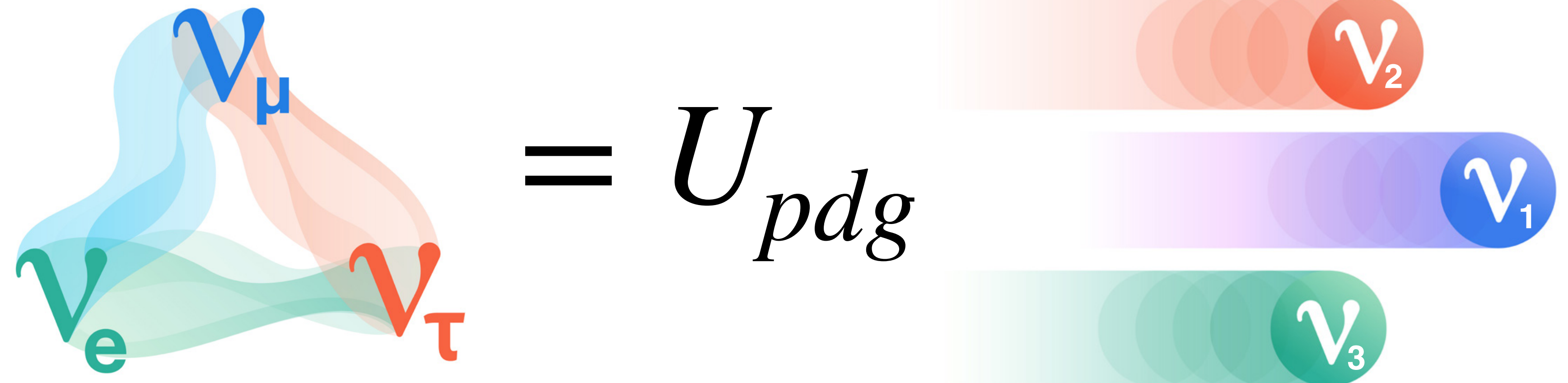
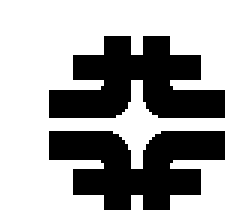


This Decades Race for the Neutrino Mass Ordering



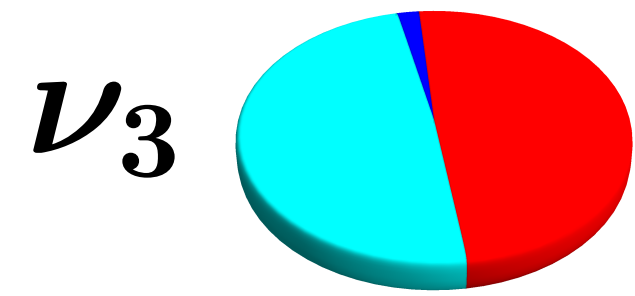
Stephen Parke: [Theory-Fermilab](#)
linktr.ee/stephen.parke



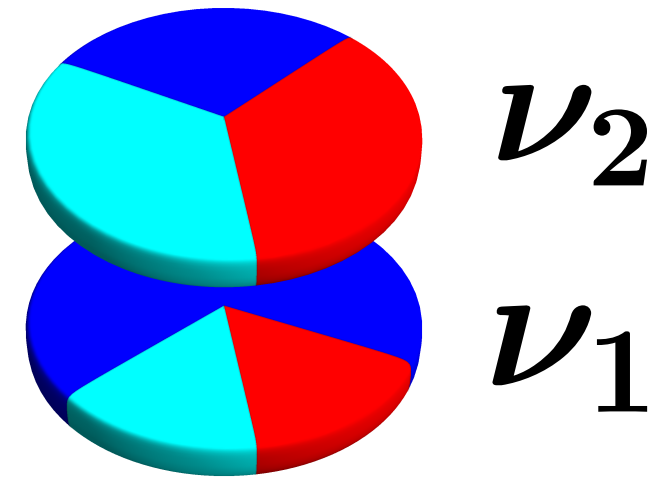


the Neutrino Mass Ordering

NO



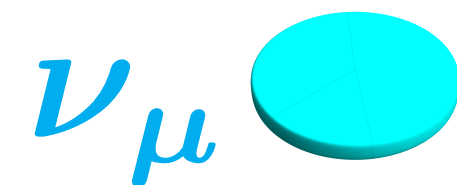
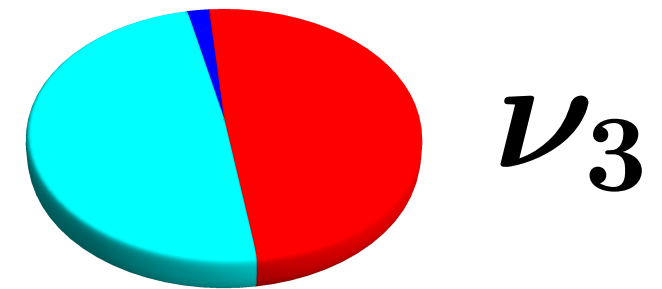
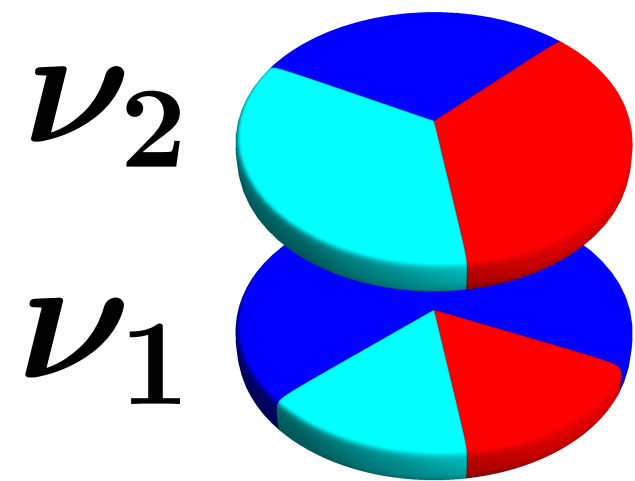
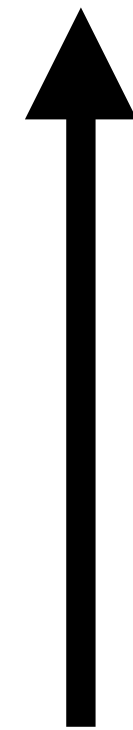
IO

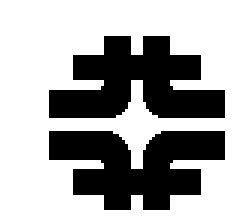


Define 1,2 & 3 such that:

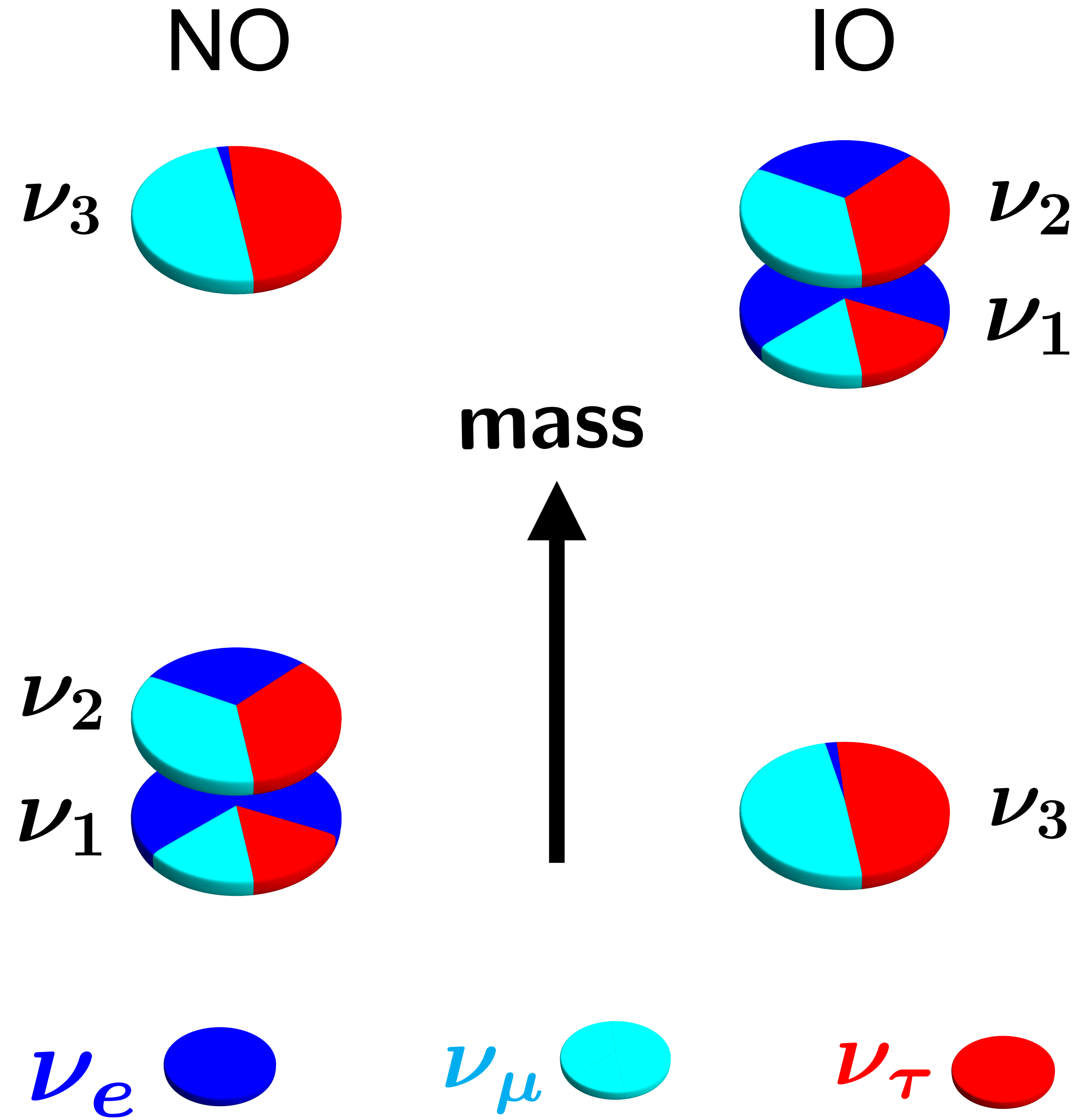
$$|U_{e1}|^2 > |U_{e2}|^2 > |U_{e3}|^2$$

mass





the Neutrino Mass Ordering

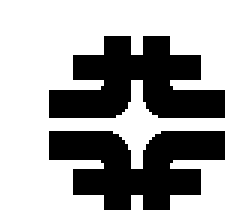


Define 1,2 & 3 such that:

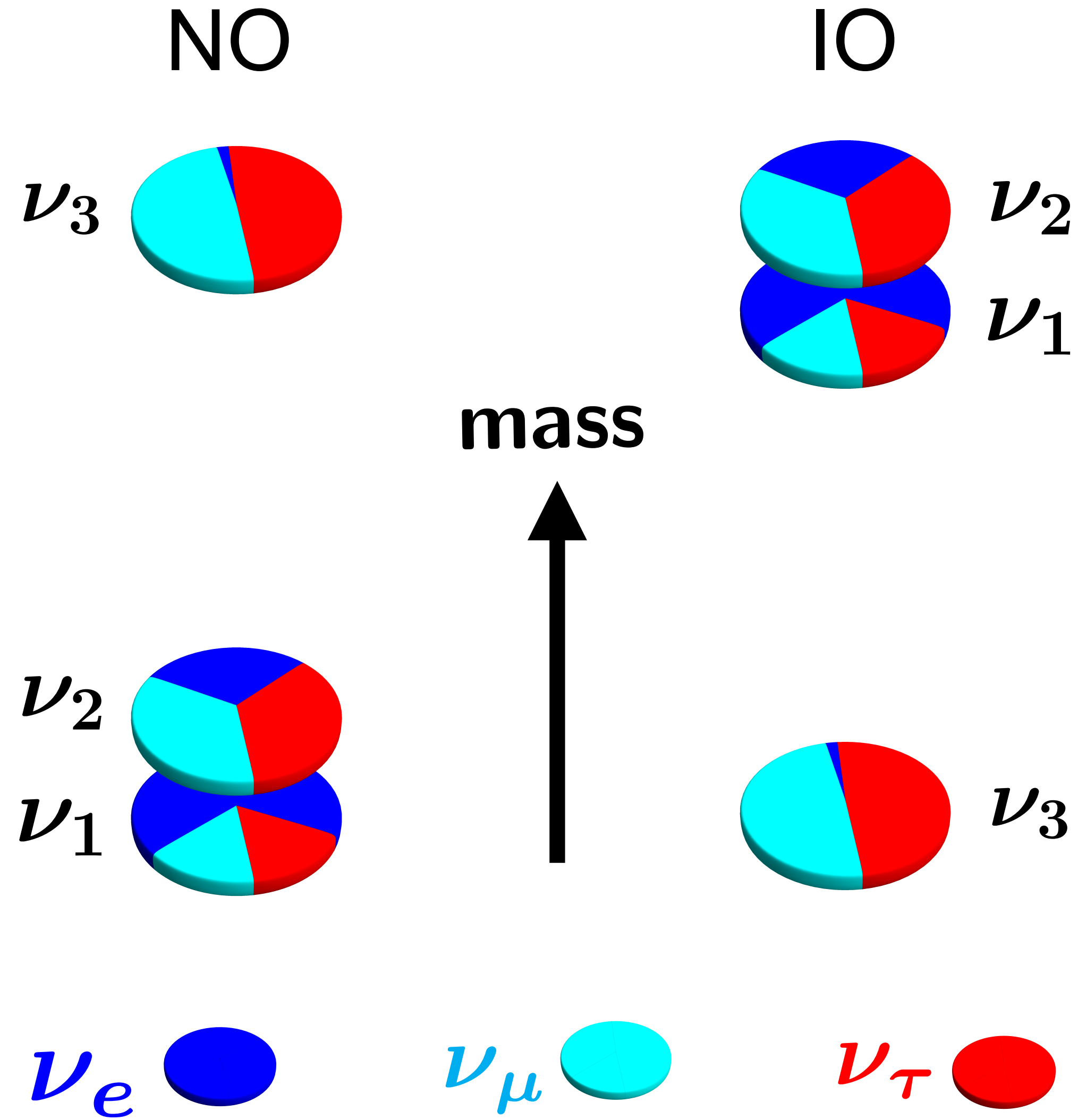
$$|U_{e1}|^2 > |U_{e2}|^2 > |U_{e3}|^2$$

SNO $m_2 > m_1$

$$|\Delta m_{21}^2| = |m_2^2 - m_1^2| = 7.5 \times 10^{-5} \text{ eV}^2$$



the Neutrino Mass Ordering



Define 1,2 & 3 such that:

$$|U_{e1}|^2 > |U_{e2}|^2 > |U_{e3}|^2$$

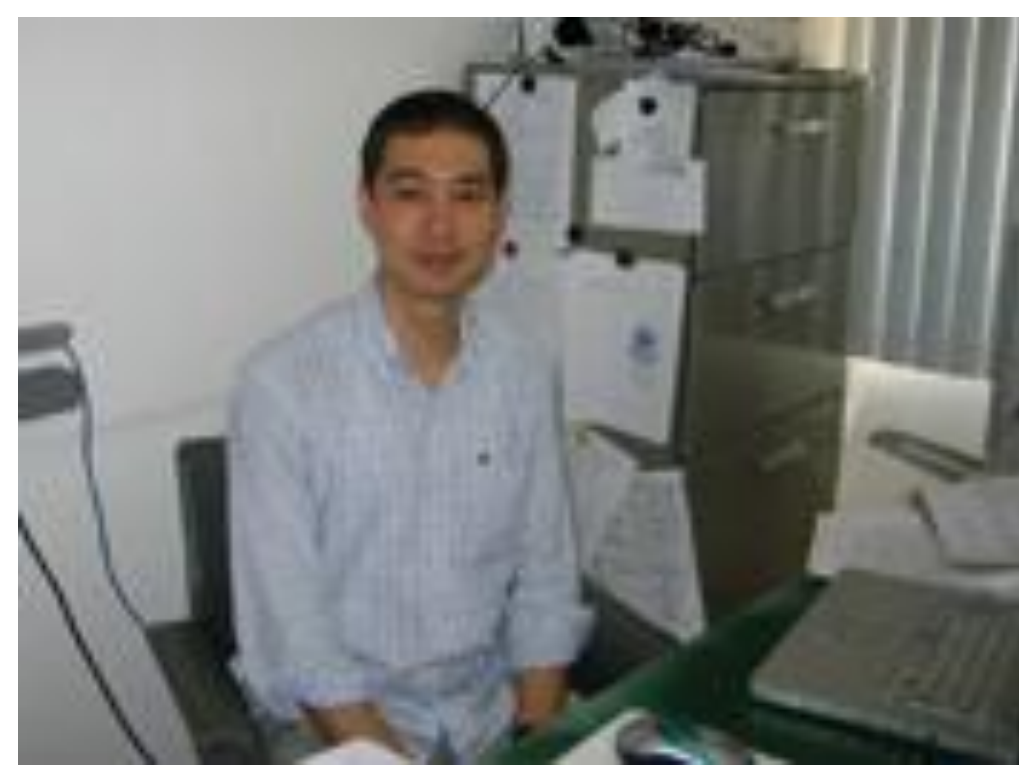
SNO $m_2 > m_1$

$$|\Delta m_{21}^2| = |m_2^2 - m_1^2| = 7.5 \times 10^{-5} \text{ eV}^2$$

$\nu_3, \nu_1/\nu_2$ Mass Ordering:

–atmospheric mass ord

$$|\Delta m_{31}^2| = |m_3^2 - m_1^2| = 2.5 \times 10^{-3} \text{ eV}^2$$



Another possible way to determine the Neutrino Mass Hierarchy

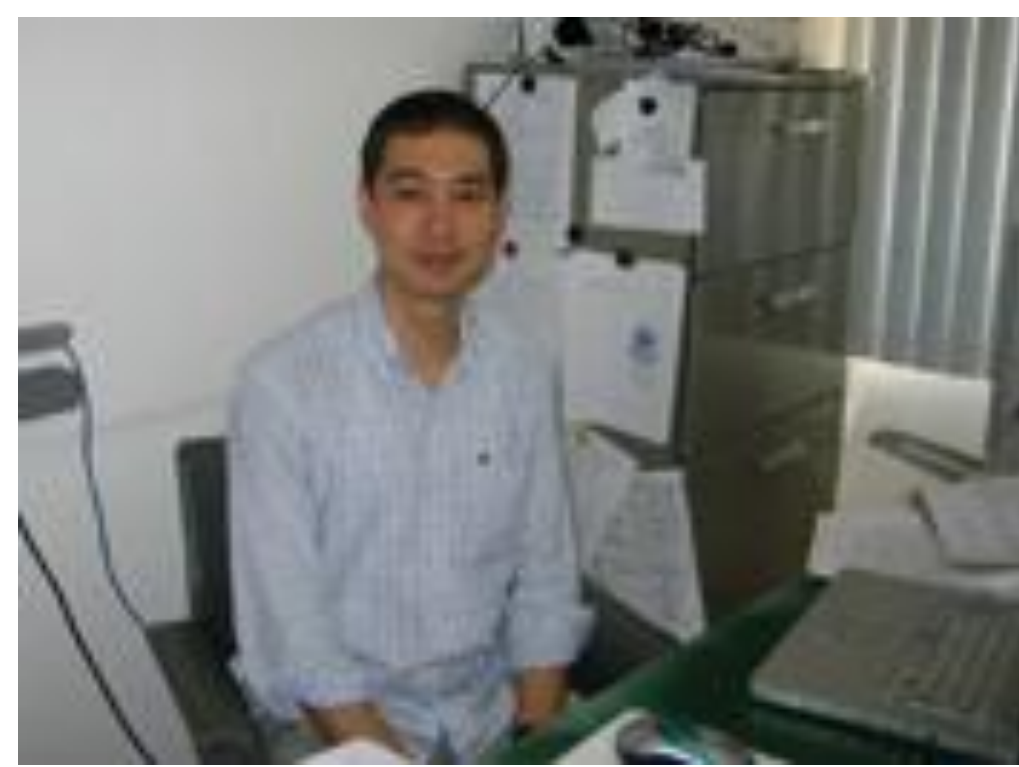


Hiroshi Nunokawa^{1,*} Stephen Parke^{2,†} and Renata Zukanovich Funchal^{3‡}

arXiv:hep-ph/0503283v1 29 Mar 2005

in PRD
NPZ'05

.....
Introduced Δm_{ee}^2 and $\Delta m_{\mu\mu}^2$ for disappearance experiments:



Another possible way to determine the Neutrino Mass Hierarchy



Hiroshi Nunokawa^{1,*} Stephen Parke^{2,†} and Renata Zukanovich Funchal^{3‡}

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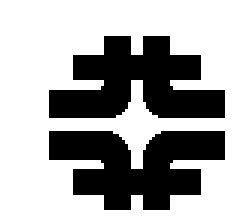
in PRD
NPZ'05

Introduced Δm_{ee}^2 and $\Delta m_{\mu\mu}^2$ for disappearance experiments:

and that $|\Delta m_{ee}^2| > |\Delta m_{\mu\mu}^2|$ implies NO

few % difference

$|\Delta m_{ee}^2| < |\Delta m_{\mu\mu}^2|$ implies IO



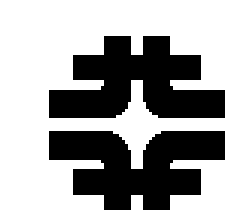
UPDATES:

The Smallness of Matter Effects in Long-Baseline Muon Neutrino Disappearance



Peter B. Denton^{1,*} and Stephen J. Parke^{2,†}

arXiv:2401.10326



UPDATES:

The Smallness of Matter Effects in Long-Baseline Muon Neutrino Disappearance



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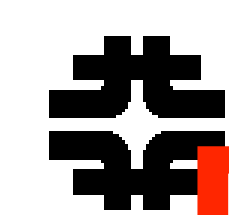
arXiv:2401.10326

A Mass Ordering Sum Rule for the Neutrino Disappearance Channels in T2K, NOvA and JUNO

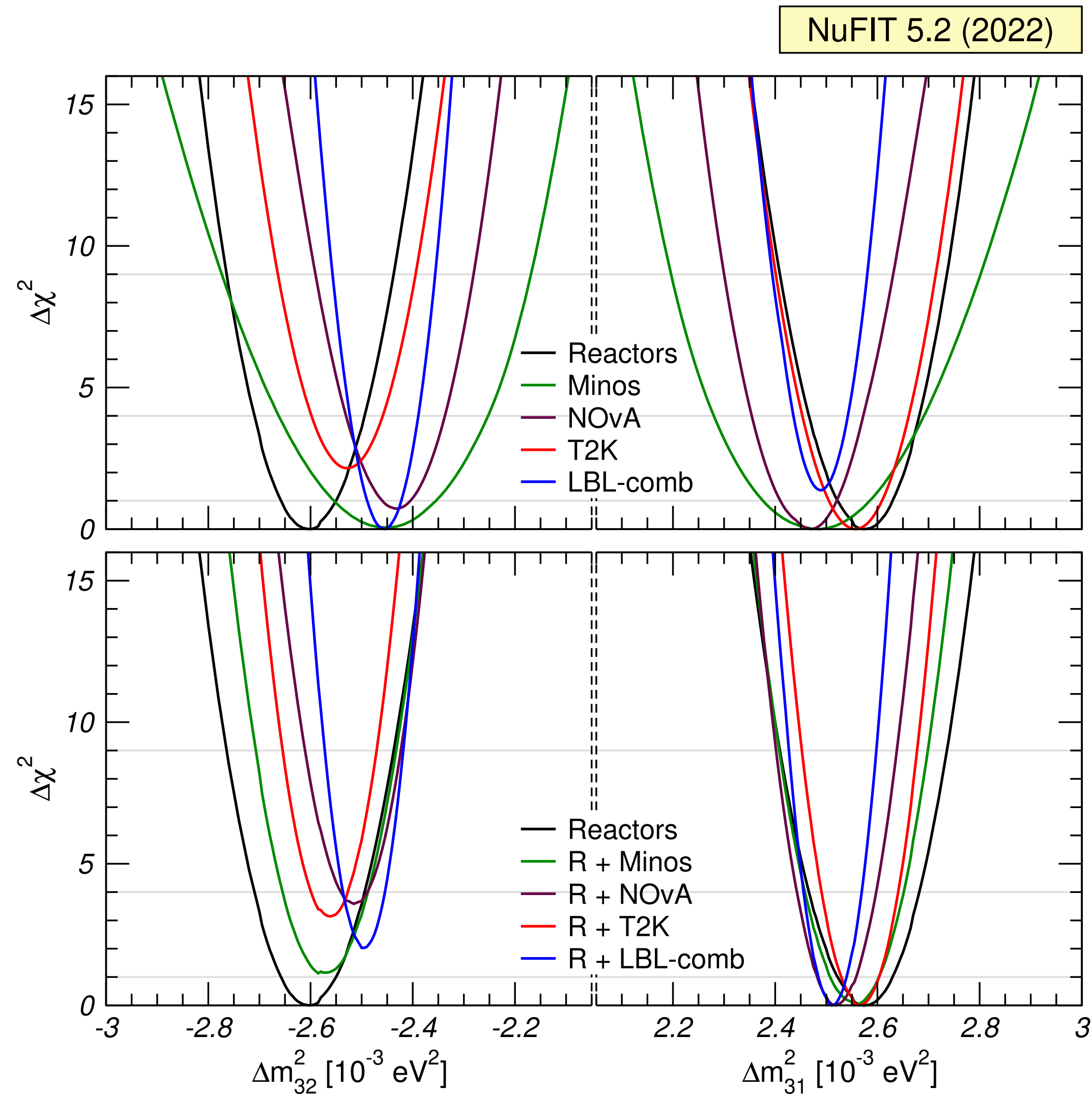
Stephen J. Parke^{*} Renata Zukanovich Funchal[†]

arXiv:2404.08733





Explain this figure + Future Prospects

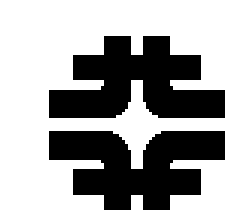


Current members:

Ivan Esteban
 Concha Gonzalez Garcia
 Michele Maltoni
 Thomas Schwetz
 Albert Zhou

Former members:

Johannes Bergström
 Alvaro Hernandez Cabezudo
 Ivan Martinez Soler
 Jordi Salvado

