Status of addition of NA61 data into PPFX Package

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11 March 2024

Outline

- Introduction
- Data Used
- G4HP Simulations
- Addition of New Reweighter
- Addition of Uncorrelated Uncertainties
- Addition of Correlated Uncertainties
- SVD decomposition for different detectors
- Comparison of G4HP and G4NuMI

Introduction

- PPFX (Package to Predict Flux) is an experiment independent package for NuMI beamline
- Constrains the hadron production model used in the beamline simulation with external measurements on thin and thick targets
- Already using data from external experiments like NA49, MIPP

NA61 Data Used

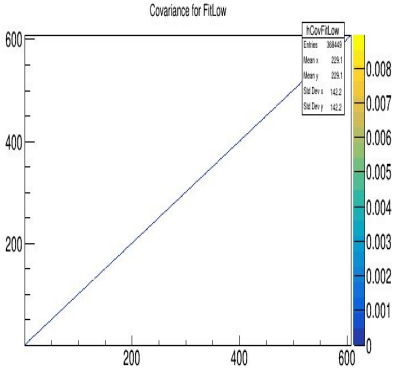
Phys. Rev. D **100**, 112004

- NA61 (the successor of NA49) is a fixed-target experiment at CERN SPS
- Measurement of multiplicities of π^+ , π^- , K+, K-, proton, Kos, Λ, $\bar{\Lambda}$ in momentum(GeV/c) and angle (mrad) bins
- Covariance matrices for different systematics
- Individual Errors (Statistical, Physics Model, Momentum Scale, Reconstruction, Fit Uncertainty, Feeddown, Normalization, Event Selection)

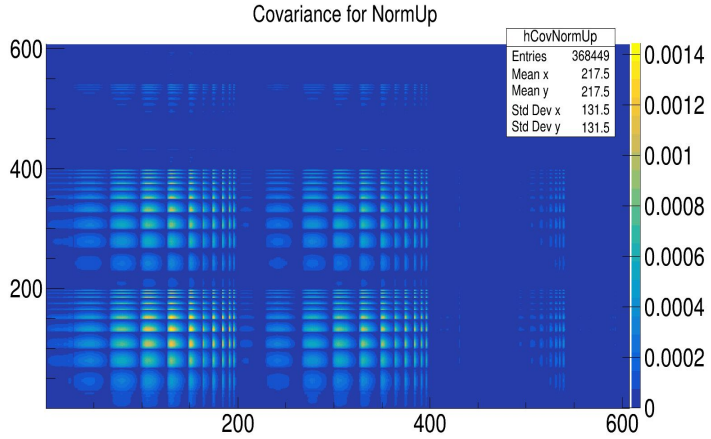
Systematic Uncertainties

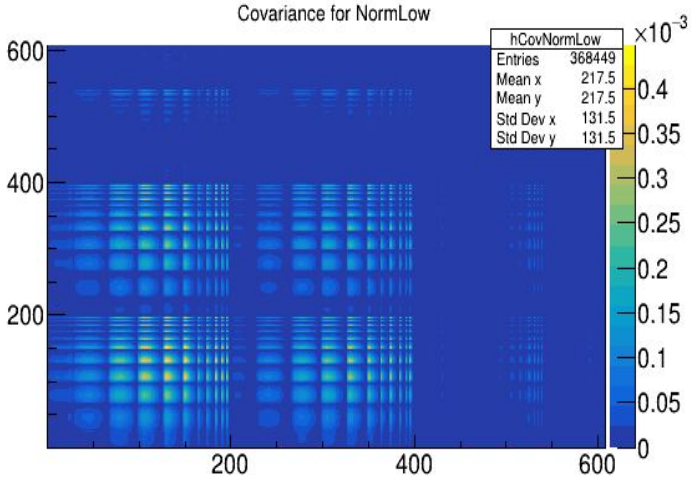
- Covariance matrices for Up and Low Uncertainties

Correlated	Uncorrelated
Momentum Scale	Statistical
Physics Model	Fit Uncertainties
Fit Uncertainty	
Feeddown	
Reconstruction	
Normalization	
Event Selection	



- **Event Selection**: Exist for Positive only
- The up and low matrices are symmetric but non identical
- We need a final Covariance Matrix
- PPFX adds uncorrelated (assign random number) and decomposition)





Plots from NA61 data

G4HP Simulations

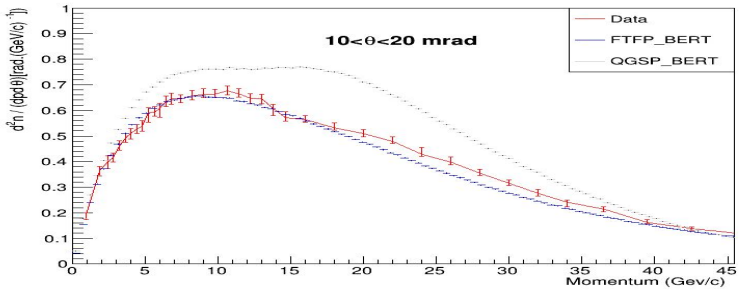
- Generated G4HP ntuples

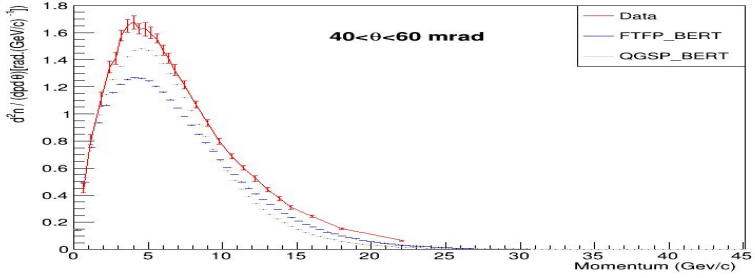
  Incident particle: Pions , **Target:** Carbon

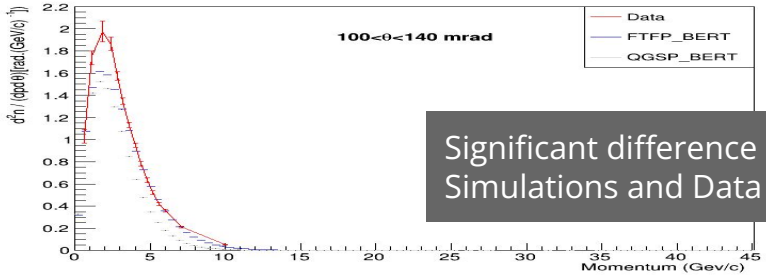
  njobs= 500 **Incident Particles**: 10000000 (10M)

- Considering the interaction: $\pi^+ + C \rightarrow \pi^+ + X$ @ 60 GeV
- Multiplicity vs momentum from G4HP ntuples
- Comparison of NA61 data with QGSP_BERT and FTFP_BERT for some angle ranges for produced pions form incident pi+ multiplicities for GEANT4 (v4.9.2.p03, NuMI simulation (G4NuMI)) by using G4HP

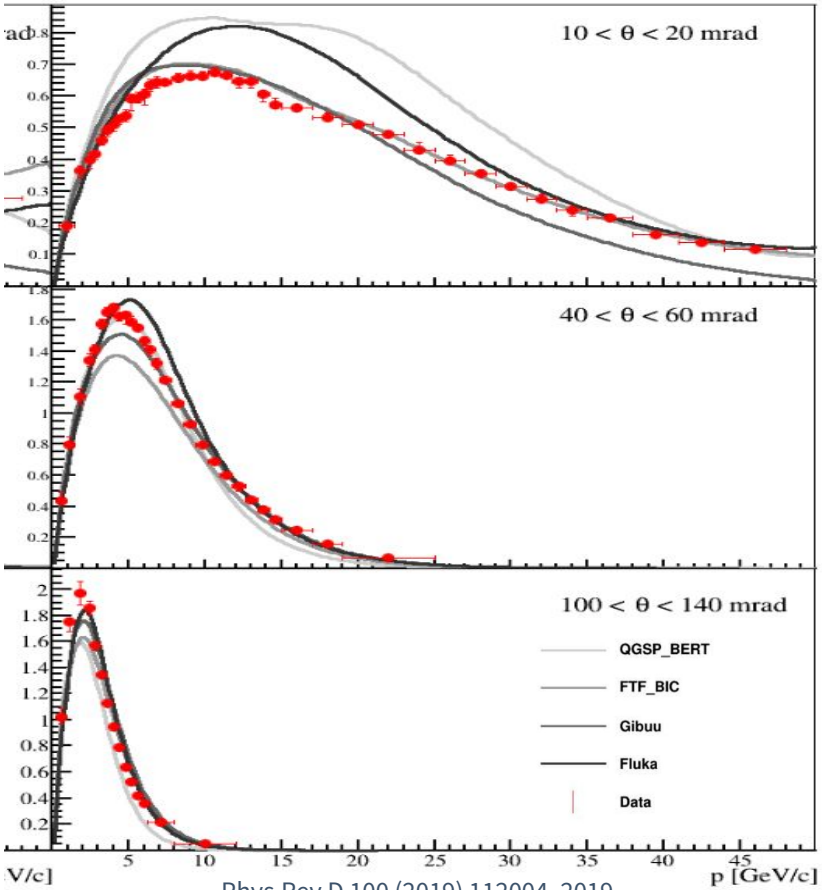
Comparison of G4HP Simulation with NA61 Data







Significant difference between Simulations and Data



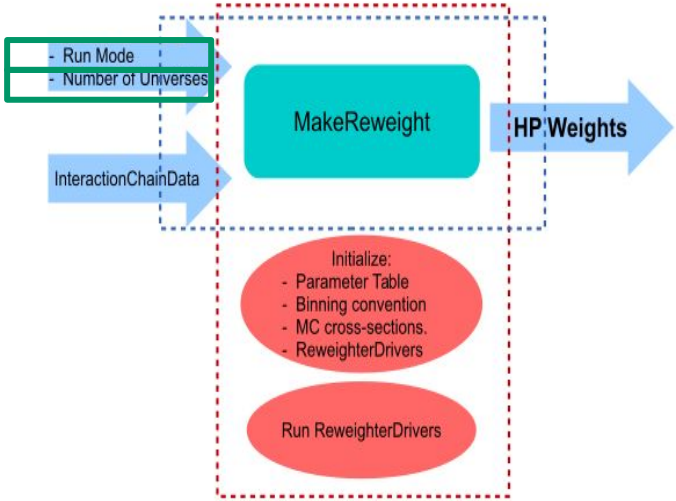
Phys.Rev.D 100 (2019) 112004, 2019

Addition of New Reweighter

- Run Mode: Switch to choose which reweighters we want to use
- Introduced **NA61On:** NA61 and NA49 data will be used for weighing simulations

  BothOff: Only NA49 data will be used

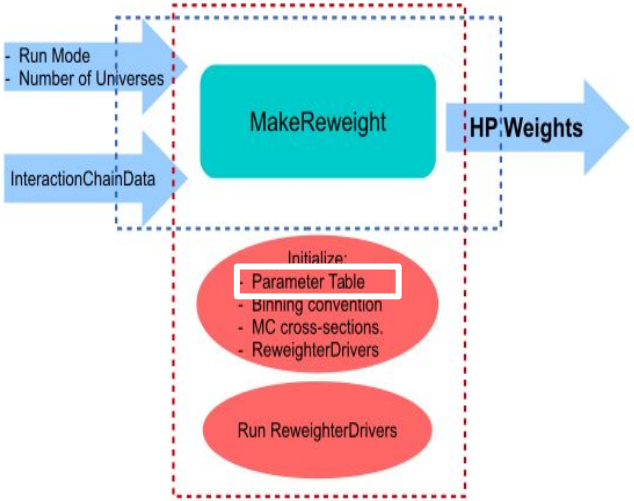
```
void ReweightDriver::ParseOptions(){
  //Parsing the file input:
  using boost::property_tree::ptree;
  ptree top;
  std::string val = "";
  read_xml(fileOptions.c_str(),top,2); // option 2 removes comment strings
  ptree& options = top.get_child("inputs.Settings");

  val = options.get<std::string>("Reweighters");
  if(val=="MIPPNuMIOn"){
              doMIPPNumi = true;
              doNA61 = false;
                }
  else if(val=="NA61On"){doMIPPNumi = false; doNA61=true;
       // std::cout<<"NA61 is on and the other (MIPP) is OFF"<<std::endl;
}
  else if(val== "BothOff"){doMIPPNumi = false; doNA61=false;
       std::cout<<"NA61 and MIPP are OFF, only the NA49 weights would act now"<<std::endl;

}
```



Leo's Thesis

Addition of New Reweighter (contd.)

- Added Uncorrelated and Correlated data to Parameter Table *ppfx/uncertainties/Parameters_default.xml*
- Statistical and Fit Uncertainties are Uncorrelated




```
<ThinTarget_pipC_pip_stat>
<!-- stats -->
<cvs>0.0612 0.128 0.168 0.176 0.193 0.206 0.209 0.214 0.222 0.232 0.232 0.229 0.242 0
0.191 0.364 0.398 0.417 0.46 0.493 0.511 0.528 0.537 0.591 0.594 0.608 0.635 0.645 0.
0.397 0.355 0.316 0.275 0.24 0.215 0.163 0.137 0.118 0.254 0.485 0.691 0.847 0.925 0.
0.822 0.764 0.701 0.604 0.438 0.39 0.305 0.244 0.168 0.0884 0.436 0.796 1.11 1.34 1.4
0.528 0.422 0.377 0.313 0.246 0.153 0.0661 0.683 1.35 1.71 1.94 1.91 1.86 1.73 1.66 1
1.97 1.86 1.57 1.34 1.13 0.946 0.785 0.636 0.526 0.419 0.358 0.213 0.0533 1.64 1.17 0
0.199 0.119 0.0352 1.63 1.54 0.809 0.451 0.255 0.137 0.043 1.65 1.25 0.503 0.227 0.07

<errs>
0.0021962451759023286 0.0027572669868094683 0.0038028923005028366 0.00383565936849867
0.004402202171314932 0.004552597667444513 0.004539914471928498 0.0045532806465659395
0.003148912009293594 0.003214907836340479 0.0032736594494413803 0.0033437147565748377
0.004324766133430981 0.004559703109791298 5 0.00474663222994158 0.004402633737434209 0
0.01060614578062344 0.011044234417585314 0.01127548434590908 0.011358433264925851 0.0
0.01262370706290746 0.012703008909306444 0.008996557315911514 0.009050496485371672 0.
0.009143703620565113 0.00908727307138275 0.008806950938392552 0.008351863032697613 0.
0.006762539746891444 0.00647852351311378 0.006171666478775915 0.00585189512029471 0.0
0.0030868935040239943 0.002468831006128268 0.005683115770803574 0.005699979513479965
```

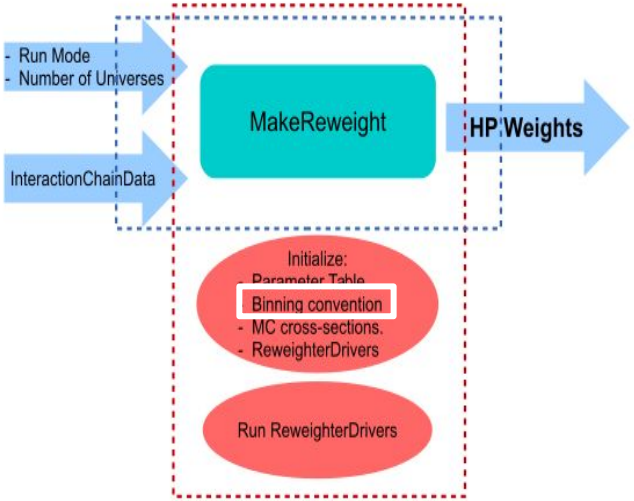
Uncorrelated Parameter

Addition of New Reweighter (contd.)

