



## **RDWIA Analysis of Final-state interactions and MicroBooNE data**

Alexis Nikolakopoulos

NuFACT 2024, Argonne National Laboratory

17<sup>th</sup> September 2024

## Collaborators

Anthony M. Kelly (Fermilab)  
Raul Gonzalez-Jimenez (USeville)  
Noemi Rocco (Fermilab)  
Josh Isaacson (Fermilab)  
Kajetan Niewczas (Ugent)  
Anna Ershova (Ecole Polytechnique)  
Federico Sanchez (UGeneva)

Thanks to, J.M Udias, Noah Steinberg, A. Papadopoulou, A. Ankowski, N. Jachowicz, Ryan Plestid (Caltech), V. Pandey

### What ?

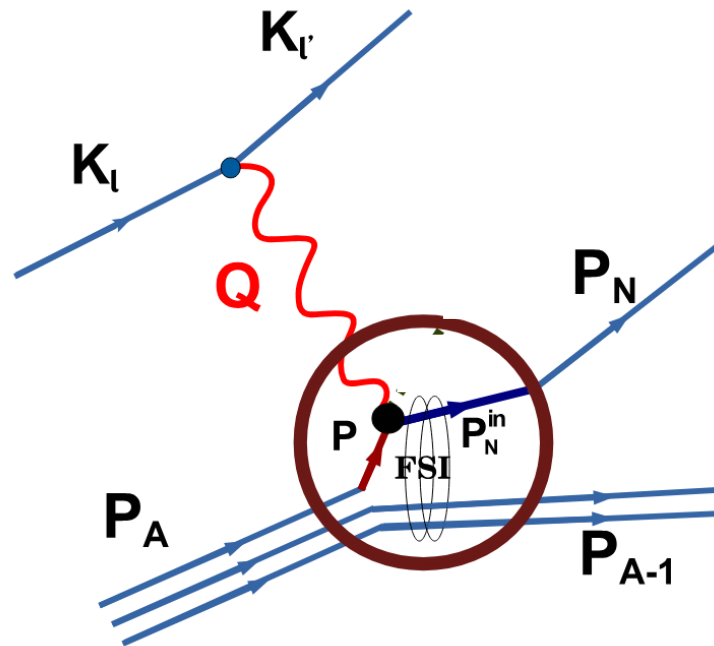
- **Distorted wave calculations with realistic nuclear spectral functions**
- **Benchmarking of cascade models with optical potentials**
- **Comparison with MicroBooNE data**

# Terminology : RDWIA, RPWIA and PWIA & ED-RMF and ROP

## -Relativistic Distorted Wave Impulse Approximation (RDWIA)

$$\mathcal{J}_{\kappa}^{m_j}(Q, P_N) = \int d\mathbf{p} \bar{\psi}(\mathbf{p} + \mathbf{q}, \mathbf{k}_N, s_N) \mathcal{O}^{\mu} \psi_{\kappa}^{m_j}(\mathbf{p})$$

Distorted wave function for final-state



## Terminology : RDWIA, RPWIA and PWIA & ED-RMF and ROP

### -Relativistic Distorted Wave Impulse Approximation (RDWIA)

$$\mathcal{J}_{\kappa}^{m_j}(Q, P_N) = \int d\mathbf{p} \bar{\psi}(\mathbf{p} + \mathbf{q}, \mathbf{k}_N, s_N) \mathcal{O}^{\mu} \psi_{\kappa}^{m_j}(\mathbf{p})$$

### - Relativistic Plane Wave Impulse Approximation (RPWIA)

$$\mathcal{J} = (2\pi)^{3/2} \bar{u}(\mathbf{k}_N, s_N) \mathcal{O}^{\mu} \psi_{\kappa}^{m_j}(\mathbf{k}_N - \mathbf{q})$$

By treating the final-state wavefunction as a plane-wave:

$$\bar{\psi}(\mathbf{p}, \mathbf{k}_N, s_N) \rightarrow (2\pi)^{3/2} \delta(\mathbf{p} - \mathbf{k}_N) \bar{u}(\mathbf{k}_N, s_N)$$

→ Neglect all final-state interactions

## Terminology : RDWIA, RPWIA and PWIA & ED-RMF and ROP

### -Relativistic Distorted Wave Impulse Approximation (RDWIA)

$$\mathcal{J}_{\kappa}^{m_j}(Q, P_N) = \int d\mathbf{p} \bar{\psi}(\mathbf{p} + \mathbf{q}, \mathbf{k}_N, s_N) \mathcal{O}^{\mu} \psi_{\kappa}^{m_j}(\mathbf{p})$$

### - Relativistic Plane Wave Impulse Approximation (RPWIA)

$$\mathcal{J} = (2\pi)^{3/2} \bar{u}(\mathbf{k}_N, s_N) \mathcal{O}^{\mu} \psi_{\kappa}^{m_j}(\mathbf{k}_N - \mathbf{q})$$

### - Plane-Wave Impulse Approximation (PWIA)

The initial state is assumed proportional to a positive-energy spinor:

$$\psi_{\kappa}^{m_j}(\mathbf{p}) \propto f(|\mathbf{p}|)u(\mathbf{p})$$

One obtains a factorized expression ('spectral function approach')

$$\frac{d\sigma(E_{\nu})}{dp_{\mu}d\Omega_{\mu}d\Omega_p dp_N} = \frac{G_F^2 \cos^2 \theta_c p_{\mu}^2 p_N^2}{(2\pi)^2} \frac{M_N^2}{E_{\nu} E_{\mu} E_N \bar{E}} L_{\mu\nu} h_{s.n.}^{\mu\nu} S(E_m, p_m)$$

## Terminology : RDWIA, RPWIA and PWIA & ED-RMF and ROP

-Relativistic Distorted Wave Impulse Approximation (RDWIA)



Remove elastic FSI

- Relativistic Plane Wave Impulse Approximation (RPWIA)



Project onto particle spinors

- Plane-Wave Impulse Approximation (PWIA)

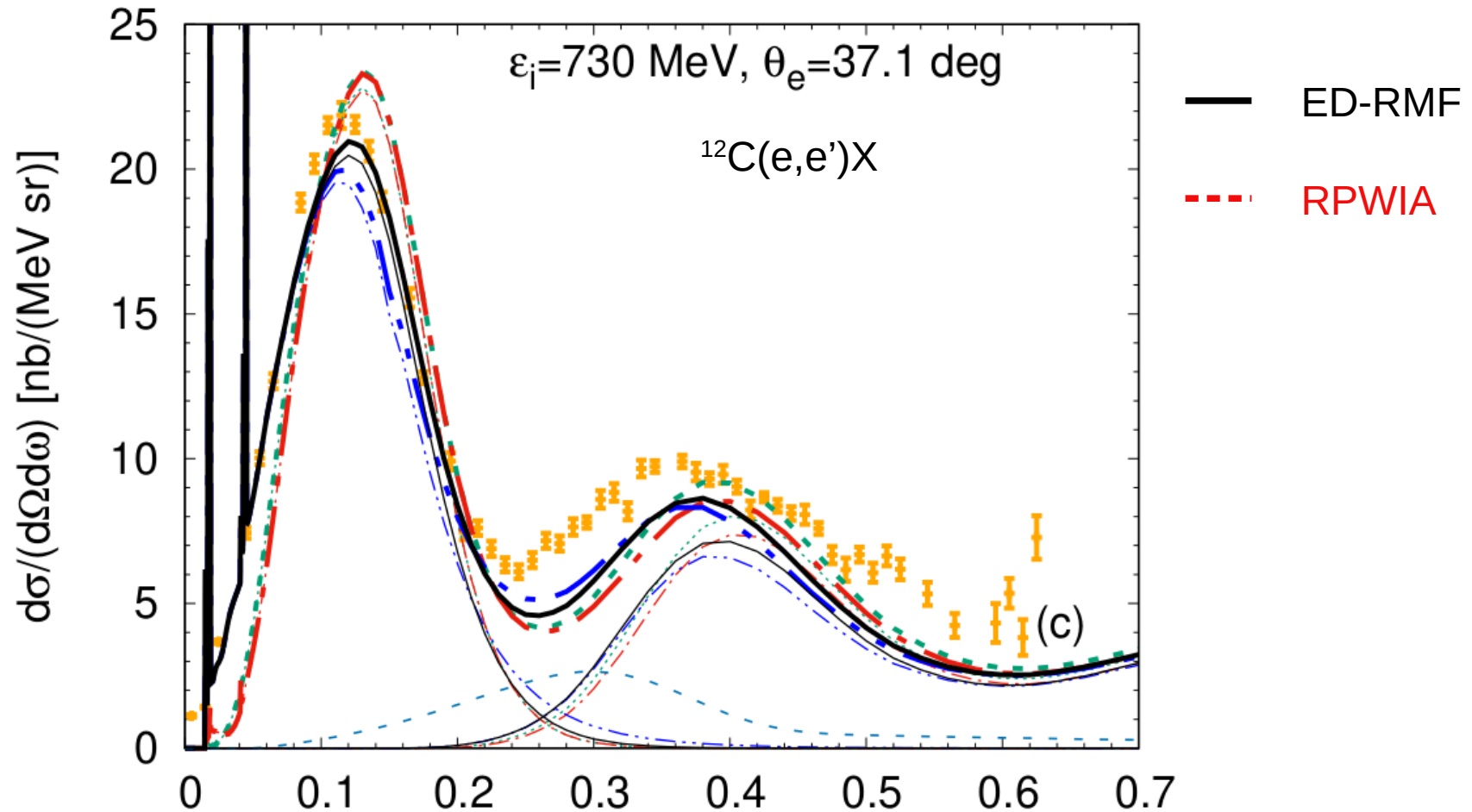
### Remember

- All results use **the same spectral function** but different final-state  
→ **can consistently check effect of FSI**
- The **difference between PWIA and RPWIA is practically negligible**  
for following results

# Terminology : RDWIA, RPWIA and PWIA & ED-RMF and ROP

## - Energy-Dependent Relativistic Mean-Field (ED-RMF)

$\bar{\psi}(\mathbf{p} + \mathbf{q}, \mathbf{k}_N, s_N)$   $\longrightarrow$  Final-state in **real** Energy-Dependent potential  
 $\rightarrow$  suitable for **FSI in inclusive** cross section

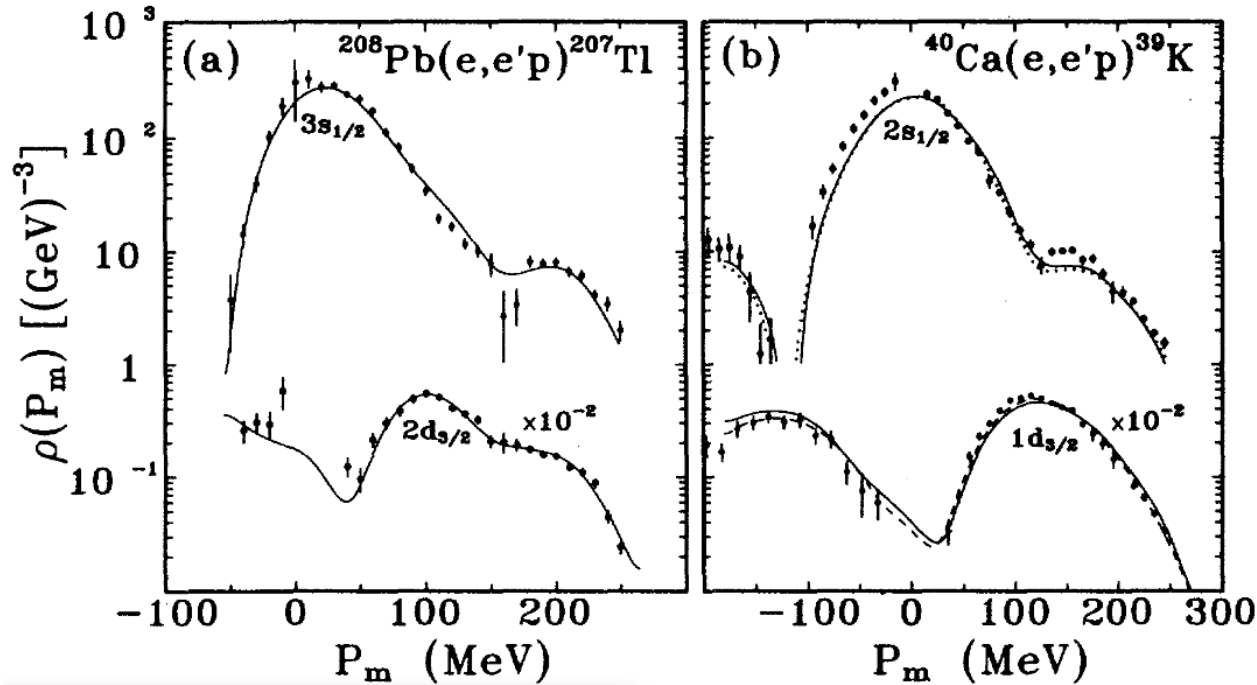


# Terminology : RDWIA, RPWIA and PWIA & ED-RMF and ROP

## - Relativistic Optical Potential (ROP)

$\bar{\psi}(\mathbf{p} + \mathbf{q}, \mathbf{k}_N, s_N)$  → Final-state in **complex** energy-dependent potential  
→ suitable for **FSI in exclusive** cross section

