

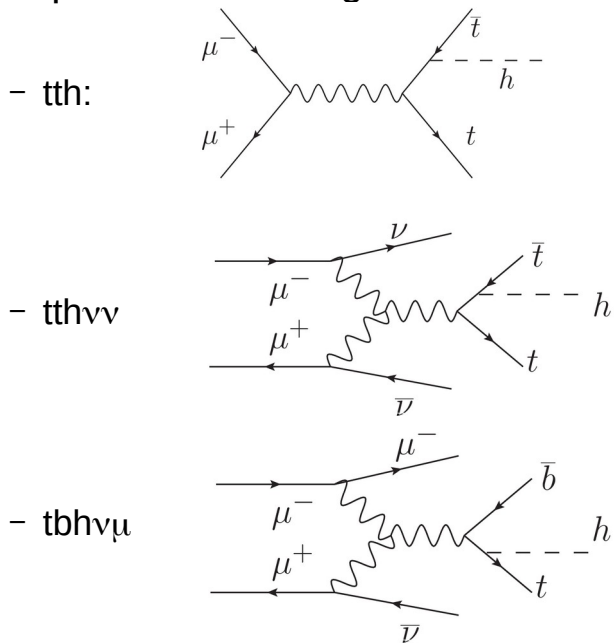
# CP Violating Top Yukawa Coupling at the Future Muon Collider

Ian Lewis  
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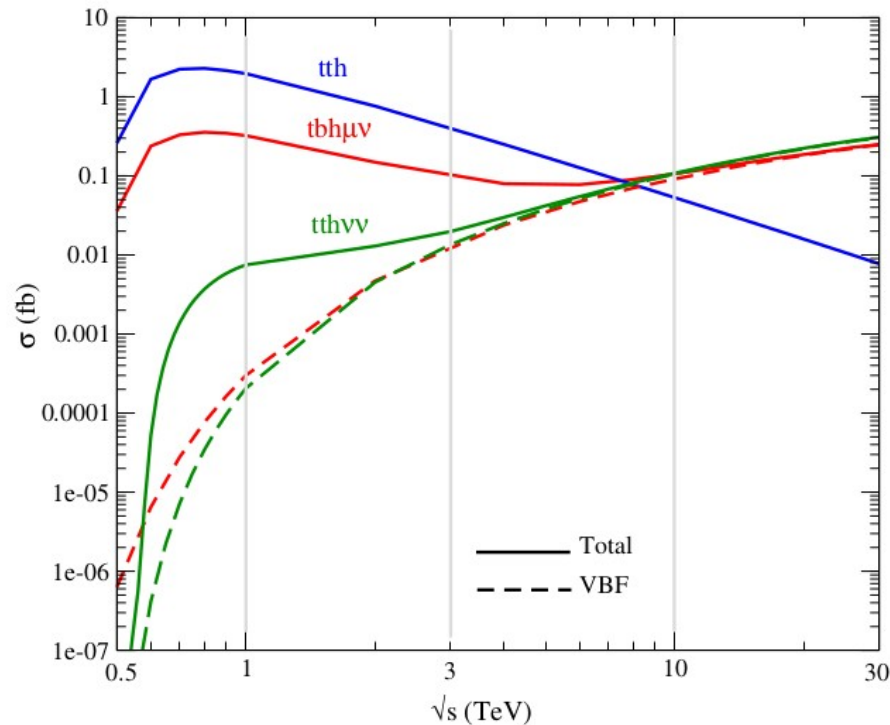
Morgan Cassidy, Cosmos Dong, KC Kong,  
Jenny Zhang, Yajuan Zheng  
arXiv:2311.07645 [hep-ph]  
JHEP 05 (2024) 176

# Direct Probes of Top Yukawa

- Directly probe top Yukawa, need processes with a Higgs and top in final state.
- Representative diagrams:



- Dotted lines are VBF-like diagrams.



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# CP violating top quark Yukawa

- The typical angular parameterization is not gauge invariant, can cause spurious growth with energy.

$$\kappa_{htt} \frac{m_t}{v} h \bar{t} (\cos \xi + i \gamma_5 \sin \xi) t$$

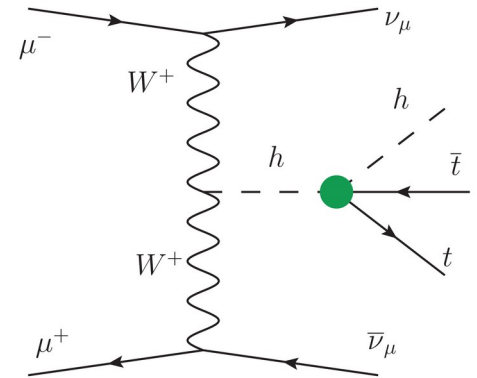
- Use SMEFT to parameterize shift in a gauge invariant way:

$$\mathcal{L}_{htt, \text{SMEFT}} = -y_t \bar{Q}_L \tilde{\Phi} t_R - c_t \Phi^\dagger \Phi \bar{Q}_L \tilde{\Phi} t_R + \text{h.c.}$$

$$\begin{aligned} &= -m_t \bar{t} t - g_{htt} h \bar{t} (\cos \xi + i \gamma_5 \sin \xi) t \\ &\quad - \frac{3}{2} g_{htt} \frac{h^2}{v} \left( 1 + \frac{h}{3v} \right) \bar{t} \left( \cos \xi - \frac{1}{\kappa_{htt}} + i \gamma_5 \sin \xi \right) t, \end{aligned}$$

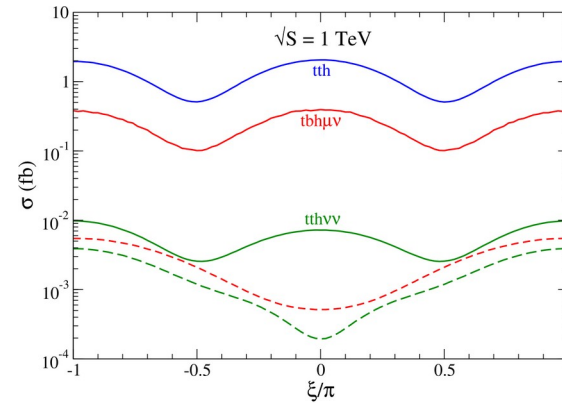
$$g_{htt} \equiv (m_t/v) \kappa_{htt}$$

- Introduces a new 4-point interaction: t-t-h-h.

