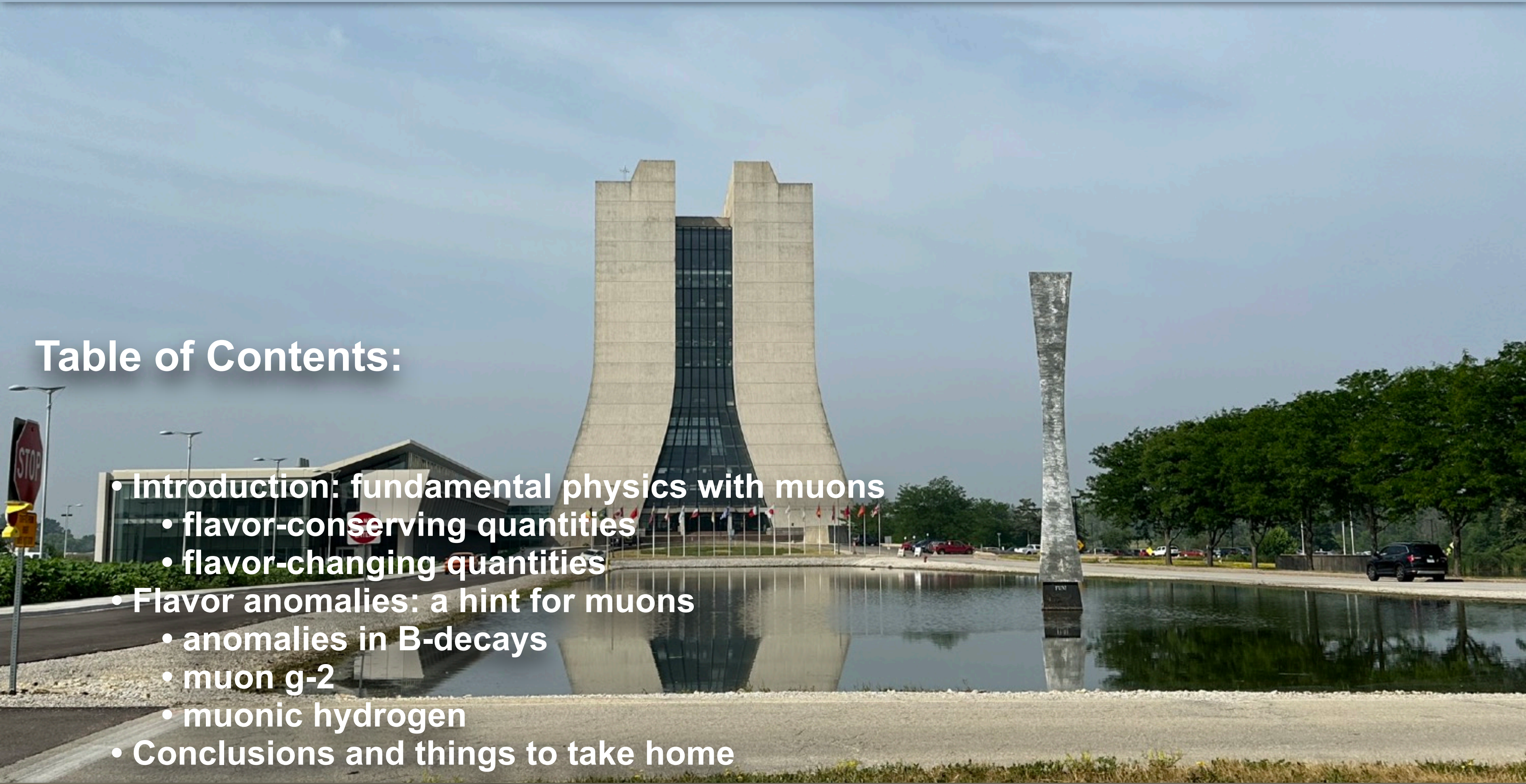
Theoretical Foundations of Flavor Physics III

Charm and Leptons



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B physics (cont.)

3. Time-dependent CP-asymmetries

★ Time-dependent CP-asymmetries probe CP-violation in $\Delta B=2$ amplitudes

- SM: CP-violation in $\Delta B=2$ and $\Delta B=1$ transitions have the same origin, this fact does not have to be true in general NP model

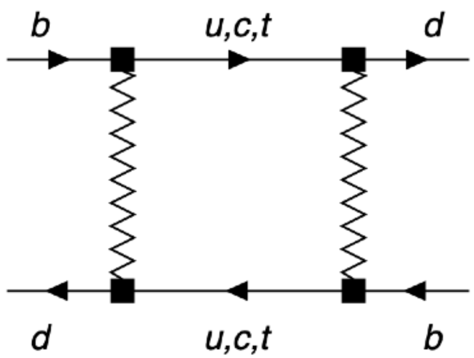
- it most conveniently can be probed in transitions that involve mixing
  - use time-dependent CP asymmetries due to the interference between B-mixing and B decay amplitudes
  - interference between the two neutral B meson evolution eigenstates generates the time-dependent CP asymmetry

$$a_{CP}(f,t) = \frac{\Gamma(B(t) \to f) - \Gamma(\overline{B}(t) \to \overline{f})}{\Gamma(B(t) \to f) + \Gamma(\overline{B}(t) \to \overline{f})}$$

- Need to develop a formalism for time-dependent decays

Time dependent decay amplitudes

★ In the SM, neutral B-mesons can mix via weak interaction diagrams

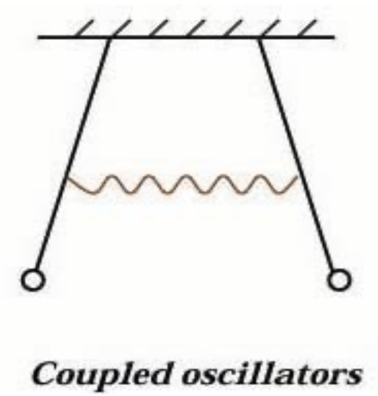


- only at one loop in the Standard Model, so can be sensitive to possible quantum effects due to new physics particles

- $\Delta B = 2$ interactions couple dynamics of B^0 and $\bar{B}^0$

- We need to study simultaneous time evolution,

$$|B(t)\rangle = \left[\begin{array}{c} a(t) \\ b(t) \end{array} \right] = a(t)|B^0\rangle + b(t)|\overline{B}^0(t)\rangle$$



- This is very similar to the case of coupled pendula in classical mechanics

Time dependent decay amplitudes

- Time dependence: coupled Schrodinger equations
  - note that CPT-invariance requires that $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$

$$i\frac{d}{dt}|B(t)\rangle = \left[M - i\frac{\Gamma}{2}\right]|B(t)\rangle \equiv \left[\begin{array}{cc} A & p^2 \\ q^2 & A \end{array}\right]|B(t)\rangle$$

Q: this Hamiltonian is clearly non-hermitian! What is goin on?

- Non-diagonal Hamiltonian: need to diagonalize the mass matrix

$$|B_L\rangle = p|B^0\rangle + q|\overline{B}^0\rangle$$
$$|B_H\rangle = p|B^0\rangle - q|\overline{B}^0\rangle$$

("switch from flavor to mass eigenstates")

- In the mass basis the mass matrix is diagonal, i.e.

$$Q^{-1}\left[M - i\frac{\Gamma}{2}\right]Q = \left(\begin{array}{cc} M_L - i\Gamma_L/2 & 0 \\ 0 & M_H - i\Gamma_H/2 \end{array}\right)$$

- … with mass and lifetime differences: $\Delta M = M_H - M_L$ & $\Delta\Gamma = \Gamma_L - \Gamma_H$

Note that $m = \frac{M_H + M_L}{2} = M_{11} = M_{22}$ & $\Gamma = \frac{\Gamma_L + \Gamma_H}{2} = \Gamma_{11} = \Gamma_{22}$

Time dependent decay amplitudes

- The transformation matrices that diagonalize the Hamiltonian are

$$Q = \begin{pmatrix} p & p \\ q & -q \end{pmatrix} \quad \text{and} \quad Q^{-1} = \frac{1}{2pq}\begin{pmatrix} q & p \\ q & -p \end{pmatrix}$$

- To find the time development of the flavor eigenstates one needs to transform the evolution equation back to the flavor basis

$$\begin{bmatrix} |B^0(t)\rangle \\ |\overline{B}^0(t)\rangle \end{bmatrix} = Q \begin{pmatrix} e^{-iM_L - \Gamma_L/2} & 0 \\ 0 & e^{-iM_H - \Gamma_H/2} \end{pmatrix} Q^{-1} \begin{bmatrix} |B^0\rangle \\ |\overline{B}^0\rangle \end{bmatrix}$$

- … which gives for the time evolution matrix in the flavor basis

$$Q \begin{pmatrix} e^{-iM_L t - \Gamma_L t/2} & 0 \\ 0 & e^{-iM_H t - \Gamma_H t/2} \end{pmatrix} Q^{-1} = \begin{pmatrix} g_+(t) & \frac{q}{p} g_-(t) \\ \frac{p}{q} g_-(t) & g_+(t) \end{pmatrix}$$

Nierste

with

$$g_+(t) = e^{-imt}\, e^{-\Gamma t/2} \left[\cosh \frac{\Delta\Gamma\, t}{4} \cos \frac{\Delta M\, t}{2} - i \sinh \frac{\Delta\Gamma\, t}{4} \sin \frac{\Delta M\, t}{2} \right],$$

$$g_-(t) = e^{-imt}\, e^{-\Gamma t/2} \left[-\sinh \frac{\Delta\Gamma\, t}{4} \cos \frac{\Delta M\, t}{2} + i \cosh \frac{\Delta\Gamma\, t}{4} \sin \frac{\Delta M\, t}{2} \right].$$

Time dependent decay amplitudes

- This procedure provides a picture of how B-states evolve due to flavor oscillations,

$$|B^0(t)\rangle = g_+(t)|B^0\rangle + \frac{q}{p} g_-(t)|\overline{B}^0\rangle$$
$$|\overline{B}^0(t)\rangle = \frac{p}{q} g_-(t)|B^0\rangle + g_+(t)|\overline{B}^0\rangle$$

with

$$\begin{aligned} g_+(t) &= e^{-imt} e^{-\Gamma t/2} \left[\cosh\frac{\Delta\Gamma\, t}{4} \cos\frac{\Delta M\, t}{2} - i \sinh\frac{\Delta\Gamma\, t}{4} \sin\frac{\Delta M\, t}{2} \right], \\ g_-(t) &= e^{-imt} e^{-\Gamma t/2} \left[-\sinh\frac{\Delta\Gamma\, t}{4} \cos\frac{\Delta M\, t}{2} + i \cosh\frac{\Delta\Gamma\, t}{4} \sin\frac{\Delta M\, t}{2} \right]. \end{aligned}$$

- The only thing left is to relate q/p, ΔM and $\Delta\Gamma$ to original parameters of H

secular equation: $$(\Delta M + i\frac{\Delta\Gamma}{2})^2 = 4\left(M_{12} - i\frac{\Gamma_{12}}{2}\right)\left(M_{12}^* - i\frac{\Gamma_{12}^*}{2}\right)$$

Re

Im

$$(\Delta M)^2 - \frac{1}{4}(\Delta\Gamma)^2 = 4|M_{12}|^2 - |\Gamma_{12}|^2 \qquad \Delta M\, \Delta\Gamma = -4\,\mathrm{Re}\,(M_{12}\Gamma_{12}^*)$$

- Finally, the ratio $$\frac{q}{p} = -\frac{\Delta M + i\,\Delta\Gamma/2}{2M_{12} - i\,\Gamma_{12}} = -\frac{2M_{12}^* - i\,\Gamma_{12}^*}{\Delta M + i\,\Delta\Gamma/2}$$

Phases and amplitudes

- The B-meson states can have an arbitrary phase, so only relative phase is physical, which implies that there are three quantities that define B-mixing

$$|M_{12}|, \qquad |\Gamma_{12}|, \qquad \text{and} \quad \phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

- … which gives for the mixing parameters

$$\Delta M \simeq 2\,|M_{12}| \quad \text{and} \quad \Delta\Gamma \simeq 2\,|\Gamma_{12}|\cos\phi$$

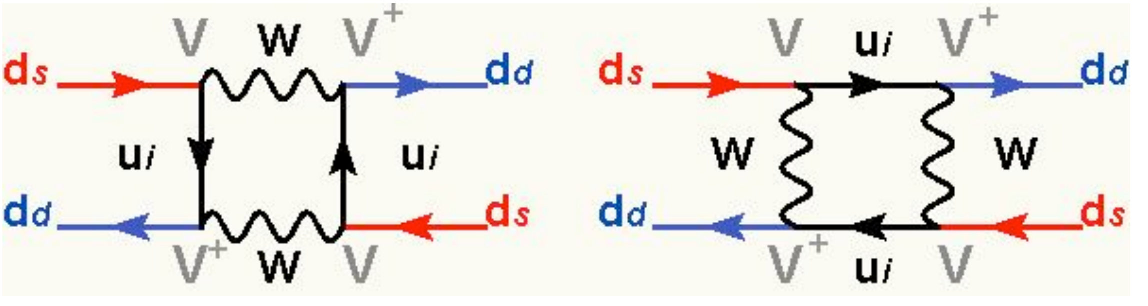
- … and, up to a good approximation, to the phase of the box diagram,

$$\frac{q}{p} = -\frac{M_{12}^*}{M_{12}} = \frac{V_{tb}^* V_{tq}}{V_{tb} V_{tq}^*} \qquad \text{and} \qquad \left|\frac{q}{p}\right|^2 = 1 - a = 1 - Im\frac{\Gamma_{12}}{M_{12}}$$

We can calculate B-mixing parameters in the SM: any sign of New Physics?

