Dark Matter Search in the Muon $g-2$ experiment at Fermilab

Byungchul Yu, University of Mississippi

On behalf of Muon g-2 collaboration

Sep. 18. 2024 (NuFact 2024)

Outline

- **Muon $g-2$ experiment at Fermilab**
  - Aim to measure the magnetic anomaly of the muon, $\boldsymbol{a_\mu}$ at 140 ppb precision ($\boldsymbol{\omega_a} = \boldsymbol{a_\mu}\frac{e}{m_\mu}\boldsymbol{B}$).
  - Measurement of $\boldsymbol{\omega_a}$ in the effort to track the time evolution of muon spin subject to magnetic field.
  - Measurement of $\boldsymbol{B}$ by using nuclear magnetic resonance (NMR) technique with a shielded water sample.
  - Please refer to these talks:
    - **Anomalous Spin Precession Frequency Analysis in the Muon g-2 Experiment at Fermilab – On Kim**
    - **Magnetic Field Analysis in the Muon g-2 Experiment – David Kessler**
    - **Beam dynamics corrections of the Muon g-2 Experiment at Fermilab – David Tarazona**
    - **A Dedicated Period of Magnetic Field Systematics Studies in the Muon g-2 Experiment at Fermilab – Matthew Bressler**

- **Dark Matter search in the Muon $g-2$ experiment at Fermilab**

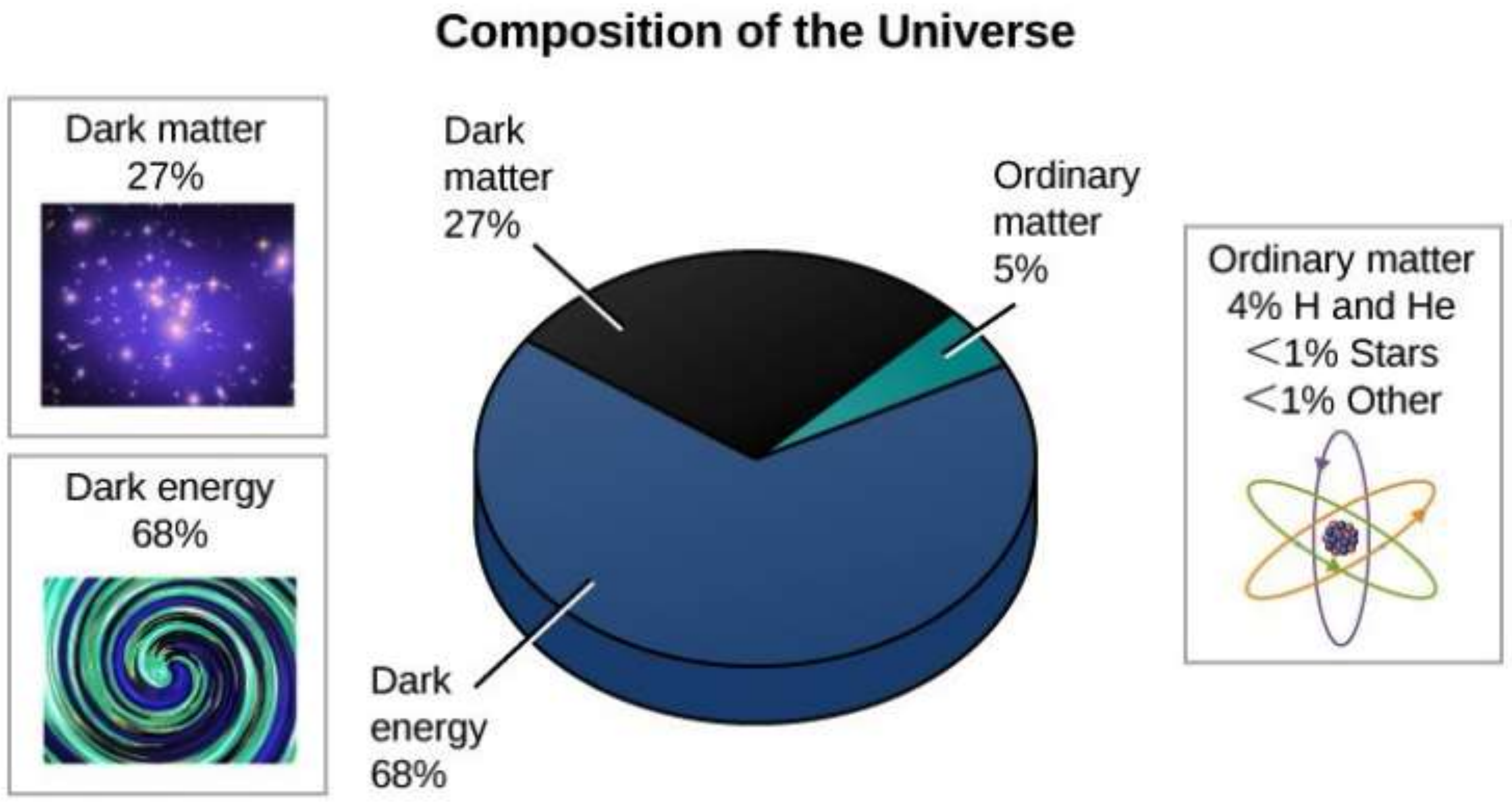
1. **Why do we search for DM in the Muon $g-2$ experiment?**
2. **Which DM models interact with muon particle in the Muon $g-2$ experiment?**
3. **How can we observe Dark Matter signal from the Muon $g-2$ data?**
4. **Timeline with the future plans**

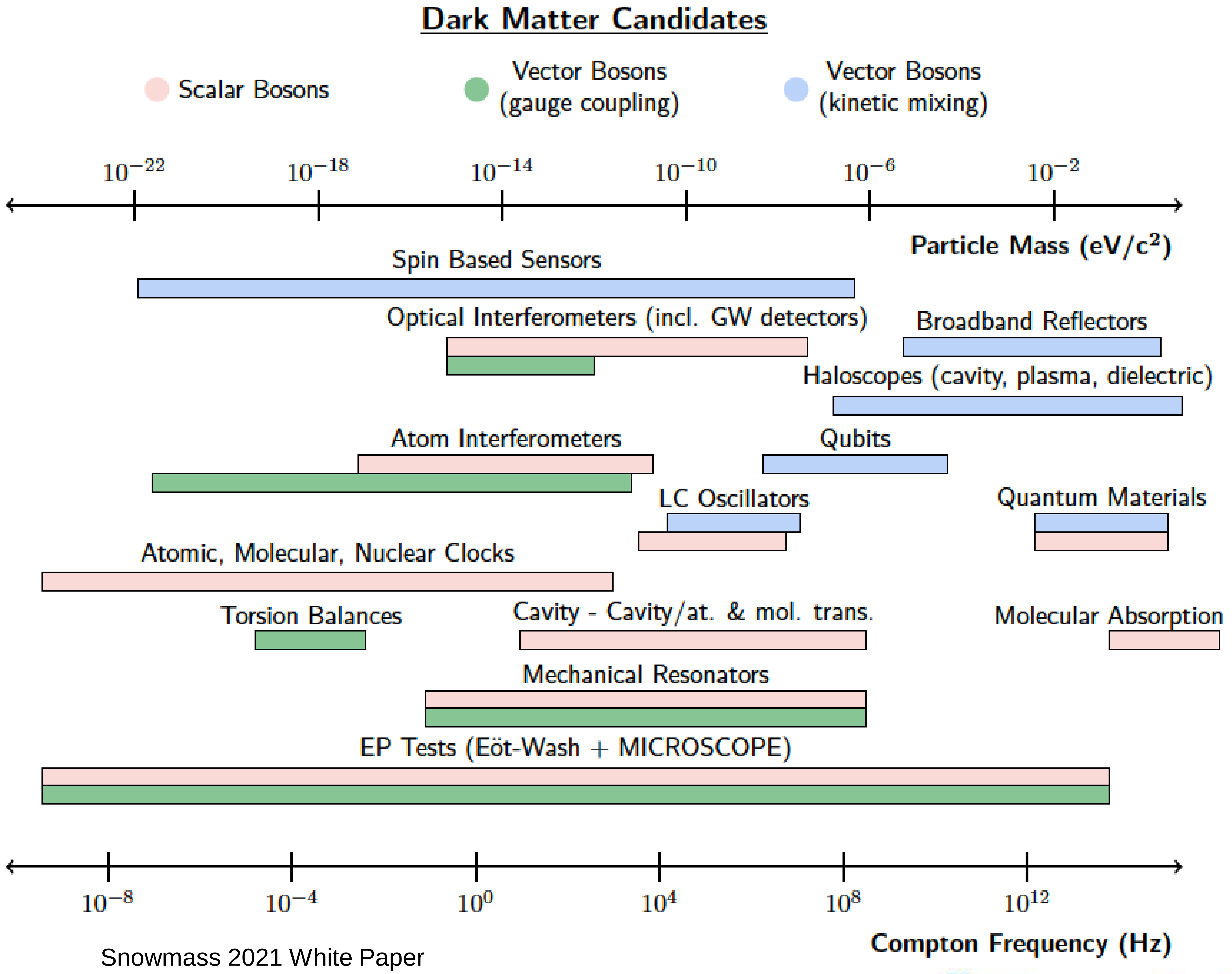
Why do we search for Dark Matter in Muon $g-2$ experiment?

Motivation

1. Dark Matter could be a major component of a complete fundamental description of nature

-> Since we don't know their identity, we need to scan all possible DM mass range in all sectors





