Inaugural
US Muon Collider
Meeting

Fermilab, August 7-9, 2024
indico.fnal.gov/e/usmc2024

High Energy Muon Collider Physics and Simulation

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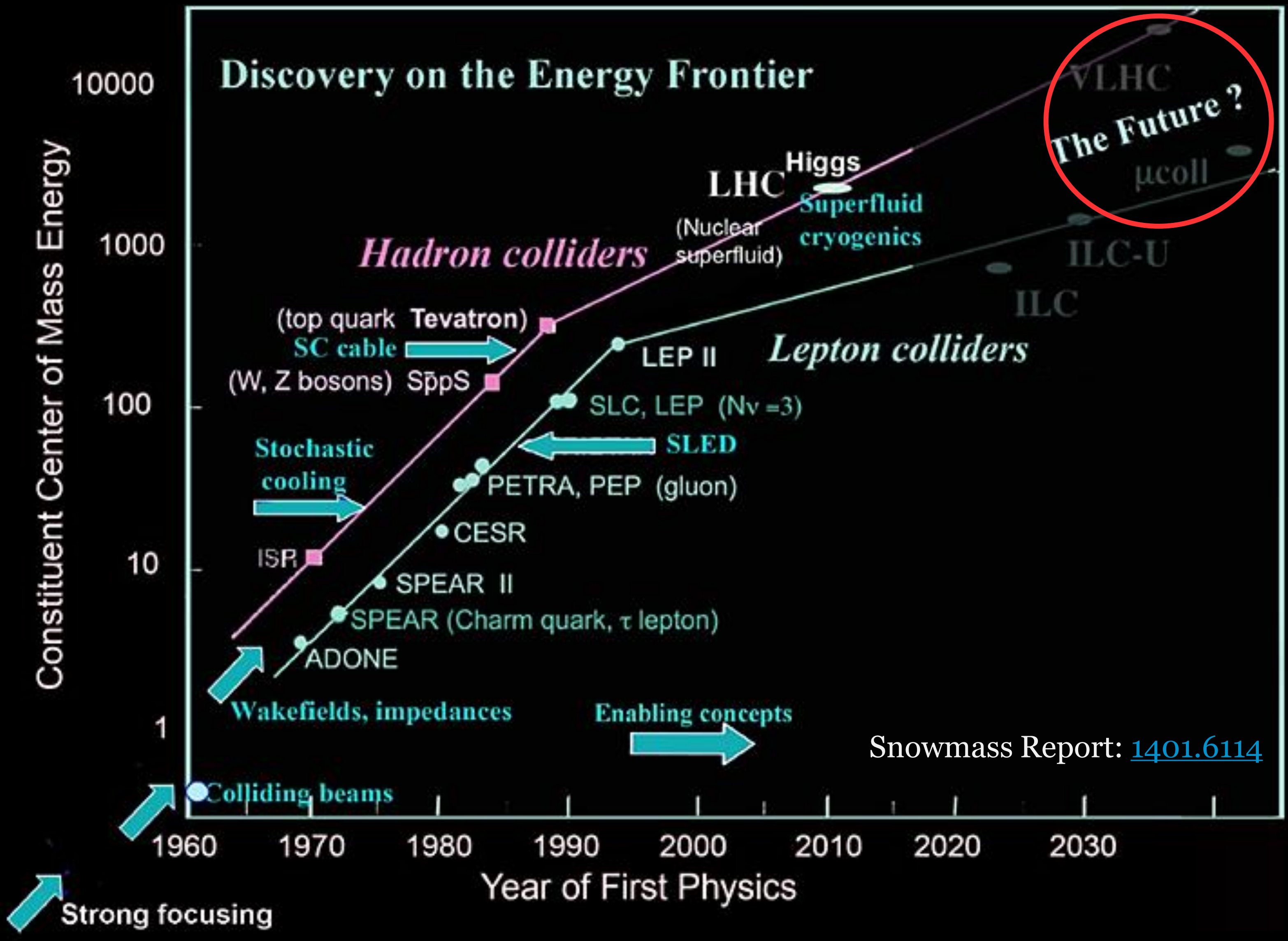
08/09/2024

High Energy Rules

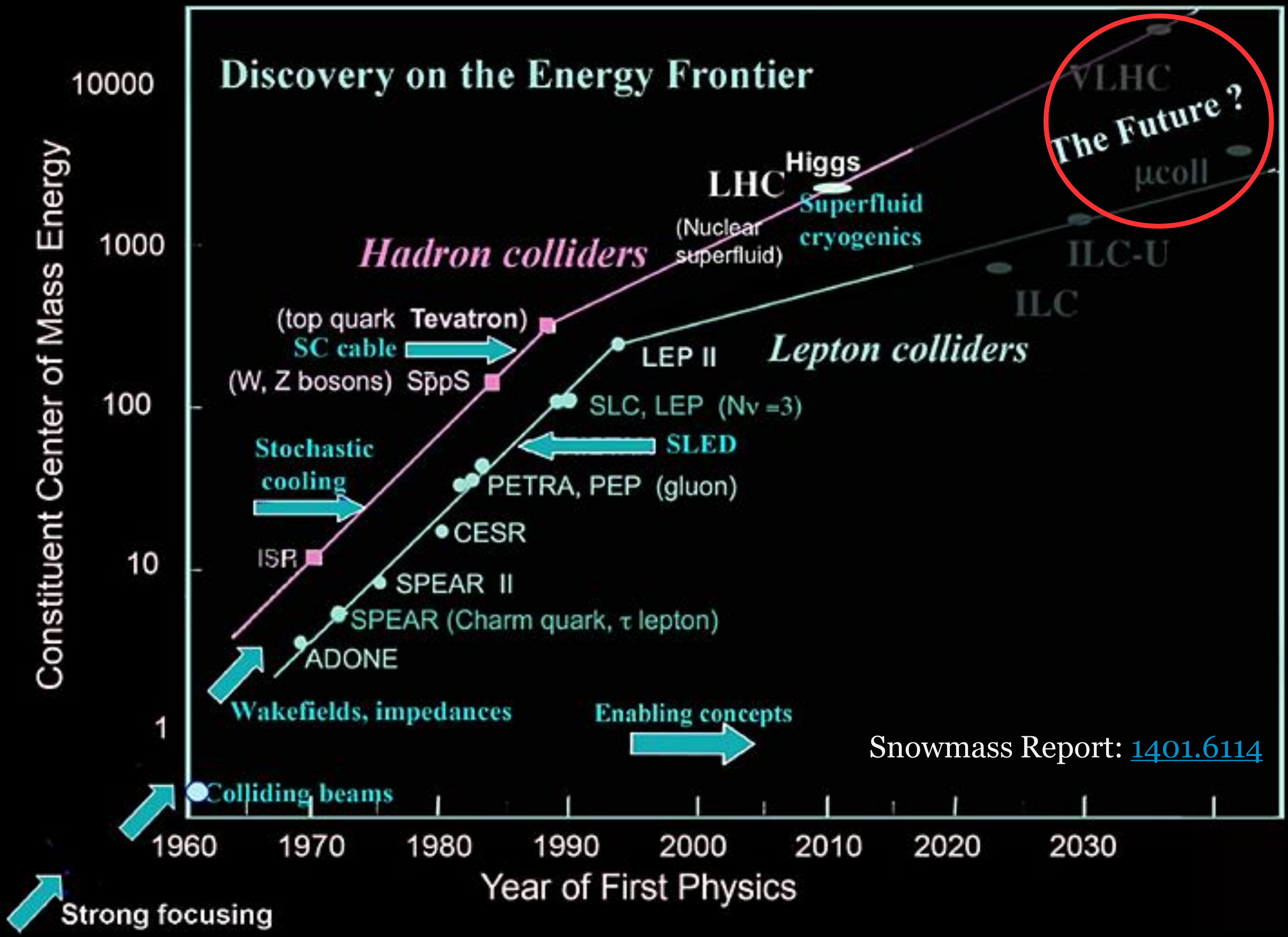


Snowmass Report: [1401.6114](https://arxiv.org/abs/1401.6114)

The forefront of tech & ambitions leads to discoveries.

The dream for high energy machines persists in our field

High Energy Rules



Snowmass Report: [1401.6114](1401.6114)

The forefront of tech & ambitions leads to discoveries.

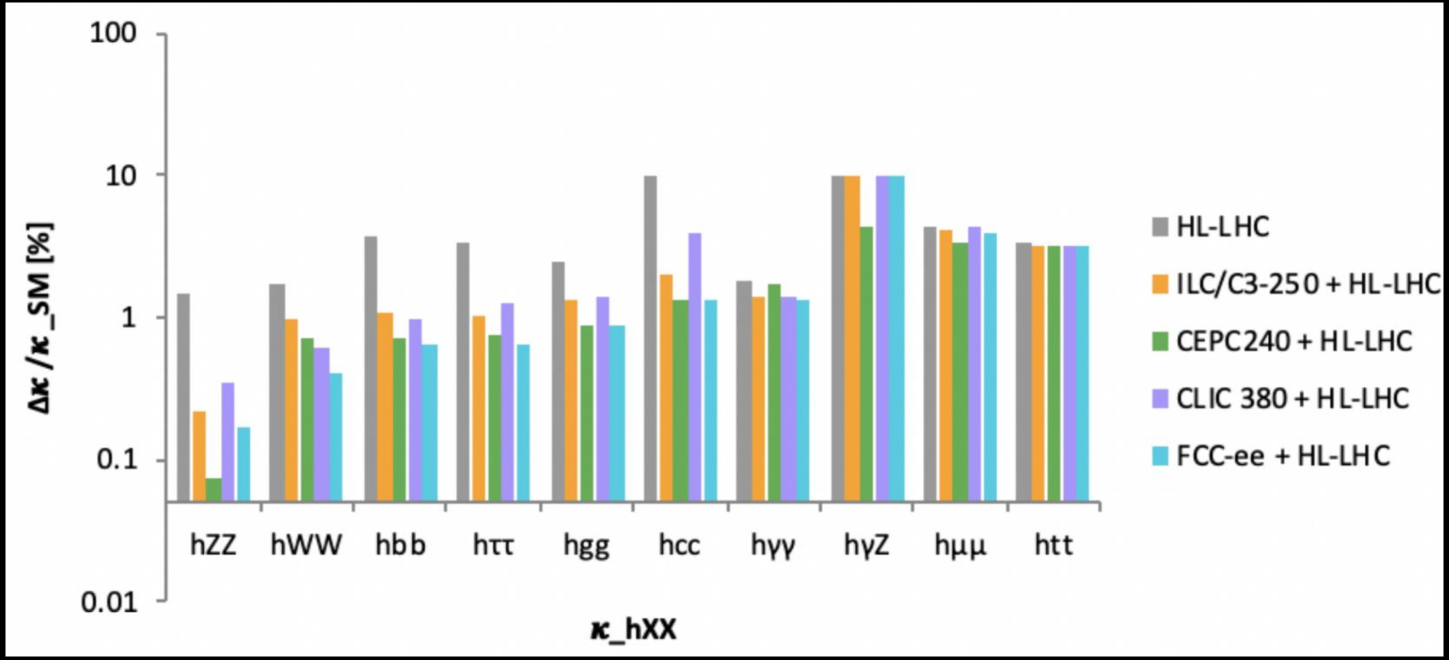
The dream for high energy machines persists in our field

People's perspectives change over time, now:

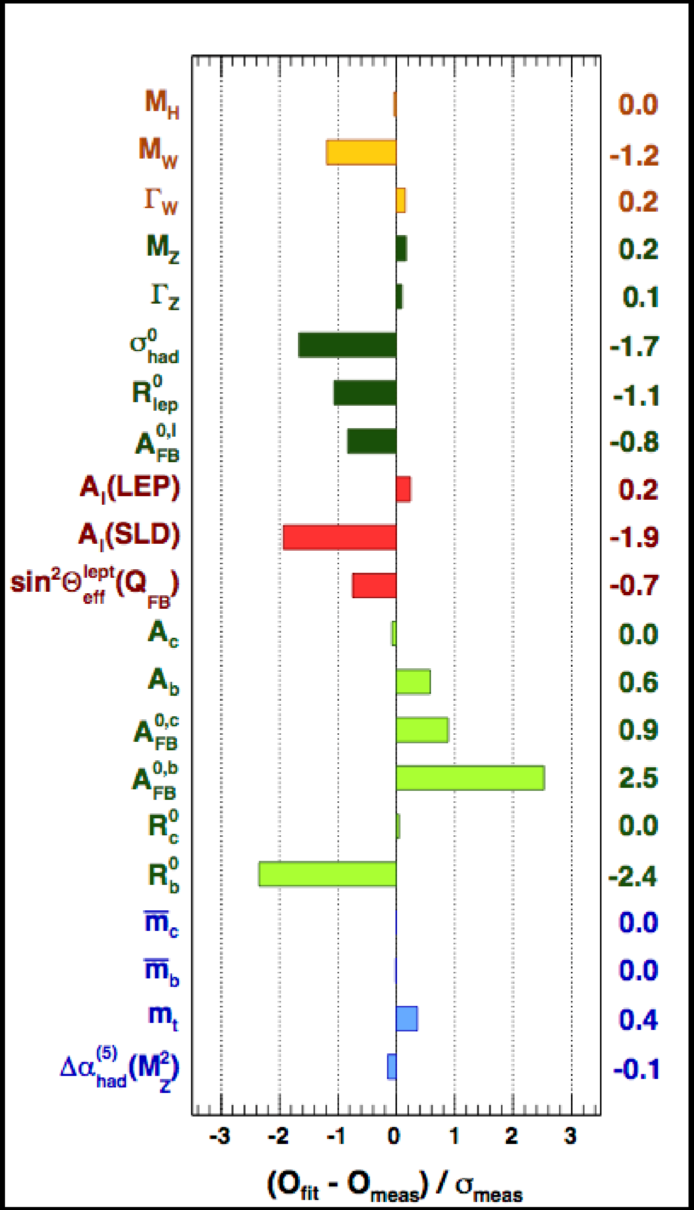
- there are excitement/call for future high energy muon collider from theory, accelerator and experimental community.
- Interesting aspects of physics to be examined.

The power of cleanness

- LEP still is a headache/treasure of theorists
- 1-4M Higgs Higgs factory v.s. 0.5B Higgs HL-LHC



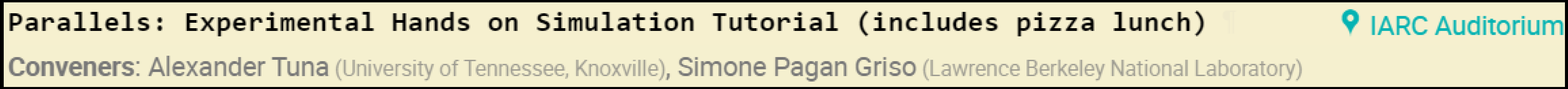
Dawson et al, [2209.07510](https://arxiv.org/abs/2209.07510)



Outline

The organizers assigned two tasks for this talk:

- Intro to basic simulation resources for Pheno studies;
  - This is not a tutorial; each simulation tool has their own well-established tutorials.
  - I give the big picture about tools so you can begin somewhere.
- Discuss subtilties that one could encounter in their pheno studies.
  - I focus on those from the theory side
  - For details of (advanced, beyond described here) detector simulation see



Parallels: Experimental Hands on Simulation Tutorial (includes pizza lunch)

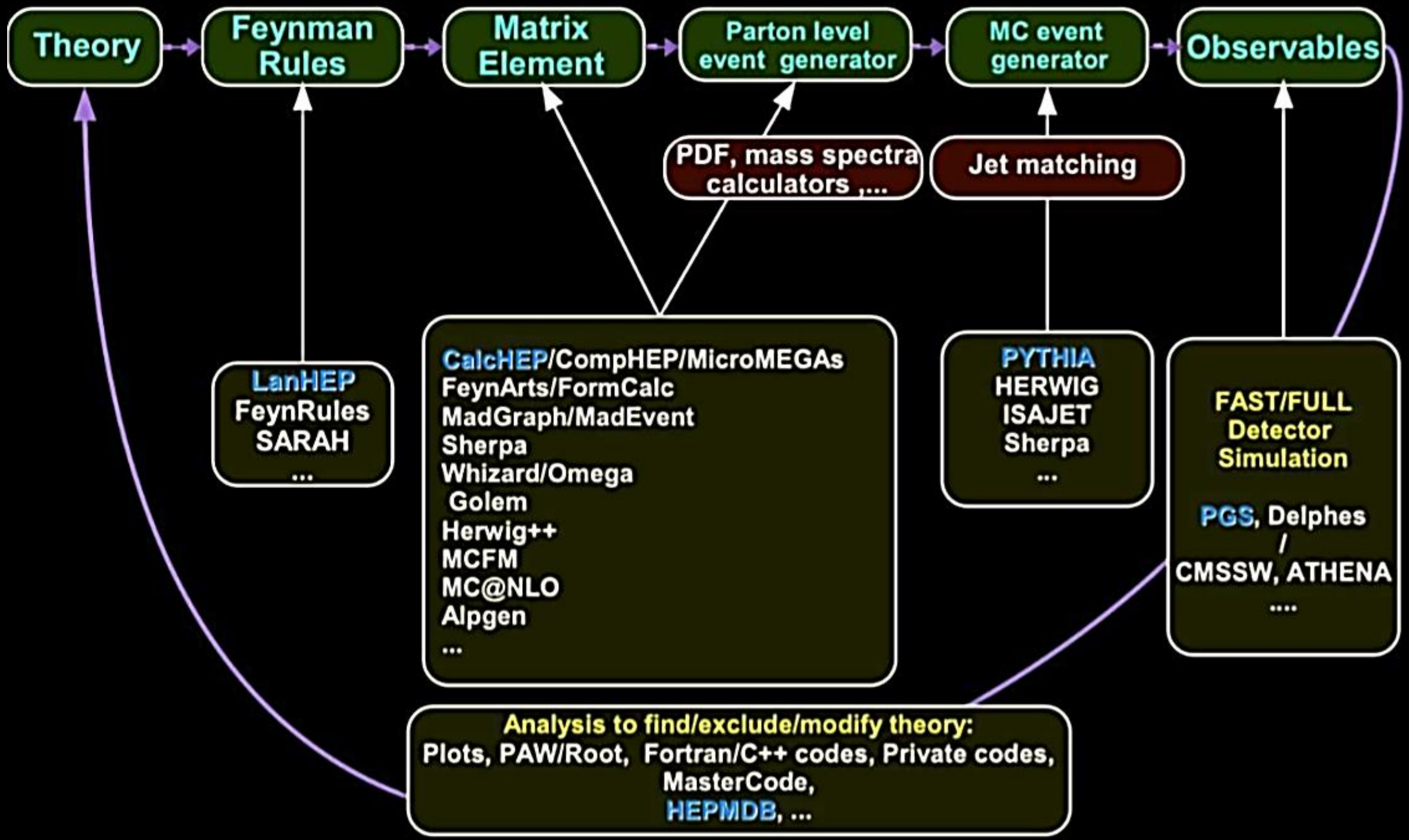
IARC Auditorium

Conveners: Alexander Tuna (University of Tennessee, Knoxville), Simone Pagan Griso (Lawrence Berkeley National Laboratory)

Logic Flow

- What are (the rates of) my signals?
- What are (the rates of) my backgrounds?
- The above intervenes as the major signal rate could have big background and hence subleading in the search sensitivity, e.g.,
  - Higgs discovered in diphoton and 4l at the LHC, which is O(500) rarer and lower signal rate than the major Higgs to 2b.