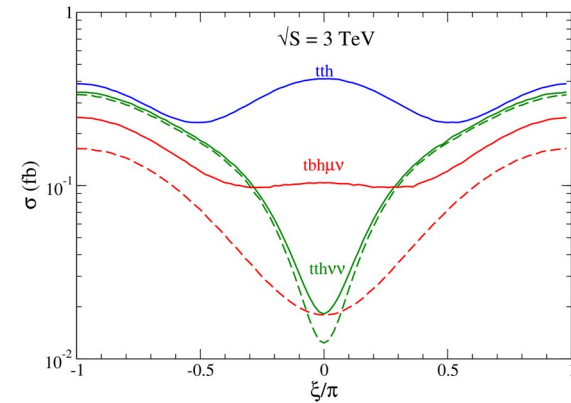


# Cross Section Dependence on Phase

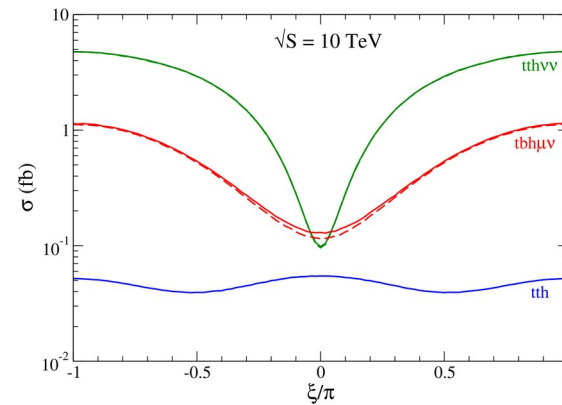
- Dashed: VBF-like diagrams.
- Very different behaviour at different energies and for different diagram types.
  - S-channel type diagrams peak at the SM value
  - VBF style diagrams are at a minimum at the SM value.
- VBF style diagrams strongly dependent on CP violating phase at high energies.
- Can parameterize cross sections in terms of the CP-violating phase.
  - Cross sections are CP-even.
  - Can only depend on powers of  $\cos \theta$



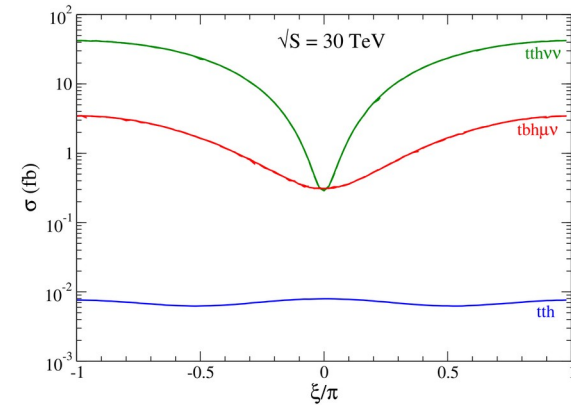
(a)



(b)



(c)



(d)

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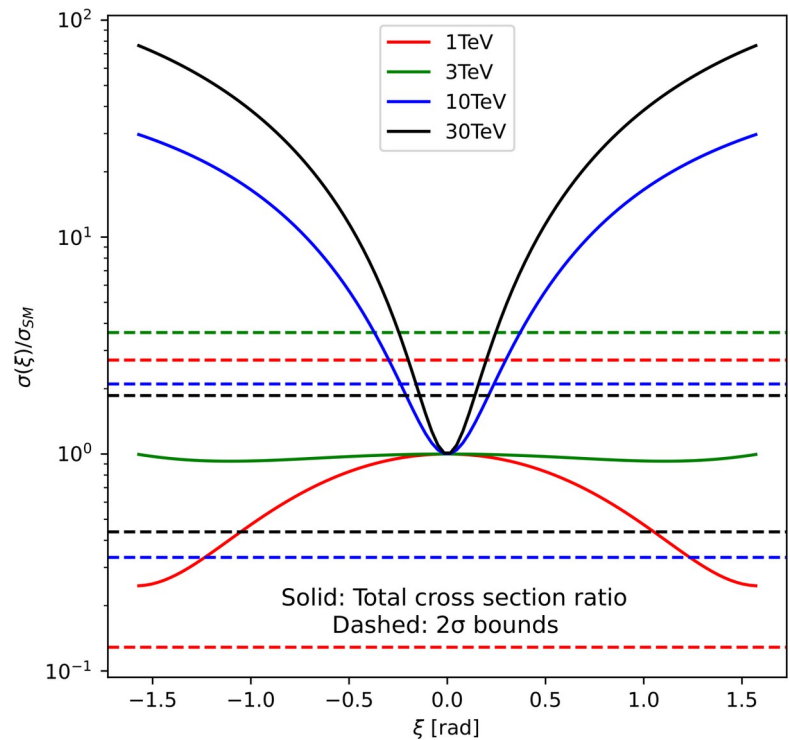
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US Muon Collider Meeting  
August 8, 2024

# $\xi$ bounds

- Benchmark luminosities:
  - 1 TeV:  $100 \text{ fb}^{-1}$
  - 3 TeV:  $1 \text{ ab}^{-1}$
  - 10 TeV:  $10 \text{ ab}^{-1}$
  - 30 TeV:  $10 \text{ ab}^{-1}$
- Signals added together and 90% b-tagging rate applied.
  - Sharp dependence on  $\xi$  at 10 TeV and 30 TeV provides strong constraints.
- Precision on top Yukawa in  $t\bar{t}H$  comparable to previous results.

[Forslund, Meade, arXiv:2203.09425](#)



Projected $1\sigma$ ( $2\sigma$ ) bounds on $ \xi $			
$\varepsilon_b$	$\sqrt{s} = 1 \text{ TeV}$	$\sqrt{s} = 10 \text{ TeV}$	$\sqrt{s} = 30 \text{ TeV}$
0.6	*	$ \xi  < 15^\circ$ ( $23^\circ$ )	$ \xi  < 8.6^\circ$ ( $14^\circ$ )
0.7	*	$ \xi  < 12^\circ$ ( $19^\circ$ )	$ \xi  < 7.2^\circ$ ( $11^\circ$ )
0.8	$ \xi  < 67^\circ$ (*) or $ \xi  > 114^\circ$ (*)	$ \xi  < 10^\circ$ ( $16^\circ$ )	$ \xi  < 6.2^\circ$ ( $9.3^\circ$ )
0.9	$ \xi  < 57^\circ$ (*) or $ \xi  > 125^\circ$ (*)	$ \xi  < 9.0^\circ$ ( $14^\circ$ )	$ \xi  < 5.4^\circ$ ( $8.0^\circ$ )

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# Thank You

# $\xi$ bounds

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- Comparing different b-tagging efficiency scenarios.
- Favorably compares to other direct probes at proposed future colliders:

Bounds on $\alpha$ at 95% CL ( $\kappa_t = 1$ )	Channel	Collider	Luminosity
$ \alpha  \lesssim 36^\circ$ [1]	dileptonic $t\bar{t}(h \rightarrow b\bar{b})$	HL-LHC	$3 \text{ ab}^{-1}$
$ \alpha  \lesssim 25^\circ$ [2]	$t\bar{t}(h \rightarrow \gamma\gamma)$ combination	HL-LHC	$3 \text{ ab}^{-1}$
$ \alpha  \lesssim 3^\circ$ [1]	dileptonic $t\bar{t}(h \rightarrow b\bar{b})$	100 TeV FCC	$30 \text{ ab}^{-1}$

