Magnetic Field Analysis in the Muon g-2 Experiment

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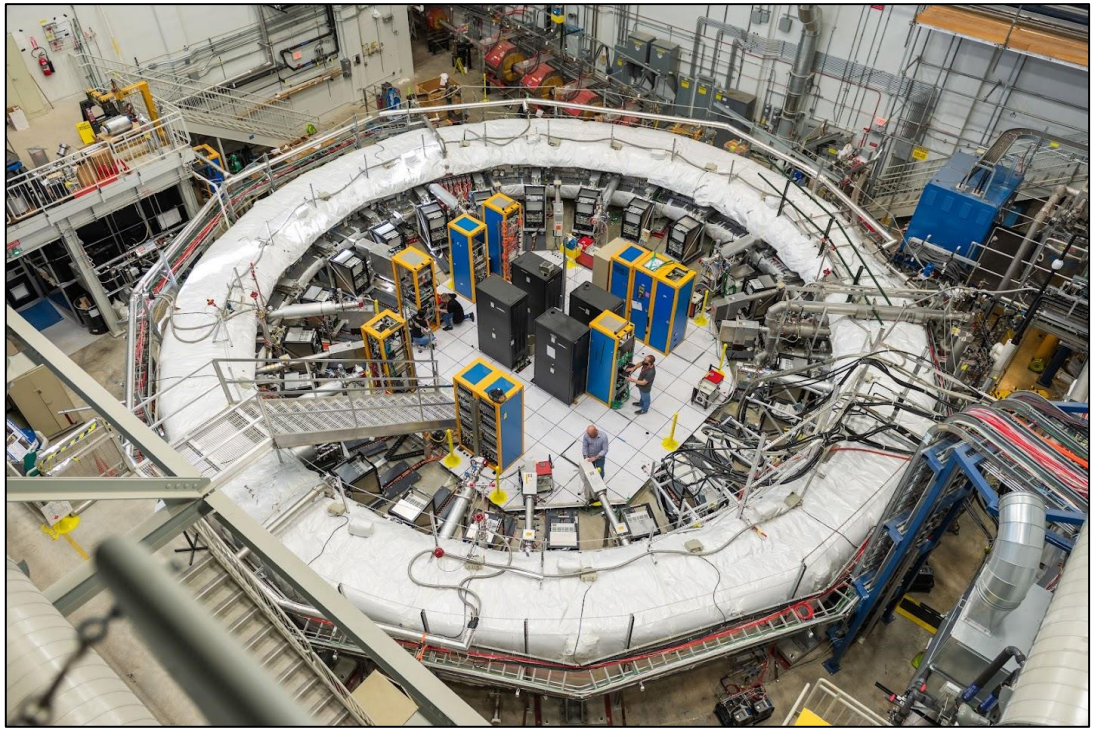
The g-2 Magnetic Field

- A **1.45-Tesla magnetic field** facilitates the Muon g-2 experiment.
- Muon anomalous precession:

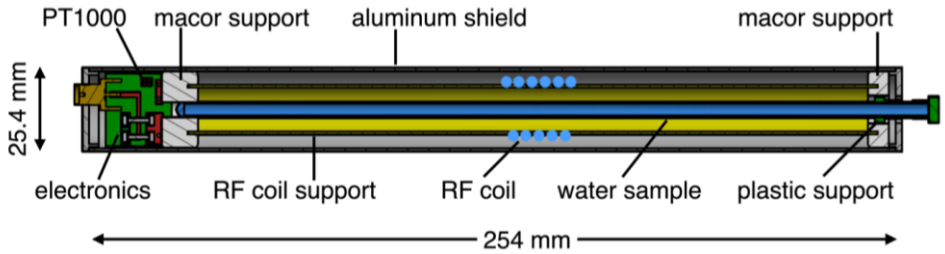
$$\omega_a = -a_\mu * \left(\frac{q}{m_\mu}\right) * B$$

- a_μ precision goal: 140 ppb.
- B precision goal: 70 ppb.

