Weak interaction induced eta production from nucleon and nuclear targets

Atika Fatima

Collaborators

M. Sajjad Athar and S. K. Singh

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Outline

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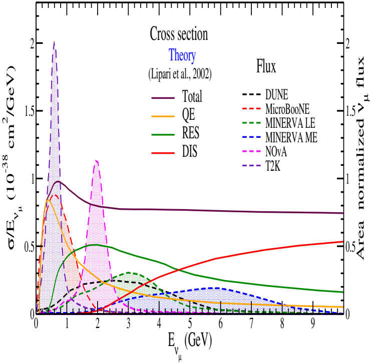
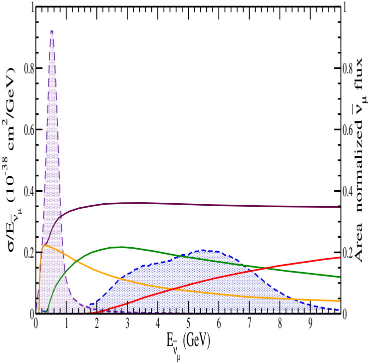
1. *Introduction*
2. *Eta production from free nucleon*
3. *Eta production from bound nucleon*
4. *Conclusion*

Introduction

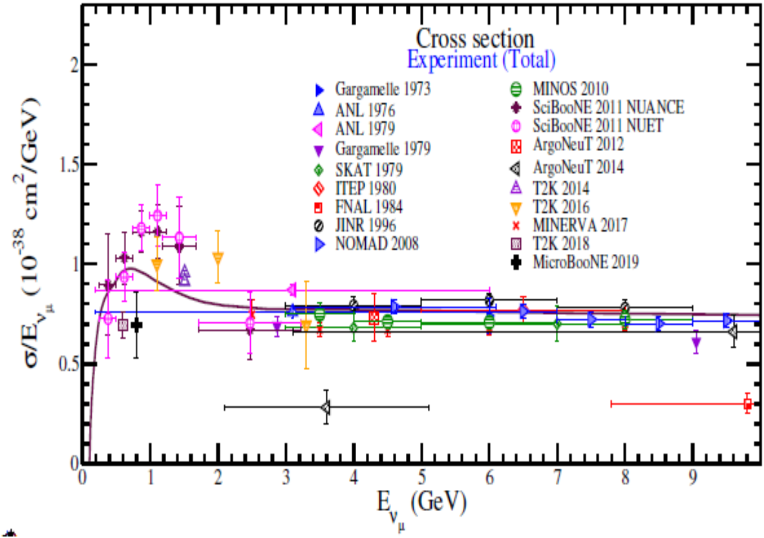
- The ongoing accelerator experiments like NOvA, MINERvA and T2K, and the upcoming DUNE experiment have (anti)neutrino peak energy in the few GeV energy region. This energy region is also important for the atmospheric neutrino experiments.





Neutrino cross section vs. neutrino energy

Cross section: theory vs experiment



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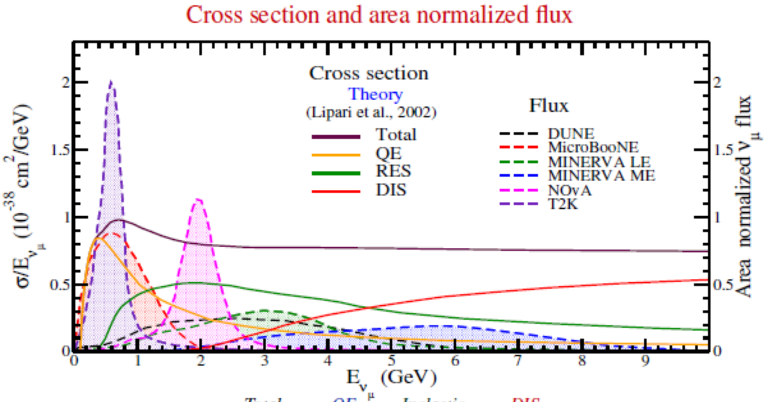

$$\sigma^{Total} = \sigma^{QE} + \sigma^{Inelastic} + \sigma^{DIS}$$

$$\sigma^{QE} = \sigma_{\nu_l n \to l^- p},\ \sigma_{\bar{\nu}_l p \to l^+ n}$$

$$\sigma^{Inelastic} = \sigma^{1\pi} + \sigma^{2\pi} + \sigma^{\eta} + \cdots + \sigma^{YK} + \sigma^{1K} + \sigma^{1Y} + \cdots$$

$$\sigma^{DIS} = \sigma_{\nu_l(\bar{\nu}_l N \to l^{\mp} X)}$$

Weak production of eta mesons

Theoretical models

- N. Dombey (1968)
- Nakamura et al. (DCC model)
- Aligarh group

Experimental studies

- BEBC Collaboration (1989)
- ICARUS Collaboration (2015)
- MicroBooNE Collaboration (2023)

MicroBooNE Collaboration has reported the flux averaged cross section for eta production (Phys. Rev. Lett. 132, 151801 (2024)).

MC generators

Most of the MC generators are using Rein-Sehgal model for the resonance model, which was initially developed for the study of single pion production.

