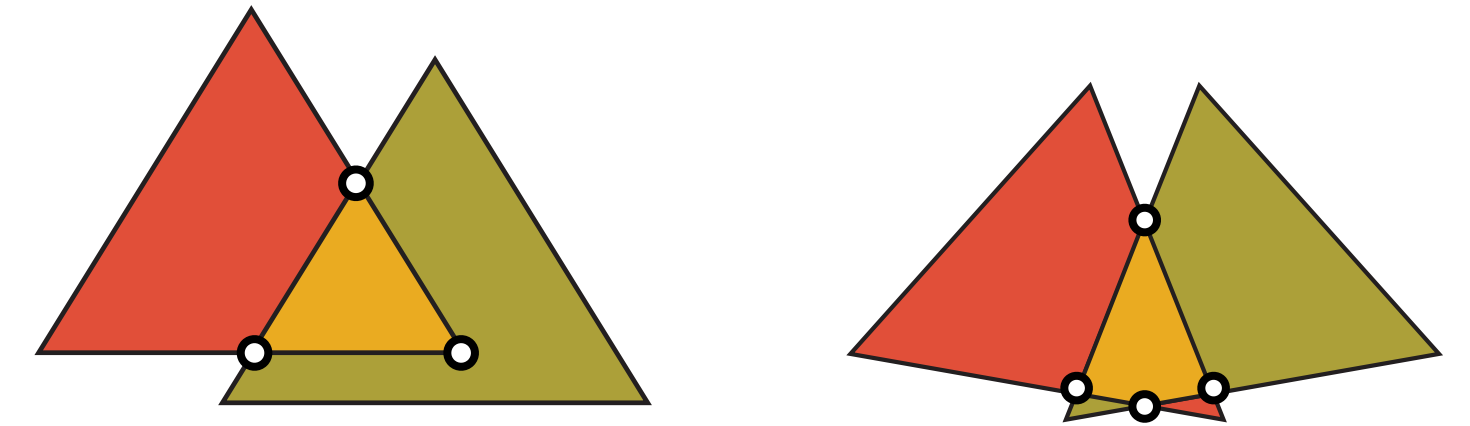
Work in Progress: Intertape current sharing

What we know:

- current sharing does not work in 2D!
- need a geometry preprocessor to compute overlapping surfaces
- coupling could work over the electric field

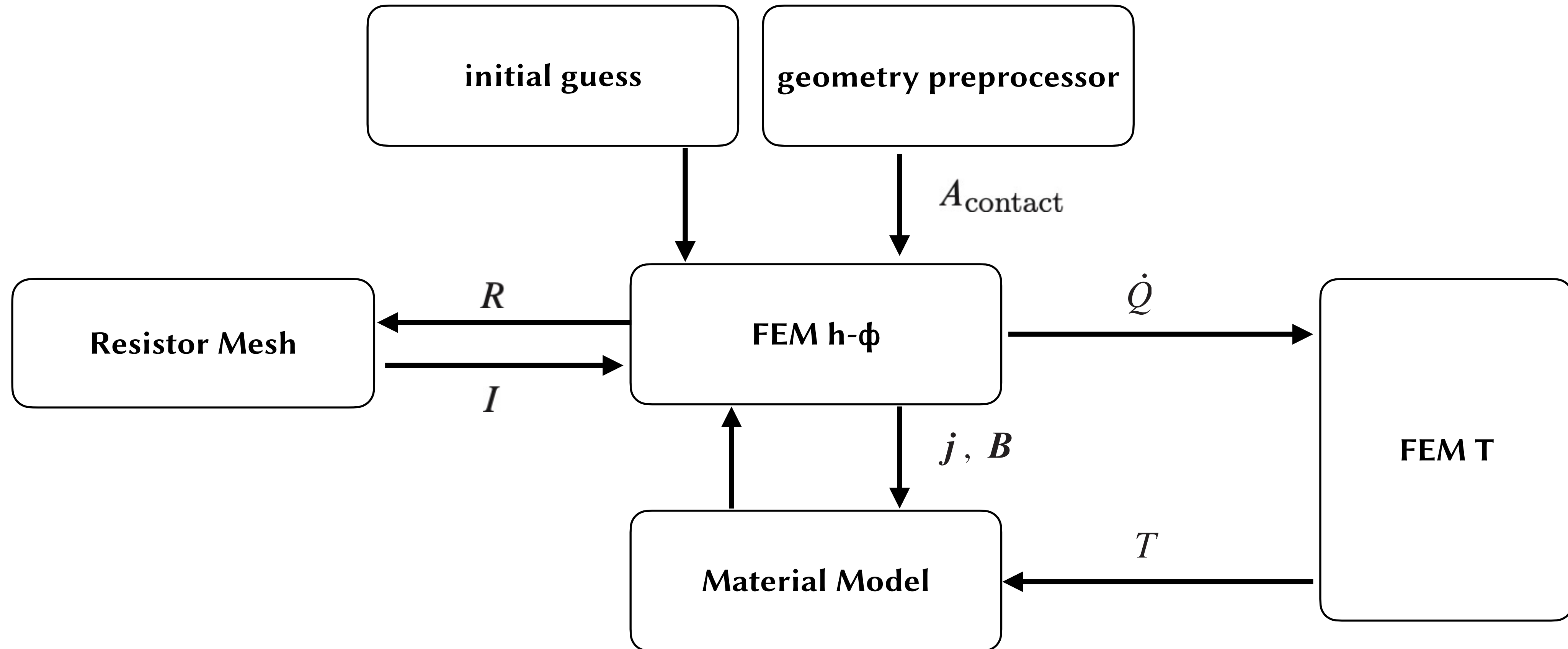
$$\int_{\Omega} \delta \mathbf{h}^T \frac{\partial (\mu \mathbf{h})}{\partial t} dV + \int_{\Omega} \delta \mathbf{h}^T \times \nabla^T \rho \nabla \times \mathbf{h} dV + \int_{\Gamma} \delta \mathbf{h}^T \mathbf{n} \times \mathbf{e} dS = 0$$

