# Constructing confidence intervals with Profiled Feldman-Cousins method for NOvA's neutrino oscillation measurement

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on behalf of the NOvA collaboration

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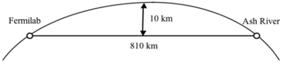
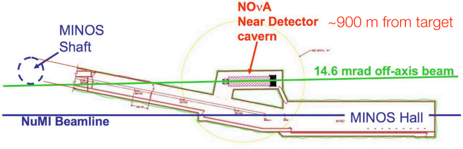
# The NOvA Experiment

- NuMI Off-axis $\nu_e$ Appearance
- Long baseline, high energy experiment
  - NuMI beam, ~900 kW beam located at Fermilab
  - Long-baseline, beam travels 810 km from Fermilab to MN
  - Off-axis, beam aimed 14.6 mrad off center to maximize 2 (
- Primary goal is study of 3-flavor neutrino oscillations
  - $\nu_\mu/\bar{\nu}_\mu$ dissappearance, $\nu_e/\bar{\nu}_e$ appearance
- Other active research areas include
  - Cosmic neutrinos
  - Sterile neutrinos
  - Beyond-standard-model physics

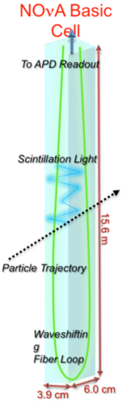



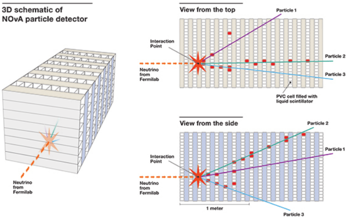
# The NOvA Detectors

Near Detector

Far Detector

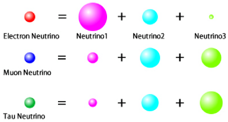



- Two detectors
  - Near detector located at Fermilab
  - Much larger Far detector located 810 km away in Minnesota
- Aside from size, both far and near detector are functionally identical
  - Alternating planes of PVC cells filled with liquid scintillator
  - Wavelength shifting fiber carries light to APD
  - Avalanche photo diode (APD) converts light to signal

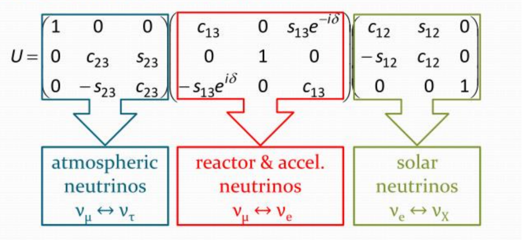


# 3-Flavor Oscillations

- Neutrino flavor states are composed of the mass eigenstates



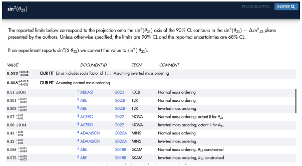

- Related by the PMNS matrix, which relies on 4 mixing parameters
  - Three mixing angles, $\theta_{12},\ \theta_{23},\ \theta_{13}$
  - One CP violating phase, $\delta_{CP}$
- Typically represented as product of 3 rotation matrices




$$(c_{ij} := \cos\theta_{ij},\ s_{ij} := \sin\theta_{ij})$$

# Open Questions

$\sin^2 \theta_{23}$ results from various experiments, from PDGLive

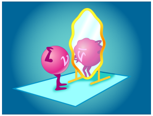


$\sin^2(\theta_{23})$ PDGID:S067P23 INSPIRE

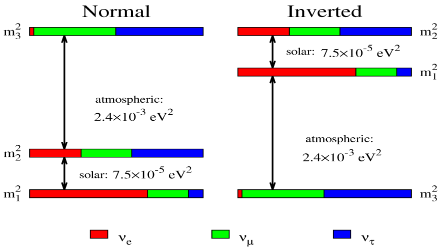
The reported limits below correspond to the projection onto the $\sin^2(\theta_{23})$ axis of the 90% CL contours in the $\sin^2(\theta_{23}) - \Delta m^2_{32}$ plane presented by the authors. Unless otherwise specified, the limits are 90% CL and the reported uncertainties are 68% CL.