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Weak η Production

- ηN couples only to nucleon resonances with isospin I = $\frac{1}{2}$
- A tool to study nucleon resonances especially $N^*(1535)$
- The $\nu/\bar{\nu}$ induced η production is interesting because
  - η is one of the important probes to search for the strange quark content of the nucleons
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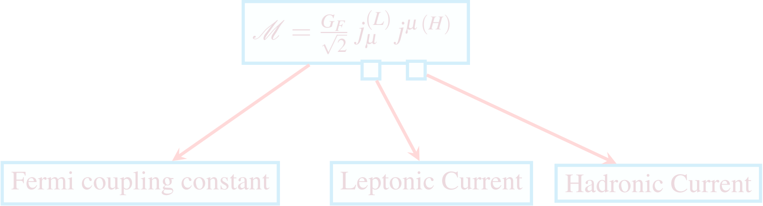
Kinematics: $\nu_l/\bar{\nu}_l(k) + N(p) \longrightarrow l^{\mp}(k') + B(p') + m(p_m)$

$$d\sigma = \frac{1}{4ME_\nu(2\pi)^5}\delta^4(k+p-k'-p'-p_m)\frac{d\vec{k}'}{(2E_l)}\frac{d\vec{p}\,'}{(2E_B)}\frac{d\vec{p}_m}{(2E_m)}\overline{\sum}\sum|\mathcal{M}|^2$$



$$\mathcal{M} = \frac{G_F}{\sqrt{2}}\, j_\mu^{(L)}\, j^{\mu\,(H)}$$

Fermi coupling constant

Leptonic Current

Hadronic Current

- Leptonic current is

$$j_\mu^{(L)} = \bar{u}(k')\gamma_\mu(1\pm\gamma_5)u(k)$$

- $j^{\mu\,(H)}$ describes hadronic matrix element for

$$W^i + N \to B' + m$$

- $j^{\mu\,(H)}$ receives contribution from
  - Resonance excitations
  - Nonresonant Born terms
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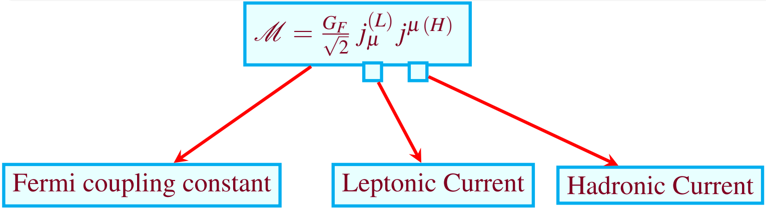

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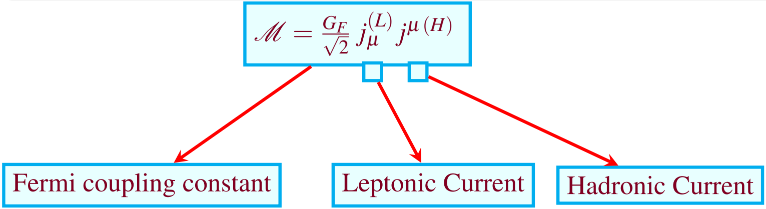

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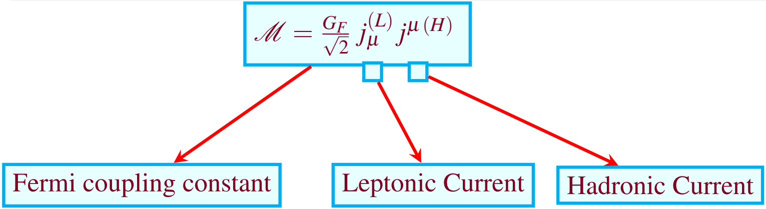

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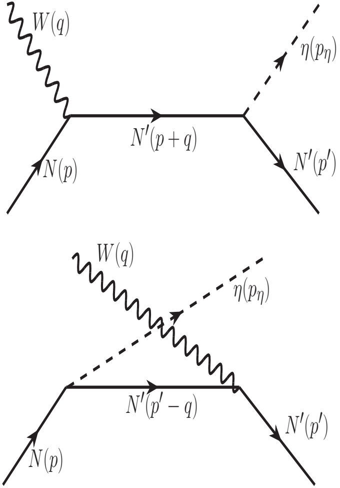
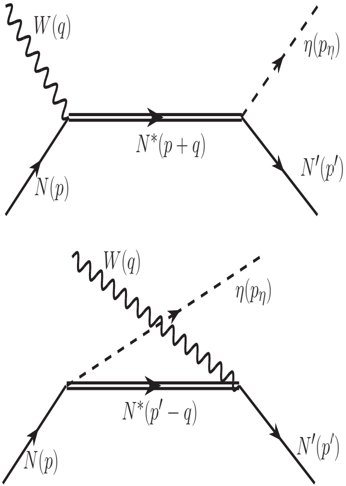
Eta production: Feynman diagrams



Born terms



Resonance excitations

- We have considered
    - Positive and negative parity spin $\frac{1}{2}$ resonances with $M_R < 2$ GeV
    - Contribution from Born diagrams

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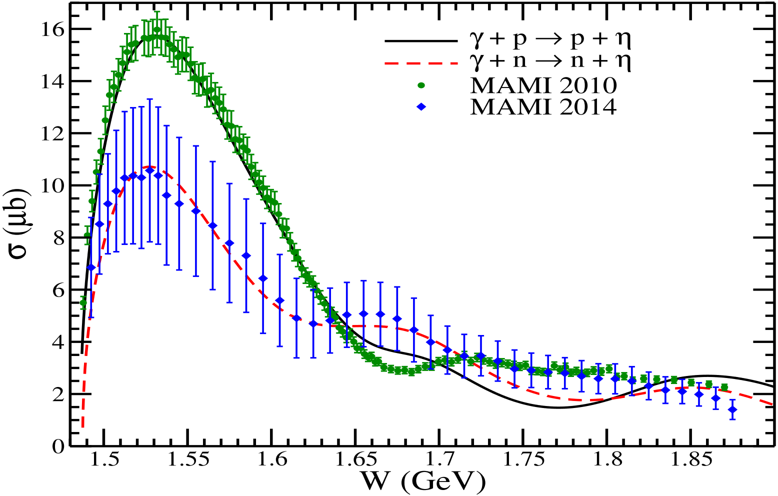
★ We make an educated guess for the Q^2 dependence which ultimately remains to be determined experimentally

