A plethora of long-range neutrino interactions probed by DUNE and T2HK

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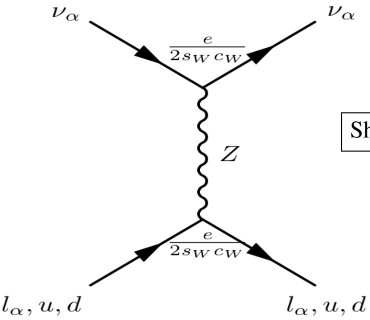
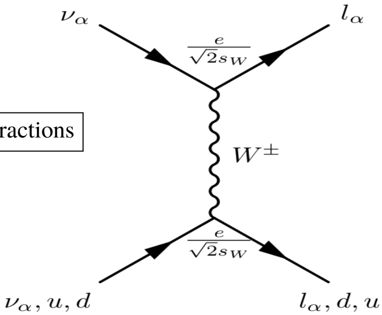
Standard neutrino-matter interactions

$$\text{Model} : SU(3)_C \times SU(2)_L \times U(1)_Y$$



Short-range interactions



$$V_{\text{NC}} = -G_F n_n / \sqrt{2}$$

$$V_{\text{CC}} = \sqrt{2} G_F n_e$$

$$\mathscr{L}_{\text{SM-NC}} = \frac{e}{2 \sin\theta_W \cos\theta_W} Z_\mu \left[-\bar{l}_\alpha \gamma^\mu P_L l_\alpha + \bar{\nu}_\alpha \gamma^\mu P_L \nu_\alpha + \bar{u} \gamma^\mu P_L u - \bar{d} \gamma^\mu P_L d \right]$$

$$\mathscr{L}_{\text{SM-CC}} = \frac{e}{\sqrt{2} \sin\theta_W} \left[W_\mu^+ \{ \bar{\nu}_\alpha \gamma^\mu P_L l_\alpha + \bar{u} \gamma^\mu P_L d \} + \text{h.c.} \right]$$

New neutrino-matter interactions from $U(1)'$ symmetries

Model : $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$

$U(1)' = U(1)_{B-L} \times U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu - L_e}$

Anomaly free charge

$$c_{\rm BL}(B-L) + c_{\mu\tau}(L_\mu - L_\tau) + c_{\mu e}(L_\mu - L_e)$$

JHEP 07 (2012) 083, JHEP 02 (2019) 082, JHEP 01 (2021) 114
arXiv: 1812.04067, SciPostPhys.6.3.038

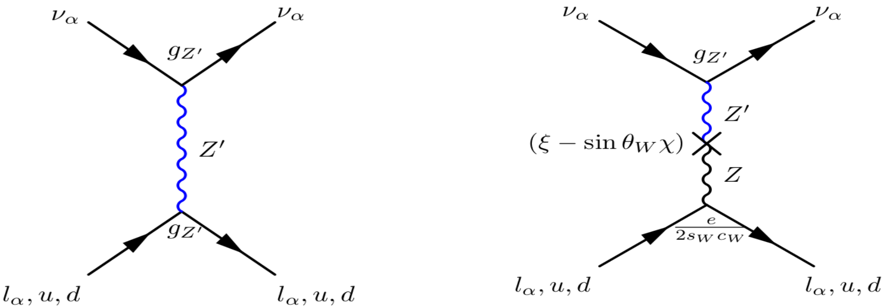
$a_u = a_d = c_{\rm BL}/3, a_e = b_e = -(c_{\rm BL} + c_{\mu e})$,
$b_\mu = -c_{\rm BL} + c_{\mu e} + c_{\mu\tau}$, and $b_\tau = -(c_{\rm BL} + c_{\mu\tau})$

Neutrino oscillation is not affected by flavor-universal symmetries — e.g. $B - L$ — hence we focus on flavor dependent ones.

$U(1)'$ symmetry	$U(1)'$ charge					
	a_u	a_d	a_e	b_e	b_μ	b_τ
$B-3L_e$	$\frac{1}{3}$	$\frac{1}{3}$	-3	-3	0	0
$L-3L_e$	0	0	-2	-2	1	1
$B-\frac{3}{2}(L_\mu+L_\tau)$	$\frac{1}{3}$	$\frac{1}{3}$	0	0	$-\frac{3}{2}$	$-\frac{3}{2}$
$L_e-\frac{1}{2}(L_\mu+L_\tau)$	0	0	1	1	$-\frac{1}{2}$	$-\frac{1}{2}$
$L_e+2L_\mu+2L_\tau$	0	0	1	1	2	2
$B_y+L_\mu+L_\tau$	$\frac{1}{3}$	$\frac{1}{3}$	0	0	1	1
$B-3L_\mu$	$\frac{1}{3}$	$\frac{1}{3}$	0	0	-3	0
$L-3L_\mu$	0	0	1	1	-2	1
$B-3L_\tau$	$\frac{1}{3}$	$\frac{1}{3}$	0	0	0	-3
$L-3L_\tau$	0	0	1	1	1	-2
L_e-L_μ	0	0	1	1	-1	0
L_e-L_τ	0	0	1	1	0	-1
$L_\mu-L_\tau$	0	0	0	0	1	-1
$B-L_e-2L_\tau$	$\frac{1}{3}$	$\frac{1}{3}$	-1	-1	0	-2