If an experiment reports $\sin^2(2\,\theta_{23})$ we convert the value to $\sin^2(\,\theta_{23})$.

| VALUE | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|
| $\mathbf{0.553^{+0.016}_{-0.024}}$ | **OUR FIT** Error includes scale factor of 1.1. Assuming inverted mass ordering | | | |
| $\mathbf{0.558^{+0.015}_{-0.021}}$ | **OUR FIT** Assuming normal mass ordering | | | |
| $0.51 \pm 0.05$ | [1] ABBASI | 2023 | ICCB | Normal mass ordering |
| $0.561^{+0.021}_{-0.032}$ | [2] ABE | 2023F | T2K | Normal mass ordering |
| $0.563^{+0.017}_{-0.032}$ | [2] ABE | 2023F | T2K | Inverted mass ordering |
| $0.57^{+0.03}_{-0.04}$ | [3] ACERO | 2022 | NOVA | Normal mass ordering; octant II for $\theta_{23}$ |
| $0.56 \pm 0.04$ | [3] ACERO | 2022 | NOVA | Inverted mass ordering; octant II for $\theta_{23}$ |
| $0.43^{+0.20}_{-0.04}$ | [4] ADAMSON | 2020A | MINS | Normal mass ordering |
| $0.42^{+0.07}_{-0.03}$ | [4] ADAMSON | 2020A | MINS | Inverted mass ordering |
| $0.588^{+0.031}_{-0.064}$ | [5] ABE | 2018B | SKAM | Normal mass ordering, $\theta_{13}$ constrained |
| $0.575^{+0.036}_{-0.073}$ | [5] ABE | 2018B | SKAM | Inverted mass ordering, $\theta_{13}$ constrained |




- Three primary questions:
  - Measurement: What are the values of the mixing parameters?
  - CP Violation: Do neutrinos and anti-neutrinos oscillate differently? If so, by how much? ($\delta_{CP}$)
  - Mass ordering: What is the sign of $m_3^2 - m_2^2 := \Delta m_{32}^2$



# Data Visualisation

- NOvA primarily measures three of the oscillation parameters:
  - $\sin^2\theta_{23}$, $\Delta m^2_{32}$, $\delta_{CP}$
- Observed data is compared to predictions generated using various combinations of the parameters (hypotheses).
  - **Likelihood** of observing our data given a chosen set of parameters as the true values.
- **Confidence Intervals** are regions of our parameter space that contain the true values of the parameters with a chosen level of confidence.

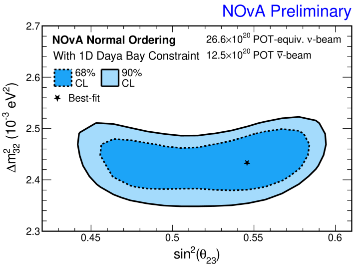



Fig: Surface plot showing the 90% and 68% confidence regions for the $\sin^2\theta_{23}$ and $\Delta m^2_{32}$ oscillation parameters

# Confidence Interval Construction

**Confidence Intervals** are regions of our parameter space that contain the true values of the parameters with a chosen level of confidence.

- Requires knowledge about the test statistic
  - **Log-Likelihood ratio**(LLR): test statistic which describes the likelihood of observing the data
- **Wilks' Theorem**: distribution of the **Log-Likelihood Ratios** converges to a $\chi^2$ distribution, given certain conditions are met
- $\chi^2$ distribution well documented, can look up critical values for given levels of confidence
  - If a given set of parameters LLR is less than or equal to the critical $\chi^2$ value, it is within that confidence interval

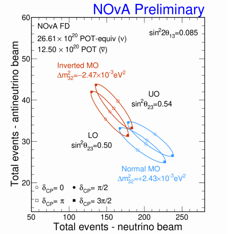
| $(1-\alpha)$ (%) | $m=1$ | $m=2$ | $m=3$ |
|---|---|---|---|
| 68.27 | 1.00 | 2.30 | 3.53 |
| 90. | 2.71 | 4.61 | 6.25 |
| 95. | 3.84 | 5.99 | 7.82 |
| 95.45 | 4.00 | 6.18 | 8.03 |
| 99. | 6.63 | 9.21 | 11.34 |
| 99.73 | 9.00 | 11.83 | 14.16 |

The level of the CI you want to draw.

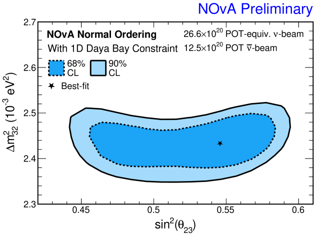
The number of dimensions.