Phenomenological Studies



Theory

Feynman Rules

Matrix Element

Parton level event generator

MC event generator

Observables

PDF, mass spectra calculators ,...

Jet matching

LanHEP
FeynRules
SARAH
...

CalcHEP/CompHEP/MicroMEGAs
FeynArts/FormCalc
MadGraph/MadEvent
Sherpa
Whizard/Omega
Golem
Herwig++
MCFM
MC@NLO
Alpgen
...

PYTHIA
HERWIG
ISAJET
Sherpa
...

FAST/FULL
Detector
Simulation

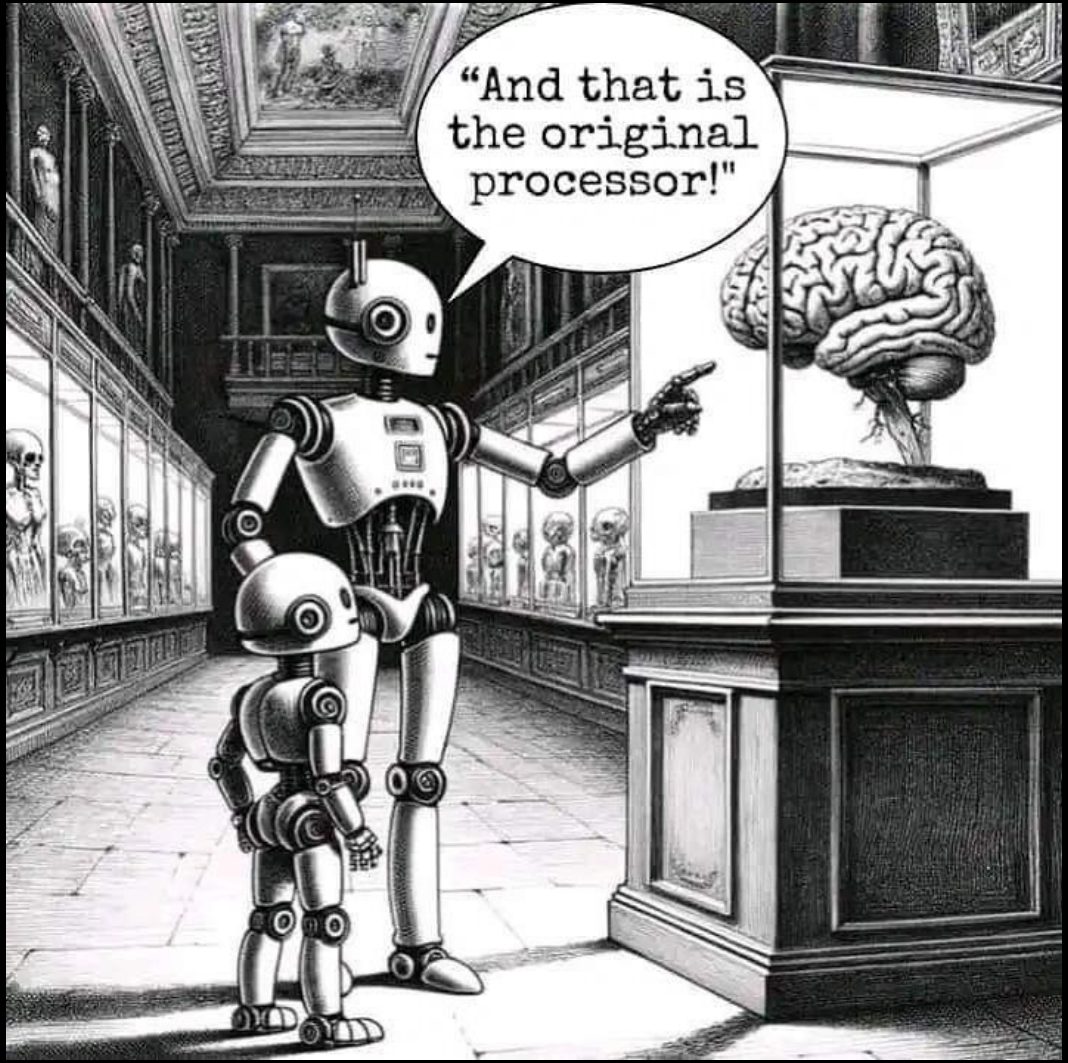
PGS, Delphes
/
CMSSW, ATHENA
....

Analysis to find/exclude/modify theory:
Plots, PAW/Root, Fortran/C++ codes, Private codes,
MasterCode,
HEPMDB, ...

Physics First (before trusting simulations due to complex nature of pheno).

Maybe then I just hit and enter and run? Machine will tell me everything

AGI hasn't arrived yet…



A valid study:

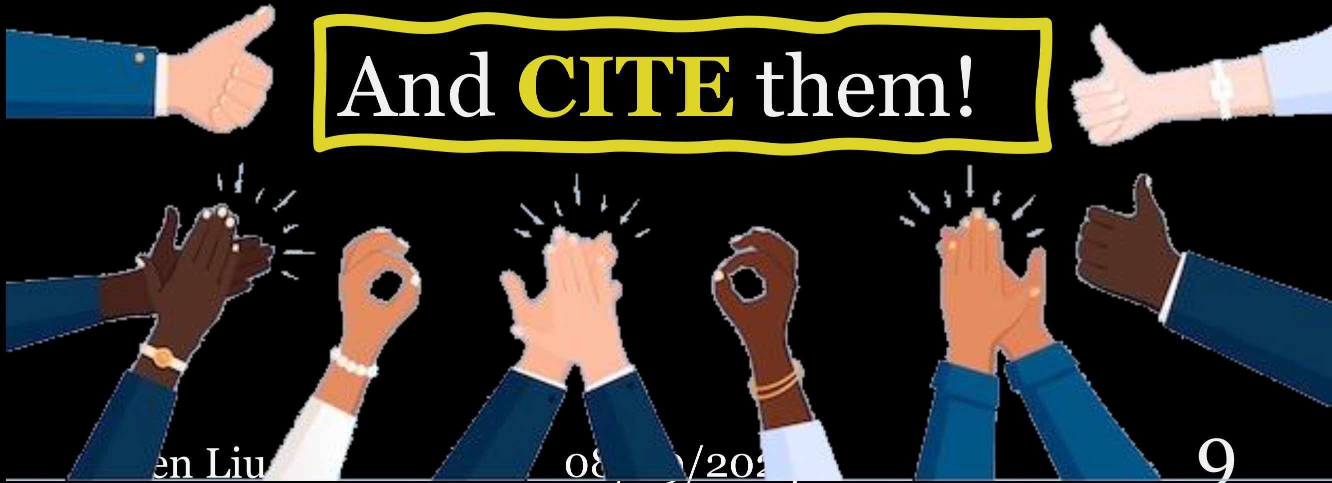
- Do I consider the signals and background correctly?
- Do I have a more accurate estimations for the above?

Key resources

- Madgraph: https://launchpad.net/mg5amcnlo
- Whizard: https://whizard.hepforge.org/

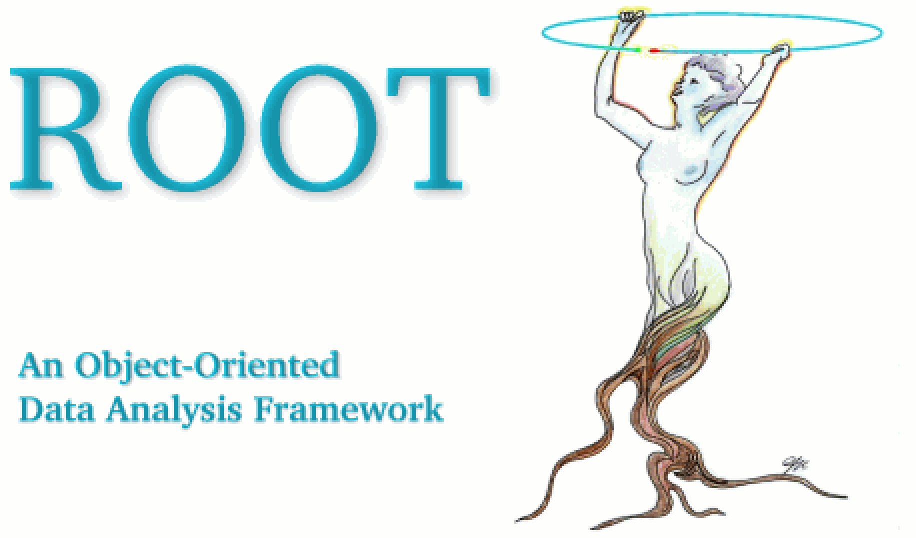
Important downflow tools

- Pythia: https://www.pythia.org/
- Delphes: https://cp3.irmp.ucl.ac.be/projects/delphes



Analysis

MAD Analysis 5



Or many homebrew version of analysis codes.

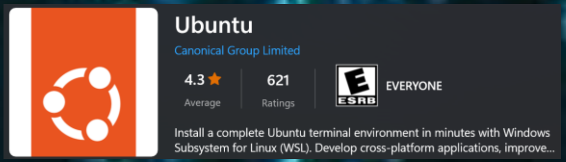
There are sample codes on the internet dealing with LHE files with pyton, c++, mathematica, etc.

For quick analysis, MadAnalysis is handy;
For more complex ones, in particular those also used by the experimental teams, ROOT.

Well, not a tutorial

Typically, people get stuck at step-0 (if no other experienced people helping you directly, or the environment is setup already for you, e.g., using clusters from collaborations)

- Simulation← Installation
  - Lots of dependencies (docker? I don’t know)
  - Read installation instructions for each software
- Installation← Terminal (X-term in Mac, Terminal in Linux…)
  - It is likely that you will HAVE TO learn it (theory friends, we have no choice here);
  - For a long time, windows users had to install dual systems or using other software to install subsystems;
  - Now windows has an app: Ubuntu in Microsoft stores;
  - I don’t know if windows powershell can be an alternative.
- But once you install them, the basic feature will run very nice and smoothly



Install a complete Ubuntu terminal environment in minutes with Windows Subsystem for Linux (WSL). Develop cross-platform applications, improve...

Recommendation

- Pick simple topologies and gain knowledge from analytic controls.
  - You can generate events yourself using Random functions in Python, Mathematica, C++, etc.
- Begin your journey of event generation and analysis with Madgraph.
- Use more complex/advanced functions, hybrid different generators, for different specific purposes.
- Always try to cross-check and understand the physics.
- Often useful to find and compare related searches (final states) to gain knowledge (e.g., compare with LHC searches and see their major background composition).

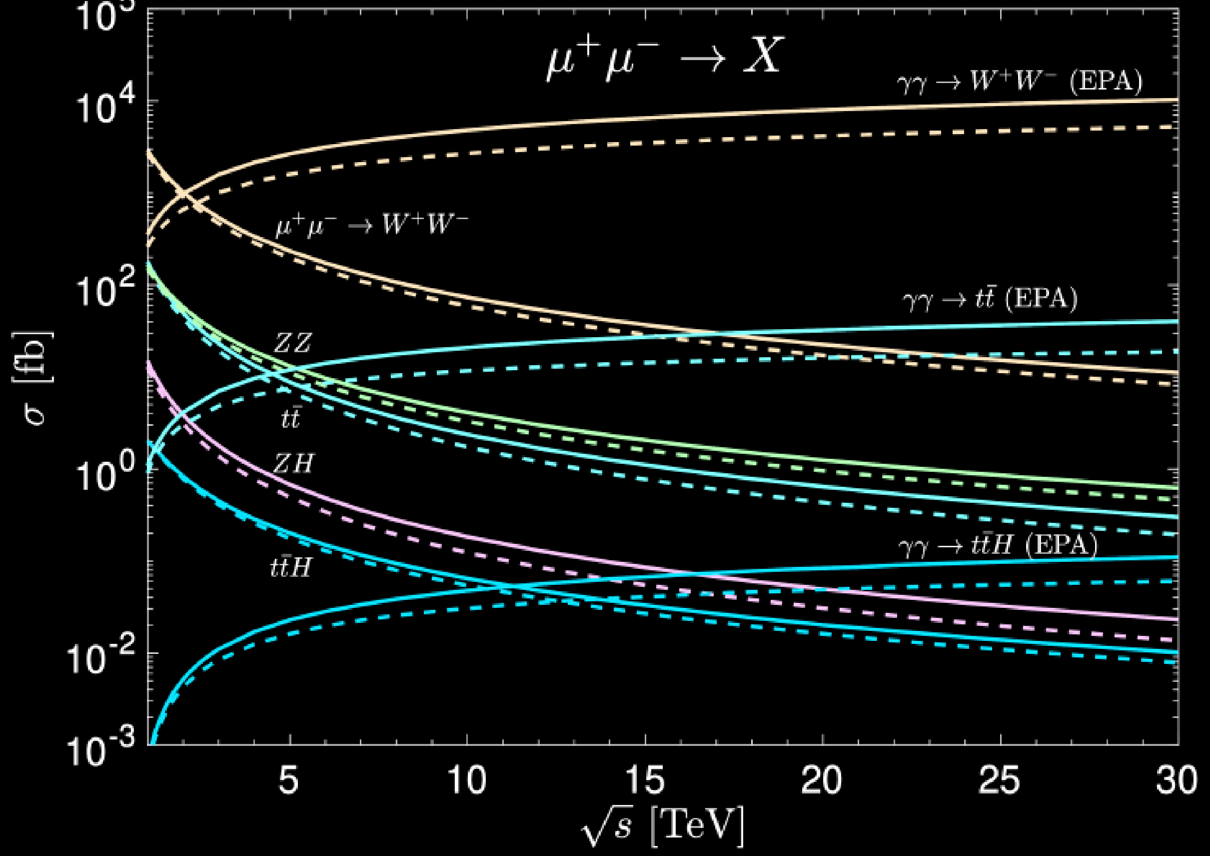
Subtilties

- Effective Photon Approximation (EPA) Weizsacker-Williams Approximation (WWA)
- →Improved WWA
- →Effective Vector Approximation
- →Electroweak PDF
- Longitudinal “enhancement”
- Physical poles from unstable particle
- Fake poles from quasi-real approximation
- Colinear divergences v.s. Forward enhancement

To avoid a dry technical introduction, let me embed them in the key Muon Collider Physics studies. You will encounter a few of them, most likely, in your studies.

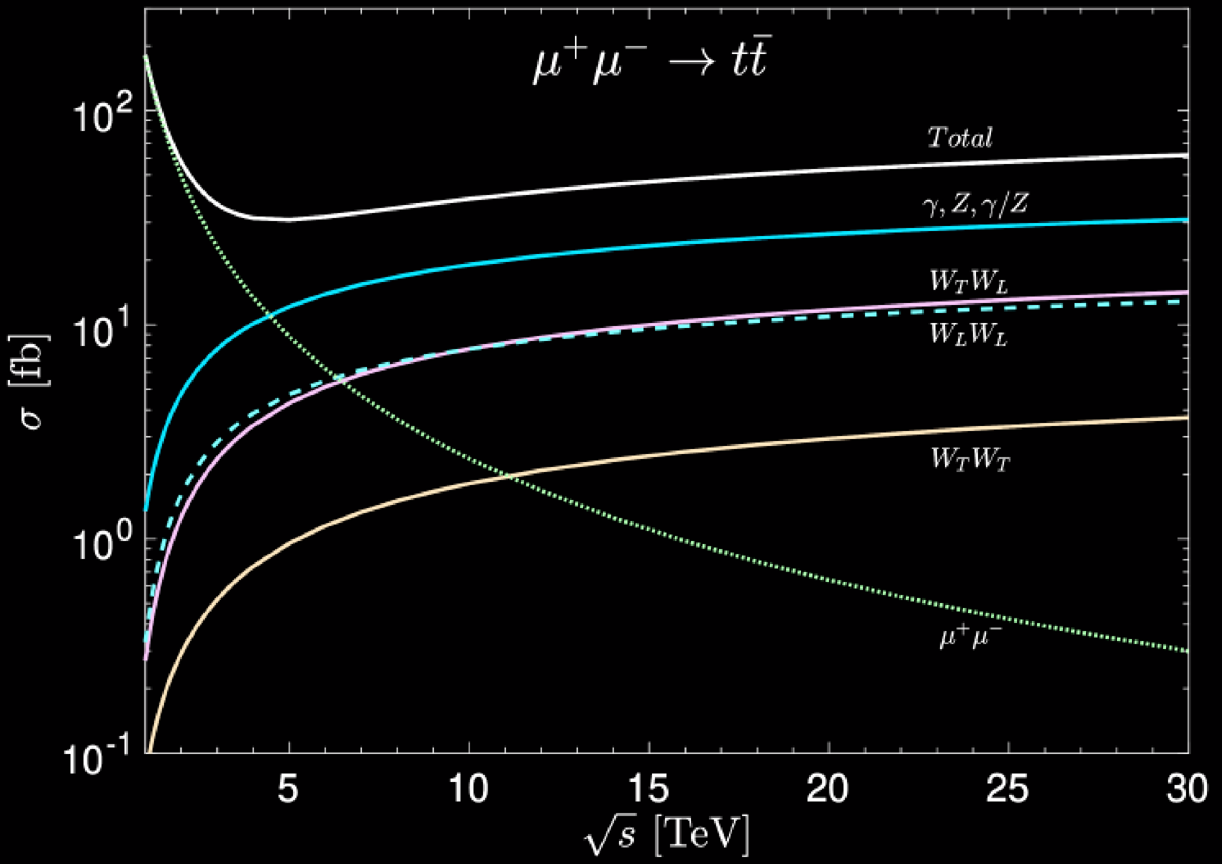
MuC is also a Vector Boson Machine

[Han, Ma, Xie, 2007.14300]



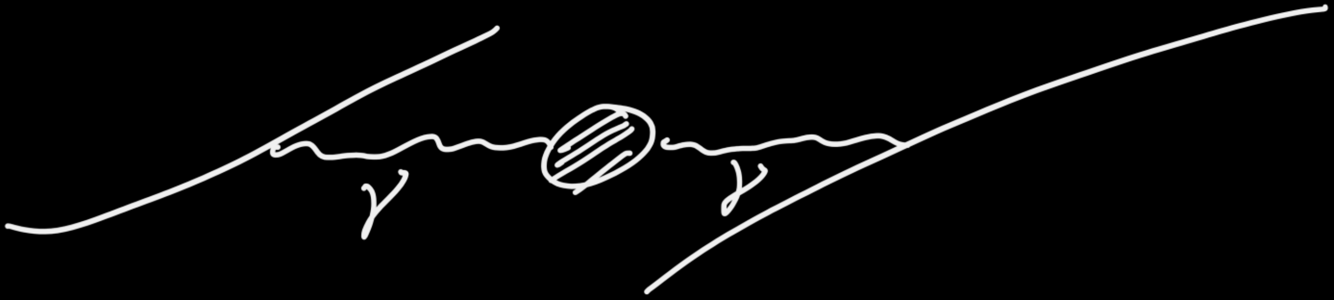
VBF dominates well above threshold due to logarithmic growth with E_{CM}

[Han, Ma, Xie, 2007.14300]



Longitudinal polarizations play a key role, making an extraordinary laboratory for EWSB

Effective Photons and Colinear Divergence



splitting kernel

Inclusive (colinear) Photon spectrum from a radiating particle: EPA or WWA

$$P_{\gamma,\ell}(x) \approx \frac{\alpha}{2\pi} P_{\gamma,\ell}(x) \ln \frac{E^2}{m_\ell^2}$$

Colinear photon emission

fixed order finite but need to be resummed.

