# QCD LECTURE 3

HCPSS 2024

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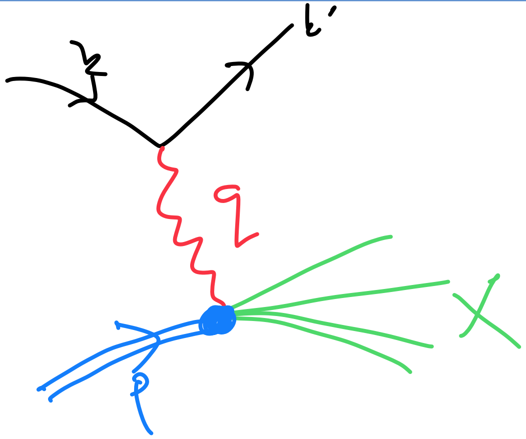
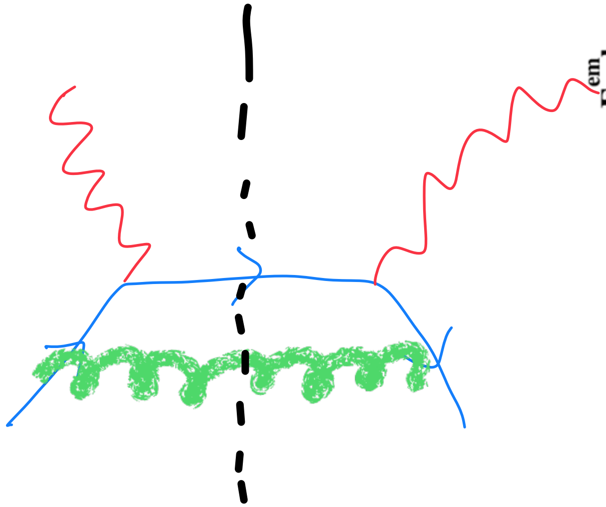
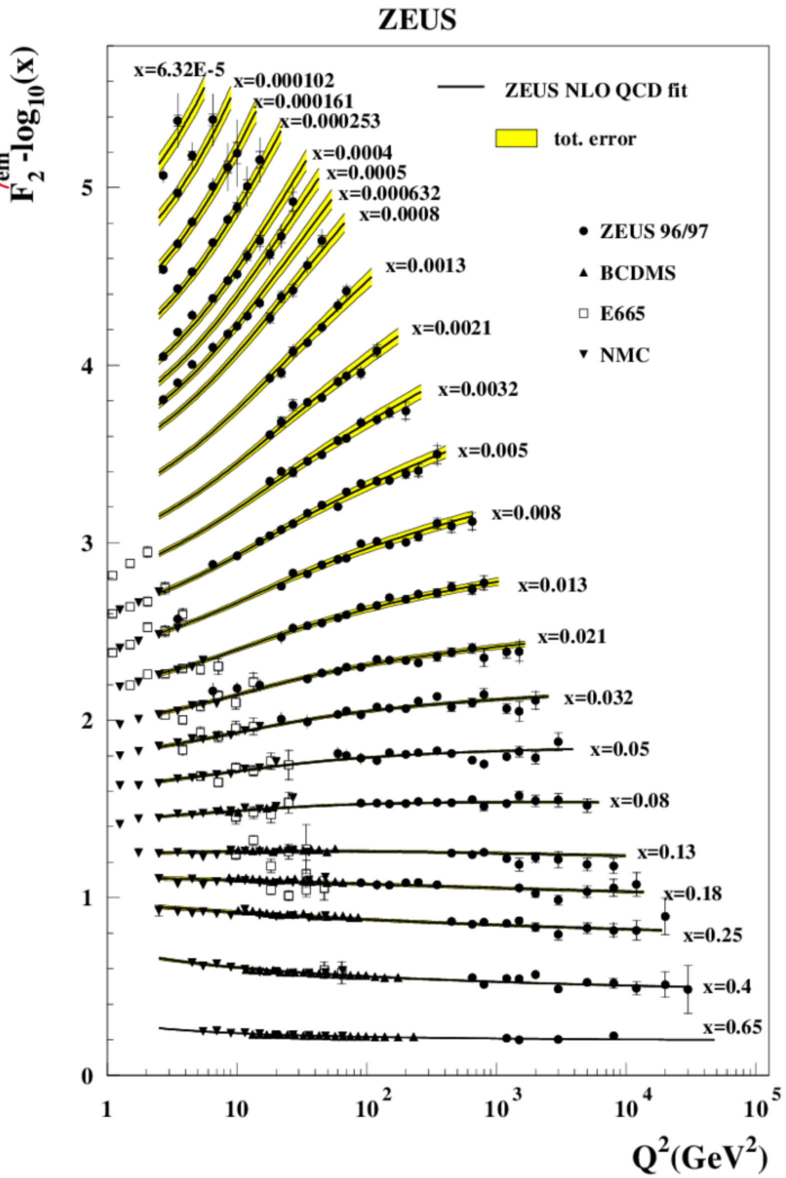
# Last time





$$P(\xi) = C_F \frac{1+\xi^2}{1-\xi}$$

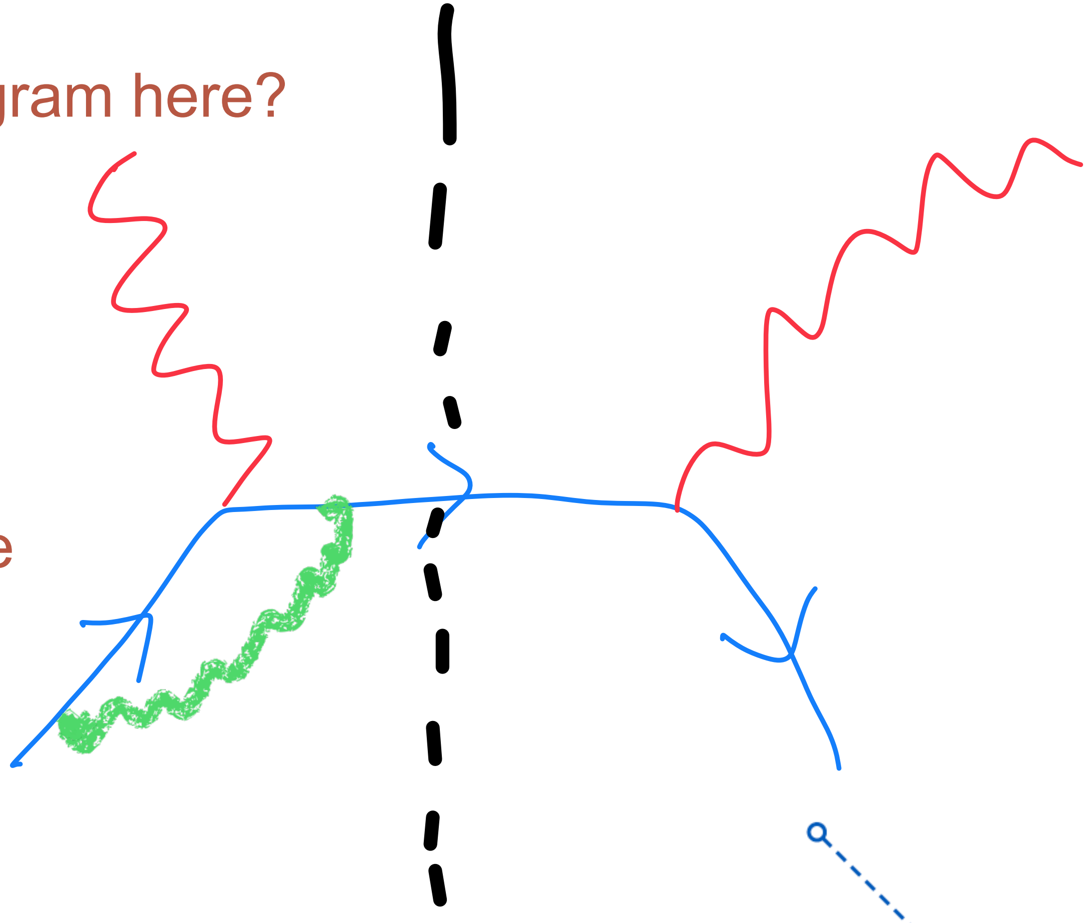
$$\hat{F}_2 = e_q^2 x \left[ \delta(1-x) + \frac{\alpha_S}{2\pi} \left( P(x) \ln\left(\frac{Q^2}{\kappa^2}\right) + C(x) \right) \right]$$



# (Almost) Total $\hat{F}_2$

WAIT! Didn't we forget that there are virtual contributions too?

Yes ("we") did. What about corrections like the diagram here?



However, since the virtual corrections share the LO topology (phase space) they will contribute a term proportional to $\delta(1-x)$

They can be accommodated by making the change

$$P(x) \to P(x) + K\delta(1-x)$$

# Fixing $K$

If we had a quiet afternoon we could calculate $K$, however we can actually get an answer even quicker by noting (or being told that) the integral over the total (modified) $P(x)$ should vanish (this ensures baryon number conservation if you're interested)

As a result we write

$$P(x) = C_F\left(\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right)$$

Where the plus distribution is defined as follows.

$$\int_0^1 dx \frac{f(x)}{(1-x)_+} = \int_0^1 dx \frac{f(x)-f(1)}{1-x}$$ And . $$\frac{1}{(1-x)_+} = \frac{1}{1-x}, \quad 0 \leq x < 1$$

# (Actual) Total $\hat{F}_2$ and $F_2$

Using our modified splitting function the result from before is now accurate to $\alpha_S$

$$\hat{F}_2 = e_q^2 x \left[ \delta(1-x) + \frac{\alpha_S}{2\pi} \left( P(x) \ln\left(\frac{Q^2}{\kappa^2}\right) + C(x) \right) \right]$$

To go from quark to proton we convolute our quark structure function with the distribution of quarks in the proton $q_0(\xi)$

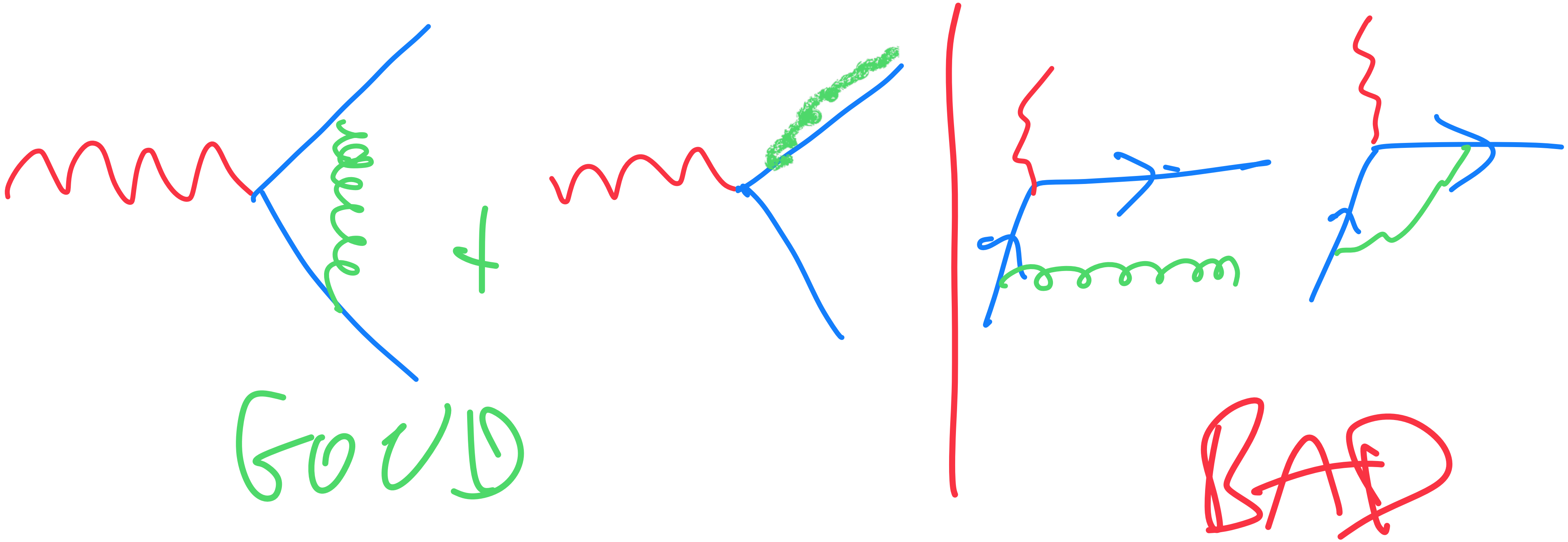
$$F_2(x, Q^2) = e_q^2 x \sum_{q,\bar{q}} \left[ q_0(x) + \frac{\alpha_S}{2\pi} \int_x^1 \frac{d\xi}{\xi} q_0(\xi) \left( P\left(\frac{x}{\xi}\right) \ln\left(\frac{Q^2}{\kappa^2}\right) + C\left(\frac{x}{\xi}\right) \right) \right]$$

What about the whopping great singularity as we formally take $\kappa^2 \to 0$.
This singularity arises when the gluon is emitted collinearly to the quark.

# Infrared safety

At this stage you might be fairly annoyed with me, or at least confused.

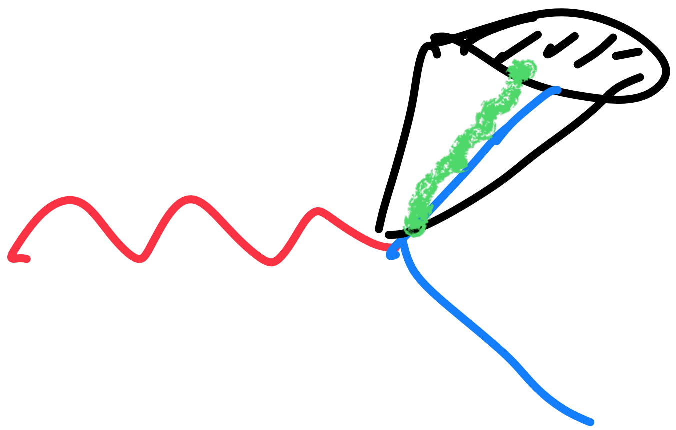
On Tuesday we saw how the process $\gamma^* \to$ hadrons, involved a cancellation between virtual and real contributions to render the whole thing finite. Why doesn't this happen here in this **very similar** process?