Barman, et al., [arXiv:2203.0817](#)

# Properties of top quark Yukawa

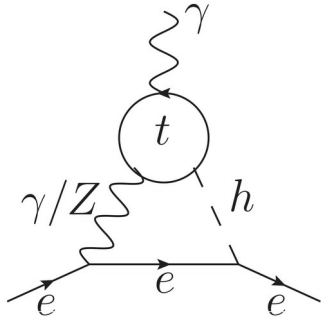
- Beyond strength of coupling, interested in properties of the Higgs boson.
  - Is it a CP eigenstate?
  - Many measurements and studies on this.
  - Measure in couplings to tau's, gauge bosons, and top quark.
  - Different couplings can have different CP properties.
- Will focus on CP properties of the top Yukawa.
  - Will parameterize the top Yukawa as:

$$\kappa_t \frac{m_t}{v} h \bar{t} (\cos \xi + i \gamma_5 \sin \xi) t$$

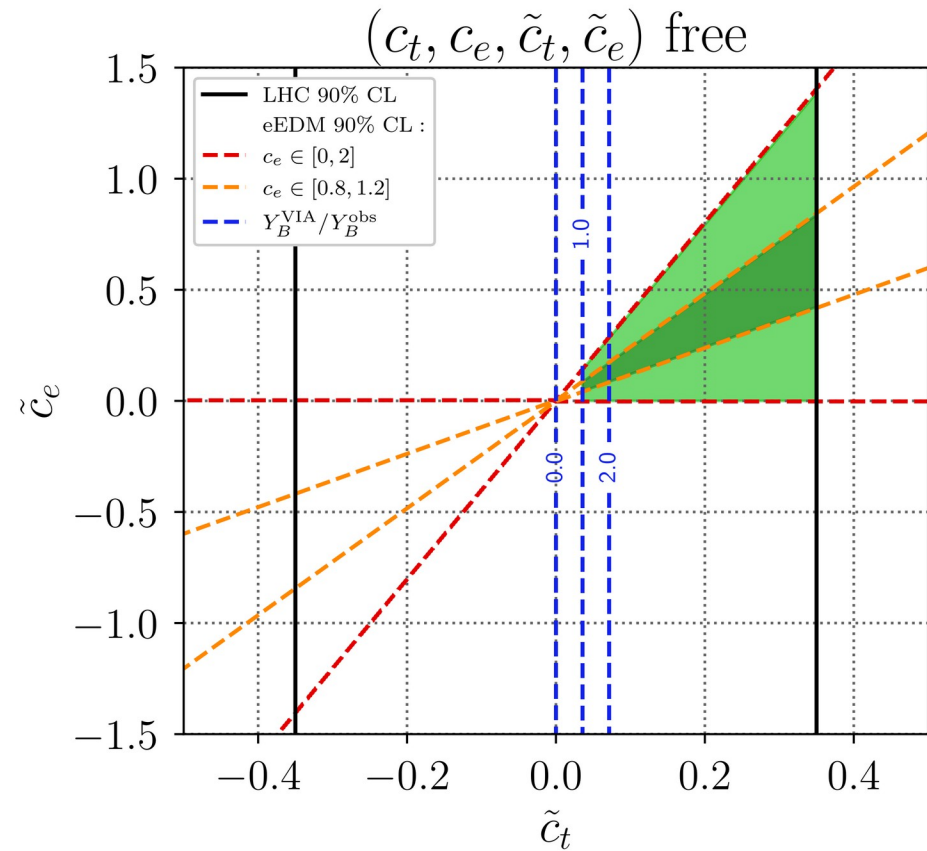
- Older constraints from ATLAS:  $|\xi| < 0.75 (43^\circ)$  [PRL 125 \(2020\) 061802](#)
- Current constraints from ATLAS:  $\xi = 11^{+52^\circ}_{-73^\circ}$ ,  $\kappa_t = 0.84^{+0.30}_{-0.46}$  [arXiv: 2303.05974\[hep-ph\]](#)
- Can be bounded very strongly by EDMs. Need direct measurements, though.



# EDM Constraints



- Very strong constraints on electron EDMs.
  - $|\xi| < 0.0014 (0.08^\circ)$  [Brod et al JHEP08 \(2022\) 294](#)
- The strong constraints depend on electron-Higgs coupling.
  - Not observed yet.
  - If we allow it to change can greatly relax constraints.
  - Hence, direct measurement of CP-violating are important
- Similar results for neutron EDMs
  - [Brod, Haisch, Zupan JHEP 11 \(2013\)](#)



[Bahl et al EPJC82 \(2022\) 604](#)

$$\mathcal{L}_{\text{yuk}} = - \sum_{f=u,d,c,s,t,b,e,\mu,\tau} \frac{y_f^{\text{SM}}}{\sqrt{2}} \bar{f} (c_f + i\gamma_5 \tilde{c}_f) f H.$$

# Direct Searches at Future colliders

- Need processes with Higgs+top in the final state to directly probe top quark Yukawa

Bounds on $\alpha$ at 95% CL ( $\kappa_t = 1$ )	Channel	Collider	Luminosity
$ \alpha  \lesssim 36^\circ$ [1]	dileptonic $t\bar{t}(h \rightarrow b\bar{b})$	HL-LHC	3 ab <sup>-1</sup>
$ \alpha  \lesssim 25^\circ$ [2]	$t\bar{t}(h \rightarrow \gamma\gamma)$ combination	HL-LHC	3 ab <sup>-1</sup>
$ \alpha  \lesssim 3^\circ$ [1]	dileptonic $t\bar{t}(h \rightarrow b\bar{b})$	100 TeV FCC	30 ab <sup>-1</sup>

Barman, et al., arXiv:2203.0817

Collider	$pp$	$pp$	$pp$	$e^+e^-$	$e^+e^-$	$e^+e^-$	$e^+e^-$	$e^-p$	$\gamma\gamma$	$\mu^+\mu^-$	$\mu^+\mu^-$	target
E (GeV)	14,000	14,000	100,000	250	350	500	1,000	1,300	125	125	3,000	(theory)
$\mathcal{L}$ (fb <sup>-1</sup> )	300	3,000	30,000	250	350	500	1,000	1,000	250	20	1,000	
$HZZ/HWW$	$4.0 \cdot 10^{-5}$	$2.5 \cdot 10^{-6}$	✓	$3.9 \cdot 10^{-5}$	$2.9 \cdot 10^{-5}$	$1.3 \cdot 10^{-5}$	$3.0 \cdot 10^{-6}$	✓	✓	✓	✓	$< 10^{-5}$
$H\gamma\gamma$	–	0.50	✓	–	–	–	–	–	0.06	–	–	$< 10^{-2}$
$HZ\gamma$	–	$\sim 1$	✓	–	–	–	$\sim 1$	–	–	–	–	$< 10^{-2}$
$Hgg$	0.12	0.011	✓	–	–	–	–	–	–	–	–	$< 10^{-2}$
$Ht\bar{t}$	0.24	0.05	✓	–	–	0.29	0.08	✓	–	–	✓	$< 10^{-2}$
$H\tau\tau$	0.07	0.008	✓	0.01	0.01	0.02	0.06	–	✓	✓	✓	$< 10^{-2}$
$H\mu\mu$	–	–	–	–	–	–	–	–	–	✓	–	$< 10^{-2}$

$$f_{CP}^{Hff} = \sin^2 \left( \alpha^{Hff} \right)$$

Gritsan et al arXiv:2205.07715

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# Measuring Top Yukawa at Muon Colliders

- Direct probe: top+Higgs production.

