T violation at a future neutrino factory

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Motivation

- Testing T violation in lepton sector has not been achieved
- CP and T violation measurements as a non-trivial check of the CPT theorem in QFT

Introduction

Framework of standard 3 flavor ν oscillation in matter

+ for ν, − for $\bar{\nu}$

$$i\frac{d}{dt}\begin{pmatrix}\nu_e(\bar{\nu}_e)\\ \nu_\mu(\bar{\nu}_\mu)\\ \nu_\tau(\bar{\nu}_\tau)\end{pmatrix} = \left[U\mathrm{diag}(0,\Delta E_{21},\Delta E_{31})U^\dagger + \mathrm{diag}(\pm A,0,0)\right]\begin{pmatrix}\nu_e(\bar{\nu}_e)\\ \nu_\mu(\bar{\nu}_\mu)\\ \nu_\tau(\bar{\nu}_\tau)\end{pmatrix}$$

Diagonalized by $\widetilde{U}$ and eigen values $\tilde{E}_j$

$$= \widetilde{U}^{(\pm)}\mathrm{diag}(\tilde{E}_1^{(\pm)},\tilde{E}_2^{(\pm)},\tilde{E}_3^{(\pm)})\widetilde{U}^{(\pm)\dagger}\begin{pmatrix}\nu_e(\bar{\nu}_e)\\ \nu_\mu(\bar{\nu}_\mu)\\ \nu_\tau(\bar{\nu}_\tau)\end{pmatrix}$$

$$U = \begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}$$

CP phase

Introduction

CP violation : if $\delta = 0$, this difference does not zero

→ **Matter effect may mimic CP violation**

$$P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

$$= -4 \sum_{j>k} \left\{ \mathrm{Re}\left[\widetilde{U}^{(+)}{}_{\beta j} \widetilde{U}^{(+)}{}^{*}_{\alpha j} \widetilde{U}^{(+)}{}^{*}_{\beta k} \widetilde{U}^{(+)}{}_{\alpha k} \right] \sin^2\left(\frac{\Delta \tilde{E}^{(+)}_{jk} L}{2} \right) \right.$$

$$\left. - \mathrm{Re}\left[\widetilde{U}^{(-)}{}_{\beta j} \widetilde{U}^{(-)}{}^{*}_{\alpha j} \widetilde{U}^{(-)}{}^{*}_{\beta k} \widetilde{U}^{(-)}{}_{\alpha k} \right] \sin^2\left(\frac{\Delta \tilde{E}^{(-)}_{jk} L}{2} \right) \right\}$$

$$+2 \sum_{j>k} \left\{ \mathrm{Im}\left[\widetilde{U}^{(+)}{}_{\beta j} \widetilde{U}^{(+)}{}^{*}_{\alpha j} \widetilde{U}^{(+)}{}^{*}_{\beta k} \widetilde{U}^{(+)}{}_{\alpha k} \right] \sin\left(\Delta \tilde{E}^{(+)}_{jk} L \right) \right.$$

$$\left. - \mathrm{Im}[\widetilde{U}^{(-)}{}_{\beta j} \widetilde{U}^{(-)}{}^{*}_{\alpha j} \widetilde{U}^{(-)}{}^{*}_{\beta k} \widetilde{U}^{(-)}{}_{\alpha k}] \sin\left(\Delta \tilde{E}^{(-)}_{jk} L \right) \right\}$$

Introduction

T violation : if $\delta = 0$, this difference is exactly zero

$\rightarrow$ **Matter effect is not important**

$\rightarrow$ **Pure CP (T) violating effect**

$$P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$$

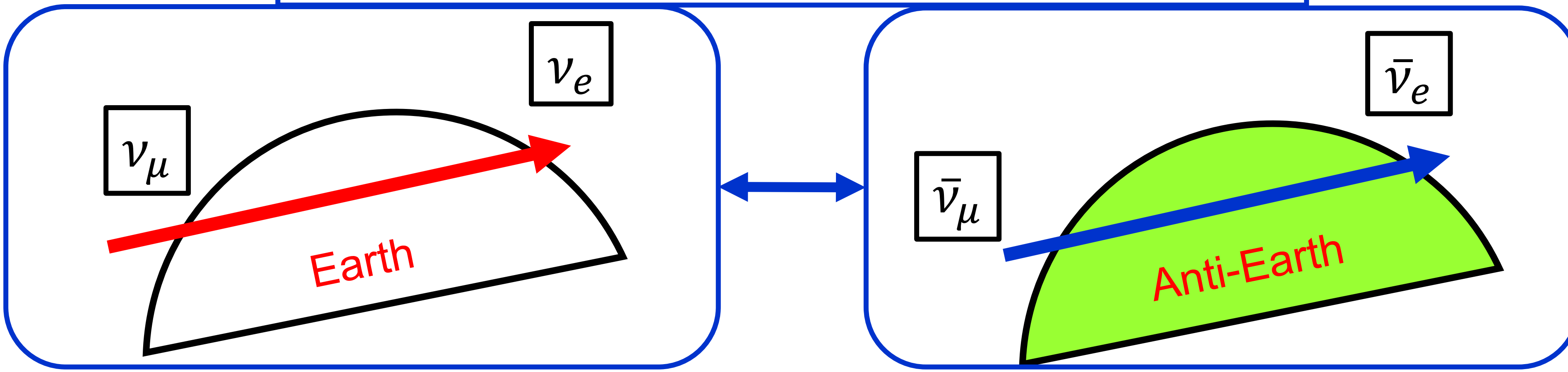
$$= -16\mathrm{Im}\left[\widetilde{U}^{(+)}{}_{\beta 2}\widetilde{U}^{(+)}{}^{*}_{\alpha 2}\widetilde{U}^{(+)}{}^{*}_{\beta 1}\widetilde{U}^{(+)}{}_{\alpha 1}\right]$$

$$\times \sin^2\left(\frac{\Delta\tilde{E}^{(+)}_{31}L}{2}\right)\sin^2\left(\frac{\Delta\tilde{E}^{(+)}_{32}L}{2}\right)\sin^2\left(\frac{\Delta\tilde{E}^{(+)}_{21}L}{2}\right)$$