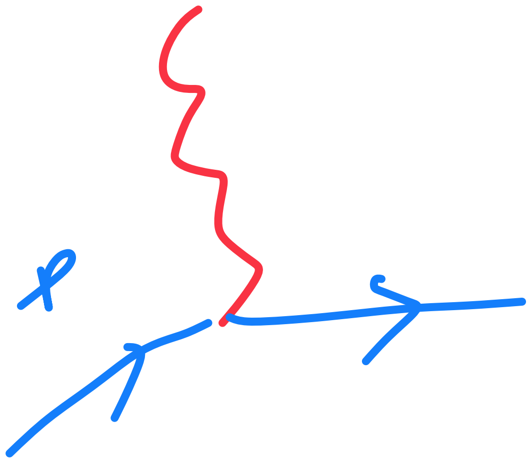


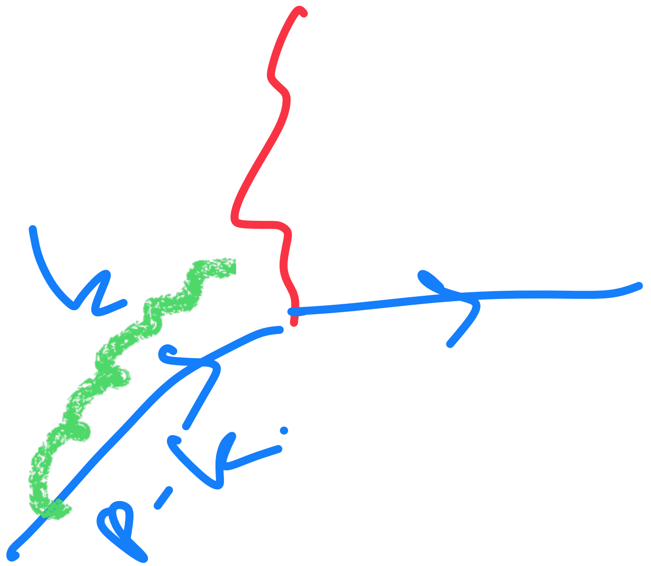
# Infrared safety

The difference lies in the fact that we can cluster the final state in the former case into a jet. The jet has good properties and is safe in the IR limits. If we talked about final state observables associated with just the quark we would run back into IR issues.

In our calculation there is no clustering, the photon CAN distinguish the difference between a quark and a quark-gluon collinear pair with the same momentum.





# Renormalization

$$F_2(x,Q^2) = e_q^2 x \sum_{q,\bar{q}} \left[ q_0(x) + \frac{\alpha_S}{2\pi}\int_x^1 \frac{d\xi}{\xi} q_0(\xi) \left( P\left(\frac{x}{\xi}\right) \ln\left(\frac{Q^2}{\kappa^2}\right) + C\left(\frac{x}{\xi}\right)\right)\right]$$

We solve the issue by renormalization. In the same way as we think of the bare coupling as being an unmeasurable theoretical construct, we think about $q_0(x)$ as a bare distribution function. We define a renormalized distribution as follows,

$$q(x,\mu_F^2) = q_0(x) + \frac{\alpha_S}{2\pi}\int_x^1 \frac{d\xi}{\xi} q_0(\xi) \left[ P\left(\frac{x}{\xi}\right) \ln\left(\frac{\mu_F^2}{\kappa^2}\right) + C\left(\frac{x}{\xi}\right)\right]$$

Then

$$F_2(x,Q^2) = e_q^2 x \sum_{q,\bar{q}} \left[ q(x,\mu_F^2) + \frac{\alpha_S}{2\pi}\int_x^1 \frac{d\xi}{\xi} q(\xi,\mu_F^2)\, P\left(\frac{x}{\xi}\right) \ln\left(\frac{Q^2}{\mu_F^2}\right) + \mathcal{O}(\alpha_s^2)\right]$$

# Factorization scale and scheme

The factorization scale $\mu_F$ plays a similar role to the renormalization scale $\mu_R$, we can think of it as the scale at which the long-range IR physics is absorbed into the proton. Some things to note:

- The distribution $q(x, \mu_F)$ cannot be calculated from first principles in perturbation theory (although possibly with the lattice) since it contains non-perturbative long-range physics.
- Effectively we have factorized the non-perturbative long-range physics and the short-range perturbative physics which depends on large momentum transfers. This type of factorization occurs beyond NLO and is vital for our ability to calculate cross sections for scattering processes.
- We chose to define our renormalized functions with the full finite piece $C(x/\xi)$ such that this was all absorbed into $q(x, \mu_F)$. This was a choice not a requirement. Like in UV physics this choice is referred to as a scheme.

# Factorization scheme.

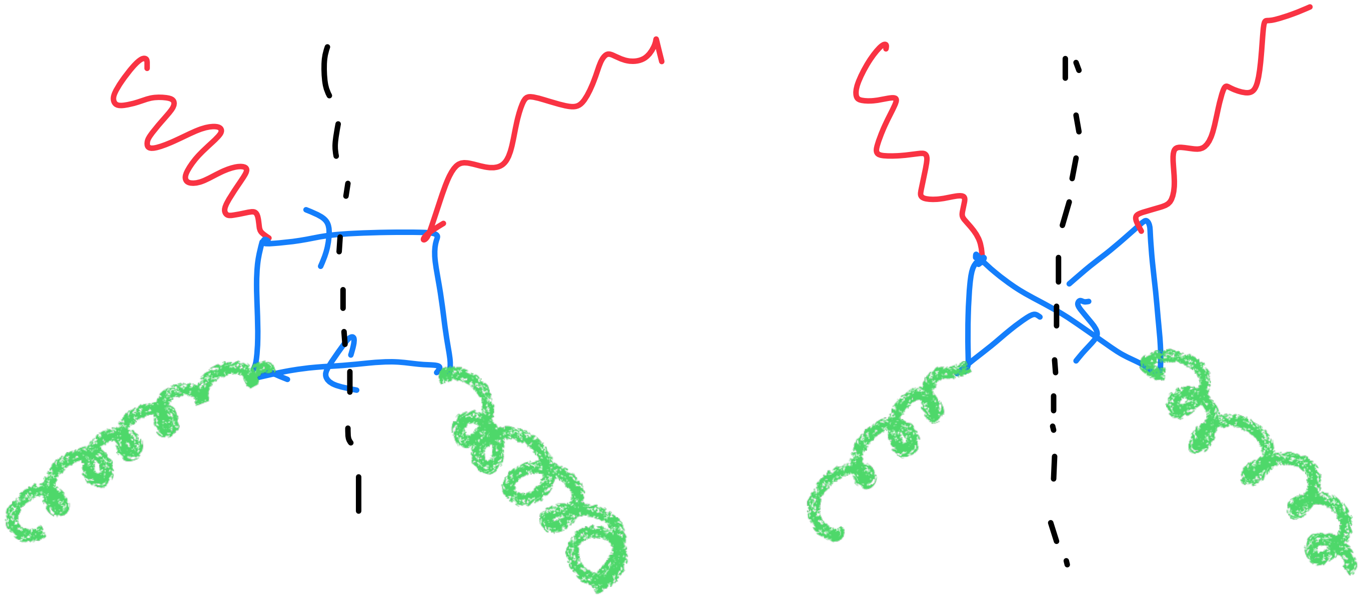
$$q(x,\mu_F^2) = q_0(x) + \frac{\alpha_S}{2\pi}\int_x^1 \frac{d\xi}{\xi} q_0(\xi)\left[P\left(\frac{x}{\xi}\right)\ln\left(\frac{\mu_F^2}{\kappa^2}\right) + C\left(\frac{x}{\xi}\right)\right]$$

The choice we made is referred to as the DIS scheme. A more common scheme for general collider physics is the $\overline{\mathrm{MS}}$ scheme in which only the divergent piece and a ubiquitous $\ln(4\pi) - \gamma_E$ is absorbed into $q_{\overline{MS}}(x,\mu_F)$

Once chosen, a scheme must be used in all cross sections, note that partonic cross sections will be different in different schemes.

# Wait! (Again)

Once again, we think we are nearly done, but QCD throws one more wrench in the machine. What about the following diagrams?

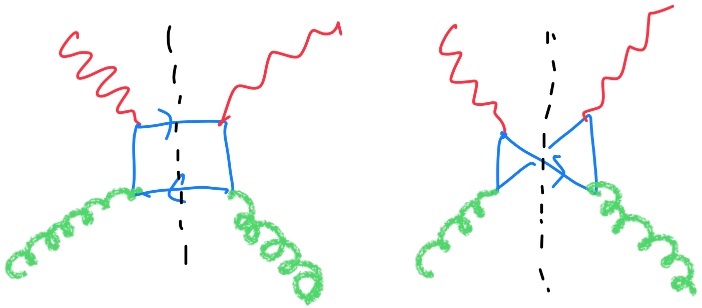


Both clearly couple a photon to a quark at $\mathcal{O}(\alpha_S)$, but the starting Parton is a gluon and not a quark!

# $\hat{F}_2^g$

The calculation proceeds in the same manner as before

$$\hat{F}_2^g(x, Q^2) = x \sum_{q,\bar{q}} e_q^2 \frac{\alpha_S}{2\pi} \left( P_{qg}(x) \ln\left(\frac{Q^2}{\kappa^2}\right) + C_g(x) \right)$$



The structure is the same as for the quark structure function, a logarithmic singularity from vanishing quark virtuality, and a finite piece. The new splitting function is

$$P_{qg}(x) = T_R(x^2 + (1-x)^2)$$

Where $T_R = 1/2$ arises from the trace over the color matrices.

(From now on we write our old splitting function $P(x) \to P_{qq}(x)$ for notational consistency)

# Finally putting it all together - DIS scheme

In the DIS scheme the gluon distribution is absorbed fully into the quark distribution function

$$q(x,\mu_F^2) = q_0(x) + \frac{\alpha_S}{2\pi}\int_x^1 \frac{d\xi}{\xi}\, q_0(\xi)\left[P_{qq}\left(\frac{x}{\xi}\right)\ln\left(\frac{\mu_F^2}{\kappa^2}\right) + C_q\left(\frac{x}{\xi}\right)\right] + \frac{\alpha_S}{2\pi}\int_x^1 \frac{d\xi}{\xi}\, g_0(\xi)\left[P_{qg}\left(\frac{x}{\xi}\right)\ln\left(\frac{\mu_F^2}{\kappa^2}\right) + C_g\left(\frac{x}{\xi}\right)\right]$$

So that $F_2(x,Q^2) = x\sum_{q,\bar{q}} e_q^2 q(x,Q^2)$ still

# Finally putting it all together - $\overline{MS}$ scheme

In the $\overline{\mathrm{MS}}$ scheme only the “divergent” pieces are sucked into the quark distribution and

$$F_2(x,Q^2) = x\sum_{q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} q(\xi,Q^2)\left[\delta\left(1-\frac{x}{\xi}\right) + \frac{\alpha_S}{2\pi} C_q^{\overline{\mathrm{MS}}}\left(\frac{x}{\xi}\right)\right] + x\sum_{q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} g(\xi,Q^2)\left(\frac{\alpha_S}{2\pi} C_g^{\overline{\mathrm{MS}}}\left(\frac{x}{\xi}\right)\right)$$

Where $C_q$ and $C_g$ are called coefficient functions.

# DIS - summary

Over the last one and a smidge lectures we've seen how structure functions can be expressed in terms of parton renormalized parton distribution functions

DIS data is therefore extremely useful in allowing us to determine the underlying nature of hadrons

We should note that there is more to the story than I've told here (a common theme). We've talked about DIS, i.e. a regime in which the proton is imaged by an extremely energetic photon. There's other regions of physics too, such as elastic scattering (when the photon doesn't disintegrate the proton) and an even more complicated region which connects the two (and is fully of resonant structures from light bound states). The understanding and matching of all these regions is an interesting and ongoing area of study. Especially in the context of neutrino scattering…

# FROM STRUCTURE FUNCTIONS TO JETS - DGLAP EQUATIONS

# Motivation

We've learnt about how DIS teaches us about the underlying probability distributions of parton's inside the proton.

We've seen how modern fits extract global information from multiple scattering experiments to allow us to determine fits for $q(x, Q^2)$ and $g(x, Q^2)$

For our final topic we would like to link back to where we started from. I.e. understand what the scale dependence of the distribution functions tell us, and then use this technology to understand jet structure, and Parton showers.

On the way we'll learn how to fit the PDFs from data too!

# DGLAP equation.

Recall our definition of the renormalized structure function (neglecting the gluon piece for now)

$$F_2(x,Q^2) = e_q^2 x \sum_{q,\bar{q}} \left[ q(x,\mu_F^2) + \frac{\alpha_S}{2\pi} \int_x^1 \frac{d\xi}{\xi} q(\xi,\mu_F^2)\, P\left(\frac{x}{\xi}\right) \ln\left(\frac{Q^2}{\mu_F^2}\right) + \ldots \right]$$

Now $F_2(x,Q^2)$ does not depend on $\mu_F$, writing $\mu_F^2 = t$ and taking the partial derivative wrt $\ln t$ we find the following equation

$$t\frac{\partial q(x,t)}{\partial t} = \frac{\alpha_S(t)}{2\pi} \int_x^1 \frac{d\xi}{\xi} P\left(\frac{x}{\xi}\right) q(\xi,t)$$

This is known as the Dokshitzer-Gribov-Lipitov-Altarelli-Parisi equation (or **DGLAP** for short!)