[Udias et al. PRC48, 2731]



- 'Standard' approach for FSI in exclusive (e,e'p) analysis in mean-field region

Including recent Jlab analyses of  $^{40}\text{Ar}$  &  $^{48}\text{Ti}$   
[PRD 107, 012005]  
[PRD 105, 112002]

→ Direct knockout accounts for ~ **50% of strength** compared to PWIA or ED-RMF calculations

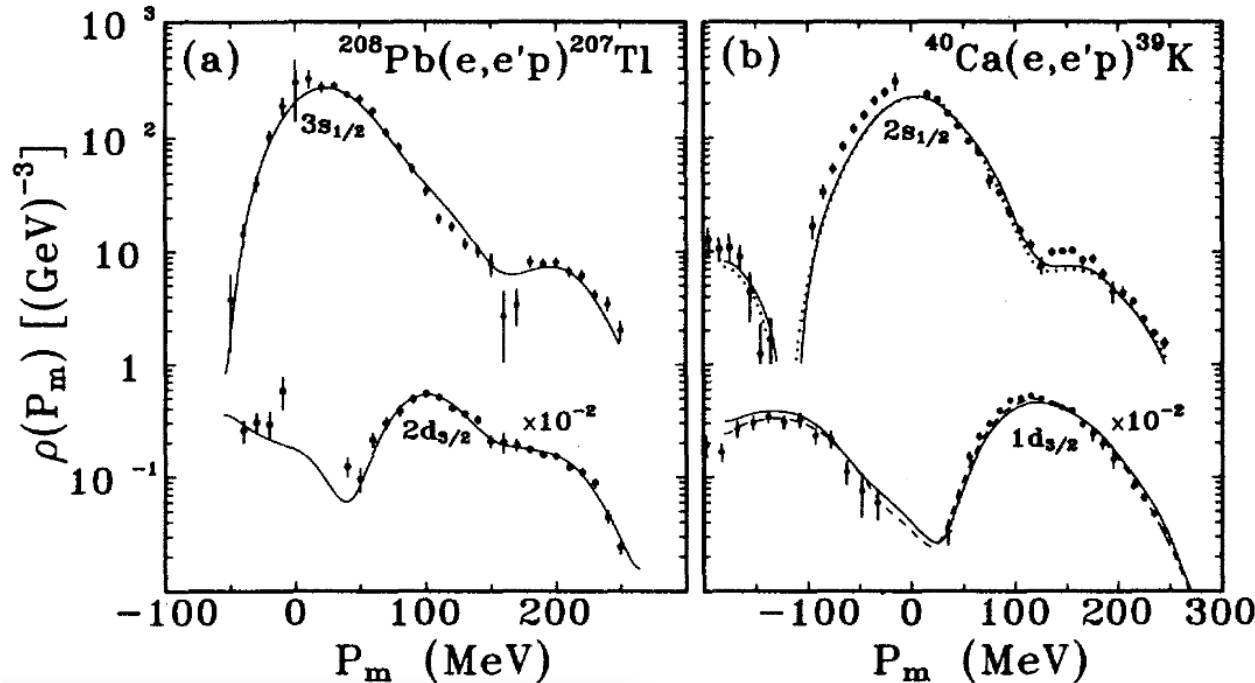


# Terminology : RDWIA, RPWIA and PWIA & ED-RMF and ROP

## - Relativistic Optical Potential (ROP)

$\bar{\psi}(\mathbf{p} + \mathbf{q}, \mathbf{k}_N, s_N)$   $\longrightarrow$  Final-state in **complex** energy-dependent potential  
 $\rightarrow$  suitable for **FSI in exclusive** cross section

[Udias et al. PRC48, 2731]



- 'Standard' approach for FSI in exclusive  $(e,e'p)$  analysis in mean-field region

Including recent Jlab analyses of  $^{40}\text{Ar}$  &  $^{48}\text{Ti}$   
[PRD 107, 012005]  
[PRD 105, 112002]

The optical potential **removes nucleon that undergoes inelastic FSI**

$\leftrightarrow$

In neutrino experiments **want to describe where the nucleon goes**

# Where do the protons go?: Intranuclear Cascade model (INC)

- ED-RMF

FSI in inclusive

-INC

FSI for relevant (semi-)exclusive channels

- ROP

FSI in single  
exclusive  
channel

Production of final-state  $|X\rangle = |p\rangle|^{39}\text{Ar}^*\rangle$

$$|\mathcal{M}|^2 \approx \left| \sum_{\alpha} \langle \Psi_0 | T_{1b} | \psi_{\alpha} \rangle \langle \psi_{\alpha} | X \rangle \right|^2, \quad \longrightarrow \quad \text{Restrict to 1-body operator}$$

$$\approx \sum_{\alpha} |\langle \Psi_0 | T_{1b} | \psi_{\alpha} \rangle|^2 |\langle \psi_{\alpha} | X \rangle|^2 \quad \longrightarrow \quad \text{Classical approximation}$$

$$\approx \sum_{\alpha} |\langle \Psi_0 | T_{1b} | \psi_{\alpha} \rangle|^2 P(X|\alpha). \quad \longrightarrow \quad \text{Intranuclear Cascade}$$

# Where do the protons go?: Intranuclear Cascade model (INC)

- ED-RMF

FSI in inclusive

-INC

FSI for relevant (semi-)exclusive channels

- ROP

FSI in single exclusive channel





Production of final-state  $|X\rangle = |p\rangle|^{39}\text{Ar}^*\rangle$

$$\begin{aligned}
 |\mathcal{M}|^2 &\approx \left| \sum_{\alpha} \langle \Psi_0 | T_{1b} | \psi_{\alpha} \rangle \langle \psi_{\alpha} | X \rangle \right|^2, && \longrightarrow \text{Restrict to 1-body operator} \\
 &\approx \sum_{\alpha} |\langle \Psi_0 | T_{1b} | \psi_{\alpha} \rangle|^2 |\langle \psi_{\alpha} | X \rangle|^2 && \longrightarrow \text{Classical approximation} \\
 &\approx \sum_{\alpha} |\langle \Psi_0 | T_{1b} | \psi_{\alpha} \rangle|^2 \underbrace{P(X|\alpha)}_{\text{INC}} && \longrightarrow \text{Intranuclear Cascade}
 \end{aligned}$$

ROP
ED-RMF
INC

Can benchmark the INC with ROP using inputs with same nuclear model  
For direct proton knockout

**Benchmarking intranuclear cascade models for neutrino scattering  
with relativistic optical potentials**

A. Nikolakopoulos ,<sup>1,2,\*</sup> R. González-Jiménez ,<sup>3</sup> N. Jachowicz,<sup>1</sup> K. Niewczas,<sup>1,4</sup> F. Sánchez ,<sup>5</sup> and J. M. Udías <sup>3</sup>

**Input to the INC**

Fully differential events from RDWIA or RPWIA  
For  $1\mu 1p$





**Cuts on the INC results**

Single proton events where proton does not lose  
Energy  $\rightarrow$  no inelastic FSI

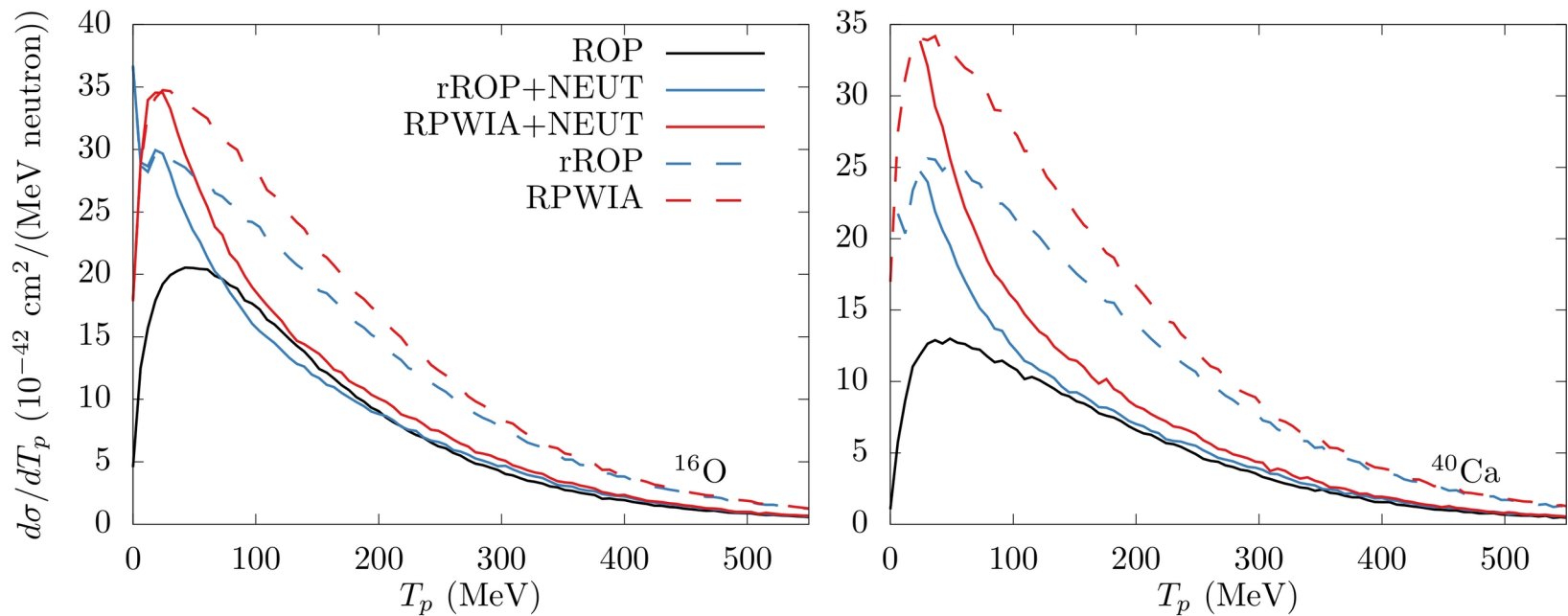


**Can be compared to ROP results**

## Benchmarking intranuclear cascade models for neutrino scattering with relativistic optical potentials

A. Nikolakopoulos <sup>1,2,\*</sup> R. González-Jiménez <sup>3</sup> N. Jachowicz,<sup>1</sup> K. Niewczas,<sup>1,4</sup> F. Sánchez <sup>5</sup> and J. M. Udías <sup>3</sup>

### Flux-folded with T2K ND flux: NEUT INC

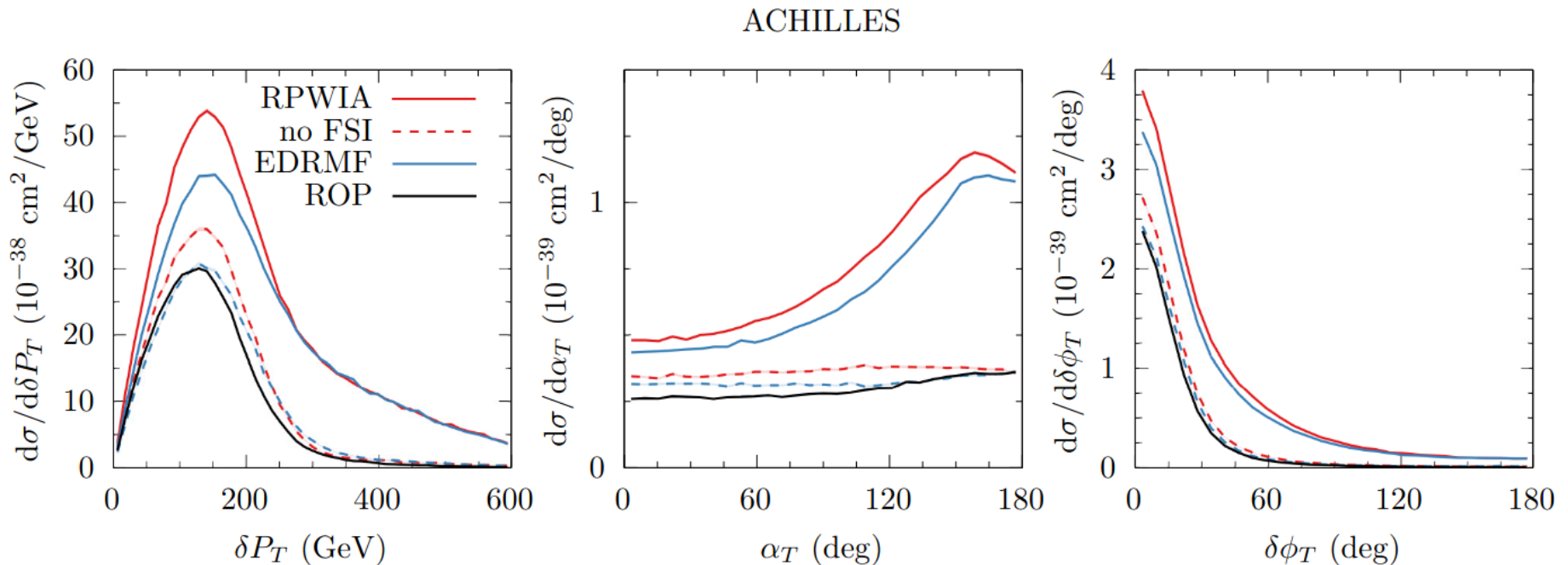


ROP and INC agree at large  $T_p$  but large disagreement for small  $T_p$

# Benchmarking INCs with RDWIA calculations for Argon

[A.N., A. Ershova, R. G-J, J. Isaacson, A.M. Kelly, K. Niewczas, N. Rocco, F. Sanchez arxiv:2406.09244]