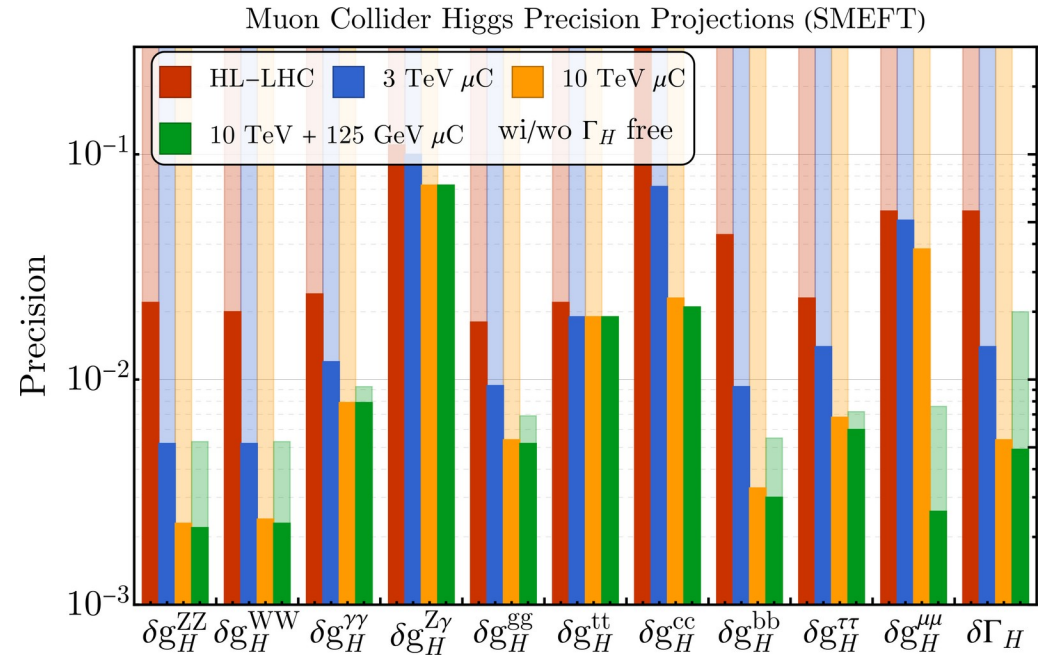
- 3 TeV:  $\Delta \sigma / \sigma = 61\%$

- 10 TeV:  $\Delta \sigma / \sigma = 53\%$

[Forslund, Meade, JHEP08 \(2022\) 185](#)

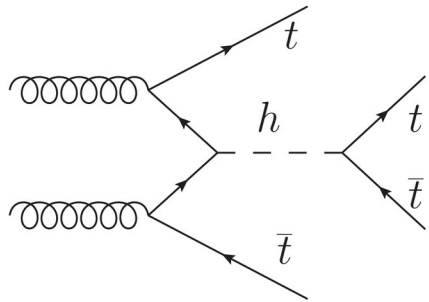
- Compare to combinations with HL-LHC.

[Muon Collider Forum Report, arXiv:2209.01318](#)



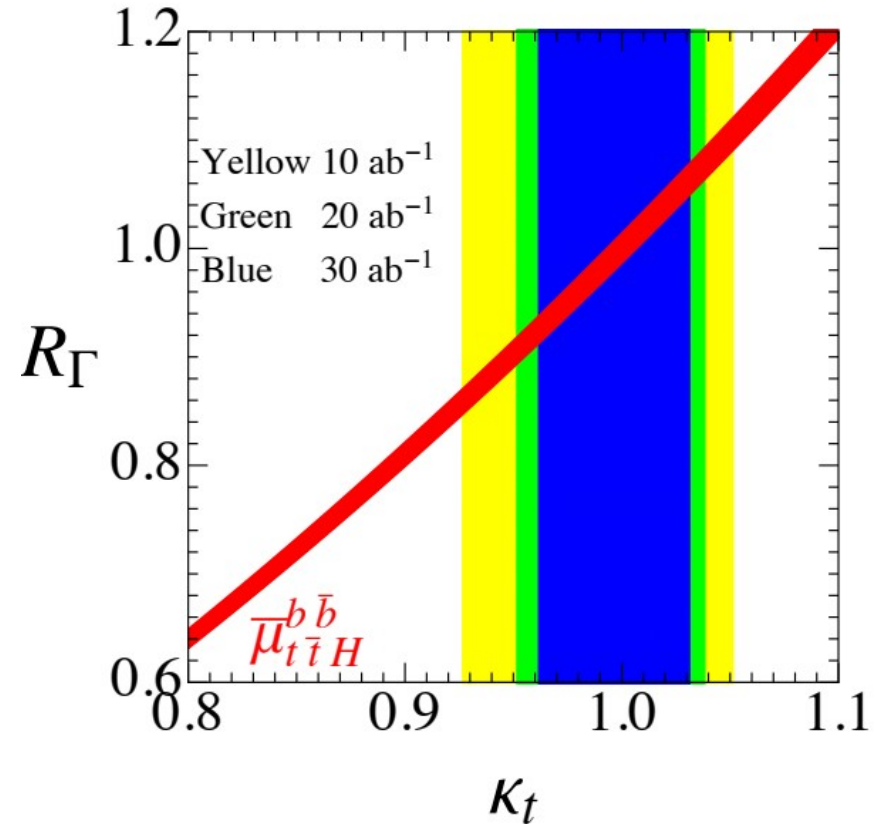
# Top Yukawa at 100 TeV

- Bands for  $K_t$  measurement from  $t\bar{t}t\bar{t}$



- Yellow:  $10 \text{ ab}^{-1}$
- Green:  $20 \text{ ab}^{-1}$
- Blue:  $30 \text{ ab}^{-1}$