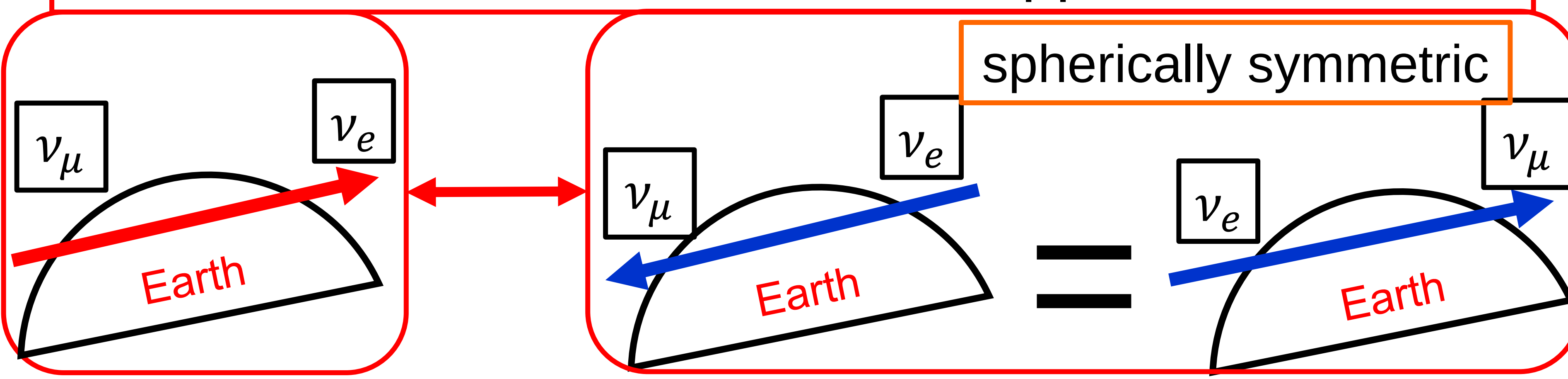
$$\propto \sin\delta$$

CP violation and T violation in matter

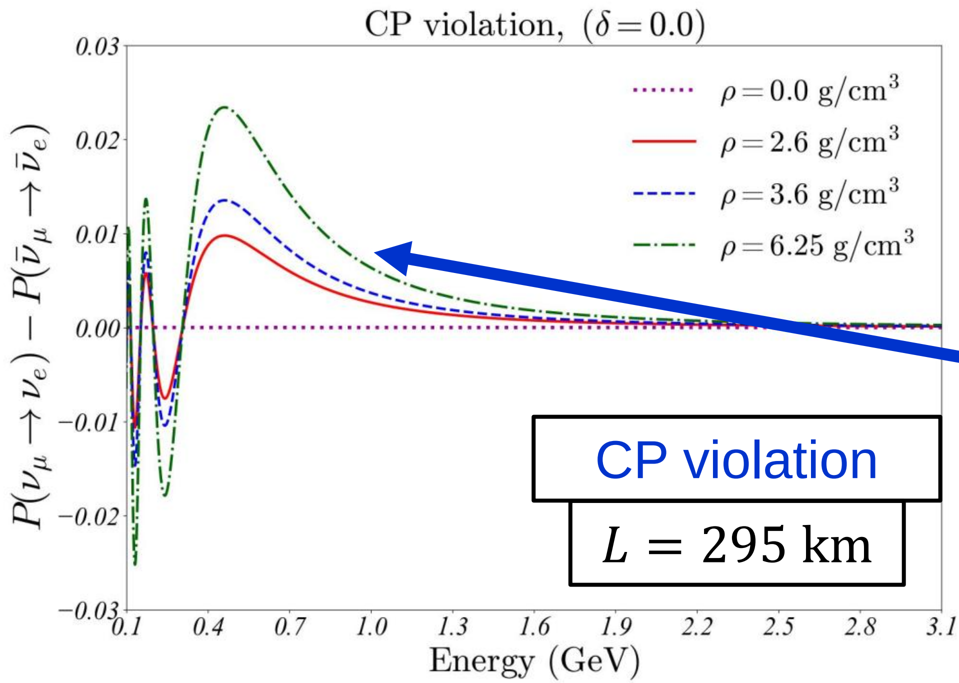
Ideal CP-test : need the anti-Earth



Ideal T-test : need ν beam from opposite direction



CP violation and T violation in matter



Assumption

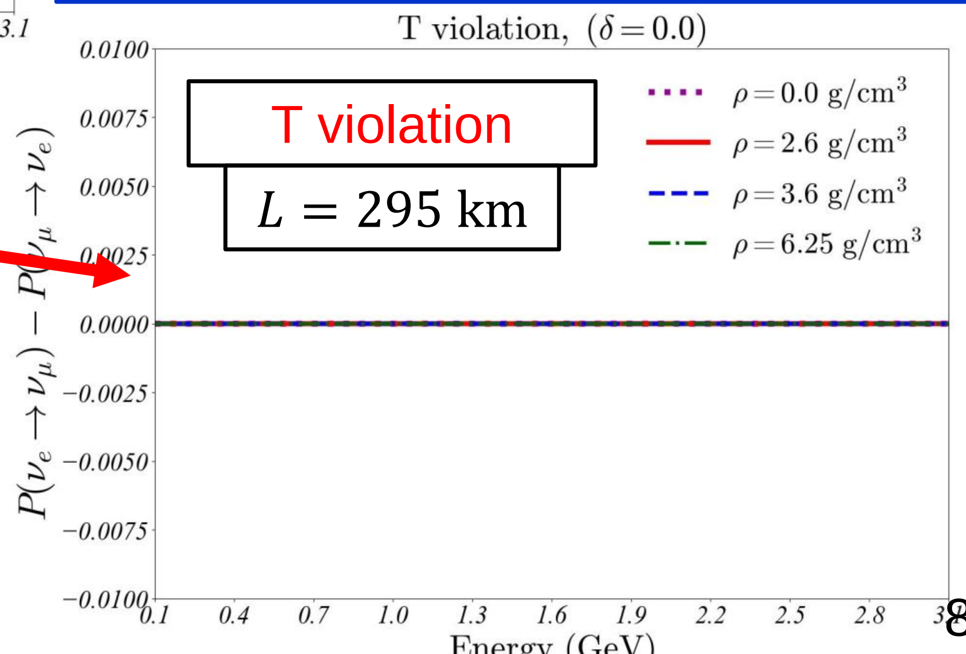
$\boldsymbol{\delta = 0^\circ}$

(No CP violation)

Normal Ordering

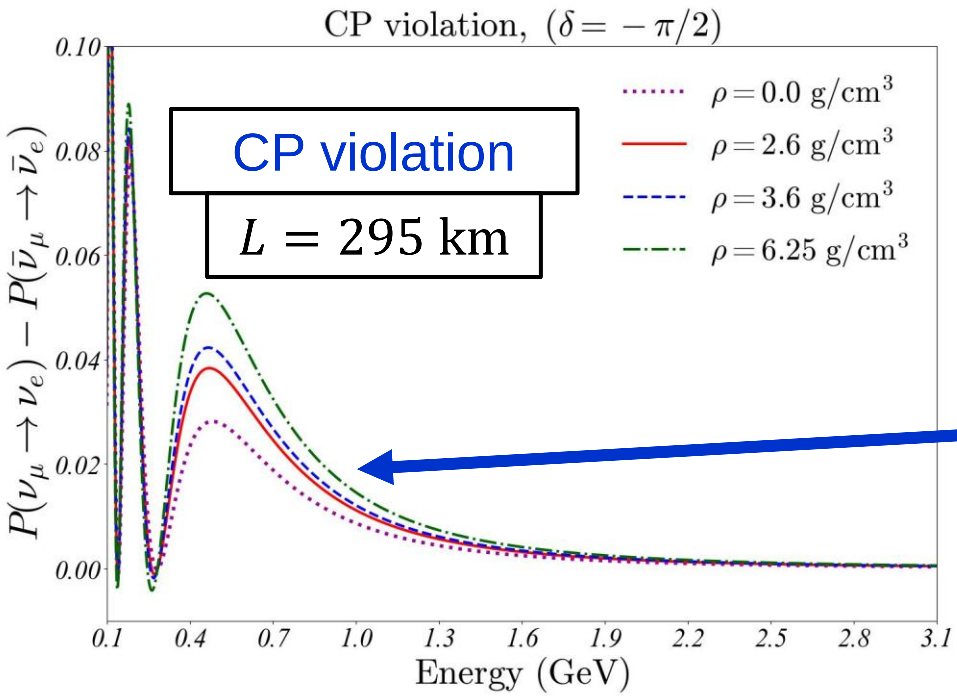
Fake CP violation exists

As long as the matter density is non-zero, i.e. not in a vacuum.



No fake CP (T) violation, regardless of matter effects.

CP violation and T violation in matter



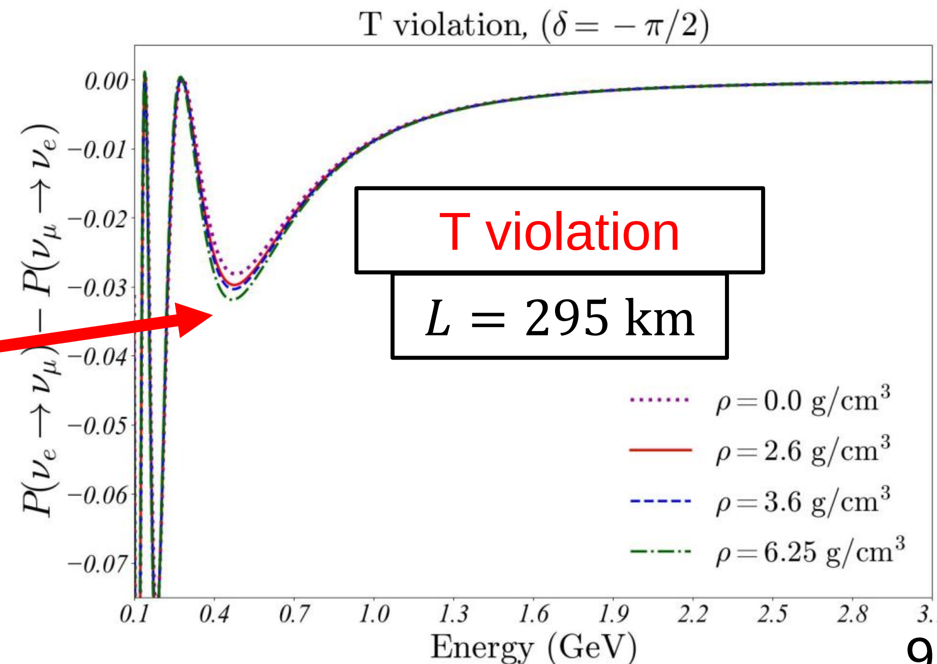
Assumption

$\boldsymbol{\delta = 270^\circ}$

(Maximally CP violation)