New neutrino-matter interactions from $U(1)'$ symmetries

$$\text{Model} : SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)' \rightarrow Z'$$



$$\mathscr{L}_{Z'} = -g_{Z'} \left(a_u\, \bar{u}\gamma^\alpha u + a_d\, \bar{d}\gamma^\alpha d + a_e\, \bar{e}\gamma^\alpha e + b_e\, \bar{\nu}_e\gamma^\alpha P_L \nu_e + b_\mu\, \bar{\nu}_\mu\gamma^\alpha P_L \nu_\mu + b_\tau\, \bar{\nu}_\tau\gamma^\alpha P_L \nu_\tau \right) Z'_\alpha$$

$$\mathscr{L}_{ZZ'} = -g_{Z'} \frac{e}{\sin\theta_W \cos\theta_W} (\xi - \sin\theta_W \chi) J'_\sigma J_3^\sigma$$

where, $J'_\sigma = \bar{\nu}_\mu \gamma_\sigma P_L \nu_\mu - \bar{\nu}_\tau \gamma_\rho P_L \nu_\tau$ and $J_3^\rho = -\frac{1}{2}\bar{e}\gamma^\sigma P_L e + \frac{1}{2}\bar{u}\gamma^\rho P_L u - \frac{1}{2}\bar{d}\gamma^\rho P_L d$

Long-range interaction (LRI): ultralight Z'

$$N_{e,☾} = N_{p,☾} = N_{n,☾}, N_{e,\oplus} = N_{p,\oplus} = N_{n,\oplus}$$
$$N_{e,\odot} = N_{p,\odot}, N_{n,\odot} = N_{e,\odot}/4$$
$$N_{e,\mathrm{MW}} = N_{p,\mathrm{MW}} \approx N_{n,\mathrm{MW}}$$
$$N_{e,\cos} = N_{p,\cos}, N_{n,\cos} = N_{e,\cos}/7$$

- Range of interaction $\sim 1/m_{Z'}$
- For ultralight Z' ($m_{Z'} = 10^{-35} - 10^{-10}$ eV)

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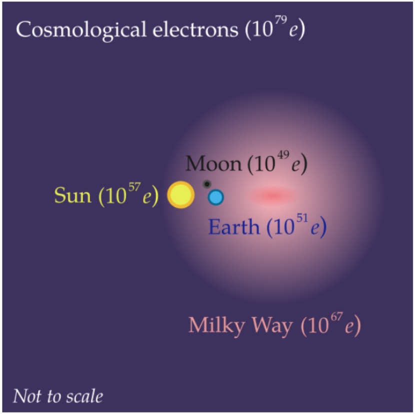
Range $\sim$ km – Gpc

Neutrinos on Earth feel the potential due to the matter inside the Moon, Sun, Milky Way, and the cosmological distribution via long-range interactions.

$$\boxed{V_f = V_f^{\oplus} + V_f^{☾} + V_f^{\odot} + V_f^{\mathrm{MW}} + V_f^{\cos}}$$



PRL 122, 061103 (2019)

Long-range interaction potential

$$\mathbf{V}_{\rm LRI} = {\rm diag}(V_{{\rm LRI},e}, V_{{\rm LRI},\mu}, V_{{\rm LRI},\tau})$$

Texture of $\mathbf{V}_{\rm LRI}$	$U(1)'$ symmetry	Texture to place limits, $\mathbf{V}_{\rm LRI} = V_{\rm LRI} \cdot {\rm diag}(\ldots)$
$\begin{pmatrix} \bullet & & \\ & 0 & \\ & & 0 \end{pmatrix}$	$B - 3L_e$	${\rm diag}(1,0,0)$
	$L - 3L_e$	${\rm diag}(1,0,0)$
	$B - \frac{3}{2}(L_\mu + L_\tau)$	${\rm diag}(1,0,0)$
	$L_e - \frac{1}{2}(L_\mu + L_\tau)$	${\rm diag}(1,0,0)$
	$L_e + 2L_\mu + 2L_\tau$	${\rm diag}(-1,0,0)$
	$B_y + L_\mu + L_\tau$	${\rm diag}(-1,0,0)$
$\begin{pmatrix} 0 & & \\ & \bullet & \\ & & 0 \end{pmatrix}$	$B - 3L_\mu$	${\rm diag}(0,-1,0)$
	$L - 3L_\mu$	${\rm diag}(0,-1,0)$
$\begin{pmatrix} 0 & & \\ & 0 & \\ & & \bullet \end{pmatrix}$	$B - 3L_\tau$	${\rm diag}(0,0,-1)$
	$L - 3L_\tau$	${\rm diag}(0,0,-1)$
$\begin{pmatrix} \bullet & & \\ & \bullet & \\ & & 0 \end{pmatrix}$	$L_e - L_\mu$	${\rm diag}(1,-1,0)$
$\begin{pmatrix} \bullet & & \\ & 0 & \\ & & \bullet \end{pmatrix}$	$L_e - L_\tau$	${\rm diag}(1,0,-1)$
$\begin{pmatrix} 0 & & \\ & \bullet & \\ & & \bullet \end{pmatrix}$	$L_\mu - L_\tau$	${\rm diag}(0,1,-1)$
	$B - L_e - 2L_\tau$	${\rm diag}(0,1,-1)$