- Flux-folded results for MicroBooNE
- ACHILLES, INCL, NEUT, and NuWro INC models
- Large set of kinematic distributions

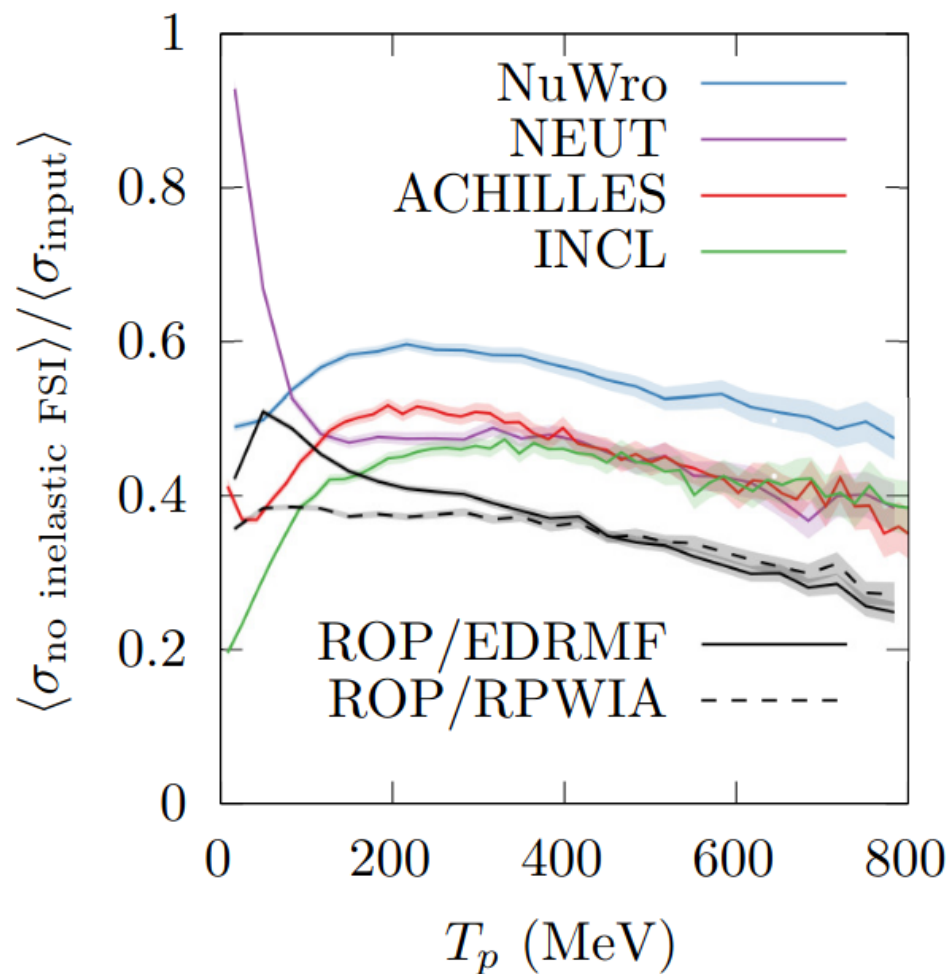


- Agreement depends on input calculation (ED-RMF  $\leftrightarrow$  RDWIA)
- Large differences between INCs at low  $T_p$  & in treatment of correlations
- No full agreement between any INC and ROP

# Benchmarking INCs with RDWIA calculations for Argon

[ arxiv:2406.09244 ]

## Comparison of $T_p$ spectrum produced in different INCs & ROP



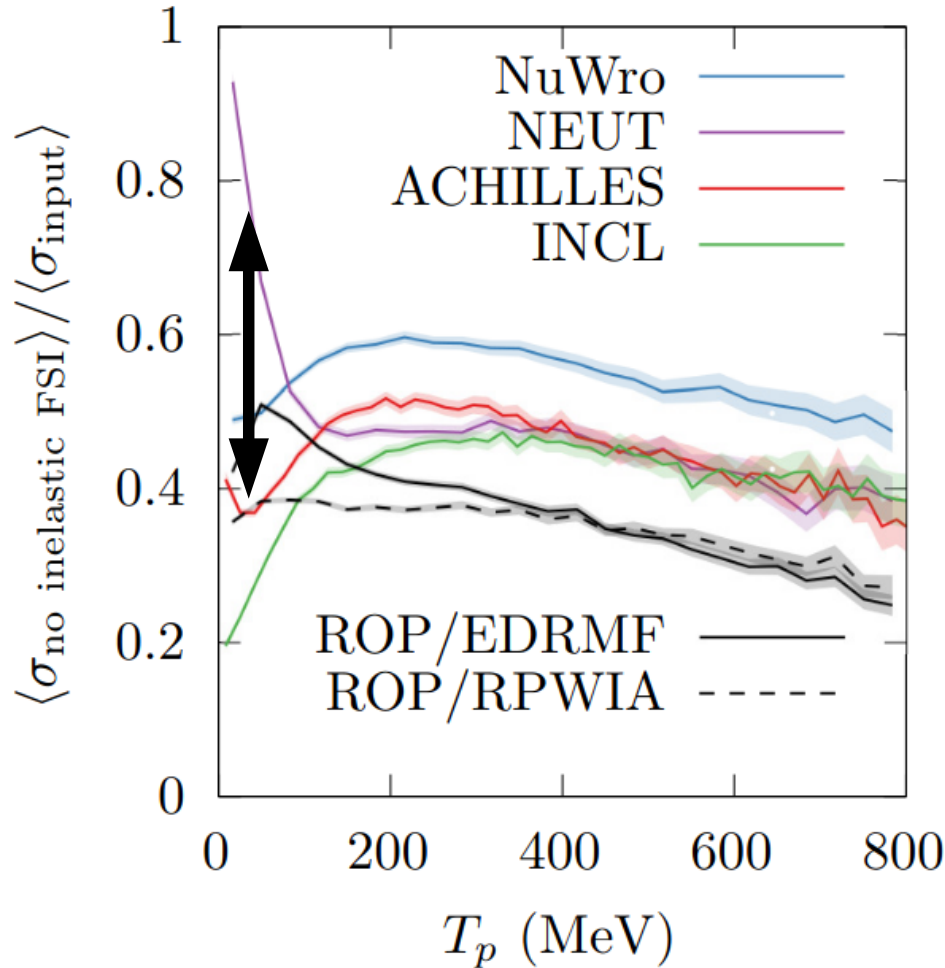
Ratio **OUT/INPUT**

→ independent of INPUT *in INC*  
= 'INC Transparency'

# Benchmarking INCs with RDWIA calculations for Argon

[ arxiv:2406.09244 ]

## Comparison of $T_p$ spectrum produced in different INCs & ROP



Ratio **OUT/INPUT**

→ independent of INPUT *in INC*  
= 'INC Transparency'

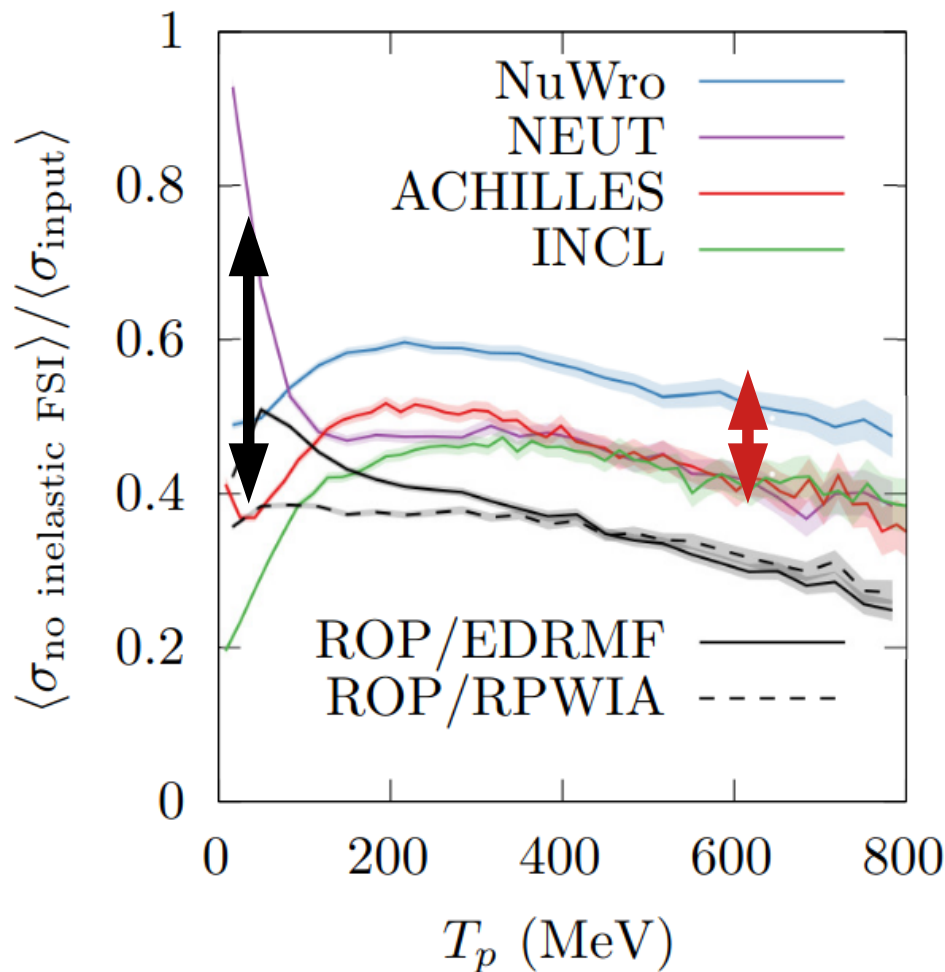
- **NEUT & ACHILLES:**
  - Low- $T_p$  differences
  - Other INCs have more rescattering at low- $T_p$  than NEUT



# Benchmarking INCs with RDWIA calculations for Argon

[ arxiv:2406.09244 ]