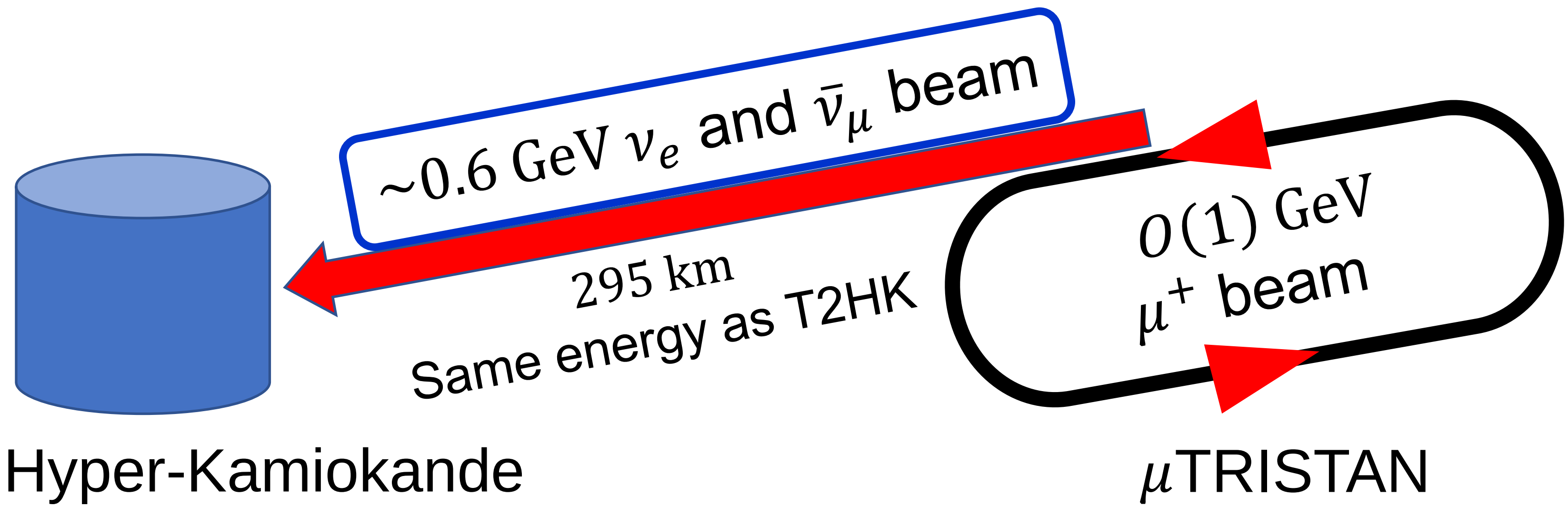
Normal Ordering

Dependence on matter density is **large**



- Dependence on matter density is **tiny**

Future neutrino factory



$P(\nu_e \to \nu_\mu)$ from μTRISTAN

$P(\nu_\mu \to \nu_e)$ from T2HK

combine

T violation $P(\nu_e \to \nu_\mu) - P(\nu_\mu \to \nu_e)$

Robust test of CP phase free from matter effects!

Probability of $\nu_e \to \nu_\mu$

At the HK, in principle, ν_μ and $\bar{\nu}_\mu$ are distinguished by neutron tagging method.
We can define the oscillation probability $P(\nu_e \to \nu_\mu)$ as

Perfect charge identification

$$P(\nu_e \to \nu_\mu) = \frac{N_{\mathrm{far}}^{\nu_e \to \nu_\mu}}{N_{\mathrm{near}}^{\nu_e \to \nu_e}}$$

No charge identification

$$P(\nu_e \to \nu_\mu) = \frac{\left(N_{\mathrm{far}}^{\nu_e \to \nu_\mu} + N_{\mathrm{far}}^{\bar{\nu}_\mu \to \bar{\nu}_\mu}\right) - N_{\mathrm{far}}^{\bar{\nu}_\mu \to \bar{\nu}_\mu}\Big|_{\mathrm{T2HK}}}{N_{\mathrm{near}}^{\nu_e \to \nu_e}}$$

Statistical analysis

Definition of χ^2_{TV}

$$\chi^2_{\mathrm{TV}} \equiv \sum_j \frac{\left[P_j^{\mathrm{TV}}(\delta_0,\ \rho_0) - P_j^{\mathrm{TV}}(\delta^{\mathrm{test}},\ \ \rho^{\mathrm{test}})\right]^2}{\left(\Delta P_j{}^{\mathrm{TV}}\right)^2}$$

$$P_j^{\mathrm{TV}}(\delta,\rho) \equiv P_j(\nu_e \to \nu_\mu) - P_j(\nu_\mu \to \nu_e)\Big|_{\mathrm{T2HK}}$$

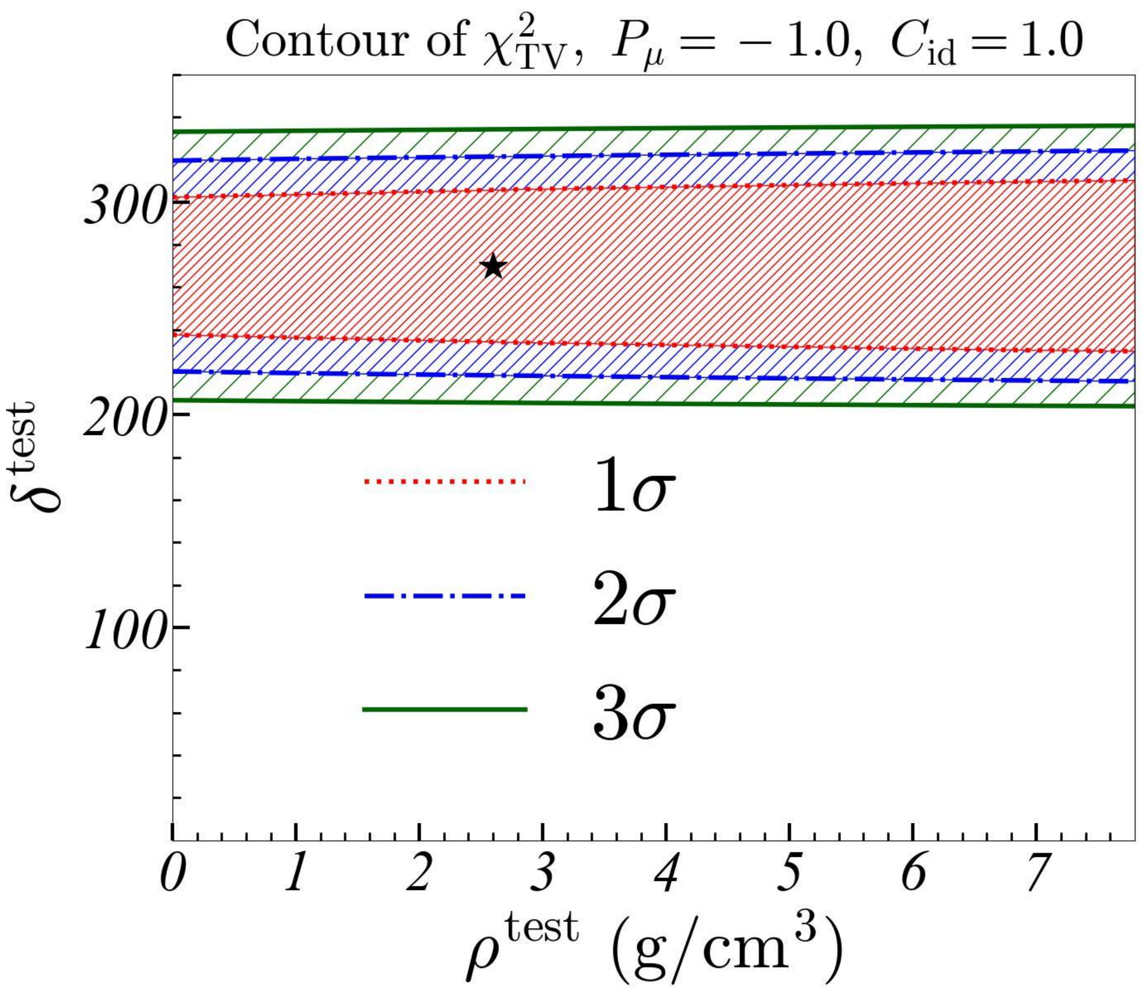
j runs over energy bins, ρ is matter density of the Earth
In this study, we consider only statistical error.
The ΔP_j^{TV} is obtained by modified oscillation probability

Reference values

$\Delta m_{21}^2/10^{-5}$ eV	$\Delta m_{31}^2/10^{-3}$ eV	θ_{12}	θ_{13}	θ_{23}
7.43	2.432	33.9°	8.49°	48.1°

The reference values are the arithmetic average of bfp in Particle Data Group 2022.
In this study, we consider only Normal Ordering.