$$\mathbf{e} = \rho \nabla \times \mathbf{h}$$



Open Questions:

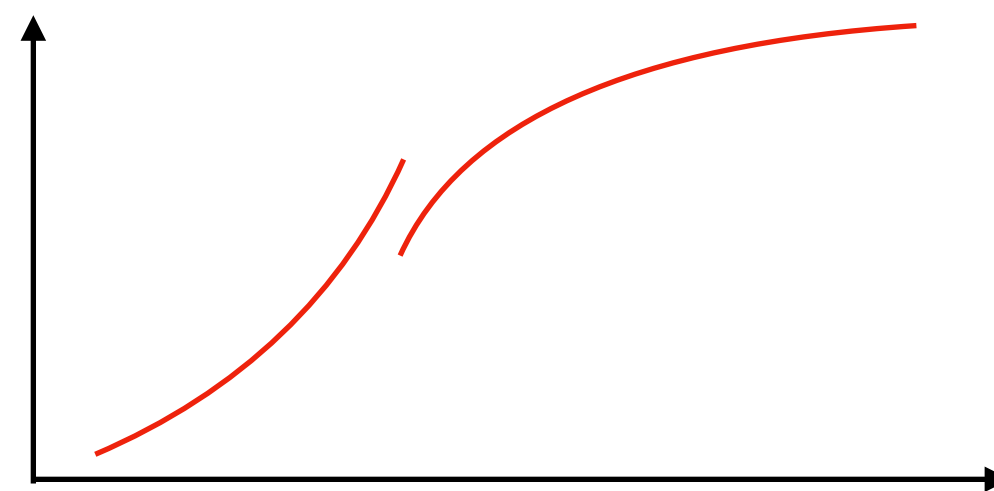
- Does contact resistivity affect the validity of cohomologies?
- Is the model complete and free of contradiction?
- Does the model need to be extended with a resistor mesh ?
 - modify current boundary conditions!
 - use same resistor mesh as for thermal model!



Material Modeling

- **High Nonlinearities for HTS Materials**

- material properties must be evaluated at every integration point
- nonlinearities require many iterations
- material curves must be smooth (consistent derivatives, continuous)
- avoid expensive functions such as exp or log