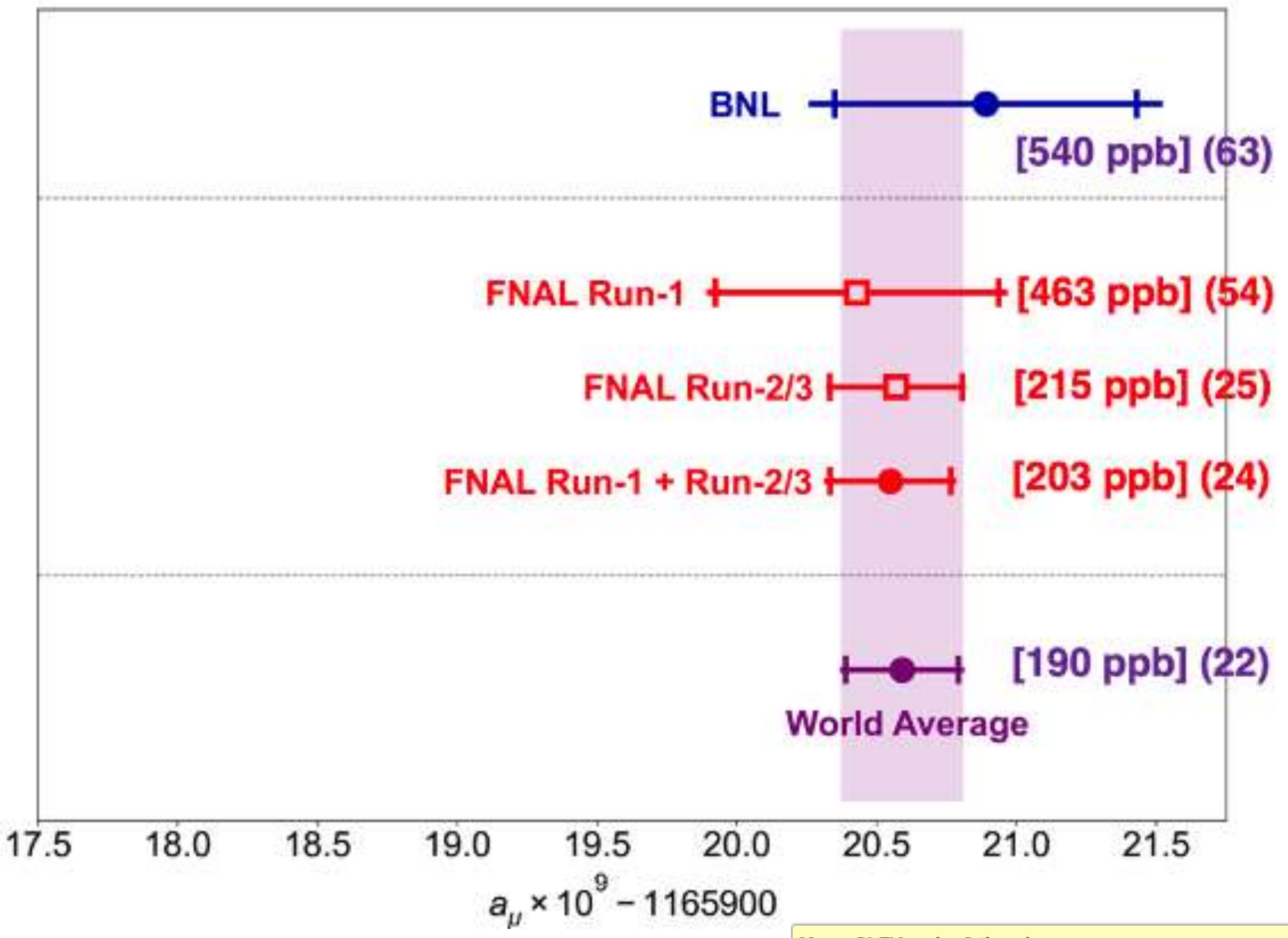
Snowmass 2021 White Paper

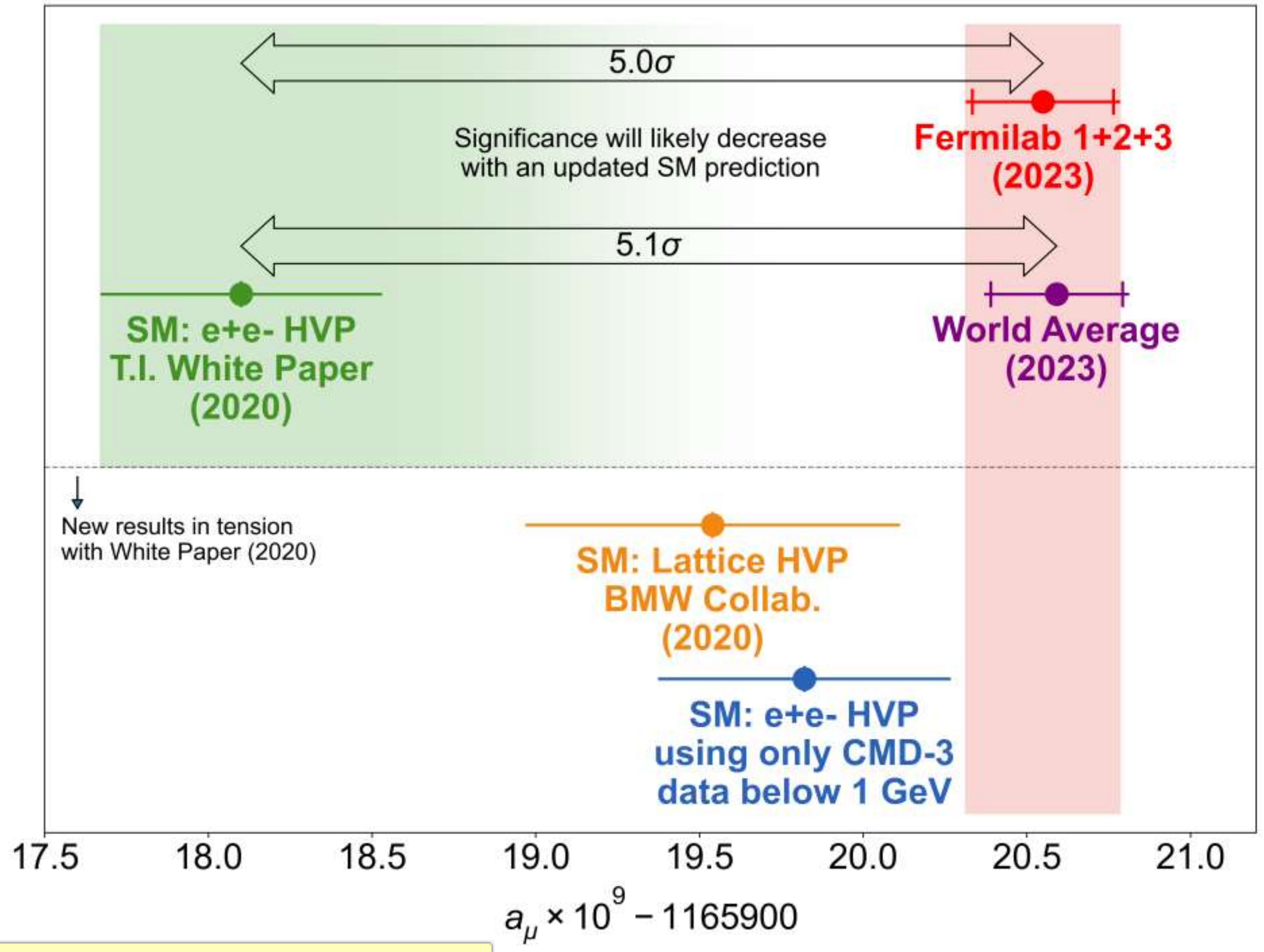
Motivation

2. Muon $g-2$ experiment may have some hints of new physics.

-> DM may be responsible for the potential discrepancy between SM prediction and experimental measurement.

-> It's a region unexplored before, potentially one of the most crucial areas to investigate due to the indications of new physics.

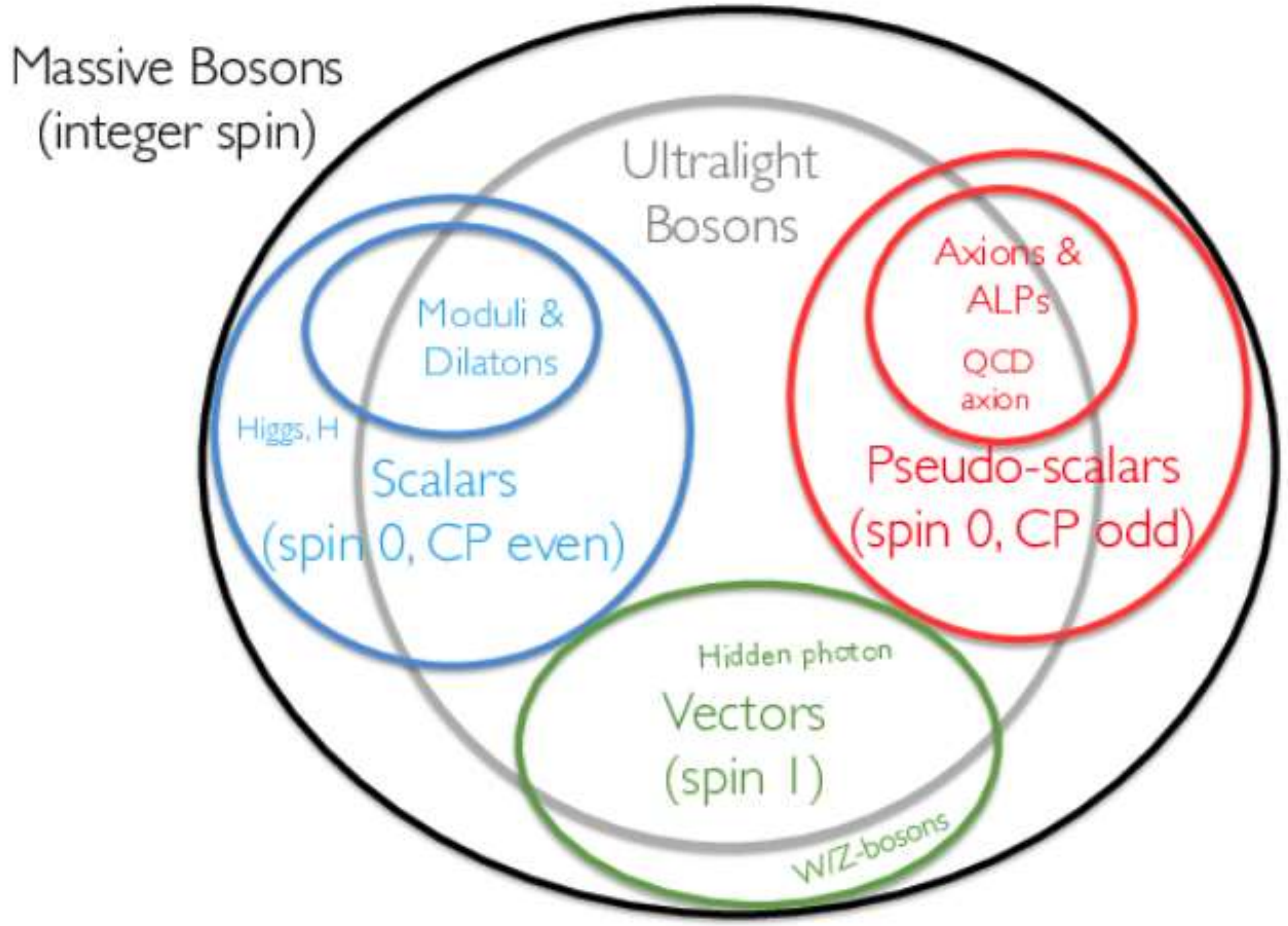




Muon CLFV and g-2 theories	Aida El-Khadra
Auditorium, #402	10:55 - 11:25

Motivation

3. Muon $g-2$ experiment enables the direct search for two ultralight DM candidates (Scalar and Pseudoscalar DM) that primarily interacts with muons.
   -> The sensitivity to these candidates greatly contributes to the area of wavelike(=ultralight bosonic) DM search from the muon sector.

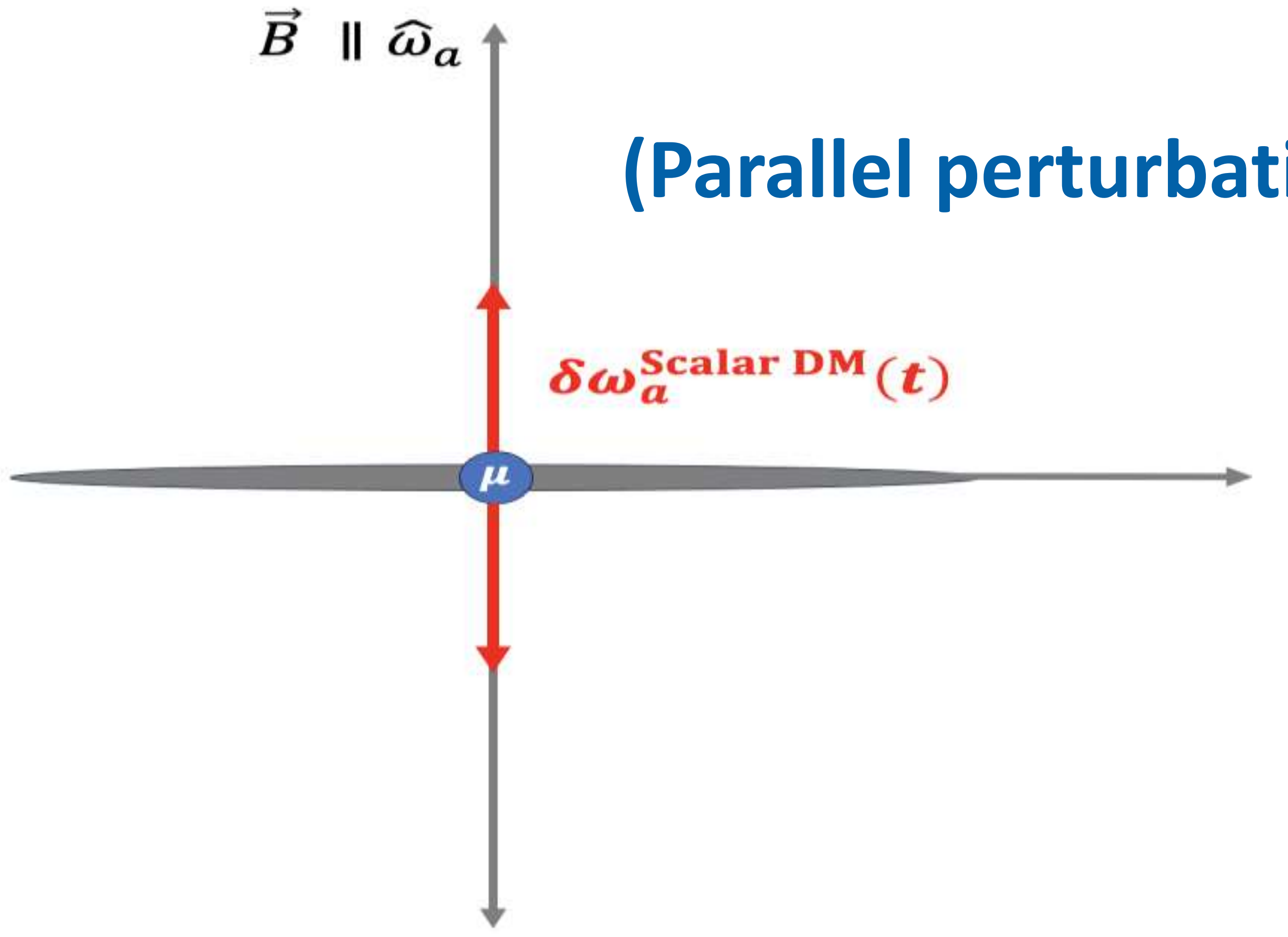


<u>This would be the first-ever direct DM search with muons in a storage ring</u>

Which Dark Matter model interact with muon particle?

Signatures of DM in the Muon $g-2$ experiment

- Two DM candidates that primarily interact with muon may alter muon spin precession in different ways
- Scalar DM may induce apparent oscillations of the muon mass at DM frequency: $\omega_a(t) = a_\mu \frac{q}{m(t)} B$



- It causes a modulation of magnitude of ω_a at m_{DM}
  (ω_a time series plot is required)

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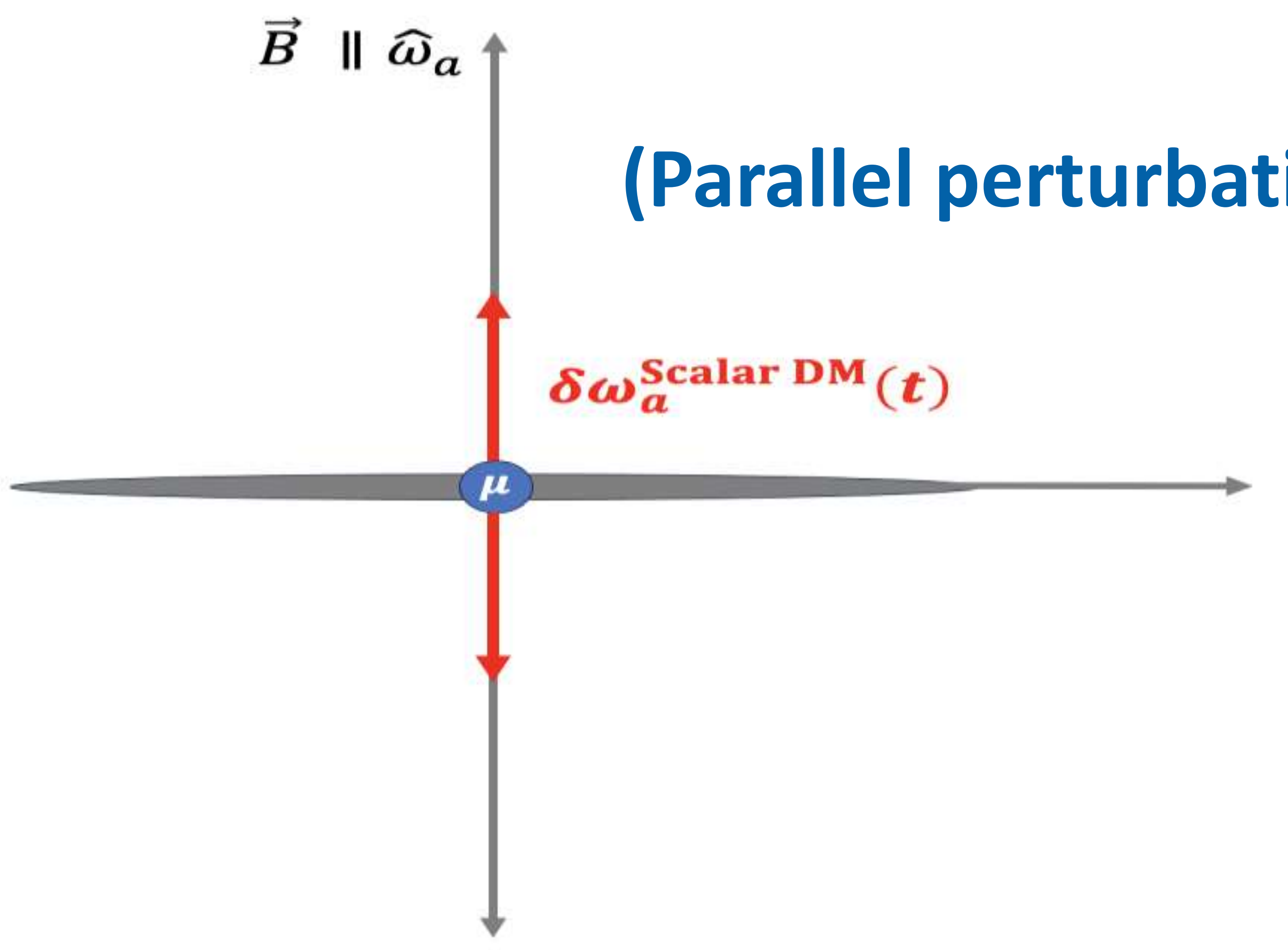