Resonance contribution

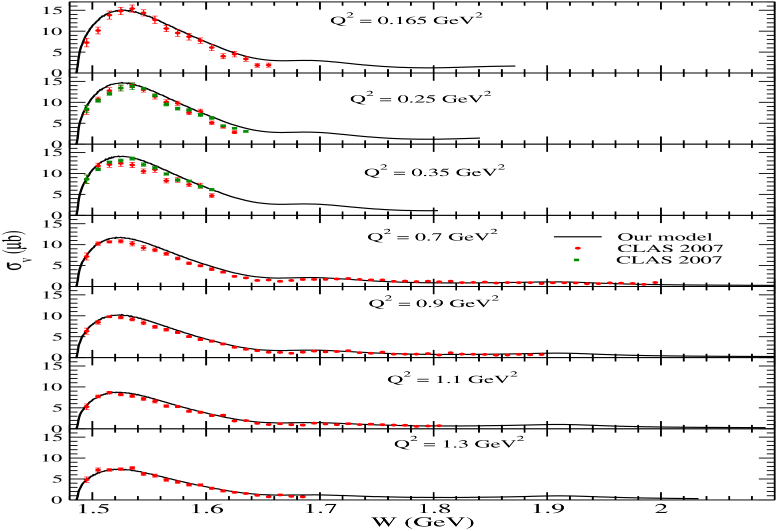
Resonance → Parameters ↓		$S_{11}(1535)$	$S_{11}(1650)$	$P_{11}(1710)$	$P_{11}(1880)$	$S_{11}(1895)$
M_R (GeV)		1.510 ± 0.01	1.655 ± 0.015	1.700 ± 0.02	1.860 ± 0.04	1.910 ± 0.02
Γ_R (GeV)		0.130 ± 0.02	0.135 ± 0.035	0.120 ± 0.04	0.230 ± 0.05	0.110 ± 0.03
$I(J^P)$		$\frac{1}{2}(\frac{1}{2}^-)$	$\frac{1}{2}(\frac{1}{2}^-)$	$\frac{1}{2}(\frac{1}{2}^+)$	$\frac{1}{2}(\frac{1}{2}^+)$	$\frac{1}{2}(\frac{1}{2}^-)$
BR (in %)	$N\pi$	32 – 52 (43)	50 – 70 (60)	5 – 20 (16)	3 – 31	2 – 18
	$N\eta$	30 – 55 (40)	15 – 35 (25)	10 – 50 (20)	1 – 55 (20)	15 – 45 (30)
	$K\Lambda$	–	5 – 15 (10)	5 – 25 (15)	(1 – 3) (2)	3 – 23 (13)
	$N\pi\pi$	4 – 31 (17)	20 – 58 (5)	14 – 48 (49)	(> 32) (44)	(17 – 74) (34)
$\lvert g_{RN\pi} \rvert$		0.1019	0.0915	0.0418	0.0466	0.0229
$\lvert g_{RN\eta} \rvert$		0.3696	0.1481	0.1567	0.1369	0.0877

σ for eta photoproduction processes



AF, MSA, SKS, Phys. Rev. D 107, 033002 (2023)

σ for eta electroproduction processes



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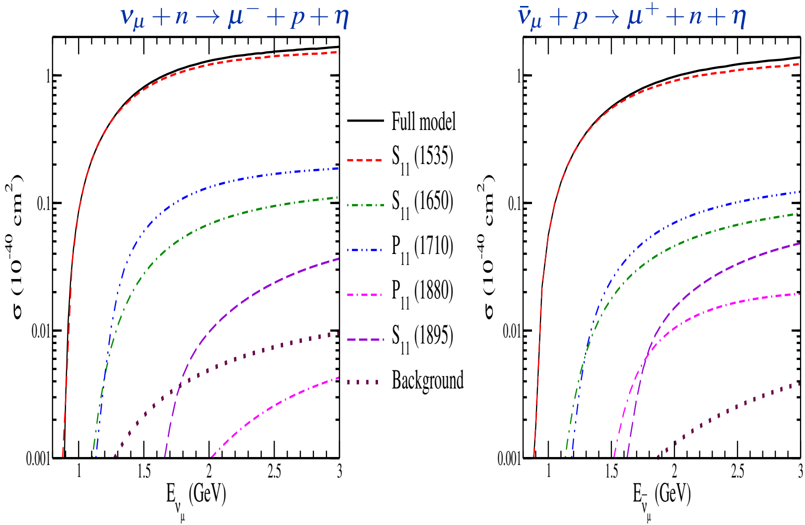
σ for CC induced eta production processes



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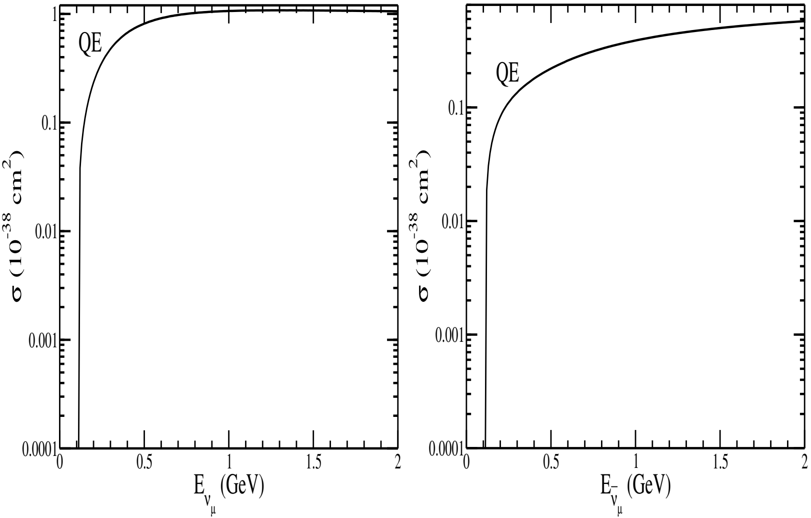
Neutrino cross section: Individual channels