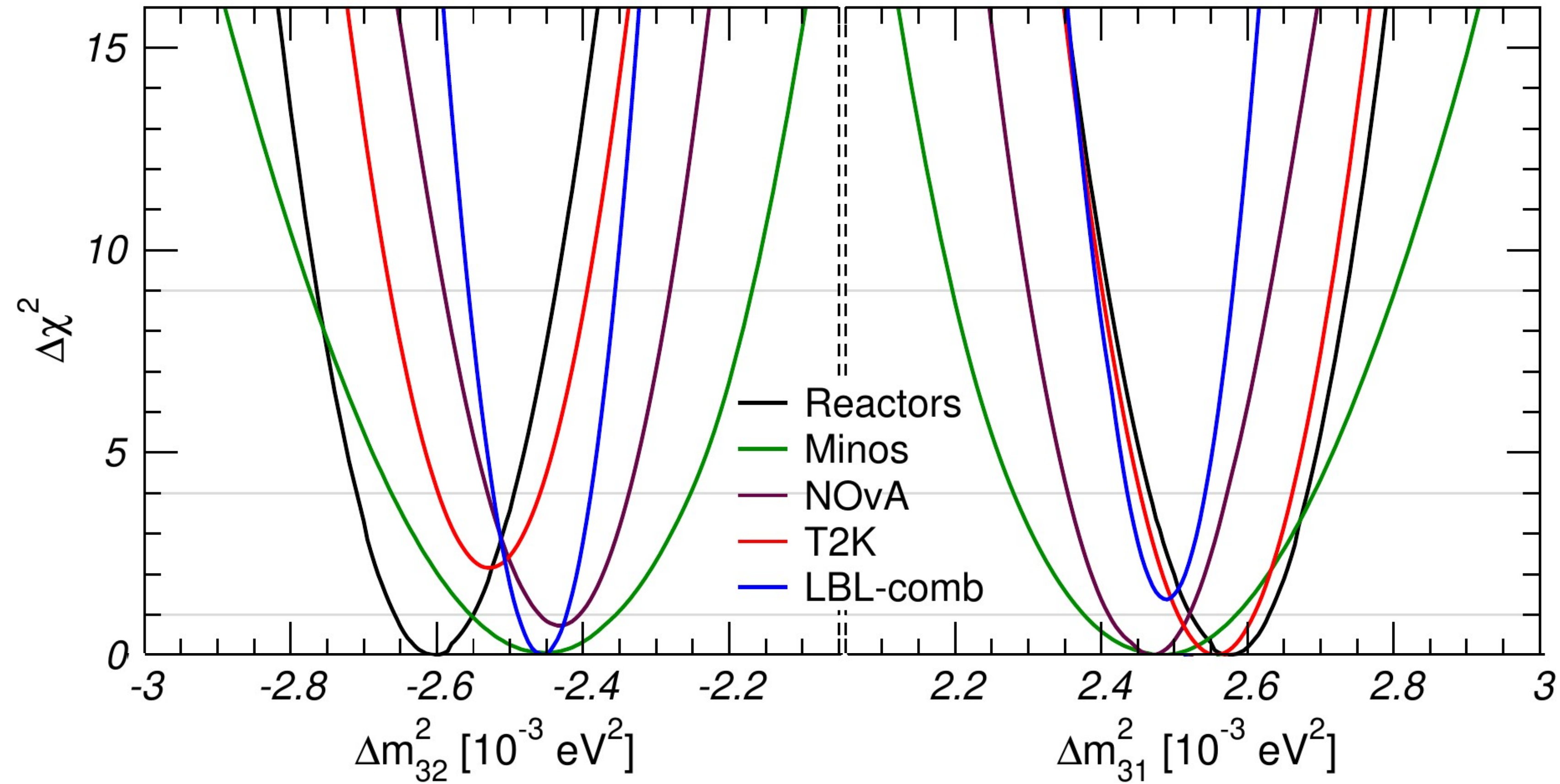


$$\frac{L}{E} \sim 500 \frac{\text{km}}{\text{GeV}} = 0.5 \frac{\text{km}}{\text{MeV}}$$

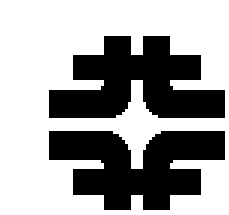
IO

NO

NuFIT 5.2 (2022)



By construction $\Delta\chi^2_{min}$ for either (or both) NO or IO at zero

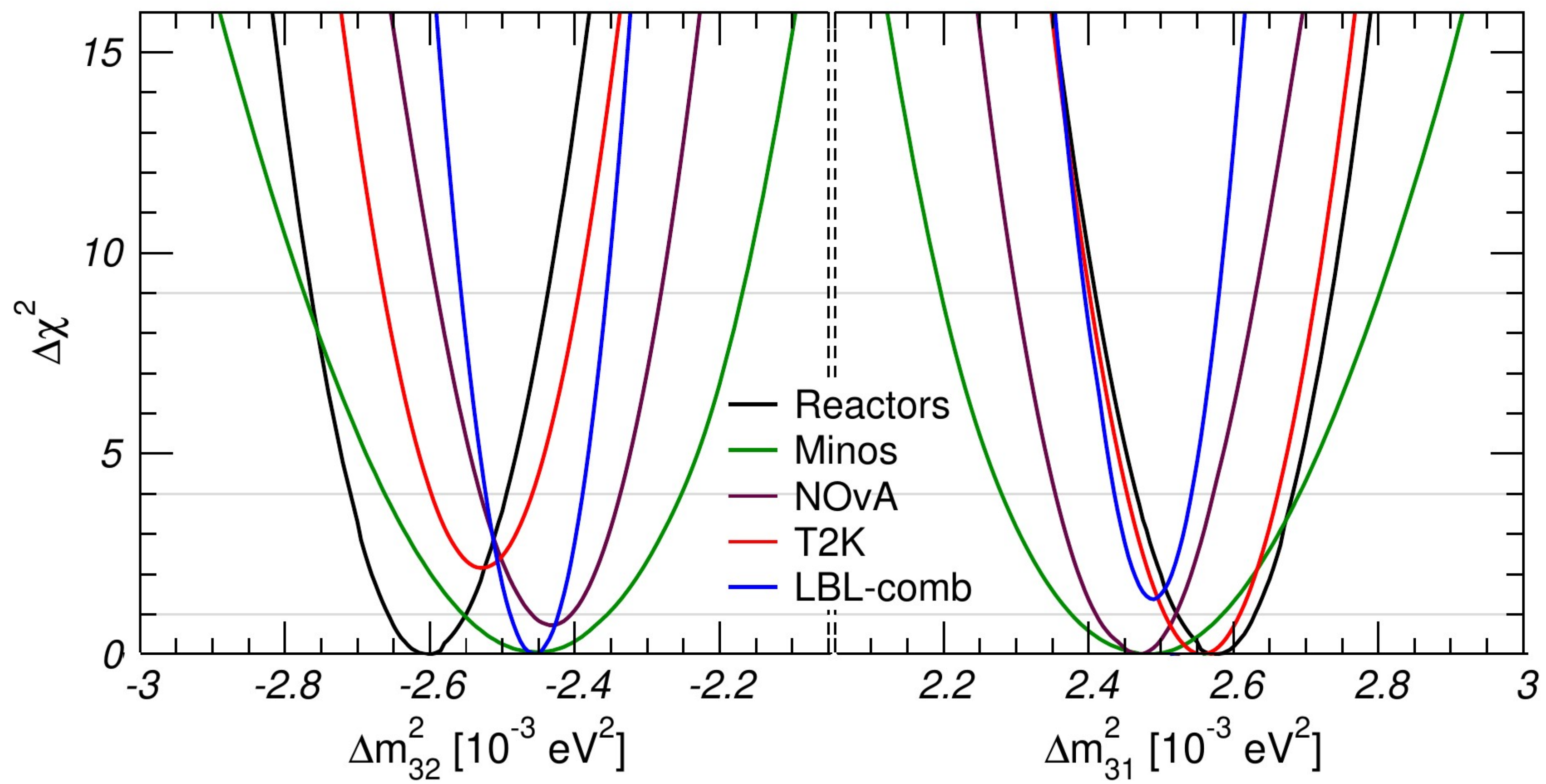


$$\frac{L}{E} \sim 500 \frac{\text{km}}{\text{GeV}} = 0.5 \frac{\text{km}}{\text{MeV}}$$

IO

NO

NuFIT 5.2 (2022)

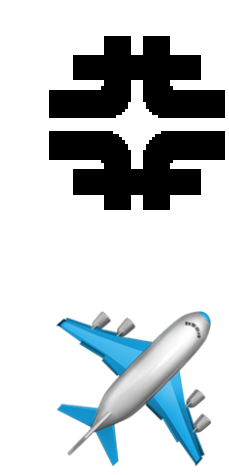


No preference
Or for NO

Except

T2K + NOvA
Combined

By construction $\Delta\chi^2_{min}$ for either (or both) NO or IO at zero

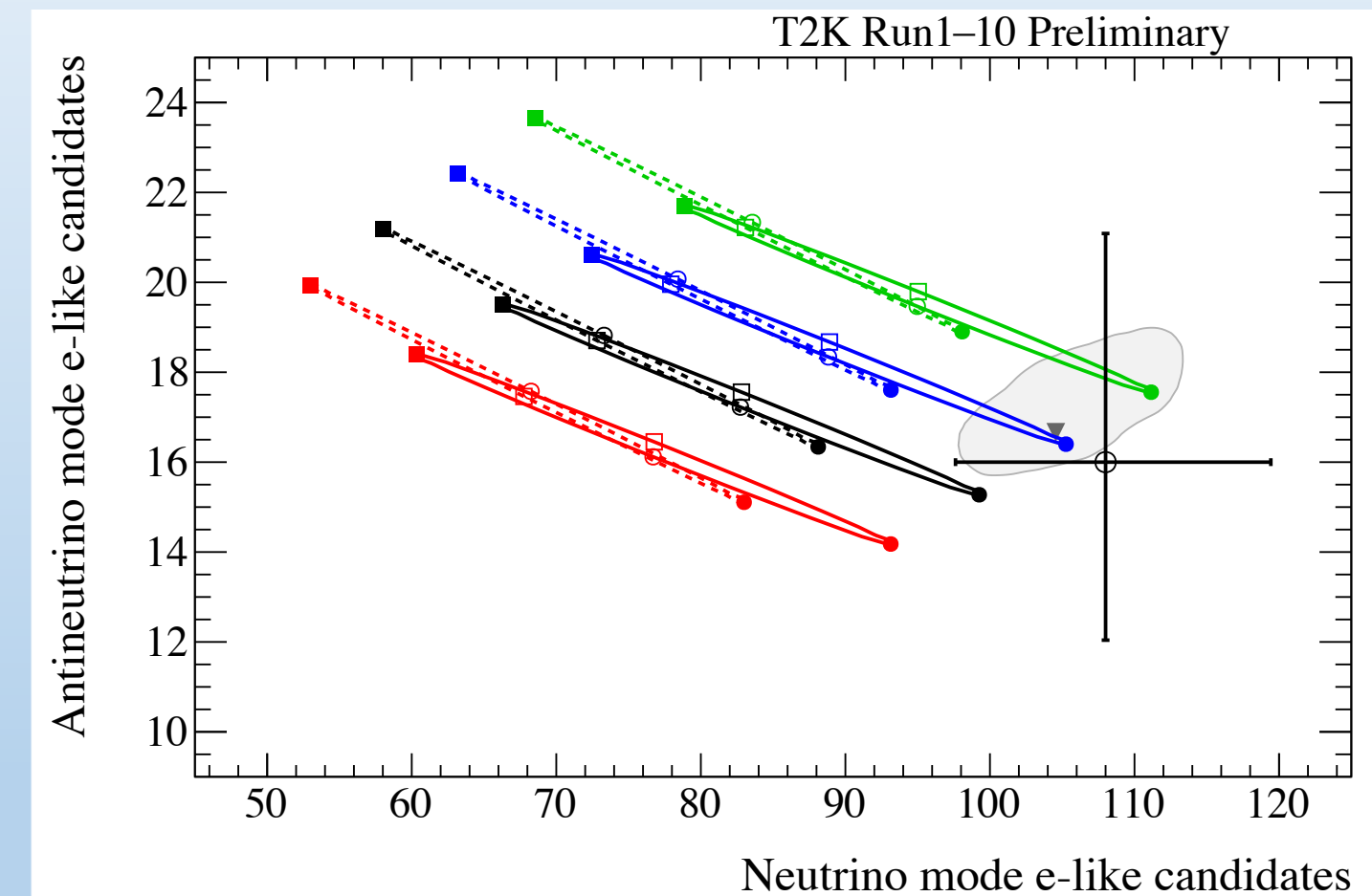


T2K & NOvA Appearance Confusion:

Number of Events proportional to Oscillation Probability

SK event samples

- O(45%) change in electron-like event rate between $\delta_{CP}=+\pi/2$ and $\delta_{CP}=-\pi/2$

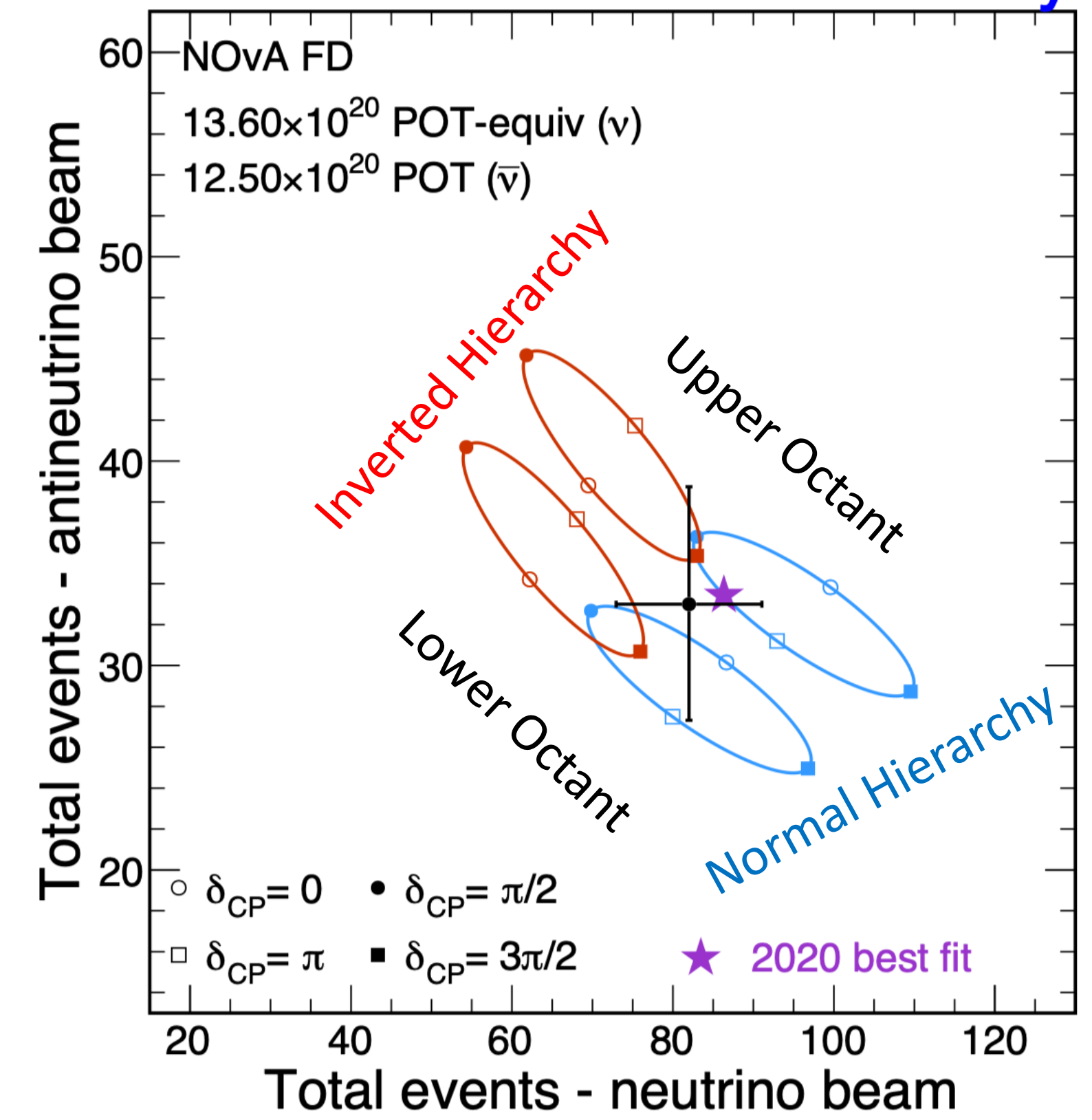


— $\sin^2\theta_{23} = 0.45, 0.50, 0.55, 0.60$ \circ $\delta_{CP} = \pi$ \square 68% syst err. at best-fit
 — $\Delta m_{32}^2 = 2.49 \times 10^{-3} \text{ eV}^2$ \blacksquare $\delta_{CP} = +\pi/2$ \blacktriangledown Best-fit
 - - - $\Delta m_{31}^2 = -2.46 \times 10^{-3} \text{ eV}^2$ \square $\delta_{CP} = 0$ \ominus Data (68% stat err.)
 \bullet $\delta_{CP} = -\pi/2$

Patrick Dunne (p.dunne12@imperial.ac.uk)

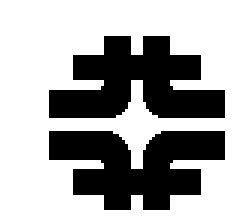
23

NOvA Preliminary



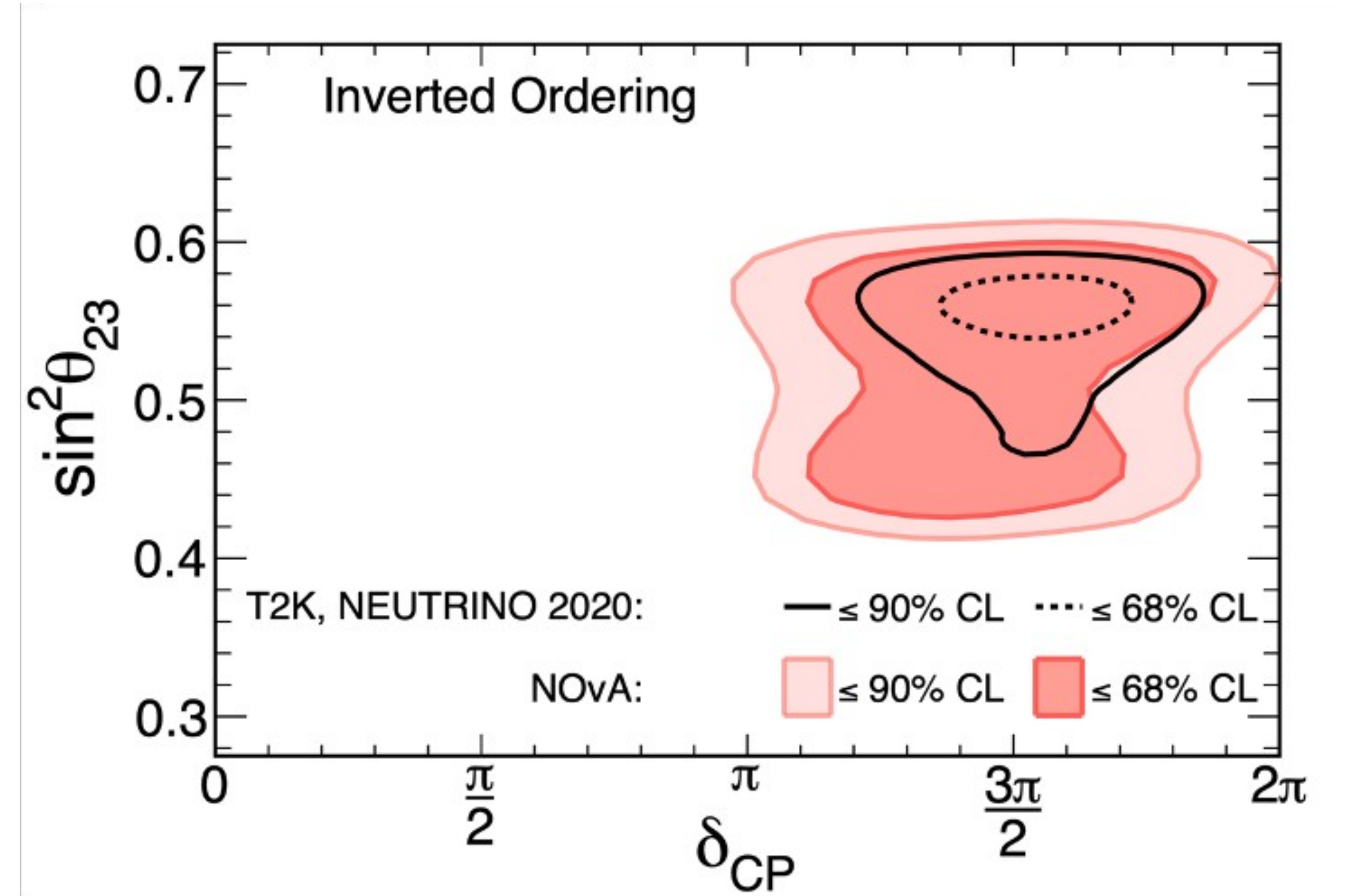
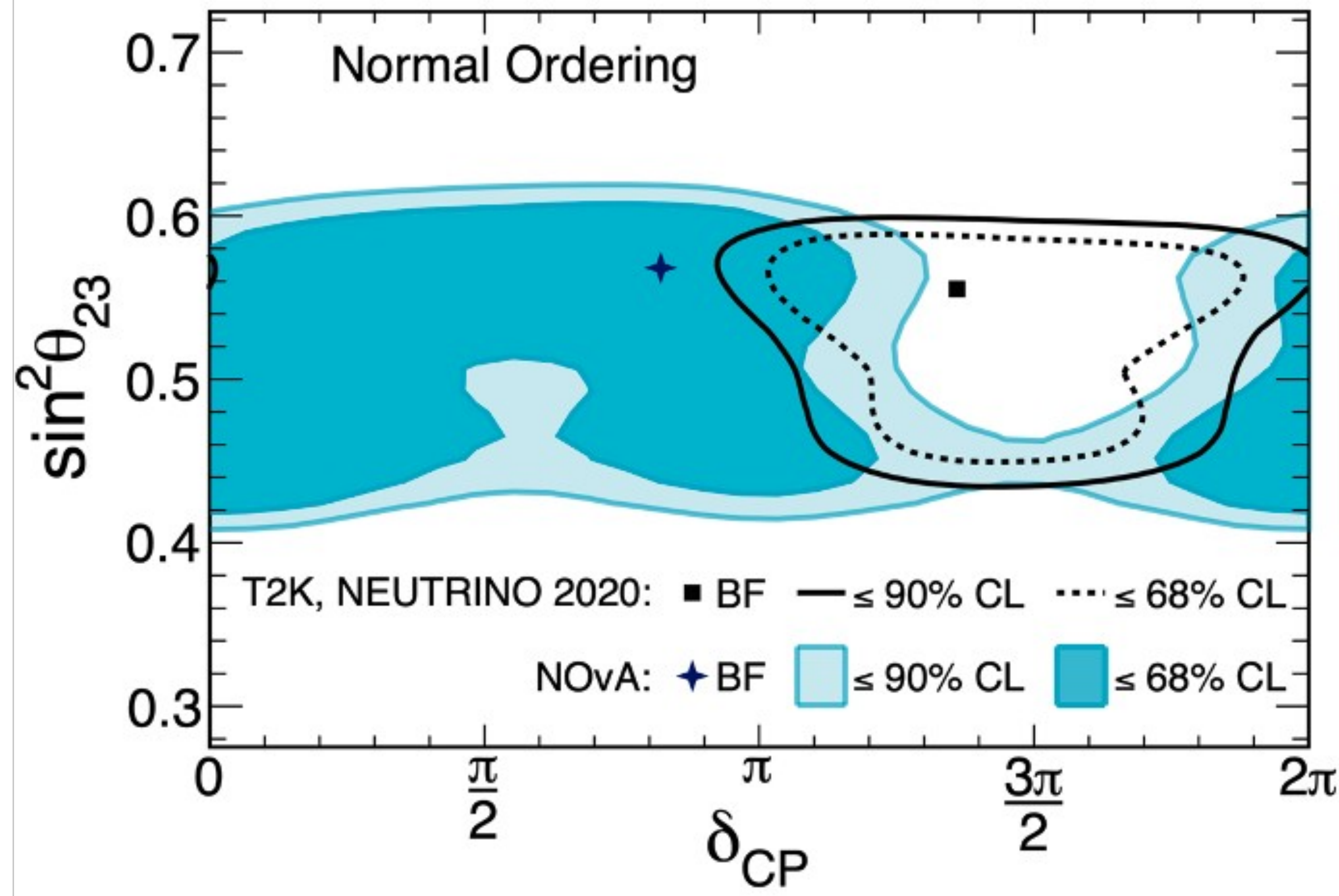
T2K NO prefer by ~ 2 units of χ^2

NOvA NO prefer by ~ 1 unit of χ^2



COMBINED

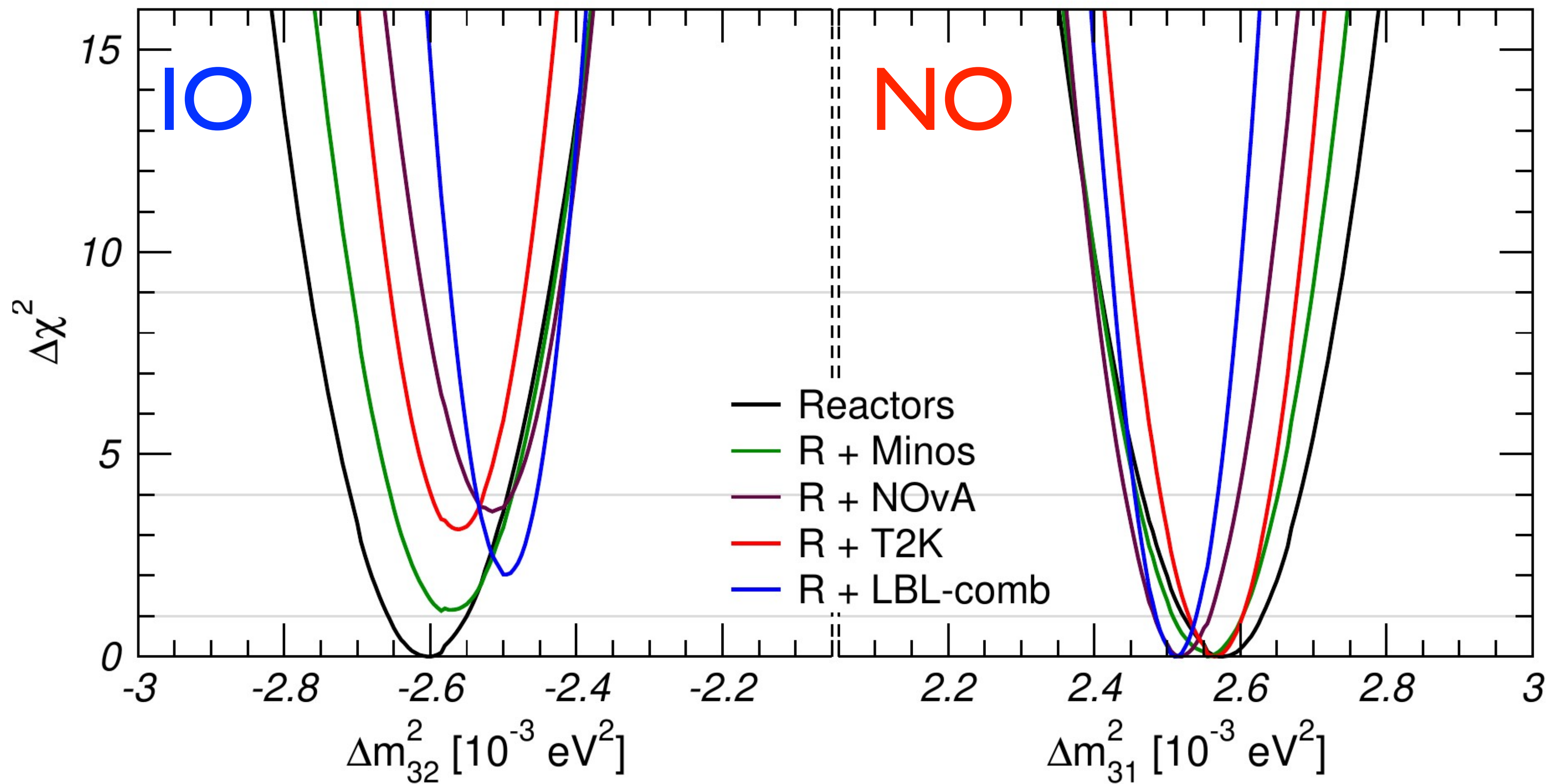
<https://doi.org/10.5281/zenodo.6683827>