- Yukawa potential, mediated by Z':

$$V_{Z',f} = G'^2 \frac{N_f}{4\pi d} e^{-m_{Z'} d},$$

$f\ (= e, p \text{ or } n)$, $\forall$ symmetries but $L_\mu - L_\tau$

- Yukawa potential, caused by $Z - Z'$ mixing:

$$V_{ZZ',n} = G'^2 \frac{e}{\sin\theta_W \cos\theta_W} \frac{N_n}{4\pi d} e^{-m_{Z'} d}$$

$$G' = \begin{cases} g_{Z'} & , \nu \text{ int. via } Z' \\ \sqrt{g_{Z'}(\xi - \sin\theta_W \chi)} & , \nu \text{ int. via } Z - Z' \text{ mixing} \end{cases}$$

- General potential regardless of the source:

$$V_{{\rm LRI},\alpha}(m_{Z'}, G') = b_\alpha \sum_{f=e,p,n} \kappa_f V_f(m_{Z'}, G')$$

- Total LRI potential, $V_{{\rm LRI},\alpha}$

$$= b_\alpha \left[\left(\kappa_e + \kappa_p \frac{N_{p,\oplus}}{N_{e,\oplus}} + \kappa_n \frac{N_{n,\oplus}}{N_{e,\oplus}} \right) V_e^{\oplus} + (\oplus \to ☾) + (\oplus \to \odot) + (\oplus \to {\rm MW}) + (\oplus \to {\rm cos}) \right].$$

The potential due to protons and neutrons are weighed relative to that of electrons.

Impact of Long-Range Interaction on Neutrino Oscillation

$$\mathbf{H} = \mathbf{H}_{\rm vac} + \mathbf{V}_{\rm mat} + \mathbf{V}_{\rm LRI}$$

$$\mathbf{H}_{\rm vac} = \frac{1}{2E}\mathbf{U}\,\mathrm{diag}(0, \Delta m^2_{21}, \Delta m^2_{31})\,\mathbf{U}^\dagger$$

$$\mathbf{V}_{\rm mat} = \mathrm{diag}(V_{\rm CC}, 0, 0),$$

$$V_{\rm CC} \approx 7.6 \cdot Y_e \cdot 10^{-14}\left(\frac{\rho_{\rm avg}}{\rm g\ cm^{-3}}\right)\,\mathrm{eV},$$

$Y_e \equiv n_e/(n_p + n_n)$, and $\rho_{\rm avg}$ is 2.848 g $\rm cm^{-3}$ for DUNE, 2.8 g $\rm cm^{-3}$ for T2HK.

$$\mathbf{V}_{\rm LRI} = \mathrm{diag}(V_{{\rm LRI},e}, V_{{\rm LRI},\mu}, V_{{\rm LRI},\tau})$$

$$\text{Oscillation probability}:\ P_{\nu_\alpha\to\nu_\beta} = \left|\sum_{i=1}^{3}\tilde{U}_{\alpha i}\exp\left[-\frac{\Delta\tilde{m}^2_{i1}L}{2E}\right]\tilde{U}^*_{\beta i}\right|^2$$

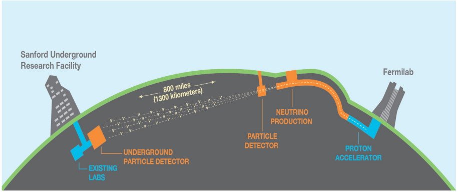
Under the approximation that $\tilde{\theta}_{12}$ saturates very quickly to 90°,

$$P_{\nu_\mu\to\nu_e} \approx \sin^2\tilde{\theta}_{23}\sin^2(2\tilde{\theta}_{13})\sin^2\left[1.27\frac{(\Delta\tilde{m}^2_{32}/\mathrm{eV}^2)(L/\mathrm{km})}{E/\mathrm{GeV}}\right]$$

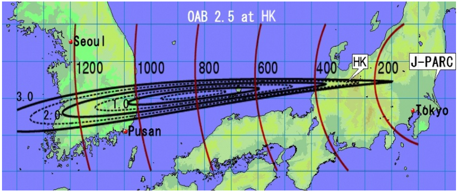
$$P_{\nu_\mu\to\nu_\mu} \approx 1 - \sin^2(2\tilde{\theta}_{23})\cos^2\tilde{\theta}_{13}\sin^2\left[1.27\frac{(\Delta\tilde{m}^2_{31}/\mathrm{eV}^2)(L/\mathrm{km})}{E/\mathrm{GeV}}\right]$$

Running param.

Details of experimental setup



DUNE



T2HK

	DUNE [1]	T2HK [2]
Detector mass	40 kt LArTPC	187 kt WC
Baseline	1285 km	295 km
Proton energy	120 GeV	80 GeV
Proton beam power	1.2 MW	1.3 MW
P.O.T./year	1.1×10^{21}	2.7×10^{21}
ν Beam type	Wide-band, on-axis	Narrow-band, off-axis (2.5°)
Run time ($\nu + \bar{\nu}$)	5 yrs + 5 yrs	2.5 yrs + 7.5 yrs

