Experimental verification of integrability in a Danilov-Nagaitsev lattice using machine learning

Applications of AI/ML in accelerators

Present uses of AI/ML methods in particle accelerators:

Surrogate models, optimization, control, virtual diagnostics, and anomaly detection.

These aim to improve design and operations based on well-known physics.

Can AI/ML be used for discovering new physics? High-energy and astrophysics fields are making progress.

Institute for Artificial Intelligence and Fundamental Interactions (IAIFI), https://aiinstitutes.org/institute-iaifi/

Synergies with Non-linear Integrable Optics

PHYSICAL REVIEW LETTERS **126,** 180604 (2021)

Editors' Suggestion

Machine Learning Conservation Laws from Trajectories

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We present AI Poincaré, a machine learning algorithm for autodiscovering conserved quantities using trajectory data from unknown dynamical systems. We test it on five Hamiltonian systems, including the gravitational three-body problem, and find that it discovers not only all exactly conserved quantities, but also periodic orbits, phase transitions, and breakdown timescales for approximate conservation laws.

DOI: 10.1103/PhysRevLett.126.180604

Many more published after this one…

Possible applications in accelerators:

1. Analyze integrability in realistic accelerator systems, in the presence of non-ideal effects such as noise, parasitic non-linearities, decoherence, etc.
2. Maximize dynamic aperture by using integrability as an analogous metric.
3. Discover new NIO systems.

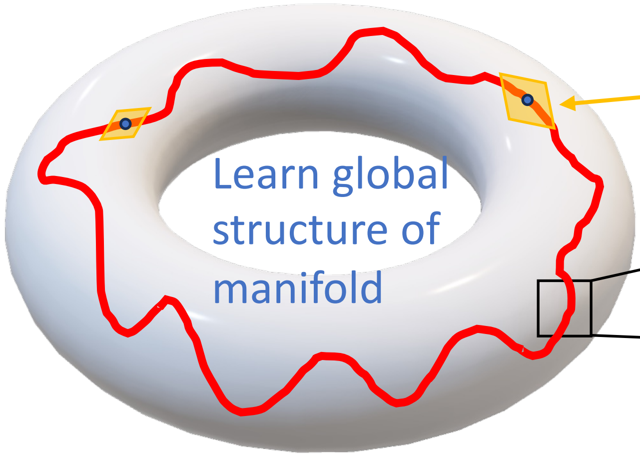
Auto-discovering invariants using AI Poincaré

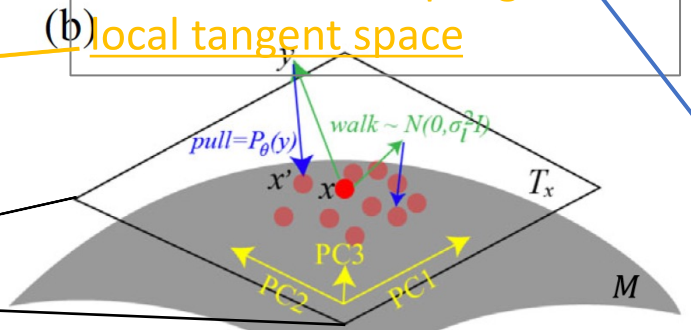
Intuition: Total number of phase-space dimensions = Number of invariants + Dimensionality of manifold

Scale, Center, filter out linear invariants using Principal Component Analysis (PCA)

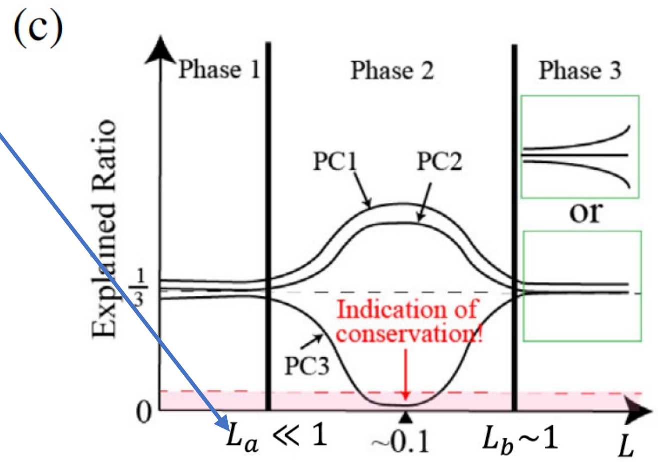
Train pull network (Neural Empirical Bayes) with length scale L, Monte Carlo sampling of local tangent space