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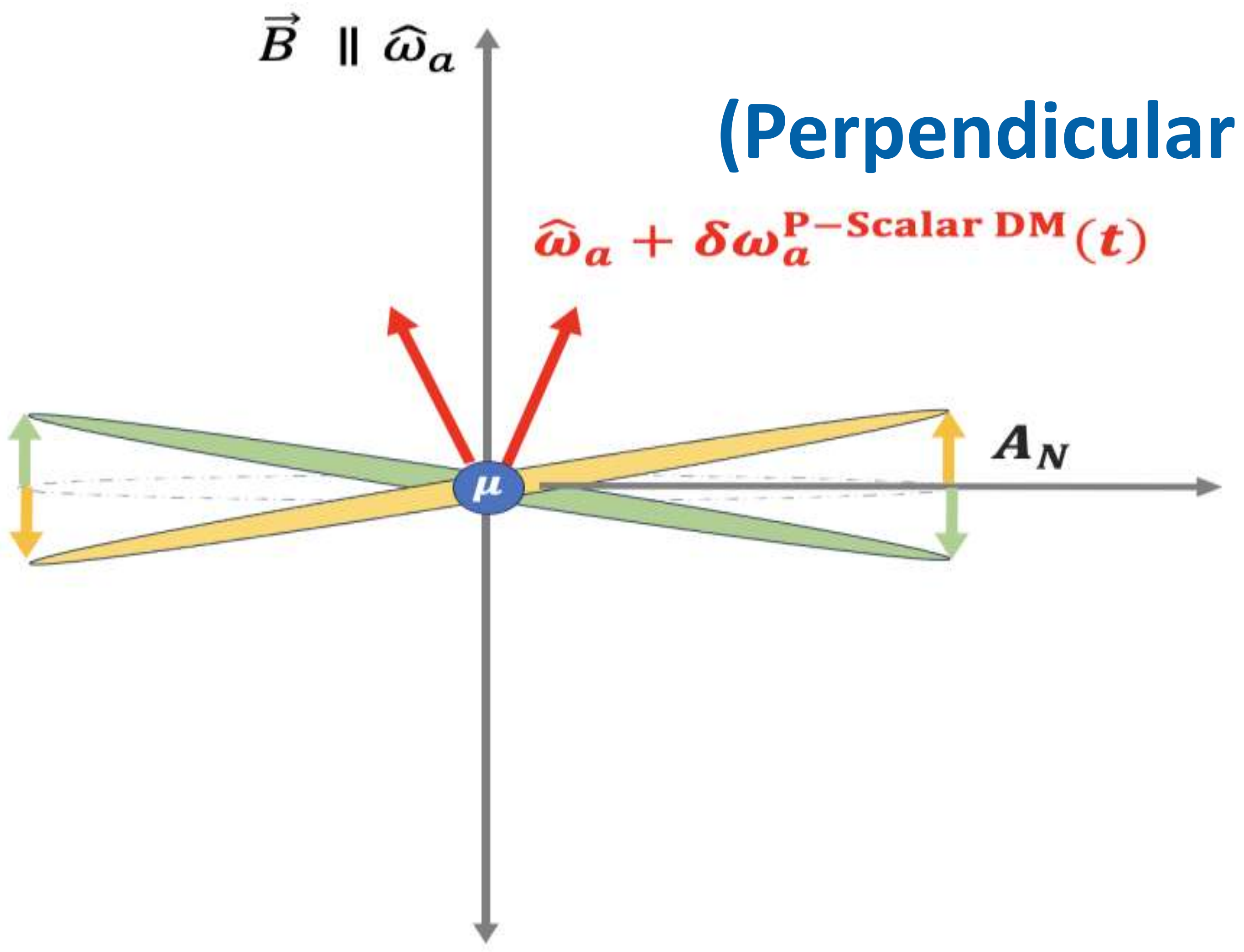
- Pseudoscalar DM may cause the muon's spin precession plane to be tilted and swing at DM frequency.



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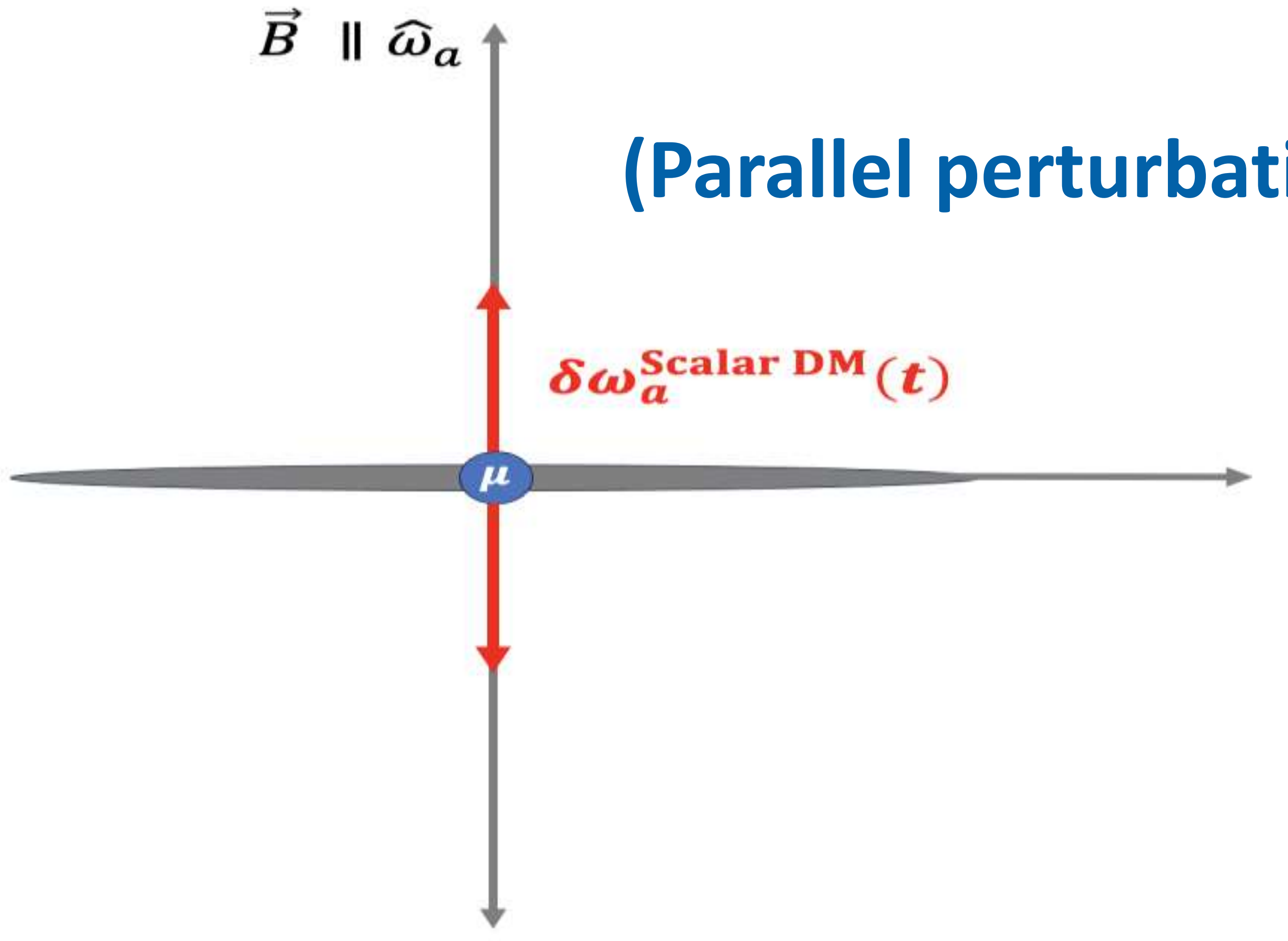

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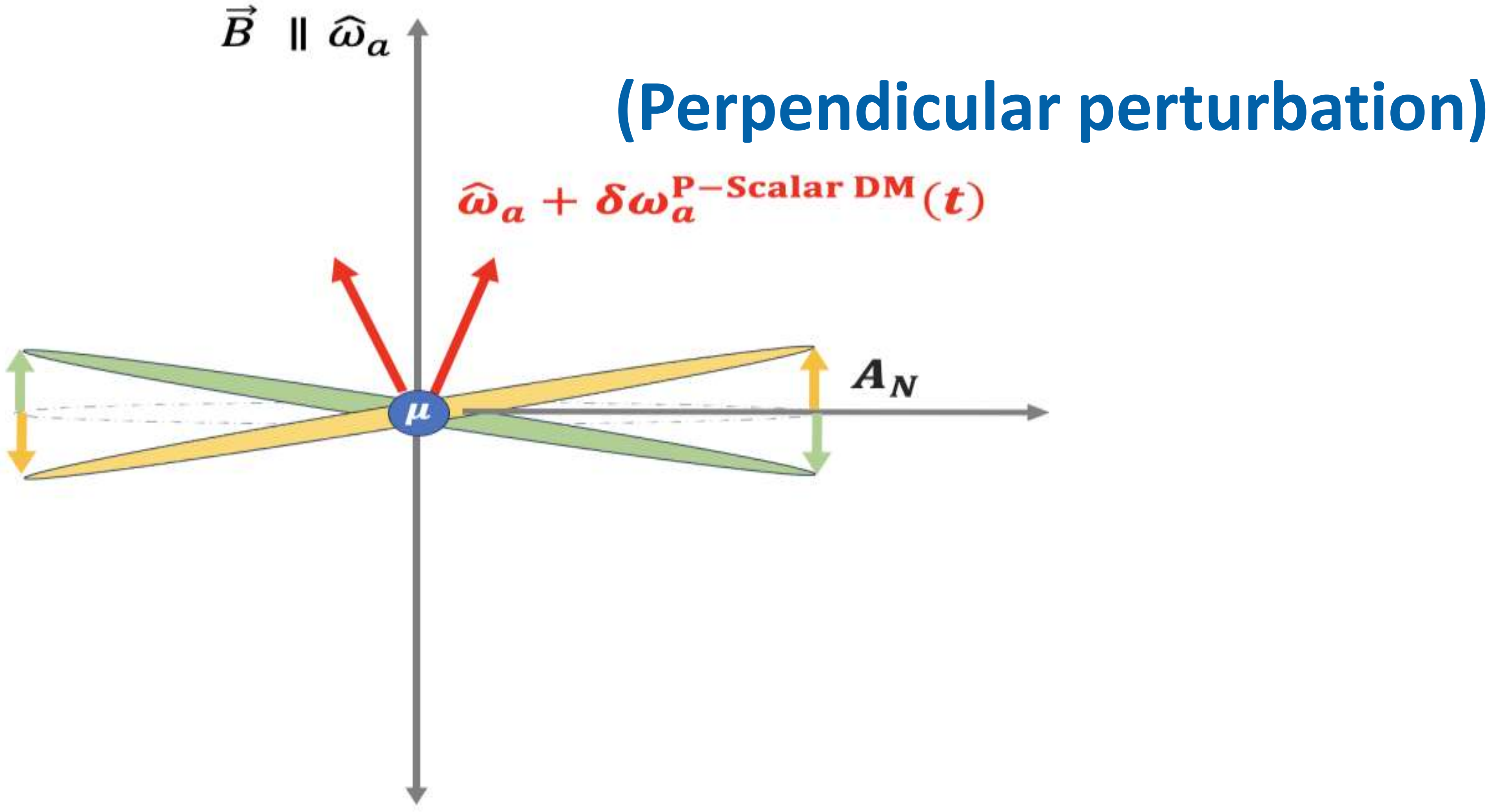

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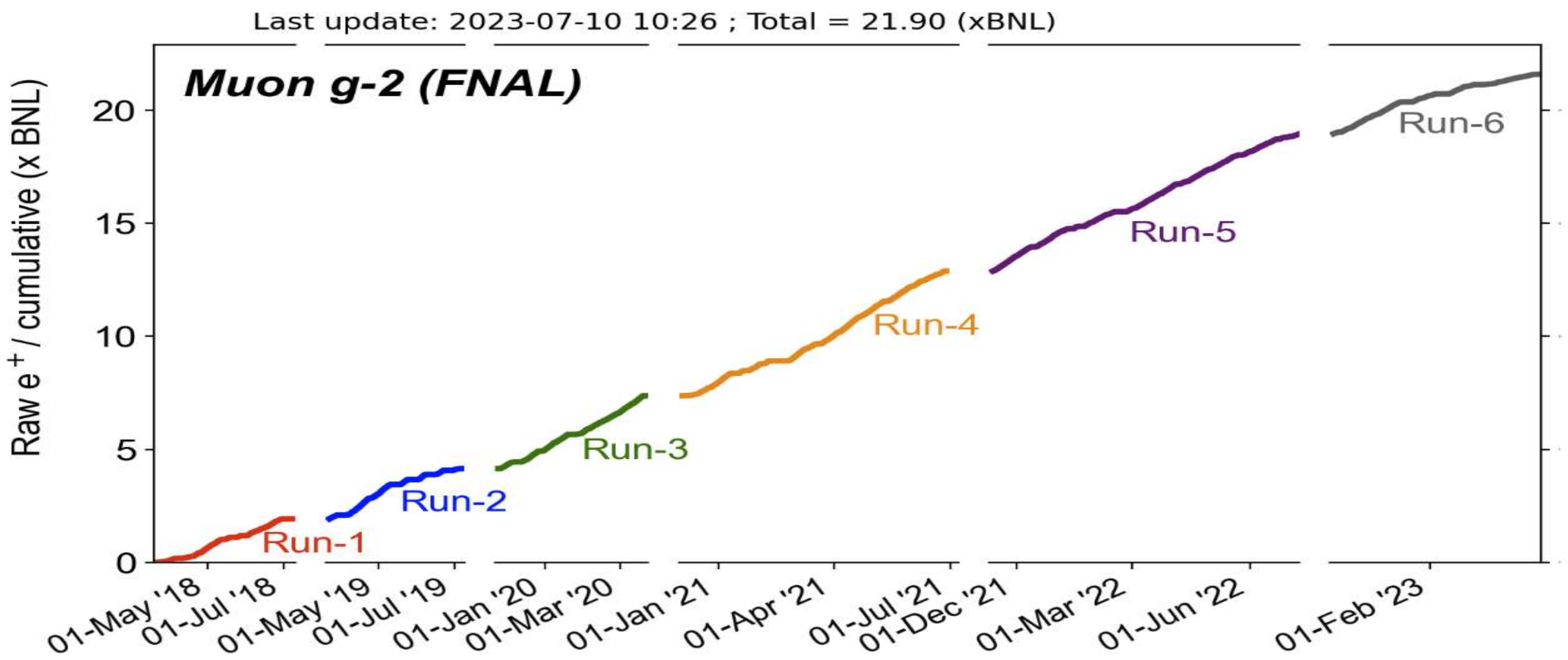
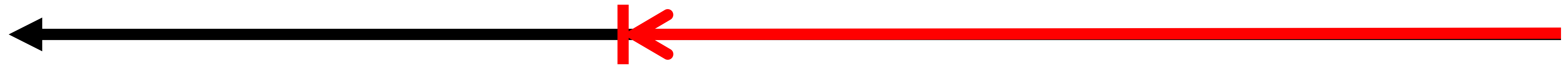
Expected DM sensitivity from experimental time scale

