

Muon $g-2$ Theory

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University of Illinois

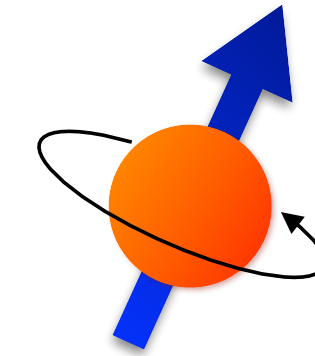
NuFact 2024

Lemont, Illinois, United States
September 16th - 21st, 2024



The 25th international workshop on Neutrinos from Accelerators

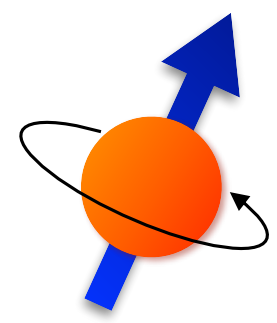
Outline



- Introduction
 - Muon g-2 Theory Initiative
 - data-driven HVP
 - lattice HVP
 - HLbL
 - Summary and Outlook
 - Appendix
- “The anomalous magnetic moment of the muon in the SM”:
1st White Paper published in 2020 (132 authors, 82 institutions)
[T. Aoyama et al, [arXiv:2006.04822](https://arxiv.org/abs/2006.04822), Phys. Repts. 887 (2020) 1-166.]
 - “Prospects for precise predictions of a_μ in the SM”:
2022 Snowmass Summer Study, [arXiv:2203.15810](https://arxiv.org/abs/2203.15810)
 - Summary statement on the status of Muon g-2 Theory
in the SM: <https://muon-gm2-theory.illinois.edu>

➡ [Lattice 2024 conference](#)

➡ [Muon g-2 TI workshop @ KEK \(9-13 Sep 2024\)](#)




Anomalous magnetic moment

The magnetic moment of charged leptons (e, μ, τ): $\vec{\mu} = g \frac{e}{2m} \vec{S}$

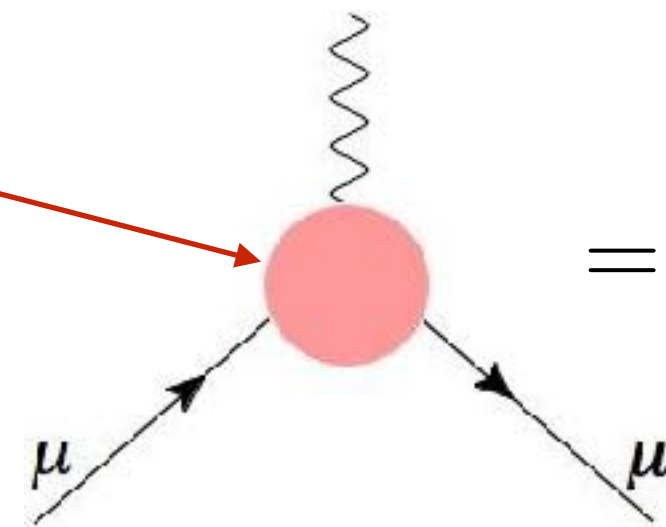
Dirac (leading order): $g = 2$

$$= (-ie) \bar{u}(p') \gamma^\mu u(p)$$

Quantum effects (loops):

$$\implies g = 2 \left(1 + \frac{\alpha}{2\pi} \right)$$


All SM particles contribute



$$= (-ie) \bar{u}(p') \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2) \right] u(p)$$

Note: $F_1(0) = 1$ and $g = 2 + 2 F_2(0)$

Anomalous magnetic moment:

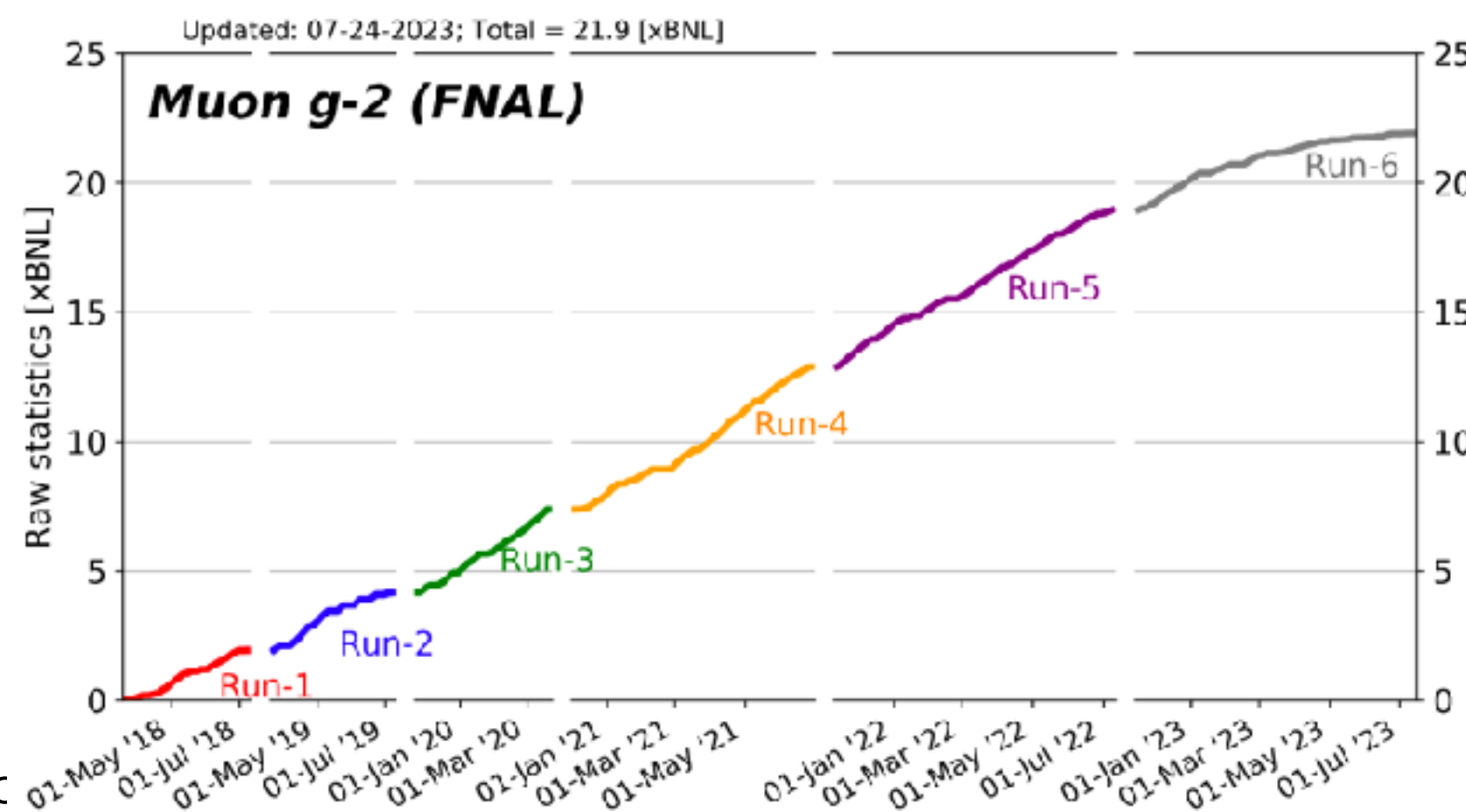
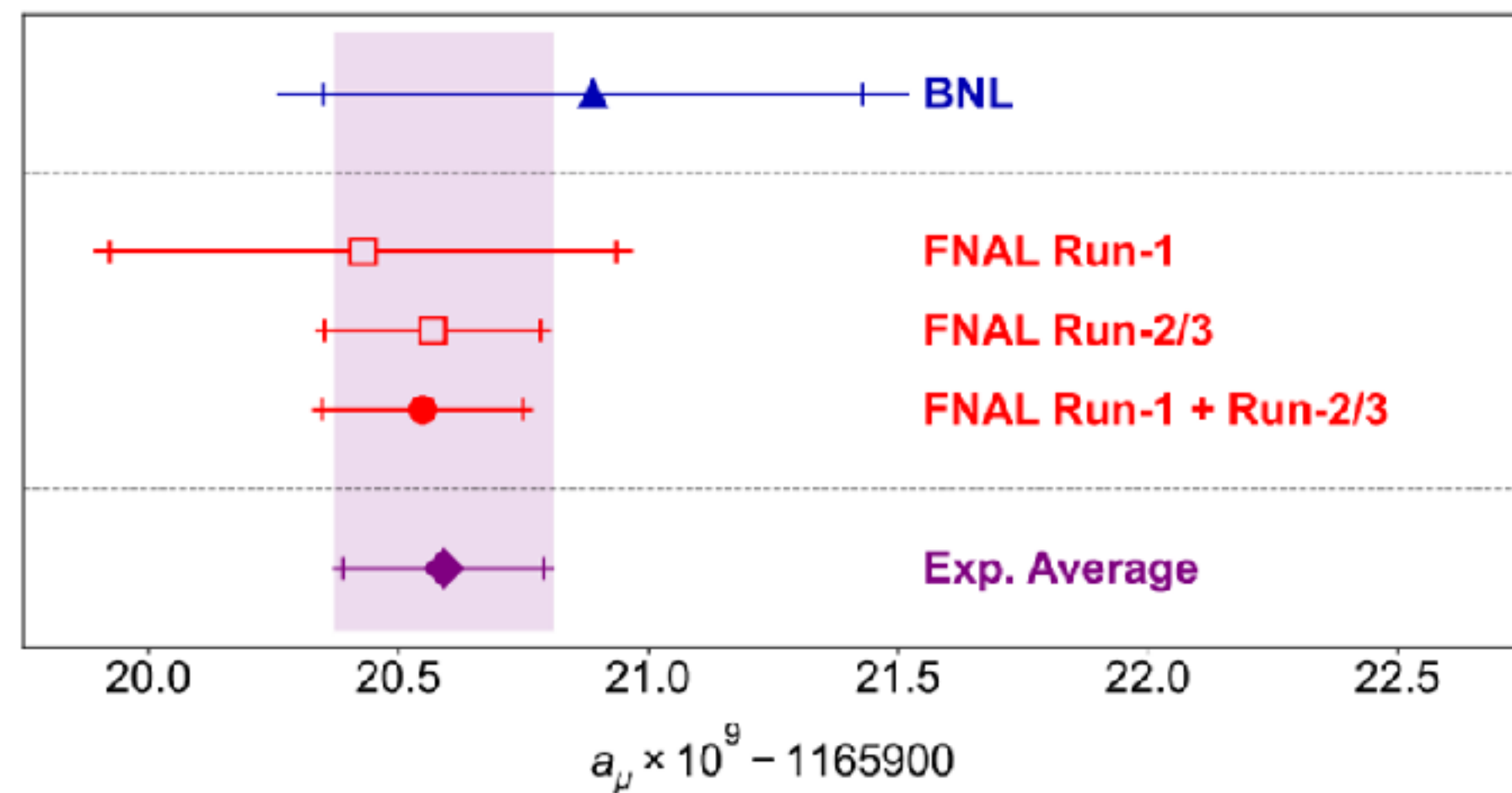
$$a \equiv \frac{g - 2}{2} = F_2(0) = \frac{\alpha}{2\pi} + O(\alpha^2) + \dots = 0.00116\dots$$

Muon g-2 experiment

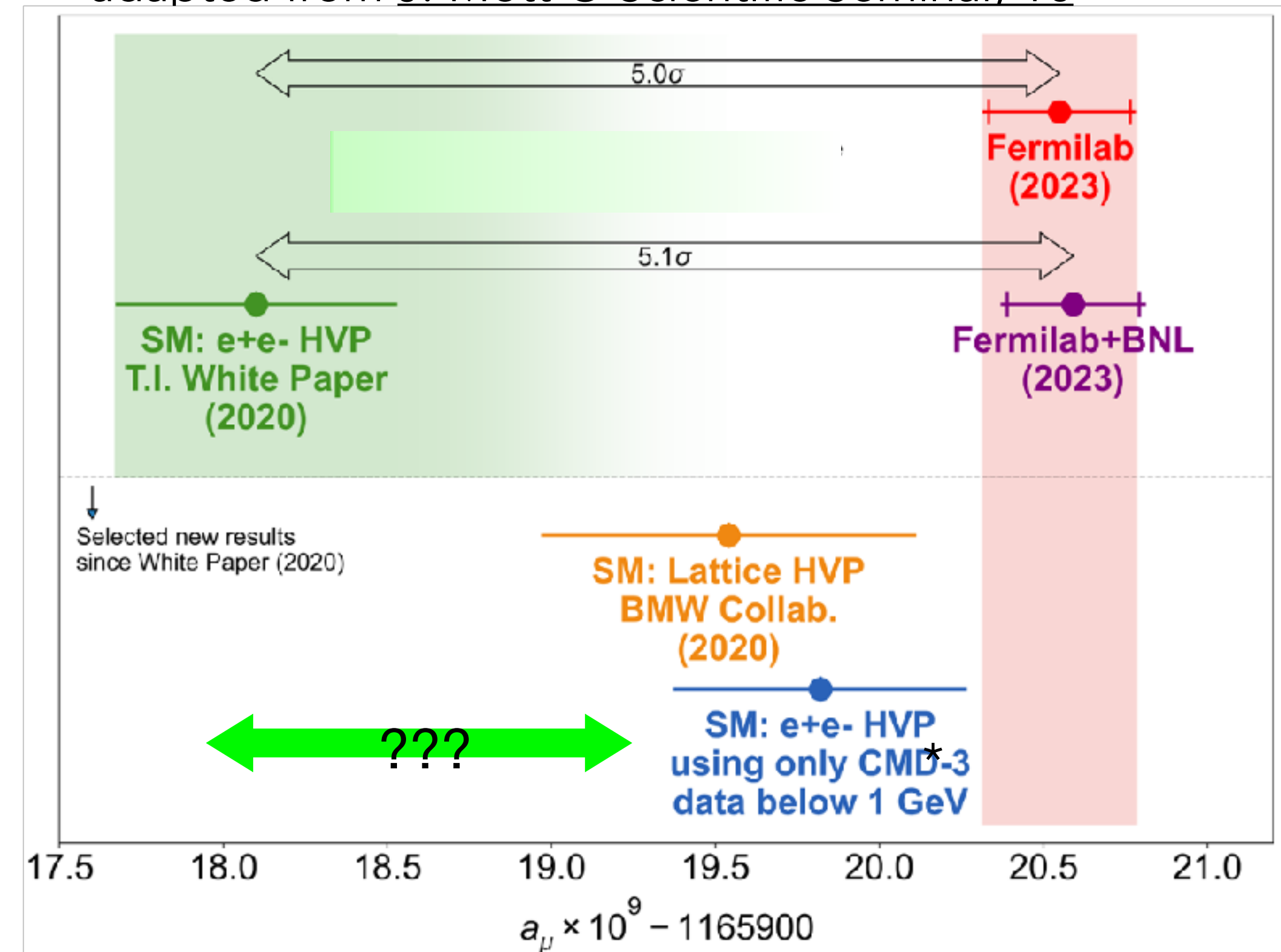
- The Fermilab experiment released the measurement result from their run 2&3 data on 10 Aug 2023. [D. Aguillard et al, 2308.06230]
- Run 6 completed summer 2023.
- Release of final measurement result expected in 2025



next talk by Sophie Middleton



adapted from J. Mott @ Scientific Seminar, 10



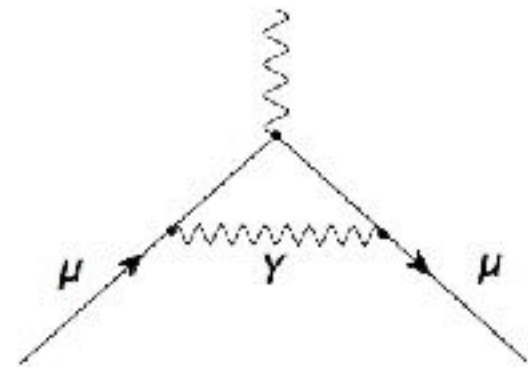
Muon g-2: SM contributions

$$a_{\mu} = a_{\mu}(\text{QED}) + a_{\mu}(\text{EW}) + a_{\mu}(\text{hadronic})$$

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$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

QED

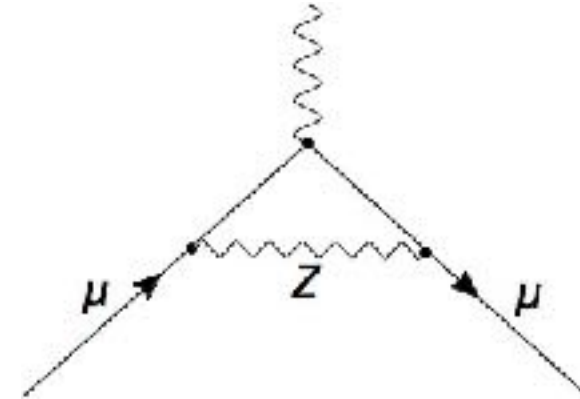


+... (5 loops)

$$116\,584\,718.9(1) \times 10^{-11}$$

0.001 ppm

EW

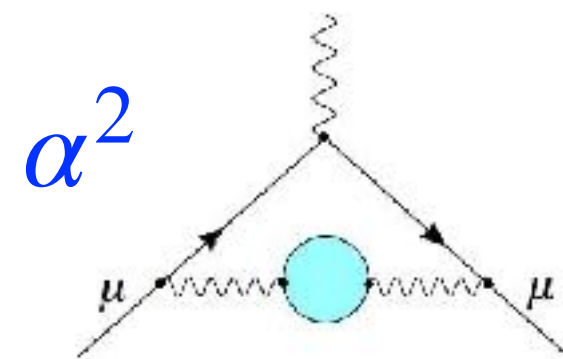


+... (2 loops)

$$153.6(1.0) \times 10^{-11}$$

0.01 ppm

HVP



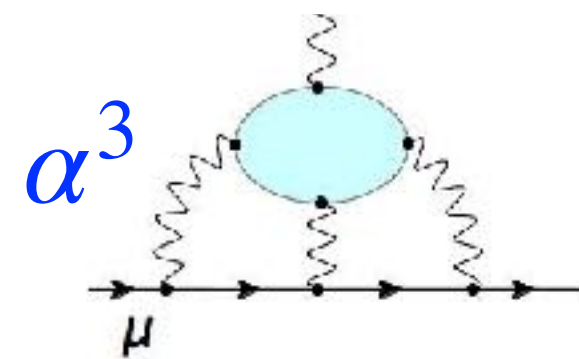
+... (NNLO)

$$6845(40) \times 10^{-11}$$

[0.6%]

0.34 ppm

HLbL



+... (NLO)

$$92(18) \times 10^{-11}$$


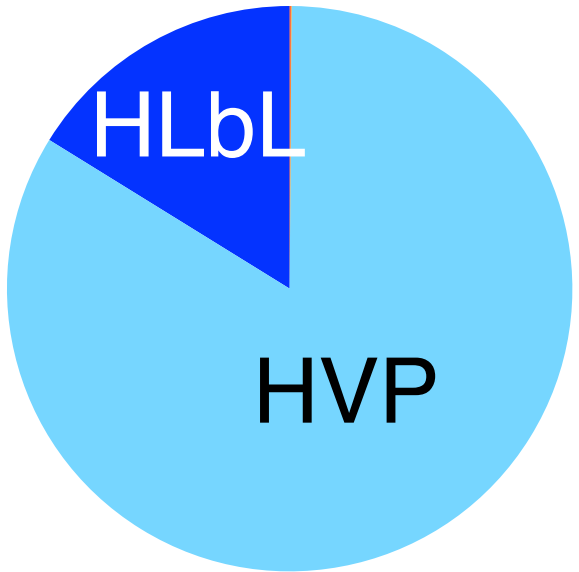
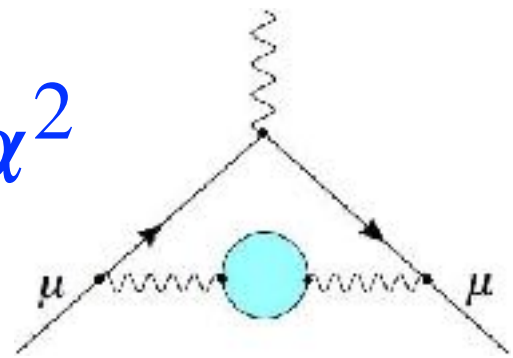
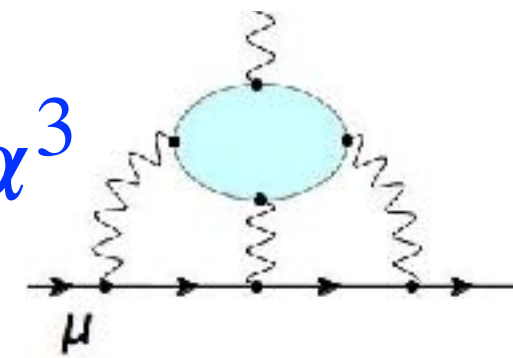
[20%]

0.15 ppm

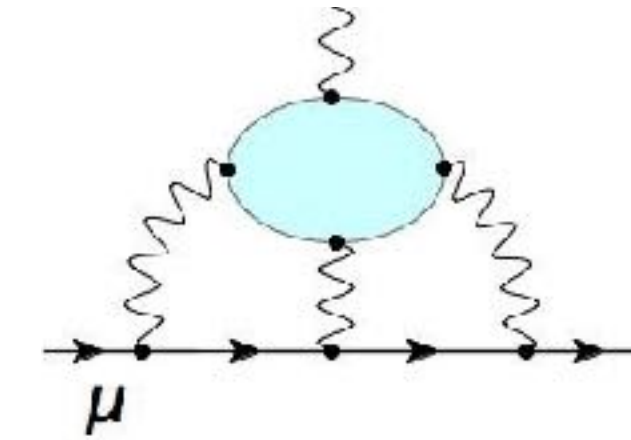
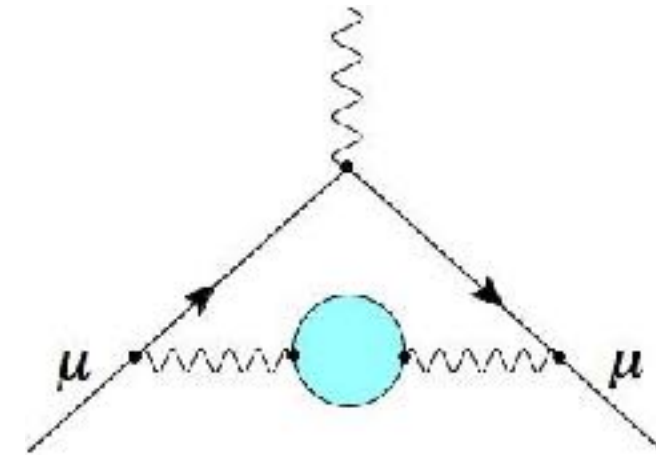
Hadronic corrections

Muon $g-2$: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

contribution	error ²		
<ul style="list-style-type: none"> ● QED ● EW ● HVP ● HLbL 		$116\,584\,718.9(1) \times 10^{-11}$	0.001 ppm
		$153.6(1.0) \times 10^{-11}$	0.01 ppm
Hadronic corrections			
HVP 	α^2 + ... (NNLO)	$6845(40) \times 10^{-11}$ [0.6%]	0.34 ppm
HLbL 	α^3 + ... (NLO)	$92(18) \times 10^{-11}$ [20%]	0.15 ppm

Muon g-2: hadronic corrections



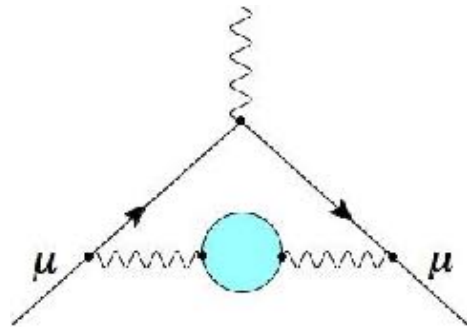
- ★ Hadronic contributions are obtained by integrating over all possible virtual photon momenta, integral is weighted towards low q^2 .
- ★ Cannot use perturbation theory to reliably compute the hadronic bubbles
- ★ Two-point & four-point functions:

$$\text{HVP: } \langle 0 | T \{ j_\mu j_\nu \} | 0 \rangle$$

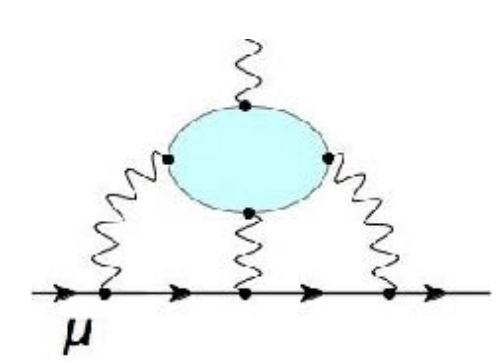
$$\text{HLbL: } \langle 0 | T \{ j_\mu j_\nu j_\rho j_\sigma \} | 0 \rangle$$

Two independent approaches

1. Dispersive, data-driven
2. Lattice QCD



Hadronic Corrections



- Dispersive, data-driven:

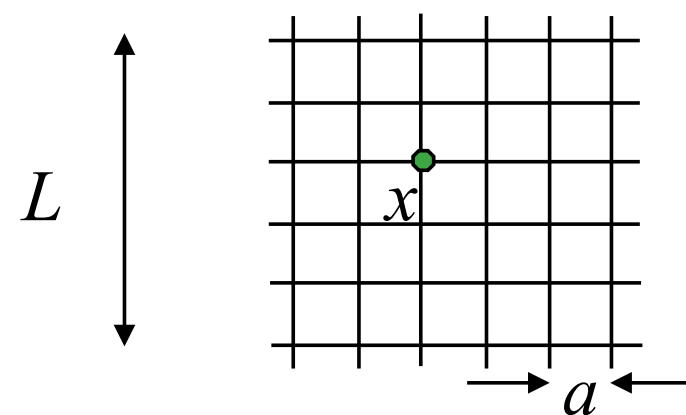
HVP: integrate hadronic cross section over CM energy:

$$\text{Im}[\text{wavy line} \text{---} \text{blue circle} \text{---} \text{wavy line}] \sim |\text{wavy line} \text{---} \text{blue arc} \text{---} \text{wavy line}|^2 \implies a_\mu^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2) \hat{\Pi}(q^2) = \frac{m_\mu^2}{12\pi^3} \int ds \frac{\hat{K}(s)}{s} \sigma_{\text{exp}}(s)$$

Many experiments (over 20+ years) have measured the e^+e^- cross sections for (almost) all channels over the needed energy range with increasing precision.

For HLbL: **new dispersive approach**

- Direct calculation using Euclidean Lattice QCD



Approximations:

discrete space-time (spacing a)

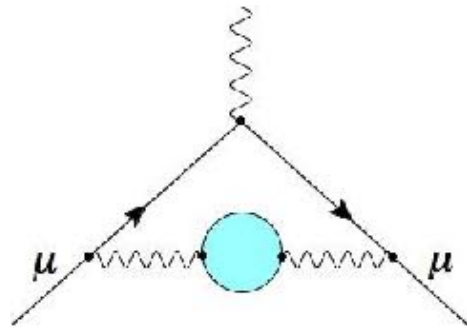
finite spatial volume (L), and time extent (T)

...

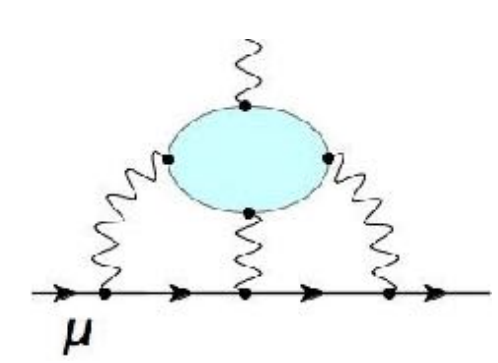
$$a_\mu^{\text{HVP,LO}} = 4\alpha^2 \int_0^\infty dt C(t) \tilde{w}(t)$$

- ab-initio* method to quantify QCD effects
- already used for simple hadronic quantities with high precision
- requires large-scale computational resources
- allows for entirely SM theory based evaluations**

Integrals are evaluated numerically using Monte Carlo methods.



Hadronic Corrections



- Dispersive, data-driven:

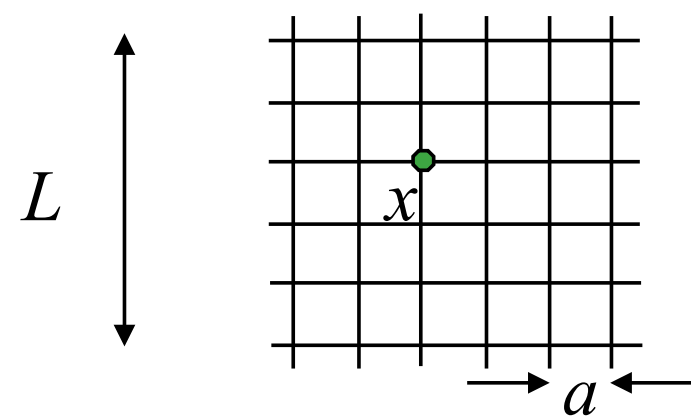
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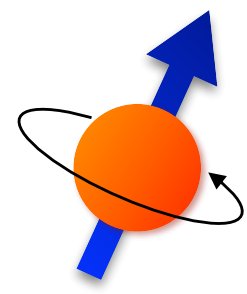
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...

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- already used for simple hadronic quantities with high precision
- requires large-scale computational resources
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Integrals are evaluated numerically using Monte Carlo methods.



Muon $g-2$ Theory Initiative

Steering Committee

- Gilberto Colangelo (Bern)
- Michel Davier (Orsay) co-chair
- Aida El-Khadra (UIUC & Fermilab) chair
- Martin Hoferichter (Bern)
- Christoph Lehner (Regensburg University) co-chair
- Laurent Lellouch (Marseille)
- Tsutomu Mibe (KEK)
J-PARC Muon $g-2$ /EDM experiment
- Lee Roberts (Boston)
Fermilab Muon $g-2$ experiment
- Thomas Teubner (Liverpool)
- Hartmut Wittig (Mainz)

<https://muon-gm2-theory.illinois.edu>

- Maximize the impact of the Fermilab and J-PARC experiments
 - **quantify and reduce the theoretical uncertainties on the SM prediction**
- assess reliability of uncertainty estimates
- summarize the theory status: White Papers
- organize workshops to bring the different communities together:
 - [First plenary workshop near Fermilab: 3-6 June 2017](#)
 - ...
 - [Virtual Spring 2024 TI workshop hosted by UIUC: 15-17, 23-24 Apr 2024](#)
 - [Seventh plenary workshop hosted by KEK/KMI \(Japan\): 9-13 Sep 2024](#)
 - Eight plenary workshop: Orsay (France), 8-12 Sep 2025
 - Ninth and tenth plenary workshops: US, UK

Near-term timeline

FNAL E989

J-PARC E34

Run 4

Run 5

Run 6



Run 1 result announced

Result from Runs 2&3

Final result from E989

Muon g-2 TI WP published

2nd WP

Theory Initiative:

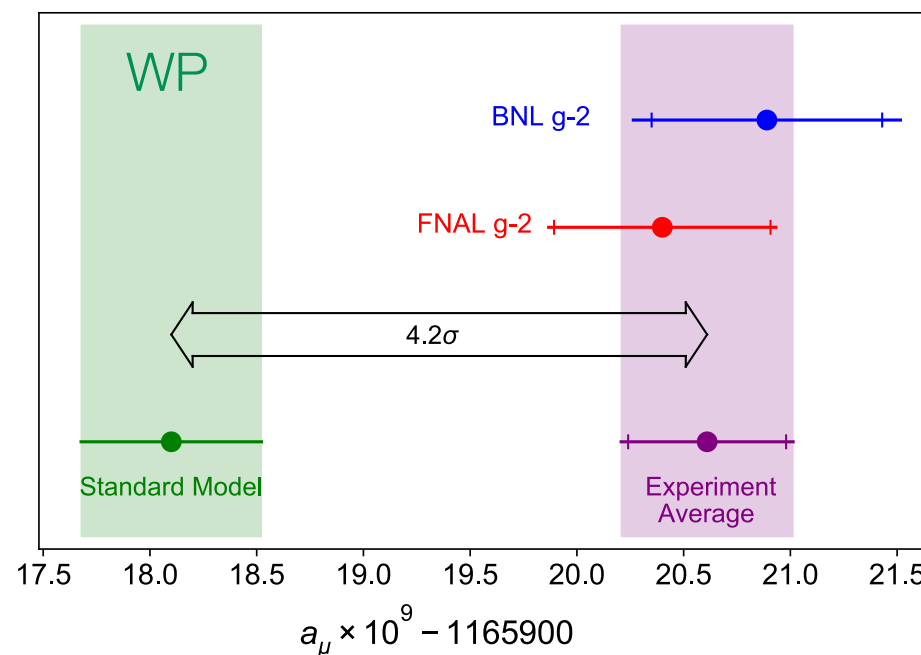
- ☆ CMD-3 seminar (virtual): 27 March 2023 at 8:00am US CDT
- ☆ 2nd CMD-3 discussion meeting
- ☆ 8/9/2023: Status of Muon g-2 Theory in SM

★ TI workshops:

Jun 2021 @ KEK (virtual)
Sep 2022 @ Higgscentre

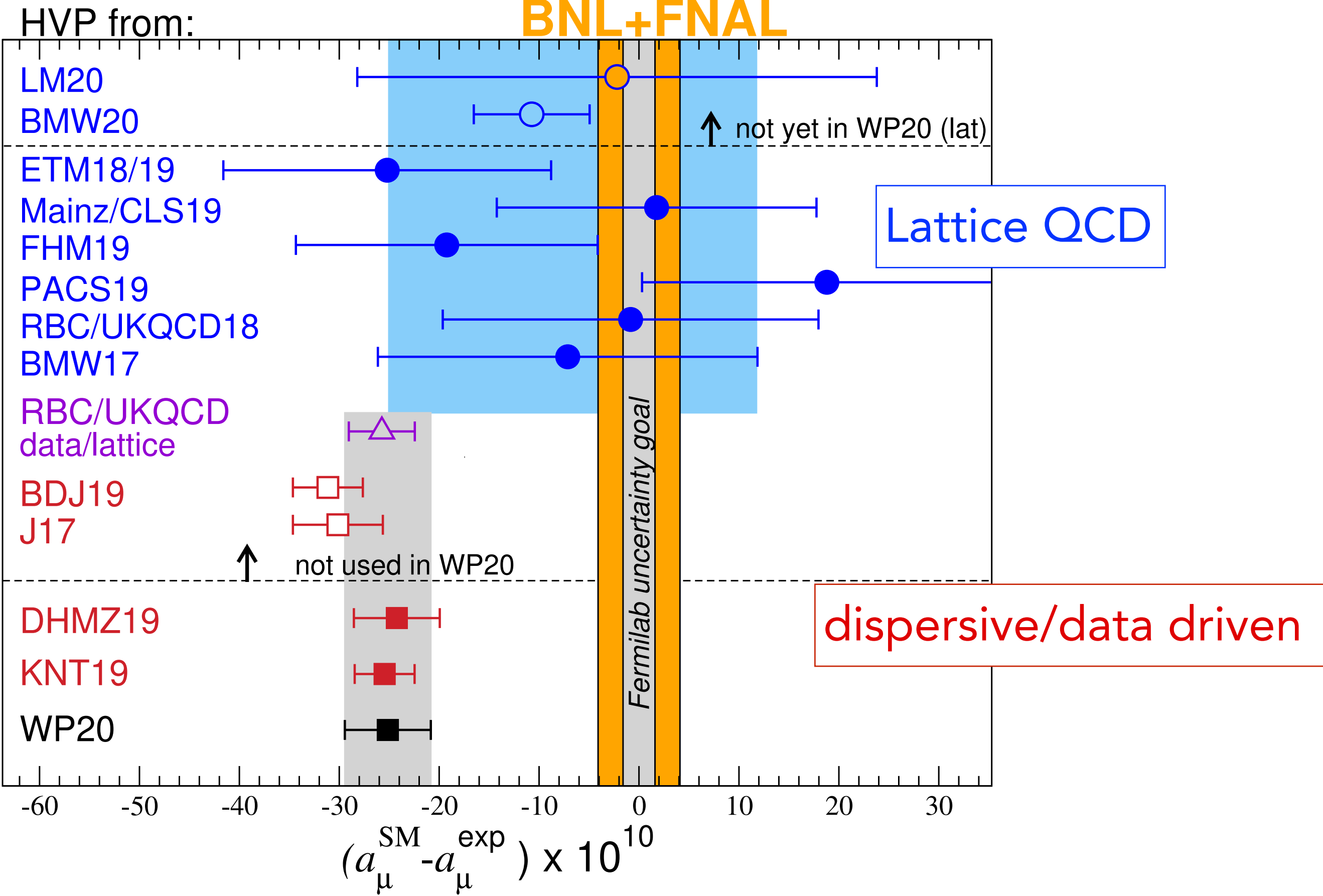
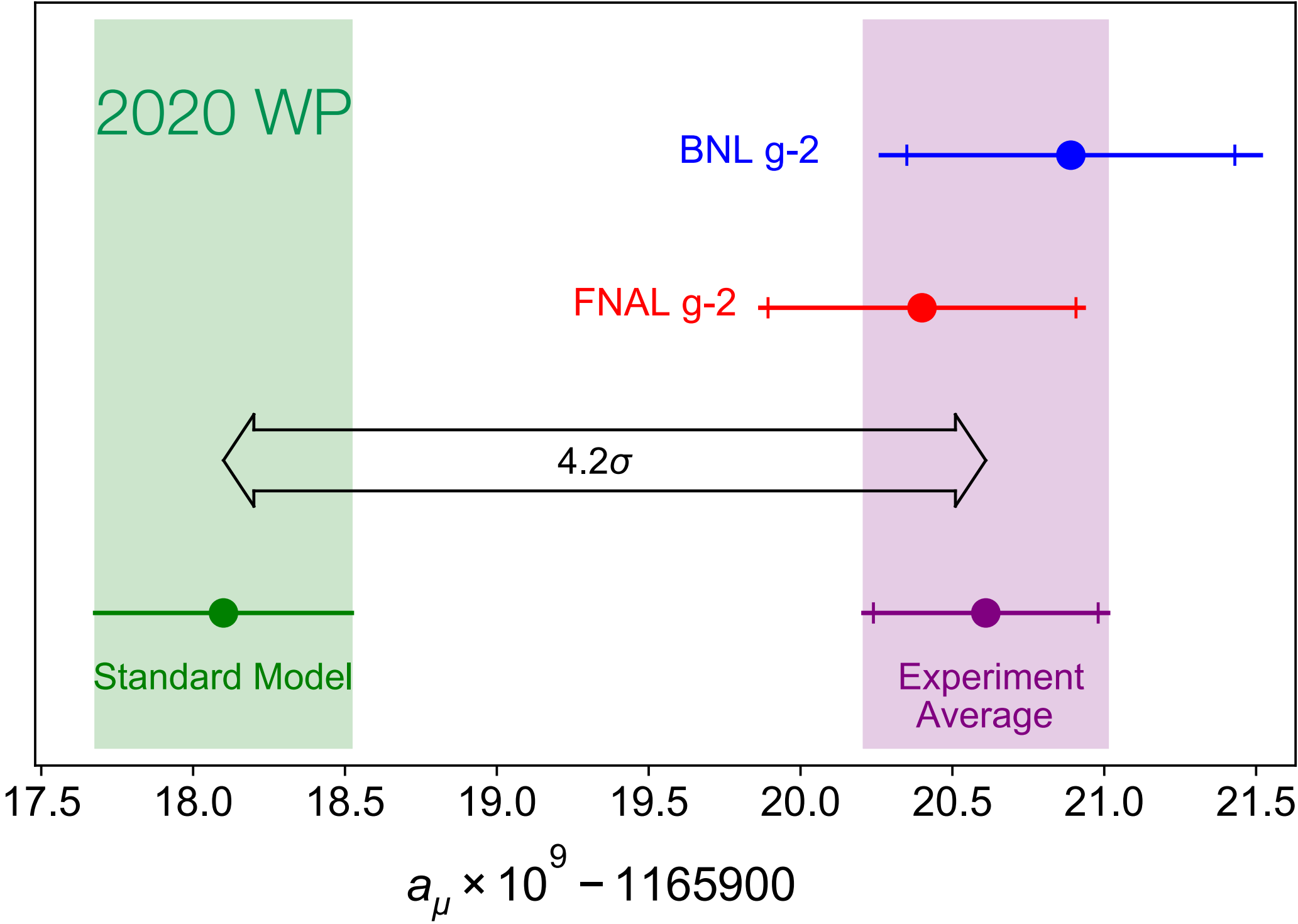
Sep 2023 @ Bern
Apr 2024 (virtual)