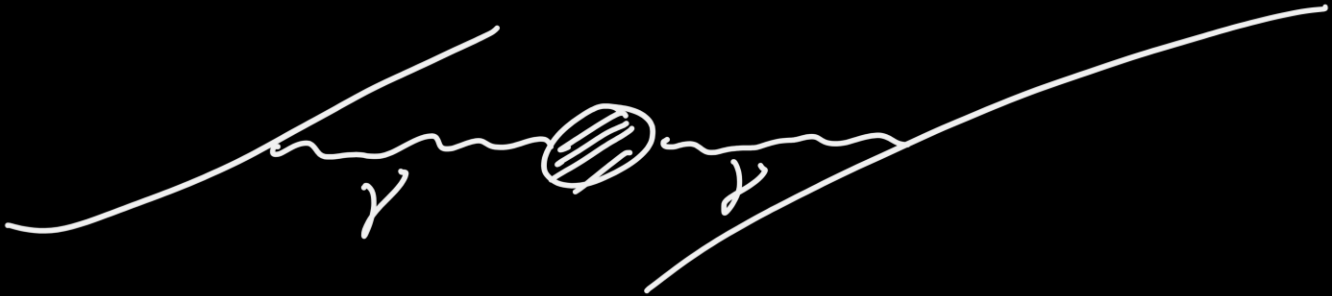
$$\propto \log \frac{E^2}{m_\mu^2} \longrightarrow \log \frac{\hat{s}}{m_\mu^2}$$

Requiring the DGLAP-like relation (since photons can radiate muons as well)

$$\frac{\mathrm{d}f_i}{\mathrm{d}\ln Q^2} = \sum_I \frac{\alpha_I}{2\pi} \sum_j P^I_{i,j} \otimes f_j$$

At leading order of solving the equation.

Effective Vectors and EW PDF



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Colinear photon emission

fixed order finite but need to be resummed.

$$\propto \log\frac{E^2}{m_\mu^2} \longrightarrow \log\frac{\hat{s}}{m_\mu^2}$$

At leading order of solving the equation.

Solving this with a fixed order of splitting kernel gets a given order PDF.

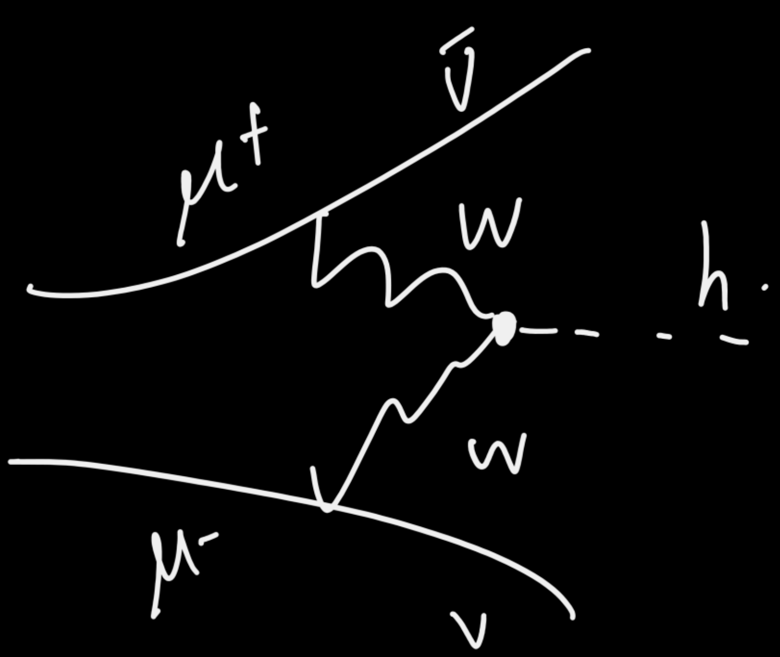
One can generalize to Effective Vector Bosons

One shall generalize to EW PDFs

Factorization

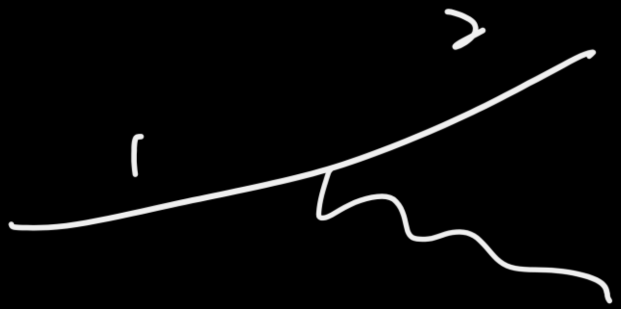
$$\sigma(AB \to F + X) = \int_{\tau_0}^{1} \mathrm{d}\tau \sum_{ij} \frac{\mathrm{d}\mathcal{L}_{ij}}{\mathrm{d}\tau}\, \hat{\sigma}(ij \to F)$$

$$\frac{\mathrm{d}\mathcal{L}_{ij}}{\mathrm{d}\tau}(\tau, \mu_f) = \frac{1}{1+\delta_{ij}} \int_{\tau}^{1} \frac{\mathrm{d}x}{x} \left[f_i(x, \mu_f) f_j(\tau/x, \mu_f) + (i \leftrightarrow j) \right]$$



OR?

Factorization? ← Quasi-real approximation



$$\frac{1}{(p_1-p_2)^2+i\epsilon}$$

Since $p_1^2 = p_2^2 = m_\mu^2$

$$(p_1-p_2)^2 = -2(p_1\cdot p_2 - m_\mu^2)$$

- For massive states, the pole will never be hit!
- But we pretend it can hits the pole so the cross section are dominant at the pole contribution and factorizes. We approximated and assumed the intermediate off-shell state to be on-shell
- The gain is to resum the log when $\hat{s} \gg m_V$, to get a better rate treatment (but there are consequences)

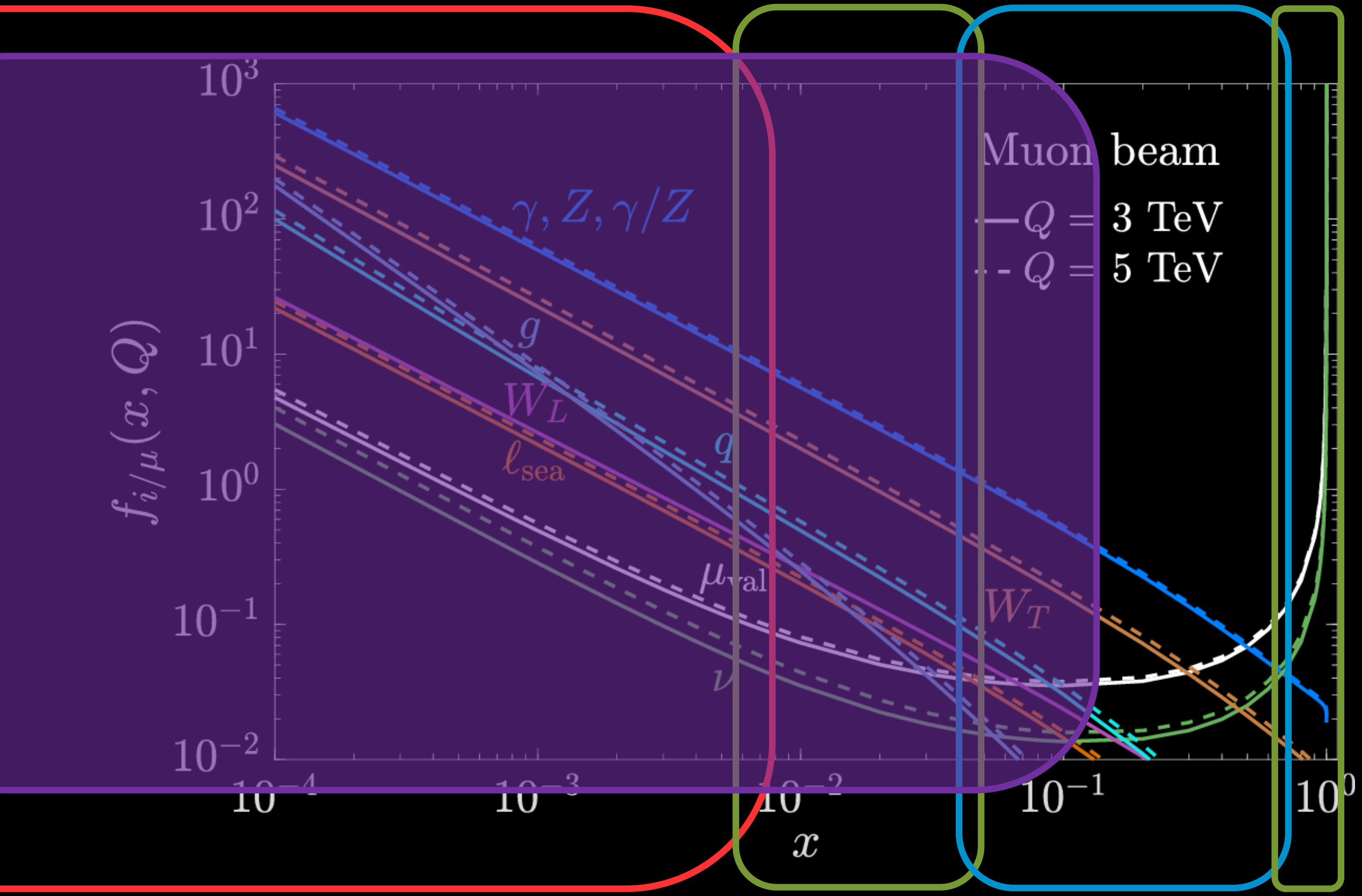
"Quasi-real Approximation"

$$\sum_\lambda \frac{\epsilon_W^\lambda \epsilon_W^{\lambda *}}{(p_2-p_1)^2 - m_W^2 + i\epsilon}$$

what's behind this factorization approximation is cross section dominate at $\frac{1}{\min(\quad)}$

which is a colinear approximation and the "mistake" one make is "merely" m_W^2

Massive Gauge Boson PDF



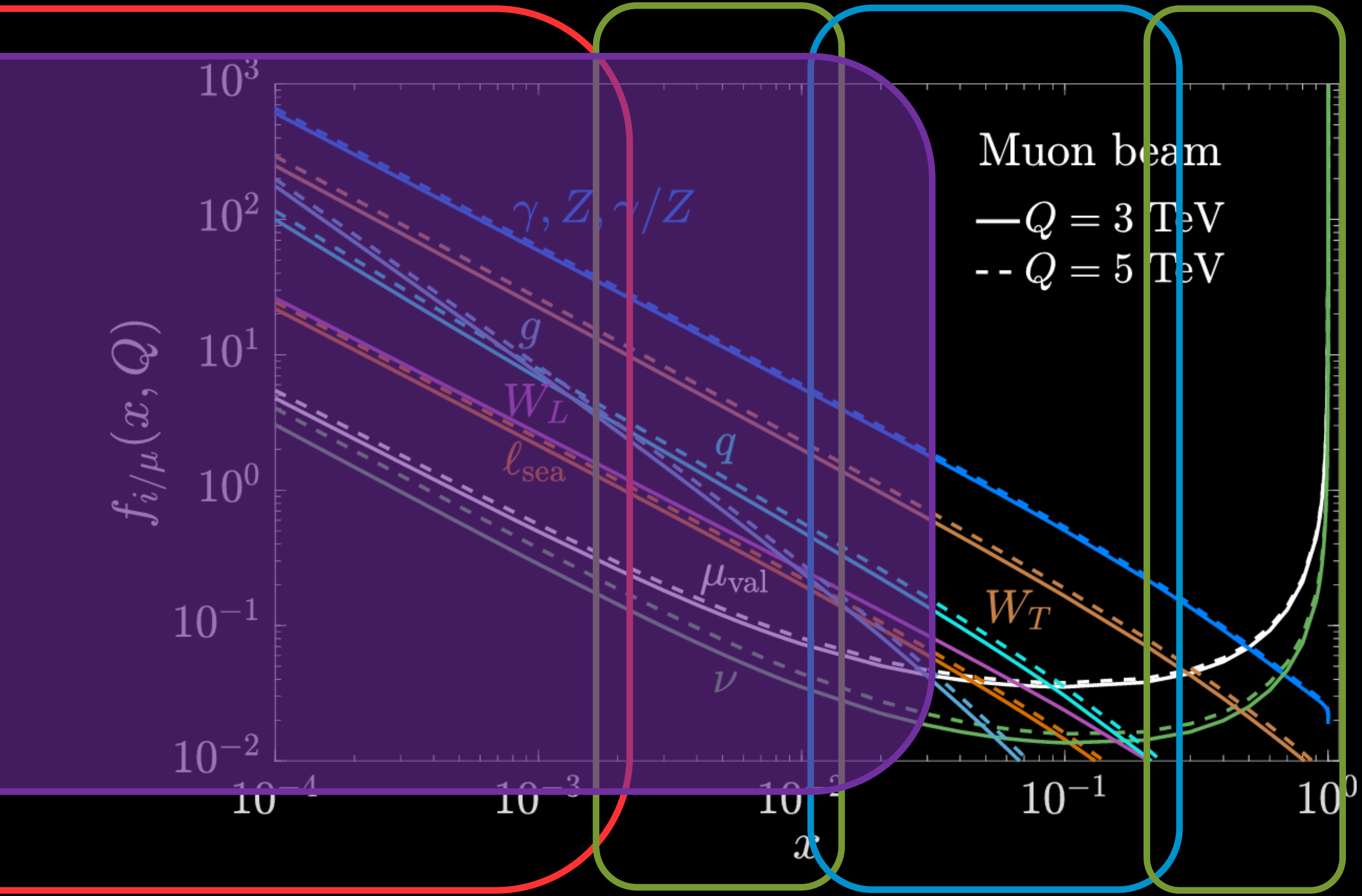
Regimes:

For $\sqrt{s} = 10\text{TeV}$ to draw the regions

- $x\sqrt{s} < m_V$
- Matching regime (very complicated and higher order inputs needed, matching scheme matters; including matching part from τ/x)
- colinear splitting enhanced regime
- **Invalid** regions to reach Q= 3 TeV (let alone 5 TeV).

For neutrinos, it is the (1-x) division of the above, due to the nature of splitting functions/EW gauge interactions.