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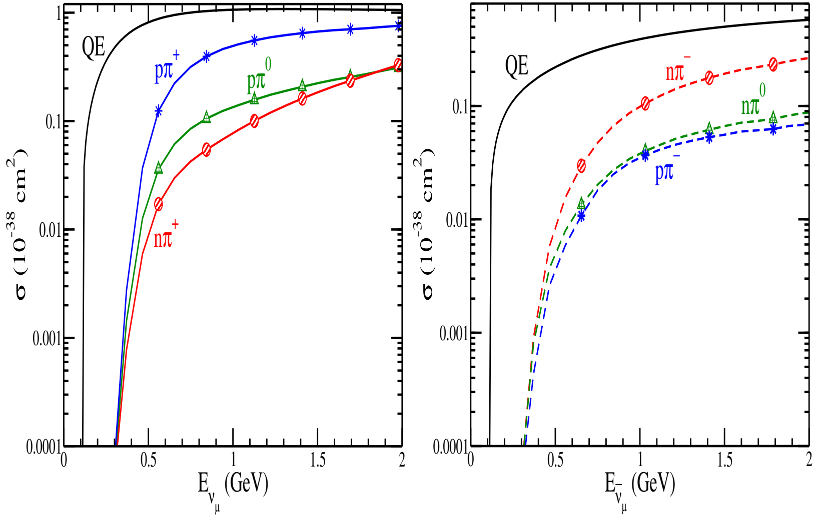

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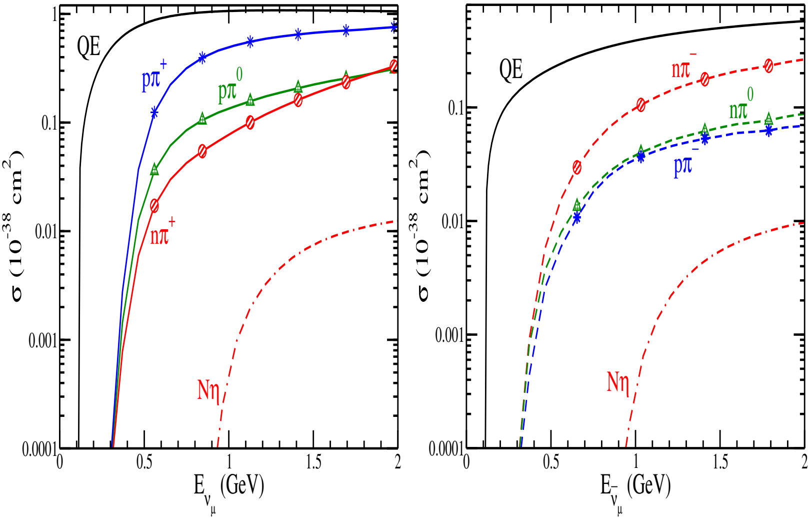

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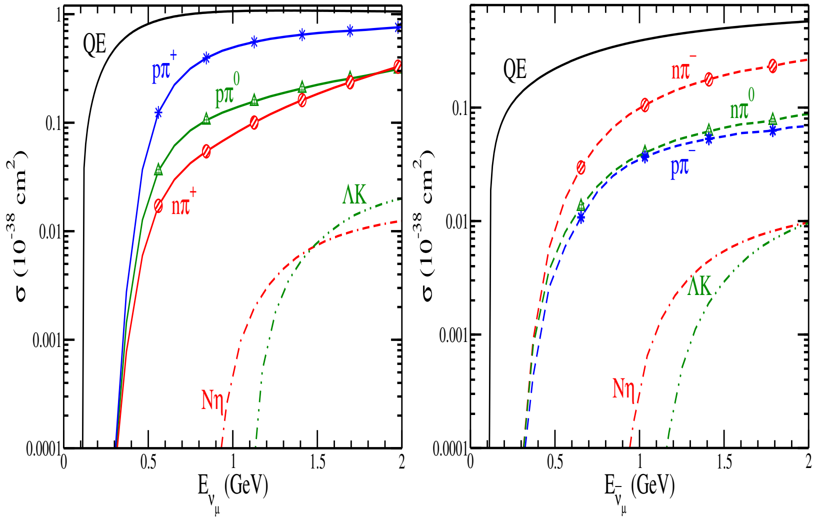

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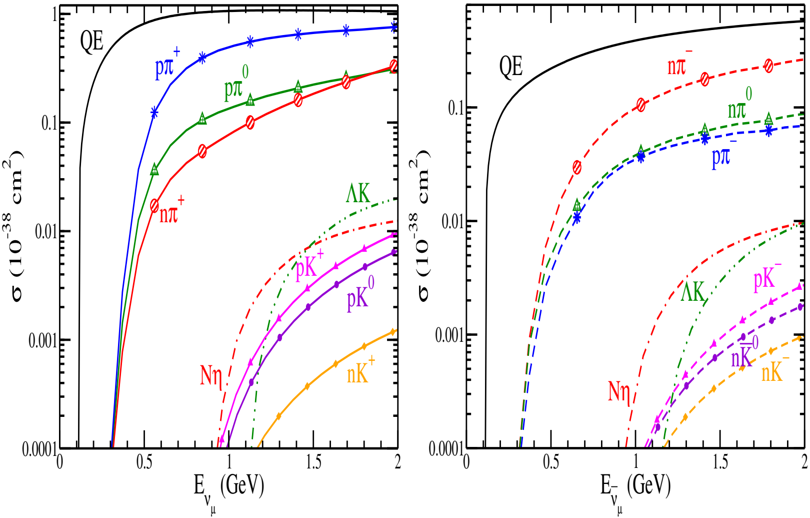