Sep 2024 @ KEK & KMI



Experiment vs SM theory

2021

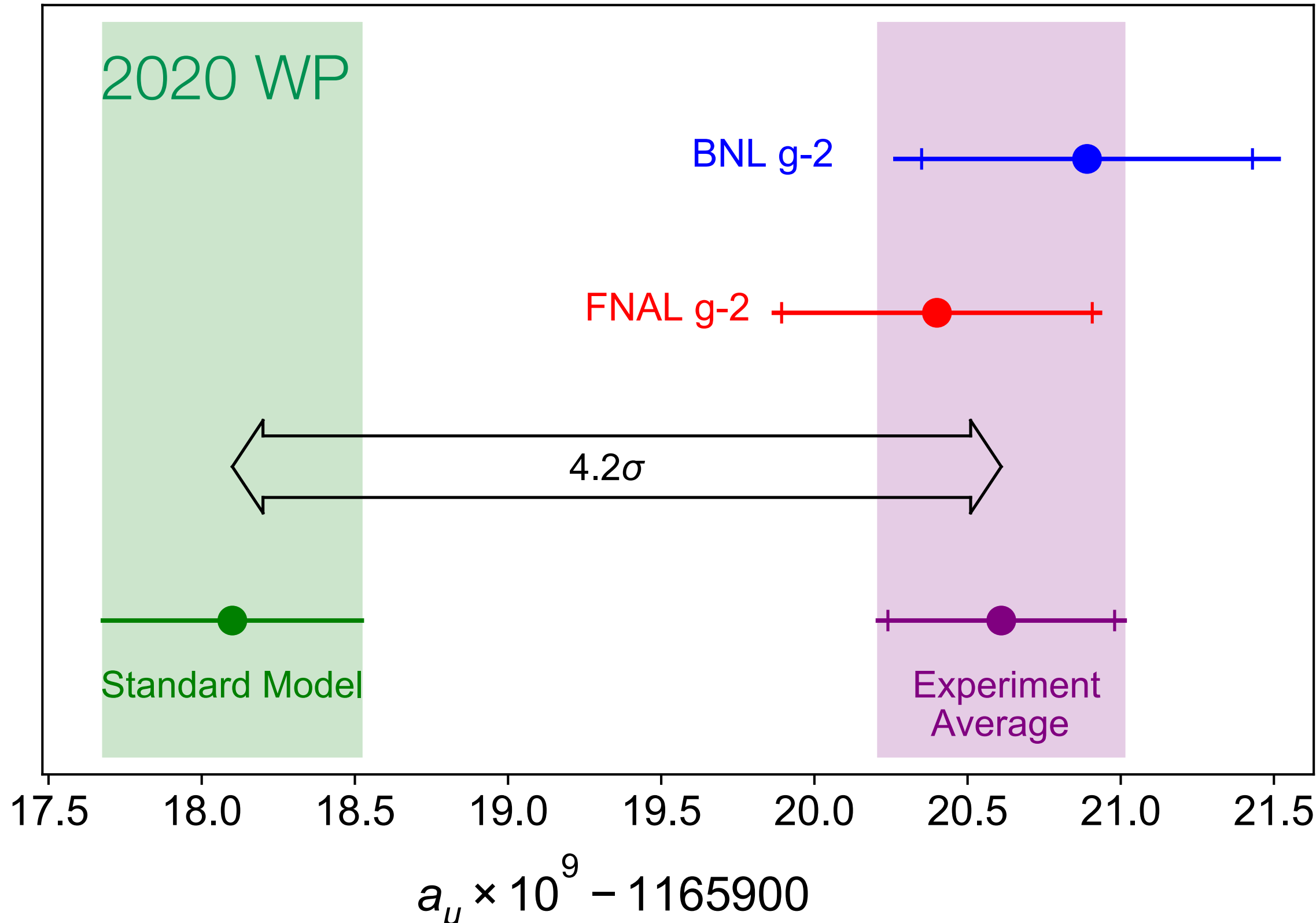


$$a_\mu^{\text{SM}} = a_\mu^{\text{HVP}} + [a_\mu^{\text{QED}} + a_\mu^{\text{Weak}} + a_\mu^{\text{HLbL}}]$$

Experiment vs SM theory

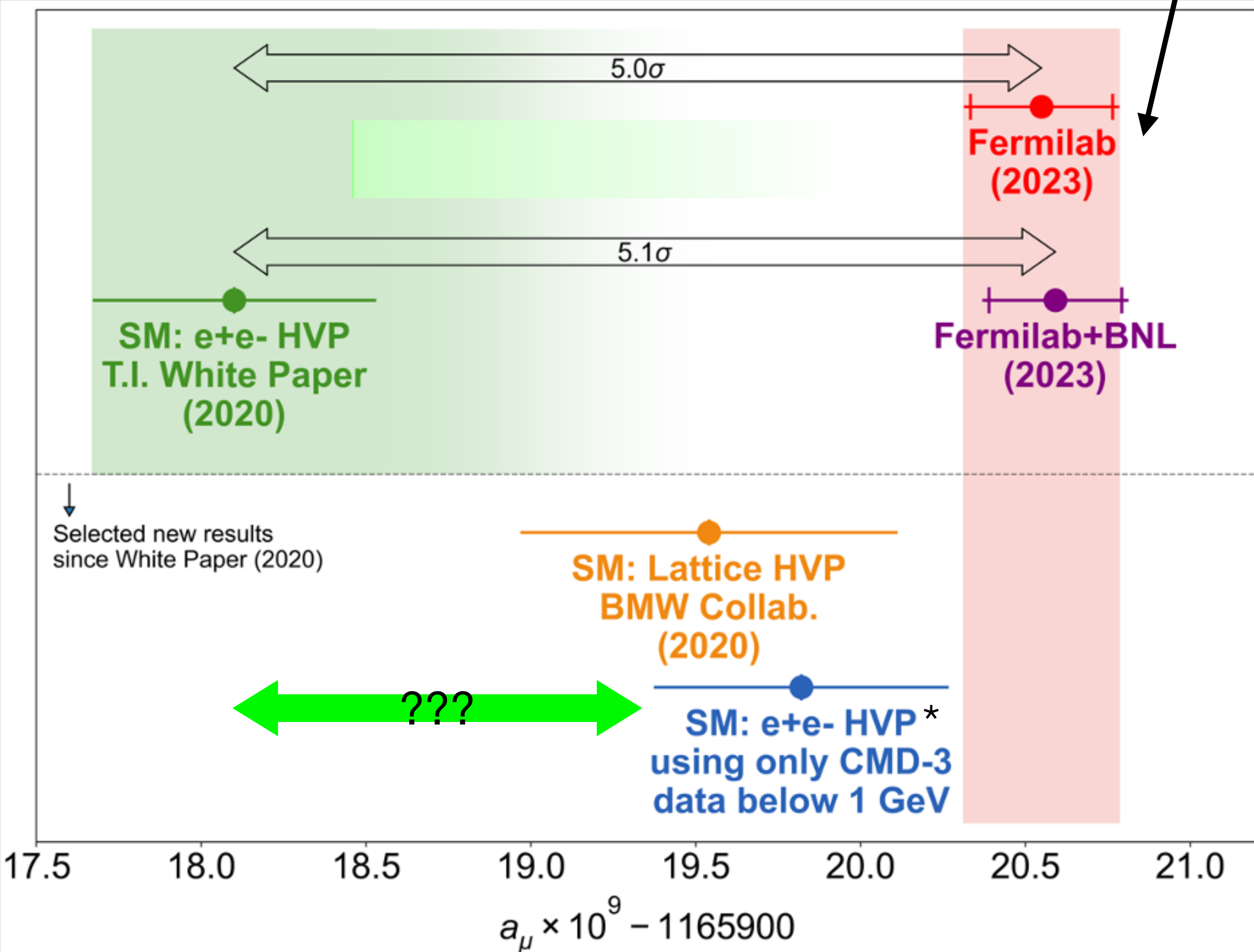
Sophie Middleton

2021



2023

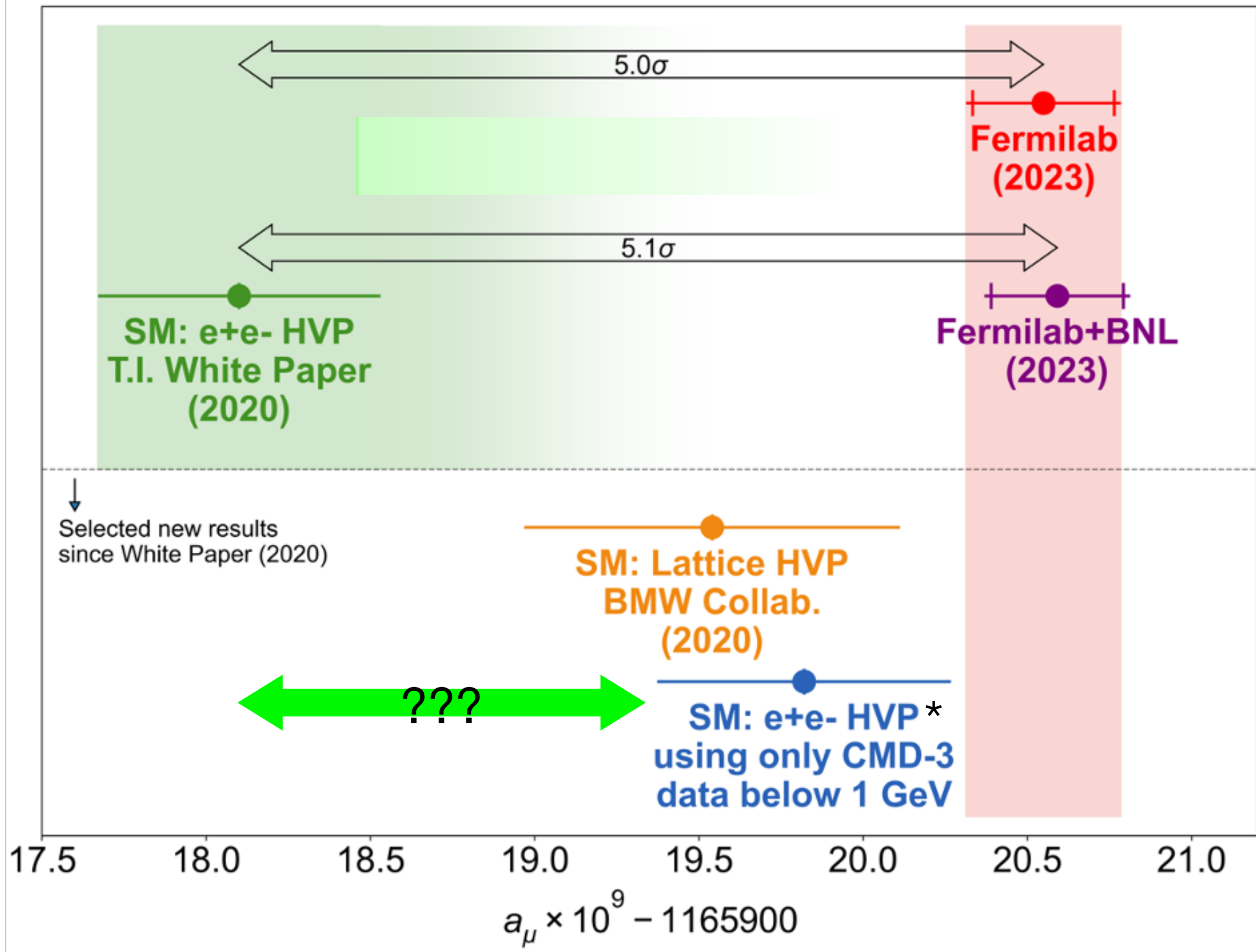
adapted from J. Mott @ Scientific Seminar, 10 Aug 2023



Experiment vs SM theory

2023

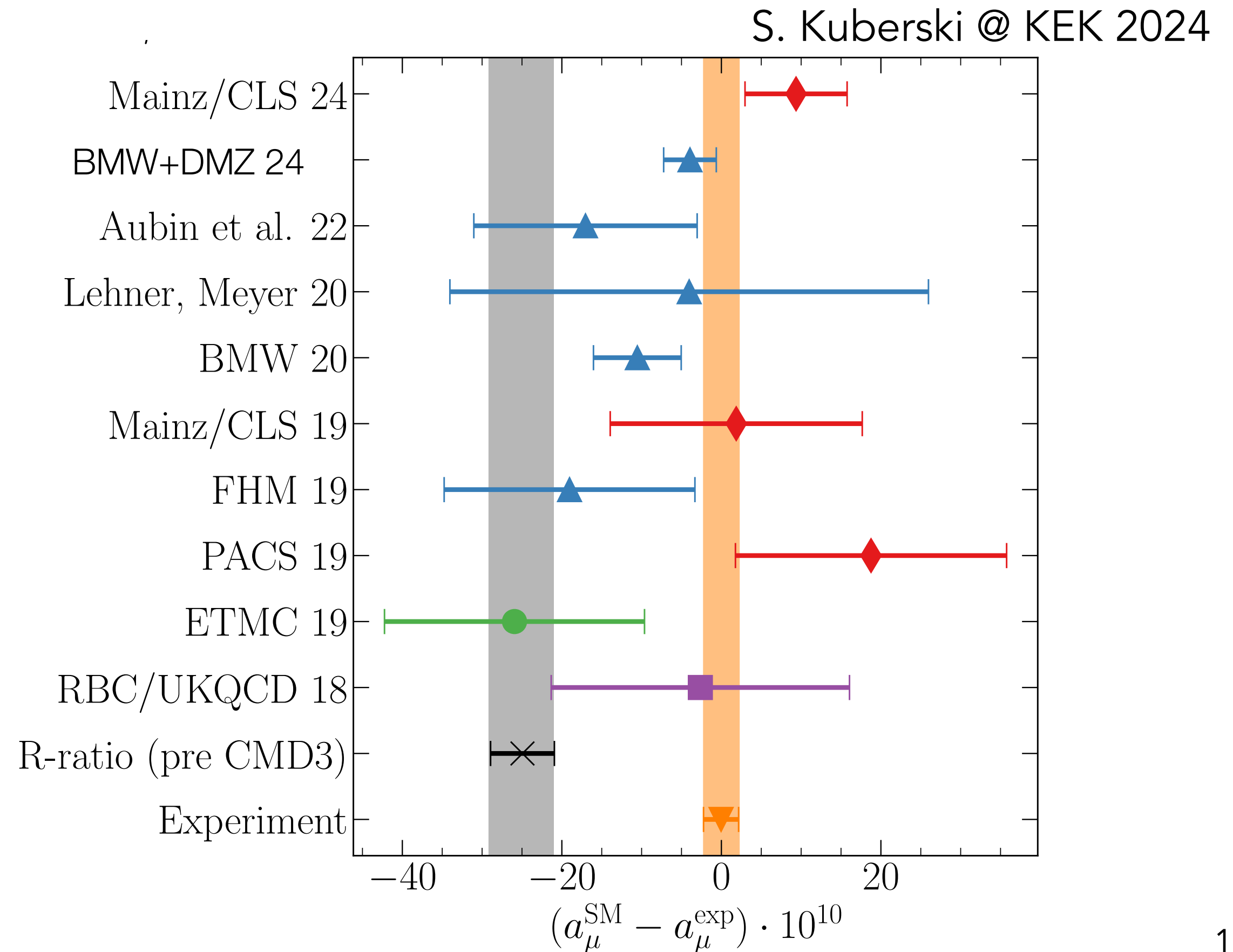
adapted from J. Mott @ Scientific Seminar, 10 Aug



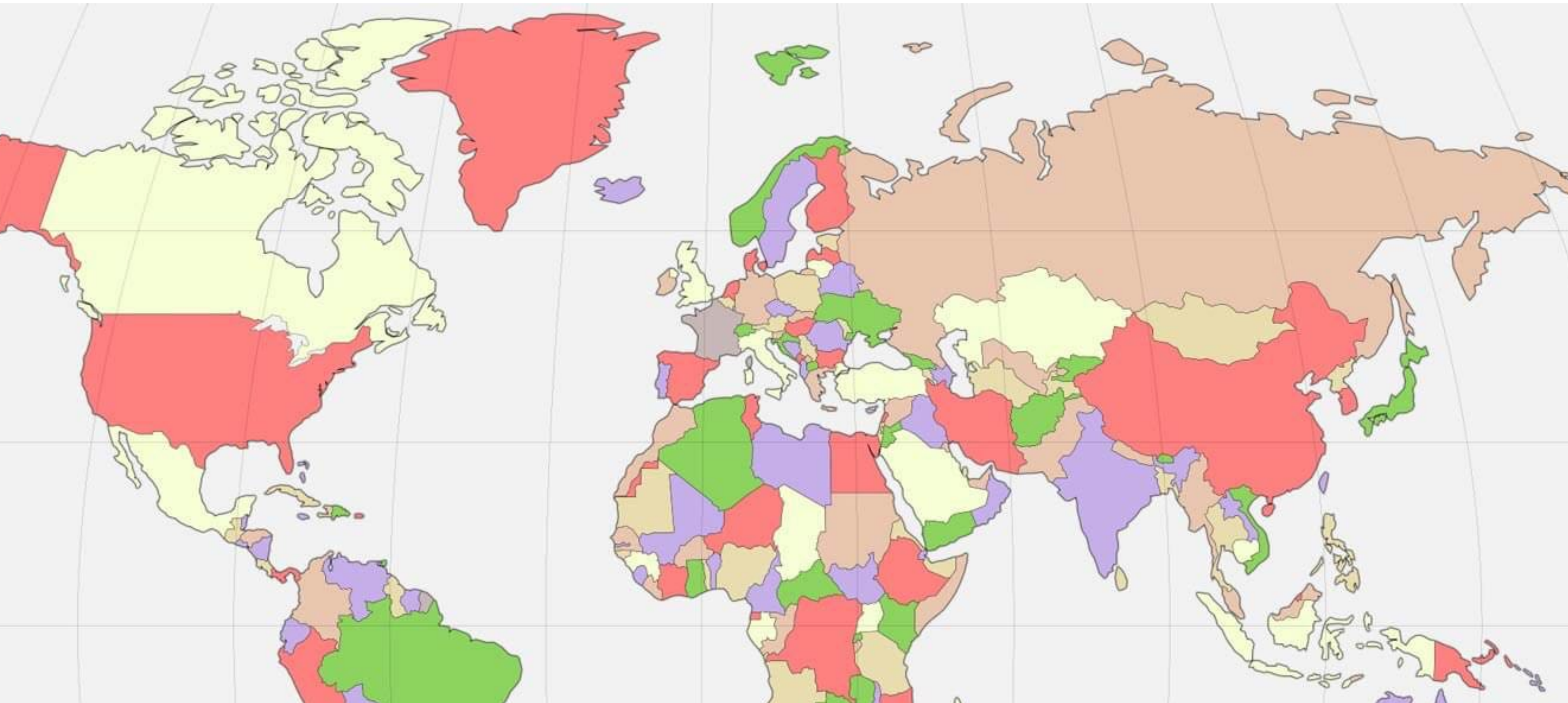
*A. Keshavarzi @ Lattice 2023

2024

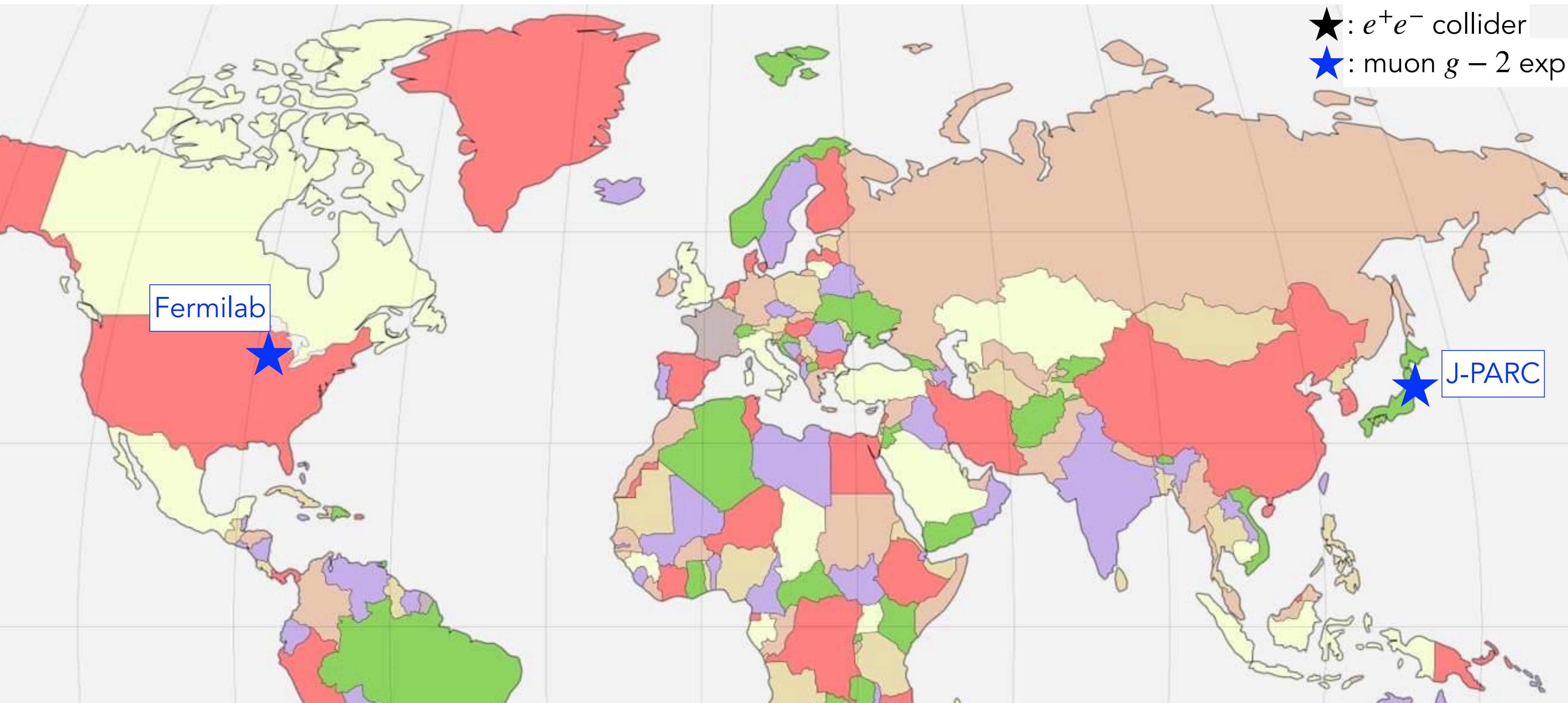
New LQCD results (all using blind analyses):
 BMW+DMZ 24 [arXiv:2407.10913]: **LQCD+R-ratio (hybrid)**
 RBC/UKQCD: Lehner@Lattice 2024
 Mainz: Kuberski @ KEK 2024
 FNAL/HPQCD/MILC: exp. fall 2024



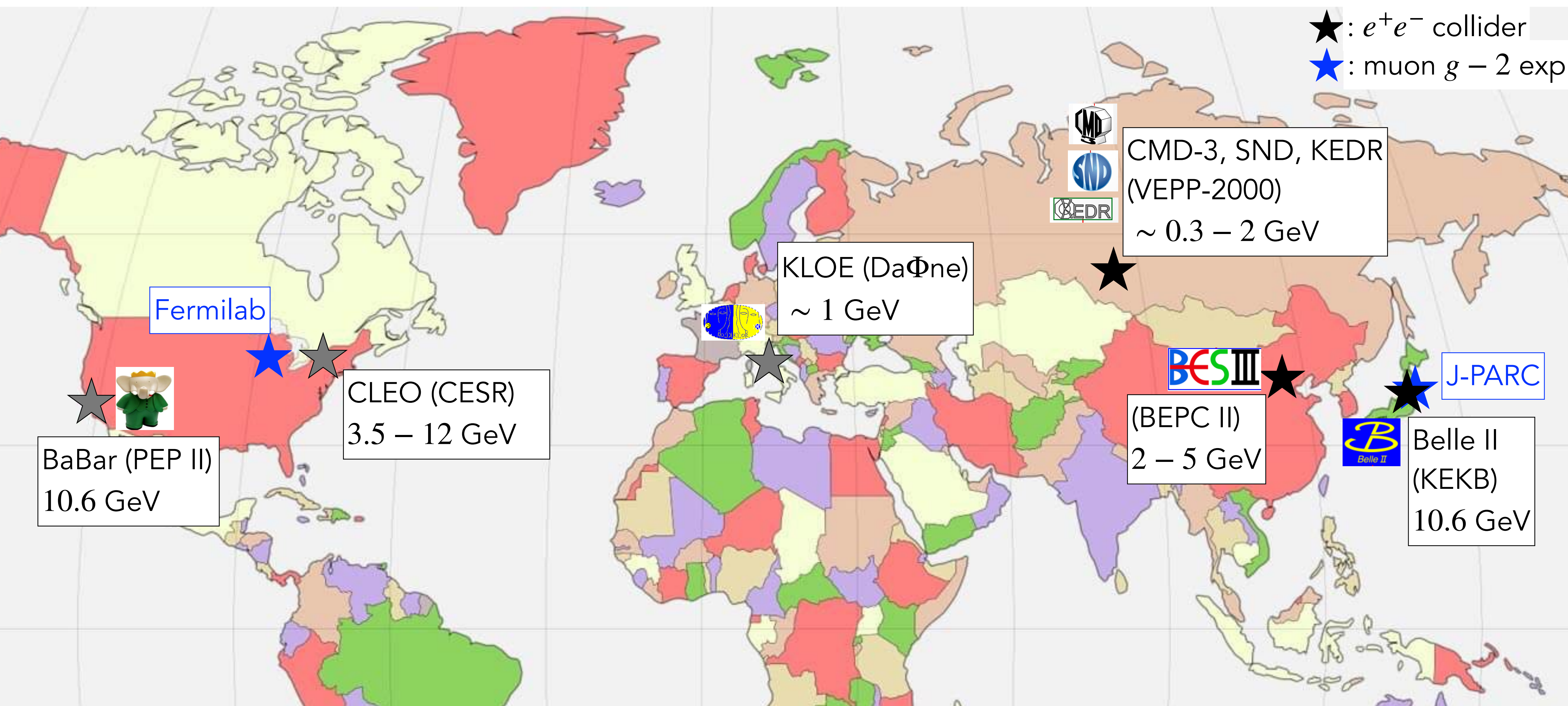
Overview of the experiments

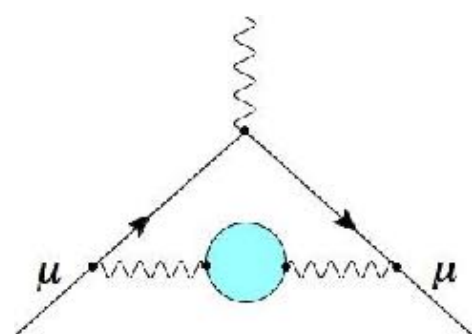


Overview of the experiments



Overview of the experiments

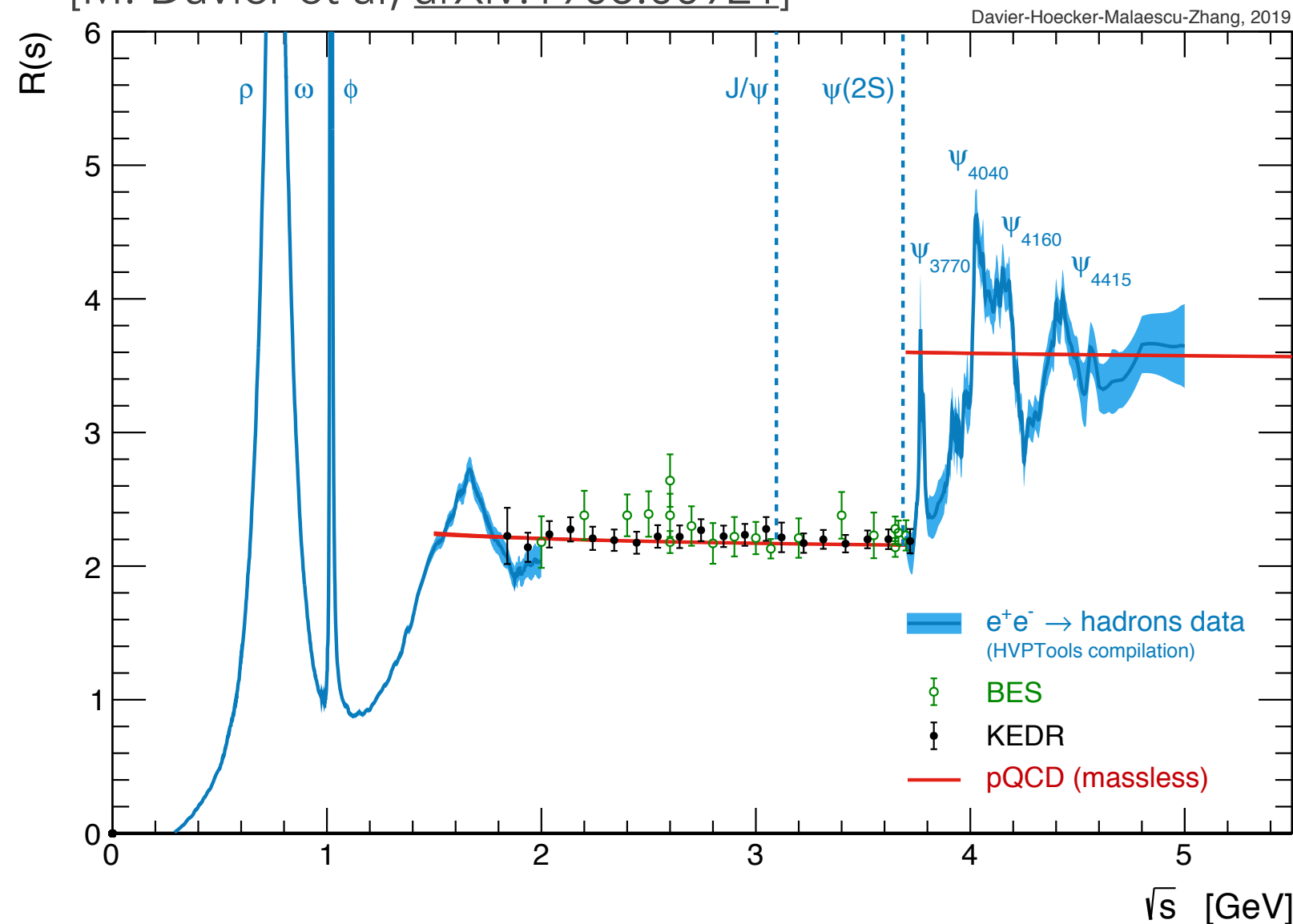




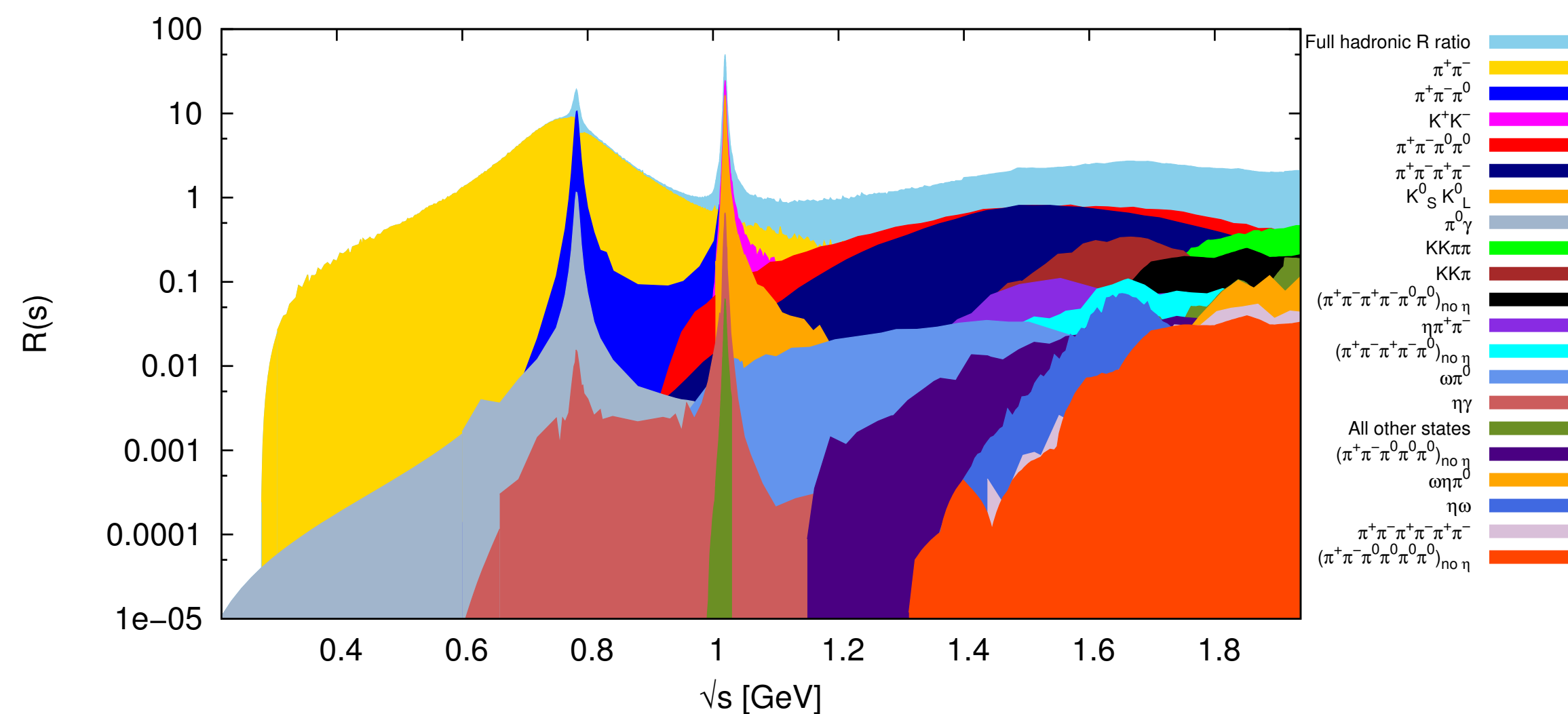
HVP: data-driven

$$a_{\mu}^{\text{HVP,LO}} = \frac{m_{\mu}^2}{12\pi^3} \int ds \frac{\hat{K}(s)}{s} \sigma_{\text{exp}}(s)$$

[M. Davier et al, arXiv:1908.00921]



[A. Keshavarzi et al, arXiv:1802.02995]

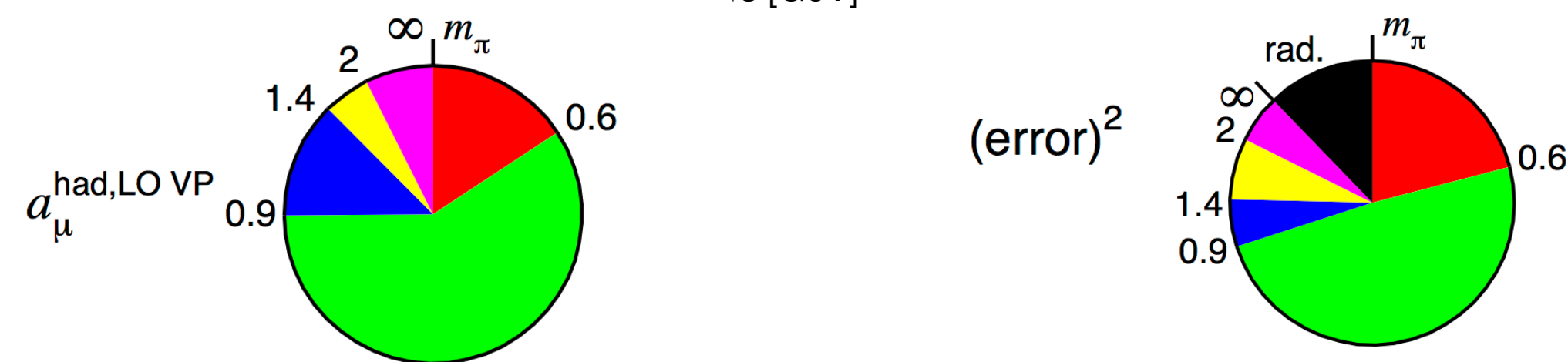


$\sigma_{\text{had}}(s)$ defined to include real & virtual photons
direct integration method: no modelling of $\sigma_{\text{had}}(s)$,
 summing up contributions from all hadronic channels

total hadronic cross section $\sigma_{\text{had}}(s)$ from > 100 data sets in
 35+ channels summed up to $\sqrt{s} \sim 2 \text{ GeV}$

$\sqrt{s} > 2 \text{ GeV}$: inclusive data + pQCD + narrow resonances

two independent compilations (DHMZ, KNT)



Tensions (of up to 3σ) between data sets:
 conservative procedure to include differences in error estimate

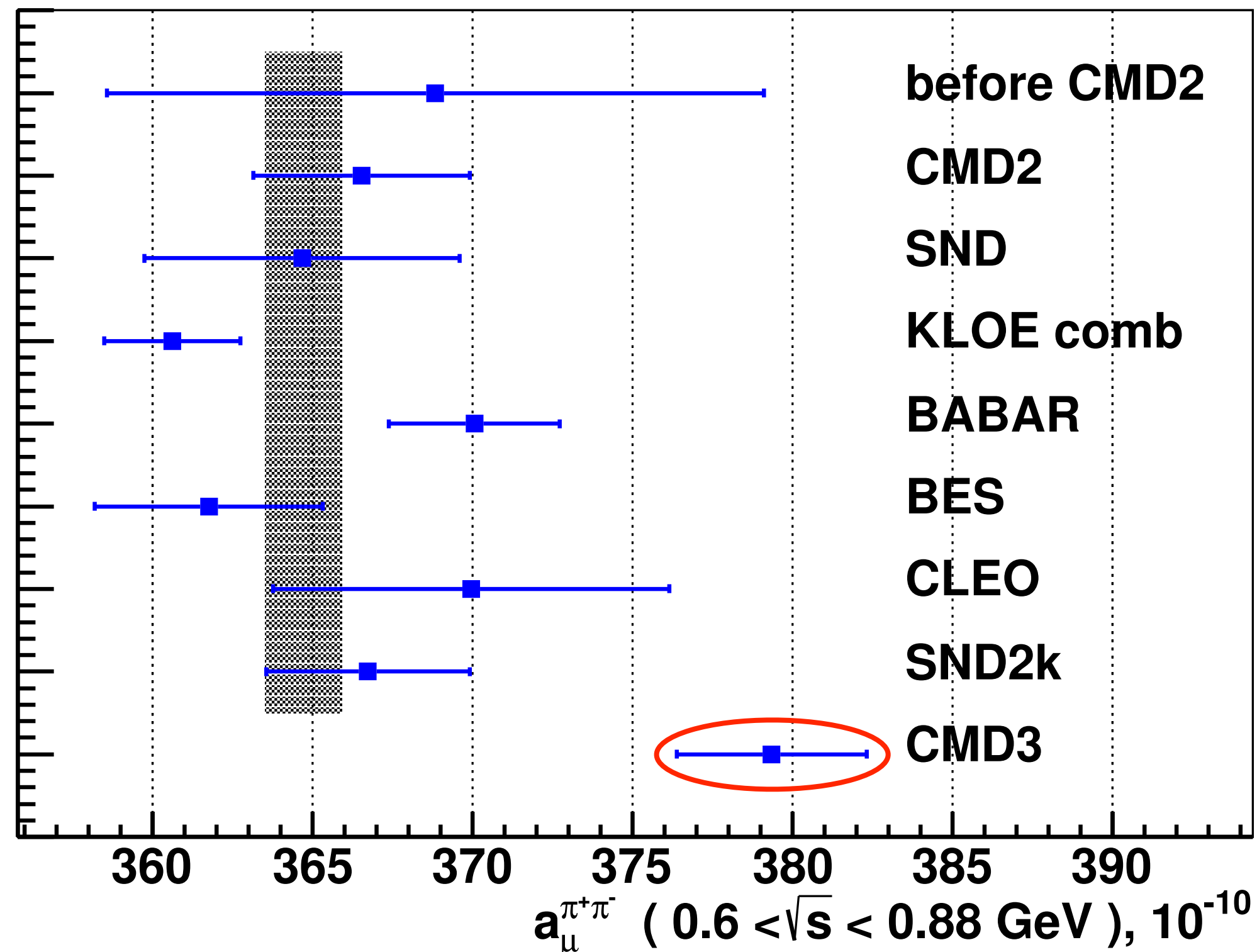
HVP: data-driven

- Since 2020:**
- 📌 20+ papers with new experimental measurements for $\sigma_{\text{had}}(s)$, for example:
 $\pi\pi, \pi\pi\pi, K\bar{K}\pi, \eta\pi\gamma, 4\pi, \pi\pi\pi\eta, \dots$
 - 📌 all ~ consistent with previous results

HVP: data-driven

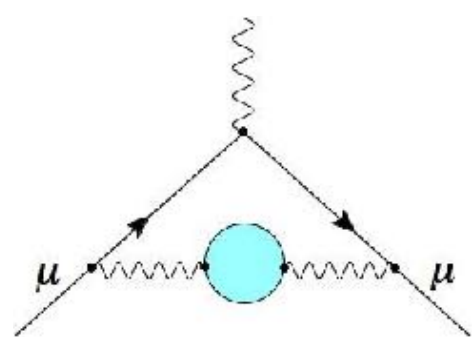
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 - 📍 all ~ consistent with previous results

Feb 2023: from CMD-3 [F. Ignatov et al, [arXiv:2302.08834](https://arxiv.org/abs/2302.08834), PRD 2024]



A new puzzle!