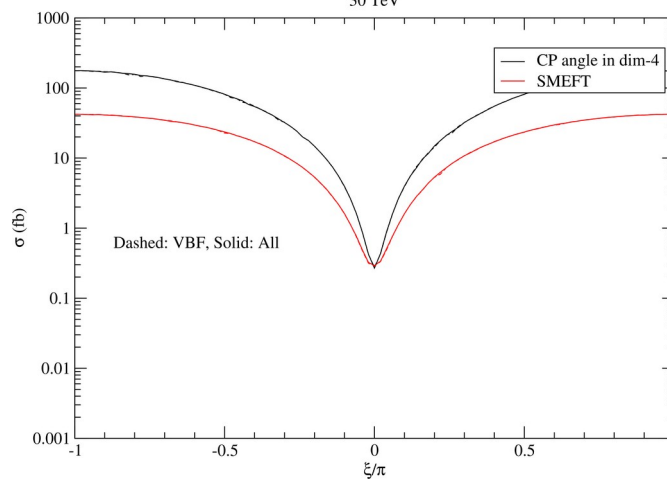
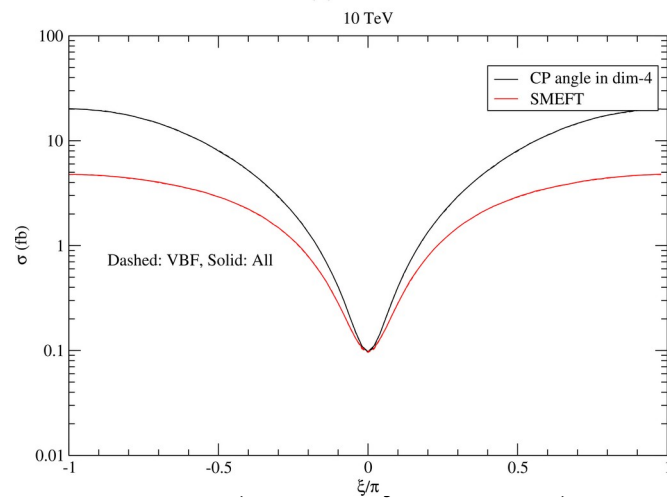
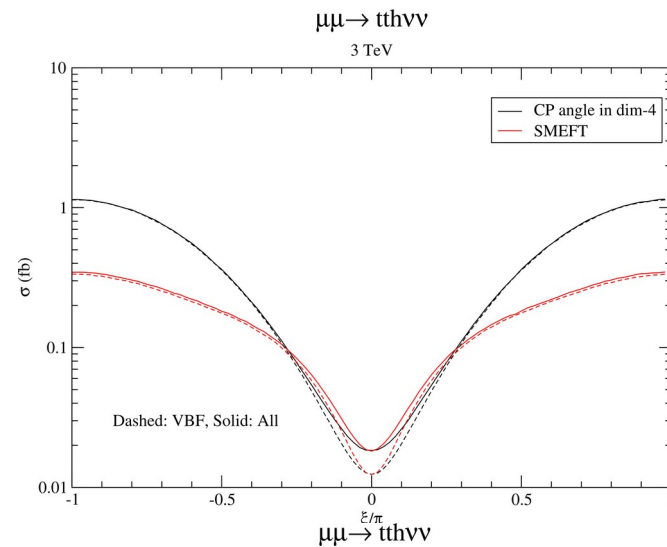
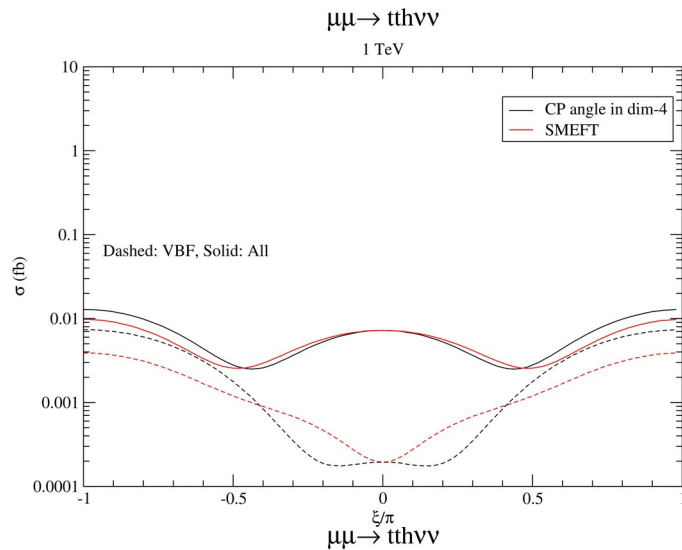
- $R_\Gamma = \Gamma_H / \Gamma_{H,SM}$



[arXiv:1606.09408](https://arxiv.org/abs/1606.09408)

# Importance of EFT

- Comparing SMEFT and dimension-4 angular parameterizations.