# DGLAP equation

We've been slightly sloppy here, working specifically at NLO. A more rigorous definition would be to write

$$t\frac{\partial q(x,t)}{\partial t} = \frac{\alpha_S(t)}{2\pi}\int_x^1 \frac{d\xi}{\xi} P_{qq}\left(\frac{x}{\xi}, \alpha_S(t)\right) q(\xi, t)$$

Where we acknowledge that the splitting function (or "evolution kernels") are really given by a perturbative expansion themselves

$$P_{qq}(z, \alpha_S) = P_{qq}^{(0)}(z) + \frac{\alpha_S}{2\pi} P_{qq}^{(1)}(z) + \dots$$

Our previous equation was the above equation expanded to first order in $\alpha_S$

# DGLAP equations

Our example before kept only the quark distribution (i.e. we temporarily set $g(x, \mu_F) = 0$ to make an easier introduction). Restoring the gluon distribution changes the DGLAP equation into a matrix equation

$$t\frac{\partial}{\partial t}\left(\begin{array}{c} q_i(x,t) \\ g(x,t) \end{array}\right) = \frac{\alpha_S(t)}{2\pi}\sum_{q_i,\overline{q}_j}\int_0^1 \frac{d\xi}{\xi} \times \left(\begin{array}{cc} P_{q_i,\overline{q}_j}(x/\xi, \alpha_S(t)) & P_{q_i,g}(x/xi, \alpha_S(t)) \\ P_{g,\overline{q}_j}(x/\xi, \alpha_S(t)) & P_{g,g}(x/xi, \alpha_S(t)) \end{array}\right)\left(\begin{array}{c} q_i(\xi,t) \\ g(\xi,t) \end{array}\right)$$

These equations are extremely important in QCD!

# DGLAP equation - Splitting functions

Entering into the DGLAP equations are the evolution kernels, each of which has its own perturbative expansion.

At LO we can interpret the splitting functions $P_{ab}^{(0)}$ as the probability of finding a Parton of type a in a parton of type b with a fraction x of the parent partons longitudinal momentum.

At LO the two other splitting functions we haven't seen yet are

$$P_{gq}^{(0)} = C_F\left(\frac{1+(1-x)^2}{x}\right),\ \text{and}$$

$$P_{gg}^{(0)}(x) = 2C_A\left(\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right) + \delta(1-x)\frac{11C_A - 4n_F T_R}{6}$$

# Solving the DGLAP equations

Solving the DGLAP equations (particularly at higher orders) is a tricky business. Here we'll look at the overall details to get a feel for how PDF fitting works, but the gory details are beyond the scope of our lecture here!

# Solving the DGLAP equations

Solving the DGLAP equations (particularly at higher orders) is a tricky business. Here we'll look at the overall details to get a feel for how PDF fitting works, but the gory details are beyond the scope of our lecture here!

In practice, individual Parton distributions such as $q(x, \mu^2)$ are not the easiest thing to solve for, instead various "singlet" and "non-singlet" linear combinations are made, some of which have much nicer properties. For example in the "non-singlet" difference $V = q_i - q_j$ the gluon drops out and we get a much simpler DE. E.g.

$$t\frac{\partial}{\partial t}V(x,t) = \frac{\alpha_S(t)}{2\pi}[P_{qq}(\xi) \otimes V(\xi,t)]$$

Where $\otimes$ is a shorthand for our convolution integral occurring in the DGLAP equations

# Solving the DGLAP equations

Other linear combinations are harder since they induce coupled differential equations, e.g. the "singlet" distribution $\Sigma(x,t) = \sum_i [q_i(x,t) + \overline{q}_i(x,t)]$,

$$t\frac{\partial \Sigma}{\partial t} = \frac{\alpha_S}{2\pi}\left[P_{qq}^{(0)} \otimes \Sigma + 2n_F P_{qg}^{(0)} \otimes g\right] \text{ and}$$

$$t\frac{\partial g}{\partial t} = \frac{\alpha_S}{2\pi}\left[P_{gq}^{(0)} \otimes \Sigma + P_{gg}^{(0)} \otimes g\right]$$

Plus higher order corrections. In practice these equations are solved numerically integrating in x-space with input distributions from data.

# Solving the DGLAP equations

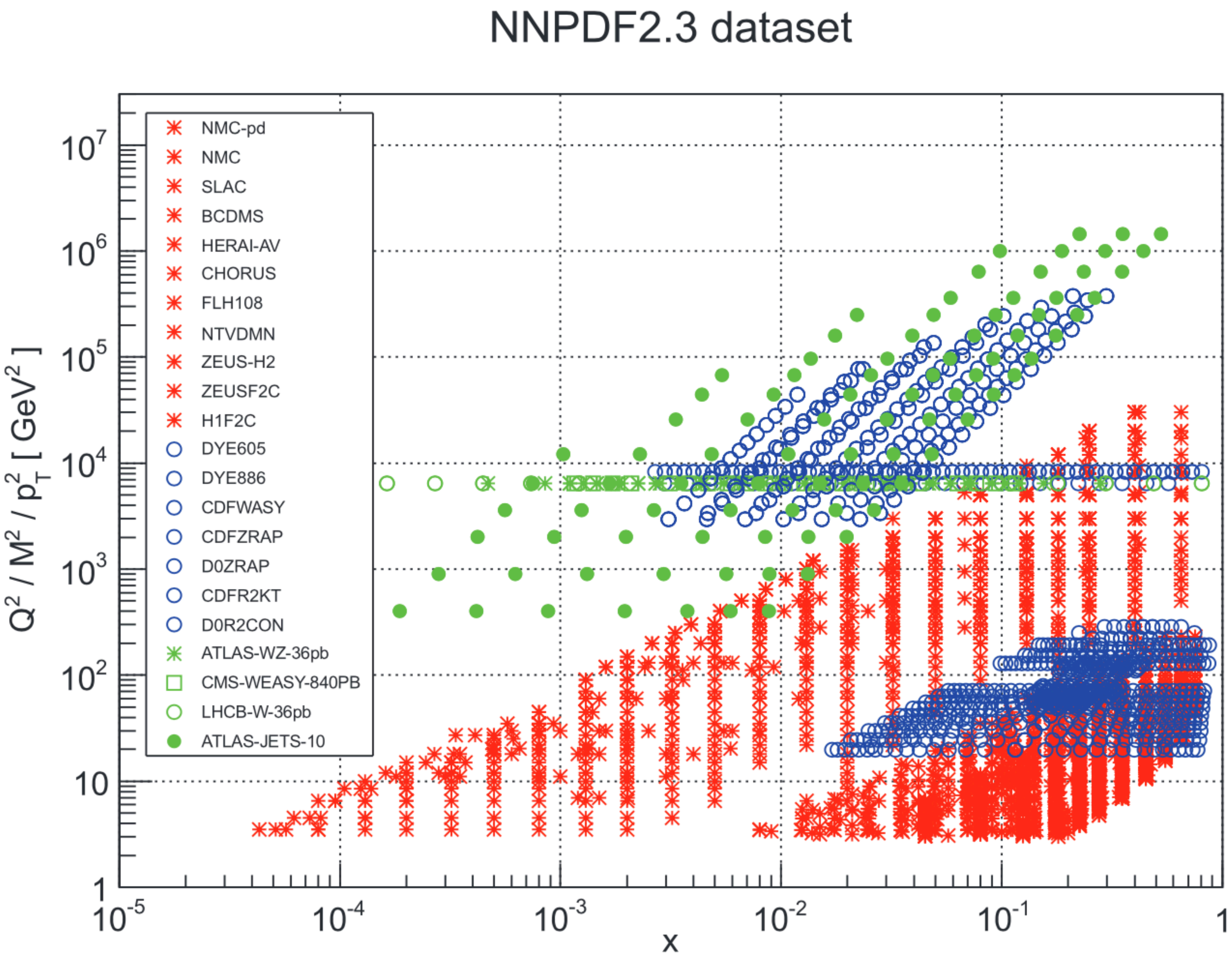
Practically you can choose some reference value $Q_0$ and parametrize the Parton distribution functions at that value, e.g.

$$q(x, Q_0^2) = Ax^a(1 + c\sqrt{x} + dx)(1 - x)^b$$

Then the DGLAP equations can be numerically solved to give the value of the PDFs at the desired scale $Q^2$.

Modern PDF fitting is a complicated and intricate business, but we have reached a level were the PDFs are pretty accurately constrained for most applications of LHC physics.

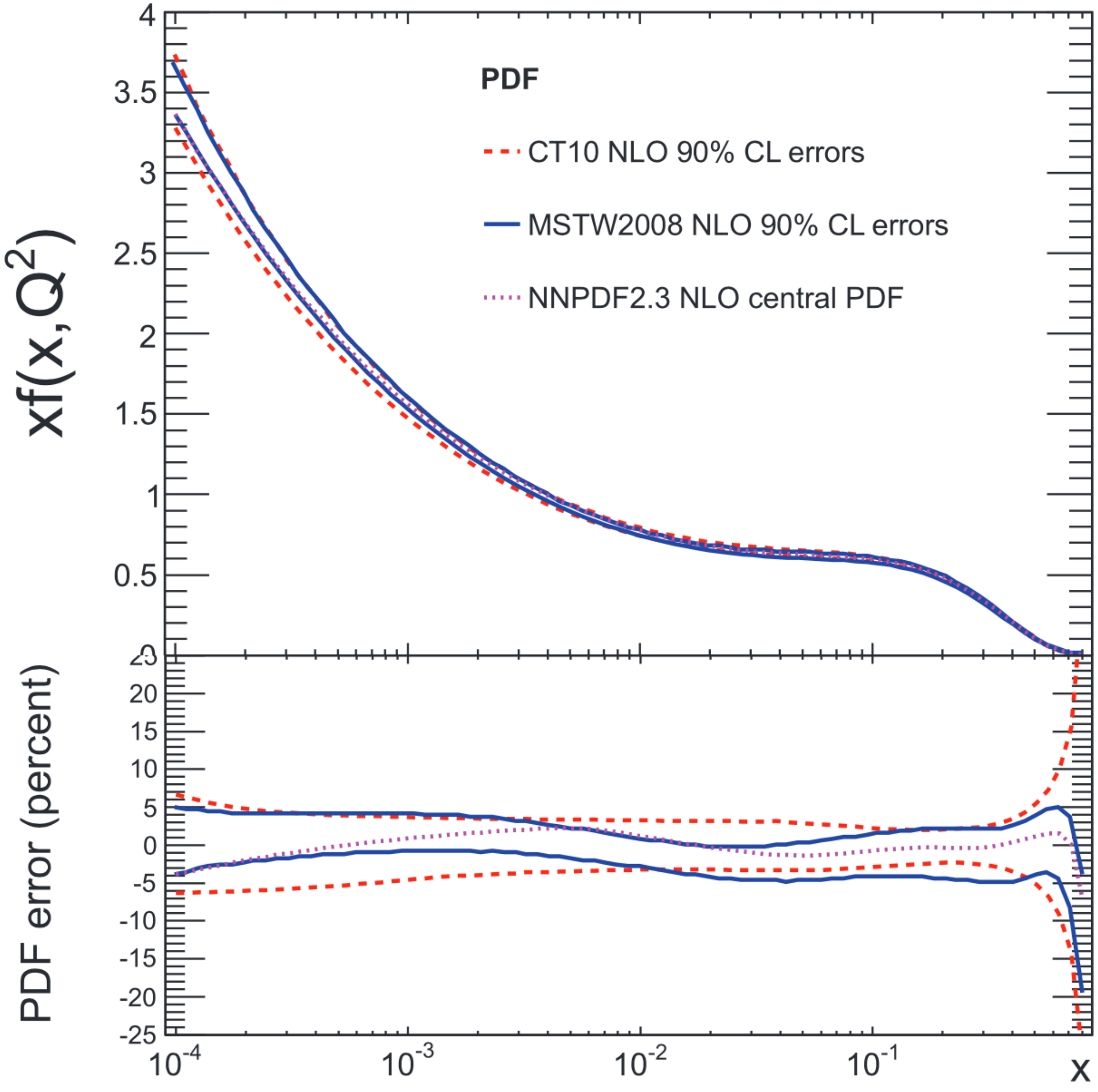
# PDF fitting and results




Modern global PDF fits use large data sets and fitting forms at a reference scale of around $Q_0 \sim 1 - 2$ GeV with around 28-30 free parameters.

# PDF comparisons

Despite using different methodologies and input data sets the three main PDF sets for LHC show good agreement with each other (figure is for u quarks at $Q = 100$ GeV).