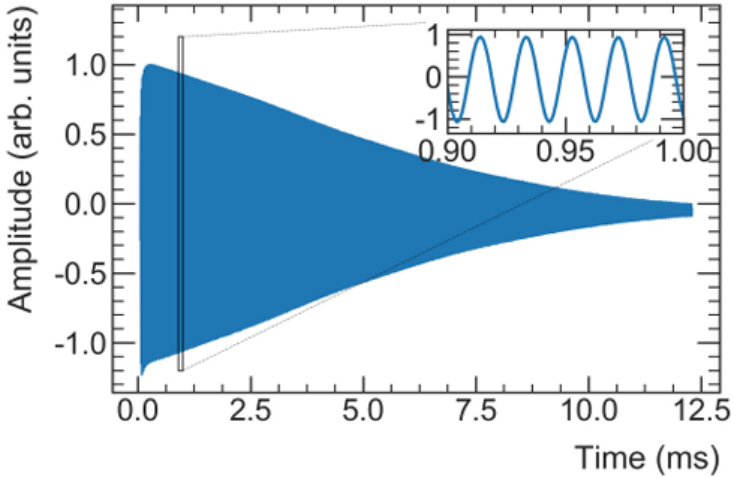


The g-2 Magnetic Storage Ring.

Measuring with Nuclear Magnetic Resonance (NMR)



g-2 Calibration Probe Diagram.



- FID signals from proton precession in sample.
- FID frequency is proportional to B.

$$\omega_p = -\frac{g_p q}{2m_p} B$$

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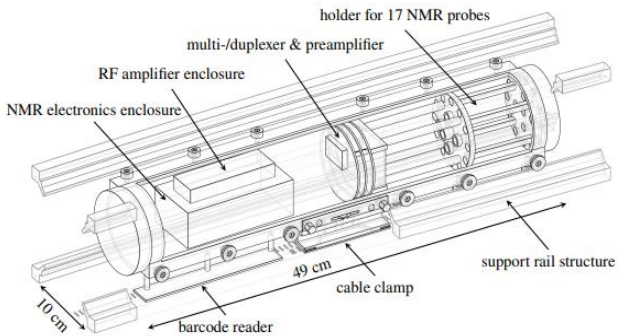
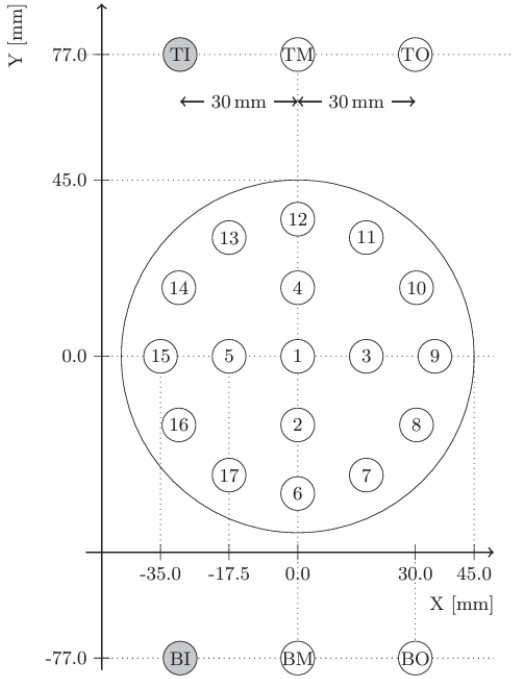
Measurement Overview

- Trolley Probes and Fixed Probes:



17 **trolley probes** map the field along the muon beam path.

378 **fixed probes** track changes in the field while muons are running.



Relative positions of trolley and fixed probes.