Result of T violation



True value

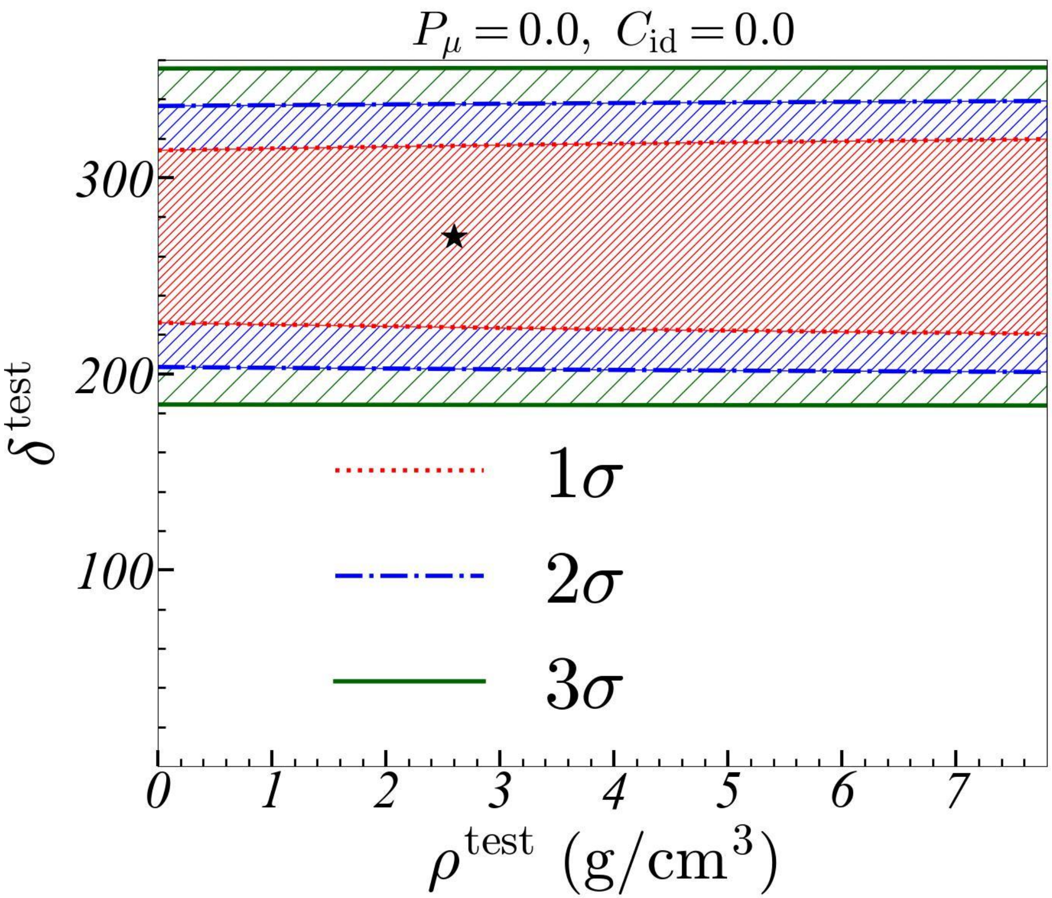
$\delta_0 = 270^\circ$

$\rho_0 = 2.6\ \mathrm{g/cm^3}$

- χ^2_{TV} only depends on δ.
- The confidence level as that for a 1 d.o.f., i.e., $\chi^2 = 1$ as 1σ.
- Perfect charge id and muon polarization, the δ is determined to be about $\pm 30^\circ$.

Free from matter density!

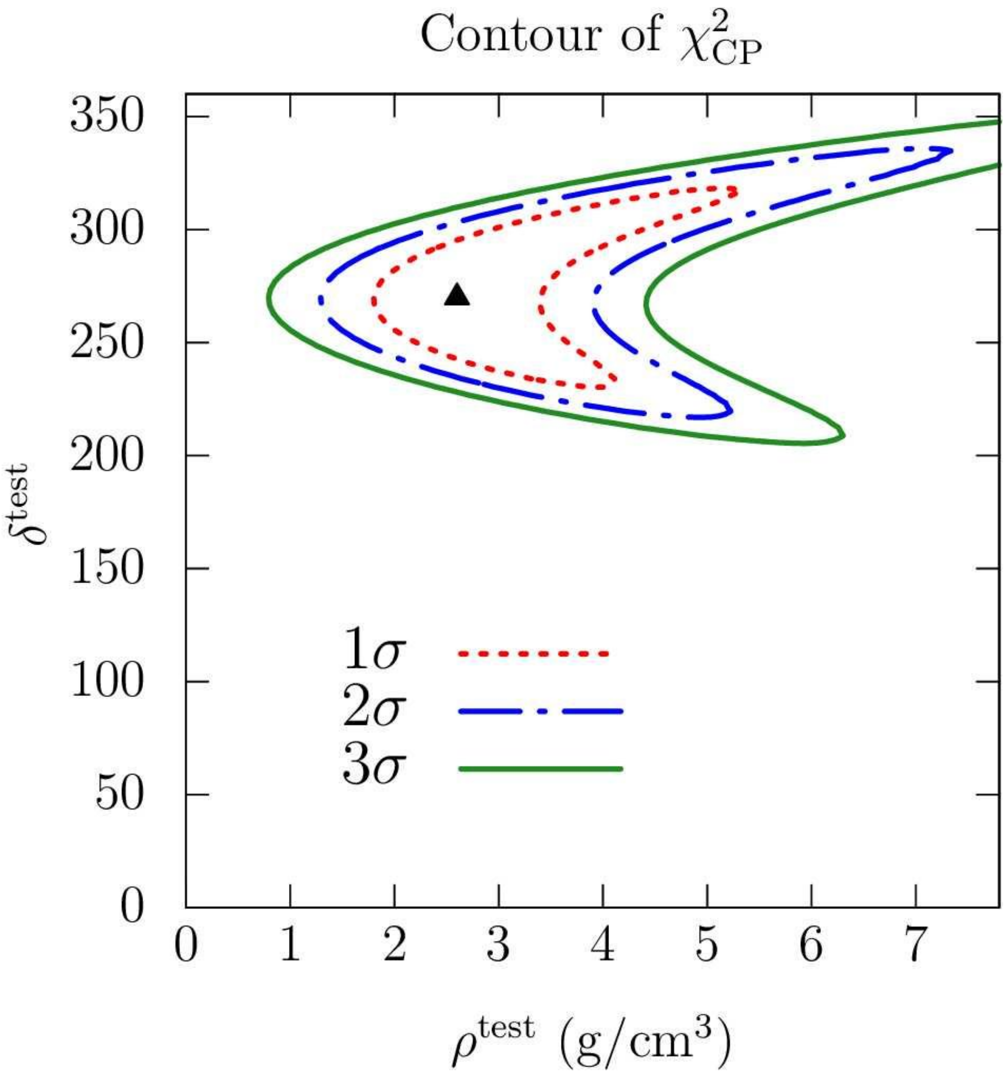
Result of T violation



Free from matter density!

- $\chi^2_{\rm TV}$ only depends on δ.
- No charge id and unpolarized muon, $\delta = 0^\circ$ and 180° (CP (or T) conserving point), can be excluded at the level of 3σ.
- With no ability of distinguish ν_μ and $\bar{\nu}_\mu$, the sensitivities do not change significantly.

Comparing with CP violation



- Define $\chi^2_{\rm CP}$ in a similar way as T violation.
- **Non-trivial ρ dependence**.
- A good knowledge of the matter density profile will be necessary.
- T violation will be an important additional information for the measurement of the CP angle δ.

Summary

● Testing T violation in the lepton sector has not been achieved, so it is important in particle physics.

● We study the possibility of measuring T violation, by combining $\nu_e \to \nu_\mu$ (μTRISTAN → HK) and $\nu_\mu \to \nu_e$ (T2HK).