Dimensionality determination using Principal Component Analysis (PCA)





Sample local hyperplane of dimension N_{dim} - N_{inv}

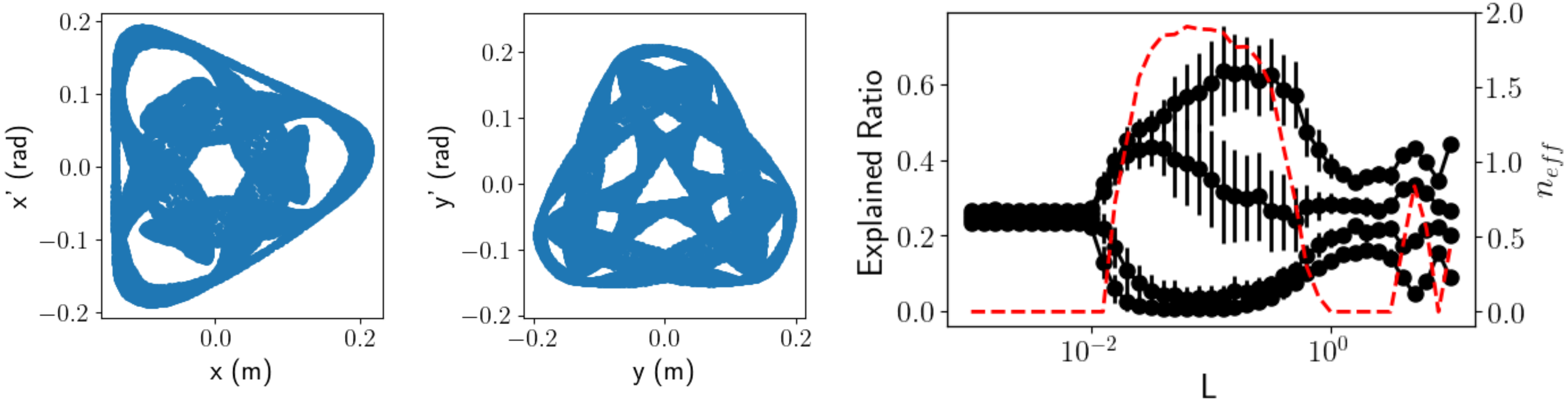


S. Saremi and A. Hyvarinen, J. Mach. Learn. Res. 20, 181:1, 2019

Ziming Liu and Max Tegmark, Phys. Rev. Lett. 126, 180604, 2021

Performance with data from simulations

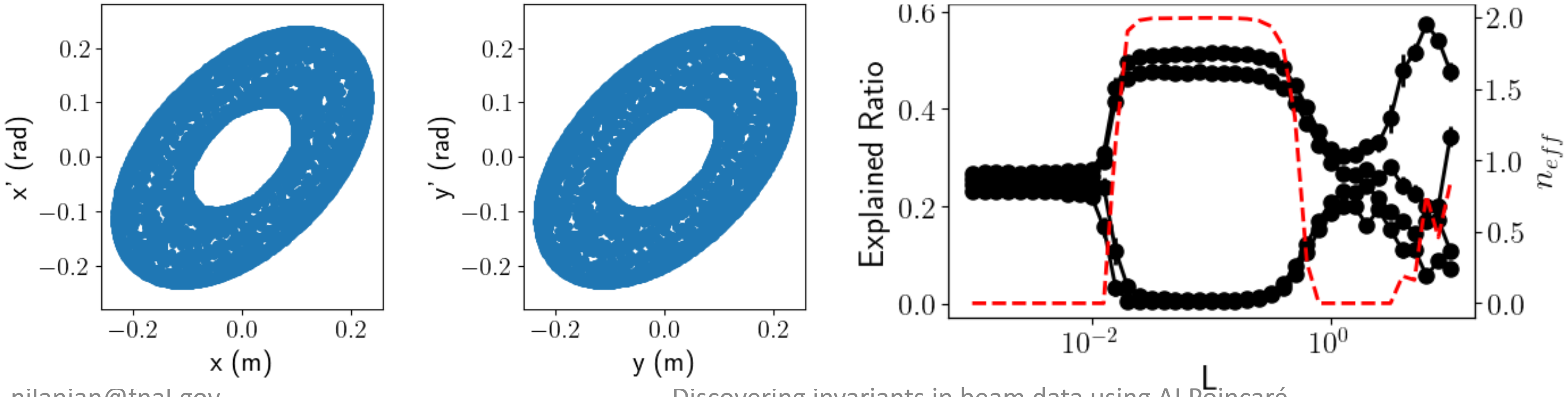
AI Poincaré can detect invariant quantities in accelerator optics.