- discrepancies between experiments now $\gtrsim (3 - 5) \sigma$ need to be understood/resolved
- [\(virtual\) scientific seminar + discussion panel on CMD-3 measurement](#)
March 27 (8:00 –11:00 am US CDT)
- [2nd CMD-3 discussion meeting](#) (20 July 2023)
- Discussions are continuing....



HVP: data-driven

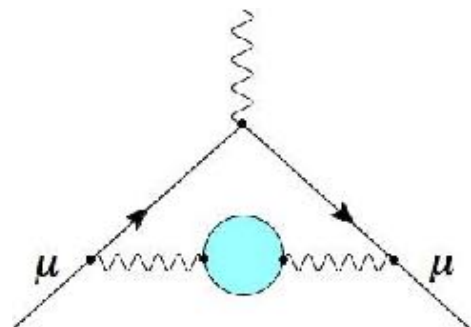
Updates presented at the 7th Muon $g-2$ Tl workshop @ KEK

Ongoing work on experimental inputs:

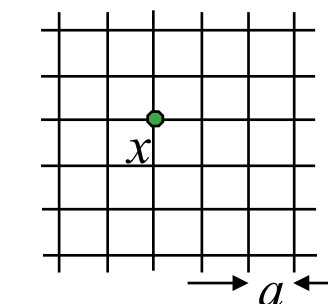
- **BaBar**: ongoing analysis of large data set (not included before) in $\pi\pi$ channel, also $\pi\pi\pi$, other channels
- **CMD3**: ongoing analyses, comparisons with CMD-2 procedures, new data expected
- **KLOE**: ongoing analysis of large data set in $\pi\pi$ channel (not included before), other channels
- **SND**: new results for $\pi\pi$ channel, other channels in progress
- **BESIII**: new results in 2021 for $\pi\pi$ channel, continued analysis also for $\pi\pi\pi$, other channels
- **Belle II**: first results for $\pi\pi\pi$ in 2024, ramping up $\pi\pi$ analysis
Better ultimate statistics than BaBar or KLOE; similar or better systematics for low-energy cross sections

Ongoing work on theoretical aspects:

- better treatment of structure dependent radiative corrections (NLO) in $\pi\pi$ and $\pi\pi\pi$ channels
- new dispersive treatment [Colangelo et al, arXiv:2207.03495]
- **Developing NNLO Monte Carlo generators** (STRONG 2020 workshop <https://agenda.infn.it/event/28089/>)
- **including τ decay data**: requires nonperturbative evaluation of IB correction [M. Bruno et al, arXiv:1811.00508]



Lattice HVP: Introduction



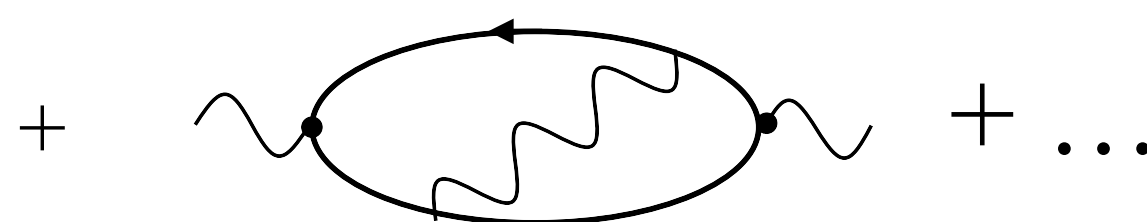
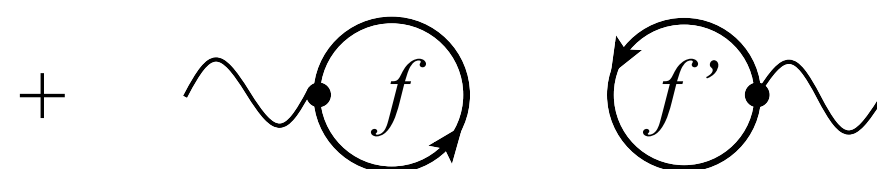
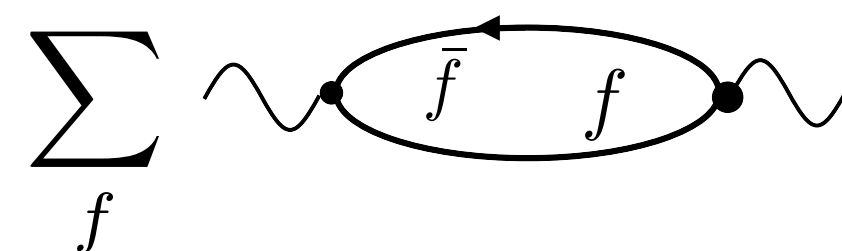
Calculate a_μ^{HVP} in Lattice QCD:

$$a_\mu^{\text{HVP,LO}} = \sum_f a_{\mu,f}^{\text{HVP,LO}} + a_{\mu,\text{disc}}^{\text{HVP,LO}}$$

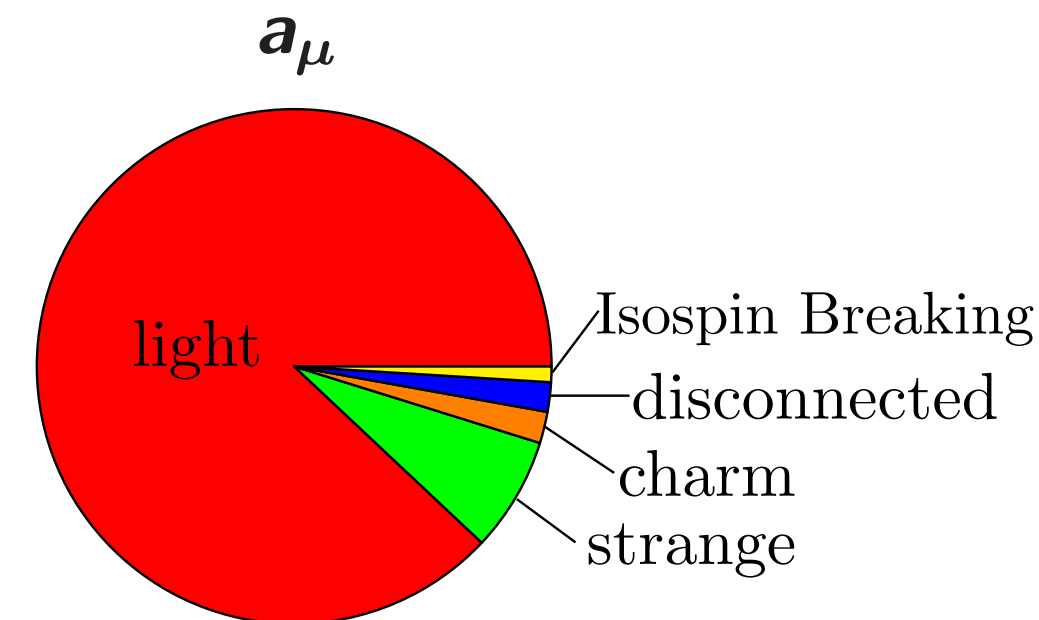
- Separate into connected for each quark flavor + disconnected contributions (gluon and sea-quark background not shown in diagrams)

Note: almost always $m_u = m_d$

$f = ud, s, c, b$

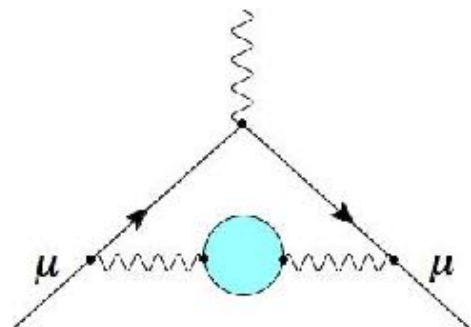


- light-quark connected contribution: $a_\mu^{\text{HVP,LO}}(ud) \sim 90\%$ of total
- s, c, b -quark contributions $a_\mu^{\text{HVP,LO}}(s, c, b) \sim 8\%, 2\%, 0.05\%$ of total
- disconnected contribution: $a_{\mu,\text{disc}}^{\text{HVP,LO}} \sim 2\%$ of total
- Isospinbreaking (QED + $m_u \neq m_d$) corrections: $\delta a_\mu^{\text{HVP,LO}} \sim 1\%$ of total

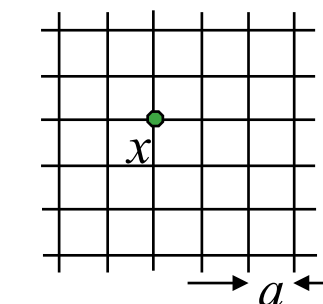


- need to add QED and strong isospin breaking ($\sim m_u - m_d$) corrections:

$$a_\mu^{\text{HVP,LO}} = a_\mu^{\text{HVP,LO}}(ud) + a_\mu^{\text{HVP,LO}}(s) + a_\mu^{\text{HVP,LO}}(c) + a_{\mu,\text{disc}}^{\text{HVP,LO}} + \delta a_\mu^{\text{HVP,LO}}$$



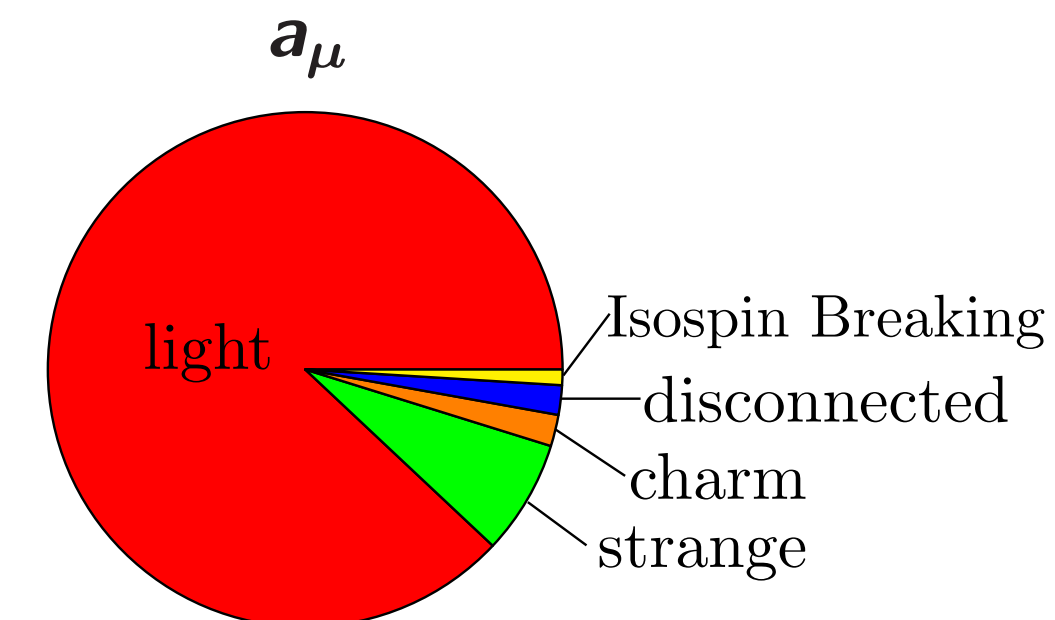
Lattice HVP: challenges



Calculate a_μ^{HVP} in Lattice QCD:

$$a_\mu^{\text{HVP,LO}} = \sum_f a_{\mu,f}^{\text{HVP,LO}} + a_{\mu,\text{disc}}^{\text{HVP,LO}}$$

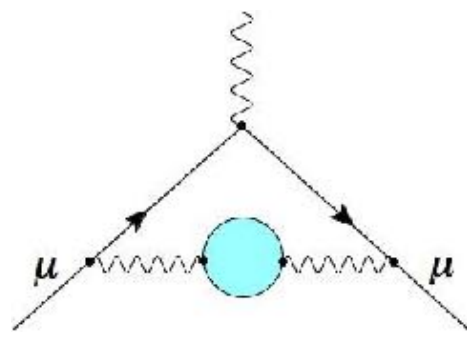
$$a_\mu^{\text{HVP,LO}} = a_\mu^{\text{HVP,LO}}(ud) + a_\mu^{\text{HVP,LO}}(s) + a_\mu^{\text{HVP,LO}}(c) + a_{\mu,\text{disc}}^{\text{HVP,LO}} + \delta a_\mu^{\text{HVP,LO}}$$



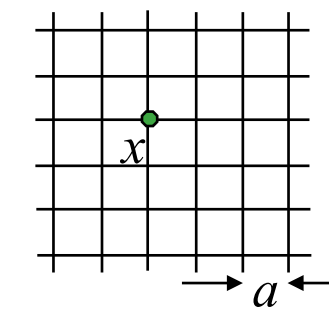
- $a_\mu^{\text{HVP,LO}}$ needed with $< 0.5\%$ precision
- subpercent statistical precision:
exponentially growing noise-to-signal in $C(t)$ as $t \rightarrow \infty$
affects light-quark contributions
- sizable finite volume effects
- sensitivity to scale setting uncertainty
- control discretization effects
- quark-disconnected diagrams: control noise
- include isospin-breaking effects

Separation of $a_\mu^{\text{HVP,LO}}$ into $a_\mu^{\text{HVP,LO}}(ud)$ and $\delta a_\mu^{\text{HVP,LO}}$ is scheme dependent.

- light-quark connected contribution:
 $a_\mu^{\text{HVP,LO}}(ud) \sim 90\%$ of total
- s, c, b -quark contributions
 $a_\mu^{\text{HVP,LO}}(s, c, b) \sim 8\%, 2\%, 0.05\%$ of total
- disconnected contribution:
 $a_{\mu,\text{disc}}^{\text{HVP,LO}} \sim 2\%$ of total
- Isospinbreaking (QED + $m_u \neq m_d$) corrections:
 $\delta a_\mu^{\text{HVP,LO}} \sim 1\%$ of total



Windows in Euclidean time



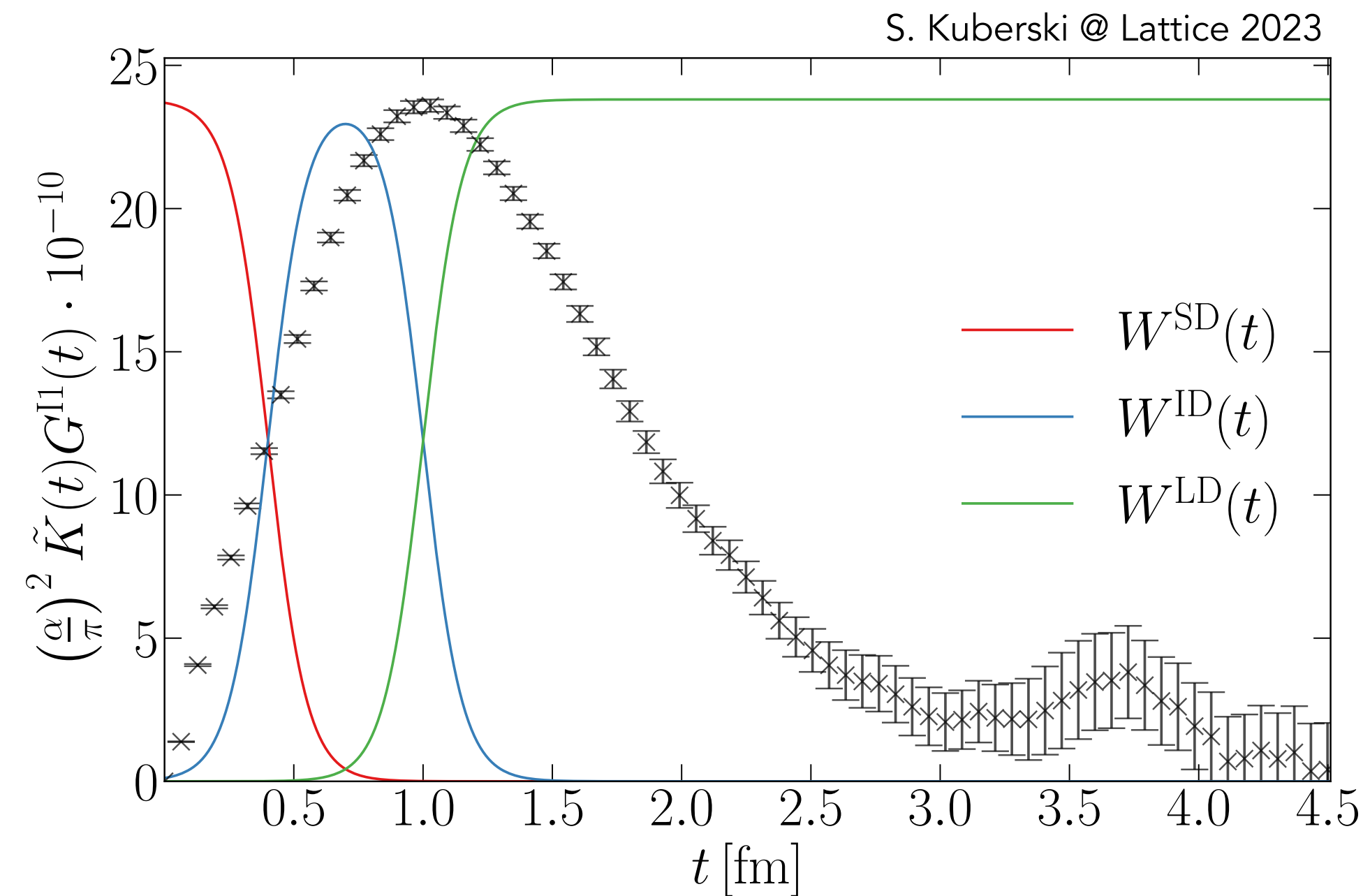
$$a_\mu^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt \tilde{w}(t) C(t)$$

- Use windows in Euclidean time to consider the different time regions separately

[T. Blum et al, arXiv:1801.07224, 2018 PRL]

$$t_0 = 0.4 \text{ fm}, t_1 = 1.0 \text{ fm}$$

- Short Distance (SD) $t : 0 \rightarrow t_0$
- Intermediate (W) $t : t_0 \rightarrow t_1$
- Long Distance (LD) $t : t_1 \rightarrow \infty$



- disentangle systematics/statistics from long distance/FV and discretization effects
- intermediate window: easy to compute in lattice QCD; compare to disperse approach
- Internal cross check: compute each window separately (in continuum, infinite volume limits,...) and

combine:

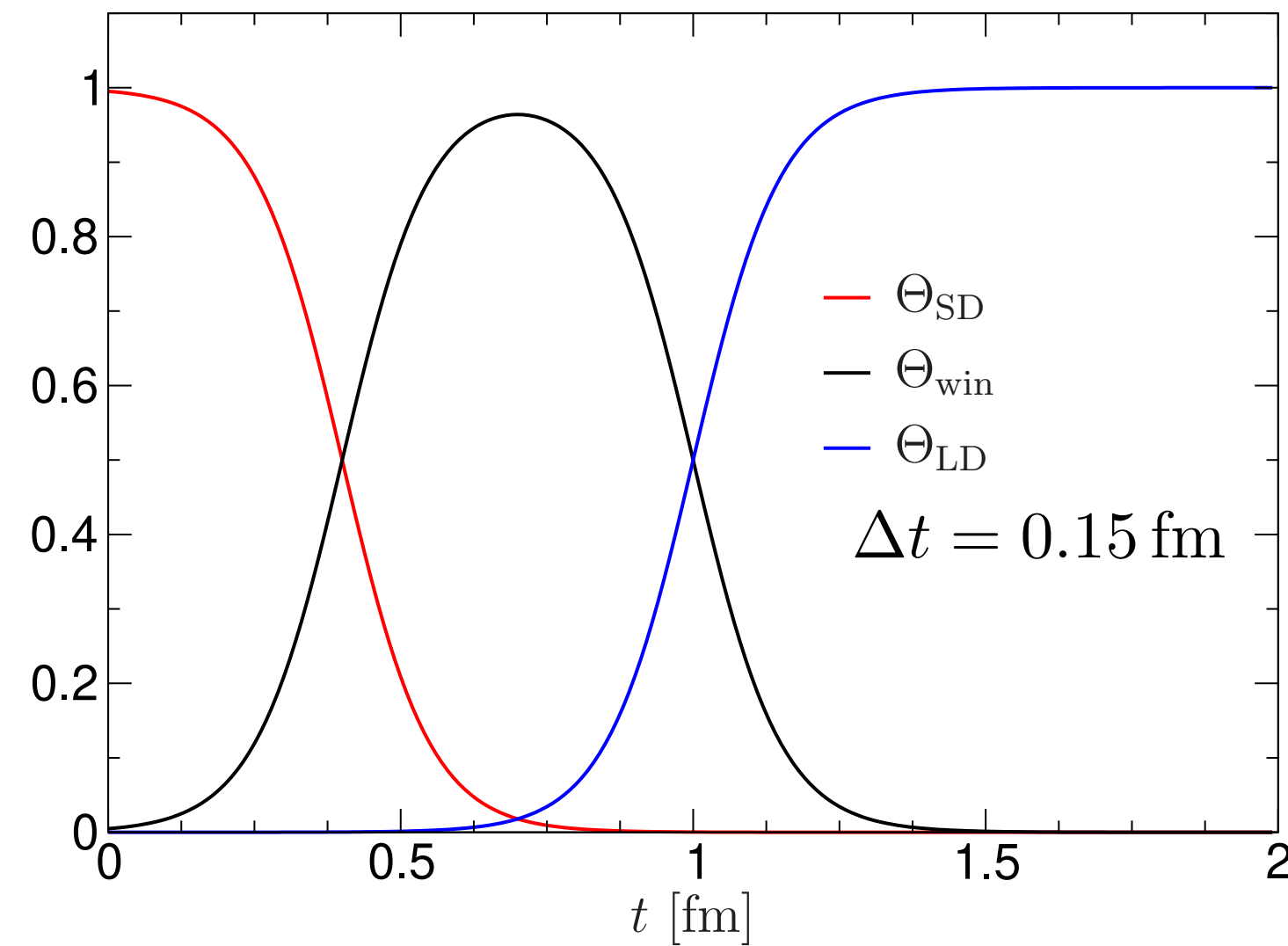
$$a_\mu = a_\mu^{\text{SD}} + a_\mu^{\text{W}} + a_\mu^{\text{LD}}$$

cross section inputs to windows observables

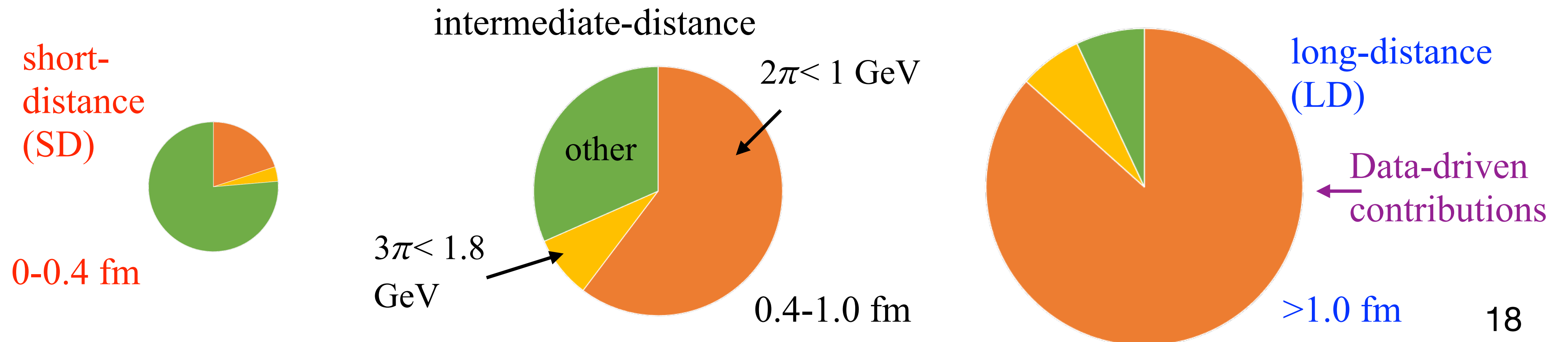
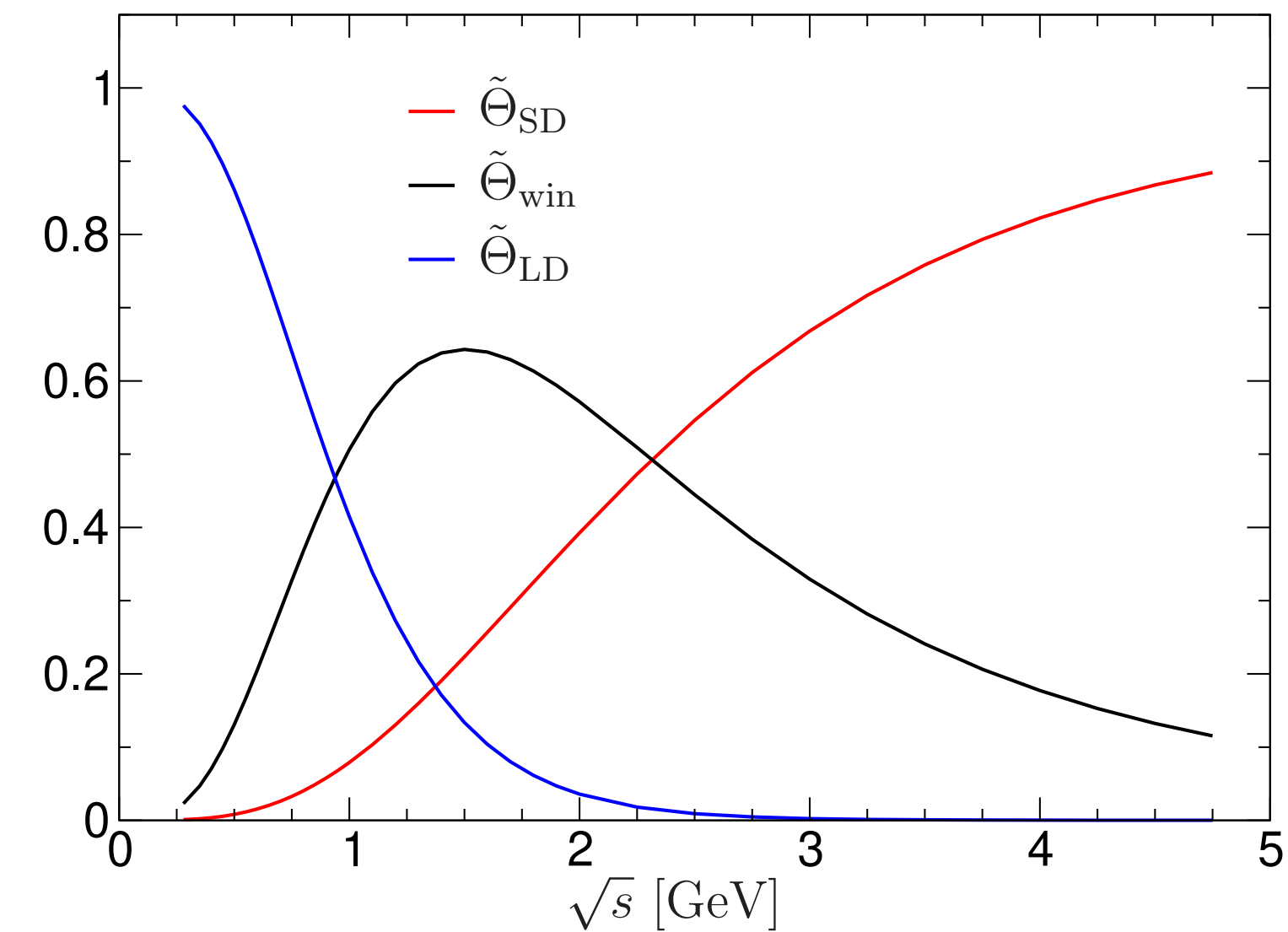
C. Davies @ Lattice 2024

Comparing data-driven and lattice HVP results

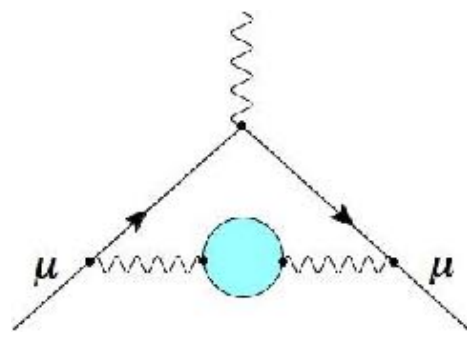
Colangelo et al
2205.12963



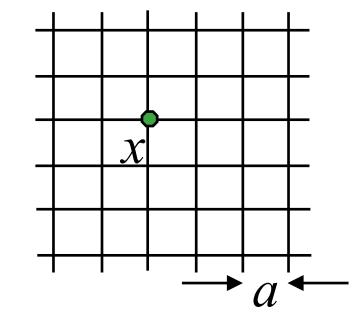
Mapping of window effects



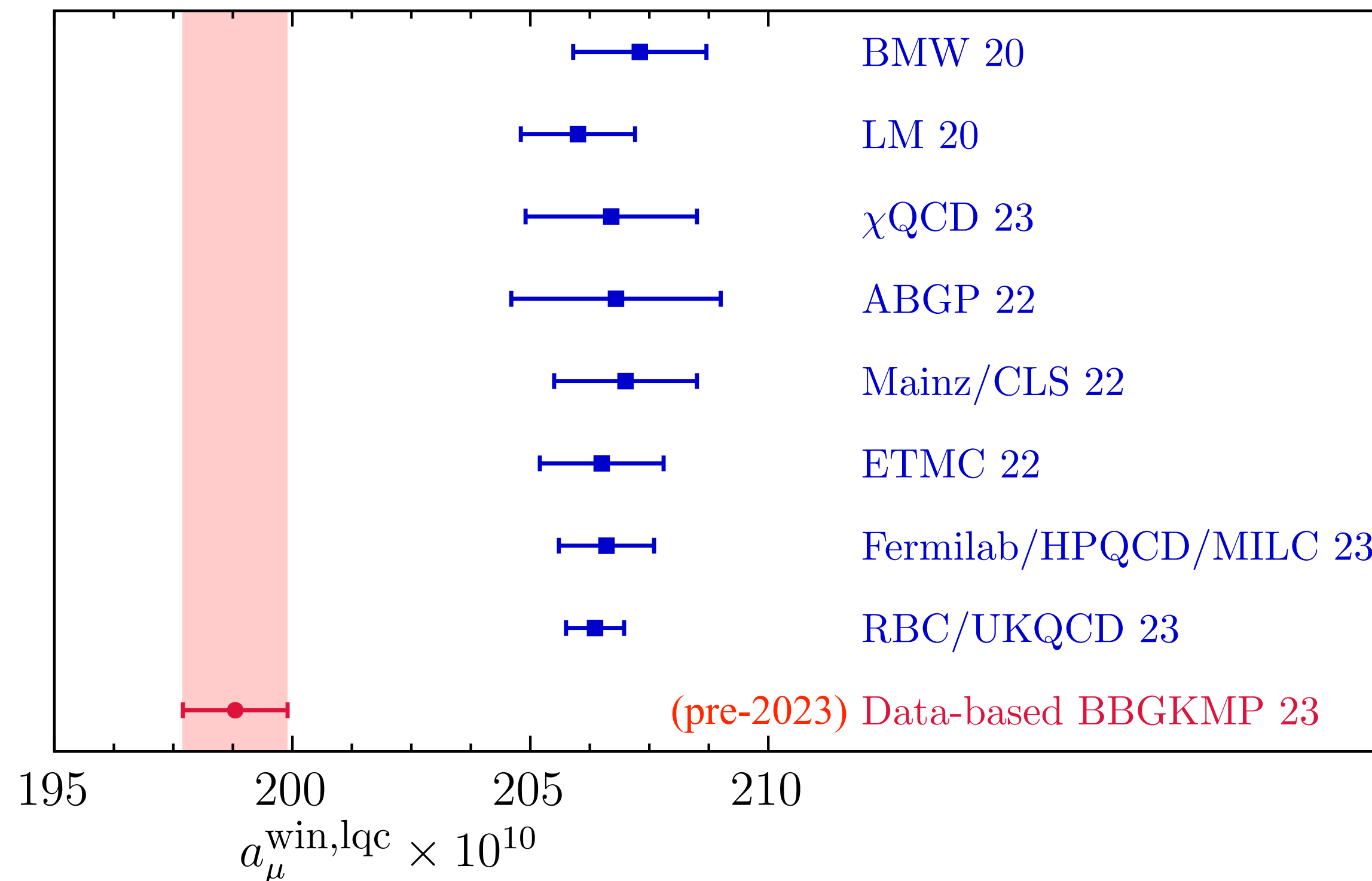
18



Lattice HVP: results for W



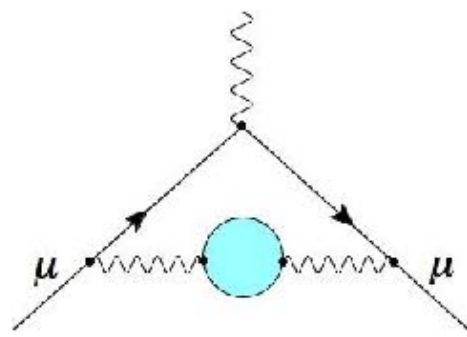
- new results in 2022-2024 for intermediate and short-distance windows.
- blind analyses by all lattice groups for results from 2023+
- focus on light-quark connected contribution to intermediate window:



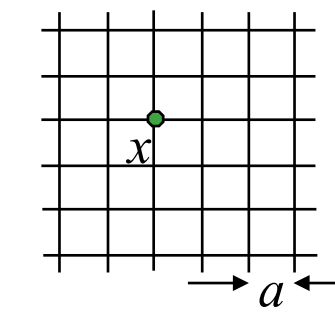
$\sim 5\sigma$ tension between LQCD and (pre-2023) data-driven evaluations for intermediate window

dispersive evaluation of light-quark connected contribution [G. Benton, et al, arXiv:2306.16808]*

*based on disp. results for IB [Hoferichter et al, arXiv:2208.08993]