● If nature has chosen $\delta = 270^\circ$, we can exclude $\delta = 0^\circ, 180^\circ$ at more than 3σ.

● Comparing with the χ^2 for T violation and CP violation, T violation would not suffer from the uncertainty in the matter density profile of the earth.

● In this study, we only consider the statistical error. A more complete analysis will be necessary to establish the feasibility.

Back up

Background subtraction

At the HK, in principle, ν_μ and $\bar{\nu}_\mu$ are distinguished by neutron tagging method.
We can define the oscillation probability $P(\nu_e \to \nu_\mu)$ as

$$P\left(\nu_e \to \nu_\mu\right) = \frac{1}{\kappa} \frac{\left(\kappa N_{\mathrm{far}}^{\nu_e \to \nu_\mu} + (1-\kappa) N_{\mathrm{far}}^{\bar{\nu}_\mu \to \bar{\nu}_\mu}\right) - (1-\kappa) N_{\mathrm{far}}^{\bar{\nu}_\mu \to \bar{\nu}_\mu}\Big|_{\mathrm{T2HK}}}{\kappa N_{\mathrm{near}}^{\nu_e \to \nu_e}}$$

$$\kappa \equiv \frac{1 + C_{\mathrm{id}}}{2}\,, C_{\mathrm{id}}: \text{charge identification efficiency}$$

In the case of $C_{\mathrm{id}} = 0.0$, we just do not perform the charge identification analysis and simply add the background events, and subtract the estimated amount by using the T2HK data, i.e. we take $\kappa = 1$ and $1 - \kappa = 1$.

Statistical error ΔP_j^{TV}

$$\left(\Delta P_j^{TV}\right)^2 = \left(\Delta P_j^{\nu_e\to\nu_\mu}\right)^2 + \left(\Delta P_j^{\nu_\mu\to\nu_e}\right)^2$$

$$= \left(P_j^{\nu_e\to\nu_\mu}\right)^2 \left[\left(\frac{\sqrt{\left(\kappa N_{\mathrm{far}}^{\nu_e\to\nu_\mu} + (1-\kappa) N_{\mathrm{far}}^{\bar\nu_\mu\to\bar\nu_\mu}\right)}}{\kappa N_{\mathrm{far}}^{\nu_e\to\nu_\mu}}\right)_j^2\right.$$

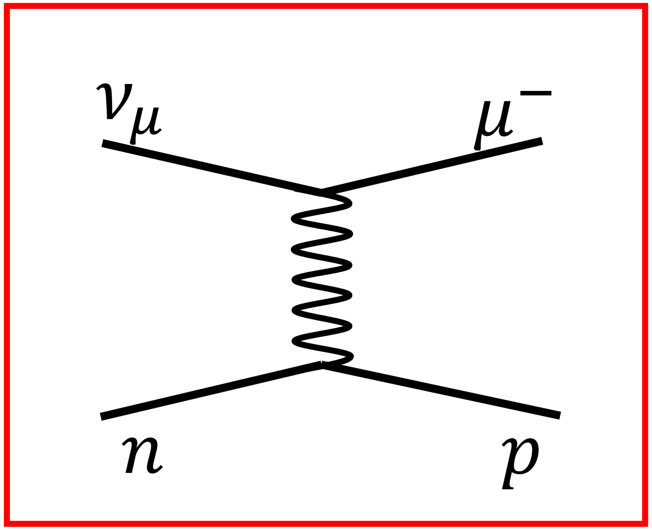
$$\left. + \left(P_j^{\nu_\mu\to\nu_e}\right)^2 \left[\left(\frac{\sqrt{N_{\mathrm{far}}^{\nu_\mu\to\nu_e}}}{N_{\mathrm{far}}^{\nu_\mu\to\nu_e}}\right)_j^2 + \left(\frac{\sqrt{N_{\mathrm{near}}^{\nu_\mu\to\nu_\mu}}}{N_{\mathrm{near}}^{\nu_\mu\to\nu_\mu}}\right)_j^2\right]\right.$$

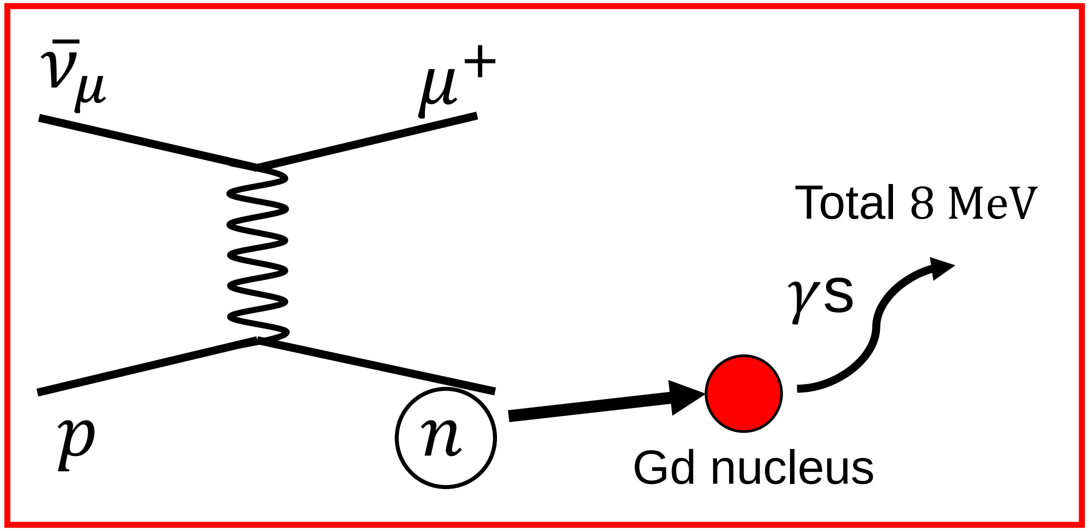
$$\sim \left(P_j^{\nu_e\to\nu_\mu}\right)^2 \left(\frac{\sqrt{\left(\kappa N_{\mathrm{far}}^{\nu_e\to\nu_\mu} + (1-\kappa) N_{\mathrm{far}}^{\bar\nu_\mu\to\bar\nu_\mu}\right)}}{\kappa N_{\mathrm{far}}^{\nu_e\to\nu_\mu}}\right)_j^2 + \left(P_j^{\nu_\mu\to\nu_e}\right)^2 \left(\frac{\sqrt{N_{\mathrm{far}}^{\nu_\mu\to\nu_e}}}{N_{\mathrm{far}}^{\nu_\mu\to\nu_e}}\right)_j^2$$