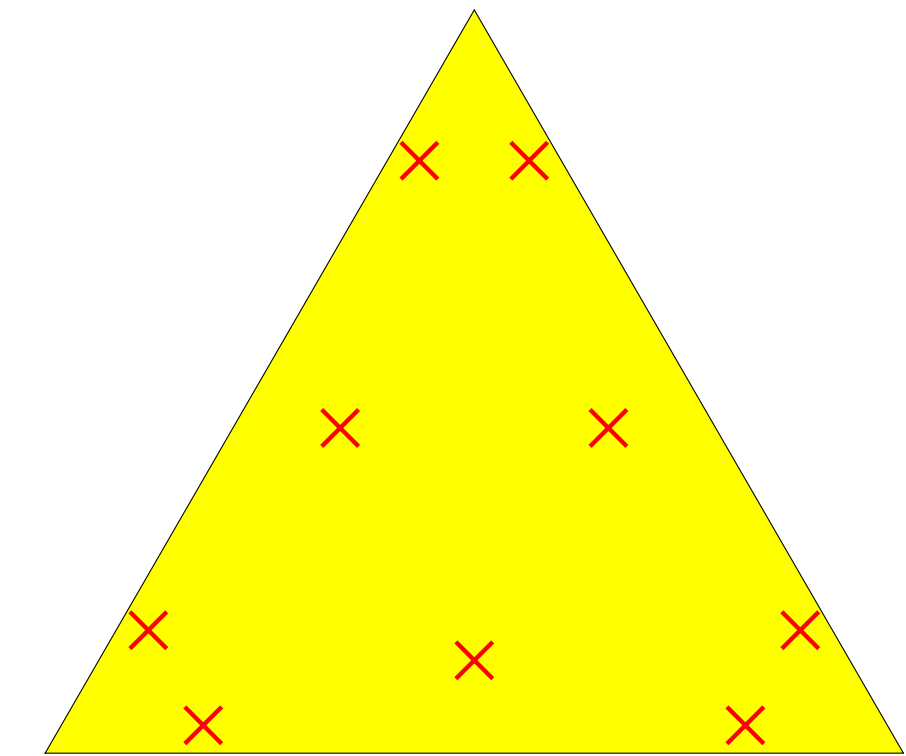
piecewise polynomials lead to convergence issues due to

- discontinuous material properties
- discontinuous derivatives

CSE can be up to ~3 x faster!!!

- **Boobytraps in modeling and coding**

- piecewise polynomials
- wasteful implementations
- validity range of functions



triangle integration points
(5th order)

bad code: wasting multiplications:

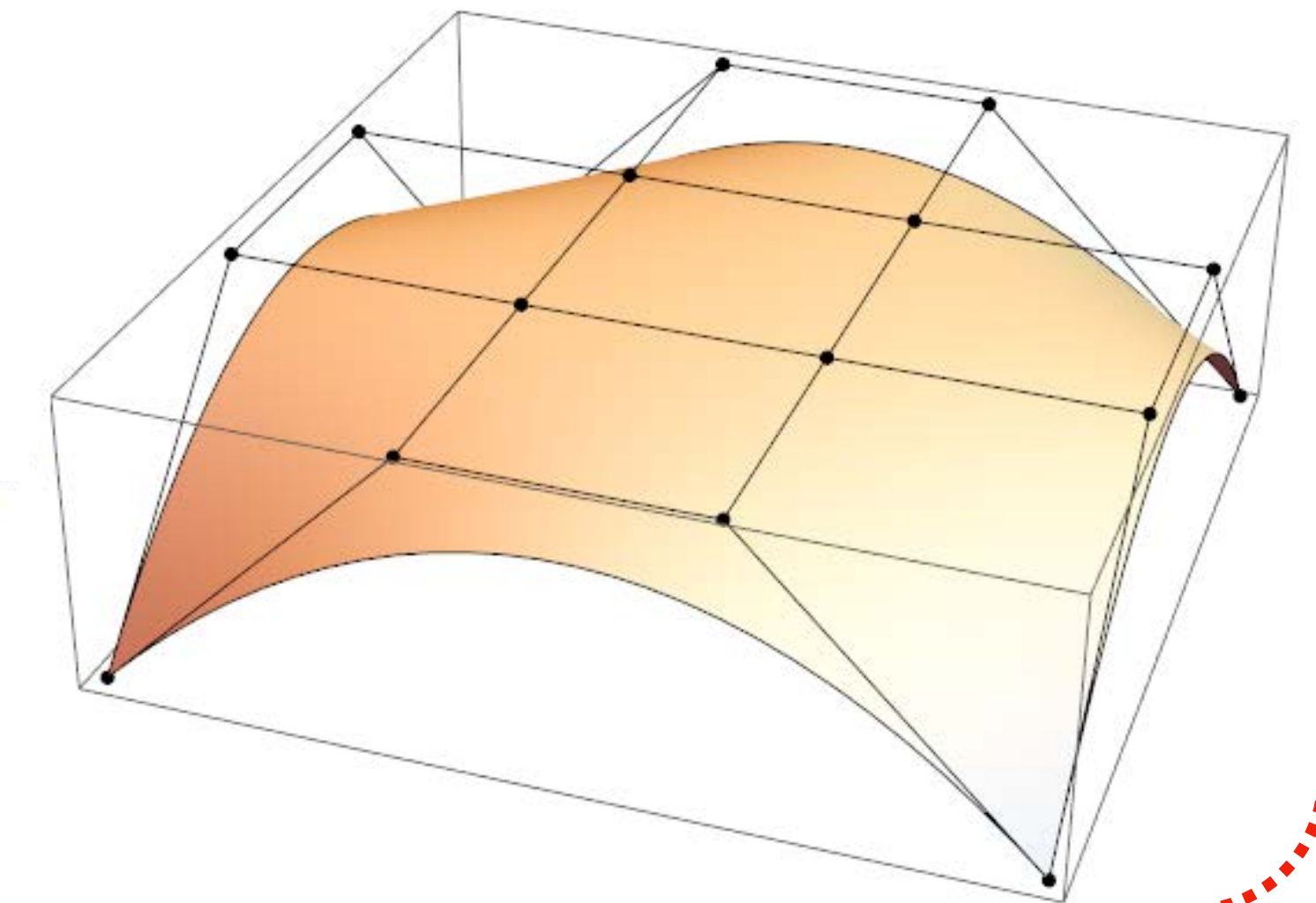
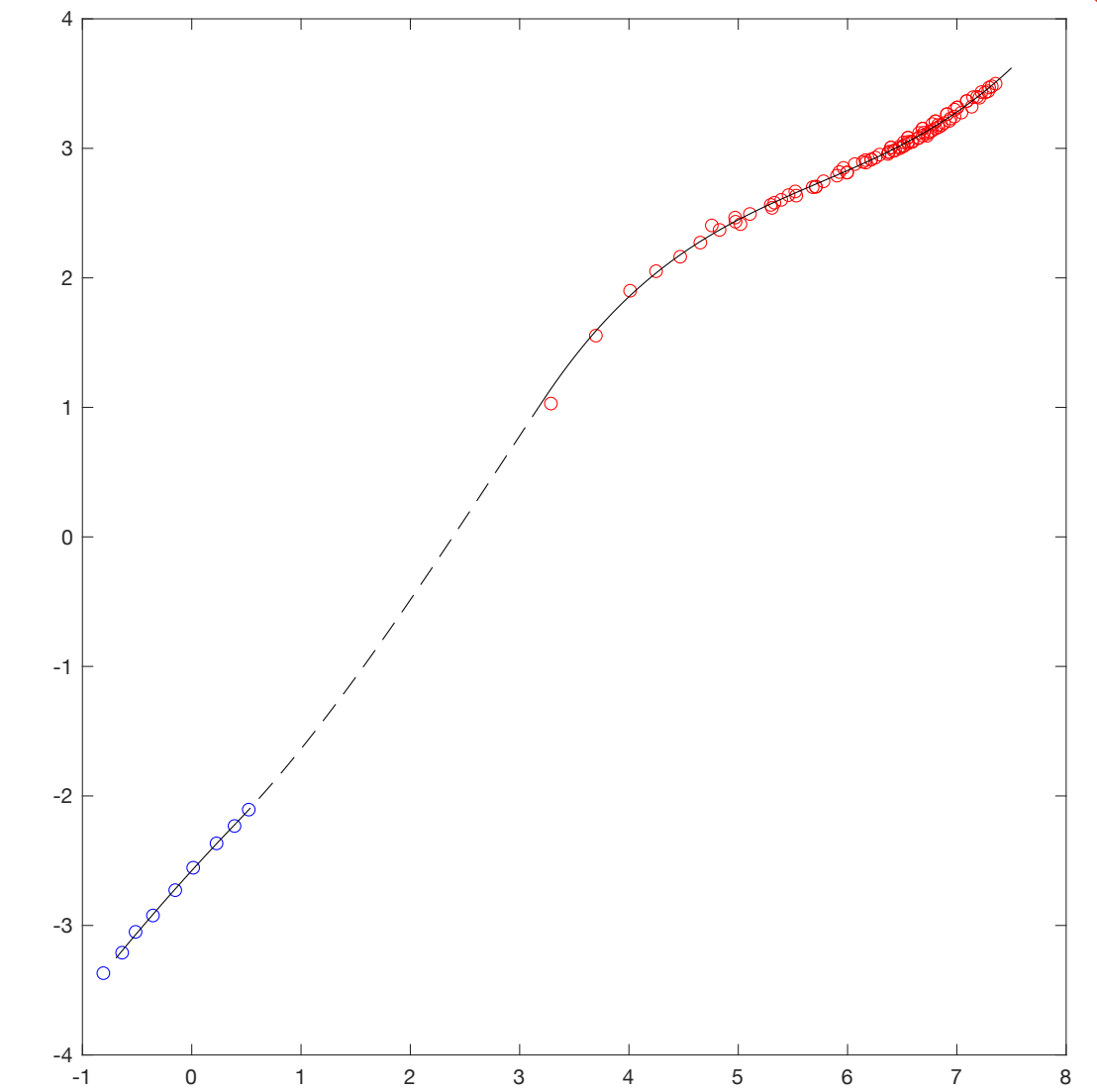