Linear lattice with thin sextupole kick.

max{n_{eff}} ~ 1.85

Local dimensionality is 2 in certain locations of the manifold.



Linear lattice with thin McMillan kick.

max{n_{eff}} ~ 1.99

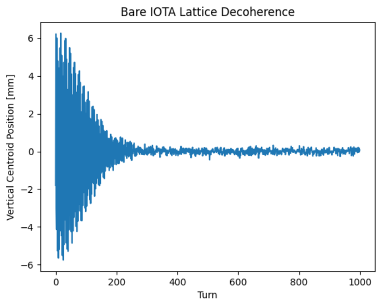
Dimensionality is 2 throughout the manifold.

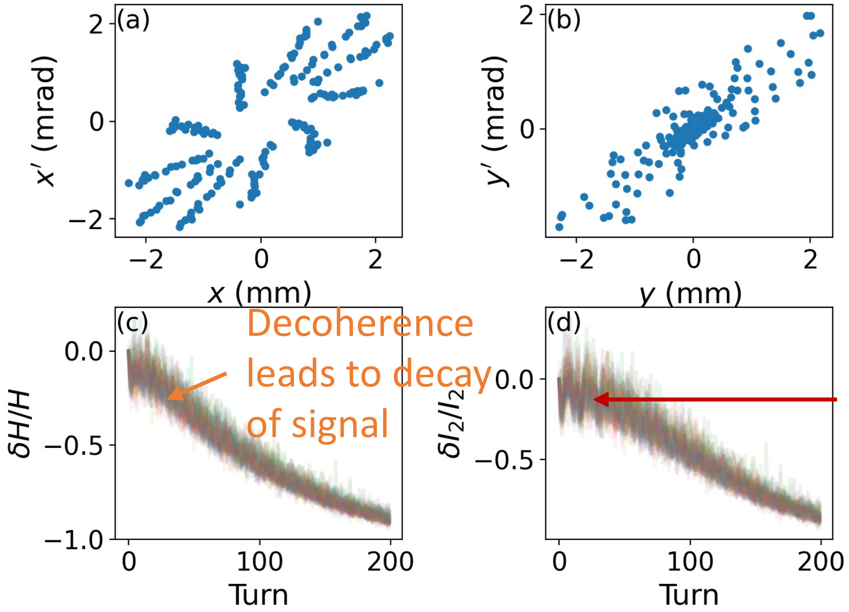
Experimental data from the DN system

Procedure

- Inject 150 MeV electrons into IOTA and capture into 1 bucket. (1 bunch)
- Let the electron beam orbit reach equilibrium.
- Apply transverse momentum kicks using electrostatic kickers.
- Measure turn-by-turn **centroid position data** on many BPMs distributed around the ring.
- Use PCA to compute turn-by-turn phase-space trajectory (x, x', y, y') at a virtual BPM.