[1] arXiv: 2103.04797
[2] arXiv: 1611.06118

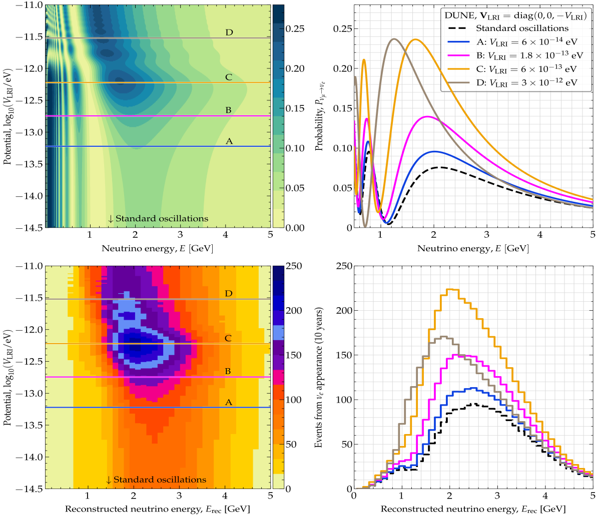
Oscillation parameters values

Parameter	Best-fit value	3σ range	Statistical treatment
θ_{12} [°]	33.45	31.27–35.87	Fixed to best fit
θ_{13} [°]	8.62 (8.61)	8.25–8.98 (8.24–9.02)	Fixed to best fit
θ_{23} [°]	42.1 (49.0)	39.7–50.9 (39.8–51.6)	Minimized over 3σ range
$\delta_{\rm CP}$ [°]	230 (278)	144–350 (194–345)	Minimized over 3σ range
$\frac{\Delta m^2_{21}}{10^{-5}\ {\rm eV}^2}$	7.42	6.82–8.04	Fixed to best fit
$\frac{\Delta m^2_{31}}{10^{-3}\ {\rm eV}^2}$	2.51 (-2.41)	2.430–2.593 (-2.506–(-2.329))	Minimized over 3σ range

Table: Values of oscillation parameters used in our analysis for normal mass ordering, NMO (inverted mass ordering, IMO).

http://www.nu-fit.org
arXiv: 2007.14792

Impact on oscillation probability and event rates

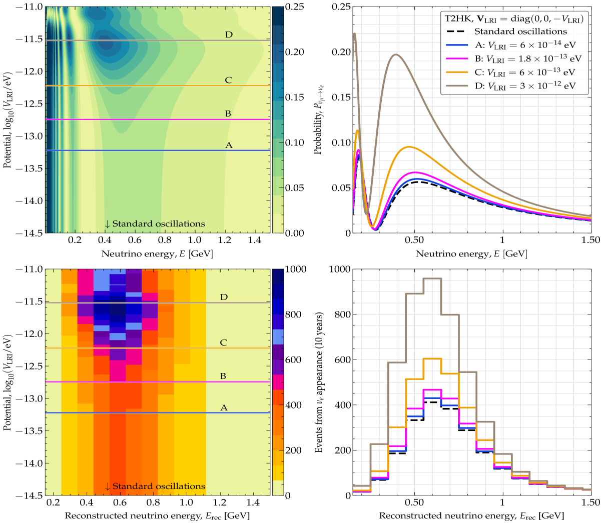


$\theta_{13} - resonace$

$$\mathbf{V}_{\mathrm{LRI}} \approx \mathbf{H}_{\mathrm{vac}} + \mathbf{V}_{\mathrm{mat}} \rightarrow \text{resonant oscillations}$$

DUNE sensitivity $\rightarrow V_{\mathrm{LRI}} \approx (\mathbf{H}_{\mathrm{vac}})_{\tau\tau} \in [3.8 \cdot 10^{-14}, 1.4 \cdot 10^{-12}]$ eV, for energy $\in [0.5, 18]$ GeV

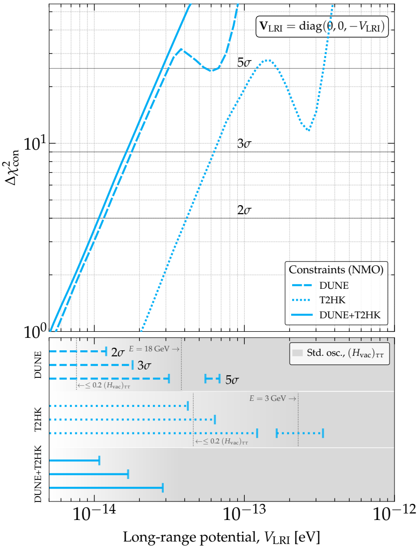
Impact on oscillation probability and event rates



$$\mathbf{V}_{\rm LRI} \approx \mathbf{H}_{\rm vac} + \mathbf{V}_{\rm mat} \to \text{resonant oscillations}$$

T2HK sensitivity $\to V_{\rm LRI} \approx (\mathbf{H}_{\rm vac})_{\tau\tau} \in [2.3 \cdot 10^{-13}, 6.8 \cdot 10^{-12}]$ eV, for energy $\in [0.1, 3]$ GeV

Results: Constraints on LRI potential

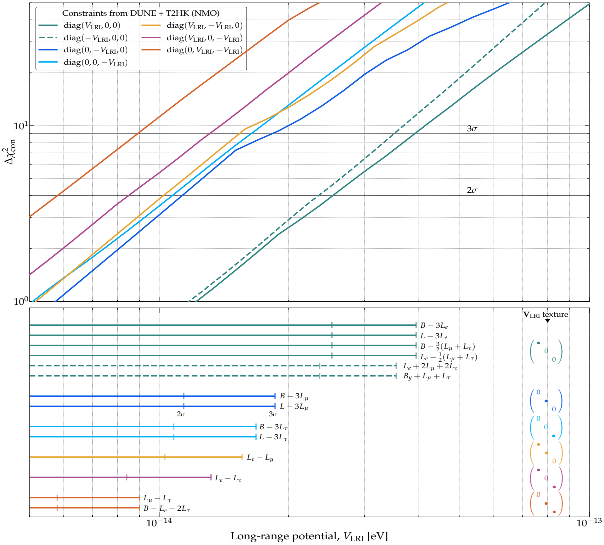


$$\Delta\chi^2_{\rm con}(V_{\rm LRI}) = \min_{\{\boldsymbol{\theta}, o\}} \left[\chi^2_{\rm test}(V_{\rm LRI} \neq 0) - \chi^2_{\rm true}(V_{\rm LRI} = 0)\right]$$

$$\boldsymbol{\theta} \to \theta_{23}\,,\, \delta_{\rm CP}\,,\, |\Delta m^2_{31}| \;\&\; o \to {\rm sign}(\Delta m^2_{31})$$

