Collimated muon beam proposal for probing neutrino charge-parity violation

in Collaboration with

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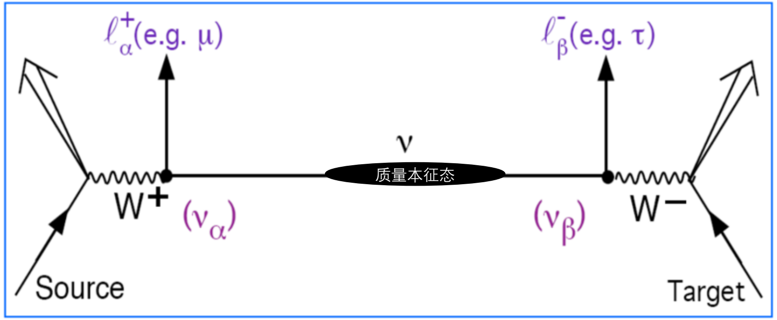
Neutrino oscillation： a quantum phenomenon

- **Oscillation: spontaneous periodic change from one neutrino flavor to another, a direct result of neutrino mixing with mass eigenstates, and is a quantum phenomenon. In a neutrino oscillation experiment, the neutrino beam is produced and detected via the weak Charged-Current (CC) interaction。**



- **Neutrino state of flavor $\alpha = e, \mu, \tau$ produced in a weak interaction can be written as superposition of mass eigenstates：**

$$|\nu_\alpha\rangle = \sum_j U^*_{\alpha j} |\nu_j\rangle$$

Neutrino Mixing Matrix or PMNS matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U^\dagger U = 1, \quad \sum_i U_{\alpha i} U^*_{\beta i} = \delta_{\alpha\beta}, \quad \sum_i U_{\alpha i} U^*_{\alpha j} = \delta_{ij}$$

Neutrino Oscillation probability

- The corresponding transition amplitude for flavor α to β can be obtained with the old-fashioned way as

$$A(\nu_\alpha \to \nu_\beta) = \langle \nu_\beta | \nu_\alpha(t,L)\rangle = \sum_{i,j} U^*_{\alpha i} U_{\beta j} e^{-iE_j t + ip_j L} \langle \nu_j | \nu_i \rangle = \sum_j U^*_{\alpha j} U_{\beta j} e^{-iE_j t + ip_j L}$$

$$E_i = \sqrt{m_i^2 + p_i^2} \simeq p_i + \frac{m_i^2}{2p_i} \simeq E + \frac{m_i^2}{2E}$$

- Highly relativistic: $\vec{p} \gg m, \quad p = E$

- The oscillation probability (for 3 flavor) is than given as

$$P(\nu_\alpha \to \nu_\beta) = |A(\nu_\alpha \to \nu_\beta)|^2 = \sum_{i,j} U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j} e^{-i(E_i - E_j)t}$$

$$= \delta_{\alpha\beta} - 4Re \sum_{j>i} [U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}] \sin^2(X_{ij})$$

$$+ 2 \sum_{j>i} Im [U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}] \sin 2X_{ij}$$

$$X_{ij} = \frac{(m_i^2 - m_j^2)L}{4E} = 1.267 \frac{\Delta m_{ij}^2}{\mathrm{eV}^2} \frac{L}{\mathrm{Km}} \frac{\mathrm{GeV}}{E}$$

$$2 \sum_{i<j} Im [U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}] \sin(2X_{ij})$$

$$= \pm 8J \sin\left(\frac{\Delta m_{21}^2 L}{4E}\right) \sin\left(\frac{\Delta m_{32}^2 L}{4E}\right) \sin\left(\frac{\Delta m_{31}^2 L}{4E}\right)$$

(+) for (e→μ), (μ→τ), (τ→e), otherwise (-)

- *Jarlskog* factor

$$J = \cos\theta_{12} \sin\theta_{12} \cos^2\theta_{13} \sin\theta_{13} \cos\theta_{23} \sin\theta_{23} \sin(\delta_{CP})$$

Jarlskog invariant PRL. 58, 1698 (1987)

CP violation in neutrino oscillation

- The oscillation probability for $\overline{\nu}_\alpha \to \overline{\nu}_\beta$ is obtained through a CP transformation on the corresponding *wave functions* of ν_α, ν_β, or simply by taking $U \to U^*$, which only changes the sign of the Imaginary part in $P(\nu_\alpha \to \nu_\beta)$.

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4Re\sum_{j>i}\left[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\right]\sin^2(X_{ij})$$
$$\pm 8J\sin\left(\frac{\Delta m^2_{21}L}{4E}\right)\sin\left(\frac{\Delta m^2_{32}L}{4E}\right)\sin\left(\frac{\Delta m^2_{31}L}{4E}\right)$$

CP transformed

$$P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) = \delta_{\alpha\beta} - 4Re\sum_{j>i}\left[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\right]\sin^2(X_{ij})$$
$$\mp 8J\sin\left(\frac{\Delta m^2_{21}L}{4E}\right)\sin\left(\frac{\Delta m^2_{32}L}{4E}\right)\sin\left(\frac{\Delta m^2_{31}L}{4E}\right)$$

$$J = \cos\theta_{12}\sin\theta_{12}\cos^2\theta_{13}\sin\theta_{13}\cos\theta_{23}\sin\theta_{23}\sin(\delta_{CP})$$

$$\mathbf{\Delta P}(\nu_\alpha \to \nu_\beta) = P(\nu_\alpha \to \nu_\beta) - P(\overline{\nu}_\alpha \to \overline{\nu}_\beta)$$
$$= \pm 16\mathbf{J}\sin\left(\frac{\mathbf{\Delta m^2_{21}L}}{\mathbf{4E}}\right)\sin\left(\frac{\mathbf{\Delta m^2_{32}L}}{\mathbf{4E}}\right)\sin\left(\frac{\mathbf{\Delta m^2_{31}L}}{\mathbf{4E}}\right) \neq \mathbf{0}$$

If $\delta_{CP} \neq 0$

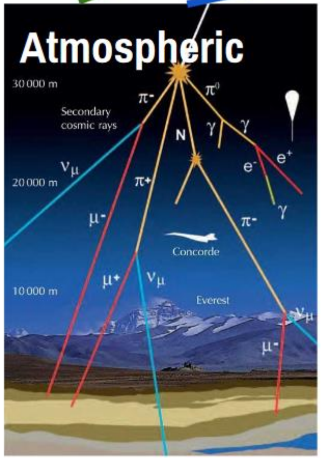
$$\mathbf{\Delta P} \neq \mathbf{0}$$

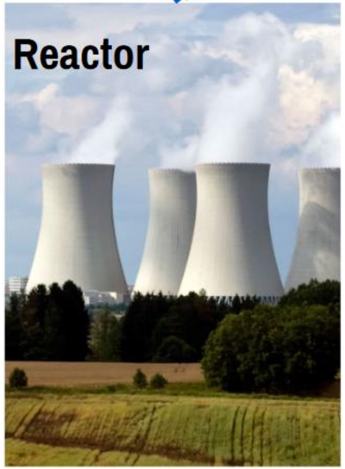
CP transformation is violated!