Maximum time scale



Whole experiment lifetime (5 years)

DM Mass (eV) $\mathbf{10^{-23}}$

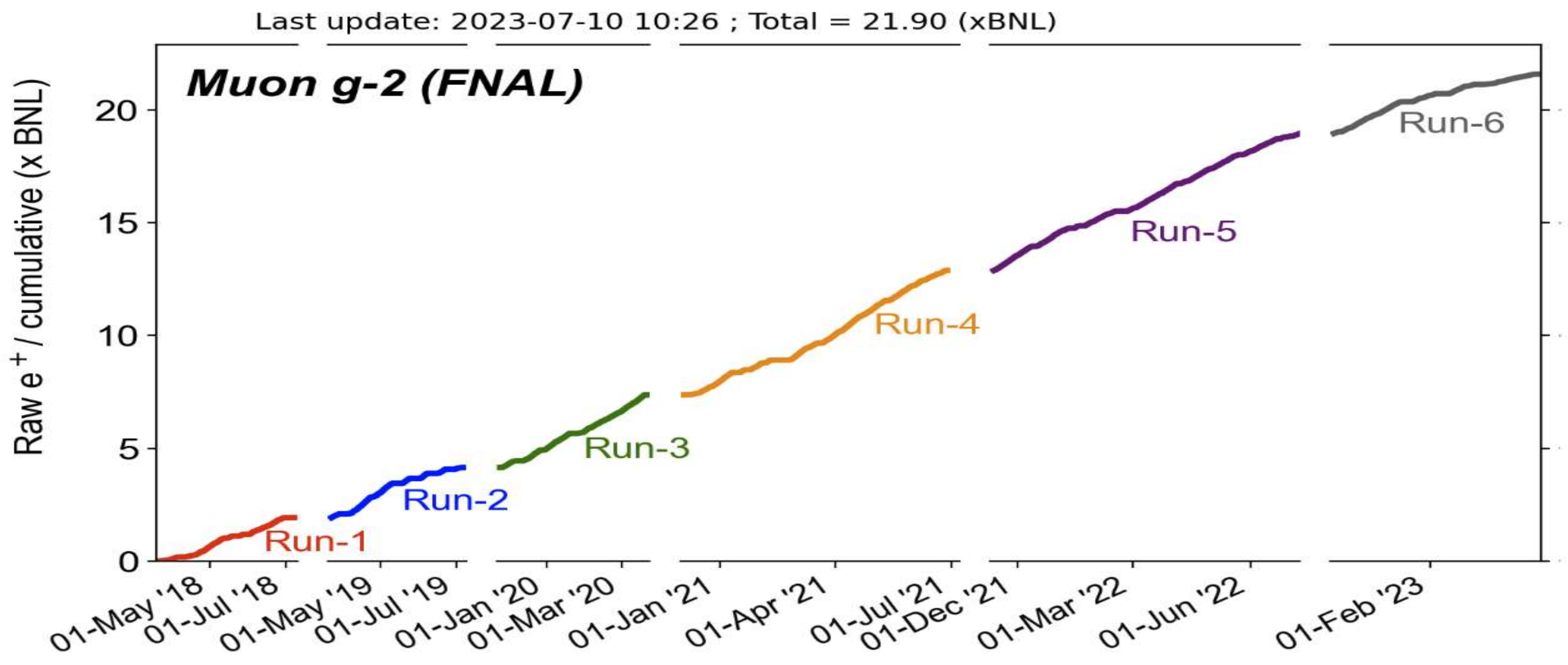


DM Freq (Hz) $\mathbf{10^{-8}}$

Expected Dark Matter mass range with its corresponding frequency

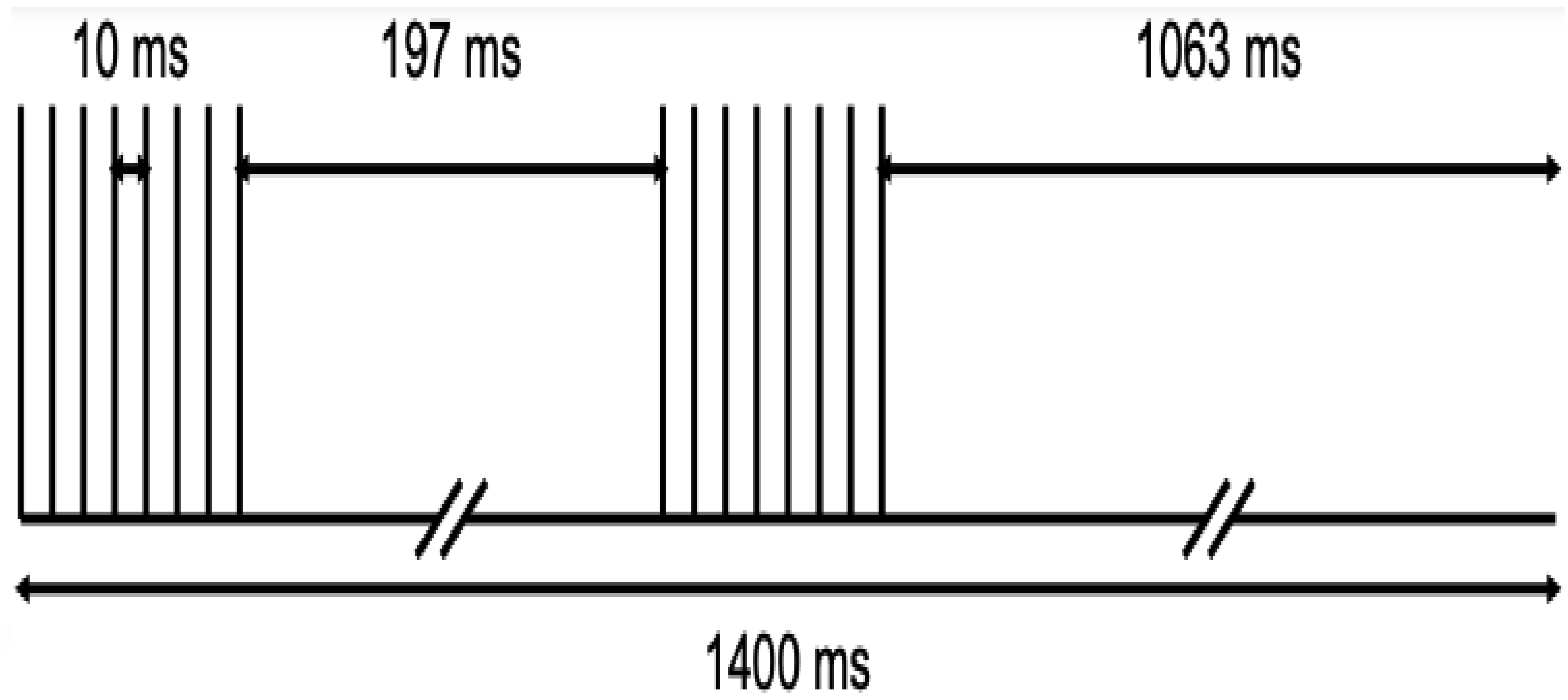
Expected DM sensitivity from experimental time scale

Maximum time scale



Whole experiment lifetime (5 years)

Minimum time scale



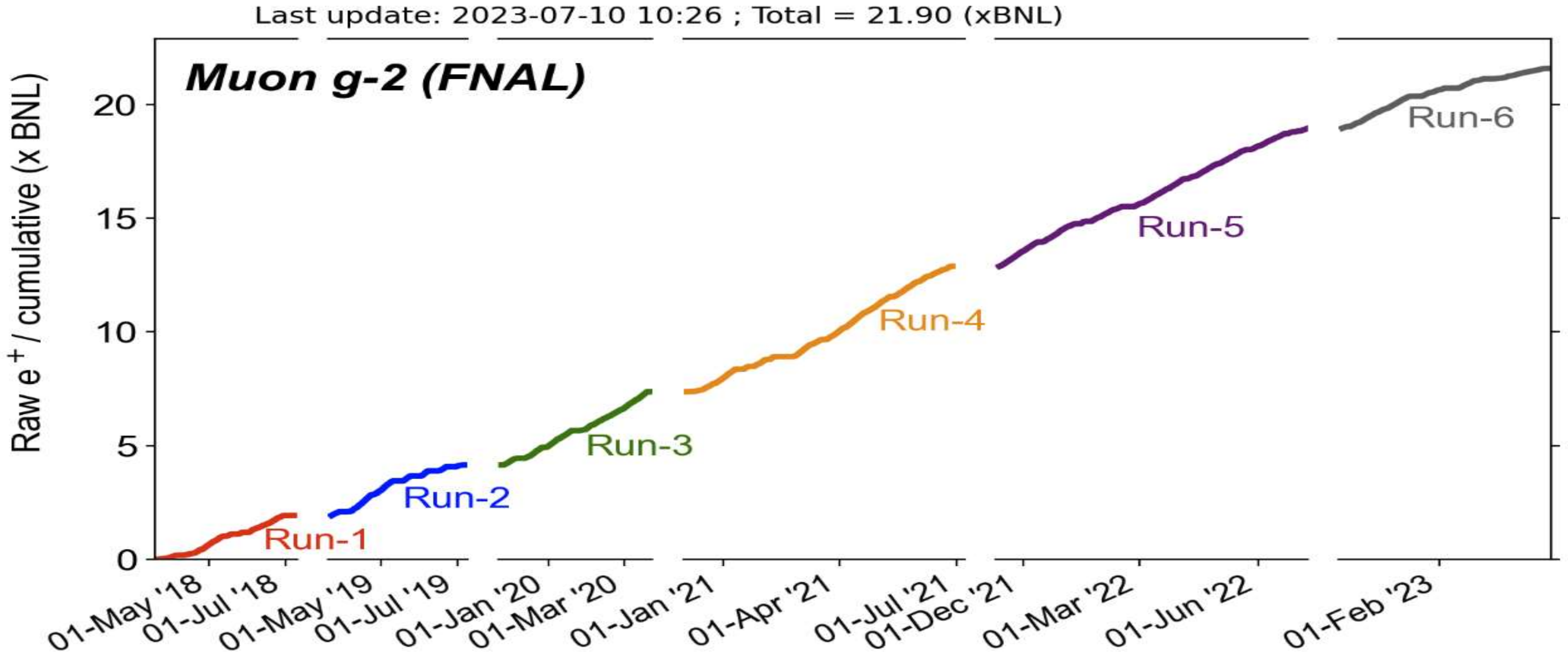
Bunch separation (10 ms)

DM Mass (eV)	10^{-23}	m_{DM} **(bunch level)**	10^{-13}
DM Freq (Hz)	10^{-8}		10^{2}

Expected Dark Matter mass range with its corresponding frequency

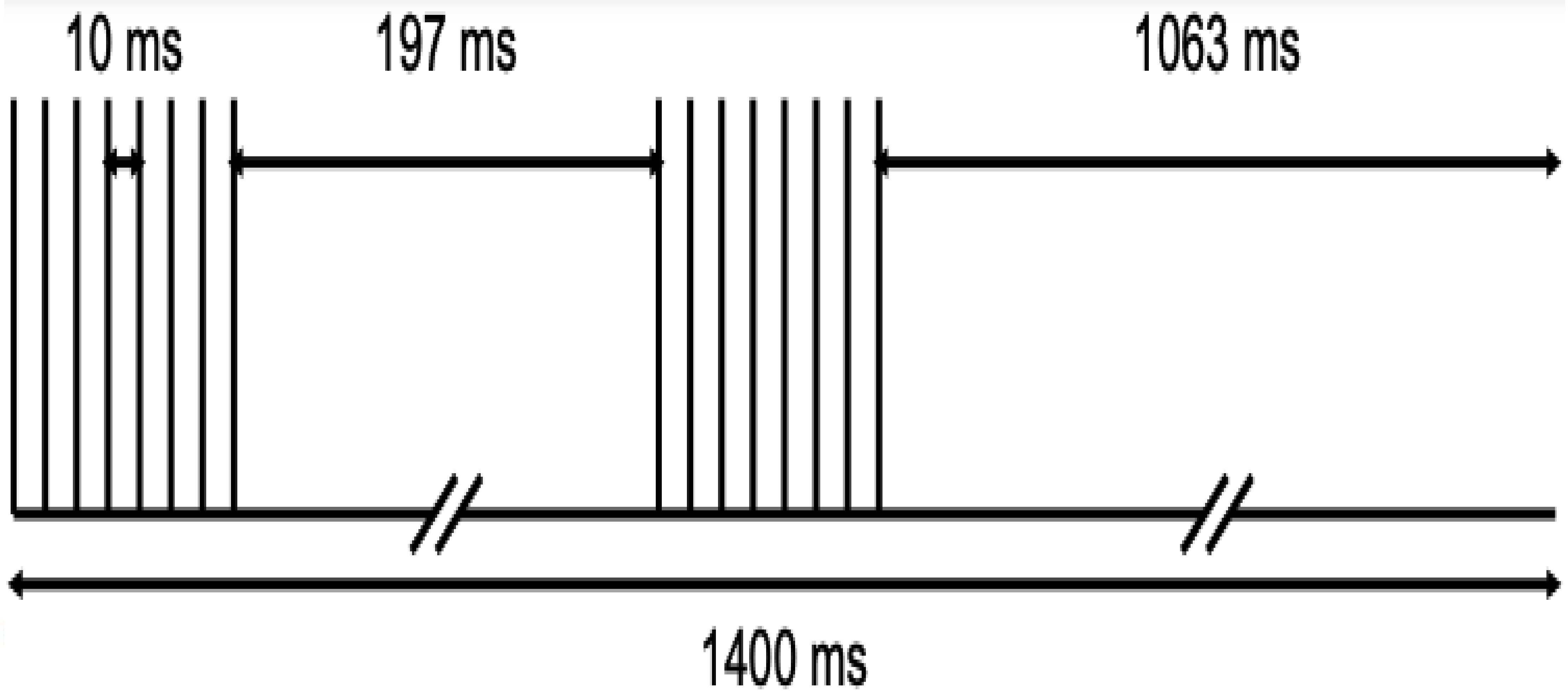
Expected DM sensitivity from experimental time scale