Massive Gauge Boson PDF

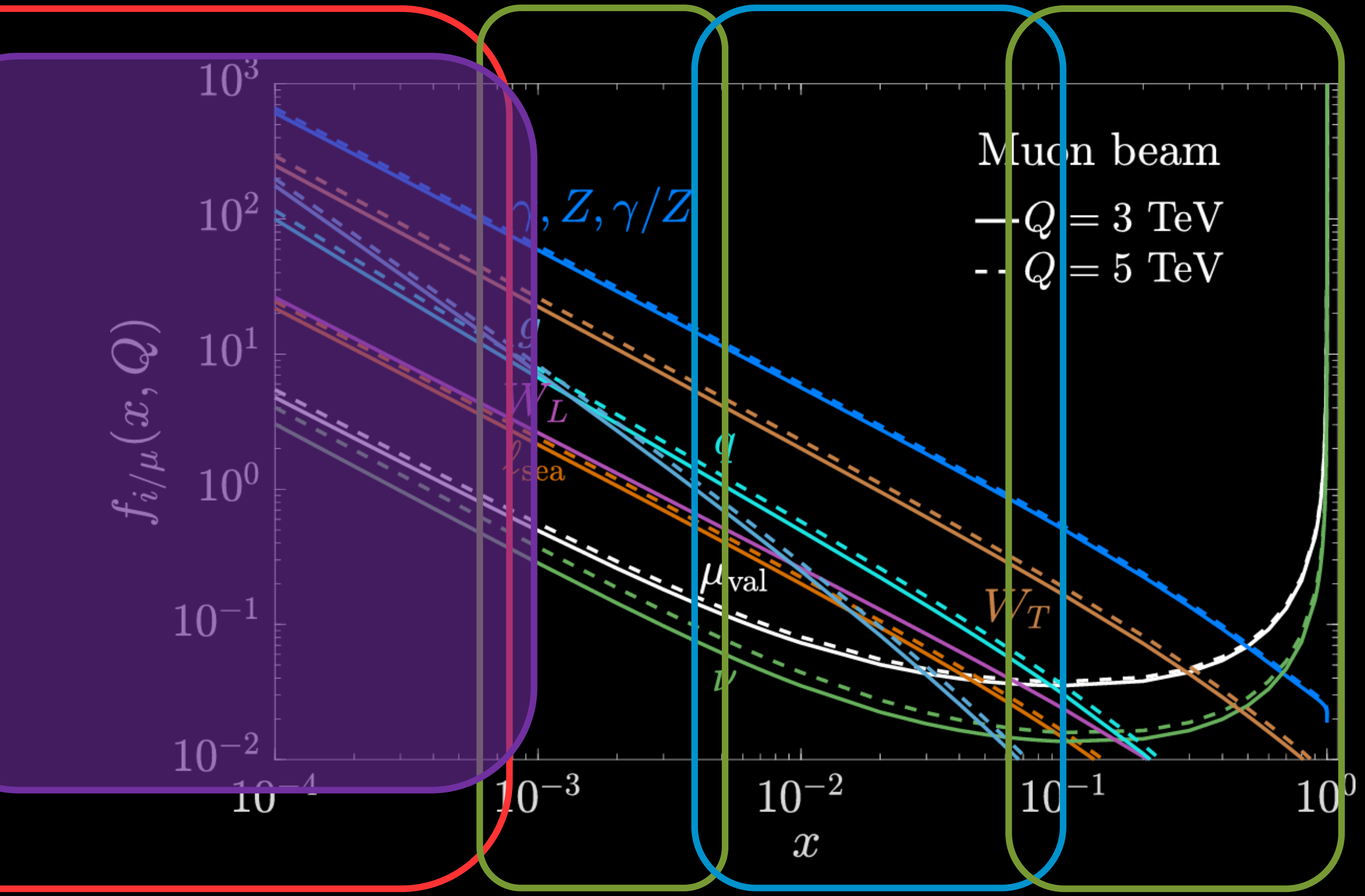


Regimes:
For $\sqrt{s} = 30\text{TeV}$ to draw the regions

- $x\sqrt{s} < m_V$
- Matching regime (very complicated and higher order inputs needed, matching scheme matters; including matching part from τ/x)
- colinear splitting enhanced regime
- **Invalid** regions to reach Q= 3 TeV (let alone 5 TeV).

For neutrinos, it is the (1-x) division of the above, due to the nature of splitting functions/EW gauge interactions.

Massive Gauge Boson PDF



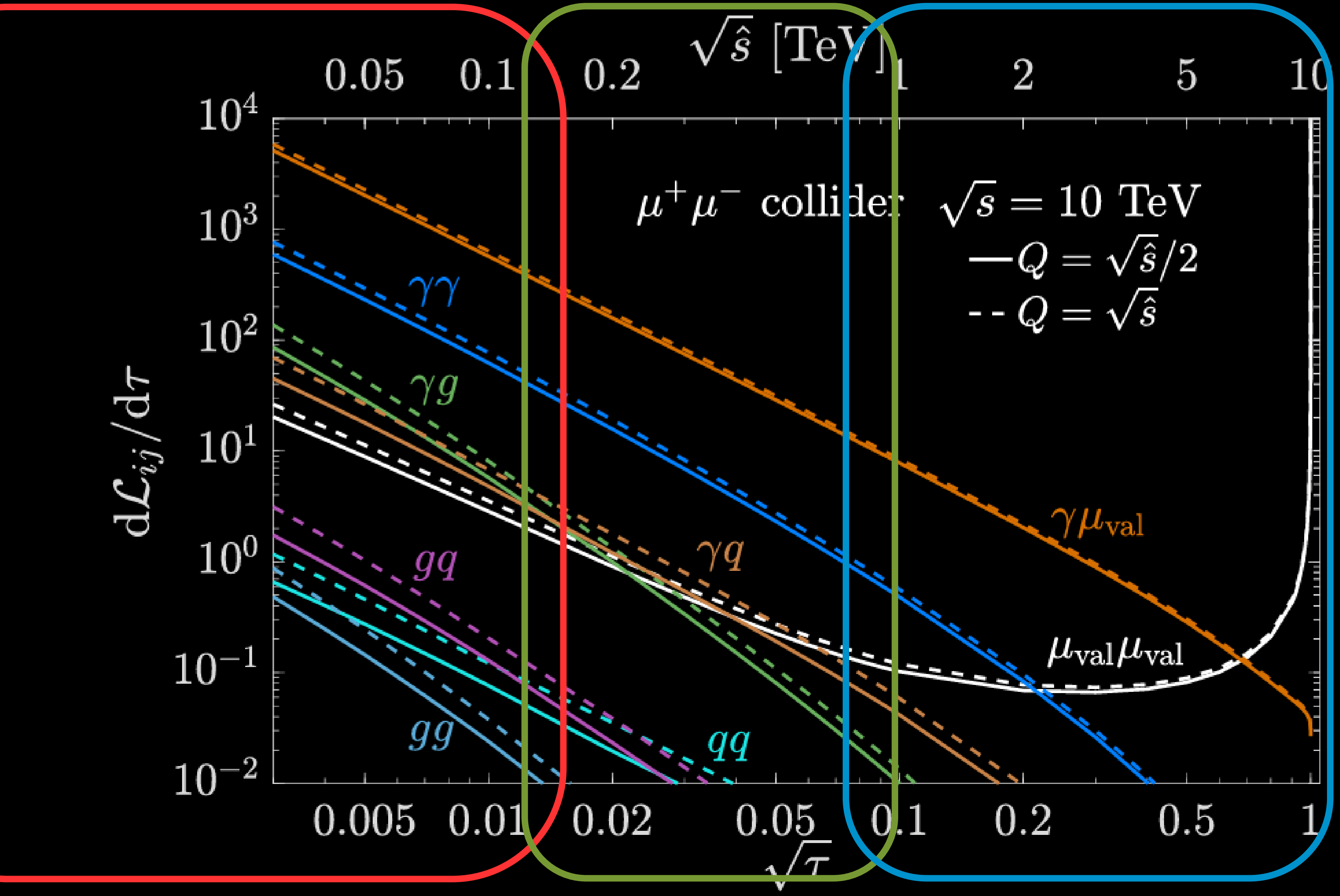
Regimes:

For $\sqrt{s} = 100$TeV to draw the regions

- $x\sqrt{s} < m_V$
- Matching regime (very complicated and higher order inputs needed, matching scheme matters; including matching part from τ/x)
- colinear splitting enhanced regime
- **Invalid** regions to reach Q= 3 TeV (let alone 5 TeV).

For neutrinos, it is the (1-x) division of the above, due to the nature of splitting functions/EW gauge interactions.

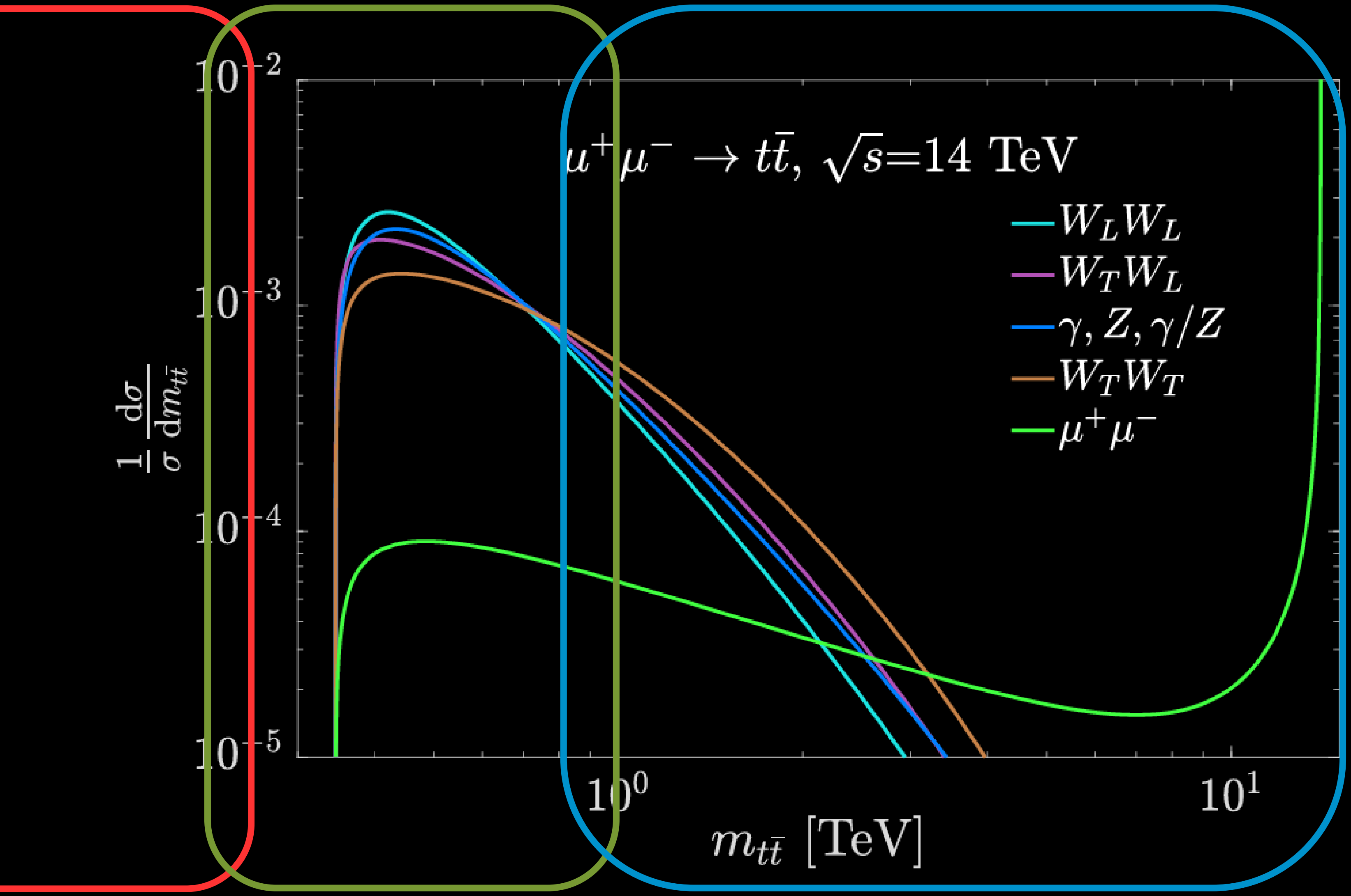
Massive Gauge Boson PDF



Regimes:

- $\sqrt{\tau s} < 2m_V$
- Matching regime (very complicated and higher order inputs needed, matching scheme matters; including matching part from τ/x)
- colinear splitting enhanced regime

Massive Gauge Boson PDF



Regimes:

- $\sqrt{\tau s} < 2m_V$
- Matching regime (very complicated and higher order inputs needed, matching scheme matters; including matching part from τ/x)
- colinear splitting enhanced regime

Muon Collider Physics

Dark Sectors
Speaker: Tien-Tien Yu

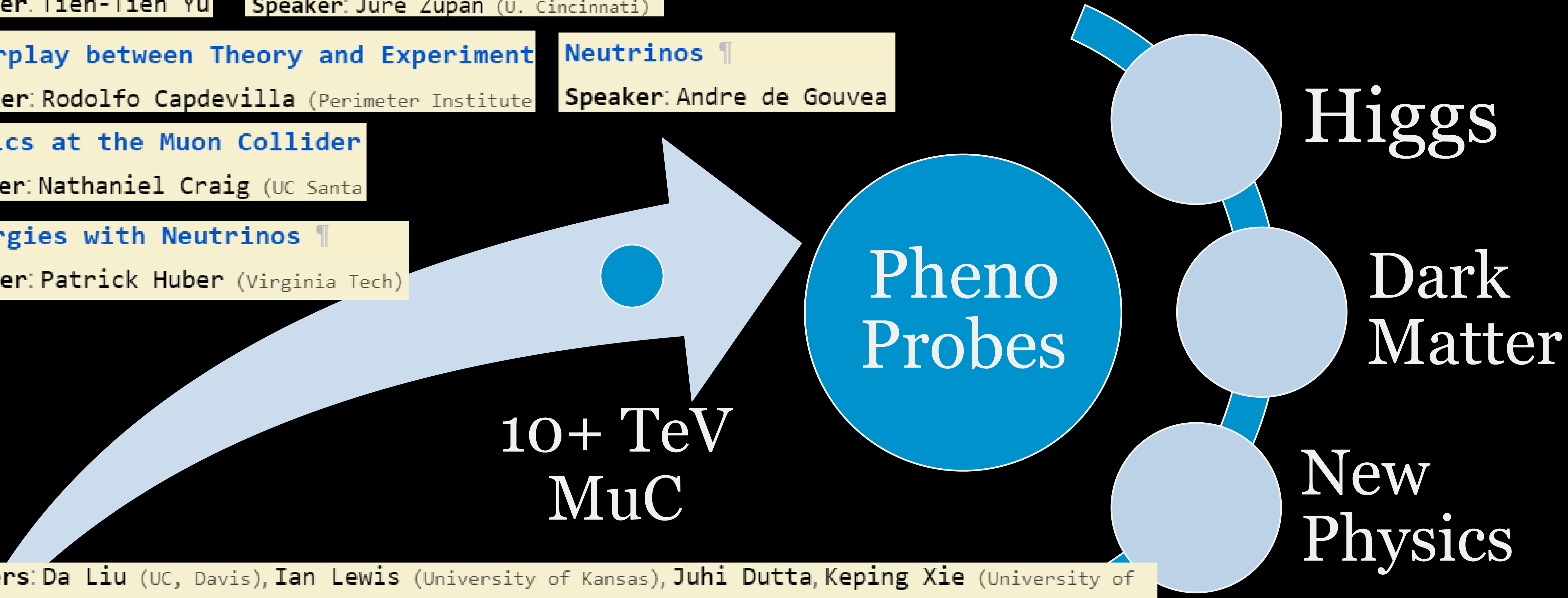
Flavor
Speaker: Jure Zupan (U. Cincinnati)

Interplay between Theory and Experiment
Speaker: Rodolfo Capdevilla (Perimeter Institute

Neutrinos ¶
Speaker: Andre de Gouvea

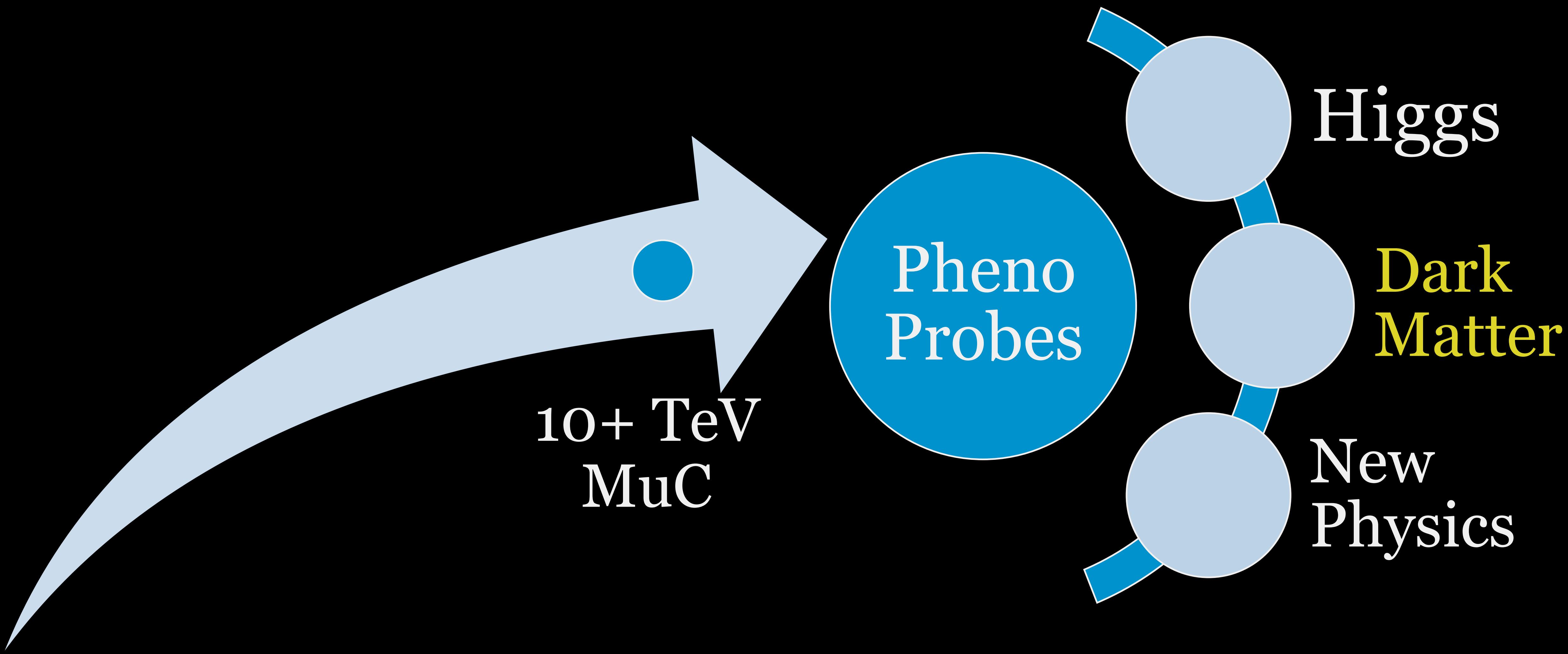
Physics at the Muon Collider
Speaker: Nathaniel Craig (UC Santa

Synergies with Neutrinos ¶
Speaker: Patrick Huber (Virginia Tech)



Speakers: Da Liu (UC, Davis), Ian Lewis (University of Kansas), Juhi Dutta, Keping Xie (University of Pittsburgh), Matheus Hostert (Harvard University), Peiran Li (University of Minnesota), Si Wang, Spencer Chang (University of Oregon), Terrance Figy (Wichita State University)