Glashow-Iliopoulos-Maiani (GIM) **mechanism**

- There are no ΔQ=2 interactions in the Standard Model…
- … but we can make them via a "two-step process" (loop diagram):



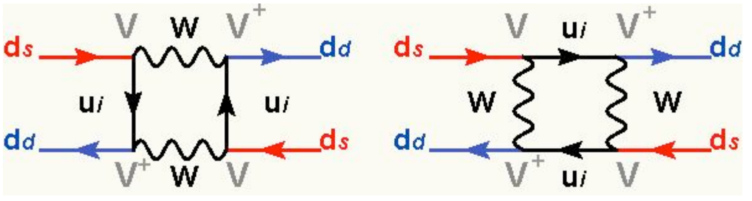
- Let's calculate them! For each internal quark type we get

$$\sim g^4 \left(V_{is} V_{id}^\dagger V_{js} V_{jd}^\dagger\right) \int \frac{d^4k}{(4\pi)^4} \frac{(\text{some gamma matrices})\ (k^2)}{(\not{k} - m_i)(\not{k} - m_j)(k^2 - m_W^2)^2}$$

Divergent: not good…

GIM-mechanism

- However, CKM matrix is unitary:
- contribution of different internal flavors comes with different signs!



top: $\left(V_{tb}V_{td}^{\dagger}V_{tb}V_{td}^{\dagger}\right) \sim (1\times A\lambda^3)(1\times A\lambda^3)$

top-charm: $\left(V_{tb}V_{td}^{\dagger}V_{cb}V_{cd}^{\dagger}\right) \sim (1\times A\lambda^3)(A\lambda^2\times(-\lambda))$

- Thus, in the limit where k >> m_i, m_j, M_W:

top: $$g^4\left(A\lambda^3\right)^2\int\frac{d^4k}{(4\pi)^4}\frac{(\text{some gamma matrices})(k^2)}{(\not{k})(\not{k})(k^2)^2}$$

top-charm: $$-g^4\left(A\lambda^3\right)^2\int\frac{d^4k}{(4\pi)^4}\frac{(\text{some gamma matrices})(k^2)}{(\not{k})(\not{k})(k^2)^2}$$

... and similarly for other quarks   Cancellation of divergences!

$$\boxed{A\propto\sum_i m_i^2(V_{is}V_{ib}^*)^2 g_k(m_i^2)}$$

Glashow-Iliopulous-Maiani

Time-dependent CP-asymmetries

★ Time-dependent CP-asymmetries probe CP-violation in ΔB=2 amplitudes

- Now we know how to deal with time-dependent rates

$$\Gamma(M(t) \to f) = \mathcal{N}_f \left|\langle f|S|M(t)\rangle\right|^2$$

$$\Gamma(\overline{M}(t) \to f) = \mathcal{N}_f \left|\langle f|S|\overline{M}(t)\rangle\right|^2$$

- … which can be calculated using the developed formalism, $\lambda_f = \frac{q}{p}\frac{\overline{A}_f}{A_f}$

$$\begin{aligned}\Gamma(M(t) \to f) \;&=\; \mathcal{N}_f\,|A_f|^2\,e^{-\Gamma t}\left\{\frac{1+|\lambda_f|^2}{2}\cosh\frac{\Delta\Gamma\,t}{2} + \frac{1-|\lambda_f|^2}{2}\cos(\Delta M\,t)\right.\\ &\left.-\mathrm{Re}\,\lambda_f\,\sinh\frac{\Delta\Gamma\,t}{2} - \mathrm{Im}\,\lambda_f\,\sin(\Delta M\,t)\right\},\end{aligned}$$

$$\begin{aligned}\Gamma(\overline{M}(t) \to f) \;&=\; \mathcal{N}_f\,|A_f|^2\,\frac{1}{1-a}\,e^{-\Gamma t}\left\{\frac{1+|\lambda_f|^2}{2}\cosh\frac{\Delta\Gamma\,t}{2} - \frac{1-|\lambda_f|^2}{2}\cos(\Delta M\,t)\right.\\ &\left.-\mathrm{Re}\,\lambda_f\,\sinh\frac{\Delta\Gamma\,t}{2} + \mathrm{Im}\,\lambda_f\,\sin(\Delta M\,t)\right\}.\end{aligned}$$

Time-dependent CP-asymmetries

★ Various time-dependent CP-asymmetries can now be formed

- The flavor-specific CP-asymmetry (aka semileptonic CP asymmetry)

$$a_{\rm fs} \equiv \frac{\Gamma(\overline{M}(t)\to f) - \Gamma(M(t)\to \overline{f})}{\Gamma(\overline{M}(t)\to f) + \Gamma(M(t)\to \overline{f})} = \frac{1-(1-a)^2}{1+(1-a)^2} = a + \mathcal{O}(a^2).$$

- CP-asymmetry for decays to CP-eigenstates (such as $f_{CP} = J/\psi K_S$, etc.)

$$a_{f_{\rm CP}}(t) = \frac{\Gamma(\overline{M}(t)\to f_{\rm CP}) - \Gamma(M(t)\to f_{\rm CP})}{\Gamma(\overline{M}(t)\to f_{\rm CP}) + \Gamma(M(t)\to f_{\rm CP})}$$

$$= -\frac{A_{CP}^{\rm dir}\cos(\Delta M\, t) + A_{CP}^{\rm mix}\sin(\Delta M\, t)}{\cosh(\Delta\Gamma\, t/2) + A_{\Delta\Gamma}\sinh(\Delta\Gamma\, t/2)} + \mathcal{O}(a)$$

where $A_{CP}^{\rm dir} = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}$, $A_{CP}^{\rm mix} = -\frac{2\,{\rm Im}\,\lambda_f}{1+|\lambda_f|^2}$ and $A_{\Delta\Gamma} = -\frac{2\,{\rm Re}\,\lambda_f}{1+|\lambda_f|^2}$

Charm physics

- How can CP-violation be observed in charm system?
  - can be observed by comparing CP-conjugated decay rates in various ways, both with and w/out time dependence

$$a_{\rm CP}(f) = \frac{\Gamma(D \to f) - \Gamma(\overline{D} \to \overline{f})}{\Gamma(D \to f) + \Gamma(\overline{D} \to \overline{f})}$$

  - can manifest itself in charm ΔC=1 transitions (direct CP-violation)

$$\Gamma(D \to f) \neq \Gamma(CP[D] \to CP[f])$$ **dCPV**

  - or in ΔC=2 transitions (indirect CP-violation): mixing $\left|D_{1,2}\right\rangle = p\left|D^0\right\rangle \pm q\left|\overline{D^0}\right\rangle$

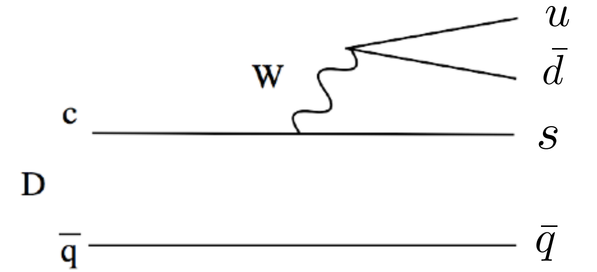
$$R_m^2 = |q/p|^2 = \left|\frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - (i/2)\Delta\Gamma}\right|^2 = 1 + A_m \neq 1$$ **CPVmix**

  - or in the interference b/w decays (ΔC=1) and mixing (ΔC=2)

$$\lambda_f = \frac{q}{p}\frac{\overline{A_f}}{A_f} = R_m e^{i(\phi+\delta)}\left|\frac{\overline{A_f}}{A_f}\right|$$ **CPVint**

★ Can be classified by SM CKM suppression of tree amplitude ($V_{us} \sim \lambda$)

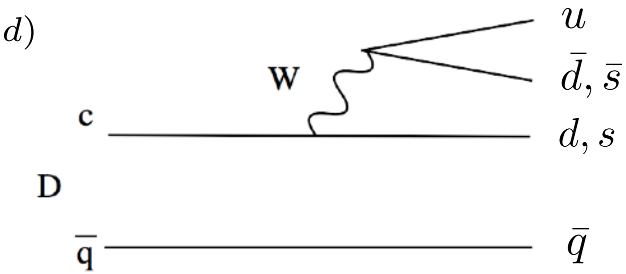
$V_{cs}V_{ud}^*$



★ Cabibbo-favored (CF: λ^0) decay

- originates from c → s u$\bar{d}$
- examples: $D^0 \to K^-\pi^+$

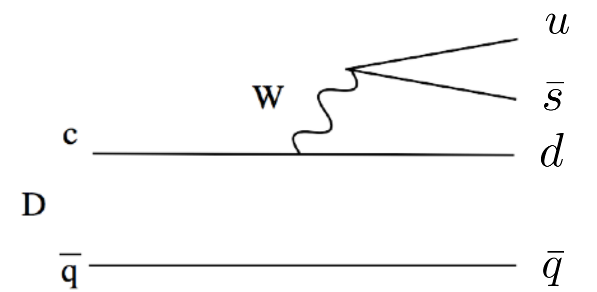
$V_{cs(d)}V_{us(d)}^*$



★ Singly Cabibbo-suppressed (SCS: λ^1) decay

- originates from c → q u$\bar{q}$
- examples: $D^0 \to \pi\pi$ and $D^0 \to KK$

$V_{cd}V_{us}^*$



★ Doubly Cabibbo-suppressed (DCS: λ^2) decay

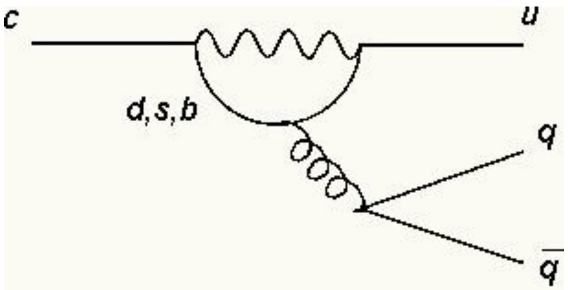
- originates from c → d u$\bar{s}$
- examples: $D^0 \to K^+\pi^-$

★ We shall concentrate on SCS decays. Why is that?

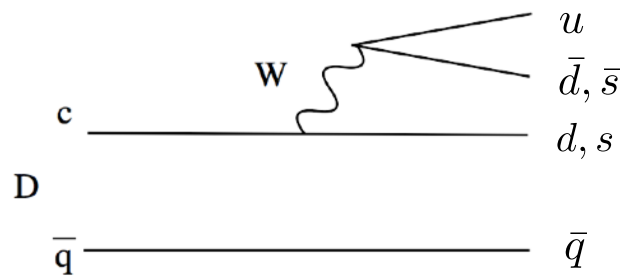
Generic expectations for sizes of CPV effects

★ Generic expectation is that CP-violating observables in the SM are small

Δc = 1 amplitudes allow to reach third -generation quarks!



"Penguin" amplitude/contraction



"Tree" amplitude

★ The Unitarity Triangle relation for charm:

$$\underbrace{V_{ud}V_{cd}^*}_{\sim\lambda} + \underbrace{V_{us}V_{cs}^*}_{\sim\lambda} + \underbrace{V_{ub}V_{cb}^*}_{\sim\lambda^5} = 0$$

With b-quark contribution neglected: only **2** generations contribute
$\Rightarrow$ **real 2x2 Cabibbo matrix**

Any CP-violating signal in the SM will be small, at most $O(V_{ub}V_{cb}^*/V_{us}V_{cs}^*) \sim 10^{-3}$

Thus, O(1%) CP-violating signal can provide a "smoking gun" signature of New Physics

2. Time-independent (direct) CP-violation

★ Direct CP-violating asymmetries probe CP-violation in ΔC=1 amplitudes

- CP-asymmetries compare partial rates of CP-conjugated decays

$$a_{CP}(f) = \frac{\Gamma(D \to f) - \Gamma(\overline{D} \to \overline{f})}{\Gamma(D \to f) + \Gamma(\overline{D} \to \overline{f})}$$ (both charged and neutral D's)

- a non-vanishing decay asymmetry requires that a decay amplitude
  - contain several components each of which has its own strong and weak phases
  - strong phases: do not change under CP transformation of the decay amplitude
  - weak phases: flip sign under CP transformation of the decay amplitude

$$A(D \to f) \equiv A_f = |A_{f1}| e^{i\delta_1} e^{i\theta_1} + |A_{f2}| e^{i\delta_2} e^{i\theta_2}$$

- Now we can form the CP-asymmetry

$$a_{CP}(f) = 2 r_f \sin(\underbrace{\theta_1 - \theta_2}_{\text{weak}}) \sin(\underbrace{\delta_1 - \delta_2}_{\text{strong}})$$ with $r_f = \left| \frac{A_{f2}}{A_{f1}} \right|$

Direct CP-violation in charm: realities of life

★IDEA: consider the DIFFERENCE of decay rate asymmetries: D →ππ vs D → KK!
For each final state the asymmetry

D^0: no neutrals in the final state!

$$a_f = \frac{\Gamma(D \to f) - \Gamma(\overline{D} \to \overline{f})}{\Gamma(D \to f) + \Gamma(\overline{D} \to \overline{f})} \quad \Rightarrow \quad a_f = a_f^d + a_f^m + a_f^i$$

direct mixing interference

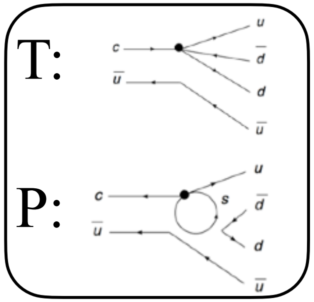
★ A reason: $a^m_{KK}=a^m_{\pi\pi}$ and $a^i_{KK}=a^i_{\pi\pi}$ (for CP-eigenstate final states), so, ideally, mixing asymmetries cancel ($r_f=P_f/A_f$)!

$$a_f^d = 2 r_f \sin\phi_f \sin\delta_f$$

★ ... and the resulting DCPV asymmetry is $\boxed{\Delta a_{CP} = a^d_{KK} - a^d_{\pi\pi} \approx 2 a^d_{KK}}$ (double!)

$$A_{KK} = \frac{G_F}{\sqrt{2}}\lambda\left[(T+E+P_{sd}) + a\lambda^4 e^{-i\gamma} P_{bd}\right]$$

$$A_{\pi\pi} = \frac{G_F}{\sqrt{2}}\lambda\left[(-(T+E)+P_{sd}) + a\lambda^4 e^{-i\gamma} P_{bd}\right]$$



★ ... so it is doubled in the limit of $SU(3)_F$ symmetry

SU(3) is badly broken in D-decays

Experimental observations

- Experimental results
  - Result 1: an observation of CP-violation in the difference…

$$\Delta a_{CP}^{dir} = a_{CP}(K^-K^+) - a_{CP}(\pi^-\pi^+) = (-15.4 \pm 2.9) \times 10^{-4}$$

LHCb 2019

  - Result 2: the individual CPV asymmetry in $D^0 \to K^+K^-$ channel

$$a_{CP}(K^-K^+) = (7.7 \pm 5.7) \times 10^{-4}$$

LHCb 2022
2209.03179v2

  - Result 3: LHCb combined the above results to obtain the CPV asymmetry in $D^0 \to \pi^+\pi^-$ channel

$$a_{CP}(\pi^-\pi^+) = (23.2 \pm 6.1) \times 10^{-4}$$

LHCb 2022
2209.03179v2

  - Wishlist: obtain the CPV asymmetries in $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ channels independently to check consistency of Δa_{CP}^{dir}

- Need confirmation from other experiments (Belle II)
- What do those results mean? New Physics? Standard Model?

Implications of experimental observations

- Check SU(3) symmetry: only need U-spin (interchange $s \leftrightarrow d$)
  - Branching ratios: $\Gamma(D^0 \to K^+K^-) = \Gamma(D^0 \to \pi^+\pi^-)$

$$\frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-)} = 2.81 \pm 0.06$$

  - CPV asymmetries: $a_{CP}(D^0 \to \pi^+\pi^-) = -\, a_{CP}(D^0 \to K^+K^-)$

$$\frac{a_{CP}(D^0 \to \pi^+\pi^-)}{a_{CP}(D^0 \to K^+K^-)} = 3.01^{+0.95}_{-5.95}$$

- In both cases: appearance of badly-broken symmetry. Also: wrong sign!