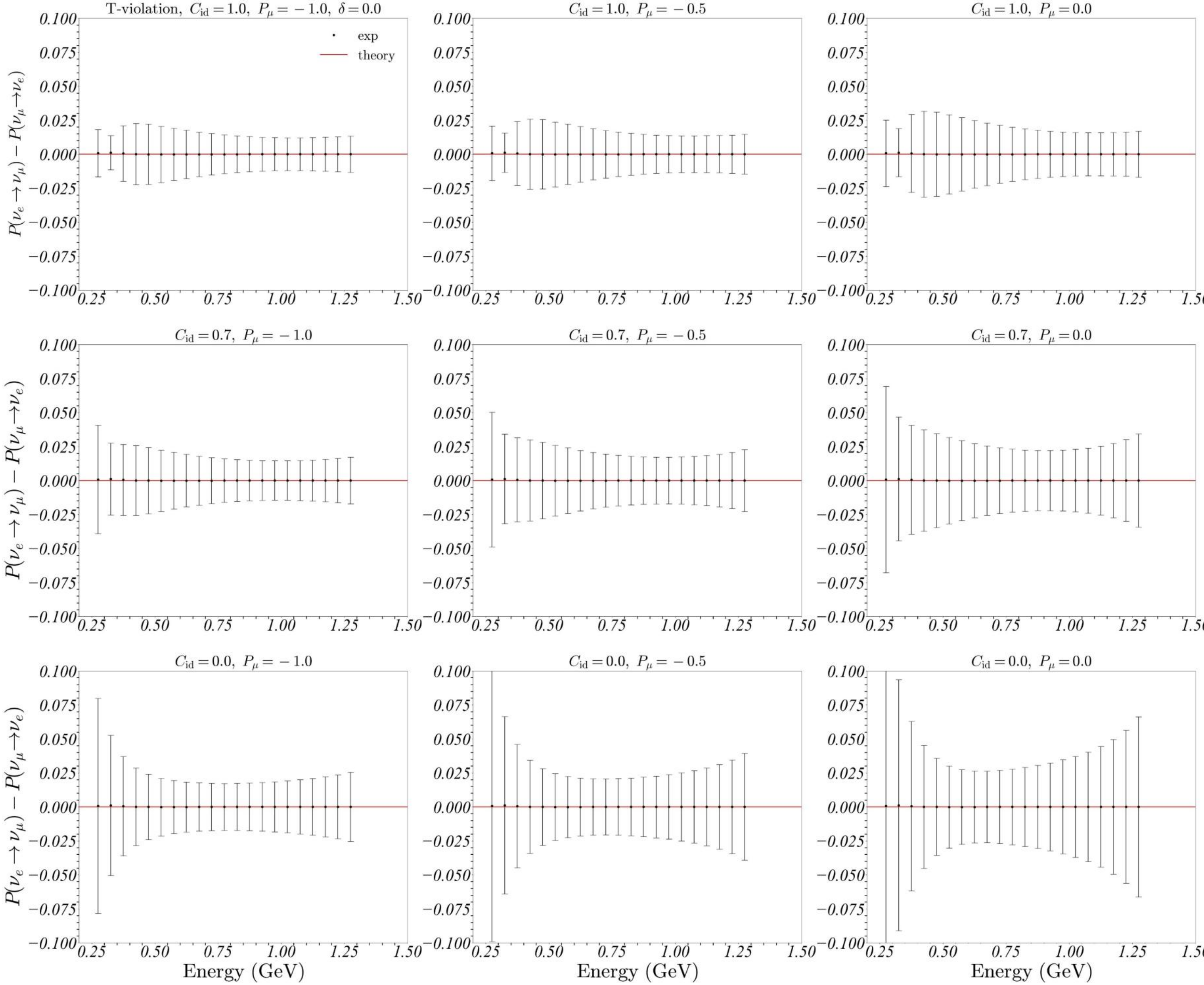
Neutron tagging

SK-Gd : efficiency ~ 70%
Hyper-K : efficiency ≥ 70%

R. Akutsu. Ph.D thesis, Tokyo University, 2019.






T-violation, $C_{\rm id}=1.0,\ P_\mu=-1.0,\ \delta=0.0$
exp
theory
$C_{\rm id}=1.0,\ P_\mu=-0.5$
$C_{\rm id}=1.0,\ P_\mu=0.0$
$C_{\rm id}=0.7,\ P_\mu=-1.0$
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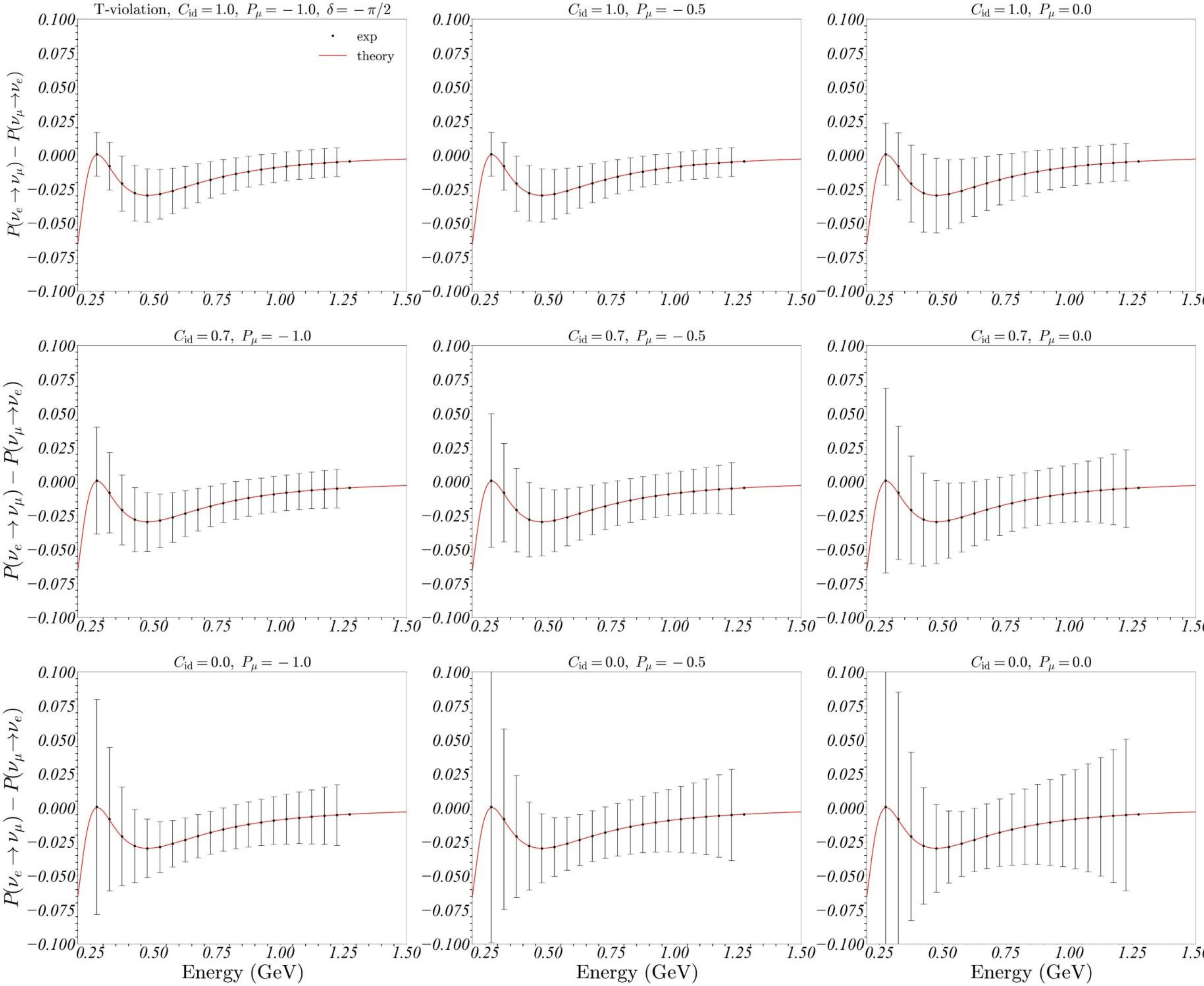
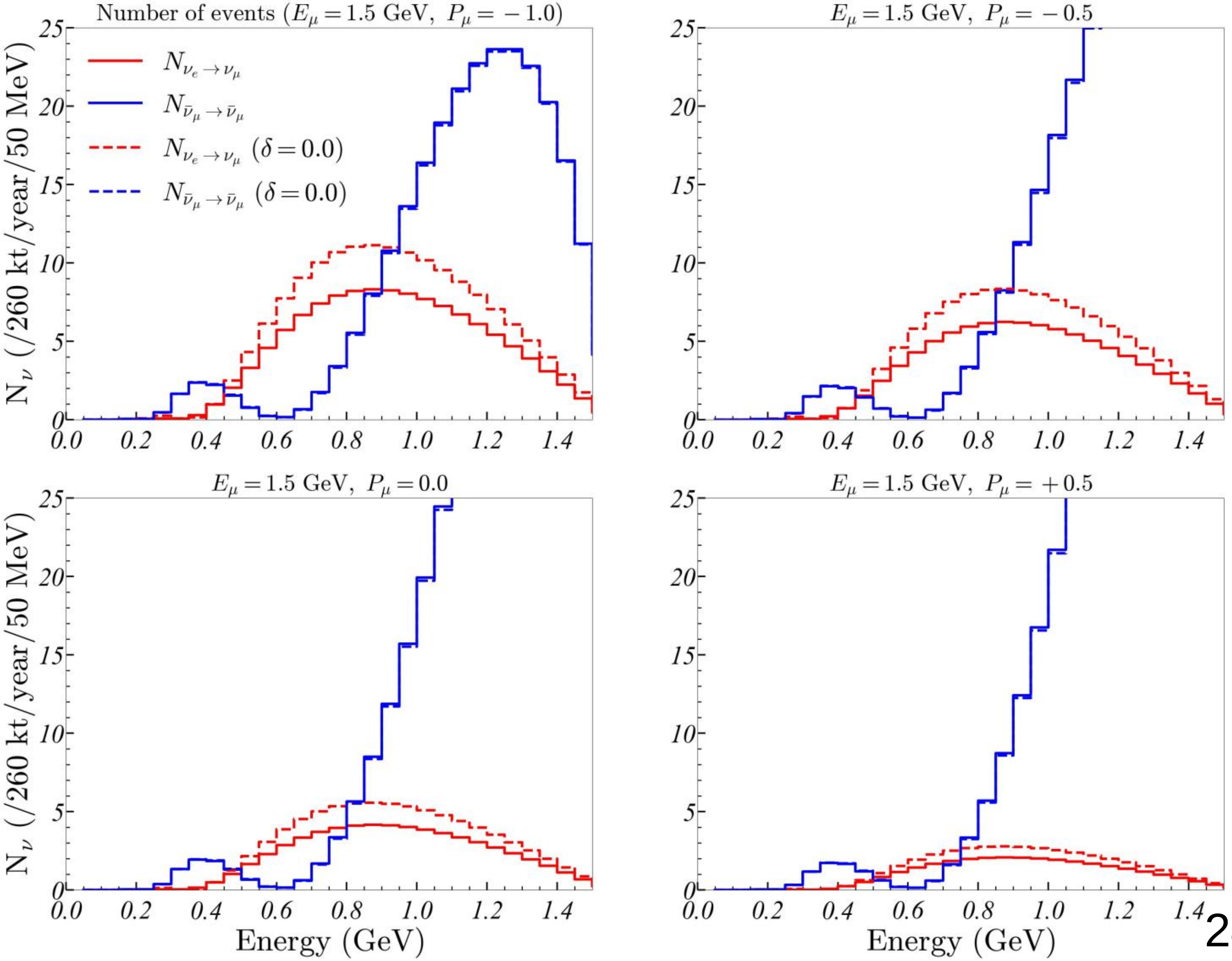
$C_{\rm id}=0.0,\ P_\mu=-1.0$
$C_{\rm id}=0.0,\ P_\mu=-0.5$
$C_{\rm id}=0.0,\ P_\mu=0.0$
$P(\nu_e\to\nu_\mu)-P(\nu_\mu\to\nu_e)$
Energy (GeV)

