

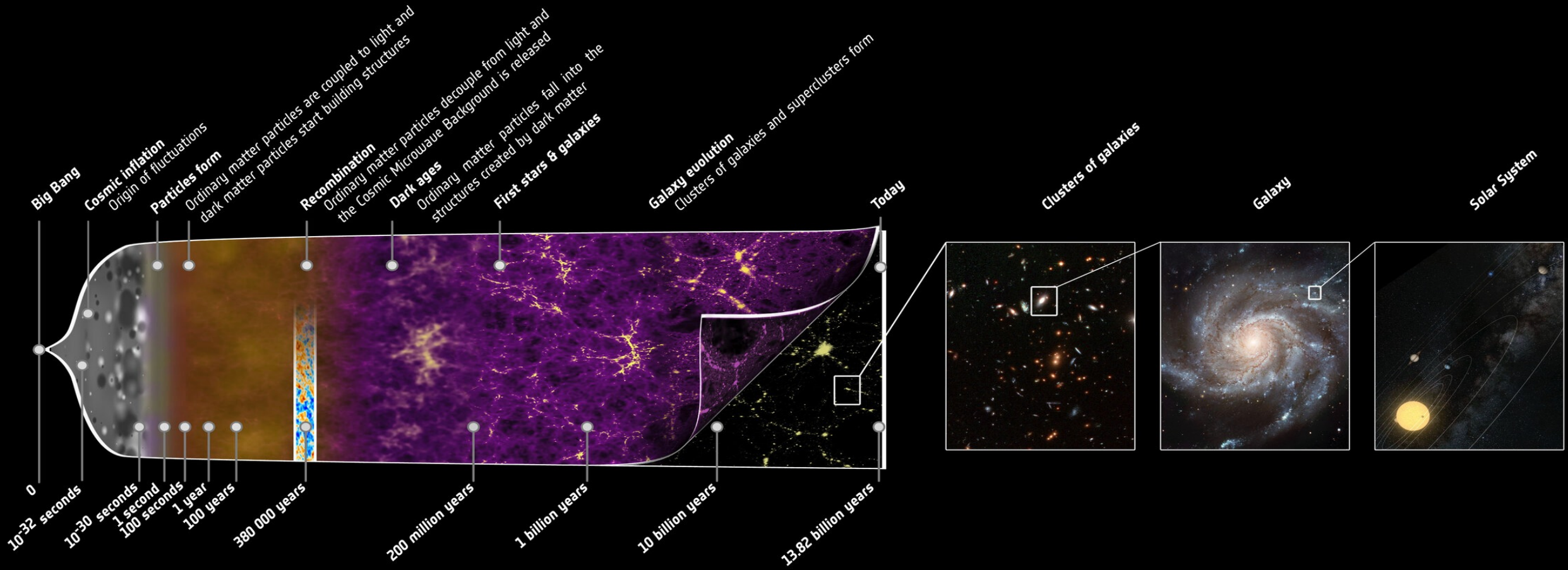
A visualization of the cosmic web, showing a complex network of filaments and nodes in shades of purple and blue. A large, dark, semi-circular shape is overlaid on the right side of the image, containing the text.

Likelihood- free Inference for Large Forward Models in Astrophysics/ Cosmology

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Fermilab, April 17, 2024

Timeline of the Universe



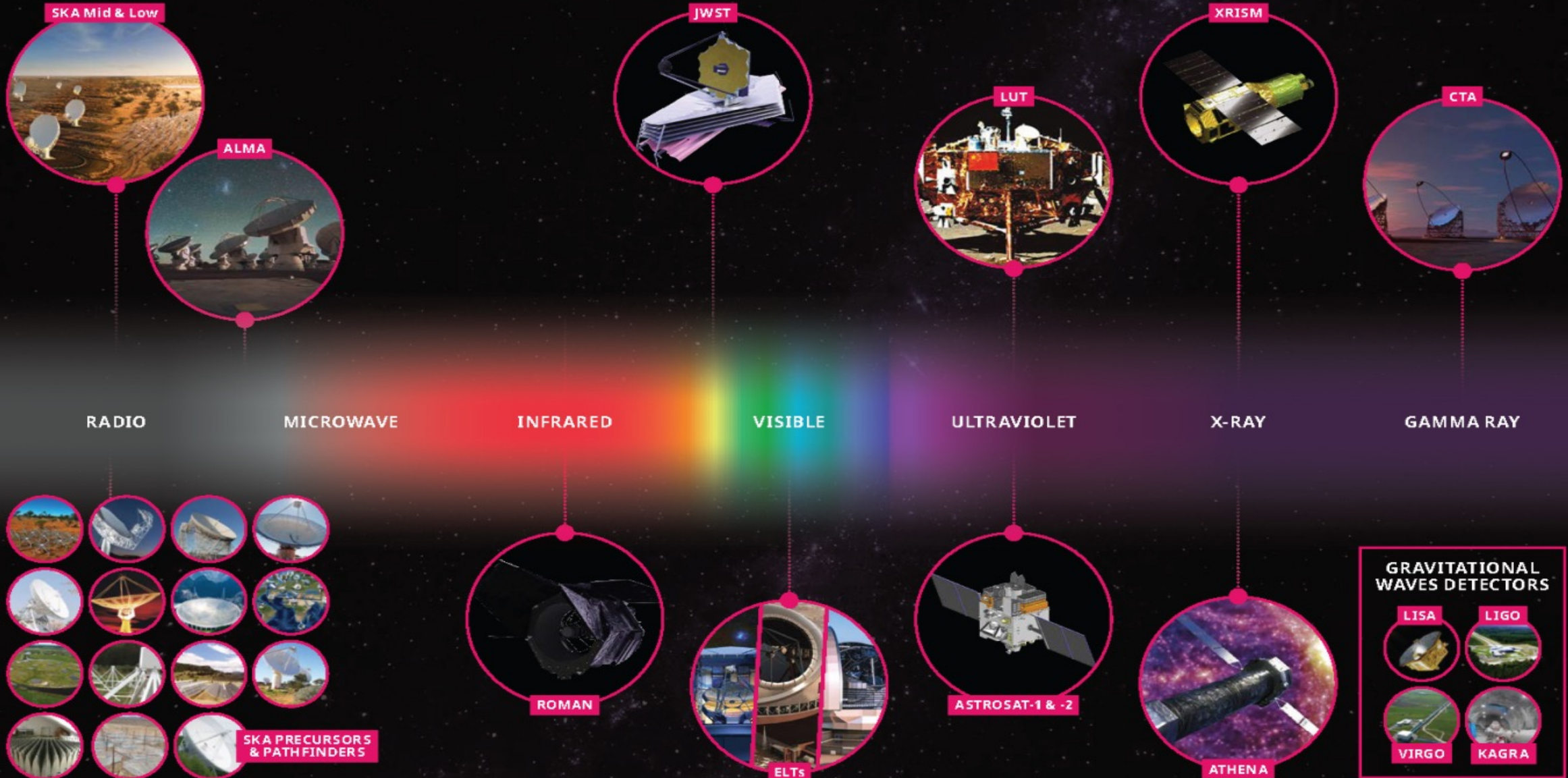
Structure Formation in the Universe



Time since the Big Bang: 0.4 billion years

ILLUSTRIS

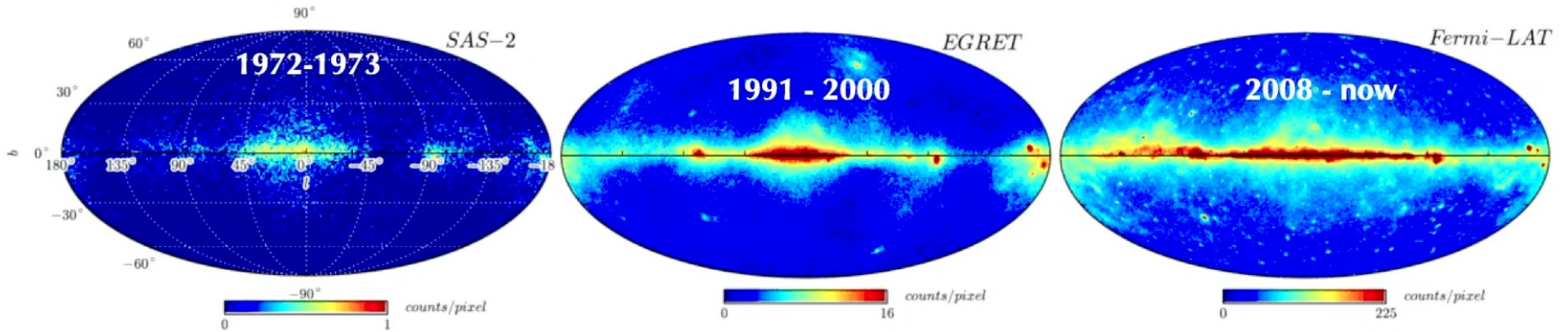
21st century cosmology



Overview

- Large Forward Models
- Inverse Problems
- Solving inverse problems with Neural Networks
- Examples

Large Forward Models



- ~ 6 Gamma-ray sources
- Diffuse emission

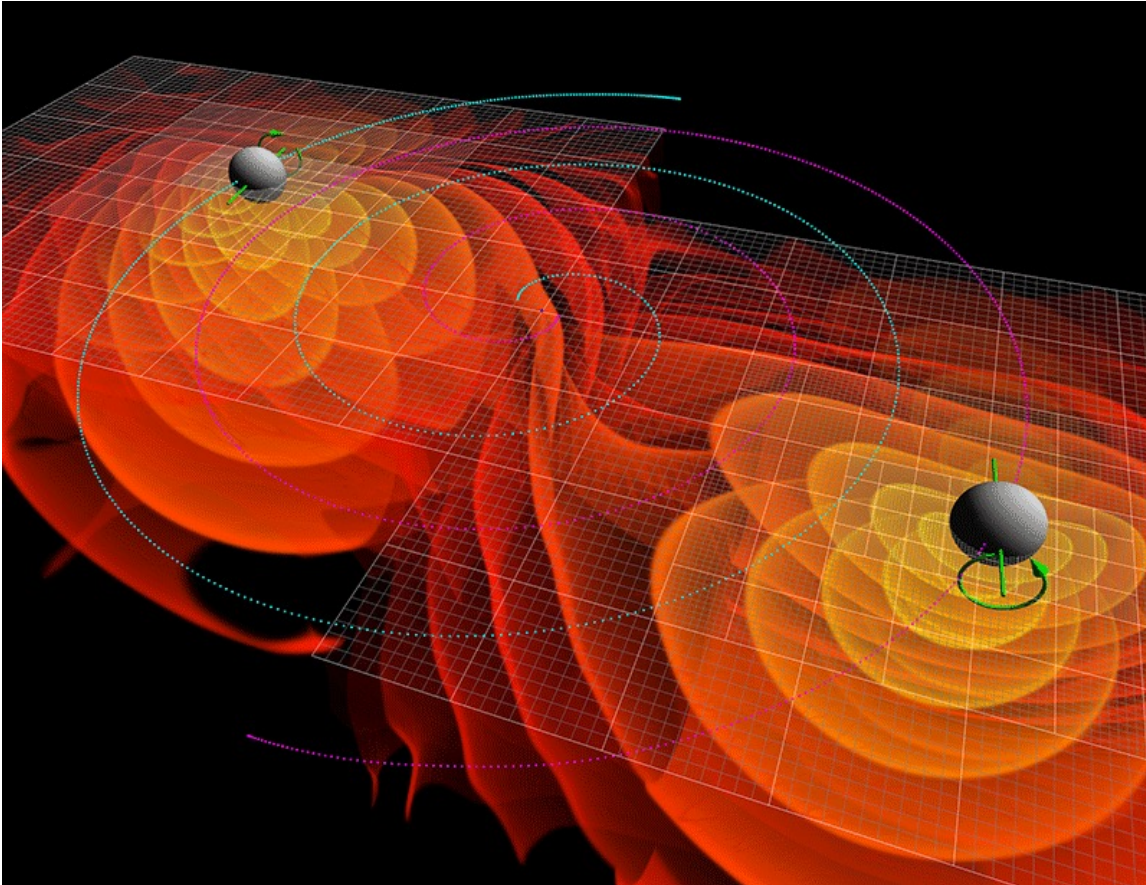
- 188 Gamma-ray sources
- Extended point sources
- Resolved gas emission

- 6658 Gamma-ray sources
- Highly resolved gas
- Many extended sources
- Fermi bubbles
- GeV excess
- Dark gas
- ...

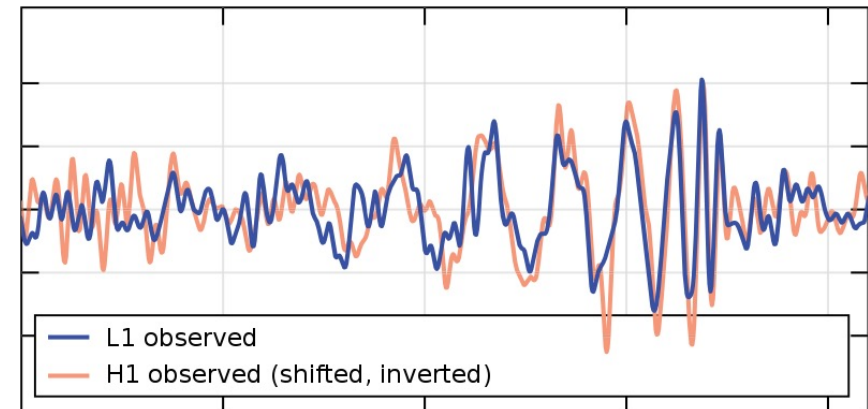
**More data leads to more objects.
More objects lead to more parameters.**

More parameters lead to more complicated statistical analyses.

Binary black hole merger ➡ Gravitational waves



Livingston, Louisiana (L1)



- Computationally expensive numerical relativity simulations.
- Many parameters (masses or merging objects, positions in the sky etc...)

Characteristics of Large Forward Models

A forward model generates samples $x, z \sim p(x|z)p(z)$. Why it could be large?