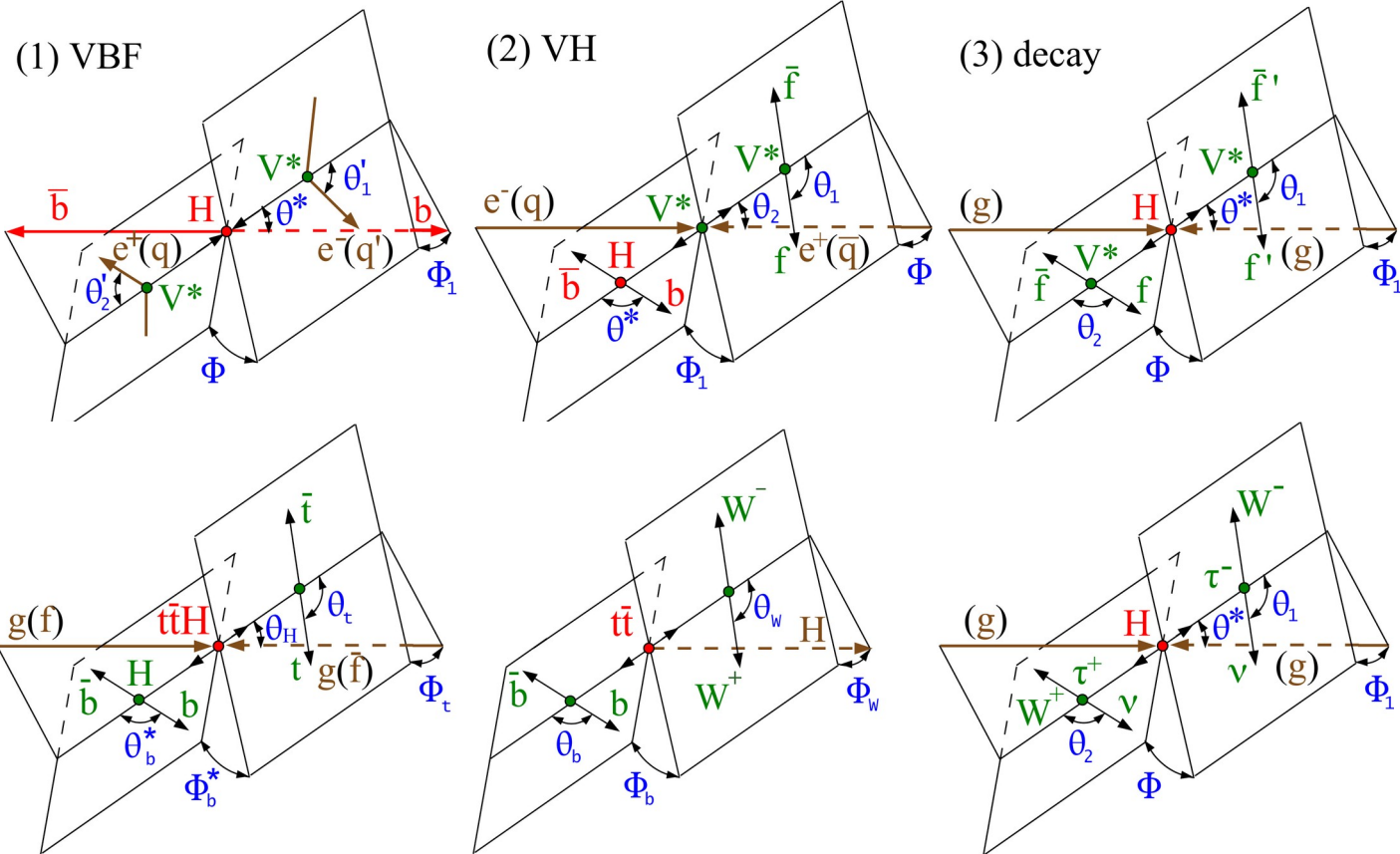


# CP Violating Variables

# Observables Sensitive to CP violation

- Cross section very sensitive to CP violating angle.
- However, to cross section is CP-even.
  - Observe a deviation, no guarantee that there is true CP violation.
- Observables sensitive to CP violation are angles between different production and decay planes and/or triple products.
- Will investigate several process by process.
- Purely theoretical investigation with no decays.
  - It is likely that first indication of a CP violating phase at a muon collider with energies of 10 TeV or larger would be a change in the total cross section.
  - Measuring CP violating observables would be needed to verify source of change in cross section.

# Typical CPV variables



Gritsan et al arXiv:2205.07715

- Create angles between production/decay planes or triple products.



# tth

- First, consider tth production.

$$\mu^+ \mu^- \rightarrow t\bar{t}h,$$

- Create an angle between the tt system and initial anti-muon-Higgs system. [Bar-Shalom, Atwood, Eilam, Soni PRD53 \(1996\) 1162](#); [Hagiwara, Yokoya, Zheng JHEP02 \(2018\) 180](#); etc.
  - Define the momentum of the planes:

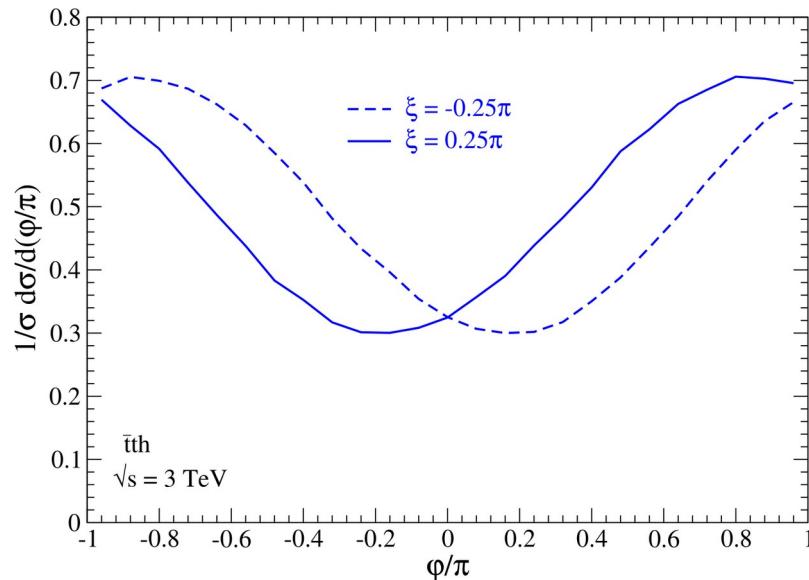
$$\vec{p}_{t\bar{t}} = \vec{p}_t + \vec{p}_{\bar{t}}.$$

$$\vec{p}_{h\mu^-} = \vec{p}_h + \vec{p}_{\mu^-}$$

- Create the observable, which is the angle between the planes:

$$\phi = \text{sign} [\vec{p}_{t\bar{t}} \cdot (\vec{p}_{\mu^-} \times \vec{p}_t)] \arccos \left[ \frac{\vec{p}_{h\mu^-} \times \vec{p}_{\mu^-}}{|\vec{p}_{h\mu^-} \times \vec{p}_{\mu^-}|} \cdot \frac{\vec{p}_{t\bar{t}} \times \vec{p}_t}{|\vec{p}_{t\bar{t}} \times \vec{p}_t|} \right]$$

# CP violating observable for $t\bar{t}h$

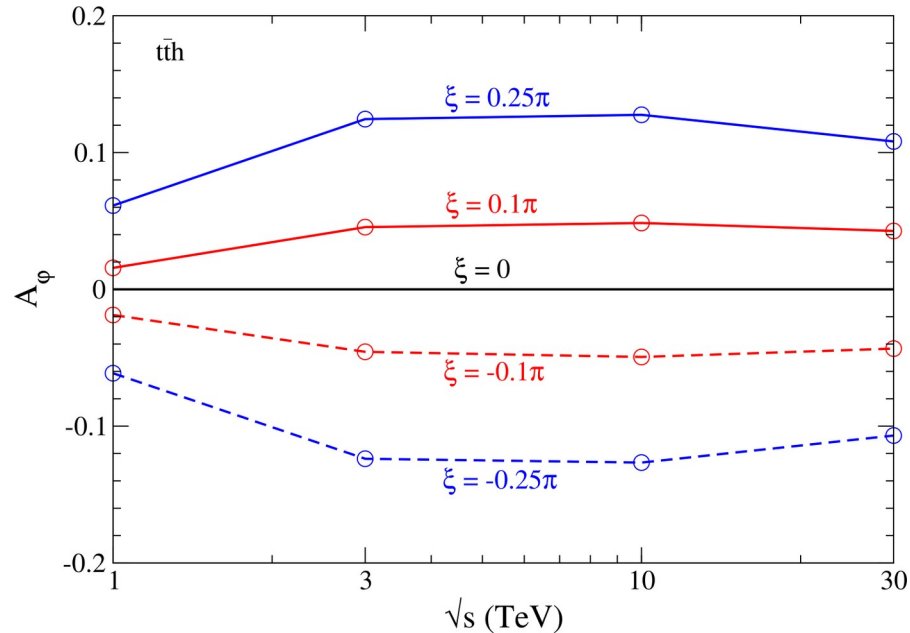


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$$\phi = \text{sign} \left[ \vec{p}_{t\bar{t}} \cdot (\vec{p}_{\mu^-} \times \vec{p}_t) \right] \arccos \left[ \frac{\vec{p}_{h\mu^-} \times \vec{p}_{\mu^-}}{|\vec{p}_{h\mu^-} \times \vec{p}_{\mu^-}|} \cdot \frac{\vec{p}_{t\bar{t}} \times \vec{p}_t}{|\vec{p}_{t\bar{t}} \times \vec{p}_t|} \right]$$

- Clearly dependent on the sign of the angle.
  - Sensitive to CPV
- Note: Show results for 3 TeV. As we will show, at this energy the CPV observables may need be necessary.
  - Cross section measurements turn out to be relatively insensitive to CPV at this energy.