- Added the binning convention used by NA61 experiment

ppfx/data/BINS/ThinTarget_pipC_pip_Bins.xml

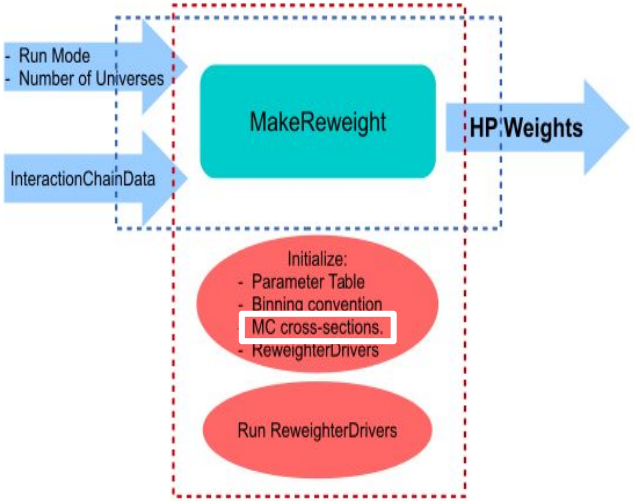
```
<bins>
<!-- List of bins -->
  <ThinTarget_pipC_pip>
  <!-- NA61 60 GeV pip inputs are theta (polar angle) [mrad]

     <ThinTarget_pipC_pip_0>
    <prange> 0.40 1.50 </prange>
    <thetarange> 0.00 10.00 </thetarange>
     </ThinTarget_pipC_pip_0>
     <ThinTarget_pipC_pip_1>
     <prange> 1.50 3.00 </prange>
     <thetarange> 0.00 10.00 </thetarange>
     </ThinTarget_pipC_pip_1>
     <ThinTarget_pipC_pip_2>
     <prange> 3.00 4.00 </prange>
     <thetarange> 0.00 10.00 </thetarange>
     </ThinTarget_pipC_pip_2>
     <ThinTarget_pipC_pip_3>
     <prange> 4.00 5.00 </prange>
     <thetarange> 0.00 10.00 </thetarange>
     </ThinTarget_pipC_pip_3>
     <ThinTarget_pipC_pip_4>
     <prange> 5.00 6.00 </prange>
     <thetarange> 0.00 10.00 </thetarange>
     </ThinTarget_pipC_pip_4>
     <ThinTarget_pipC_pip_5>
     <prange> 6.00 7.00 </prange>
     <thetarange> 0.00 10.00 </thetarange>
     </ThinTarget_pipC_pip_5>
     <ThinTarget_pipC_pip_6>
     <prange> 7.00 8.00 </prange>
     <thetarange> 0.00 10.00 </thetarange>
     </ThinTarget_pipC_pip_6>
```



Addition of New Reweighter (contd.)

- *Added a new file consisting of multiplicity value in each bin* *ppfx/data/NA61/pipC_pip_mc.xml*
- cvmc has been evaluated from the G4HP simulations

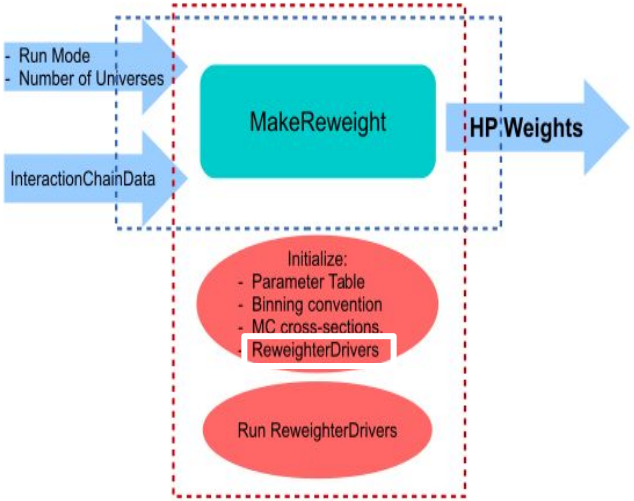
```
<mcbins>
  <pipC_pip_mc>
   <pipC_pip_mc_0>
    <cvmc> 0.051440957741781076  </cvmc>
    <prange> 0.40 1.50 </prange>
    <thetarange> 0.00 10.00 </thetarange>
   </pipC_pip_mc_0>
     <pipC_pip_mc_1>
     <cvmc> 0.12588079233084856    </cvmc>
     <prange> 1.50 3.00 </prange>
     <thetarange> 0.00 10.00 </thetarange>
     </pipC_pip_mc_1>
     <pipC_pip_mc_2>
     <cvmc> 0.1781363167542372    </cvmc>
     <prange> 3.00 4.00 </prange>
     <thetarange> 0.00 10.00 </thetarange>
     </pipC_pip_mc_2>
     <pipC_pip_mc_3>
     <cvmc>0.20136830972422048     </cvmc>
     <prange> 4.00 5.00 </prange>
     <thetarange> 0.00 10.00 </thetarange>
     </pipC_pip_mc_3>
     <pipC_pip_mc_4>
     <cvmc> 0.21648411033042347    </cvmc>
     <prange> 5.00 6.00 </prange>
     <thetarange> 0.00 10.00 </thetarange>
     </pipC_pip_mc_4>
     <pipC_pip_mc_5>
     <cvmc> 0.2301189154050759    </cvmc>
     <prange> 6.00 7.00 </prange>
     <thetarange> 0.00 10.00 </thetarange>
     </pipC_pip_mc_5>
```



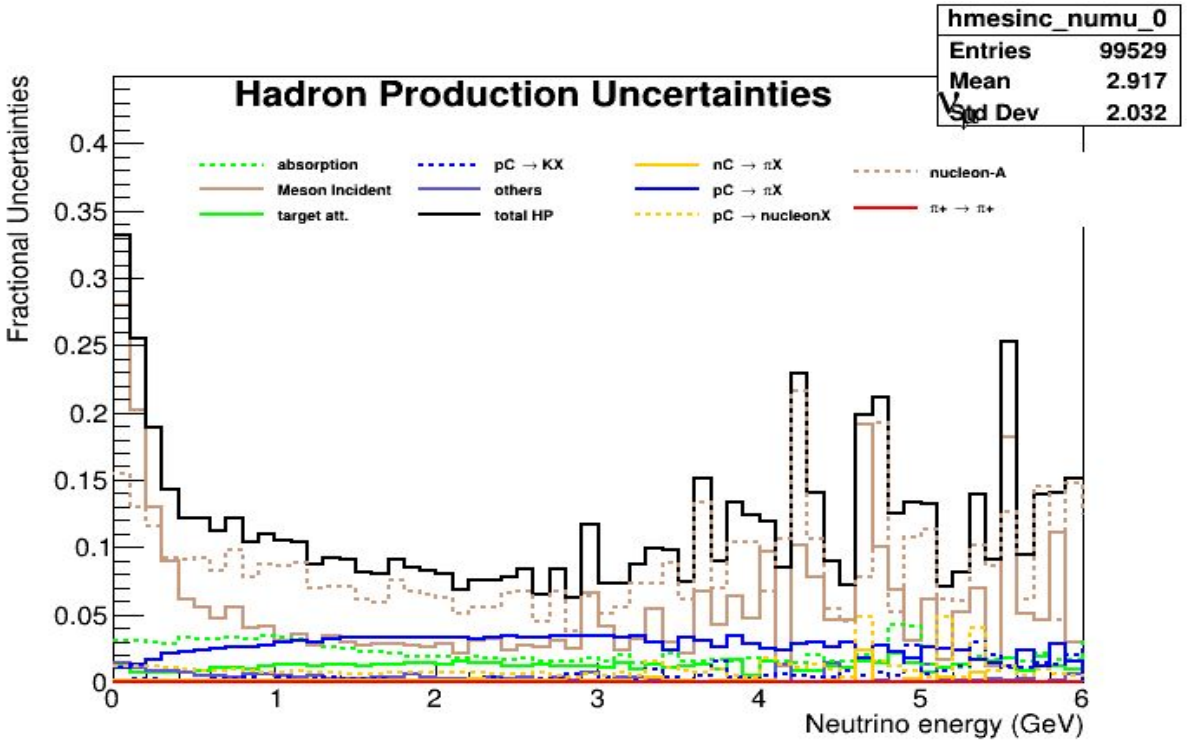
Addition of New Reweighter (contd.)

Added a new Reweighter driver

```
double ThinTargetpipCpipReweighter::calculateWeight(const InteractionData& aa){

  double wgt = 1.0;
  double low_value = 1.e-18;     //BHU, Copied it, but how do we get this value

  bool right_inc = aa.Inc_pdg == 211;
  bool right_prod = aa.Prod_pdg == 211;

  ThinTargetpipCpipBins*  Thinbins =  ThinTargetpipCpipBins::getInstance();
  ThinTargetpipCpipMC*  mc =  ThinTargetpipCpipMC::getInstance();
 //Getting the MC:

 double pipC_pip_cv = mc->getMCval(aa.Prod_P, aa.Theta, aa.Inc_pdg, aa.Prod_pdg);    //BHU.Edit this
//  std::cout<<"The central value from mc is: "<<pipC_pip_cv<< std::endl;

 if(pipC_pip_cv<1.e-18){
 std::cout<<"LOW MC VAL: "<<pipC_pip_cv<<std::endl;

 //std::cout<<"The central value from mc is: "<<pipC_pip_cv<< std::endl;

 return 1.0;}
 //getting data bin value
   int bin      = Thinbins->pipC_pip_BinID(aa.Prod_P,aa.Theta, aa.Inc_pdg, aa.Prod_pdg);
 // std::cout<<"The bin number from data bins is: "<<bin<<std::endl;
 if(bin<0 || bin>200){
 std::cout<<"BINID from data is less than ZERO or greater than 200, incident mom is:" <<aa.Inc_P<<"Theta value
    return 1.0;}
//Calculating the weight
 if(aa.Prod_pdg == 211 && bin>=0 && aa.Inc_pdg==211)
  wgt = vbin_data_pipC_pip[bin]/pipC_pip_cv;
 std::cout<<"The bin content from data for this particular bin is:"<< vbin_data_pipC_pip[bin] <<std::endl;
 std::cout<<"Weight by NA61 reweighter using Statistical only is : "<<wgt<<std::endl;
```



Muon Neutrino Corrected Flux (NA49 data)

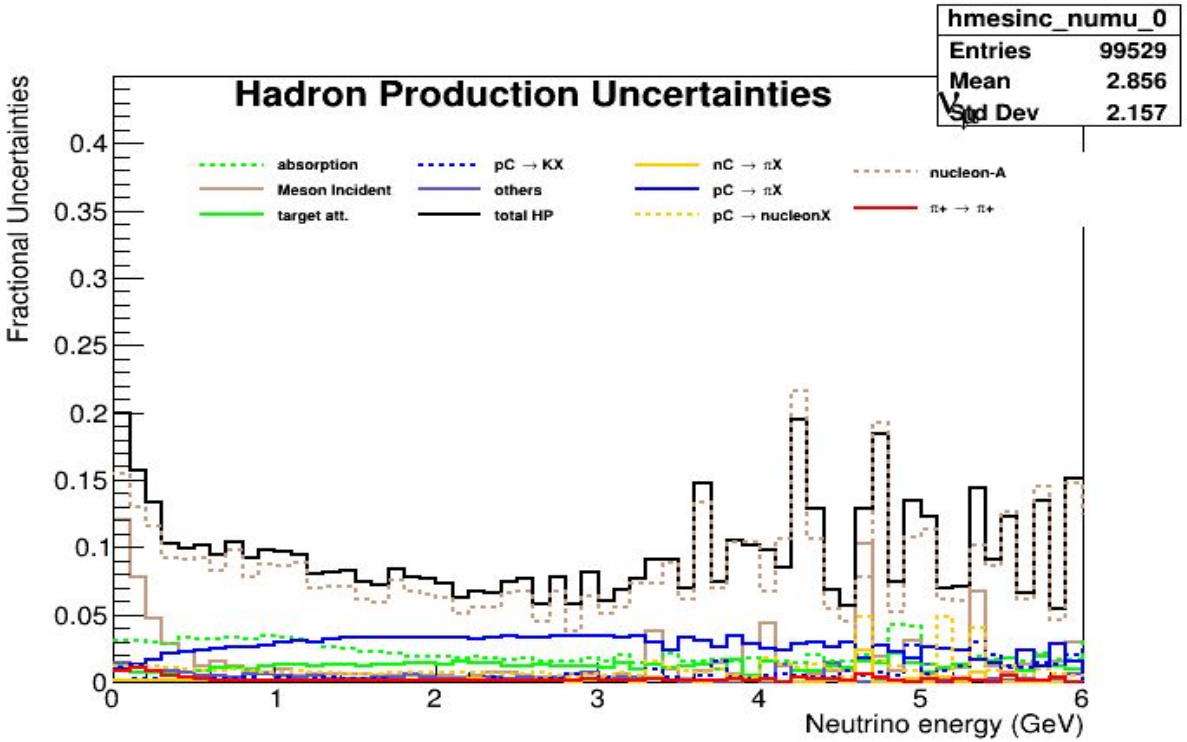


- Use of NA49 data only
- $\pi^+ + C \longrightarrow \pi^+$ is a part of Meson Interaction reweighter
- 40% uncertainty is assumed
- Whenever correction using

  NA61 data: pip -> pip, in a New reweighter

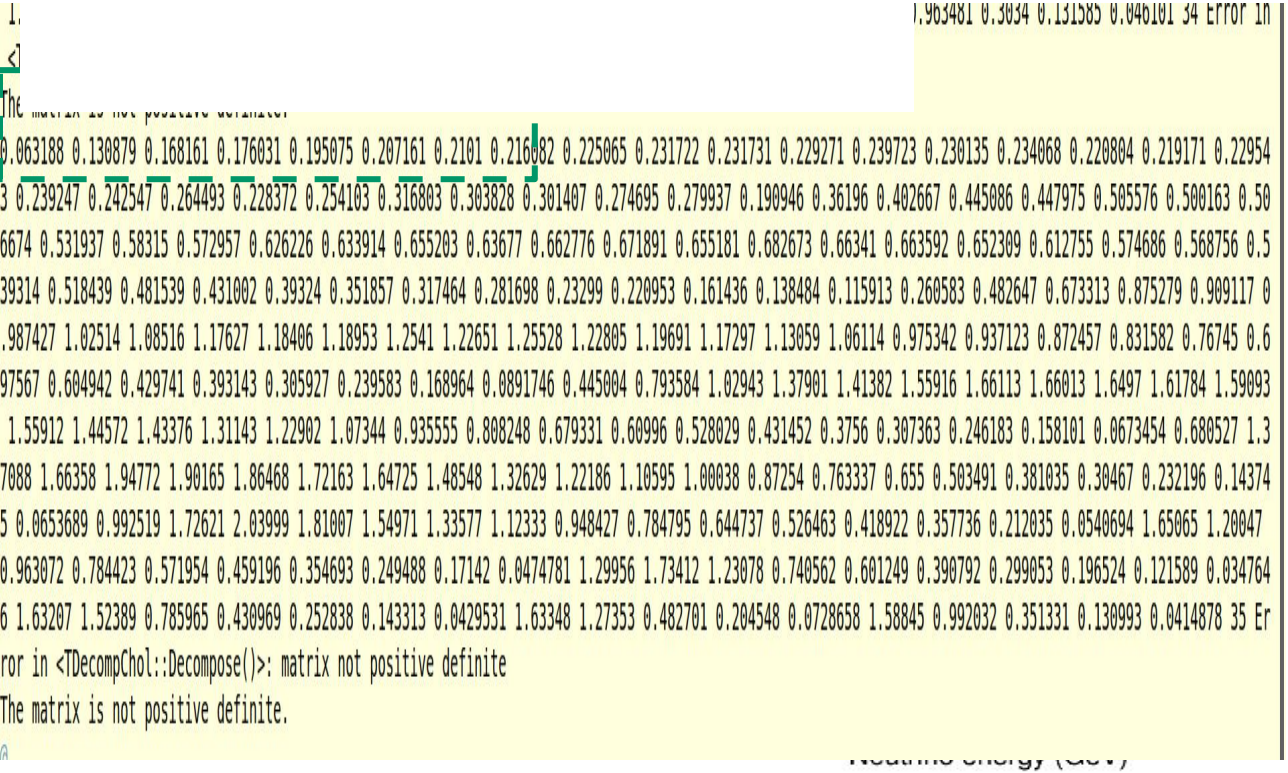
  NA49 data: pip->pip, MesonIncident reweighter

Muon Neutrino Corrected Flux (NA61 data)



- Considering Statistical and Fit Uncertainties
- Significant reduction in Meson Incident Contribution and total uncertainty
- No correlated systematics considered yet
- π^+ ---> π^+ interaction has now been constrained by data