- U-spin sum rule: $$\frac{a_{CP}(D^0 \to \pi^+\pi^-)}{a_{CP}(D^0 \to K^+K^-)} \frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-)} = -1$$

… but it appears that experimentally $= +\,0.93^{+0.62}_{-0.41}$

S. Schacht, JHEP 03 (2023) 205

Theoretical troubles

ΔA_{CP} within the Standard Model and beyond

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Implications on the first observation of charm CPV at LHCb

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The Emergence of the $\Delta U = 0$ Rule in Charm Physics

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Revisiting CP violation in $D \to PP$ and VP decays

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Calculating CP-asymmetries?

- Effective Hamiltonian for singly Cabibbo-suppressed (SCS) decays
  - drop all "penguin" operators (Q_i for $i \geq 3$) as C_i are small, $\lambda_q = V_{uq} V_{cq}^*$,

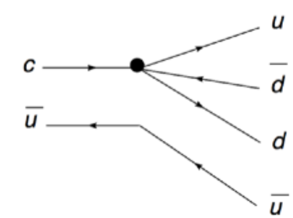
$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\sum_{q=d,s} \lambda_q \left(C_1 Q_1^q + C_2 Q_2^q \right) - \cancel{\lambda_b \sum_{i=3,\ldots,6,8g} C_i Q_i} \right]$$

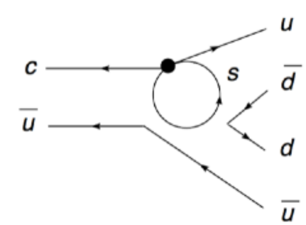
$$Q_1^q = (\bar{u} \Gamma_\mu q)(\bar{q} \Gamma^\mu c), \qquad Q_2^q = (\bar{q} \Gamma_\mu q)(\bar{u} \Gamma^\mu c)$$

  - recall that $\sum_{q=d,s,b} \lambda_q = 0 \;\text{ or }\; \lambda_d = -(\lambda_s + \lambda_b)$ and $\mathcal{O}^q \equiv \frac{G_F}{\sqrt{2}} \sum_{i=1,2} C_i Q_i^q, \quad \text{with} \;\; q = d, s.$

without QCD

with QCD





Amplitudes?

How to compute decay amplitudes?

- A_{CP}: need to compute/fit/derive hadronic decay amplitudes
  - matrix elements of 4-fermion operators (factorization?)

$$
\begin{aligned}
A_{\pi\pi} &= \langle \pi^+\pi^- | \mathcal{H} | D^0 \rangle \\
&= \frac{G_F}{\sqrt{2}} V_{ud} V_{cd}^* \langle \pi^+\pi^- | \left(\bar{u}d\right)_L \left(\bar{d}c\right)_L | D^0 \rangle \\
&\sim \frac{G_F}{\sqrt{2}} V_{ud} V_{cd}^* \langle \pi^+ | \left(\bar{u}d\right)_L 0 \rangle \langle \pi^- | (\bar{d}c)_L | D^0 \rangle \\
&\sim\sim \frac{G_F}{\sqrt{2}} V_{ud} V_{cd}^* \; f_\pi F_{D\to\pi} m_D^2
\end{aligned}
$$

No imaginary part?

  - need a better approach (but can retain some elements)! Recall $R_{DCS/CF}$

Falk, Nir, AAP
JHEP 12 (1999) 019

$$
\begin{aligned}
A_R &= \frac{g_H}{m_D^2 - m_{K_H}^2 + i\Gamma_{K_H} m_D} \langle K_H | H_{eff} | D^0 \rangle \,, \\
B_R &= \frac{g_H}{m_D^2 - m_{K_H}^2 + i\Gamma_{K_H} m_D} \langle K_H | H_{eff} | \overline{D}^0 \rangle \,,
\end{aligned}
$$

Resonances? FSI? Both?

Resonance enhancement of decay amplitudes

- A_{CP}: need to compute/fit/derive hadronic decay amplitudes
  - parameterize $D \to KK$ and $D \to \pi\pi$ decay amplitudes
  - use isospin decomposition, as possible nearby resonances are classified according to isospin, etc.

Schacht, Soni
PLB 825 (2022) 136855

$$A(D^0 \to \pi^+\pi^-) = \frac{1}{\sqrt{6}}\lambda_{sd} A^{\pi\pi}_{\frac{3}{2},2} + \frac{1}{\sqrt{3}}\left(\lambda_{sd} A^{\pi\pi}_{\frac{1}{2},0} - \frac{\lambda_b}{2} B^{\pi\pi}_{\frac{1}{2},0}\right)$$

$$A(D^0 \to K^+K^-) = \frac{1}{2}\lambda_{sd} A^{KK}_{\frac{3}{2},1} + \frac{1}{2}\left(\lambda_{sd} A^{KK}_{\frac{1}{2},1} - \frac{\lambda_b}{2} B^{KK}_{\frac{1}{2},1}\right) + \frac{1}{2}\left(\lambda_{sd} A^{KK}_{\frac{1}{2},0} - \frac{\lambda_b}{2} B^{KK}_{\frac{1}{2},0}\right)$$

… and similarly for other D-decays, where $\lambda_{sd} = (\lambda_s - \lambda_d)/2$ and $A^{ff}_{\Delta I\, I}$ ($B^{ff}_{\Delta I\, I}$) are CP-even (CP-odd)

- Resonance enhancement of decay amplitudes (model)
  - choose model and resonances that provide enhancement (I=0): f_0 states

$$A^{ff}_{\frac{1}{2},0} = g_{f_0 \to ff} M^{sd}_{f_0} R(m_{f_0}, \Gamma_{f_0}, m_D, \ldots)$$

$$B^{ff}_{\frac{1}{2},0} = g_{f_0 \to ff} M^{b}_{f_0} R(m_{f_0}, \Gamma_{f_0}, m_D, \ldots)$$

Resonance enhancement of decay amplitudes

- Resonance enhancement of decay amplitudes (model)
  - choose model and resonances that provide enhancement (I=0): f_0 states

$$A^{ff}_{\frac{1}{2},0} = g_{f_0\to ff} M^{sd}_{f_0} \underline{R(m_{f_0}, \Gamma_{f_0}, m_D, ...)}$$

$$B^{ff}_{\frac{1}{2},0} = g_{f_0\to ff} M^{b}_{f_0} \underline{R(m_{f_0}, \Gamma_{f_0}, m_D, ...)}$$

possible interference among different f_0 states

  - ...where $g_{f_0\to ff}$ describes f_0 coupling to KK or $\pi\pi$ and

$$M^{sd}_{f_0} = \langle f_0 | \mathcal{O}^{\Delta I=1/2}_{sd} | D^0 \rangle \qquad M^{b}_{f_0} = \langle f_0 | \mathcal{O}^{\Delta I=1/2}_{b} | D^0 \rangle$$

  - there are nearby f_0 resonances

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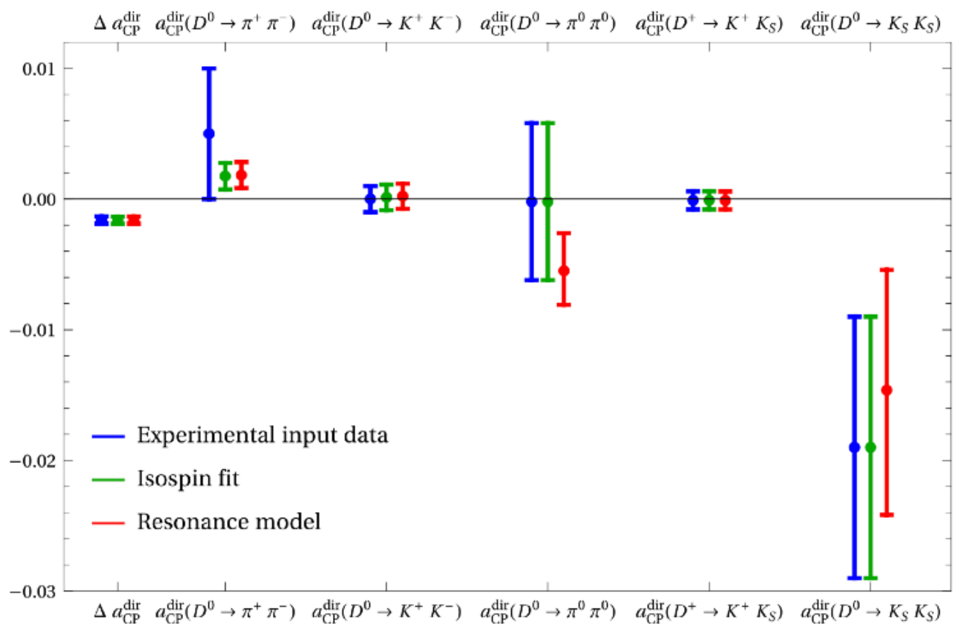
Employed experimental data for scalar unflavored resonances close to the D^0 mass.

Resonance	$I^G(J^{PC})$	mass m [MeV]	Γ [MeV]	Ref.
$f_0(1710)$	$0^+(0^{++})$	1704 ± 12	123 ± 18	[5]
$f_0(1790)$	$0^+(0^{++})$	1790^{+40}_{-30}	270^{+60}_{-30}	[53,54]

Note: other f_0 states? E.g., $f_0(2020)$: $m_{f_0(2020)} = 1982^{+54.1}_{-3.0}$ MeV, $\Gamma_{f_0(2020)} = 436 \pm 50$ MeV

Resonance enhancement of decay amplitudes

- Resonance enhancement of decay amplitudes (model)



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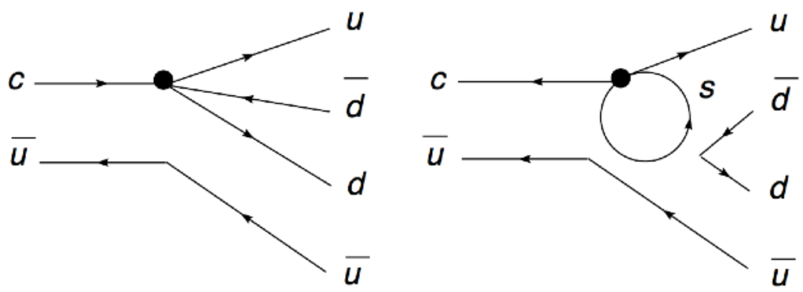
- Note: compatibility of the result depends on how many resonances are included in the fit

★ These asymmetries are notoriously difficult to compute

★ In the Standard Model

- need to estimate size of penguin/penguin contractions vs. tree



- unknown penguin contributions
  - SU(3) analysis: some ME are enhanced?
    Golden & Grinstein PLB 222 (1989) 501; Pirtshalava & Uttayarat 1112.5451
  - could expect large $1/m_c$ corrections (E/PE/PA/…)
    Isidori et al PLB 711 (2012) 46; Brod et al 1111.5000
  - flavor-flow diagrams
    Broad et al 1203.6659; Bhattacharya et al PRD 85 (2012) 054014; Cheng & Chiang 1205.0580; 1909.03063; Gronau, Rosner

★ General comments on SU(3)/flavor flow — type analyses
- fit both SM and (possible) NP parts of the amplitudes: can one claim SM-only?
- many parameters: can one claim $O(10^{-4})$ precision if rates are known to $O(10^{-2})$?

★ Need direct calculations of amplitudes/CPV-asymmetries

- QCD sum rule calculations of Δa_{CP} Khodjamirian, AAP; Lenz, Piscopo, Rusov
- SU(3) breaking analyses of $D \to PV, VV$
- constant (but slow) lattice QCD progress in $D \to \pi\pi, \pi\pi\pi$ Hansen, Sharpe

★ Other observables can be constructed for baryons, e.g.

$$A\left(\Lambda_c \to N\pi\right) = \bar{u}_N\left(p,s\right)\left[A_S + A_P\gamma_5\right]u_{\Lambda_c}\left(p_\Lambda, s_\Lambda\right)$$

These amplitudes can be related to "asymmetry parameter" $\alpha_{\Lambda_c} = \dfrac{2\,\mathrm{Re}\left(A_S^* A_P\right)}{\left|A_S\right|^2 + \left|A_P\right|^2}$

... which can be extracted from $\dfrac{dW}{d\cos\vartheta} = \dfrac{1}{2}\left(1 + P\alpha_{\Lambda_c}\cos\vartheta\right)$

Same is true for $\overline{\Lambda}_c$-decay

If CP is conserved $\alpha_{\Lambda_c} \overset{CP}{\Rightarrow} -\bar{\alpha}_{\Lambda_c}$, thus CP-violating observable is

$$\boxed{A_f = \frac{\alpha_{\Lambda_c} + \overline{\alpha}_{\Lambda_c}}{\alpha_{\Lambda_c} - \overline{\alpha}_{\Lambda_c}}}$$

FOCUS[2006]: $A_{\Lambda\pi}$=-0.07±0.19±0.24

Things to take home: charm

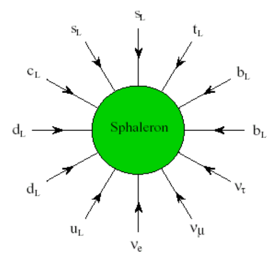
- Computation of charm amplitudes is a difficult task
  - no dominant heavy dof, as in beauty decays
  - light dofs give no contribution in the flavor SU(3) limit
  - D-mixing is a second order effect in SU(3) breaking (x,y ~ 1% in the SM)
- For indirect CP-violation studies
  - constraints on Wilson coefficients of generic operators are possible, point to the scales much higher than those directly probed by LHC
  - consider new parameterizations that go beyond the "superweak" limit
- For direct CP-violation studies
  - unfortunately, large DCPV signal is no more; need more results in individual channels, especially including baryons
  - hit the "brown muck": future observation of DCPV does not give easy interpretation in terms of fundamental parameters
  - need better calculations: lattice?
- Lattice calculations can, in the future, provide a result for a_{CP}!
- Need to give more thought on how large SM CPV can be…

Leptons

Fundamental physics: building the Universe

★ Standard Model satisfies Sakharov's conditions for baryogenesis

- ✓ Baryon (and lepton) number - violating processes
  to ***generate*** asymmetry



$\Delta B = 3,\ \Delta L = 3,$
$B - L$ conserved

- ✓ Universe that evolves out of thermal equilibrium
  to ***keep*** asymmetry from ***being washed out***
- ✓ "Microscopic CP-violation"
  to ***keep*** asymmetry from ***being compensated in the "anti-world"***

- but there are still issues preventing it to succeed (not enough CP-violation via CKM mechanism, order of the phase transition, …)

★ What about New Physics?