Muon Collider Physics



Physics Driver

10+ TeV MuC

Pheno Probes

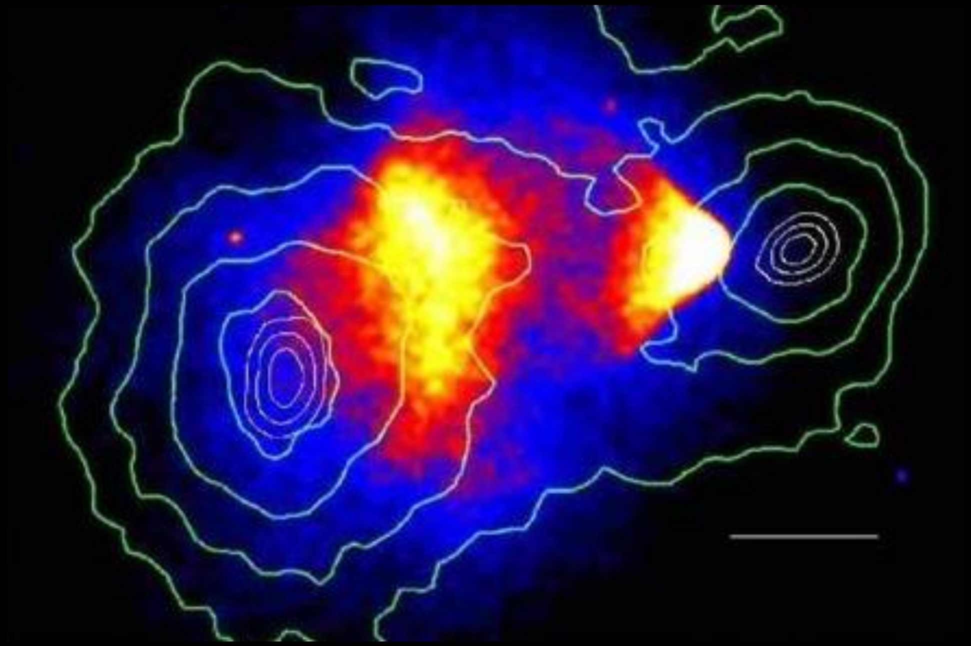
Higgs

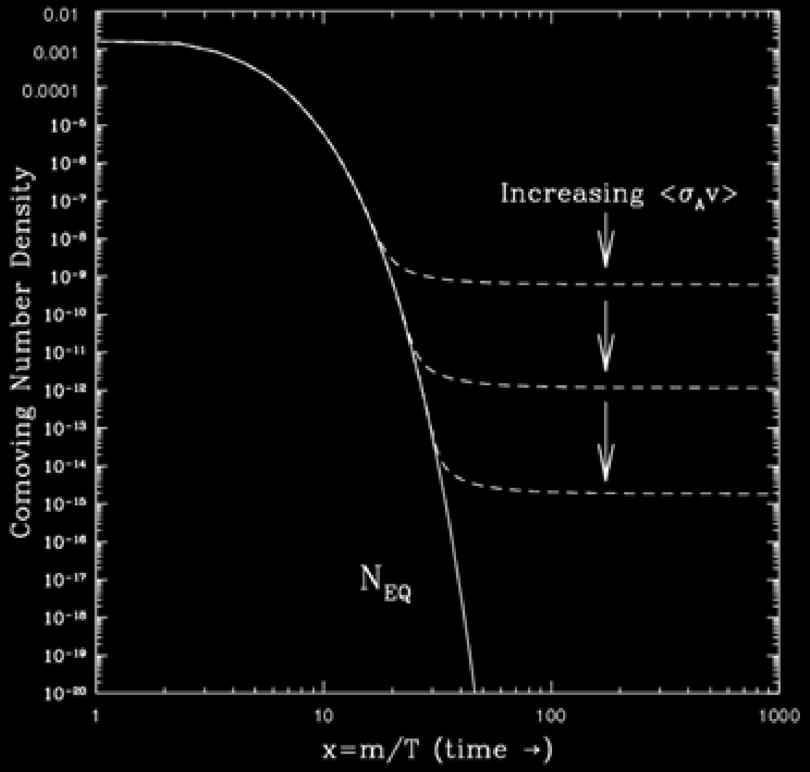
Dark Matter

New Physics

WIMP Dark Matter

Compelling, simple, predictive explanation for thermal, cold dark matter





$$\Omega h^2 \simeq 0.1 \times (\frac{2 \times 10^{-26} \text{cm}^3/\text{sec}}{< \sigma_{\text{eff}} v >_{\text{freeze-out}}})$$

$$< \sigma_{\text{eff}} v >_{\chi\bar{\chi} \to VV} \simeq \frac{\pi \alpha_\chi^2}{m_\chi^2}$$

There is a scale…

Han, ZL, Wang, Wang, 2009.11287, 2203.07351

Basic Pheno Considerations

"non-trivial" to consider muon collider reaches

- Minimal signature
  - Mass splitting O(few hundred MeV)
  - Decay products soft
  - Transition between states fast (<mm for most of the cases)
- Missing ET (at LHC)→Missing Mass (at MuC)
- The interplay between different channels:
  - DY-type dominance but large background
  - VBF-type log-growth but limited available energy
- Photon initial state process important
  - Needs to use photon PDF or Weizsacker-Williams approximation
  - (small) Hacked Madgraph to implement (now a function in Madgraph)
  - Additional divergences often-appear
- Beam induced background (BIB)
  - Affects detector coverage
  - Affects photon, muon threshold
  - Affects disappearing track considerations

Missing Mass signature:

- Simple and inclusive (hence also most conservative)
- Mono-photon
- VBF-dimuon
- Mono-muon

Disappearing track signature:

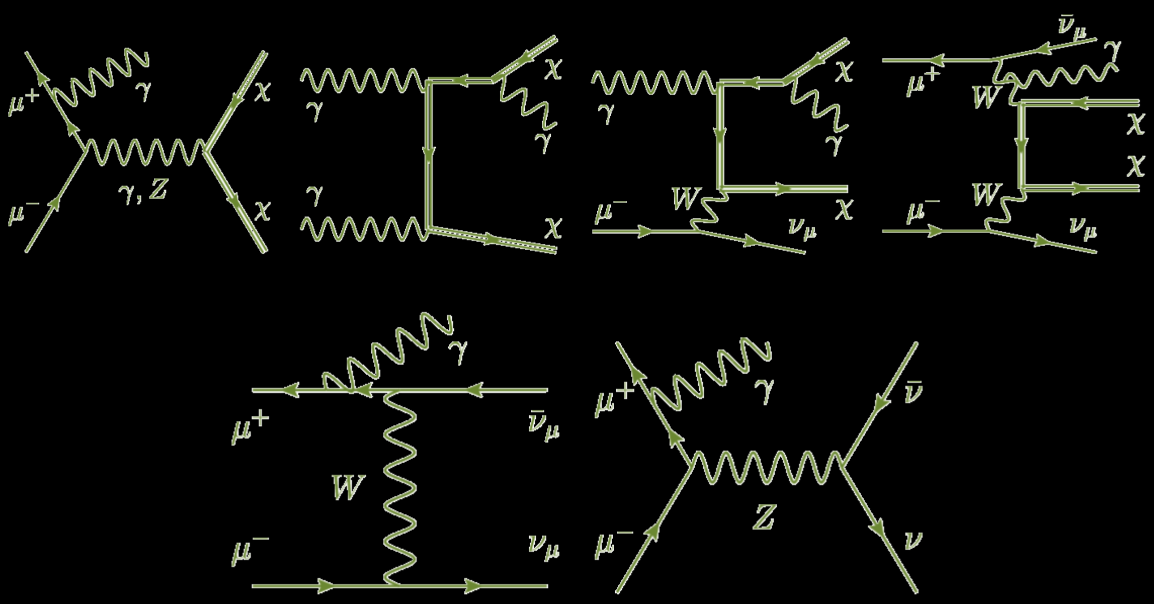
- Exclusive but challenging
- Most useful for Wino and Higgsinos
- Great potential

$$\sqrt{s} = 3,\ 6,\ 10,\ 14,\ 30 \text{ and } 100 \text{ TeV}$$

$$\mathcal{L} = 1,\ 4,\ 10,\ 20,\ 90,\ \text{and } 1000 \text{ ab}^{-1}$$

Mono-Photon

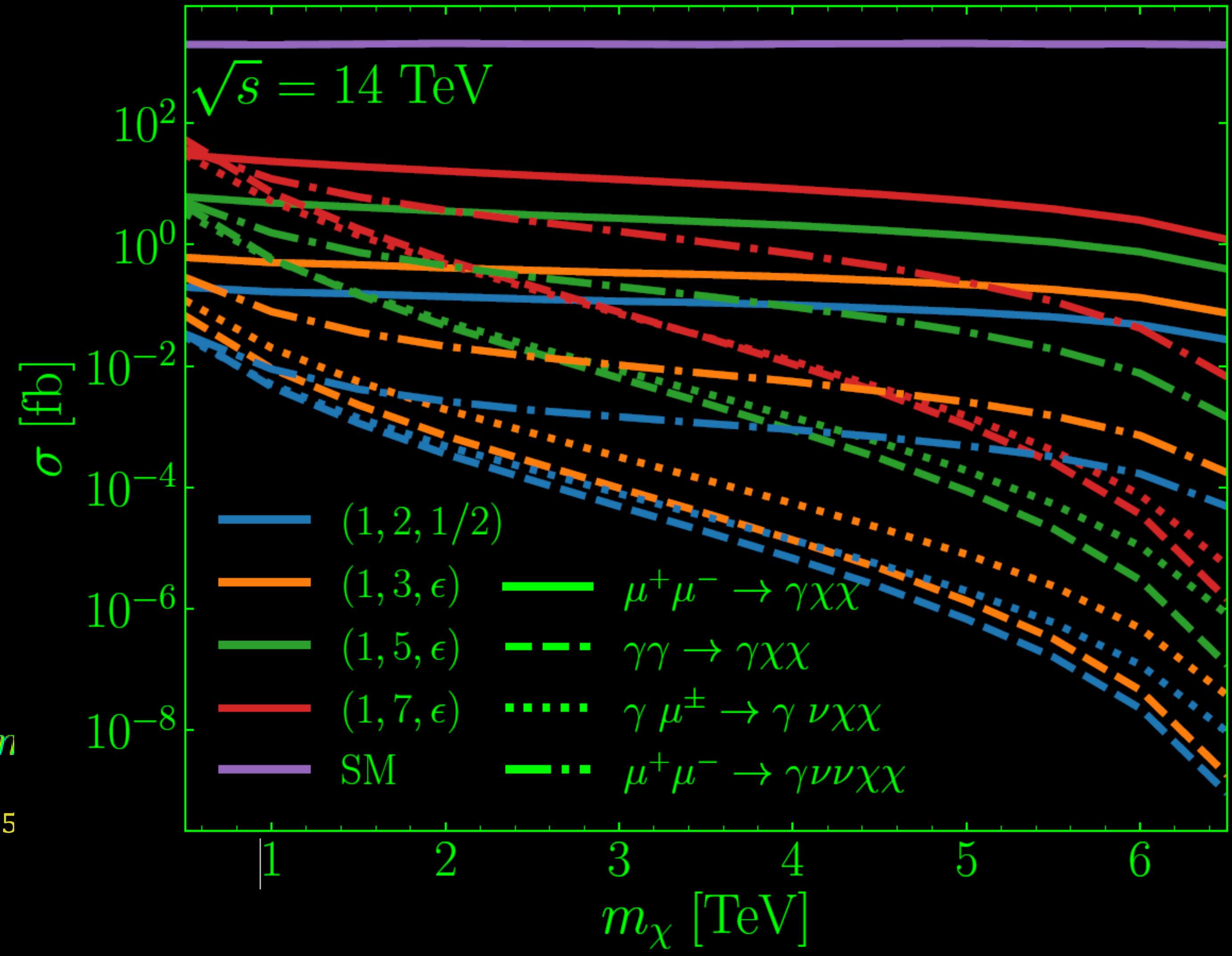
All combinations of components of the EW multiplet are included, so-long as they respect the underlying gauge symmetries



$$10^\circ < \theta_\gamma < 170^\circ$$

$$E_\gamma > 50\ \text{GeV}, \quad m^2_{\text{missing}} \equiv (p_{\mu^+} + p_{\mu^-} - p_\gamma)^2 > 4$$

Rate grows with n-plets as roughly $n^{2\sim3}$ (DY) and $n^{4\sim5}$
Doublet and Triplet very hard to probe

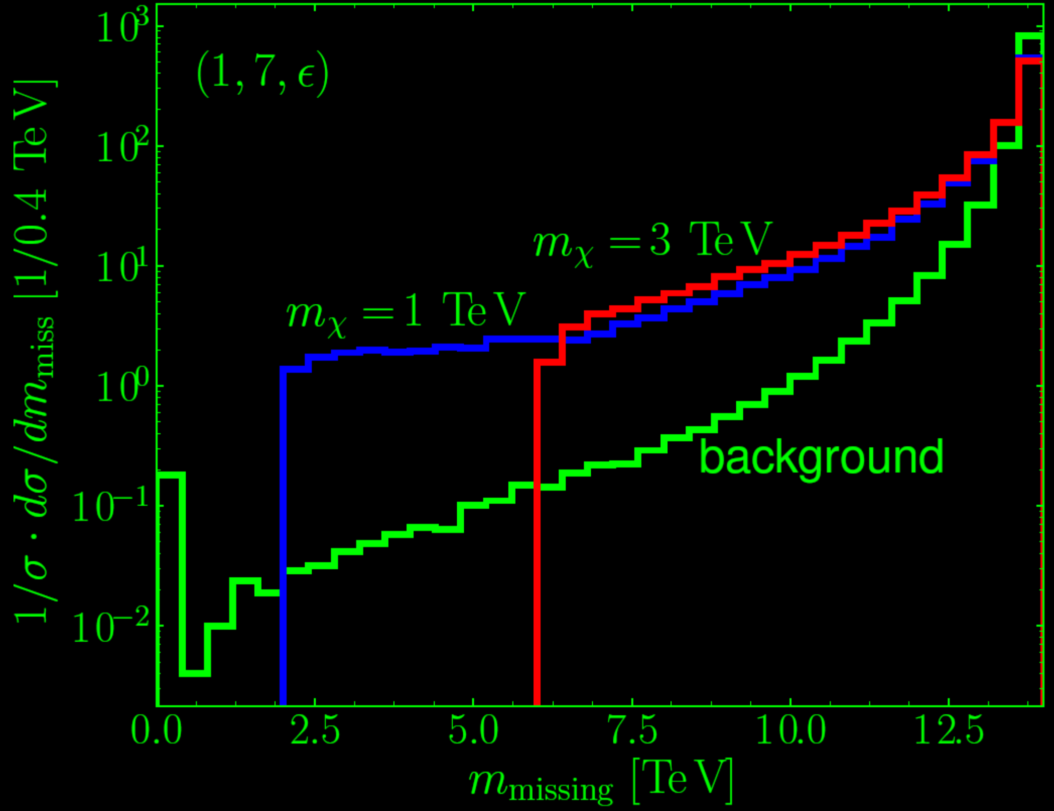


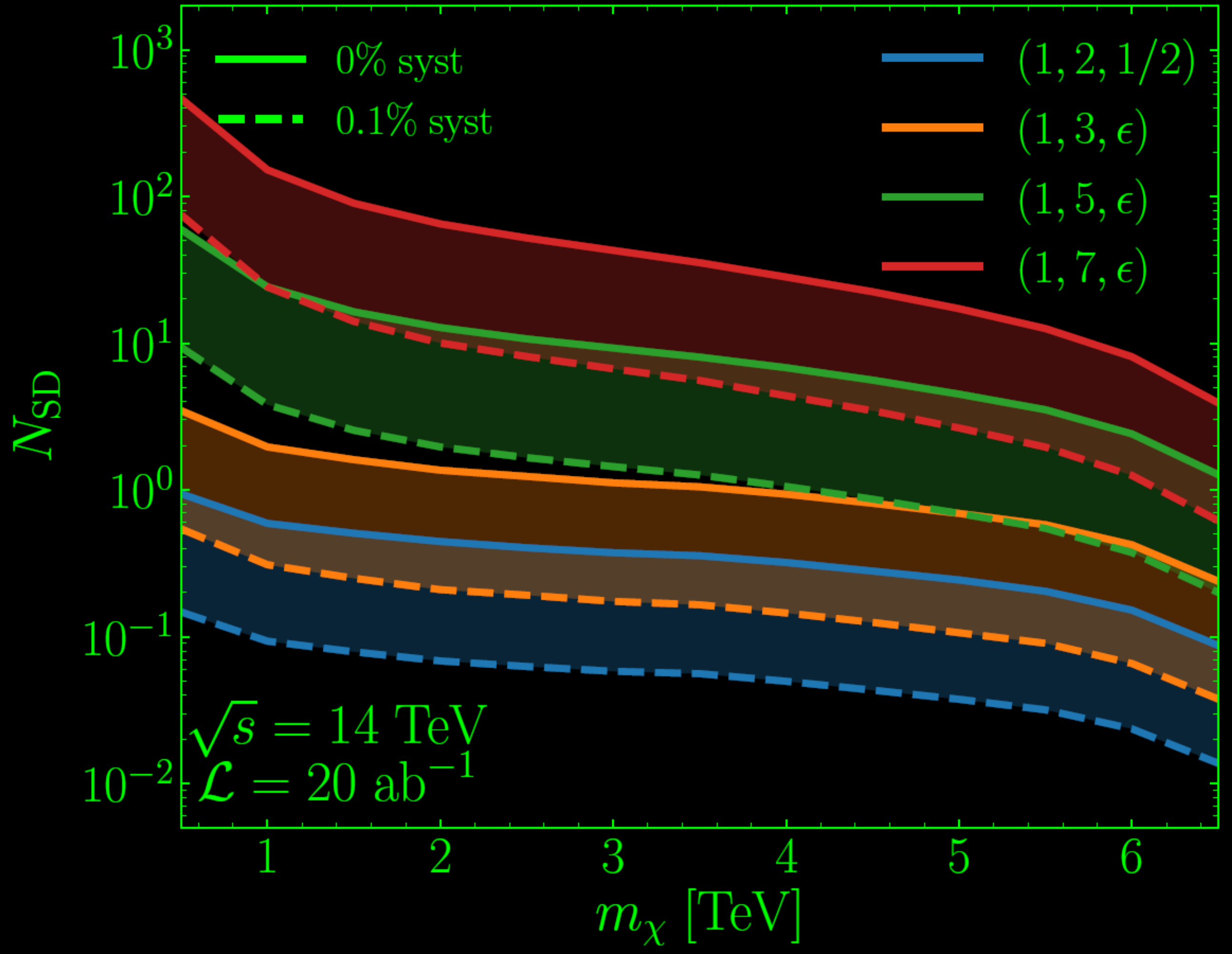
Mono-photon

Signal-background ratio 10^-3
At lepton colliders systematics controlled to this level should be achievable but requires theory & experimental work