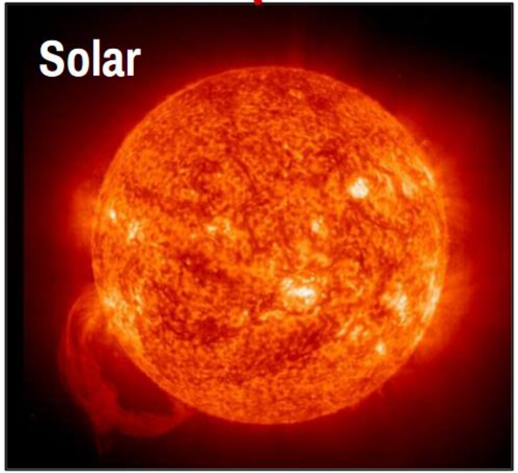
Neutrino Sources and Mixing Parameters

$c_{ij} = \cos\theta_{ij}$, and $s_{ij} = \sin\theta_{ij}$.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$





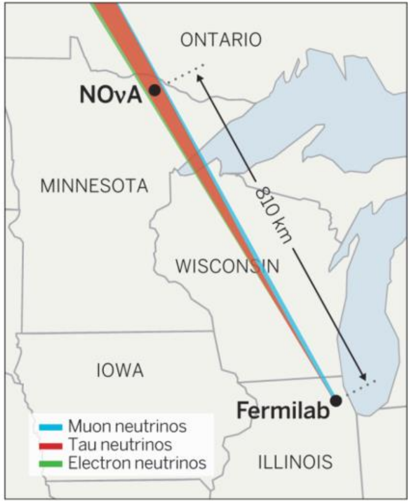


Neutrino Experiments and Oscillation parameters

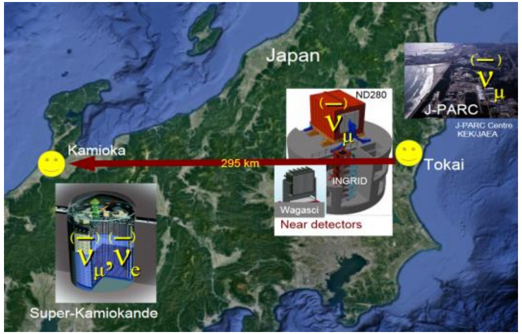
K. ENGMAN/SCIENCE 345, 6204

◆ **Parameters to be determined**

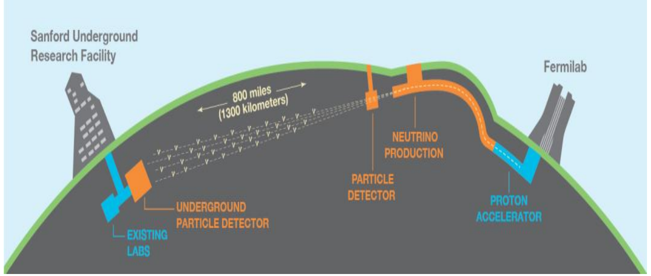
1. Three mixing angles: $\theta_{13}, \theta_{12}, \theta_{23} \neq 0$
1. Two mass differences: $\Delta m^2_{12}, \Delta m^2_{13}, \Delta m^2_{23}$
2. One Dirac phase :δ_{cp}



T2K



DUNE: Deep Underground Neutrino Experiment

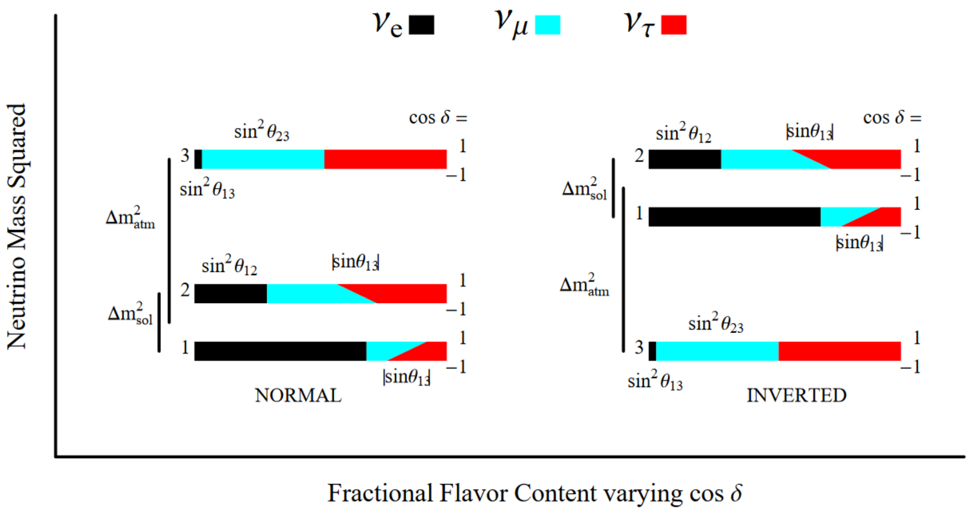


Mass Order: normal or inverted ?

$$\Delta m_{12}^2 = \Delta m_{\text{solar}}$$

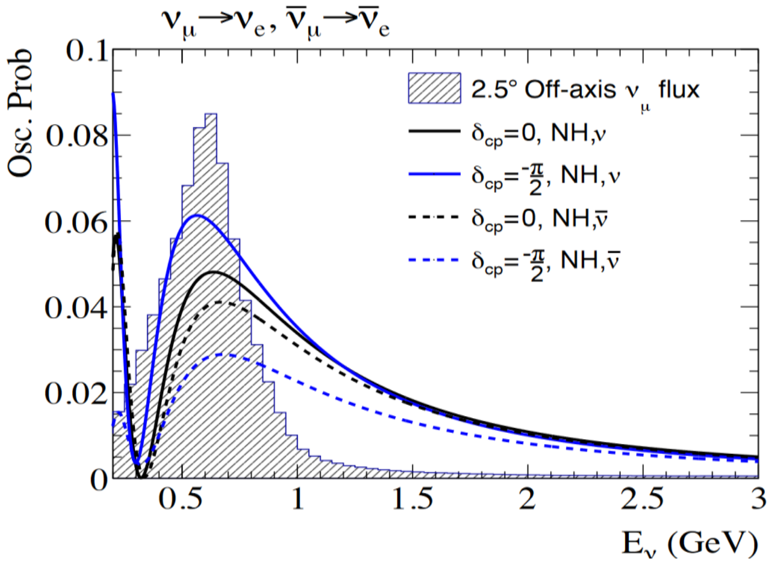
$$\Delta m_{m32}^2 \approx \Delta m_{31}^2 = \Delta m_{atmo}^2$$