Maximum time scale



Whole experiment lifetime (5 years)

Minimum time scale



Bunch separation (10 ms)

DM Mass (eV) 10^{-23} 10^{-15} 10^{-13}

m_{DM} (bunch level)

m_{DM} (sub-run level $\sim \mathcal{O}$ (1) seconds)

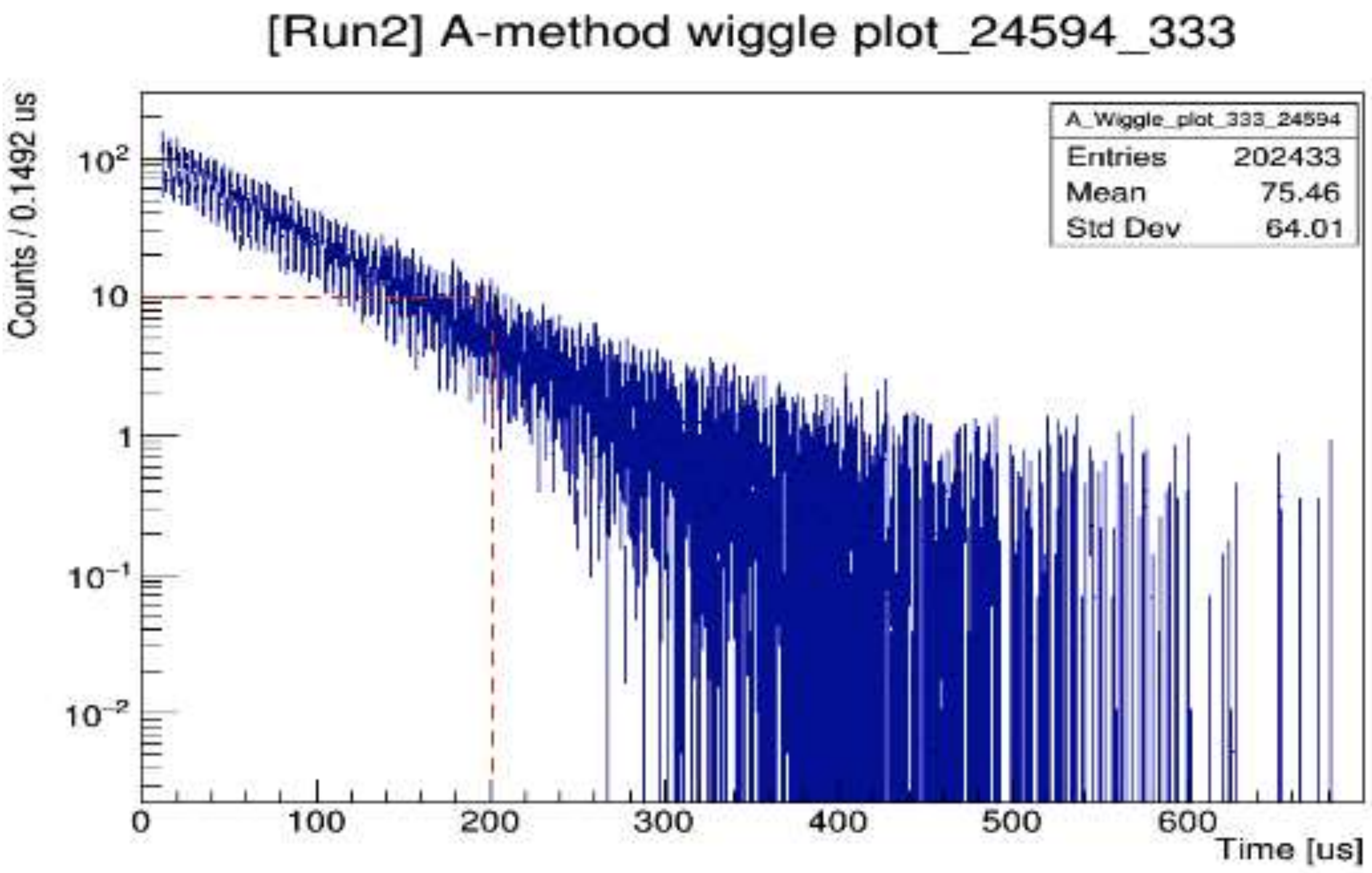
DM Freq (Hz) 10^{-8} 10^{0} 10^{2}

Expected Dark Matter mass range with its corresponding frequency

How can we observe the DM signals from the Muon $g-2$ data?

Data preparation: Extracting ω_a from wiggle plots

- From the Muon $g-2$ data, specific dataset (ω_a time series) for the scalar DM search needs to be produced.
- This requires the generation of "wiggle plot", showing the number of decay positrons as a function of time.
- The ω_a is determined by the frequency of the modulation in the wiggle plot.



$$N(t) = N_0\, e^{-\frac{t}{\gamma\tau}}[1 + A\cos(\omega_a t + \phi)],$$

Where N_0: Normalization constant

τ : Average lifetime of muon

A : Asymmetry (Measurement of how much positrons emission direction is correlated with the muon spin)

ω_a: Anomalous spin precession frequency

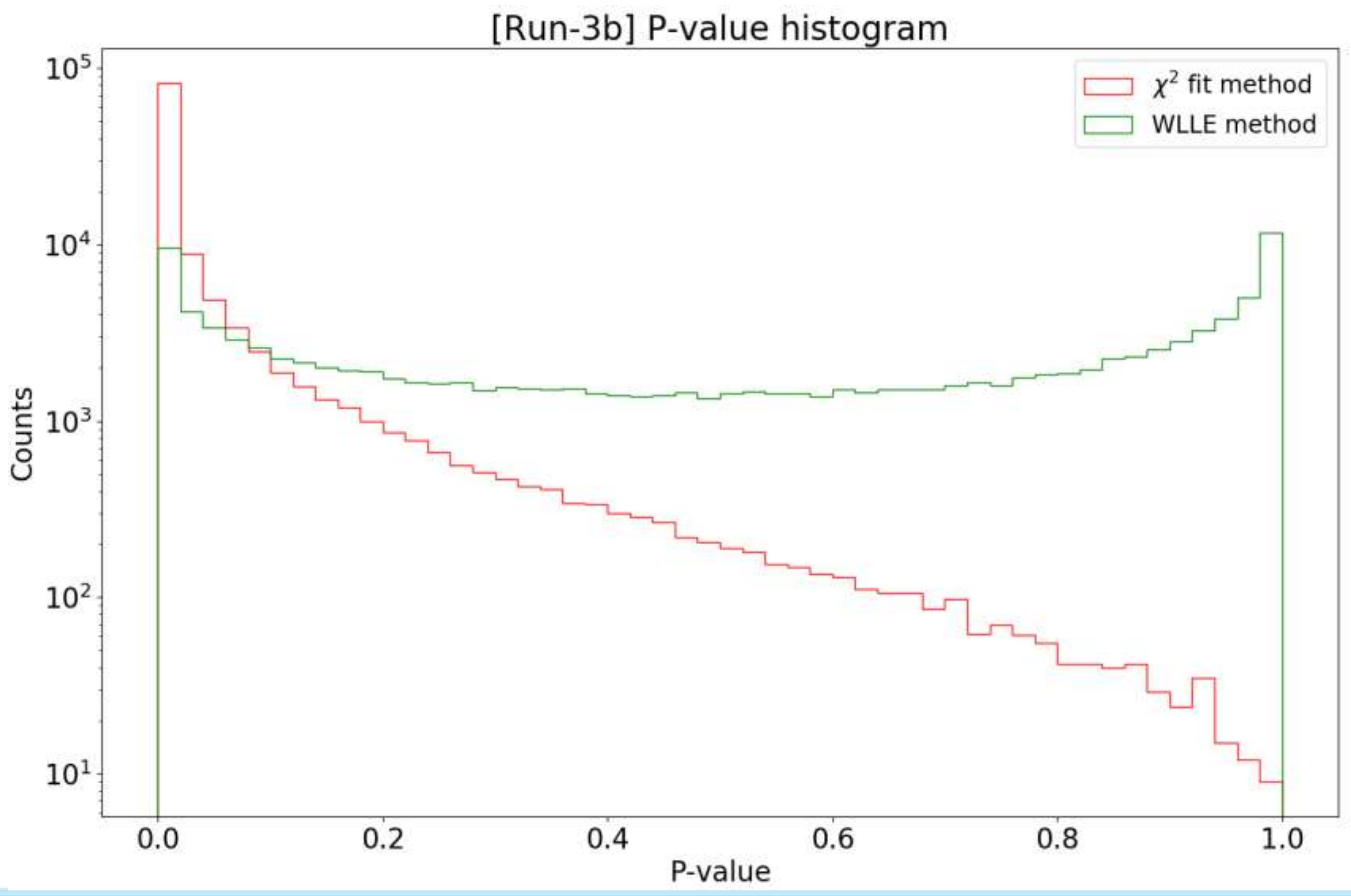
ϕ : $g-2$ phase

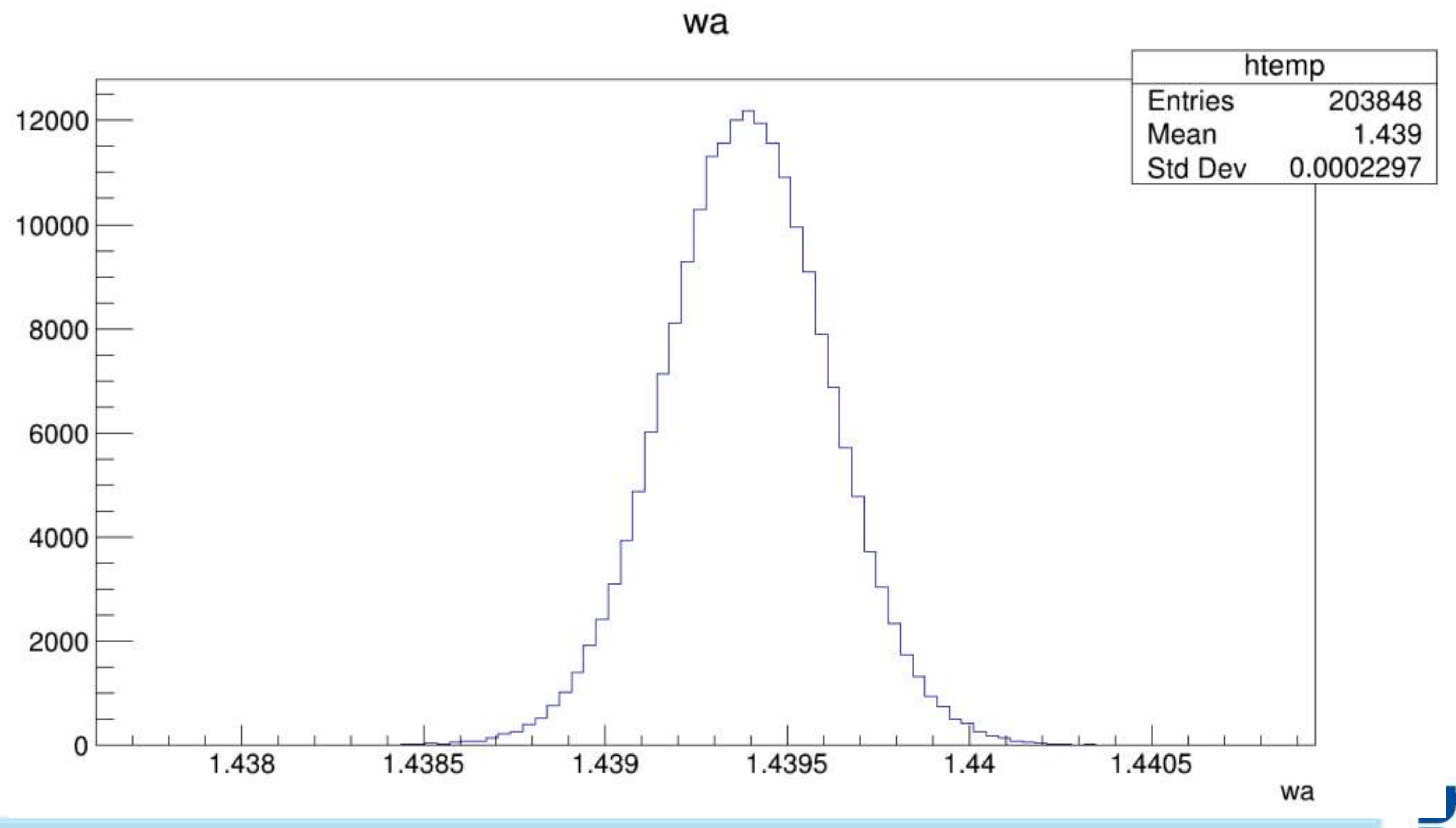
Weighted Log Likelihood Estimation (WLLE)

- Poor statistics in sub-run level wiggle plots lead to a significant bias on the fit results from the χ^2 method.
- WLLE takes into account both the **Poisson nature of the count data** and **their uncertainty through the assigned weights**. (assuming the Weighted Poisson Distribution)

$$\mathcal{L}(\lambda|x_1 \cdots x_n) = \prod_{i=1}^{N} \left(\frac{e^{-\lambda} \lambda^{x_i}}{x_i\,!} \right)^{w_i} \rightarrow \text{approximation}: \quad -\log \mathcal{L} = -\sum_{i=1}^{n} S_i \log \frac{e^{-n_{\text{eff}}} {n_{\text{eff}}}^{x_i}}{N\,!},$$

where the effective counts $n_{\text{eff}} = \frac{\left(\sum_j w_j\right)^2}{\sum_j w_j^2}$ (not necessarily be an integer) and the scale factor $S_i = \frac{\sum_j w_i^2}{\sum_j w_i}$