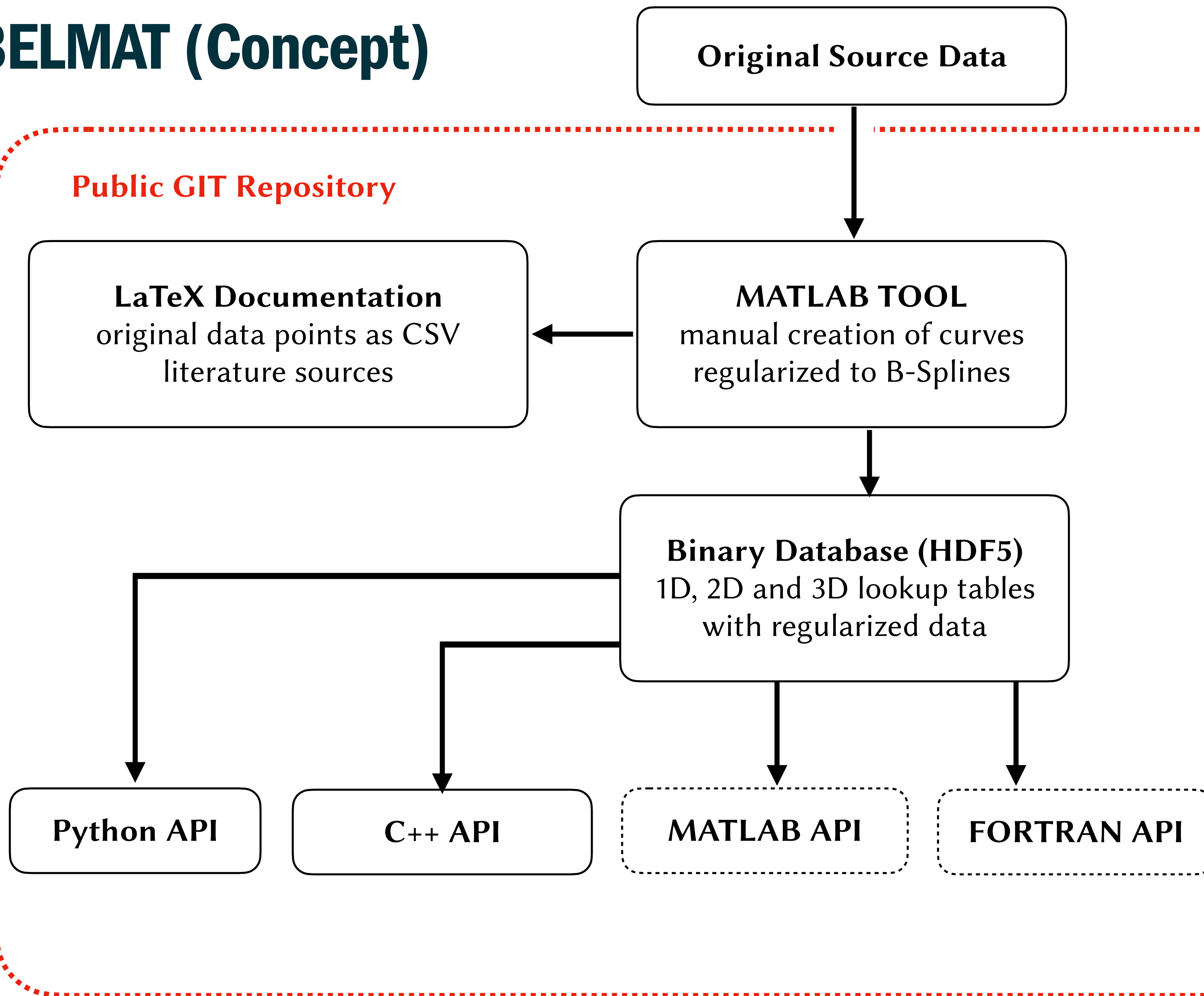
$$y = 10^{(a + b * \log(T) + c * \log(T)^2 + d * \log(T)^3 + \dots)}$$