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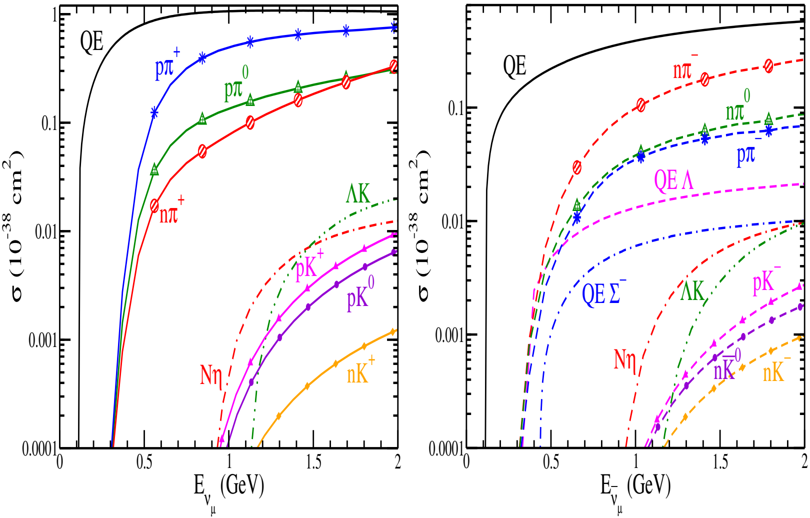

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Inside the nucleus

Local Fermi gas model

Cross section is evaluated as a function of local Fermi momentum($p_F(r)$) and integrated over the size of whole nucleus.

Inside the nucleus, the neutrino interacts with a neutron or proton whose local density in the medium is $\rho_n(r)$ or $\rho_p(r)$, respectively. Corresponding local Fermi momenta for neutron and proton are

$$p_{F_n} = [3\pi^2\rho_n(r)]^{\frac{1}{3}};\ \ p_{F_p} = [3\pi^2\rho_p(r)]^{\frac{1}{3}}$$

Differential scattering cross section

$$\left(\frac{d\sigma}{dE_\eta d\Omega_\eta}\right)_{\nu A} = \int d\vec{r}\,\rho_n(r)\left(\frac{d\sigma}{dE_\eta d\Omega_\eta}\right)_{\nu N}$$

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Mass and width modifications

- Inside the nuclear medium, the properties of the resonances like its mass and decay width gets modified.
- In the literature, other than $\Delta(1232)$ resonance, these modifications have only been studied for $S_{11}(1535)$ resonance by Oset et al. (Phys. Rev. C 44, 738 (1991)).
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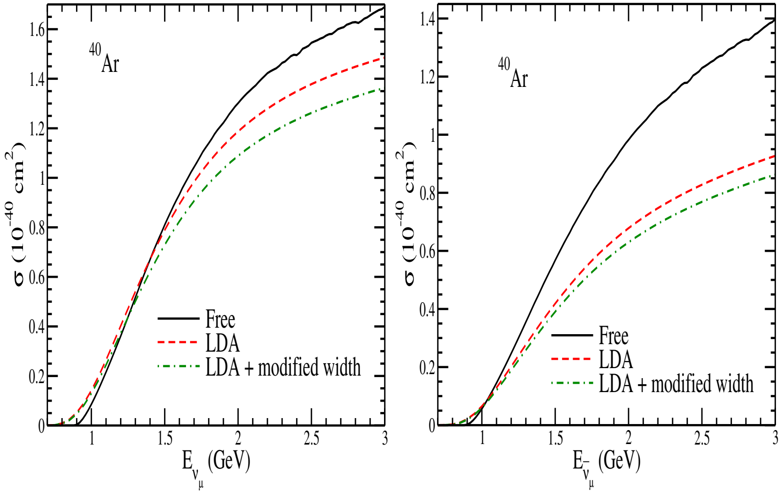
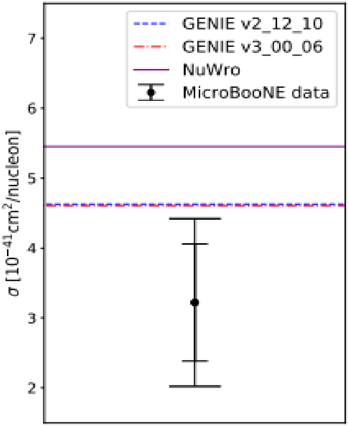
σ per interacting nucleon for the CC neutrino (left) and antineutrino (right) from ^{40}Ar nuclear target



AF, MSA, SKS, Paper in preparation

MicroBooNE η production result

$$\langle\sigma\rangle = (3.22 \pm 0.84 \pm 0.86) \times 10^{-41}\ \mathrm{cm}^2/\mathrm{nucleon}$$



Phys. Rev. Lett. 132, 151801 (2024)

- $\langle\sigma\rangle_{\mathrm{free}} = 1.87 \times 10^{-41}\ \mathrm{cm}^2/\mathrm{nucleon}$
- $\langle\sigma\rangle_{^{40}\mathrm{Ar}} = 1.78 \times 10^{-41}\ \mathrm{cm}^2/\mathrm{nucleon}$

- GENIE $v2\_12\_10$: $4.63 \times 10^{-41}\ \mathrm{cm}^2/\mathrm{nucleon}$
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- NEUT v5.4.0: $11.9 \times 10^{-41}\ \mathrm{cm}^2/\mathrm{nucleon}$

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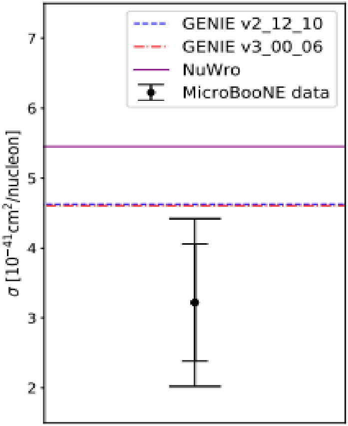

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Conclusion

- The study of η production is important:
  - in modelling the neutrino event generators
  - to understand the axial-vector response of the hadronic sector
- The model is first applied to the eta production in the electromagnetic sector and a good agreement with the experimental data is found.
- The effect of nuclear medium and width modification in the case of $S_{11}(1535)$ resonance are studied, and found to be significant in the case of both neutrino and antineutrino induced reactions.
- In order to reduce the systematics, a better understanding of the neutrino-nucleon cross sections for the individual reactions are required.