Decoherence of centroid position due to tune spread





Fast oscillations indicate violation of invariance

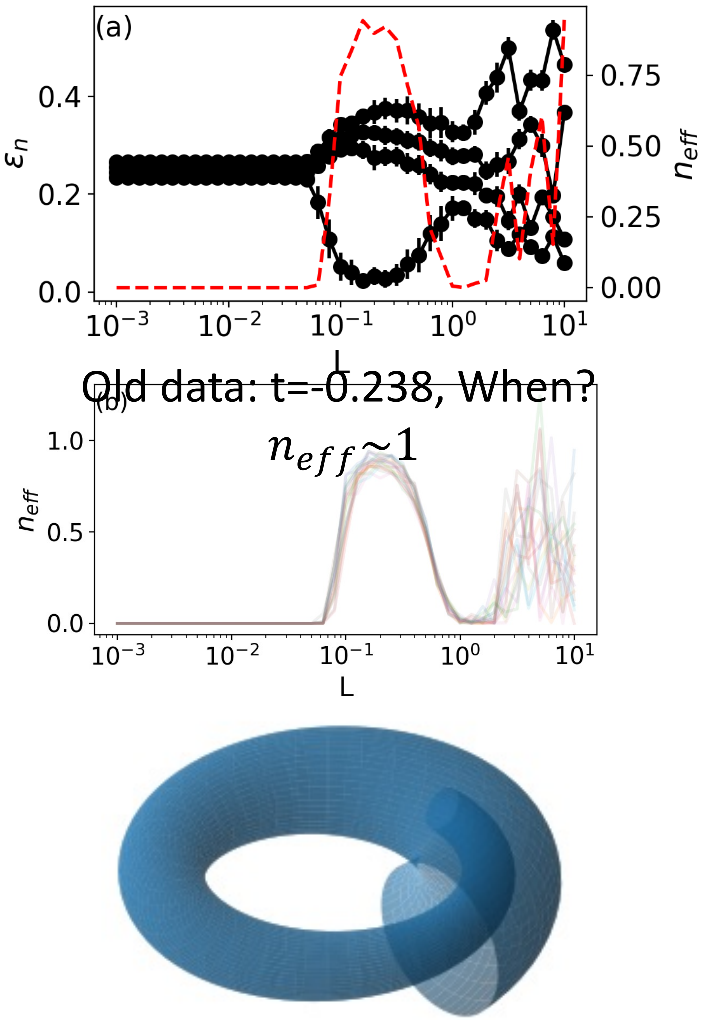
New data: t=-0.238, 10/23/23.
66 datasets with same kick amplitude.

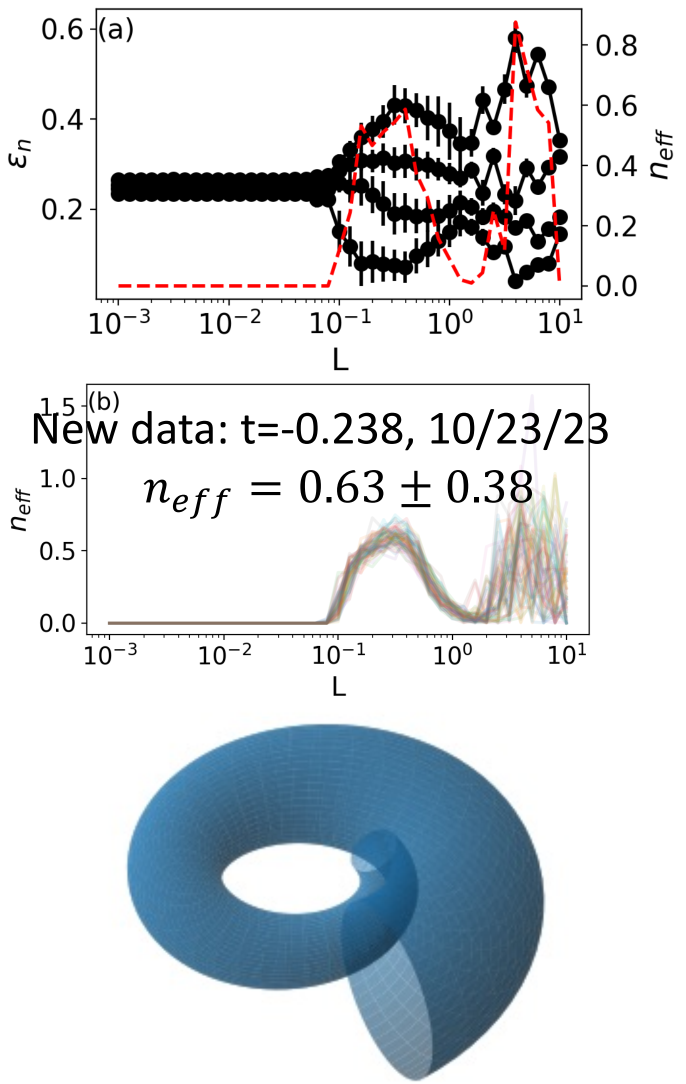
Applying AI Poincaré to experimental data