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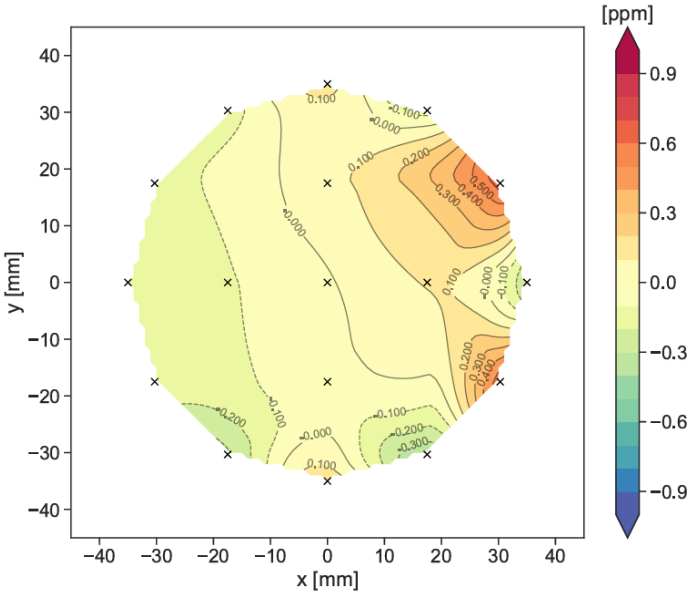
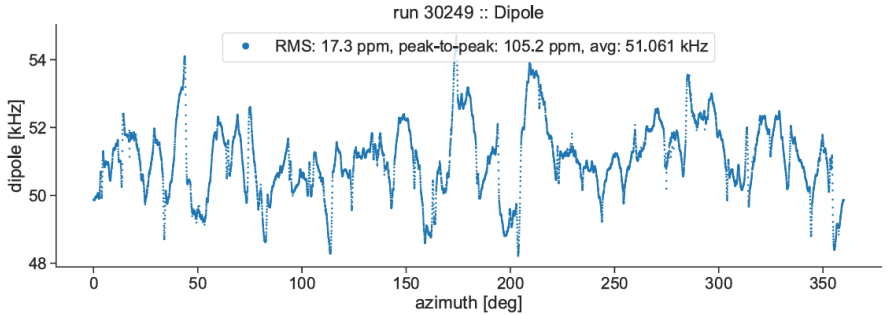
Trolley Runs



Trolley readings are fit to 2D multipole expansions.

Trolley runs were performed every 2-3 days, with muon beam off.



Each trolley run covers the circumference of the storage ring.

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Analysis Overview

- Tasks and Challenges:
  - Interpolation – Combining trolley and FP data.
  - Calibration – Trolley perturbs field while present.
  - Transient Fields – Too fast for trolley/FP to see.

➢ The Objective: $\tilde{\omega}_p{}'$

Shielded proton

Spherical water sample at 34.7° Celsius

In field averaged over time/space muon distribution

Using $\widetilde{\omega}_p{}'$

- Shielded proton frequency $\widetilde{\omega}_p{}'$ combines with ω_a, alongside known fundamental constants, to produce a_μ.

$$a_\mu = \left\{\frac{\omega_a}{\widetilde{\omega}_p{}'}\right\} * \frac{\mu_p{}'}{\mu_e(H)} * \frac{\mu_e(H)}{\mu_e} * \frac{m_\mu}{m_e} * \frac{g_e}{2}$$

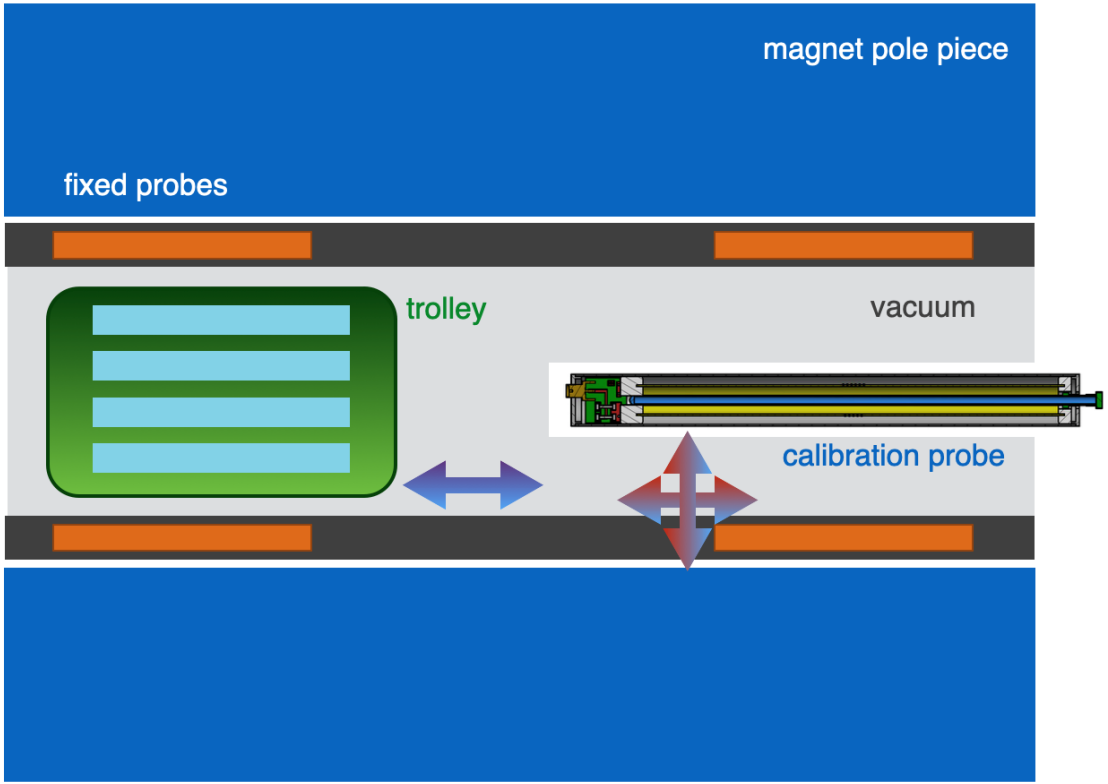
Ratio from our collaboration's analysis.