- no new strongly-interacting particles so far at the LHC (SUSY?)
- neutrinos oscillations: ν's have mass and so CLFV transitions are guaranteed
- use sphaleron mechanism: baryogenesis via leptogenesis   Fukugita, Yanagida
- new sources of CP-violation in the lepton sector

Should we look for New Physics in the charged lepton sector? Muons?

Fundamental physics with muons: flavor violation

★ Example of the common origin of the neutrino masses and CLFV transitions

- consider a model with a triplet Higgs, e.g., a left-right model

$$-\mathcal{L}_{\text{Yukawa}} = \overline{\psi}'_{iL}\left(G_{ij}\phi + H_{ij}\widetilde{\phi}\right)\psi'_{jR} + \frac{i}{2}F_{ij}\left(\psi'^{T}_{iL}C\tau_2\Delta_L\psi'_{jL} + \psi'^{T}_{iR}C\tau_2\Delta_R\psi'_{jR}\right) + \text{h.c.}$$

with $\psi'_{iL,R} = \begin{pmatrix} \nu'_{iL,R} \\ e'_{iL,R} \end{pmatrix}$ and $\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$

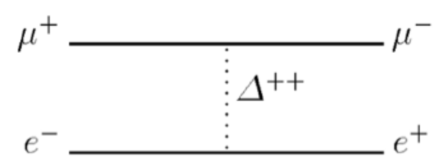
- this Lagrangian leads to the Majorana masses for the neutrinos

Pati, Salam; Mohapatra, Pati; Senjanovic, et al, Schechter and Valle; K. Kiers et al

$$-\mathcal{L}_{\text{Majorana}} = \frac{1}{2\sqrt{2}}\left(\overline{\nu'^c_L}F v_L e^{i\theta_L}\nu'_L + \overline{\nu'^c_R}F v_R \nu'_R\right) + \text{h.c.}$$

- ... and both $\Delta L_\mu = 1$ (FCNC decays) and $\Delta L_\mu = 2$ (muonium oscillations) transitions

$$\mathcal{H}_\Delta = -\frac{g_{ee}g^*_{\mu\mu}}{8M^2_\Delta}(\overline{\mu}_L\gamma_\alpha e_L)(\overline{\mu}_L\gamma^\alpha e_L) + H.c.$$



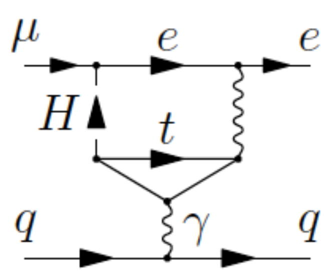
Chang, Keung (89); Schwartz (89);
Conlin, AAP (21); Han, Tang, Zhang (21)

Fundamental physics with muons: flavor violation

★ Leptonic FCNC could be generated by New Physics

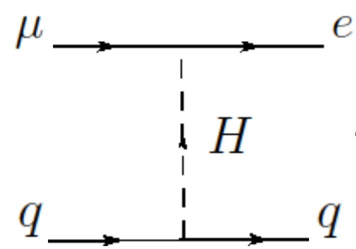
- ★ Ex.1 FCNC Higgs decays H → μe, τe, etc.: $Y_{ij} = \frac{m_i}{v}\delta_{ij} + \frac{v^2}{\sqrt{2}\Lambda^2}\hat{\lambda}_{ij}$ Harnik, Kopp, Zupan
- ★ FCNC Higgs model & muon conversion/quarkonium decays

e.g.



$\sim \mathcal{O}(\alpha^2)$

Barr-Zee type



$\sim \mathcal{O}(m_u \text{ or } m_d)$

tree level
(note suppression of light quark couplings)

Process	Coupling	Bound
$h \to \mu e$	$\sqrt{\lvert Y^h_{\mu e}\rvert^2 + \lvert Y^h_{e\mu}\rvert^2}$	$< 5.4 \times 10^{-4}$
$\mu \to e\gamma$	$\sqrt{\lvert Y^h_{\mu e}\rvert^2 + \lvert Y^h_{e\mu}\rvert^2}$	$< 2.1 \times 10^{-6}$
$\mu \to eee$	$\sqrt{\lvert Y^h_{\mu e}\rvert^2 + \lvert Y^h_{e\mu}\rvert^2}$	$\lesssim 3.1 \times 10^{-5}$
$\mu\,\mathrm{Ti} \to e\,\mathrm{Ti}$	$\sqrt{\lvert Y^h_{\mu e}\rvert^2 + \lvert Y^h_{e\mu}\rvert^2}$	$< 1.2 \times 10^{-5}$

Calibbi, Signorelli

Fundamental physics with muons: flavor violation

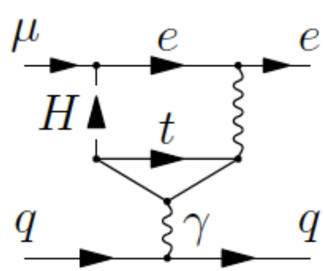
★ Leptonic FCNC could be generated by New Physics

★ Ex.1 FCNC Higgs decays H → μe, τe, etc.: $Y_{ij} = \frac{m_i}{v}\delta_{ij} + \frac{v^2}{\sqrt{2}\Lambda^2}\hat{\lambda}_{ij}$

Harnik, Kopp, Zupan

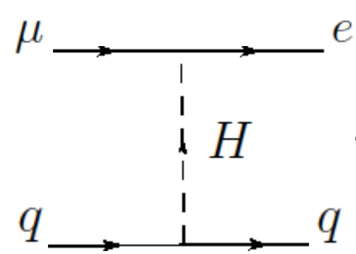
★ FCNC Higgs model & muon conversion/quarkonium decays

e.g.



$\sim \mathcal{O}(\alpha^2)$

Barr-Zee type



$\sim \mathcal{O}(m_u \text{ or } m_d)$

tree level
(note suppression of light quark couplings)

★ Ex.2 Exceptional couplings of (flavor-diagonal) NP to third generation $\mathcal{H}_{\rm NP} = G\bar{b}'_L\gamma^\lambda b'_L\bar{\tau}'_L\gamma_\lambda\tau'_L$ -> flavor "anomalies"

Glashow, Guadagnoli, Lane

★ Ex.3 Leptoquarks -> flavor "anomalies"

★ Leptons and New Physics: choose muons (long lifetime and large mass)

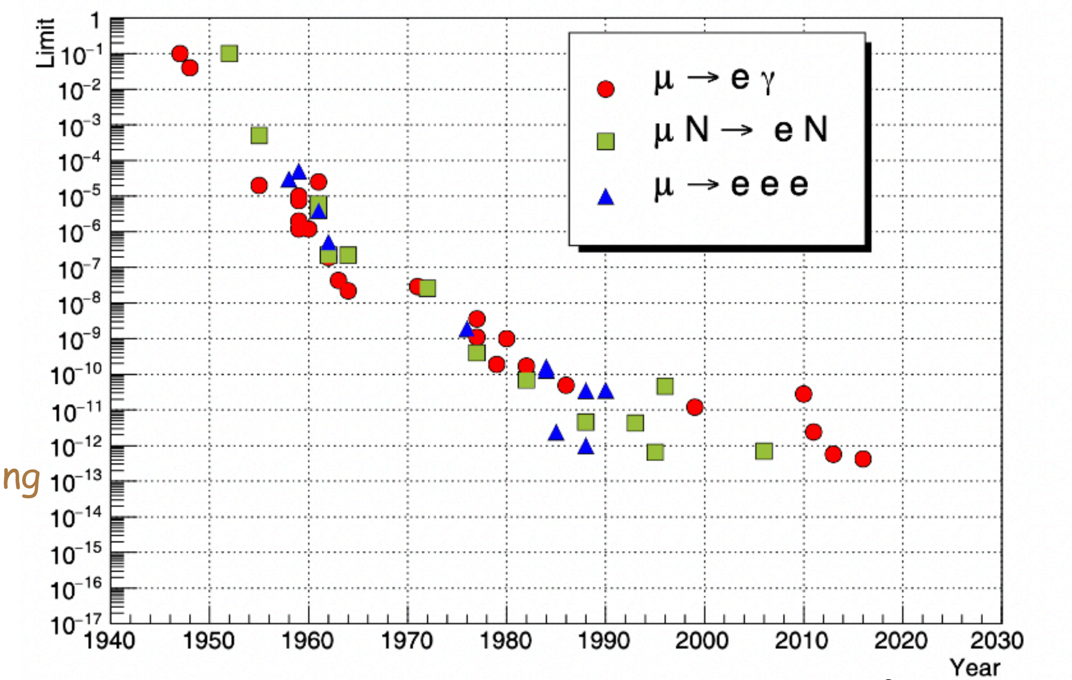
Fundamental physics with muons: flavor violation

★ Muons can help solving the most fundamental problems in particle physics!

★ Possible experimental searches of LFCNC

- lepton-flavor violating processes
  - $\mu \to e\gamma$, $\tau \to e\gamma$, etc.
  - $\mu \to eee$, $\tau \to \mu ee$, etc.
  - $\mu^+e^- \to e^-\mu^+$ (muonium oscillations)
  - $Z^0 \to \mu e$, τe, etc.
  - $H \to \mu e$, τe, etc.
  - K^0 (B^0, D^0, ...) $\to \mu e$, τe, etc.
  - $\mu^- + (A, Z) \to e^- + (A, Z)$
- lepton number and lepton-flavor violating processes
  - $(A, Z) \to (A, Z\pm2) + e^{\mp}e^{\mp}$
  - $\mu^- + (A, Z) \to e^+ + (A, Z-2)$

LORENZO CALIBBI and GIOVANNI SIGNORELLI



★ Decays are highly suppressed in the Standard Model: $Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_i U^*_{\mu i} U_{ei} \frac{m^2_{\nu_i}}{M_W^2}\right|^2 < 10^{-54}$

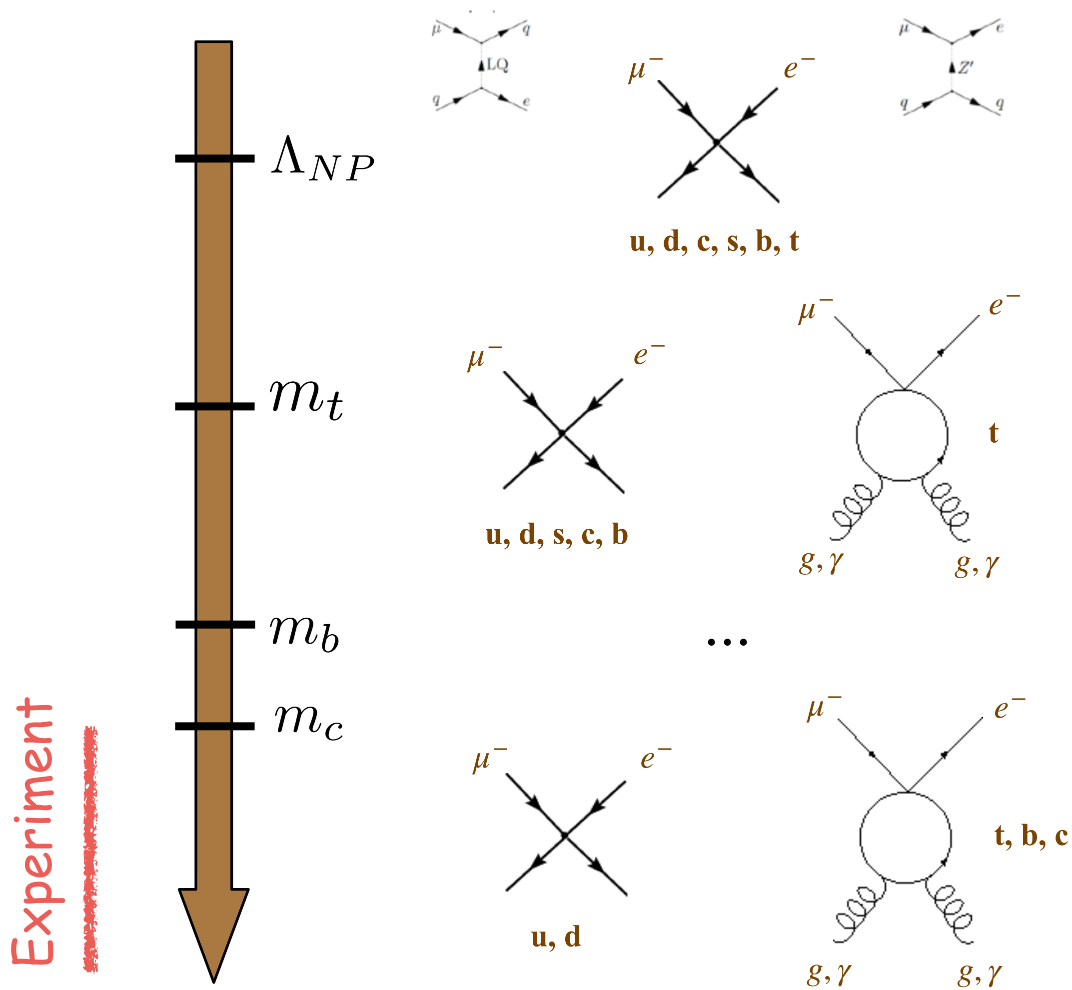
★ But: no trivial FCNC vertices in the Standard Model: sensitive tests of New Physics!

(Number of possible models) > (number of model builders). How do we proceed?

Flavor violation and effective Lagrangians

★ Modern approach to flavor physics calculations: effective field theories

★ It is important to understand ALL relevant energy scales for the problem at hand



New Physics generates lepton FCNC

Scales associated with heavy SM particles (quarks, leptons)

...

heavy quarks decouple

...