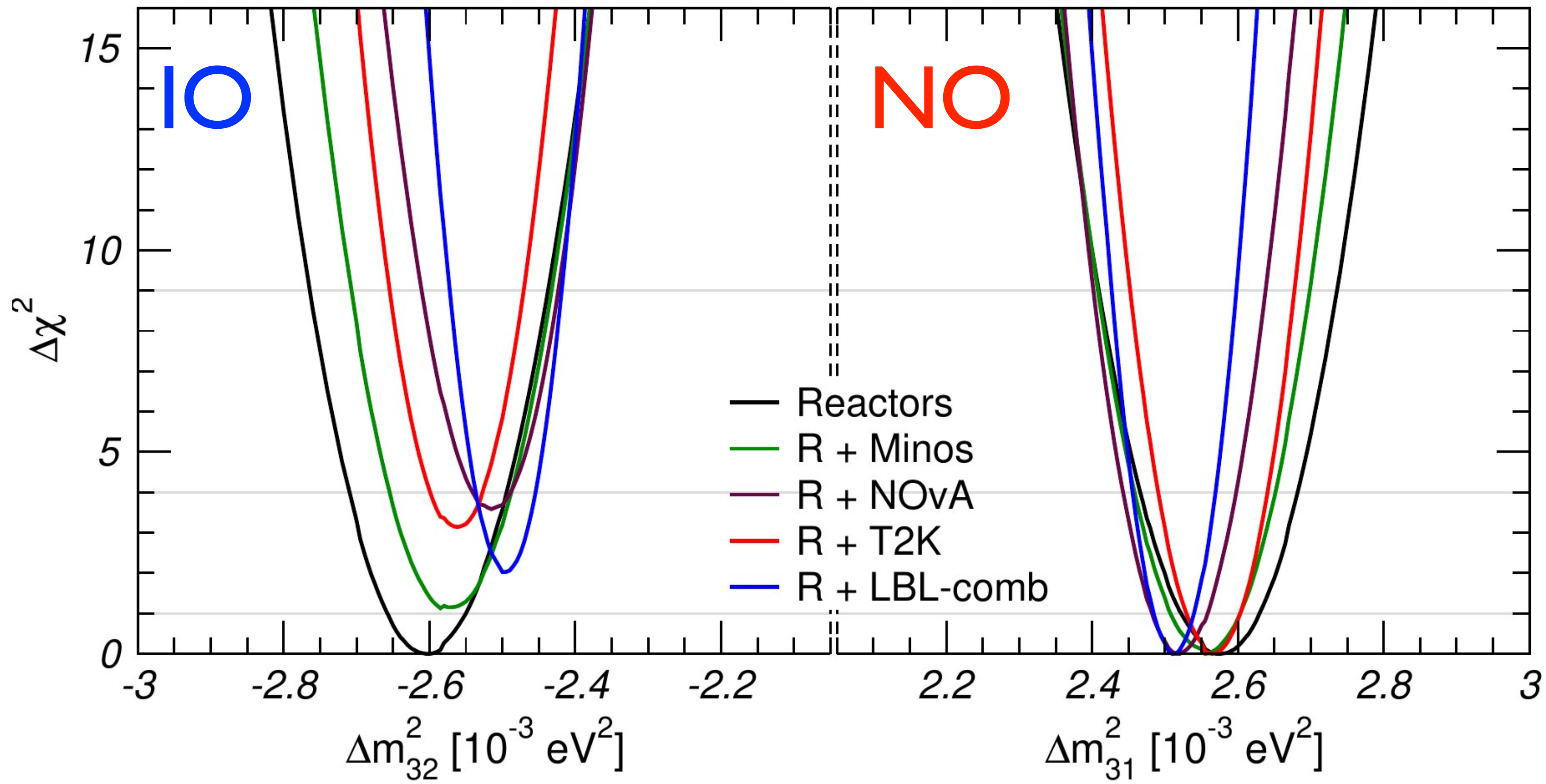
IO prefer by ~ 1.6 unit of $\Delta\chi^2$

Kelly, Machado, SP, Perez, Zukanovich 2007.08526 plus other papers

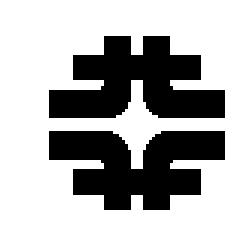
NuFIT 5.2 (2022)



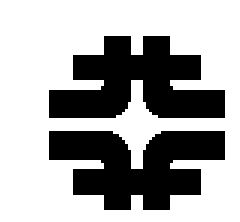
NuFIT 5.2 (2022)



Reactors + LBL
All
Prefer NO:
Even T2K + NOvA



Sum Rule:

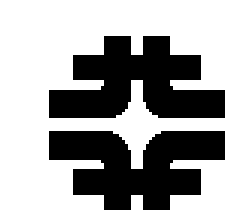


For these Experiments there is a ‘Mass Ordering Sum Rule:’

$$\left(|\Delta m_{32}^2|_{DB}^{IO} - |\Delta m_{32}^2|_{\mu dis}^{IO} \right) + \left(|\Delta m_{31}^2|_{\mu dis}^{NO} - |\Delta m_{31}^2|_{DB}^{NO} \right) = (2.4 - 0.9 \widehat{\cos \delta})\% |\Delta m_{ee}^2|$$

$$\widehat{\cos \delta} \equiv (\cos \delta^{NO} + \cos \delta^{IO})/2$$

Unchanged if 31 ↔ 32 in either or both MO's

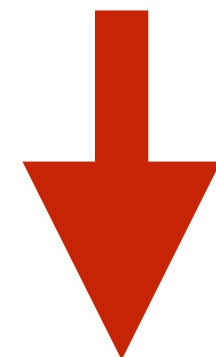


For these Experiments there is a ‘Mass Ordering Sum Rule:’

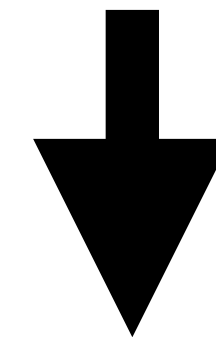
$$\left(|\Delta m_{32}^2|_{DB}^{IO} - |\Delta m_{32}^2|_{\mu dis}^{IO} \right) + \left(|\Delta m_{31}^2|_{\mu dis}^{NO} - |\Delta m_{31}^2|_{DB}^{NO} \right) = (2.4 - 0.9 \widehat{\cos \delta})\% |\Delta m_{ee}^2|$$



If IO then ≈ 0



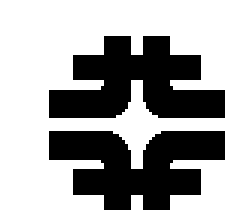
If NO then ≈ 0



+1.5 to +3.3 %

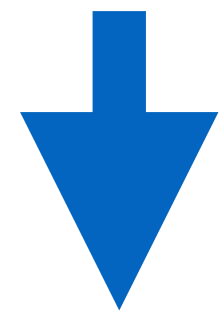
$$\widehat{\cos \delta} \equiv (\cos \delta^{NO} + \cos \delta^{IO})/2$$

Unchanged if 31 ↔ 32 in either or both MO's

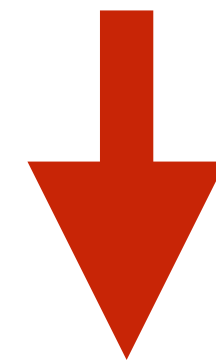


For these Experiments there is a ‘Mass Ordering Sum Rule:’

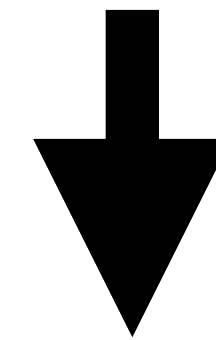
$$\left(|\Delta m_{32}^2|_{DB}^{IO} - |\Delta m_{32}^2|_{\mu dis}^{IO} \right) + \left(|\Delta m_{31}^2|_{\mu dis}^{NO} - |\Delta m_{31}^2|_{DB}^{NO} \right) = (2.4 - 0.9 \widehat{\cos \delta})\% |\Delta m_{ee}^2|$$



If IO then ≈ 0



If NO then ≈ 0

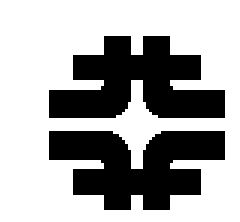


+1.5 to +3.3 %

Valid for some but not all
ICECUBE, KM3Net/Orca.
Needs tweak for JUNO

$$\widehat{\cos \delta} \equiv (\cos \delta^{NO} + \cos \delta^{IO})/2$$

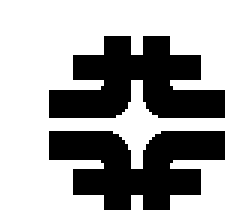
Unchanged if 31 \leftrightarrow 32 in either or both MO's



$$\left(|\Delta m_{32}^2|_{DB}^{IO} - |\Delta m_{32}^2|_{\mu dis}^{IO} \right) + \left(|\Delta m_{31}^2|_{\mu dis}^{NO} - |\Delta m_{31}^2|_{DB}^{NO} \right) = (2.4 - 0.9 \widehat{\cos \delta})\% |\Delta m_{ee}^2|$$



	$ \Delta m_{32}^2 _{DB}^{IO} - \Delta m_{32}^2 _{\mu dis}^{IO}$	$ \Delta m_{31}^2 _{\mu dis}^{NO} - \Delta m_{31}^2 _{DB}^{NO}$
NO	$(2.4 - 0.9 \widehat{\cos \delta})\%$	≈ 0
IO	≈ 0	$(2.4 - 0.9 \widehat{\cos \delta})\%$



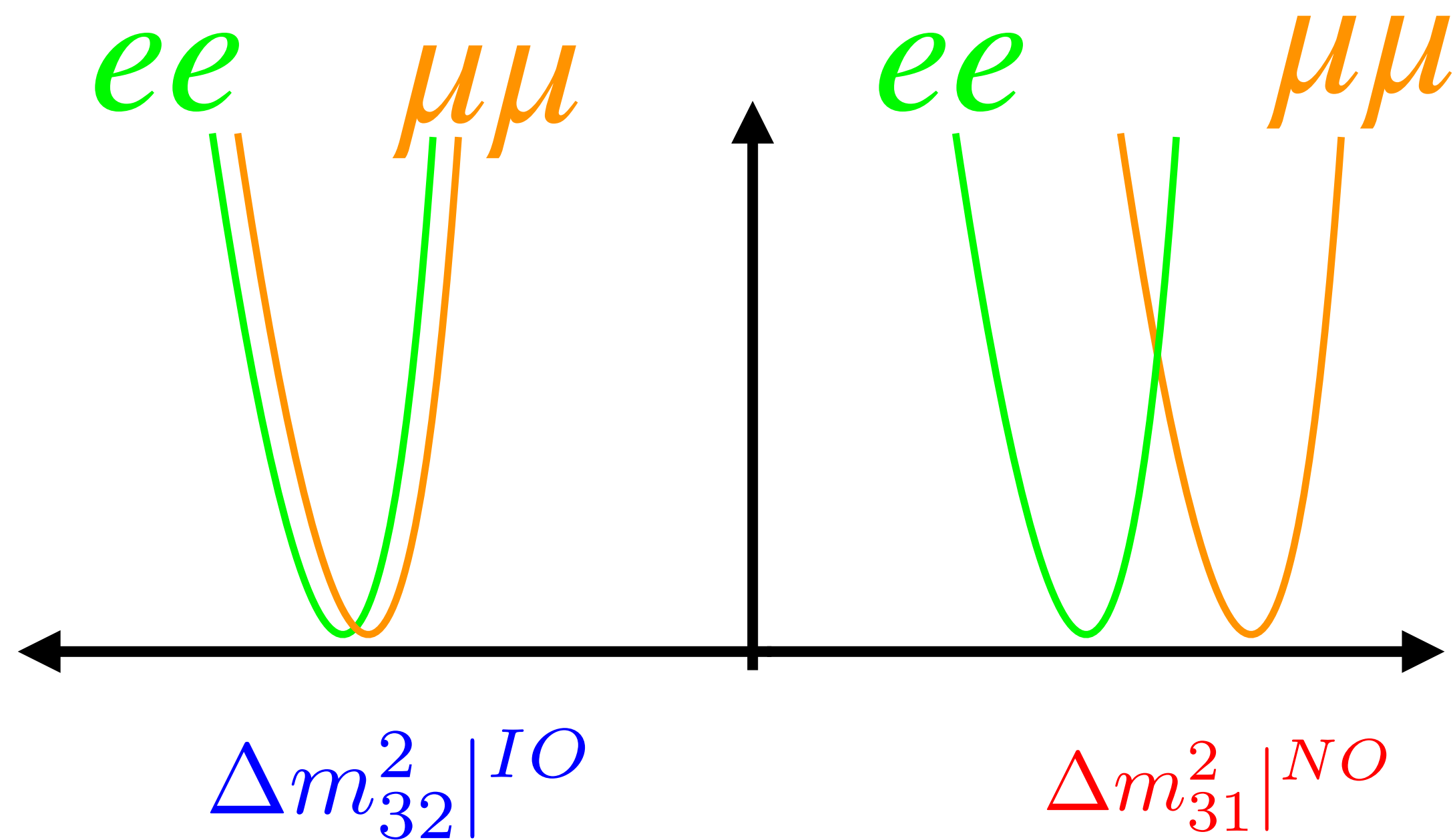
$$\left(|\Delta m_{32}^2|_{DB}^{IO} - |\Delta m_{32}^2|_{\mu dis}^{IO} \right) + \left(|\Delta m_{31}^2|_{\mu dis}^{NO} - |\Delta m_{31}^2|_{DB}^{NO} \right) = (2.4 - 0.9 \widehat{\cos \delta})\% |\Delta m_{ee}^2|$$

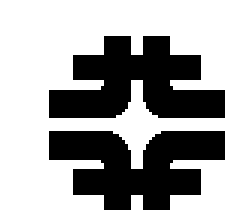


	$ \Delta m_{32}^2 _{DB}^{IO} - \Delta m_{32}^2 _{\mu dis}^{IO}$	$ \Delta m_{31}^2 _{\mu dis}^{NO} - \Delta m_{31}^2 _{DB}^{NO}$
NO	$(2.4 - 0.9 \widehat{\cos \delta})\%$	≈ 0
IO	≈ 0	$(2.4 - 0.9 \widehat{\cos \delta})\%$

IO

$\Delta\chi^2$





$$\left(|\Delta m_{32}^2|_{DB}^{IO} - |\Delta m_{32}^2|_{\mu dis}^{IO} \right) + \left(|\Delta m_{31}^2|_{\mu dis}^{NO} - |\Delta m_{31}^2|_{DB}^{NO} \right) = (2.4 - 0.9 \widehat{\cos \delta})\% |\Delta m_{ee}^2|$$

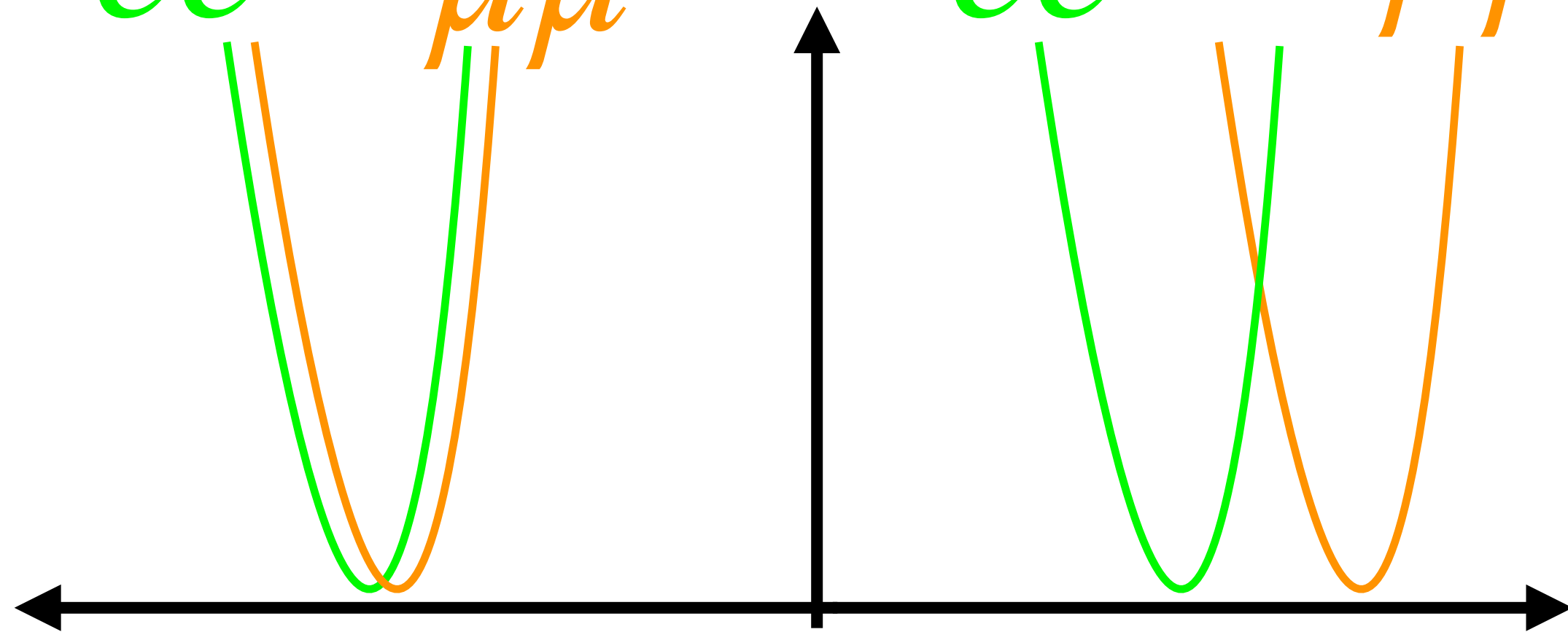


	$ \Delta m_{32}^2 _{DB}^{IO} - \Delta m_{32}^2 _{\mu dis}^{IO}$	$ \Delta m_{31}^2 _{\mu dis}^{NO} - \Delta m_{31}^2 _{DB}^{NO}$
NO	$(2.4 - 0.9 \widehat{\cos \delta})\%$	≈ 0
IO	≈ 0	$(2.4 - 0.9 \widehat{\cos \delta})\%$