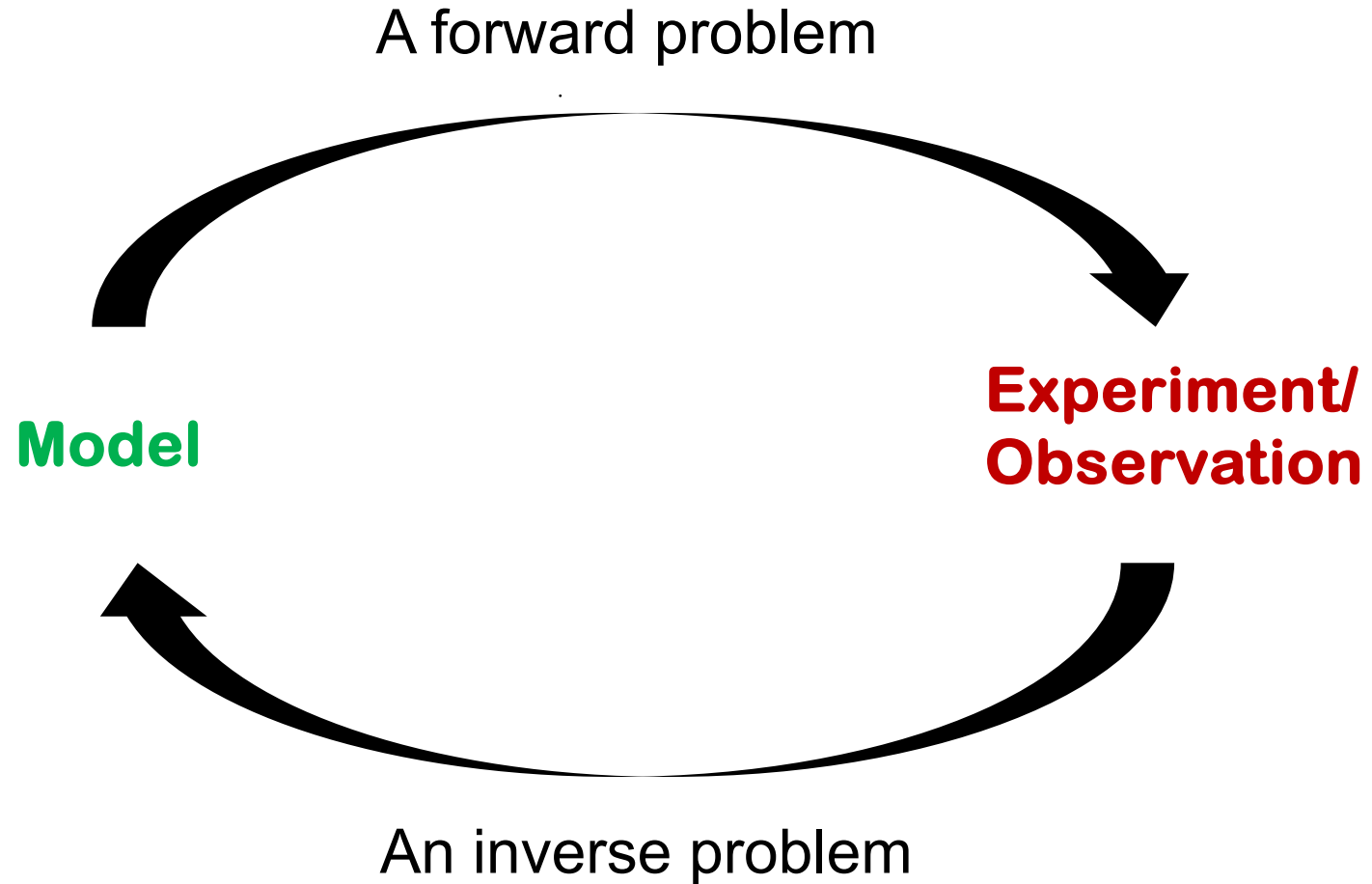
 simulator

- 1. Model Complexity:** Number of parameters with complex interactions between them
- 2. Amount of Data:** Different data sources, volume of data
- 3. Computational Recourses:** Computational power, memory or time to run

- **In Physics and Astronomy**

Forward models are usually principled. They implement specific physical laws. Parameters z typically have an interpretation in terms of specific physical quantities.

Inverse Problems



Bayes Theorem

Posterior

Likelihood

Prior

$$p(\mathbf{z}|\mathbf{x}) = \frac{p(\mathbf{x}|\mathbf{z})p(\mathbf{z})}{p(\mathbf{x})}$$

where $p(\mathbf{x}) = \int d\mathbf{z} p(\mathbf{x}|\mathbf{z})p(\mathbf{z})$

Evidence

Likelihood-based Inference

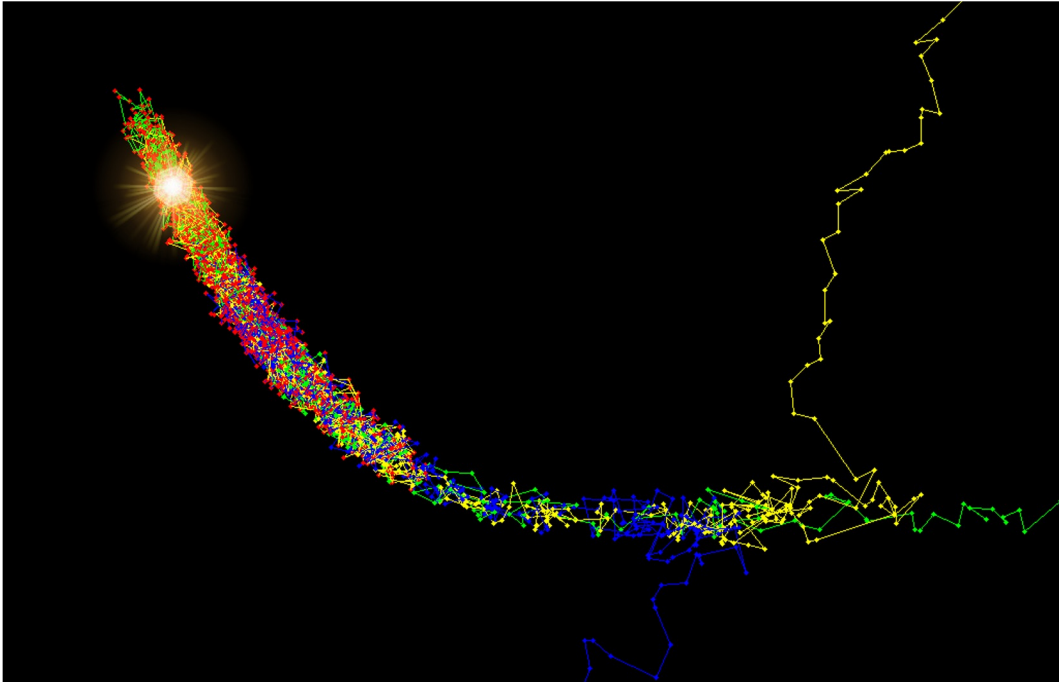


Image credit: Wikipedia

Ex: Metropolis Hastings Algorithm

$$p(\mathbf{z}|\mathbf{x}) = \frac{p(\mathbf{x}|\mathbf{z})p(\mathbf{z})}{p(\mathbf{x})}$$

- MC methods sample from the **joint high-dimensional posterior for all parameters**

$$\mathbf{z} \sim p(\mathbf{z}|\mathbf{x}), \mathbf{z} \in \mathbb{R}^D$$

D : Number of parameters

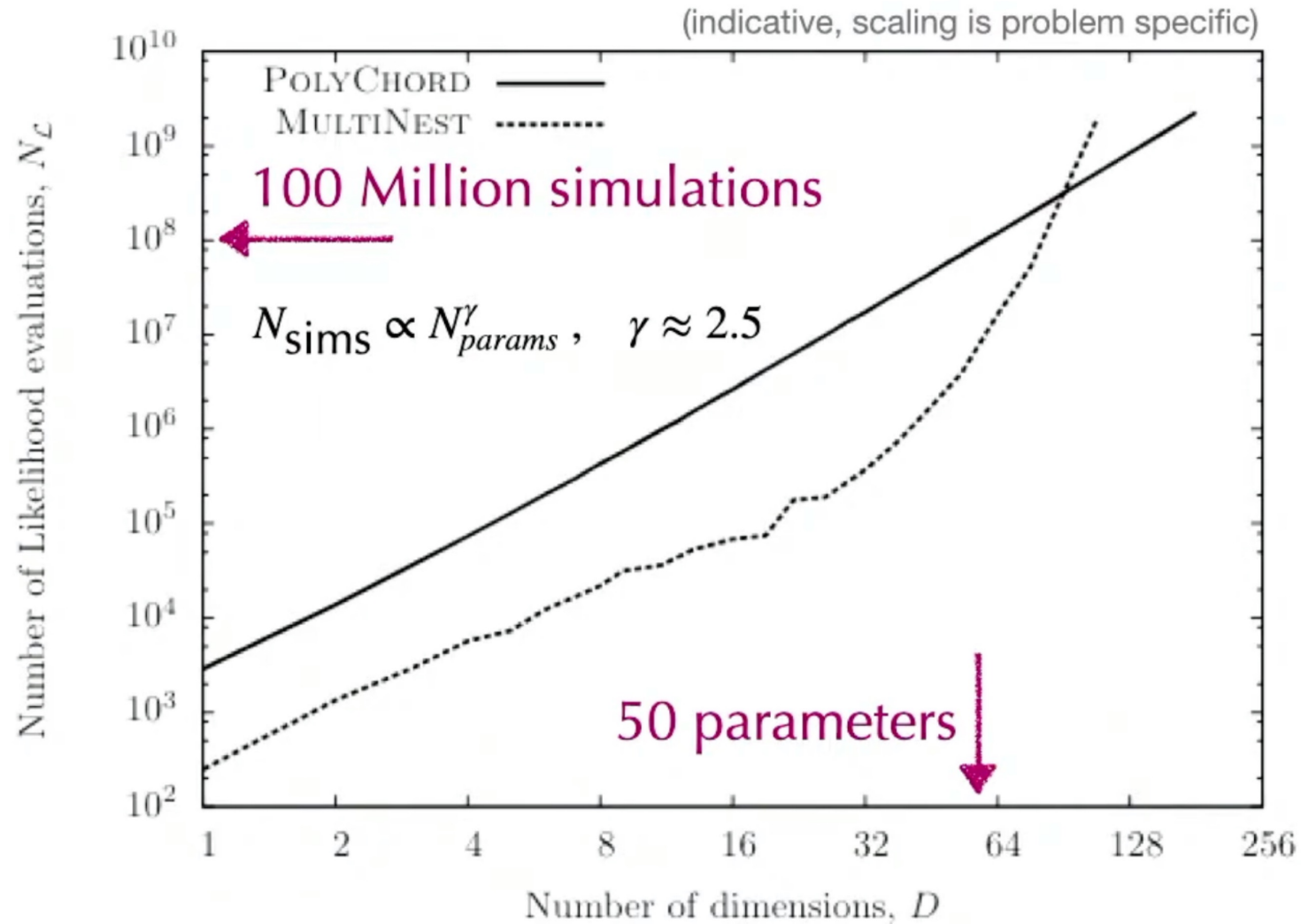
- **Obtain marginals by projection onto parameters of interest**

$$\mathbf{z} \equiv (Z_1, Z_2, \dots, Z_D)^T \rightarrow (z_i, z_j)^T \in \mathbb{R}^2$$

Expensive!

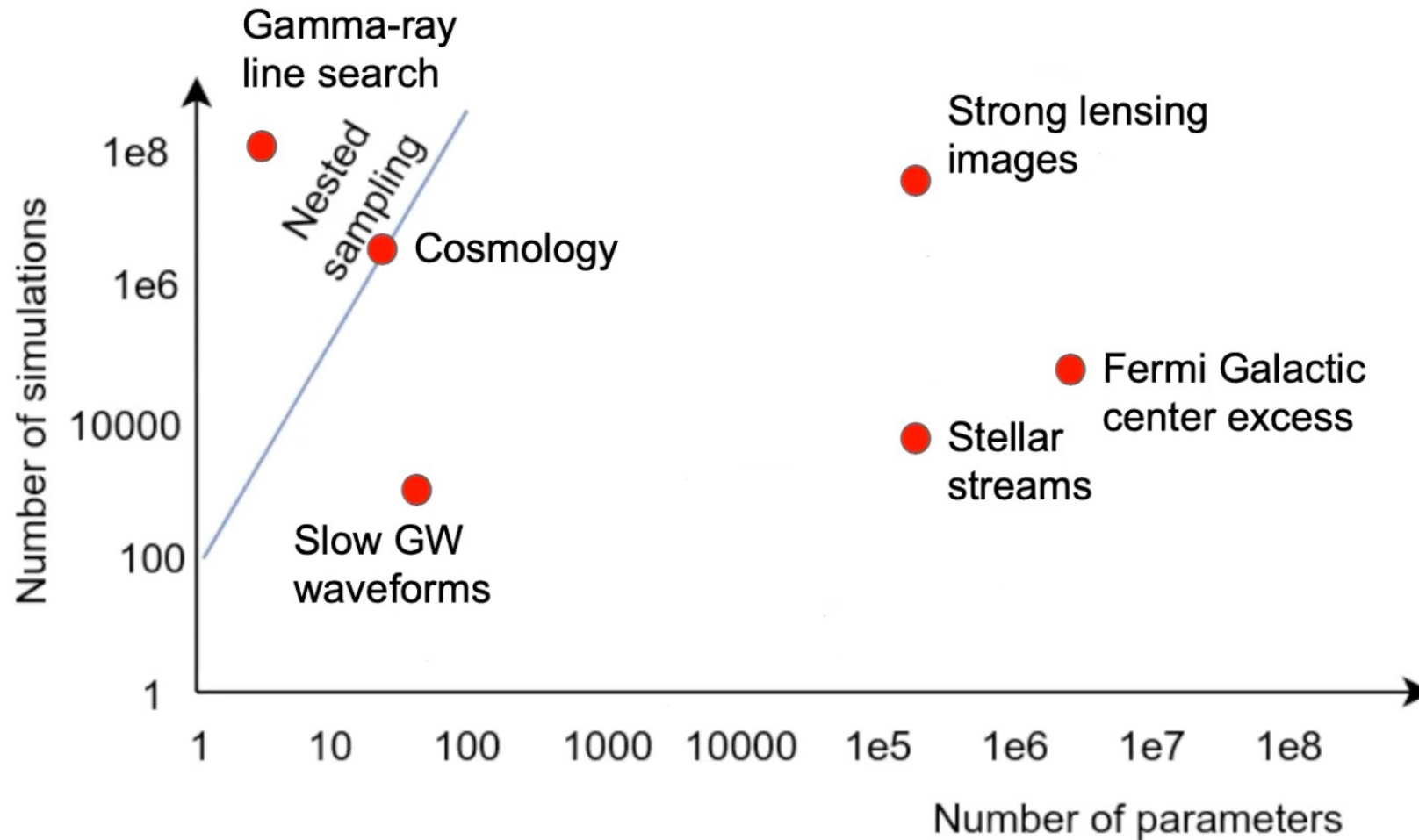
- Require an explicit likelihood

Likelihood-based Inference



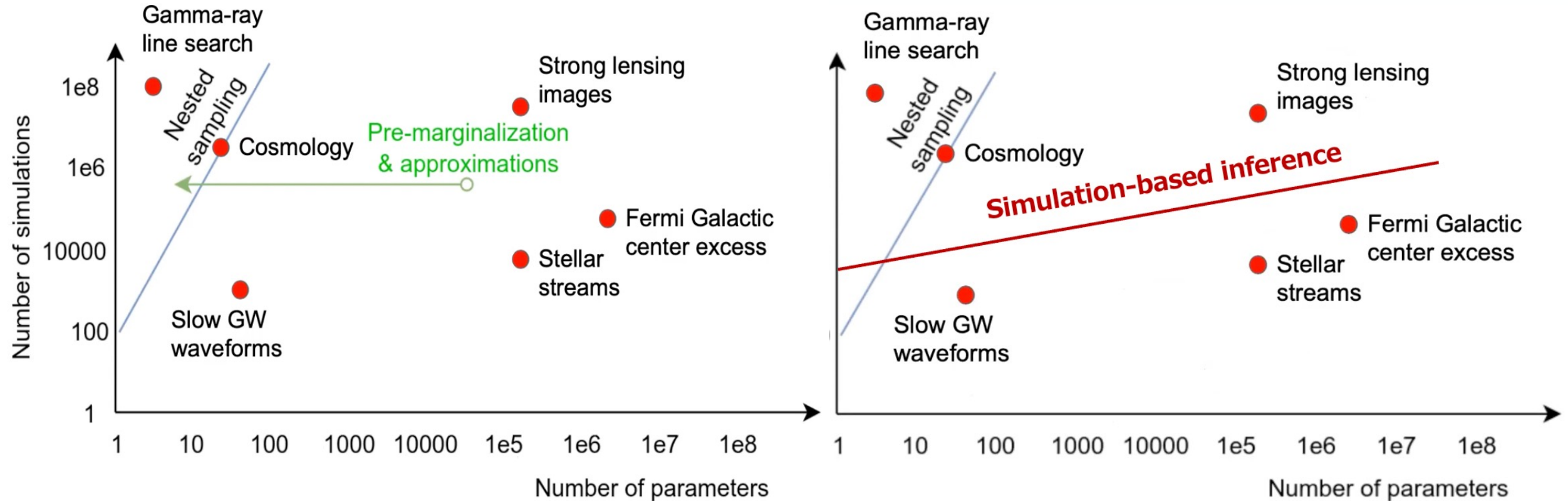
[Handley, Hosbon,
Lasenby 1911.01429]

Likelihood-based Inference vs high dimensional models



Slide credit: Christoph Weniger

Making problems tractable



Slide credit: Christoph Weniger

A thought experiment

Simulated images:



Observed data:



1, 3, 2, **1**, 5, 4, 3, 1, 6, 7, 9, ...