## Comparison of $T_p$ spectrum produced in different INCs & ROP



### Ratio **OUT/INPUT**

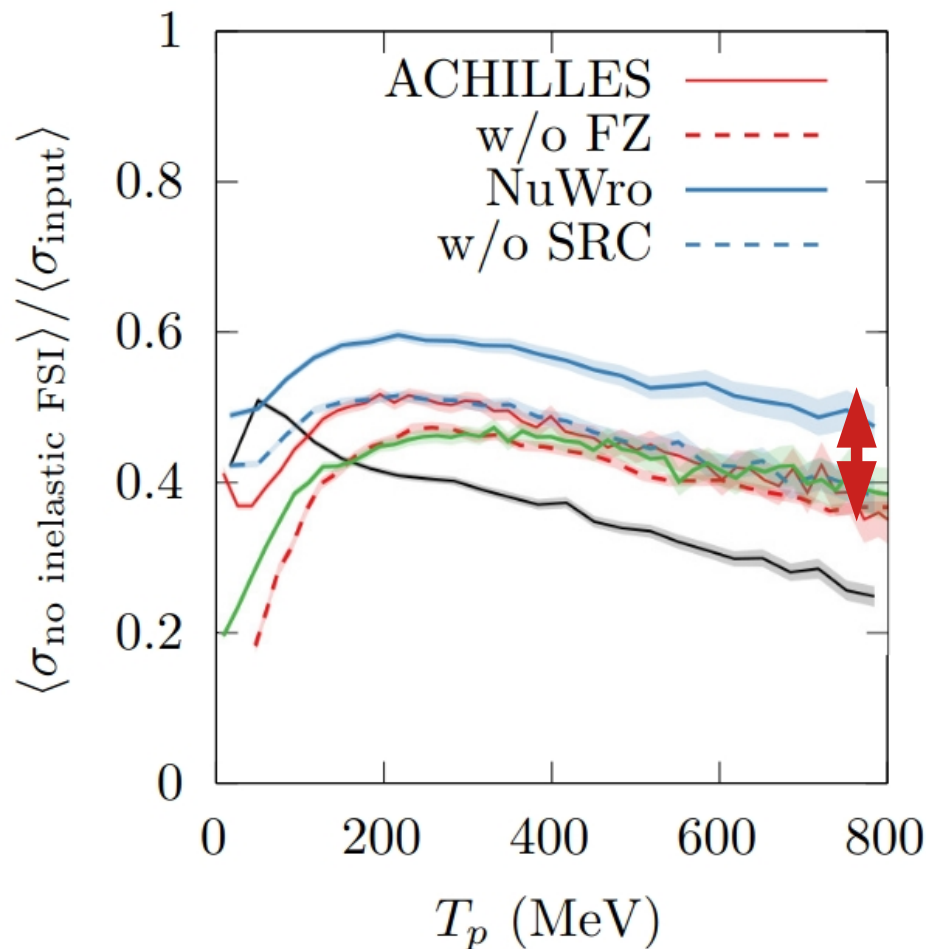
→ independent of INPUT *in INC*  
= 'INC Transparency'

- **NEUT & ACHILLES:**
  - Low- $T_p$  differences
  - Other INCs have more rescattering at low- $T_p$  than NEUT
- **NuWro:**
  - Treatment of SRCs in NuWro

# Benchmarking INCs with RDWIA calculations for Argon

[ arxiv:2406.09244 ]

## Comparison of $T_p$ spectrum produced in different INCs & ROP



### Ratio **OUT/INPUT**

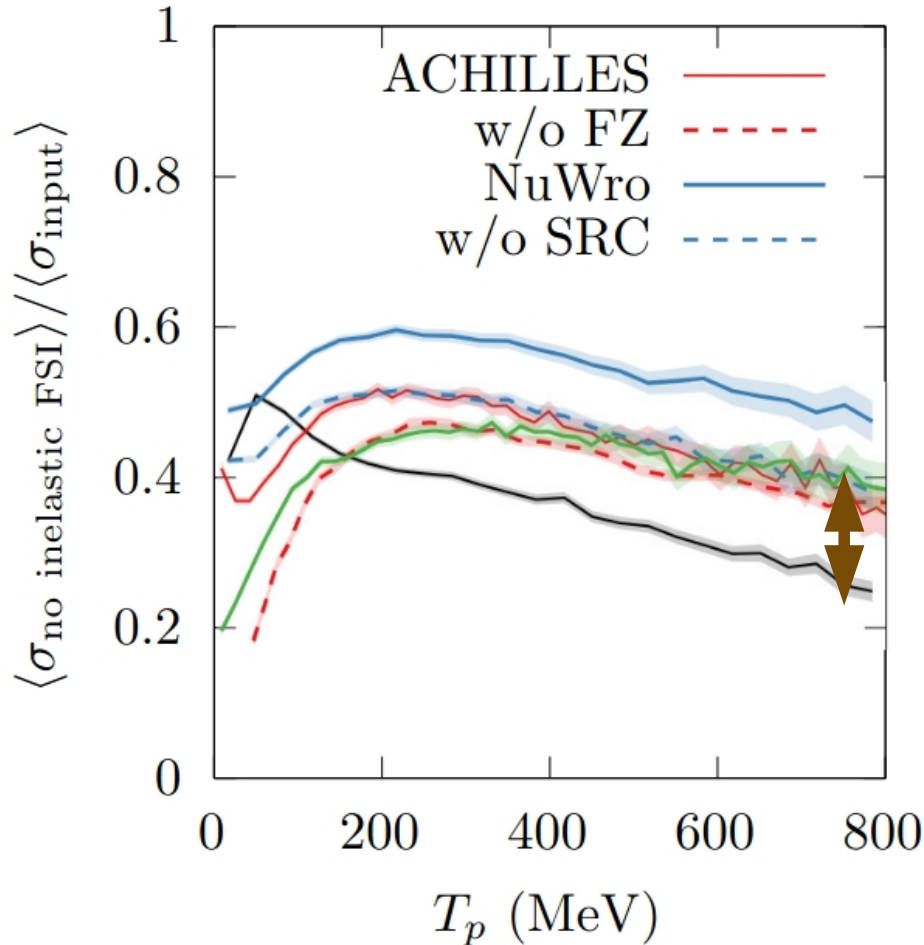
→ independent of INPUT *in INC*  
= 'INC Transparency'

- **NEUT & ACHILLES:**
  - Low- $T_p$  differences
  - Other INCs have more rescattering at low- $T_p$  than NEUT
- **NuWro:**
  - Treatment of SRCs in NuWro

# Benchmarking INCs with RDWIA calculations for Argon

[ arxiv:2406.09244 ]

## Comparison of $T_p$ spectrum produced in different INCs & ROP



### Ratio **OUT/INPUT**

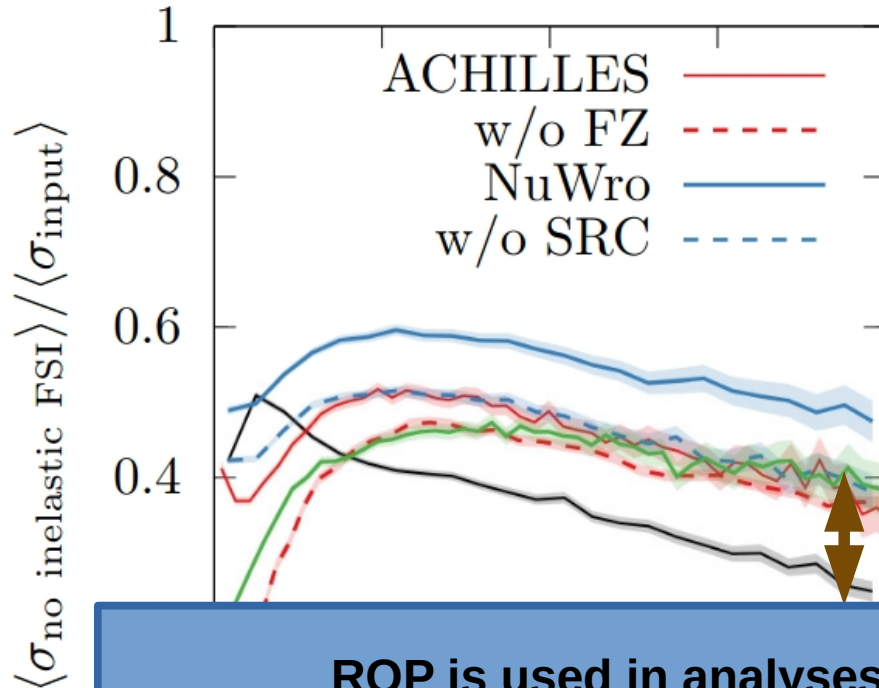
→ independent of INPUT *in INC*  
= 'INC Transparency'

- **NEUT & ACHILLES:**
  - Low- $T_p$  differences
  - Other INCs have more rescattering at low- $T_p$  than NEUT
- **NuWro:**
  - Treatment of SRCs in NuWro
- **Consistent discrepancy between ROP and INC!**
  - = discrepancy with (e,e'p) analyses ?

# Benchmarking INCs with RDWIA calculations for Argon

[ arxiv:2406.09244 ]

## Comparison of $T_p$ spectrum produced in different INCs & ROP



Ratio **OUT/INPUT**

→ independent of INPUT *in INC*  
= 'INC Transparency'

- **NEUT & ACHILLES:**
  - Low- $T_p$  differences
  - Other INCs have more rescattering at low- $T_p$  than NEUT

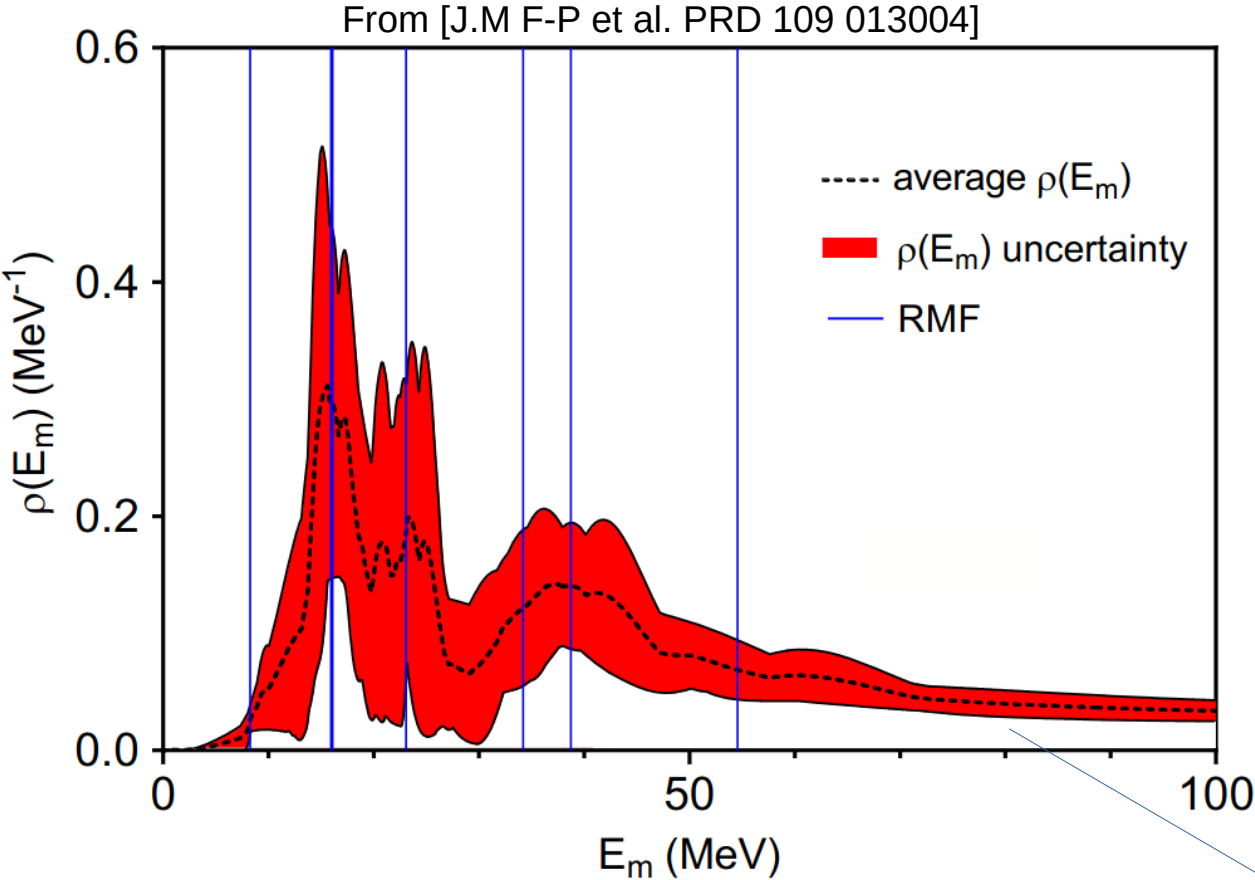
ROP is used in analyses to determine spectral function

No definite benchmark/uncertainty on INC or ROP for lepton scattering

→ New (e,e'p) datasets with  $E_m$  cuts ?