# PARTON SHOWERS

# Motivation

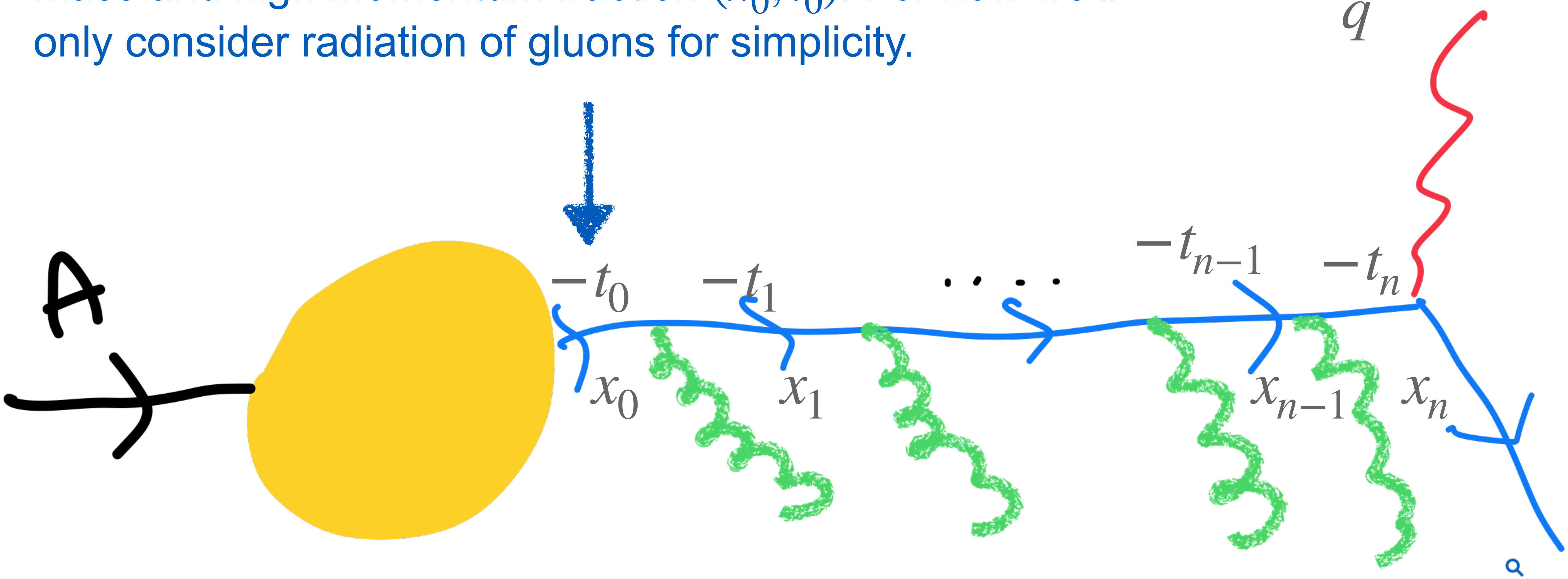
The DGLAP is an extremely important equation in QCD, since it allows us to take Parton distribution functions extracted a given input scale (e.g. from low energy data or DIS) and evolve the distributions to a hard scale relevant for our particular purposes.

However, it will play a second important (albeit related) role for us, since it will allow us to write Monte Carlo codes to describe radiation patterns from initial and final state partons.

This will lead us to a greater understanding of the theoretical properties of QCD jets, through what's known as a **parton shower.**

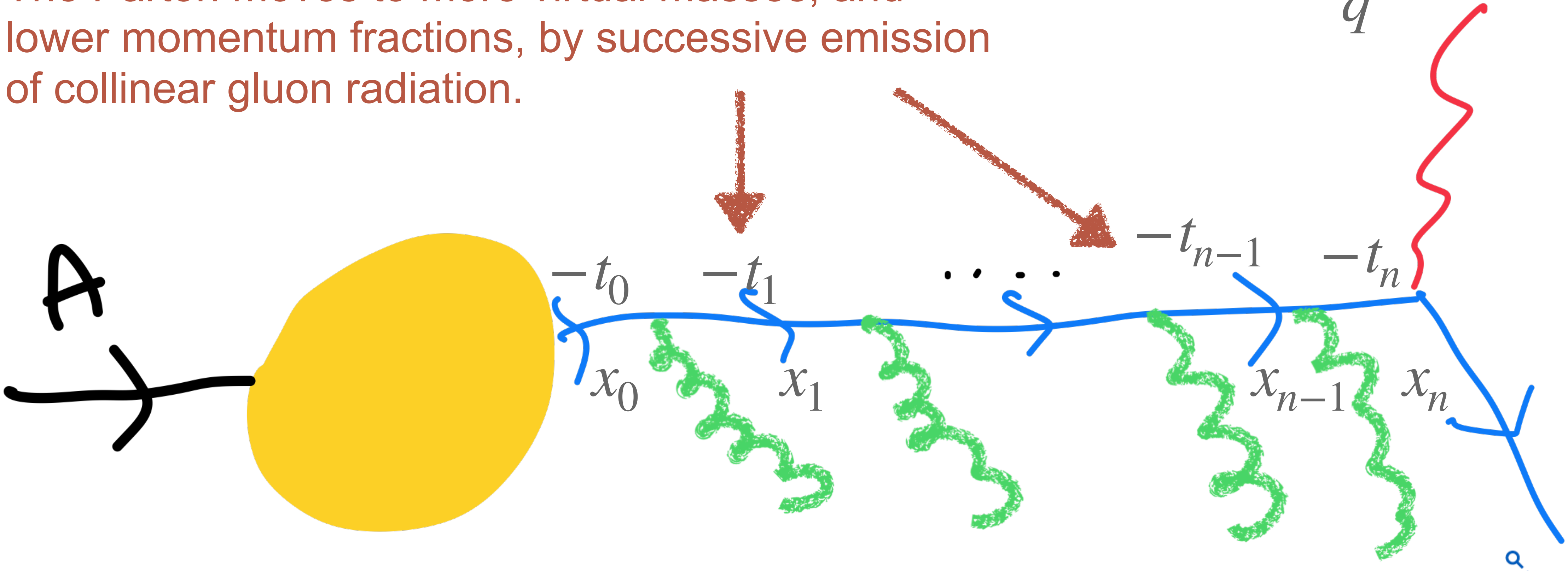
# Initial state Branching of partons

Incoming Hadron A starts of with a quark with low virtual mass and high momentum fraction $(x_0, t_0)$. For now we'll only consider radiation of gluons for simplicity.



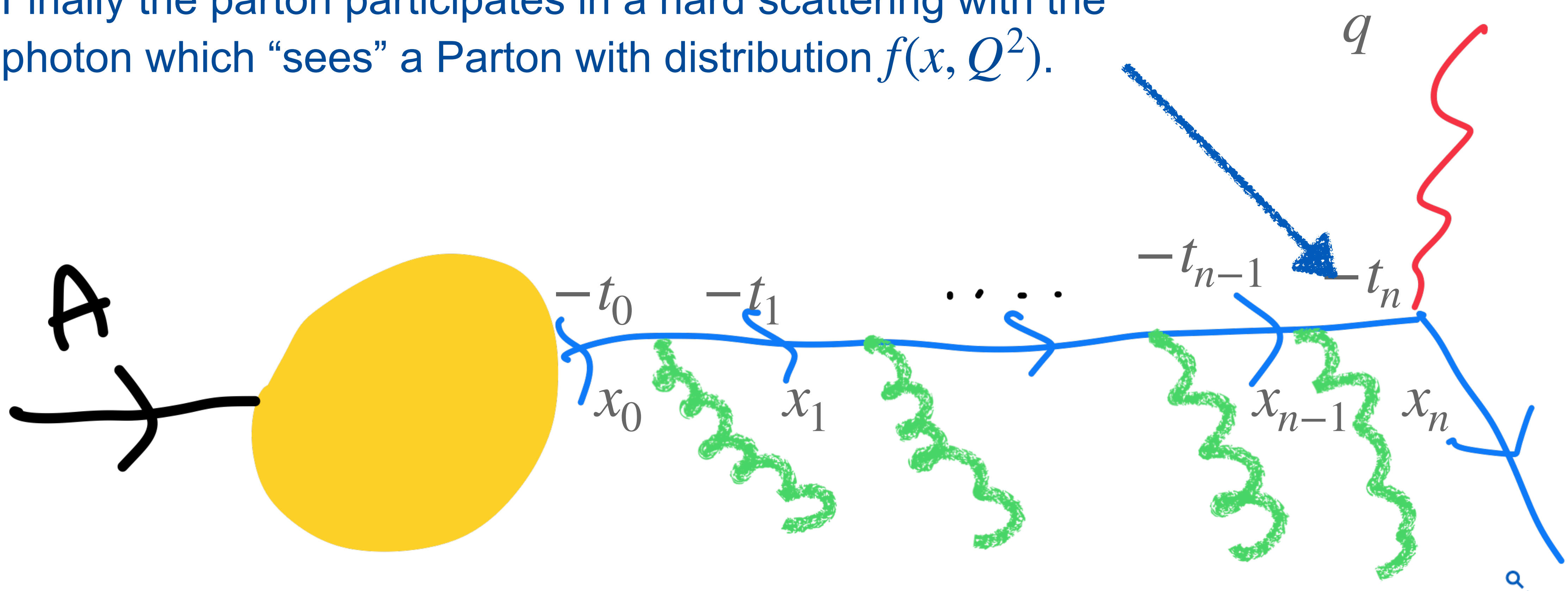
# Initial state Branching of partons

The Parton moves to more virtual masses, and lower momentum fractions, by successive emission of collinear gluon radiation.

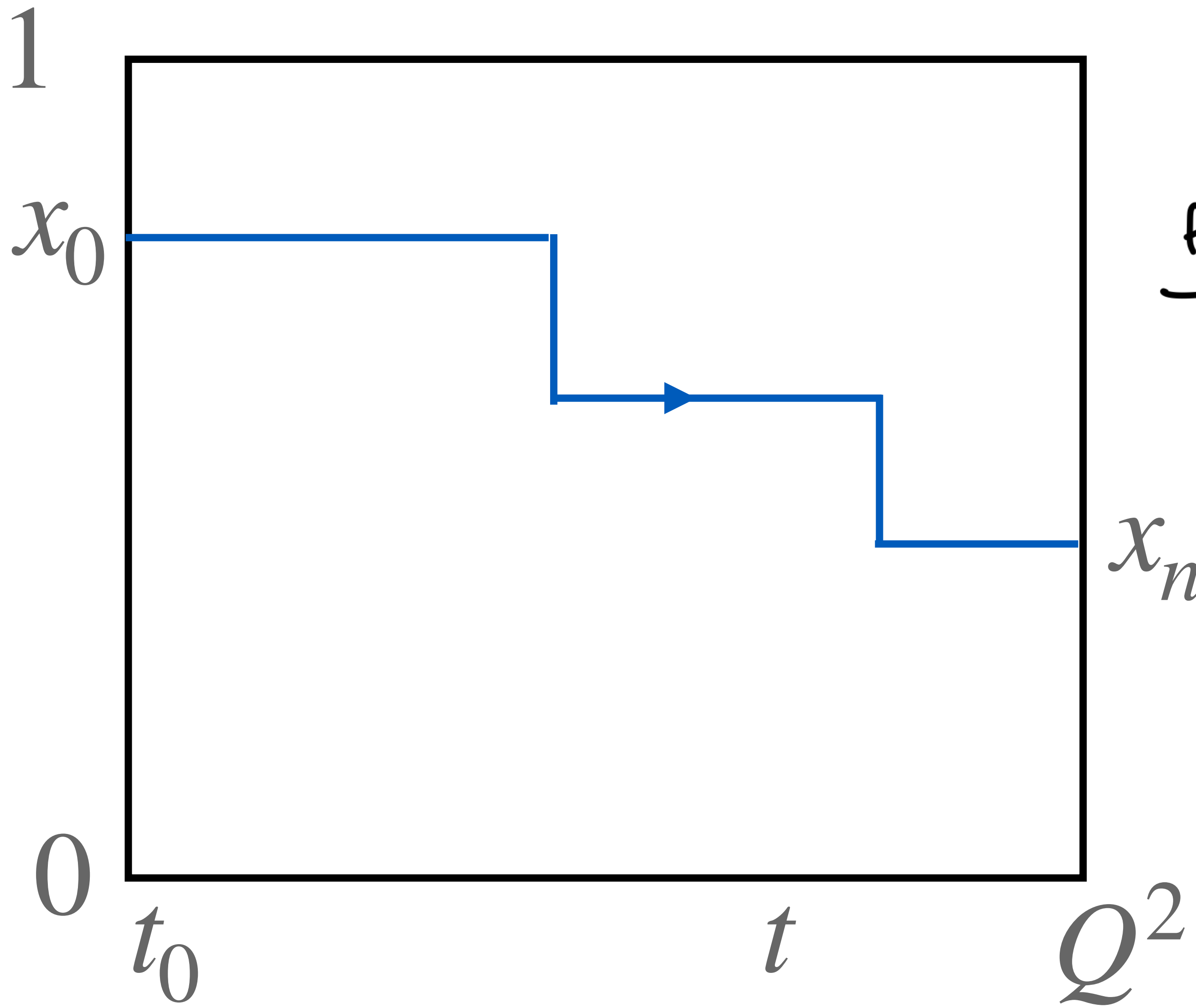


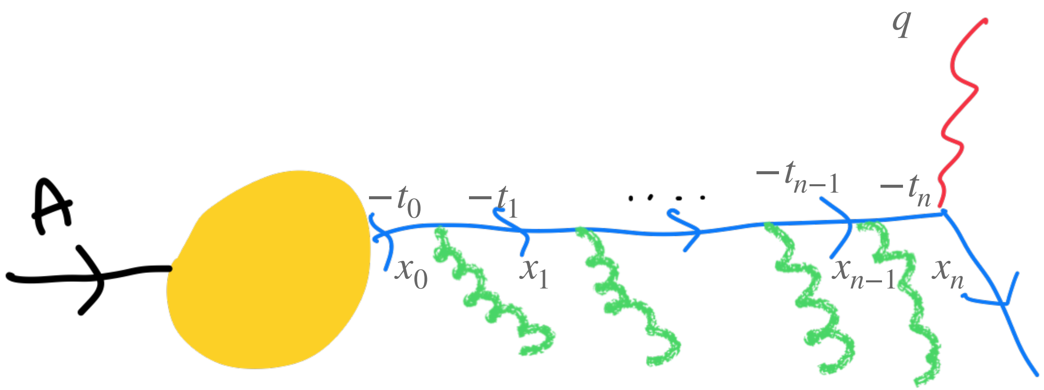
# Initial state Branching of partons

Finally the parton participates in a hard scattering with the photon which “sees” a Parton with distribution $f(x, Q^2)$.