6, 2, 5, **8**, 6, 8, 4, 3, 2, 1, 3, ...

3, 4, 2, **3**, 1, 7, 8, 9, 5, 3, 2, ...

4, 2, 1, **4**, 6, 8, 6, 4, 3, 2, 4, ...

1, 3, 2, **9**, 5, 4, 3, 1, 6, 7, 9, ...

6, 2, 5, **8**, 6, 8, 4, 3, 1, 3, 4, ...

2, 3, 4, **1**, 1, 7, 8, 9, 5, 3, 2, ...

4, 2, 1, **2**, 6, 8, 6, 4, 3, 2, 4, ...

1, 3, 2, **4**, 5, 4, 3, 1, 6, 7, 9, ...

?

Red (position of moon):
parameter of interest

Black:
Nuisance parameters
(parametrizing *all* possible
background images)

A thought experiment

Simulated images:



Observed data:



1, 3, 2, **1**, 5, 4, 3, 1, 6, 7, 9, ...

6, 2, 5, **8**, 6, 8, 4, 3, 2, 1, 3, ...

3, 4, 2, **3**, 1, 7, 8, 9, 5, 3, 2, ...

4, 2, 1, **4**, 6, 8, 6, 4, 3, 2, 4, ...

1, 3, 2, **9**, 5, 4, 3, 1, 6, 7, 9, ...

6, 2, 5, **8**, 6, 8, 4, 3, 1, 3, 4, ...

2, 3, 4, **1**, 1, 7, 8, 9, 5, 3, 2, ...

4, 2, 1, **2**, 6, 8, 6, 4, 3, 2, 4, ...

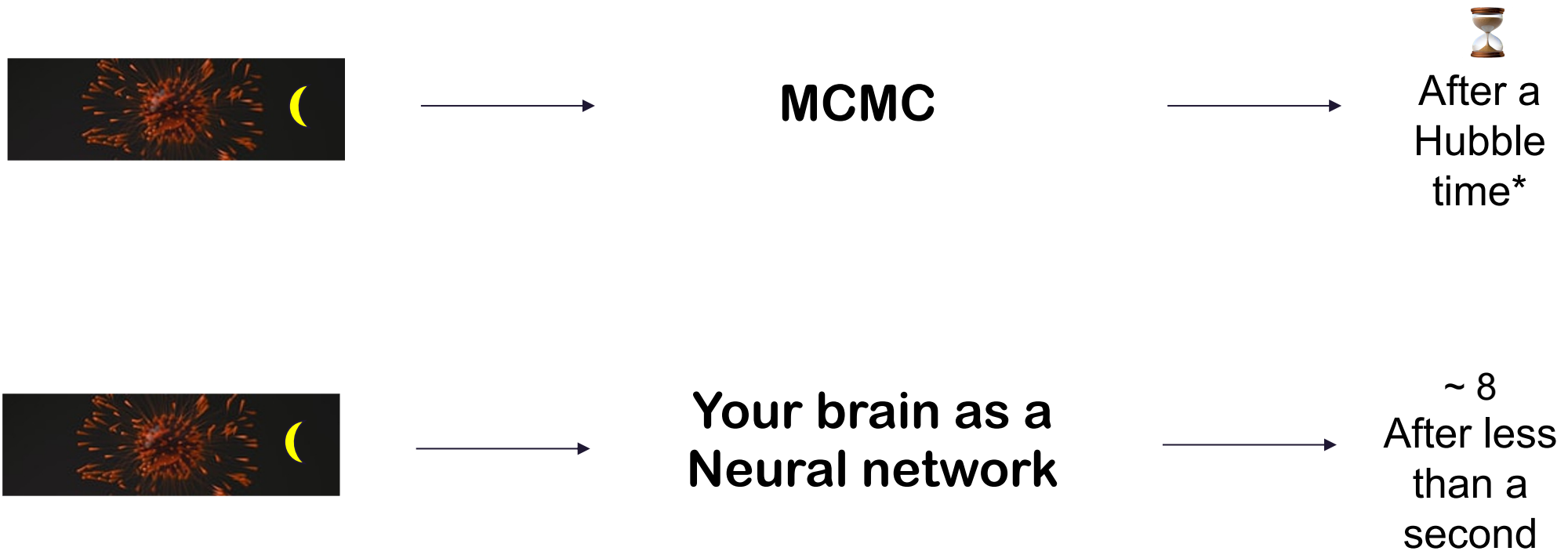
1, 3, 2, **4**, 5, 4, 3, 1, 6, 7, 9, ...

-, -, -, **8**, -, -, -, -, -, -, -, ...

Red (position of moon):
parameter of interest

Black:
Nuisance parameters
(parametrizing *all* possible
background images)

A thought experiment



You learned from a few examples to recognize the pattern related to the parameter of interest.

Simulation efficient: Just 8 examples are enough!

How can we achieve efficiency and precision?

Neural simulation-based inference (SBI)

Numerous methods exist with various pros/cons

General goal: obtain *neural network approximator* for one of the following:

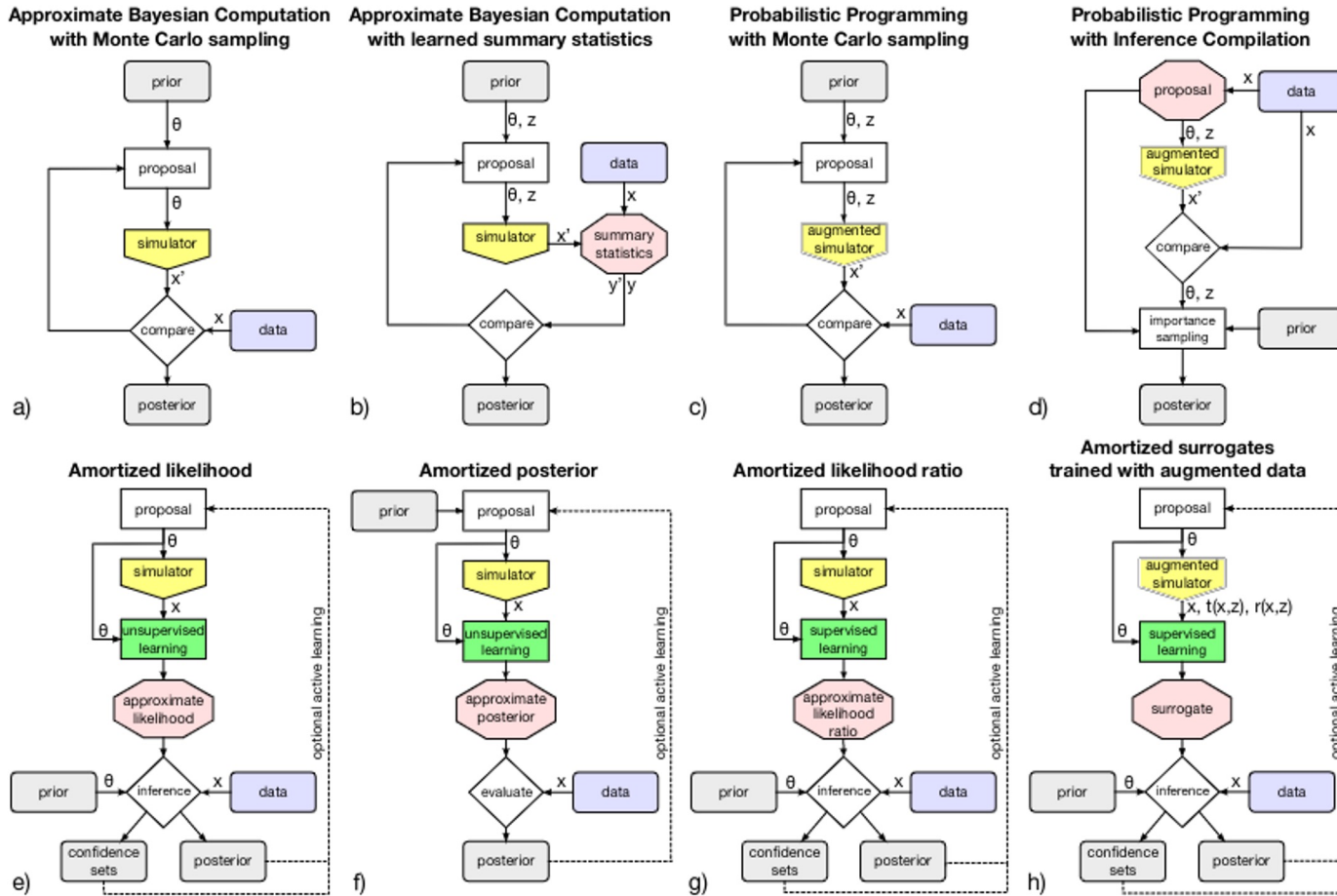


Fig. 3. Overview of different approaches to simulation-based inference.

[Cranmer, Brehmer, Louppe 1911.01429]

- Posterior* $p(\mathbf{z}|\mathbf{x})$
- Likelihood* $p(\mathbf{x}|\mathbf{z})$
- Ratios of posteriors and priors = ratios of likelihood and evidence

$$\frac{p(\mathbf{z}|\mathbf{x})}{p(\mathbf{z})} = \frac{p(\mathbf{x}|\mathbf{z})}{p(\mathbf{x})}$$


- Various variations of the above quantities ...

Neural Ratio Estimation (NRE)

Estimating the joint-vs-marginal ratio estimates the posterior

$$r(\mathbf{x}, \mathbf{z}) = \frac{p(\mathbf{x}, \mathbf{z})}{p(\mathbf{x})p(\mathbf{z})} = \frac{p(\mathbf{x}|\mathbf{z})p(\mathbf{z})}{p(\mathbf{x})p(\mathbf{z})} = \frac{p(\mathbf{z}|\mathbf{x})}{p(\mathbf{z})}$$

Bayes theorem



Strategy: We train a neural network $d_\phi(\mathbf{x}, \mathbf{z})$ as a binary classifier to be able to distinguish

- dependent sample-parameter pairs: $\mathbf{x}, \mathbf{z} \sim p(\mathbf{x}|\mathbf{z}) p(\mathbf{z})$ with class label $Y=1$ from
- independent ones: $\mathbf{x}, \mathbf{z} \sim p(\mathbf{x}) p(\mathbf{z})$ with class label $Y=0$.

[Cranmer, Begy, Louppe 1903.04057]

Neural Ratio Estimation (NRE)

We want the output of the NN to be the probability of class $Y=1$, i.e.,

$$d_\varphi(\mathbf{x}, \mathbf{z}) = p(\mathbf{y} = \mathbf{1} | \mathbf{x}, \mathbf{z}) = \frac{p(\mathbf{x}, \mathbf{z} | Y=1) p(Y=1)}{p(\mathbf{x}, \mathbf{z} | Y=1) p(Y=1) + p(\mathbf{x}, \mathbf{z} | Y=0) p(Y=0)} = \frac{p(\mathbf{x}, \mathbf{z})}{p(\mathbf{x}, \mathbf{z}) + p(\mathbf{x}) p(\mathbf{z})}$$


The corresponding loss function is the binary cross-entropy loss function:

$$L[d_\varphi(\mathbf{x}, \mathbf{z})] = - \int d\mathbf{x} d\mathbf{z} p(\mathbf{x}, \mathbf{z}) [-\ln(d_\varphi(\mathbf{x}, \mathbf{z}))] + p(\mathbf{x}) p(\mathbf{z}) [\ln(1 - d_\varphi(\mathbf{x}, \mathbf{z}))]$$

$z = \{\text{cats, donkeys}\}$


$\mathbf{x}, \mathbf{z} \sim p(\mathbf{x}, \mathbf{z}) \propto p(\mathbf{x} | \mathbf{z}) p(\mathbf{z})$
Draw labeled images

Cat Donkey Cat



$\mathbf{x}, \mathbf{z} \sim p(\mathbf{x}) p(\mathbf{z})$
Draw images with random labels