Muon neutrino corrected flux CHOL



Statistical, Fit as Uncorrelated and rest all errors as correlated

CHOLESKY decomposition NO difference, decomposition does not take pla

Covariance Matrix

```
Cov_matrix = np.add(np.add(np.add(np.add(np.add(PhyMatrix_low,NormMatrix_low),MomMatrix_low),FeedMatrix_up),RecMatri
```

```
print(Cov_matrix)
```

```
[[2.75494576e-05 7.16396823e-05 4.66064358e-05 ... 7.04499022e-05
  3.72444066e-05 3.50955634e-05]
 [7.16396823e-05 1.98094645e-04 1.28110935e-04 ... 1.94432532e-04
  1.02513388e-04 9.82931572e-05]
 [4.66064358e-05 1.28110935e-04 8.47558060e-05 ... 1.28598211e-04
  6.78976593e-05 6.29847332e-05]
 ...
 [7.04499022e-05 1.94432532e-04 1.28598211e-04 ... 1.96869006e-04
  1.04075637e-04 9.60615581e-05]
 [3.72444066e-05 1.02513388e-04 6.78976593e-05 ... 1.04075637e-04
  6.12470402e-05 5.12171005e-05]
 [3.50955634e-05 9.82931572e-05 6.29847332e-05 ... 9.60615581e-05
  5.12171005e-05 4.92050651e-05]]
```

```
print("The dimensions of final matrix are", Cov_matrix.shape)
print("The trace of final matrix is:", np.trace(Cov_matrix))
```

```
The dimensions of final matrix are (200, 200)
The trace of final matrix is: 0.12593386695214556
```

```
eigenvalues, eigenvectors = np.linalg.eig(Cov_matrix)
positive_eigenvalues = np.all(eigenvalues > 0)
print("Are all eigenvalues positive:", positive_eigenvalues)
if (positive_eigenvalues==False):
    number = np.sum(eigenvalues<0)
    print("The number of negative eigenvalues for this matrix:",number )
print(eigenvalues)
```

```
Are all eigenvalues positive: False
The number of negative eigenvalues for this matrix: 98
```

- Selected the matrix with higher error for each systematic
- Added all of them
- Symmetric matrices
- Not positive definite

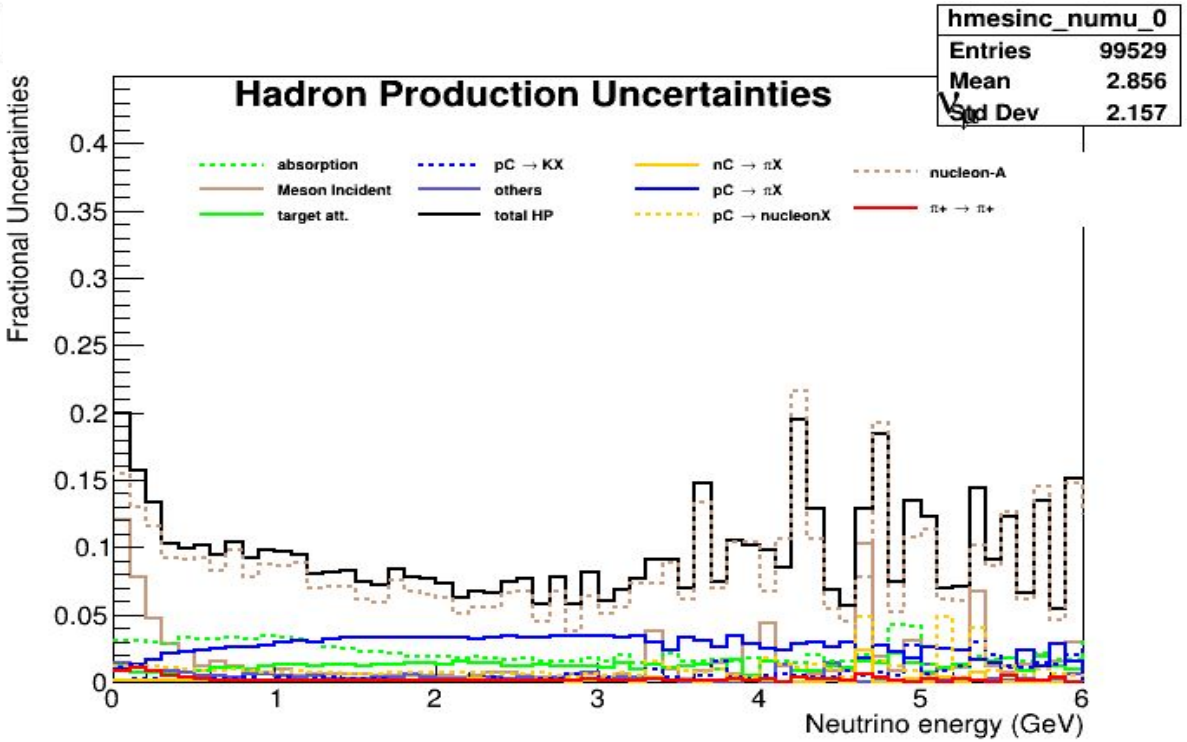
Decomposition of Covariance Matrix

- Replaced Cholesky decomposition by Singular Value Decomposition (SVD)
- SVD is valid for all types of matrices
- Decomposes original atrix (A) as:
- $\mathbf{A}(mxn) = \mathbf{U}(mxm) * \mathbf{S}(mXn) * \mathbf{V}(nxn)$
- U, V are orthogonal matrices
- S is a diagonal matrix, elements are eigenvalues

SVD Decomposition

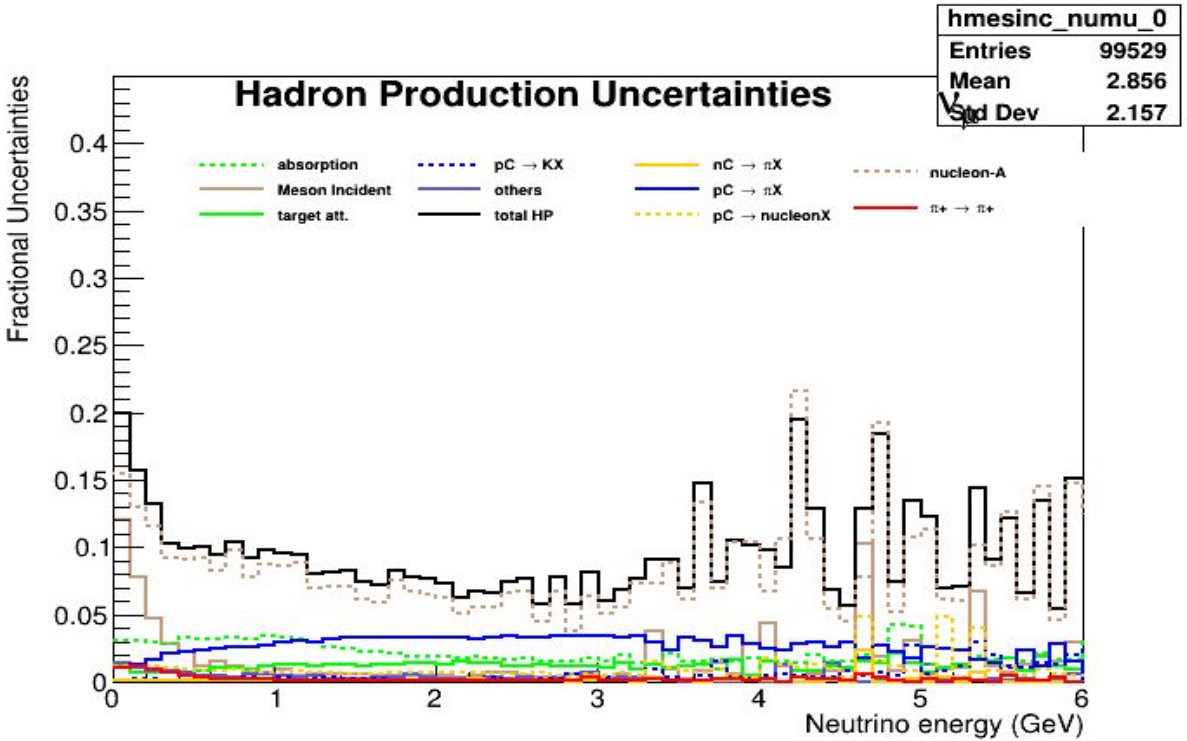
```
TDecompSVD *decomp;

for(size_t ii=0;ii<covariance_matrices.size();++ii){

  decomp=new TDecompSVD(covariance_matrices[ii],0.0);

  bool isDecomposed=decomp->Decompose();
  TMatrixD U = decomp->GetU();
  TVectorD S = decomp->GetSig();
  TMatrixD Vt = decomp->GetV();


  int nmat = S.GetNoElements();
  TVectorD vsigma(nmat);
  for(int jj=0;jj<nmat;jj++){
    vsigma[jj]= cvfactor*(r3->Gaus(0.0,1.0));
  }
  TMatrixD S_matrix(nmat, nmat);
  S_matrix.Zero();
  for (Int_t i = 0; i < nmat; ++i) {
    S_matrix(i, i) = S[i];
   }


  TVectorD vecDShift = U * (S_matrix * (Vt * vsigma));


  const boost::interprocess::flat_map<std::string, double>& tb = (correlated_par_tables[ii]).getMap();
  boost::interprocess::flat_map<std::string, double>::const_iterator it_tb = tb.begin();

  for(;it_tb != tb.end();++it_tb){
    std::string tmp_name = it_tb->first;
    std::string snID = tmp_name.substr((it_tb->first).rfind("_")+1,(it_tb->first).length());
    std::stringstream ssID(snID);
    int nID;
    ssID >> nID;
    double new_val = it_tb->second + vecDShift[nID];
    Parameter p(it_tb->first,new_val);
```

Muon Neutrino Corrected Flux (NA61 data)



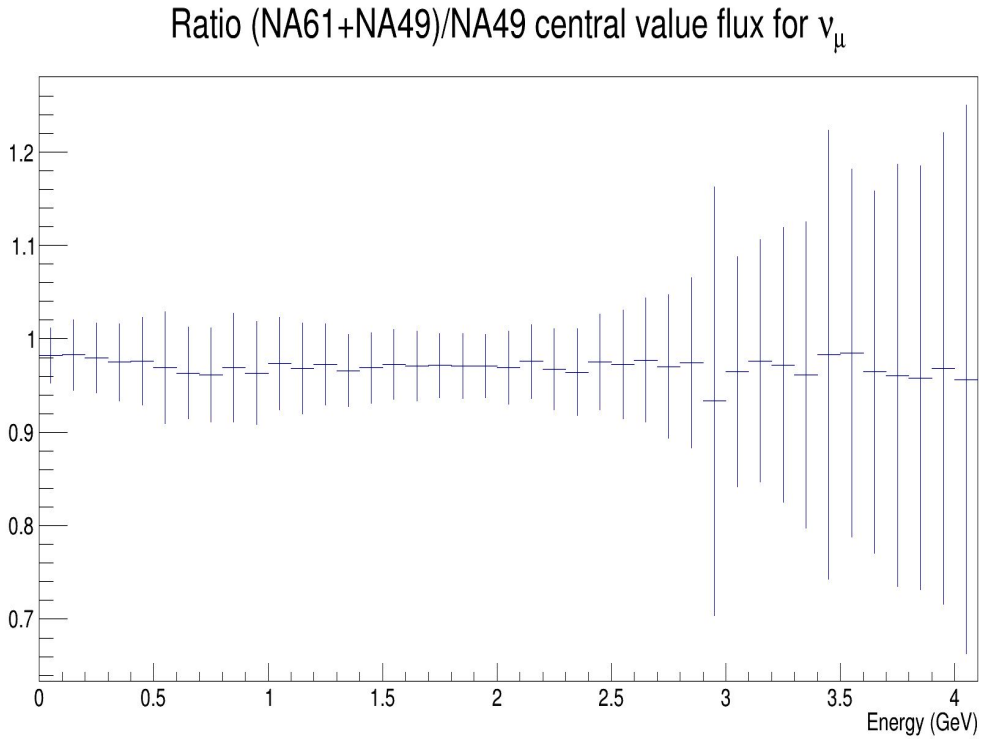
- Considering Statistical and Fit Uncertainties
- Significant reduction in Meson Incident Contribution and total uncertainty
- No correlated systematics considered yet
- π^+ ---> π^+ interaction has now been constrained by data

Muon neutrino corrected flux (NA61 data)



- Statistical, Fit as Uncorrelated and rest all errors as correlated
- **SVD decomposition**
- Added all uncertainties, but not a significant difference on addition

Central value Comparison



Noticeable difference at peak between central values

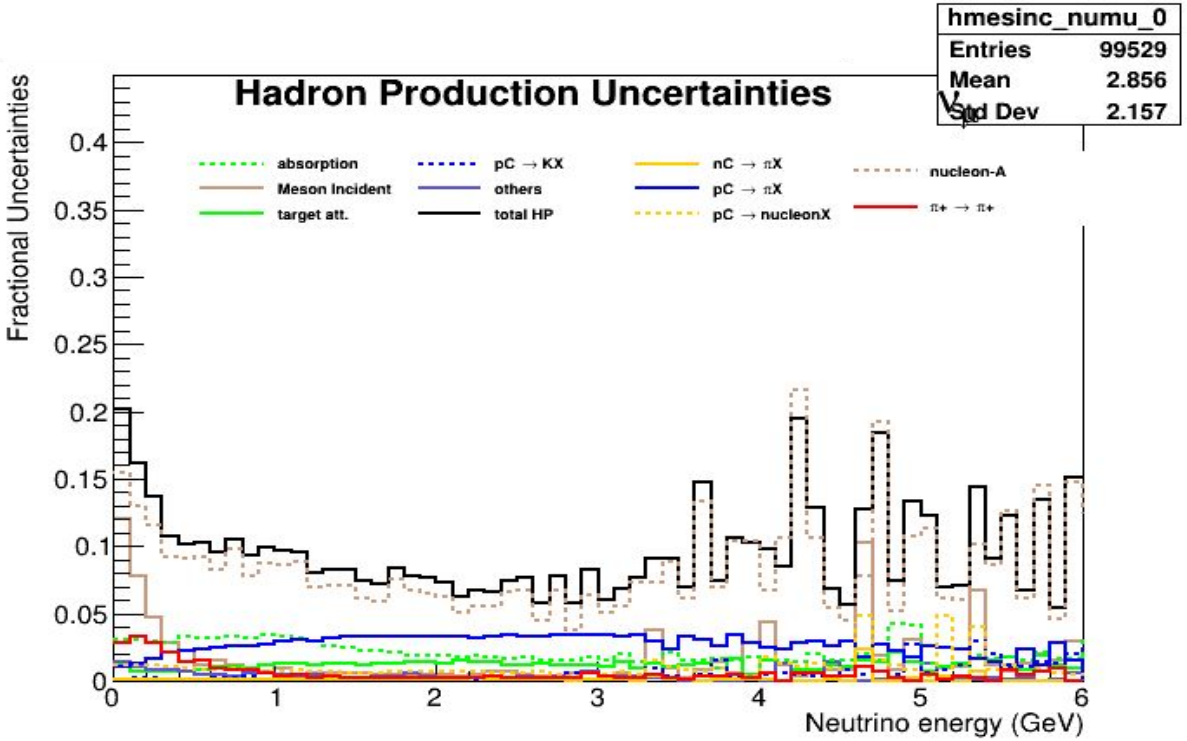
Compared central values for different Universes, different seed, same in all cases (not shown here)