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Thank you!

BACKUP

Non-linear sigma model

- This is an effective field theory (EFT).
- EFT is a low energy approximation to some underlying, more fundamental theory. Low is defined with respect to some energy scale.
- The basic idea consists of writing down the most general possible Lagrangian, including all terms consistent with assumed symmetry principles, and then calculating matrix elements with this Lagrangian within some perturbative scheme.

Meson-Meson Interaction

The lowest order Lagrangian with the minimal number of derivatives describing the interaction of the Goldstone bosons

$$\mathscr{L} = \frac{f_\pi^2}{4} Tr(D_\mu U^\dagger D^\mu U)$$

U is $SU(3)$ matrix containing the Goldstone boson fields

$$U(x) = \exp\left(i\frac{\Phi(x)}{f_\pi} \right),$$

$SU(3)$ representation of pseudoscalar fields

$$\Phi(x) = \sum_{k=1}^{8} \phi_k(x)\lambda_k = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

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Interaction of pseudoscalar fields with baryons

We consider the octet of $\frac{1}{2}^{+}$ baryons. With each member of the octet we associate a complex, four-component Dirac field

$$B(x) = \sum_{k=1}^{8} \frac{1}{\sqrt{2}} b_k(x)\lambda_k = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix},$$

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The lowest-order chiral Lagrangian for the baryon octet in the presence of an external current may be written in terms of the SU(3) matrix B as,

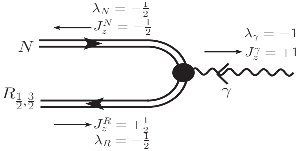
$$\begin{aligned}\mathscr{L}_{MB}^{(1)} &= \mathrm{Tr}\left[\bar{B}\left(i\not{D}-M\right)B\right]-\frac{D}{2}\mathrm{Tr}\left(\bar{B}\gamma^{\mu}\gamma_5\{u_\mu,B\}\right)\\ &- \frac{F}{2}\mathrm{Tr}\left(\bar{B}\gamma^{\mu}\gamma_5[u_\mu,B]\right),\end{aligned}$$

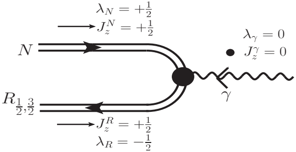
covariant derivative of B:

$$D_\mu B = \partial_\mu B + [\Gamma_\mu, B],$$

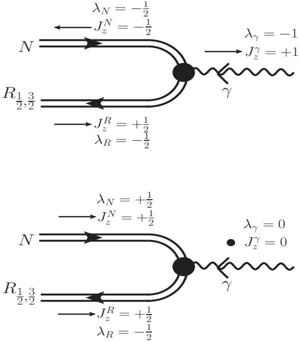
$$\Gamma^\mu = \frac{1}{2}\left[u^\dagger(\partial^\mu - ir^\mu)u + u(\partial^\mu - il^\mu)u^\dagger\right]$$

Electromagnetic form factors and helicity amplitudes





Electromagnetic form factors and helicity amplitudes



$$A_{\frac{1}{2}}^{p,n} = \sqrt{\frac{2\pi\alpha}{M}\frac{(M_R \mp M)^2 + Q^2}{M_R^2 - M^2}}\left[\frac{Q^2}{4M^2}F_1^{R^+,R^0} + \frac{M_R \pm M}{2M}F_2^{R^+,R^0}\right]$$

$$S_{\frac{1}{2}}^{p,n} = \mp\sqrt{\frac{\pi\alpha}{M}\frac{(M \pm M_R)^2 + Q^2}{M_R^2 - M^2}\frac{(M_R \mp M)^2 + Q^2}{4M_R M}}\left[\frac{M_R \pm M}{2M}F_1^{R^+,R^0} - F_2^{R^+,R^0}\right]$$

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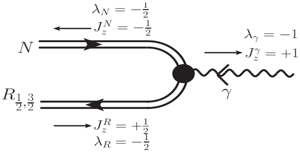
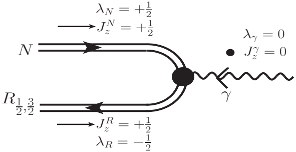
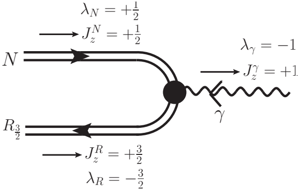




In case of spin $\frac{3}{2}$ resonances



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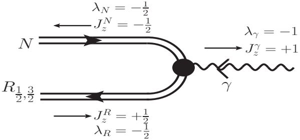

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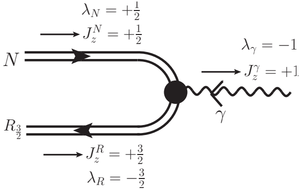

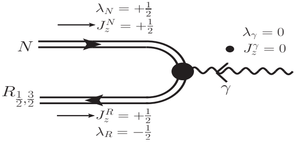

$$A^{p,n}_{\frac{3}{2}} = \sqrt{\frac{\pi\alpha}{M}\frac{(M_R \mp M)^2+Q^2}{M_R^2-M^2}}\left[\frac{C_3^{R+,R0}}{M}(M \pm M_R) \pm \frac{C_4^{R+,R0}}{M^2}\frac{M_R^2-M^2-Q^2}{2} \pm \frac{C_5^{R+,R0}}{M^2}\frac{M_R^2-M^2+Q^2}{2}\right]$$

$$A^{p,n}_{\frac{1}{2}} = \sqrt{\frac{\pi\alpha}{3M}\frac{(M_R \mp M)^2+Q^2}{M_R^2-M^2}}\left[\frac{C_3^{R+,R0}}{M}\frac{M^2+MM_R+Q^2}{M_R} - \frac{C_4^{R+,R0}}{M^2}\frac{M_R^2-M^2-Q^2}{2} - \frac{C_5^{R+,R0}}{M^2}\frac{M_R^2-M^2+Q^2}{2}\right]$$