# Asymmetry Parameter



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- Define asymmetry to maximally utilize data:

$$A_\phi = \frac{\sigma(\phi > 0) - \sigma(\phi < 0)}{\sigma(\phi > 0) + \sigma(\phi < 0)}$$

# tbh $\nu\mu$ and tth $\nu\nu$

- Consider both processes:

$$\mu^+ \mu^- \rightarrow t\bar{t}h\nu\bar{\nu} \equiv t\bar{t}h\nu_\ell\bar{\nu}_\ell \quad (\ell = e, \mu \text{ and } \tau),$$

$$\mu^+ \mu^- \rightarrow tbh\mu\nu \equiv t\bar{b}h\mu^-\nu_\mu + \bar{t}bh\mu^+\bar{\nu}_\mu,$$

- Many more possible momentum to create observables.

[Bar-Shalom, Atwood, Eilam, Soni PRD53 \(1996\) 1162](#); [The Higgs Hunters Guide, Gunion, He, PRL76 \(1996\) 4468](#)

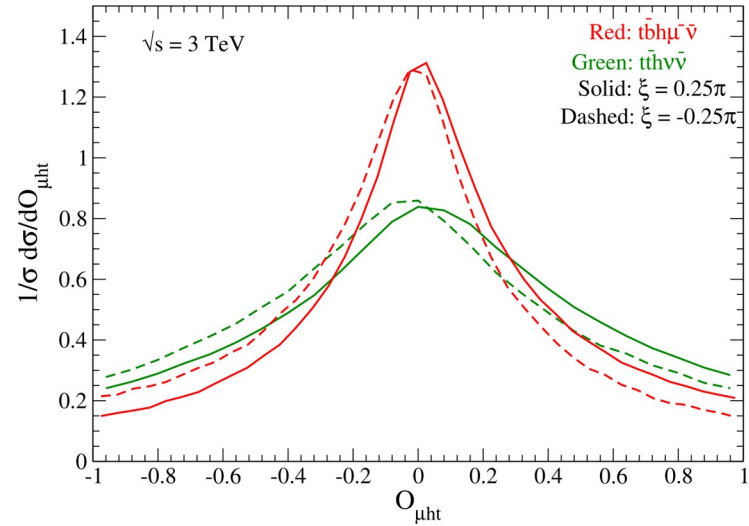
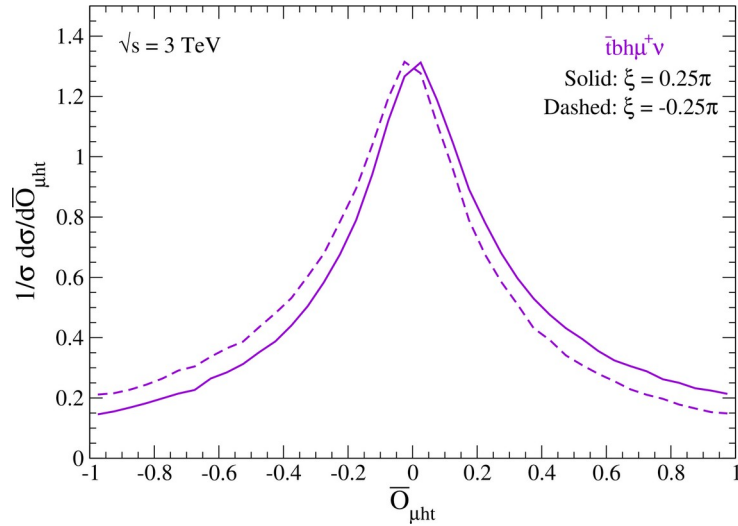
- For both cases, create a triple product between the final state top quark, Higgs, and initial state muon:

$$\mathcal{O}_{\mu ht} \equiv \frac{(\vec{p}_{\mu^-} \times \vec{p}_h) \cdot \vec{p}_t}{|\vec{p}_{\mu^-} \times \vec{p}_h| |\vec{p}_t|},$$

- For tbh $\nu\mu$  with consider process with top quark and anti-top quark separately.
  - For anti-top quark can define conjugate triple product.

$$\bar{\mathcal{O}}_{\mu ht} \equiv \frac{(\vec{p}_{\mu^+} \times \vec{p}_h) \cdot \vec{p}_{\bar{t}}}{|\vec{p}_{\mu^+} \times \vec{p}_h| |\vec{p}_{\bar{t}}|}.$$

# tbh $\nu\mu$ and tth $\nu\nu$



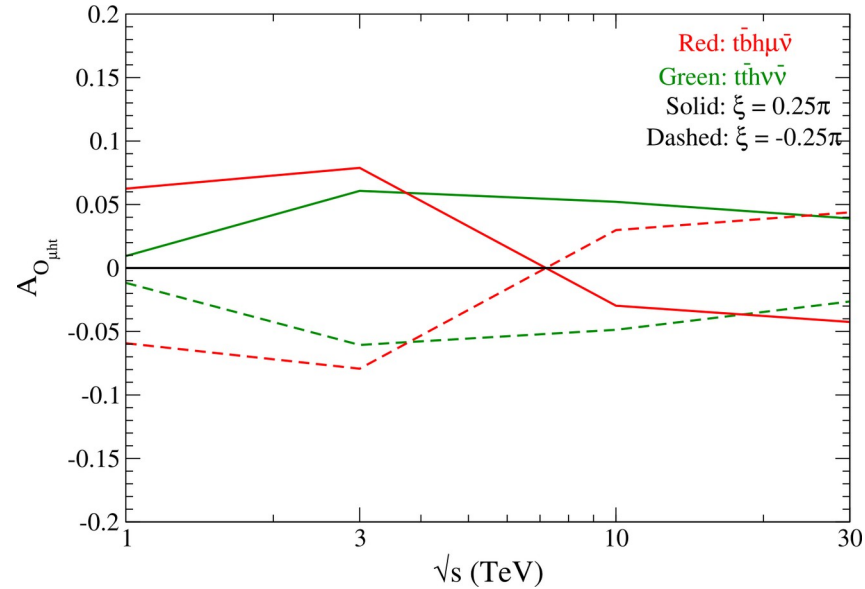
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- CPV observables:

$$\mathcal{O}_{\mu ht} \equiv \frac{(\vec{p}_{\mu^-} \times \vec{p}_h) \cdot \vec{p}_t}{|\vec{p}_{\mu^-} \times \vec{p}_h| |\vec{p}_t|}; \quad \overline{\mathcal{O}}_{\mu ht} \equiv \frac{(\vec{p}_{\mu^+} \times \vec{p}_h) \cdot \vec{p}_{\bar{t}}}{|\vec{p}_{\mu^+} \times \vec{p}_h| |\vec{p}_{\bar{t}}|}.$$

- Separation for signs of CP violating angle.
- tbh $\nu\mu$   $\alpha$  and conjugate process have similar behavior.
- tbh $\nu\mu$  is more peaked near zero than tth $\nu\nu$
- Separation is relatively small, look at asymmetry parameters.