- The dips in the test statistic are due to the degeneracies between $V_{\rm LRI}$, θ_{23}, & $\delta_{\rm CP}$ in DUNE and between $V_{\rm LRI}$ & sign(Δm^2_{31}) in T2HK.
- Combining DUNE and T2HK lifts the degeneracies improving the bounds.
- The limits lie around 20% of $(H_{\rm vac})_{\tau\tau}$.

Results: Ultimate constraints on the LRI potential



- The symmetries which mostly affect $\mu - \tau$ sector are tightly constrained.

  $\Downarrow$

  Primary contribution from $\nu_\mu \to \nu_\mu$ & $\bar{\nu}_\mu \to \bar{\nu}_\mu$ (Higher event rates)

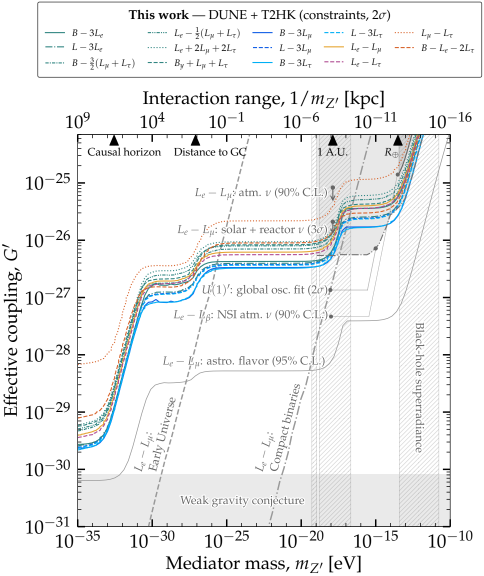
- The symmetries which mostly affect electron sector are weakly constrained.

  $\Downarrow$

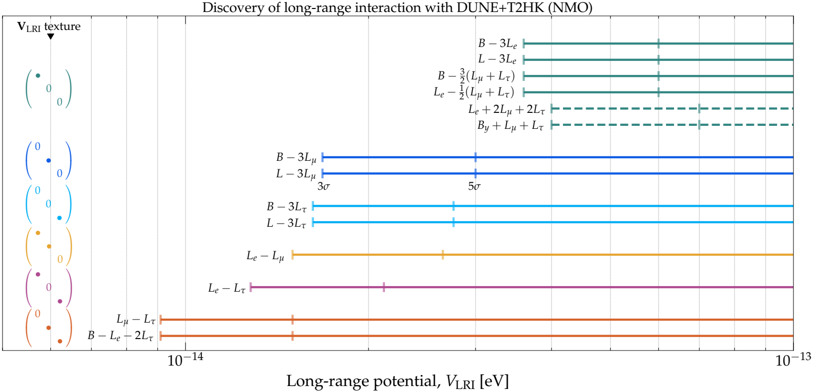
  Primary contribution from $\nu_\mu \to \nu_e$ & $\bar{\nu}_\mu \to \bar{\nu}_e$ (Lower event rates)

- DUNE and T2HK may constrain the new matter potential to a level comparable to the standard-oscillation terms, roughly 10^{-14}–10^{-13} eV, regardless of what is the $U(1)'$ symmetry responsible for inducing the new interaction.

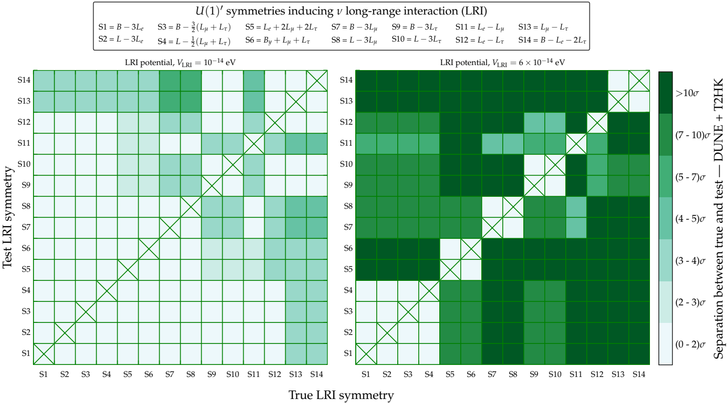
Results: Constraints on G' vs. $m_{Z'}$ plane



Results: Discovery strength of long-range potential



$$\Delta\chi^2_{\rm disc}(V_{\rm LRI}) = \min_{\{\boldsymbol{\theta},o\}}\left[\chi^2_{\rm test}(V_{\rm LRI}=0) - \chi^2_{\rm true}(V_{\rm LRI}\neq 0)\right]$$



- The separation is clearer between symmetries whose matter potential matrices, $\mathbf{V}_{\rm LRI}$, have different textures.
- The separation is blurred between symmetries whose matter potentials have similar texture, *e.g.*, between $B - 3L_e$ and $L_e + 2L_\mu + 2L_\tau$.
- Separation is null between symmetries whose matter potentials have equal texture, *e.g.*, between $B - 3L_e$ and $L - 3L_e$.
- $B - L_e - 2L_\tau$ and $L_\mu - L_\tau$ are easily separable from others.