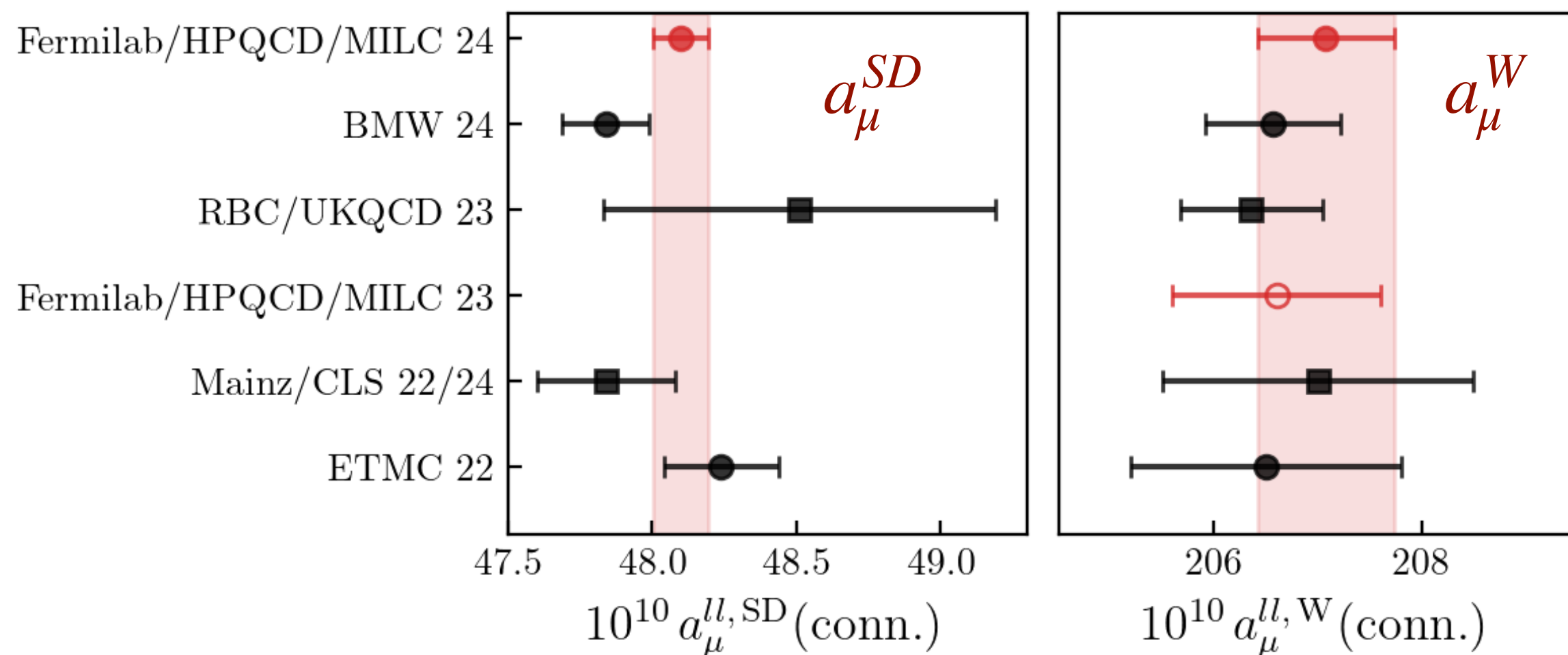


Lattice HVP: windows



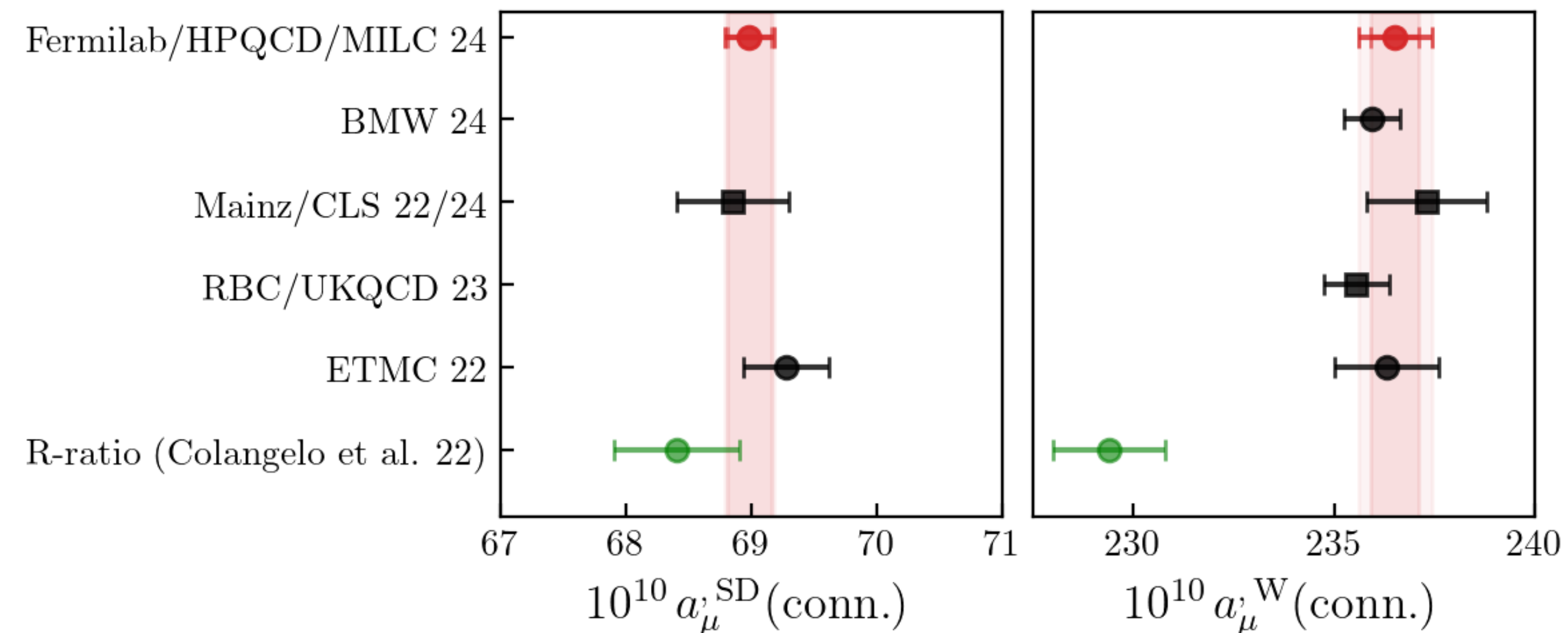
update: Fermilab/HPQCD/MILC 2024

light-quark connected



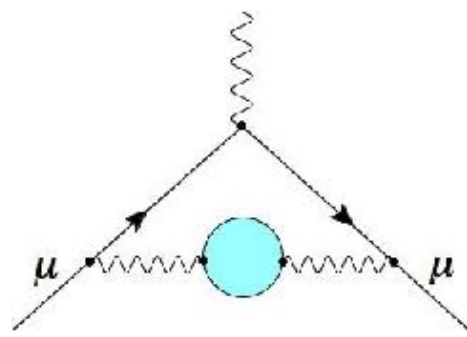
S. Lahert talk @ KEK workshop:
Unblinded results for (all) contributions to a_μ^{SD} and a_μ^W (including correlations).

total (u, d, s, c) + disc + IB

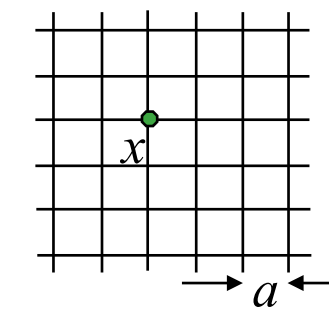


- ▶ W:
 - Consistent w/ all previous determinations
 - Leading uncertainty: scale setting (w_0 fm).
- ▶ SD: Good agreement with other groups.

▶ appendix for updates from BMW 2024 for W, SD windows



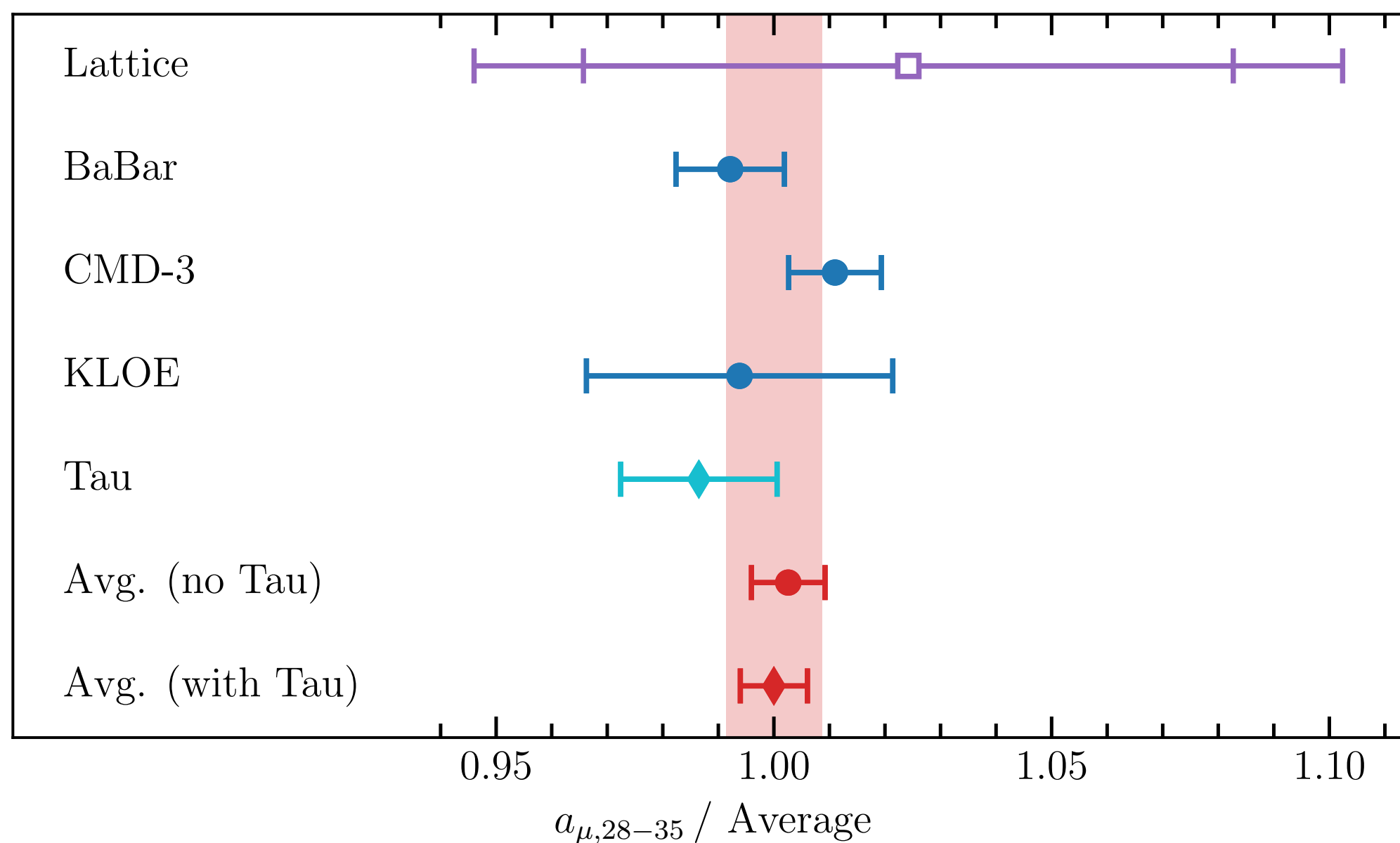
Lattice HVP: long-distance window



update: BMW+DMZ 2024

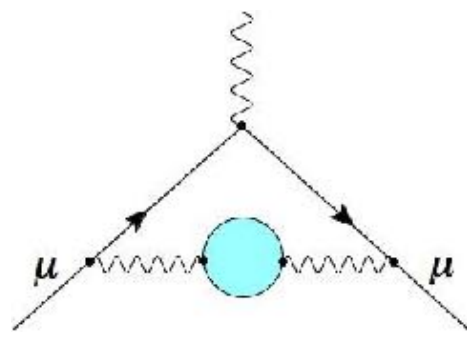
BMW+DMZ 24 [A. Boccaletti et al, arXiv:2407.10913]

- statistical/systematic errors at long distances, $t \gtrsim 2.5$ fm still large:

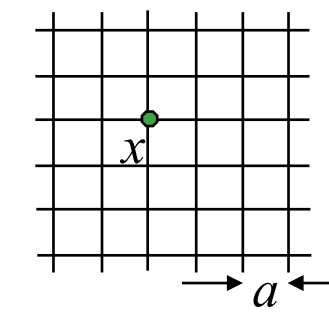


Laurent Lellouch @ [Lattice @ CERN](#):

- Partial tail $a_{\mu,28-35}^{\text{LO-HVP}}$ for comparison with lattice dominated by cross section below ρ peak: $\sim 70\%$ for $\sqrt{s} \leq 0.63$ GeV
- cross section measurements compatible for $\sqrt{s} < 0.5$ GeV



Lattice HVP: long-distance window

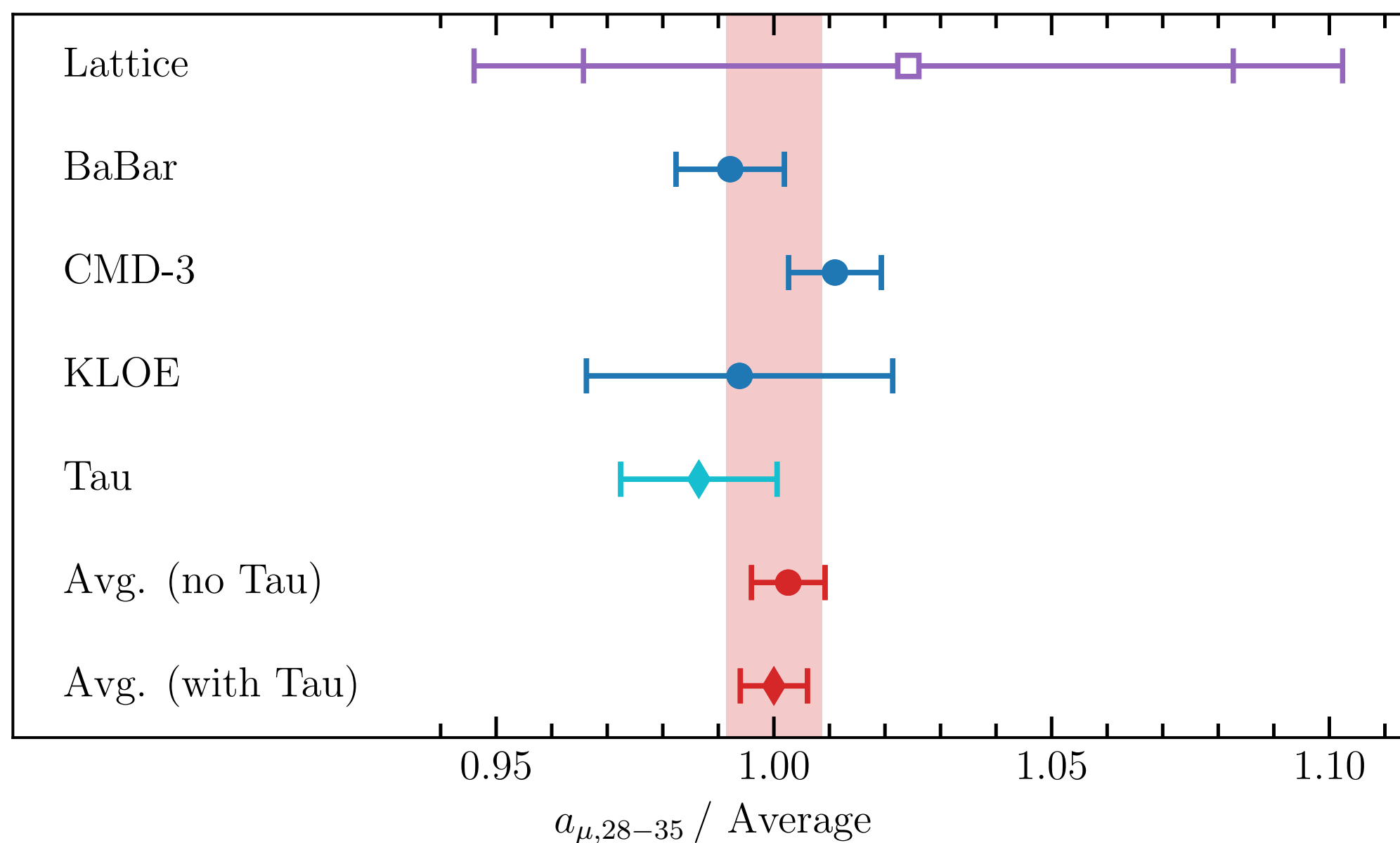


update: BMW+DMZ 2024

● BMW+DMZ 24 [A. Boccaletti et al, arXiv:2407.10913]

- statistical/systematic errors at long distances, $t \gtrsim 2.5$ fm still large:

[Laurent Lellouch [Lattice @ CERN](#)]



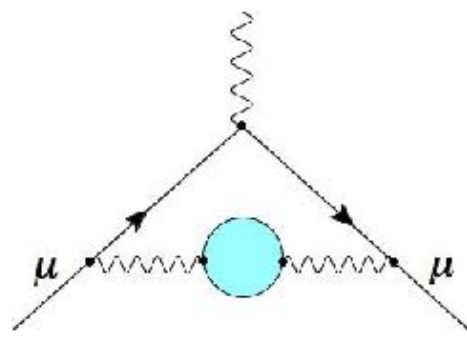
- All data-driven result agree very well
- Final number: average w/ τ , and systematic = full difference τ /no- τ added linearly
- Weighted average taken w/ and w/out τ : $\chi^2/\text{dof} = 1.1$ for both
- Final number: average w/ τ , PDG factor, and systematic = full difference τ /no- τ added linearly
- Excellent agreement w/ lattice, but uncertainty reduced by factor ~ 15
- Final number: average w/ τ , and systematic = full difference τ /no- τ added linearly
- Only $\lesssim 5\%$ of final result for a_μ
- Contributes $\sim 65\%$ to total squared uncertainty uncertainty improvement: $5.5 \rightarrow 3.3$

$$a_{\mu,28-\infty}^{\text{LO-HVP}} = 27.59(17)(9)[26]$$

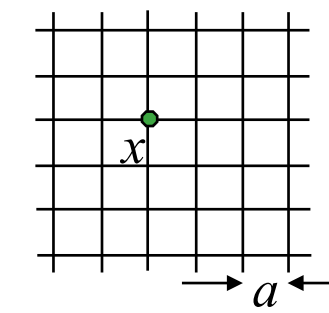
$$a_{\mu,28-35}^{\text{LO-HVP}} = 18.12(11)(5)[16]$$

➡ **hybrid evaluation:** combine lattice QCD calculation of one-sided window [Davies et al, arXiv:2207.04765]

$a_\mu(t_1 = 2.8 \text{ fm})$ with data-driven evaluation of long tail, $t > 2.8 \text{ fm}$.



Lattice HVP: long-distance window

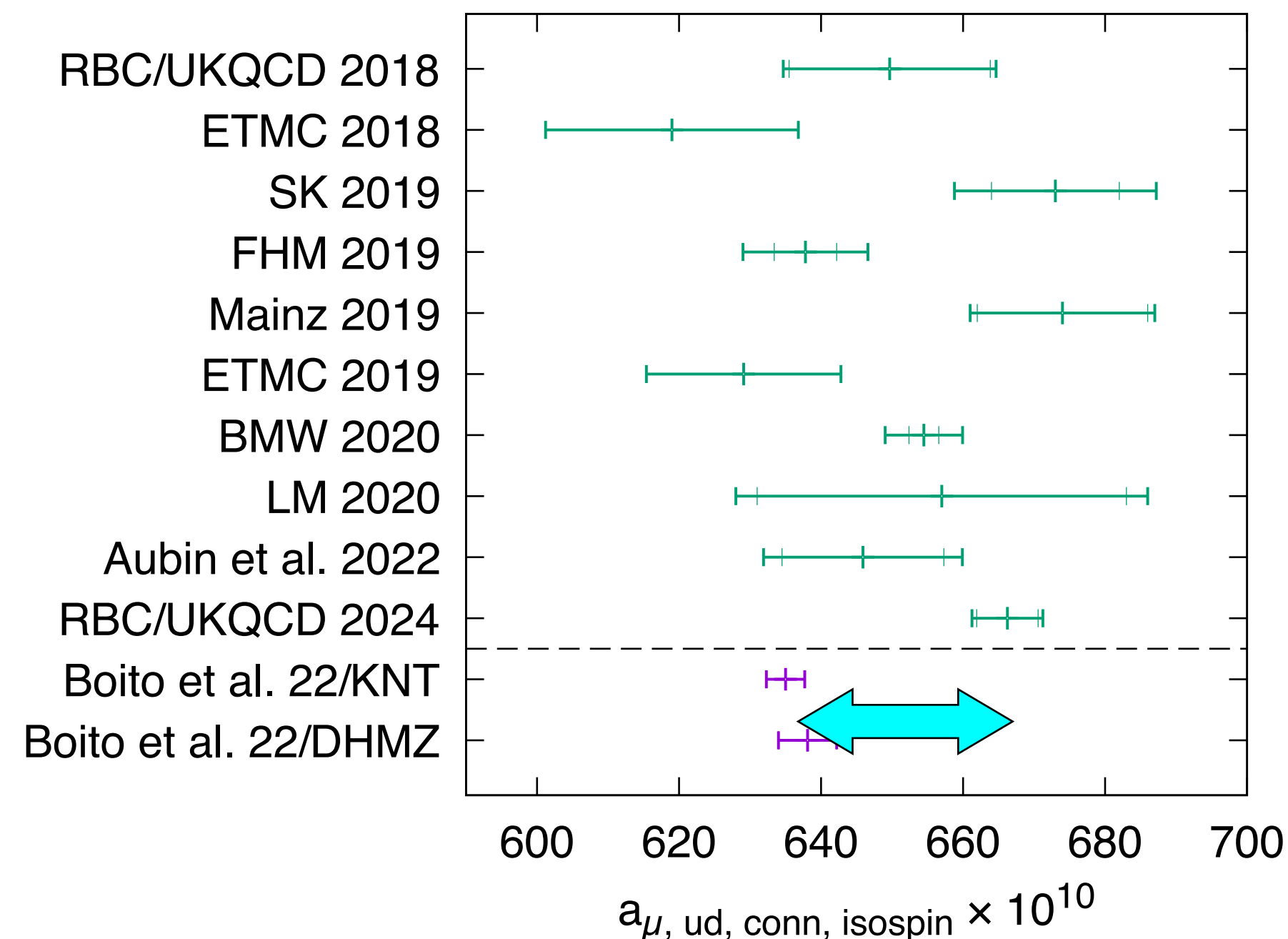
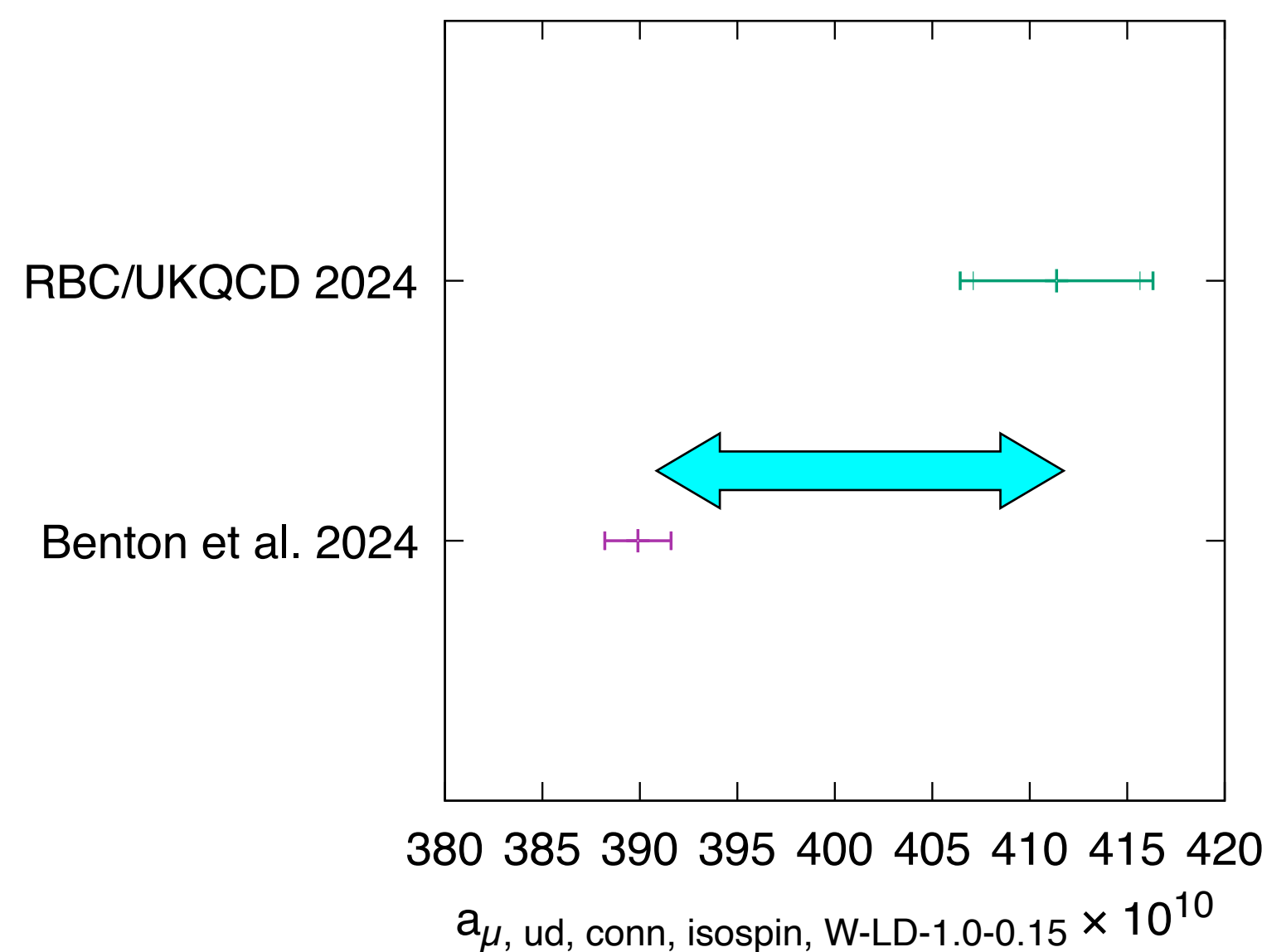
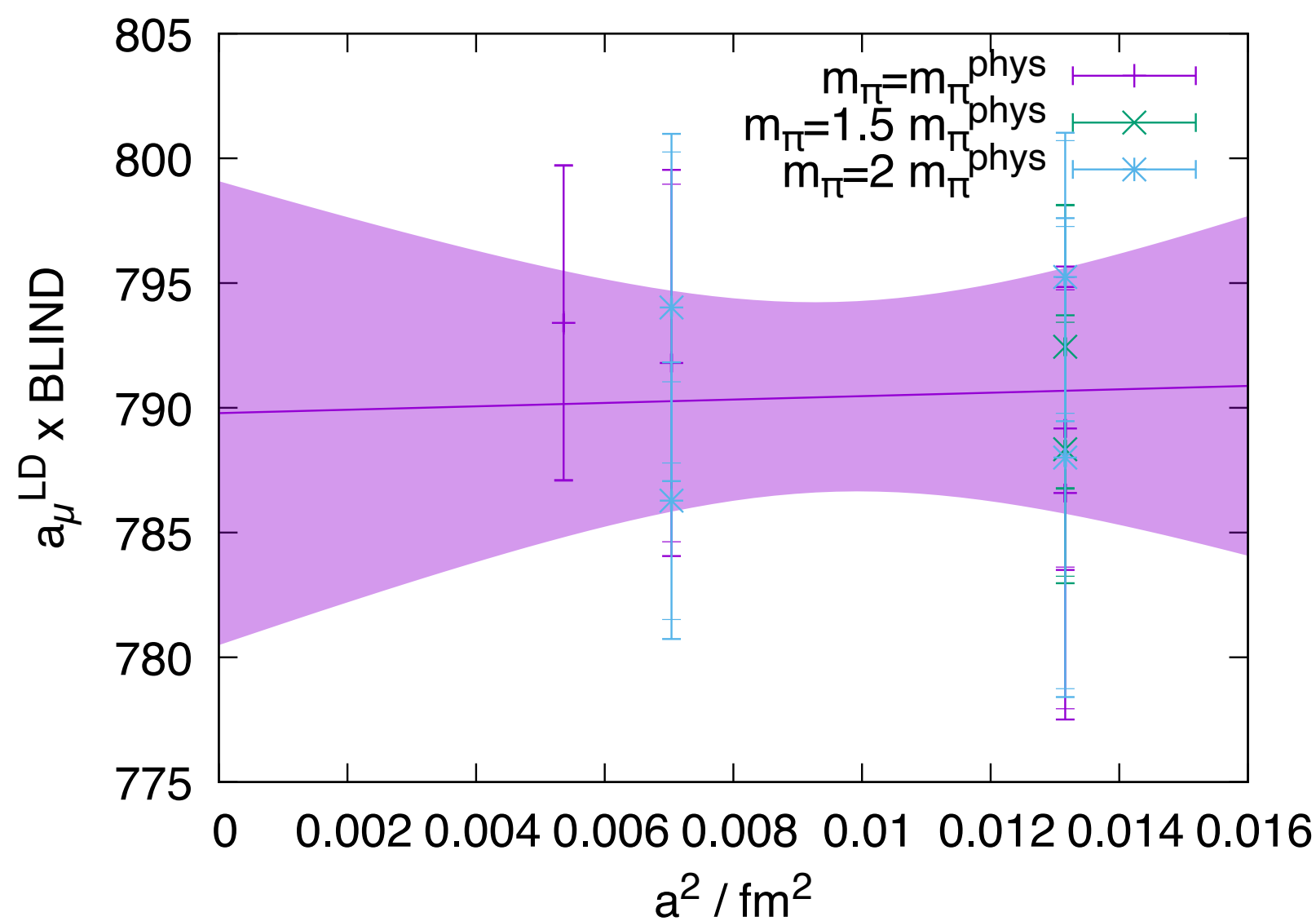


update: RBC/UKQCD 2024

long-distance window a_μ^{LD} and full $a_\mu^{ll}(\text{conn.})$

Unblinded results in BMW20 isospin-symmetric world

C. Lehner @ Lattice 2024

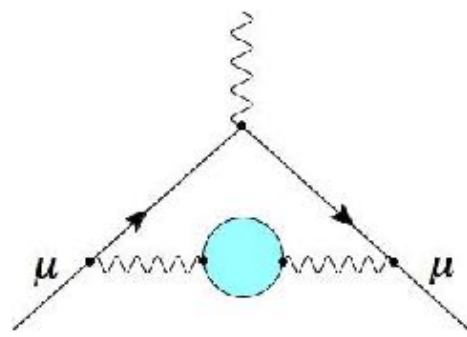


- “BMW20 world”: fixed $w_0 = 0.1716$ fm scale uncertainty not included
- paper in progress

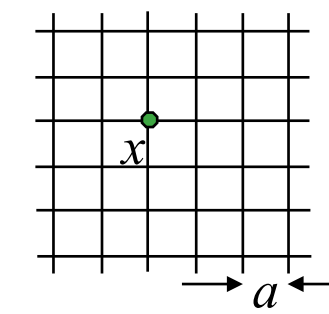
Result for $a_\mu^{iso\ lqc}$ with 7.5/1000 precision.

$$a_\mu^{LD\ iso\ lqc} = 411.4(4.3)_{\text{stat.}}(2.3)_{\text{syst.}} \times 10^{-10},$$

$$a_\mu^{iso\ lqc} = 666.2(4.3)_{\text{stat.}}(2.5)_{\text{syst.}} \times 10^{-10}.$$



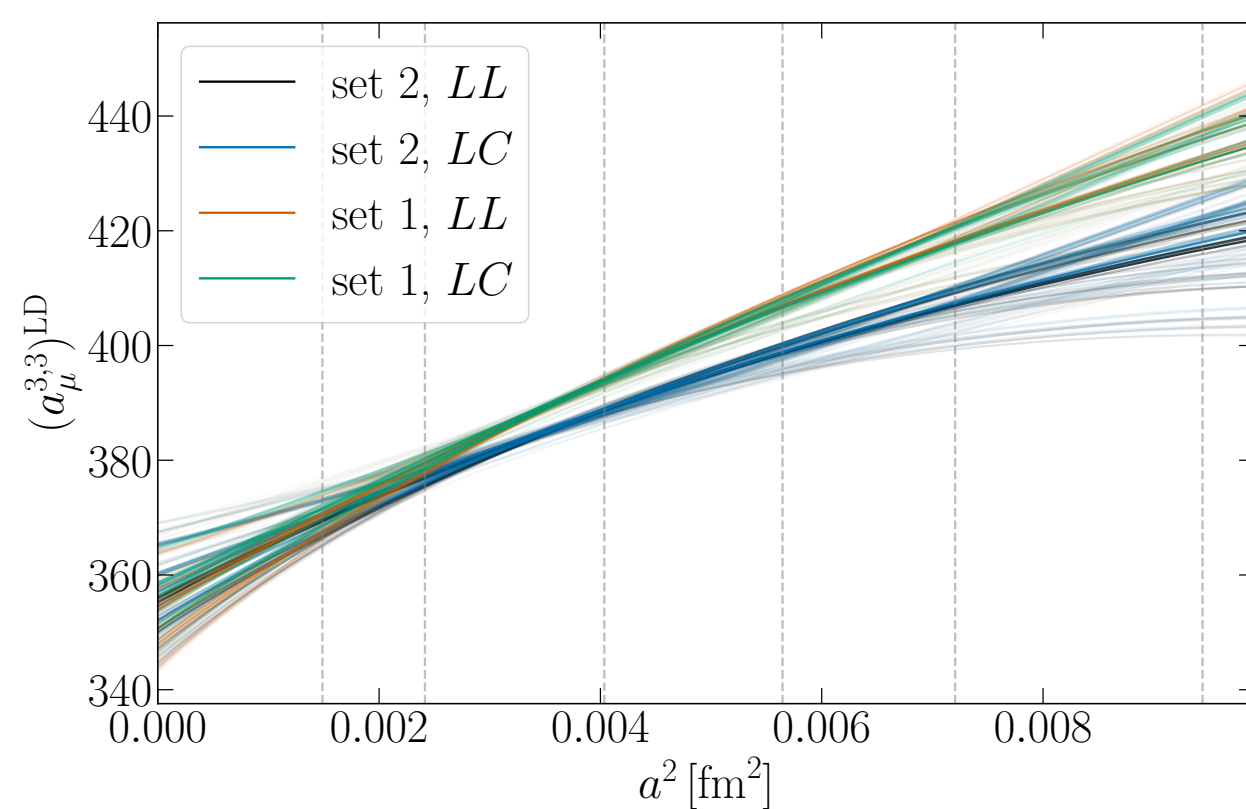
Lattice HVP: long-distance window



update: Mainz 2024

S. Kuberski @ KEK 2024

$(a_\mu^{\text{hvp}})^{\text{LD}}$ IN THE ISOVECTOR CHANNEL: CUTOFF DEPENDENCE



- Dependence of $(a_\mu^{3,3})^{\text{LD}}$ on a^2 at physical quark masses.
- Four sets of data (colors) differ by $O(a^2)$.
- Each line represents a fit in the model average.
- Include terms à la $[\alpha_s(1/a)]^{0.395} a^2$ [Husung, 2401.04303].

$(a_\mu^{\text{hvp}})^{\text{LD}}$: STATUS AND OUTLOOK

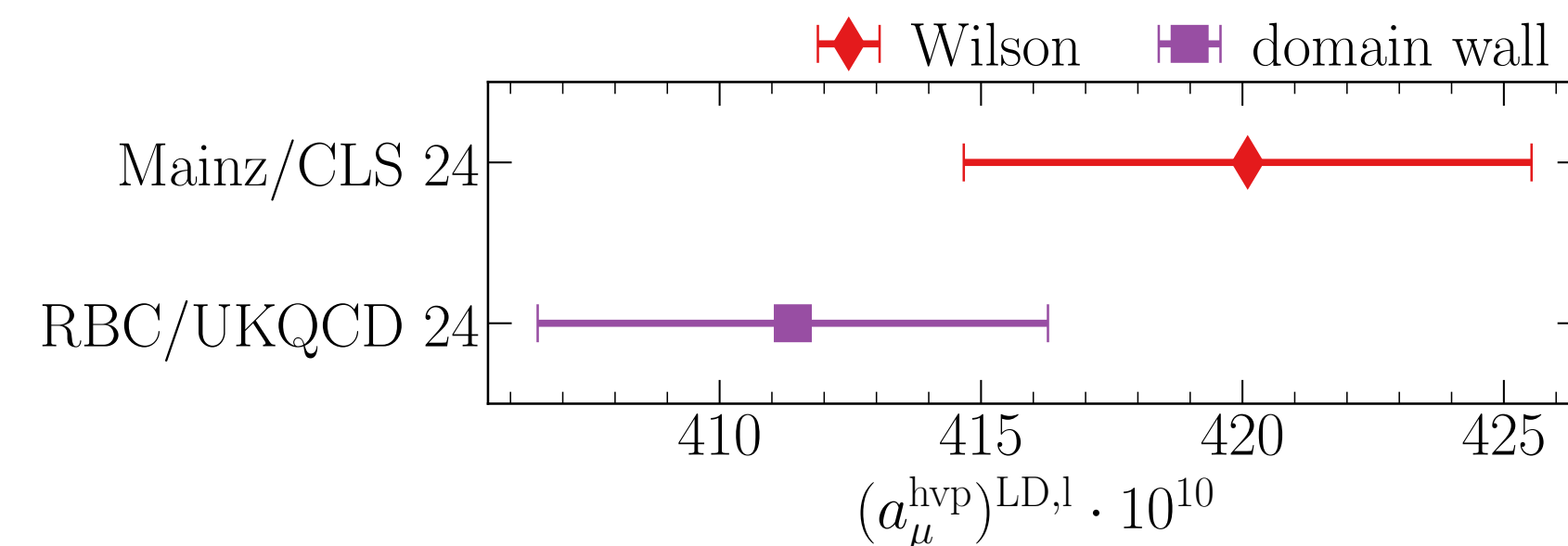
Achievements

- High statistical precision at m_π^{phys} and excellent control of the m_π dependence.
- Large span of lattice spacings to control the continuum extrapolation.
- Compute full isoQCD $(a_\mu^{\text{hvp}})^{\text{LD}}$ to 1.3% precision (statistics dominated).