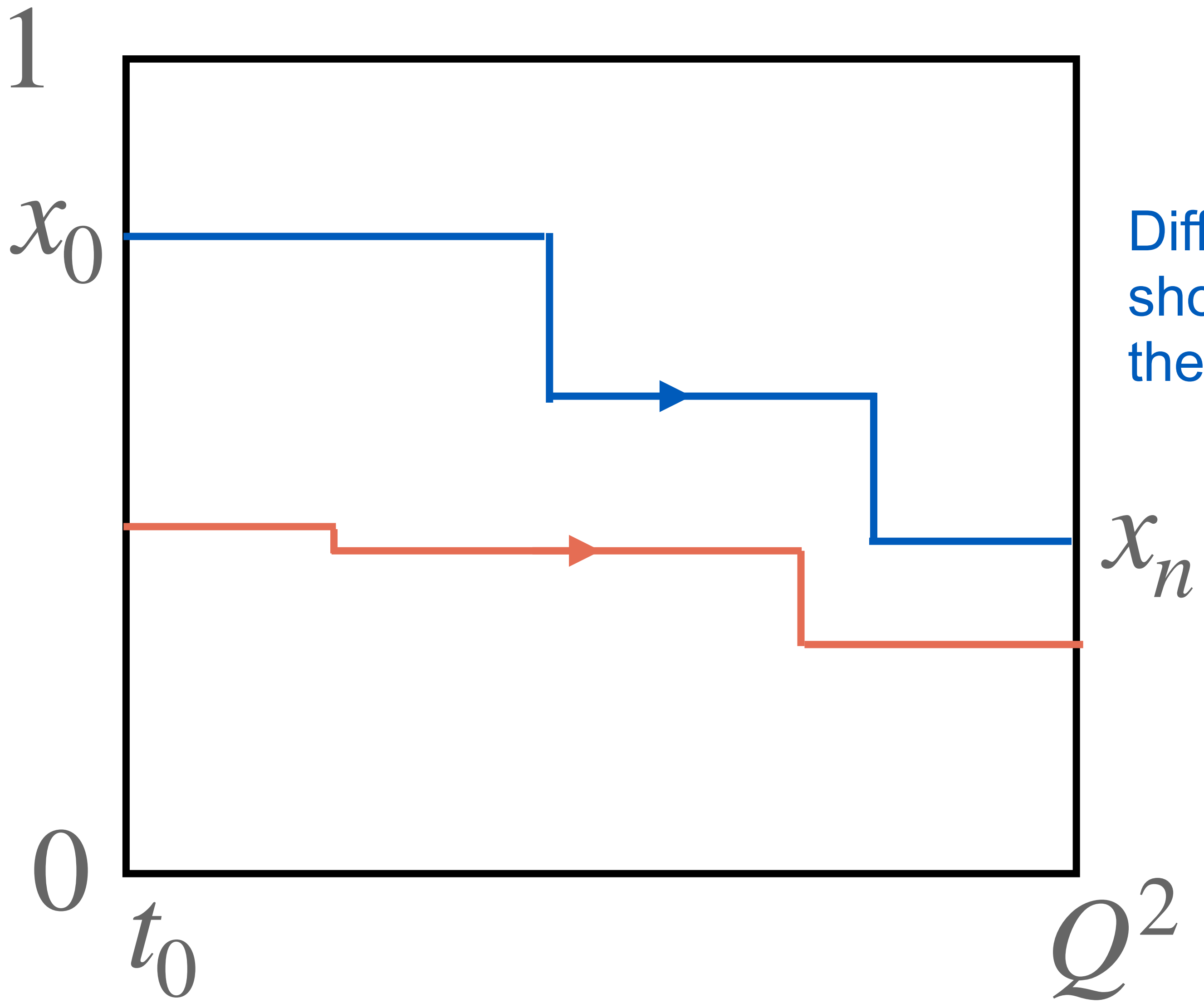
# Branching in (t,x) space



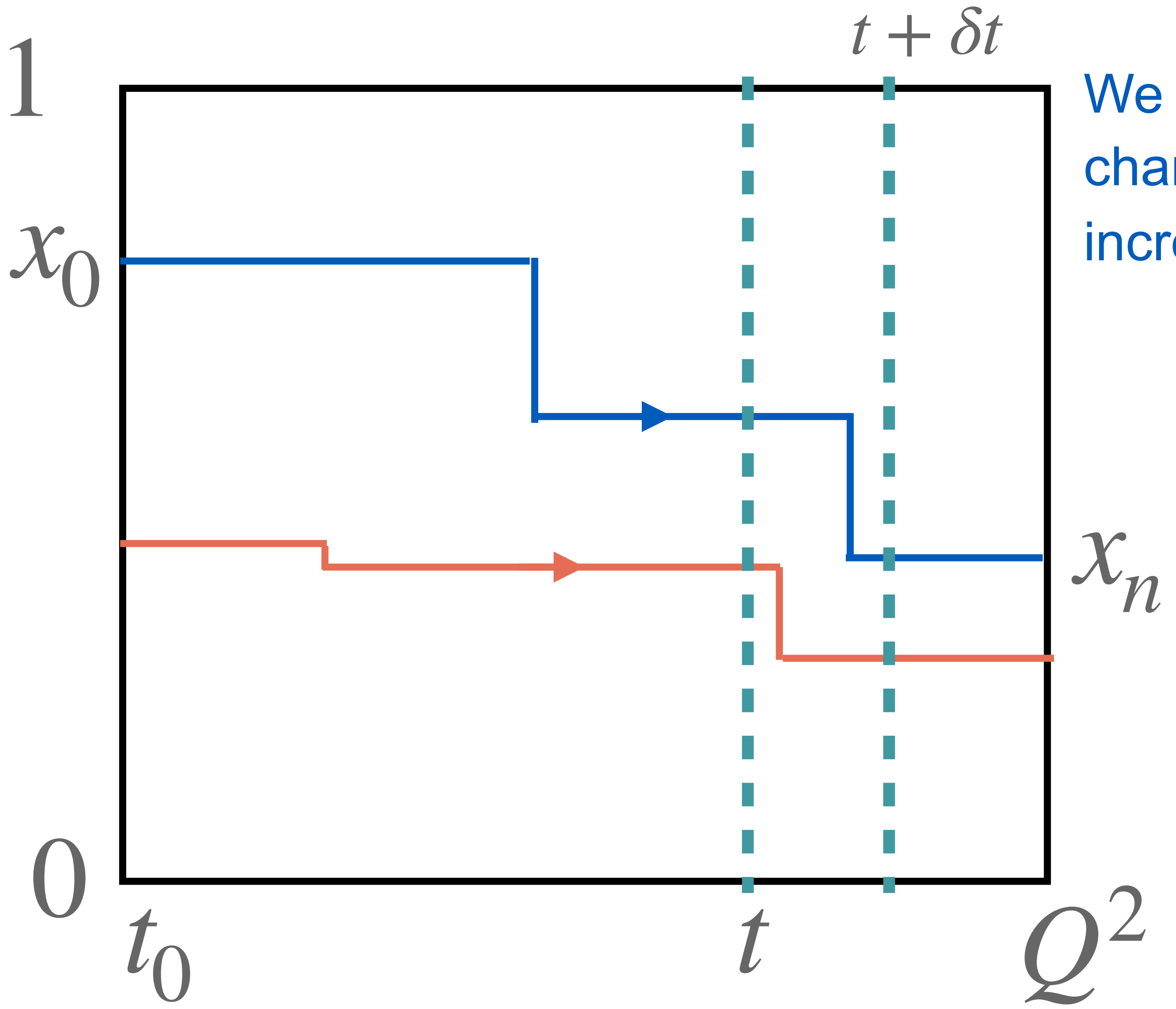

We can represent our branching history pictorially by thinking of it as a path through $(t, x)$ space. Moving from left to right and higher to lower as a series of steps

# Branching in (t,x) space



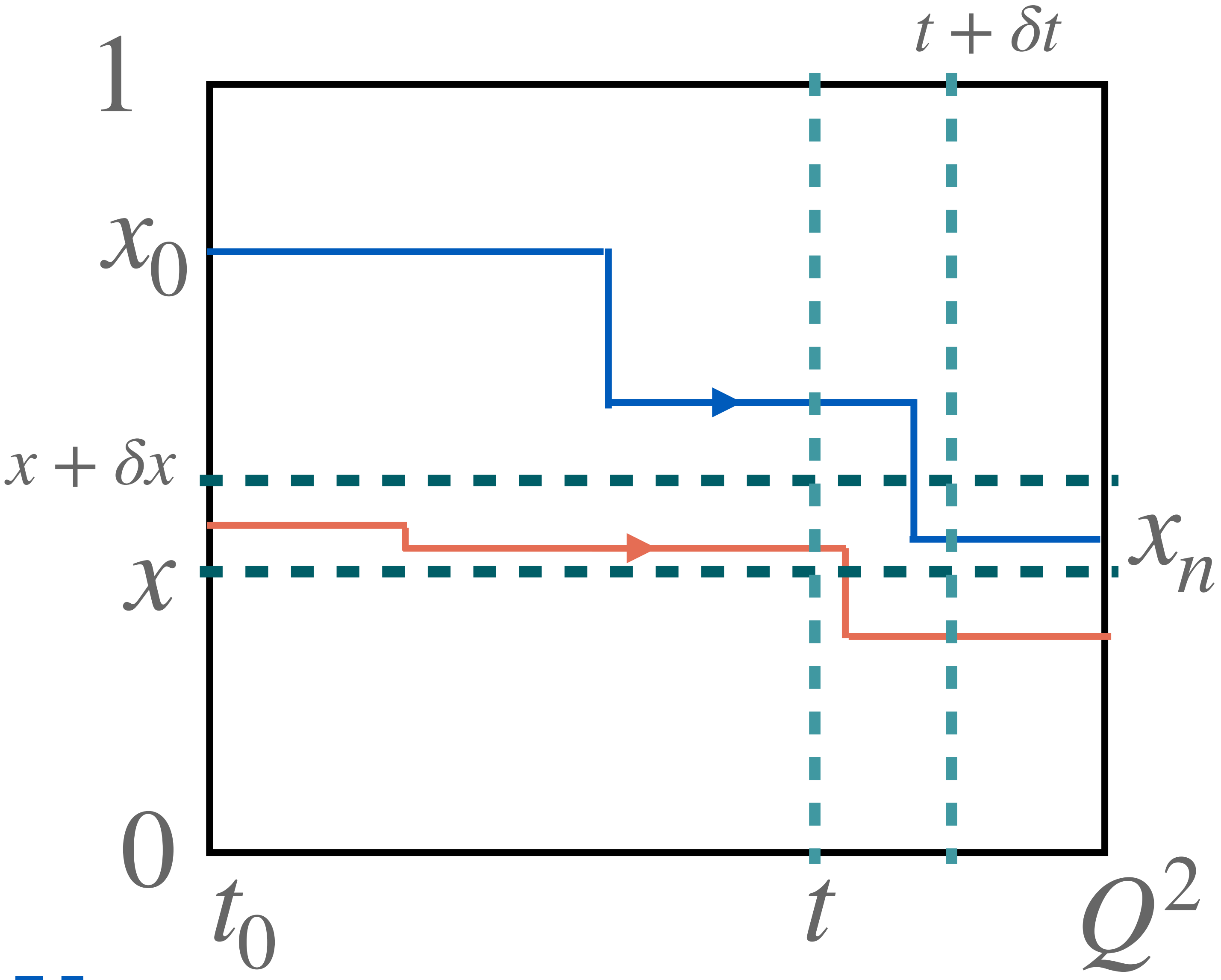

Different branching histories are shown as different paths through the space.

# Branching in (t,x) space




We would like to know the change in the PDFs when $t$ is increased to $t + \delta t$

# Branching in (t,x) space

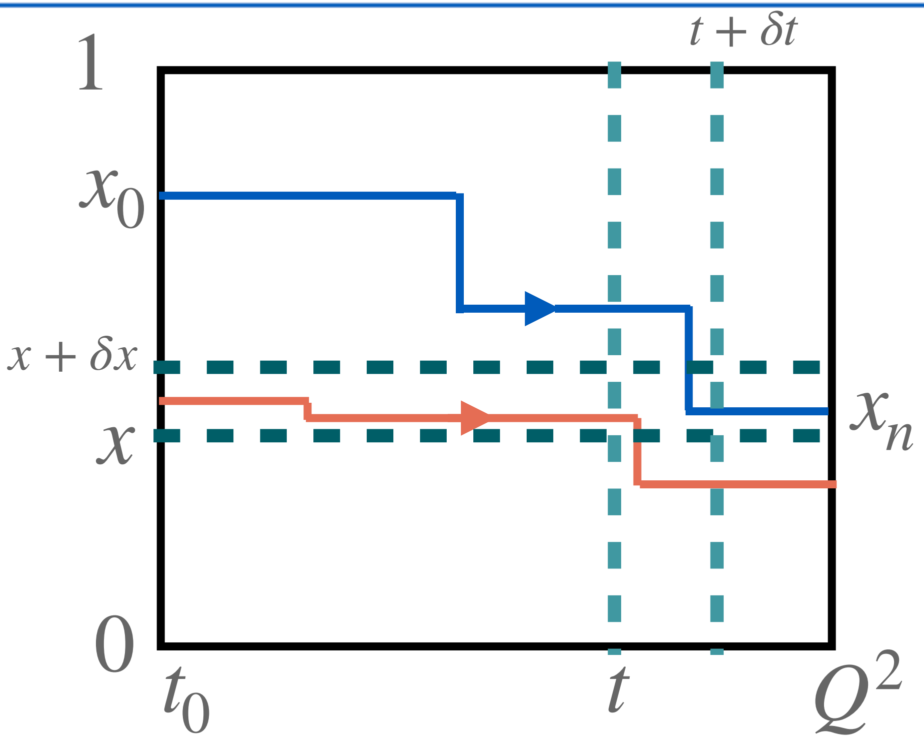



We would like to know the change in the PDFs when $t$ is increased to $t + \delta t$

We need to know the number of paths entering and leaving our element.

In this figure for instance one path enters and one path leaves.