# Wilks' Theorem Conditions

- ∼ Large sample size
  - Neutrino events notoriously rare, less of a problem with enough data
- ∼ **Nested hypotheses**: Null hypothesis is a special case of the alternative (i.e. fixed parameters)
  - ✓ Measuring parameters under normal **or** inverted mass ordering
  - ✗ Determining mass ordering; normal ordering is not a subset of inverted ordering
- ✗ Ellipsoidal distributions of the uncertainty in the parameters
  - Two primary modes of failure
    - Uncertainties crossing physical boundaries
    - Degeneracies in the model
- Using the critical $\chi^2$ values for the desired confidence level will result in incorrect intervals




Probabilites under 4 non-nested hypotheses

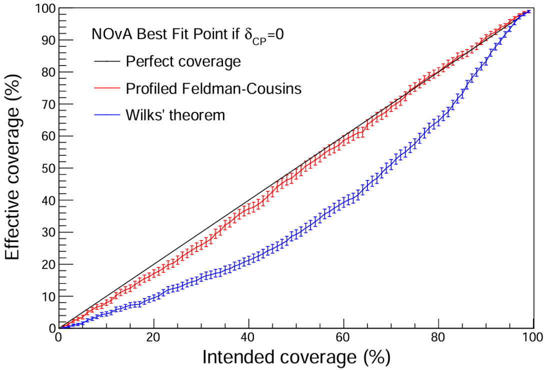



Confidence regions with non-ellipsoidal shape

# Feldman and Cousins

Solution: **Feldman-Cousins Technique**

- A method for empirically generating log-likelihood distributions
- Confidence interval construction using this generated distribution more accurate



# Traditional Feldman and Cousins

Developed by Gary Feldman and Robert Cousins[a]

- Generate **pseudoexperiments** at each grid point in the parameter space
  - Pseudoexperiments(PSEs): Mock data generated using model predictions via monte carlo methods
- Determine the log-likelihood for each PSE, resulting in a log-likelihood distribution **for each grid point**
- New critical $\chi^2$ value for that grid point obtained at the point in the distribution that corresponds to the desired confidence level

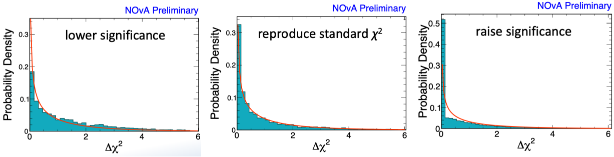



Fig: Examples of how different grid points in the parameter space can vary from the traditional $\chi^2$ distribution

[a]Feldman and Cousins, "Unified approach to the classical statistical analysis of small signals".

# Profiled Feldman-Cousins

Traditional Feldman-Cousins has no way to handle **nuisance parameters** and becomes less effective with parameter spaces that contain large amounts of systematic parameters

- **Profiled Feldman and Cousins**[a]
  - Used by the NOvA collaboration to handle nuisance parameters
  - Nuisance parameters: any parameter that is not a parameter of interest
    - Systematic uncertainties, NOvA has ∼70
    - Oscillation parameters not actively being plotted
  - Solution: **profile** over nuisance parameters
    - Profiling: Fix nuisance parameters to observed best fit values during pseudoexperiment generation
    - Profiled values differ for each combination of the parameters of interest

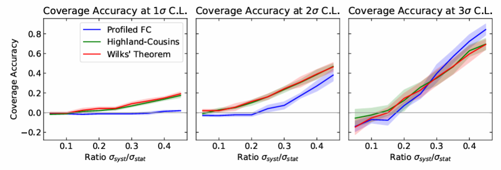
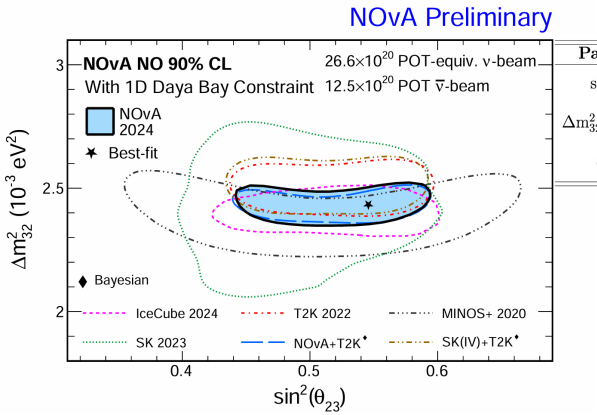



Fig: Coverage accuracy as the amount of systematic uncertainty grows (toy model simulation)

[a] Acero et al., "The Profiled Feldman-Cousins technique for confidence interval construction in the presence of nuisance parameters".

# 3Flavor Analysis 2024

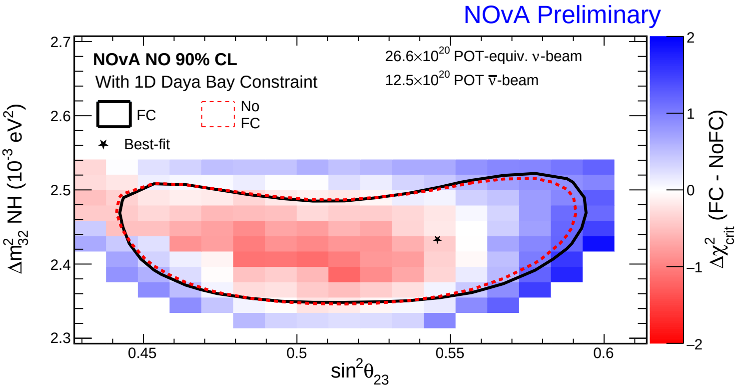
- NOvA recently presented results representing a total of over 10 years of data, and almost double (96%) the $\nu_\mu$ beam exposure since last analysis