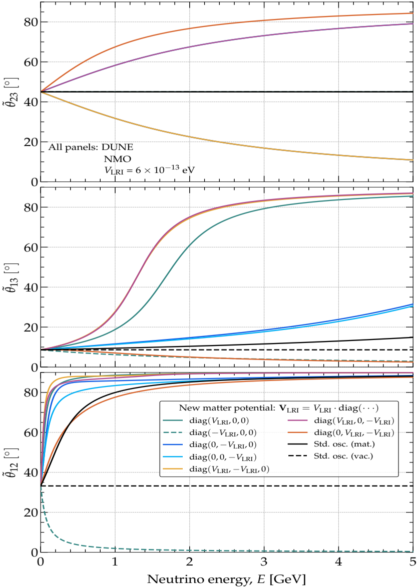
Summary

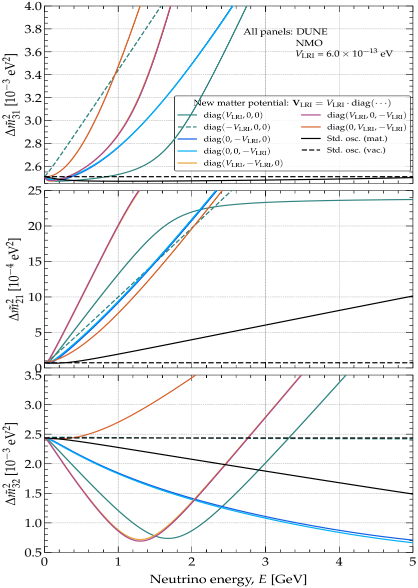
- The high precision detectors and well characterized beams in the long-baseline experiments like DUNE and T2HK allow them to identify even a tiny deviation from the Standard Model.
- For the first time, we explore the sensitivity of DUNE and T2HK to a wide variety of symmetries, built from combinations of lepton and baryon numbers, each of which induces new flavor-dependent neutrino-matter interactions that affect oscillations differently.
- We interpret their sensitivity in the context of long-range neutrino interactions, mediated by a new neutral boson lighter than 10^{-10} eV, and sourced by the vast amount of nearby and distant matter in the Earth, Moon, Sun, Milky Way, and beyond.
- We find ample sensitivity: for all symmetries, DUNE and T2HK may constrain the existence of the new interaction even if it is supremely feeble, may discover it, and, in some cases, may identify the symmetry responsible for it.

Thank you for your attention!

Backup Slides

Running of oscillation parameters in presence of LRI

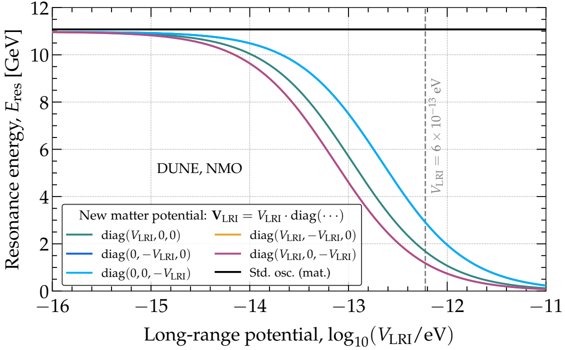




Back Illu. sym

Resonance energy in presence of LRI

$\tilde{\theta}_{13}$ becomes 45° at resonance energy



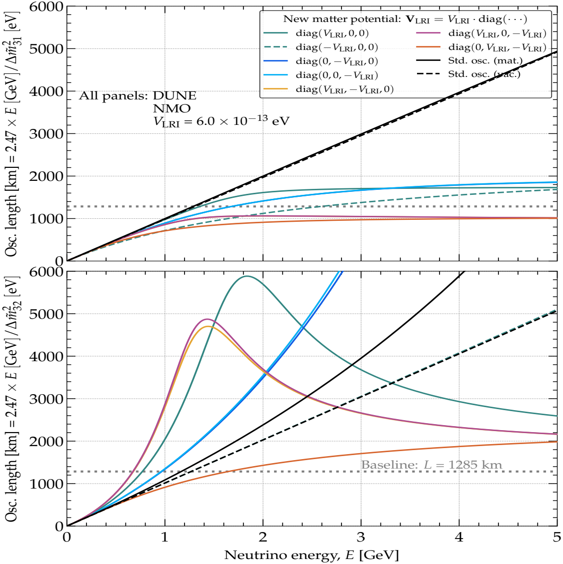
Under one-mass-scale-dominance (OMSD, $\Delta m^2_{31} L/4E >> \Delta m^2_{21} L/4E$) approximation,

$$E_{\rm res}^{\rm LRI} \simeq \left[E_{\rm res}^{\rm SI}\right]_{\rm OMSD} \cdot V_{\rm CC} \cdot \left[\frac{1 - (\alpha s_{12}^2 c_{13}^2 / \cos 2\theta_{13})}{V_{\rm CC} - \frac{1}{2}(V_{{\rm LRI},\mu} + V_{{\rm LRI},\tau} - 2V_{{\rm LRI},e})}\right]$$

$$\left[E_{\rm res}^{\rm SI}\right]_{\rm OMSD} = \frac{\Delta m^2_{31} \cos 2\theta_{13}}{2V_{CC}}$$