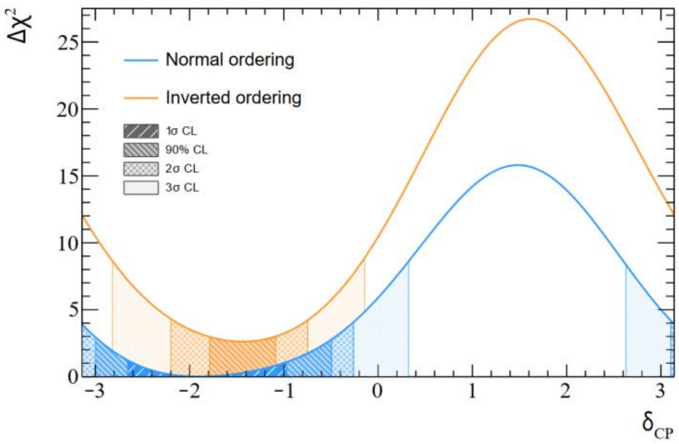
$\boldsymbol{m_1, m_2, m_3} = ?$



PRD. 69 (2004) 117301

Probing CP phase: *T2K Experiment*

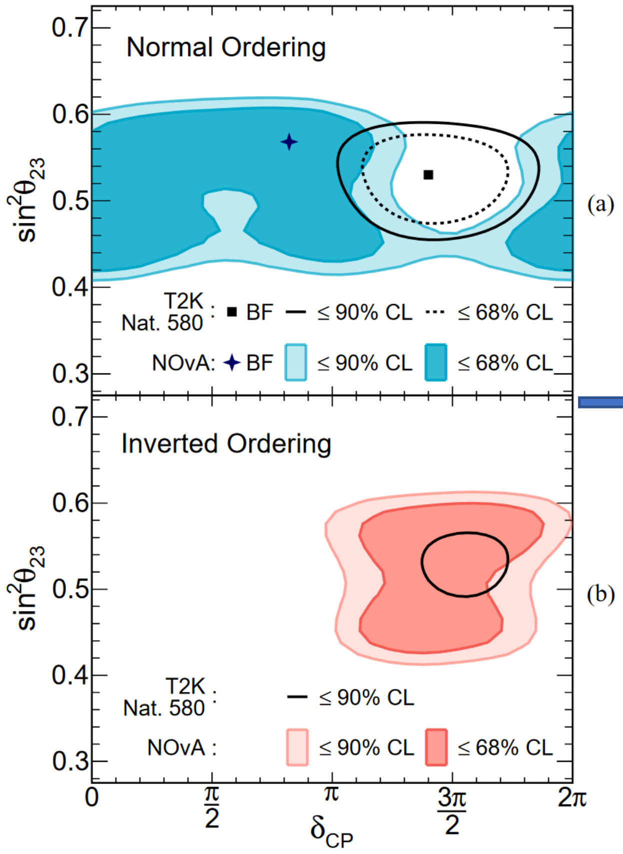




Eliminates $\boldsymbol{\delta_{CP}} = \frac{\boldsymbol{\pi}}{\mathbf{2}}$ at 3 sigma level

T2K Collaboration
Eur.Phys.J.C 83 (2023) 9, 782

Probing CP phase: *NOνA Experiment*



Phys. Rev. D 106, 032004 (2022)

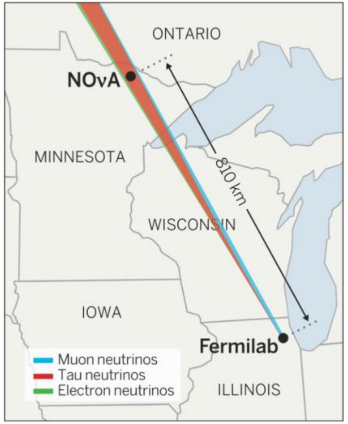
FIG. 6. The 68% and 90% confidence level contours in $\sin^2\theta_{23}$ vs. δ_{CP} in the (a) normal mass ordering and (b) inverted mass ordering [95]. The cross denotes the NOvA best-fit point and colored areas depict the 90% and 68% FC corrected allowed regions for NOvA. Overlaid black solid-line and dashed-line contours depict allowed regions reported by T2K [91][3].

Non-standard (NC&CC) interactions of the neutrinos

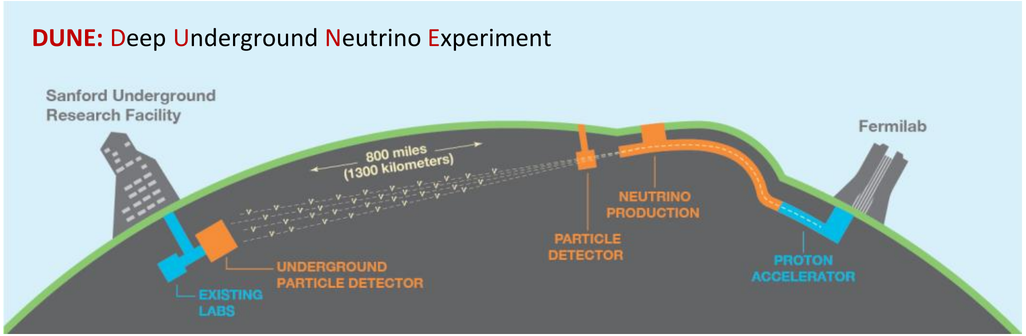
$$\mathcal{L}_{NC-NSI} = -2\sqrt{2}G_F\varepsilon^{fC}_{\alpha\beta}(\overline{\nu_\alpha}\gamma^\mu P_L\nu_\beta)(\overline{f}\gamma_\mu P_C f)$$

$$\mathscr{L}_{CC\text{-}NSI} = -2\sqrt{2}G_F \sum_{f,f',\alpha,\beta,P} \epsilon^{f,f',P}_{\alpha\beta}[\bar{\nu}_\beta\gamma^\mu P_L l_\alpha][\bar{f}\gamma_\mu P f']$$

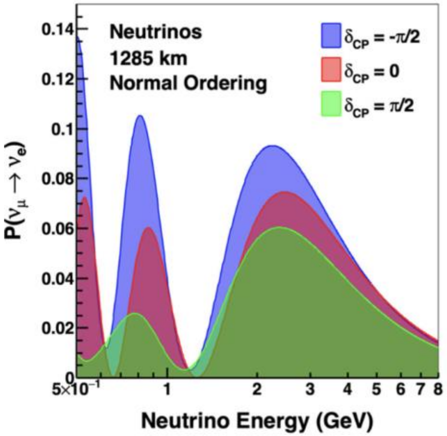
arXiv:2401.02901, Daya Bay

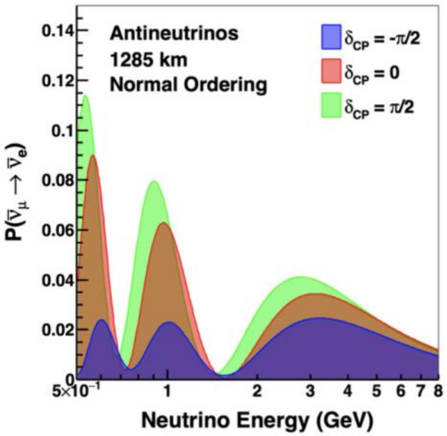


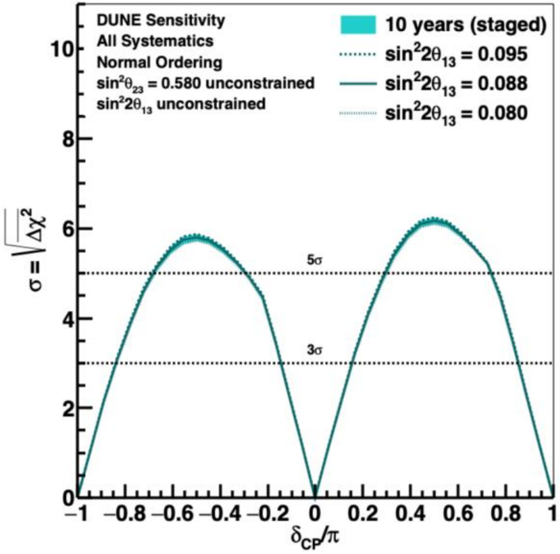
Probing CP phase: DUNE simulation



Under construction, to be completed in 2029 and start data taking in 2035 to 2040