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$$\times \left[\frac{C_3^{R+,R0}}{M}M_R + \frac{C_4^{R+,R0}}{M^2}M_R^2 + \frac{C_5^{R+,R0}}{M^2}\frac{M_R^2+M^2+Q^2}{2}\right]$$

Non-linear sigma model

Also referred to as the Weinberg-Lagrangian

$$\mathscr{L}_{NLSM} = \bar{\psi}(i\not{\partial} + \gamma^\mu V_\mu + \gamma^\mu \gamma_5 A_\mu - M)\psi + \frac{f_\pi^2}{4} Tr(\partial_\mu U^\dagger \partial^\mu U)$$

with

- $V_\mu = \frac{1}{2}\left[\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger\right]$
- $A_\mu = \frac{i}{2}\left[\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger\right]$
- $\xi = e^{i\frac{\vec{\tau}\cdot\vec{\Phi}(x)}{2f\pi}} \implies U = \xi\xi$
- $\vec{\Phi}$ is identified with the meson field

$$\Phi(x) = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

In order to take into account the external fields (gauge bosons) into account, ∂^μ is replaced by D^μ as

$$\begin{aligned} D^\mu U &\equiv \partial^\mu U - ir^\mu U + iU l^\mu, \\ D^\mu U^\dagger &\equiv \partial^\mu U^\dagger + iU^\dagger r^\mu - il^\mu U^\dagger \end{aligned}$$

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Hadronic current for Born terms

$$J^{\mu}_{N(s)} = F_s(s)\,\frac{D-3F}{2\sqrt{3}f_\eta}\bar{u}_N(p')\not{p}_\eta\gamma_5\frac{\not{p}+\not{q}+M}{(p+q)^2-M^2}\mathscr{O}^{\mu}_{V}u_N(p)$$

$$J^{\mu}_{N(u)} = F_u(u)\,\frac{D-3F}{2\sqrt{3}f_\eta}\bar{u}_N(p')\mathscr{O}^{\mu}_{V}\frac{\not{p}-\not{p}_\eta+M}{(p-p_\eta)^2-M^2}\not{p}_\eta\gamma_5u_N(p),$$

- D and F are the axial-vector couplings of the baryon octet
- f_η is the eta decay constant
- $F_s(s)$ and $F_u(u)$ are form factors at the strong vertex, introduced to take into account the hadronic structure
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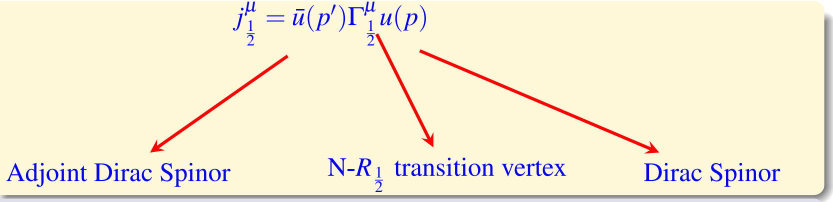
Hadronic current for resonance excitations

$$
\begin{aligned}
j^{\mu}\Big|_{s} &= F_s^{*}(s)\,\frac{g_{RN\eta}}{f_\eta}\bar{u}(p')\not{p}_\eta\gamma_5\Gamma_s\left(\frac{\not{p}+\not{q}+M_R}{s-M_R^2+iM_R\Gamma_R}\right)\Gamma^{\mu}_{\frac{1}{2}\pm}u(p),\\
j^{\mu}\Big|_{u} &= F_u^{*}(u)\,\frac{g_{RN\eta}}{f_\eta}\bar{u}(p')\Gamma^{\mu}_{\frac{1}{2}\pm}\left(\frac{\not{p}'-\not{q}+M_R}{u-M_R^2+iM_R\Gamma_R}\right)\not{p}_\eta\gamma_5\Gamma_s u(p)
\end{aligned}
$$

Spin $\frac{1}{2}$ resonance



$$j^{\mu}_{\frac{1}{2}} = \bar{u}(p')\Gamma^{\mu}_{\frac{1}{2}}u(p)$$

Adjoint Dirac Spinor

N-$R_{\frac{1}{2}}$ transition vertex

Dirac Spinor

Transition vertex

- Positive parity state $$\Gamma^{\mu}_{\frac{1}{2}^+} = V^{\mu}_{\frac{1}{2}} - A^{\mu}_{\frac{1}{2}}$$
- Negative parity state $$\Gamma^{\mu}_{\frac{1}{2}^-} = \left[V^{\mu}_{\frac{1}{2}} - A^{\mu}_{\frac{1}{2}}\right]\gamma_5$$

$$V^{\mu}_{\frac{1}{2}} = \left[\frac{f_1(Q^2)}{(2M)^2}\left(Q^2\gamma^{\mu} + \not{q}q^{\mu}\right) + \frac{f_2(Q^2)}{2M}i\sigma^{\mu\alpha}q_{\alpha}\right]\gamma_5$$

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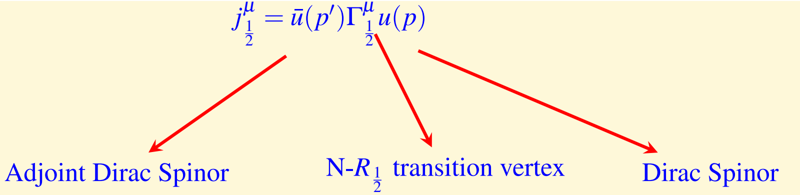