IO

$\Delta\chi^2$

ee *$\mu\mu$*



$\Delta m_{32}^2|^{IO}$

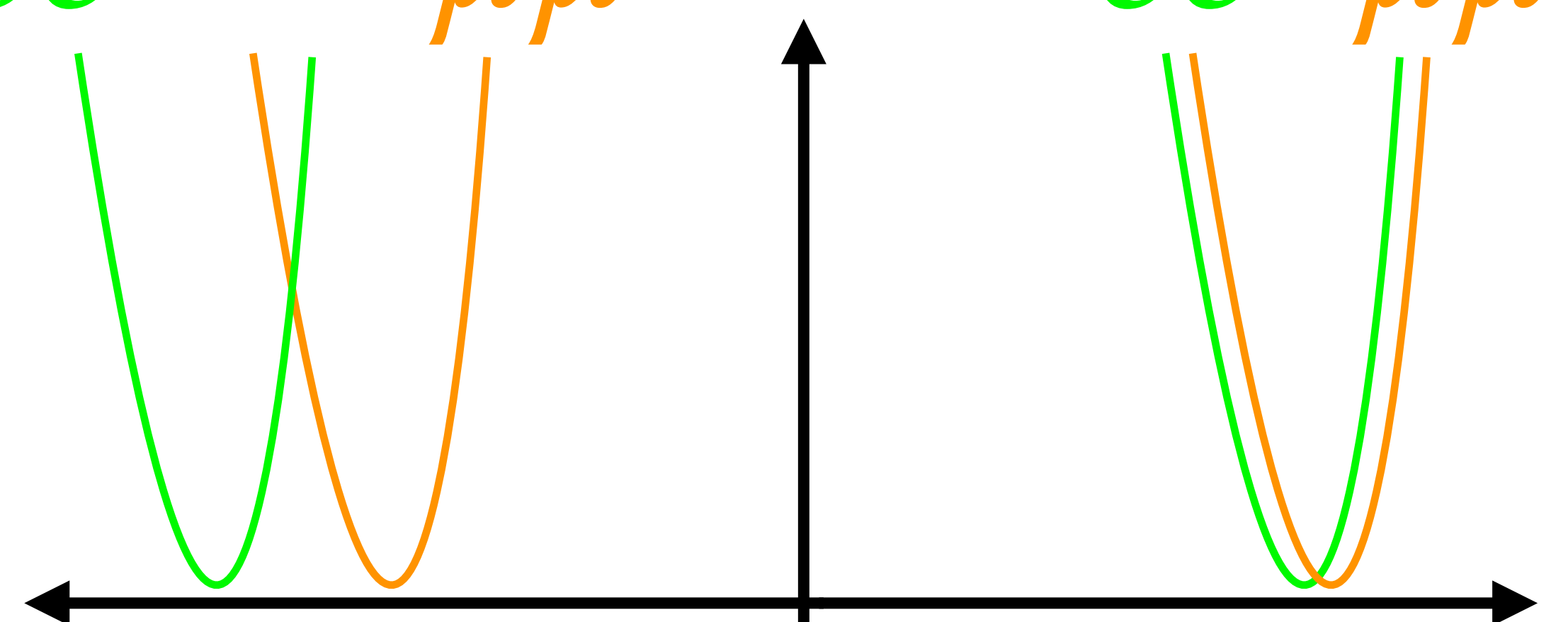
ee *$\mu\mu$*

$\Delta m_{31}^2|^{NO}$

NO

$\Delta\chi^2$

ee *$\mu\mu$*

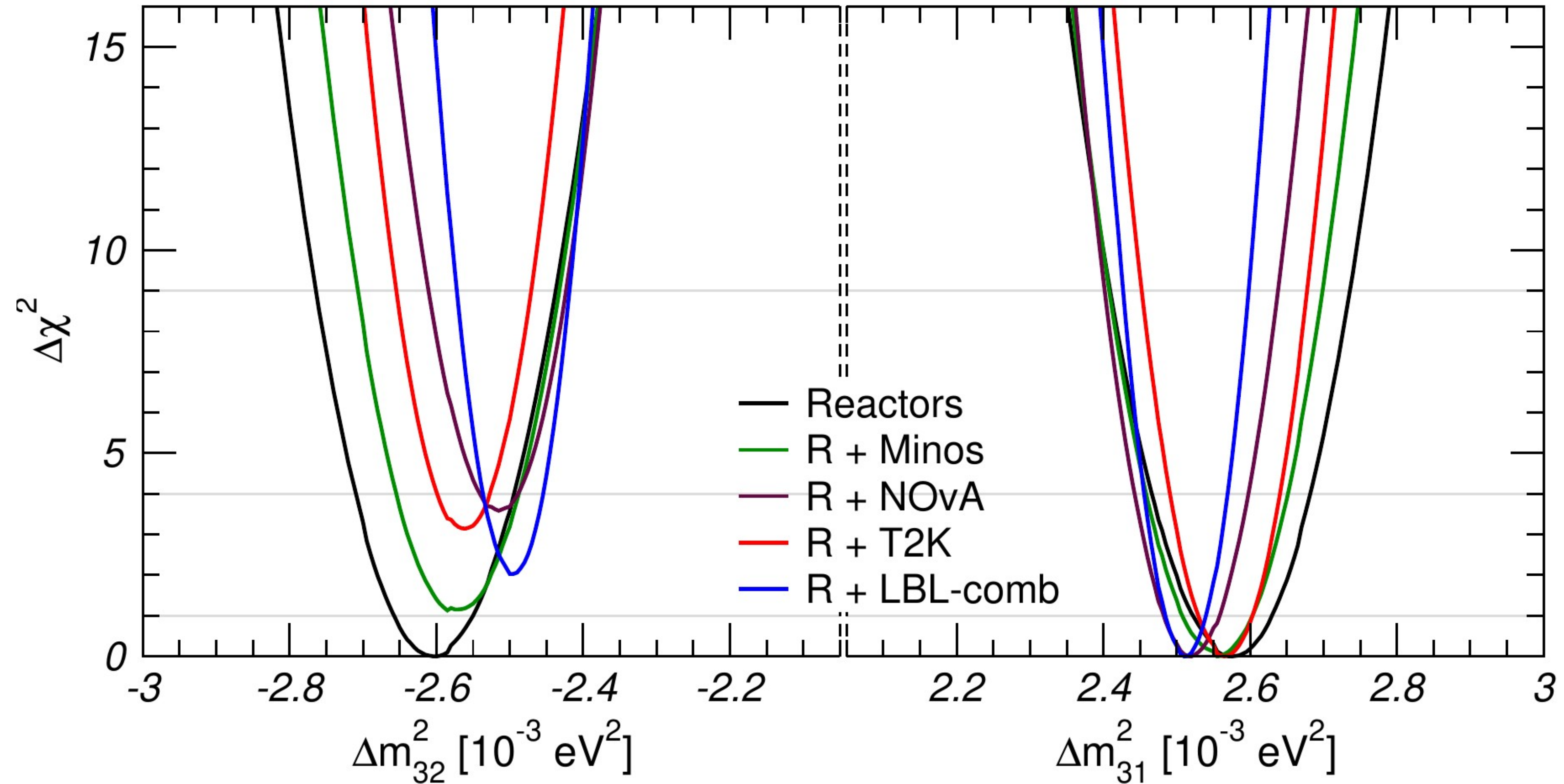


$\Delta m_{32}^2|^{IO}$

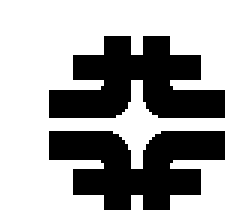
$\Delta m_{31}^2|^{NO}$

IO

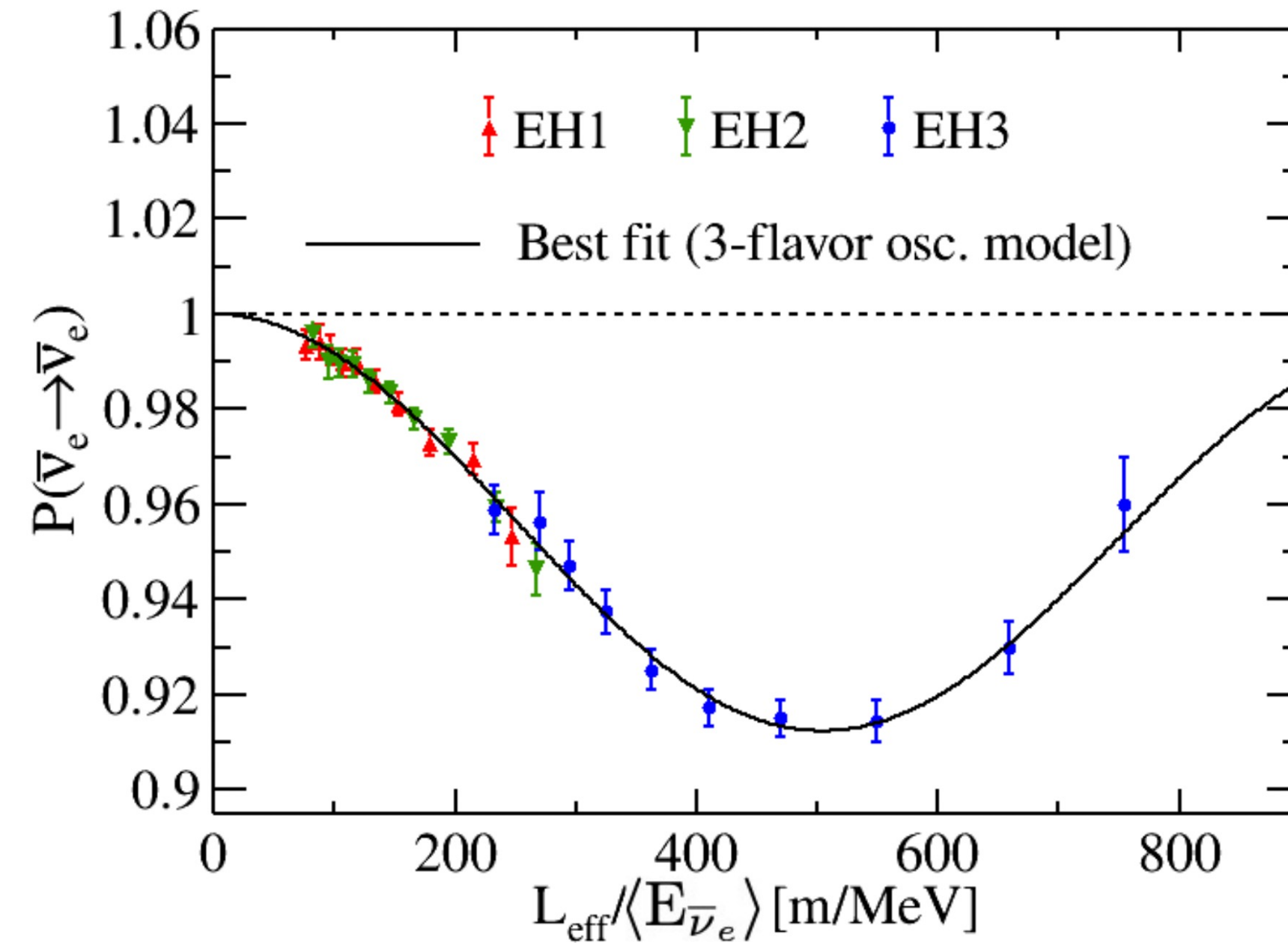
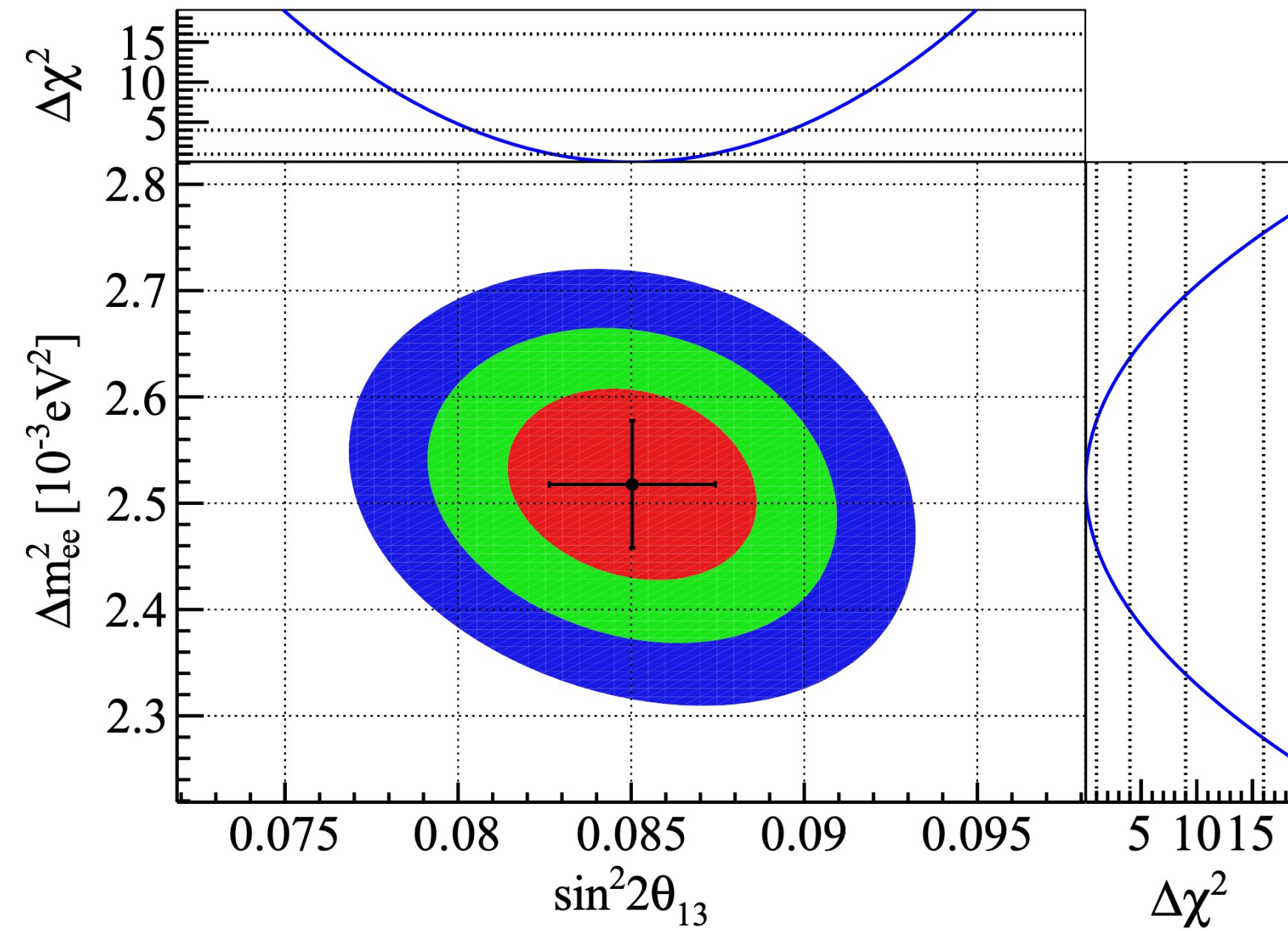
NO



$$\left(|\Delta m_{32}^2|_{DB}^{IO} - |\Delta m_{32}^2|_{\mu dis}^{IO} \right) + \left(|\Delta m_{31}^2|_{\mu dis}^{NO} - |\Delta m_{31}^2|_{DB}^{NO} \right) = (2.4 - 0.9 \widehat{\cos \delta}) \% |\Delta m_{ee}^2|$$



Improved $\sin^2 2\theta_{13}$ and Δm_{32}^2



Best-fit results:

$$\chi^2/\text{ndf} = 559/518$$

$$\sin^2 2\theta_{13} = 0.0851^{+0.0024}_{-0.0024} \quad (2.8\% \text{ precision})$$

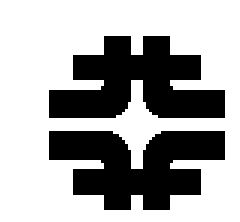
Normal hierarchy:

$$\Delta m_{32}^2 = + (2.466^{+0.060}_{-0.060}) \times 10^{-3} \text{ eV}^2 \quad (2.4\% \text{ precision})$$

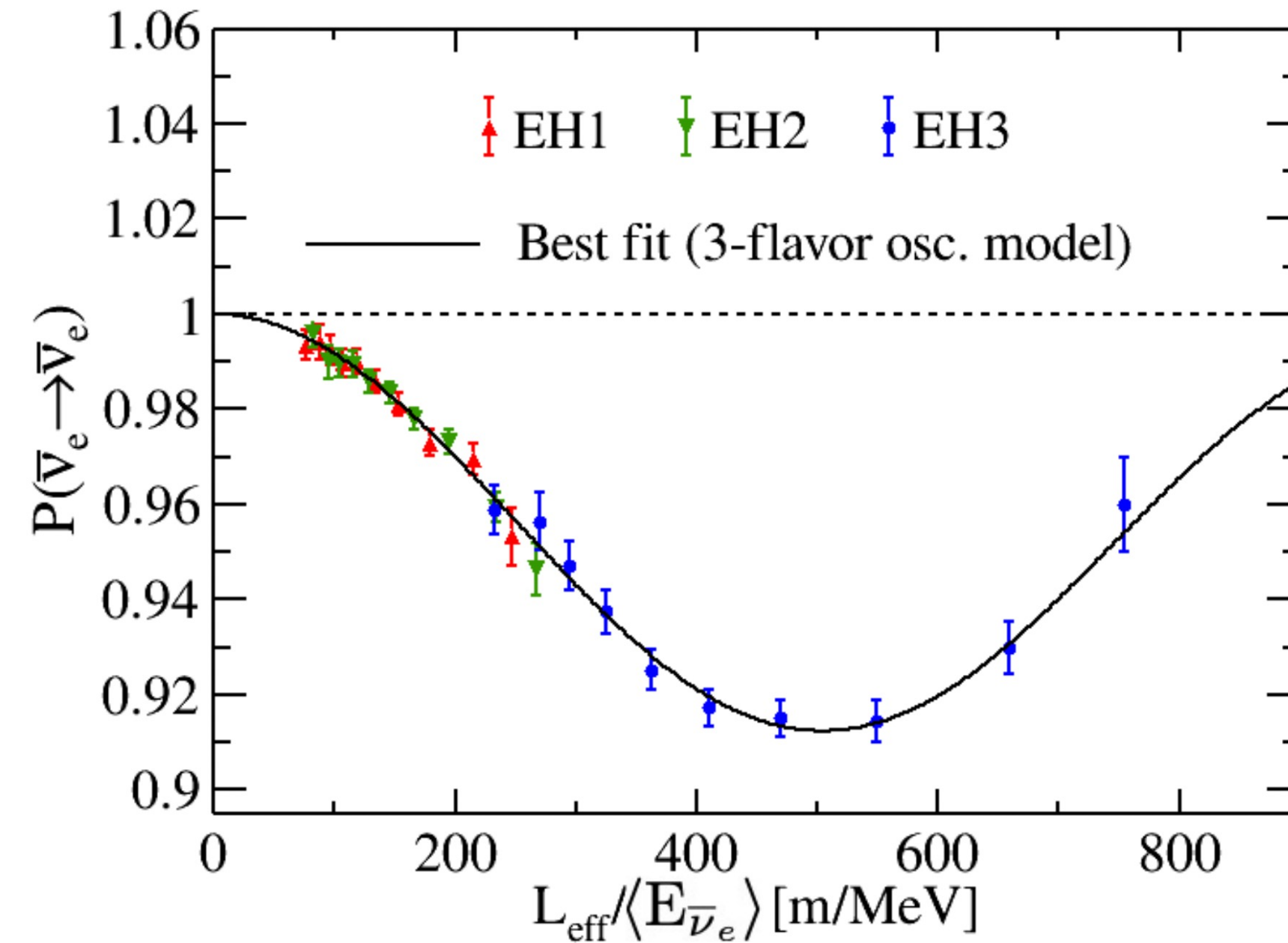
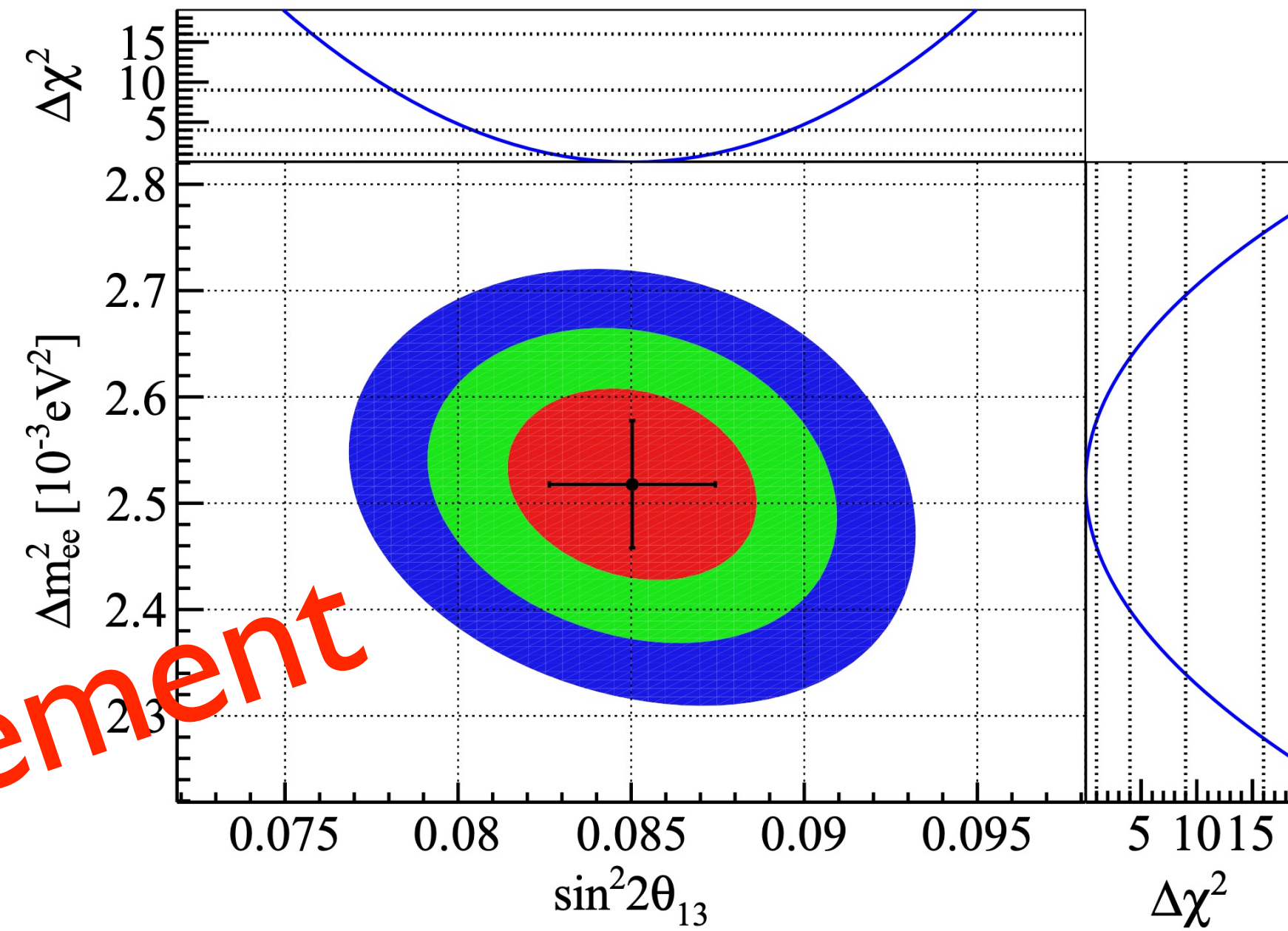
Inverted hierarchy:

$$\Delta m_{32}^2 = - (2.571^{+0.060}_{-0.060}) \times 10^{-3} \text{ eV}^2 \quad (2.3\% \text{ precision})$$

18



Improved $\sin^2 2\theta_{13}$ and Δm_{32}^2



Measurement

Best-fit results:

$$\chi^2/\text{ndf} = 559/518$$

$$\pm c_{12}^2 \Delta m_{21}^2$$

$$\sin^2 2\theta_{13} = 0.0851^{+0.0024}_{-0.0024} \quad (2.8\% \text{ precision})$$

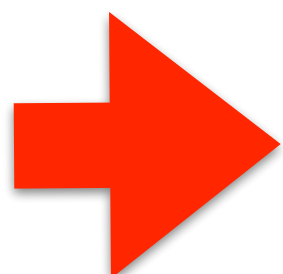
Normal hierarchy:

$$\Delta m_{32}^2 = + (2.466^{+0.060}_{-0.060}) \times 10^{-3} \text{ eV}^2 \quad (2.4\% \text{ precision})$$

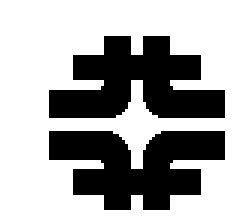
Inverted hierarchy:

$$\Delta m_{32}^2 = - (2.571^{+0.060}_{-0.060}) \times 10^{-3} \text{ eV}^2 \quad (2.3\% \text{ precision})$$

18



$$|\Delta m_{32}^2|^{IO} - |\Delta m_{32}^2|^{NO} = + 2c_{12}^2 \Delta m_{21}^2 = 0.105 \times 10^{-3} \text{ eV}^2$$



$$|\Delta m_{\mu\mu}^2| \approx |\Delta m_{32}^2| \frac{NO}{IO} \pm (s_{12}^2 + s_{13} \cos \delta \frac{NO}{IO}) \Delta m_{21}^2 = |\Delta m_{31}^2| \frac{NO}{IO} \mp (c_{12}^2 - s_{13} \cos \delta \frac{NO}{IO}) \Delta m_{21}^2$$

T2K:

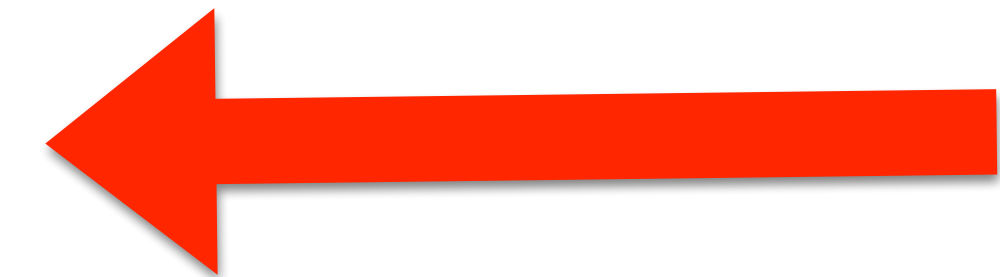
Parameter	MIXING CONSTRAINT	
	Normal ordering	Inverted ordering
δ_{CP} (rad.)	$-1.97^{+0.97}_{-0.62}$	$-1.44^{+0.56}_{-0.59}$
$\sin^2 \theta_{13}/10^{-3}$	—	—
$\sin^2 \theta_{23}$	$0.561^{+0.019}_{-0.038}$	$0.563^{+0.017}_{-0.032}$
$\Delta m_{32}^2/10^{-3}$ (eV ²)	$2.494^{+0.041}_{-0.058}$	—
$ \Delta m_{31}^2 /10^{-3}$ (eV ²)	—	$2.463^{+0.042}_{-0.056}$

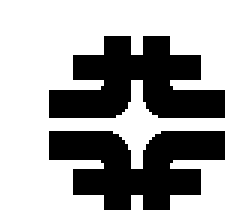
± 0.05 (2%)

$$|\Delta m_{31}^2|^{IO} - |\Delta m_{32}^2|^{NO} = -(\cos 2\theta_{12} - 2s_{13} \widehat{\cos \delta}) \Delta m_{21}^2$$

$$|-2.463| - +2.494 \approx -(0.4 - 0.30 \widehat{\cos \delta}) \times 0.075$$

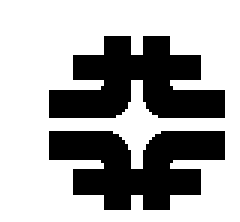
$$-0.031 = - \begin{pmatrix} 0.008 & \widehat{\cos \delta} = 1 \\ 0.030 & \widehat{\cos \delta} = 0 \\ 0.053 & \widehat{\cos \delta} = -1 \end{pmatrix}$$





Matter Effect:

Daya Bay: $\frac{E_\nu}{12 \text{ GeV}} < 10^{-3}$ irrelevant

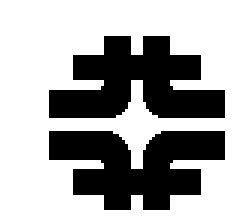


Matter Effect:

Daya Bay: $\frac{E_\nu}{12 \text{ GeV}} < 10^{-3}$ irrelevant

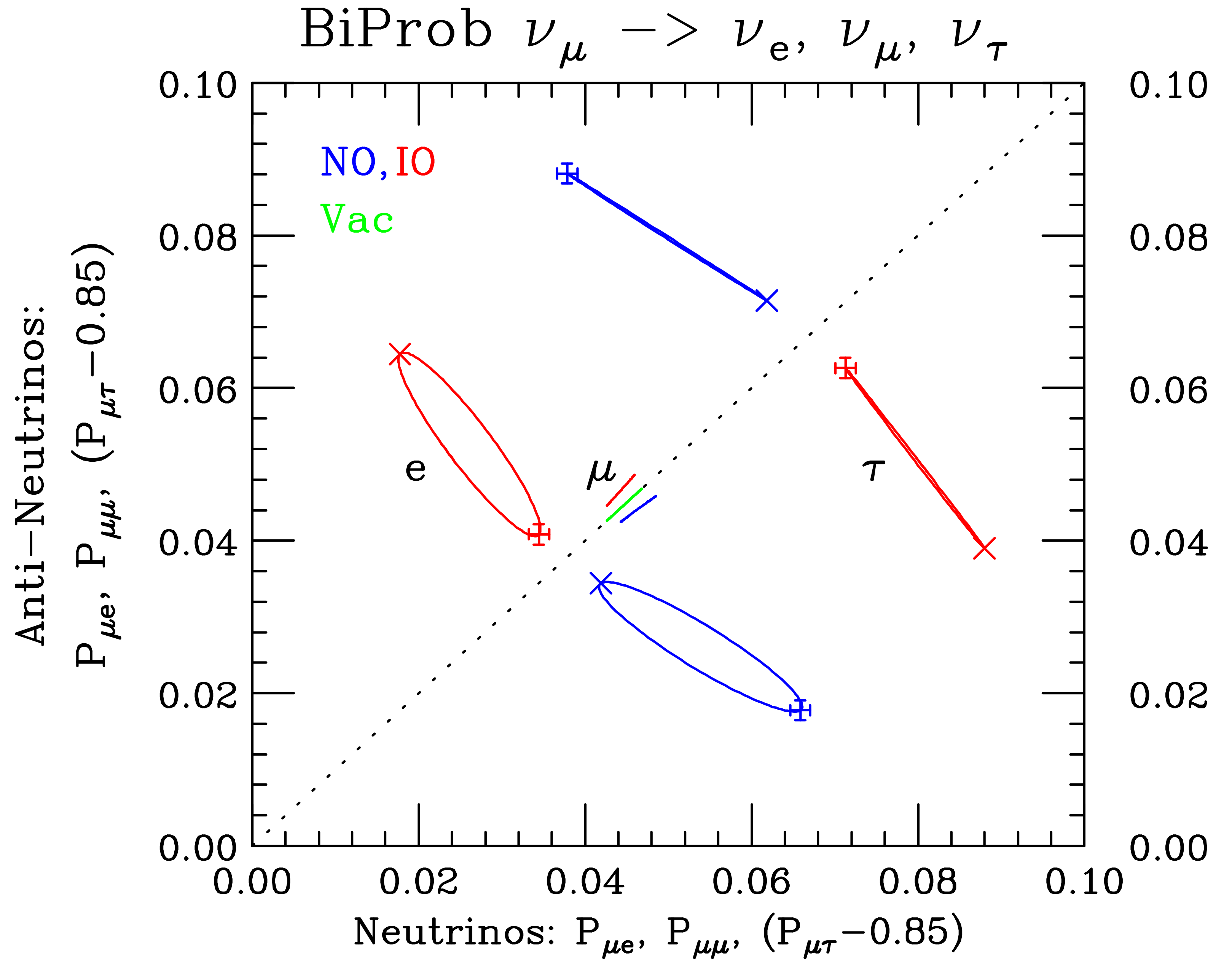
NOvA Disappearance: $\frac{E_\nu}{12 \text{ GeV}} \approx 0.2$

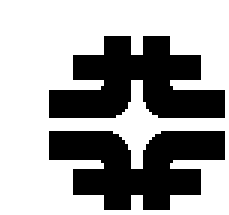
But further suppressed by s_{13}^2 and $(1 - 2|U_{\mu 3}|^2)$
Combined approx. 0.002 !



DUNE:
3 GeV

arXiv:2401.10326





ν_e Disappearance:

$|\Delta m_{ee}^2|$ same for both orderings

Daya Bay:

ν_μ Disappearance:

$|\Delta m_{\mu\mu}^2|$ same for both orderings

NOvA, T2K:

$$|\Delta m_{32}^2|_{DB}^{IO} = |\Delta m_{31}^2|_{DB}^{NO} + \cos 2\theta_{12} \Delta m_{21}^2$$

$$\cos 2\theta_{12} \approx 0.40$$

$$|\Delta m_{32}^2|_{\mu dis}^{IO} = |\Delta m_{31}^2|_{\mu dis}^{NO} - (\cos 2\theta_{12} - 2 \sin \theta_{13} \widehat{\cos \delta}) \Delta m_{21}^2$$

$$\cos 2\theta_{12} - 2 \sin \theta_{13} \cos \delta \approx 0.40 - 0.30 \cos \delta$$

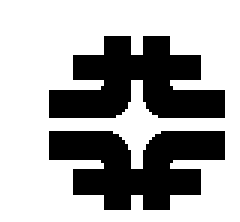
If IO then 0

If NO then 0

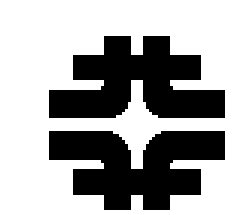
$$\left(|\Delta m_{32}^2|_{DB}^{IO} - |\Delta m_{32}^2|_{\mu dis}^{IO} \right) + \left(|\Delta m_{31}^2|_{\mu dis}^{NO} - |\Delta m_{31}^2|_{DB}^{NO} \right) = (2.4 - 0.9 \widehat{\cos \delta})\% |\Delta m_{ee}^2|$$

1.5 to 3.3 %

Unchanged if 31 ↔ 32 in either or both MO's



NEXT STEP: JUNO



JUNO



Fig. 1. Map of the local area around the experimental site of JUNO, located on the South-West part of the Guangzhou city in China.