(Knowing this ratio is why we aim for $\widetilde{\omega}_p{}'$.)

Precisely-known ratios from other experiments.

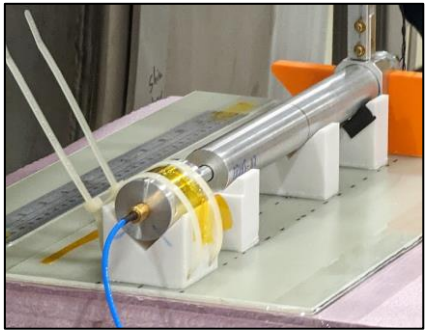
Trolley Calibration

- Each trolley probe compared to calibration probe.
- Determines field perturbation from trolley structure.
- Alignment and Rapid Swapping procedure.
- Nine campaigns over the course of g-2 operations.



Absolute Calibration

- Getting $\tilde{\omega}_p'$ (*shielded spherical-water…*) from $\omega_p(cali\ probe)$.
- Tests performed with Argonne magnet to quantify each effect.



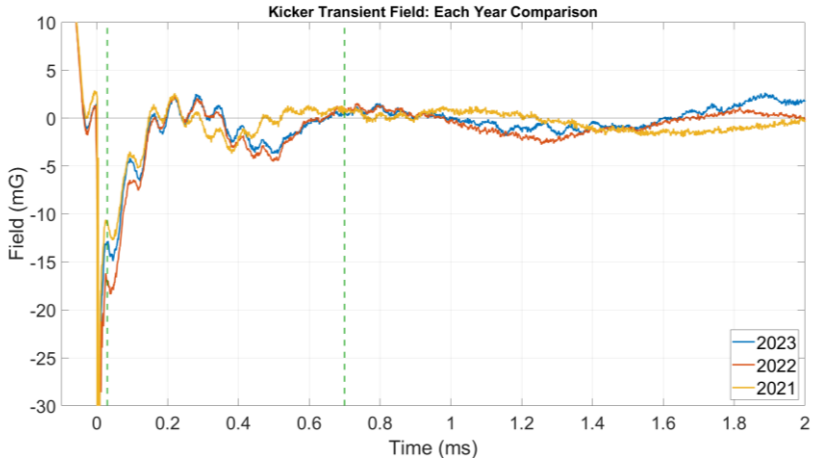
$$\tilde{\omega}_p'(T_r) = \left[1 + \delta^T(T_r - T) + \left(\epsilon - \frac{1}{3}\right)\chi_{H_2O}(T) - \delta_{probe}\right]\omega_p(cali\ probe, T)$$

Temperature Correction ~100 ppb

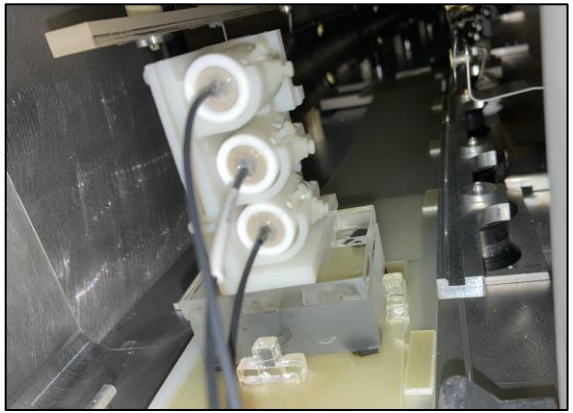
Shape and Susceptibility Effects ~1500 ppb