Missing mass:

- Sharp kinematic features
- Signal-background separation
- Signal parameter determination

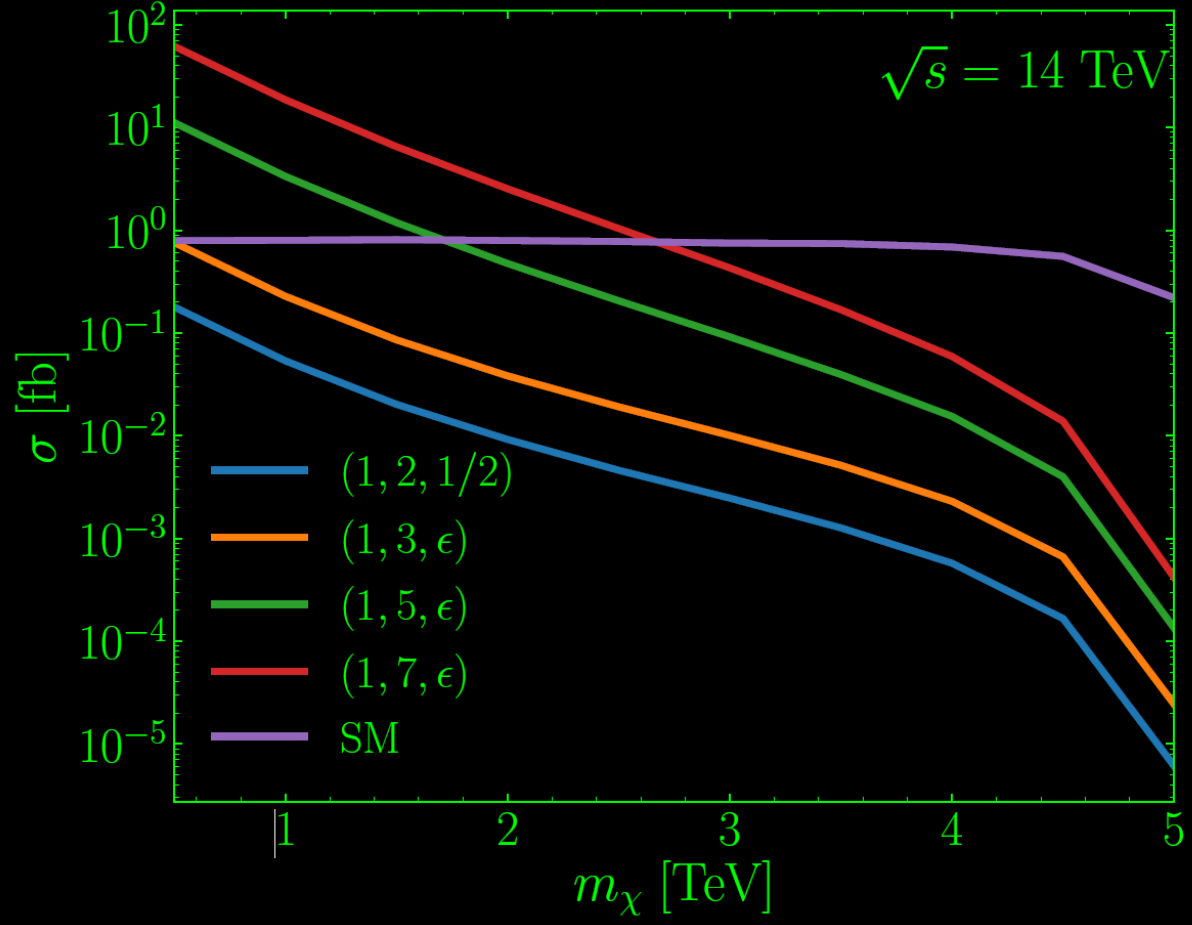




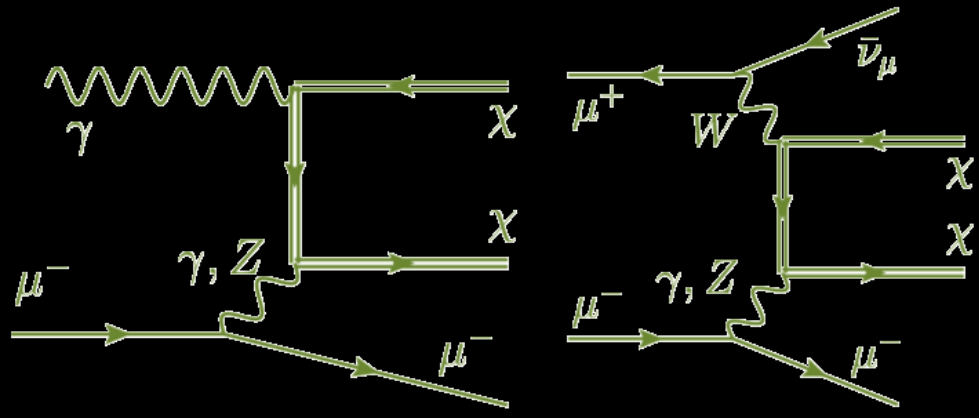
Unique Mono-Muon Channel

Apparent “Charge Violation” channel

(very different from the LHC)



Signature: Energetic mono muon

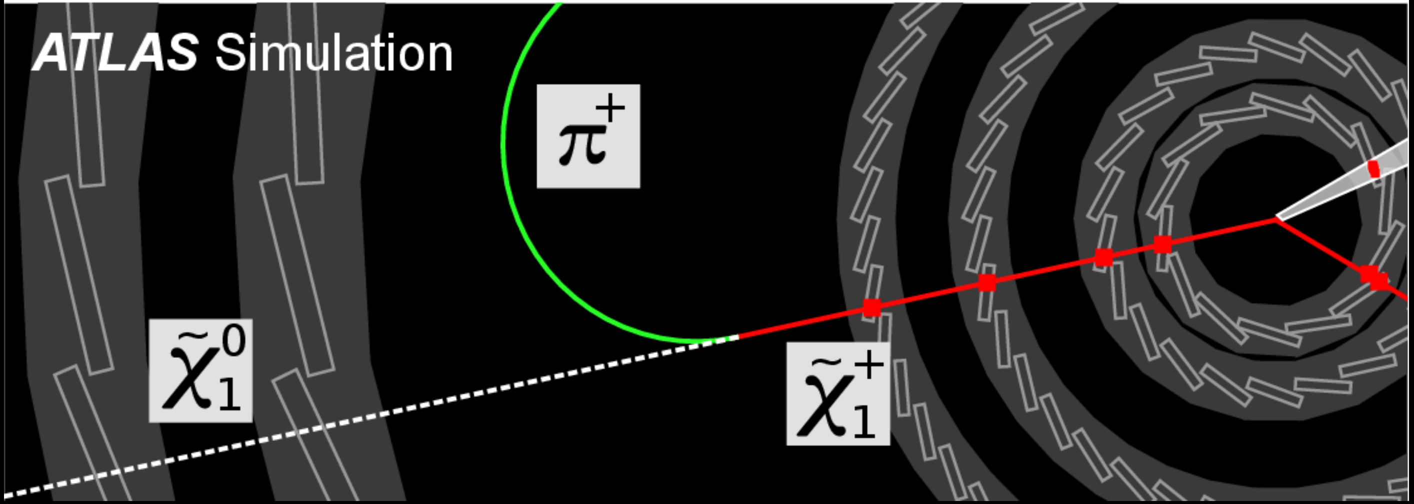


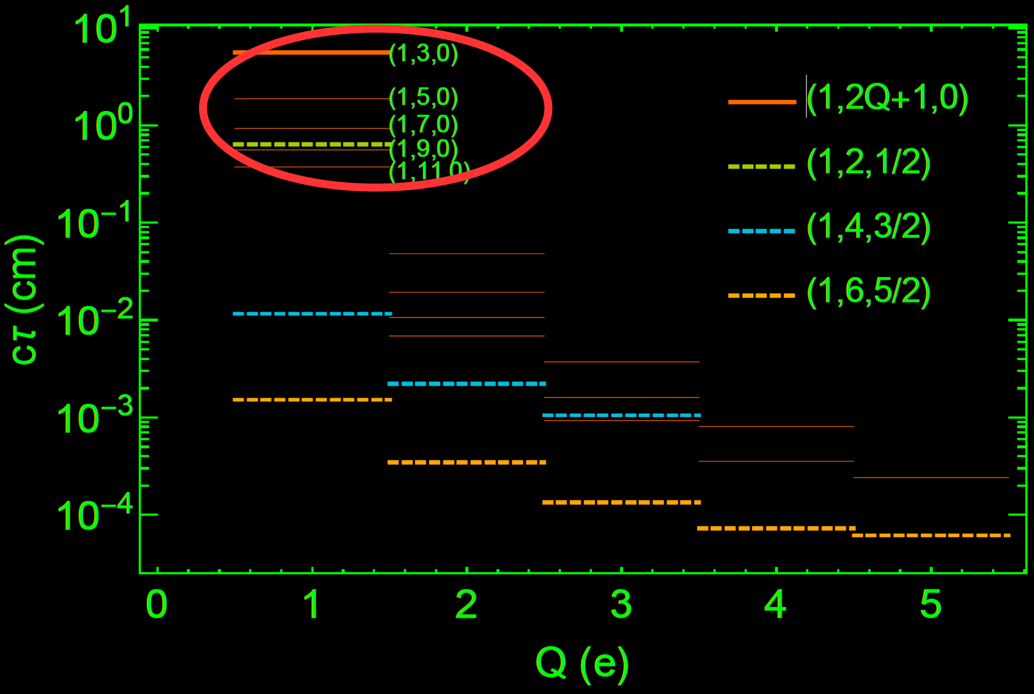
Muon pairs   muon + missing mass

One charge is missed due to the soft (non-reconstructable) decays of the charged states

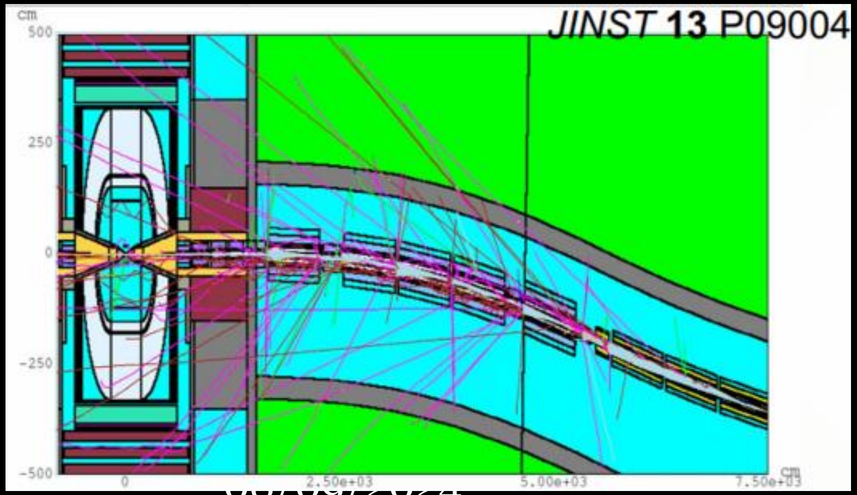
Unique and powerful channel for low-rate channels.

Disappearing Tracks: next to minimal signatures





- Only useful for searches using charge 1 states
- Still, all higher charged states will cascade back to charge 1 states promptly
- Use all the production rates of charged states
- Mono-photon+disappearing tracks
- Beam Induced Background



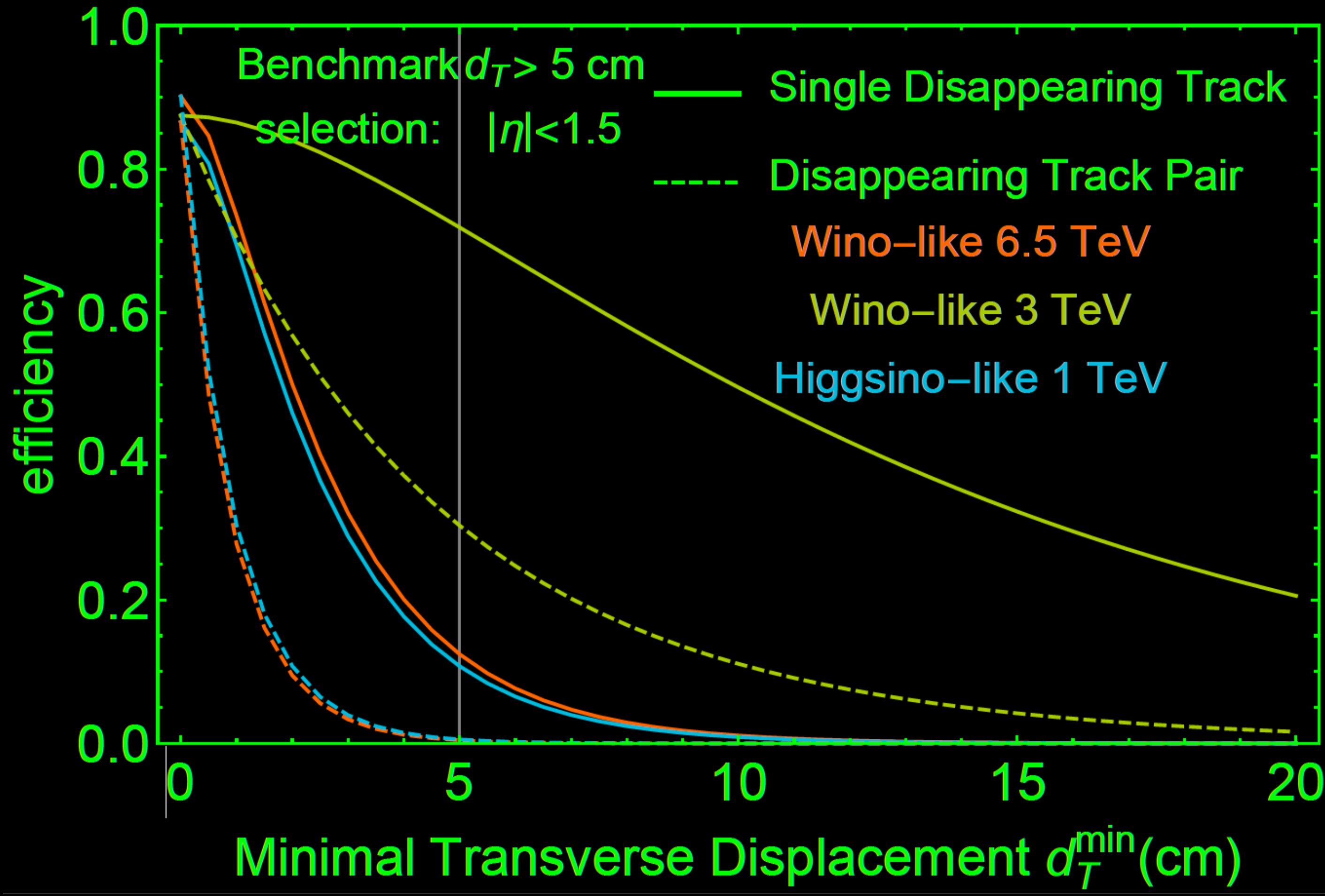
Also see a recent optimization work looking for soft pions, achieving sensivity to Higgsino at 3TeV MuC, Capdevilla, Meloni, Zurita, 2405.08858