Scales associated with experiment

★ Effective Lagrangians parameterize New Physics without specifying a model

- write out all terms consistent with symmetries and according to power counting
- match to possible UV completions/compute experimental observables

$$
\begin{aligned}
\mathcal{L}_{\ell_1\to\ell_2\nu_2\bar\nu_1} = -\frac{4G_F}{\sqrt{2}}\Big[& g^S_{RR}\left(\overline{\ell_2}_R\nu_{\ell_2 L}\right)\left(\overline{\nu_{\ell_1}}_L\ell_{1R}\right) + g^S_{RL}\left(\overline{\ell_2}_R\nu_{\ell_2 L}\right)\left(\overline{\nu_{\ell_1}}_R\ell_{1L}\right) \\
& + g^S_{LR}\left(\overline{\ell_2}_L\nu_{\ell_2 R}\right)\left(\overline{\nu_{\ell_1}}_L\ell_{1R}\right) + g^S_{LL}\left(\overline{\ell_2}_L\nu_{\ell_2 R}\right)\left(\overline{\nu_{\ell_1}}_R\ell_{1L}\right) \\
& + g^V_{RR}\left(\overline{\ell_2}_R\gamma^\alpha\nu_{\ell_2 R}\right)\left(\overline{\nu_{\ell_1}}_R\gamma_\alpha\ell_{1R}\right) + g^V_{RL}\left(\overline{\ell_2}_R\gamma^\alpha\nu_{\ell_2 R}\right)\left(\overline{\nu_{\ell_1}}_L\gamma_\alpha\ell_{1L}\right) \\
& + g^V_{LR}\left(\overline{\ell_2}_L\gamma^\alpha\nu_{\ell_2 L}\right)\left(\overline{\nu_{\ell_1}}_R\gamma_\alpha\ell_{1R}\right) + g^V_{LL}\left(\overline{\ell_2}_R\gamma^\alpha\nu_{\ell_2 R}\right)\left(\overline{\nu_{\ell_1}}_L\gamma_\alpha\ell_{1L}\right) \\
& + \frac{g^T_{RL}}{2}\left(\overline{\ell_2}_R\sigma_{\alpha\beta}\nu_{\ell_2 L}\right)\left(\overline{\nu_{\ell_1}}_R\sigma^{\alpha\beta}\ell_{1L}\right) + \frac{g^T_{LR}}{2}\left(\overline{\ell_2}_L\sigma_{\alpha\beta}\nu_{\ell_2 R}\right)\left(\overline{\nu_{\ell_1}}_L\sigma^{\alpha\beta}\ell_{1R}\right) + h.c.\Big],
\end{aligned}
$$

- which for $\mu \to e\nu\bar\nu$ (muon decay) leads to

$$
\begin{aligned}
\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}\left[F\left(\frac{m_e^2}{m_\mu^2}\right) + 4\eta\frac{m_e}{m_\mu}G\left(\frac{m_e^2}{m_\mu^2}\right) - \frac{32}{3}\frac{m_e^2}{m_\mu^2}\left(\rho-\frac{3}{4}\right)\left(1-\frac{m_e^4}{m_\mu^4}\right)\right] \\
\times\left(1+\frac{3}{5}\frac{m_\mu^2}{m_W^2}\right)\left[1+\frac{\alpha(m_\mu)}{2\pi}\left(\frac{25}{4}-\pi^2\right)\right],
\end{aligned}
$$

- where ρ and η are the Michel parameters

$$
\begin{aligned}
\rho &= \frac{3}{16}\left[\left|g^S_{RR}\right|^2 + \left|g^S_{LL}\right|^2 + \left|g^S_{RL}-2g^T_{RL}\right|^2 + \left|g^S_{LR}-2g^T_{LR}\right|^2 + \frac{3}{4}\left(\left|g^V_{RR}\right|^2+\left|g^V_{LL}\right|^2\right)\right], \\
\eta &= \frac{1}{2}\mathrm{Re}\left[g^V_{RR}g^{S*}_{LL} + g^V_{LL}g^{S*}_{RR} + g^V_{RL}\left(g^{S*}_{LR}+6g^{T*}_{LR}\right) + g^V_{LR}\left(g^{S*}_{RL}+6g^{T*}_{RL}\right)\right],
\end{aligned}
$$

Flavor violation and effective Lagrangians

★ Systematic approach: Standard Model Effective Field Theory (SMEFT)

- effective Lagrangian

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} Q^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} Q_i^{(6)} + \ldots$$

with the Weinberg operator $Q^{(5)}$

$$Q^{(5)} = \epsilon_{jk}\epsilon_{mn} H^j H^m \left(L_p^k\right)^T \mathcal{C} L_r^n$$

and lots (59+5) of $Q_i^{(6)}$ operators

- the strategy of identifying an NP model involves fitting C_i from experimental data and/or matching of $\mathscr{L}$ to UV-completed NP models

TABLE 2.3 Operators with H^n, sets X^3, H^6, H^4D^2, and ψ^2H^3.

X^3		H^6 and H^4D^2		ψ^2H^3+ h.c.	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_H	$\left(H^\dagger H\right)^3$	Q_{eH}	$\left(H^\dagger H\right)\left(\overline{L}_p e_r H\right)$
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{H\Box}$	$\left(H^\dagger H\right)\Box\left(H^\dagger H\right)$	Q_{uH}	$\left(H^\dagger H\right)\left(\overline{Q}_p u_r \widetilde{H}\right)$
Q_W	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	Q_{HD}	$\left(H^\dagger D^\mu H\right)^*\left(H^\dagger D_\mu H\right)$	Q_{dH}	$\left(H^\dagger H\right)\left(\overline{Q}_p d_r H\right)$
$Q_{\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				

TABLE 2.4 Operators with H^n, sets X^2H^2, ψ^2XH, and ψ^2H^2D.

X^2H^2		ψ^2XH+ h.c.		ψ^2H^2D	
Q_{HG}	$H^\dagger H G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$\left(\overline{L}_p\sigma^{\mu\nu}e_r\right)\tau^I H W_{\mu\nu}^I$	$Q_{Hl}^{(1)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{L}_p\gamma^\mu L_r\right)$
$Q_{H\widetilde{G}}$	$H^\dagger H \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$\left(\overline{L}_p\sigma^{\mu\nu}e_r\right) H B_{\mu\nu}$	$Q_{Hl}^{(3)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu^I H\right)\left(\overline{L}_p\tau^I\gamma^\mu L_r\right)$
Q_{HW}	$H^\dagger H W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$\left(\overline{Q}_p\sigma^{\mu\nu}T^A u_r\right)\widetilde{H} G_{\mu\nu}^A$	Q_{He}	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{e}_p\gamma^\mu e_r\right)$
$Q_{H\widetilde{W}}$	$H^\dagger H \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$\left(\overline{Q}_p\sigma^{\mu\nu}u_r\right)\tau^I\widetilde{H} W_{\mu\nu}^I$	$Q_{Hq}^{(1)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{Q}_p\gamma^\mu Q_r\right)$
Q_{HB}	$H^\dagger H B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$\left(\overline{Q}_p\sigma^{\mu\nu}u_r\right)\widetilde{H} B_{\mu\nu}$	$Q_{Hq}^{(3)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu^I H\right)\left(\overline{Q}_p\tau^I\gamma^\mu Q_r\right)$
$Q_{H\widetilde{B}}$	$H^\dagger H \widetilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$\left(\overline{Q}_p\sigma^{\mu\nu}T^A d_r\right) H G_{\mu\nu}^A$	Q_{Hu}	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{u}_p\gamma^\mu u_r\right)$
Q_{HWB}	$H^\dagger\tau^I H W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$\left(\overline{Q}_p\sigma^{\mu\nu}d_r\right)\tau^I H W_{\mu\nu}^I$	Q_{Hd}	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{d}_p\gamma^\mu d_r\right)$
$Q_{H\widetilde{W}B}$	$H^\dagger\tau^I H \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$\left(\overline{Q}_p\sigma^{\mu\nu}d_r\right) H B_{\mu\nu}$	Q_{Hud}	$i\left(\widetilde{H}^\dagger D_\mu H\right)\left(\overline{u}_p\gamma^\mu d_r\right)$

TABLE 2.5 Four-fermion operators, classes $(\overline{L}L)(\overline{L}L)$, $(\overline{R}R)(\overline{R}R)$, and $(\overline{L}L)(\overline{R}R)$.

$(\overline{L}L)(\overline{L}L)$		$(\overline{R}R)(\overline{R}R)$		$(\overline{L}L)(\overline{R}R)$	
Q_{ll}	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{L}_s\gamma^\mu L_t\right)$	Q_{ee}	$\left(\overline{e}_p\gamma^\mu e_r\right)\left(\overline{e}_s\gamma^\mu e_t\right)$	Q_{le}	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{e}_s\gamma^\mu e_t\right)$
$Q_{qq}^{(1)}$	$\left(\overline{Q}_p\gamma^\mu Q_r\right)\left(\overline{Q}_s\gamma^\mu Q_t\right)$	Q_{uu}	$\left(\overline{u}_p\gamma^\mu u_r\right)\left(\overline{u}_s\gamma^\mu u_t\right)$	Q_{lu}	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{u}_s\gamma^\mu u_t\right)$
$Q_{qq}^{(3)}$	$\left(\overline{Q}_p\gamma^\mu\tau^I Q_r\right)\left(\overline{Q}_s\gamma^\mu\tau^I Q_t\right)$	Q_{dd}	$\left(\overline{d}_p\gamma^\mu d_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$	Q_{ld}	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$
$Q_{lq}^{(1)}$	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{Q}_s\gamma^\mu Q_t\right)$	Q_{eu}	$\left(\overline{e}_p\gamma^\mu e_r\right)\left(\overline{u}_s\gamma^\mu u_t\right)$	Q_{qe}	$\left(\overline{Q}_p\gamma^\mu Q_r\right)\left(\overline{e}_s\gamma^\mu e_t\right)$
$Q_{lq}^{(3)}$	$\left(\overline{L}_p\gamma^\mu\tau^I L_r\right)\left(\overline{Q}_s\gamma^\mu\tau^I Q_t\right)$	Q_{ed}	$\left(\overline{e}_p\gamma^\mu e_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$	$Q_{qu}^{(1)}$	$\left(\overline{Q}_p\gamma^\mu Q_r\right)\left(\overline{u}_s\gamma^\mu u_t\right)$
		$Q_{ud}^{(1)}$	$\left(\overline{u}_p\gamma^\mu u_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$	$Q_{qu}^{(8)}$	$\left(\overline{q}_p\gamma^\mu T^A q_r\right)\left(\overline{u}_s\gamma^\mu T^A u_t\right)$
		$Q_{ud}^{(8)}$	$\left(\overline{u}_p\gamma^\mu T^A u_r\right)\left(\overline{d}_s\gamma^\mu T^A d_t\right)$	$Q_{qd}^{(1)}$	$\left(\overline{q}_p\gamma^\mu q_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$
				$Q_{qd}^{(8)}$	$\left(\overline{Q}_p\gamma^\mu T^A Q_r\right)\left(\overline{d}_s\gamma^\mu T^A d_t\right)$

TABLE 2.6 Four-fermion operators, classes $(LR)(RL)$, and B (baryon-number) violating.

$(\overline{L}R)(\overline{R}L)$		B-violating	
Q_{ledq}	$\left(\left(\overline{L}_p^j e_r\right)\left(\overline{d}_s Q_t^j\right)\right.$	Q_{duq}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[\left(d_p^\alpha\right)^T C u_r^\beta\right]\left[\left(Q_s^{\gamma j}\right)^T C L_t^k\right]$
$Q_{quqd}^{(1)}$	$\left(\left(\overline{Q}_p^j u_r\right)\epsilon_{jk}\left(\overline{Q}_s^k d_t\right)\right.$	Q_{qqu}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[\left(Q_p^{\alpha j}\right)^T C Q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$\left(\left(\overline{Q}_p^j T^A u_r\right)\epsilon_{jk}\left(\overline{Q}_s^k T^A d_t\right)\right.$	$Q_{qqq}^{(1)}$	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\epsilon_{mn}\left[\left(Q_p^{\alpha j}\right)^T C Q_r^{\beta k}\right]\left[(Q_s^{\gamma m})^T C L_t^n\right]$
$Q_{lequ}^{(1)}$	$\left(\left(\overline{L}_p^j e_r\right)\epsilon_{jk}\left(\overline{Q}_s^k u_t\right)\right.$	$Q_{qqq}^{(3)}$	$\epsilon^{\alpha\beta\gamma}\left(\tau^I\epsilon\right)_{jk}\left(\tau^I\epsilon\right)_{mn}\left[\left(Q_p^{\alpha j}\right)^T C Q_r^{\beta k}\right]\left[(Q_s^{\gamma m})^T C L_t^n\right]$
$Q_{lequ}^{(3)}$	$\left(\left(\overline{L}_p^j\sigma_{\mu\nu}e_r\right)\epsilon_{jk}\left(\overline{Q}_s^k\sigma^{\mu\nu}u_t\right)\right.$	Q_{duu}	$\epsilon^{\alpha\beta\gamma}\left[\left(d_p^\alpha\right)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

Flavor violation and effective Lagrangians

★ Radiative FCNC decays of leptons $\ell_1 \to \ell_2 + \gamma$

- the most general amplitude is

$$A_{\ell_1 \to \ell_2 \gamma}(p, p') = \frac{i}{m_{\ell_1}} \overline{u}_{\ell_2}(p') \left[A_L P_L + A_R P_R \right] \sigma_{\mu\nu} q^\nu u_{\ell_1}(p) \epsilon^{*\mu},$$

- which leads to the decay rate

$$\Gamma(\ell_1 \to \ell_2 \gamma) = \frac{m_{\ell_1}}{16\pi} \left(|A_L|^2 + |A_R|^2 \right)$$

with $$A_R = A_L^* = \sqrt{2}\, \frac{v m_i^2}{\Lambda^2} \left(c_W C_{eB}^{fi} - s_W C_{eW}^{fi} \right) \equiv \sqrt{2}\, \frac{v m_i^2}{\Lambda^2} C_\gamma^{fi}$$

Effective coupling (example)	Bounds on Λ (TeV) (for $\lvert\mathcal{C}_{ij}^6\rvert = 1$)	Bounds on $\lvert\mathcal{C}_{ij}^6\rvert$ (for $\Lambda = 1$ TeV)	Observable
$\mathcal{C}_{e\gamma}^{\mu e}$	6.3×10^4	2.5×10^{-10}	$\mu \to e\gamma$
$\mathcal{C}_{e\gamma}^{\tau e}$	6.5×10^2	2.4×10^{-6}	$\tau \to e\gamma$
$\mathcal{C}_{e\gamma}^{\tau\mu}$	6.1×10^2	2.7×10^{-6}	$\tau \to \mu\gamma$
$\mathcal{C}_{\ell\ell,ee}^{\mu eee}$	207	2.3×10^{-5}	$\mu \to 3e$
$\mathcal{C}_{\ell\ell,ee}^{e\tau ee}$	10.4	9.2×10^{-5}	$\tau \to 3e$
$\mathcal{C}_{\ell\ell,ee}^{\mu\tau\mu\mu}$	11.3	7.8×10^{-5}	$\tau \to 3\mu$
$\mathcal{C}_{(1,3)H\ell}^{\mu e}, \mathcal{C}_{He}^{\mu e}$	160	4×10^{-5}	$\mu \to 3e$
$\mathcal{C}_{(1,3)H\ell}^{\tau e}, \mathcal{C}_{He}^{\tau e}$	≈ 8	1.5×10^{-2}	$\tau \to 3e$
$\mathcal{C}_{(1,3)H\ell}^{\tau\mu}, \mathcal{C}_{He}^{\tau\mu}$	≈ 9	$\approx 10^{-2}$	$\tau \to 3\mu$

Teixeira; Feruglio, Paradisi, Pattori

Other interesting modes that probe similar couplings: $\ell_1 \to \ell_2\gamma\gamma$, $\ell_1 \to 3\ell_2$, and others