Cat Cat Cat

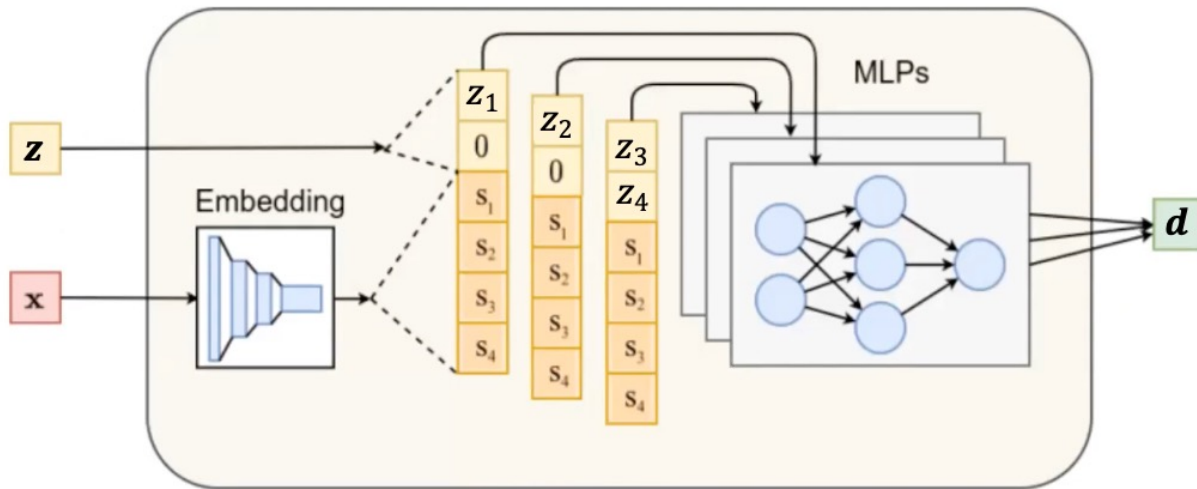


Minimizing this function w.r.t. the network parameters φ yields

$$d_{\varphi}^*(\mathbf{x}, \mathbf{z}) = \frac{p(\mathbf{x}, \mathbf{z})}{p(\mathbf{x}, \mathbf{z}) + p(\mathbf{x})p(\mathbf{z})} \quad \text{and thus} \quad r(\mathbf{x}, \mathbf{z}) = \frac{d_{\varphi}^*(\mathbf{x}, \mathbf{z})}{d_{\varphi}^*(\mathbf{x}, \mathbf{z}) - 1} \quad [\text{Cranmer, Brehmer, Louppe 1911.01429}]$$

Using the result of the NN we have access to posteriors

$$p(\mathbf{z}|\mathbf{x}) = r(\mathbf{x}, \mathbf{z}) \cdot p(\mathbf{z})$$



- Estimates: $p(z_1|\mathbf{x}), p(z_2|\mathbf{x}), p(z_3, z_4|\mathbf{x})$
- Does **not** estimate $p(z_1, z_2|\mathbf{x}), p(z_1, z_2, z_3|\mathbf{x})$

Pros

- **Implicit likelihood** → **Cherry-picking marginal posteriors!**
- No need to retrain to look at a different observation (**Amortization**)
- Ability to focus on a single observation through truncation
- Easy network architecture

Cons

- Difficult to sample from high-dim posteriors

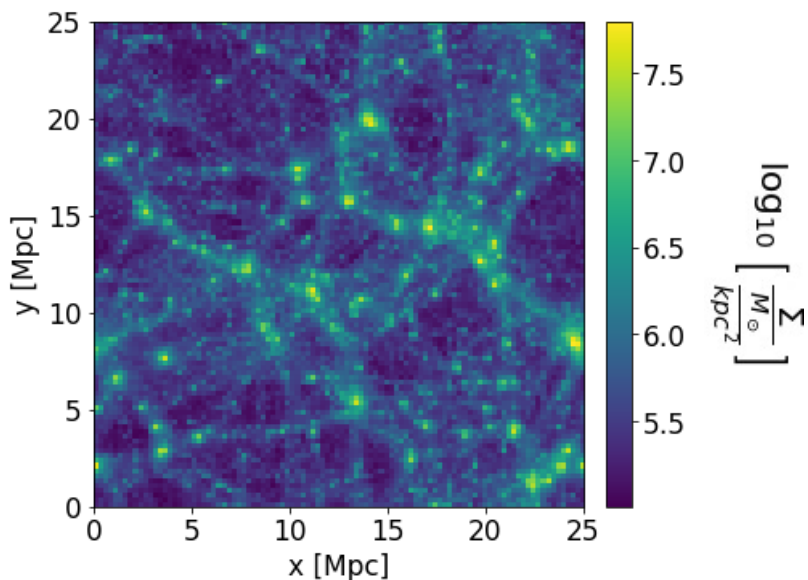
Example 1:


A toy model for N-body simulations

[[AD](#), Camila Correa, Christoph Weniger 2206.11312]

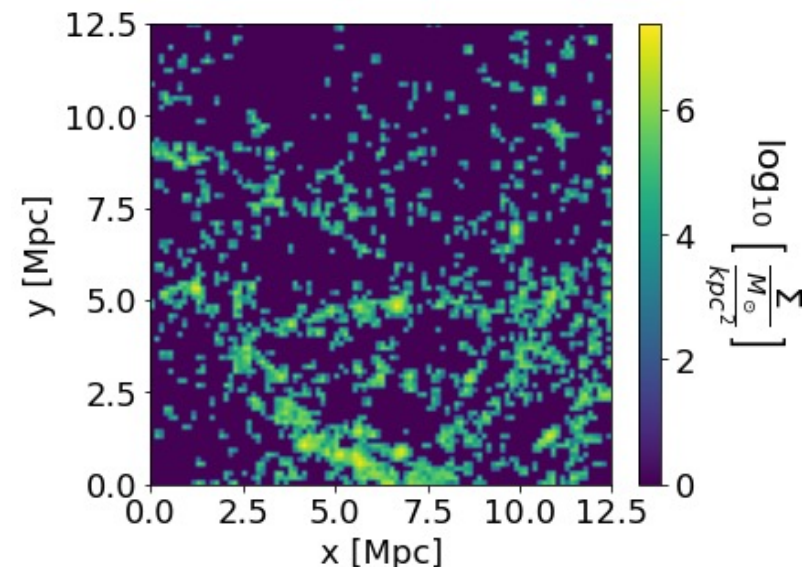
- **Goal:** reconstruction of **halo clustering** and **halo mass function** of DM-only cosmological simulations generated by the **EAGLE project**
- **Approach:** an **analytical halo model** based on a **toy implementation** of two body correlation functions
- **The EAGLE project**

(25 Mpc)³ box with 376³ particles

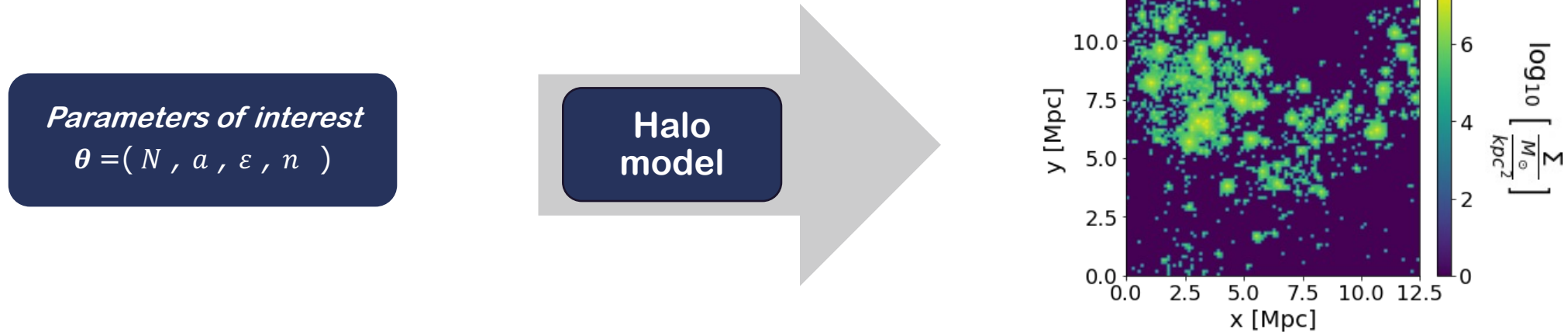


FOF halo finder

Particle data

$M_h \in (10^9, 10^{12}) M_\odot$, (12.5 Mpc)³ box



- **The toy halo model**



- The **first parameter of interest** is the number, N , of the haloes
- The masses of the haloes, $M_h \in (10^9, 10^{12}) M_\odot$, can be sampled from a halo mass function

$$\frac{dn}{dM} \propto M^{-a}$$

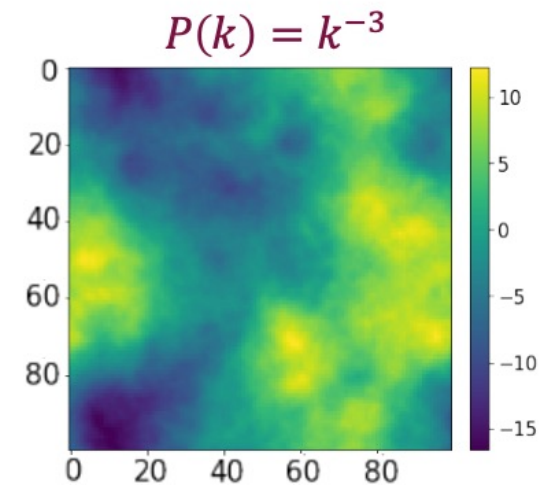
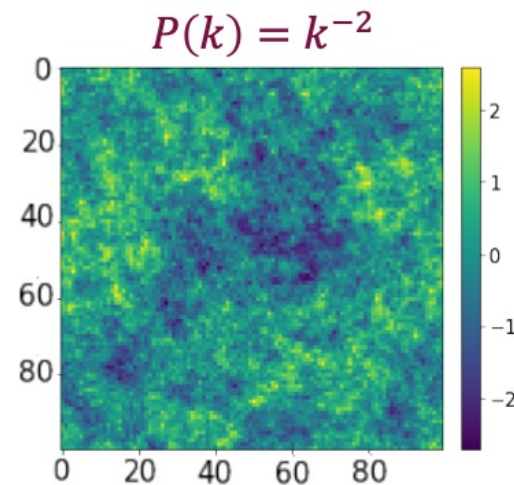
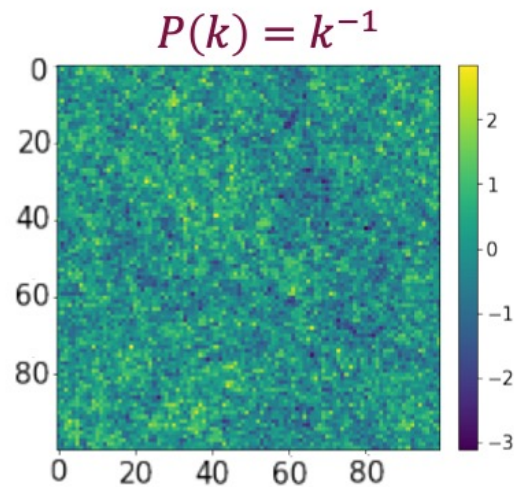
- The **second parameter of interest** is the slope, a , of the halo mass function

- **Adding Clustering to the Model**

- We will sample the positions according to distributions generated from 2D realizations of gaussian random fields, δ
- The gaussian fields will be specified by a power-law power spectrum

$$P(k) \propto \frac{1}{k^n}$$

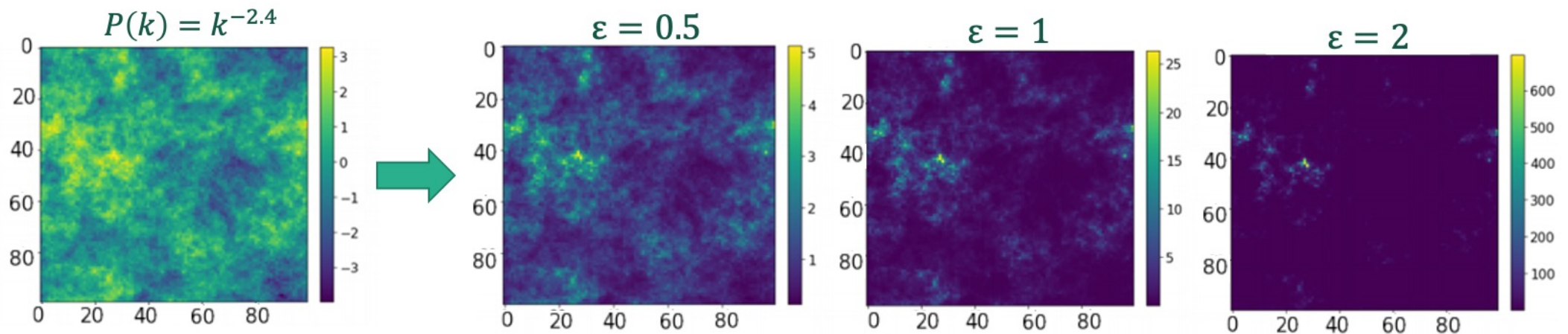
- The slope of the power spectrum, n , is the **third parameter of interest** of our model



- **Adding Clustering to the Model**

- We transform the field δ to a probability distribution function to sample from it:

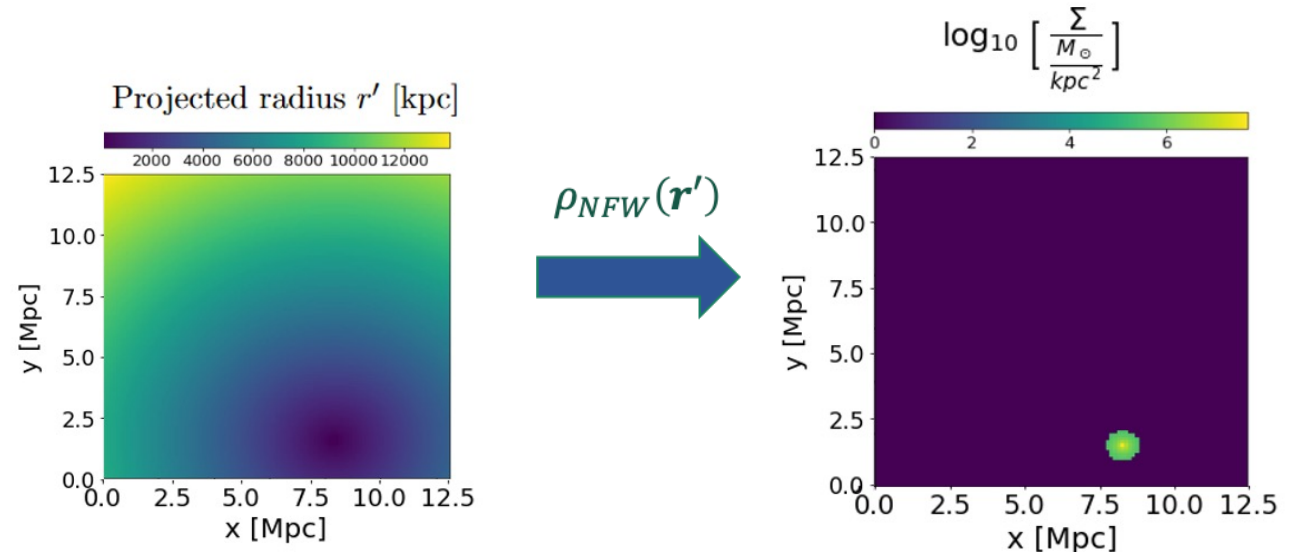
- We first multiply δ with a **fourth parameter of interest** ε ,
- We exponentiate $\delta \cdot \varepsilon$,
- We normalize the field $f = e^{\delta \cdot \varepsilon}$, s.t., its values sum to 1