Minimal transverse displacement

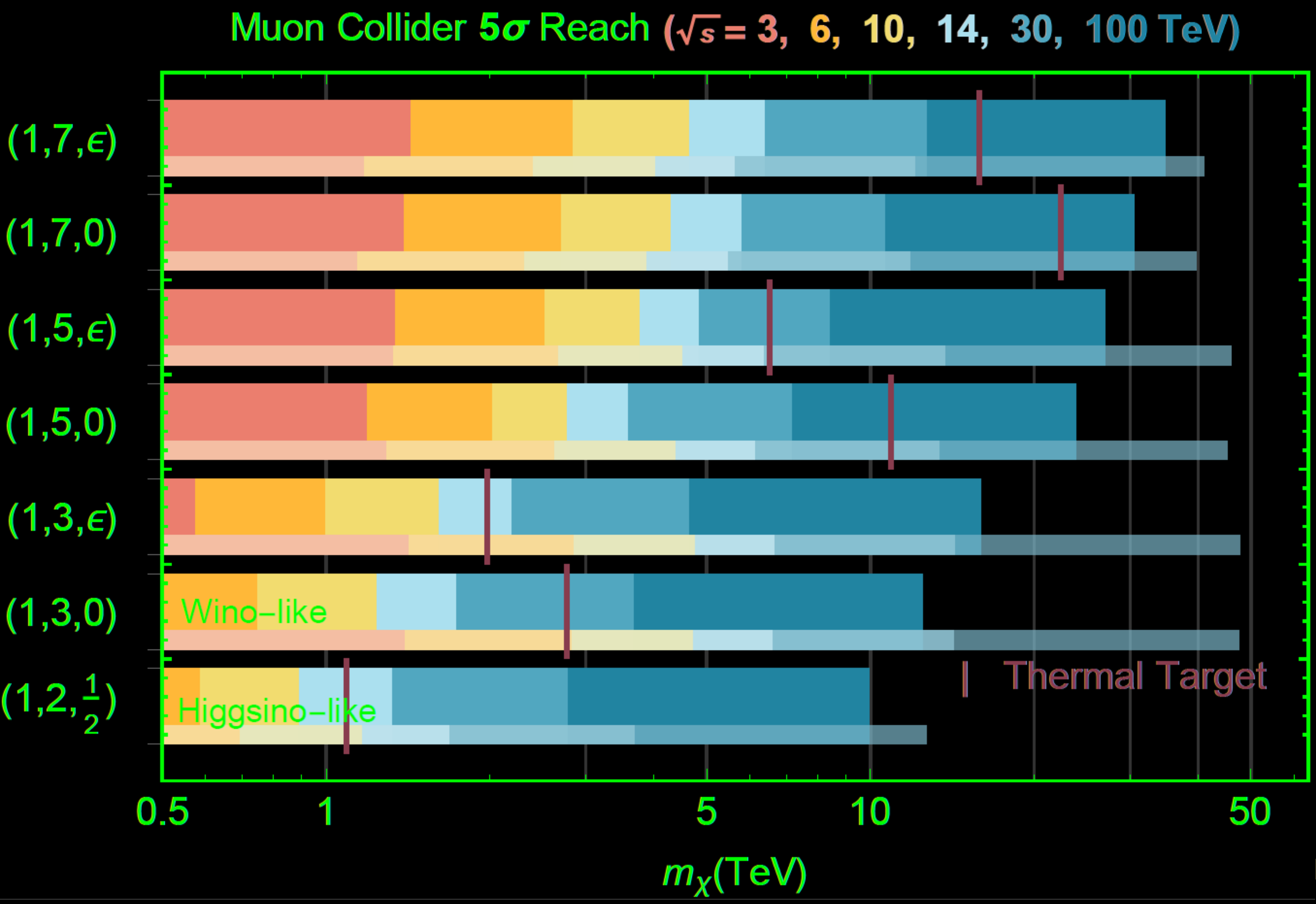
- Only use the central tracks, |eta|<1.5
- Current design have first layer of pixel detector at 3cm (new discussion about 2cm)
- We assume at least two-hits can be measured at 5cm
- Show both pair reconstruction or single reconstruction results
- Requiring 50 signal events for discovery

$$d_T^{\rm min} = 5 \text{ cm with } |\eta_\chi| < 1.5$$

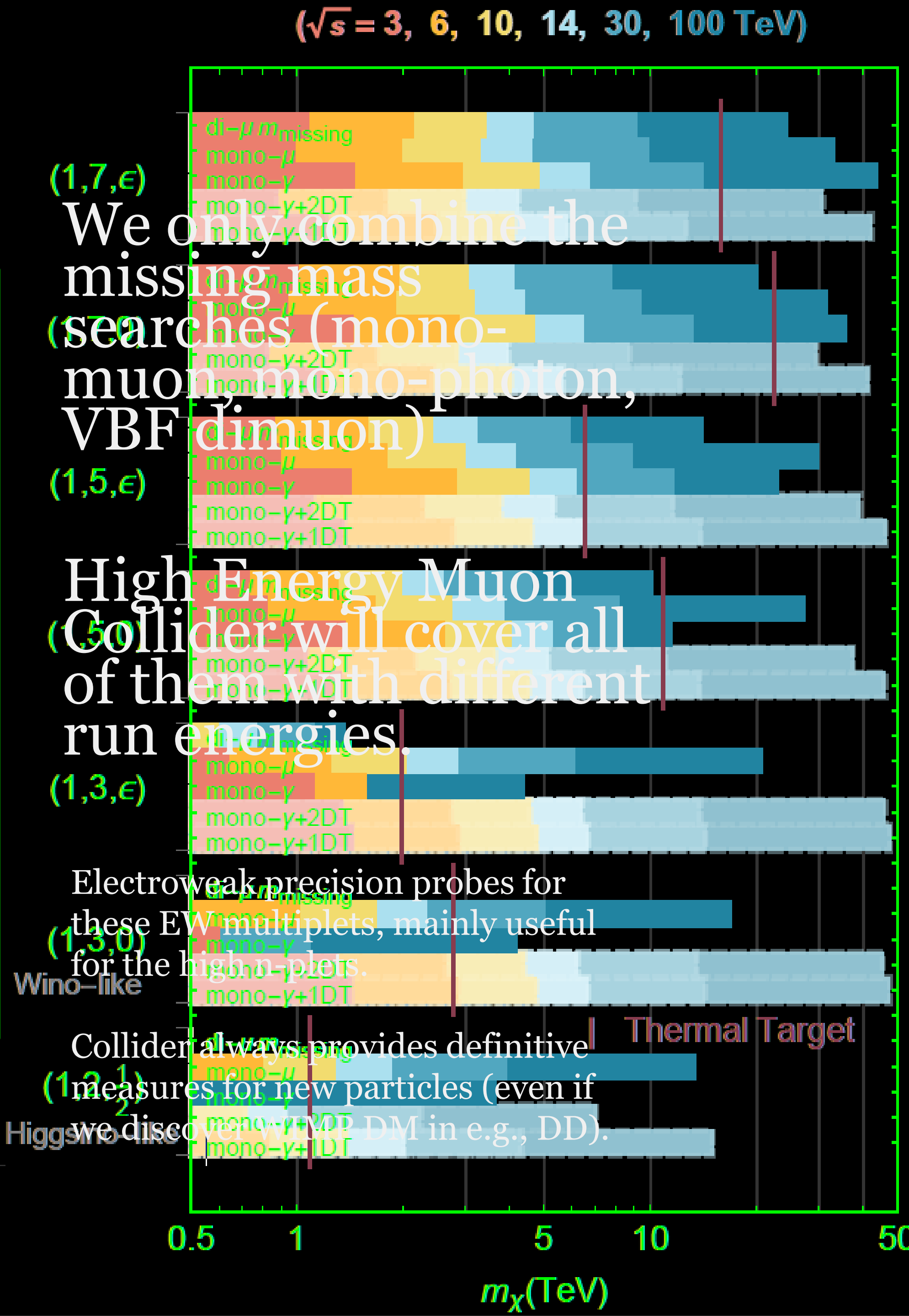
$$\epsilon_\chi(\cos\theta, \gamma, d_T^{\rm min}) = \exp\left(\frac{-d_T^{\rm min}}{\beta_T \gamma c\tau}\right)$$



WIMP discovery Machine



High Energy Muon Collider will cover all of them with different run energies.



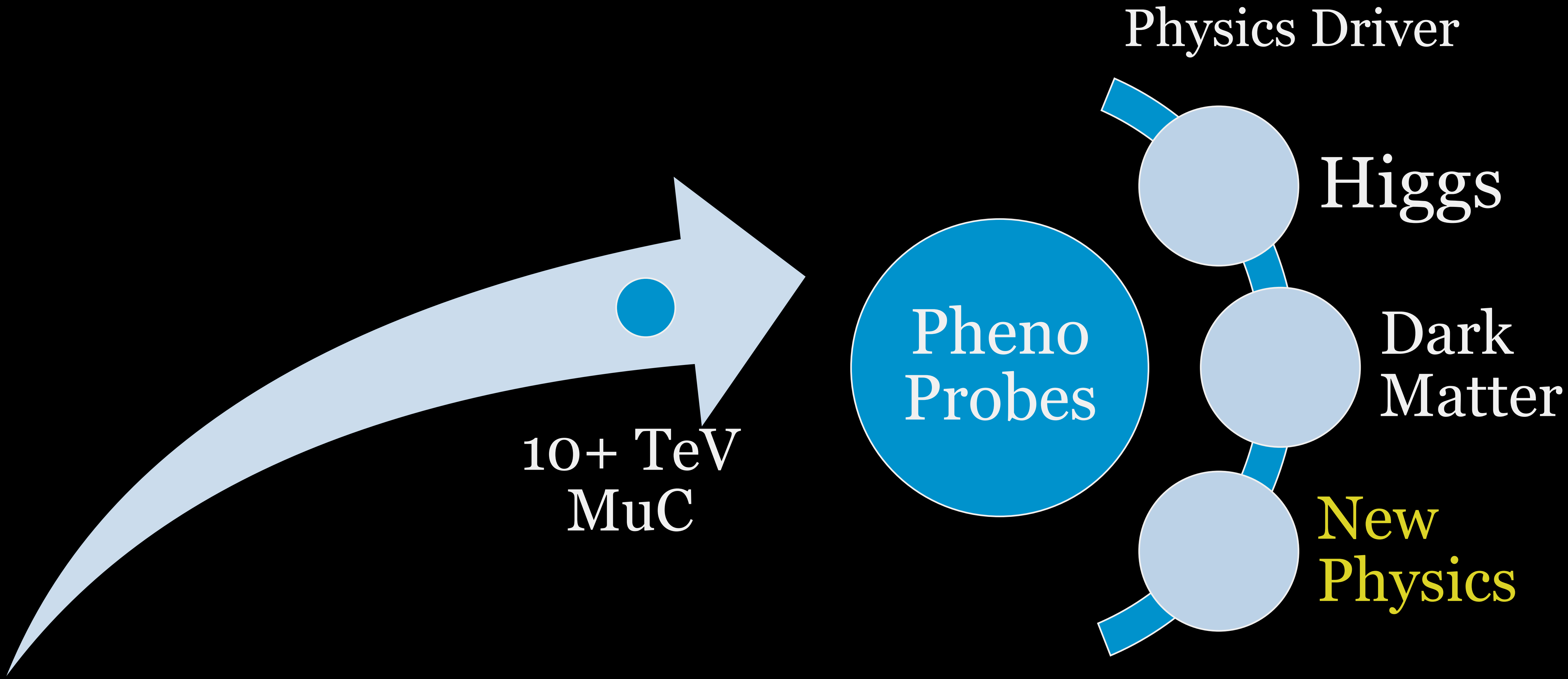
We only combine the missing mass searches (mono-muon, mono-photon, VBF dimuon)

High Energy Muon Collider will cover all of them with different run energies.

Electroweak precision probes for these EW multiplets, mainly useful for the high n-plets.

Collider always provides definitive measures for new particles (even if we discover WIMP DM in e.g., DD).

The Muon Shot



Physics Driver

10+ TeV
MuC

Pheno
Probes

Higgs

Dark
Matter

New
Physics

Neutrino is a puzzling sector

- In SM, neutrino is massless. While the experiments have confirmed its tiny mass <0.1 eV.
- Seesaw mechanism
- We choose to work in a simple scenario. Suppose

  there is a heavy neutral lepton. We can parametrize

  its mass m_N and mixing angle with SM neutrino $U_\ell = sin\theta_\ell$.

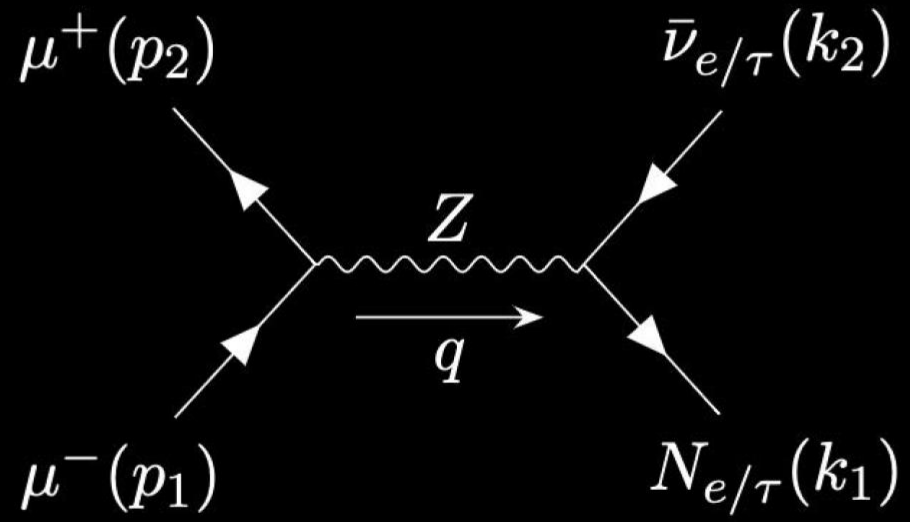
$$\mathcal{L} = \mathcal{L}_W + \mathcal{L}_Z + \mathcal{L}_H$$

$$\mathcal{L}_W = \frac{gU_l}{\sqrt{2}}\left(W_\mu \bar{l}_L \gamma^\mu N + h.c.\right)$$

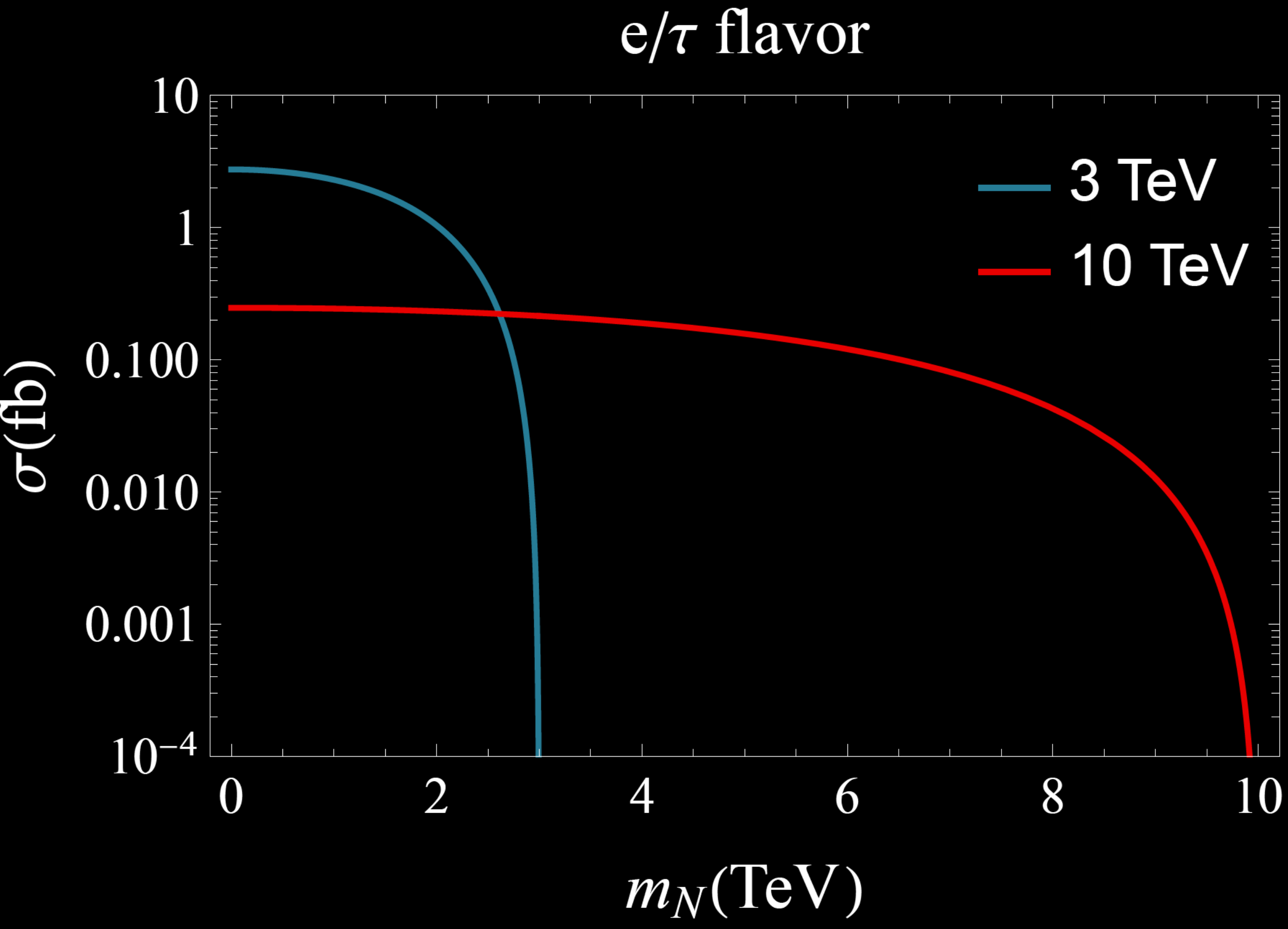
$$\mathcal{L}_Z = -\frac{gU_l}{2\cos\theta_w} Z_\mu \left(\bar{\nu}_L \gamma^\mu N + \bar{N}\gamma^\mu \bar{\nu}_L\right)$$

$$\mathcal{L}_H = -\frac{U_l m_N}{v} h \left(\bar{\nu}_L N + \bar{N}\nu_L\right)$$

S-channel production (e/ μ /τ flavored)



- 1/s suppressed;
- Flat rate until near the threshold s/2
- $O(fb)$ cross section;