Ratio is less than one, so we get more precise flux when we use more data

Fit Uncertainty with other systematics

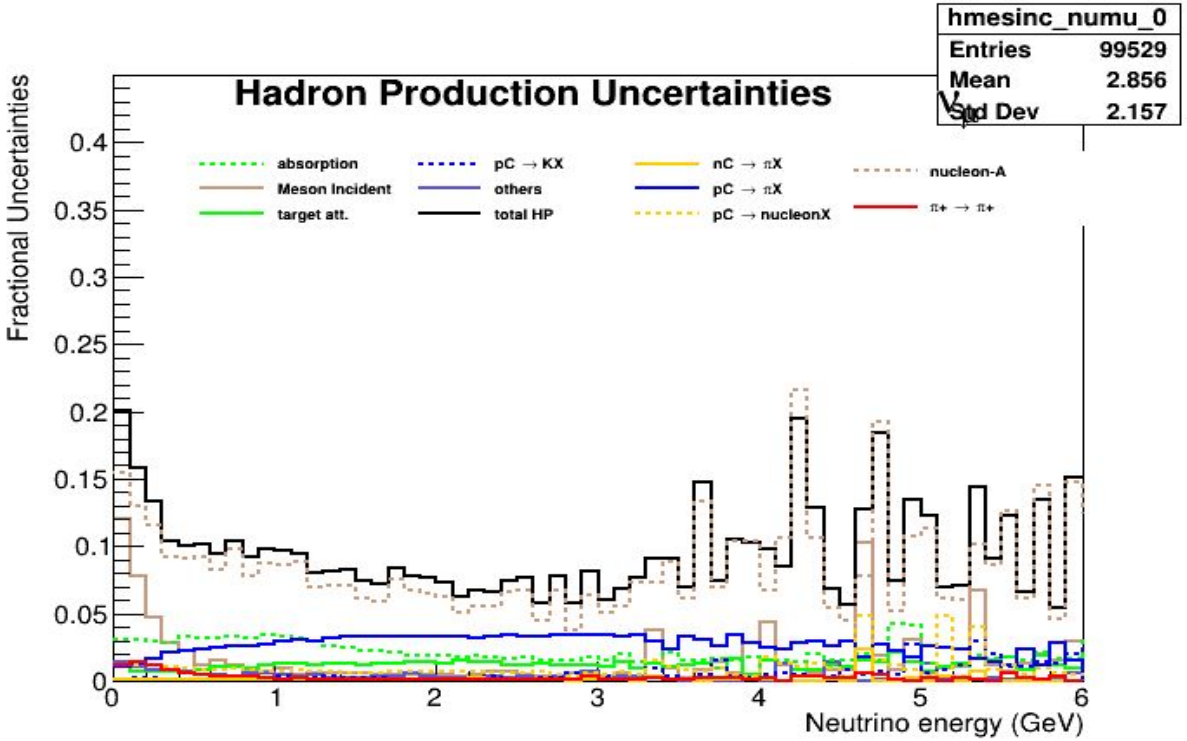
- Considering Fit Uncertainty too as a correlated systematic
- Adding it to generate the final covariance matrix, not considering as uncorrelated
- The covariance matrix now becomes positive definite
- Both SVD and Cholesky decomposition are applicable

Cholesky Decomposition



- Statistical uncertainty as uncorrelated and all systematics as uncorrelated
- As the matrix was positive definite, now decomposed

Singular Value Decomposition



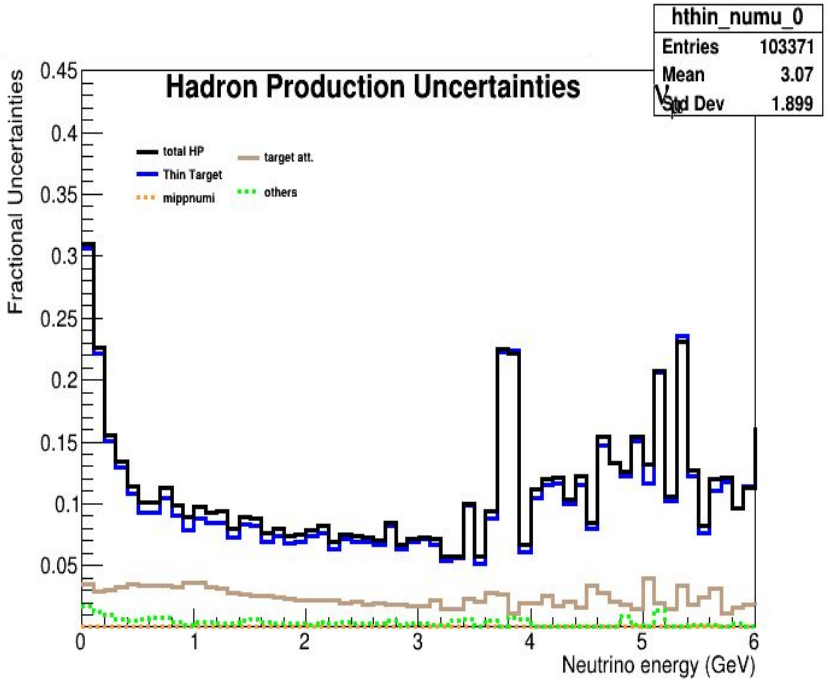
- Statistical uncertainty as uncorrelated and all systematics as uncorrelated
- Uncertainty is lower, if we use SVD decomposition

SVD Decomposition for different detectors

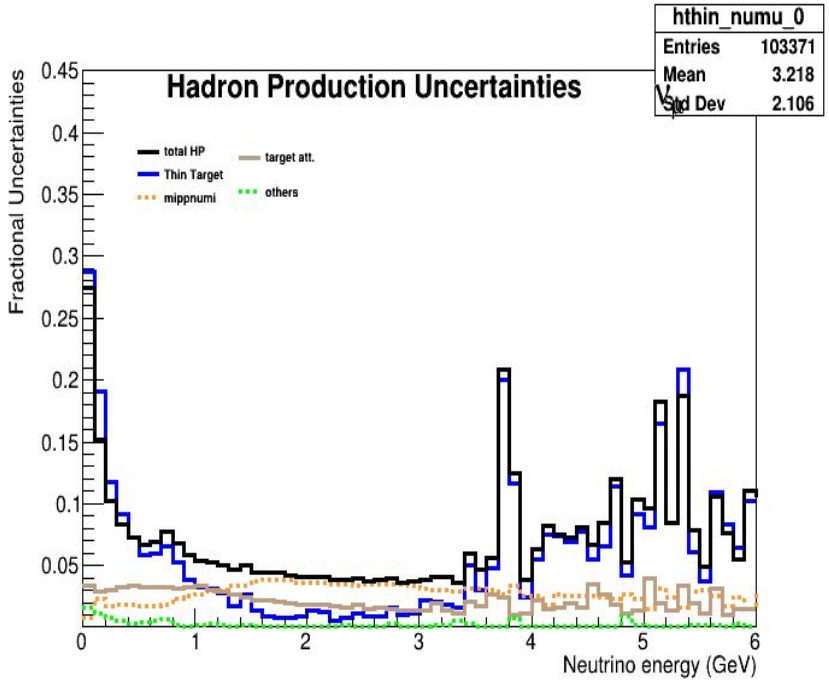
- PPFX is currently using MIPP data (positive definite matrices), using cholesky decomposition
- Can be handled by SVD decomposition as well
- Using SVD decomposition for existing MIPP data in the PPFX package
- Results for NOvA ND, MINOS and MINERvA

Muon Neutrino Flux (NOvA)

Cholesky

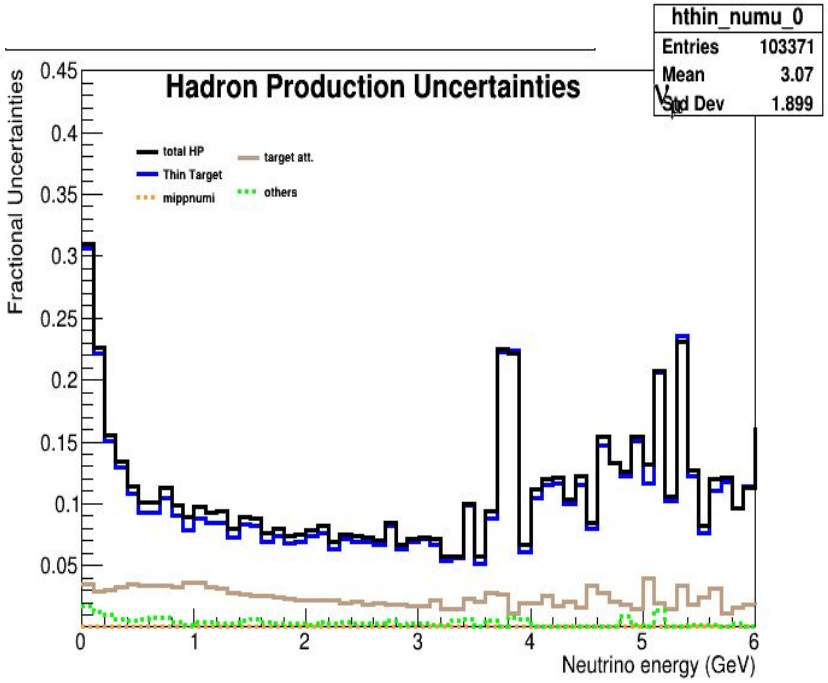


MIPP OFF



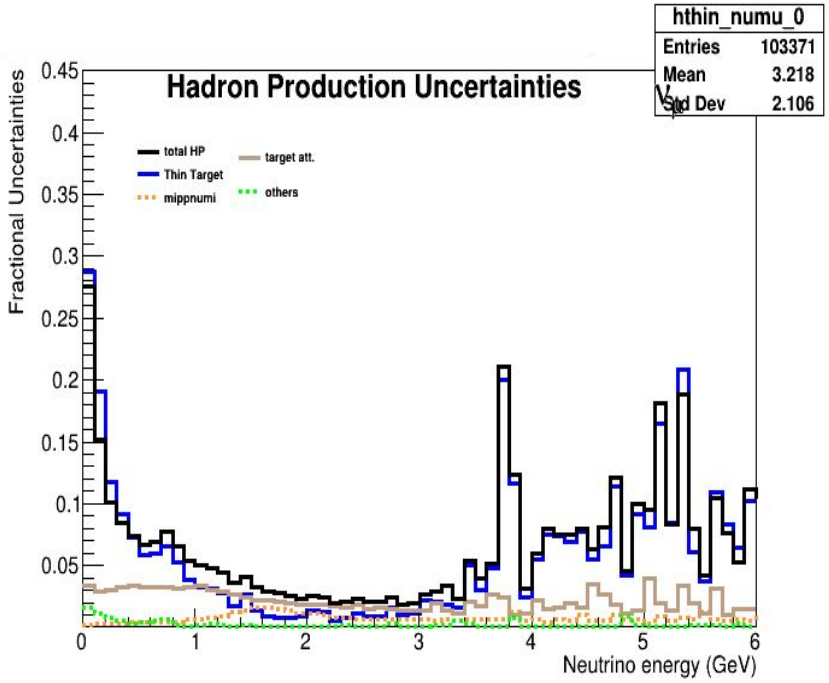
MIPP ON

Muon Neutrino Flux (NOvA)



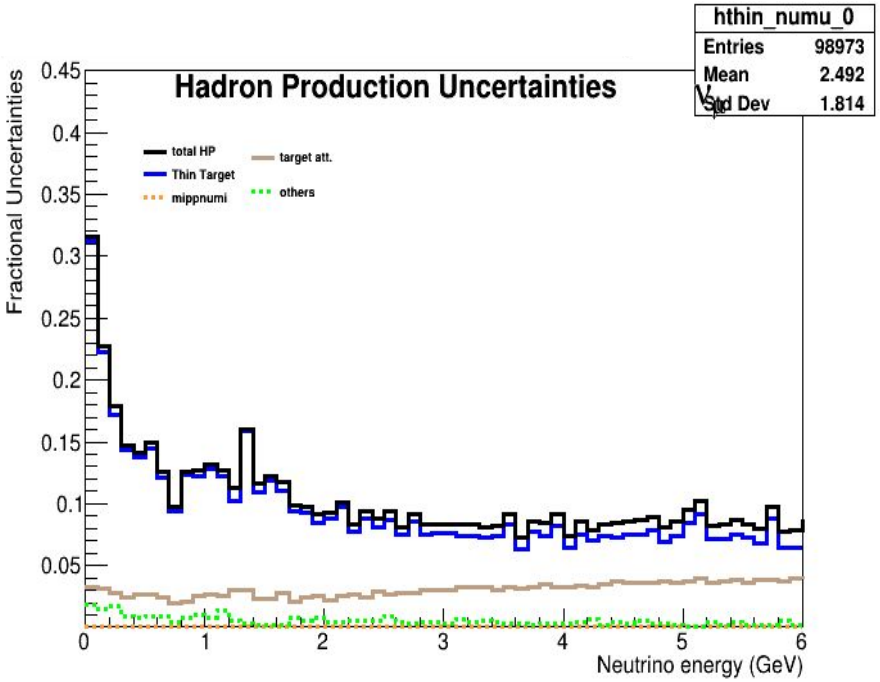
MIPP OFF

SVD

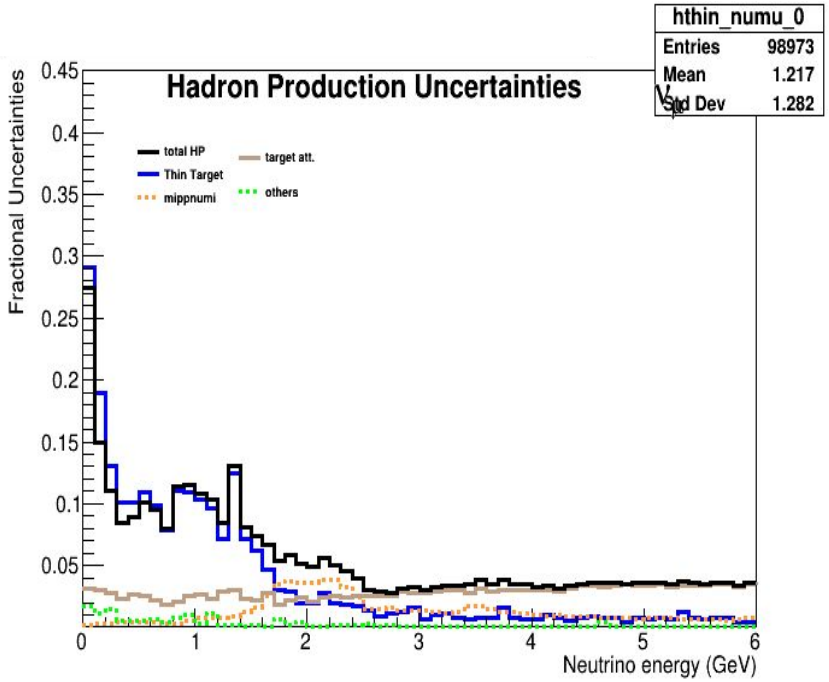


MIPP ON

Muon Neutrino Flux (Minerva) SVD



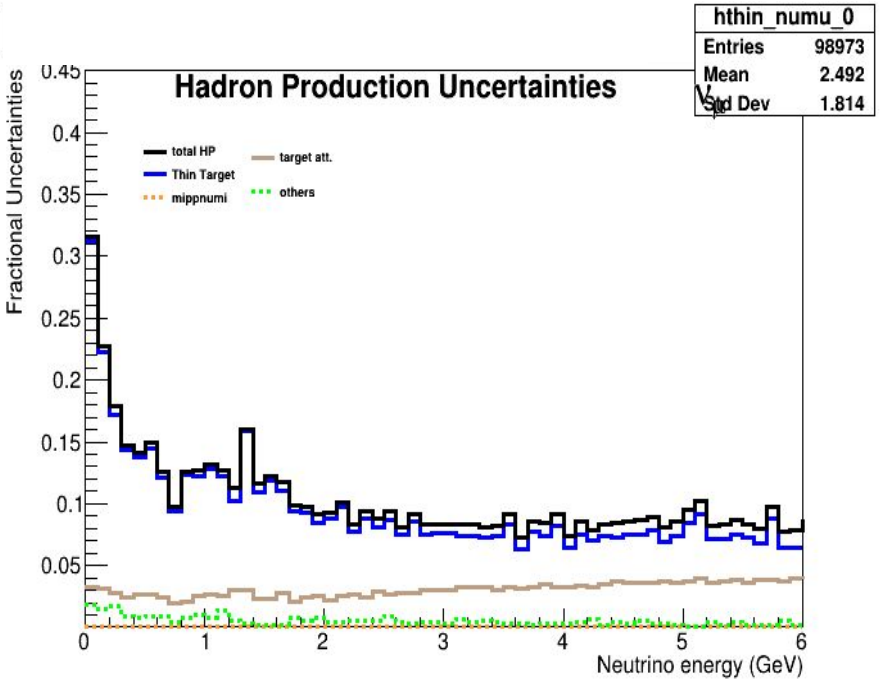
MIPP OFF



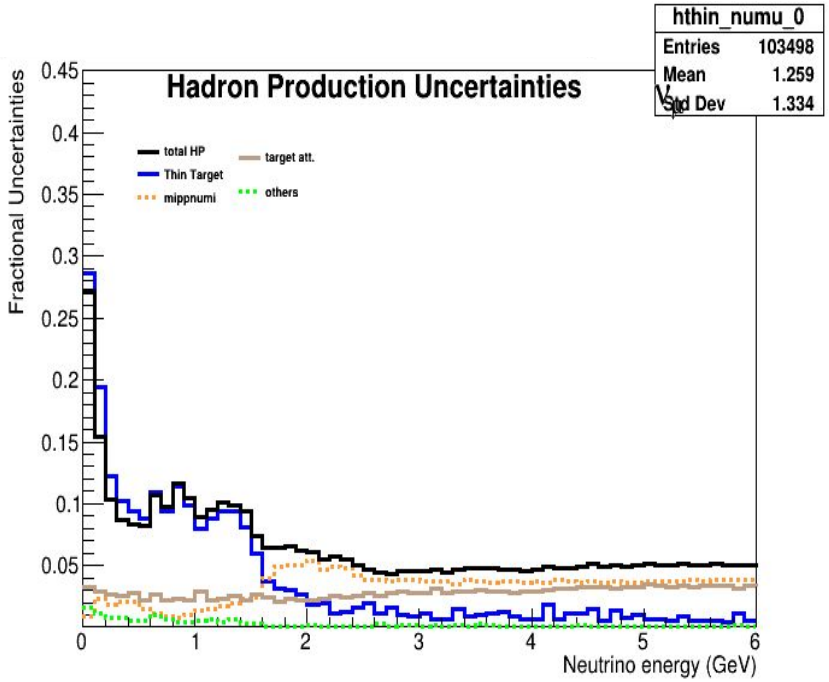
MIPP ON

Muon Neutrino Flux (Minerva)