Flavor violation and effective Lagrangians

★ Similarly, for purely leptonic FCNC decays $\ell_i \to \ell_j \ell_k \ell_l$

- the most general decay rate is [with (a): $\ell_1 \to 3\ell_2$ and (b): $\tau^\pm \to e^\pm \mu^+ \mu^-$ and $\tau^\pm \to \mu^\pm e^+ e^-$]

$$
\begin{aligned}
\Gamma(\ell_i \to \ell_j \ell_k \ell_l) &= \frac{\kappa_c m_{\ell_1}^5}{32\,(192\pi^3)\Lambda^4}\Big[X_\gamma + 4\left(|C_{VLL}|^2 + |C_{VRR}|^2 + |C_{VLR}|^2 + |C_{VRL}|^2\right) \\
&+ |C_{SLL}|^2 + |C_{SRR}|^2 + |C_{SLR}|^2 + |C_{SRL}|^2 + +48\left(|C_{TL}|^2 + |C_{TR}|^2\right)\Big]
\end{aligned}
$$

with

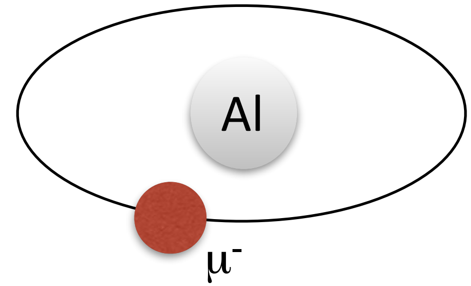
$$
\begin{aligned}
X_\gamma^{(a)} &= -\frac{16ev}{m_i}\mathrm{Re}\left[C_{\gamma L}^*\left(2C_{VLL} + C_{VLR} - \frac{1}{2}C_{SLR}\right) + C_{\gamma R}^*\left(2C_{VRR} + C_{VRL} - \frac{1}{2}C_{SRL}\right)\right] \\
&+ \frac{64e^2v^2}{m_i^2}\left(\log\frac{m_i^2}{m_f^2} - \frac{11}{4}\right)\left(|C_{\gamma L}|^2 + |C_{\gamma R}|^2\right), \\
X_\gamma^{(a)} &= -\frac{16ev}{m_i}\mathrm{Re}\left[C_{\gamma L}^*\left(C_{VLL} + C_{VLR}\right) + C_{\gamma R}^*\left(C_{VRR} + C_{VRL}\right)\right] \\
&+ \frac{32e^2v^2}{m_i^2}\left(\log\frac{m_i^2}{m_f^2} - 3\right)\left(|C_{\gamma L}|^2 + |C_{\gamma R}|^2\right).
\end{aligned}
$$

TABLE 3.1 Matching of SM EFT Wilson coefficients in leptonic LFV decays of leptons. Here $X = L$ or R.

	Class (a): $\ell_i \to 3\ell_j$	Class (b): $\ell_i \to \ell_j 2\ell_k$	Class (c): $\ell_i^\pm \to \ell_j^\mp \ell_k^\pm \ell_k^\pm$
C_{VLL}	$2\left[(2s_W^2-1)\left(C_{\ell H}^{(1)ji} + C_{\ell H}^{(3)ji}\right) + C_{\ell\ell}^{jikk}\right]$	$(2s_W^2-1)\left(C_{\ell H}^{(1)ji} + C_{\ell H}^{(3)ji}\right) + C_{\ell\ell}^{jijj}$	$2C_{\ell\ell}^{kikj}$
C_{VRR}	$2\left(2s_W^2 C_{eH}^{ji} + C_{ee}^{jijj}\right)$	$2s_W^2 C_{eH}^{ji} + C_{ee}^{jikk}$	$2C_{ee}^{kikj}$
C_{VLR}	$2s_W^2\left(C_{\ell H}^{(1)ji} + C_{\ell H}^{(3)ji}\right) + C_{\ell e}^{jijj}$	$2s_W^2\left(C_{\ell H}^{(1)ji} + C_{\ell H}^{(3)ji}\right) + C_{\ell e}^{jikk}$	$C_{\ell e}^{kikj}$
C_{VRL}	$(2s_W^2-1)C_{eH}^{ji} + C_{\ell e}^{jjji}$	$(2s_W^2-1)C_{eH}^{ji} + C_{\ell e}^{jkki}$	$C_{\ell e}^{kjki}$
C_{SLR}	$-2\left[(2s_W^2-1)C_{eH}^{ji} + C_{\ell e}^{jjji}\right]$	$-2C_{\ell e}^{jkki}$	$-2C_{\ell e}^{kjki}$
C_{SRL}	$-2\left[2s_W^2\left(C_{\ell H}^{(1)ji} + C_{\ell H}^{(3)ji}\right) + C_{\ell e}^{jijj}\right]$	$-2C_{\ell e}^{jikk}$	$-2C_{\ell e}^{kikj}$
C_{SXX}	0	0	0
C_{TX}	0	0	0
$C_{\gamma L}$	$\sqrt{2}C_\gamma^{ij*}$	$\sqrt{2}C_\gamma^{ij*}$	0
$C_{\gamma R}$	$\sqrt{2}C_\gamma^{ji}$	$\sqrt{2}C_\gamma^{ji}$	0

Fundamental physics with muons: flavor violation

★ Employ bound states: μ conversion experiment



★ take low energy muons (~ 30 MeV) a stop them in a target A(Z,A-Z): muons cascade to atomic 1s state

★ Binding energy and orbit radius for muonic hydrogen-like state

$$E_b = -\frac{Z^2 m e^4}{8n^2} \sim \frac{Z^2 m}{n^2}$$

$$r = \frac{n^2}{Z\pi m e^2} \sim \frac{n^2}{Zm}$$

muonic atom is 200x stronger bound
radius is 200x smaller

★ The radial wave function for the hydrogen-like system: $R_{nl} \sim r^\ell Z^{3/2}$
overlap probability: $p \sim r^{2\ell} Z^3$

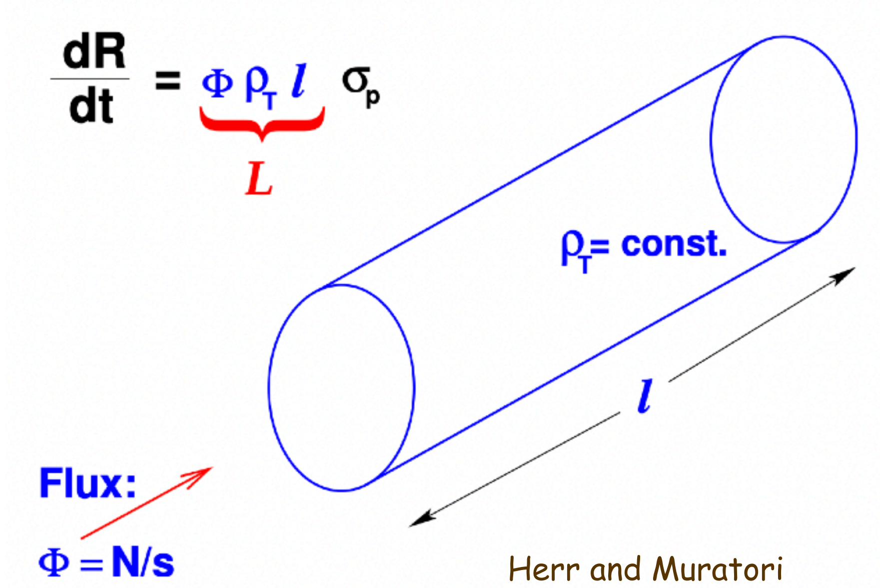
large overlap for an s-wave and high-Z nucleus

Measure $R_{\mu e} = \frac{\Gamma\left[\mu^- + (A,Z) \to e^- + (A,Z)\right]}{\Gamma\left[\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right]}$ to probe NP

Rare processes: luminosity

★ Need a lot of muons: high luminosity experiments

– Number of events/second



	Energy (GeV)	$\mathcal{L}$ $cm^{-2}s^{-1}$
SPS ($p\bar{p}$)	315x315	6 10^{30}
Tevatron ($p\bar{p}$)	1000x1000	50 10^{30}
HERA (e^+p)	30x920	40 10^{30}
LHC (pp)	7000x7000	10000 10^{30}
LEP (e^+e^-)	105x105	100 10^{30}
PEP (e^+e^-)	9x3	3000 10^{30}
KEKB (e^+e^-)	8x3.5	10000 10^{30}
eRHIC		10^{33}-10^{35}

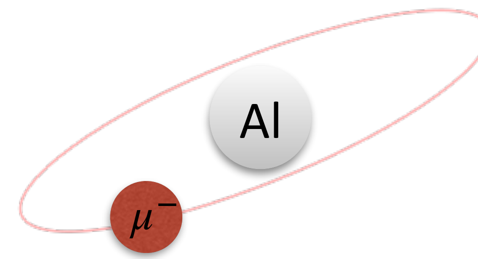
– ... or another way

$$L = \Phi \rho_T \ell = N \rho_T \frac{\ell}{t} = N \rho_T v$$

Fundamental physics with muons: flavor violation

- How effective is this approach compared to scattering/decays?
  - let's compute effective luminosity
  - recall that

$$L = \Phi \rho_T \ell = N \rho_T \frac{\ell}{t} = N \rho_T v$$



  - in this "experiment" probability density is given by the 1s wave function
  - ... and we need to take into account the fact that muon decays
  - Then Luminosity = (density)(velocity)(flux of muons)(lifetime)

$$L_{\text{eff}} = |\psi(0)|^2 \times \alpha Z \times \Phi_\mu \times \tau_\mu = \frac{m_\mu^3 Z^4 \alpha^4}{\pi} \Phi_\mu \tau_\mu$$

  - For Al target (Z=13), flux of $\Phi_\mu = 10^{10}$ muons/sec and $\tau_\mu = 2\ \mu$sec

Bernstein, Czarnecki

$$L_{\text{eff}} = 10^{48} \text{cm}^{-2}\ \text{sec}^{-1}$$

Recall the luminosity of the modern flavor experiment $L \sim 10^{34} - 10^{35}$ cm^{-2}sec^{-1}!

Bound states: muon conversion

★ Different nuclei are sensitive to a variety of New Physics scenarios, also

Nucleus	$R_{\mu e}(Z)$ / $R_{\mu e}(Al)$	Bound lifetime	Atomic Bind. Energy(1s)	Conversion Electron Energy	Prob decay >700 ns
Al(13,27)	1.0	.88 μs	0.47 MeV	104.97 MeV	0.45
Ti(22,~48)	1.7	.328 μs	1.36 MeV	104.18 MeV	0.16
Au(79,~197)	~0.8-1.5	.0726 μs	10.08 MeV	95.56 MeV	negligible

J. Miller, 2006

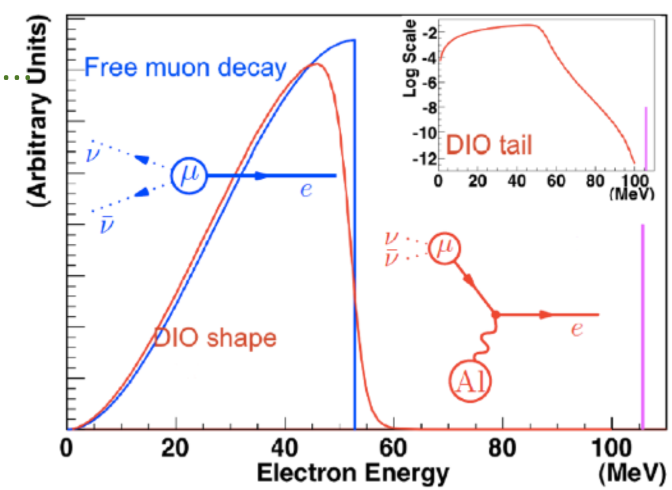
★ The experiment is tricky

- ✓ Muon conversion gives monoenergetic electrons...
- ✓ ... yet, there are other sources of electrons as well!

$$\begin{cases} \mu^- \to e^- + \bar{\nu}_e + \nu_\mu & \text{- decay (40\%)} \\ \mu^- + Al \to X + \nu_\mu & \text{- capture (60\%)} \\ \mu^- + Al \to e^- + Al & \text{- conversion} \end{cases}$$

$$\text{SINDRUM II (PSI), 2006 :} \qquad R_{\mu e} < 7 \times 10^{-13}$$

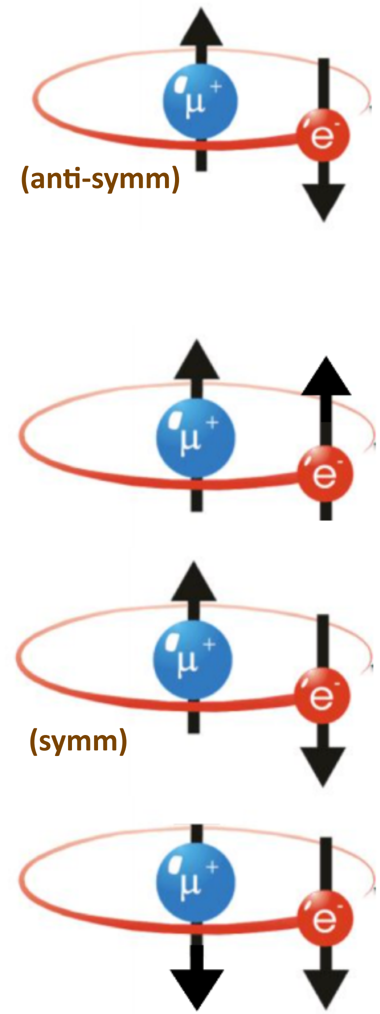
$$\text{M2e goal :} \qquad R_{\mu e} < \text{a few} \times 10^{-17}$$



Czarnecki, Marciano, Tormo

Muonium: just like hydrogen, but simpler!

- Muonium: a bound state of μ^+ and e^-
  - $(\mu^+\mu^-)$ bound state is a *true muonium*
- Muonium lifetime $\tau_{M_\mu} = 2.2\ \mu s$
  - main decay mode: $M_\mu \to e^+e^-\bar{\nu}_\mu\nu_e$
  - annihilation: $M_\mu \to \bar{\nu}_\mu\nu_e$
- Muonium’s been around since 1960's
  - used in chemistry
  - QED bound state physics, etc.
  - New Physics searches (oscillations)



Spin-0 (singlet) paramuonium

Spin-1 (triplet) orthomuonium

Hughes (1960)

The masses of singlet and triplet are almost the same!