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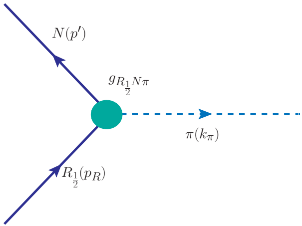
Axial vector form factors and strong coupling $g_{R_{\frac{1}{2}}N\pi}$



$$\mathscr{L}_{R_{\frac{1}{2}}N\pi} = \frac{g_{R\frac{1}{2}N\pi}}{m_\pi}\bar{\Psi}_{R_{\frac{1}{2}}}\,\Gamma^{\mu}_{\frac{1}{2}}\,\partial_\mu\phi^i T_i\Psi$$

$\mathscr{R}N\pi$ coupling strength

Nucleon field

Field for spin $\frac{1}{2}$ resonances

$$\Gamma_{R_{\frac{1}{2}}\to\pi N} = \frac{\mathscr{C}}{4\pi}\left(\frac{g_{R\frac{1}{2}N\pi}}{m_\pi}\right)^2 (M_R \pm M)^2\,\frac{E_N \mp M}{M_R}|\vec{q}_{\rm cm}|$$

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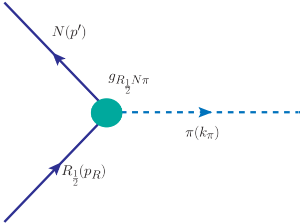

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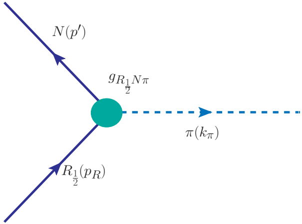

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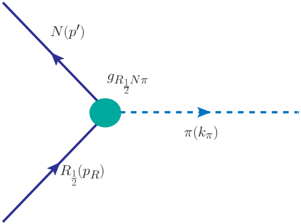

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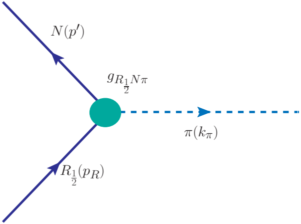
Axial vector form factors and strong coupling $g_{R_{\frac{1}{2}}N\pi}$



$$\mathscr{L}_{R_{\frac{1}{2}}N\pi} = \frac{g_{R\frac{1}{2}N\pi}}{m_\pi}\bar{\Psi}_{R_{\frac{1}{2}}}\;\Gamma^\mu_{\frac{1}{2}}\;\partial_\mu\phi^i T_i \Psi$$

$\mathscr{R}N\pi$ coupling strength

Field for spin $\frac{1}{2}$ resonances

Nucleon field

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$N-R_{\frac{1}{2}}$ transition vector form factors

- ★ Isospin symmetry relates weak vector form factors with electromagnetic form factors

$$f^V_{1,2}(Q^2) = F^{R+}_{1,2}(Q^2) - F^{R0}_{1,2}(Q^2).$$

- ★ EM form factors are derived from the helicity amplitudes extracted from the real and/or virtual photon scattering experiments

$$\begin{aligned} A^{p,n}_{\frac{1}{2}} &= \sqrt{\frac{2\pi\alpha}{M}\frac{(M_R \mp M)^2+Q^2}{M_R^2-M^2}}\left[\frac{Q^2}{4M^2}F_1^{R^+,R^0} + \frac{M_R \pm M}{2M}F_2^{R^+,R^0}\right] \\ S^{p,n}_{\frac{1}{2}} &= \mp\sqrt{\frac{\pi\alpha}{M}\frac{(M \pm M_R)^2+Q^2}{M_R^2-M^2}\frac{(M_R \mp M)^2+Q^2}{4M_RM}}\left[\frac{M_R \pm M}{2M}F_1^{R^+,R^0} - F_2^{R^+,R^0}\right] \end{aligned}$$

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Parameterization of the helicity amplitudes

$$\mathscr{A}_\alpha(Q^2) = \mathscr{A}_\alpha(0)(1 + a_1 Q^2)e^{-b_1 Q^2}$$

Resonance	Helicity amplitude	Proton target			Neutron target		
		$\mathscr{A}_\alpha(0)$	a_1	b_1	$\mathscr{A}_\alpha(0)$	a_1	b_1
$S_{11}(1535)$	$A_{\frac{1}{2}}$	95.0	0.85	0.85	−78.0	1.75	1.75
	$S_{\frac{1}{2}}$	−2.0	1.9	0.81	32.5	0.4	1.0
$S_{11}(1650)$	$A_{\frac{1}{2}}$	33.3	0.45	0.72	26.0	0.1	2.5
	$S_{\frac{1}{2}}$	2.5	1.88	0.96	3.8	0.4	0.71
$P_{11}(1710)$	$A_{\frac{1}{2}}$	55.0	1.0	1.05	−45.0	−0.02	0.95
	$S_{\frac{1}{2}}$	4.4	2.18	0.88	−31.5	0.35	0.85
$P_{11}(1880)$	$A_{\frac{1}{2}}$	−60.0	0.4	1.0	−45.0	−0.02	0.95
	$S_{\frac{1}{2}}$	0.4	0.75	0.5	−31.5	0.35	0.85
$S_{11}(1895)$	$A_{\frac{1}{2}}$	−15.0	1.45	0.6	26.0	0.1	2.5
	$S_{\frac{1}{2}}$	−3.5	0.88	0.6	3.8	0.4	0.71

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$N - R_{\frac{1}{2}}$ transition axial-vector form factors

- Experimentally, the information regarding the axial vector form factors is scarce
- PCAC and PDDAC relates g_1 with g_3
- Generalized GT relation gives $g_1(0)$ in terms of $g_{RN\pi}$
- $g_{RN\pi}$ is obtained using partial decay width of the $R \rightarrow N\pi$