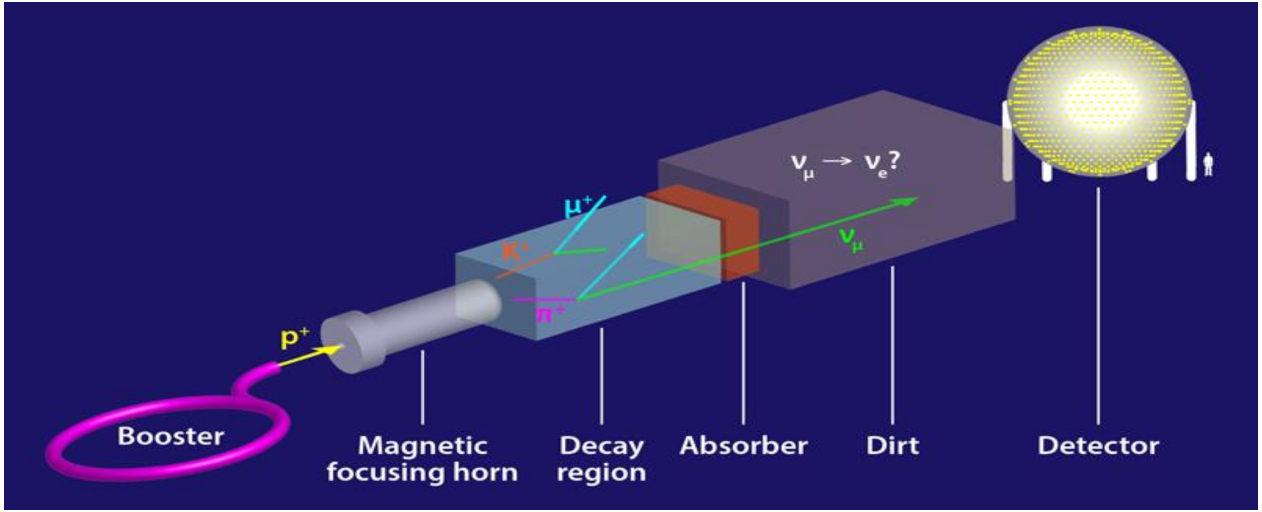




EPJC. 80 (2020) 10, 978

Accelerator neutrinos for Oscillation experiments

➢ **Conventional muon sources: accelerated proton-on-target**



Limitations

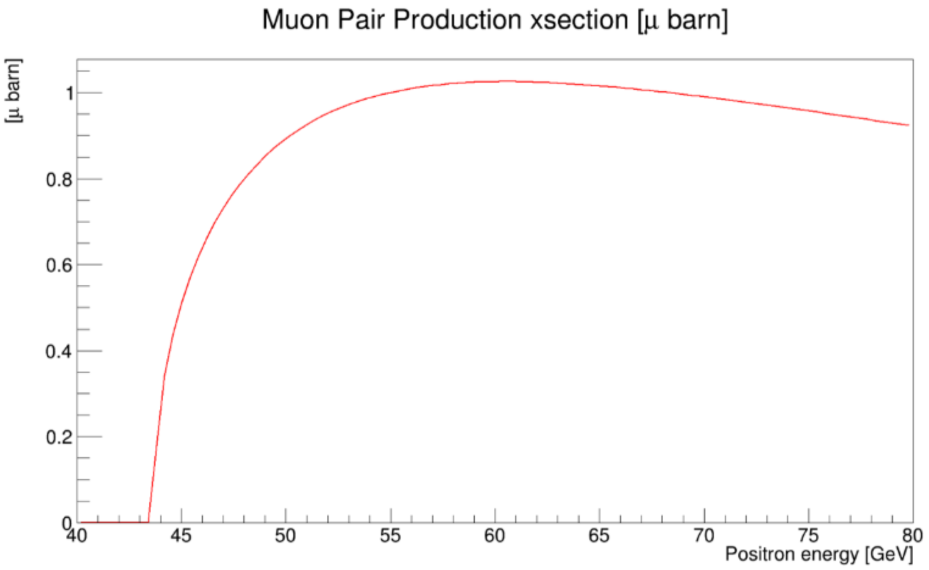
- Lower neutrino flux
- Limited neutrino energy spectrum
- Background contamination

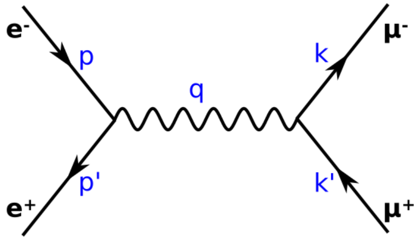
APS Physics 11 (2018) 122

Positron driven muon sources

Low EMittance Muon Accelerator (LEMMA)