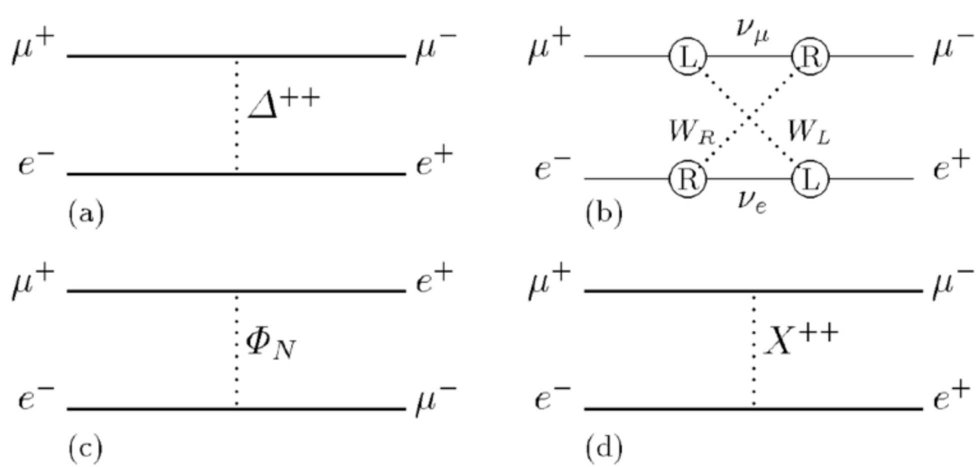
★ Lepton-flavor violating interactions can change $M_\mu \to \overline{M}_\mu$

– … just like $B^0\bar{B}^0$ mixing, but simpler!

Pontecorvo (1957)
Feinberg, Weinberg (1961)

- Such transition amplitudes are tiny in the Standard Model
  – … but there are plenty of New Physics models where it can happen

Clark, Love; Cvetic et al,
Li, Schmidt; Endo, Iguro, Kitahara;
Fukuyama, Mimura, Uesaka; …



$$\sim \left(\bar{\mu}\Gamma e\right)\left(\bar{\mu}\Gamma e\right)$$

effective operator

– theory: compute transition amplitudes for ALL New Physics models!

– experiment: produce M_μ but look for the decay products of $\overline{M}_\mu$

Combined evolution = flavor oscillations

★ Lepton-flavor violating interactions can change $M_\mu \to \overline{M}_\mu$

– … just like $B^0\bar{B}^0$ mixing, but simpler!

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Feinberg, Weinberg (1961)

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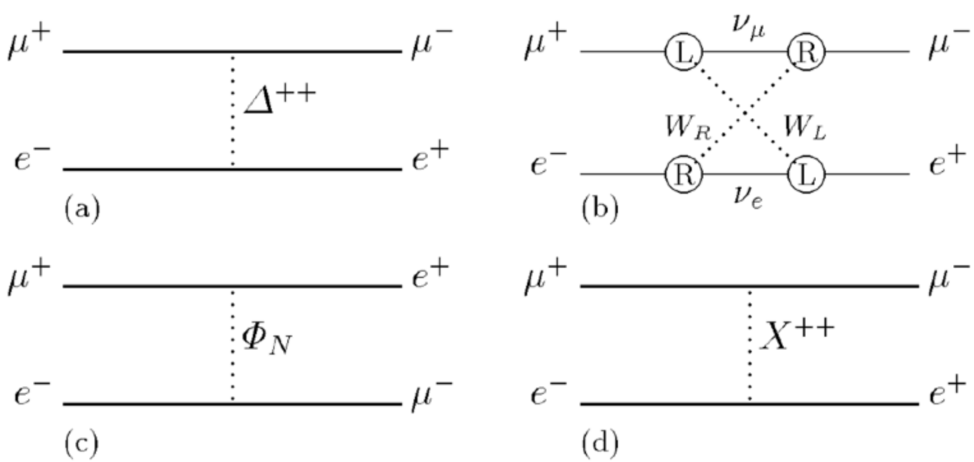


$$\sim \left(\bar{\mu}\Gamma e\right)\left(\bar{\mu}\Gamma e\right)$$

effective operator

– theory: compute transition amplitudes for ALL New Physics models!

– experiment: produce M_μ but look for the decay products of $\overline{M}_\mu$

- Mass difference comes from the dispersive part

$$x = \frac{1}{2M_M\Gamma}\mathrm{Re}\left[2\langle\overline{M}_\mu\left|\mathcal{H}_{\rm eff}\right|M_\mu\rangle + \cancel{\langle\overline{M}_\mu\left|i\int d^4x\ \mathrm{T}\left[\mathcal{H}_{\rm eff}(x)\mathcal{H}_{\rm eff}(0)\right]\right|M_\mu\rangle}\right]$$

  - consider only $\Delta L_\mu = 2$ Lagrangian contributions (largest?)

$$\mathcal{L}_{\rm eff}^{\Delta L_\mu=2} = -\frac{1}{\Lambda^2}\sum_i C_i^{\Delta L=2}(\mu)Q_i(\mu)$$

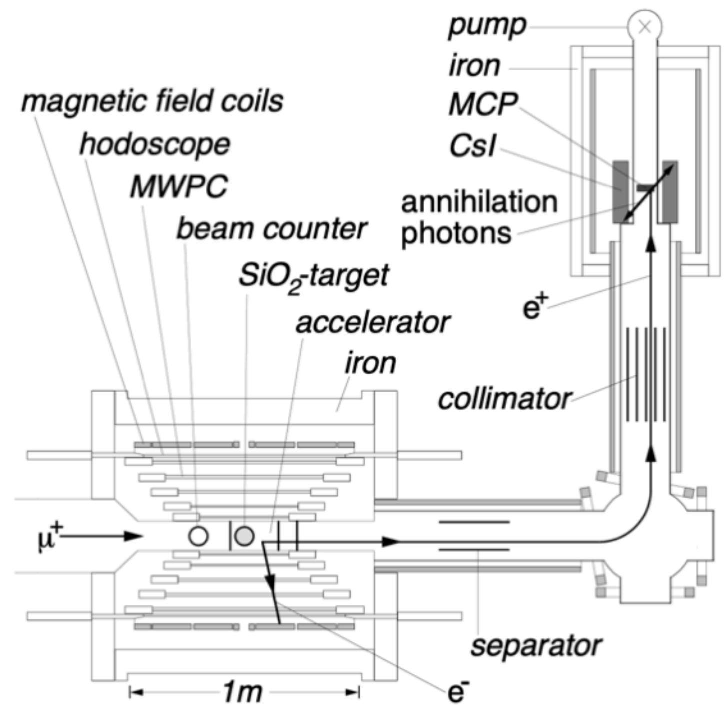
  - leading order: all heavy New Physics models are encoded in (the Wilson coefficients of) the five dimension-6 operators

$$\begin{aligned}
Q_1 &= (\overline{\mu}_L\gamma_\alpha e_L)(\overline{\mu}_L\gamma^\alpha e_L), \quad Q_2 = (\overline{\mu}_R\gamma_\alpha e_R)(\overline{\mu}_R\gamma^\alpha e_R), \\
Q_3 &= (\overline{\mu}_L\gamma_\alpha e_L)(\overline{\mu}_R\gamma^\alpha e_R), \quad Q_4 = (\overline{\mu}_L e_R)(\overline{\mu}_L e_R), \\
Q_5 &= (\overline{\mu}_R e_L)(\overline{\mu}_R e_L).
\end{aligned}$$

  - matrix elements for both singlet and triplet states: easy (QED only)

Experimental setup and constraints

- Similar experimental set ups for different experiments
  - example: MACS at PSI
  - idea: form M_μ by scattering muon (μ^+) beam on SiO_2 target
- A couple of "little inconveniences":
  - ➡ how to tell f apart from $\bar{f}$?
    - $M_\mu \to f$ decay: $M_\mu \to e^+ e^- \bar{\nu}_\mu \nu_e$
    - $\overline{M}_\mu \to \bar{f}$ decay: $\overline{M}_\mu \to e^+ e^- \bar{\nu}_e \nu_\mu$
    - $\bar{f}$: fast e^- (~53 MeV), slow e^+ (13.5 eV)
  - ➡ oscillations happen in magnetic field
    - ... which selects M_μ vs. $\overline{M}_\mu$



Muonium-Antimuonium Conversion Spectrometer (MACS)

L. Willmann, et al. PRL 82 (1999) 49

The most recent experimental data comes from 1999! Time is ripe for an update!

Experimental constraints

- We can put constraints on the Wilson coefficients of effective operators from the 1999 MACS data (assume single operator dominance)
  - presence of the magnetic field

$$P(M_\mu \to \overline{M}_\mu) \leq 8.3 \times 10^{-11}/S_B(B_0)$$

  - no separation of spin states: average

$$P(M_\mu \to \overline{M}_\mu)_{\rm exp} = \sum_{i=P,V} \frac{1}{2S_i+1} P({M_\mu}^i \to {\overline{M}_\mu}^i)$$

  - set Wilson coefficients to one, set constraints on the scale probed

Operator	Interaction type	$S_B(B_0)$ (from [9])	Constraints on the scale Λ, TeV
Q_1	$(V-A)\times(V-A)$	0.75	5.4
Q_2	$(V+A)\times(V+A)$	0.75	5.4
Q_3	$(V-A)\times(V+A)$	0.95	5.4
Q_4	$(S+P)\times(S+P)$	0.75	2.7
Q_5	$(S-P)\times(S-P)$	0.75	2.7
Q_6	$(V-A)\times(V-A)$	0.75	0.58×10^{-3}
Q_7	$(V+A)\times(V-A)$	0.95	0.38×10^{-3}

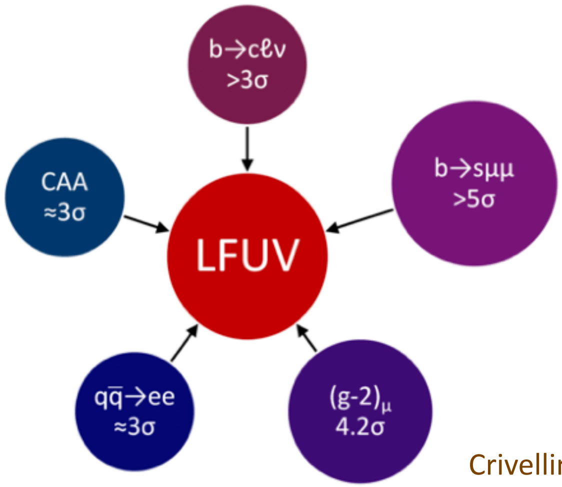
R. Conlin and AAP, Phys.Rev.D 102 (2020) 9, 095001

Muons and recent experimental anomalies

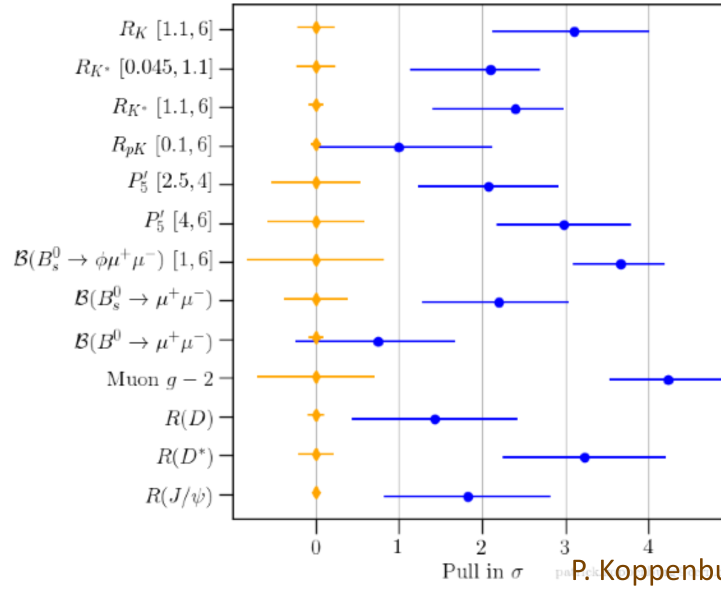
Are there any indications that NP might affect processes with muons?

Muons and recent experimental anomalies

★ Many experimental anomalies involve interactions with muons and taus

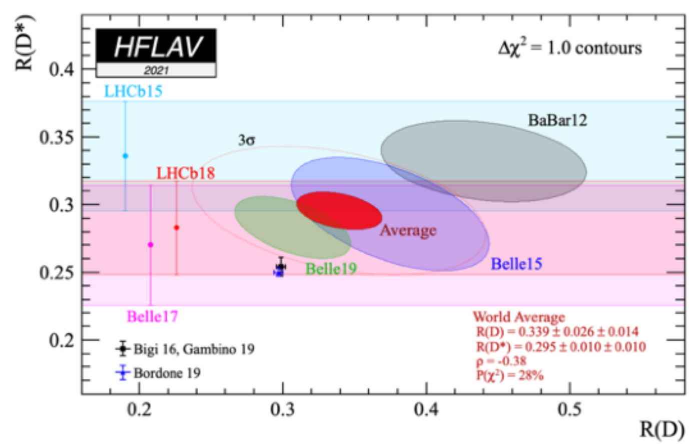


Crivellin, Hoferichter



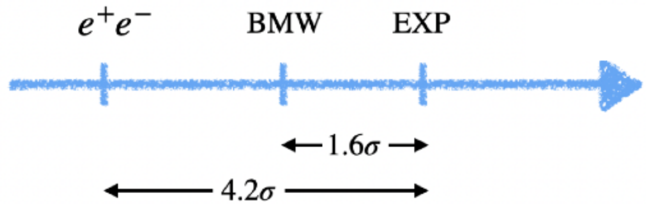
P. Koppenburg

- other lepton-flavor conserving processes
  - magnetic properties: muon g-2
    - currently a discrepancy theory/exp
  - electric properties: muon EDM
    - probes CP-violation in leptons
  - muonic hydrogen
    - proton size/QED/New Physics

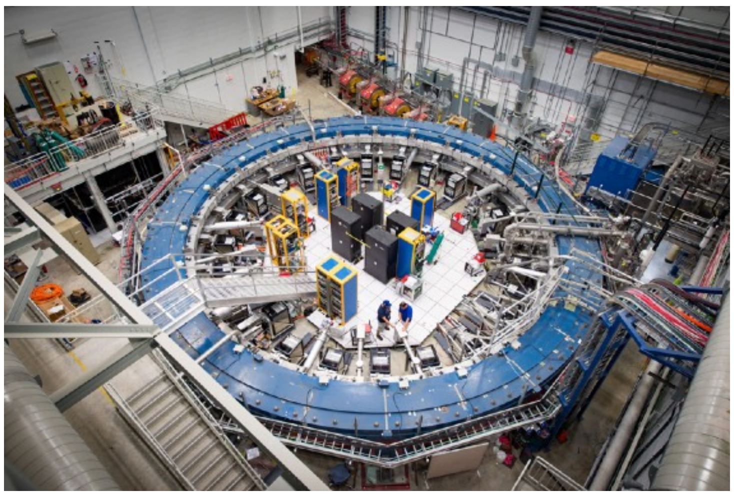


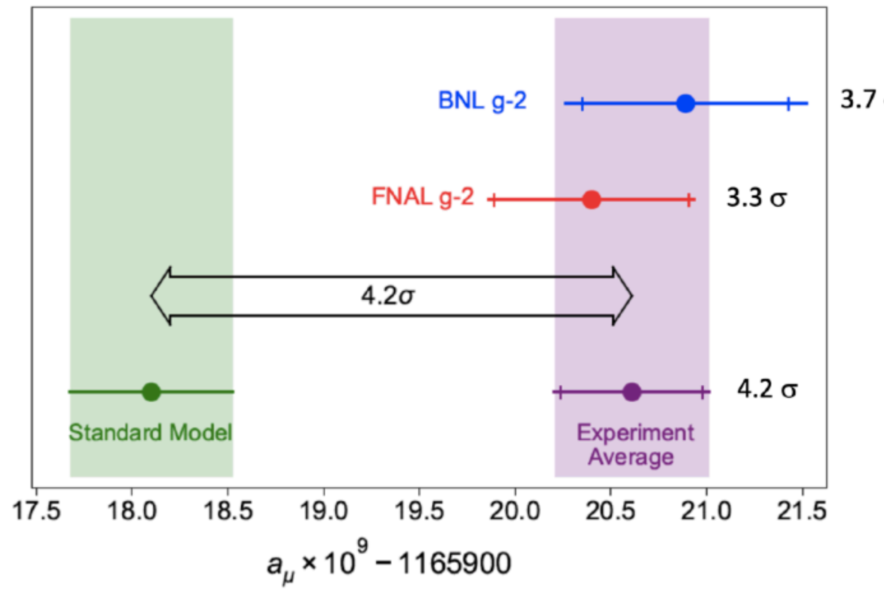
★ Muon's magnetic properties (g-2): $a_\mu = (g-2)/2$ with $\vec{\mu} = g\dfrac{e}{2m}\vec{s}$

discrepancy



Di Luzio, et al.