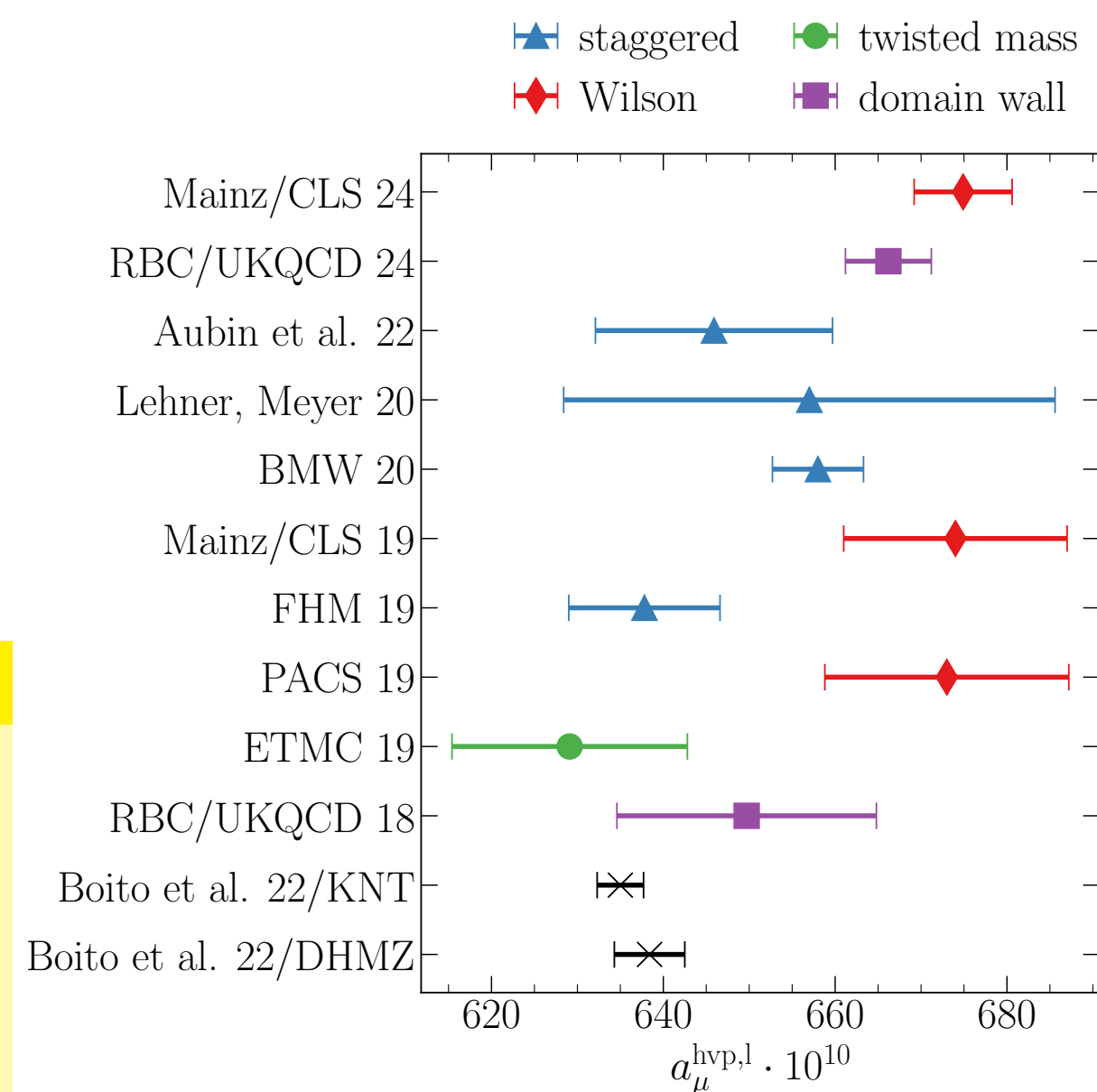
Outlook

- More data at fine lattice spacing and m_π^{phys} is being computed.
- Strong scale dependence in the long-distance regime:
 - ▶ We observe a strong scheme dependence: due to differences in the scale setting?
 - ▶ The global status of gradient flow scales is unsatisfactory [FLAG23].

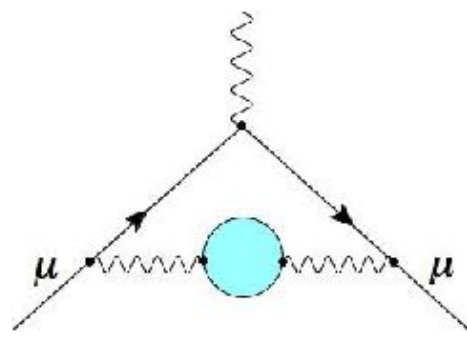
Comparison LD window (light):



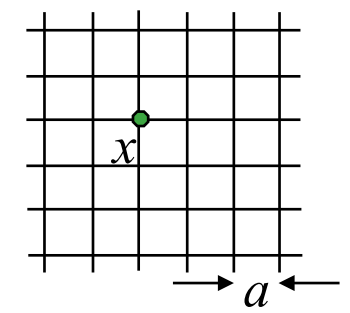
Comparison for full light-quark connected:



- Compute contributions to a_μ^{hvp} in isoQCD (Mainz world) by combinations with $(a_\mu^{\text{hvp}})^{\text{SD}}$ and $(a_\mu^{\text{hvp}})^{\text{ID}}$.
- We (will) publish the derivatives w.r.t. the input that defines our scheme. See [Portelli] for a comparison of schemes.
- $a_\mu^{\text{hvp,l}}$ determined to 0.8% precision
- Excellent compatibility of Mainz/CLS 19 with Mainz/CLS 24.

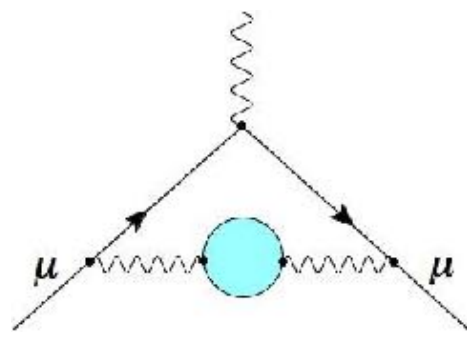


Lattice HVP: outlook

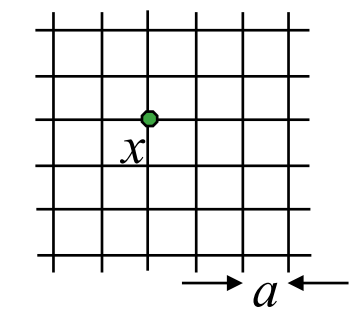


For total HVP and long-distance window:

- expect unblinded lattice results from FNAL/HPQCD/MILC (fall 2024) and ETM (~2025)
 - ⇒ consolidated (?) lattice HVP for White Paper 2



Lattice HVP: outlook

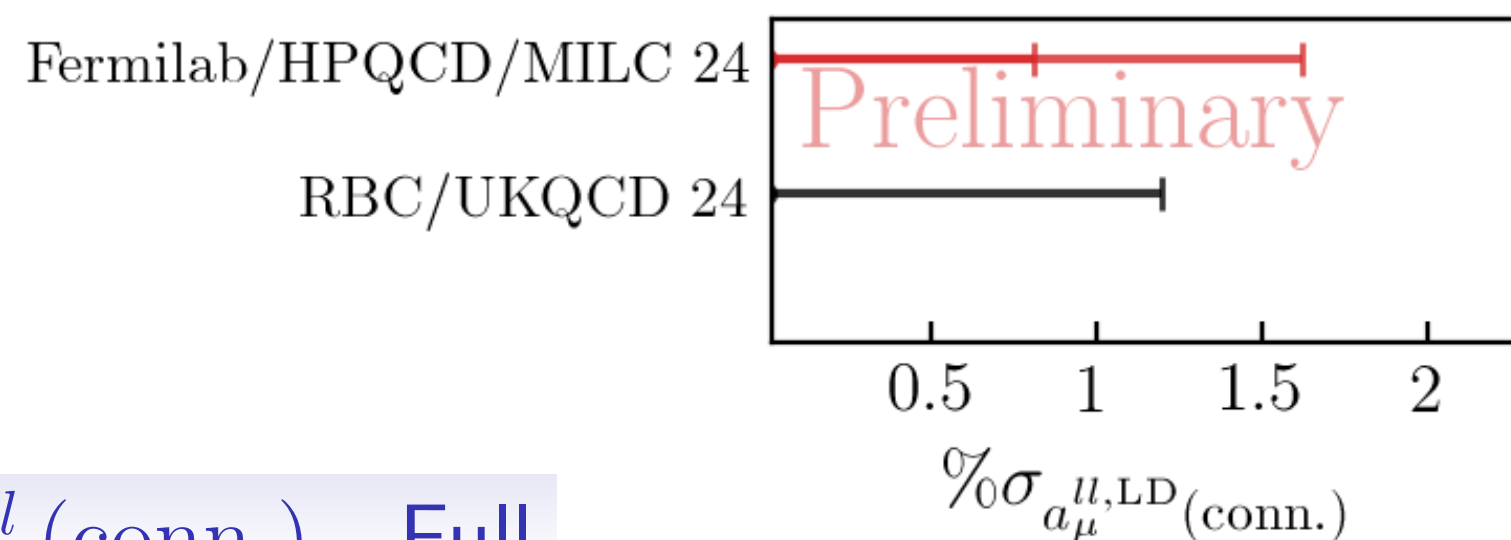


update: Fermilab/HPQCD/MILC 2024

M. Lynch @ Lattice 2024
S. Lahert @ KEK 2024

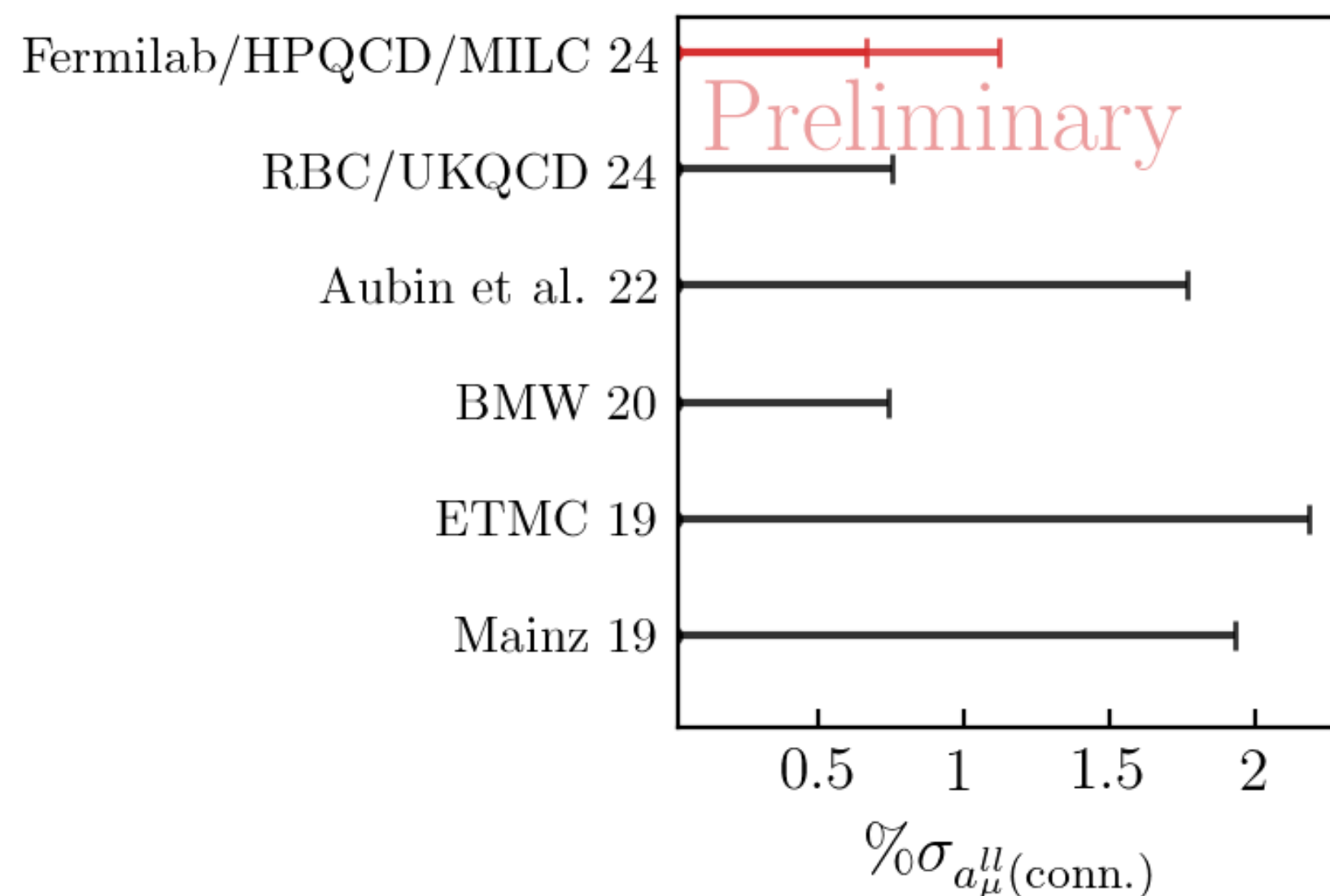
🕒 long-distance window a_μ^{LD} and full $a_\mu^{ll}(\text{conn.})$

$a_\mu^{ll}(\text{conn.})$ - LD window



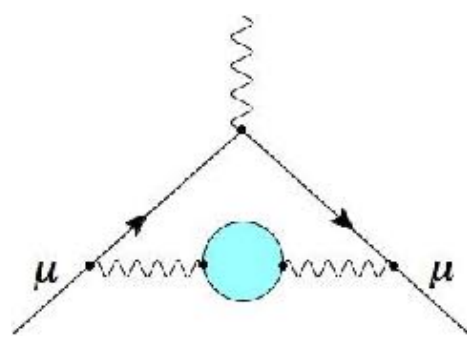
- Inner error w/o scale setting (w_0 fm) uncertainty
- Scale setting is now dominant error contributor.

$a_\mu^{ll}(\text{conn.})$ - Full

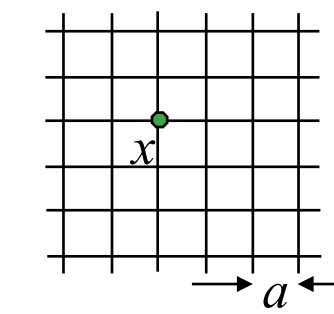


We expect further improvements in stat., sys. uncertainties from...

- Generation of correlator data at a lattice spacing of 0.04 fm is in progress.
- Improved scale setting via M_Ω .
- Joint fit analysis with multiple vector current discretizations
- Direct finite volume study: $L \sim 5.5$ fm \rightarrow $L \sim 11$ fm (at $a = 0.09$ fm) to replace EFT-based FV error estimates.
- Calculation of two-pion contributions to vector-current correlation functions at finer lattice spacings.



Lattice HVP: full window

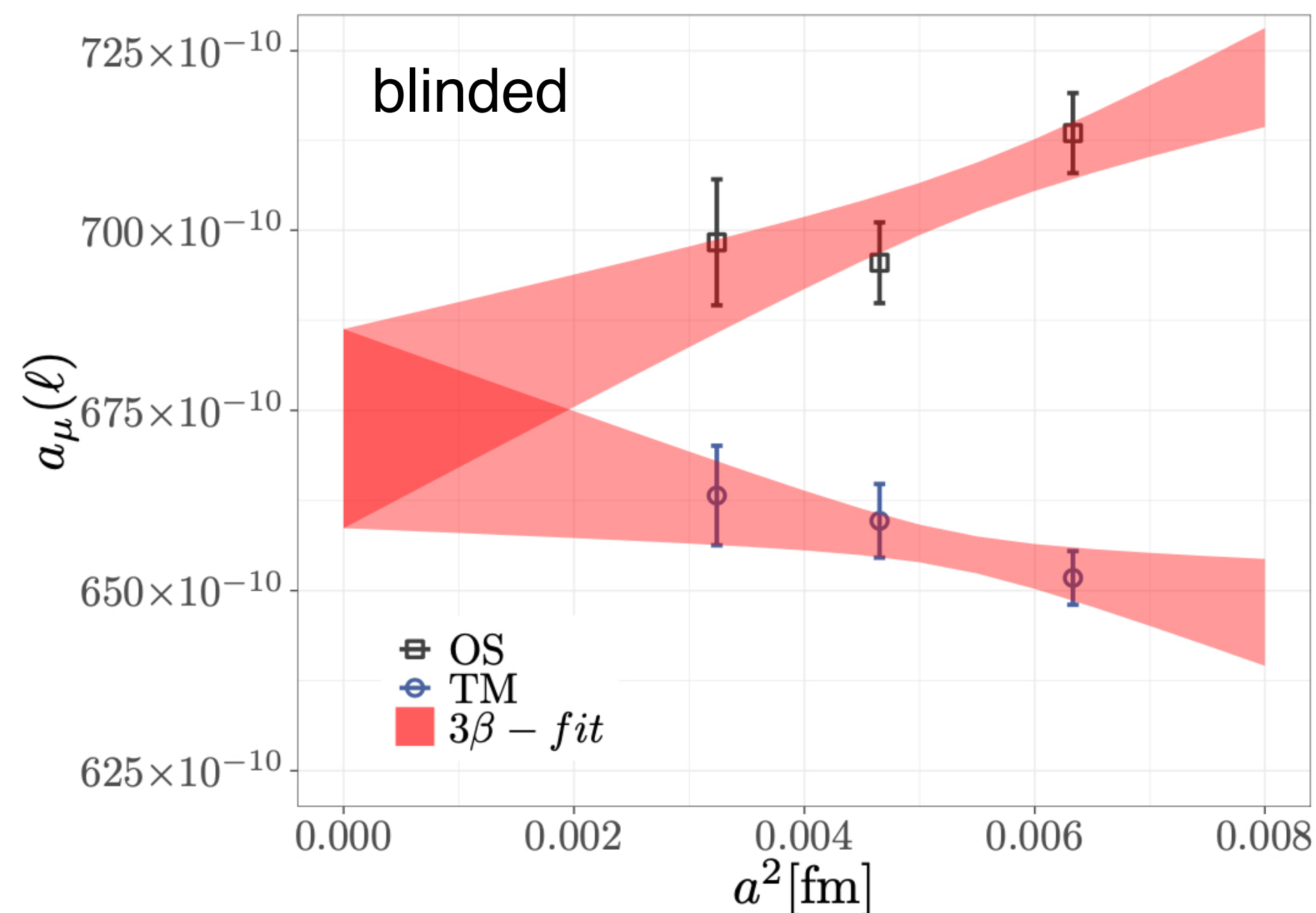


update: ETMC 2024

M. Garofalo @ Lattice 2024

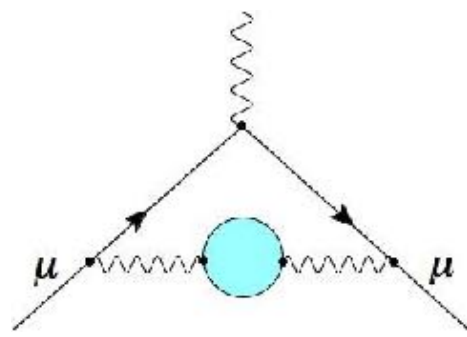
U. Wenger @ KEK 2024

ℓ -quark connected [Preliminary]

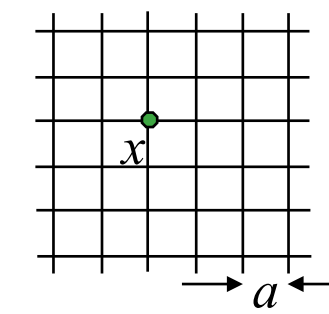


- Data interpolated to $L = 5.46$ fm:
 \Rightarrow using GS/MLLGS approach
- Correction to isoQCD point:
 \Rightarrow calculation in progress
- Significant reduction in uncertainty possible:
 - \Rightarrow additional final lattice spacing
 - \Rightarrow higher statistics in progress
 - \Rightarrow EFT model for TM lattice artefacts ($\rho\pi\gamma$ -model)

also prelim. results for strange & charm



Lattice HVP: outlook

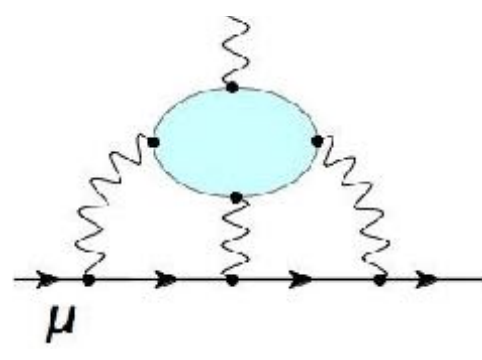


For total HVP and long-distance window:

- expect unblinded lattice results from FNAL/HPQCD/MILC (fall 2024) and ETM (~2025)
 - consolidated (?) lattice HVP for White Paper 2
- Including $\pi\pi$ states for refined long-distance computation (Mainz, RBC/UKQCD, FNAL/MILC)
- smaller lattice spacings to test continuum extrapolations crucial
 - Slides from Lattice 2024 conference
 - Slides from Muon g-2 TI workshop @ KEK

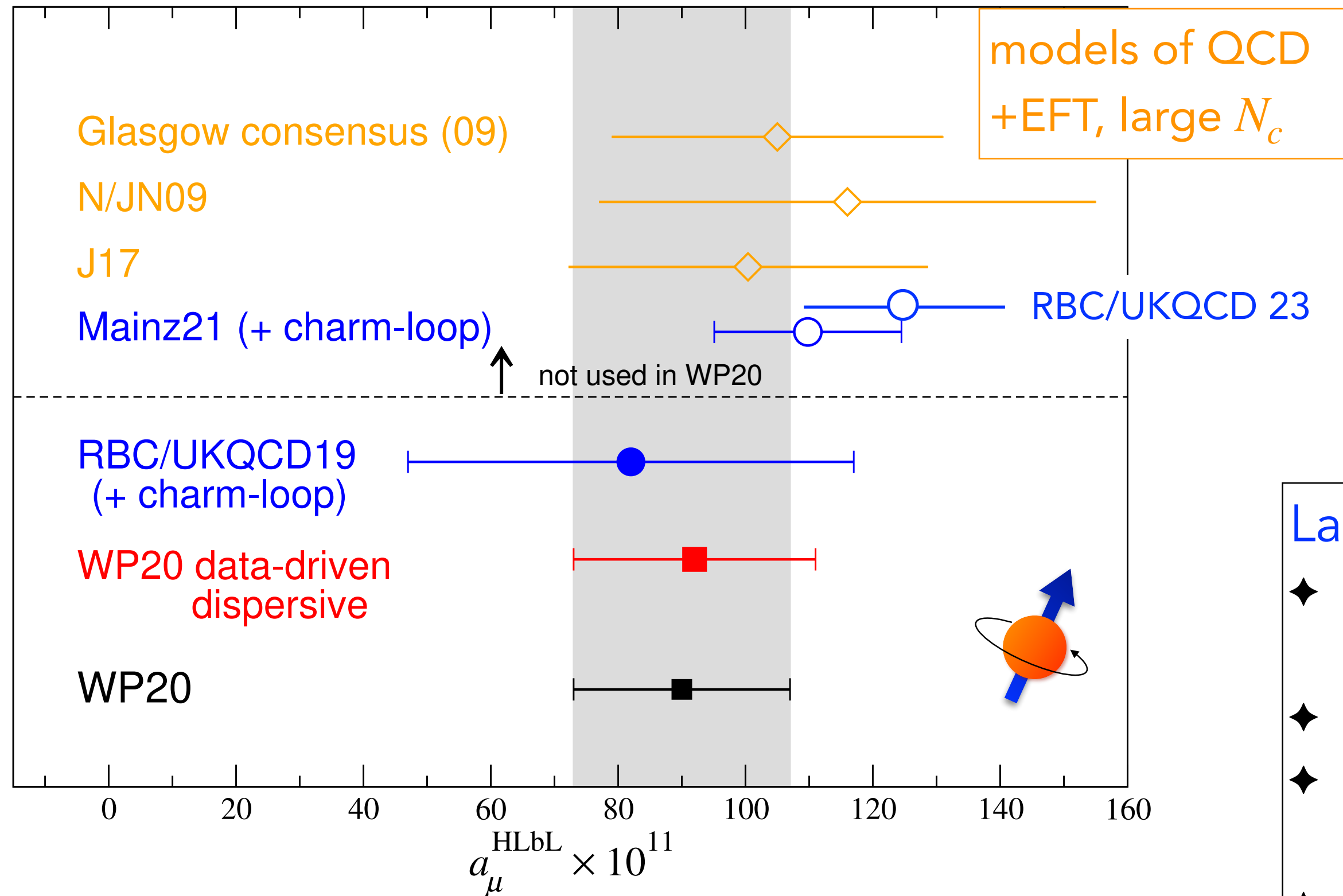
More windows:

- Use linear combinations of finer windows to locate the tension (if it persists) in \sqrt{s}
[Colangelo et al, arXiv:12963]
- One-sided windows (excluding the long-distance region $t \gtrsim 2.5$ fm) to test data-driven evaluations [Davies et al, arXiv:2207.04765]



Hadronic Light-by-Light: Summary

$$a_{\mu}^{\text{HLbL}}$$



Dispersive approach:

[Colangelo et al, 2014; Pauk & Vanderhaegen 2014; ...]

- ◆ model independent
- ◆ significantly more complicated than for HVP
- ◆ provides a framework for data-driven evaluations
- ◆ can also use lattice results as inputs
- ◆ current 20% uncertainty \Rightarrow ~10% feasible

Lattice QCD+QED:

- ◆ Independent calculations by four groups (Mainz, RBC/UKQCD, ETM, BMW)
- ◆ consistent with each other and with previous calculations
- ◆ Lattice groups are continuing to improve their calculations, adding more statistics, lattice spacings, physical mass ensembles
- ◆ ongoing LQCD calculations of π, η, η' transition form factors to determine pseudo scalar pole contributions [Mainz, ETMC, BMW, RBC/UKQCD]

Summary

- ★ consistent results from independent, precise LQCD calculations for light-quark connected contribution to intermediate window a_μ^W ($\sim 1/3$ of $a_\mu^{\text{HVP,LO}}$) $\Rightarrow \sim 5 \sigma$ tension with (pre-2023) data-driven results
- ★ **2 new LQCD** results for long-distance contribution with $\sim 0.8 \%$ precision, **more coming soon!**
 - \Rightarrow check consolidation, develop lattice HVP average for White Paper 2
- ★ Programs and plans in place for:
 - 👤 **data-driven HVP:**
 - new analyses from BaBar, KLOE, CMD3, SND, Belle II, ... will shed light on current discrepancies
 - improved treatment of structure dependent radiative corrections (NLO) in $\pi\pi$ and $\pi\pi\pi$ channels
 - 👤 **lattice HVP: if no tensions** between independent lattice results, $\sim 0.5 \%$ possible
 - 👤 **dispersive HLbL and lattice HLbL:** no puzzles, steady progress, $\sim 10 \%$
- ★ **Need to understand tensions between data-driven and lattice HVP if they persist**
- ★ **including τ decay data in data-driven approach:**
 - requires nonperturbative evaluation of IB correction [M. Bruno et al, arXiv:1811.00508]
- \Rightarrow **continued coordination by Theory Initiative: 2nd WP in progress**

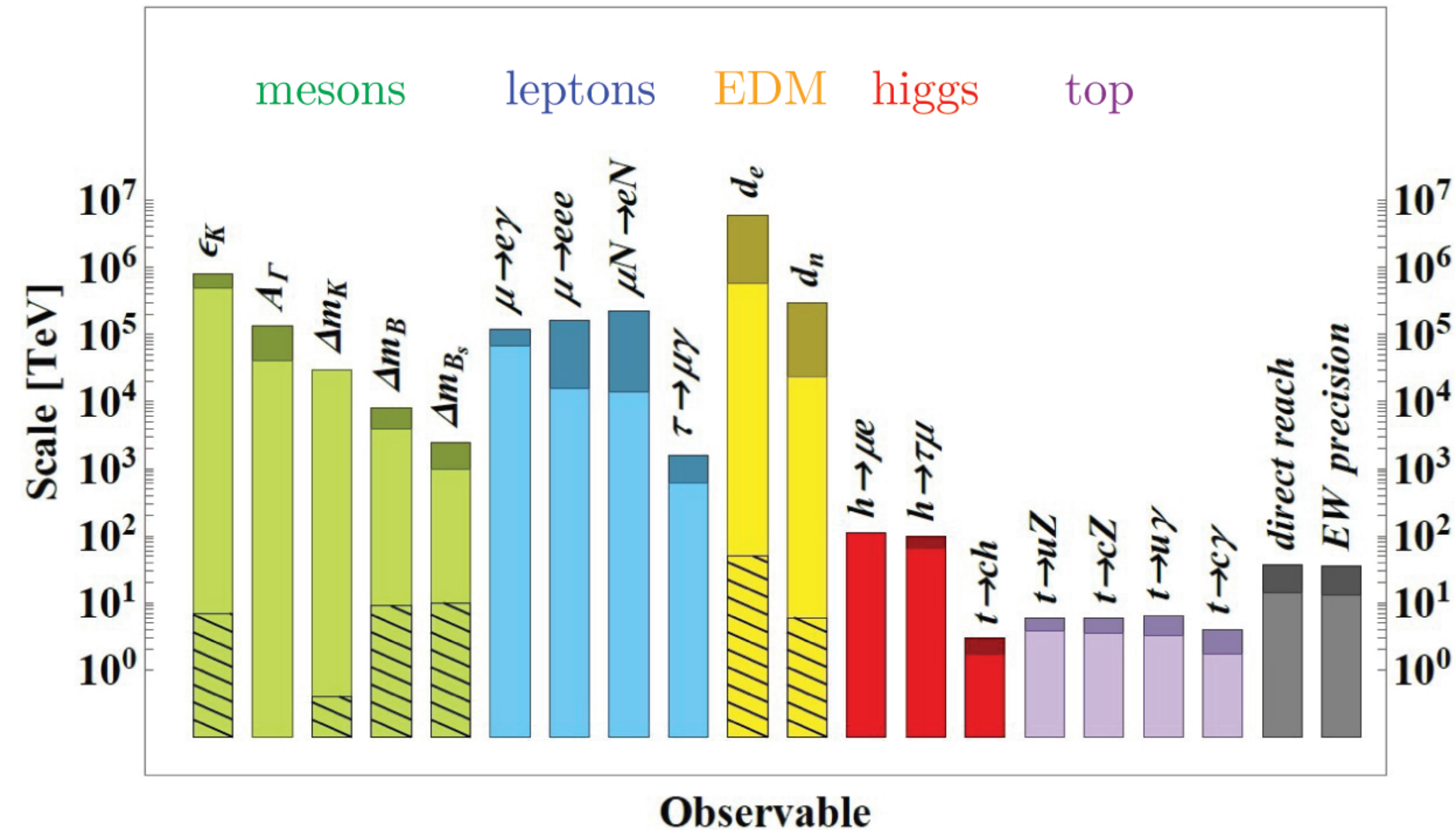
Beyond the SM possibilities

- a_μ is loop-induced, conserves CP & flavor, flips chirality.
- Generically expect: $a_\mu^{\text{NP}} \sim a_\mu^{\text{EW}} \times \frac{M_W^2}{\Lambda^2} \times \text{couplings}$
 - the difference between exp-WP20 is large: $\Delta a_\mu = 249 (48) \times 10^{-11} > a_\mu(\text{EW})$
- Will likely be different if using (consolidated) lattice HVP average.
- Tensions between data-driven and lattice HVP results:

- Can new physics hide in the low-energy $\sigma(e^+e^- \rightarrow \pi\pi)$ cross section?
 - **No** [Luzio, et al, arXiv:2112.08312]
- New boson at $\sim 1\text{GeV}$ decays into $\mu^+\mu^-$, e^+e^- , affects $\sigma(e^+e^- \rightarrow \pi\pi)$ indirectly [L. Darmé et al, arXiv:2112.09139]
- Neutral, long-lived hadrons, heretofore undetected? [Farrar, arXiv:2206.13460]
- Z' at $< 1\text{ GeV}$, coupling to 1st gen matter particles [Coyle, Wagner, arXiv:2305.02354]

Charged Lepton Flavor Violation

[Snowmas RPF report, arXiv:2210.04765]



Sensitive probe of New Physics

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{gauge}}^{\text{SM}} + \mathcal{L}_{\text{Higgs}}^{\text{SM}} + \sum_i \frac{1}{\Lambda^{d-4}} C_i \mathcal{O}_i^{d>4}$$

e.g. for $\mu \rightarrow e$

$$\mathcal{L}_{\mu e} = -\frac{4G_F}{\sqrt{2}} \sum_{X=L,R} \left[m_\mu C_{D,X} \bar{e} \sigma^{\alpha\beta} P_X \mu F_{\alpha\beta} + C_{S,XX} \bar{e} P_X \mu \bar{e} P_X e \right. \\ \left. + \sum_{Y \in L,R} C_{V,XY} \bar{e} \gamma^\alpha P_X \mu \bar{e} \gamma_\alpha P_Y e + \sum_{N=p,n} (C_{S,X}^N \bar{e} P_X \mu \bar{N} N + C_{V,X}^N \bar{e} \gamma^\alpha P_X \mu \bar{N} \gamma_\alpha N) \right]$$

[Altmannshofer + Zupan, 2203.07726, Snowmas RPF5 TG, 2209.00142]

Bounds on $\mu \rightarrow e \gamma, \mu \rightarrow eee$, etc.. imply $\Lambda_{\text{NP}} \gtrsim 10^4 \text{ TeV}$ in tension with implications from $\Delta a_\mu = 249(48) \times 10^{-11} > a_\mu(\text{EW})$ obtained using WP20 SM prediction.

LQCD results may imply a much smaller Δa_μ (wait for White Paper 2).

Outlook

★ Experimental program beyond 2025:

- 📍 J-PARC: Muon $g-2$ /EDM
- 📍 CERN: MUonE
- 📍 Fermilab: future muon campus experiments?
- 📍 Belle II, BESIII, Novosibirsk,...
- 📍 Chiral Belle (?)