# RDWIA calculations with spectral functions

See: [J. M. Franco-Patino et al. PRD 109, 013004] & [R. Gonzalez-Jimenez et al. PRC 105, 025502]



mean field

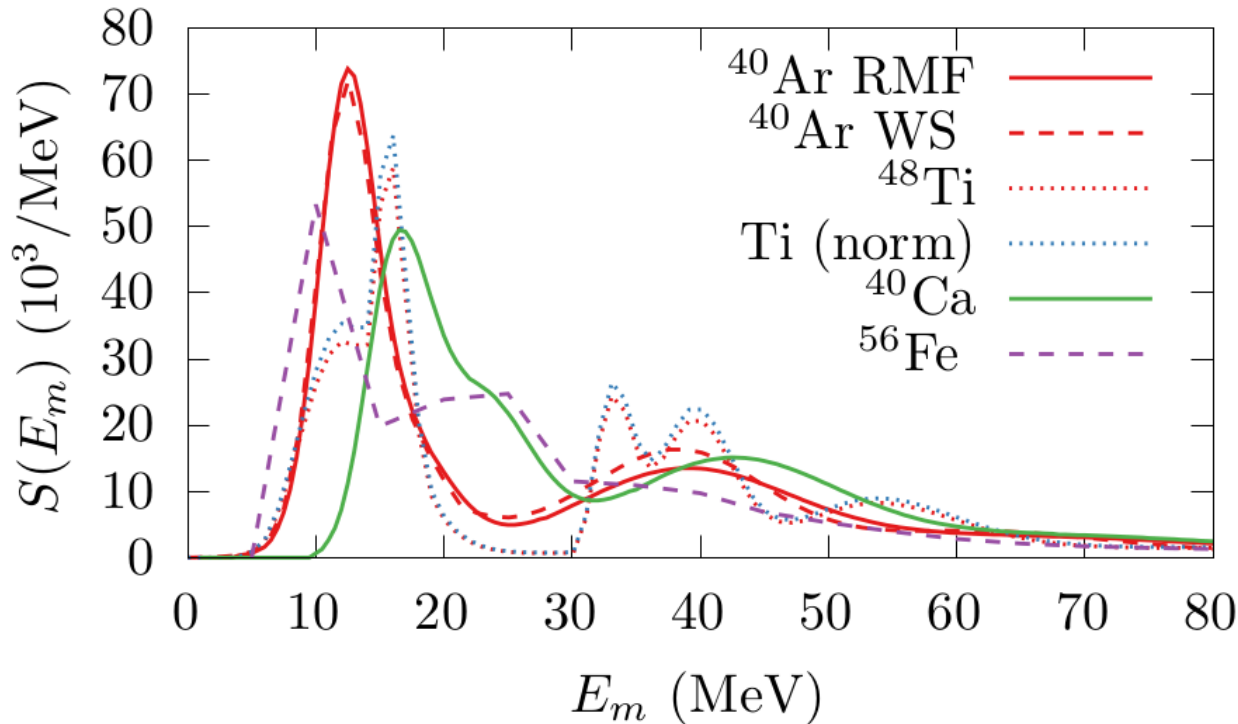
mean field + src

$$\delta(E_i - E_f) \sum_f |\mathcal{M}_{if}| = L_{\mu\nu} \sum_{\kappa} (2J_{\kappa} + 1) \delta(E_m - E_{\kappa}) H_{\kappa}^{\mu\nu}(Q, P_N) \rightarrow L_{\mu\nu} \left\{ \sum_{\kappa} N_{\kappa} \rho_{\kappa}(E_m) H_{\kappa}^{\mu\nu}(Q, P_N) + \rho_{corr}(E_m) H_{corr}^{\mu\nu}(Q, P) \right\}$$

# RDWIA calculations with spectral functions for MicroBooNE

$$L_{\mu\nu} \left\{ \sum_{\kappa} N_{\kappa} \rho_{\kappa}(E_m) H_{\kappa}^{\mu\nu}(Q, P_N) + \rho_{corr}(E_m) H_{corr}^{\mu\nu}(Q, P) \right\}$$

Choices of  $N_{\kappa}$  and  $\rho(E_m)$

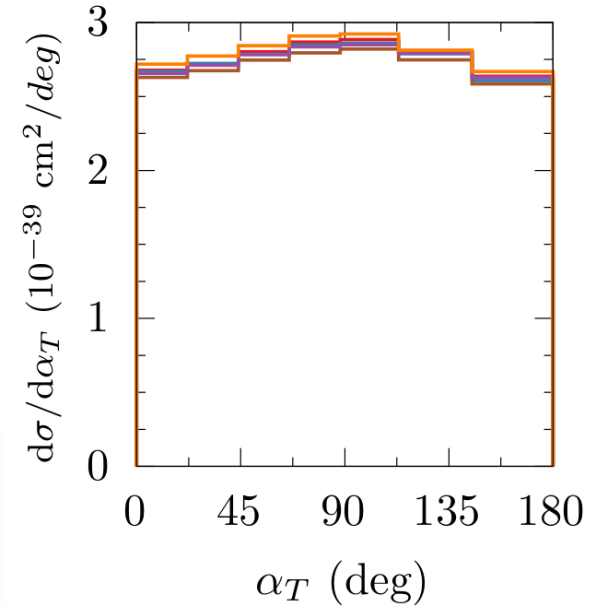
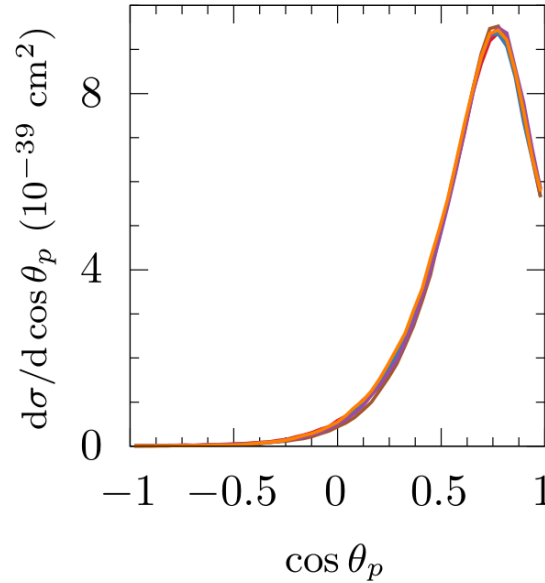
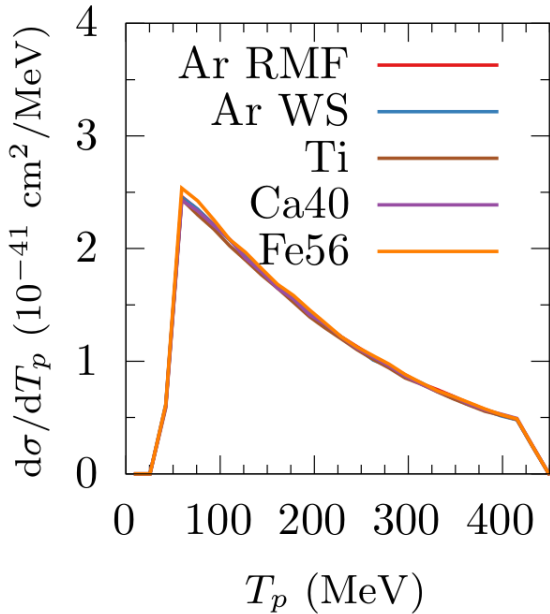


- $^{40}\text{Ar}$  spectral functions  
[Butkevich PRC 85, 065501]  
& [Jlab, PRD 107, 012005]
- $^{48}\text{Ti}$  from Jlab  
[PRD 107, 012005]
- $^{56}\text{Fe}$   
[Benhar et al. NPA 579, 493]
- $^{40}\text{Ca}$   
[Butkevich PRC 85, 065501]

Large variation in  $E_m$  profiles to check sensitivity of observables

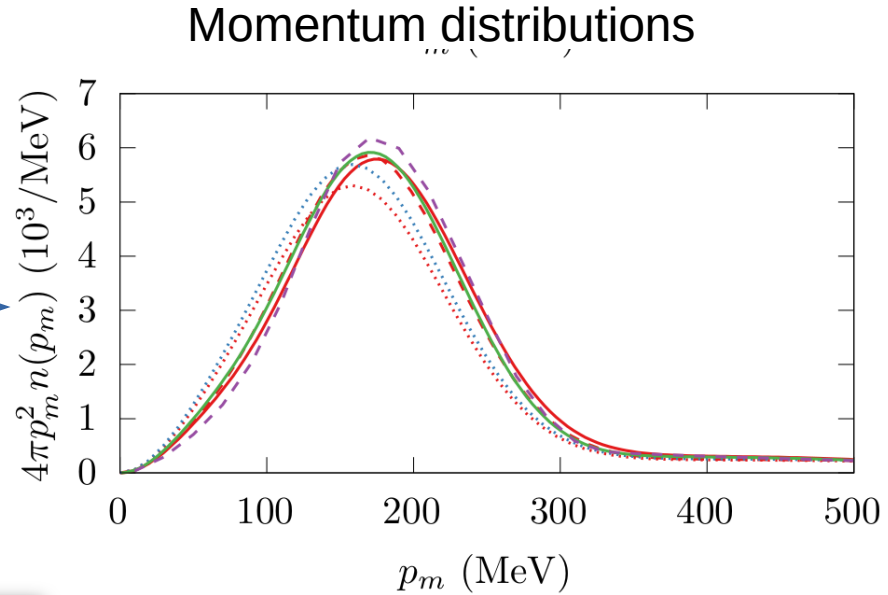
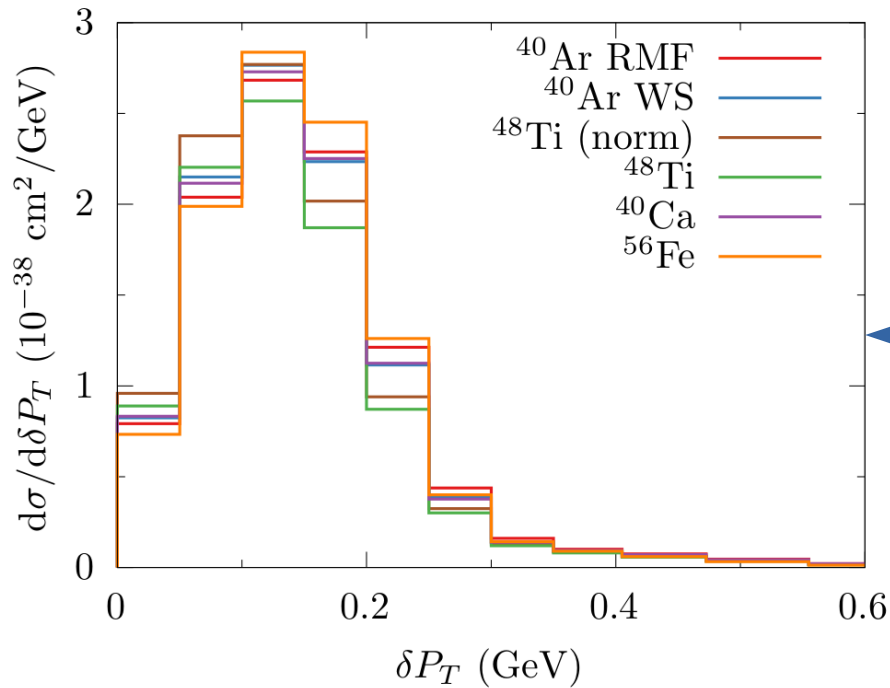
# Sensitivity to variations in the spectral functions: PWIA calculations

## Observables for MicroBooNE flux-averaged signal



**Observables that do not correlate  $p_p$  and  $p_\mu$  in flux-averaged data  
Cannot distinguish between these spectral functions**

# Sensitivity to variations in the spectral functions: PWIA calculations



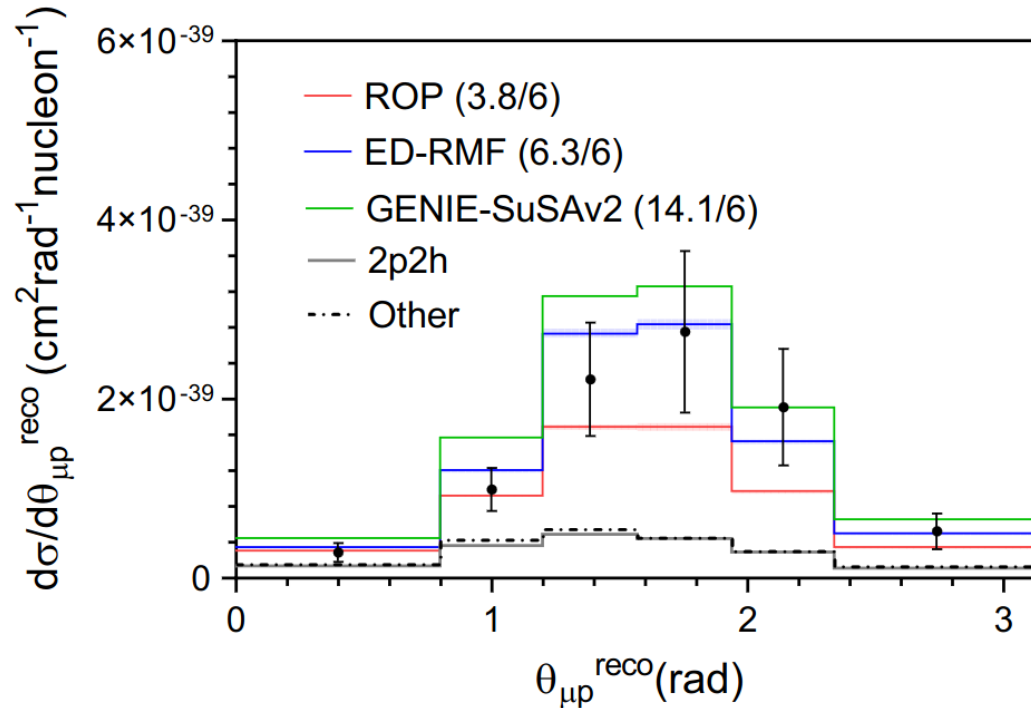
**$-\delta P_T$  is sensitive to momentum distribution**