# Asymmetry Parameters



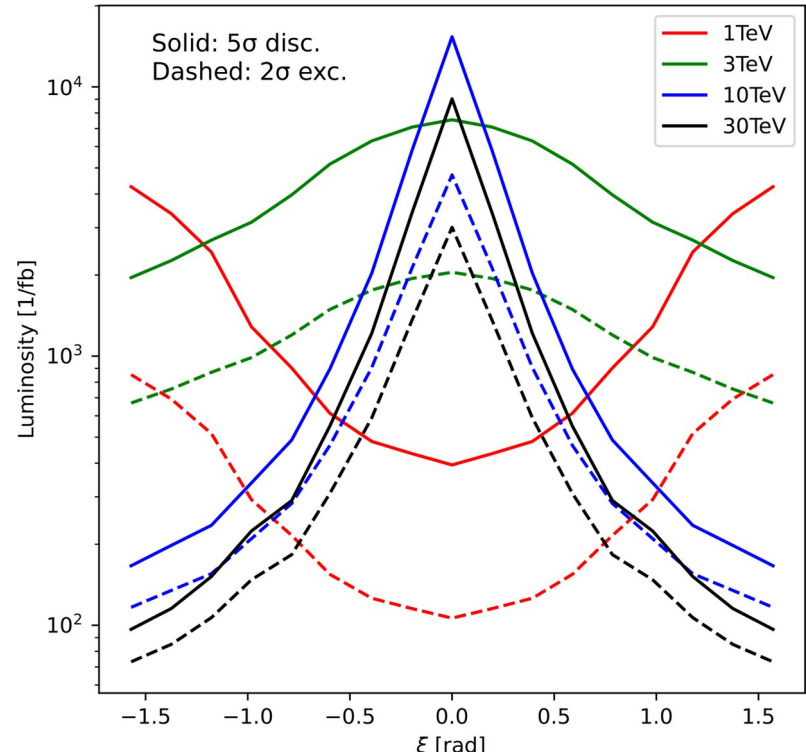
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- Define asymmetry parameter:

$$A_{O_{\mu ht}} = \frac{\sigma(\mathcal{O}_{\mu ht} > 0) - \sigma(\mathcal{O}_{\mu ht} < 0)}{\sigma(\mathcal{O}_{\mu ht} > 0) + \sigma(\mathcal{O}_{\mu ht} < 0)}$$

# Luminosity needed for Discovery

- Solid: Luminosity needed for  $5\sigma$  discovery.
- Dashed: Luminosity needed for  $2\sigma$  exclusion.
- All signals added together
  - Differences between energies due to differences in cross dependence on  $\xi$
- 90% b-tagging efficiency applied.
- Luminosity benchmarks:
  - 1 TeV:  $100 \text{ fb}^{-1}$
  - 3 TeV:  $1 \text{ ab}^{-1}$
  - 10 TeV:  $10 \text{ ab}^{-1}$
  - 30 TeV:  $10 \text{ ab}^{-1}$



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# Cross Section Dependence

- At the SM values, strong destructive interference in  $t\bar{t}h\nu\nu$  and  $tbh_{\mu\nu}$  at high energies.

$$\sigma(X) = C_X^4 \cos^4 \xi + C_X^3 \cos^3 \xi + C_X^2 \cos^2 \xi + C_X^1 \cos \xi + C_X^0$$

- Interference between Higgs radiated from gauge boson or top quarks.

$$\sigma_{tbh_{\mu\nu}}(\xi = 0) = 0.31 \text{ fb} \quad \sigma_{t\bar{t}h\nu\nu}(\xi = 0) = 0.31 \text{ fb}.$$

$$\sigma_{tbh_{\mu\nu}}(\xi = \pm\pi) = 3.5 \text{ fb} \quad \sigma_{t\bar{t}h\nu\nu}(\xi = \pm\pi) = 43 \text{ fb}.$$

- Similar effect in single top+Higgs production at the LHC.

- Small SM cross section, but very large enhancement for wrong sign Yukawas.

- Total cross section very sensitive to CP angle.

Process	$\sqrt{s}$ (TeV)	$C^0$ (fb)	$C^1$ (fb)	$C^2$ (fb)	$C^3$ (fb)	$C^4$ (fb)
$t\bar{t}h$	1	0.511	0.0465	1.523	-	-
	3	0.233	0.0134	0.173	-	-
	10	0.0397	$1.48 \cdot 10^{-3}$	0.0142	-	-
	30	$6.33 \cdot 10^{-3}$	$1.69 \cdot 10^{-4}$	$1.53 \cdot 10^{-3}$	-	-
$tbh_{\mu\nu}$	1	0.108	$8.29 \cdot 10^{-3}$	0.312	-	-
	3	0.122	-0.0717	0.0537	-	-
	10	0.537	-0.503	0.0967	-	-
	30	1.66	-1.57	0.224	-	-
$t\bar{t}h\nu\bar{\nu}$	1	$2.57 \cdot 10^{-3}$	$-5.83 \cdot 10^{-4}$	$6.12 \cdot 10^{-3}$	$-6.88 \cdot 10^{-4}$	$-1.70 \cdot 10^{-4}$
	3	0.184	-0.119	$4.25 \cdot 10^{-3}$	-0.0468	$-4.50 \cdot 10^{-3}$
	10	2.92	-2.06	-0.468	-0.280	-0.0149
	30	23.9	-20.4	-2.52	-0.712	0.0446

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# Conclusions

- The possibility of a muon collider has gained renewed interest.
- Studied CP-violating top Yukawa at muon colliders.
  - At high energies, VBF diagrams dominate.
  - Due to strong destructive interference in SM, total rates very sensitive CP violating angle.
  - Studied several CP violating observables.
- Performed a collider study showing a pure rate measurement can be sensitive to a CP-violating top Yukawa.
  - Results at 10 and 30 TeV compare favorably to other direct tests at future colliders.
- Once a discovery is made, will need to verify that this is from CP violation.
  - Investigating different observables that could give confirmation of a change of rate comes from CP-violation
- At a 3 TeV muon collider, rate measurements insensitive to a CP violating angle.
  - True CP violating observables may be needed to find a CP-violating top Yukawa at this energy.
- At energies above 30 TeV,  $\log(E^2/M_w^2)$  enhancement from VBF becomes very large
  - Need to use EW pdfs [Chen, Han, Tweedie, JHEP 11 \(2017\) 093](#); [Ruiz, Costantini, Maltoni, Mattelaer, 2111.02442](#); [Han, Ma, Xie, 2203.11129](#); etc. etc.