Applying the algorithm to data from all 200 turns.

Results depend on dataset.

Manifold traced by beam centroid is not a perfect torus due to decoherence.





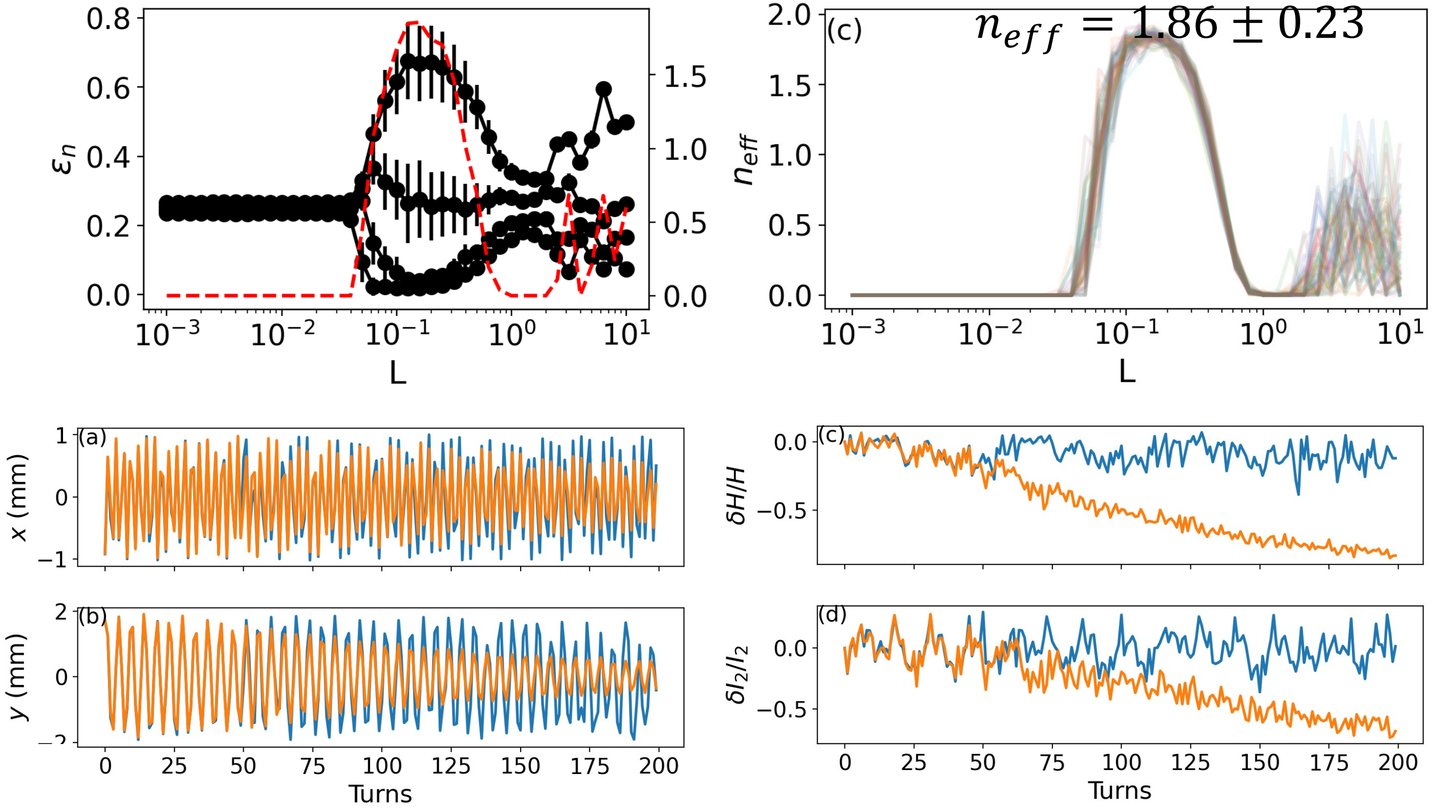
Applying AI Poincaré to first 50 turns

Applying the algorithm to the first 50 turns, yields two invariants.

We verify that the manifold learnt by the pull network in AI Poincaré is of the correct structure by asking it to pull the experimental data back to the manifold and computing the theoretical invariants.

New data: t=-0.238, 10/23/23

$n_{eff} = 1.86 \pm 0.23$



The inferred manifold approximately conserves both theoretical invariants.

Training a neural network to model the invariants for a fixed DN magnet strength

Using a modified version of AI Poincaré 2.0.

Ziming Liu, Varun Madhavan, and Max Tegmark, Phys. Rev. E 106, 045307, 2022.

Constant value on the manifold