# $\delta f_{in}$



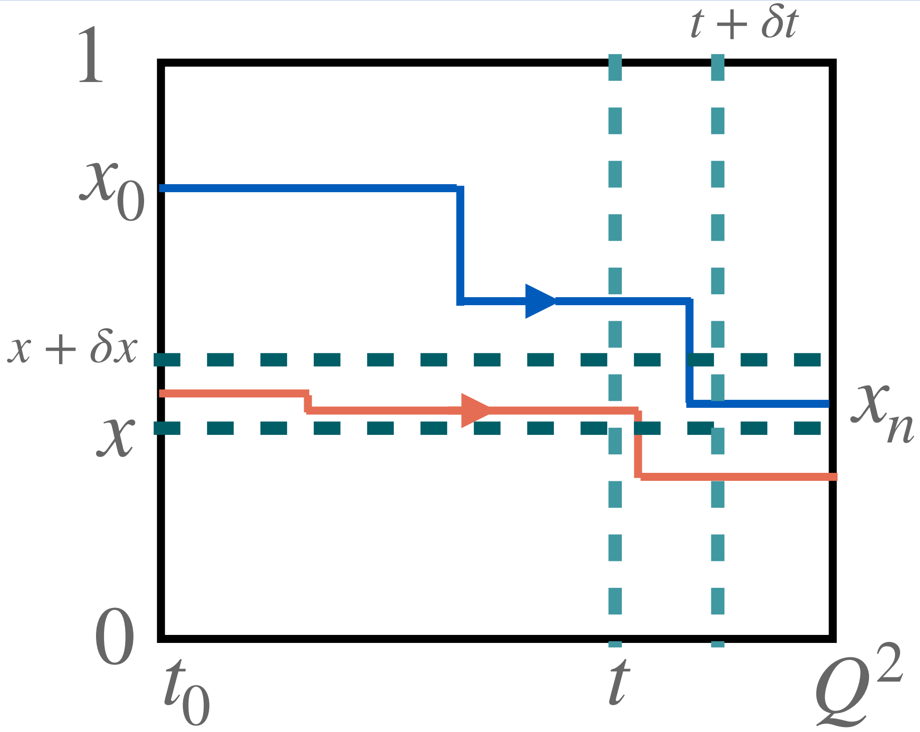

To find the number of paths in we integrate the branching probability times the Parton density overall higher momentum fractions $x' = x/z$

$$\delta f_{in}(x,t) = \frac{\delta t}{t}\int_x^1 dx'dz\frac{\alpha_S}{2\pi}\hat{P}(z)f(x',t)\delta(x - zx')$$

$$= \frac{\delta t}{t}\int_0^1 \frac{dz}{z}\frac{\alpha_S}{2\pi}\hat{P}(z)f(x/z,t)$$

Where $\hat{P}(z) = C_F(1+z^2)/(1-z)$ is the unregularized splitting function.

# $\delta f_{out}$




To find the number of paths leaving we integrate over all **lower** momentum fractions $x' = xz$

$$\delta f_{out}(x,t) = \frac{\delta t}{t} f(x,t) \int_x^1 dx' dz \frac{\alpha_S}{2\pi} \hat{P}(z) \delta(x' - zx)$$

$$= \frac{\delta t}{t} f(x,t) \int_0^1 dz \frac{\alpha_S}{2\pi} \hat{P}(z)$$

# $\delta f$

The net change in population is then

$$\delta f(x,t) = \delta f_{in} - \delta f_{out} = \frac{\delta t}{t}\int_0^1 dz \frac{\alpha_S}{2\pi}\hat{P}(z)\left[\frac{1}{z}f(x/z,t) - f(x,t)\right]$$

Which we can tidy up by using the plus prescription $P(z) = (\hat{P}(z))_+$

$$t\frac{\partial f(x,t)}{\partial t} = \int_x^1 \frac{dz}{z}\frac{\alpha_S}{2\pi}P(z)f(x/z,t)$$

# DGLAP Again!

Our equation

$$t\frac{\partial f(x,t)}{\partial t} = \int_x^1 \frac{dz}{z}\frac{\alpha_S}{2\pi}P(z)f(x/z,t)$$

Is nothing other than our old friend the DGLAP equation.

Recall we included only gluon emission from a quark line, if we want to generalized to include different types of partonic emissions we get a selection of coupled equations

$$t\frac{\partial f_i(x,t)}{\partial t} = \int_x^1 \frac{dz}{z}\frac{\alpha_S}{2\pi}P(z)_{ij}f(x/z,t)$$

# Sudakov form factor

In order to use the DGLAP equations to describe radiation of partons its convenient to introduce the following **Sudakov Form Factor**

$$\Delta(t) = \exp\left[-\int_{t_0}^{t}\frac{dt'}{t'}\int dz\frac{\alpha_S}{2\pi}\hat{P}(z)\right]$$

Note the (re)appearance of our hat, the spitting function here is **unregularized** $(\hat{P}(z) = C_F(1+z^2)/(1-z))$

Using the unregularized form will make a smoother transition to a numerical setup for a Monte Carlo approach.

# Sudakov form factor

$$\Delta(t) = \exp\left[-\int_{t_0}^{t}\frac{dt'}{t'}\int dz\frac{\alpha_S}{2\pi}\hat{P}(z)\right]$$

Starting from our equation : $t\dfrac{\partial f(x,t)}{\partial t} = \displaystyle\int_0^1 dz\frac{\alpha_S}{2\pi}\hat{P}(z)\left[\frac{1}{z}f(x/z,t) - f(x,t)\right]$

And re-writing in terms of $\Delta(t)$, we obtain

$$t\frac{\partial f(x,t)}{\partial t} = \int_0^1 dz\frac{\alpha_S}{2\pi}\hat{P}(z)\frac{1}{z}f(x/z,t) + \frac{f(x,t)}{\Delta(t)}t\frac{\partial \Delta(t)}{\partial t}$$

Or

$$t\frac{\partial}{\partial t}\left(\frac{f(x,t)}{\Delta}\right) = \frac{1}{\Delta}\int_0^1 dz\frac{\alpha_S}{2\pi}\hat{P}(z)\frac{1}{z}f(x/z,t)$$

# Sudakov Form Factor

$$t\frac{\partial}{\partial t}\left(\frac{f(x,t)}{\Delta}\right) = \frac{1}{\Delta}\int_0^1 dz \frac{\alpha_S}{2\pi}\hat{P}(z)\frac{1}{z}f(x/z,t)$$

This equation is of the same form as the DGLAP equations, but with two changes

1) $f \rightarrow f/\Delta$

2) $P(z) \rightarrow \hat{P}(z)$

# Integrating our equation

$$t\frac{\partial}{\partial t}\left(\frac{f(x,t)}{\Delta}\right) = \frac{1}{\Delta}\int_0^1 dz \frac{\alpha_S}{2\pi}\hat{P}(z)\frac{1}{z}f(x/z,t)$$

We can understand the equation in a more physical form if we write it as an integral equation, starting from some initial parton distribution boundary condition $f(x, t_0)$

$$f(x,t) = \Delta(t)f(x,t_0) + \int_{t_0}^{t}\frac{dt'}{t'}\frac{\Delta(t)}{\Delta(t')}\int\frac{dz}{z}\frac{\alpha_S}{2\pi}\hat{P}(z)f(x/z,t')$$

No branching happens between scales $t_0$ and t

Contribution from all paths which have their last branching scale at t'

# Sudakov Form factor

$$\Delta(t) = \exp\left[-\int_{t_0}^{t}\frac{dt'}{t'}\int dz\frac{\alpha_S}{2\pi}\hat{P}(z)\right]$$

Given this interpretation it makes sense to think about $\Delta(t)$ as the probability of evolving from the scale $t_0$ to $t$ **without** branching.

$$f(x,t) = \Delta(t)f(x,t_0) + \int_{t_0}^{t}\frac{dt'}{t'}\frac{\Delta(t)}{\Delta(t')}\int\frac{dz}{z}\frac{\alpha_S}{2\pi}\hat{P}(z)f(x/z,t')$$

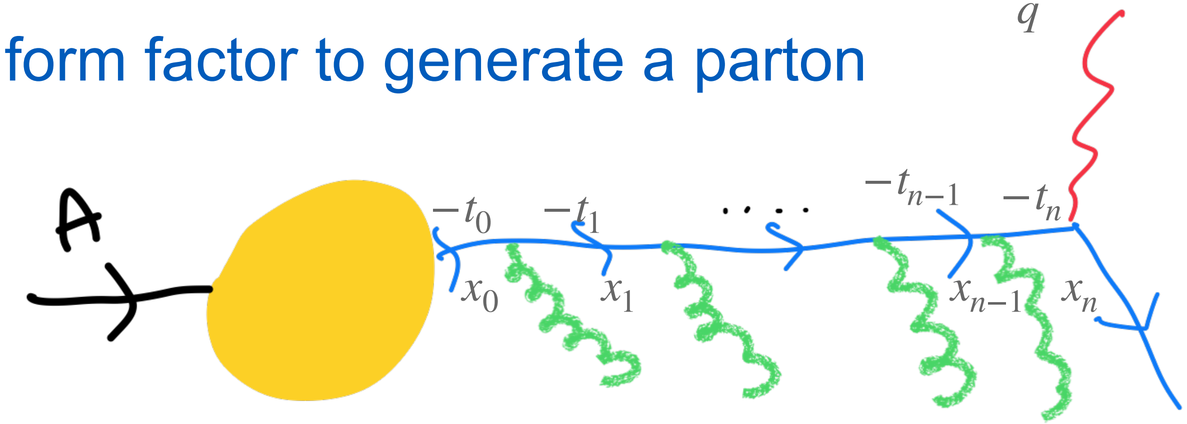
Pre-factor determines probability that no branching took place.

$\Delta(t)/\Delta(t')$ is the probability of going from $t'$ to $t$ without branching.

# A parton Shower

Lets sketch out how we can use the Sudakov form factor to generate a parton shower.

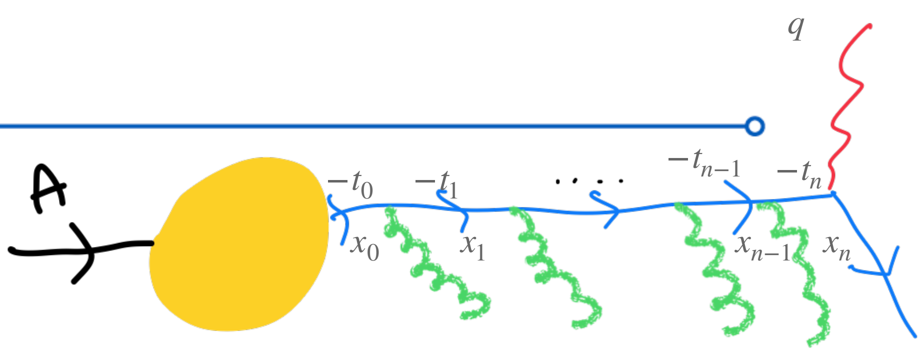
We'll think about how to generate a final state shower first.




To do this we begin with a starting scale defined by the hard process $Q^2 = t_1$

We want to generate the probability of evolving downwards to a lower scale $t_2$ without branching.

# A parton Shower



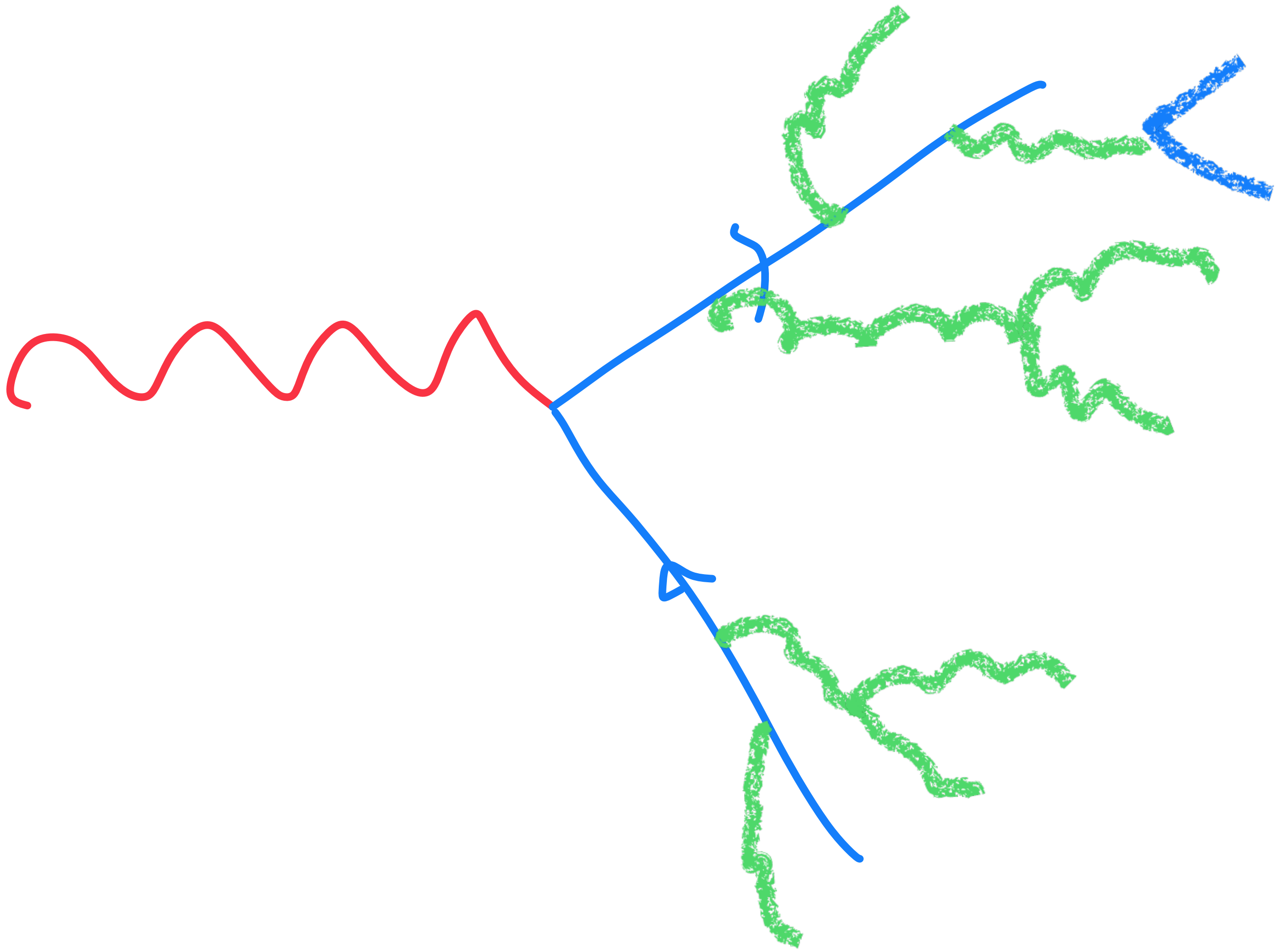

We can generate a probability distribution for $t_2$ by solving the equation $\frac{\Delta(t_1)}{\Delta(t_2)} = R$, where $R$ is a random number generated between [0,1].