D. Alesini *et al*
arXiv:1905.05747

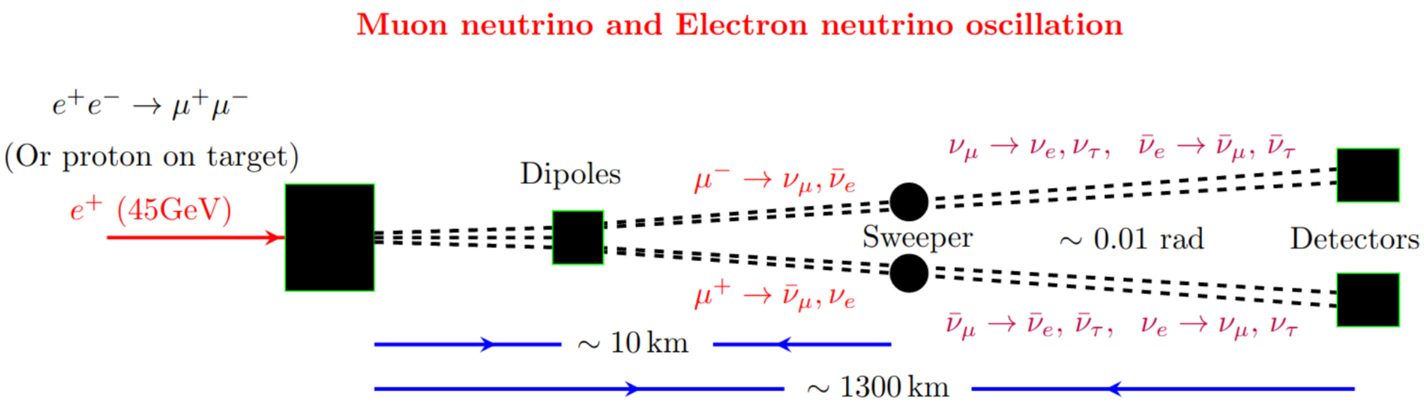




Positron driven muon sources for neutrino oscillation

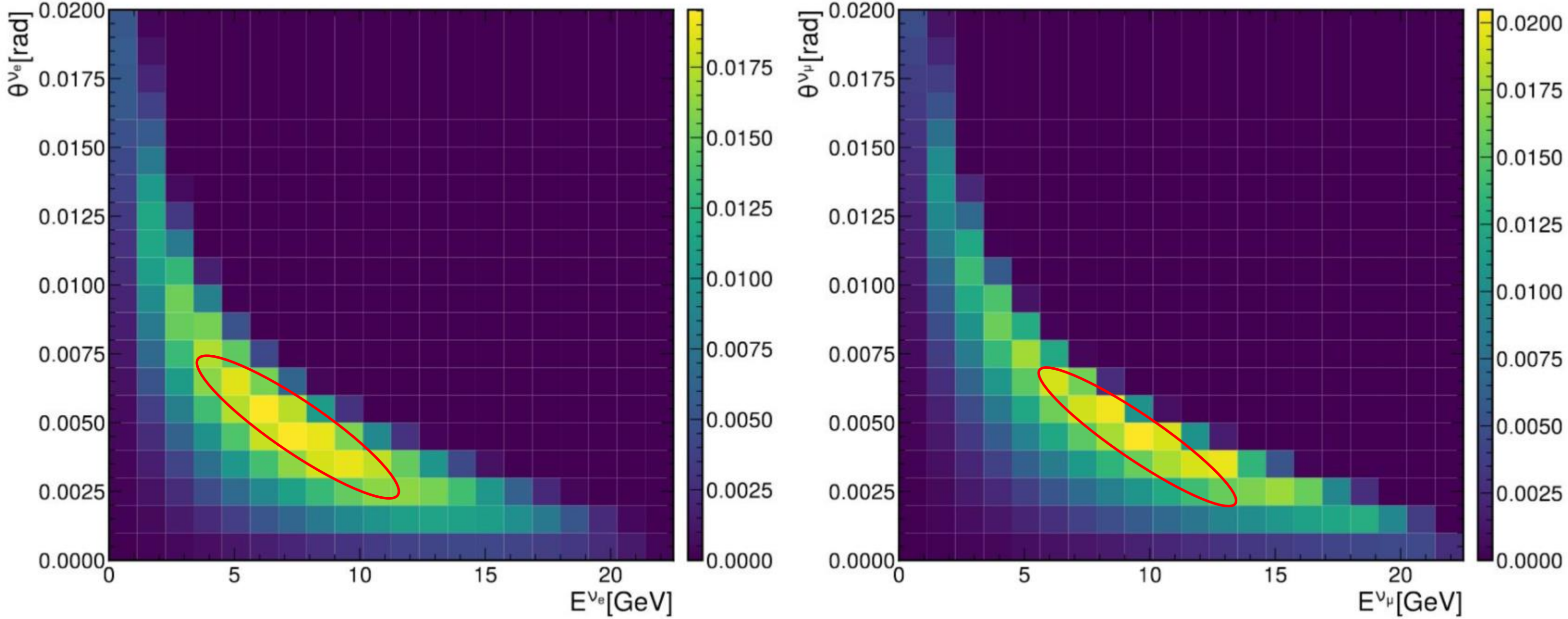
arXiv:2301.02493

A. Ruzi & Qiang Li, et al



- **Collimated and manipulable** muon beams, which lead to a larger acceptance of neutrino sources in the far detector side.
- **Symmetric μ+ and μ– beams,** and thus symmetric neutrino and antineutrino sources, ideally useful for measuring neutrino CP violation.

Neutrino Energy profile



5~10 GeV energy range > tau threshold

Oscillation probability in Matter

Series expansion of oscillation probability: ***JHEP* 04 (2004) 078**

$$P_{\alpha\beta} = P_{\alpha\beta}(\Delta m^2_{21}, \Delta m^2_{31}, \theta_{12}, \theta_{13}, \theta_{23}, \delta_{\rm CP}; E, L, V(x)), \quad \alpha, \beta = e, \mu, \tau$$

$$H \simeq \frac{1}{2E} U \,\mathrm{diag}(0, \Delta m^2_{21}, \Delta m^2_{31}) U^\dagger + \mathrm{diag}(V, 0, 0).$$

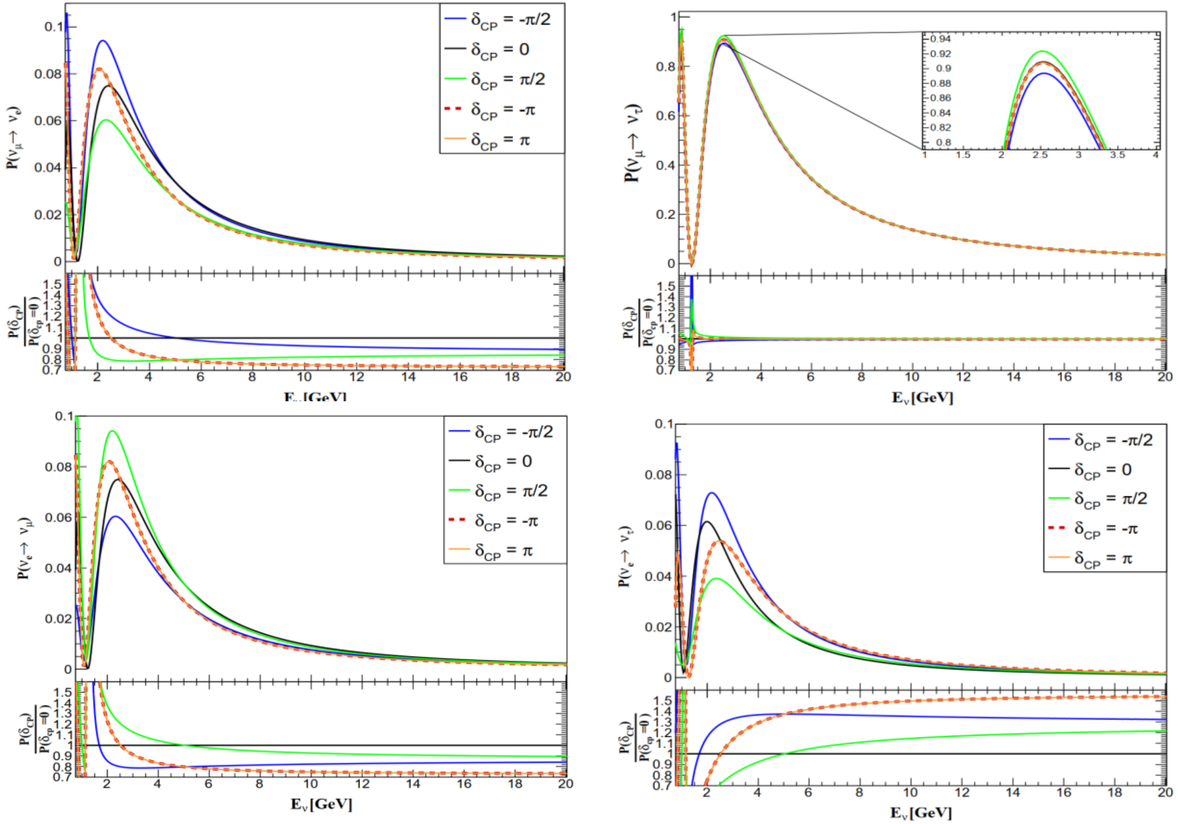
$$V(x) \simeq 7.56 \times 10^{-14} \left(\frac{\rho(x)}{\mathrm{g/cm^3}}\right) Y_e(x) \;\; \mathrm{eV}$$

$$Y_e(x) = 0.5$$

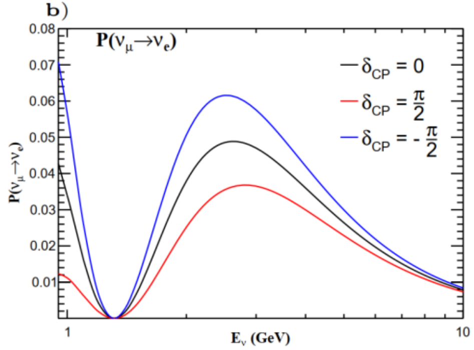
$Y_e(x)$ is the number of electrons per nucleon. For the matter of the Earth.