- We sample the positions of the haloes according to this distribution

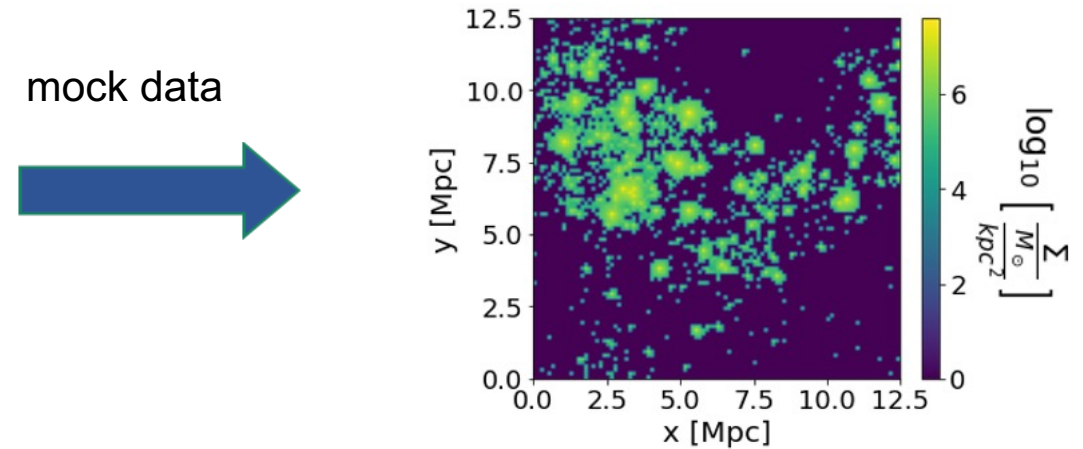
- **The toy halo model**

- We calculate the logarithmic surface density



- We add all the images of the individual haloes together to obtain the total surface density field

- We add poisson noise to the final image



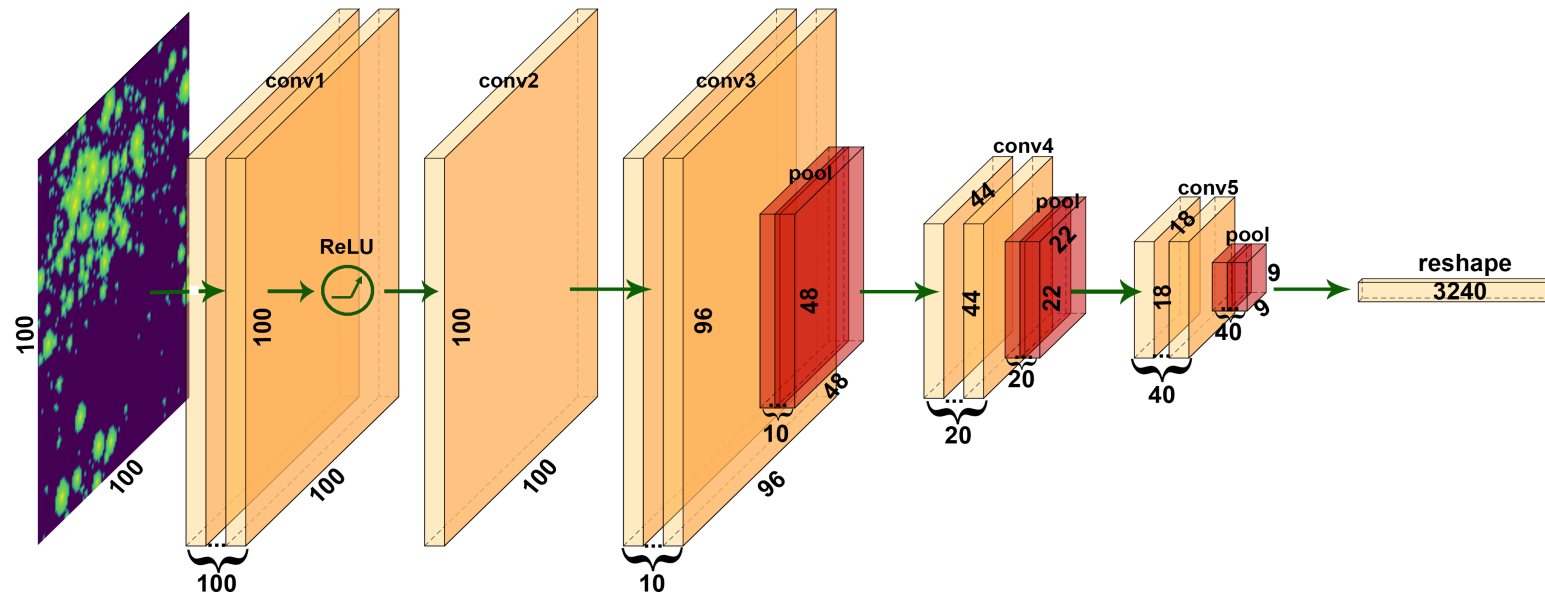
Thousands of nuisance parameters: positions of haloes, masses

Training NRE

Parameters of interest:

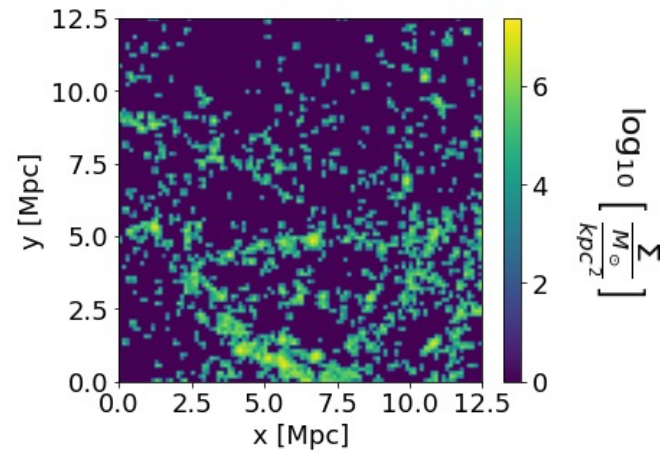
- N : Number of halos, where $N \in (100, 2100)$
- a : Inner slope of the halo mass function, $a \in (1, 3)$
- ε : Exponent of the density field, where $\varepsilon \in (0, 2)$
- n : Slope of the power spectrum, where $n \in (0, 10)$

- We define a **CNN**:



- We **train** using 200.000 mock images

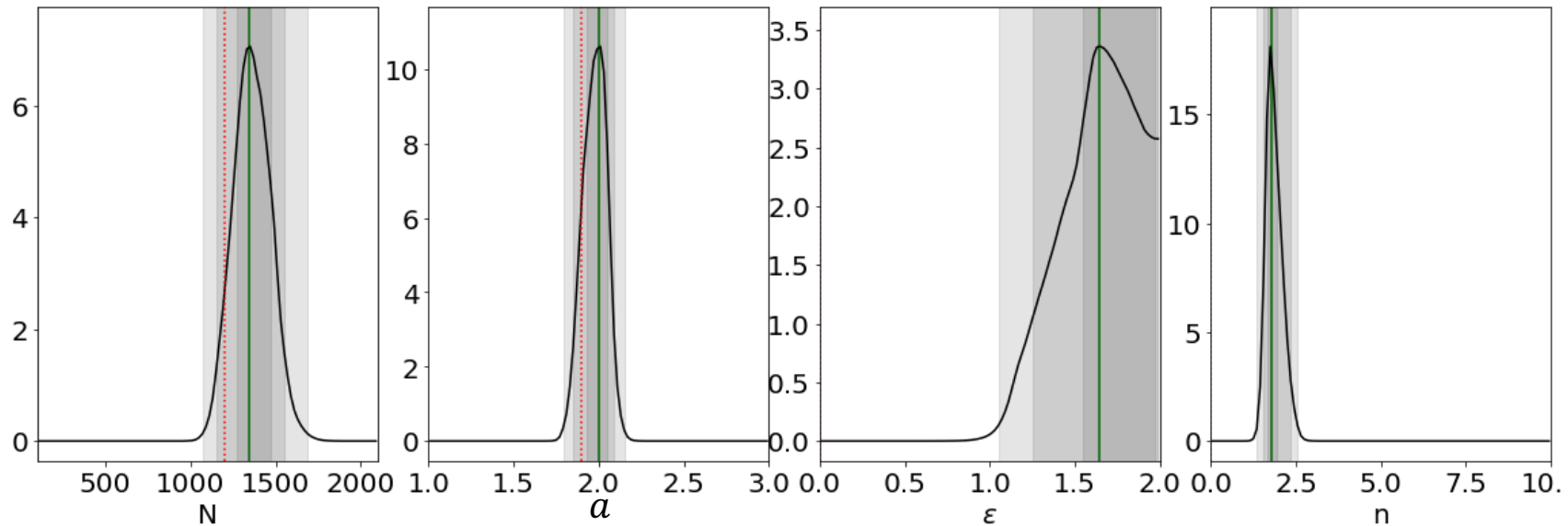
Results on actual N body simulations



trained NN

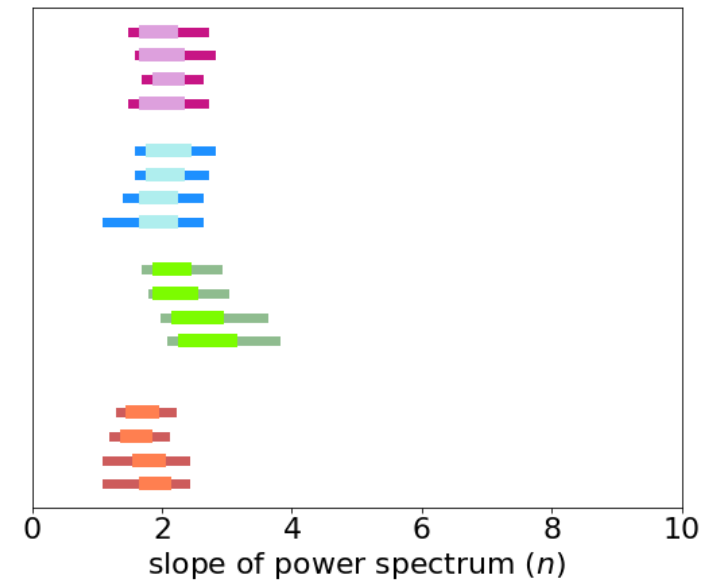
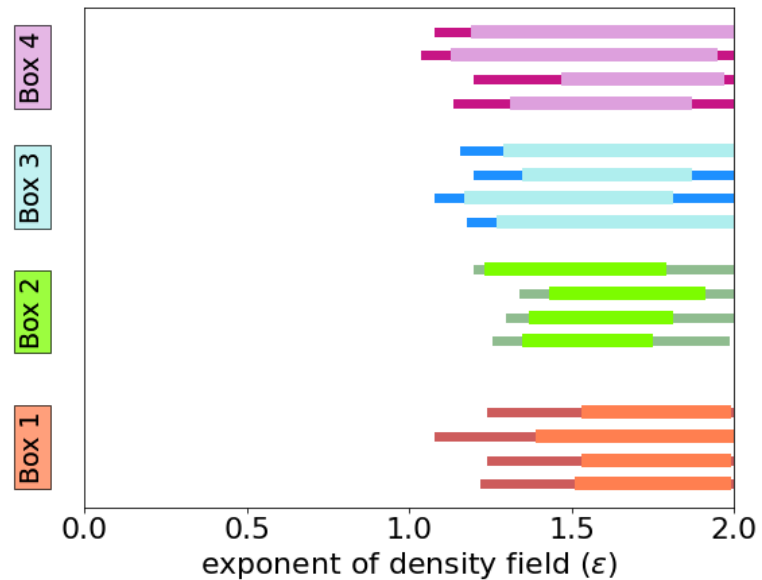
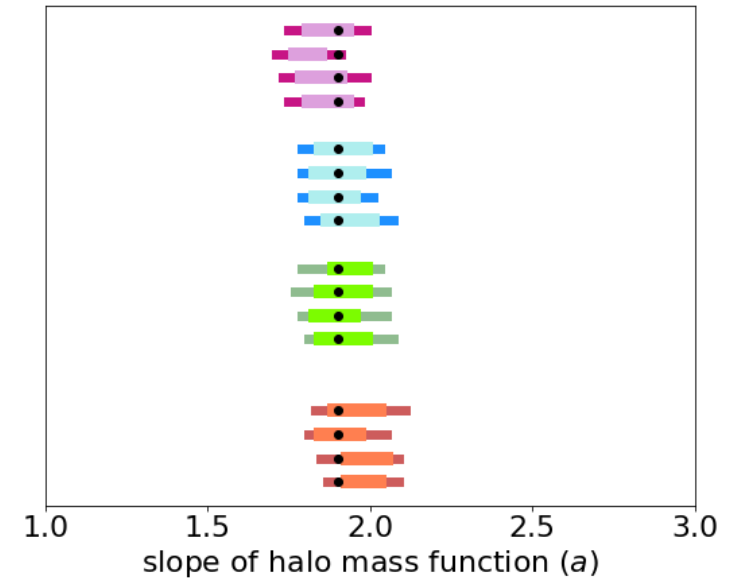
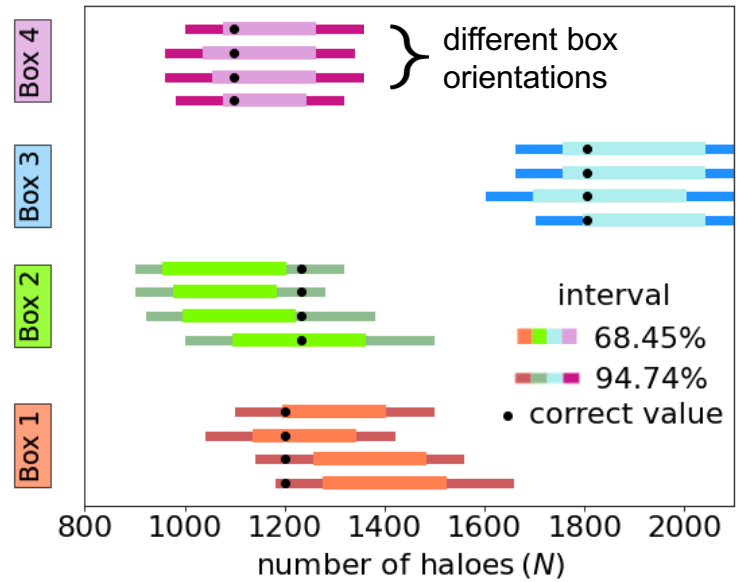
— mode