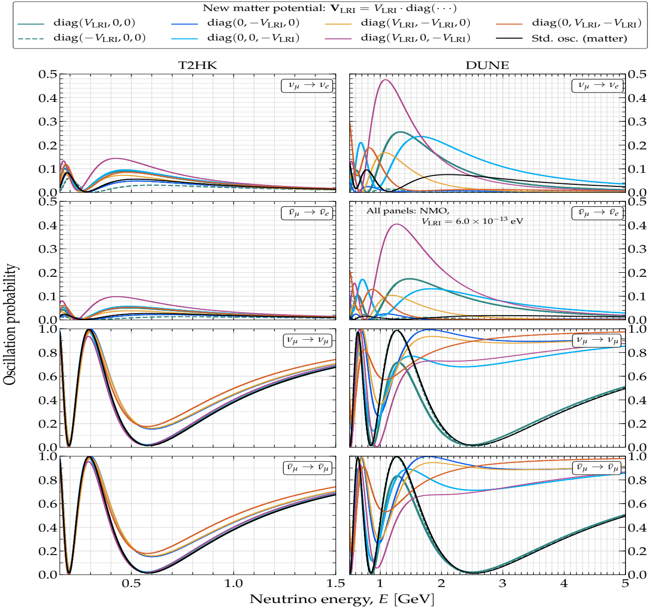
Illu. sym

Running of oscillation length in presence of LRI



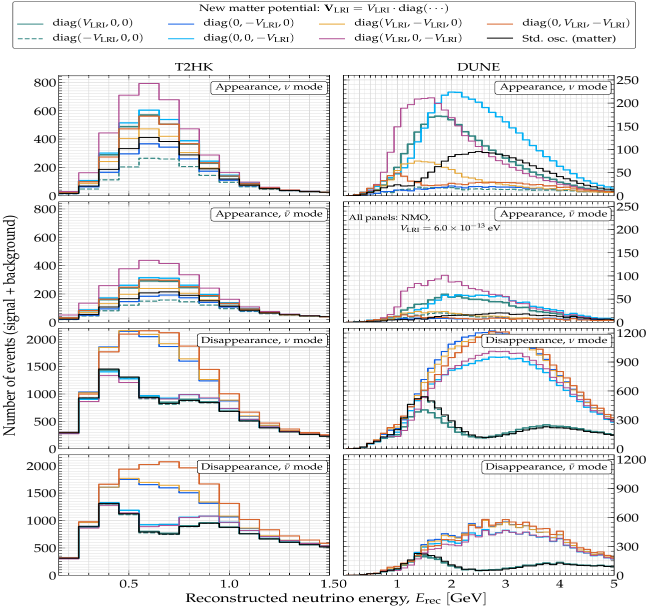
Illu. sym

Neutrino oscillation probabilities in presence of LRI



Illu. sym

Event rates in presence of LRI



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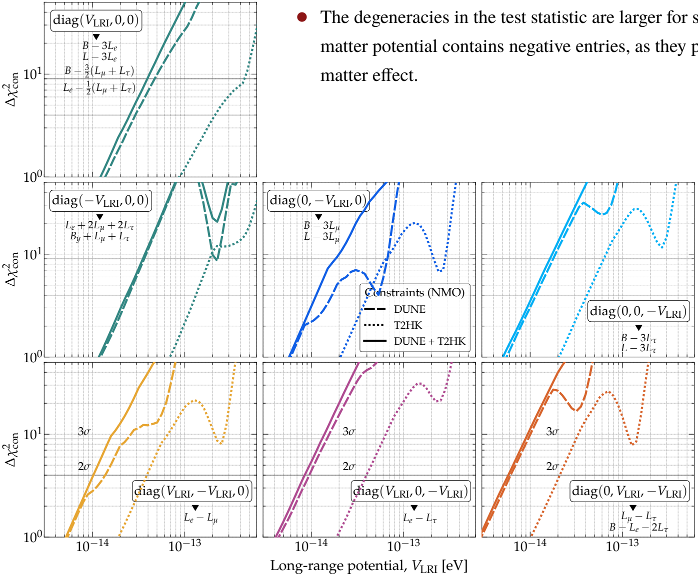
Results: Constraints on LRI potential



- The degeneracies in the test statistic are larger for symmetries whose matter potential contains negative entries, as they partially cancels the matter effect.

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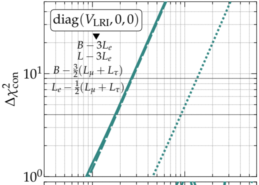
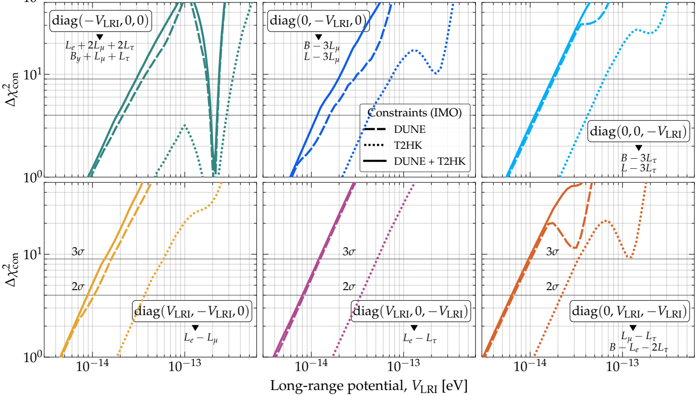


- The effect of degeneracies in the test statistic are milder compared to NMO, due to balanced ν-$\bar{\nu}$ contribution in case of IMO lifting $\theta_{23}-\delta_{\rm CP}$ degeneracy.
- The symmetries with texture diag($-V_{\rm LRI}$, 0, 0) being the exception suffer deeper dip in the test statistic in case of IMO compared to NMO.