- Almost **universal** for the realistic spectral functions considered
- **Titanium analysis is the outlier!**



## Sensitivity to variations in the spectral functions

Checking detailed dependence on SF for  $^{40}\text{Ar}$  in [J.M Franco-P et al. PRD 109 013004]

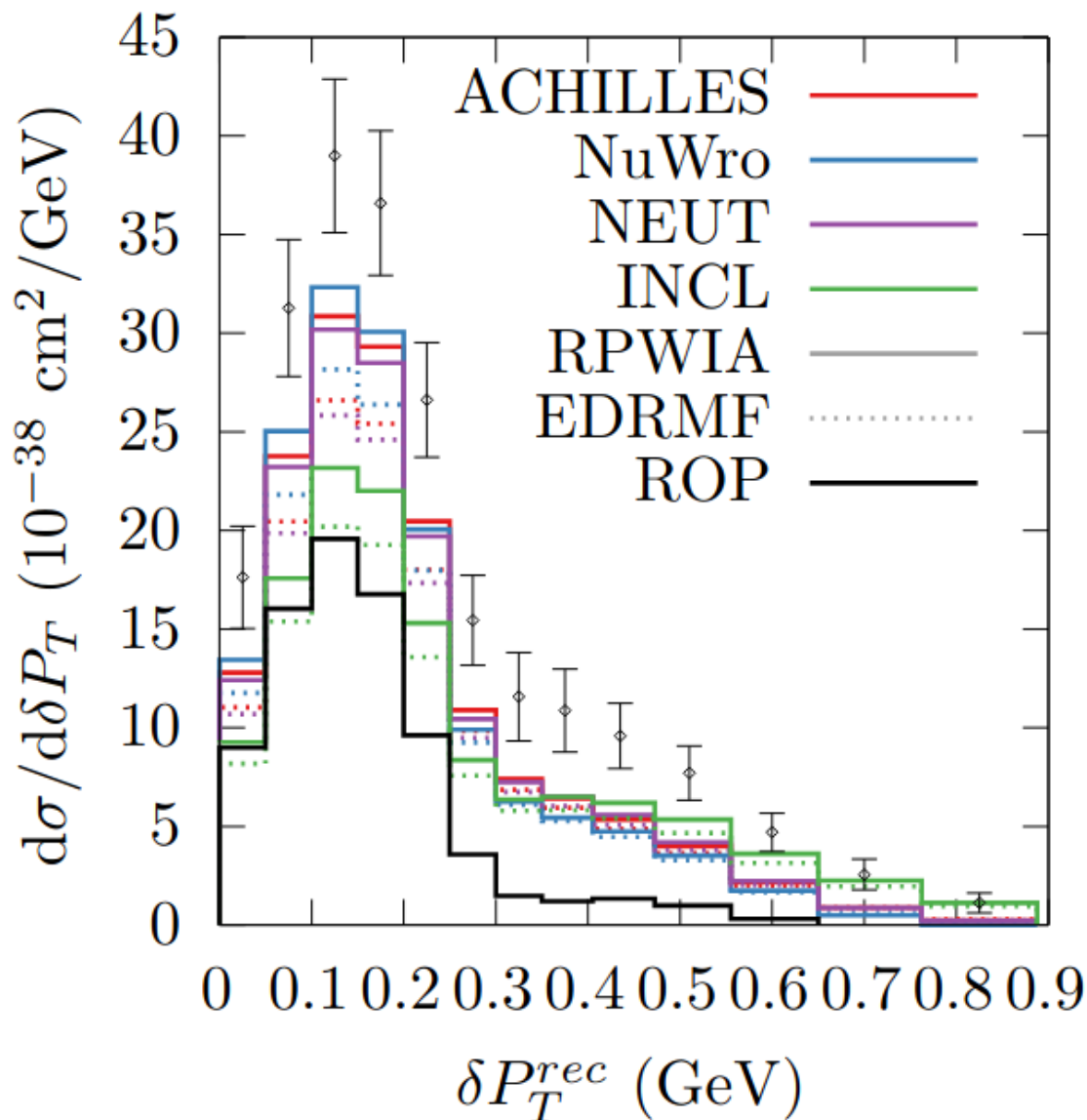


We conclude that for MicroBooNE data the  $^{40}\text{Ar}$  RMF choice is realistic enough  
→ Subdominant to FSI effects

Data not sensitive to missing-energy profile  
But reconstructed energy is → [R. Gonzalez-Jimenez et al. PRC 105 025502]

# RDWIA calculations for MicroBooNE data

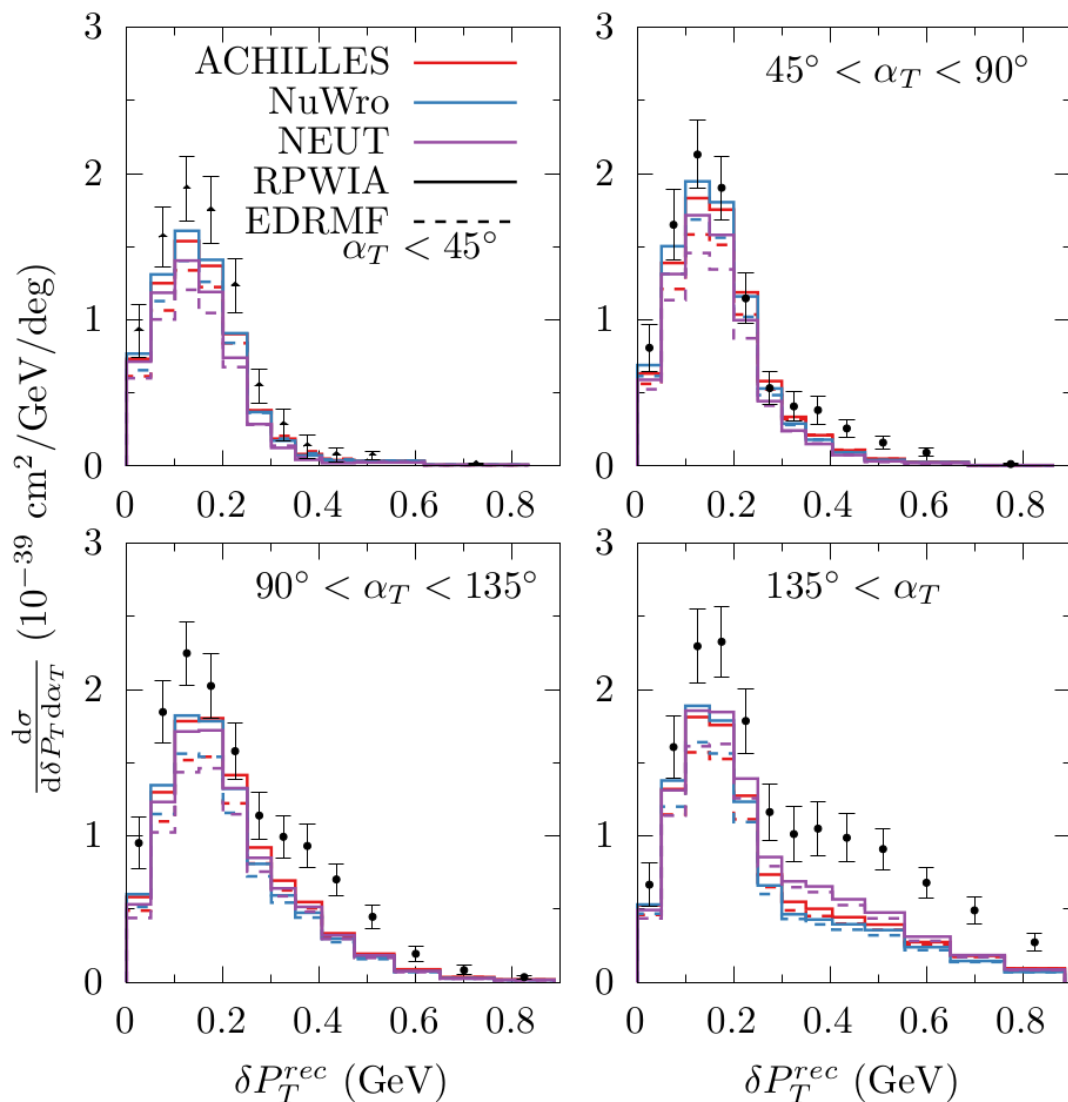
[ arxiv:2406.09244 ]



- Differences between INC smaller with kinematic cuts MicroBooNE
- RPWIA  $\rightarrow$  ED-RMF consistent  $\sim$ **10%** reduction
- Overall underprediction of data **expected** : no higher energy interactions (2p2h, SPP, ...)
- **Underprediction at low- $dP_T$** 
  - $\rightarrow$  Axial form factor ?
  - $\rightarrow$  **Interference with 2-body ?**
  - $\rightarrow$  Remove correlations ?

# RDWIA calculations for MicroBooNE data

[arxiv:2406.09244]



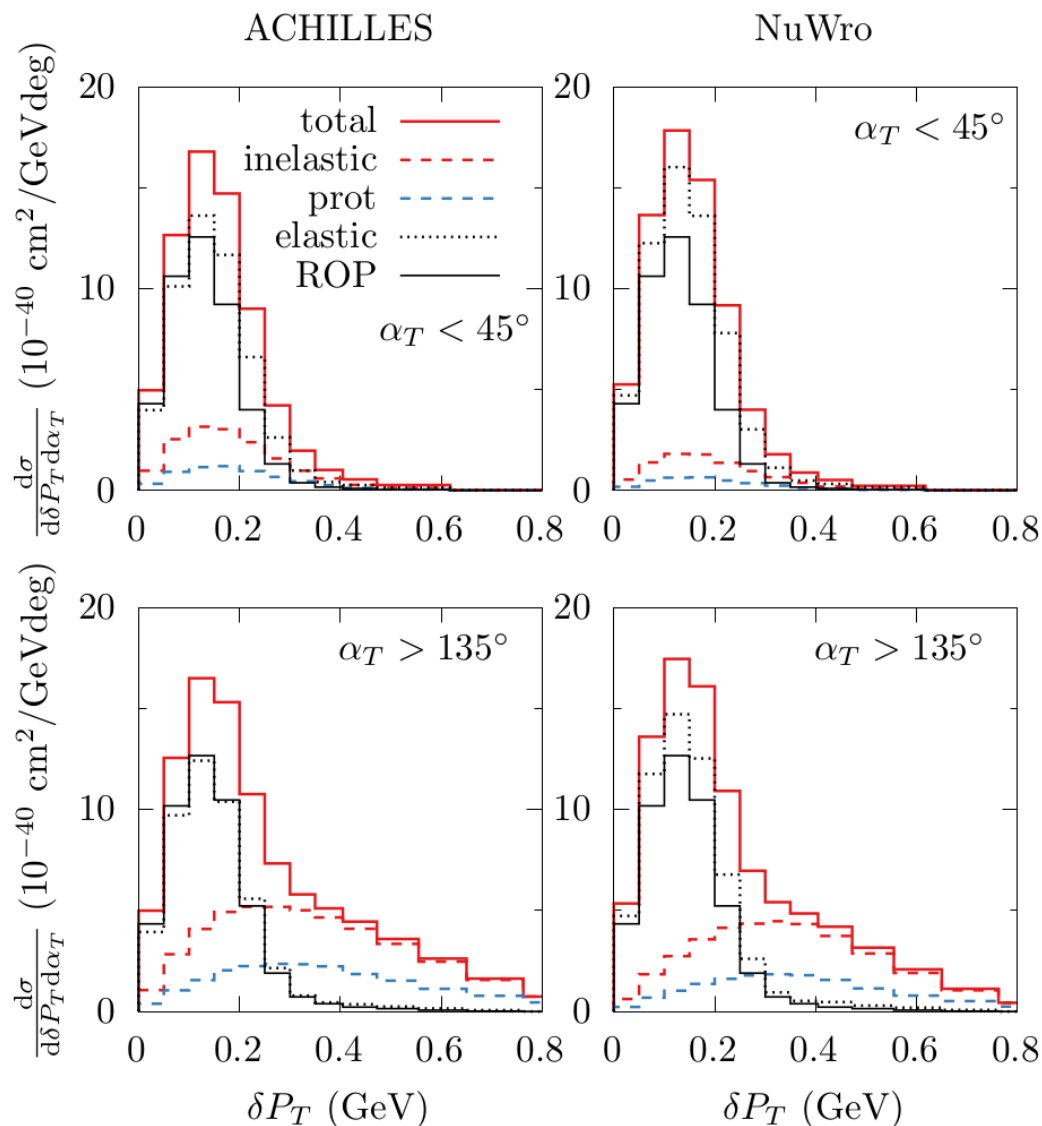
Double differential in  $dP_T$  and  $\alpha_T$   
 → effect of FSI is clear