Gradients of invariants must be orthogonal to tangent (flow) vectors. [Used in AI Poincaré 2.0]

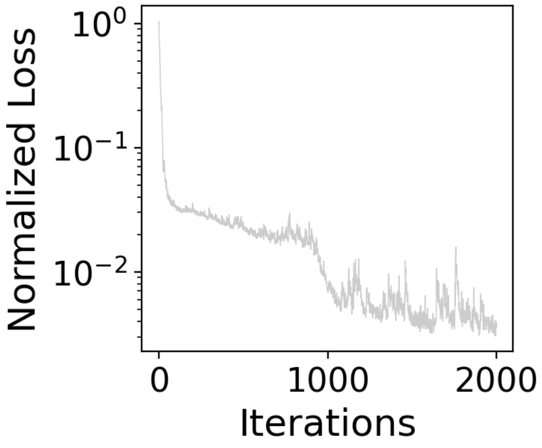
$$L(\theta) = \sum_{p=1}^{P} \left\{ \frac{\alpha_1}{M} \sum_{i=1}^{M} \left| U_i(\vec{Z}_p) - \bar{U}_i \right|^2 + \frac{\alpha_2}{M(N-M)} \sum_{i=1,k=0}^{i=M,k=N-M} \left| \hat{\nabla} U_i(\vec{Z}_p) \cdot \hat{t}_k(\vec{Z}_p) \right|^2 \right\} + \sum_{q=1}^{Q} \left\{ \frac{2\alpha_3}{M(M-1)} \sum_{i=1,j=2}^{i=M-1,j=M} \left| \hat{\nabla} U_i(\vec{Z}_q) \cdot \hat{\nabla} U_j(\vec{Z}_q) \right|^2 + \alpha_4 \left\| \hat{\nabla} U_1(\vec{Z}_q) - \hat{\nabla} J(\vec{Z}_q) \right\|_2^2 \right\} + \frac{\alpha_5}{M} \sum_{i=1}^{M} \left| U_i(\vec{O}) \right|^2$$

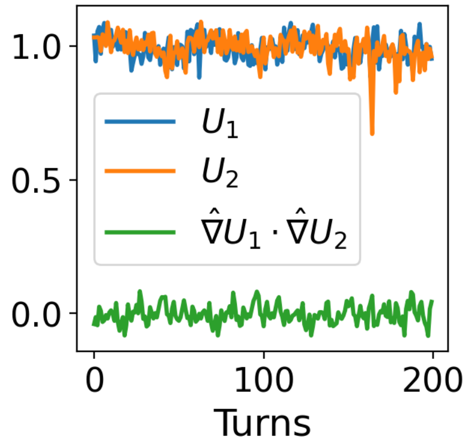
Gradients of invariants must be orthogonal – Independence requirement [Used in AI Poincaré 2.0]

First invariant similar to the Courant-Snyder invariant.