Once we have a $t_2$ we have a scale for the next branching, so need to generate a value of the momentum fraction $z = x_2/x_1$ which we do by solving

$$\int_{\epsilon}^{x_2/x_1} dz \frac{\alpha_S}{2\pi} P(z) = R' \int_{\epsilon}^{1-\epsilon} dz \frac{\alpha_S}{2\pi} P(z)$$

Where R’ is another random number. (Finally you generate a random azimuthal angle between [0,2$\pi$] and you’re all set!).

# A parton shower



After each branching we start the algorithm again, for both of the two daughter particles.

What we have done is effectively made an algorithm that will (randomly) generate QCD splittings with the correct probability distribution.

This idea forms the basis of the parton showers used in event generators.

# A couple of caveats

What we have discussed so far has a couple of technical caveats worth mentioning.

First, what we discussed was for a single splitting, so our formulas are relevant for radiating gluons from quarks, but need to be modified to allow for gluon splitting and quark-antiquark production from a gluon.

Secondly, we have to be careful, since there is a soft singularity at $z \to 1$ in the splitting function I neglected. In general our current setup generates a collinear parton shower, but does not correctly model the other interesting IR region, related to soft emissions of gluons.

# A couple of caveats

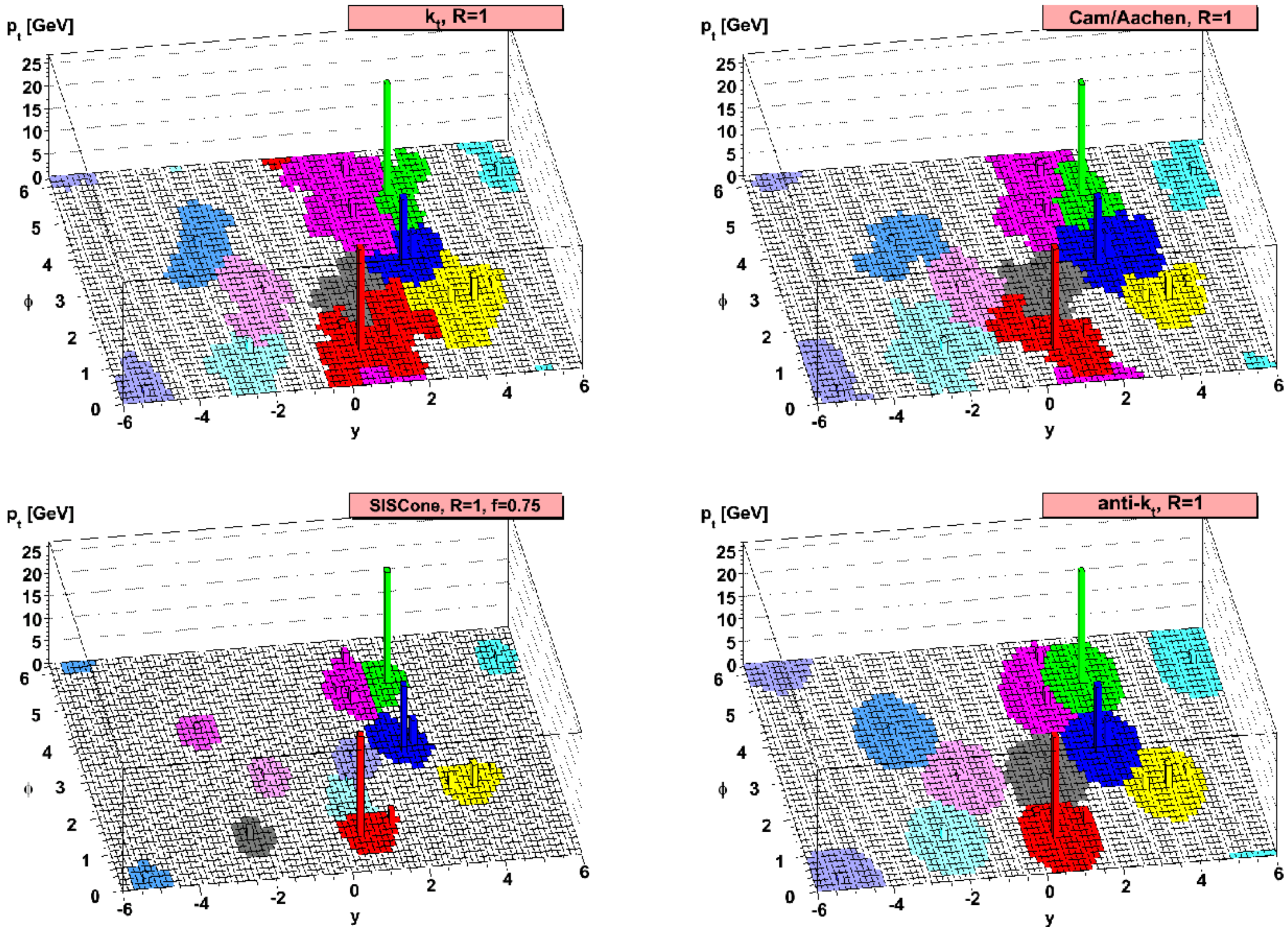
Solving both of these issues is beyond the scope of this lecture. But the methodology is the same in both cases, a modified Sudakov form factor $\hat{\Delta}(t)$. Using the modified Sudakov will result in a corrected shower.

First, what we discussed was for a single splitting, so our formulas are relevant for radiating gluons from quarks, but need to be modified to allow for gluon splitting and quark-antiquark production from a gluon.

Including multiple parton species in our collinear setup isnt too bad,  in that instance we just sum over allowed splittings in the exponent

$$\Delta_i(t) = \exp\left[-\sum_j \int_{t_0}^{t} \int dz \frac{\alpha_S}{2\pi} \hat{P}_{ji}(z)\right]$$

# Jets recap.




At the LHC there are a few different options for jet clustering.

All take in a cone size $R$ which sets the size of the jet in the $(y, \phi)$ plane

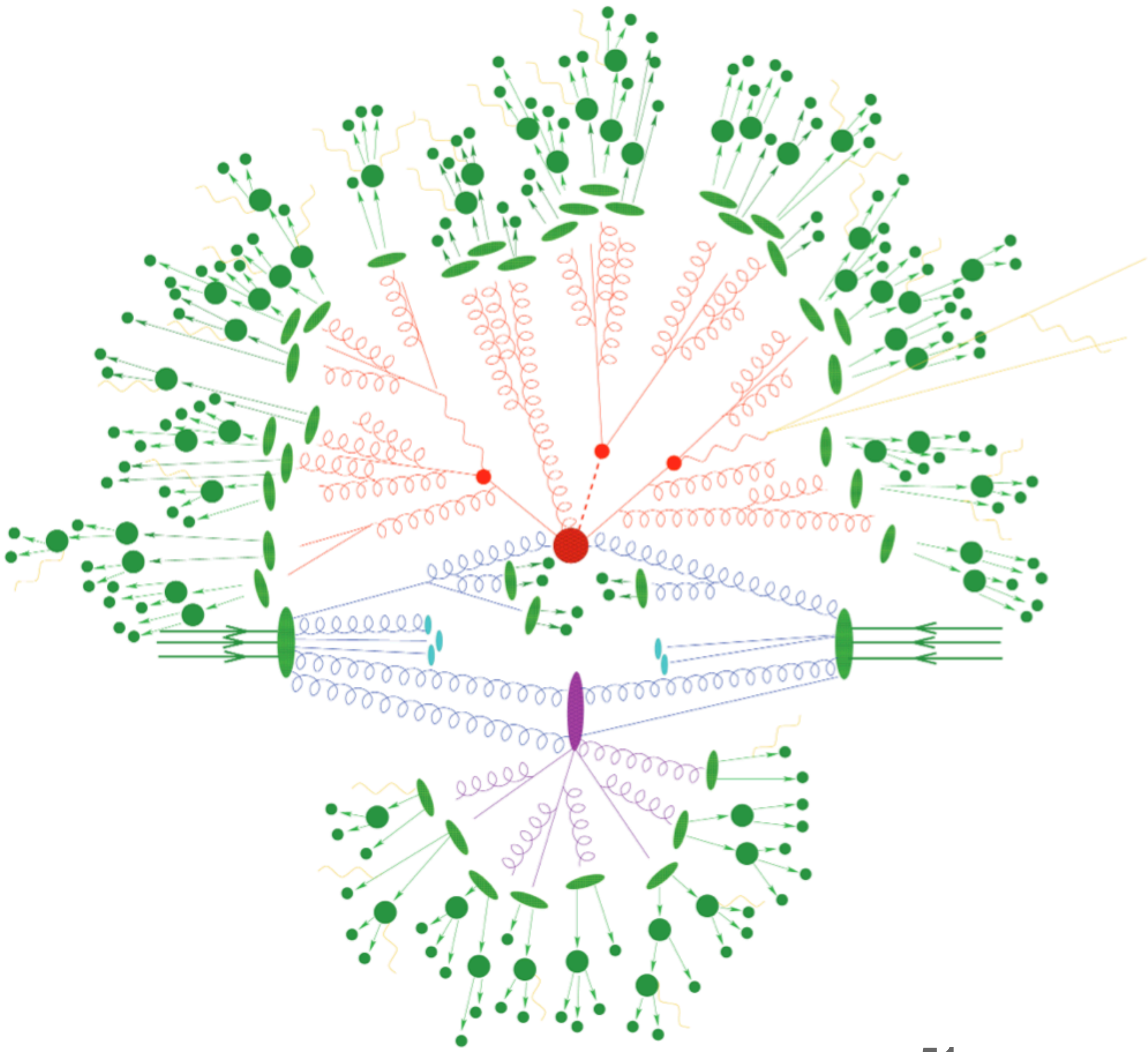
They combine partons based on comparing the measure:

$$d_{ij} = \min(k_{T,i}^{2p}, (k_{T,j}^{2p})\frac{\Delta_{ij}^2}{R^2}$$

Where $p = \{1,0,-1\}$ defines the $k_T$ , C/A or anti-$k_T$ algorithm.

# An event at the LHC.

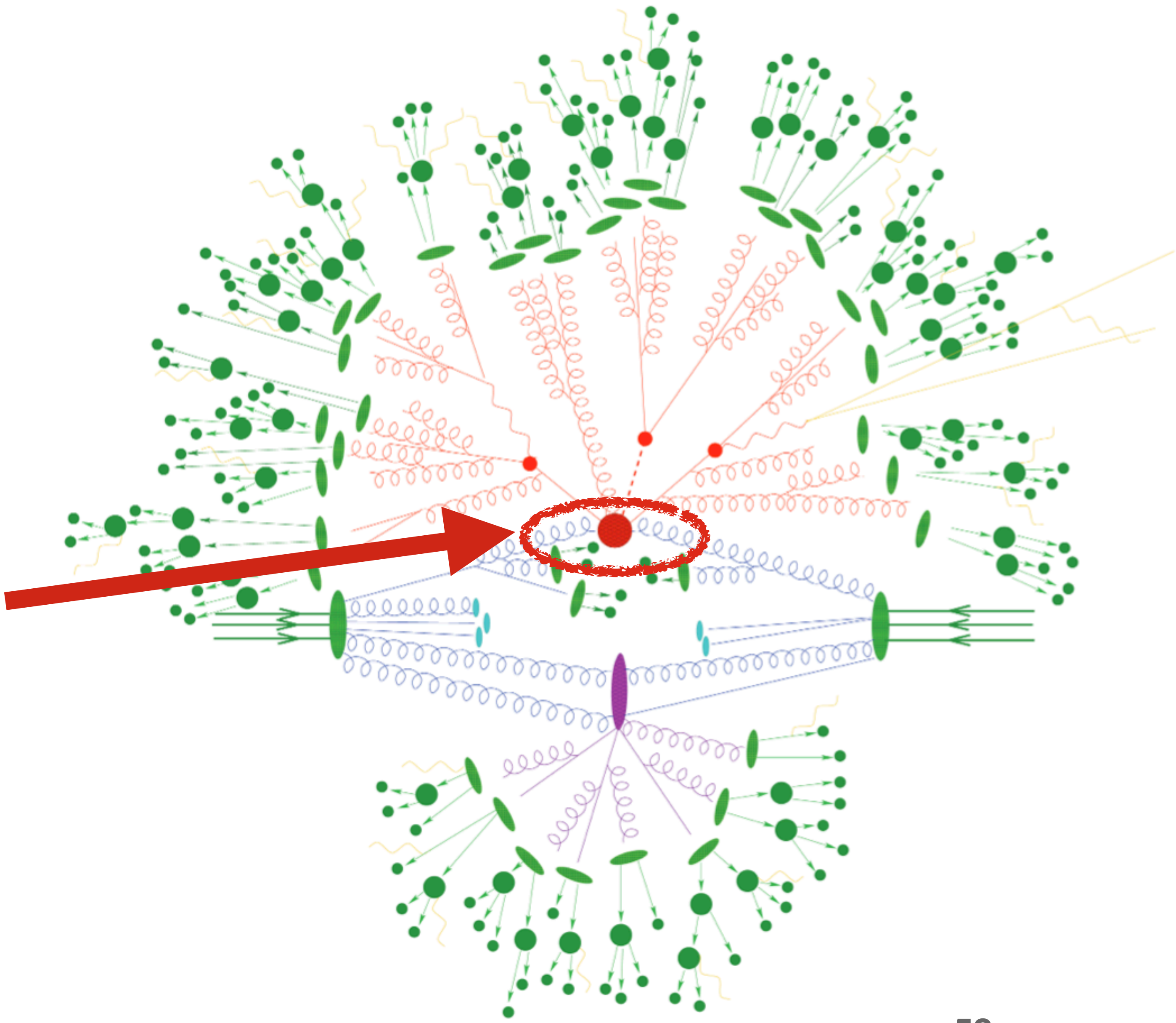
Although brief in detail, our lectures have tried to cover the relevant physics to describe an LHC event sketched out in the (famous) figure
(Taken from the black book, a famous Sherpa figure)