**Picture remains:**

- 10% reduction in MF region in ED-RMF
- **Underprediction high  $\alpha_T$**   
 → **expected**
- **Low- $\alpha_T$  and  $dP_T$  ???**

# RDWIA calculations for MicroBooNE data

[arxiv:2406.09244]



**-Composition of signal**  
→ **INC dependent**  
→ **Significant contribution of Inelastic events**

**Even at low  $\alpha_T$**   
**Significant rescattering remains**

→ Could be removed with  
**electron scattering with  $E_m$  cut**

## Conclusions and outlook

- **Detailed comparisons of ACHILLES, INCL, NEUT & NuWro INCs with ROP**
  - No full agreement of any INC with the optical potential
  - Differences in low- $T_p$  region and due to treatment of SRC's

### What to do ?

- (e,e'p) over large hadron phase space with cut on  $E_m$  ?
- Assessment of the classical approximation underlying the INC

- **RDWIA results with INC & realistic spectral functions for scattering on  $^{40}\text{Ar}$**

- Constructed consistently with description for (e,e'p) and (e,e')
- Small sensitivity to variation in *realistic* spectral functions
- RDWIA leads to ~ 10 % reduction compared to typical PWIA
- General underprediction of data also in the low- $dP_T$  region

### What to do ?

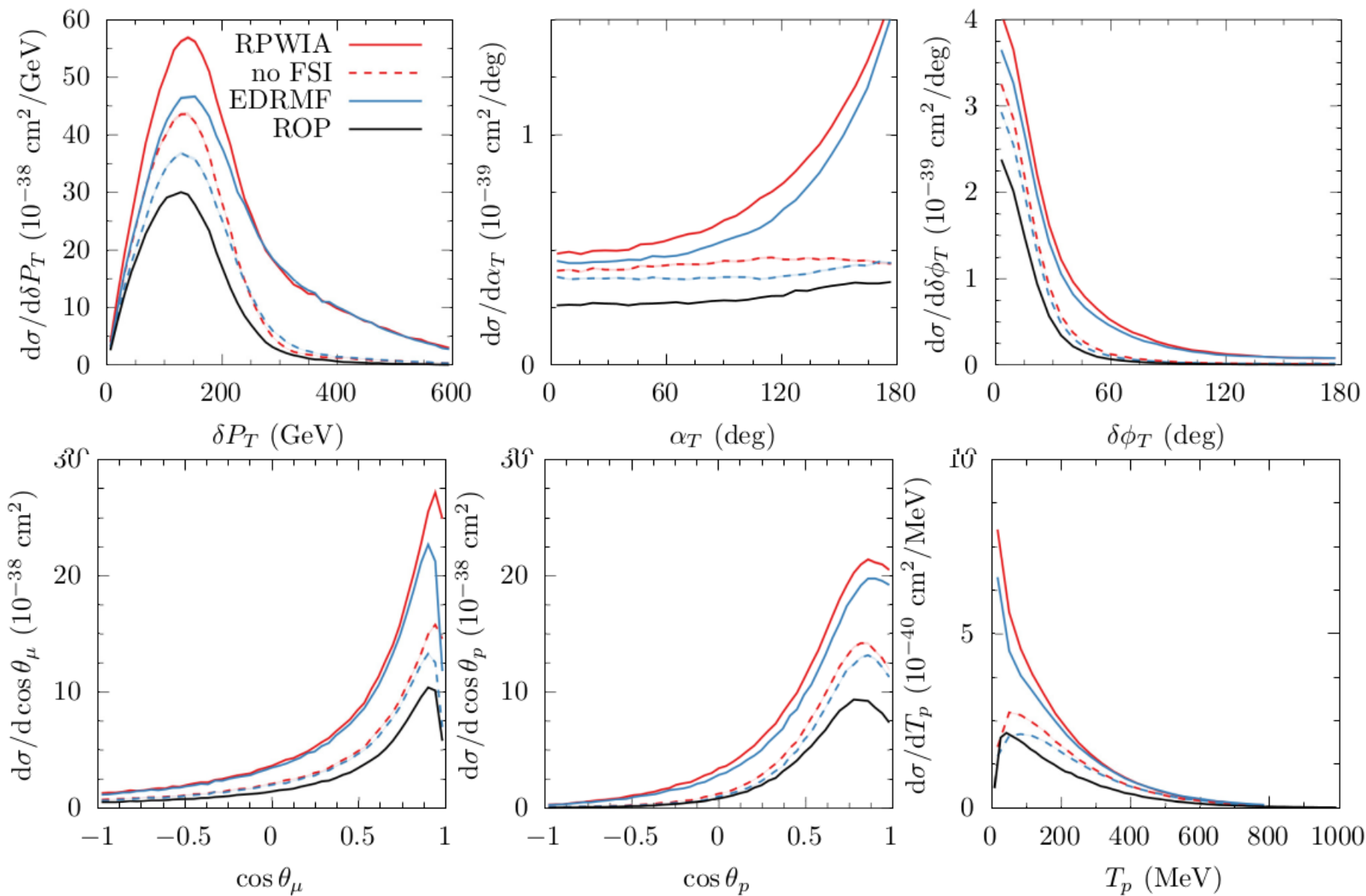
- Include interference with 2-body currents  
e.g [T Franco-Munoz et al. PRC 108 064608]  
[Lovato et al arxiv:2312.12545]
- Measurements sensitive to the missing-energy distribution ?  
e.g. [Baudis et al arxiv:2310.15633]

**Other stuff**

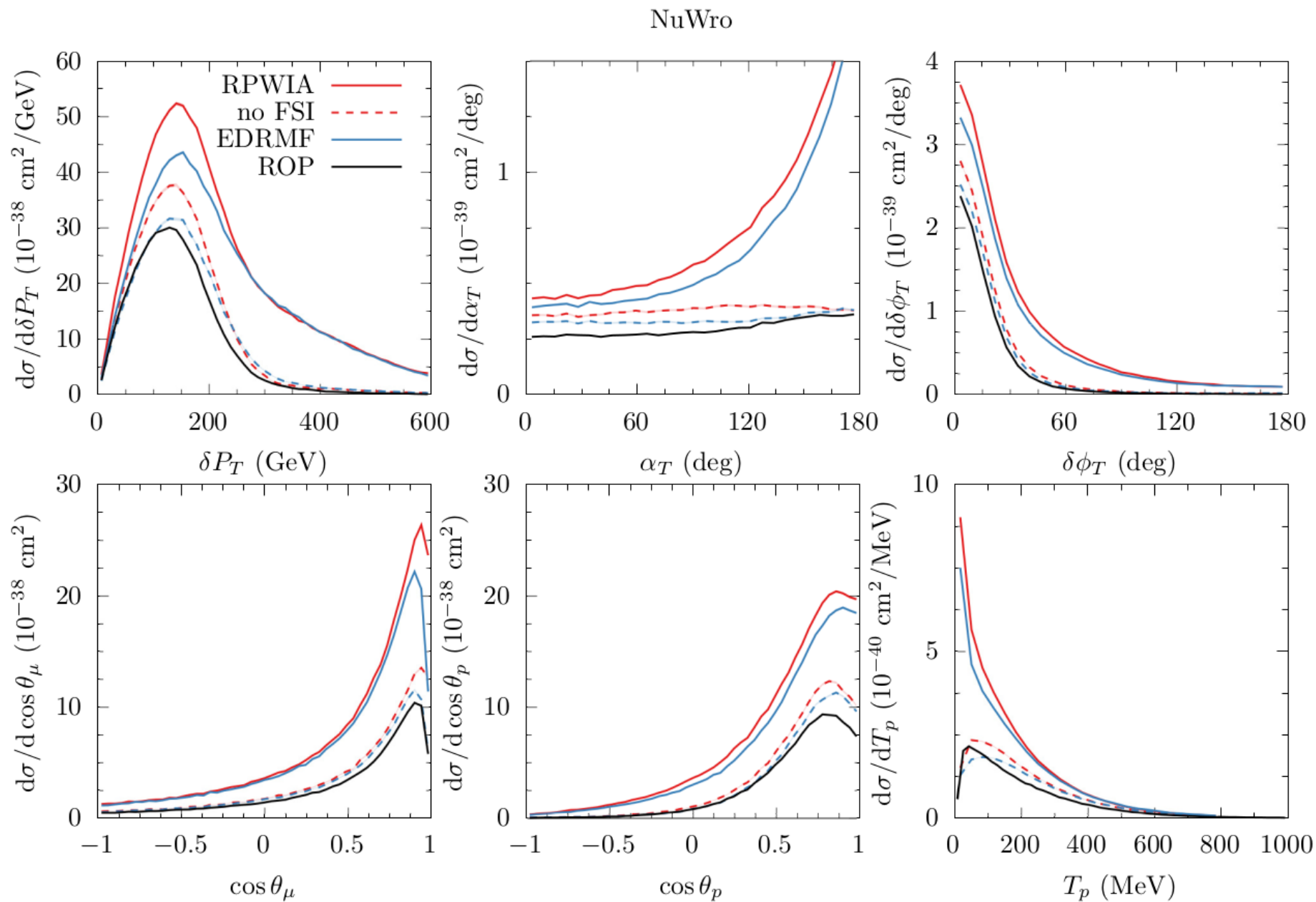


# NuWro with SRC effect in Mean-free path

NuWro



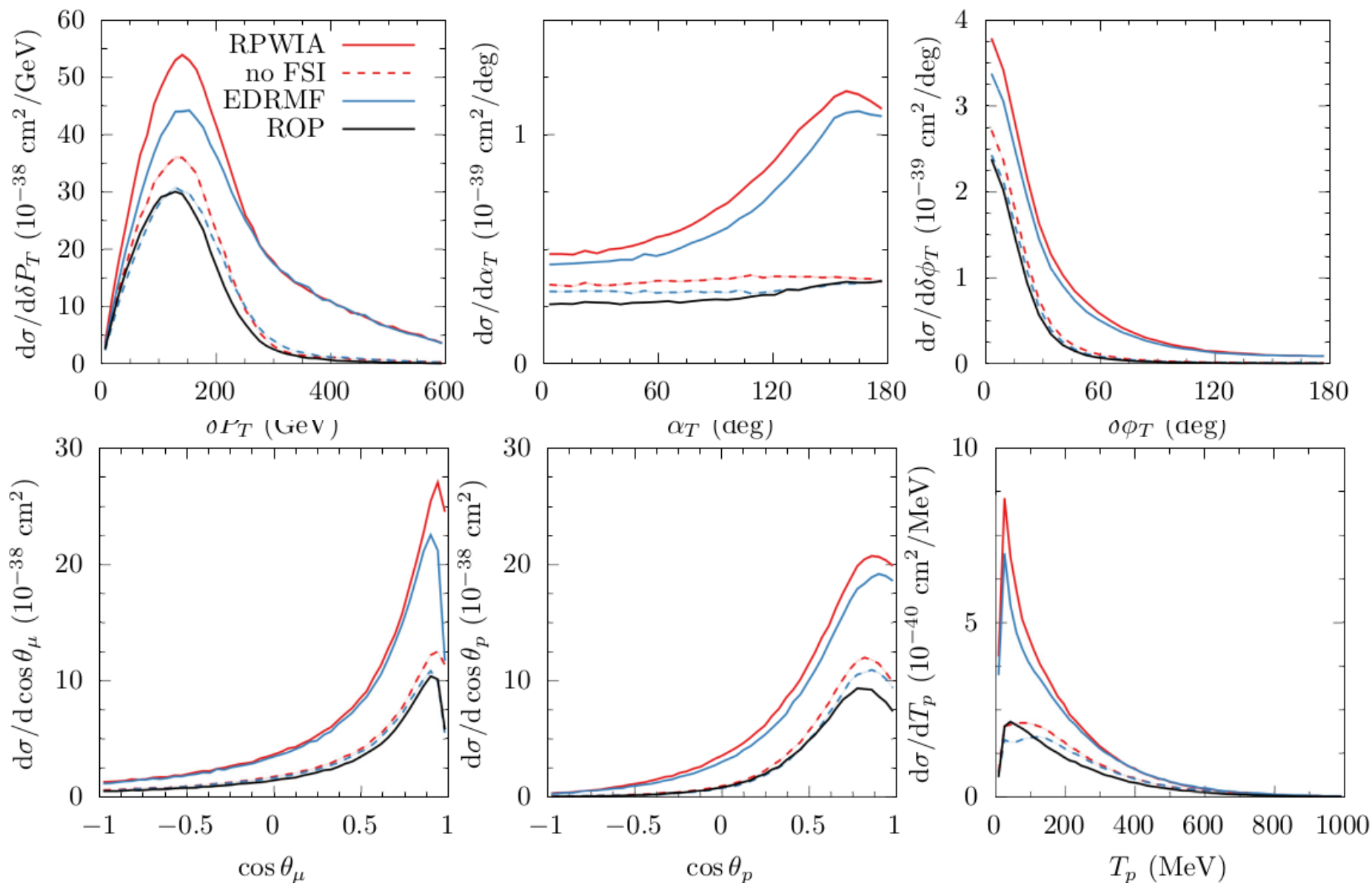
# NuWro without SRC effect in Mean-free path