good code: Common Subexpression Elimination (CSE)

$$\log T = \log(T)$$

$$y = 10^{(a + \log T * (b + \log T * (c + \log T * (d + \dots))))}$$

BELMAT (Concept)



Usecase Example

- implemented in SparseLizard: ASC 2022
- implemented in BELFEM: SUST 2023

$H-\phi$ Formulation in Sparselizard Combined With Domain Decomposition Methods for Modeling Superconducting Tapes, Stacks, and Twisted Wires

N. Riva, A. Halbach, M. Lyly, C. Messe, J. Ruuskanen, and V. Lahtinen

Abstract—The growing interest in the modeling of superconductors has led to the development of effective numerical methods and software. One of the most utilized approaches for magnetostatic simulations in applied superconductivity is the H -formulation. However, due to the large number of degrees of freedom (DOFs) present when modeling large and complex systems (e.g. large coils for fusion applications, electrical machines, and medical applications) using the standard H -formulation on a desktop machine becomes infeasible. The H -formulation solves the Faraday's law formulated in terms of the magnetic field intensity H using edge elements in the whole modeling domain. For this reason, a very high resistivity is assumed for the non-conducting domains, leading to an ill-conditioned system matrix and therefore long computation times. In contrast, the $H-\phi$ formulation uses the H -formulation in the conducting regions, and the ϕ formulation (magnetic scalar potential) in the surrounding non-conducting domains, drastically reducing DOFs and computation time. In this work, we use the $H-\phi$ formulation in 2D for the magnetothermal (AC losses and quench) analysis of stacks of REBCO tapes. The same approach is extended to a 3D case for the AC loss analysis of a twisted superconducting wire. All the results obtained by simulations in Sparselizard are compared with results obtained with COMSOL. Our custom tool allows us to distribute the simulations over hundreds of CPUs using domain decomposition methods, considerably reducing the simulation times without compromising accuracy.

Index Terms—HTS, REBCO, modeling, AC Loss, quench, $H-\phi$ -formulation, cloud, DDM

1. INTRODUCTION

WHEN modeling superconducting materials, the electrical resistivity is generally modeled using the power law constitutive relationship [1], which may include a complex critical current density dependence [2]. The highly nonlinear properties and strong anisotropic field dependence of the critical current density could lead to a very large computation time. Moreover, the high aspect ratio of the superconducting tapes (especially in the case of High-Temperature Superconductors (HTS)) leads to a large number of elements and degrees of freedom (DOFs) in the finite element mesh. This research is supported by Quantest, Type One Energy, PFC MIT, CPS and LLNL. M. Lyly acknowledges support from the Academy of Finland project 32487.

A. Halbach, M. Lyly, and V. Lahtinen are with Quantest Oy (Tampere, Finland) (e-mail: alexander.halbach@quantest.com, mila.lyly@quantest.com, vesa.lahtinen@quantest.com).

M. Lyly and J. Ruuskanen are with Tampere University (Tampere, Finland) (e-mail: mila.lyly@utu.fi, jarmo.ruuskanen@utu.fi).

N. Riva is with MIT Plasma Science and Fusion Center (MA, USA) (e-mail: nriva@mit.edu).

C. Messe and J. Ruuskanen are with Lawrence Berkeley National Laboratory (CA, USA) (e-mail: cmesse@lbl.gov).

freedom (DOFs). A widely used method is the H -formulation [3]. However, due to the large scale of systems such as electrical machines [4] and fusion devices [5], the computational limits are rapidly reached with the H -formulation [6], [7]. Moreover, the use of the H -formulation in nonconducting domains leads to unnecessary large number of DOFs due to the vectorial nature of the magnetic field intensity H and to numerical instabilities due to the imposed high resistivity to avoid eddy currents in such domains, leading to an ill-conditioned matrix. The development of approaches more efficient than the H -formulation to be implemented in commercial and in-house software is of paramount importance to improve the computational efficiency of the models. Recently, several works have led to drastic improvements in computational efficiency using the $A-H$ [8]–[10], the $T-A$ (similar to the $A-H$) [11]–[13], and the $H-\phi$ formulations [14]–[17].