Generalized Lomb-Scargle (GLS) periodogram

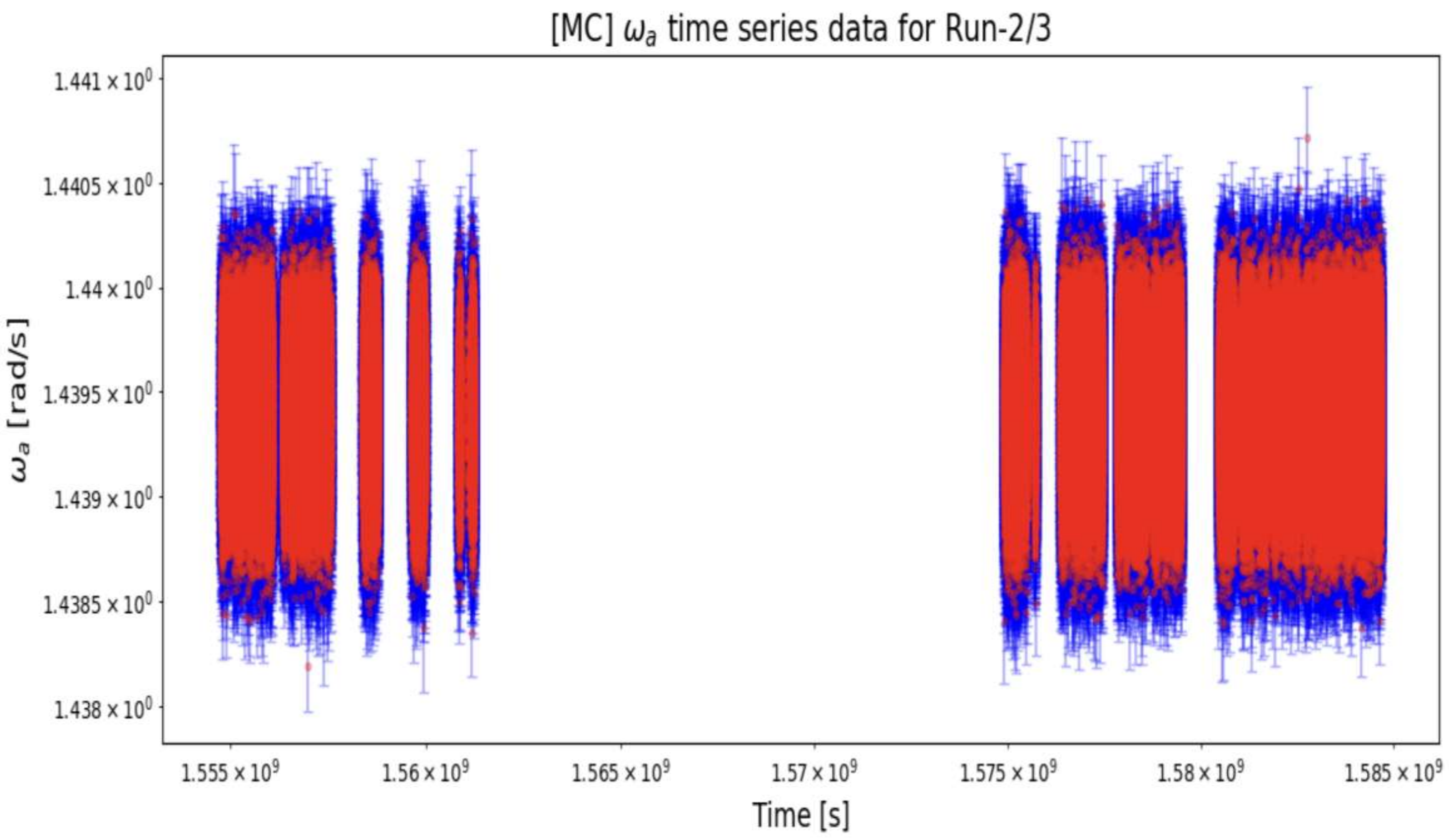
- The GLS periodogram is designed for analyzing unevenly spaced time series data for detecting and characterizing periodic signals.

- This method includes a floating mean in sinusoidal wave model, effectively for fitting with an adjustable baseline and optimizing both the frequency and amplitude:

$$\chi^2 = \sum_i \left(\frac{(y_i - f(t_i; \vec{p}))}{\sigma_i} \right)^2$$

, where $f(t_i; \vec{p}) = a \cos \omega t + b \sin \omega t + c$

$$\boldsymbol{P_N(\omega) = \frac{1}{2\sigma_0} \left[\frac{\left\{ \sum_{i=1}^{N} w_i (y_i - \bar{y}) \cos \omega (t_i - \tau) \right\}^2}{\sum_{i=1}^{N} w_i \cos^2 \omega (t_i - \tau)} + \frac{\left\{ \sum_{i=1}^{N} w_i (y_i - \bar{y}) \sin \omega (t_i - \tau) \right\}^2}{\sum_{i=1}^{N} w_i \sin^2 \omega (t_i - \tau)} \right]}$$



- One of the key advantages of GLS method is the ability to customize the frequency domain settings, unlike FFT, which is limited by the sampling rate of the data.

- This flexibility allows for a precise tuning of the frequency resolution and range, which is essential to ensure that potential dark matter signals are not missed.

Frequency grid optimization

- The structure of the ω_a time series data fundamentally determines the frequency grid optimization.

$$\left(f_{\min} = \frac{1}{\beta_{\min} \times T},\quad f_{\max} = \frac{\beta_{\max}}{(2 \times dT_{\text{ave}})},\ df = \frac{1}{\alpha \times T}\right),\ \text{where}\ \ \beta_{\min} = 1\sim10\ ,\beta_{\max} = 1\sim10,\ \alpha = 2$$

Time series data structure		Frequency grid	
Total time span (T)	2.99e7 s ≈ 346 days	Minimum frequency ($f_{\min}$)	3.3e-8 Hz
Average time spacing(dT_{ave})	45.98 s	Maximum frequency ($f_{\max}$)	1.1e-1 Hz
Time sample size (N_T)	6.51e5	Frequency spacing (df)	1.7e-8 Hz
-	-	Trial frequency size (N_{trial})	6.51e6

- There is a risk of missing peaks in the periodogram if we arbitrary reset the frequency resolution.
- Need to keep using $df < 10^{-8}$ Hz over the whole frequency range because the width of the potential signal depends predominantly on T, which in our case is 10^{-8} Hz.

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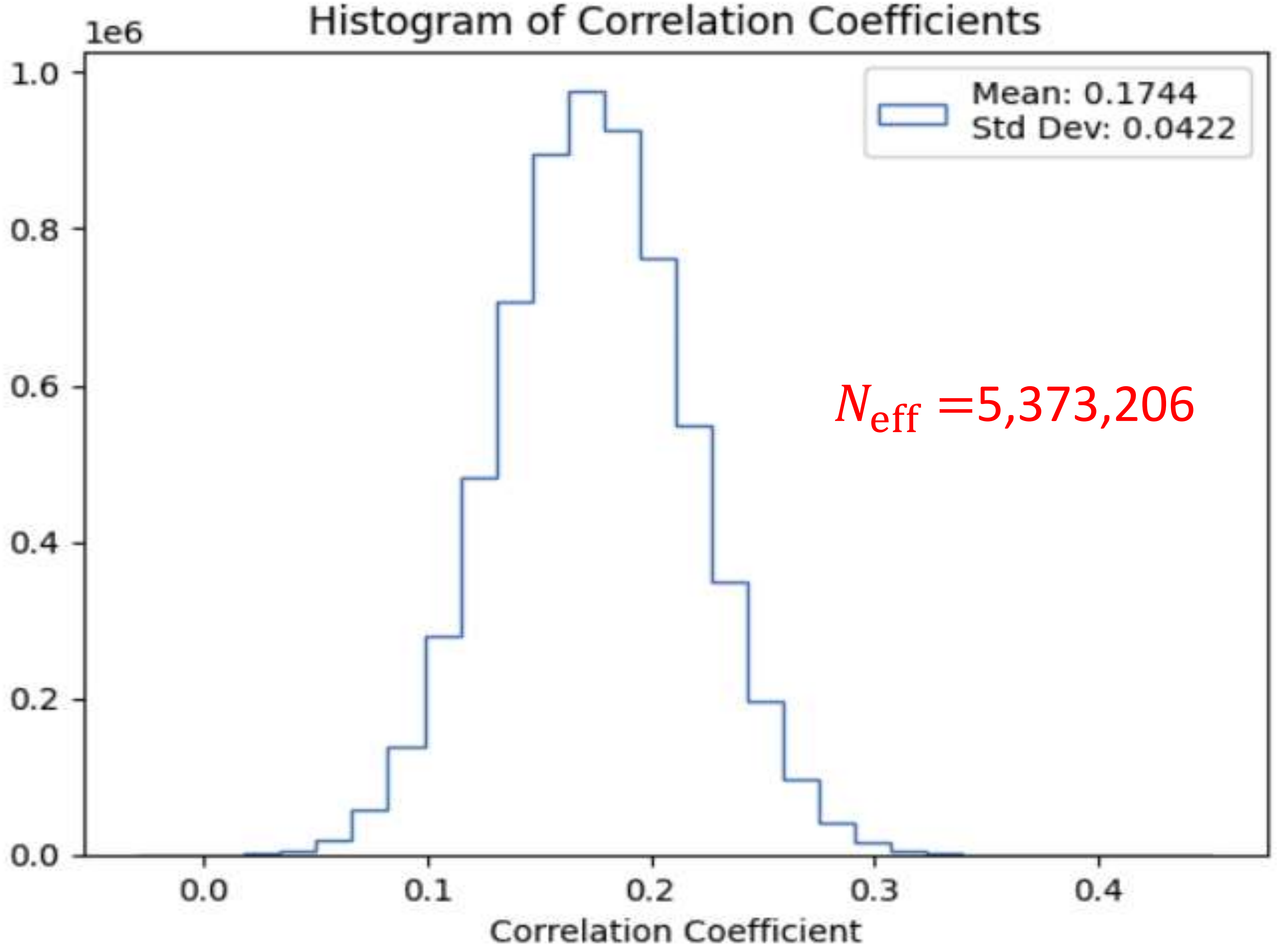
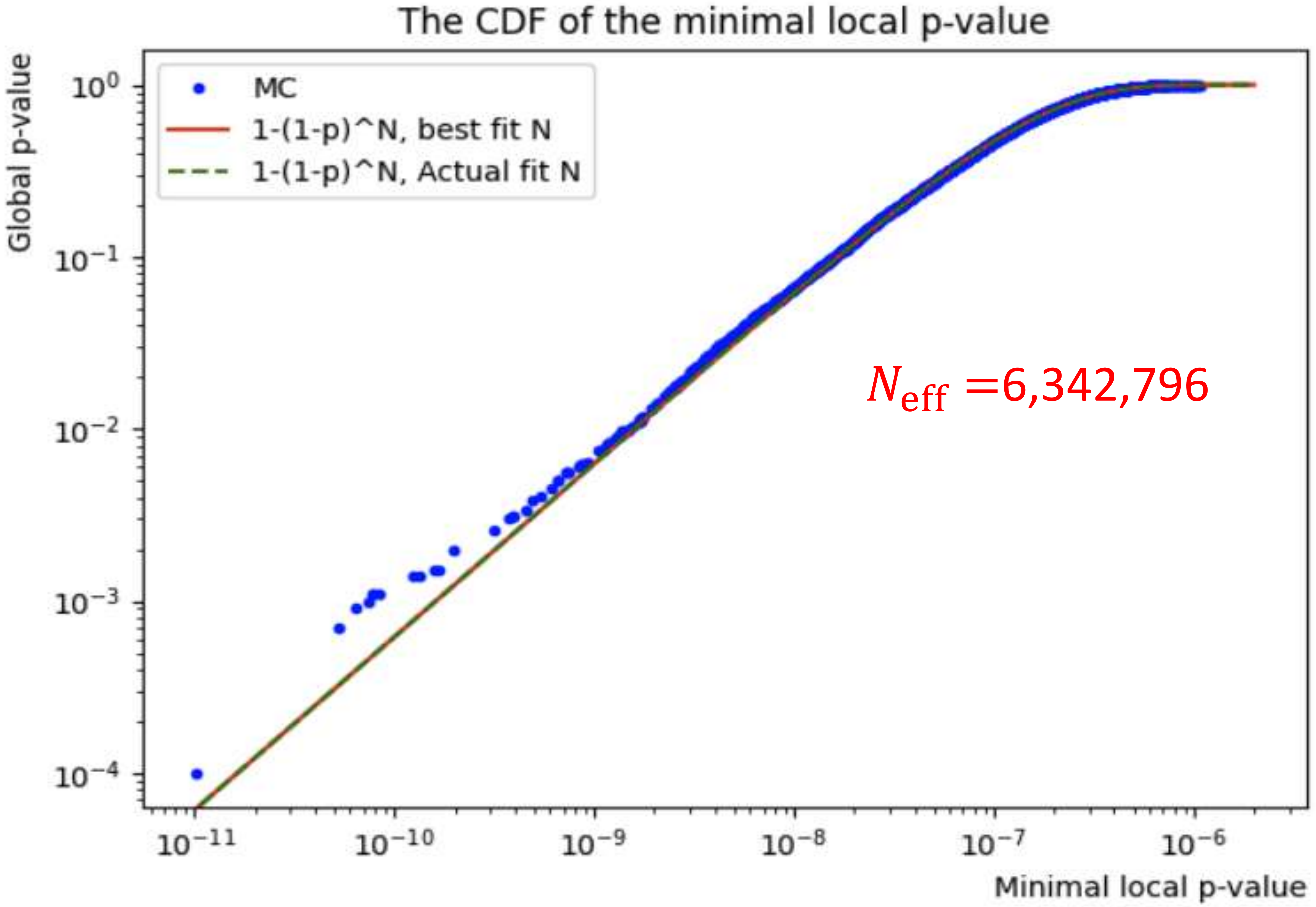
Effective number of independent frequency (N_{eff})

Global *p*-value	False Alarm Probability (FAP)
$p_G \approx 1 - (1 - p_{\text{min}\_L,\,i})^{N_{\text{eff}}}$	$P_{FA} \approx 1 - (1 - p_{L,\,i})^{N_{\text{eff}}}$