Cholesky



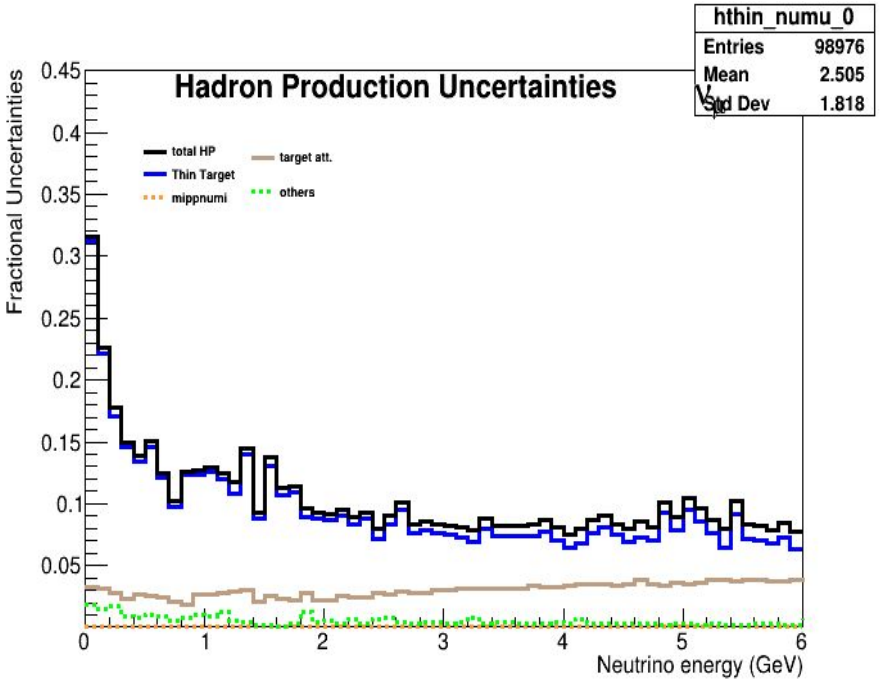
MIPP OFF



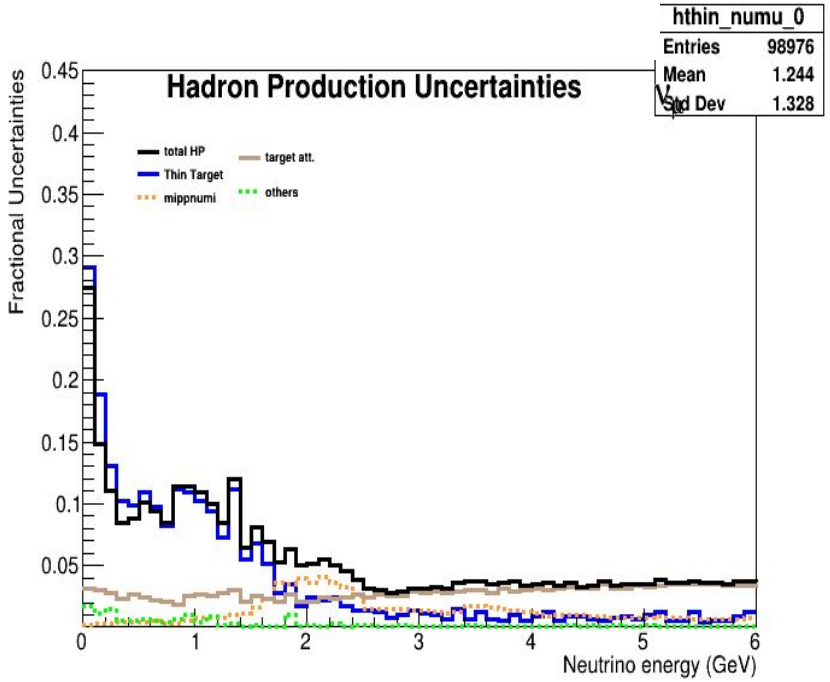
MIPP ON

Muon Neutrino Flux (MINOS)

SVD



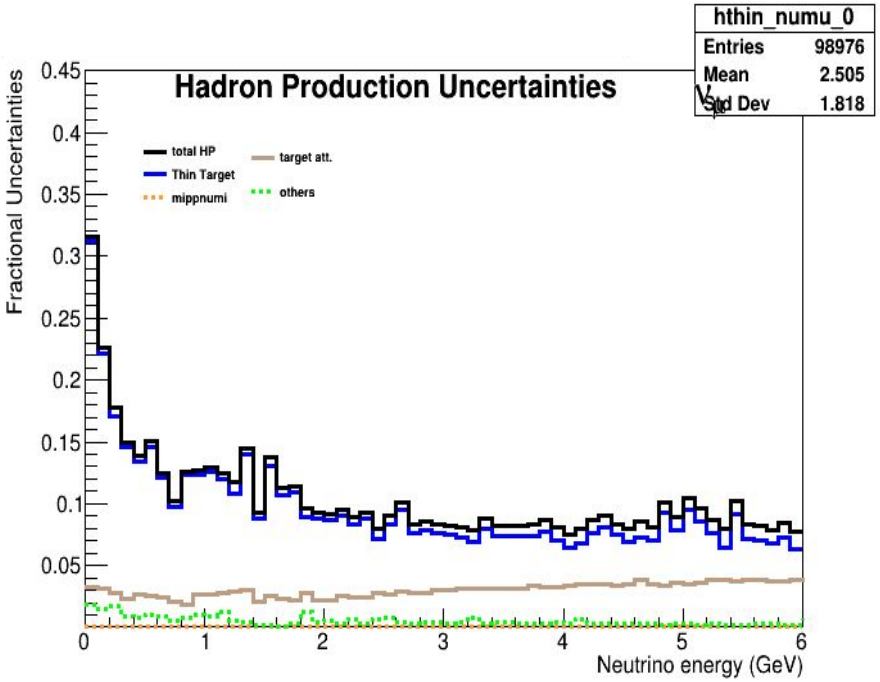
MIPP OFF



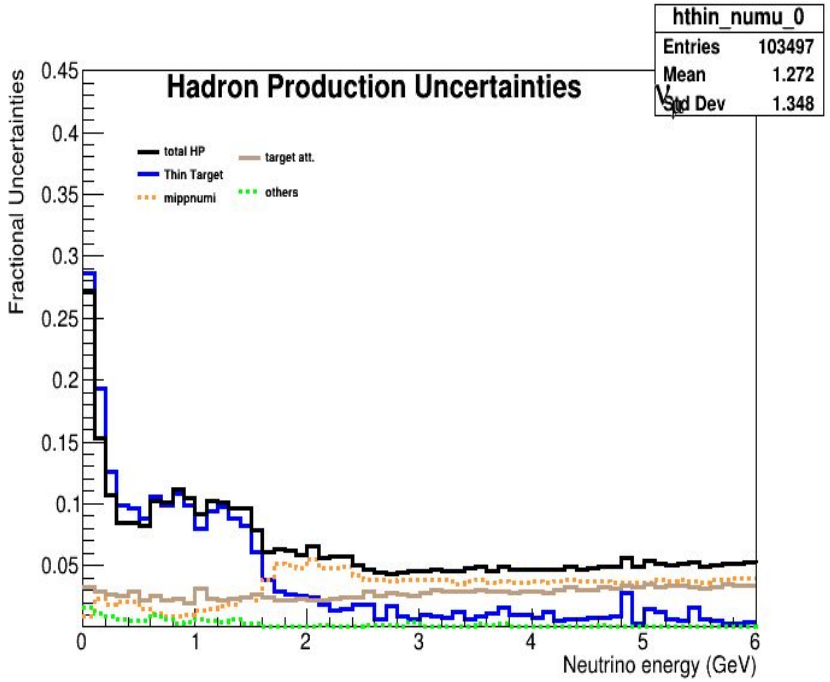
MIPP ON

Muon Neutrino Flux (MINOS)

Cholesky

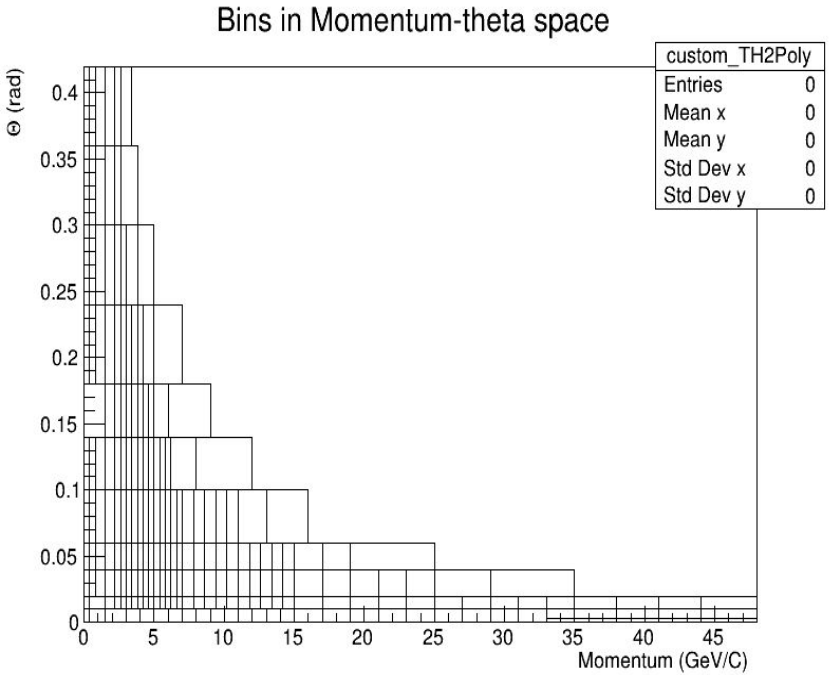


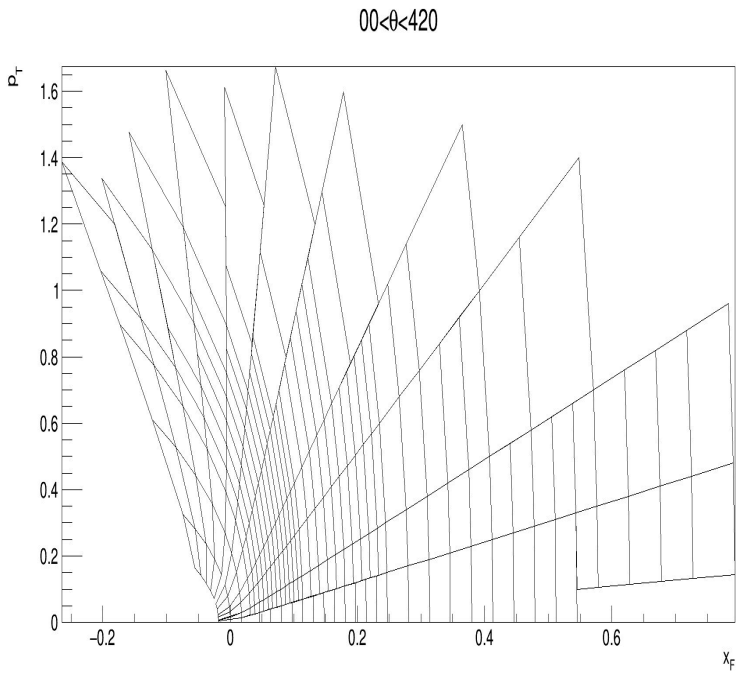
MIPP OFF



MIPP ON

Visualising NA61 data bins





Bin Centers **3 <θ< 10** **10<θ< 20** **10<θ< 20**

Visualising NA61 data bins

- The bins in (x_F-p_T) phase space have been added in the same sequence as given in data (p-Θ)
- If we have the values of coordinates in the (p-Θ) space we will get the corresponding values in (x_F-p_T) phase space and the bin number

```
Enter X coordinate: 0.180388
Enter Y coordinate: 0.109998
Bin number for (0.180388, 0.109998) is 11
```

Summary

- Addition of data (only for $\pi^+ + C \longrightarrow \pi^+$), leads to significant reduction in flux uncertainty
- The contribution from Meson Incident ~ <1% after using data from NA61
- Adding the entire dataset will further reduce the uncertainty
- Comparing SVD and Cholesky decomposition, uncertainties are very low for SVD decomposition
- Which method should be used?

Thank you!

Back up

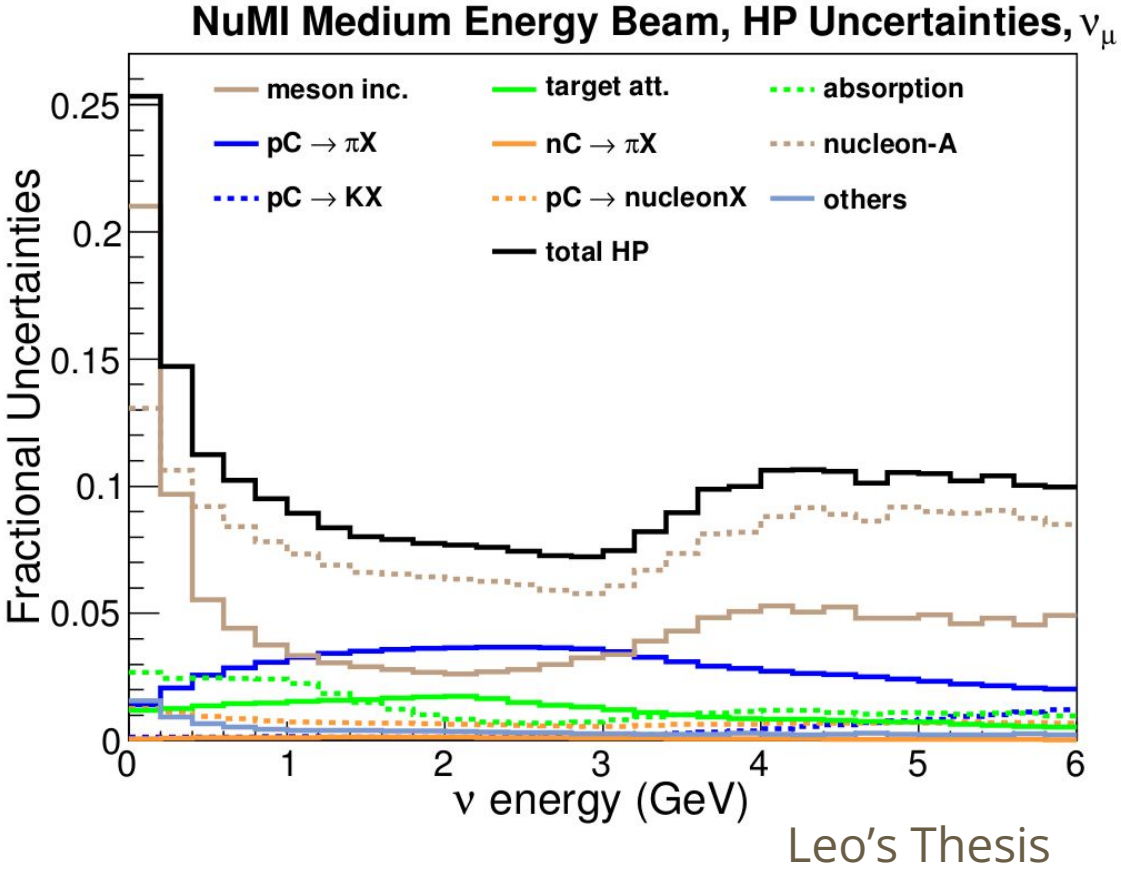


FIG. 7.7: Fractional Uncertainties for ME Gen2 - thin ν_μ flux in NOvA.