This paper aims at addressing the current challenges of 3D modeling 2G HTS using the $H-\phi$ formulation combined with domain decomposition methods (DDM) [18], [19], enabling massive parallel computation and drastically reduced simulation time. The presented case studies are chosen to represent fusion-energy inspired industrially relevant cases in AC loss and quench modeling.

In section II, we briefly describe the formulation and its implementation in Sparselizard and we describe the utilized custom DDM tool. In section III, we present validating results using simple 2D models, and in section IV, we move on to more complex 3D models, demonstrating the virtues of our DDM-based tool. Finally, in section V, we draw conclusions.

II. $H-\phi$ FORMULATION AND IMPLEMENTATION

A. Formulation

The $H-\phi$ formulation where current constraints are imposed using cohomology cuts is well-known in computational electromagnetics [20]–[23] and the mathematical main ideas in an electromagnetic context can be traced back to Kotega's curly works on making cuts for scalar potentials [24]. Eventually, it was brought to the context of superconductor AC loss simulations by Lahtinen, Stenwall et al. [14], [15].

The finite element formulation is obtained by developing the weak form of Faraday's law of induction and the Gauss law for the magnetic field. Ohm's law is used as transport law. The magnetic field strength H is discretized in the conducting regions of the computational domain with Nédélec

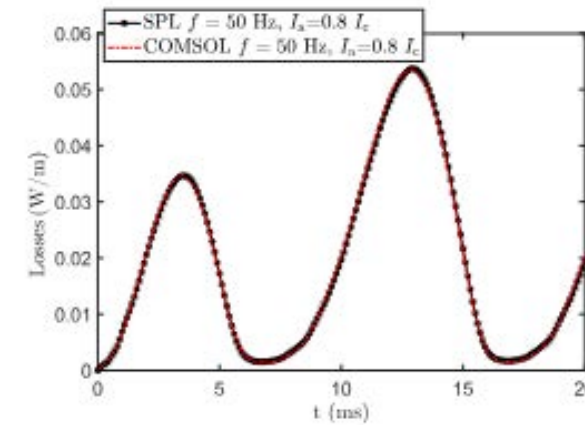


Fig. 5. Instantaneous losses computed on the entire assembly (NbTi wires + copper), with Sparselizard (continuous line + markers) and COMSOL (dashed line).

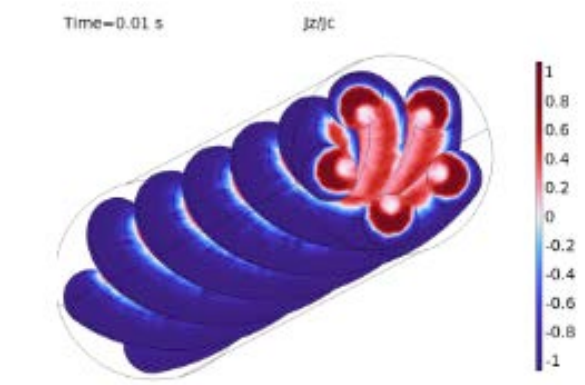


Fig. 6. Normalized current density at $t = 10$ ms.

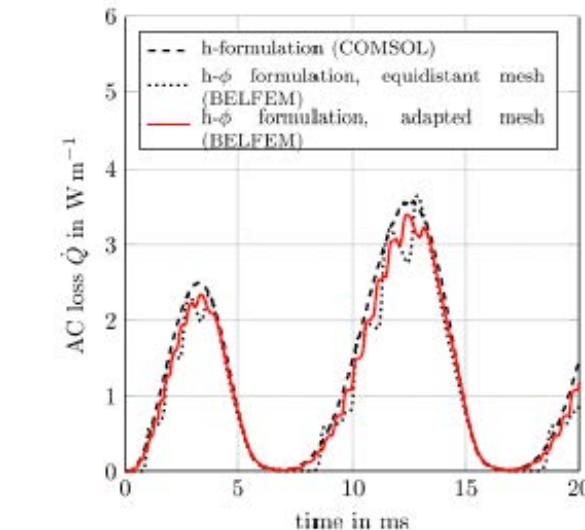


Figure 12. AC losses for the tape stack at $I = 90$ A per tape at $f = 50$ Hz.