- - - correct value

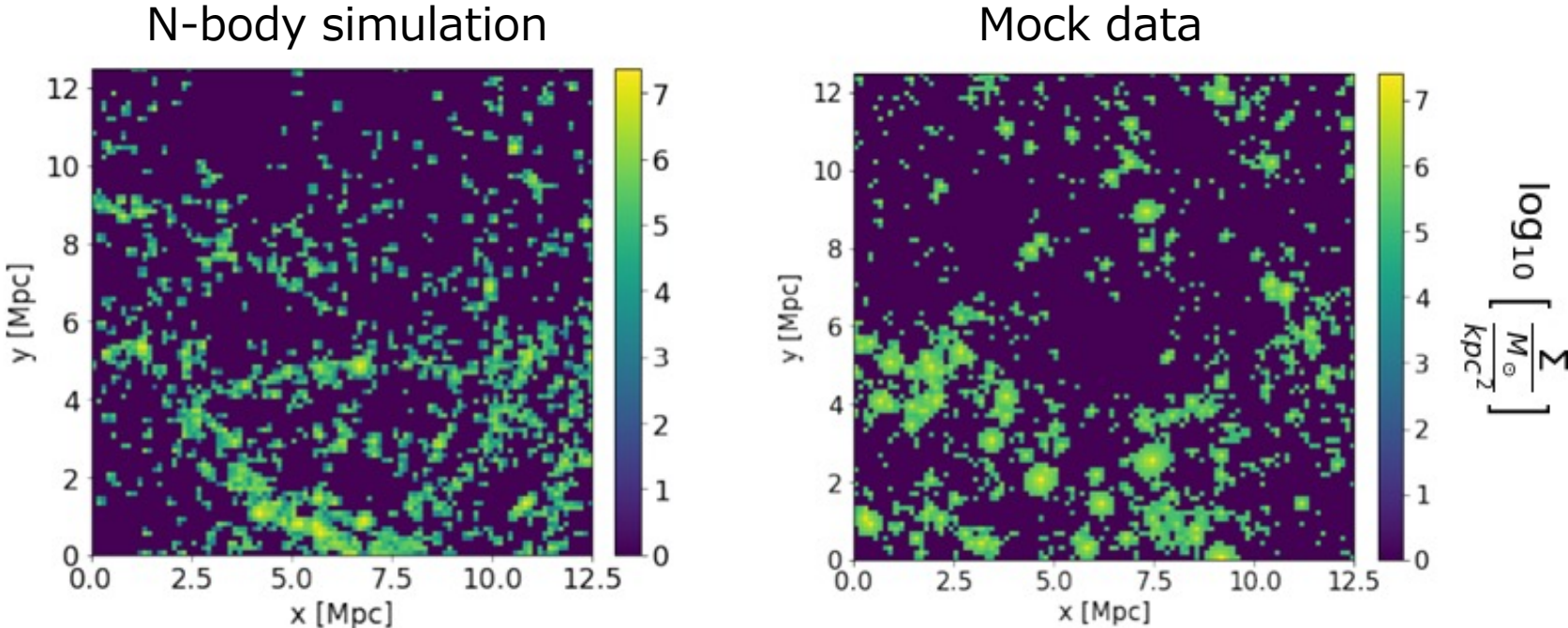


Results on actual N body simulations

- 4 simulation boxes and 3 rotations of them



N-body simulation image vs mock image



Neural Posterior Estimation (NPE)

Approximate $p(\mathbf{z}|\mathbf{x}_0)$ with a NN $q_\phi(\mathbf{z}|\mathbf{x}_0)$. How can we compare the two pdfs?

$$D_{KL}(p||q_\phi) = \int d\mathbf{z} p(\mathbf{z}|\mathbf{x}_0) \ln \left[\frac{p(\mathbf{z}|\mathbf{x}_0)}{q_\phi(\mathbf{z}|\mathbf{x}_0)} \right] = \mathbb{E}_{\mathbf{z} \sim p(\mathbf{z}|\mathbf{x}_0)} \ln \left[\frac{p(\mathbf{z}|\mathbf{x}_0)}{q_\phi(\mathbf{z}|\mathbf{x}_0)} \right]$$

Problem: How to avoid sampling from the intractable posterior?

Solution: Amortize (average) over simulation data. The loss is now the expected KL divergence, where we have exploited Bayes theorem, i.e., $p(\mathbf{x}|\mathbf{z})p(\mathbf{z}) = p(\mathbf{z}|\mathbf{x})p(\mathbf{x})$.

$$\mathbb{E}_{\mathbf{x} \sim p(\mathbf{x})} \left[D_{KL}(p||q_\phi) \right] = \mathbb{E}_{\mathbf{x} \sim p(\mathbf{x})} \mathbb{E}_{\mathbf{z} \sim p(\mathbf{z}|\mathbf{x})} \left[\ln \frac{p(\mathbf{z}|\mathbf{x})}{q_\phi(\mathbf{z}|\mathbf{x})} \right] = \mathbb{E}_{\mathbf{x}, \mathbf{z} \sim p(\mathbf{x}, \mathbf{z})} \left[\ln \frac{p(\mathbf{z}|\mathbf{x})}{q_\phi(\mathbf{z}|\mathbf{x})} \right]$$

$\mathbf{x}, \mathbf{z} \sim p(\mathbf{x}, \mathbf{z}) = p(\mathbf{x}|\mathbf{z})p(\mathbf{z})$

Neural Posterior Estimation (NPE)

$$\mathbb{E}_{\mathbf{x} \sim p(\mathbf{x})} [D_{KL}(p || q_\phi)] = \mathbb{E}_{\mathbf{x}, \mathbf{z} \sim p(\mathbf{x}, \mathbf{z})} \left[\ln \frac{p(\mathbf{z} | \mathbf{x})}{q_\phi(\mathbf{z} | \mathbf{x})} \right] = \overset{\text{tractable}}{-\mathbb{E}_{\mathbf{x}, \mathbf{z} \sim p(\mathbf{x}, \mathbf{z})} \ln q_\phi(\mathbf{z} | \mathbf{x})} + \overset{\text{intractable but constant}}{\mathbb{E}_{\mathbf{x}, \mathbf{z} \sim p(\mathbf{x}, \mathbf{z})} \ln p(\mathbf{z} | \mathbf{x})}$$

Training inference networks with forward KL leads to posteriors that minimize the posterior entropy:

$$\mathcal{L} = -\mathbb{E}_{\mathbf{x}, \mathbf{z} \sim p(\mathbf{x}, \mathbf{z})} \ln q_\phi(\mathbf{z} | \mathbf{x}) \longrightarrow \varphi^* \text{ (optimum parameters)} \quad [\text{Papamakarios, Murray 1605.06376}]$$

Pros

- **Implicit likelihood → Cherry-picking marginal posteriors!**
- No need to retrain to look at a different observation (**Amortization**)
- Ability to focus on a single observation with multi-round inference
- Obtain samples and evaluate the posterior

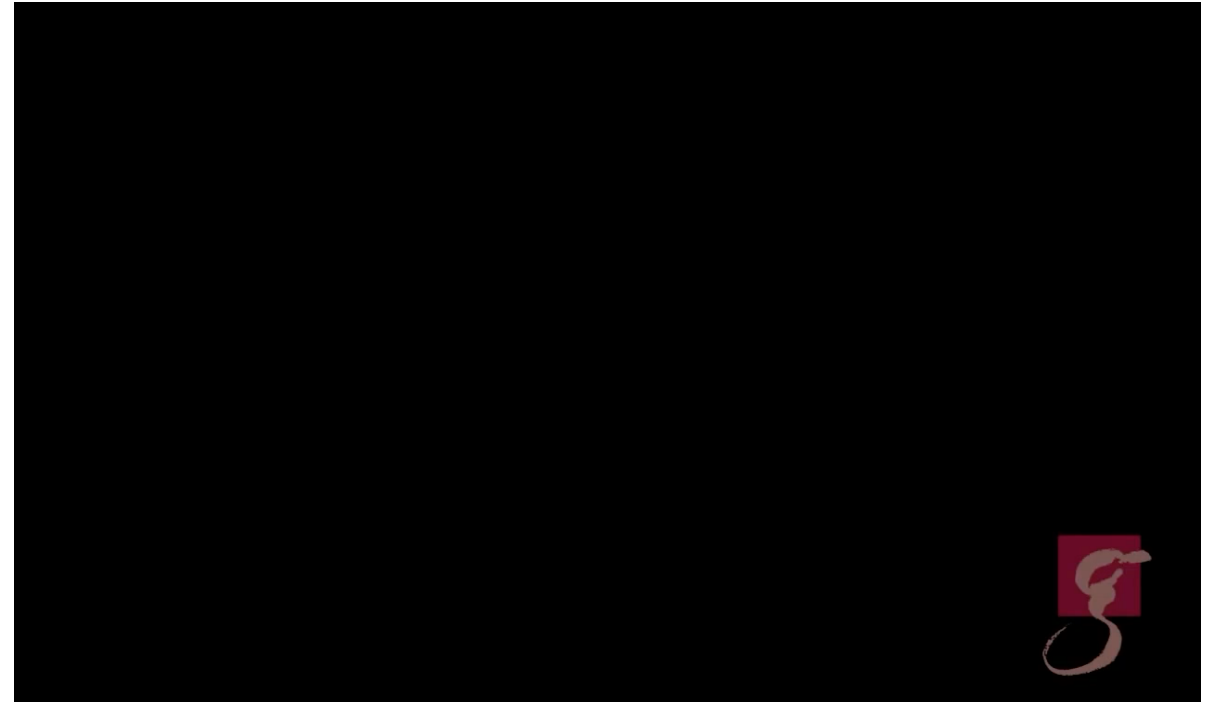
Cons

- q_ϕ should be a PDF, i.e. normalizing flow or Gaussian mixture model, **sometimes difficult to train**

Example 2:
Reconstruction of Gravitational Wave
Backgrounds

[[AD](#), Daniel G. Figueroa, Bryan Zaldivar 2309.08430]

- **Goal:** Reconstructing the **spectral shape** $\Omega_{\text{GW}}(f)$ of an unknown **Stochastic Gravitational Wave Background (SGWB)** with **LISA**:
 - will consist of a constellation of **three satellites** forming a nearly equilateral triangle with ~ 2.5 million km length arms
 - will perform **three correlated interferometric measurements** (X, Y and Z data streams)
 - X, Y and Z data streams can be **transformed into three uncorrelated ones** (A, E, and T), which diagonalise the signal and noise covariance matrices
 - will cover **frequencies** from 3×10^{-5} Hz to 0.5 Hz



- **Approach:** A model for data as close as possible to the real data-taking procedure of LISA

- We assume that the LISA mission will last 4 years.
- The data will be collected 75% of the time and will be clean of noise glitches and transient signals.
- Due to the need for regular operational breaks, the data will be collected in 94 time chunks (N_c).

- **Data**

α, β : given channels

$$D_{i,j}^{\alpha\beta} = \overset{\text{GW signal}}{\boxed{S_{i,j}}} + \overset{\text{Noise}}{\boxed{\mathcal{N}_{i,j}^{\alpha\beta}}}$$

$i \in \{1, 2, \dots, N_c\}$
time chunks

$$f_j \in \{3 \times 10^{-3}, 0.5\}$$

➤ **Data**

$$D_{i,j}^{\alpha\beta} = \overset{\text{GW signal}}{S_{i,j}} + \overset{\text{Noise}}{\mathcal{N}_{i,j}^{\alpha\beta}} \quad \text{where}$$

$$S_{i,j} = \left| \frac{G_1(0, \sqrt{\Omega_{\text{GW}}(f_j)}) + iG_2(0, \sqrt{\Omega_{\text{GW}}(f_j)})}{\sqrt{2}} \right|^2$$

Gs samples from Gaussian distributions

$$\mathcal{N}_{i,j}^{\alpha\beta} = \left| \frac{G_3\left(0, \sqrt{\Omega_{\text{noise}}^{\alpha\beta}(f_j)}\right) + iG_4\left(0, \sqrt{\Omega_{\text{noise}}^{\alpha\beta}(f_j)}\right)}{\sqrt{2}} \right|^2$$

➤ For each frequency f_j , we obtain the mean across time chunks for each channel

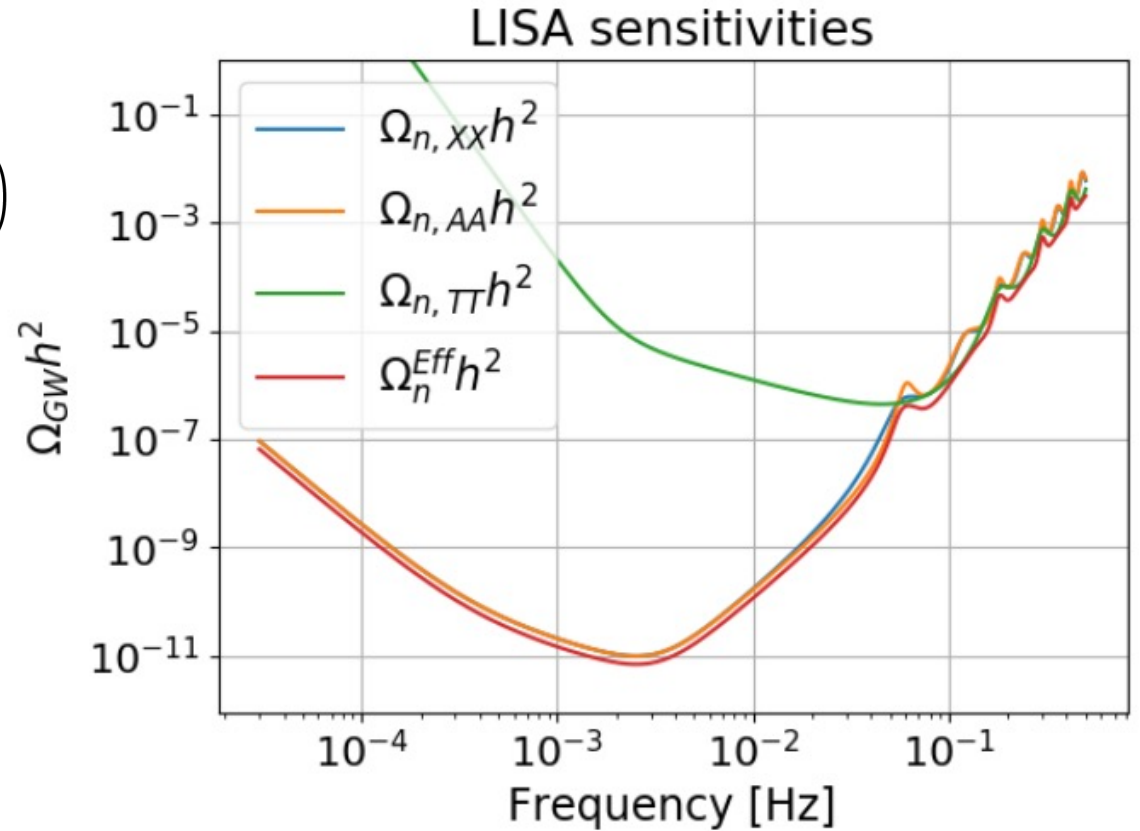
$$\bar{D}_j^{\alpha\beta} = \frac{1}{N_c} \sum_{i=1}^{N_c} D_{i,j}^{\alpha\beta}$$