NA61

Production cross-section (σ_{prod}): Processes in which new hadrons are produced

$$\sigma_{tot} = \sigma_{inelactic} + \sigma_{elastic}$$

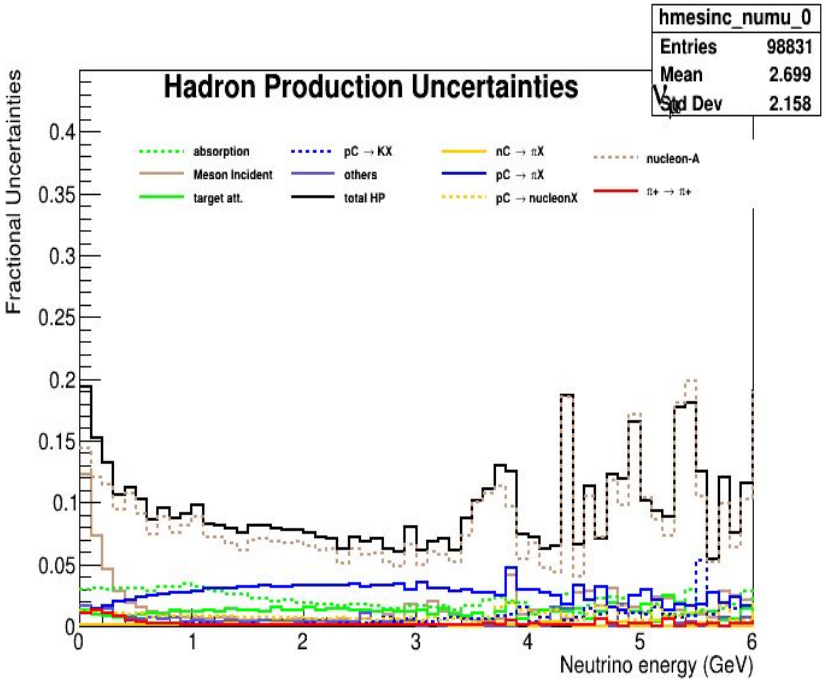
$$\sigma_{inel} = \sigma_{prod} + \sigma_{qe}$$

Differential Production Multiplicity: Yield of particles produced per production interaction per unit momentum per radian in each kinematic bin k

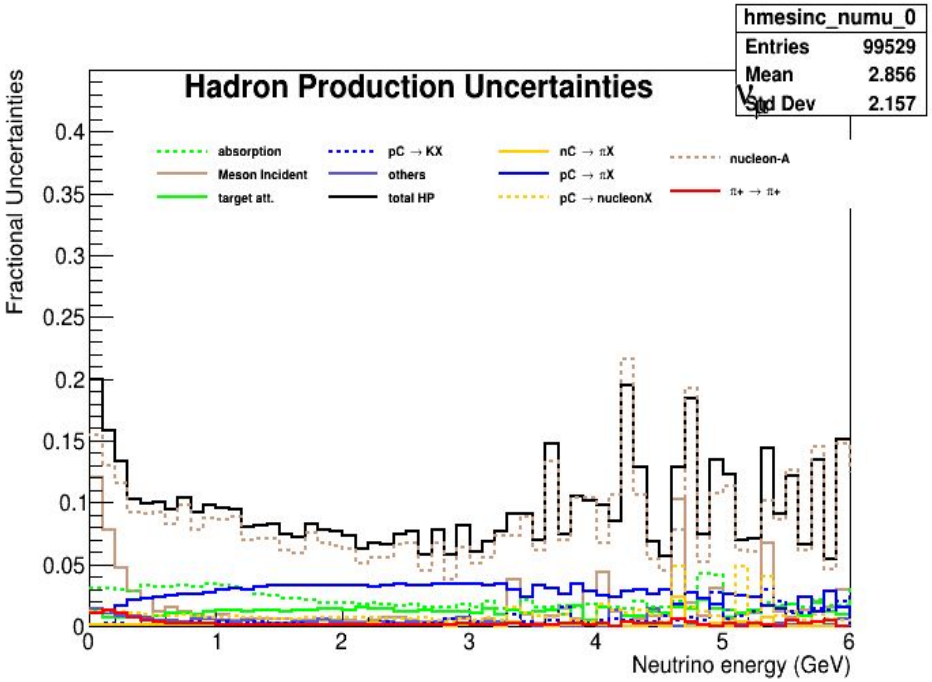
$$\frac{d^2\sigma_k}{dpd\theta} = \sigma_{prod} \frac{d^2n_k}{dpd\theta}$$

Differential Cross section Differential production multiplicity

Targets

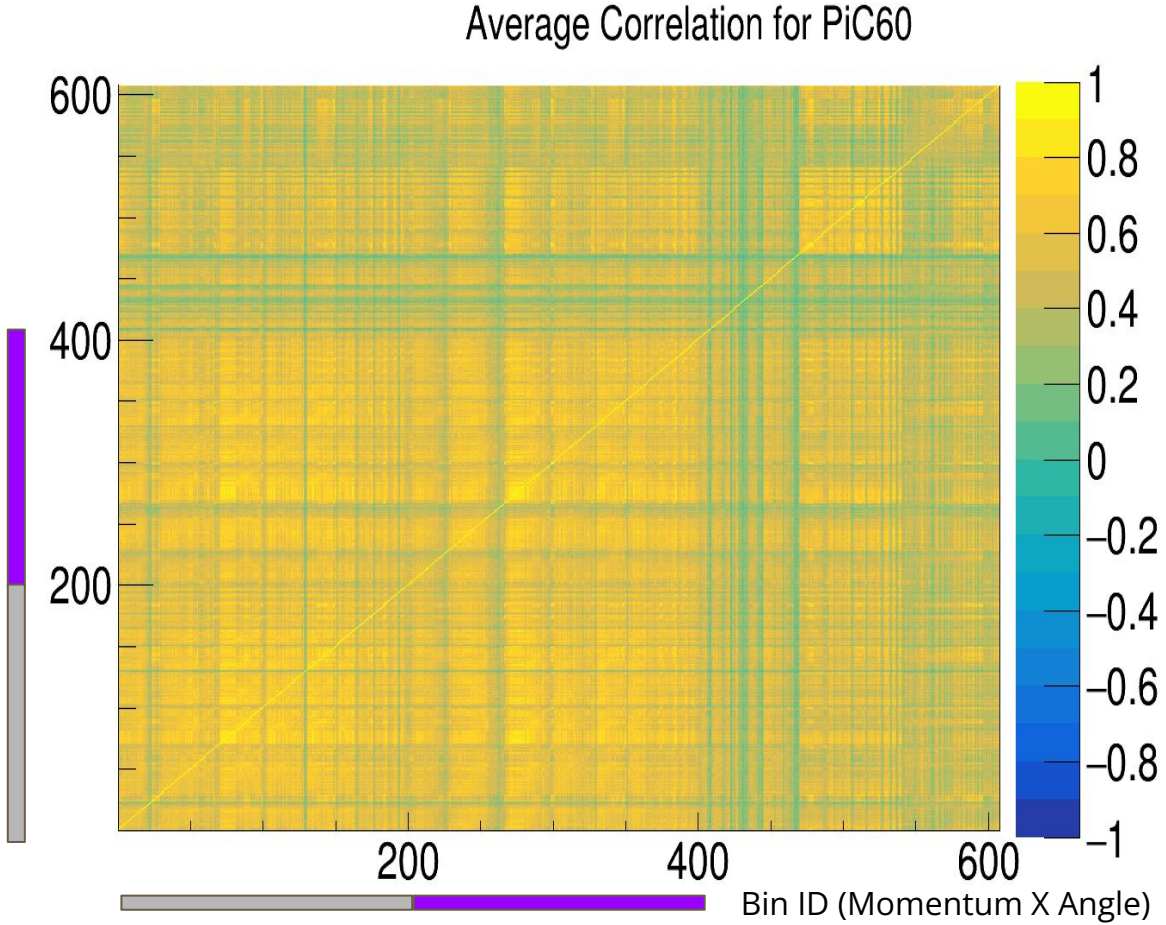


New target
σx = σy = 1.5 mm



Old target
σx = σy = 1.4 mm

Data from NA61 (Correlation Matrix)



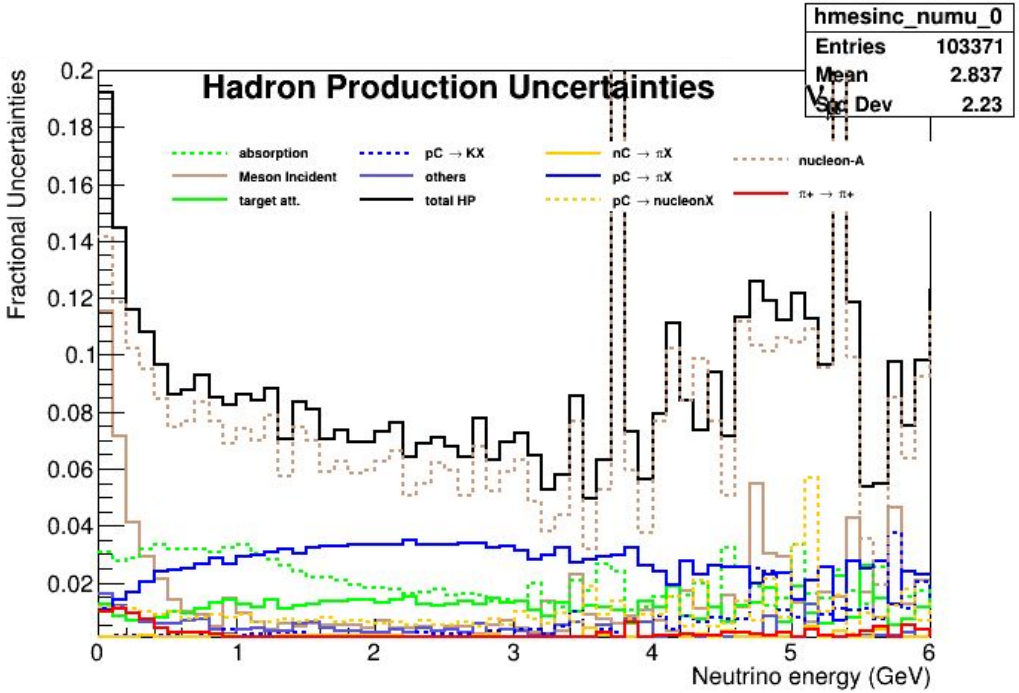
Bins	Particle
200	Pi+
200	Pi-
34	K+
35	K-
72	Proton
34	Ks0
21	^
11	anti-^
607	

Impact of Number of Universes

- Considering Statistical and Fit Uncertainties
- Two different Random number generators
- Seed for random number generator:
  NA49 data: 0
  NA61 data: 300
- Number of Universes (N)

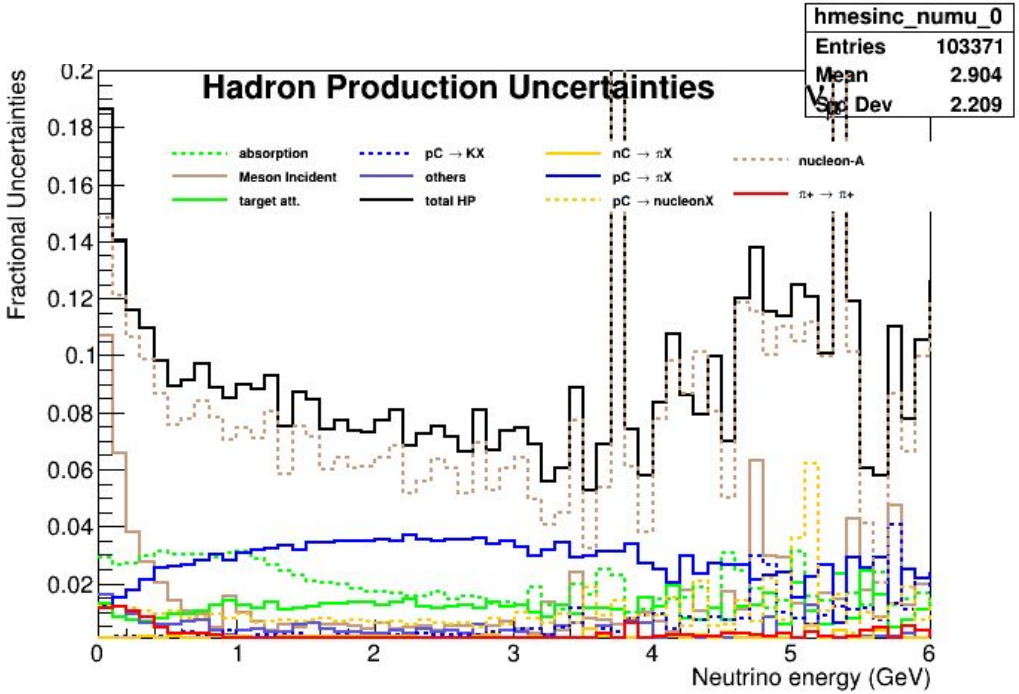
Muon Neutrino corrected flux (N= 100)

Neutrino Flavor: ν_μ



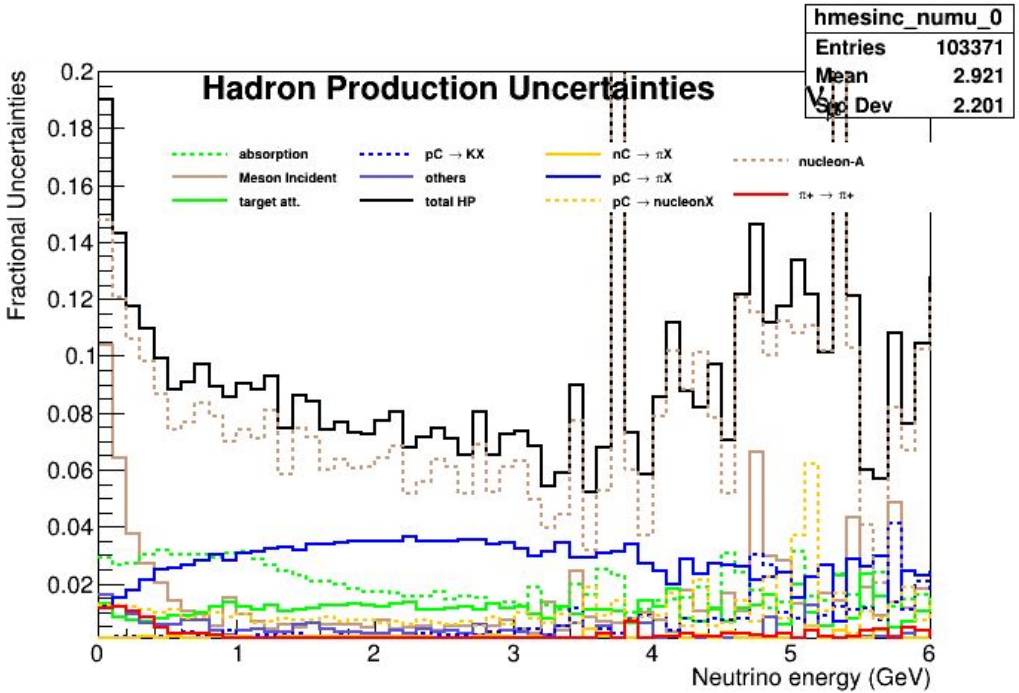
Muon Neutrino corrected flux (N= 200)