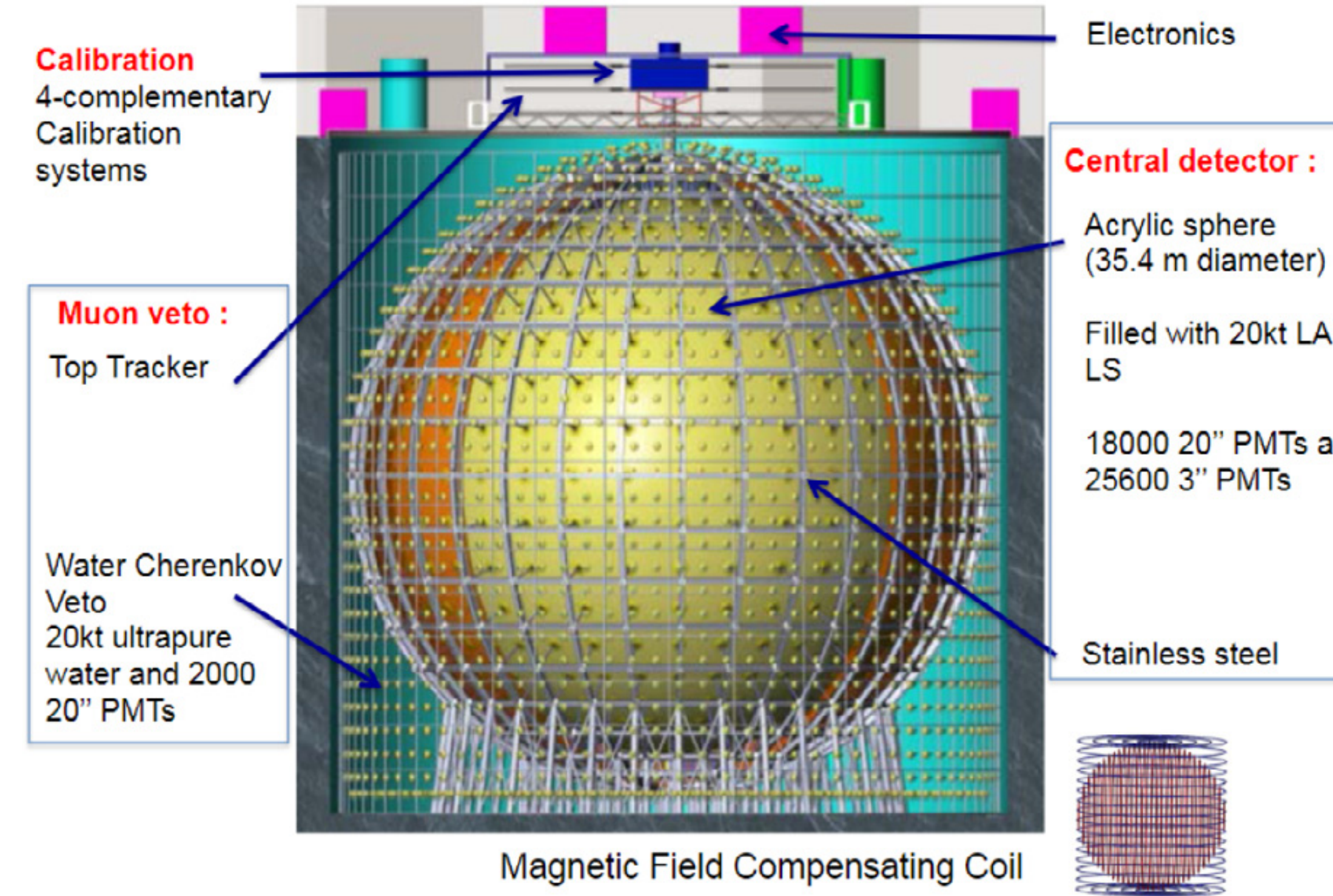
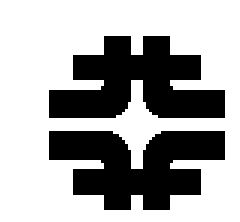


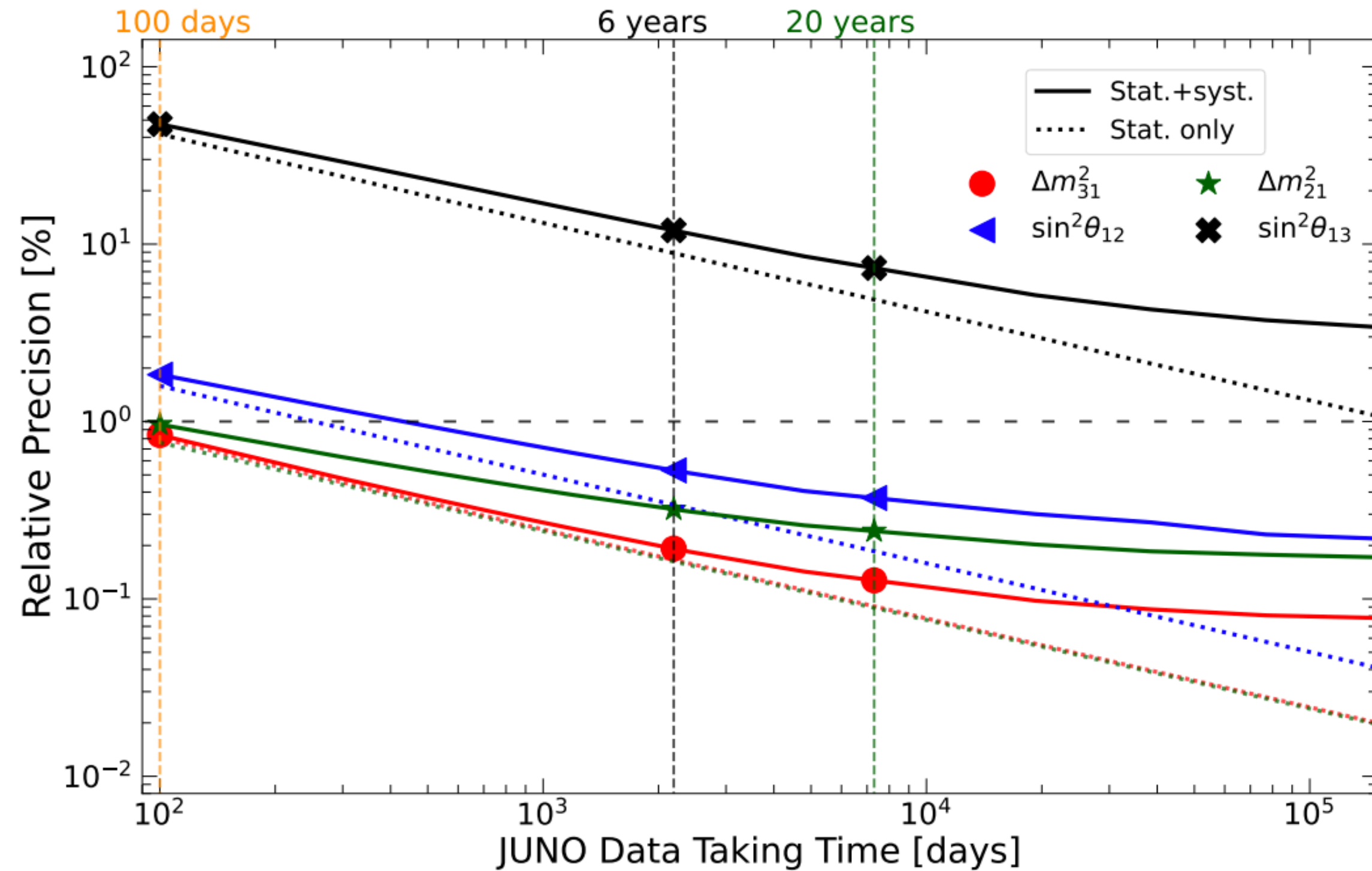
Fig. 4. Schematic view of the JUNO detector.

Reactor	YJ-C1	YJ-C2	YJ-C3	YJ-C4	YJ-C5	YJ-C6	TS-C1	TS-C2	DB	HZ
Power (GW_{th})	2.9	2.9	2.9	2.9	2.9	2.9	4.6	4.6	17.4	17.4
Baseline (km)	52.74	52.82	52.41	52.49	52.11	52.19	52.77	52.64	215	265



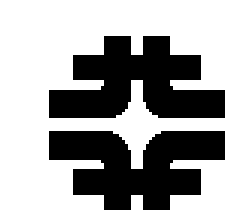
Time Evolution of JUNO measurements

JUNO_update_2204.13249



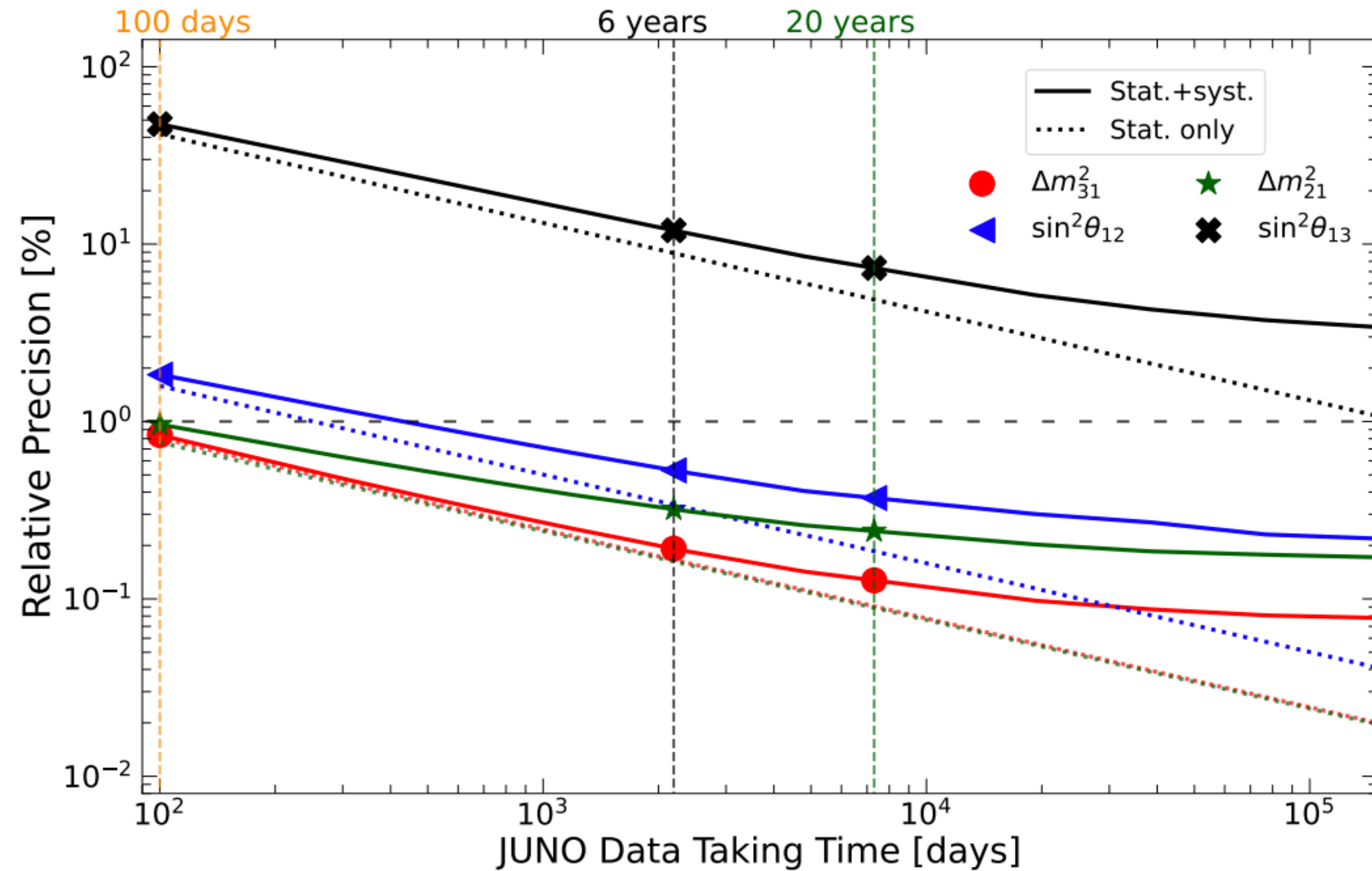
Time	% on Δm_{atm}^2
100 days	1.0
4 years	0.3
8 years	0.2
12 Years	0.15





Time Evolution of JUNO measurements

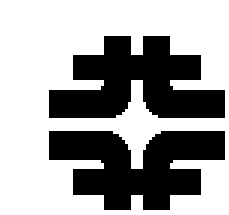
JUNO_update_2204.13249



Time	% on Δm_{atm}^2
100 days	1.0
4 years	0.3
8 years	0.2
12 Years	0.15



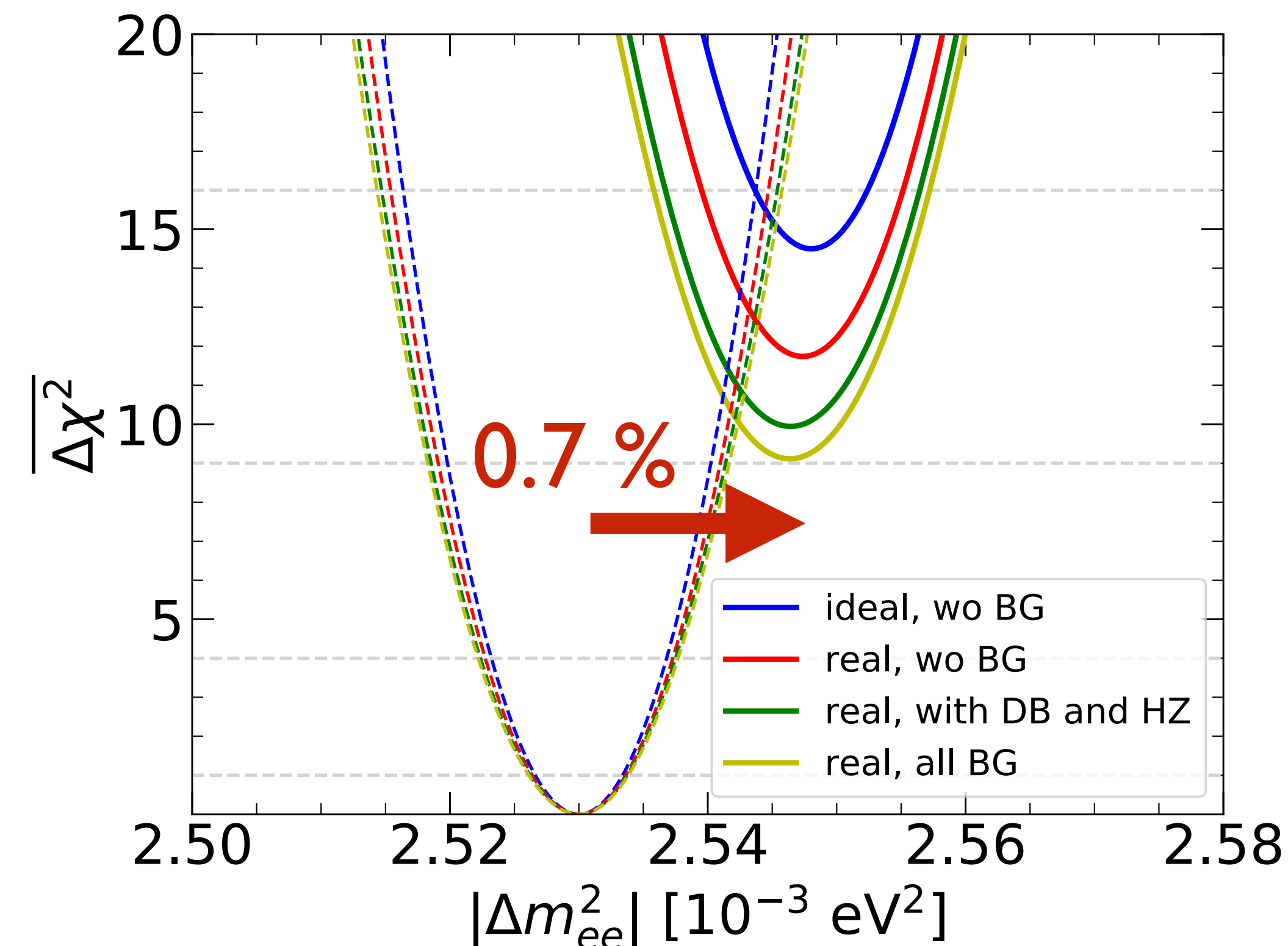
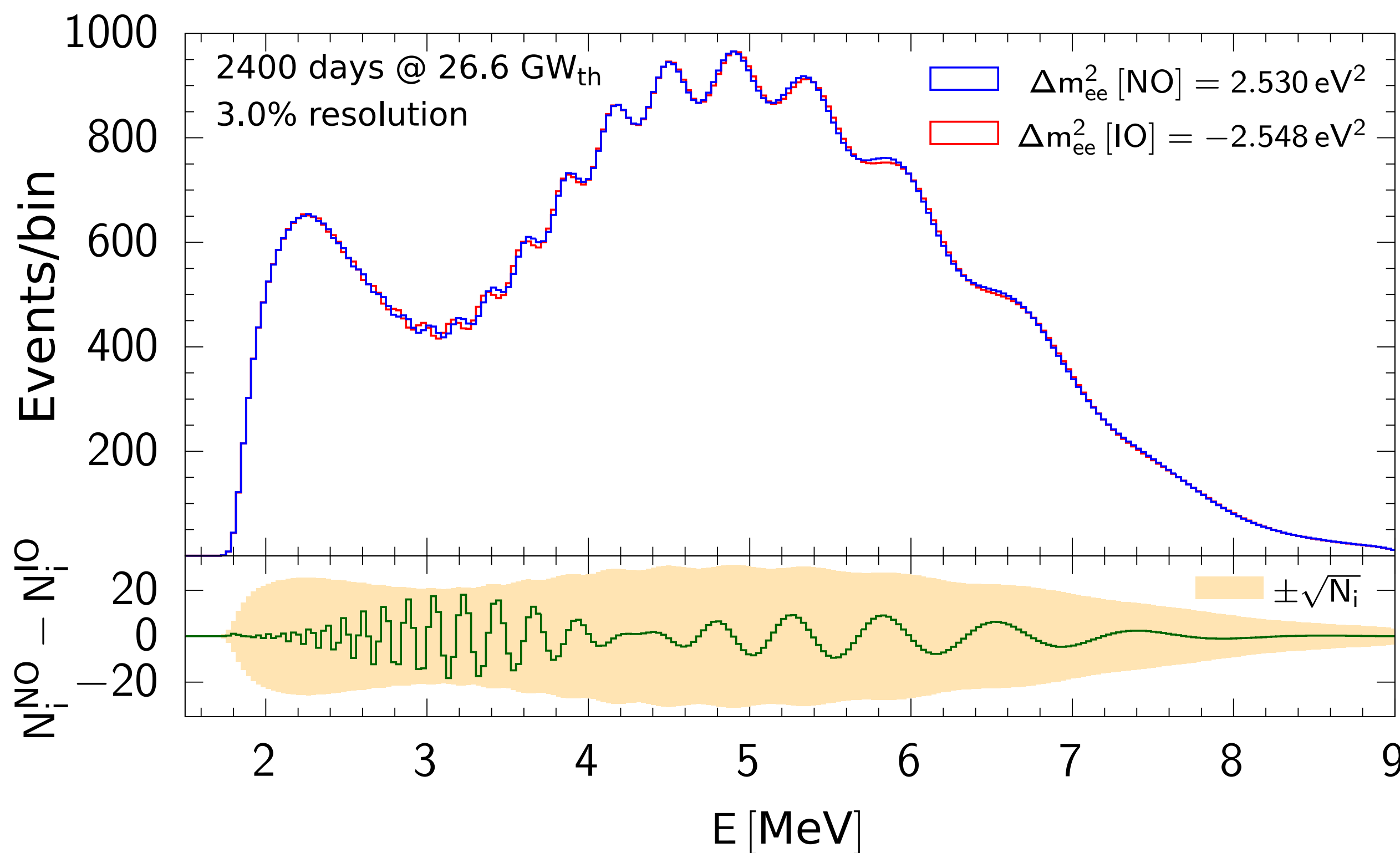
500+ years



JUNO Events Spectra



Real Baseline Distribution + Backgrounds



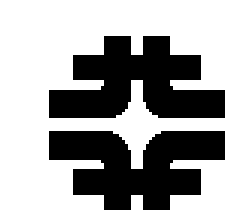
8 years, 26.6 GW_{th}
baseline exactly 52.5 km
3.0 % resolution

Forero, SP, Ternes, Zukanovich 2107.12410

If $|\Delta m_{32}^2| (IO) = |\Delta m_{32}^2| (NO)$, then $|\Delta m_{ee}^2| (IO) = 2.428$

If $|\Delta m_{31}^2| (IO) = |\Delta m_{31}^2| (NO)$, then $|\Delta m_{ee}^2| (IO) = 2.578$

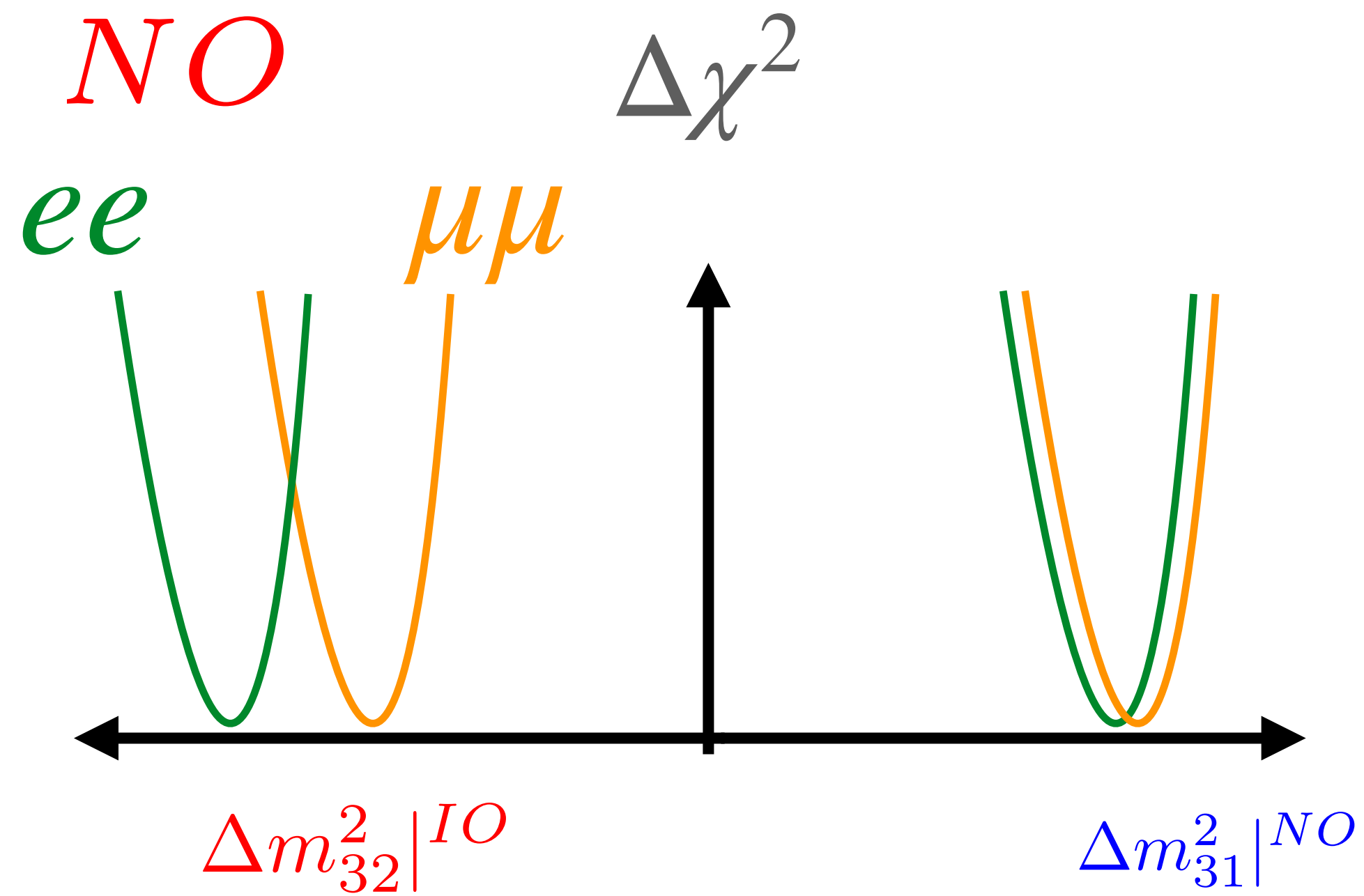
If $|\Delta m_{32}^2| (IO) = |\Delta m_{31}^2| (NO)$, then $|\Delta m_{ee}^2| (IO) = 2.503$



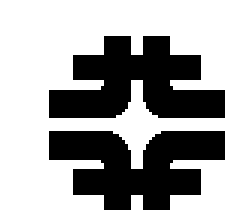
For JUNO: $|\Delta m_{ee}^2|^{IO} = 1.007 |\Delta m_{ee}^2|^{NO}$

$$\left(|\Delta m_{32}^2|_{Ju}^{IO} - |\Delta m_{32}^2|_{\mu dis}^{IO} \right) + \left(|\Delta m_{31}^2|_{\mu dis}^{NO} - |\Delta m_{31}^2|_{Ju}^{NO} \right) = (3.3 - 0.9 \widehat{\cos \delta})\% |\Delta m_{ee}^2|$$

and experimental uncertainty on $|\Delta m_{ee}^2|$ drops to $<1\%$. (Daya Bay 2.4%).