★ Data-driven/dispersive program beyond 2025:

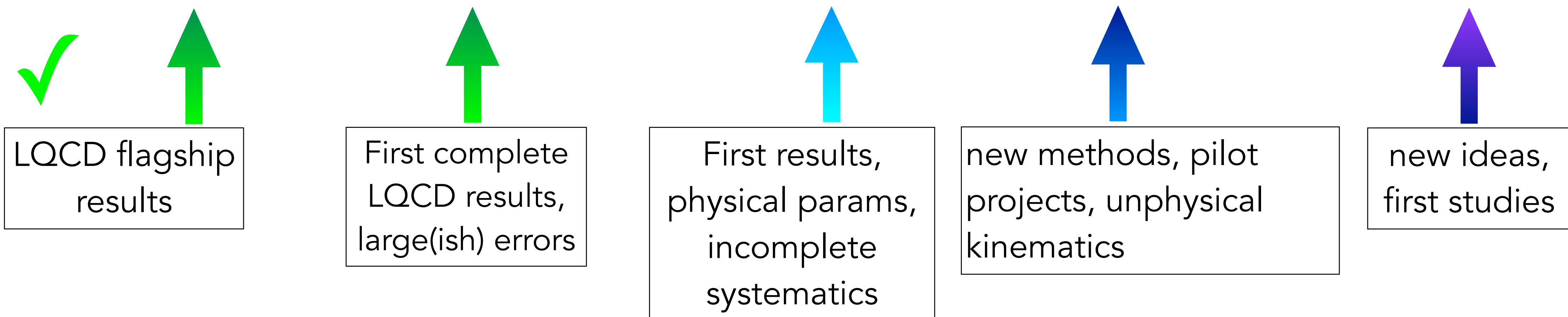
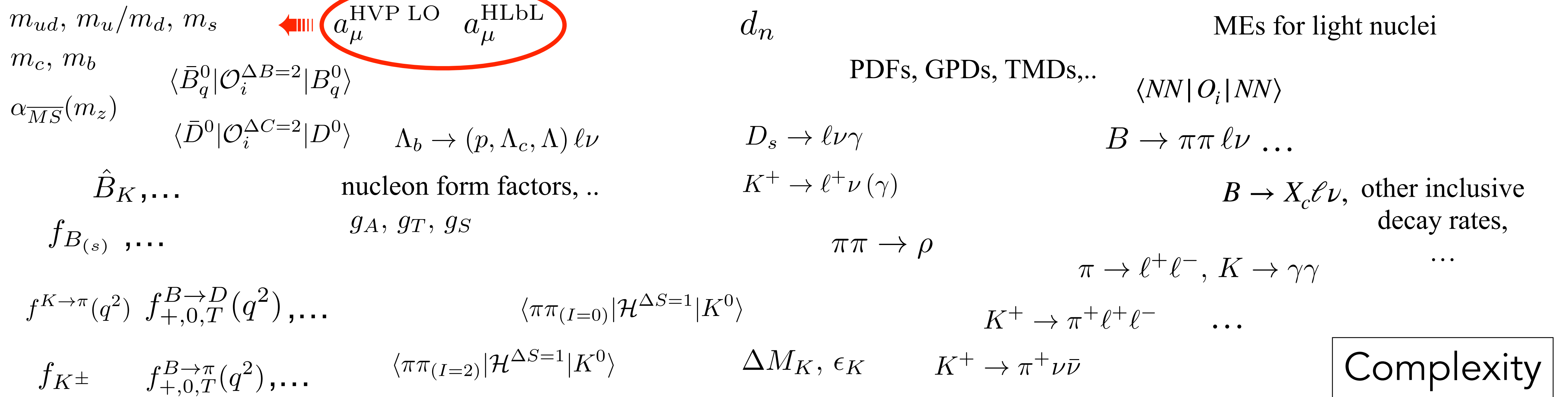
- 📍 development of NNLO MC generators
- 📍 for HLbL, improved experimental/lattice inputs together with further development of dispersive approach

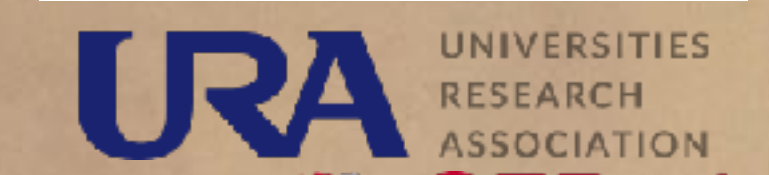
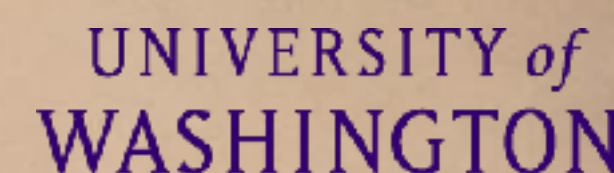
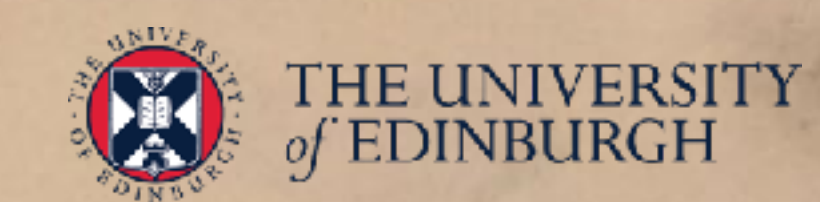
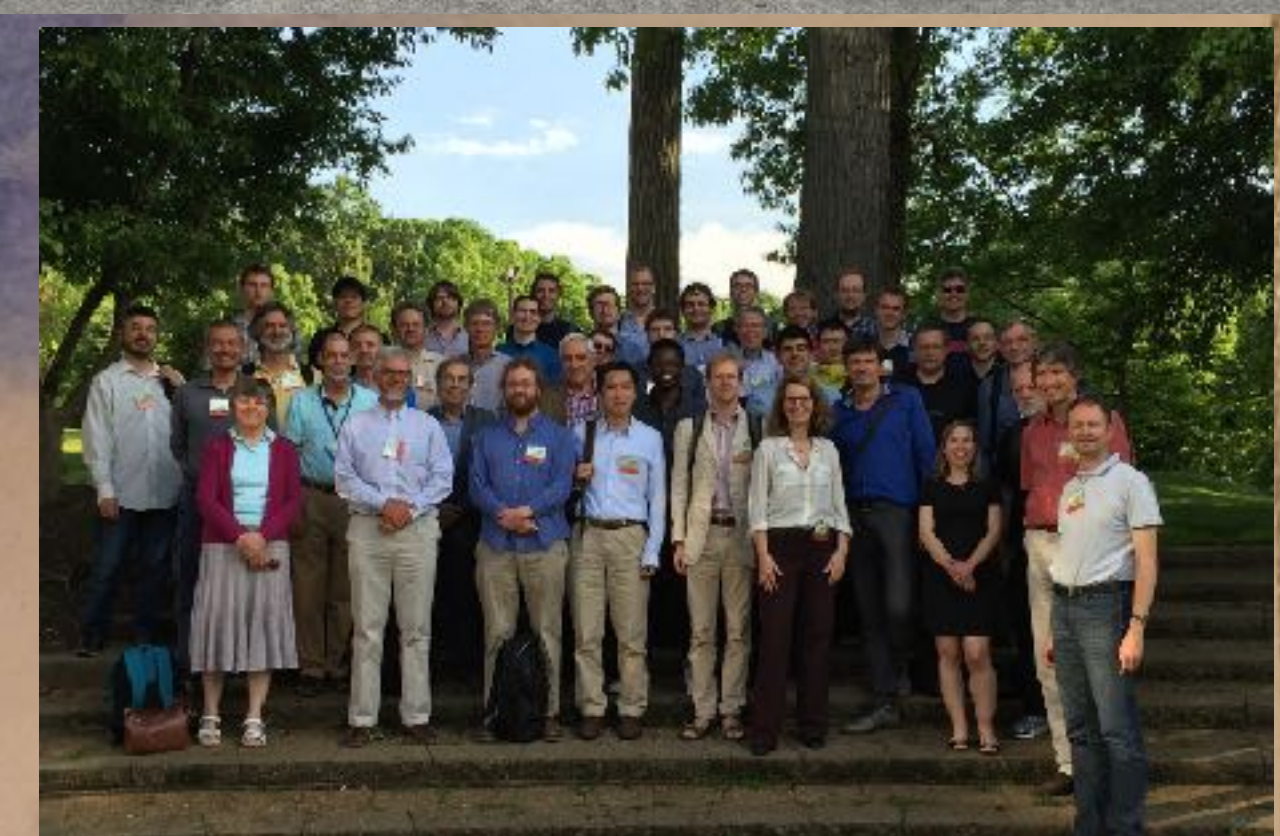
★ MUonE will provide a space-like determination of HVP

★ Lattice QCD beyond 2025:

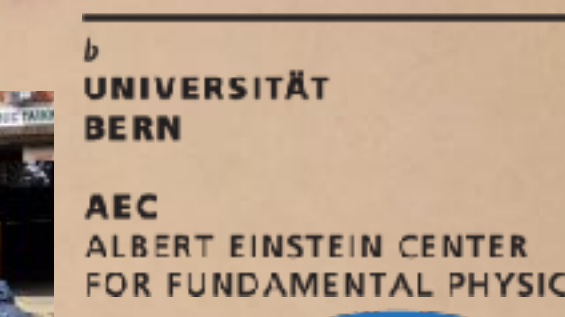
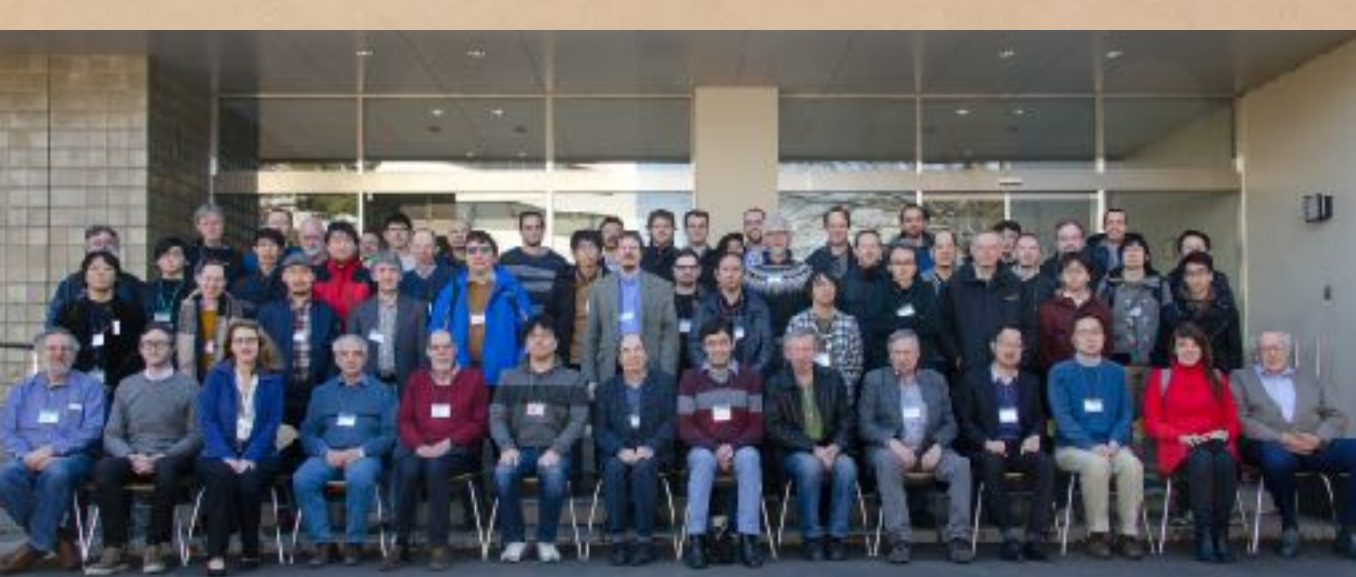
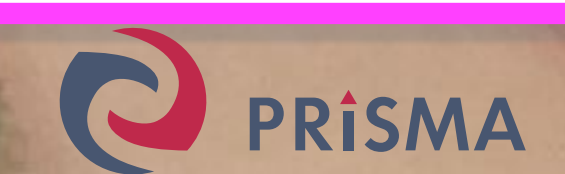
- 📍 access to future computational resources (coming Exascale) will enable improvements of all errors (statistical and systematic)
- 📍 concurrent development of better methods and algorithms (gauge-field sampling, noise reduction) will accelerate progress
- 📍 **beyond $g-2$** : a rich program relevant for all areas of HEP

Outlook





Thank you!

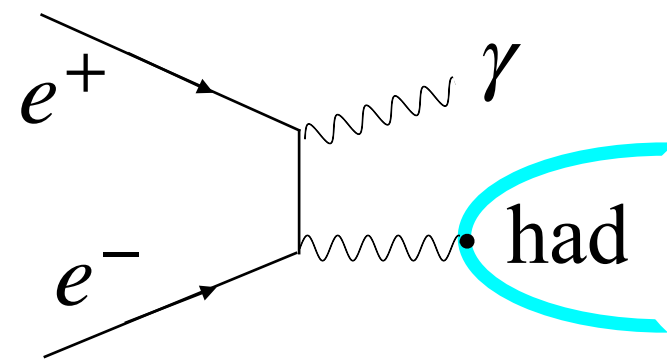


Appendix

Experimental Inputs to HVP

★ two exp. approaches

- “Direct scan”: change CM energy of e^+e^- beams
- “Radiative Return”: with fixed e^+e^- CM energy, select events with initial state radiation (ISR)

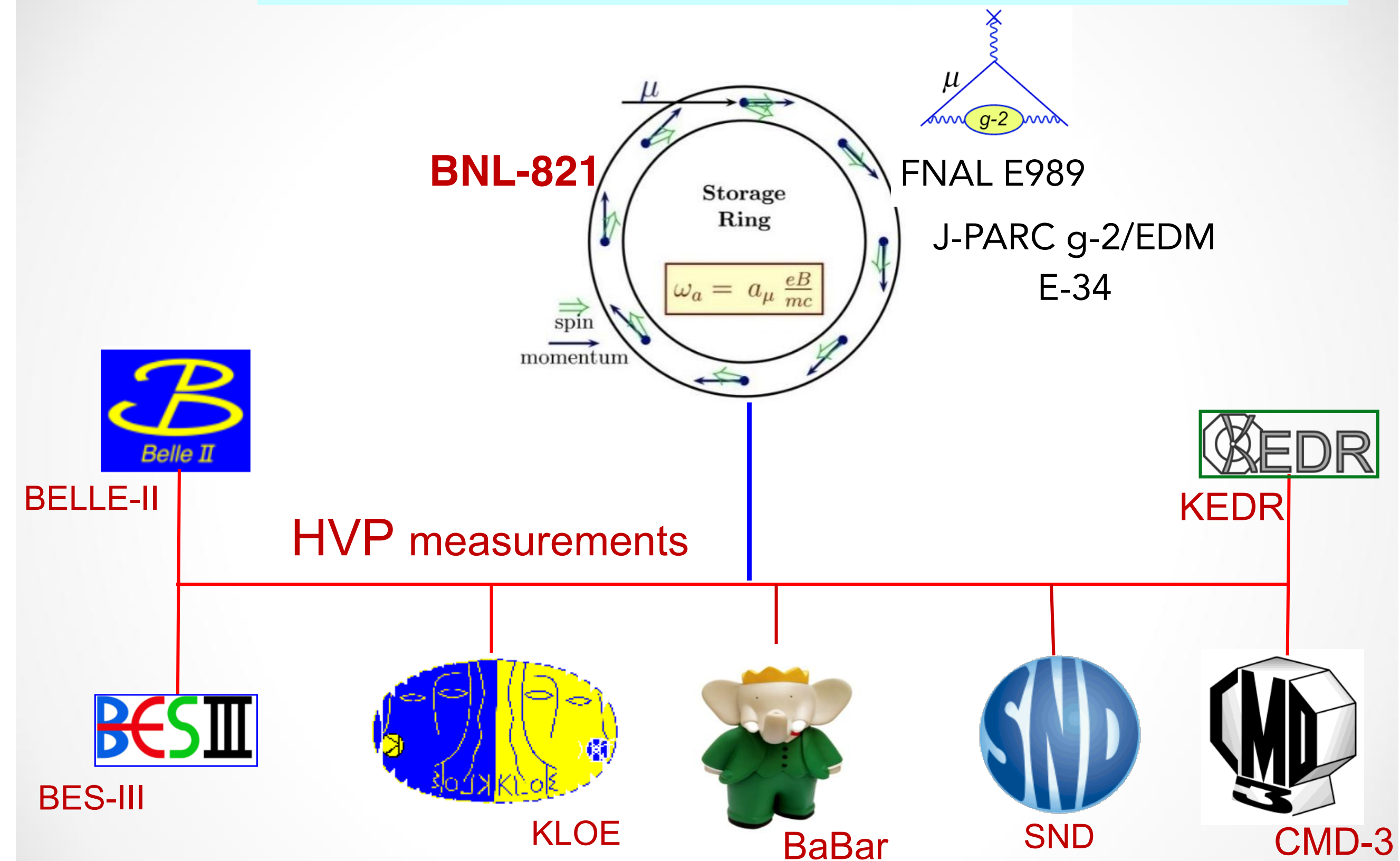


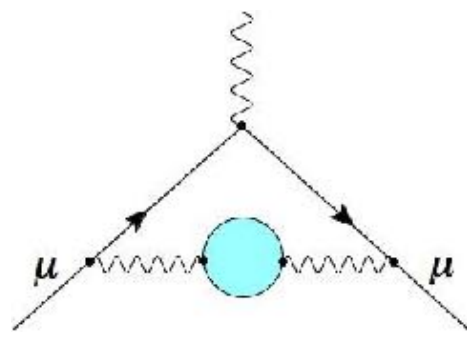
★ complemented by:

- MC generators for $\sigma_{\text{had}}(s)$ (e.g. PHOKARA)
- detailed studies of radiative corrections (now known through NLO)

S. Serednyakov (for SND) @ HVP KEK workshop

e^+e^- facilities involved in HVP measurement





HVP: data-driven

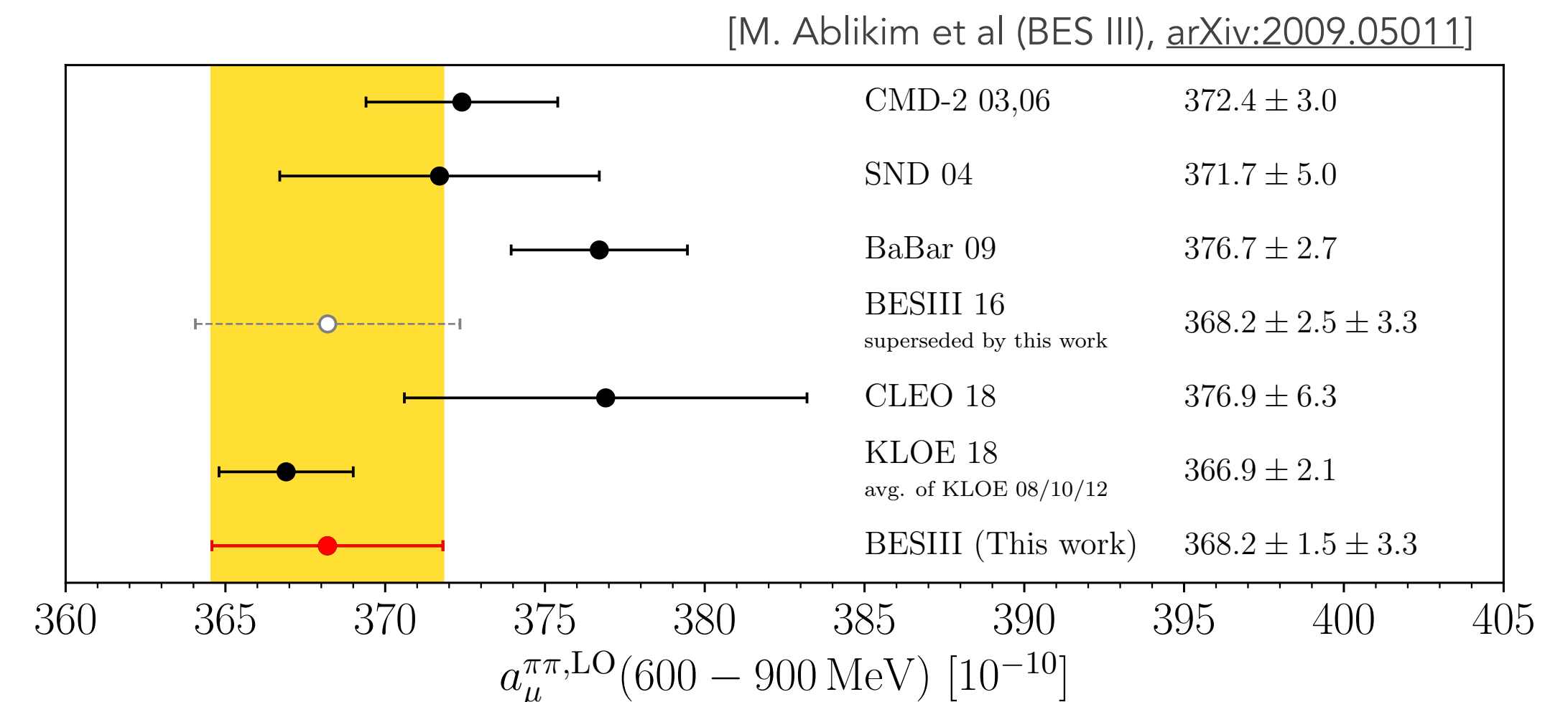
2020 White Paper [T. Aoyama et al, [arXiv:2006.04822](https://arxiv.org/abs/2006.04822), Phys. Repts. 887 (2020) 1-166.]

Conservative merging procedure to obtain a realistic assessment of the underlying uncertainties:

- account for tensions between data sets
- account for differences in methodologies for compilation of experimental inputs and treatment of correlations between systematic errors
- include results using constraints from unitarity & analyticity in $\pi\pi$ and $\pi\pi\pi$ channels
[Colangelo et al, 2018; Anantharayan et al, 2018; Davier et al, 2019; Hoferichter et al, 2019]
- Full NLO radiative corrections [Campanario et al, 2019]

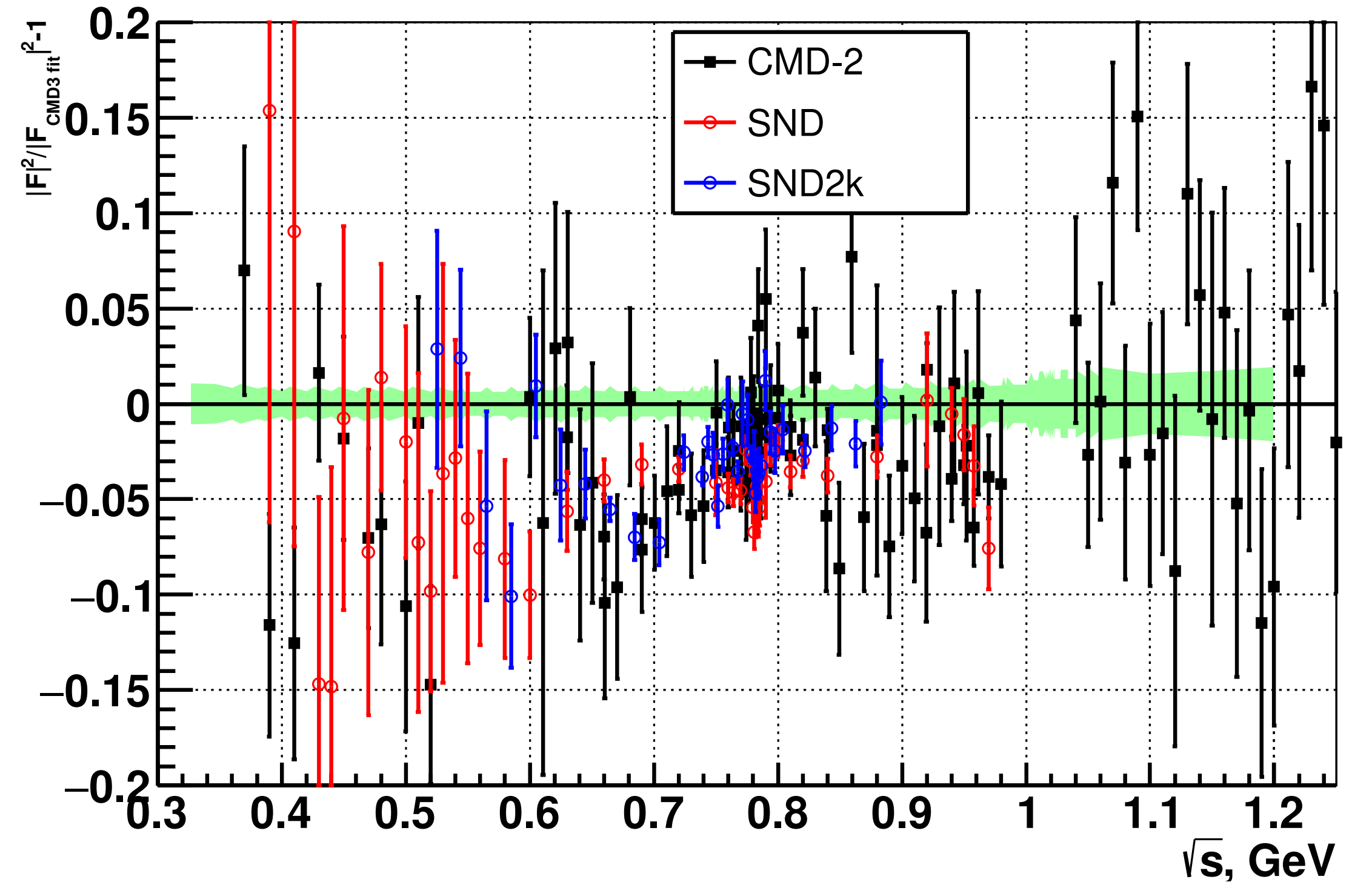
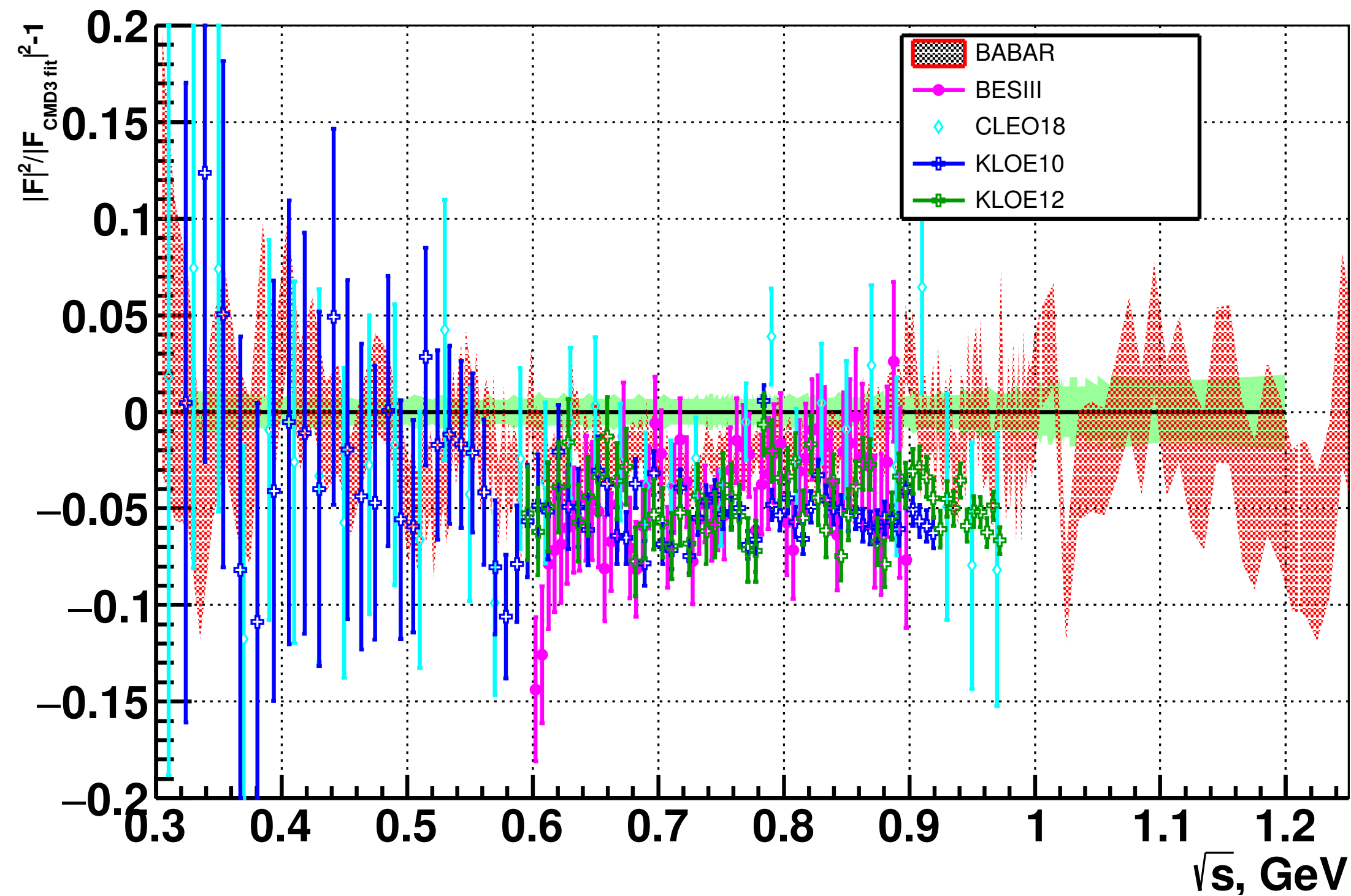
$$a_{\mu}^{\text{HVP,LO}} = 693.1 (2.8)_{\text{exp}} (0.7)_{\text{DV+pQCD}} (2.8)_{\text{BaBar-KLOE}} \times 10^{-10}$$

$$= 693.1 (4.0) \times 10^{-10}$$

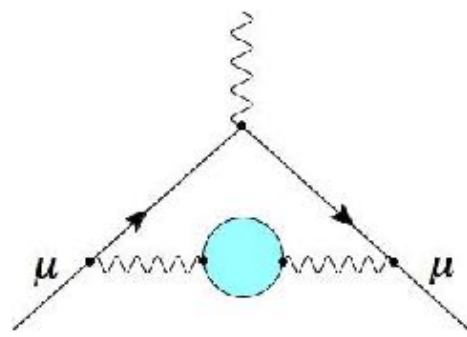


cross section comparisons

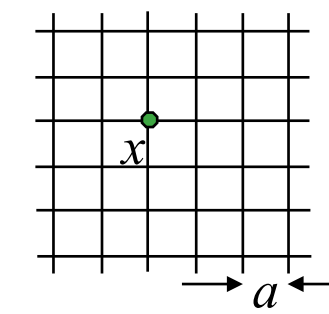
[CMD-3, F. Ignatov et al, [arXiv:2302.08834](https://arxiv.org/abs/2302.08834), PRD2024]



- For $\sqrt{s} \lesssim 0.6$ GeV: good consistency between cross section measurements
- For 0.6 GeV $\lesssim \sqrt{s} \lesssim 1$ GeV: significant differences between measurements



Lattice HVP: Introduction



Leading order HVP contribution:

$$a_{\mu}^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2) \hat{\Pi}(q^2)$$

[B. Lautrup, A. Peterman, E. de Rafael, Phys. Rep 1972; E. de Rafael, Phys. Let. B 1994; T. Blum, PRL 2002]

- Calculate $a_{\mu}^{\text{HVP,LO}}$ in Lattice QCD

Start with correlation function of EM currents: $C(t) = \frac{1}{3} \sum_{i,x} \langle j_i^{\text{EM}}(x,t) j_i^{\text{EM}}(0,0) \rangle$ $j_{\mu}^{\text{EM}} = \sum_f q_f \bar{\psi}_f(x,t) \gamma_{\mu} \psi_f(x,t)$
 $f = u, d, s, c, \dots$

Fourier transform yields $\hat{\Pi}(Q^2) = 4\pi^2 \int_0^{\infty} dt C(t) \left[t^2 - \frac{4}{Q^2} \sin^2 \left(\frac{Qt}{2} \right) \right]$

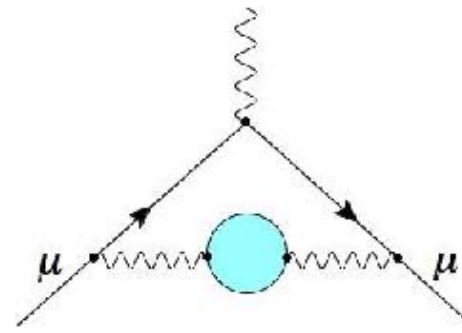
[D. Bernecker and H. Meyer, arXiv:1107.4388, EPJA 2011]

so that $a_{\mu}^{\text{HVP,LO}}$ can be obtained as an integral over Euclidean time, aka time momentum representation (TMR):

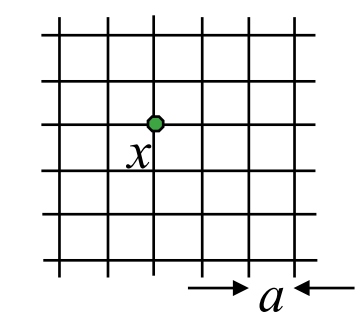
$$a_{\mu}^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^{\infty} dQ^2 w(Q^2) \hat{\Pi}(Q^2) = 4\alpha^2 \int_0^{\infty} dt C(t) \int_0^{\infty} dQ^2 w(Q^2) \left[t^2 - \frac{4}{Q^2} \sin^2 \left(\frac{Qt}{2} \right) \right]$$



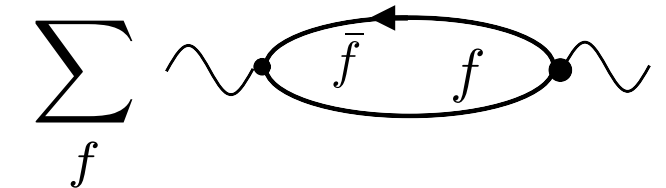
$$a_{\mu}^{\text{HVP,LO}} = 4\alpha^2 \int_0^{\infty} dt C(t) \tilde{w}(t)$$



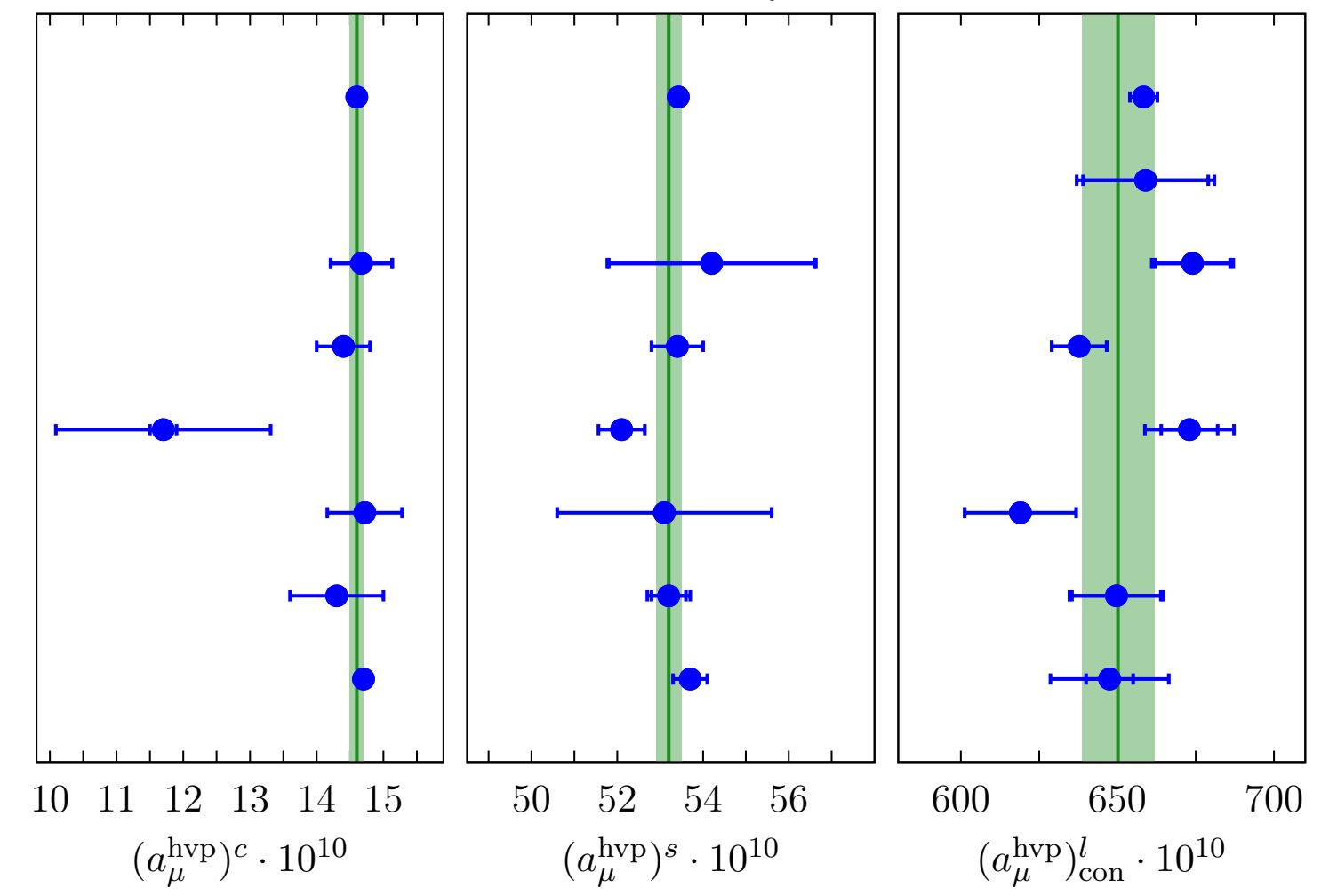
Lattice HVP: summary of contributions



H. Wittig @ Lattice HVP workshop

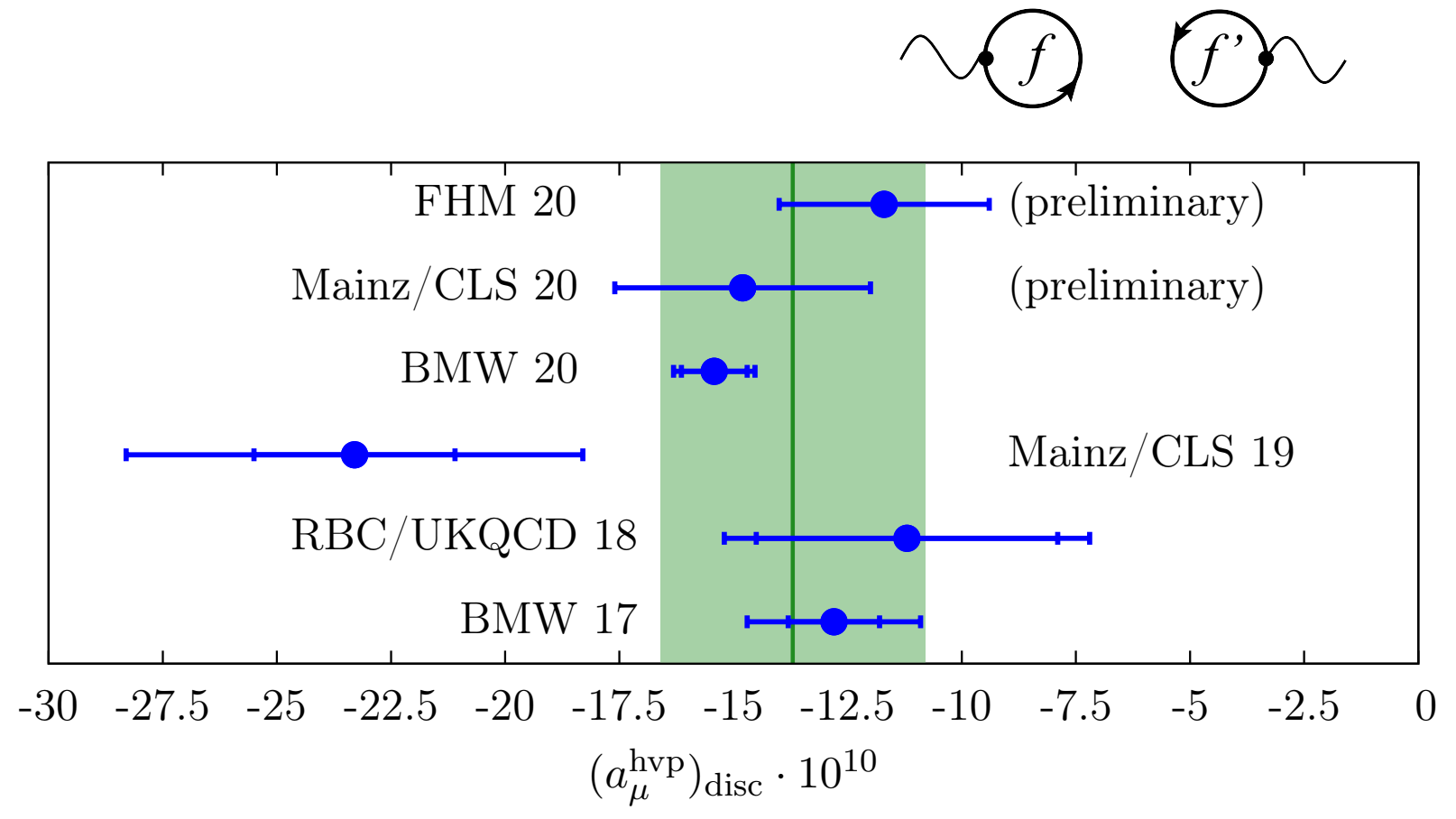


- Charm, strange contributions already well determined.
- Mild tensions for light contribution

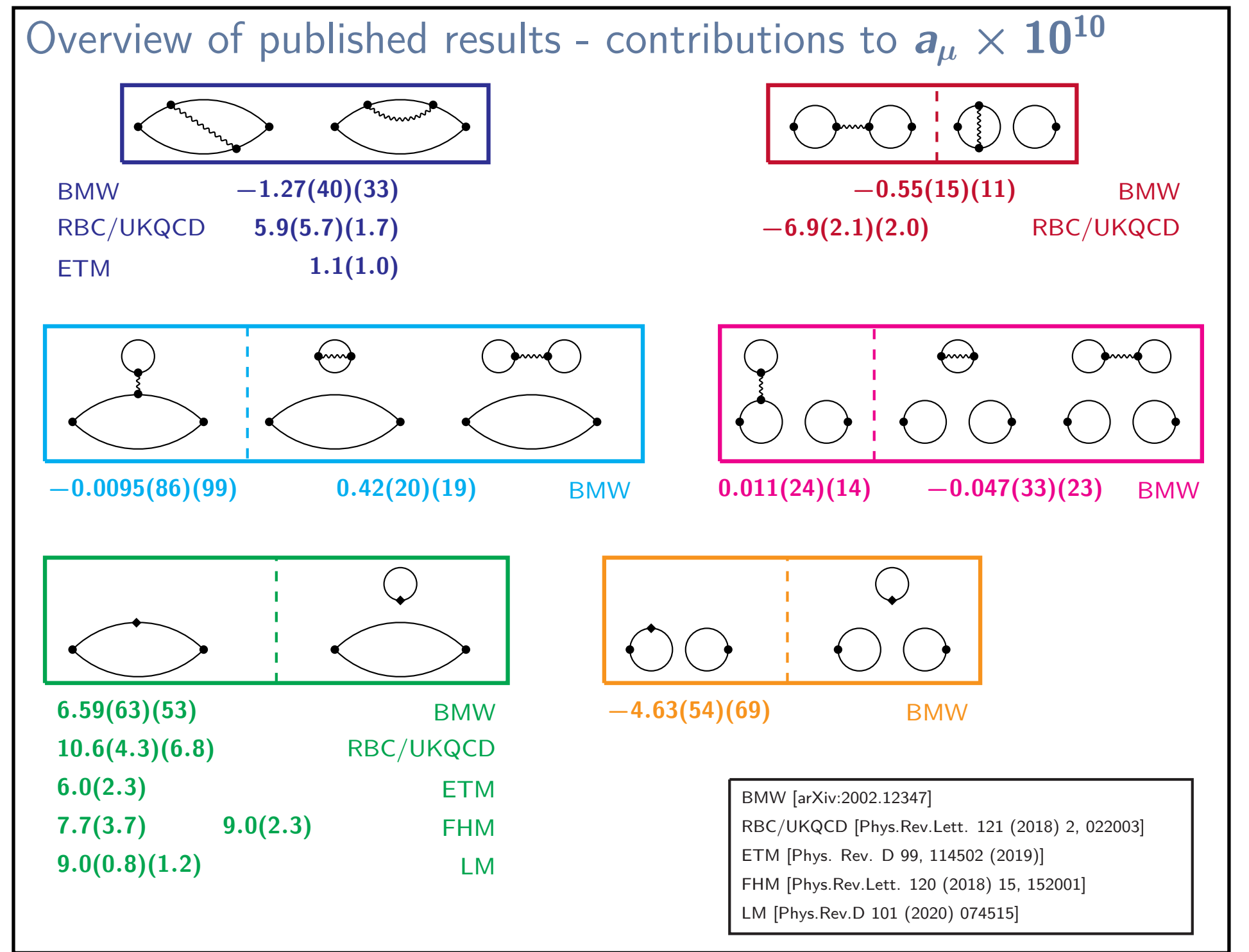


- BMW 20
- Aubin et al. 19
- Mainz/CLS 19
- FHM 19
- PACS 19
- ETMC 19
- RBC/UKQCD 18
- BMW 17