Experimental Parameters	Values
Stored Muons	1×10^{20}
E_μ[GeV]	22.5 GeV
Run time	5 years
Matter density	$2.8\ g/cm^3$
Base line length	1300 Km
Target mass (Detector)	40 Kt Liquid Argon

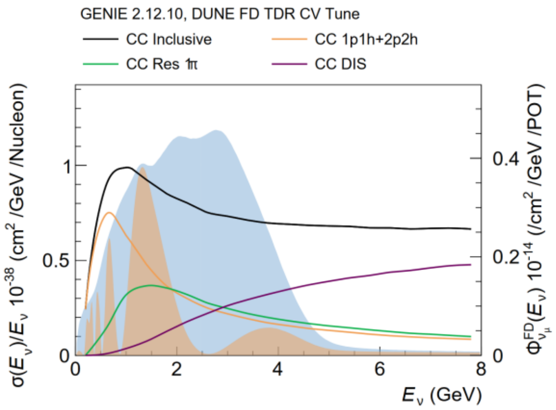
Matter effects on the Oscillation probability



Oscillations in Vacuum

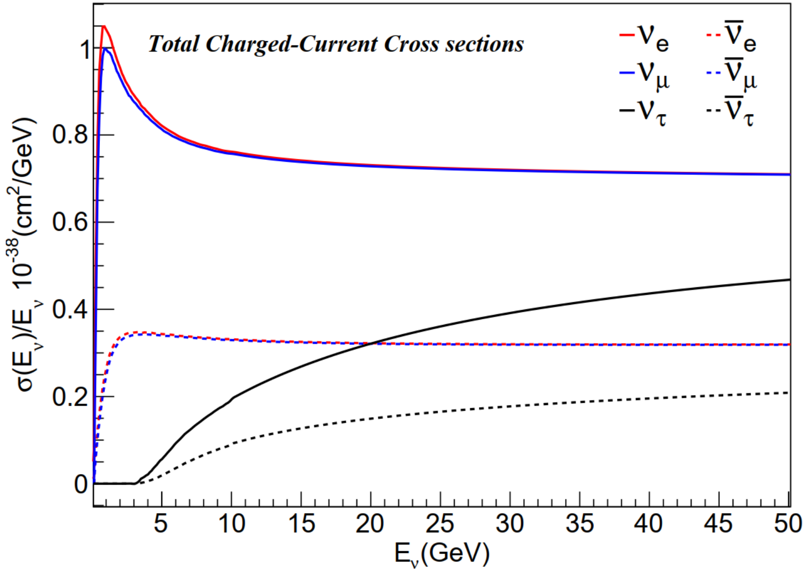


Neutrino CC interactions inside detector



DUNE Collaboration • B. Abi (Oxford U.) et al.

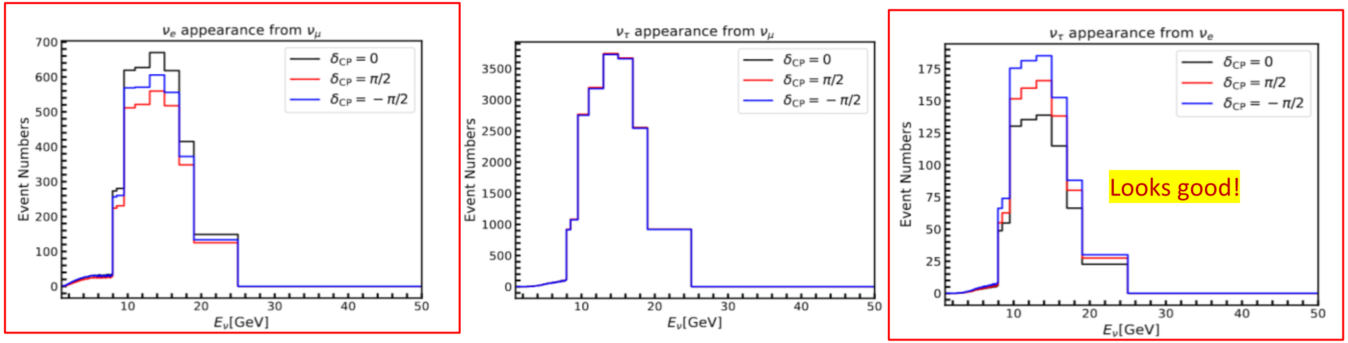
Cross sections are true results obtained from GENIE simulation: *an event generator for neutrino nucleon interactions*



Event spectrum

- **Positron source:** positron bunch density $\mathbf{10^{12}}$**/bunch** with crossing frequency as $\mathbf{10^5/sec}$, which means $\mathbf{10^{17}/sec\ e^+}$ on target. Eventually, we have **muon production rates** as $\frac{dN_\mu}{dt} \sim 10^{12}$**/sec or** $\mathbf{10^{19}}$**/year.**