- **Noise power spectra**

➤ We model the LISA noise spectra in the A, E, and T channel basis,

$$\Omega_{noise}^{\alpha\beta}(f, A_P, A_{acc}) \left\{ \begin{array}{l} \Omega_{noise}^{AA}(f, A_P, A_{acc}) = \Omega_{noise}^{EE}(f, A_P, A_{acc}) \\ \Omega_{noise}^{TT}(f, A_P, A_{acc}) \end{array} \right.$$

[Flauger et al. 2009.11845]



- **GW power spectrum**

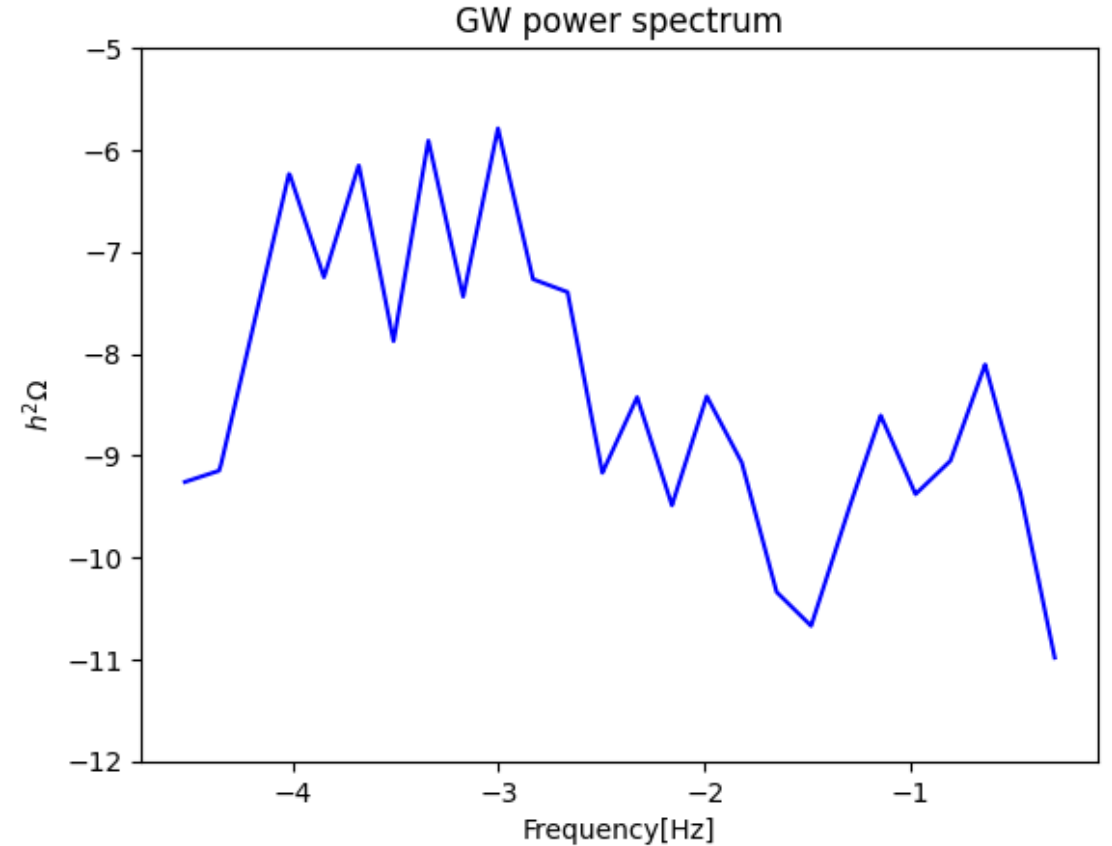
- We define a GW signal with an unknown frequency profile by approximating it **in a piece-wise manner as power-laws** in 27 intervals (27 bins).

$$\Omega_{\text{GW}}(f; \mathbf{s}) = \Omega_p^{(*)} \cdot \begin{cases} \vdots \\ F_{-2}(f) & \text{if } f_{-2} \leq f < f_1 \\ F_{-1}(f) & \text{if } f_{-1} \leq f < f_0 \\ F_0(f) & \text{if } f_0 \leq f < f_1 \\ F_{+1}(f) & \text{if } f_1 \leq f < f_2 \\ F_{+2}(f) & \text{if } f_2 \leq f < f_3 \\ \vdots \end{cases} \quad \text{with}$$

$$F_0(f) \equiv \left(\frac{f}{f_0}\right)^{\gamma_0},$$

$$F_{-1}(f) \equiv F_0(f_0) \left(\frac{f}{f_0}\right)^{\gamma_{-1}}, \quad F_{-2}(f) \equiv F_{-1}(f_{-1}) \left(\frac{f}{f_{-1}}\right)^{\gamma_{-2}}, \quad \dots$$

$$F_{+1}(f) \equiv F_0(f_1) \left(\frac{f}{f_1}\right)^{\gamma_1}, \quad F_{+2}(f) \equiv F_{+1}(f_2) \left(\frac{f}{f_2}\right)^{\gamma_2}, \quad \dots$$



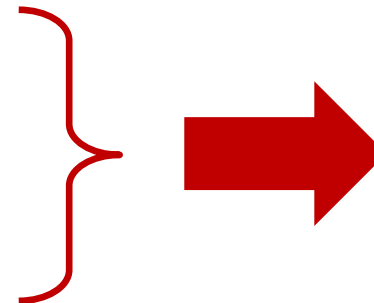
1 Amplitude: $\Omega_p^{(*)}$ and 27 slopes: $\dots, \gamma_{-1}, \gamma_0, \gamma_1, \dots$

➤ Given a dataset $\mathbf{D}_0 = \{D_0^{\text{AA}}, D_0^{\text{EE}}, D_0^{\text{TT}}\}$ we want to reconstruct and plot,

$$\Omega_{\text{GW}}(f; \mathbf{s}) = \Omega_p^{(*)} \cdot \begin{cases} \vdots \\ F_{-2}(f) & \text{if } f_{-2} \leq f < f_1 \\ F_{-1}(f) & \text{if } f_{-1} \leq f < f_0 \\ F_0(f) & \text{if } f_0 \leq f < f_1 \\ F_{+1}(f) & \text{if } f_1 \leq f < f_2 \\ F_{+2}(f) & \text{if } f_2 \leq f < f_3 \\ \vdots \end{cases} \quad \text{with} \quad \begin{cases} F_0(f) \equiv \left(\frac{f}{f_0}\right)^{\gamma_0}, \\ F_{-1}(f) \equiv F_0(f_0) \left(\frac{f}{f_0}\right)^{\gamma_{-1}}, & F_{-2}(f) \equiv F_{-1}(f_{-1}) \left(\frac{f}{f_{-1}}\right)^{\gamma_{-2}}, \dots \\ F_{+1}(f) \equiv F_0(f_1) \left(\frac{f}{f_1}\right)^{\gamma_1}, & F_{+2}(f) \equiv F_{+1}(f_2) \left(\frac{f}{f_2}\right)^{\gamma_2}, \dots \end{cases}$$

i.e., we need to sample from $p(\Omega_p^{(*)}, \dots, \gamma_{-1}, \gamma_0, \gamma_1, \dots, A_p, A_{\text{acc}} | \mathbf{D}_0)$

- **30 parameters of interest:** 2 noise + 28 GW parameters
- **thousands of nuisance parameters:** G's
- **non-gaussian likelihood**

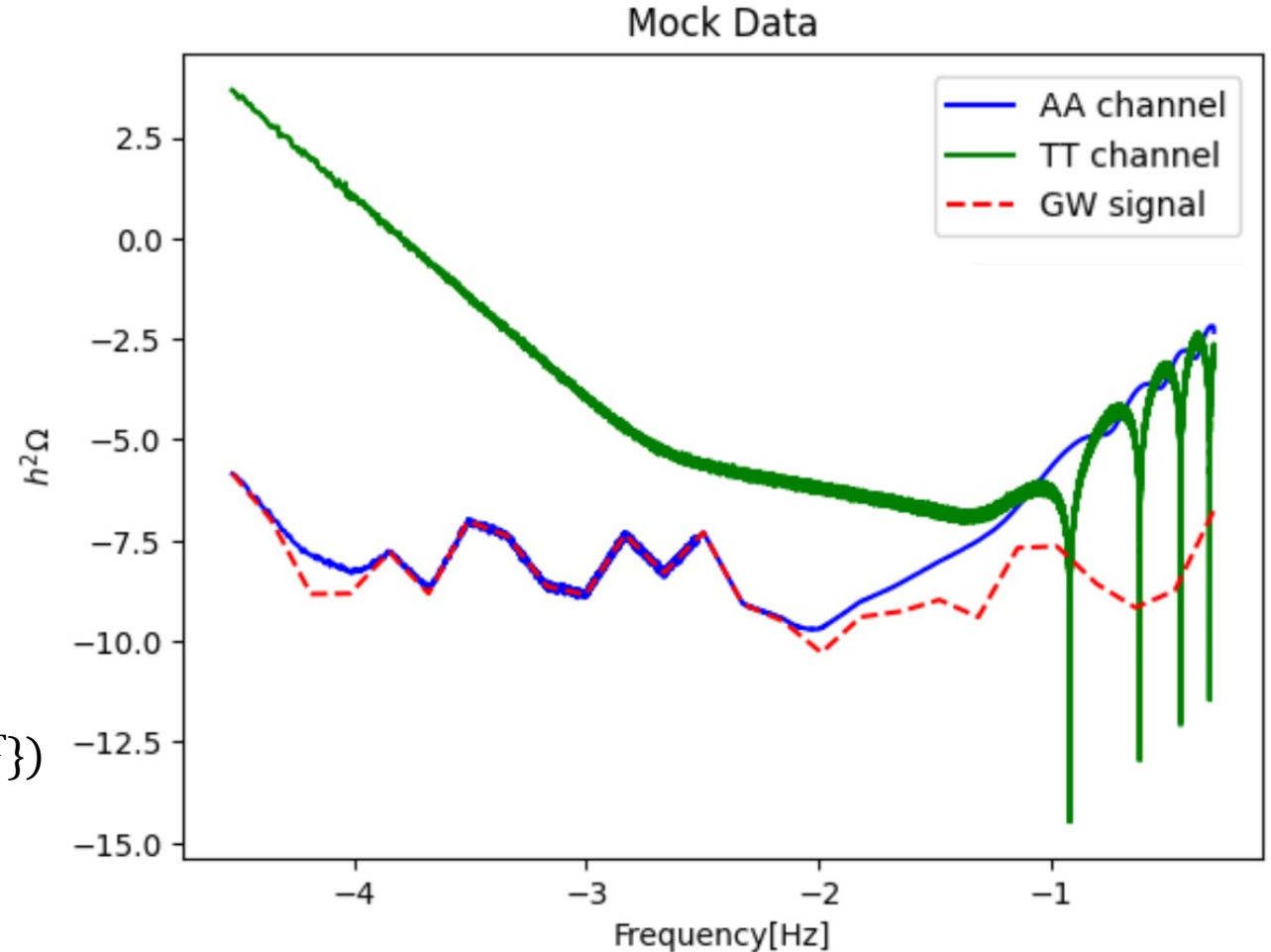


**GWBackFinder
with NPE**

GWBackFinder pipeline

- Generate training data in the 3 channels by varying the parameters of interest ($\mathcal{O}(10^5)$ training data)
- Train a normalizing flow to approximate **instantly** the posterior $p(\Omega_p(*), \dots, \gamma_{-1}, \gamma_0, \gamma_1, \dots, A_P, A_{acc} | \{D^{AA}, D^{EE}, D^{TT}\})$ of every possible observation $\{D^{AA}, D^{EE}, D^{TT}\}$.

Amortized!



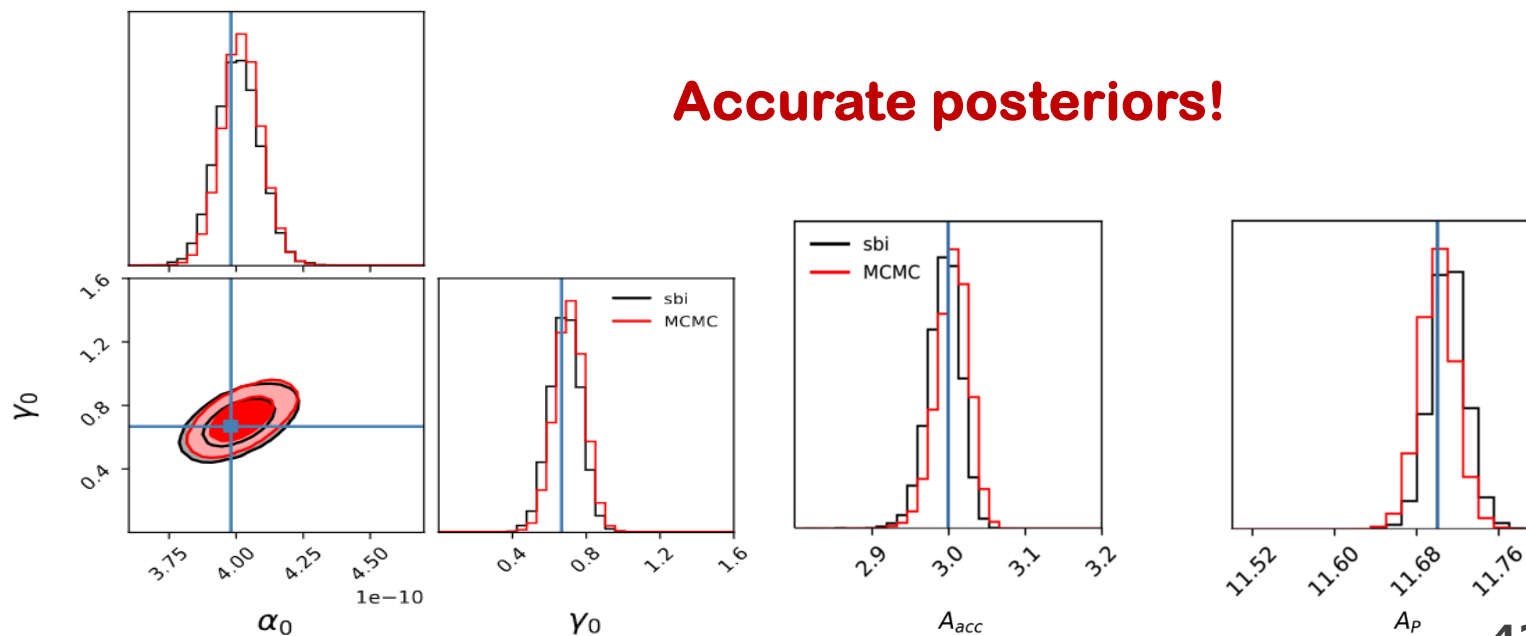
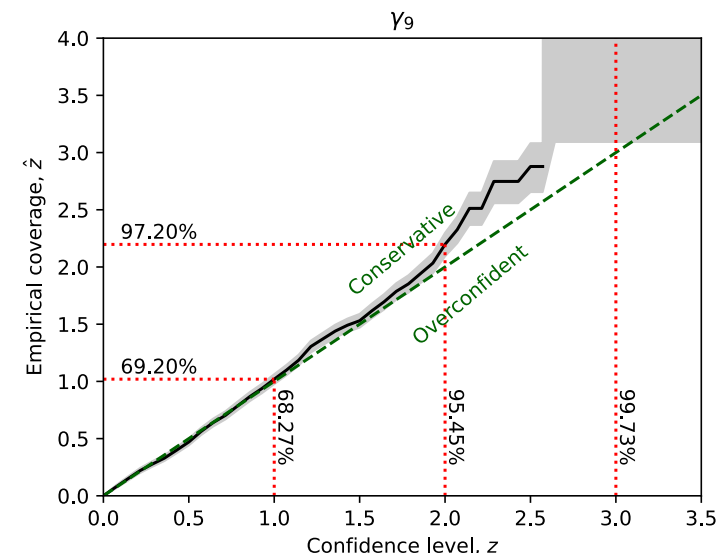
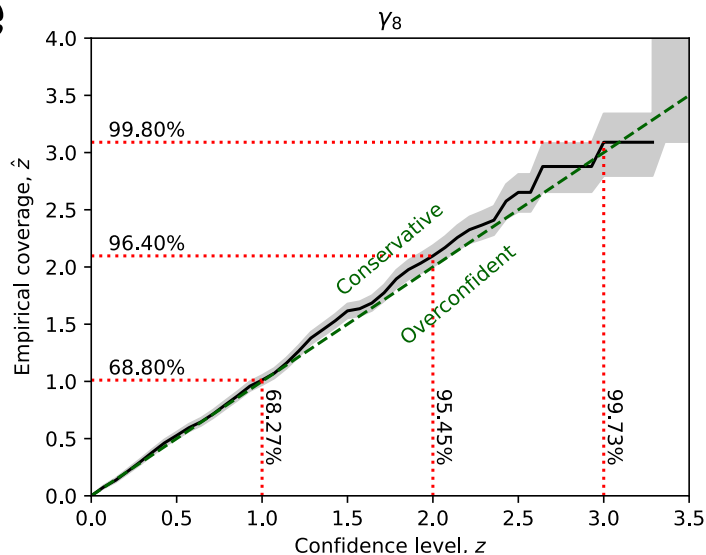
GWBackFinder pipeline