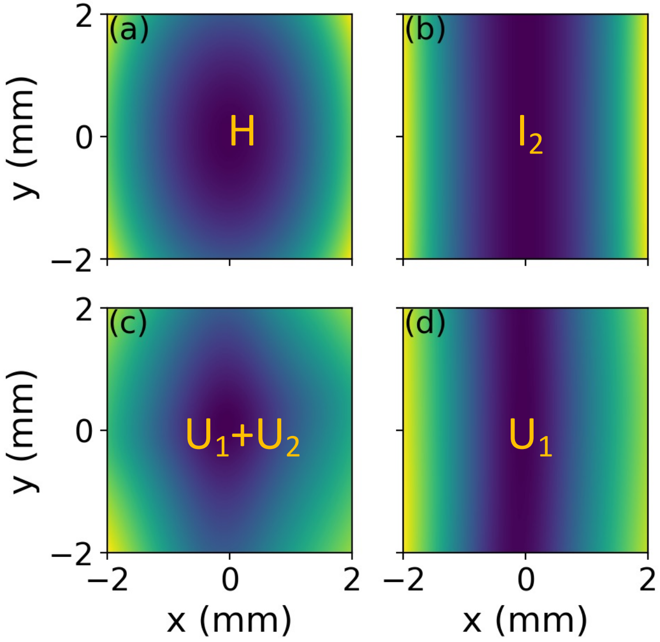
0 at origin

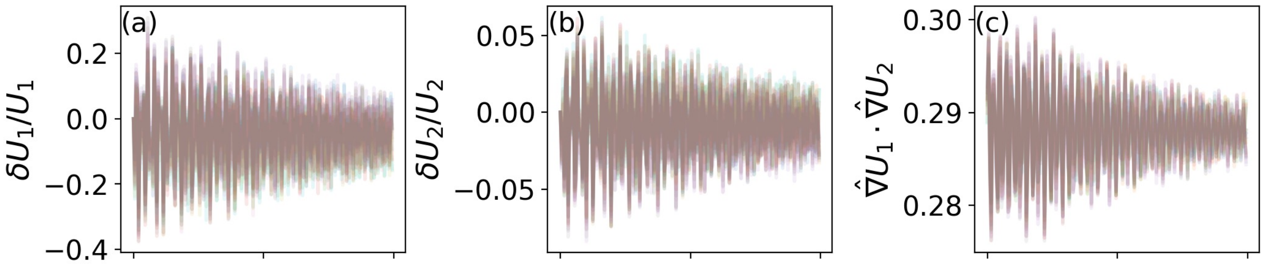
An example solution



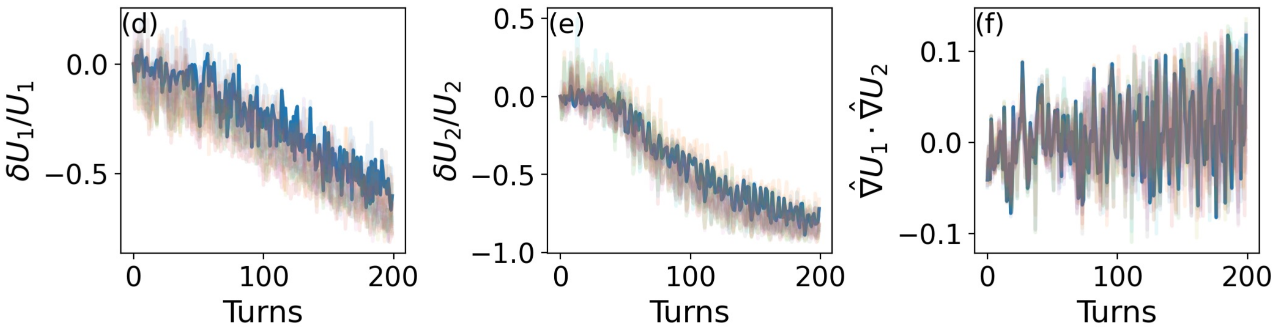


The model converges to approximately the same solution for multiple training runs.