N_{eff} =5,373,206

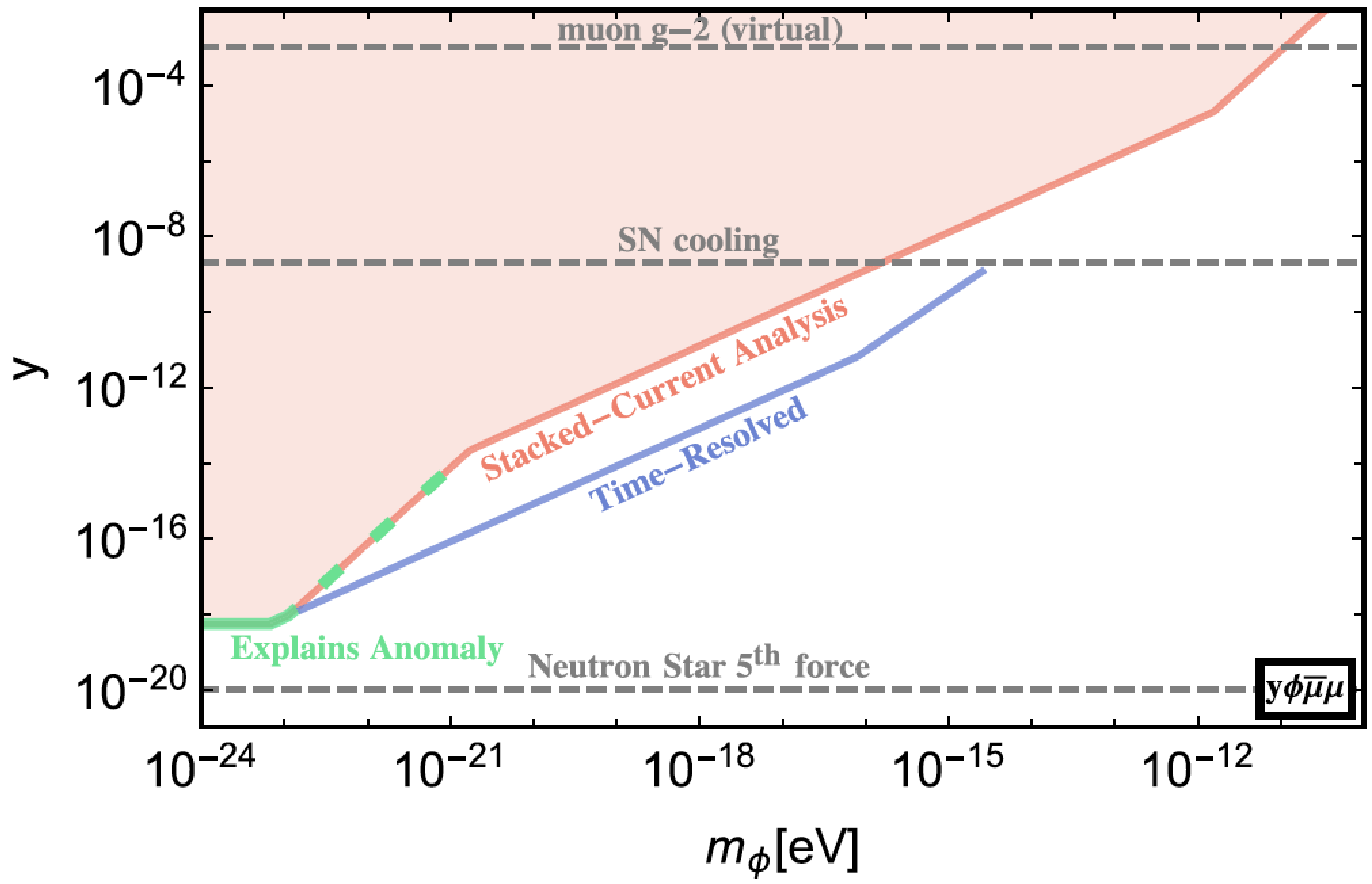


N_{eff} =6,342,796

- To solidify our confidence, we are planning to implement a bootstrapping technique for a comprehensive final assessment.

Projected sensitivity plot for Scalar DM

Scalar DM (Yukawa coupling)



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<u>Scalar DM search from Muon g-2 data will improve the projected sensitivity plot</u>

Rough timeline with future plans

- **Remaining tasks in the data analysis framework will be completed ($\sim$ Dec).**
- **Systematic study is currently working progress in parallel ($\sim$ Dec).**
- **Scalar DM search analysis for Run-2/3 Muon $g - 2$ data will be completely done ($\sim$2025 May).**
- **Pseudoscalar DM search data extraction will be launching in parallel (2025 $\sim$).**

Thank you for your attention!
Q & A

Back up slides

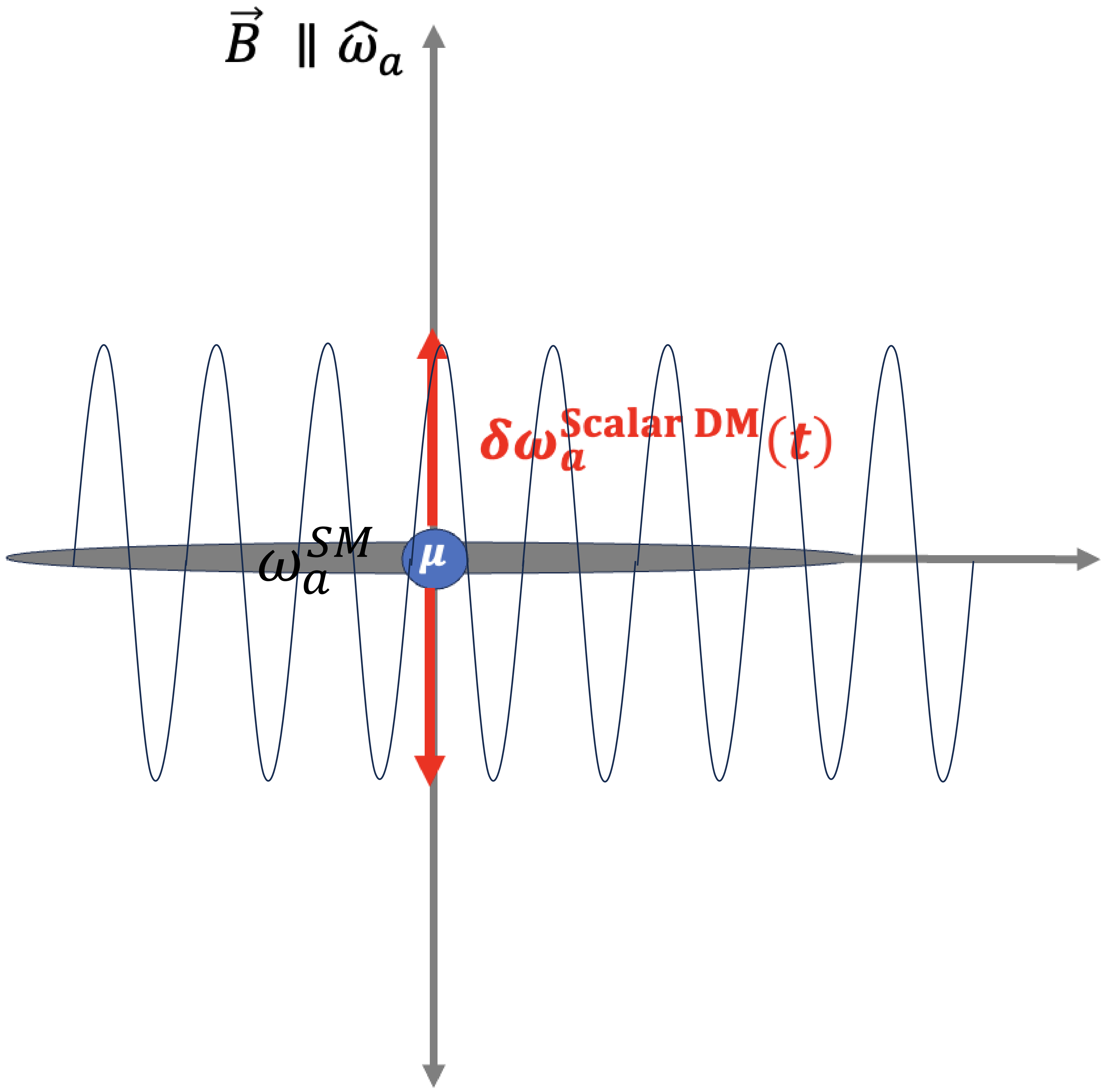
Physics signature

- Ultralight scalar field DM : $\phi(t) = \phi_0 \cos(m_\phi t)$
  - Scalar coupling $\mathcal{L} \supset y\phi\bar{\mu}\mu$
- Pseudoscalar axion-like DM

- Ultralight Scalar DM may lead to induce apparent oscillations of the m_μ in a parallel direction to $\boldsymbol{\omega_a}$:

$$\omega_a(t) = a\frac{q}{m(t)}B = \omega_a^{SM} + \delta\omega_a^{\text{scalar DM}}(t)$$

, where $m(t) \rightarrow m_0(1 + \phi'_0 \cos m_{\text{DM}} t)$



- Therefore, it causes a modulation of magnitude of ω_a at m_{DM}.
- Since DM mass is unknown, ω_a can be modulated at any frequency.
- Need to search for a wide spectrum of DM mass as much as as we can based on the estimation of DM sensitivity.

Data analysis framework for the Muon $g-2$ data

Looking at the entire parameter space

- False Alarm Probability (P_{FA}) is the probability of getting at least one PSD as extreme as or more than an observed one among all frequencies.
- Several approaches to obtain P_{FA}:

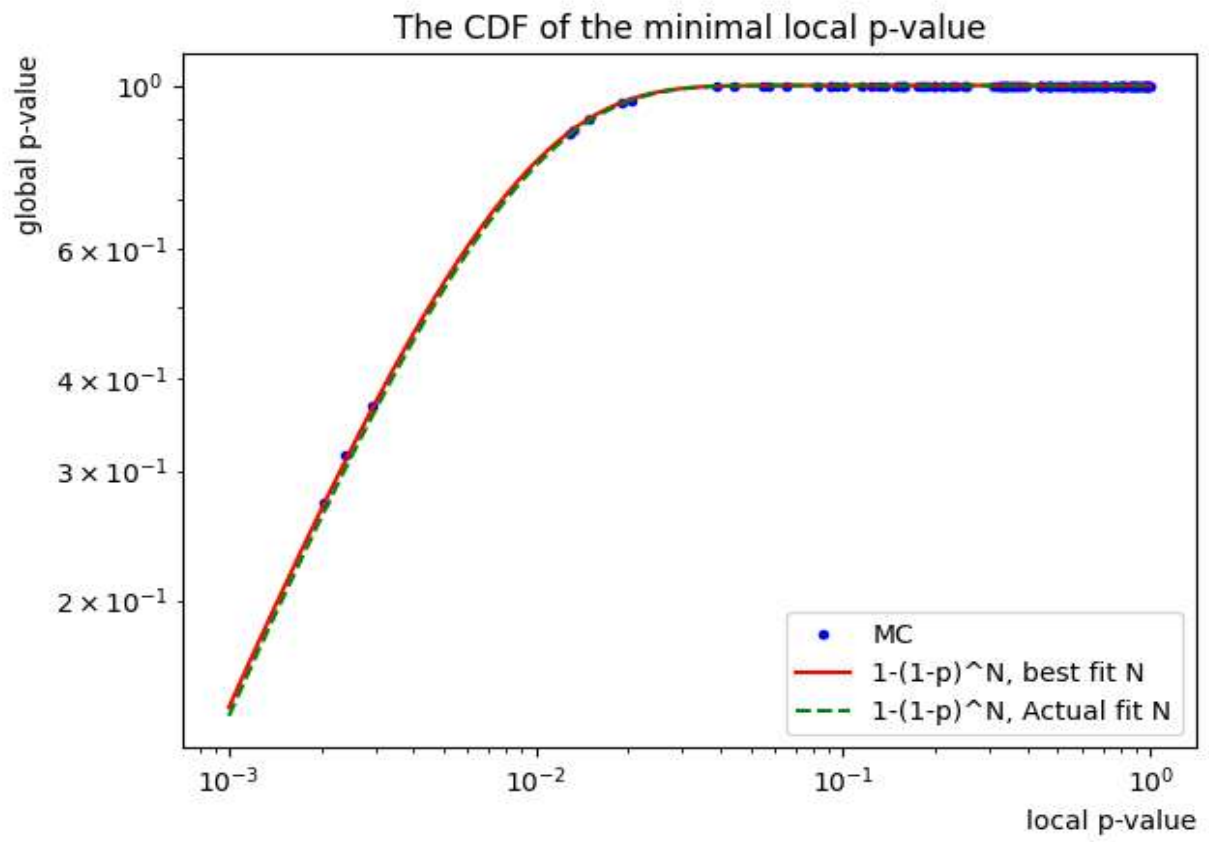
1. Independent frequency method: find out the effective number of independent frequencies (N_{eff}).

e^{-P}: local p-value (probability of getting a PSD as extreme as an observed one at a particular frequency)
$1-e^{-P}$: cumulative probability (probability of getting a PSD as common as an observed one at a particular frequency)
$[1-e^{-P}]^N$: Probability of getting at least one PSD as common as an observed one among all frequencies.
$1-[1-e^{-P}]^N$: Probability of getting at least one PSD as extreme as an observed one among all frequencies.
Note that, in the periodogram, the PSD at one frequency is closely correlated with the value at adjacent frequencies in a way that to express analytically.

$$P_{\mathrm{FA}} \approx 1-[1-e^{-P}]^{N_{\mathrm{eff}}}$$

Eventually, we will determine the global p-value (p_{G}), from the FAP

$$p_{\mathrm{G}} \approx 1-[1-e^{-P_{\mathrm{MAX}}}]^{N_{\mathrm{eff}}}$$



Looking at the entire parameter space

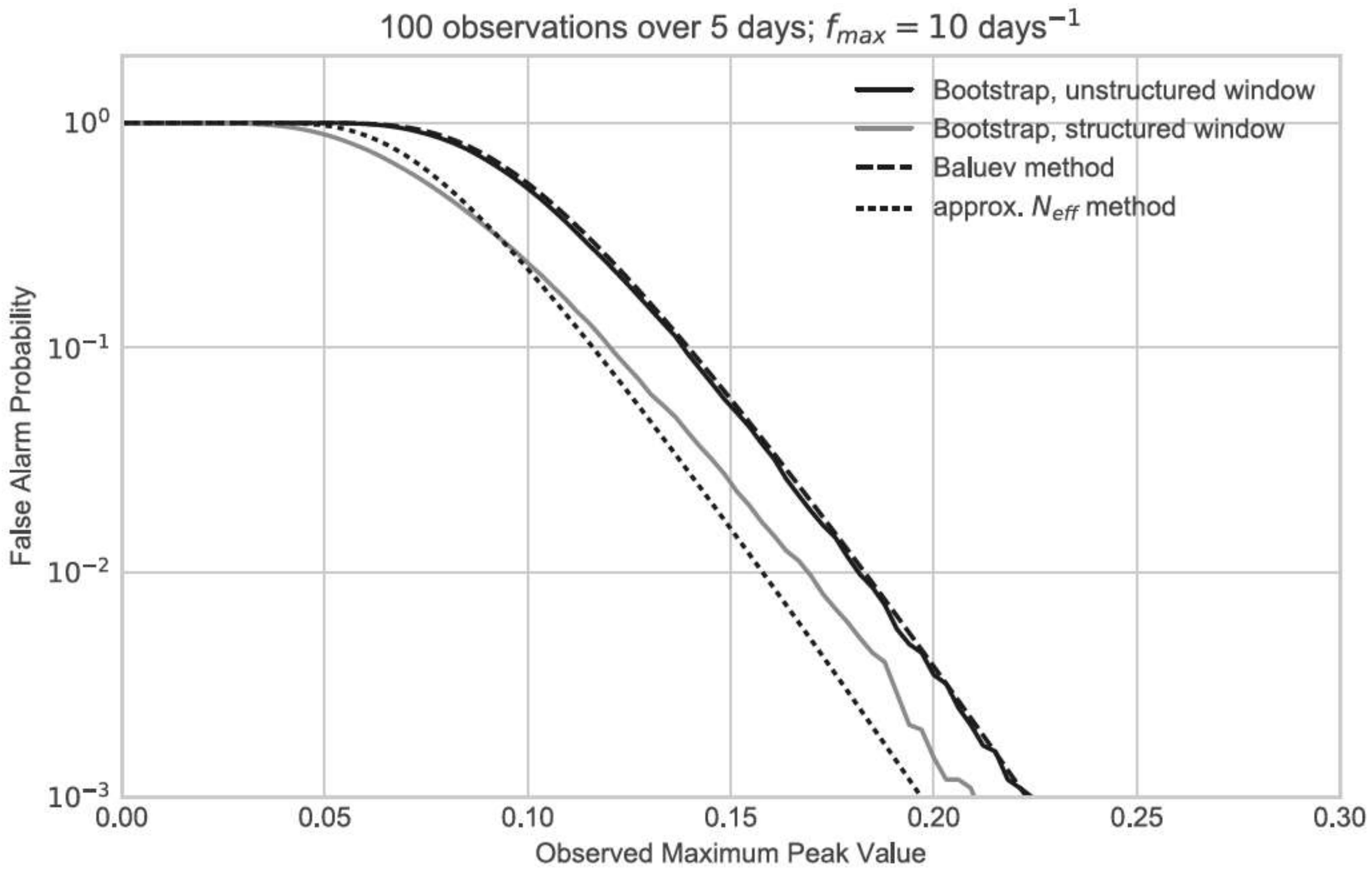
2. Baluev Method: All relevant information about correlation and mutual dependence of adjacent frequencies is contained in the window function.