n(μ) = 1.e20, L = 1300 Km, Detector Mass = 4万吨液氩, 运行5年

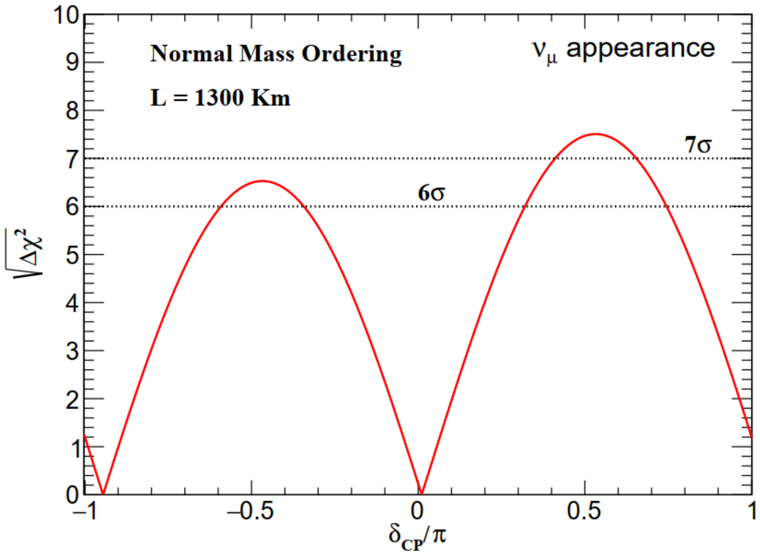


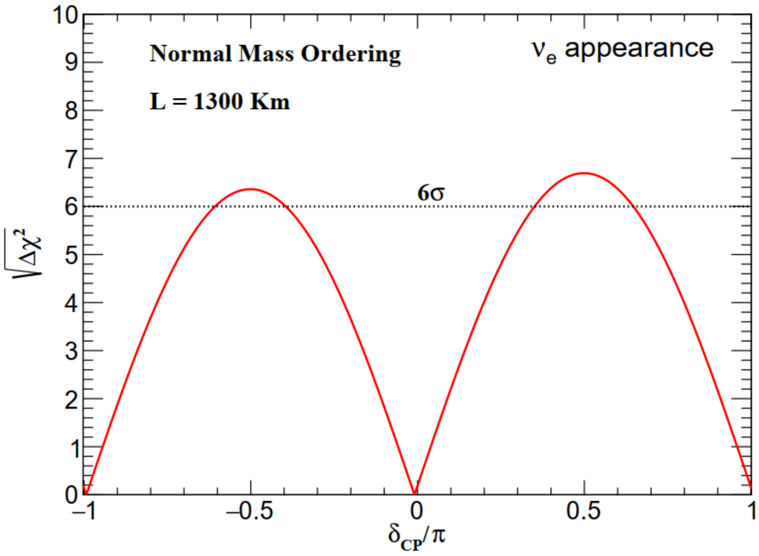
- $\boldsymbol{\nu_\mu \to \nu_e}$**:** the basic channel used by many neutrino oscillation experiment and shows fairly good sensitivity on $\boldsymbol{\delta_{CP}}$ here
- $\boldsymbol{\nu_\mu \to \nu_\tau}$**:** gives the largest tau neutrino events, but poor sensitivity on $\boldsymbol{\delta_{CP}}$
- $\boldsymbol{\nu_e \to \nu_\tau}$**:** gives fairly good sensitivity too!

Sensitivity on δ_{CP}

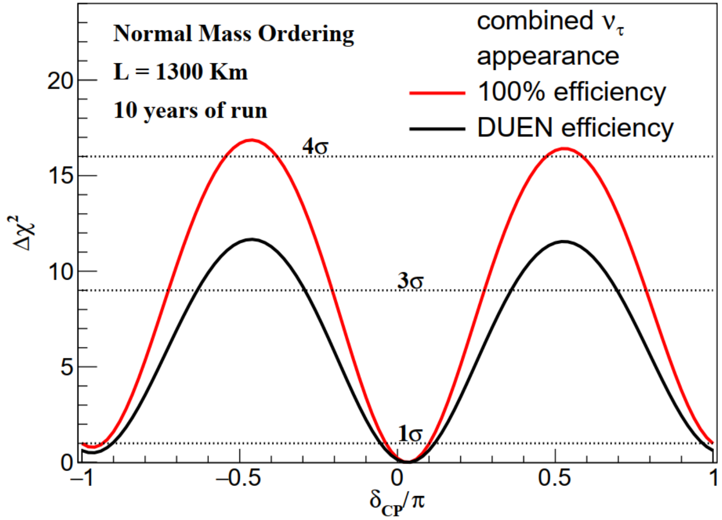
$$\chi^2 = \frac{\left(N(\delta_{\rm CP}^{true}) - \overline{N}(\delta_{\rm CP}^{true})\right)^2}{(N+\overline{N})(\delta_{\rm CP}=0,\pi)}$$

N^{true}: Events produced using $\delta_{CP} = \frac{\pi}{2}$.





Significance (2)



Now formally accepted by Nature Communications Physics
orcid.org/0000-0002-9569-8231

Summary

- Neutrino oscillation is one of the observed physical phenomenon beyond Standard Model, still contains undiscovered physics.
- CP violation in neutrino oscillation still demands compelling data from super-beam experiments.
- LEMMA approach may provide better Muon sources in the super-beam experiments, **HyperK and DUNE.**

Thanks a lot for your attention!

Back Ups

PMNS matrix

- **The PMNS matrix is usually expressed by 3 rotation matrices and three complex phases:**

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos\theta_{23} & sin\theta_{23} \\ 0 & -sin\theta_{23} & cos\theta_{23} \end{pmatrix} \begin{pmatrix} cos\theta_{13} & 0 & sin\theta_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -sin\theta_{13}e^{i\delta_{CP}} & 0 & cos\theta_{13} \end{pmatrix} \begin{pmatrix} cos\theta_{12} & sin\theta_{12} & 0 \\ -sin\theta_{12} & cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\beta} & 0 \\ 0 & 0 & e^{i\gamma} \end{pmatrix}$$

Quantifies the CP violation effect in the neutrino oscillation because $\boldsymbol{\theta_{13} \neq 0}$

Two flavor oscillation

Majorana phase, can ignored (for now)

- **Ignoring the Majorana phases, we find that, when multiplied out, the PMNS matrix becomes**

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix}$$

$c_{ij} = \cos\theta_{ij}, \; s_{ij} = \sin\theta_{ij}$

PhysRevLett.51.1945

Neutrino oscillation in vacuum

$$P(\nu_\mu \to \nu_\tau) \simeq \sin^2(2\theta_{23})\cos^4(\theta_{13})\sin^2\left(1.27\frac{\Delta m^2_{32}L}{E_\nu}\right) \pm 1.27\Delta m^2_{21}\frac{L}{E_\nu}\sin^2\left(1.27\frac{\Delta m^2_{32}L}{E_\nu}\right)\times 8J_{\rm CP},$$

$$P(\nu_\mu \to \nu_e) \simeq \sin^2(2\theta_{13})\sin^2(\theta_{23})\sin^2\left(1.27\Delta m^2_{32}\frac{L}{E_\nu}\right) \mp 1.27\Delta m^2_{21}\frac{L}{E_\nu}\sin^2\left(1.27\Delta m^2_{32}\frac{L}{E_\nu}\right)\times 8J_{\rm CP},$$

$$P(\nu_e \to \nu_\tau) \simeq \sin^2(2\theta_{13})\cos^2(\theta_{23})\sin^2\left(1.27\Delta m^2_{32}\frac{L}{E_\nu}\right) \mp 1.27\Delta m^2_{21}\frac{L}{E_\nu}\sin^2\left(1.27\Delta m^2_{32}\frac{L}{E_\nu}\right)\times 8J_{\rm CP},$$