FNAL (g-2): $a_\mu(\mathrm{Exp}) = 116592061(41) \times 10^{-11}$

$a_\mu(\mathrm{Theory}) = 116591810(43) \times 10^{-11}$

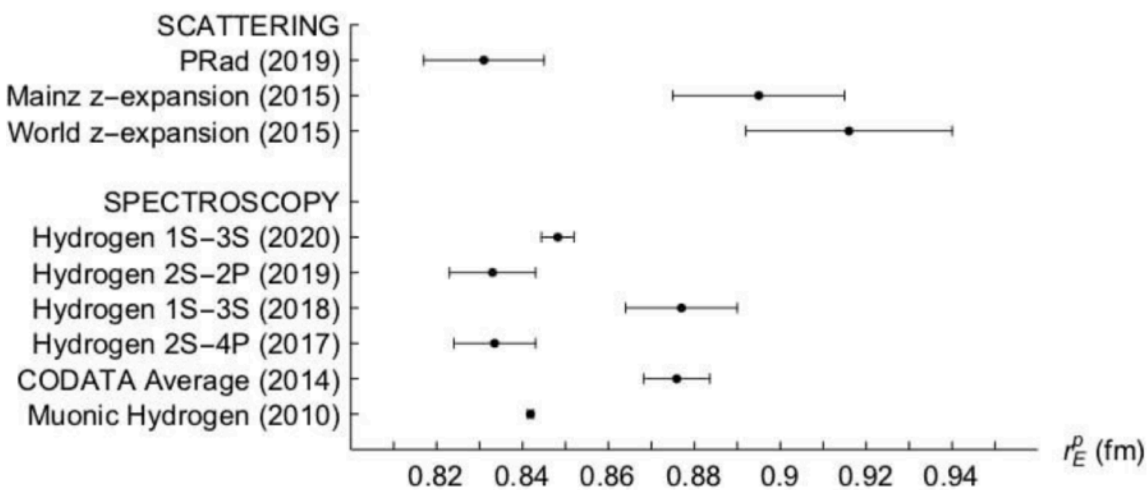
$a_\mu(\mathrm{BMW}) = 116591954(55) \times 10^{-11}$

Are there possible New Physics particles that are responsible for this difference?

Muons and recent experimental anomalies

★ Proton's radius from muonic hydrogen: possible New Physics?

★ Level splittings (e.g. Lamb shift) are sensitive to the charge radius of the proton



★ They are also sensitive to QED radiative corrections

★ Are there possible light New Physics particles that are responsible for this difference?

Barger et al, PRL 106 (2011) 153001

Remove proton radius issue from the problem: atomic physics with muonium?

Leptons: things to take home

- There is no indication from high energy studies where the NP show up
  - this makes indirect searches the most valuable source of information
- Muonium is the simplest atom: atomic physics
  - level splitting (Lamb shift): probe NP w/out QCD complications

MuSEUM experiment (J-PARC)

- Muons are ideal tools to probe fundamental physics
  - flavor-conserving quantities (g-2, EDM)
  - flavor-changing neutral current decays
  - flavor oscillations (muonium-antimuonium conversion)
  - muon transitions already probe the LHC energy domain and can do better!
  - all studies are complimentary to each other
- New experimental facilities are needed (AMF?)

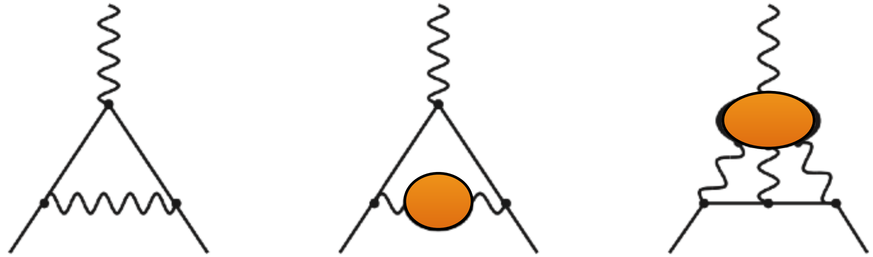
Prospects for precise predictions of a_μ in the Standard Model
G. Colangelo, et. al., arXiv:2203.15810 [hep-ph]

Snowmass2021 Whitepaper: Muonium to antimuonium conversion
A.-Y. Bai, …, AAP, …, arXiv:2203.11406 [hep-ph]


EXTRA!

Theoretical issues with g-2

- Independent lattice computations of HVP
- Data-driven estimates of hadronic vacuum polarization (HVP)
  - discrepancy between KLOE and BaBar data used in HVP

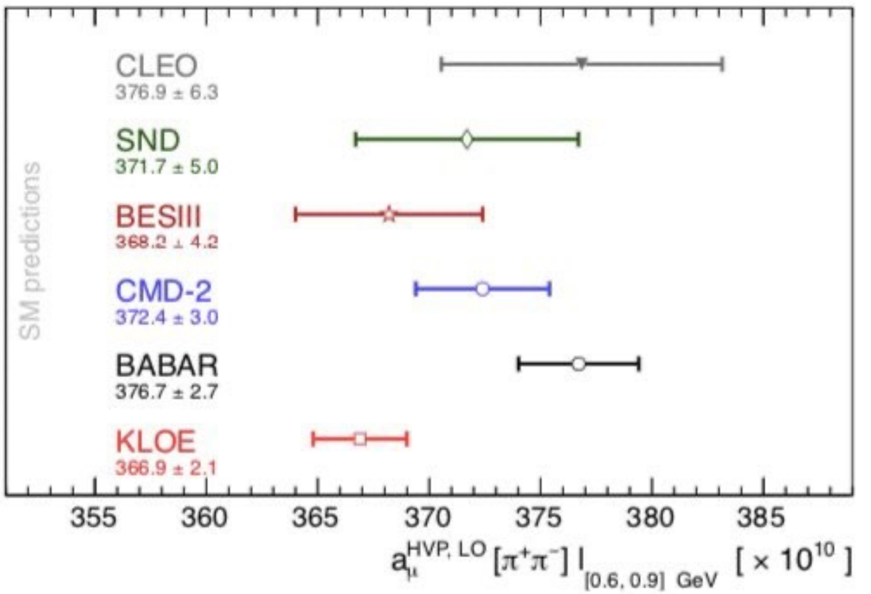


Leading order   Vacuum polarization   Light-by-light scattering

$$a_\mu^{hvp} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s} K(s) R(s)$$

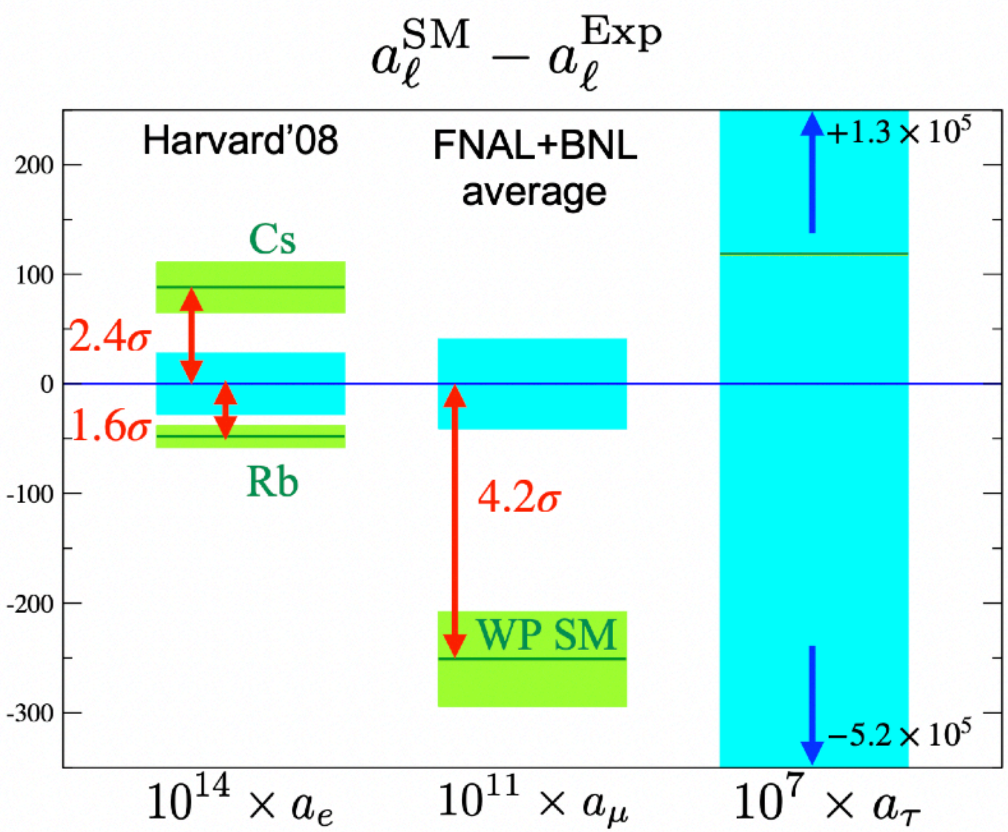
$$\frac{1}{12\pi} R(s) = \frac{1}{12\pi} \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)m_\mu^2}{x^2 m_\mu^2 + (1-x)s}$$



  - need radiative return Belle II data to eliminate the discrepancy
  - τ-decay data is not currently used: Belle II + lattice?

Summary of leptonic anomalous magnetic moments



Sensitivity to heavy new physics:

$$a_\ell^{\rm NP} \sim \frac{m_\ell^2}{\Lambda^2}$$

$$(m_\mu/m_e)^2 \sim 4 \times 10^4$$

Cs: α from Berkeley group [Parker et al, Science 360, 6385 (2018)]

Rb: α from Paris group [Morel et al, Nature 588, 61–65(2020)]

A. El-Khadra (talk at LP21)

Conversion probability

★ Calculation of muon conversion probability involves interesting interplay of particle and nuclear physics - and QED!

Measure $$R_{\mu e} = \frac{\Gamma\left[\mu^- + (A,Z) \to e^- + (A,Z)\right]}{\Gamma\left[\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right]}$$ to probe NP

★ Lepton wave functions are taken as solutions of Dirac equation

- with usual substitutions $u_1(r) = r\,g(r)$ and $u_2(r) = r\,f(r)$

$$\frac{d}{dr}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} -\kappa/r & W - V + m_i \\ -(W - V - m_i) & \kappa/r \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\psi = \psi^\mu_\kappa = \begin{pmatrix} g(r)\chi^\mu_\kappa(\theta,\phi) \\ i f(r)\chi^\mu_{-\kappa}(\theta,\phi) \end{pmatrix}$$

★ ... with Dirac equation in a potential $V(r) = -e\int_r^\infty E(r')dr'$

$$E(r) = \frac{Ze}{r^2}\int_0^r r'^2 \rho^{(p)}(r')dr'$$

SINDRUM II (PSI), 2006 :	$R_{\mu e} < 7 \times 10^{-13}$
M2e goal :	$R_{\mu e} <$ a few $\times 10^{-17}$

Conversion probability

★ Calculation of muon conversion probability involves interesting interplay of particle and nuclear physics

★ Nuclear averages are often done as an approximation. For a general quark operator Q

$$\langle N|Q|N\rangle = \int d^3r \, [Z \rho_p(r) \langle p|Q|p\rangle + (A-Z)\, \rho_n(r) \langle n|Q|n\rangle]$$

p(n) densities

$$\rho_{p(n)}(r) = \frac{\rho_0}{1+\exp[(r-c)/z]}, \quad \int d^3 \rho_{p(n)}(r) = 1$$

★ Matrix elements of light quark currents are easily computed

- since $(m_\mu - m_e) << m_N$ we can neglect space components of the quark current

$$\langle p|\bar{u}\gamma^0 u + c_d \bar{d}\gamma^0 d|p\rangle = 2 + c_d$$

$$\langle n|\bar{u}\gamma^0 u + c_d \bar{d}\gamma^0 d|n\rangle = 1 + 2c_d$$

count number of quarks

★ Gluonic contribution can be removed removed using anomaly equation or can be computed

Conversion probability

★ Calculation of muon conversion probability involves interesting interplay of particle and nuclear physics

★ Nuclear averages are often done as an approximation. For a gluonic Rayleigh operator

$$\langle N|\frac{\beta_L}{4\alpha_s}G^a_{\mu\nu}G^{a\mu\nu}|N\rangle = -\frac{9}{2}\left[ZG^{(g,p)}\rho^{(p)} + (A-Z)G^{(g,n)}\rho^{(n)}\right],$$

where $G^{(g,\mathcal{N})} = \langle\mathcal{N}|\frac{\alpha_s}{4\pi}G^a_{\mu\nu}G^{a\mu\nu}|\mathcal{N}\rangle \approx -189 \text{ MeV}$

★ The (coherent) conversion rate is

$$\Gamma_{(}\mu^- + (A,Z) \to e^- + (A,Z)) = \frac{4a_N^2}{\Lambda^4}\left(|c_1|^2 + |c_3|^2\right)$$

with $a_N = G^{(g,p)}S^{(p)} + G^{(g,n)}S^{(n)}$

The overlap integrals $S^{(p,n)}$ with muon and electron wave functions are

$$S^{(p)} = \frac{1}{2\sqrt{2}}\int_0^\infty drr^2 Z\rho^{(p)}(g_e^- g_\mu^- - f_e^- f_\mu^-),$$

$$S^{(n)} = \frac{1}{2\sqrt{2}}\int_0^\infty drr^2 (A-Z)\rho^{(n)}(g_e^- g_\mu^- - f_e^- f_\mu^-).$$