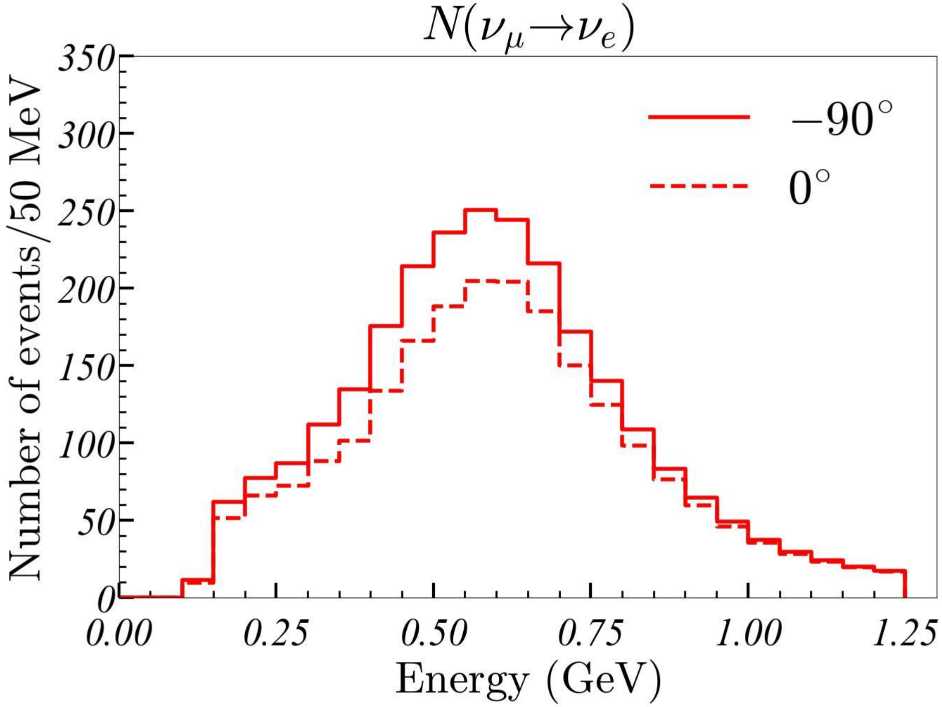

T-violation, $C_{\rm id} = 1.0,\ P_\mu = -1.0,\ \delta = -\pi/2$
exp
theory
$C_{\rm id} = 1.0,\ P_\mu = -0.5$
$C_{\rm id} = 1.0,\ P_\mu = 0.0$
$C_{\rm id} = 0.7,\ P_\mu = -1.0$
$C_{\rm id} = 0.7,\ P_\mu = -0.5$
$C_{\rm id} = 0.7,\ P_\mu = 0.0$
$C_{\rm id} = 0.0,\ P_\mu = -1.0$
$C_{\rm id} = 0.0,\ P_\mu = -0.5$
$C_{\rm id} = 0.0,\ P_\mu = 0.0$
$P(\nu_e \to \nu_\mu) - P(\nu_\mu \to \nu_e)$
Energy (GeV)

Number of events

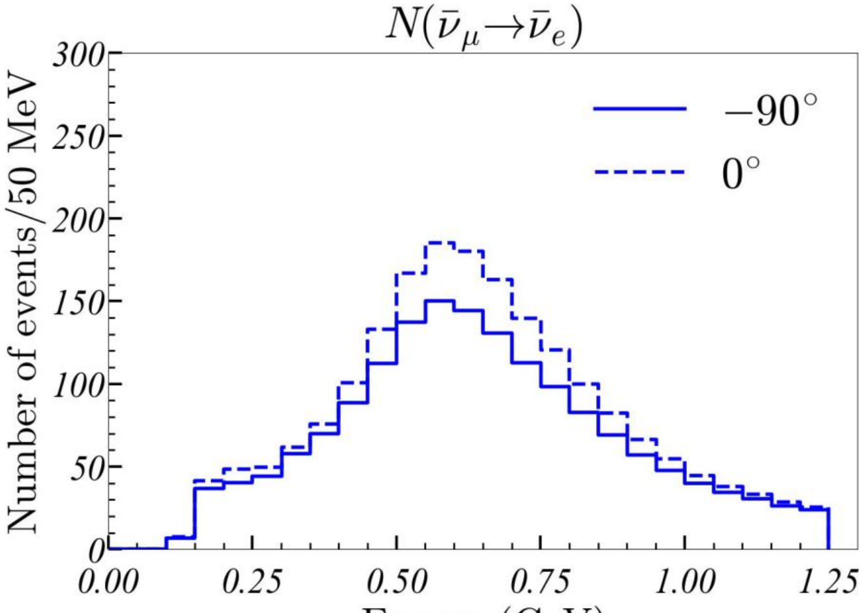
μTRISTAN
$N_\mu \sim 10^{21}$ muons/year