Neutrino Flavor: ν_μ



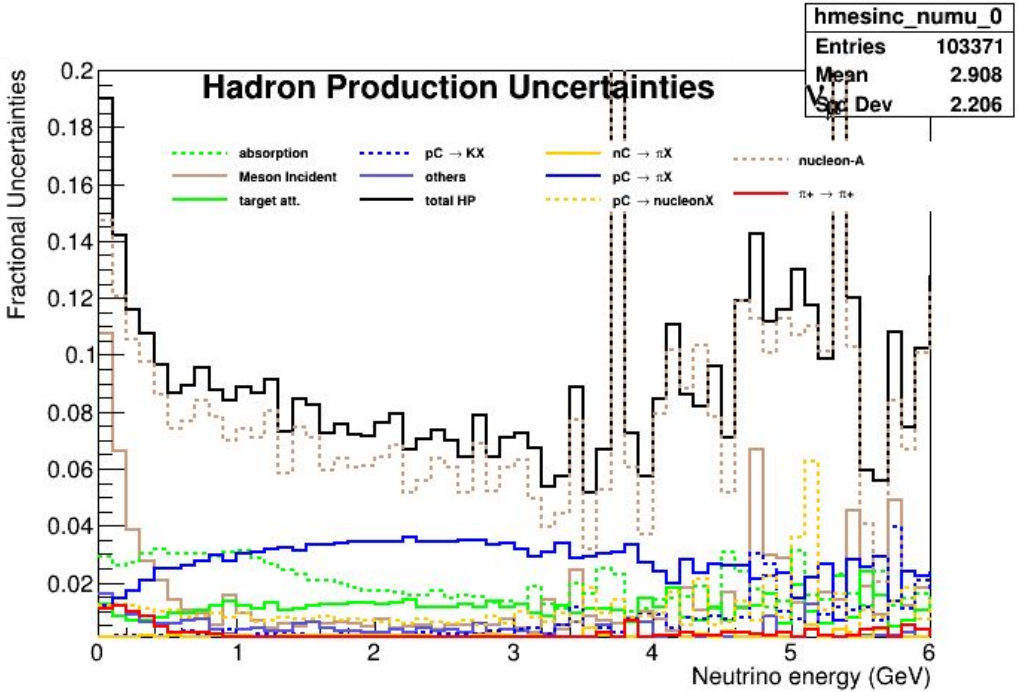
Muon Neutrino corrected flux (N= 400)

Neutrino Flavor: ν_μ



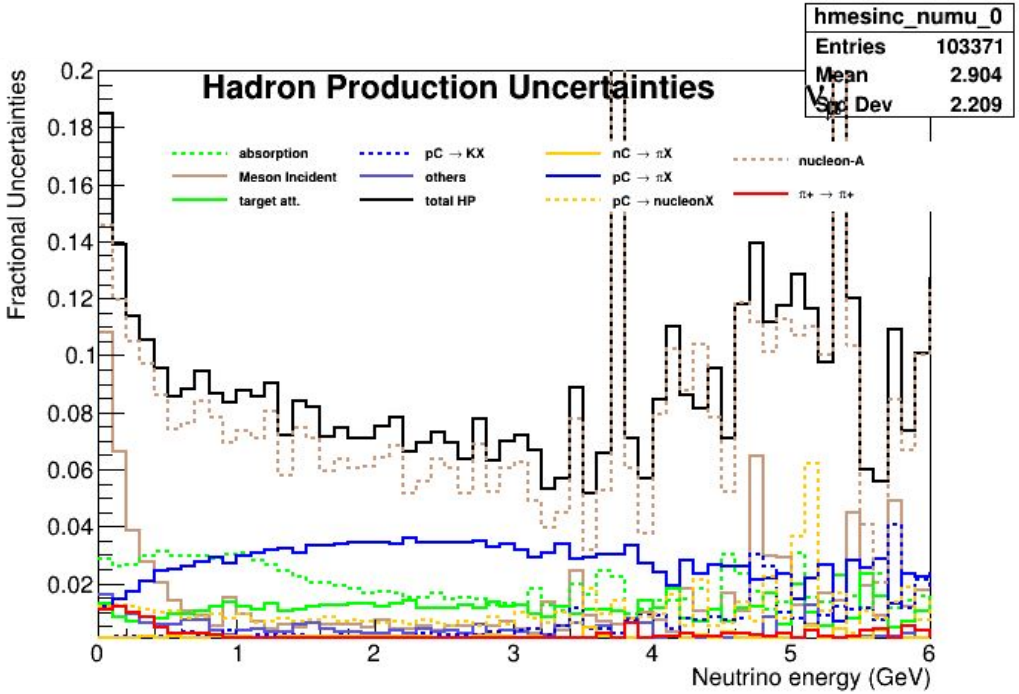
Muon Neutrino corrected flux (N= 500)

Neutrino Flavor: ν_μ



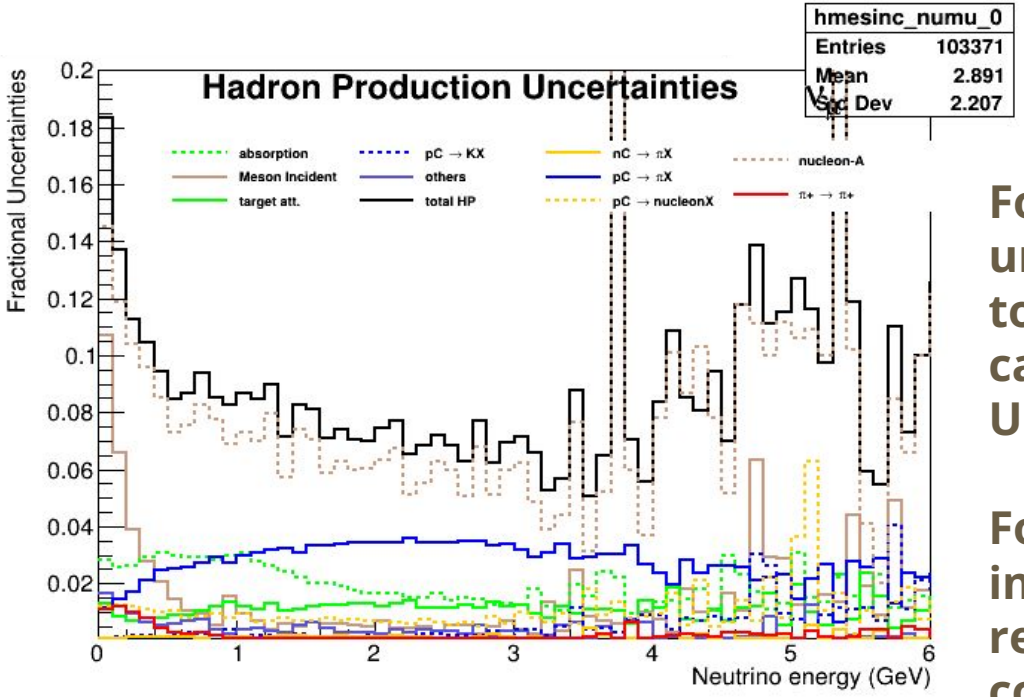
Muon Neutrino corrected flux (N= 600)

Neutrino Flavor: ν_μ



Muon Neutrino corrected flux (N= 700)

Neutrino Flavor: ν_μ

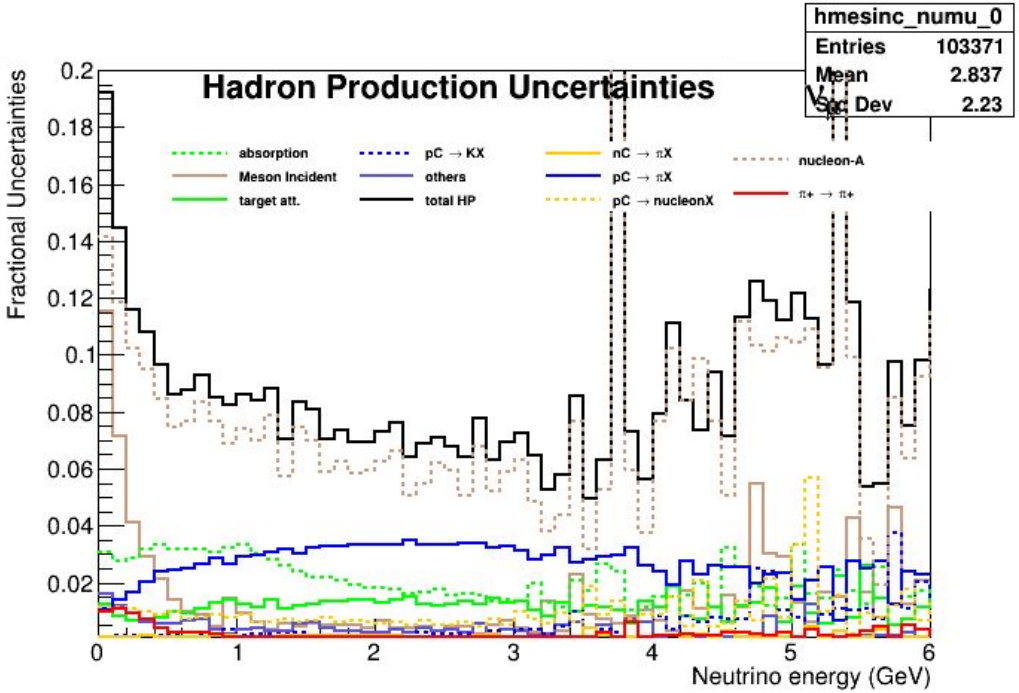


For small number of universes (<500), total error is least in case of 100 Universes

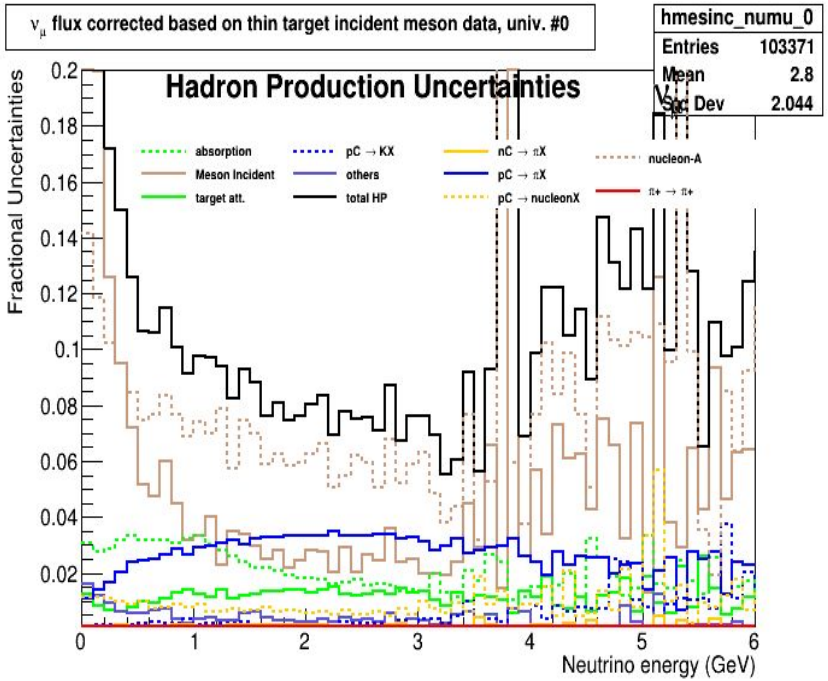
For n>500, as n increases total error reduces, but computationally challenging

Muon Neutrino corrected flux (N= 100)

Neutrino Flavor: ν_μ

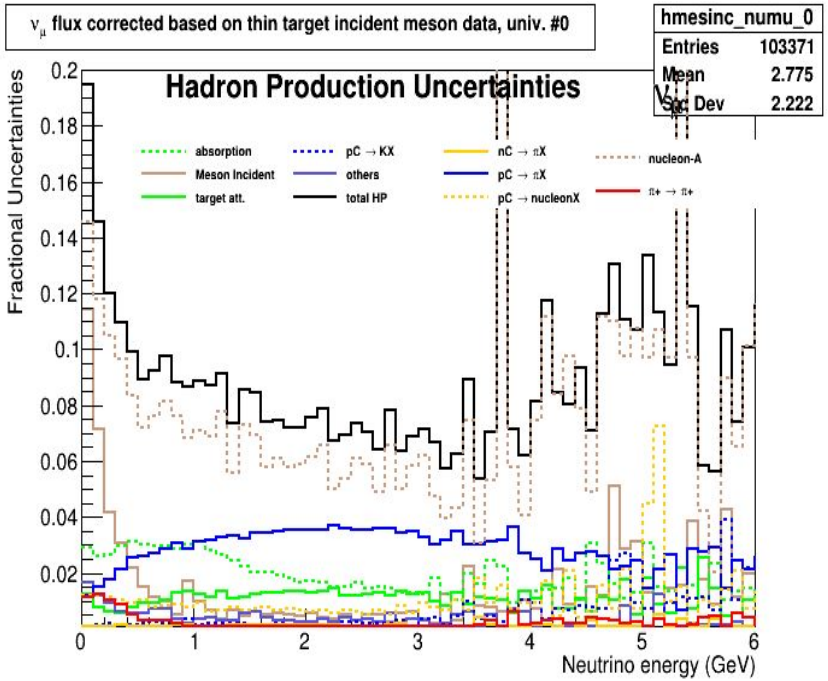


Muon Neutrino Corrected Flux (NA49)



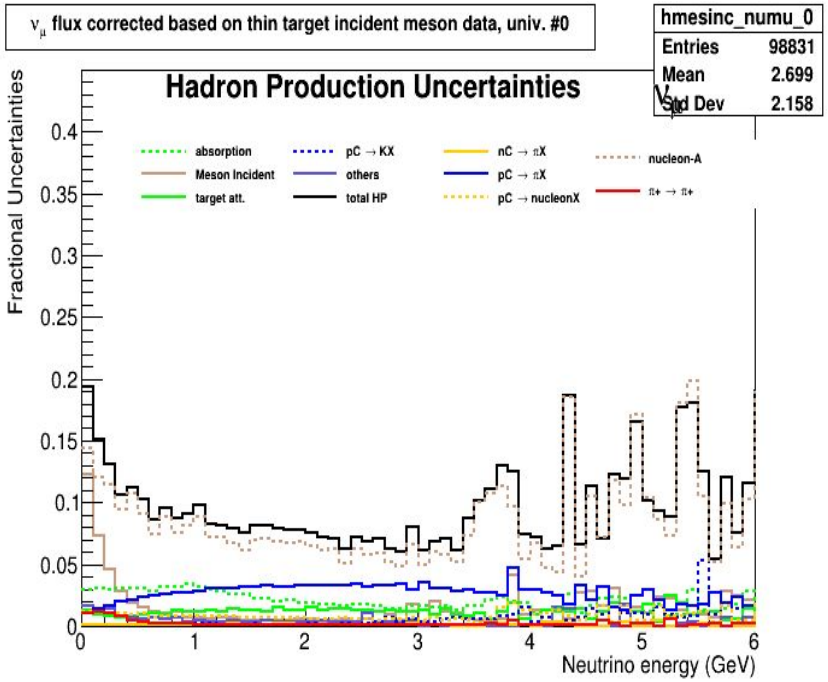
- Use of NA49 data only
- $\Pi^+ + C \longrightarrow \pi^+$ is a part of Meson Interaction reweighter
- 40% uncertainty is assumed

Muon Neutrino Corrected Flux (NA61 Statistical)