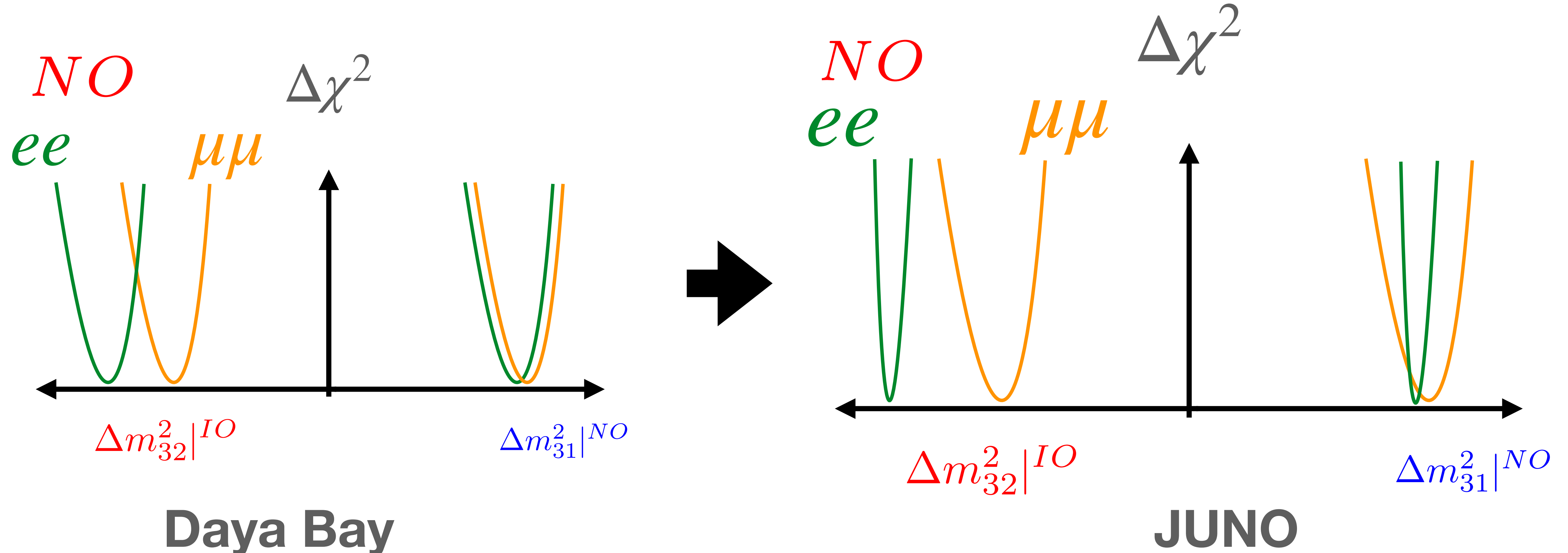
Daya Bay

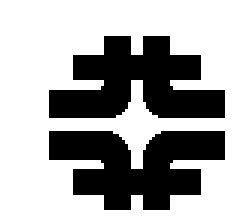


For JUNO: $|\Delta m_{ee}^2|^{IO} = 1.007 |\Delta m_{ee}^2|^{NO}$

$$\left(|\Delta m_{32}^2|_{Ju}^{IO} - |\Delta m_{32}^2|_{\mu dis}^{IO} \right) + \left(|\Delta m_{31}^2|_{\mu dis}^{NO} - |\Delta m_{31}^2|_{Ju}^{NO} \right) = (3.3 - 0.9 \widehat{\cos \delta})\% |\Delta m_{ee}^2|$$

and experimental uncertainty on $|\Delta m_{ee}^2|$ drops to $<1\%$. (Daya Bay 2.4%).

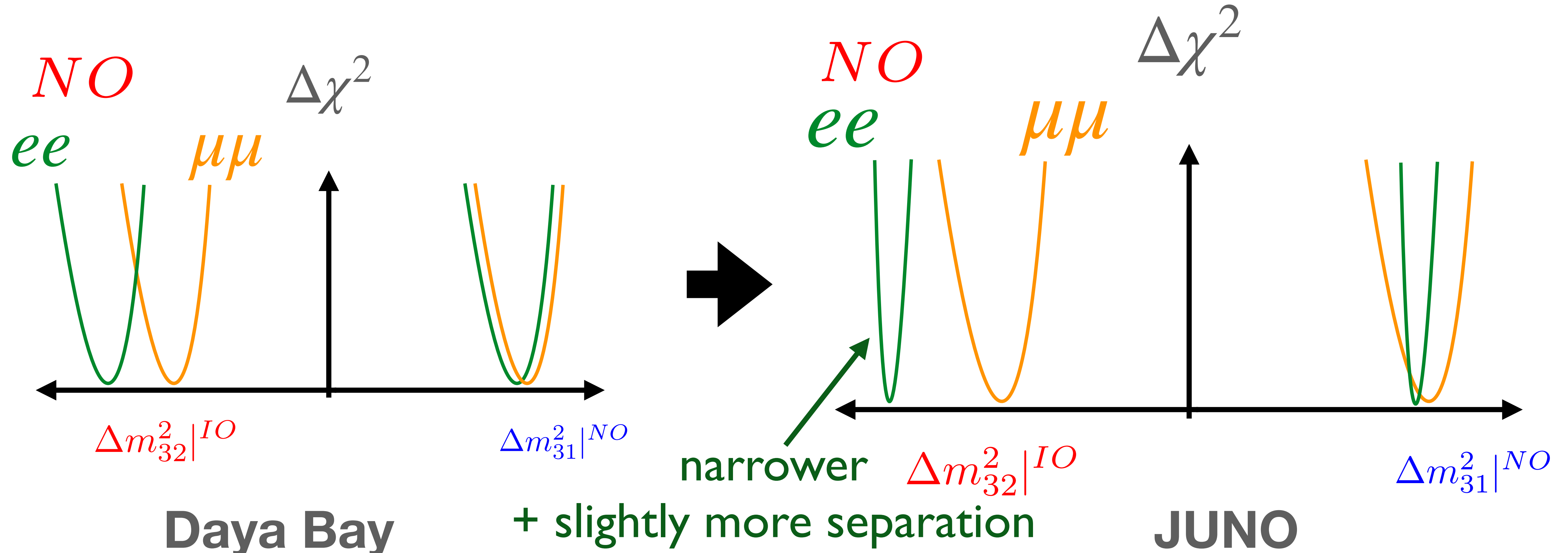


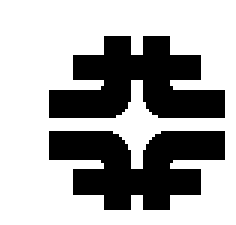


For JUNO: $|\Delta m_{ee}^2|^{IO} = 1.007 |\Delta m_{ee}^2|^{NO}$

$$\left(|\Delta m_{32}^2|_{Ju}^{IO} - |\Delta m_{32}^2|_{\mu dis}^{IO} \right) + \left(|\Delta m_{31}^2|_{\mu dis}^{NO} - |\Delta m_{31}^2|_{Ju}^{NO} \right) = (3.3 - 0.9 \widehat{\cos \delta})\% |\Delta m_{ee}^2|$$

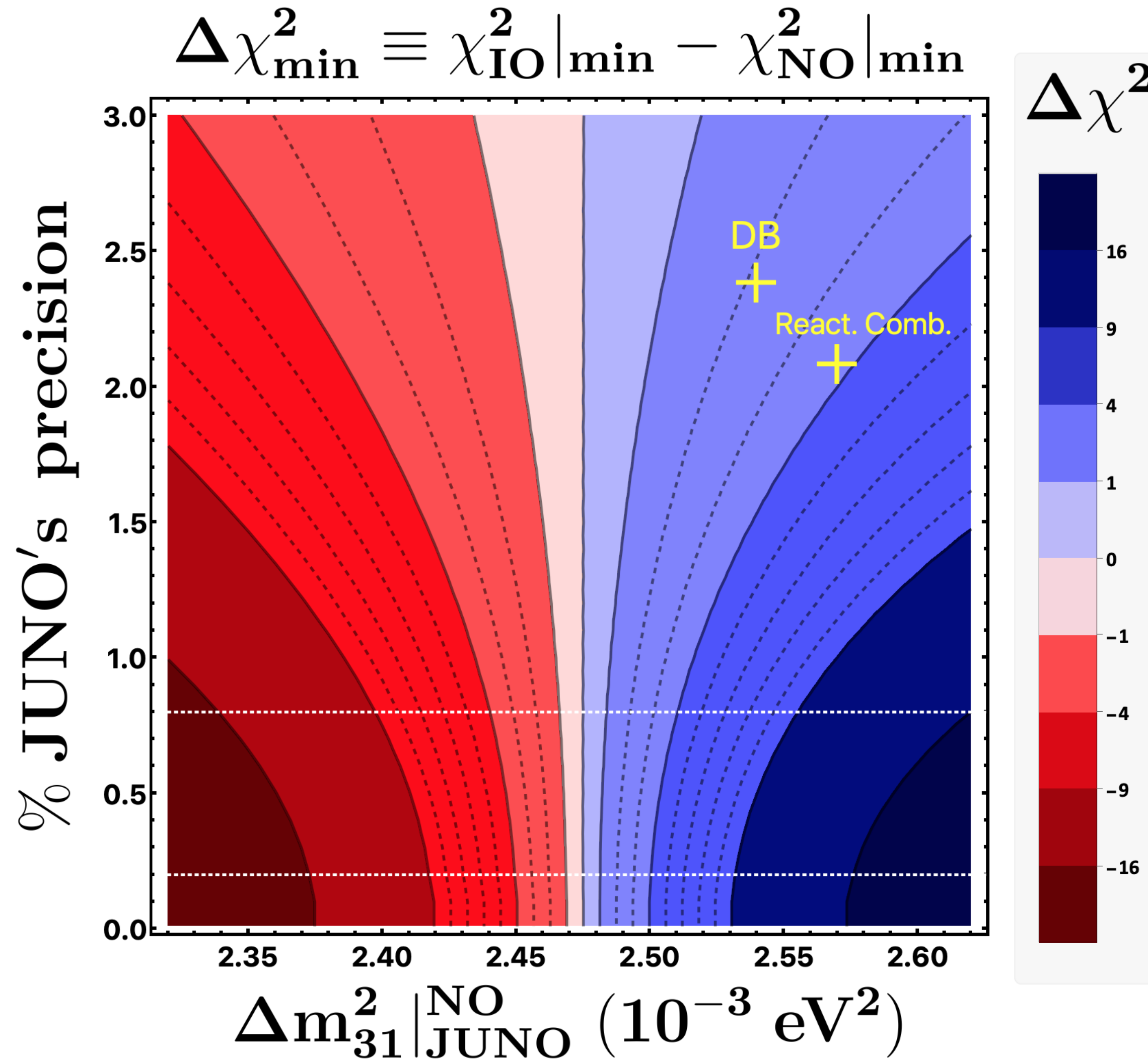
and experimental uncertainty on $|\Delta m_{ee}^2|$ drops to $<1\%$. (Daya Bay 2.4%).





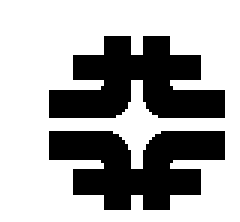
A Mass Ordering Sum Rule for the Neutrino Disappearance Channels in T2K, NOvA and JUNO

arXiv:2404.08733



100 days

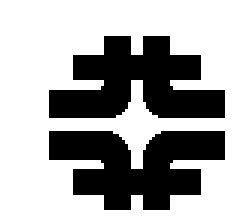
6 years



Further Synergies:

JUNO-ICECUBE UPGRADES 1911.06745

JUNO-KM3NET 2108.06293



Further Synergies:

JUNO-ICECUBE UPGRADES 1911.06745

JUNO-KM3NET 2108.06293

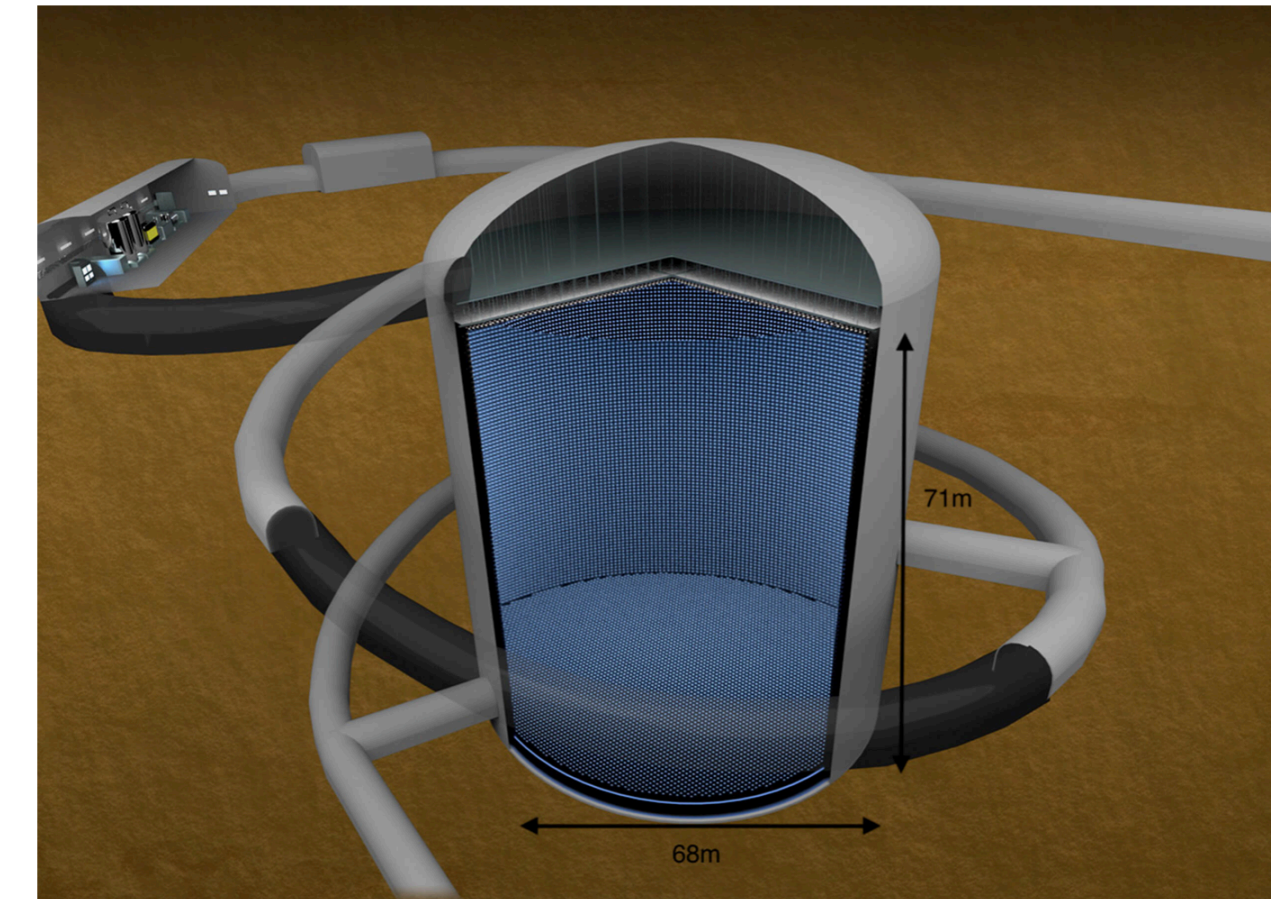
Single Experiments:

JUNO 1507.05613

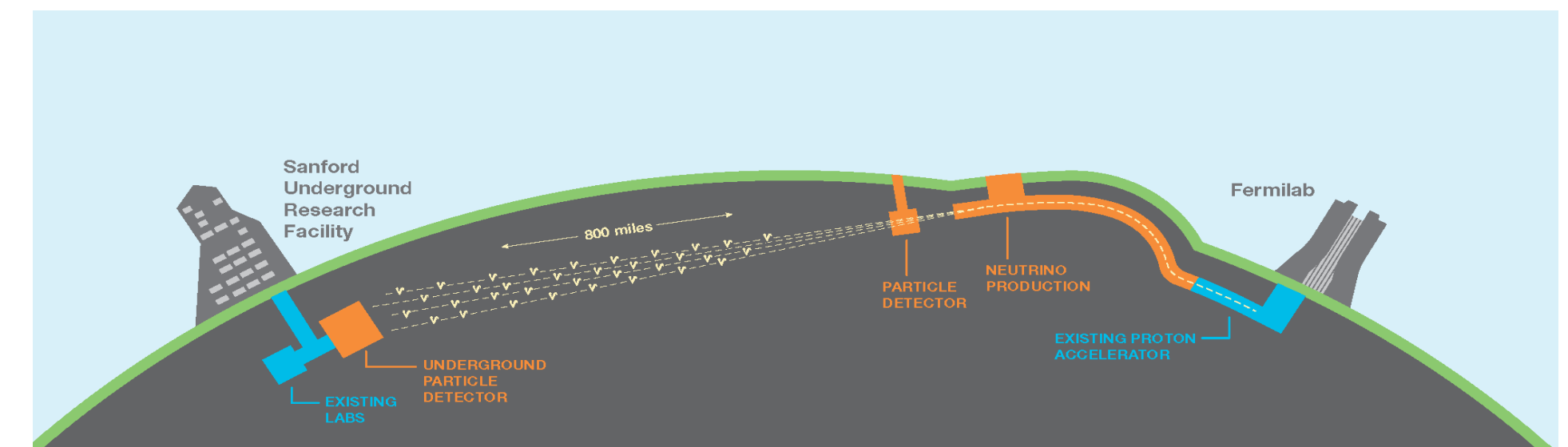
no MO update in
JUNO 2204.13249

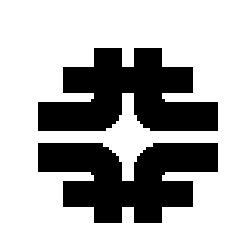
See also
FPTZ:2107.12410

HyperK:



DUNE:

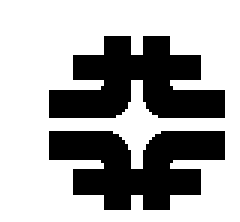




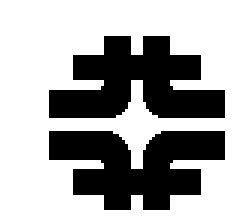
Summary:



- **Circa Nu 2026:** Global fits, including JUNO's precision Δm^2 measurement may give us Neutrino Mass Ordering $> 3\sigma$.
 - Precision Disappearance Δm^2 measurements will make significant contributions (NPZ '05)
- **Circa Nu 202x:** Synergies of JUNO with ICECUBE/PINGU, KM3NET
- **Circa Nu 203x:** JUNO, HK and DUNE will each have Neutrino Mass Ordering $> 3\sigma$ in a single experiment
- **A Year Later:** DUNE $> 5\sigma$ for Neutrino Mass Ordering



Extras

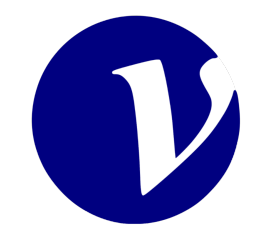
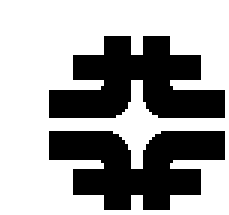


$$\begin{aligned} |\Delta m_{ee}^2| &\equiv |\Delta m_{32}^2|^{\frac{NO}{IO}} \pm c_{12}^2 \Delta m_{21}^2 \\ &\equiv |\Delta m_{31}^2|^{\frac{NO}{IO}} \mp s_{12}^2 \Delta m_{21}^2 \end{aligned}$$

Daya Bay (10^{-3} eV^2)

$$\begin{aligned} |\Delta m_{32}^2|^{IO} - |\Delta m_{32}^2|^{NO} &= + 2c_{12}^2 \Delta m_{21}^2 \\ | - 2.571 | - + 2.466 &\approx + 2 \times 0.7 \times 0.075 = 0.105 \\ &\pm 0.060 \end{aligned}$$

Perfect agreement !



ν_e Disappearance:

$|\Delta m_{ee}^2|$ same for both orderings

Daya Bay:

ν_μ Disappearance:

$|\Delta m_{\mu\mu}^2|$ same for both orderings

NOvA, T2K:

$$\pm = \text{NO} / \text{IO}$$

$$\Delta m_{32}^2 = \pm |\Delta m_{ee}^2| - \cos^2 \theta_{12} \Delta m_{21}^2$$

$$\Delta m_{31}^2 = \pm |\Delta m_{ee}^2| + \sin^2 \theta_{12} \Delta m_{21}^2$$

$$-\Delta m_{32}^2|_{DB}^{IO} = \Delta m_{31}^2|_{DB}^{NO} + \cos 2\theta_{12} \Delta m_{21}^2$$

$$\Delta m_{32}^2 = \pm |\Delta m_{\mu\mu}^2| - \sin^2 \theta'_{12} \Delta m_{21}^2$$

$$\Delta m_{31}^2 = \pm |\Delta m_{\mu\mu}^2| + \cos^2 \theta'_{12} \Delta m_{21}^2$$

$$-\Delta m_{32}^2|_{\mu dis}^{IO} = \Delta m_{31}^2|_{\mu dis}^{NO} - \cos 2\theta'_{12} \Delta m_{21}^2$$

$$\cos 2\theta_{12} \approx 0.40$$

If IO then 0

$$\cos 2\theta'_{12} = \cos 2\theta_{12} - 2s_{13} \cos \delta \approx 0.40 - 0.30 \cos \delta$$

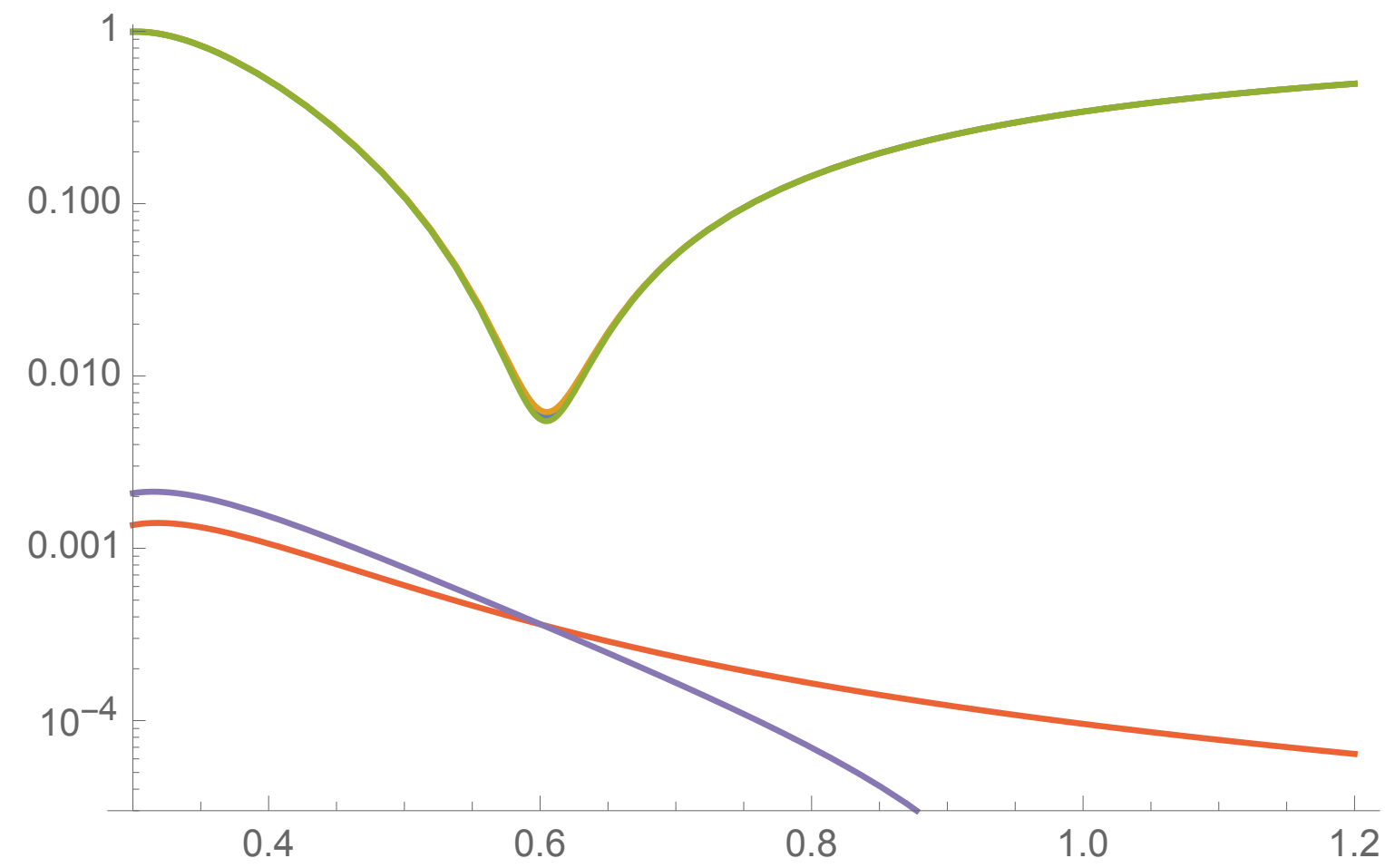
If NO then 0

$$(\Delta m_{32}^2|_{\mu dis}^{IO} - \Delta m_{32}^2|_{DB}^{IO}) + (\Delta m_{31}^2|_{\mu dis}^{NO} - \Delta m_{31}^2|_{DB}^{NO}) = (2.4 - 0.9 \cos \delta)\% \Delta m_{ee}^2$$