# An event at the LHC.

Although brief in detail, our lectures have tried to cover the relevant physics to describe an LHC event sketched out in the (famous) figure
(Taken from the black book, its a famous Sherpa figure)

Hard scattering, described by perturbative QCD at high scales, calculated in terms of free ingoing and outgoing partons $\hat{\sigma}(x_1, x_2)$



# An event at the LHC.

Although brief in detail, our lectures have tried to cover the relevant physics to describe an LHC event sketched out in the (famous) figure
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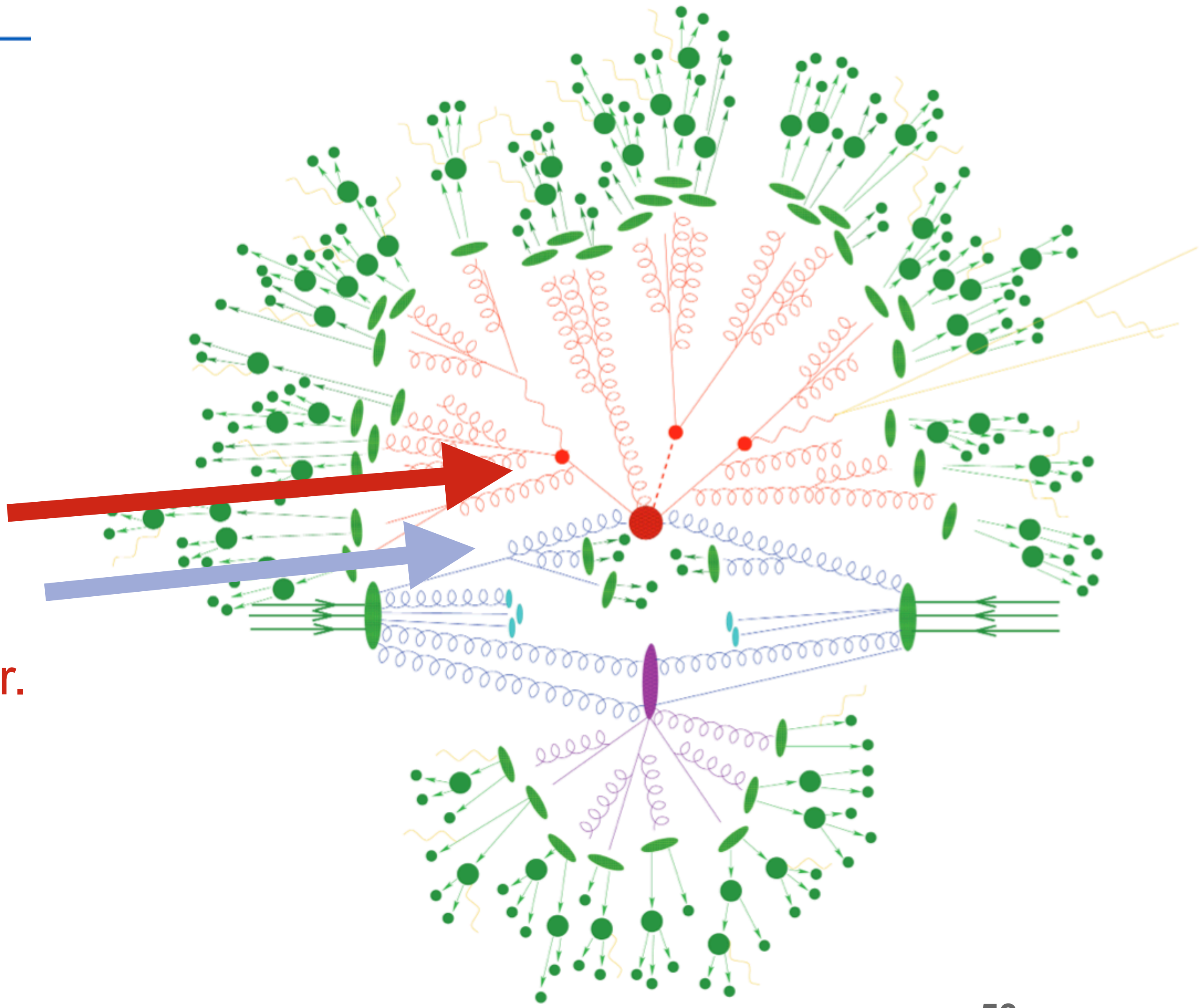
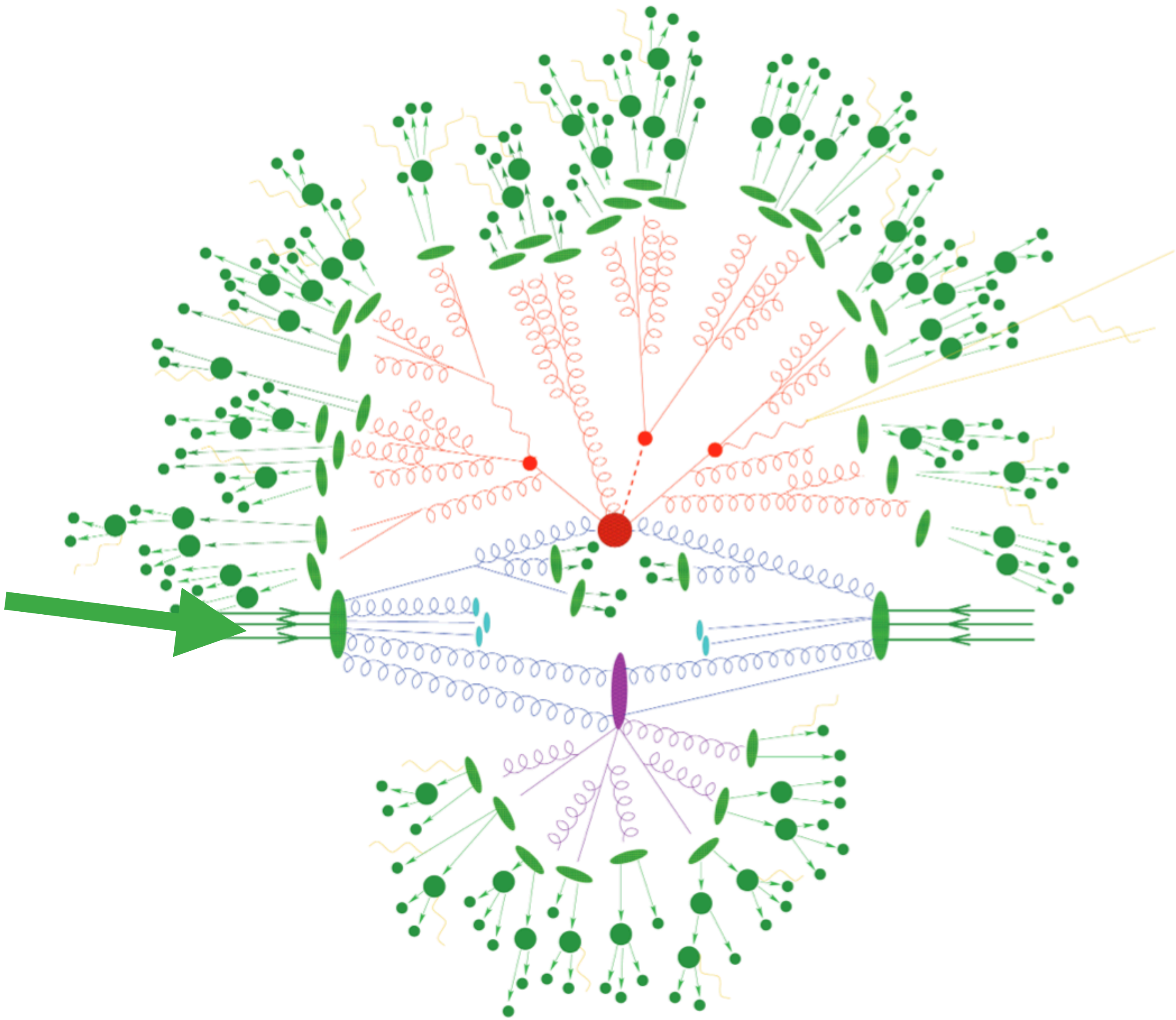
Evolution of partons using the DGLAP equations resulting in emission of initial and final state radiation through the parton shower.



# An event at the LHC.

Although brief in detail, our lectures have tried to cover the relevant physics to describe an LHC event sketched out in the (famous) figure
(Taken from the black book, a famous Sherpa figure)

Initial state proton, studied and understood from knowledge of DIS and other experiments to understand parton distribution functions.

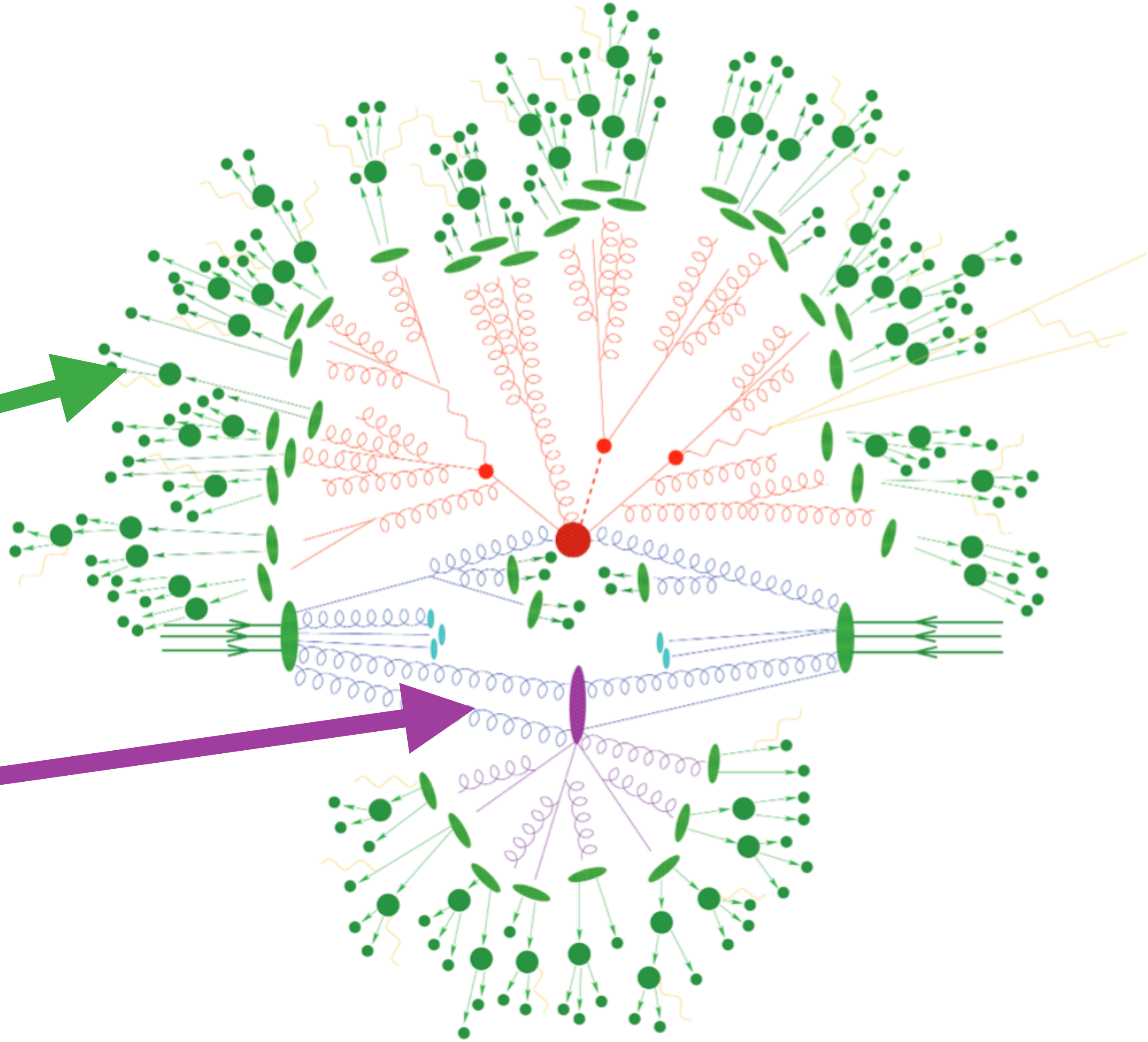


# An event at the LHC.

We didnt have a chance to discuss all the gory details however..

Hadronization model to turn shower products into observable particles after showering reaches cutoff scale $t_0$

Underlying event which models interaction between proton remnants.



# QCD Summary

Perturbative QCD is a huge topic, I've tried to boil down some of the fundamental topics that come up again and again.

The QCD coupling is not particularly small at LHC collier energies. Higher order predictions are crucial to understand data.

Renormalization of both the strong coupling and the parton distribution functions introduce arbitrary scales $\mu_R$ and $\mu_F$

Since nature doesn't depend on these scales the invariance of our predictions upon variation of the scales allows us to constrain the type of behavior which is acceptable theoretically.

# QCD summary

We saw examples of this through the running of $\alpha_S$, the DGLAP evolution of the parton distribution functions, and constructing Sudakov form factors for parton showers.