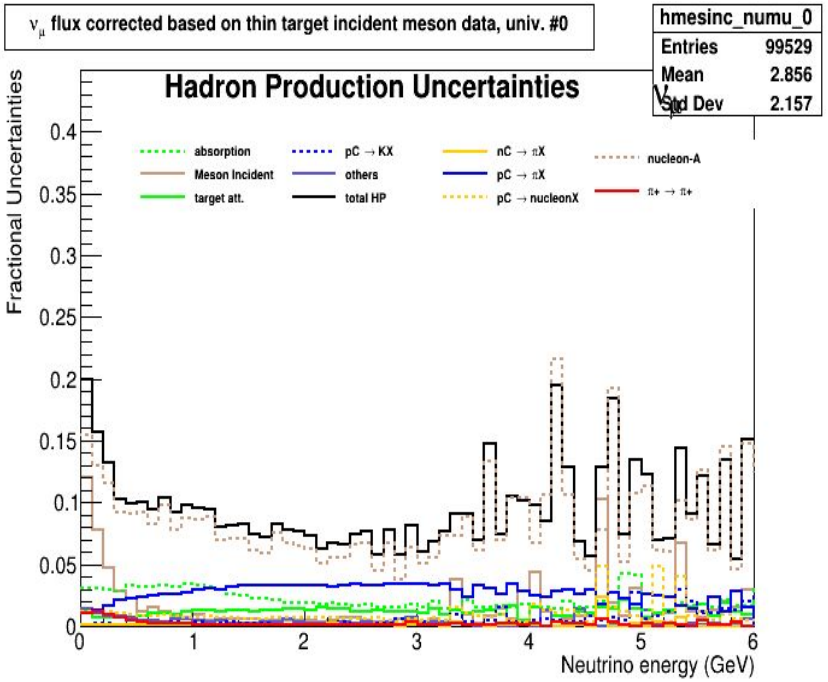


- Significant reduction in Meson Incident Contribution and total uncertainty
- No systematics considered yet
- Change in other interactions (pC→π, absorption) unexpected
- Same Random Number Generator for all the reweighters in PPFX

Targets (Stat+Fit+Systematic_SVD)

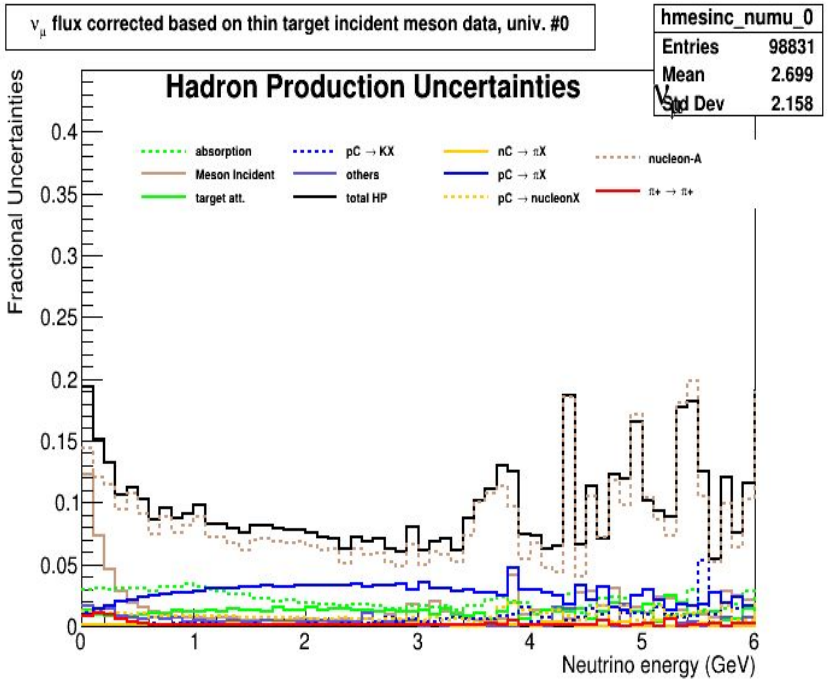


New target
σx = σy = 1.5 mm

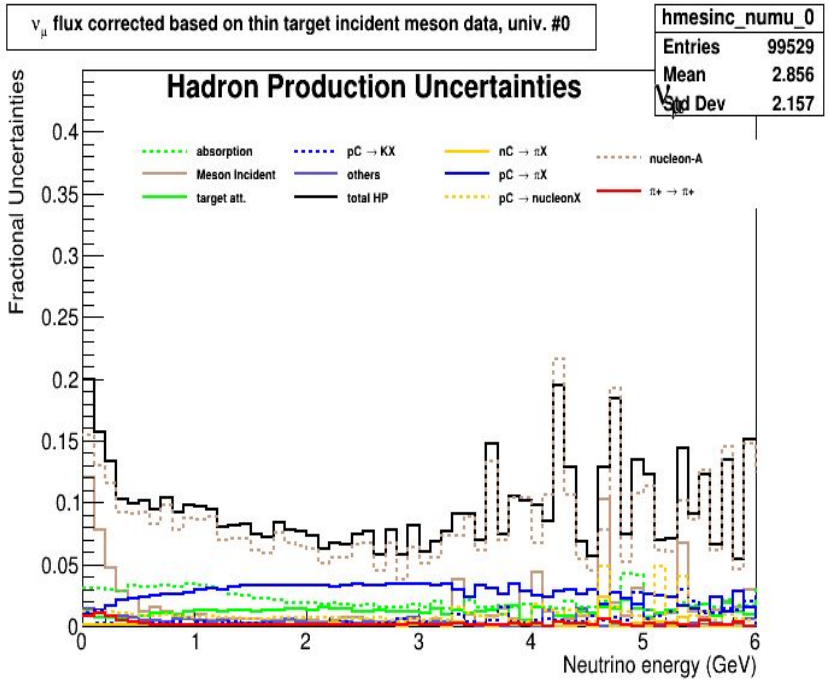


Old target
σx = σy = 1.4 mm

Targets (Stat+Fit+Systematic_Cholesky)



New target
σx = σy = 1.5 mm



Old target
σx = σy = 1.4 mm

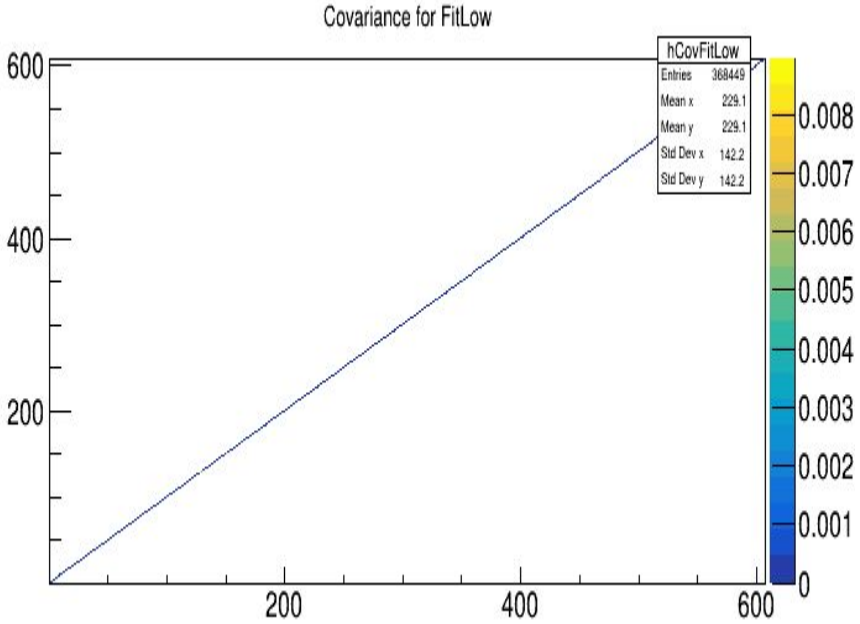
Use of Uncertainties

If we add Fit Uncertainty to Correlated

Matrix become positive definite, we can use both the decomposition methods

Fit Uncertainty is a diagonal matrix

Have submitted jobs for all the cases, to see the impact and compare



Systematic Uncertainties

- **Selection Uncertainties**: For same kinematics, data tracks are composed of 5% less clusters than tracks in MC.

  MC correction by artificially decreasing number of clusters in MC tracks by 5%

  Lead to Higher Multiplicity, upper bound of selection Uncertainty

SVD

5*5 matrix from MIPP data

Decomposed matrix

Correlated variables:

U(S(Vt*Z))

Z: Vector of Random numbers

```
bhumika@hP:~/Simulations/outputs/MC_for_PPFX/Jun/crosschecks$ ./secsvd
Initial matrix A:

5x5 matrix is as follows

     |      0    |      1    |      2    |      3    |      4    |
-----------------------------------------------------------------
   0 |   0.001052   0.0003842   0.0002488   0.0002102   0.0001141
   1 |  0.0003842   0.0002495   0.0001212   0.0001023   5.558e-05
   2 |  0.0002488   0.0001212   0.0001046   6.628e-05     3.6e-05
   3 |  0.0002102   0.0001023   6.628e-05   7.463e-05    3.04e-05
   4 |  0.0001141   5.558e-05     3.6e-05    3.04e-05   2.201e-05

Initial matrix U is:

5x5 matrix is as follows

     |      0    |      1    |      2    |      3    |      4    |
-----------------------------------------------------------------
   0 |    -0.8786      0.4629     -0.1096     0.03243     0.02717
   1 |    -0.3603     -0.8032     -0.4614     0.09253     0.06078
   2 |    -0.2268     -0.2852       0.786      0.4867       0.112
   3 |    -0.1905     -0.2203      0.3789      -0.865      0.1531
   4 |    -0.1024      -0.104      0.1174    -0.07289     -0.9796

Initial matrix V transpose is:

5x5 matrix is as follows

     |      0    |      1    |      2    |      3    |      4    |
-----------------------------------------------------------------
   0 |    -0.8786      0.4629     -0.1096     0.03243     0.02717
   1 |    -0.3603     -0.8032     -0.4614     0.09253     0.06078
   2 |    -0.2268     -0.2852       0.786      0.4867       0.112
   3 |    -0.1905     -0.2203      0.3789      -0.865      0.1531
   4 |    -0.1024      -0.104      0.1174    -0.07289     -0.9796
```

```
Initial matrix Sigma:

Vector (5)  is as follows

     |        1  |
------------------
   0 |0.0013328
   1 |0.000106342
   2 |3.61329e-05
   3 |2.1074e-05
   4 |6.53295e-06

The Number of elements are5
The Number of rows are5
The vector Z[i] is:

Vector (5)  is as follows

     |        1  |
------------------
   0 |0.998933
   1 |-0.434764
   2 |0.781796
   3 |-0.0300528
   4 |0.824264

Correlated Random Variables:

Vector (5)  is as follows

     |        1  |
------------------
   0 |0.00132047
   1 |0.000567138
   2 |0.000375262
   3 |0.000298676
   4 |0.000166543
```

Cholesky

5*5 matrix from MIPP data

Decomposed matrix

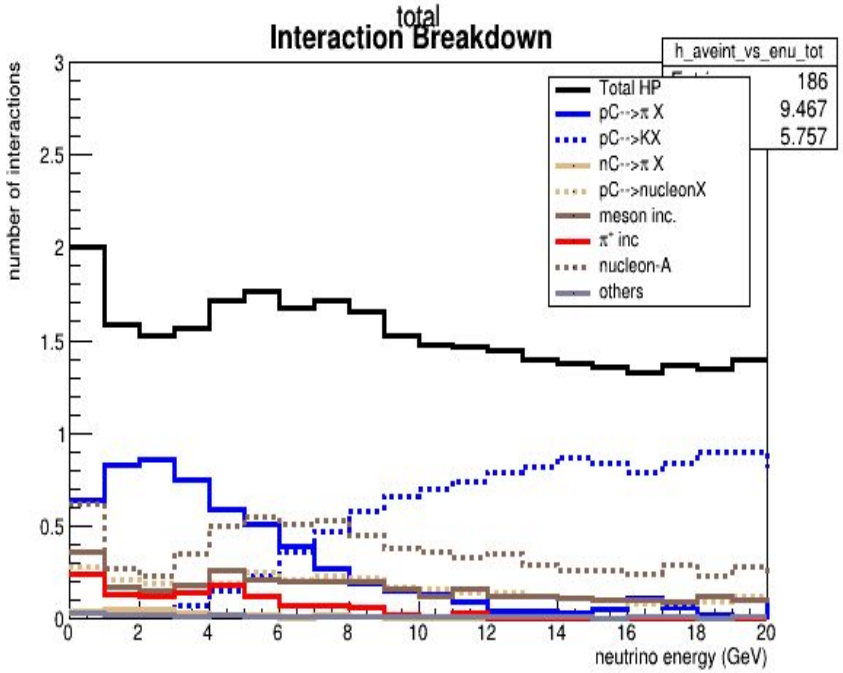
Correlated variables:

U*Z

Z: Vector of Random numbers

```
Initial matrix A:

5x5 matrix is as follows

     |      0    |      1    |      2    |      3    |      4    |
-----------------------------------------------------------------------
   0 |   0.001052   0.0003842   0.0002488   0.0002102   0.0001141
   1 |  0.0003842   0.0002495   0.0001212   0.0001023   5.558e-05
   2 |  0.0002488   0.0001212   0.0001046   6.628e-05     3.6e-05
   3 |  0.0002102   0.0001023   6.628e-05   7.463e-05    3.04e-05
   4 |  0.0001141   5.558e-05     3.6e-05    3.04e-05   2.201e-05

The decomposed matrix from Cholesky is (U):

5x5 matrix is as follows

     |      0    |      1    |      2    |      3    |      4    |
-----------------------------------------------------------------------
   0 |    0.03244     0.01185     0.007672     0.006479     0.003519
   1 |           0     0.01045       0.0029     0.002449      0.00133
   2 |           0           0     0.006113     0.001549    0.0008412
   3 |           0           0            0     0.004925    0.0006172
   4 |           0           0            0            0     0.002603

Correlated Random Variables (Cholesky):

Vector (5)  is as follows

     |        1  |
-------------------
   0 |0.0359551
   1 |-0.00125254
   2 |0.00542594
   3 |0.000360766
   4 |0.00214527
```

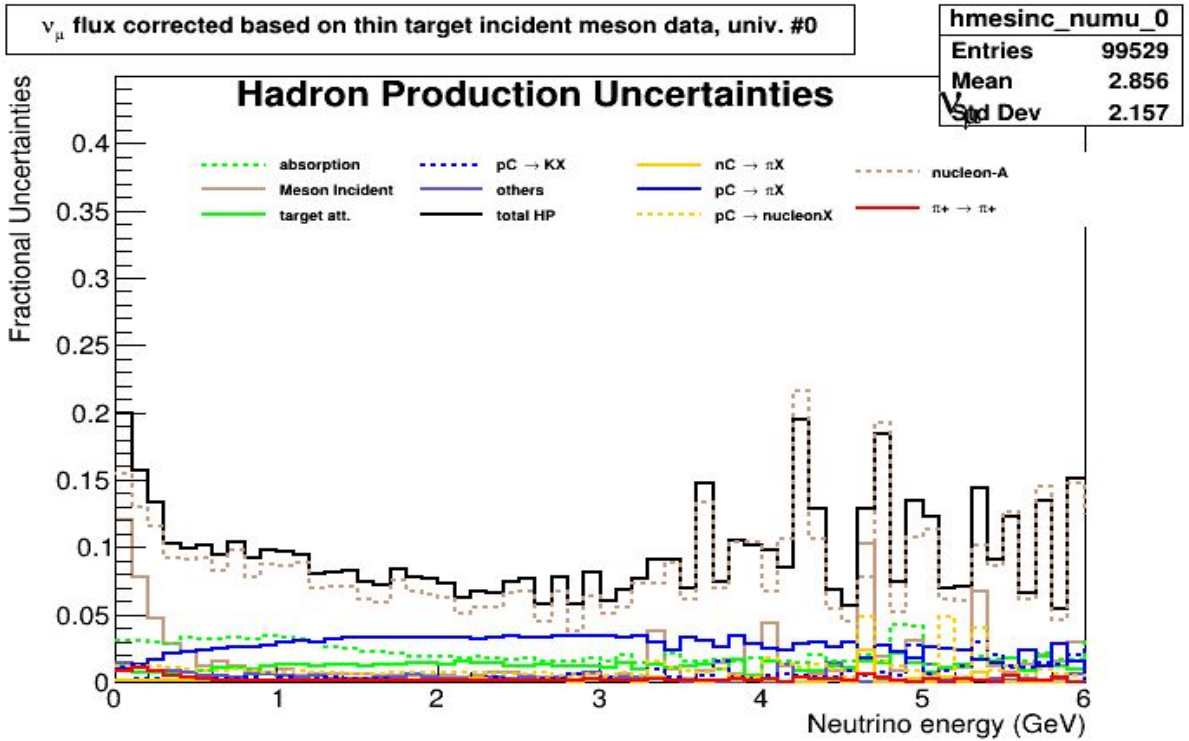
Interaction Breakdown



NA61

- Worked to get the Interaction Brakdown
- Will support existing work
- This plot is for few files only, running on local system, time consuming
- Will share results along with these plots

Muon Neutrino Corrected Flux (NA61 data)



- Statistical Uncertainties only
- Correlated errors not considered yet