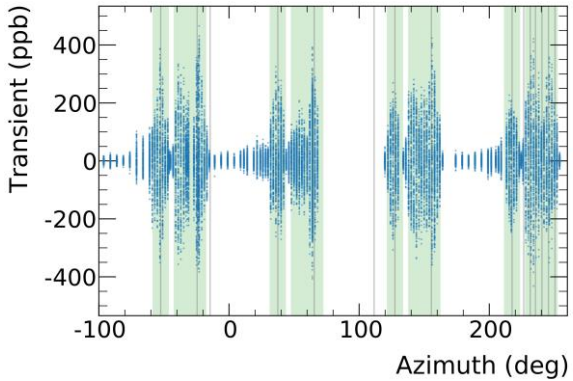
Material Effects ~10 ppb

Transient Fields

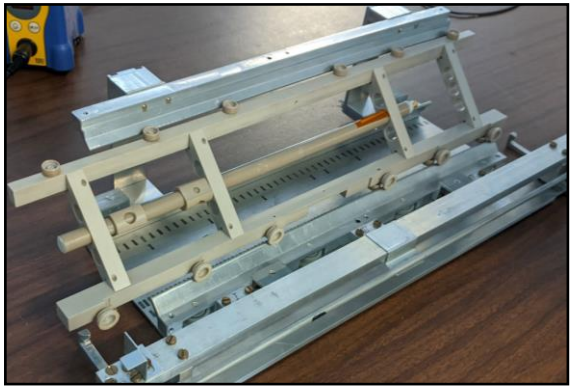


Kicker transient field and Faraday Effect magnetometer.





Electrostatic Quadrupoles transient field and NMR magnetometer.



Uncertainty Comparisons

Run-1 Uncertainties Table

Quantity	Correction terms (ppb)	Uncertainty (ppb)
ω_a^m (statistical)	···	434
ω_a^m (systematic)	···	56
C_e	489	53
C_p	180	13
C_{ml}	−11	5
C_{pa}	−158	75
$f_{\text{calib}}\langle\omega_p(x,y,\phi)\times M(x,y,\phi)\rangle$	···	56
B_k	−27	37
B_q	−17	92
$\mu'_p(34.7°)/\mu_e$	···	10
m_μ/m_e	···	22
$g_e/2$	···	0
Total systematic	···	157
Total fundamental factors	···	25
Totals	544	462

Run-2/3 Uncertainties Table

Quantity	Correction (ppb)	Uncertainty (ppb)
ω_a^m (statistical)	···	201
ω_a^m (systematic)	···	25
C_e	451	32
C_p	170	10
C_{pa}	−27	13
C_{dd}	−15	17
C_{ml}	0	3
$f_{\text{calib}}\cdot\langle\omega'_p(\vec{r})\times M(\vec{r})\rangle$	···	46
B_k	−21	13
B_q	−21	20
$\mu'_p(34.7°)/\mu_e$	···	11
m_μ/m_e	···	22
$g_e/2$	···	0
Total systematic for $\mathcal{R}'_\mu$	···	70
Total external parameters	···	25
Total for a_μ	622	215

Original field goal achieved!

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Upcoming Improvements

Run-2/3 Field Uncertainties:

Description	Uncertainty (ppb) Run-2	Run-3a	Run-3b
Calibration probe		8.9	
Trolley calibration		17.8	
Spatial field maps	37.2	38.5	38.1
Tracking	17.3	16.5	17.8
Muon weighting	13.4	7.9	6.9
Transient booster		7	
Transient kicker	13.3		
Transient ESQ	19.5		
Subtotal uncorrelated	15.4	10.7	16.0
Subtotal correlated	51.3	52.0	50.6

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Upcoming Run-4/5/6 results will include these updates:

Cross-calibration with He3 and Japan probes.
Post-Run6 calibration campaign.
Precision garage- and collimator-region measurements.

Reduced uncertainty (better beam centering).

Kicker transient spatial analysis.
ESQ transient off-diagonal measurements.

And more!