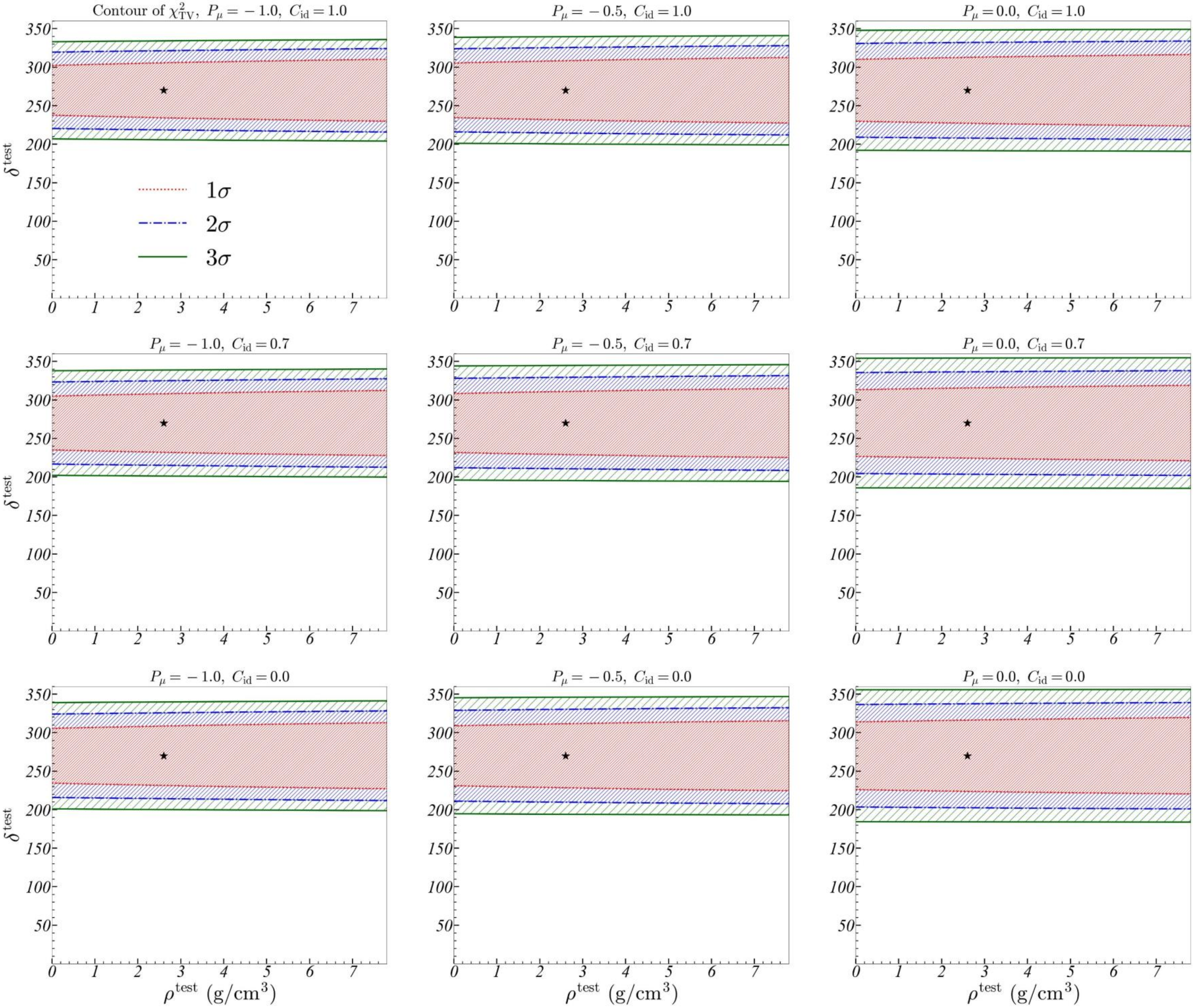
Number of events

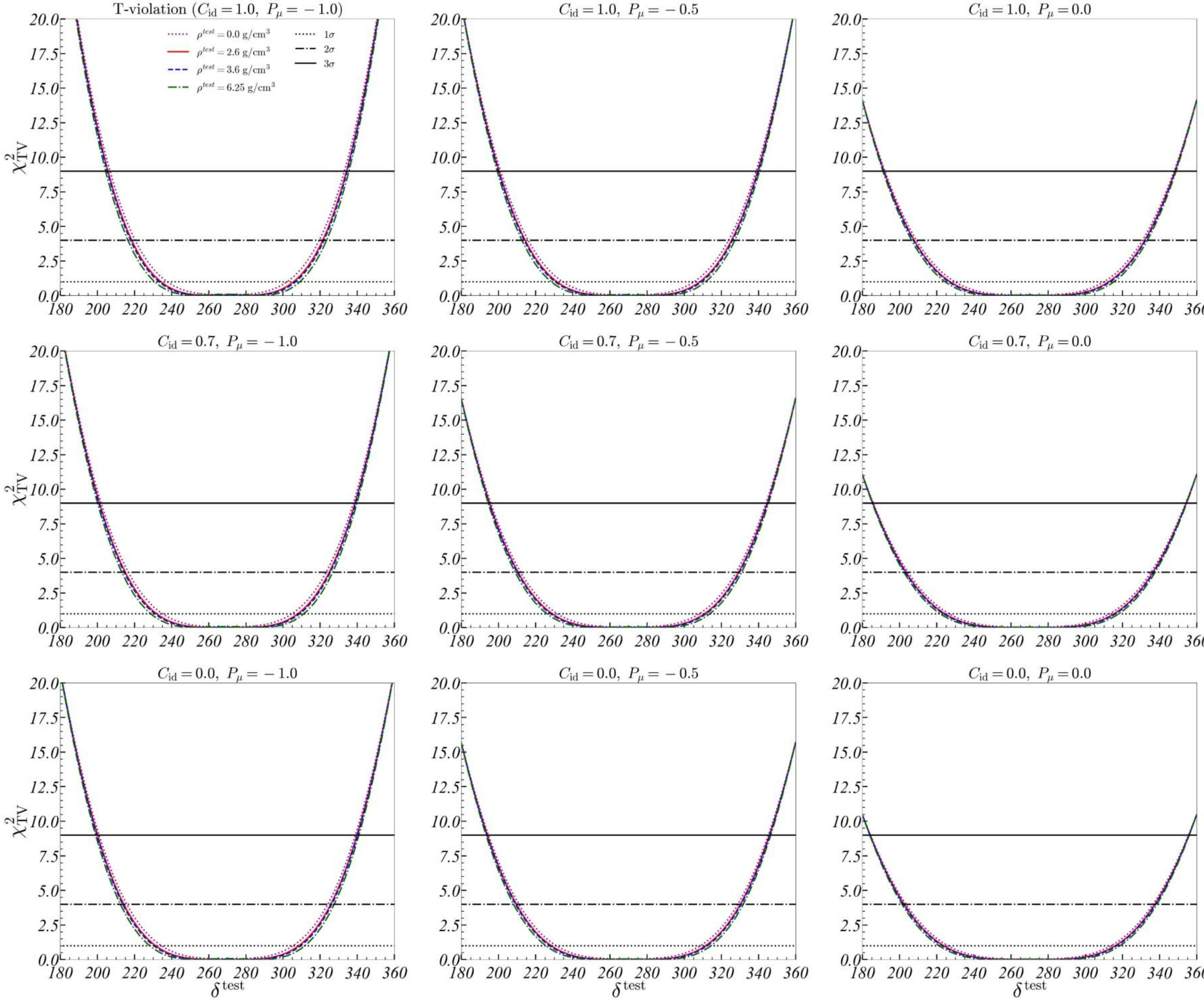


T2HK
$\sim 10^{21}$ POT



K. Abe et al. [Hyper-Kamiokande Proto-], 2015


Contour of $\chi^2_{\rm TV}$, $P_\mu = -1.0$, $C_{\rm id} = 1.0$
$P_\mu = -0.5$, $C_{\rm id} = 1.0$
$P_\mu = 0.0$, $C_{\rm id} = 1.0$
$P_\mu = -1.0$, $C_{\rm id} = 0.7$
$P_\mu = -0.5$, $C_{\rm id} = 0.7$
$P_\mu = 0.0$, $C_{\rm id} = 0.7$
$P_\mu = -1.0$, $C_{\rm id} = 0.0$
$P_\mu = -0.5$, $C_{\rm id} = 0.0$
$P_\mu = 0.0$, $C_{\rm id} = 0.0$
1σ
2σ
3σ
$\delta^{\rm test}$
$\rho^{\rm test}$ (g/cm^3)



T-violation ($C_{\mathrm{id}} = 1.0,\ P_\mu = -1.0$)
$C_{\mathrm{id}} = 1.0,\ P_\mu = -0.5$
$C_{\mathrm{id}} = 1.0,\ P_\mu = 0.0$
$C_{\mathrm{id}} = 0.7,\ P_\mu = -1.0$
$C_{\mathrm{id}} = 0.7,\ P_\mu = -0.5$
$C_{\mathrm{id}} = 0.7,\ P_\mu = 0.0$
$C_{\mathrm{id}} = 0.0,\ P_\mu = -1.0$
$C_{\mathrm{id}} = 0.0,\ P_\mu = -0.5$
$C_{\mathrm{id}} = 0.0,\ P_\mu = 0.0$
$\rho^{test} = 0.0$ g/cm^3
$\rho^{test} = 2.6$ g/cm^3
$\rho^{test} = 3.6$ g/cm^3
$\rho^{test} = 6.25$ g/cm^3
1σ
2σ
3σ
χ^2_{TV}
δ^{test}

Possible CPT test?

Although we do not try in this study, one would be able to perform a similar analysis for CPT violation,

$$P_j^{CPT} = P_j(\nu_e \to \nu_\mu) - P_j(\bar{\nu}_\mu \to \bar{\nu}_e)\Big|_{\mathrm{T2HK}} = P_j^{TV} + P_j^{CP}$$

Under our assumptions, this quantity would not measure anything. The analysis of this kind will be a quite important fundamental test of symmetry in physical laws of the Universe. Nevertheless, this will provide us with a quite important test of our underlying assumptions such as the three-neutrino scheme as well as quantum field theory.