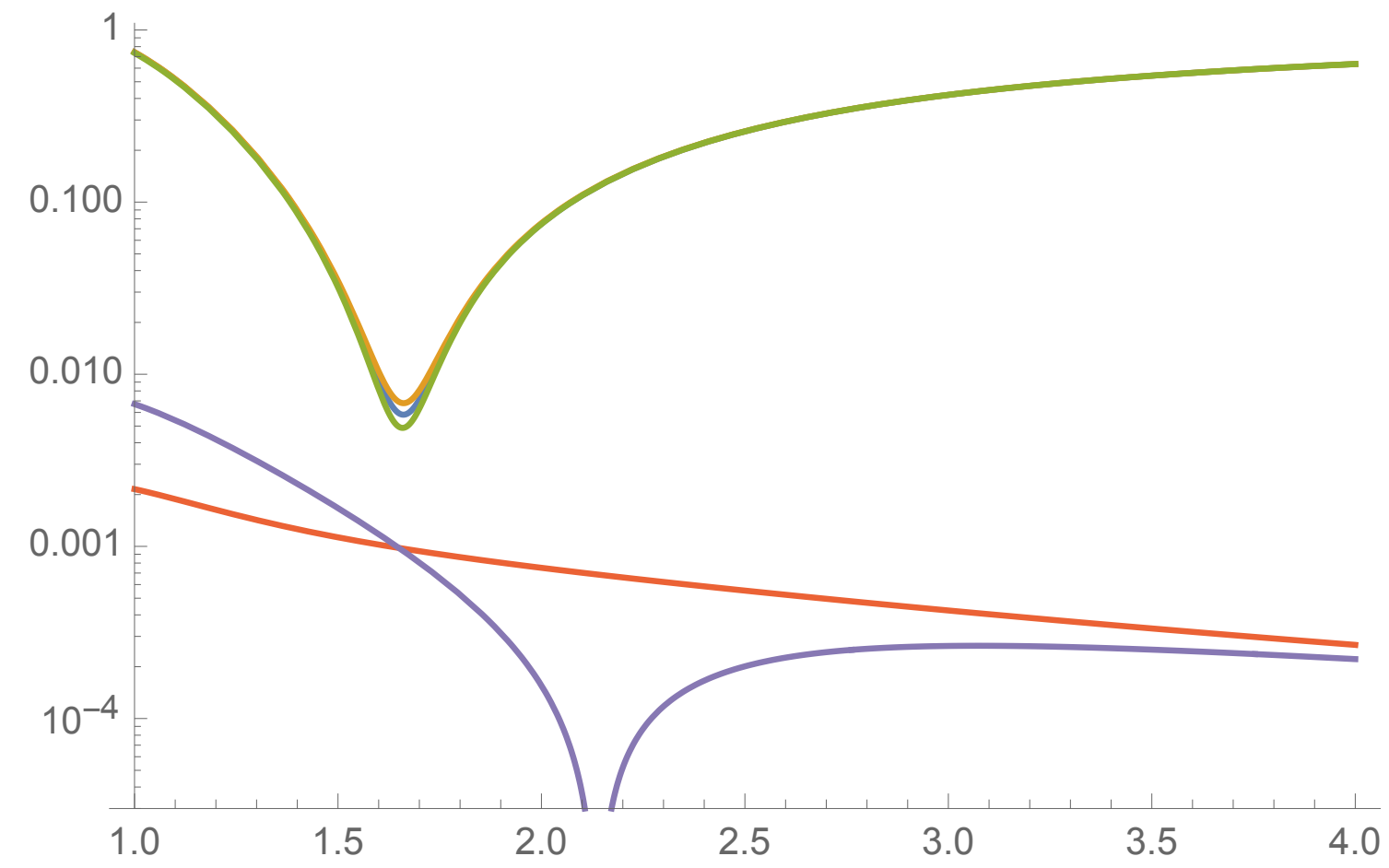
1.5 to 3.3 %

Unchanged if $31 \leftrightarrow 32$ in either or both MO's

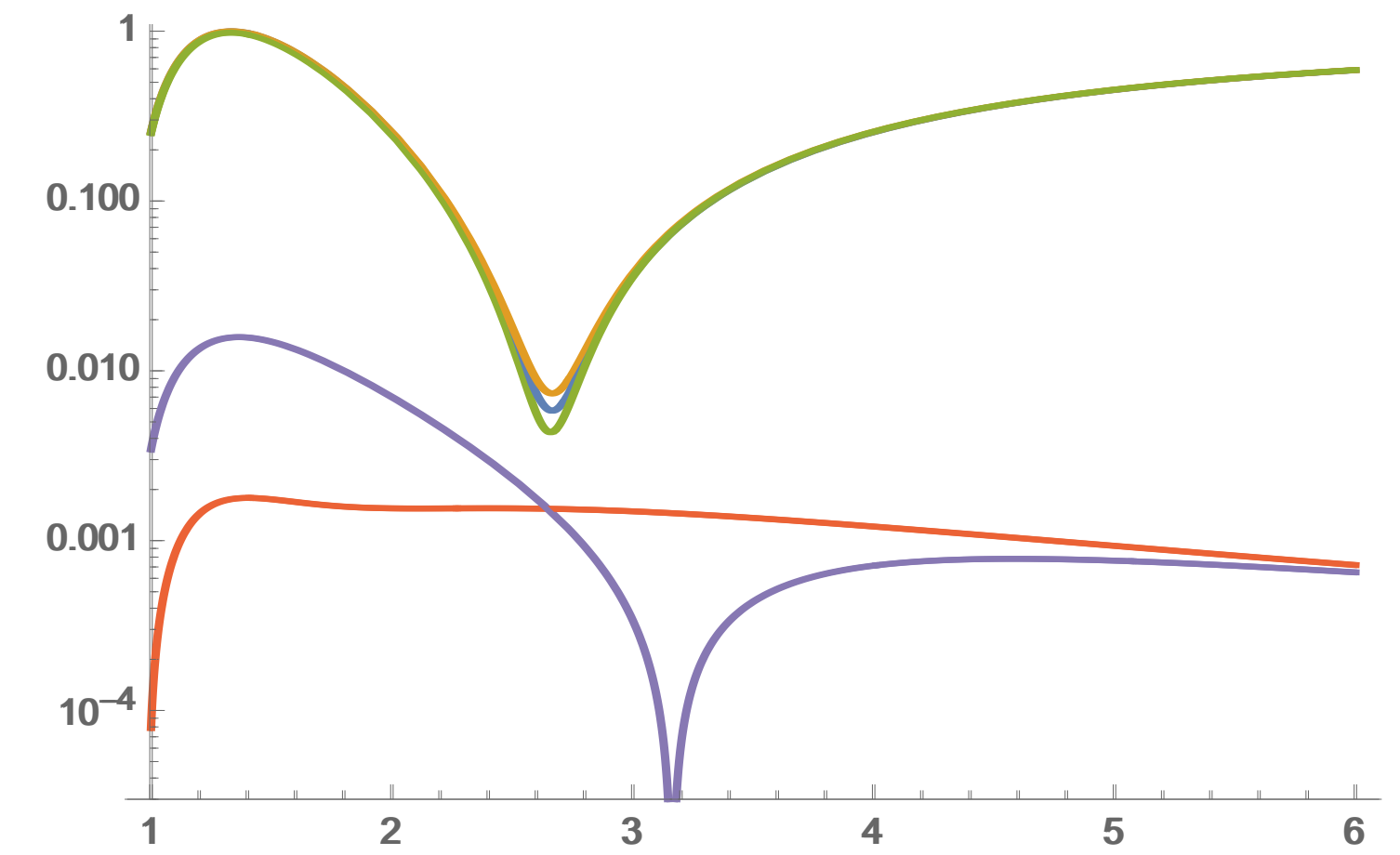
Vacuum v Matter:



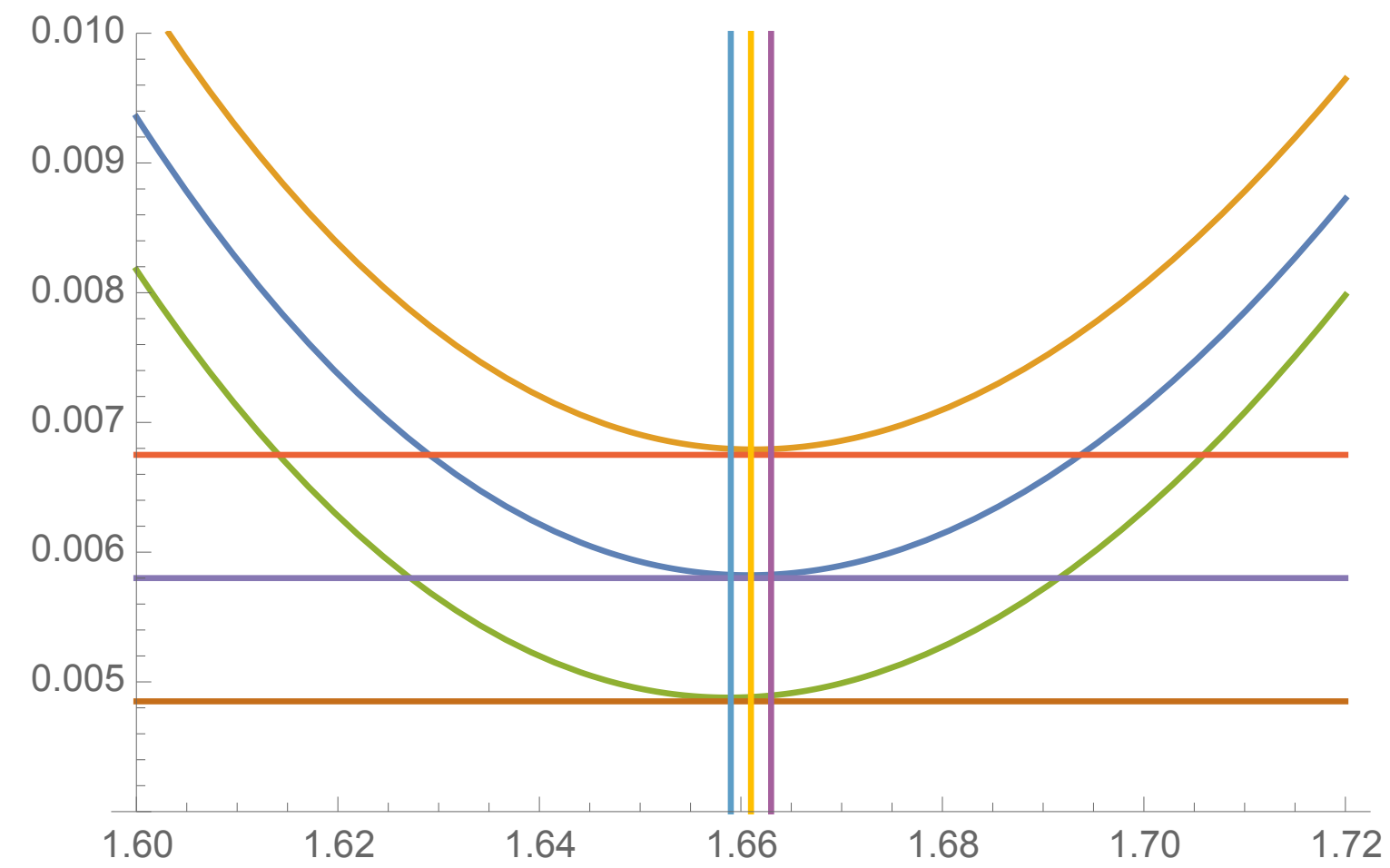
T2K

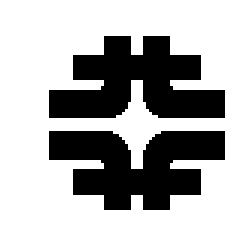


NOvA



DUNE





$$|\Delta m_{\mu\mu}^2| \approx |\Delta m_{32}^2| \frac{NO}{IO} \pm (s_{12}^2 + s_{13} \cos \delta \frac{NO}{IO}) \Delta m_{21}^2 = |\Delta m_{31}^2| \frac{NO}{IO} \mp (c_{12}^2 - s_{13} \cos \delta \frac{NO}{IO}) \Delta m_{21}^2$$



NOvA:

Parameter	Normal ord.		Inverted ord.	
	UO	LO	UO	LO
$\Delta m_{32}^2 (10^{-3} \text{ eV}^2)$	$+2.41 \pm 0.07$	$+2.39$	-2.45	-2.44
$\sin^2 \theta_{23}$	$0.57^{+0.03}_{-0.04}$	0.46	0.56	0.46
$\delta_{CP} (\pi)$	$0.82^{+0.27}_{-0.87}$	0.07	1.52	1.41

± 0.07

$$|\Delta m_{32}^2|^{IO} - |\Delta m_{32}^2|^{NO} = (2s_{12}^2 + s_{13} \cos \delta^{NO} + s_{13} \cos \delta^{IO}) \Delta m_{21}^2$$

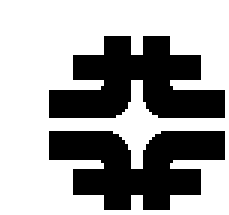
$$\text{UO} \quad | -2.45 | \quad - \quad +2.41 \quad \approx \quad (0.6 + 0.15 \cos \delta^{NO} + 0.15 \cos \delta^{IO}) \times 0.075$$

$$0.04 \approx 0.045 - 0.008$$

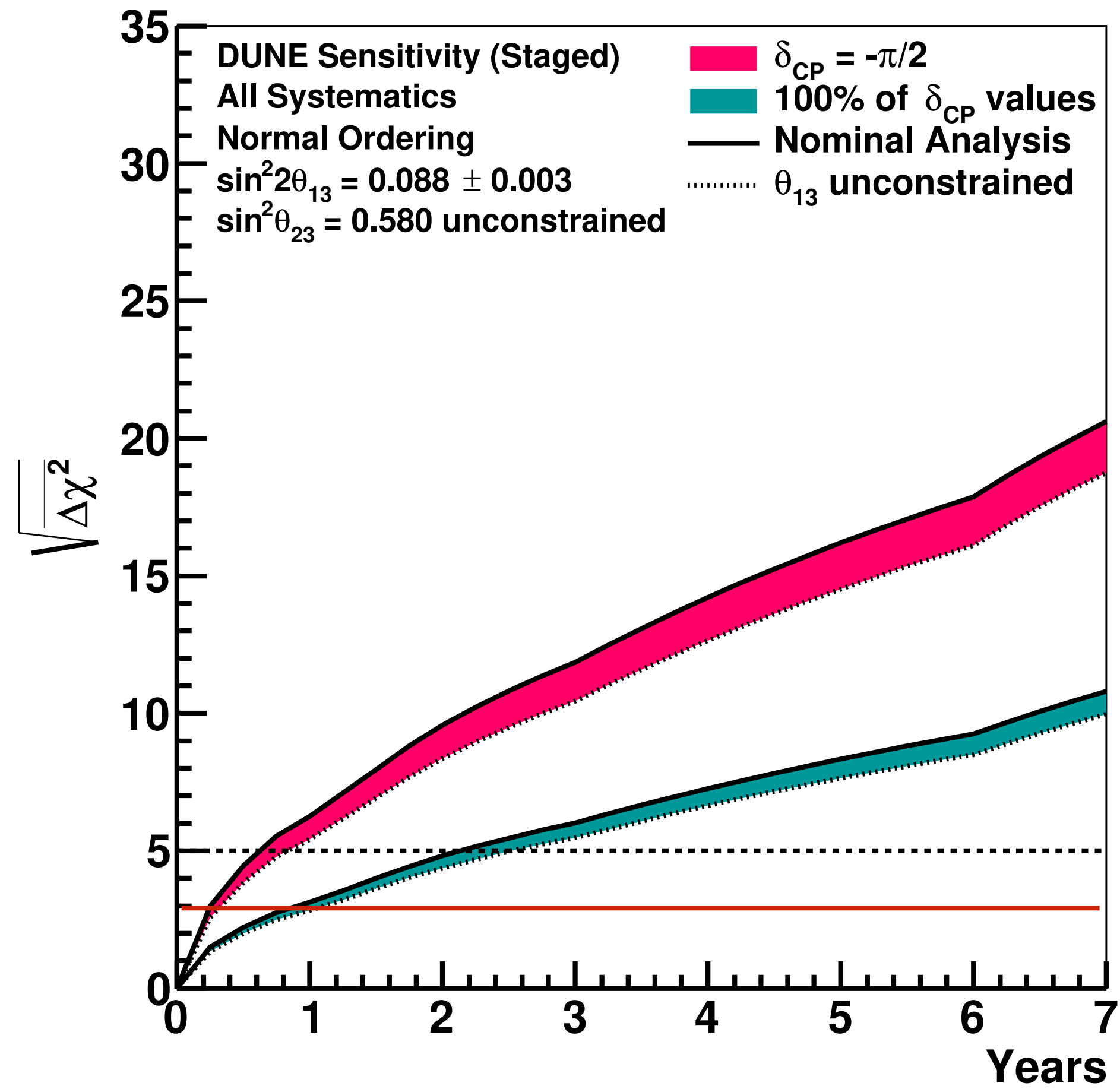
$$\text{LO} \quad | -2.44 | \quad - \quad +2.39 \quad \approx \quad (0.6 + 0.15 \cos \delta^{NO} + 0.15 \cos \delta^{IO}) \times 0.075$$

$$0.05 \approx 0.045 + 0.007$$

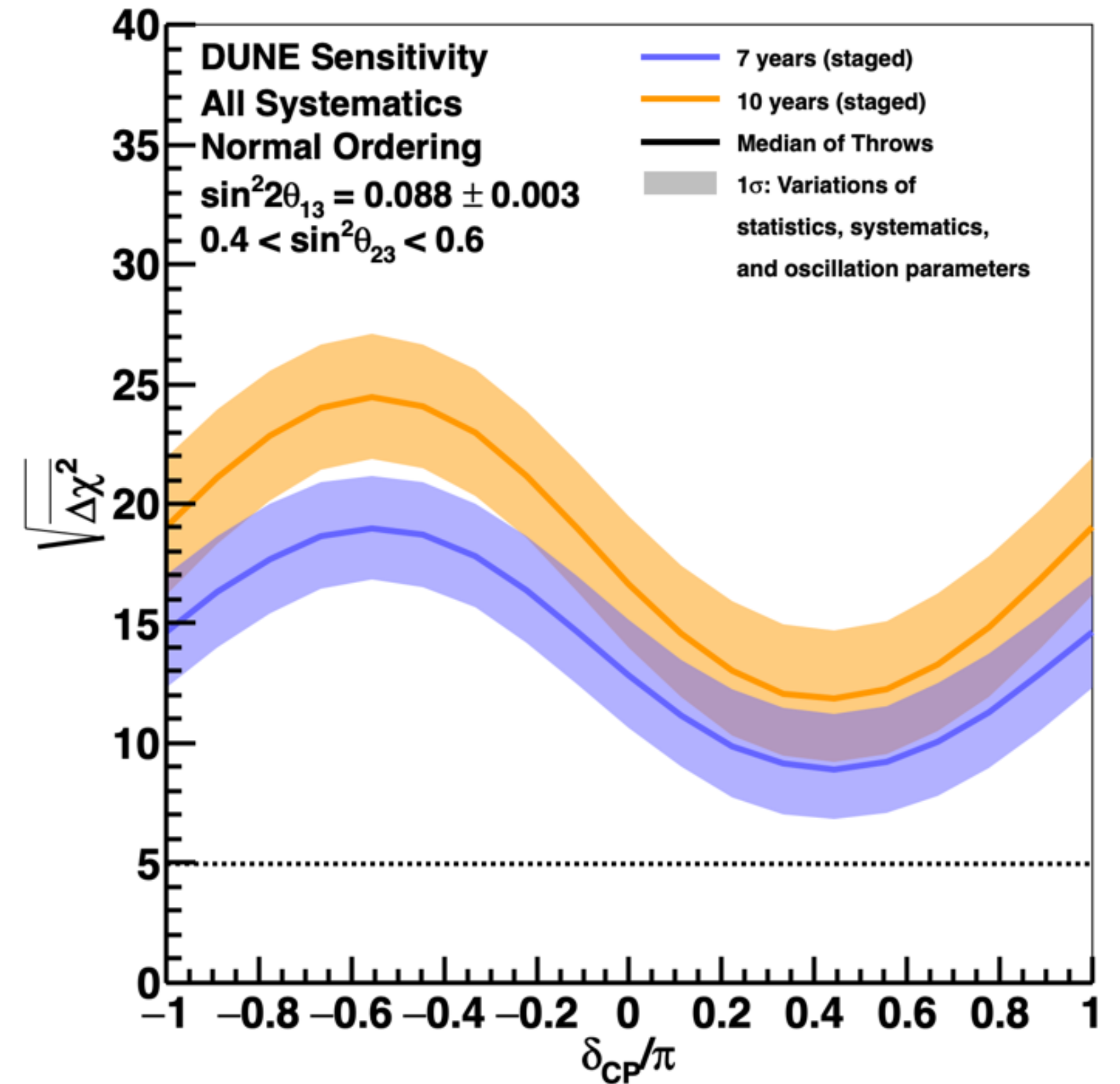
agrees to the accuracy provided !



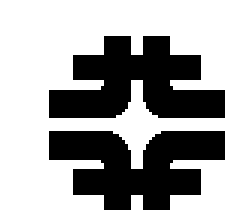
Mass Ordering Sensitivity



Mass Ordering Sensitivity



one (two) year $> 3 \sigma$ ($> 5 \sigma$) for all values of δ_{CP}



Another possible way to determine the Neutrino Mass Hierarchy

Hiroshi Nunokawa^{1,*} Stephen Parke^{2,†} and Renata Zukanovich Funchal^{3‡}

arXiv:hep-ph/0503283v1 29 Mar 2005

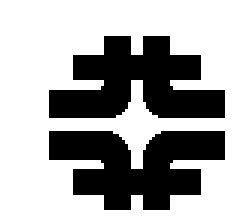
in PRD
NPZ'05

Introduced Δm_{ee}^2 and $\Delta m_{\mu\mu}^2$ for disappearance experiments:

and that $|\Delta m_{ee}^2| > |\Delta m_{\mu\mu}^2|$ implies NO

few % difference

$|\Delta m_{ee}^2| < |\Delta m_{\mu\mu}^2|$ implies IO



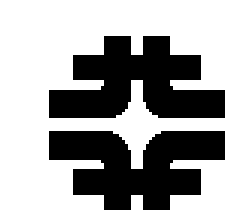
$\bar{\nu}_e$ disappearance at an $L/E \sim 0.5 \text{ km/MeV}$

$$\Delta_{ij} = \frac{\Delta m_{ij}^2 L}{4E}$$

$$\begin{aligned} P(\nu_e \rightarrow \nu_e) &= 1 - P_{\odot} - \sin^2 2\theta_{13} (\cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}) \\ &\approx 1 - P_{\odot} - \sin^2 2\theta_{13} (\sin^2 \Delta_{3i} + (-1)^i \mathcal{O}(\Delta_{21})) \quad i = 1 \text{ or } 2 \\ &\approx 1 - P_{\odot} - \sin^2 2\theta_{13} (\sin^2 \Delta_{ee} + \mathcal{O}(\Delta_{21}^2)) \quad \text{note "2"} \end{aligned}$$

$$\Delta_{21} = \left(\frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right) \Delta_{31} = 0.03 \frac{\pi}{2} = \frac{1}{20} \text{ and therefore } \Delta_{21}^2 = \frac{1}{400}$$

$$P_{\odot} = \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21} = 0.002 \text{ when } \Delta_{31} = \frac{\pi}{2}$$



$\bar{\nu}_e$ disappearance at an $L/E \sim 0.5 \text{ km/MeV}$

$$\Delta_{ij} = \frac{\Delta m_{ij}^2 L}{4E}$$

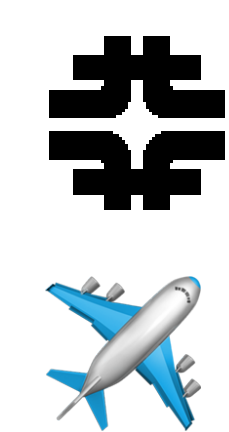
$$\begin{aligned} P(\nu_e \rightarrow \nu_e) &= 1 - P_{\odot} - \sin^2 2\theta_{13} (\cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}) \\ &\approx 1 - P_{\odot} - \sin^2 2\theta_{13} (\sin^2 \Delta_{3i} + (-1)^i \mathcal{O}(\Delta_{21})) \quad i = 1 \text{ or } 2 \\ &\approx 1 - P_{\odot} - \sin^2 2\theta_{13} (\sin^2 \Delta_{ee} + \mathcal{O}(\Delta_{21}^2)) \quad \text{note "2"} \end{aligned}$$

$$\Delta_{21} = \left(\frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right) \Delta_{31} = 0.03 \frac{\pi}{2} = \frac{1}{20} \text{ and therefore } \Delta_{21}^2 = \frac{1}{400}$$