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Statistical method: Poisson χ^2

$$\chi^2_{e,c}(V_{\rm LRI}, \boldsymbol{\theta}, o) = \min_{\{\xi_s, \{\xi_{b,c,k}\}\}} \Bigg\{ 2 \sum_{i=1}^{N_e} \Big[N^{\rm test}_{e,c,i}(V_{\rm LRI}, \boldsymbol{\theta}, o, \xi_s, \{\xi_{b,c,k}\}) - N^{\rm true}_{e,c,i} \left(1 + \ln \frac{N^{\rm test}_{e,c,i}(V_{\rm LRI}, \boldsymbol{\theta}, o, \xi_s, \{\xi_{b,c,k}\})}{N^{\rm true}_{e,c,i}} \right) \Big] + \xi_s^2 + \sum_k \xi^2_{b,c,k} \Bigg\}$$

$e = \{\text{T2HK}, \text{DUNE}\}$, $c = \{\text{app}\ \nu, \text{app}\ \bar{\nu}, \text{disapp}\ \nu, \text{disapp}\ \bar{\nu}\}$

$\boldsymbol{\theta} \to \theta_{23}\,, \delta_{\rm CP}\,, |\Delta m^2_{31}|$ & $o \to {\rm sign}(\Delta m^2_{31})$

ξ_s and $\xi_{b,c,k}$ $\to$ systematic uncertainties on the signal and the k-th background contribution to the detection channel c

$$N^{\rm true}_{e,c,i} = N^{s,\rm true}_{e,c,i} + N^{b,\rm true}_{e,c,i}\,,$$

$$N^{\rm test}_{e,c,i}(V_{\rm LRI}, \boldsymbol{\theta}, o, \xi_s, \{\xi_{b,c,k}\}) = N^s_{e,c,i}(V_{\rm LRI}, \boldsymbol{\theta}, o)(1+\pi^s_{e,c}\xi_s) + \sum_k N^b_{e,c,k,i}(\boldsymbol{\theta}, o) \left(1 + \pi^b_{e,c,k}\xi_{b,c,k}\right)$$

$\pi^s_{e,c}$ and $\pi^b_{e,c,k}$ are normalization errors

Expts.	Normalization errors [%]							
	Signal (π^s)				Background (π^b)			
	ν App.	$\bar{\nu}$ App.	ν Disapp.	$\bar{\nu}$ Disapp.	$\nu_e, \bar{\nu}_e$ CC	$\nu_\mu, \bar{\nu}_\mu$ CC	$\nu_\tau, \bar{\nu}_\tau$ CC	NC
DUNE	2	2	5	5	5	5	20	10
T2HK	5	5	3.5	3.5	10	10	–	10

$$\mathscr{L}_{ZZ'} = -\frac{1}{2}\sin\chi \hat{Z}'_{\mu\nu}\hat{B}^{\mu\nu} + \delta\hat{M}^2 \hat{Z}'_{\mu}\hat{Z}^{\mu}$$

Diagonalization of the kinetic terms and mass terms redefines the fields in terms of physical fields as,

$$\begin{aligned}
A^\mu &= \hat{c}_W\, B^\mu + \hat{s}_W\, W^{3\,\mu} \\
Z_1^\mu &= \cos\xi \left(\hat{Z}^\mu - \hat{s}_W\, \sin\chi\, \hat{Z}'_\mu\right) + \sin\xi\, \cos\chi\, \hat{Z}'_\mu \\
Z_2^\mu &= \cos\xi\, \cos\chi\, \hat{Z}'_\mu - \sin\xi \left(\hat{Z}^\mu - \hat{s}_W\, \sin\chi\, \hat{Z}'_\mu\right)
\end{aligned}$$

where, $s_W = \sin\theta_W\,,\ c_W = \cos\theta_W\,.$

A^μ is the mass-less photon and Z_1^μ ($\equiv Z$), Z_2^μ ($\equiv Z'$) are the massive gauge bosons.

In the physical basis, relevant neutrino-matter interaction terms in the limit, $\chi << 1\,, \delta\hat{M}^2 << \hat{M}_Z^2$,

$$\begin{aligned}
\mathscr{L}_A &= -e\,(J_{\text{EM}})_\mu\, A^\mu, \text{ with } J^\mu_{\text{EM}} = J^{\mu 3}_W + \frac{j^\mu_Y}{2} \\
\mathscr{L}_{Z_1} &= -\left(\frac{e}{s_W\, c_W}\,\left((J^3_W)_\mu - s^2_W\,(J_{\text{EM}})_\mu\right) + g'\,\xi\,(J_{Z'})_\mu\right) Z_1^\mu \\
\mathscr{L}_{Z_2} &= -\left(g'\,(J_{Z'})_\mu - (\xi - s_W\,\chi)\frac{e}{s_W\, c_W}\,\left((J^3_W)_\mu - s^2_W\,(J_{\text{EM}})_\mu\right) - e\, c_W\, \chi\,(J_{\text{EM}})_\mu\right) Z_2^\mu
\end{aligned}$$