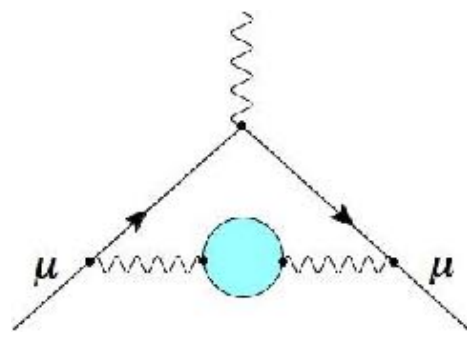
Consistent results with increasing precision



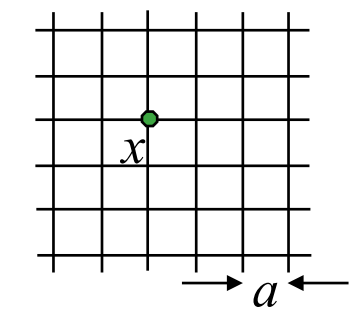
V. Gülpers @ Lattice HVP workshop



- Some tensions between lattice results for individual contributions.
- Large cancellations between individual contributions:
 $\delta a_\mu^{\text{IB}} \lesssim 1\%$



Lattice HVP: long-distance tail



$$C(t) = \frac{1}{3} \sum_{i,x} \langle j_i^{\text{EM}}(x,t) j_i^{\text{EM}}(0,0) \rangle$$

- Start with spectral decomposition: $C(t) = \sum_{n=0}^{\infty} |A_n|^2 e^{-E_n t}$

♦ bounding method:

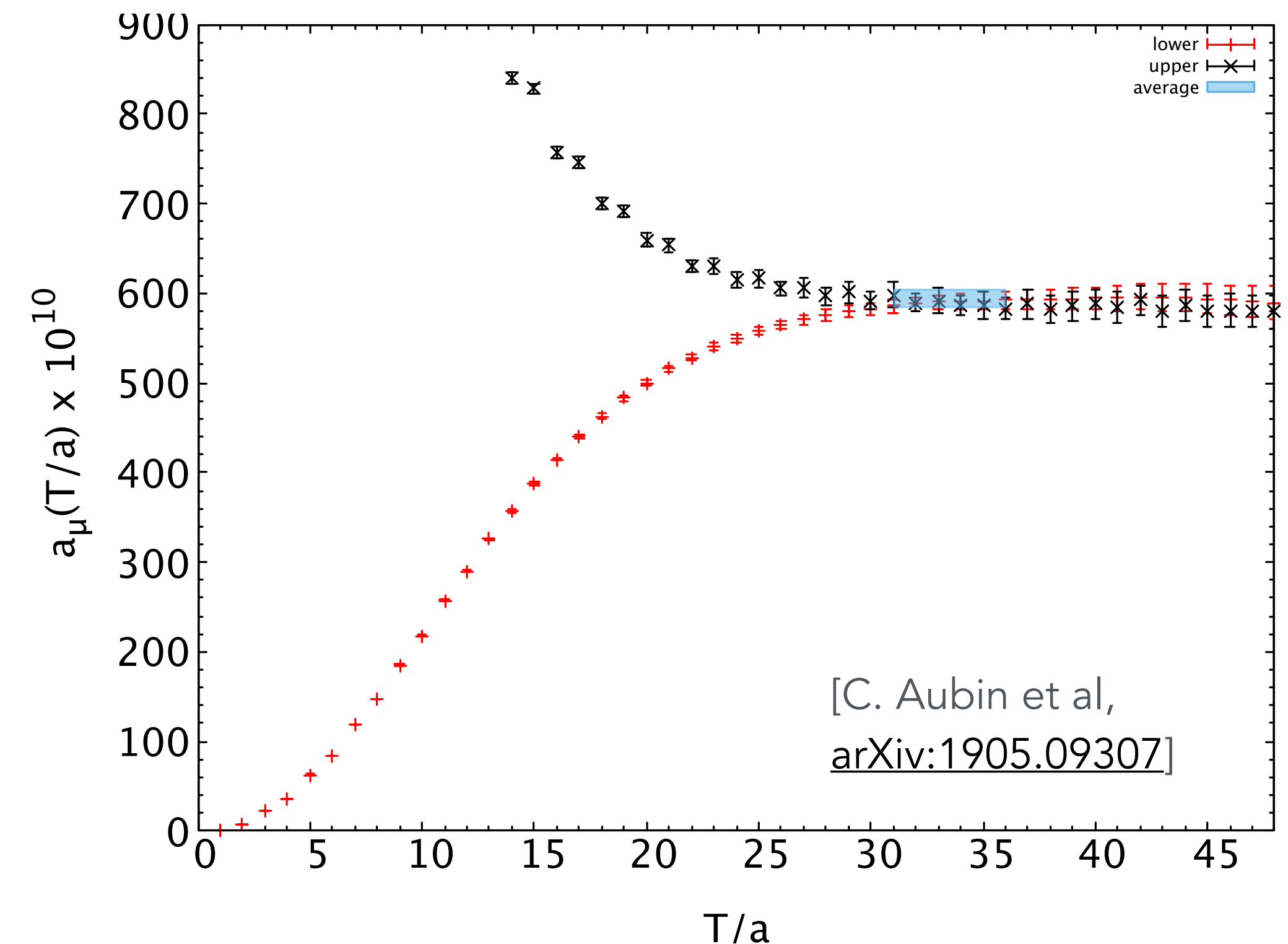
[Borsanyi et al, [PRL 2018](#), Blum et al, [PRL 2018](#)]

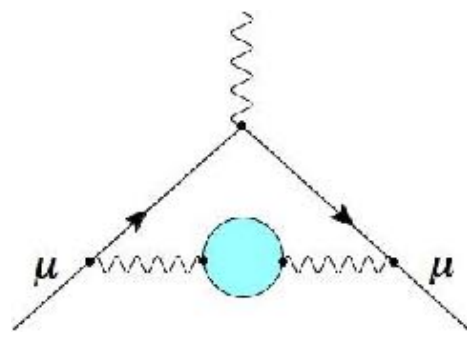
$$\text{for } t > t_c: 0 \leq C(t_c) e^{-\bar{E}_{t_c}(t-t_c)} \leq C(t) \leq C(t_c) e^{-E_0(t-t_c)}$$

\bar{E}_{t_c} : effective mass of $C(t)$ at t_c

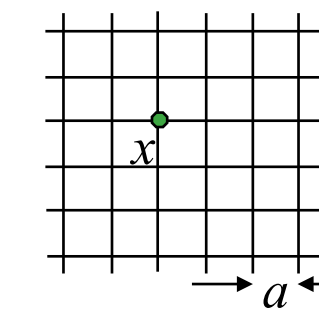
E_0 : ground state energy

replace $G(t > t_c)$ with upper and lower bound, vary t_c





Lattice HVP: long-distance tail (again)



$$C(t) = \frac{1}{3} \sum_{i,x} \langle j_i^{\text{EM}}(x,t) j_i^{\text{EM}}(0,0) \rangle$$

- Start with spectral decomposition: $C(t) = \sum_{n=0}^{\infty} |A_n|^2 e^{-E_n t}$

- obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states

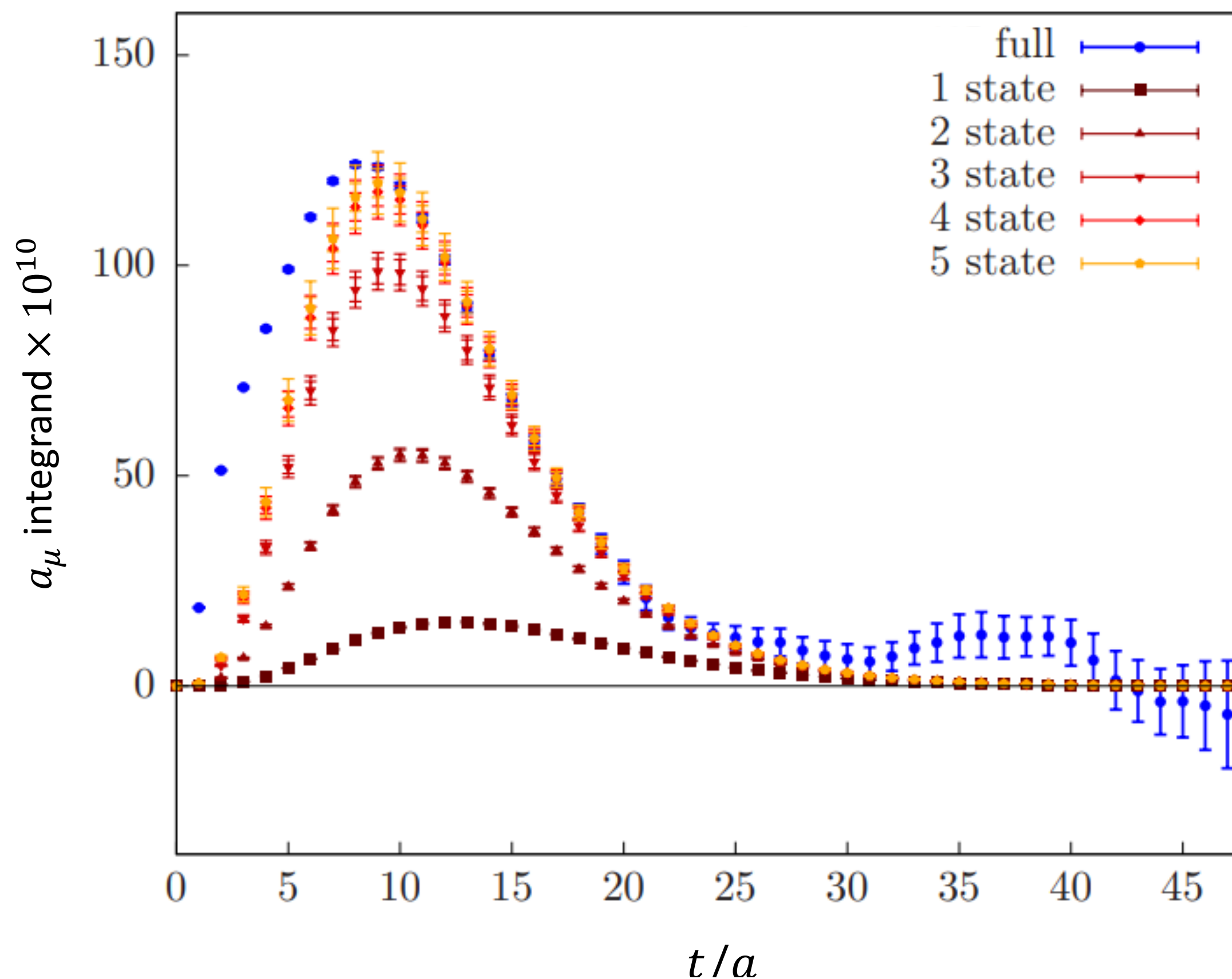
- use to reconstruct $C(t > t_c)$

- can be used to improve bounding method:

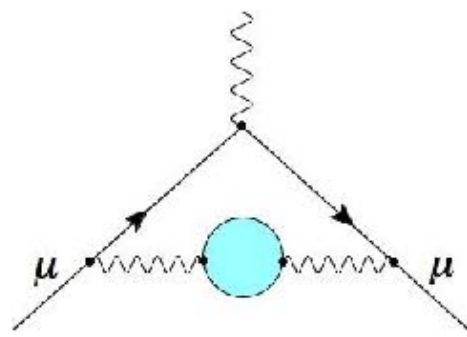
$$C(t) \rightarrow C(t) - \sum_{n=0}^N A_n^2 e^{-E_n t}$$

use E_{N+1} in upper bound

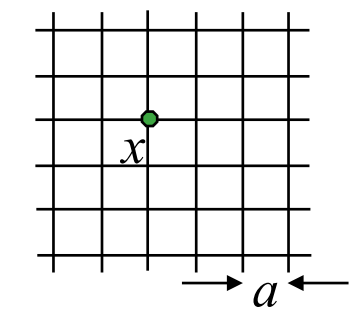
- yields big reduction in stat. errors (compared with bounding method)



J. Mckeon @ Lattice 2024
(RBC/UKQCD)
See also:
[Bruno et al, RBC/UKQCD,
arXiv:1910.11745]



Lattice HVP: long-distance tail



$$C(t) = \frac{1}{3} \sum_{i,x} \langle j_i^{\text{EM}}(x,t) j_i^{\text{EM}}(0,0) \rangle$$

- Start with spectral decomposition: $C(t) = \sum_{n=0}^{\infty} |A_n|^2 e^{-E_n t}$

- obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states

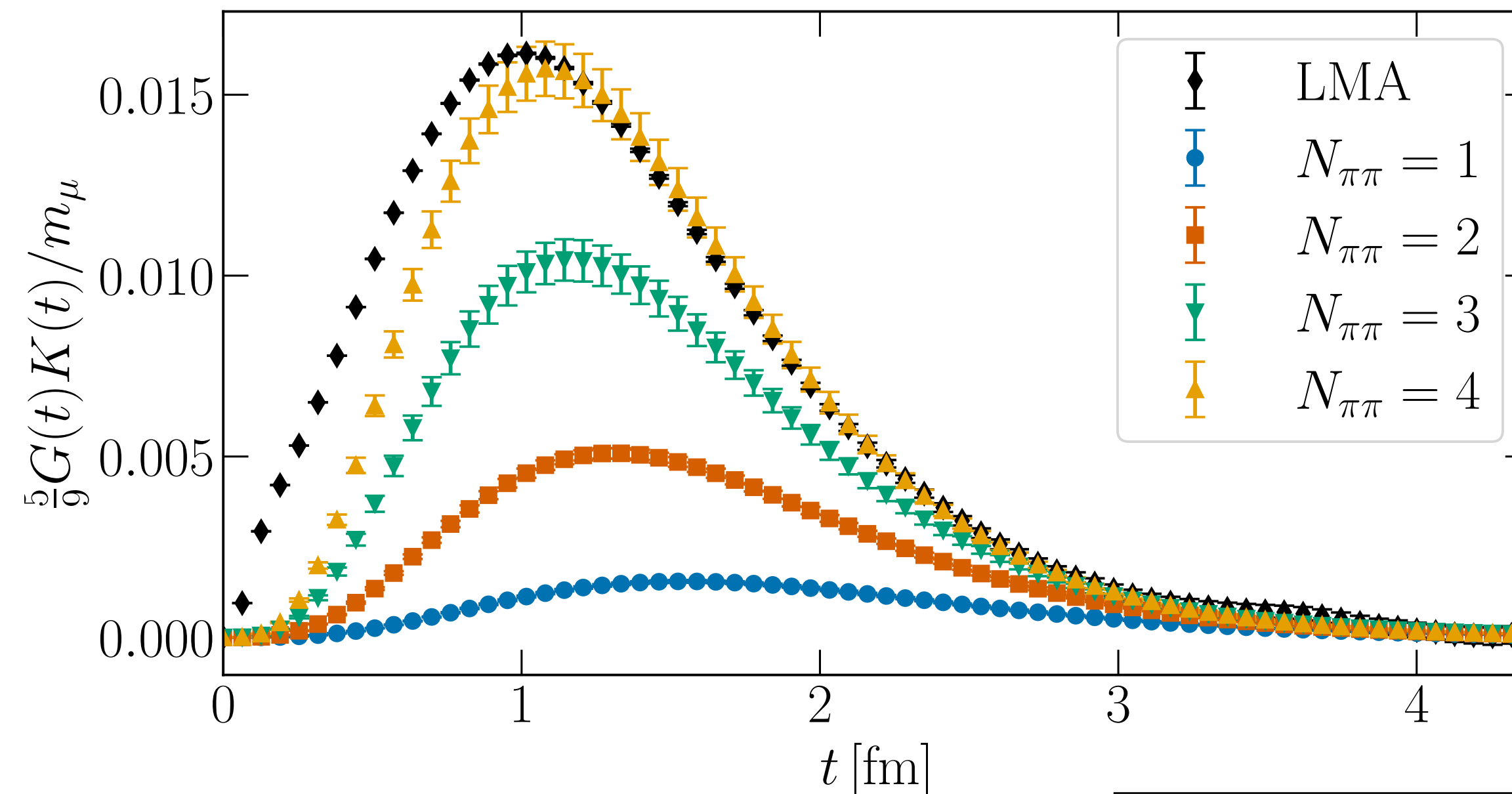
- use to reconstruct $C(t > t_c)$

- can be used to improve bounding method:

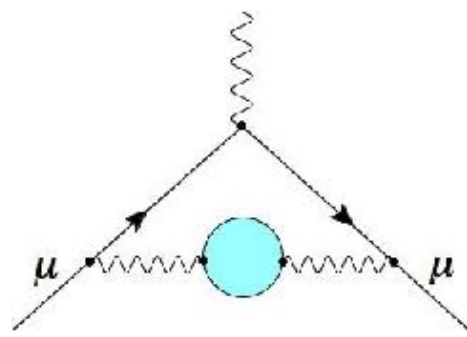
$$C(t) \rightarrow C(t) - \sum_{n=0}^N A_n^2 e^{-E_n t}$$

use E_{N+1} in upper bound

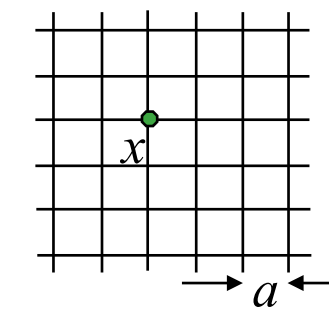
- yields $\sim \times 2$ reduction in stat. errors



S. Kuberski & N. Miller @ Lattice 2024 (Mainz)
See also: A. Gerardin et al, [PRD 2019](#)



Lattice HVP: long-distance tail



$$C(t) = \frac{1}{3} \sum_{i,x} \langle j_i^{\text{EM}}(x,t) j_i^{\text{EM}}(0,0) \rangle$$

- Start with spectral decomposition: $C(t) = \sum_{n=0}^{\infty} |A_n|^2 e^{-E_n t}$

- obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states

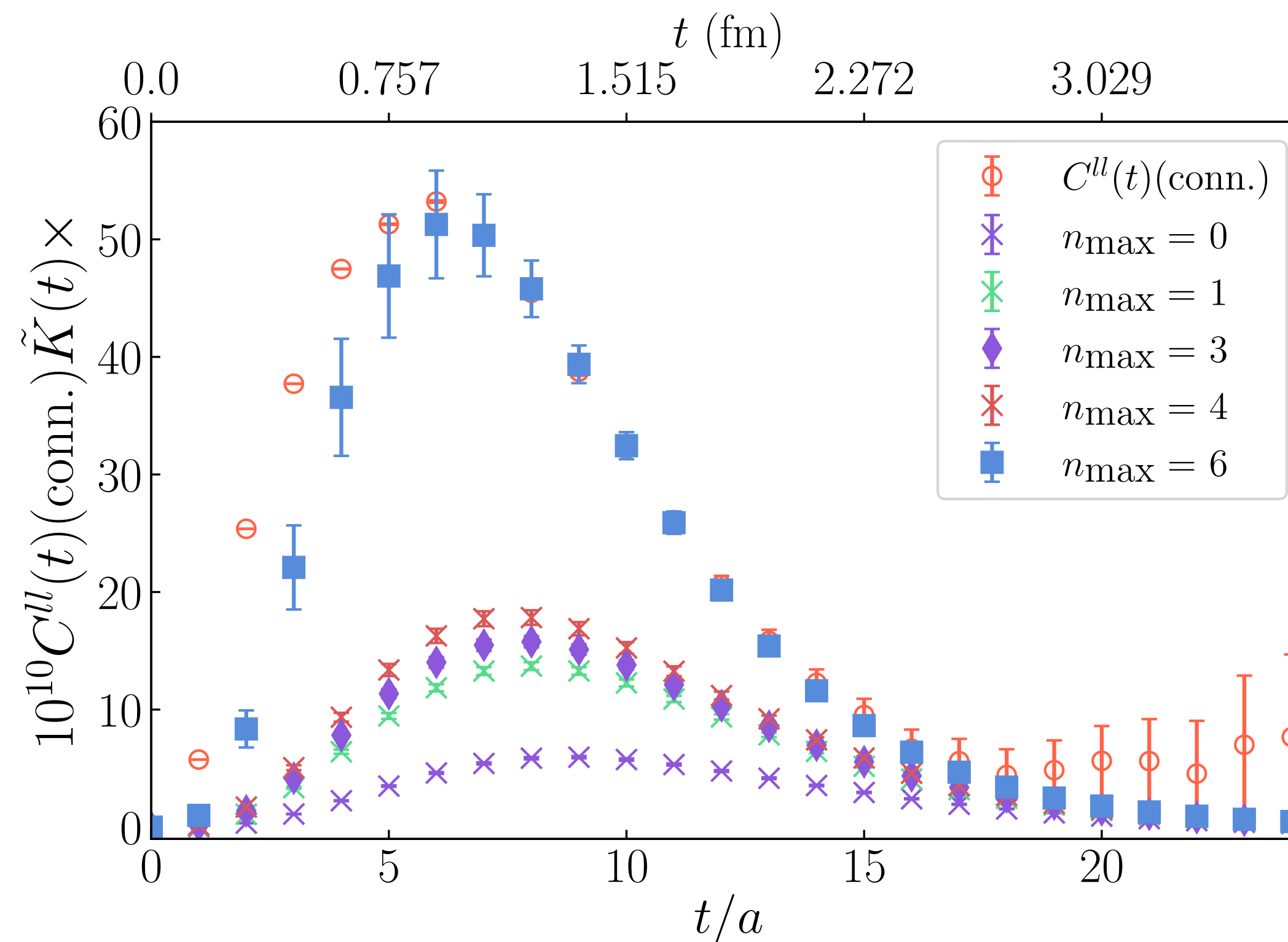
- use to reconstruct $C(t > t_c)$

- can be used to improve bounding method:

$$C(t) \rightarrow C(t) - \sum_{n=0}^N A_n^2 e^{-E_n t}$$

use E_{N+1} in upper bound

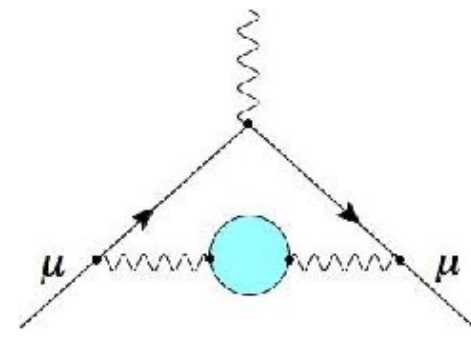
- yields $\sim \times 2.5$ reduction in stat. errors (compared with bounding method)



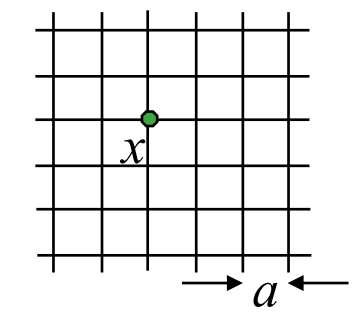
[Lahert, et al, arXiv:2409.00756]
see also:
[Lahert et al, arXiv:2112.11647]

First LQCD calculation
with staggered multi-pion
operators

see also:
[Frech for BMW @ Lattice 2024]



Long-distance tail (ud)



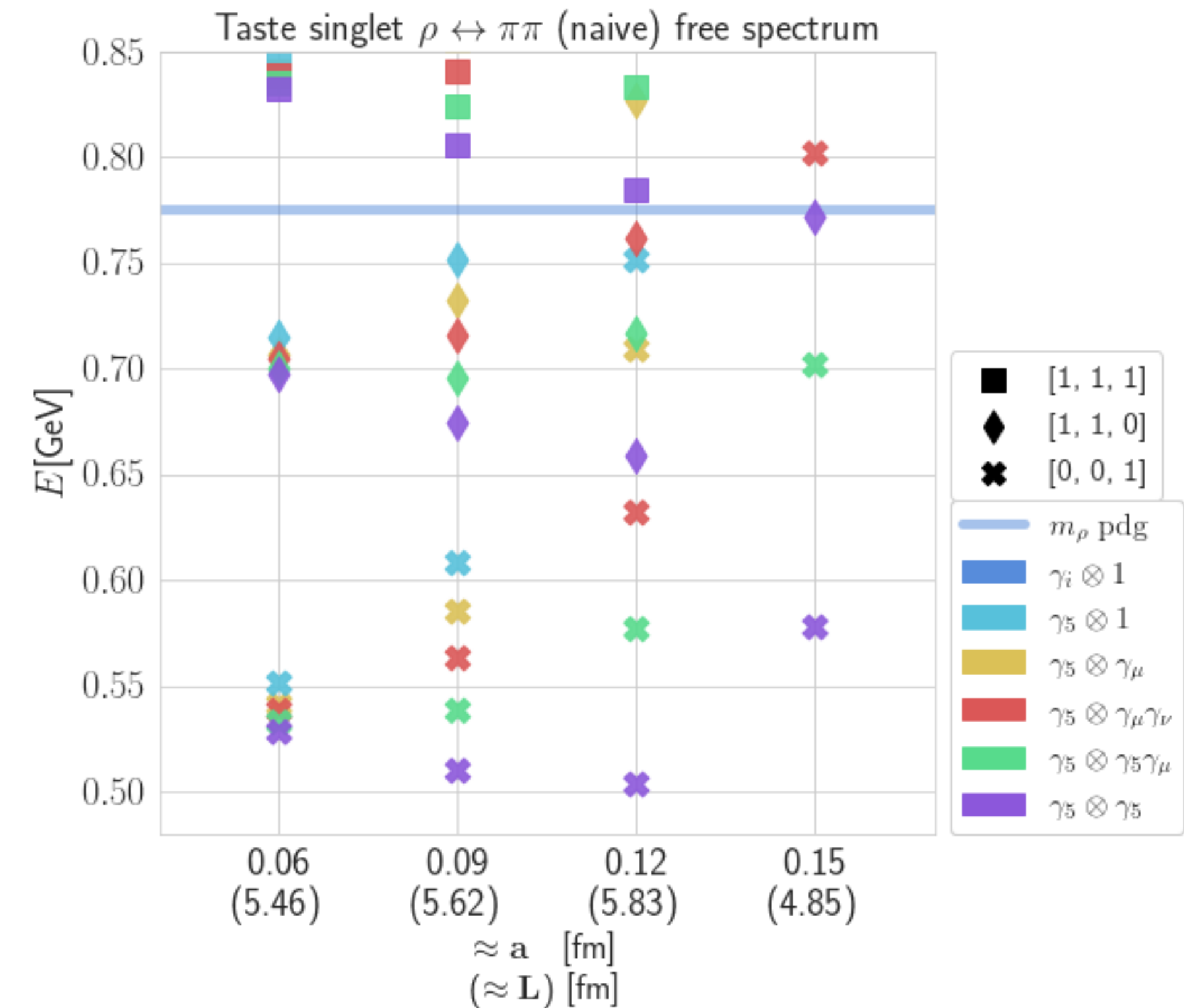
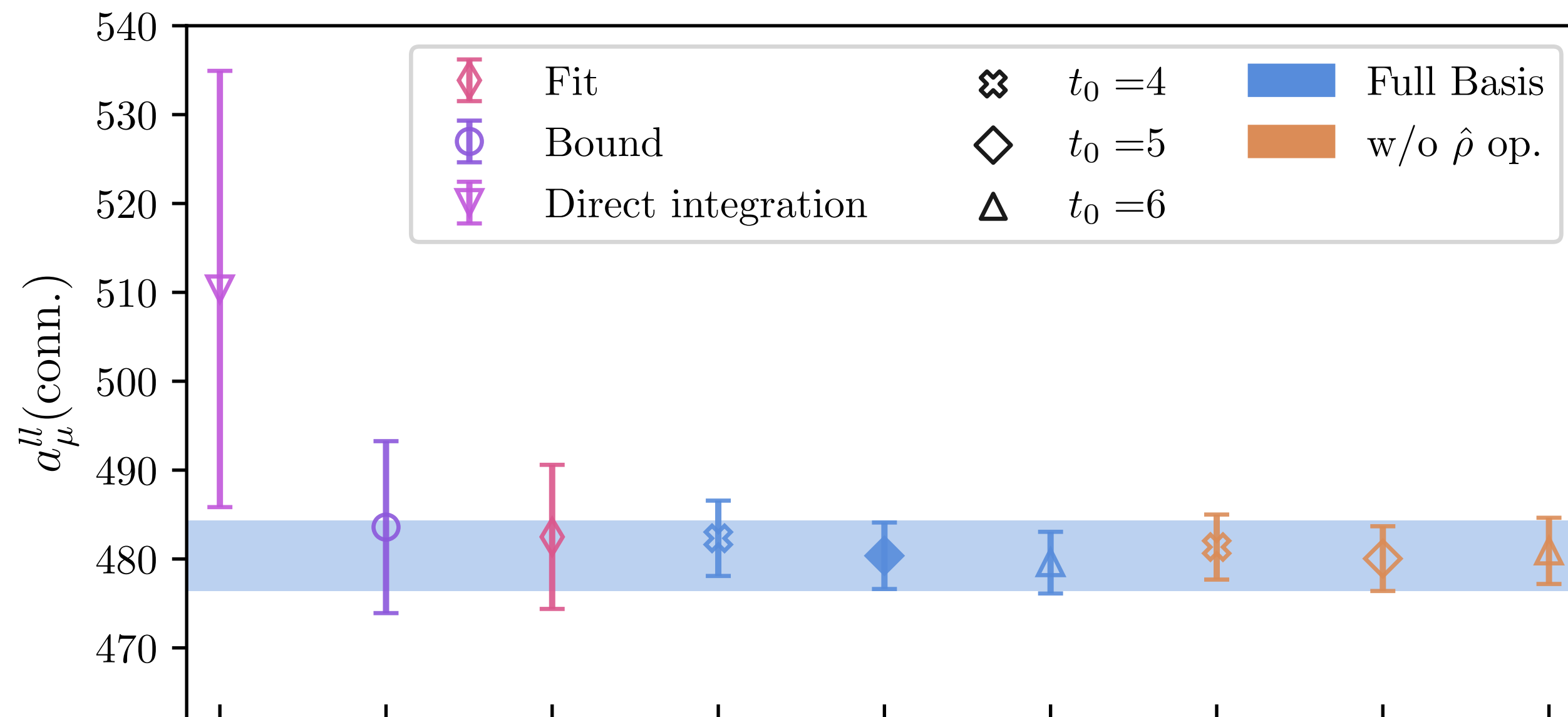
[Shaun Lahert et al, arXiv: 2409.00756]

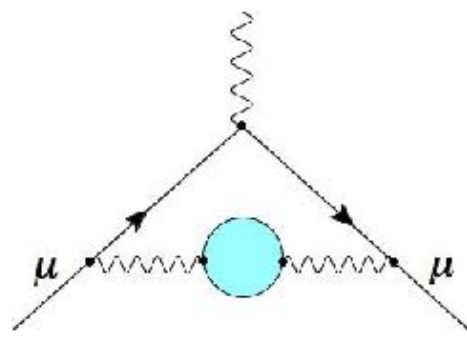
$$C(t) = \frac{1}{3} \sum_{i,x} \langle j_i^{\text{EM}}(x,t) j_i^{\text{EM}}(0,0) \rangle$$

• Spectral reconstruction: $C(t) = \sum_{n=0}^{\infty} |A_n|^2 e^{-E_n t}$

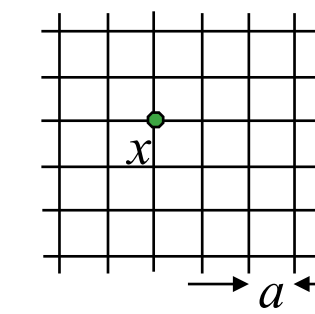
- ♦ obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states
- ♦ First LQCD calculation with staggered multi-pion operators
- ♦ Construct matrix of correlators (2,3,4-point functions)
- ♦ Use GEVP to obtain energies and amplitudes for $\pi\pi$ states
- ♦ Reconstruct vector-current correlator

$$\mathbf{C}(t) = \begin{pmatrix} C(t)_{J,\vec{J} \rightarrow J,\vec{J}} & C(t)_{J,\vec{J} \rightarrow \pi\pi} \\ C(t)_{\pi\pi \rightarrow J,\vec{J}} & C(t)_{\pi\pi \rightarrow \pi\pi} \end{pmatrix}$$





Lattice HVP: results



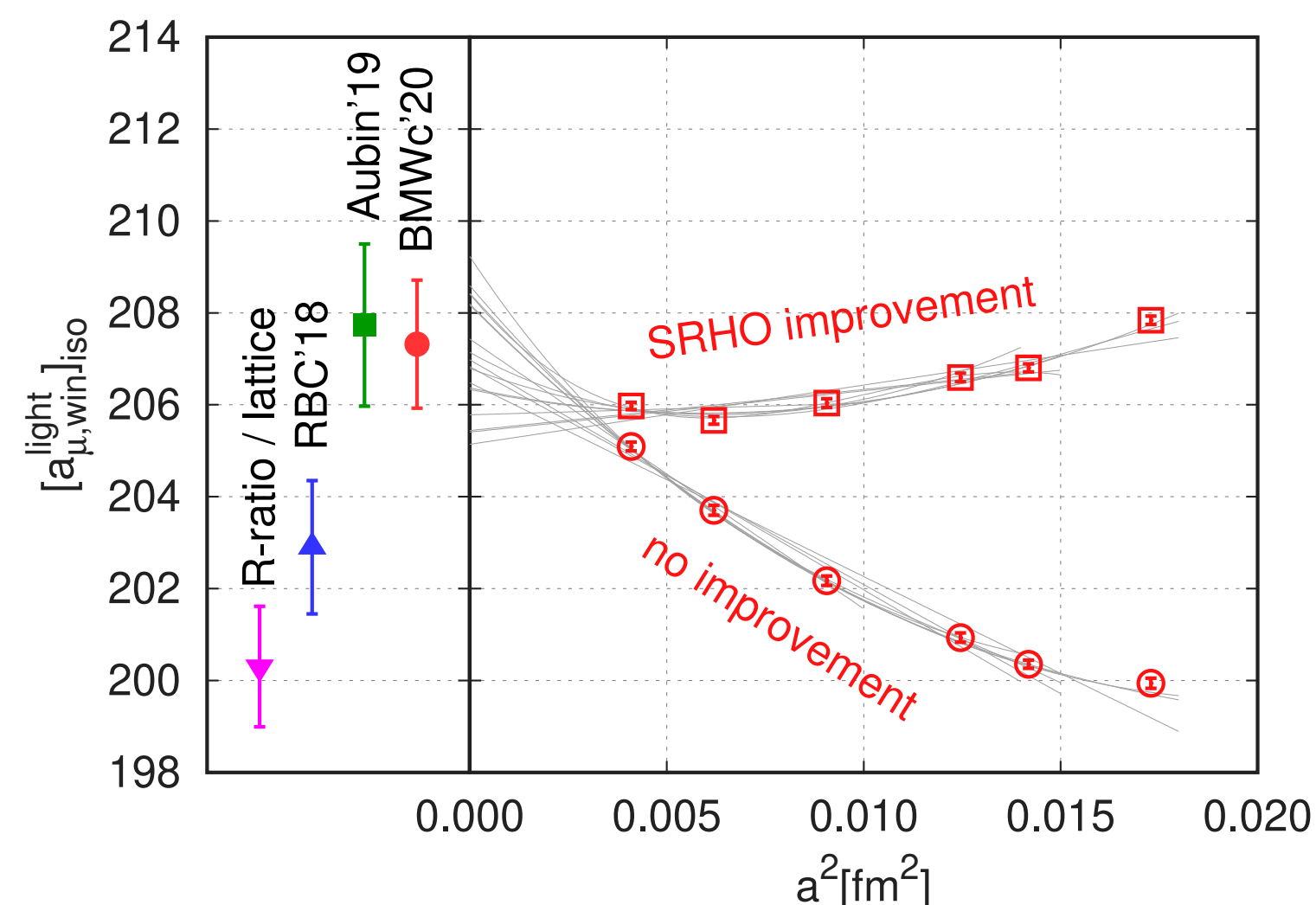
In 2020 WP:

- Lattice HVP average at 2.6 % total uncertainty: $a_{\mu}^{\text{HVP,LO}} = 711.6(18.4) \times 10^{10}$
- BMW 20 [Sz. Borsanyi et al, arXiv:2002.12347, 2021 Nature] first LQCD calculation with sub-percent (0.8 %) error in tension with data-driven HVP (2.1 σ)

- Further tensions for intermediate window:

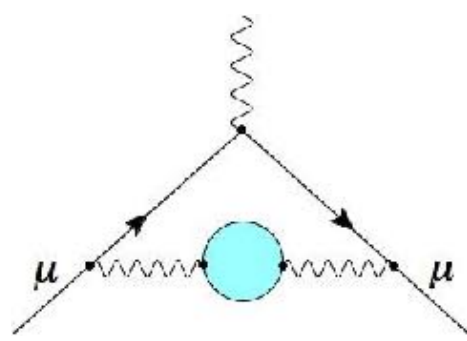
-3.7 σ tension with data-driven evaluation

-2.2 σ tension with RBC/UKQCD18

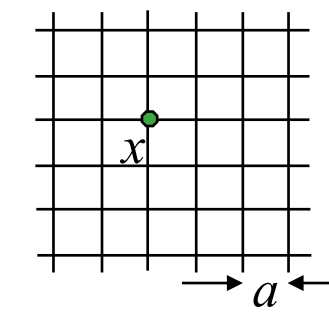


Staggered fermions:

- taste-breaking effects (which yield taste splittings) are significant (sometimes dominant) source of discretization errors
- possible to use EFT schemes (ChPT, Chiral Model, MLLGS) to correct for taste-splitting effects before taking continuum extrapolation: continuum limit should not be affected



Lattice HVP: intermediate window (W)

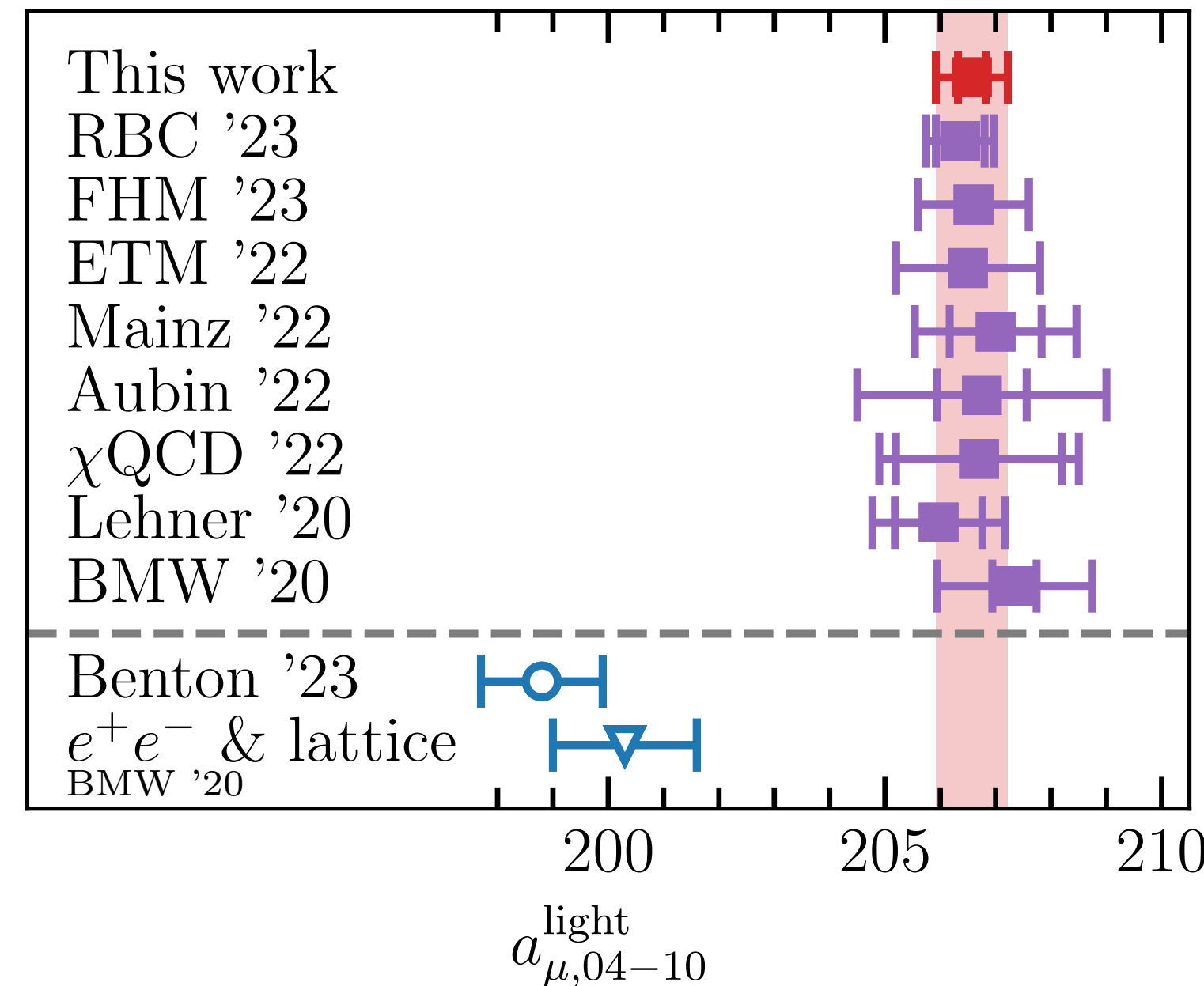
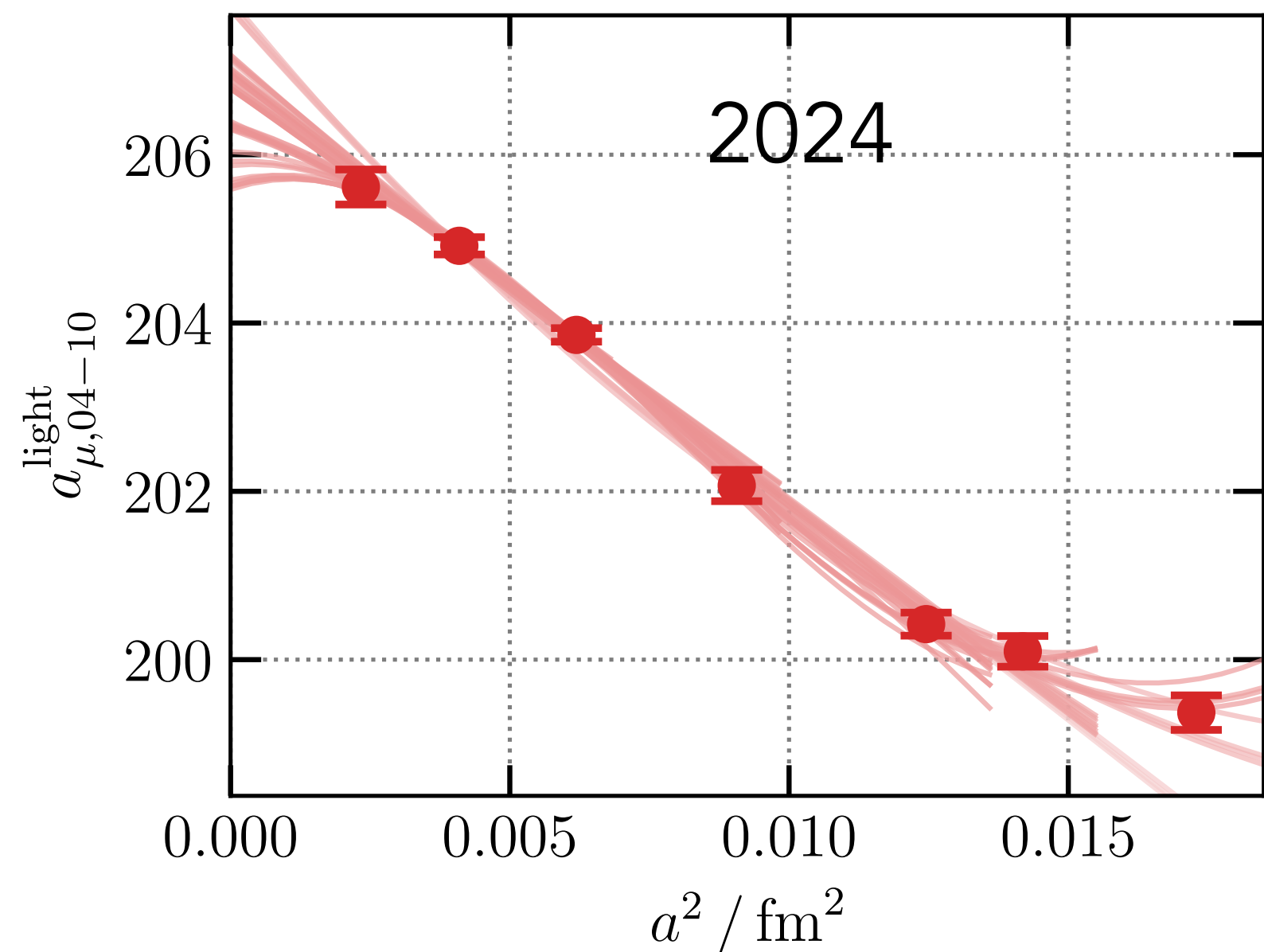
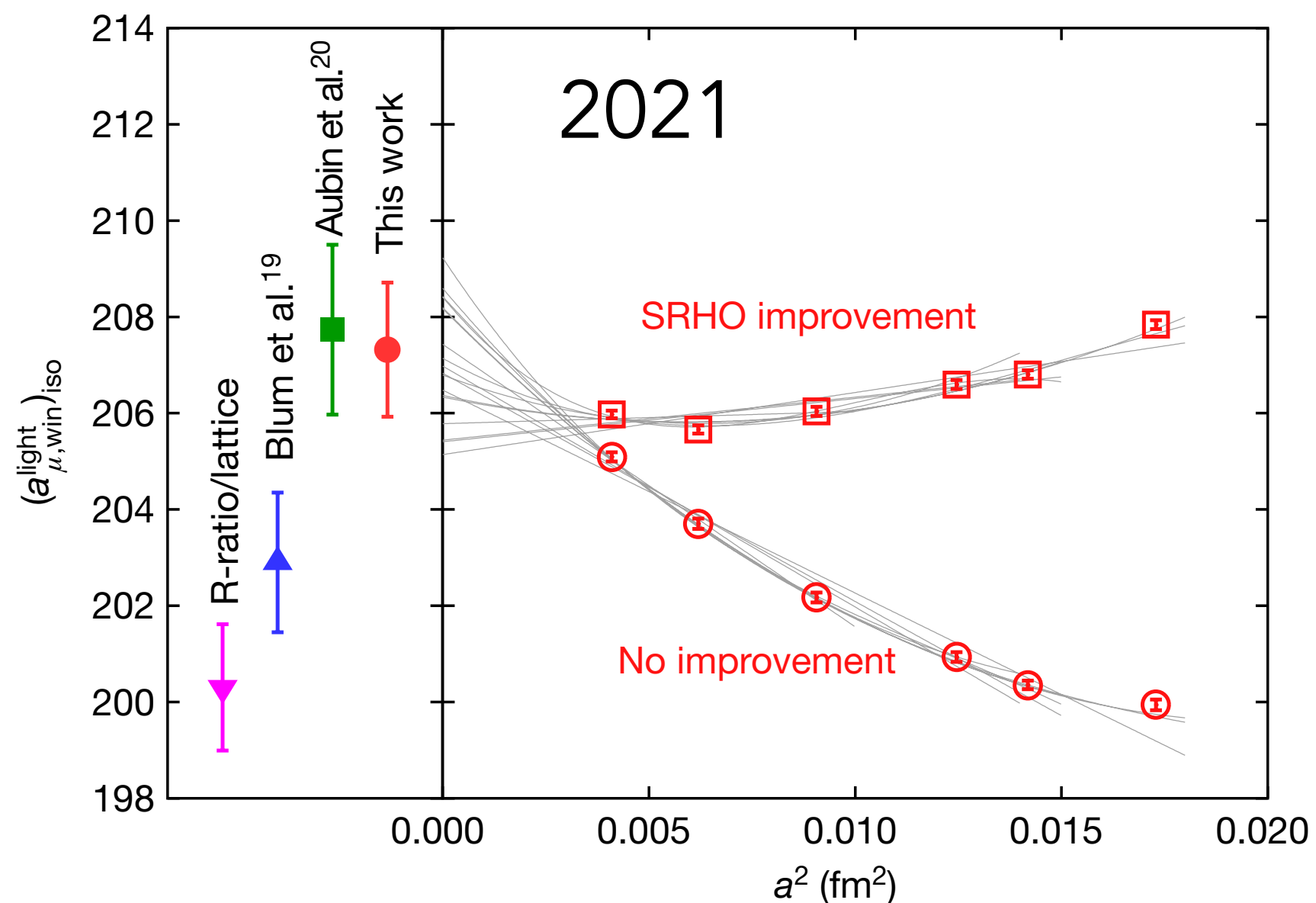


update: BMW+DMZ 2024

intermediate window, a_μ^W

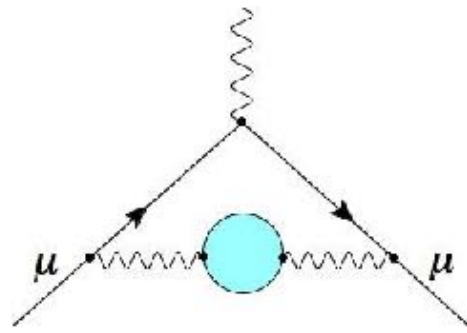
BMW 20 [Sz. Borsanyi et al, arXiv:2002.12347, 2021 Nature]

BMW-DMZ 24 [A. Boccaletti et al, arXiv:2407.10913]

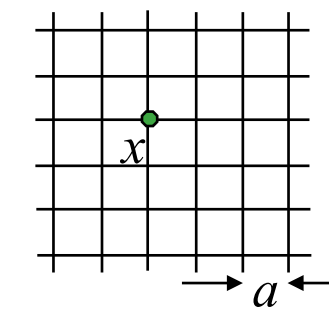


- Improvement: correct lattice data for discretization effects due to taste-splittings before taking continuum limit.

Continuum extrapolations obtained only from data not corrected for taste-splittings.



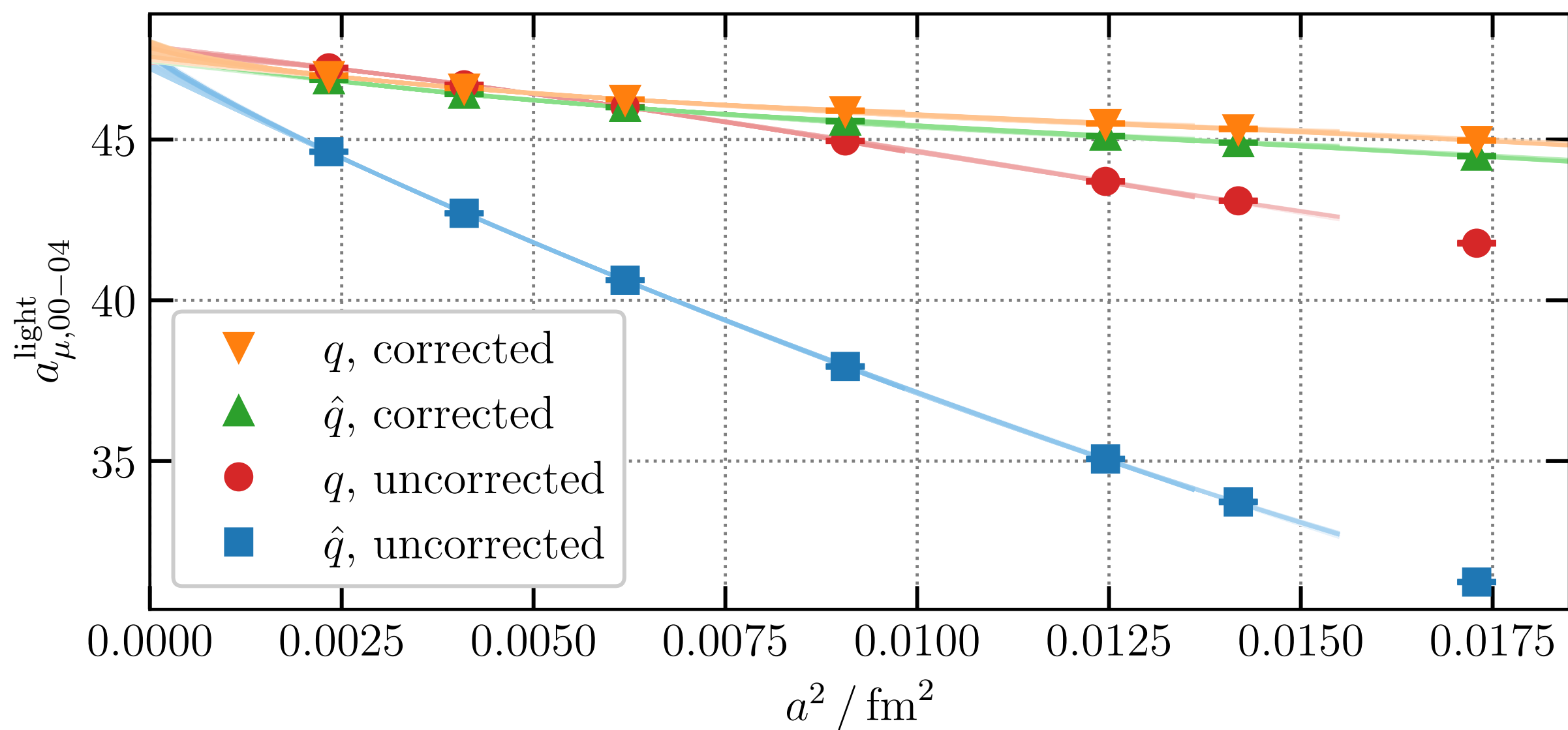
lattice HVP: short-distance window (SD)



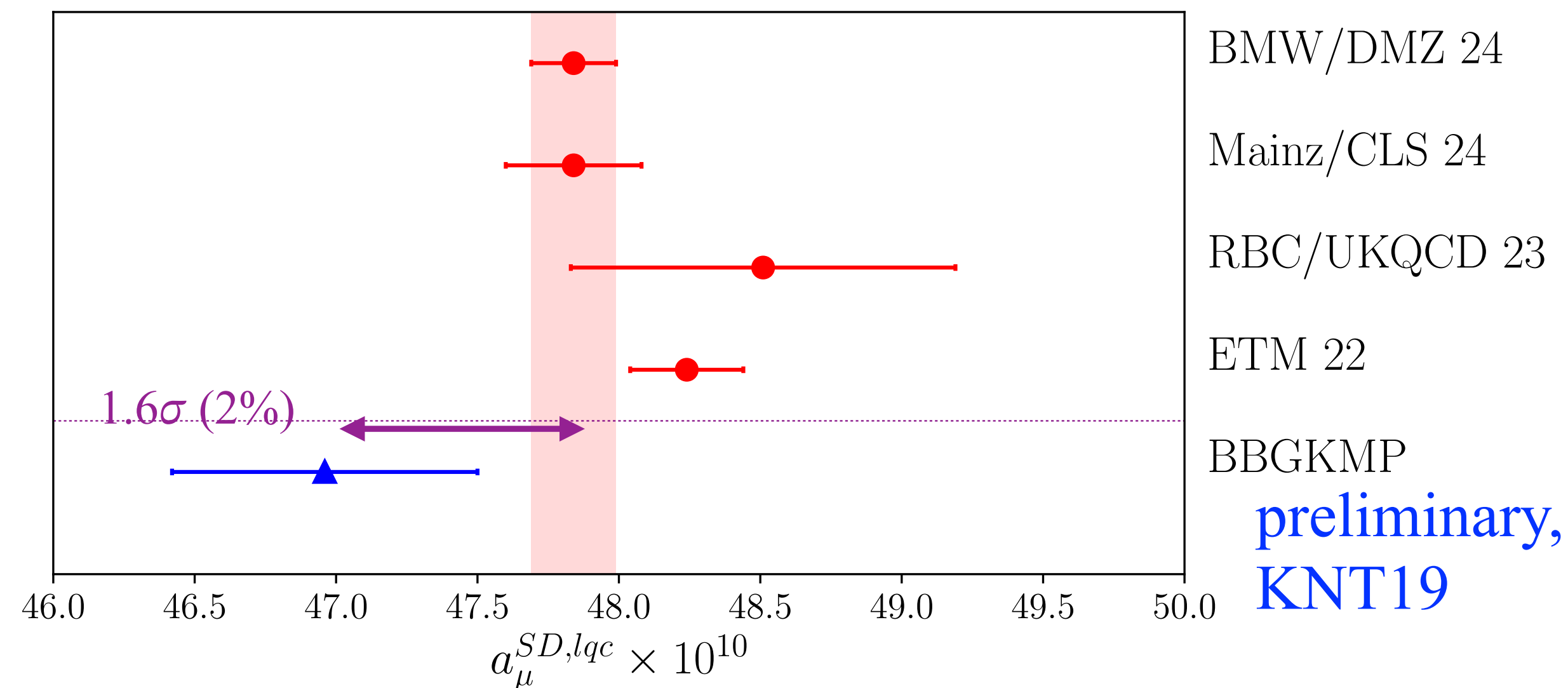
update: BMW+DMZ 2024

short-distance window, a_μ^{SD}

BMW 24 [A. Boccaletti et al, arXiv:2407.10913]



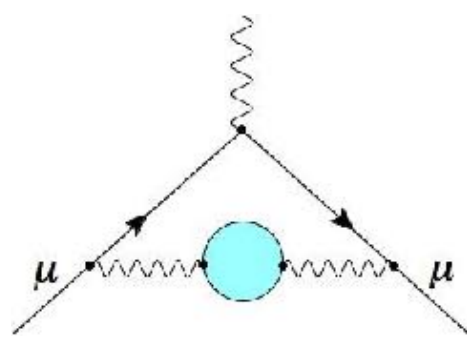
C. Davies @ Lattice 2024



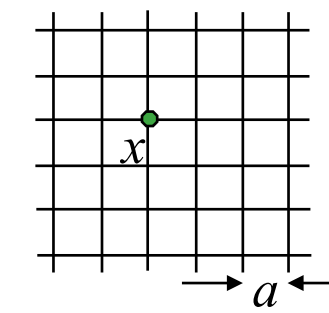
- corrected: remove log-enhanced discretization effects at tree-level

- \hat{q} :
$$\left[t^2 - \frac{4}{(aQ)^2} \sin^2 \left(\frac{aQt}{2} \right) \right] \rightarrow \left[t^2 - \frac{4}{(a\hat{Q})^2} \sin^2 \left(\frac{aQt}{2} \right) \right]$$

small tension in SD with pre-2023 data-driven evaluation

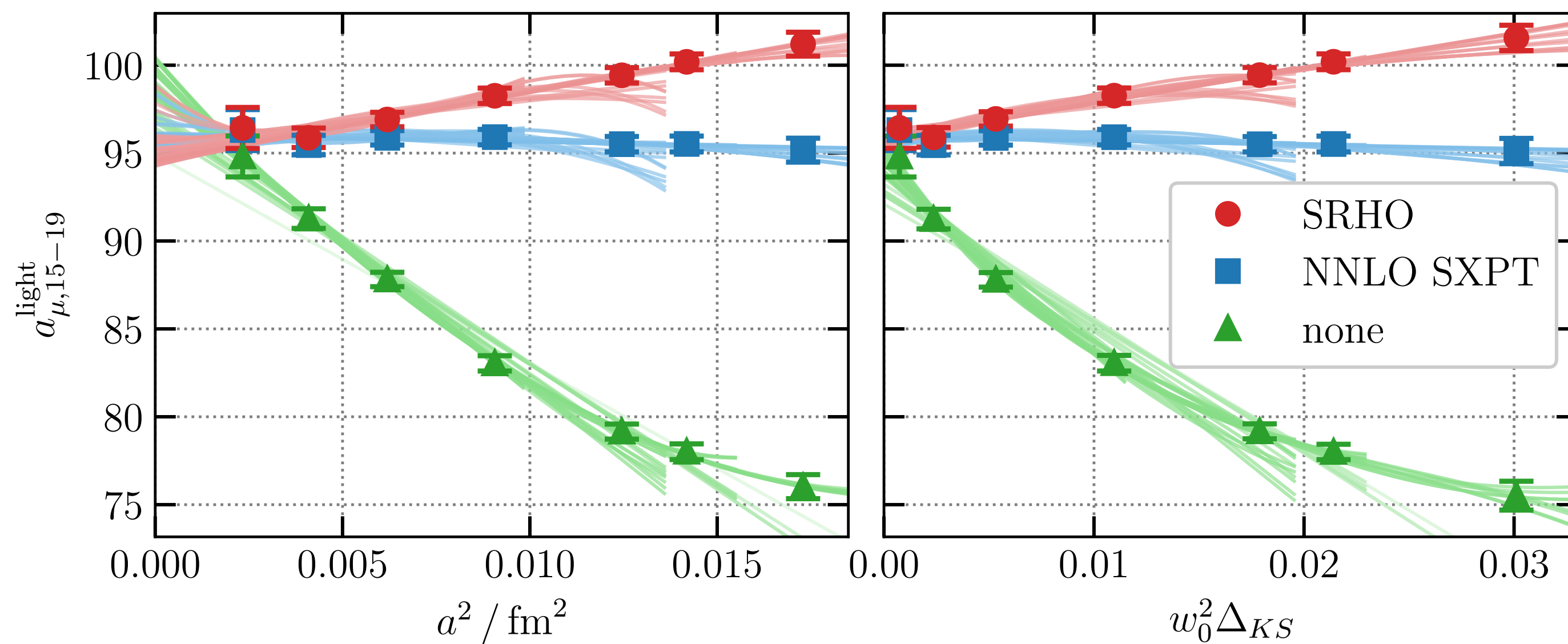


Lattice HVP: 2nd window (W2)

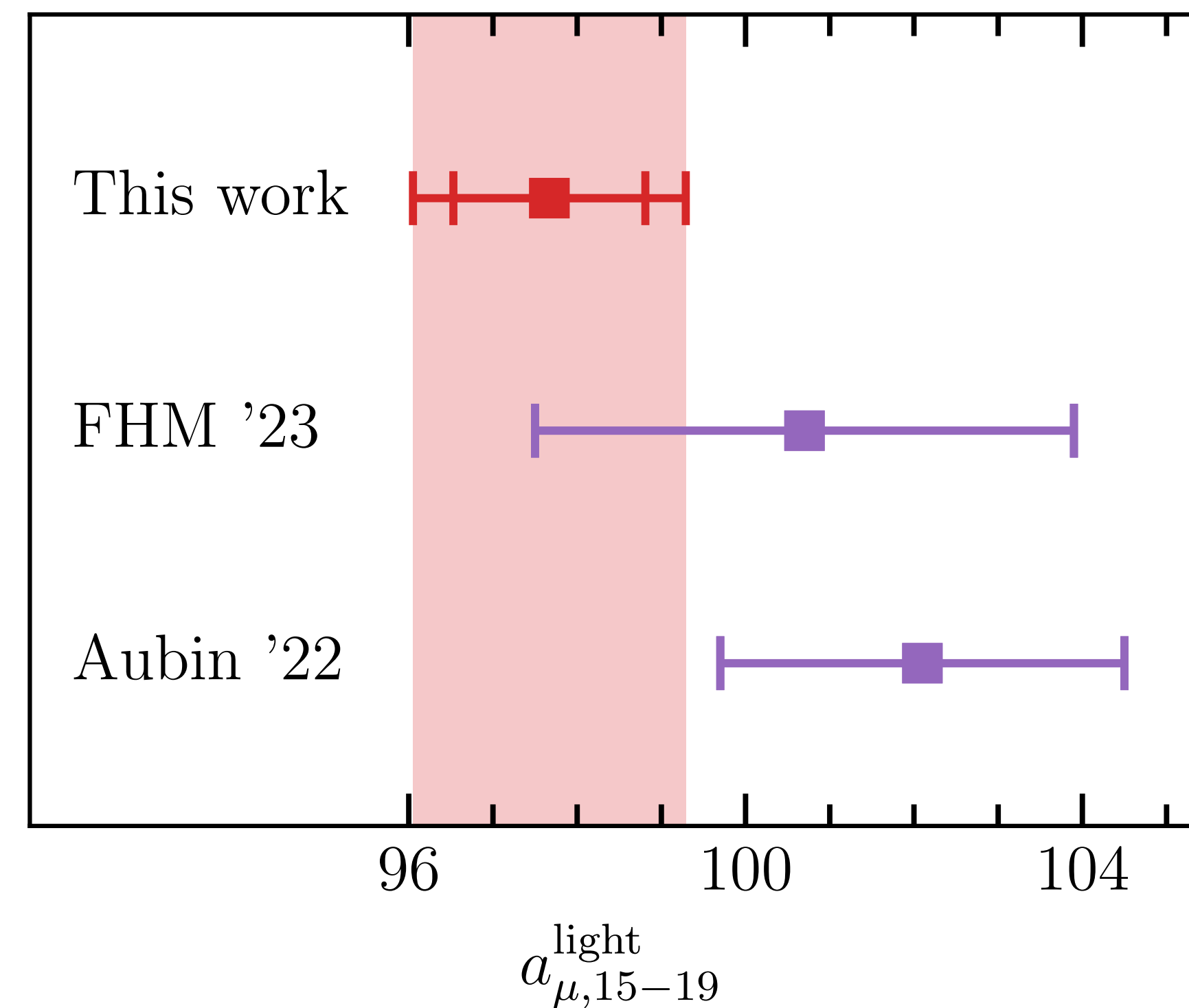


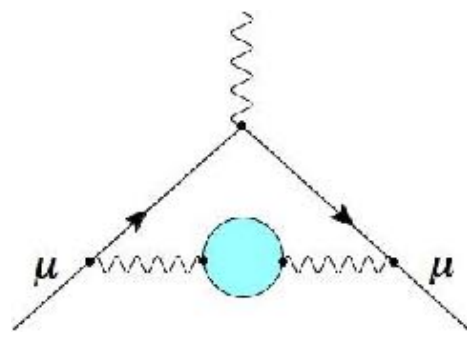
update: BMW+DMZ 2024

[A. Boccaletti et al, arXiv:2407.10913]

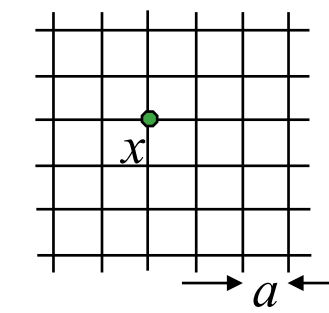


Continuum extrapolations of data with "No improvement" (green) excluded from model average.





Lattice HVP: windows



update: Fermilab/HPQCD/MILC 2024

FNAL/HPQCD/MILC @ Lattice 2024:

Shaun Lahert (Utah) & Michael Lynch (UIUC)

Shaun Lahert (Utah)

David Clarke (Utah)

Jake Sitison (U Colorado)

Craig McNeile (Plymouth)

$$a_\mu^{ll}(\text{conn.})$$

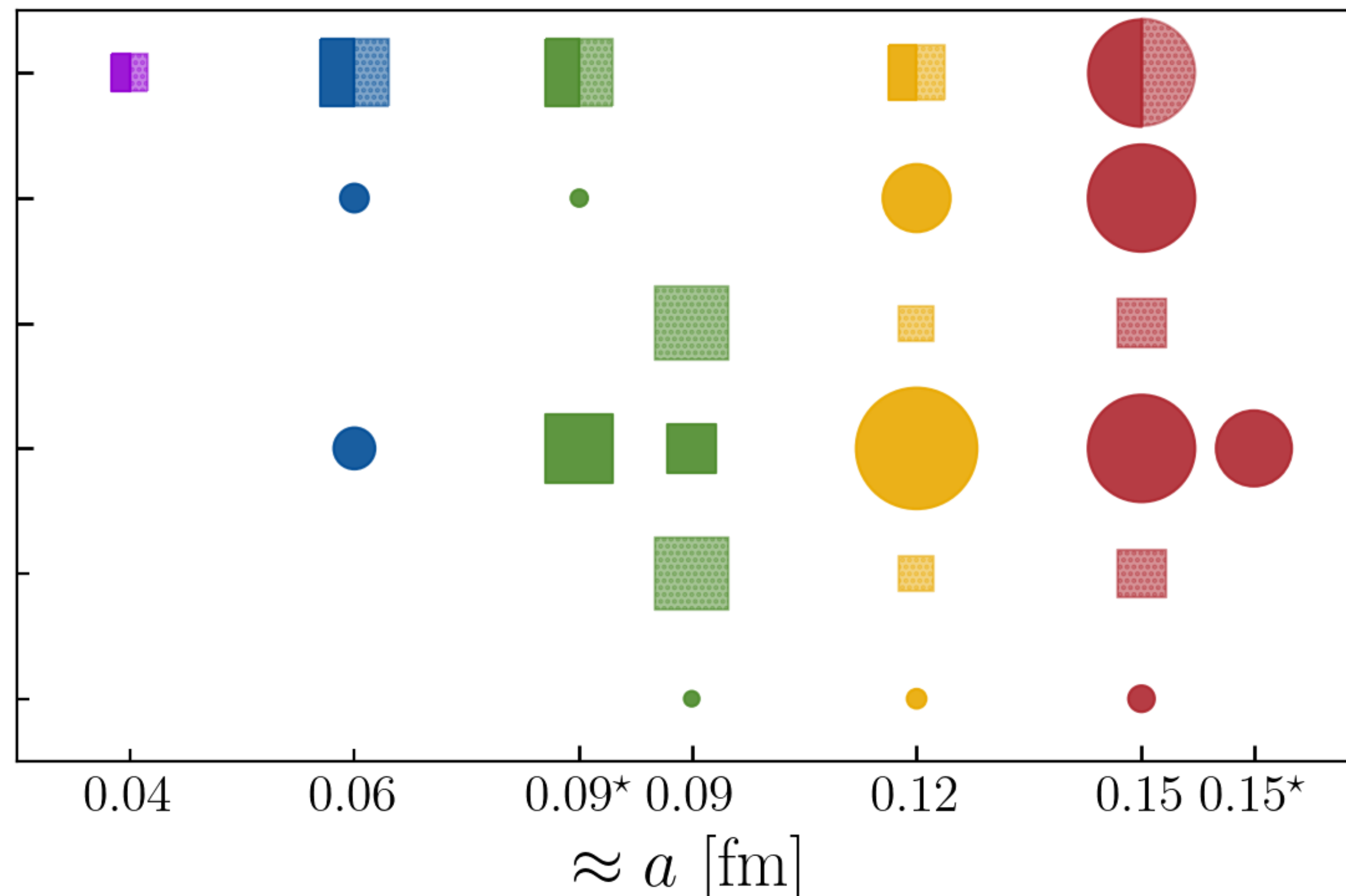
$$a_\mu^{ss/cc}(\text{conn.})$$

$$a_\mu^{ls}(\text{disc.})$$

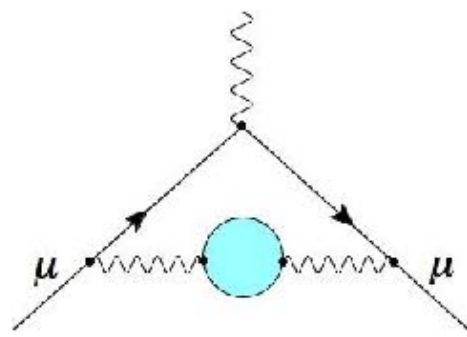
$$\Delta a_\mu^{ud}(\text{SIB conn.})$$

$$\Delta a_\mu^{ud}(\text{SIB disc.})$$

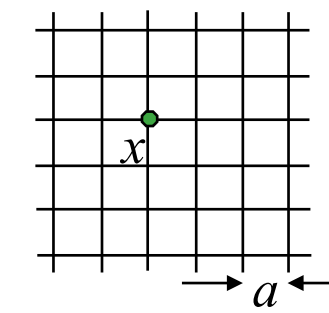
$$\Delta a_\mu^{ls}(\text{QED})$$



- ▶ Solid color (local current) hatched (one-link)
- ▶ Squares: low-mode improved.
- ▶ Size \sim statistics



Lattice HVP: long-distance window

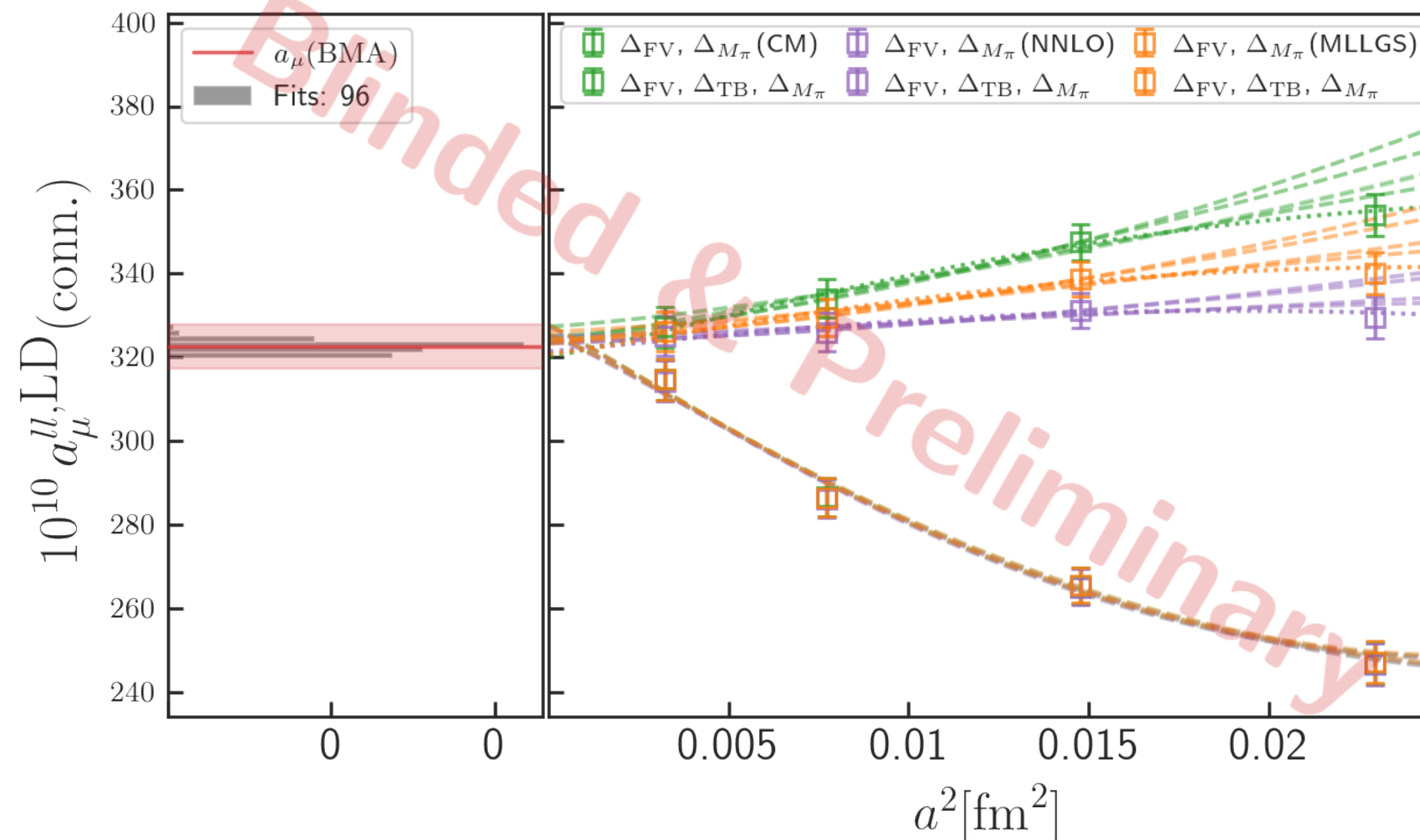


update: Fermilab/HPQCD/MILC 2024

Michael Lynch @ Lattice 2024

long-distance window a_μ^{LD} and full $a_\mu^{ll}(\text{conn.})$

$a_\mu^{ll}(\text{conn.})$ - LD window



$a_\mu^{ll}(\text{conn.})$ - Full