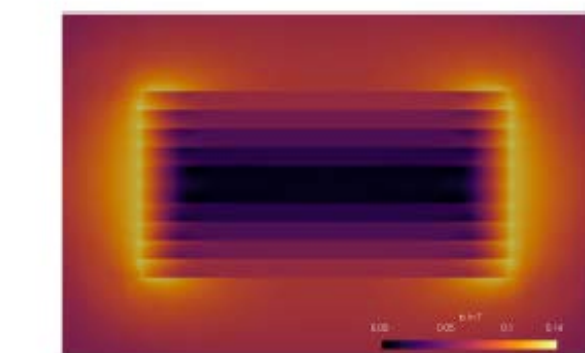


Figure 13. Magnetic flux density at 15 ms with $I = 90$ A per tape at $f = 50$ Hz.

```

main.cpp
/private/tmp/main.cpp
1 int main( int argc,
2 char * argv[] )
3 {
4 // Material selection.
5 std::string database = "materials.hdf5" ; // database file
6 std::string label = "copper" ; // material to compute
7 uint rrr = 50 ; // purity value
8
9 // create the material
10 belmat::material mat( database, label, rrr );
11
12 double B = 0.0 ; // magnetic flux density B
13 double T = 0.0 ; // temperature in K
14
15 std::cout << mat.label() << std::endl ;
16 std::cout << " density @298.15K: " << mat.density() << std::endl ;
17 while( T < 300 )
18 {
19 std::cout << " " << T ;
20
21 if( mat.has_cp() )
22 {
23 std::cout << " " << mat.cp( T ) ;
24 }
25 if( mat.has_k() )
26 {
27 std::cout << " " << mat.k( T, B ) ;
28 }
29 if( mat.has_rho() )
30 {
31 std::cout << " " << mat.rho( T, B ) ;
32 }
33 std::cout << std::endl ;
34 T += 5.0 ;
35 }

```

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BELFEM: a special purpose finite element code for the magnetodynamic modeling of high-temperature superconducting tapes

Christian Messe¹, Nicolò Riva², Sofia Viarengo³, Gregory Giard⁴ and Frédéric Sirois⁵

¹Lawrence Berkeley National Laboratory, Accelerator Technology and Applied Physics Division, Berkeley, CA, United States of America

²Massachusetts Institute of Technology, Plasma Science and Fusion Center, Cambridge, MA, United States of America

³MAATEP Group, Dipartimento Energia "Guglielmo Ferraris", Politecnico di Torino, Turin, Italy

⁴Politecnico di Montréal, Department of Electrical Engineering, Montréal QC H3T 1J4, Canada

E-mail: cmesse@lbl.gov

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Abstract

Predicting the performance and reliability of high-temperature superconducting (HTS) cables and magnets is a critical component of their research and development process. Novel mixed finite element formulations, particularly the $H-\phi$ -formulation with thin-shell simplification, present promising opportunities for more efficient simulation of larger geometries. To make these new methods accessible in a flexible tool, we are developing the Berkeley Lab Finite Element Framework (BELFEM). This paper provides an overview of the relevant formulations, discusses the current state of the art and discusses the main aspects of the BELFEM code structure. We validate a first 2D thin-shell implementation in BELFEM against selected benchmarks computed in COMSOL. Multiphysics analysis and compare the performance of our code with a comparable formulation in GetDP. We also outline the next steps in the development process, paving the way for more advanced and robust modeling capabilities.

Keywords: HTS, FEM, modeling, $H-\phi$ -formulation, code development

(Some figures may appear in colour only in the online journal)

1. Introduction

High-temperature superconducting (HTS) cables and magnets play a crucial role in nuclear fusion applications [1], particle acceleration [2], medical devices [3], and even spacecraft engines [4]. To ensure their safe operation, it is necessary to analyze and understand their electrodynamic and

thermal performance. Here, advanced finite element methods are essential and very powerful tools: not only do they solve the Maxwell's equations in their magnetodynamic form and the heat transfer equation that describe the physical behavior of these devices, they are also able to realistically model the highly nonlinear behavior of the used materials. The computational cost of these methods, however, remains an important challenge. In recent years, significant progress has been made in the development of so called *mixed formulations*, which promise to be more efficient than traditional finite element

* Author to whom any correspondence should be addressed.



- developing a finite-element framework tailored to HTS cable & magnet development needs
- demonstrated proof of concept in 2D
- first performance tests very promising
- Work in progress: 3D tape model for quenching (goal: winter 2024)
- Work in progress: Intertape current sharing (goal: spring 2025)
- Work in progress: Material Database