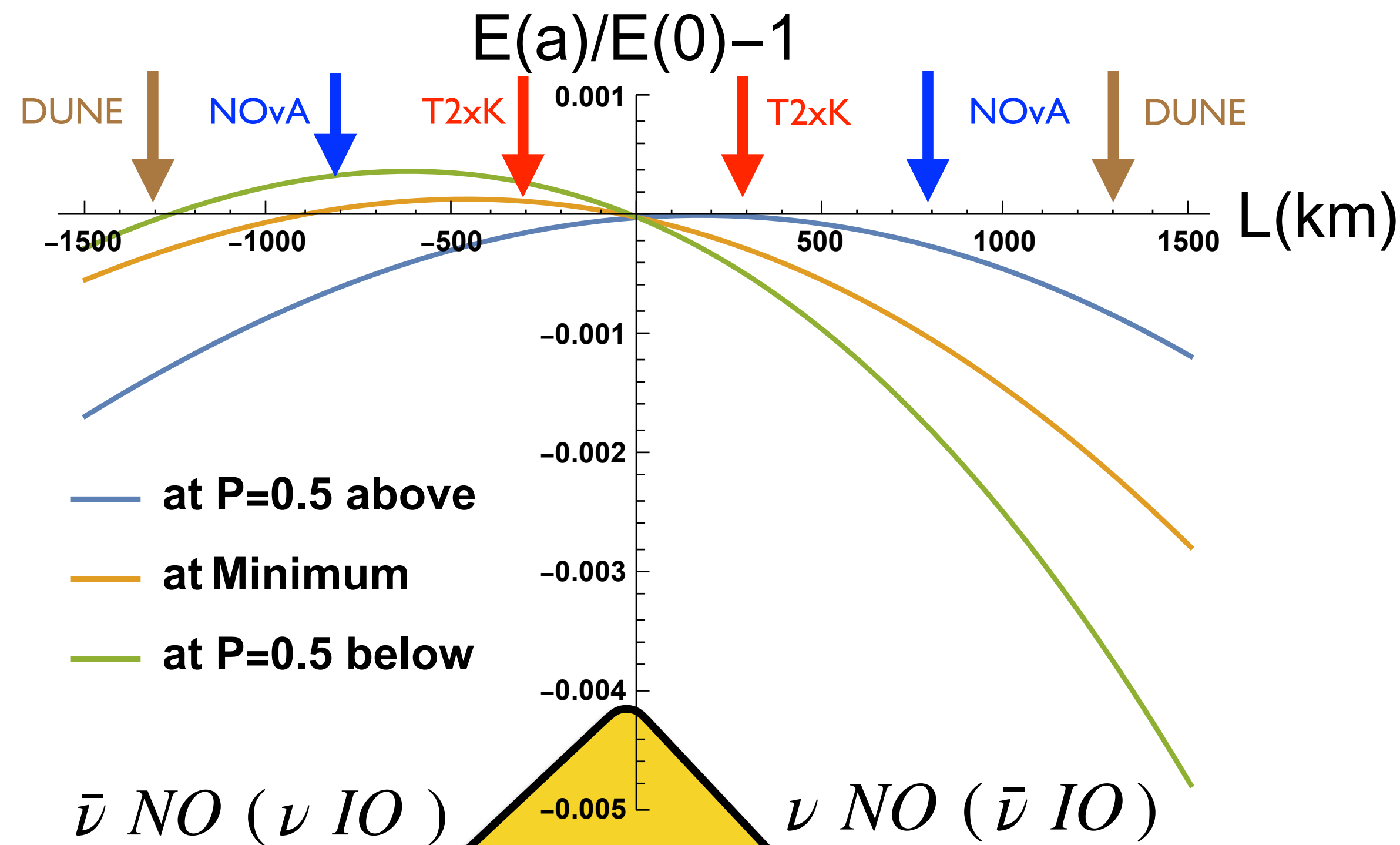
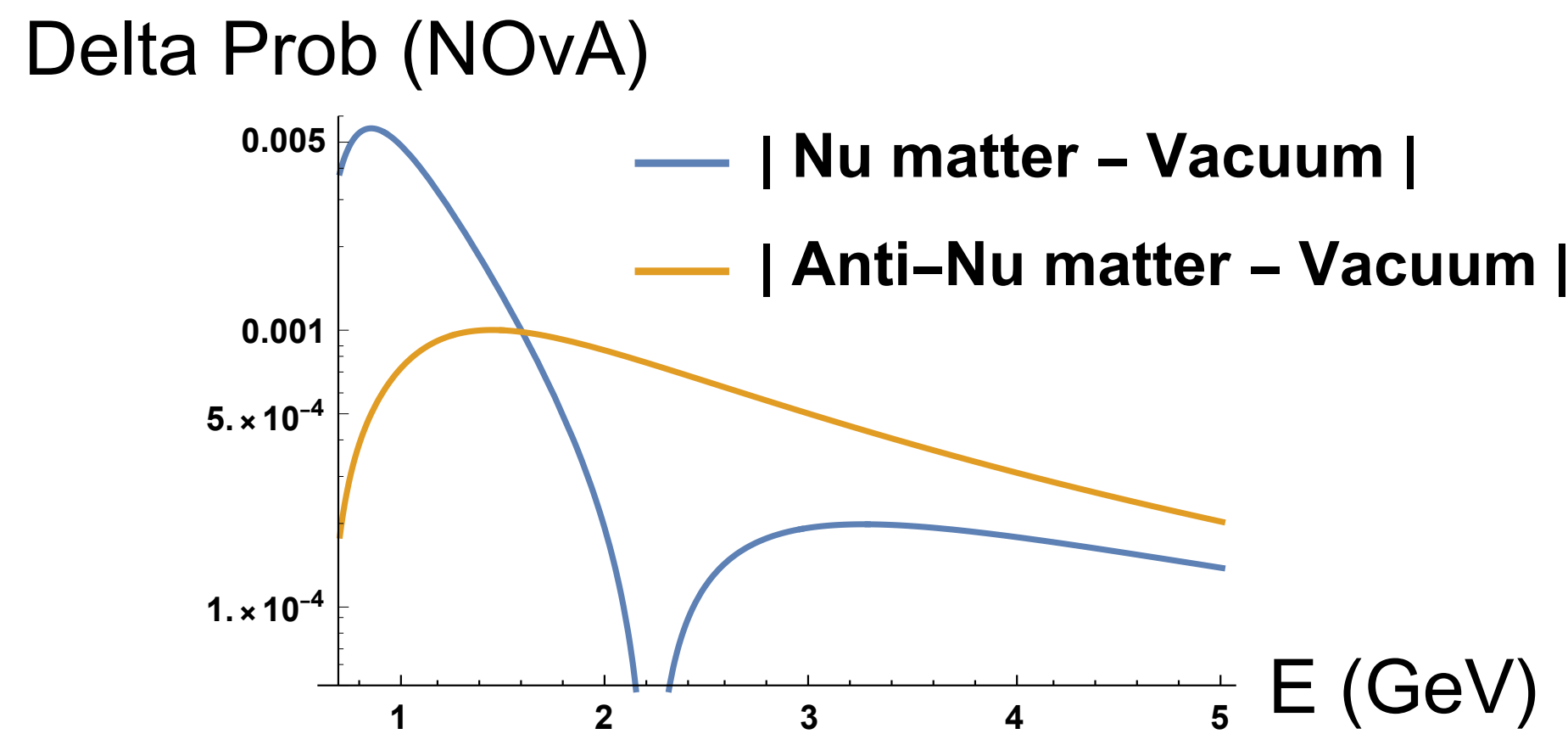
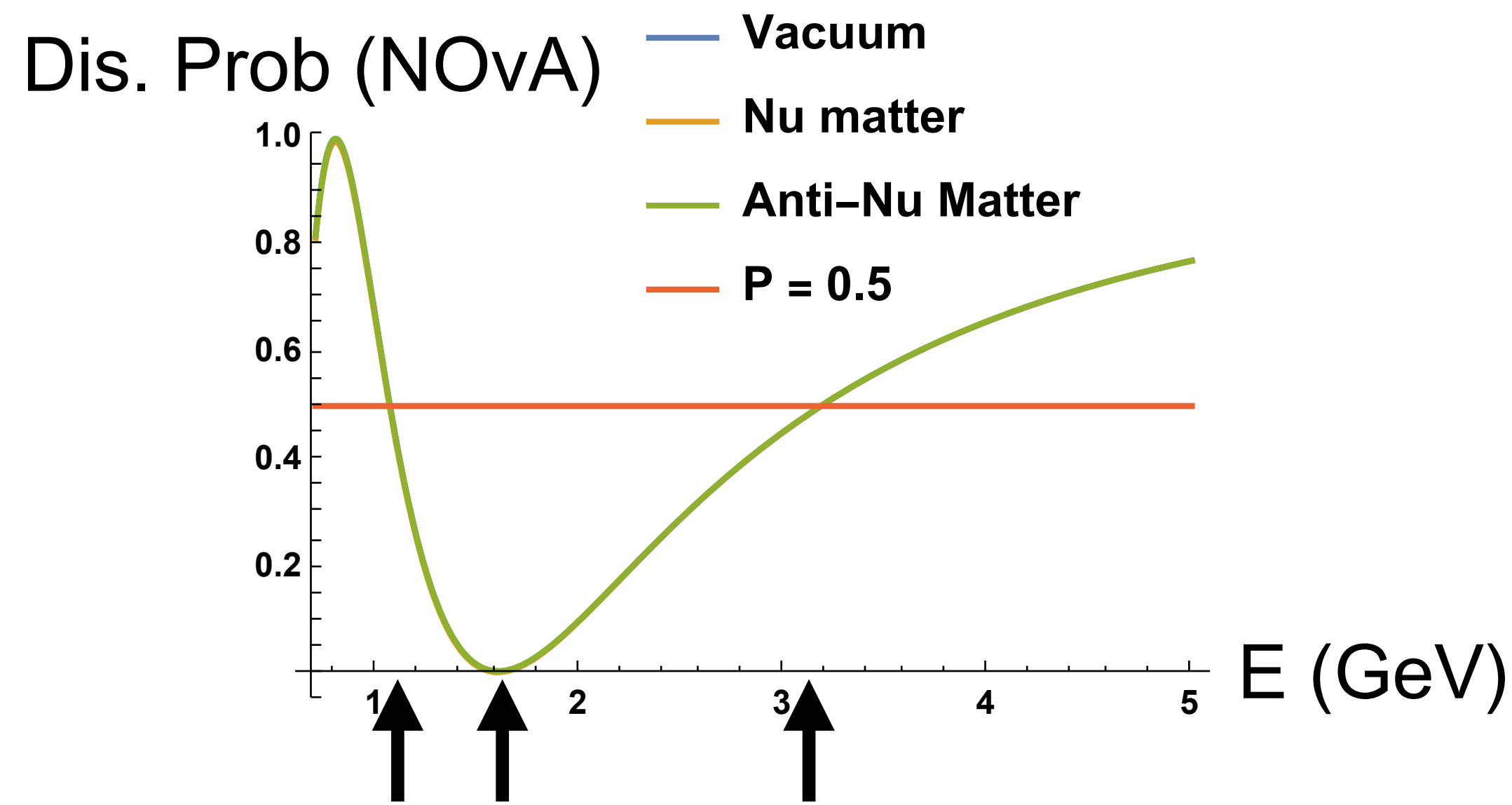
$$\Delta m_{ee}^2 \equiv \cos^2 \theta_{12} \Delta m_{31}^2 + \sin^2 \theta_{12} \Delta m_{32}^2 = m_3^2 - (c_{12}^2 m_1^2 + s_{12}^2 m_2^2)$$

ν_e average of Δm_{31}^2 and Δm_{32}^2

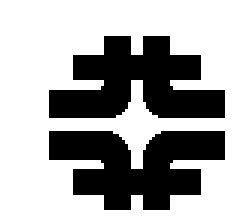
$$P_{\odot} = \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21} = 0.002 \text{ when } \Delta_{31} = \frac{\pi}{2}$$



NOvA matter effects in the disappearance channel:

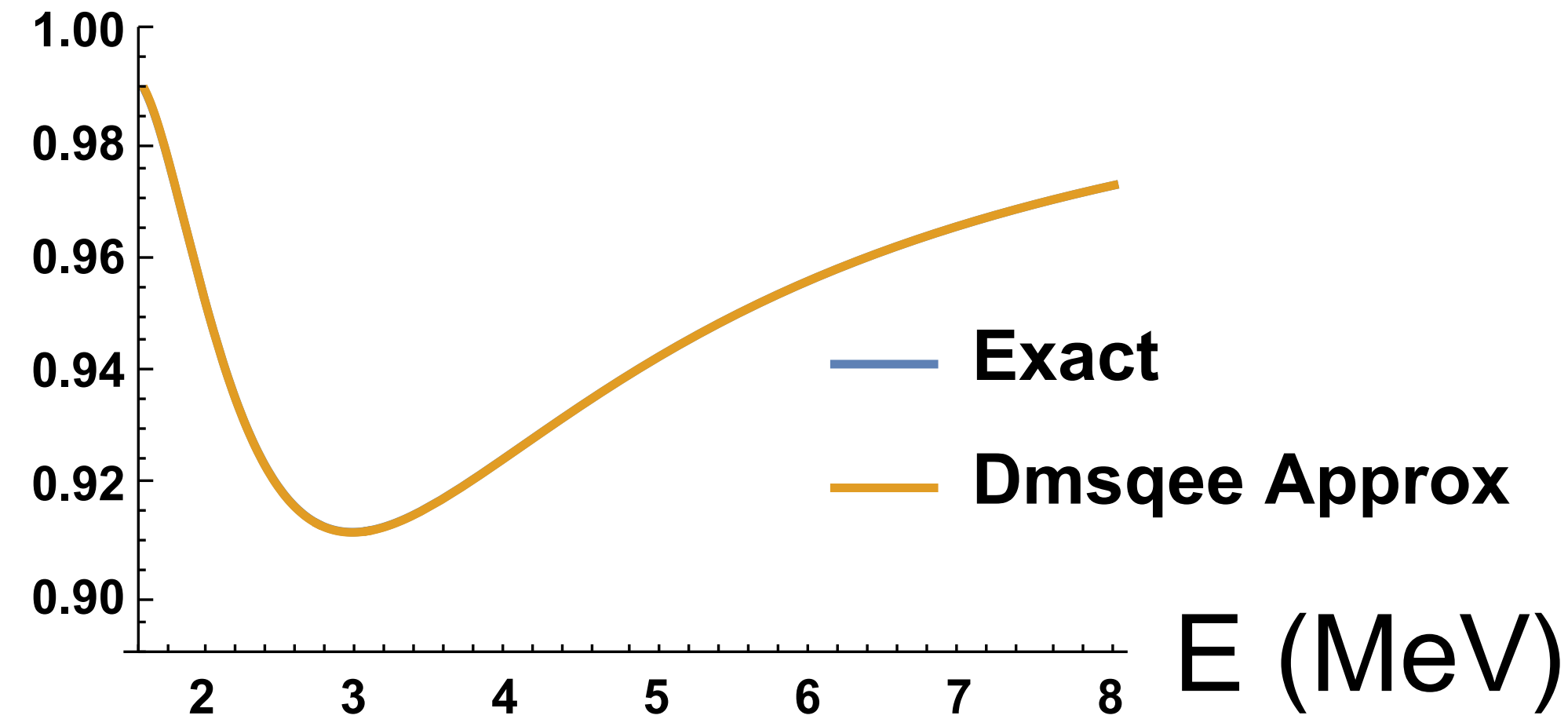


-0.5%

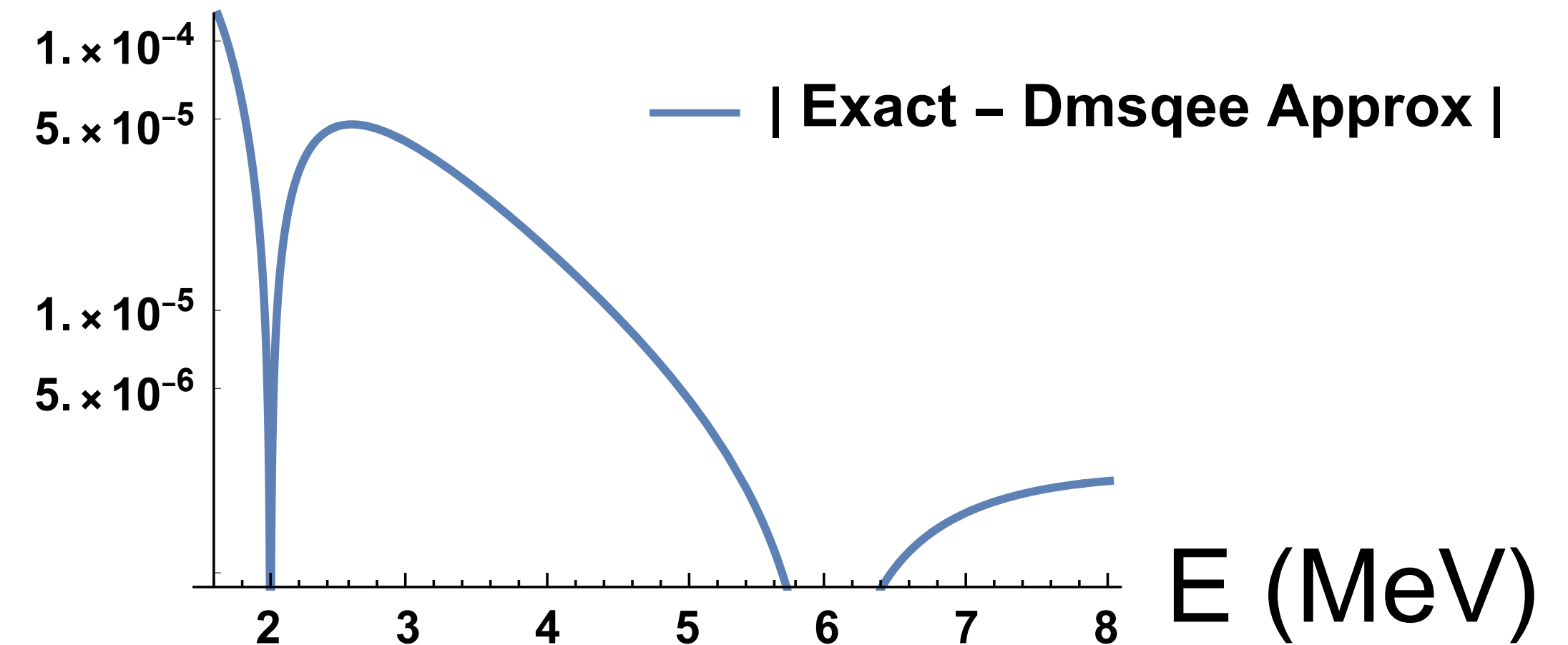


DAYA BAY OSCILLATION PROBABILITY:

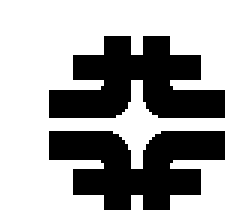
Dis. Prob (Daya Bay)



Delta Prob (Daya Bay)



$$P_{\text{NPZ}}(\nu_e \rightarrow \nu_e) \approx 1 - c_{13}^4 \sin^2 2\theta_{12} \sin^2 \left(\frac{\Delta m_{21}^2 L}{4E} \right) - \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{ee}^2 L}{4E} \right)$$



ν_μ disappearance at an $L/E \sim 500 \text{ km/GeV}$

$$\Delta m_{\mu\mu}^2 \equiv \frac{|U_{\mu 1}|^2 \Delta m_{31}^2 + |U_{\mu 2}|^2 \Delta m_{32}^2}{|U_{\mu 1}|^2 + |U_{\mu 2}|^2} = m_3^2 - \frac{|U_{\mu 1}|^2 m_1^2 + |U_{\mu 2}|^2 m_2^2}{|U_{\mu 1}|^2 + |U_{\mu 2}|^2}$$
$$\approx \Delta m_{ee}^2 - (\cos 2\theta_{12} - \sin \theta_{13} \cos \delta) \Delta m_{21}^2$$

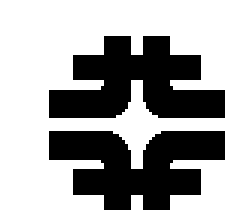
($\sin 2\theta_{12} \tan \theta_{23} \approx 1$)

ν_μ average of Δm_{31}^2 and Δm_{32}^2

$|\Delta m_{ee}^2| > |\Delta m_{\mu\mu}^2|$ implies NO

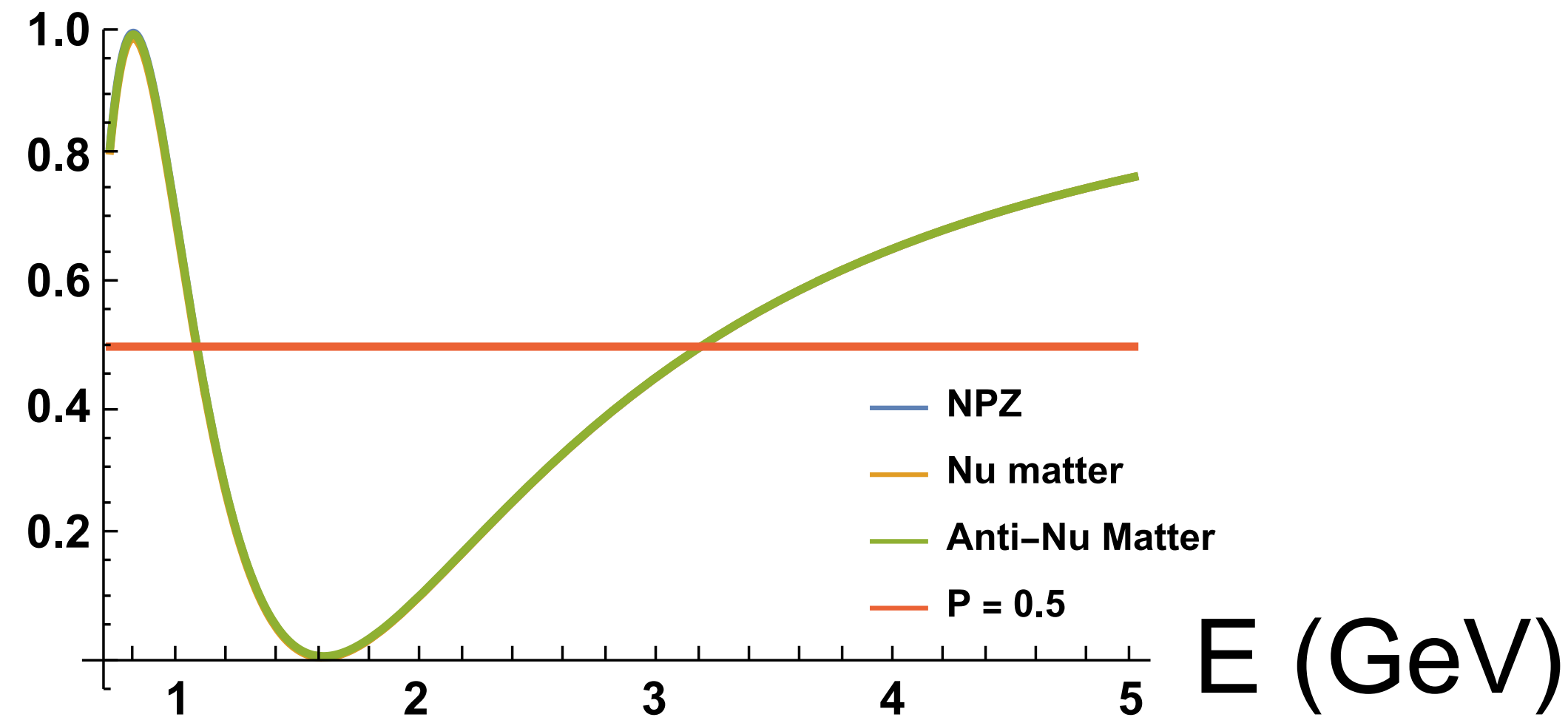
$|\Delta m_{ee}^2| < |\Delta m_{\mu\mu}^2|$ implies IO

THIS IS IGNORING MATTER EFFECTS:

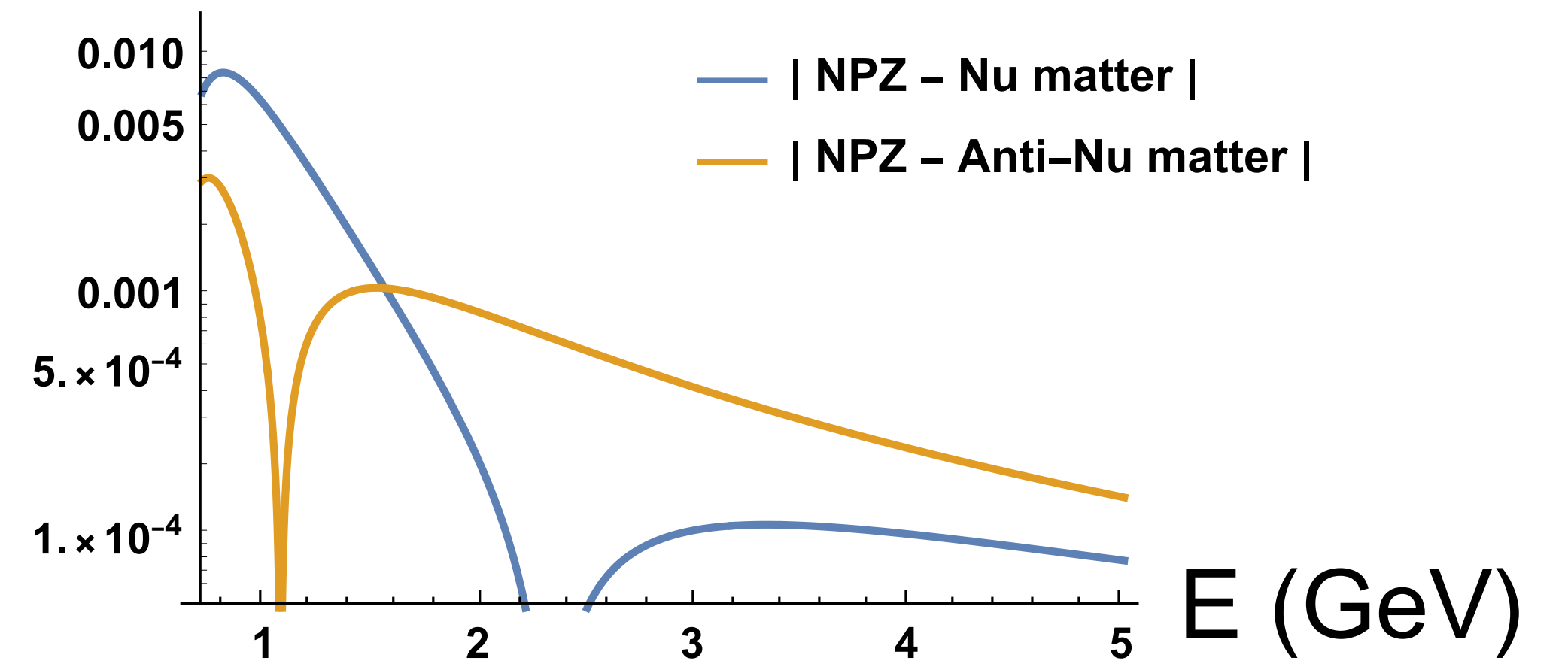


$$P_{\text{NPZ}}(\nu_\mu \rightarrow \nu_\mu) \approx 1 - 4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2) \sin^2 \left(\frac{\Delta m_{\mu\mu}^2 L}{4E} \right)$$

Dis. Prob (NOvA)

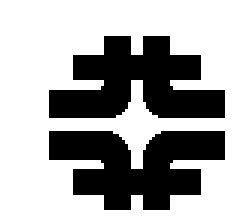


Delta Prob (NOvA)

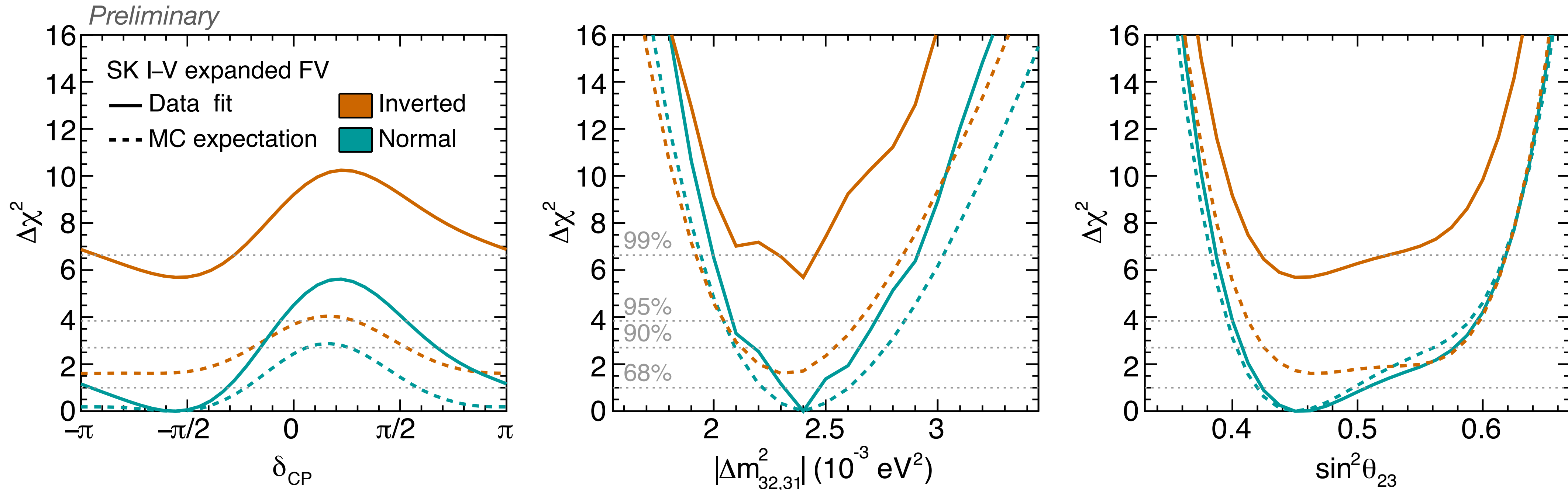


For Disappearance channel, only $|U_{\mu 3}|^2$ and $|\Delta m_{\mu\mu}^2|$ are measurable around first oscillation minimum.

To extract Δm_{32}^2 , for 1% precision one needs $\mp \sin^2 \theta_{12} \Delta m_{21}^2$
and for 0.5% level also $\mp \sin \theta_{13} \cos \delta \Delta m_{21}^2$



SK I-V Atmospheric Oscillation Results



SK 2023 best fit: **Normal ordering**, $\delta_{CP} \sim -\pi/2$, $\Delta m^2_{32} \sim 2.4 \times 10^{-3} \text{ eV}^2$, $\sin^2\theta_{23} \sim 0.45$

Mass ordering: $\Delta\chi^2_{\text{I.O.} - \text{N.O.}} \sim 5.7^*$

With reactor constraint: $\sin^2\theta_{13} = 0.0220 \pm 0.0007$