| Parameter | Best-fit |
|---|---|
| $\sin^2(\theta_{23})$ | $0.546^{+0.032}_{-0.075}$ |
| $\Delta m^2_{32}\ (10^{-3}\,\mathrm{eV}^2)$ | $2.433^{+0.035}_{-0.036}$ |
| $\delta_{CP}\ (\pi)$ | 0.875 |

# 3Flavor Analysis 2024

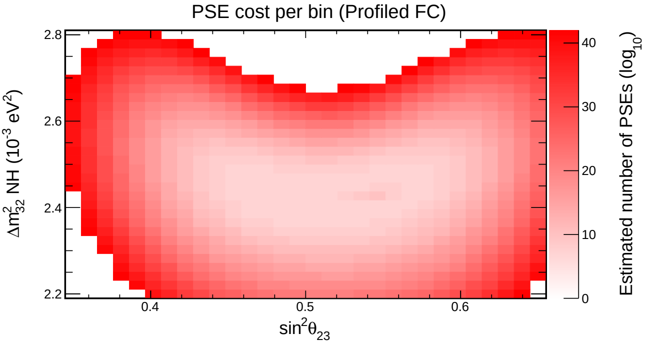
- Frequentist plots must be FC corrected in order to accurately report findings
- FC Corrections alter the confidence regions




Comparison between non-corrected and corrected confidence regions, blue areas mark where the critical $\chi^2$ value grows, and can include otherwise excluded bins into the confidence region, where the red areas are the opposite, marking a decrease in the critical $\chi^2$ and potentially excluding bins from the region

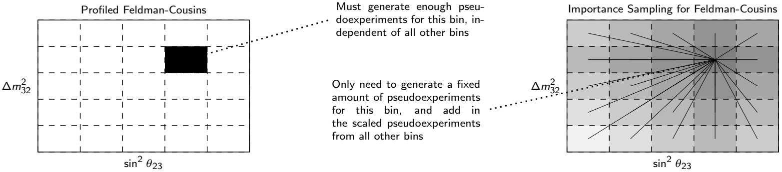
# Computational Cost

- FC Corrections require generating and fitting pseudoexperiments(PSEs) **for each bin**
- Number of PSEs required depends upon desired precision and initial likelihood of the bin
- Required use of parrallelization (via the DIY C++ package)
- Ran at National Energy Research Scientific Computing center (NERSC)
  - Ran on extremely powerful supercomputer, Perlmutter, utilizing up to 13,056 CPU cores at a time
    - Even still, cost to correct full plot was unaffordable

# Potential Cost Improvement - Importance Sampling

- Method to reduce computational cost during Feldman-Cousins[a]
- Implements a weighting to be applied to pseudoexperiments at other grid points, to be used in the current grid point
- Would reduce computational cost, especially at high significance levels




[a] Berns, "Importance sampling method for Feldman-Cousins confidence intervals".

# Conclusion

- NOvA continues to measure the values of the neutrino 3-flavor mixing parameters, as well as shedding light into the the neutrino mass ordering problem

- Neutrino experiments pose a unique statistical probelm due to the nature of it's parameter space and the difficulty inherent in the detection of neutrinos

- One of the solutions to this statistical problem is the Profiled Feldman-Cousins technique, and allows for more accurate confidence intervals

- This method is computationally expensive, and we continue to explore optimisation techniques, such as the Importance Sampling method

# References


Acero, M. A. et al. "The Profiled Feldman-Cousins technique for confidence interval construction in the presence of nuisance parameters". In: (2022). arXiv: 2207.14353 [hep-ex]. URL: https://arxiv.org/abs/2207.14353.

Berns, Lukas. "Importance sampling method for Feldman-Cousins confidence intervals". In: *Physical Review D* 109.9 (May 2024). ISSN: 2470-0029. DOI: 10.1103/physrevd.109.092002. URL: http://dx.doi.org/10.1103/PhysRevD.109.092002.

Feldman, Gary J. and Robert D. Cousins. "Unified approach to the classical statistical analysis of small signals". In: *Physical Review D* 57.7 (Apr. 1998), 3873–3889. ISSN: 1089-4918. DOI: 10.1103/physrevd.57.3873. URL: http://dx.doi.org/10.1103/PhysRevD.57.3873.

# Thank you!

Questions?