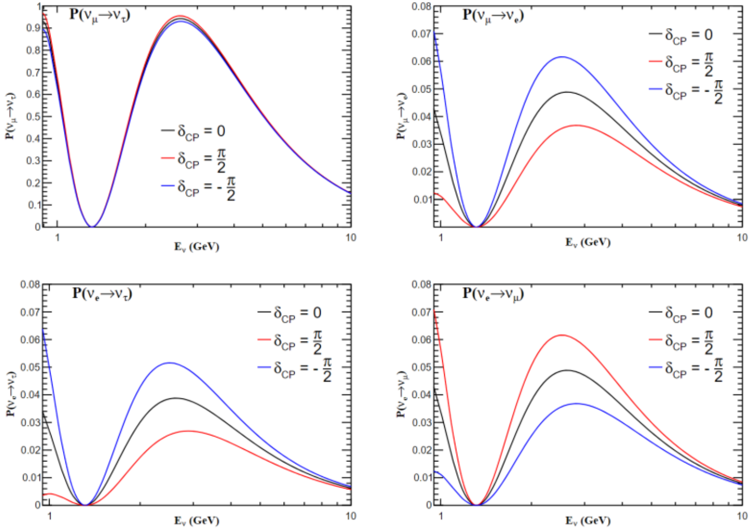
$$P(\nu_e \to \nu_\mu) \simeq \sin^2(2\theta_{13})\sin^2(\theta_{23})\sin^2\left(1.27\Delta m^2_{32}\frac{L}{E_\nu}\right) \pm 1.27\Delta m^2_{21}\frac{L}{E_\nu}\sin^2\left(1.27\Delta m^2_{32}\frac{L}{E_\nu}\right)\times 8J_{\rm CP},$$

$$P(\nu_\mu \to \nu_\tau) = 0.2916 \pm 0.0026\sin\delta_{\rm CP}\ (0.5093 \pm 0.0048\sin\delta_{\rm CP}),$$

$$P(\nu_\mu \to \nu_e) = 0.0151 \mp 0.0026\sin\delta_{\rm CP}\ (0.0264 \mp 0.0048\sin\delta_{\rm CP}),$$

$$P(\nu_e \to \nu_\mu) = 0.0151 \pm 0.0026\sin\delta_{\rm CP}\ (0.0264 \pm 0.0048\sin\delta_{\rm CP}),$$

$$P(\nu_e \to \nu_\tau) = 0.0119 \mp 0.0026\sin\delta_{\rm CP}\ (0.0209 \mp 0.0048\sin\delta_{\rm CP}).$$



Neutrino oscillation in matter

$$P_{\mu\tau} = \sin^2 2\theta_{23}\, \sin^2\Delta - \alpha\, c_{12}^2\, \sin^2 2\theta_{23}\, \Delta\, \sin 2\Delta + \alpha^2\, c_{12}^4\, \sin^2 2\theta_{23}\, \Delta^2\, \cos 2\Delta$$
$$- \frac{1}{2A}\alpha^2\, \sin^2 2\theta_{12}\, \sin^2 2\theta_{23}\left(\sin\Delta\, \frac{\sin A\Delta}{A}\, \cos(A-1)\Delta - \frac{\Delta}{2}\, \sin 2\Delta\right)$$
$$+ \frac{2}{A-1}\, s_{13}^2\, \sin^2 2\theta_{23}\left(\sin\Delta\, \cos A\Delta\, \frac{\sin(A-1)\Delta}{A-1} - \frac{A}{2}\Delta\, \sin 2\Delta\right)$$
$$+ 2\,\alpha\, s_{13}\, \sin 2\theta_{12}\, \sin 2\theta_{23}\, \sin\delta_{\mathrm{CP}}\, \sin\Delta\, \frac{\sin A\Delta}{A}\, \frac{\sin(A-1)\Delta}{A-1}$$
$$- \frac{2}{A-1}\,\alpha\, s_{13}\, \sin 2\theta_{12}\, \sin 2\theta_{23}\, \cos 2\theta_{23}\, \cos\delta_{\mathrm{CP}}\, \sin\Delta\left(A\sin\Delta - \frac{\sin A\Delta}{A}\, \cos(A-1)\Delta\right)$$

$$P_{e\mu} = \alpha^2\, \sin^2 2\theta_{12}\, c_{23}^2 \frac{\sin^2 A\Delta}{A^2} + 4\, s_{13}^2\, s_{23}^2 \frac{\sin^2(A-1)\Delta}{(A-1)^2}$$
$$+ 2\,\alpha\, s_{13}\, \sin 2\theta_{12}\, \sin 2\theta_{23} \cos(\Delta - \delta_{\mathrm{CP}})\, \frac{\sin A\Delta}{A}\, \frac{\sin(A-1)\Delta}{A-1}$$

$$P_{e\tau} = \alpha^2\, \sin^2 2\theta_{12}\, s_{23}^2 \frac{\sin^2 A\Delta}{A^2} + 4\, s_{13}^2\, c_{23}^2 \frac{\sin^2(A-1)\Delta}{(A-1)^2}$$
$$- 2\,\alpha\, s_{13}\, \sin 2\theta_{12}\, \sin 2\theta_{23} \cos(\Delta - \delta_{\mathrm{CP}})\, \frac{\sin A\Delta}{A}\, \frac{\sin(A-1)\Delta}{A-1}$$

Akhmedov， Johansson， Lindner， J. High Energy Phys. 2004-05-05

Event simulation in GLoBES

$$n_i^c = N/L^2 \int_{E_i-\Delta E_i/2}^{E_i+\Delta E_i/2} dE' \int_0^\infty dE \; \Phi^c(E)\, P^c(E)\, \sigma^c(E)\, R^c(E,E')\, \epsilon^c(E')$$

- N: renormalization factor.
- L : baseline length.
- E: energy of incoming neutrino.
- E': reconstructed energy.
- Φ^c: incoming neutrino flux in specific channel.
- $P^c(E)$: oscillation probability.
- $\sigma(E)$: cross section of neutrino-nucleus interaction inside detector.
- $R^c(E,E')$: Energy resolution function.
- $\epsilon^c(E')$: Post smearing efficiency , or energy efficiency.

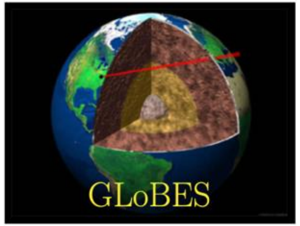
$$R^c(E,E') = \frac{1}{\sigma(E)\sqrt{2\pi}} e^{-\frac{(E-E')^2}{2\sigma^2(E)}}$$

$$\sigma(E) = \alpha \cdot E + \beta \cdot \sqrt{E} + \gamma$$

What to do with GLoBES

Neutrino Physics

- **Theory and pheno**
  - **Standard and non-standard oscillation (*goal of SK, HK and DUNE*)**
    - ✓ Sensitivity on CP phase (being worked out)
    - ✓ Modification of PMNS-matrix
    - ✓ Search for sterile neutrino and do sensitivity check on the new mixing parameters
    - ✓ Neutrino Global fit (precision measurements of mixing parameters and Δm^2)
  - **Neutrino mass problem (Hard)**
    - ✓ Origin of neutrino mass (EFT approach: Weinberg operator)
    - ✓ Solving mass ordering problem (matter effects can help)
- **Software**
  - **GLoBES: neutrino oscillation simulator**
    - ✓ Simulation of neutrino experiment (Nuclear, accelerator and atmospheric neutrino)
    - ✓ χ^2 analysis: projections on $\theta_{ij}, \delta_{CP}, \Delta m_{ij}^2$
  - **Genie: Neutrino event generator**
    - ✓ Cross section calculation
    - ✓ Detector simulation
    - ✓ Neutrino-target experiment, νN DIS
    - ✓ Elastic and DIS Dark matter-Nucleon cross section and event generation



Version from May 5, 2020 for GLoBES 3.2.18