- Test coverage on mock data
- Compare analysis with MCMC for mock data using a non-gaussian likelihood:

$$\ln \mathcal{L} = \frac{1}{3} \ln \mathcal{L}_G + \frac{2}{3} \ln \mathcal{L}_{LN}$$

$\mathcal{O}(10^5)$ likelihood evaluations to obtain the posterior of only one observation

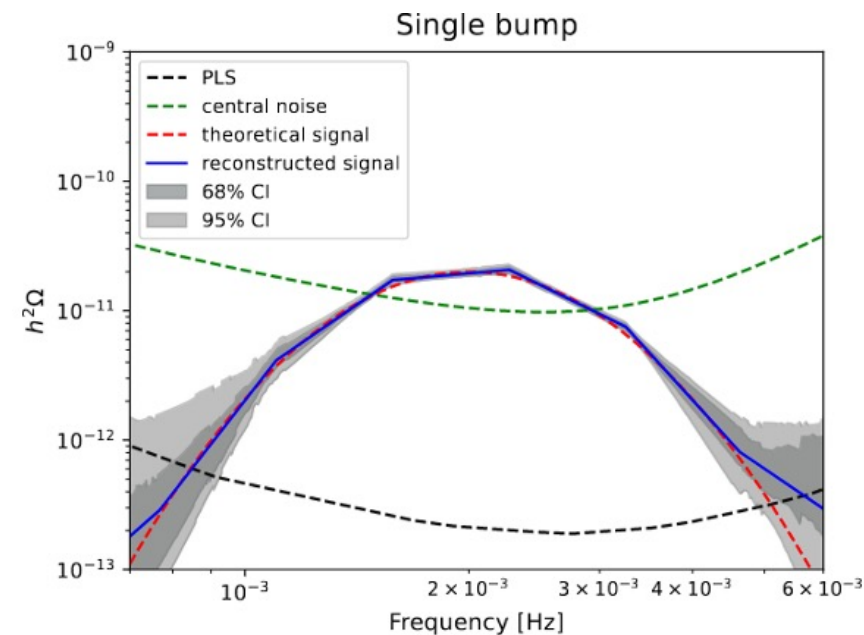
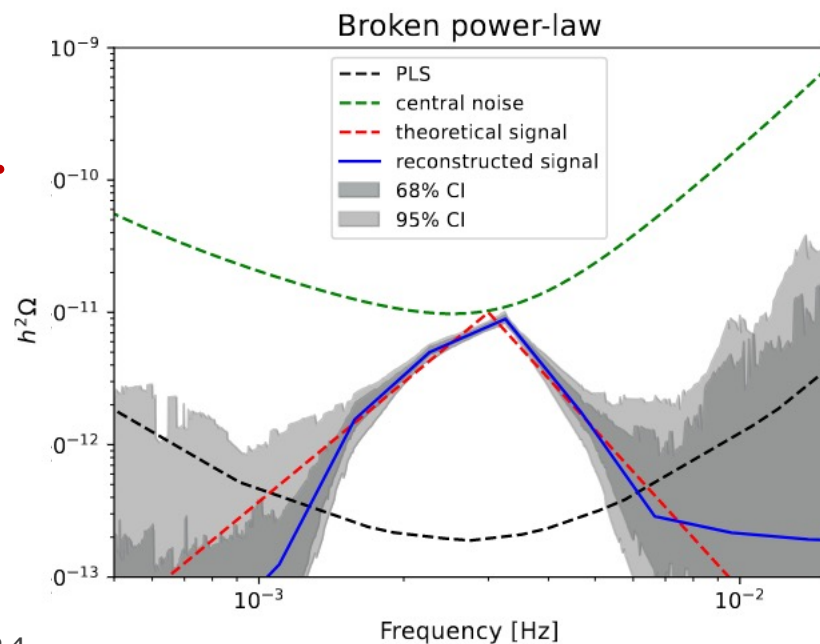
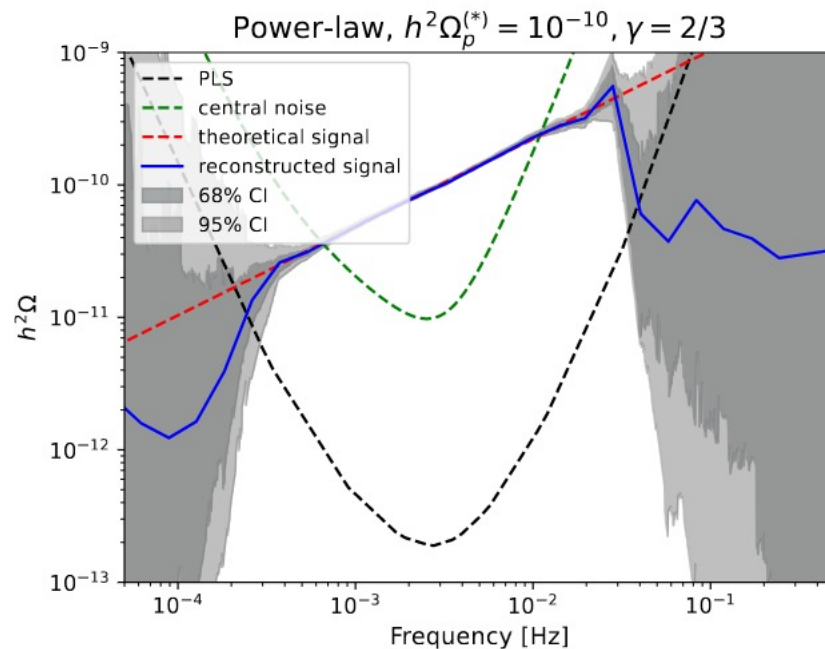
slow & NOT Amortized !



GWBackFinder pipeline

- Reconstruct **template** signals, e.g.,
 - single power-law
 - broken power-law
 - single bump signal

Accurate & fast reconstruction!



GWBackFinder pipeline

- Reconstruct **template** signals in the presence of **foregrounds**:

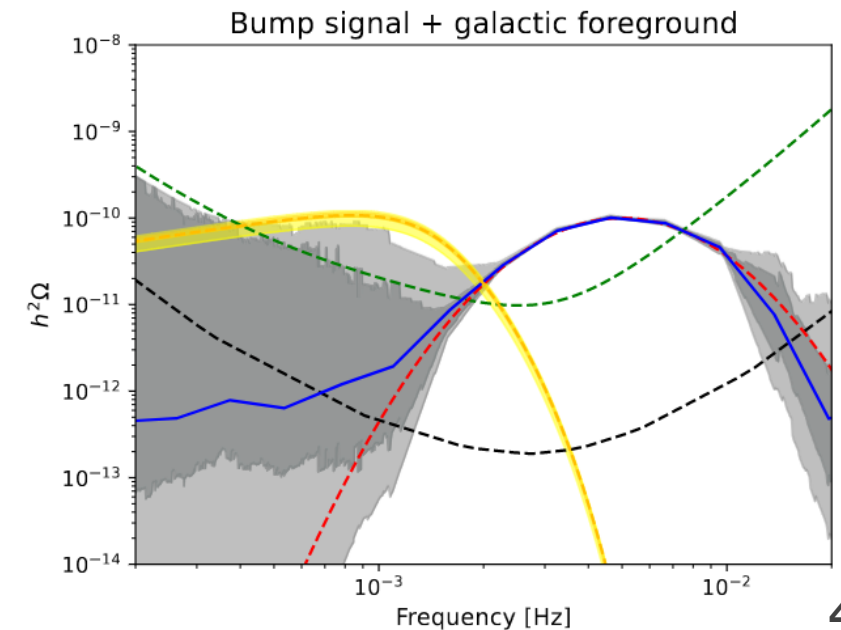
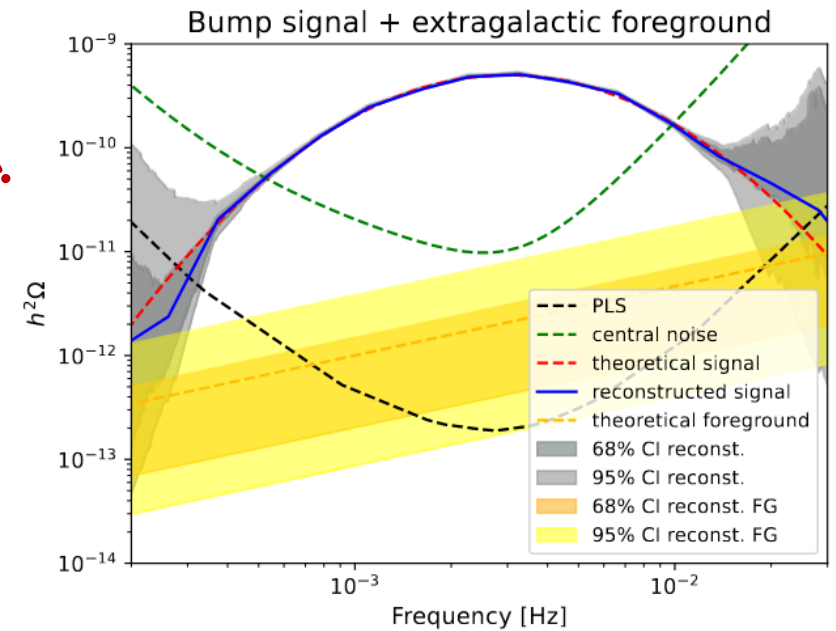
➤ Extragalactic:

$$\Omega_{\text{EF}}(f) = 10^{\alpha_{\text{EF}}} \left(\frac{f}{0.001 \text{ Hz}} \right)^{2/3}$$

➤ Galactic:

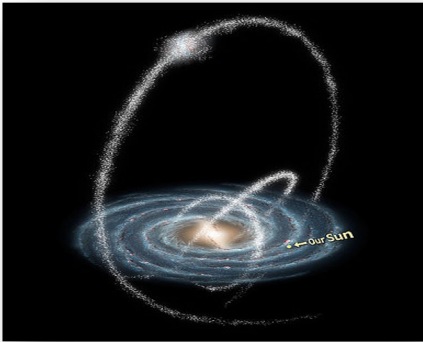
$$\Omega_{\text{GF}}(f) = 10^{\alpha_{\text{GF}}} f^{2/3} \exp\left(-f^{a_1} - a_2 f \sin(a_3 f)\right) \cdot \{1 + \tanh[a_4(f_k - f)]\}$$

Accurate & fast reconstruction!



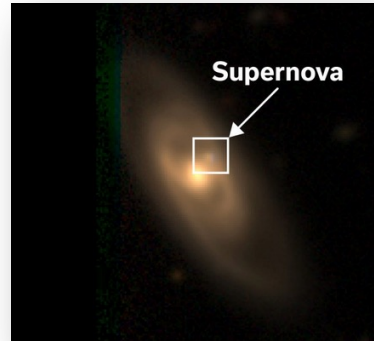
SBI appears to be broadly applicable

Stellar streams



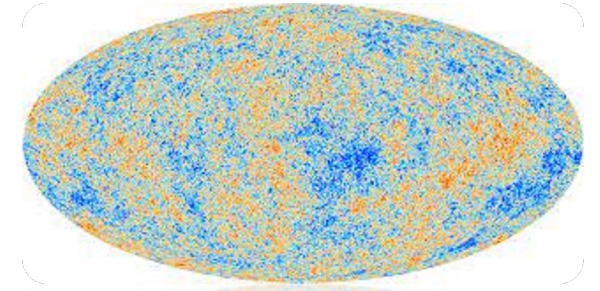
arxiv: 1911.01429

Supernova cosmology



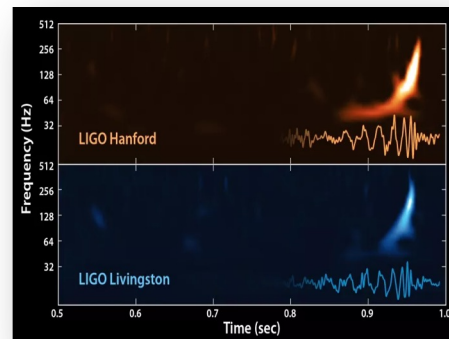
arxiv: 2209.06733

CMB physics



arxiv: 2111.08030

Gravitational waves



arxiv: 2308.06318

Strong Lensing



arxiv: 2211.04365

Conclusions

- Large forward models are omnipresent in physics/astronomy and pose challenges for analysing current and future data.
- Deep learning opens many new powerful ways for tackling these inference problems.
- There is a large variety of different methods and algorithms, both simulation- and likelihood-based, which differ in applicability, scaling and requirements for the forward model.
- There are many open research questions, both in physics/ astronomy and computer science.

Thank you!