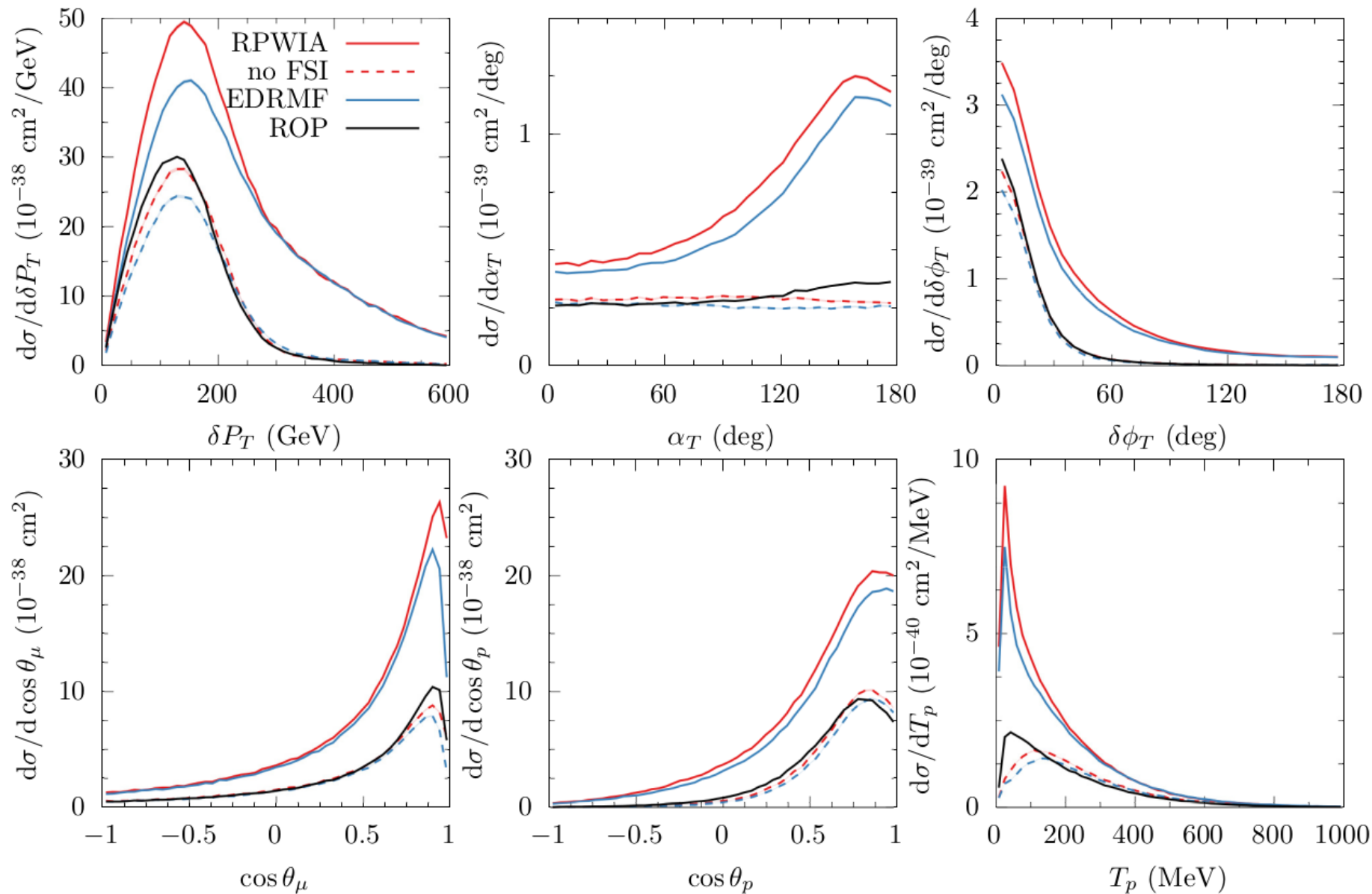
# ACHILLES with Formation time

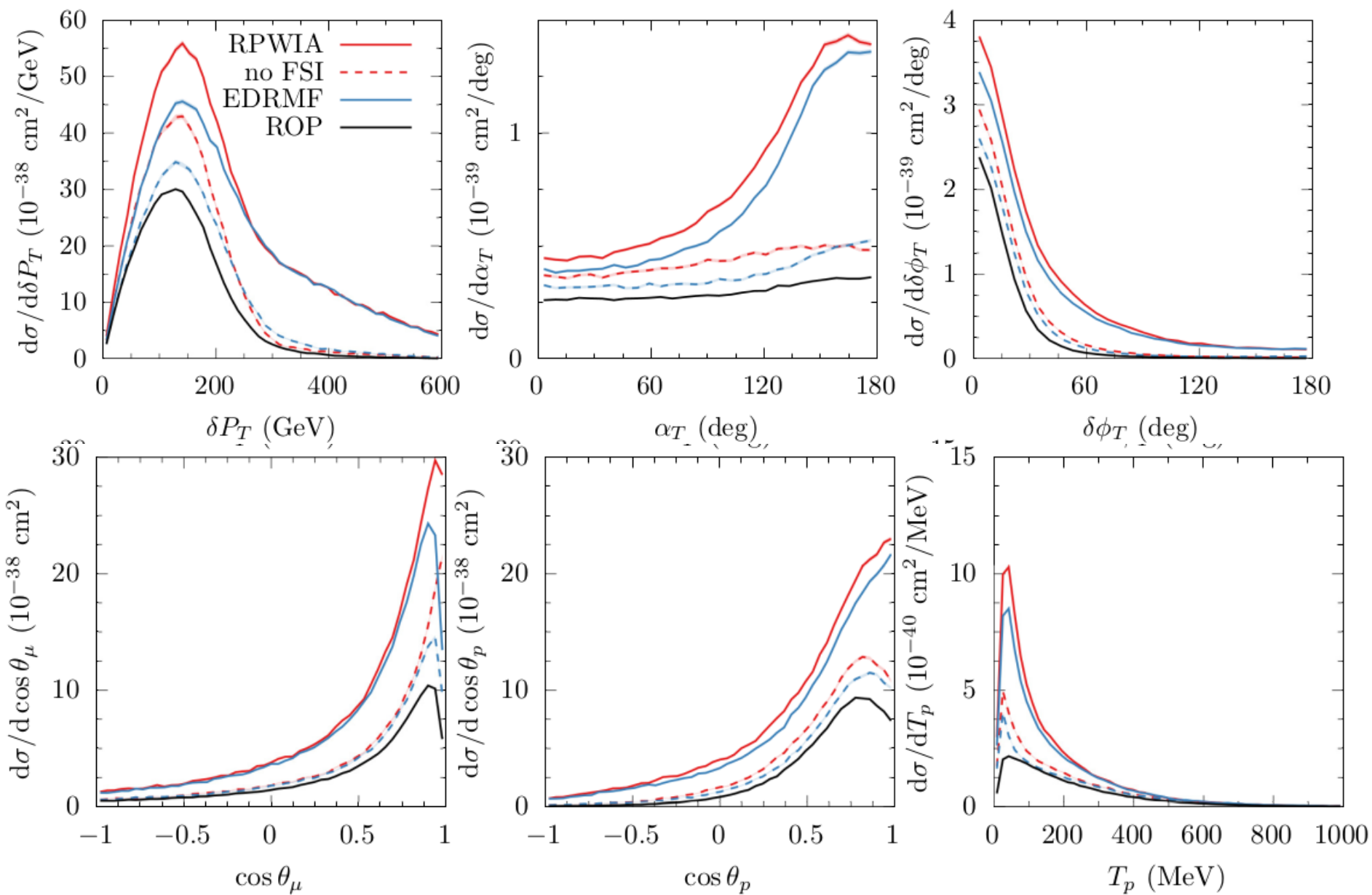
ACHILLES



# ACHILLES without Formation time

## ACHILLES

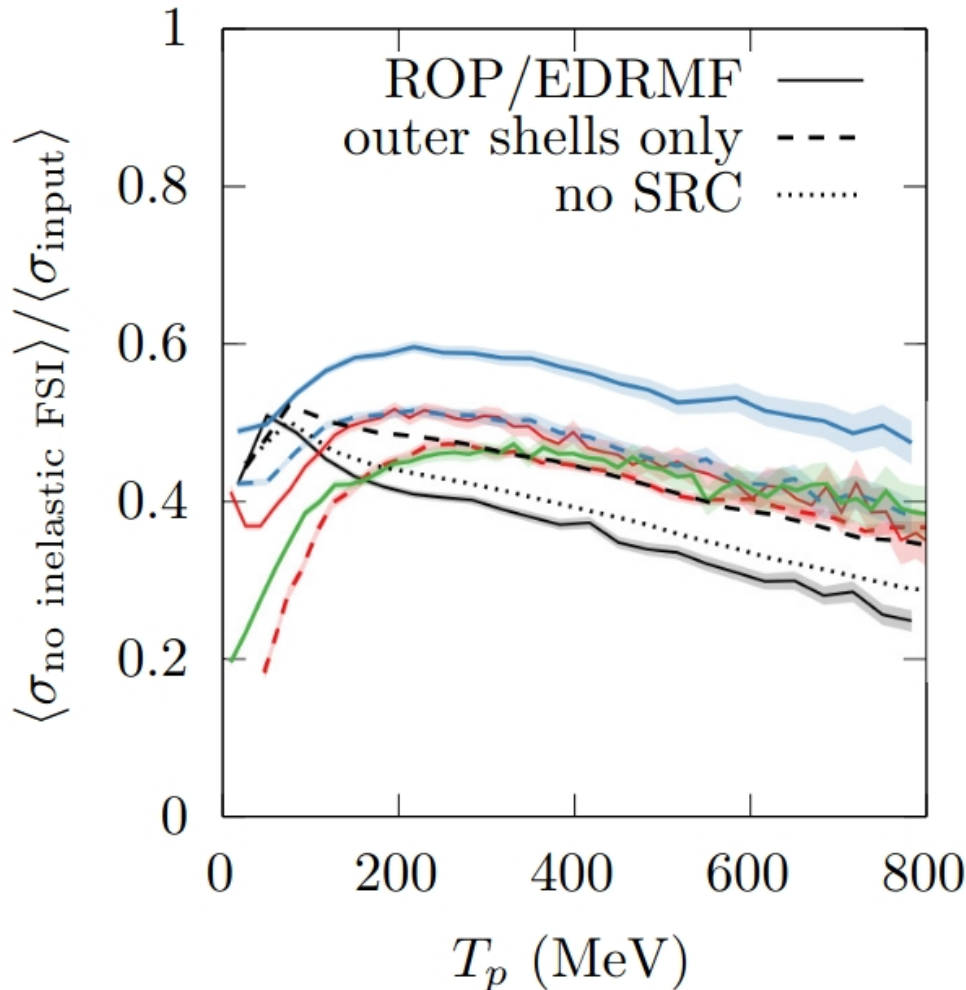




# Benchmarking INCs with RDWIA calculations for Argon

[ arxiv:2406.09244 ]

## Comparison of $T_p$ spectrum produced in different INCs & ROP



ratio **OUT/INPUT**

independent of INPUT *in INC*  
'INC Transparency'

**In ROP:**

**Innermost shells suffer more FSI than outer shells**

→ **Aligns with intuition**  
**the nucleon travels longer**

**SRC in this approximation**

→ **Suffer a lot of FSI**

**Outer shells agree with INCs**

**Full nucleus doesn't**