$$P_{\mathrm{FA}} \approx 1 - P_{\mathrm{single}}(z)e^{-\tau(z)}$$

Where, $\tau(z) \approx W(1-z)^{(N-4)/2}\sqrt{z}$

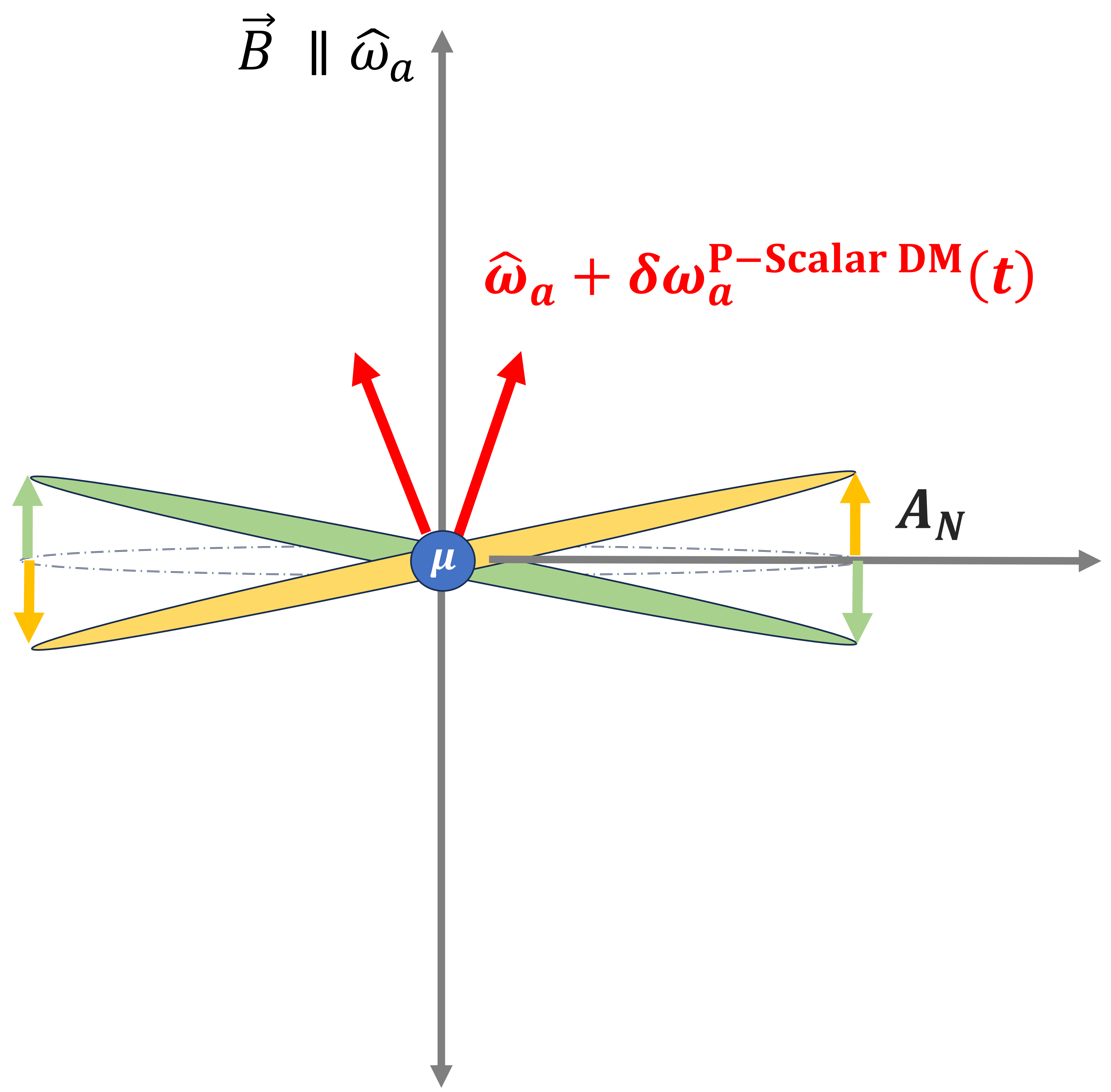
and $W = f_{\max}\sqrt{4\pi\ \mathrm{var}(t)}$

3. Bootstrap Method: the statistic is computed repeated on my random resamplings of the data in order to approximate the distribution of that statistic. This method produce most robust estimate of FAP, but computationally intensive.



Pseudoscalar DM signature (Perpendicular perturbation)

- **Ultralight Pseudoscalar DM works as an anomalous magnetic field that interacts only with the muon spin, making the muon's spin precession plane tilted and swing at DM frequency.**
- The decay positron is preferentially emitted to the spin direction, leading to the "up-down number asymmetry".
- Up-down number asymmetry refers to a difference in the number of decay positrons accepted at the crystals in a calorimeter's upper and lower parts.
- **Therefore, it causes a modulation of A_N (Amplitude of up-down number asymmetry) at m_{DM}**



Pseudoscalar DM exclusion plots

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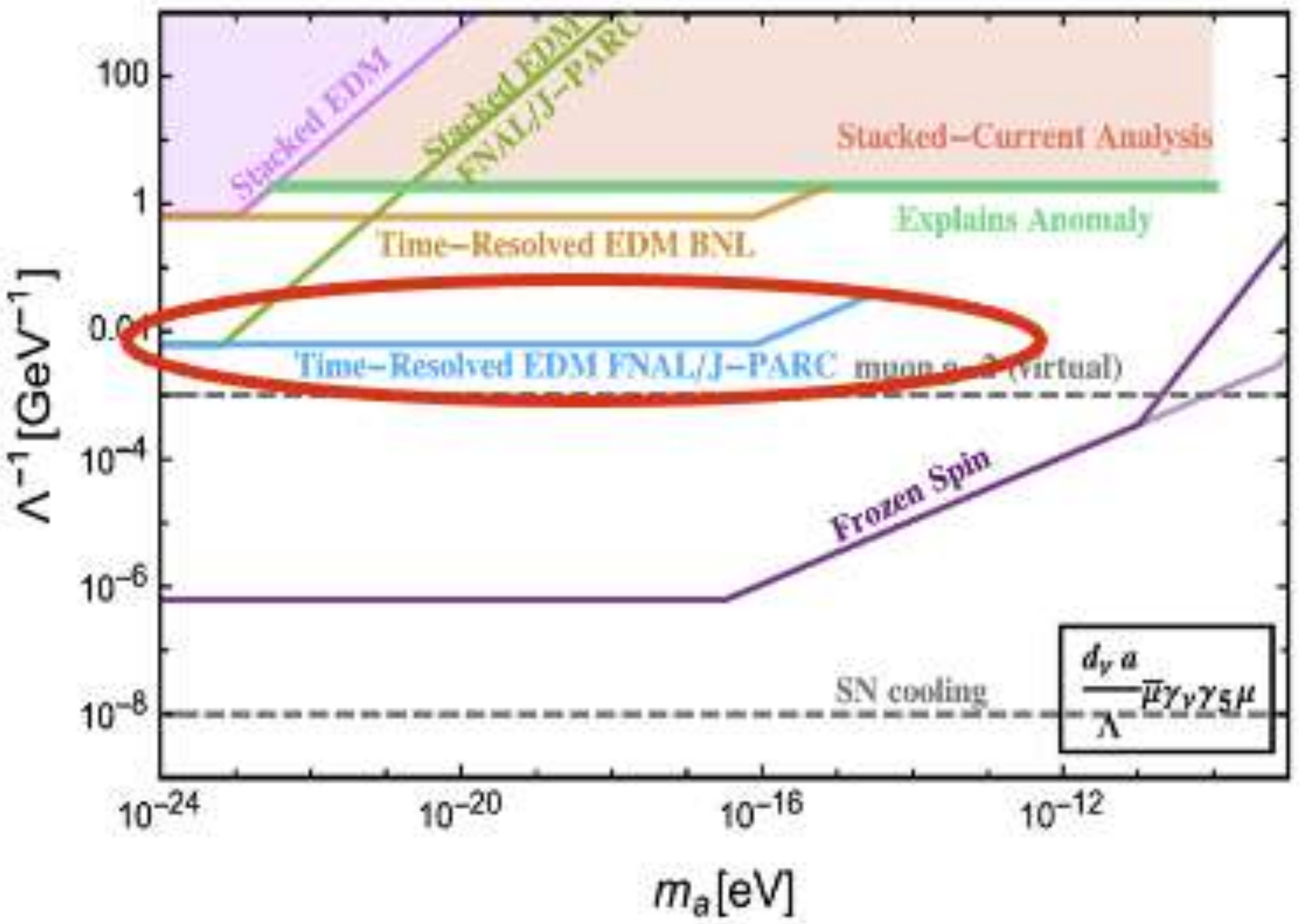
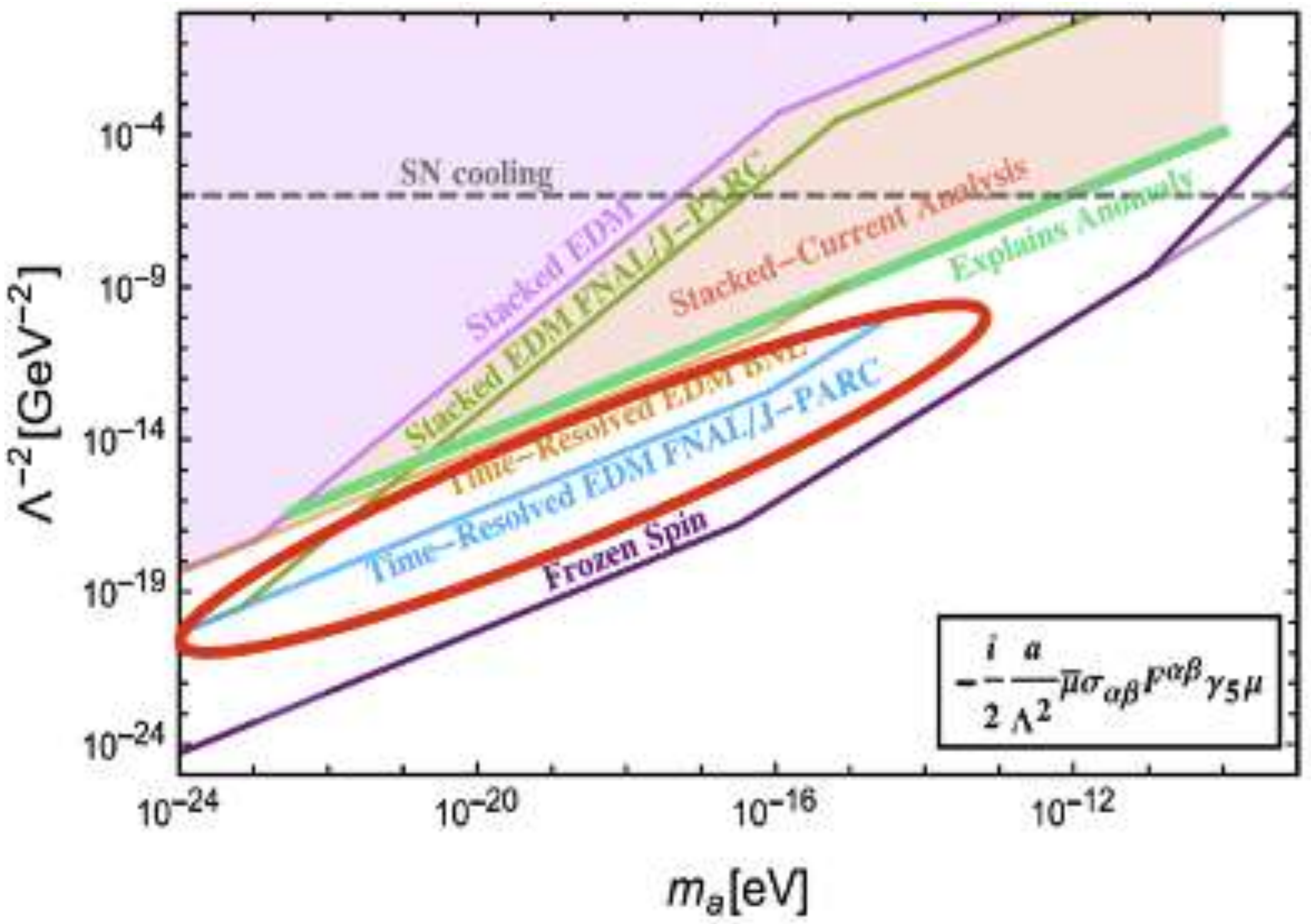


Axion-like DM (gradient)



Axion-like DM (EDM)