Old data: t=-0.238, When?

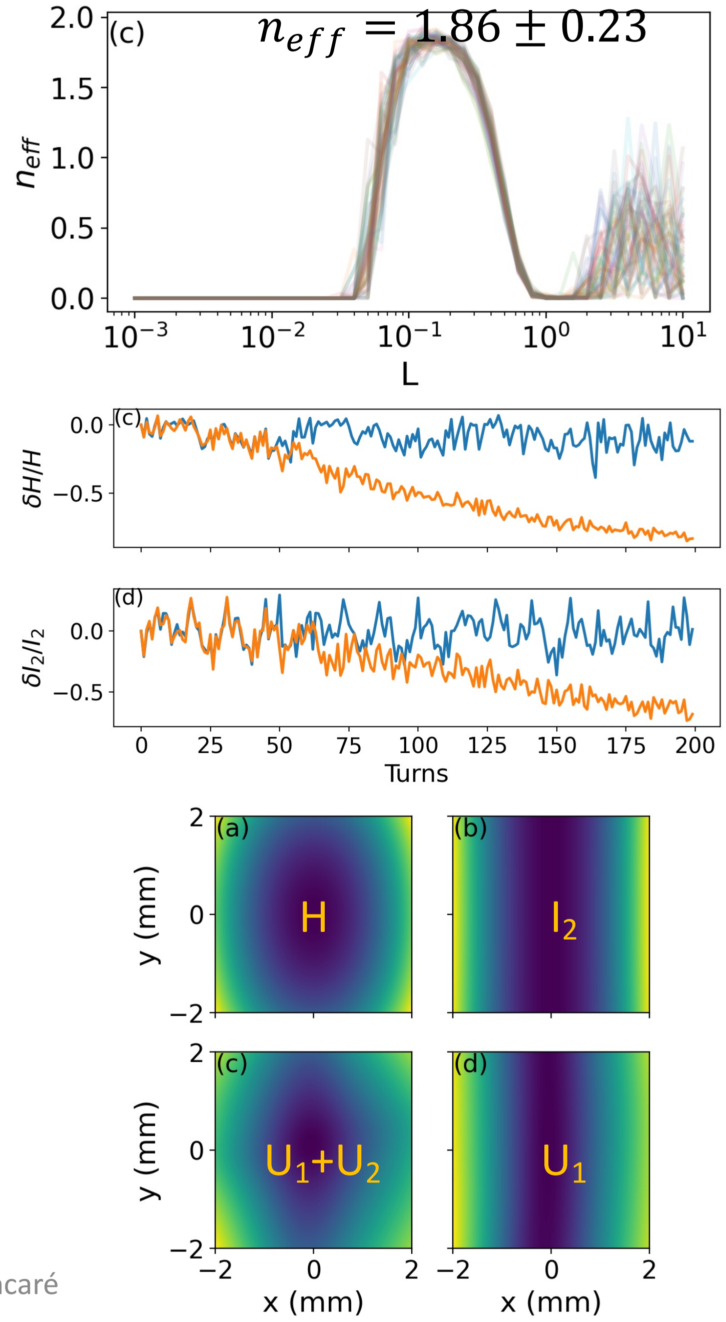


Does not work uniformly on new data sets!

Conclusion

- AI Poincaré can verify the presence of invariants in experimental data.
- The structure of the manifold inferred from the data is consistent with theory.
- We can reconstruct approximate invariant functions from the data, but the process doesn't work on all datasets and only qualitative agreement can be seen within the sampled domain.

New data: t=-0.238, 10/23/23



Next Steps

- Finish writing proceeding and draft poster.
- Determine which datasets are more amenable to this analysis and why?
- Develop statistics and quantitative predictions.
- Generalize the invariant network to multiple DN magnet strengths.
- Consider publication?