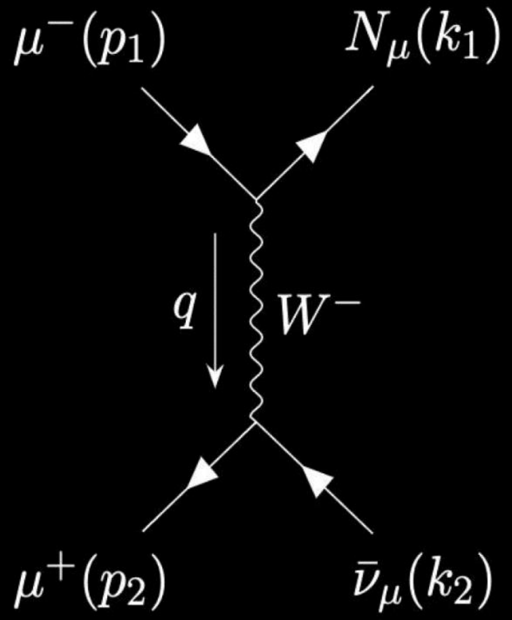


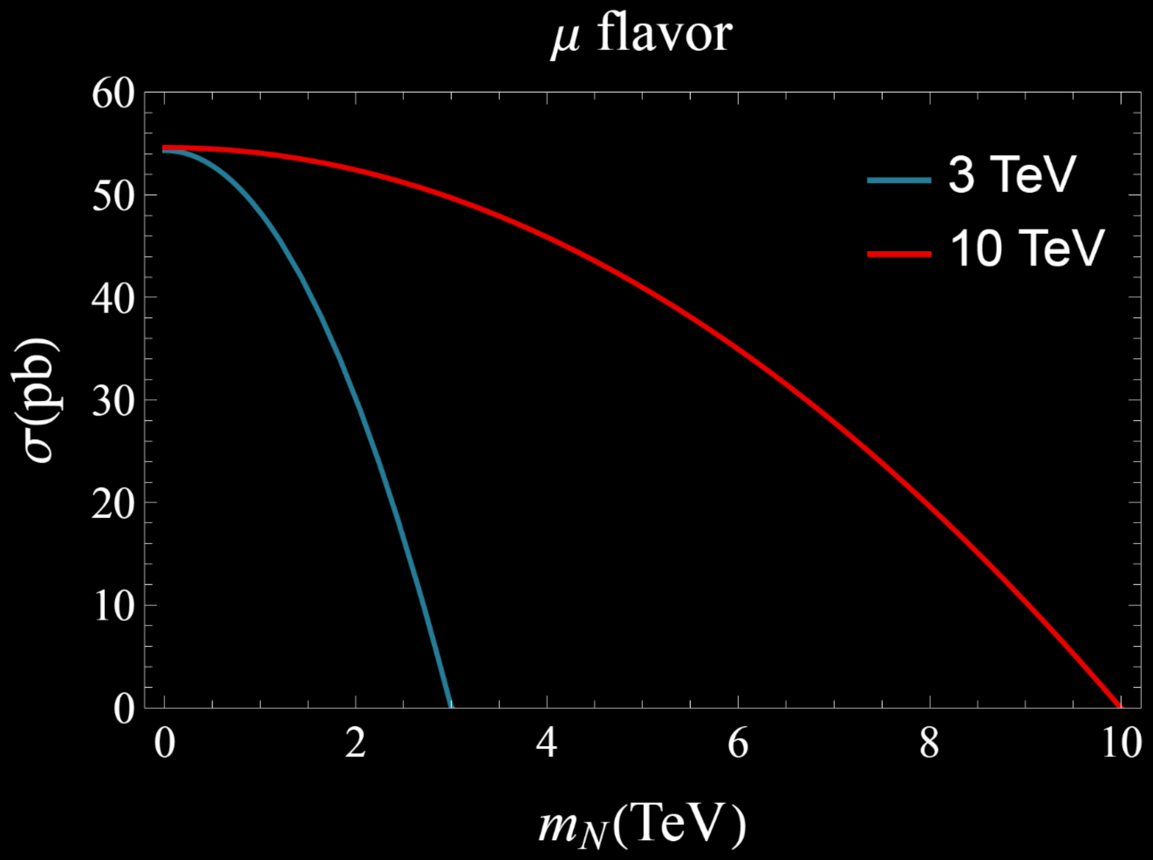
Peiran Li, Kun-Feng Lyu, ZL, 2301.07117

Muon Flavor

Production dominated by t-channel

$$\mu^+ + \mu^- \rightarrow N_\mu + \bar{\nu}_\mu$$

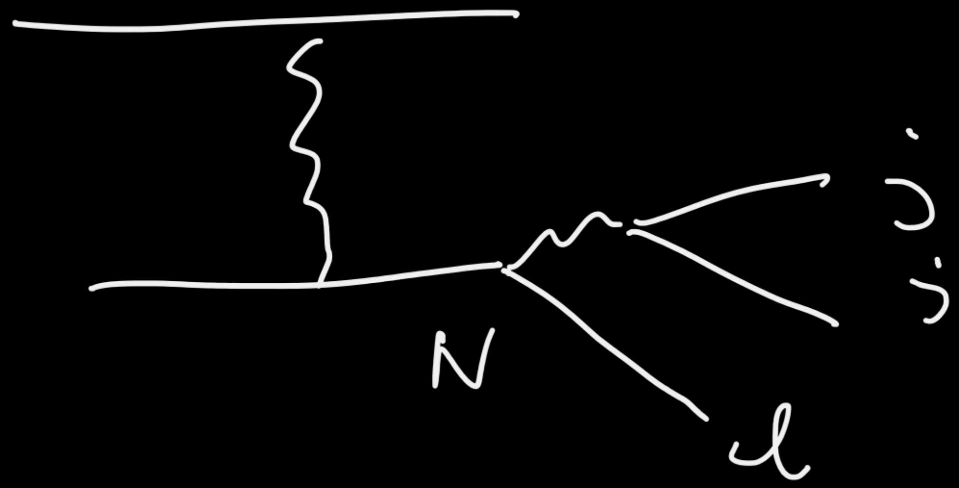




Type	Signal process	$\sigma/\lvert U_\mu\rvert^2$ (w. conj. channel) $m_N = 1$ TeV
t-channel	$\mu^+\mu^- \longrightarrow N_\mu\bar{\nu}_\mu$	20.28 pb
VBF	$\mu^+\mu^- \longrightarrow \mu^+\mu^- N_\mu\bar{\nu}_\mu$	~ 1 pb
VBF	$\mu^+\mu^- \longrightarrow \bar{\nu}_\mu\nu_\mu N_\mu\bar{\nu}_\mu$	~ 0.1 pb

Decay selection $m_N > O(100)$ GeV

- $\boxed{N_\mu \to W^+ + \mu^-}$
- $N_\mu \to Z + \nu_\mu$
- $N_\mu \to H + \nu_\mu$



$$\boxed{N_\mu \to W^+ + \mu^-, \qquad W \to jj}$$

$$\mu^+ + \mu^- \to N_\mu + \bar{\nu}_\mu \to \boxed{jj + \mu^- + \bar{\nu}_\mu}$$

The dijets almost come from onshell W/Z boson.

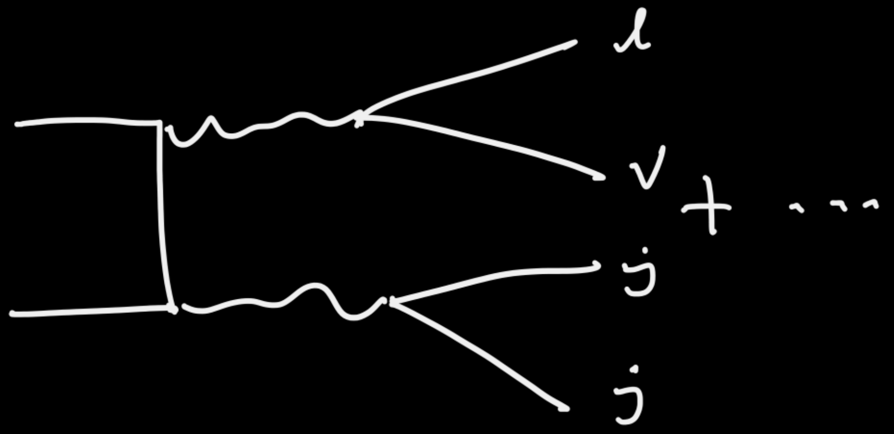
We focus on the final states of W and μ and reconstruct its invariant mass distribution.

Including the charge conjugation process

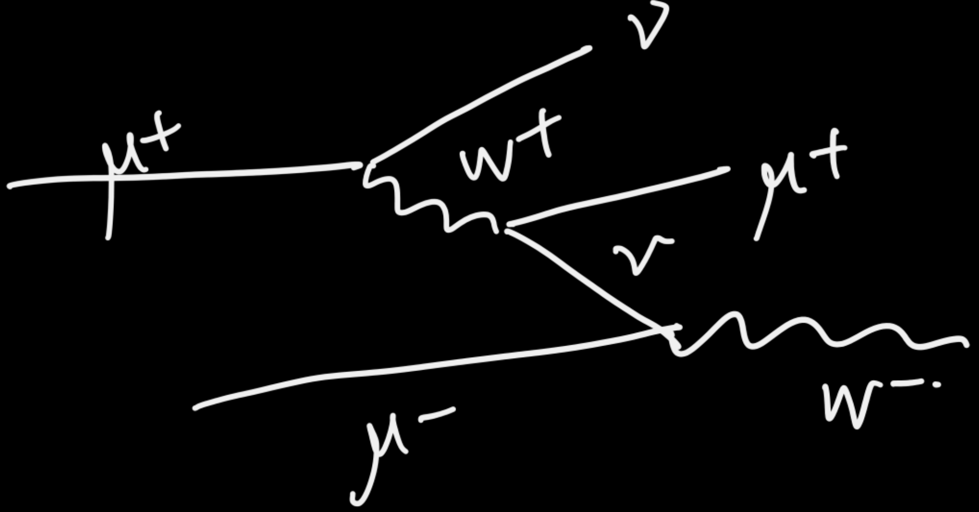
10TeV Background

Type	Background process	σ (w. conj. channel)	Pre-selection cut (PSC)
t-channel	$\mu^+\mu^- \longrightarrow W^+\mu^-\bar{\nu}_\mu$	0.214 pb	PSC
t-channel	$\mu^+\mu^- \longrightarrow Z\mu^+\mu^-$	0.464 pb	PSC & missing μ^+
VBF	$\mu^+\mu^- \longrightarrow \mu^+\mu^-W^+\mu^-\bar{\nu}_\mu$	0.401 pb	PSC & missing $\mu^+\mu^-$
VBF	$\mu^+\mu^- \longrightarrow \bar{\nu}_\mu\nu_\mu W^+\mu^-\bar{\nu}_\mu$	0.0686 pb	PSC

Need special treatment

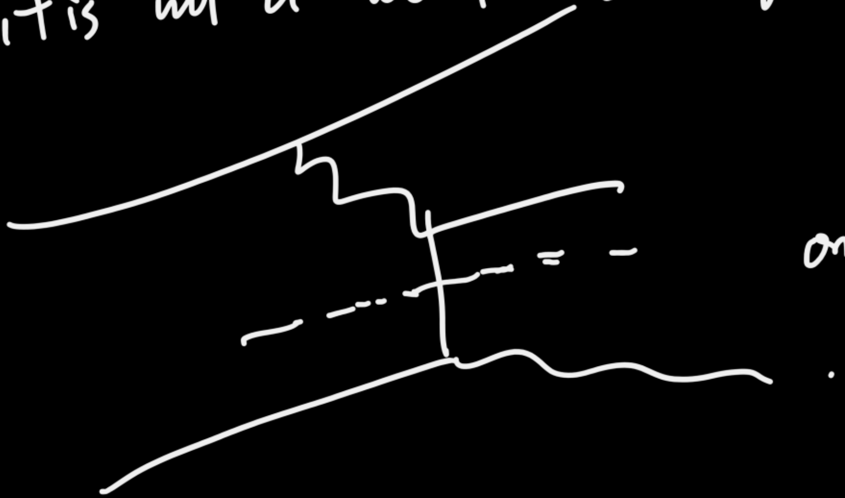


Physical Poles from Unstable Particle Scattering



$\mu^+\mu^- \to W^- \mu^+ \bar{\nu}$

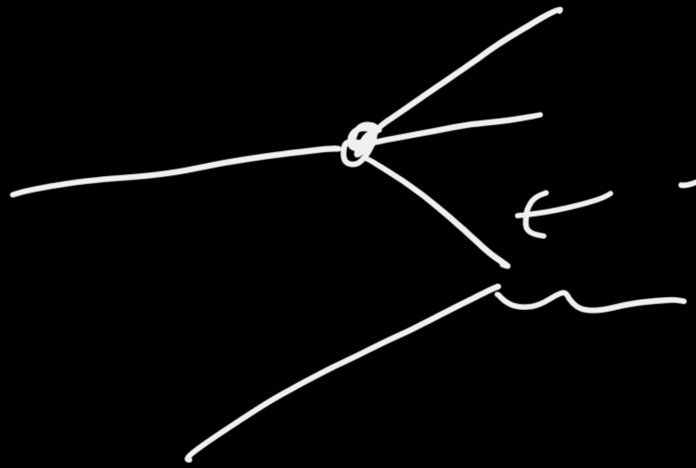
But this is not normal. QFT divergences.
it is not a "local" divergence, it comes from



If you calculate it, analytically or any honest generator, you will get ∞

And this is not a part of the ν-PDF. Neither, as muon decays

So, what regulates it?



← this is the asymptotic state that participate in the interactions

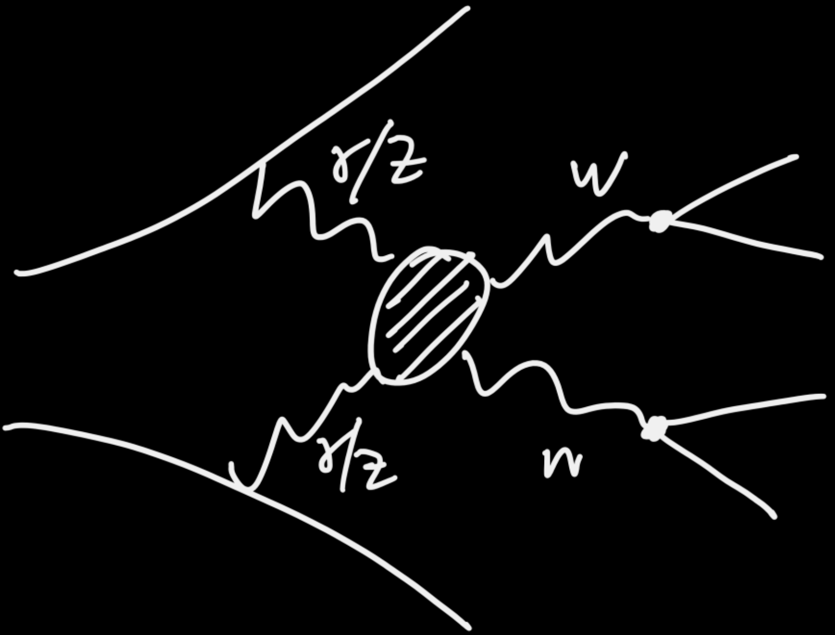
the regulator is beam size. , finite width, etc.

(there was a debate in 1990s but there is a better formalism IMO, I'm developing it)

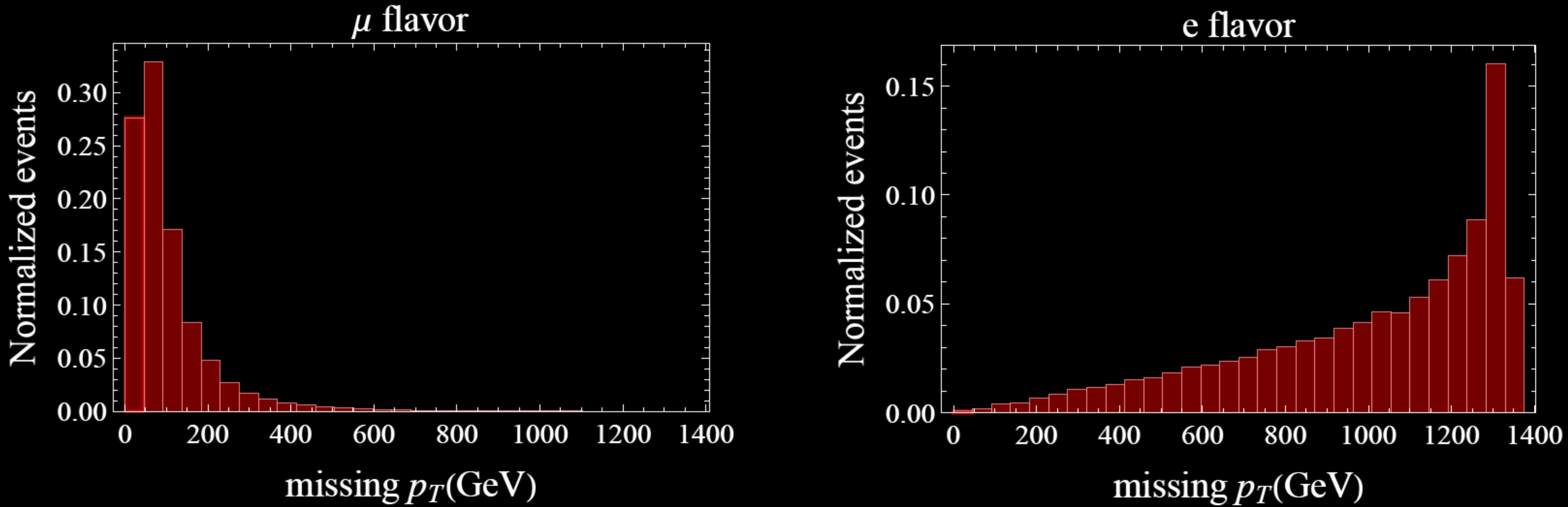
10TeV Background

Type	Background process	σ (w. conj. channel)	Pre-selection cut (PSC)
t-channel	$\mu^+\mu^- \longrightarrow W^+\mu^-\bar{\nu}_\mu$	0.214 pb	PSC
t-channel	$\mu^+\mu^- \longrightarrow Z\mu^+\mu^-$	0.464 pb	PSC & missing μ^+
VBF	$\mu^+\mu^- \longrightarrow \mu^+\mu^-W^+\mu^-\bar{\nu}_\mu$	0.401 pb	PSC & missing $\mu^+\mu^-$
VBF	$\mu^+\mu^- \longrightarrow \bar{\nu}_\mu\nu_\mu W^+\mu^-\bar{\nu}_\mu$	0.0686 pb	PSC

Need special treatment:
Effective Photons
(or EW PDF)

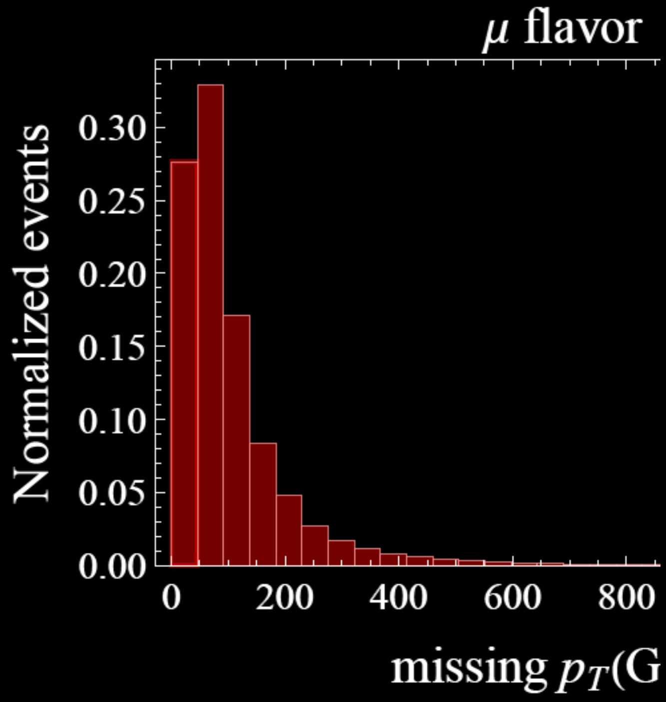


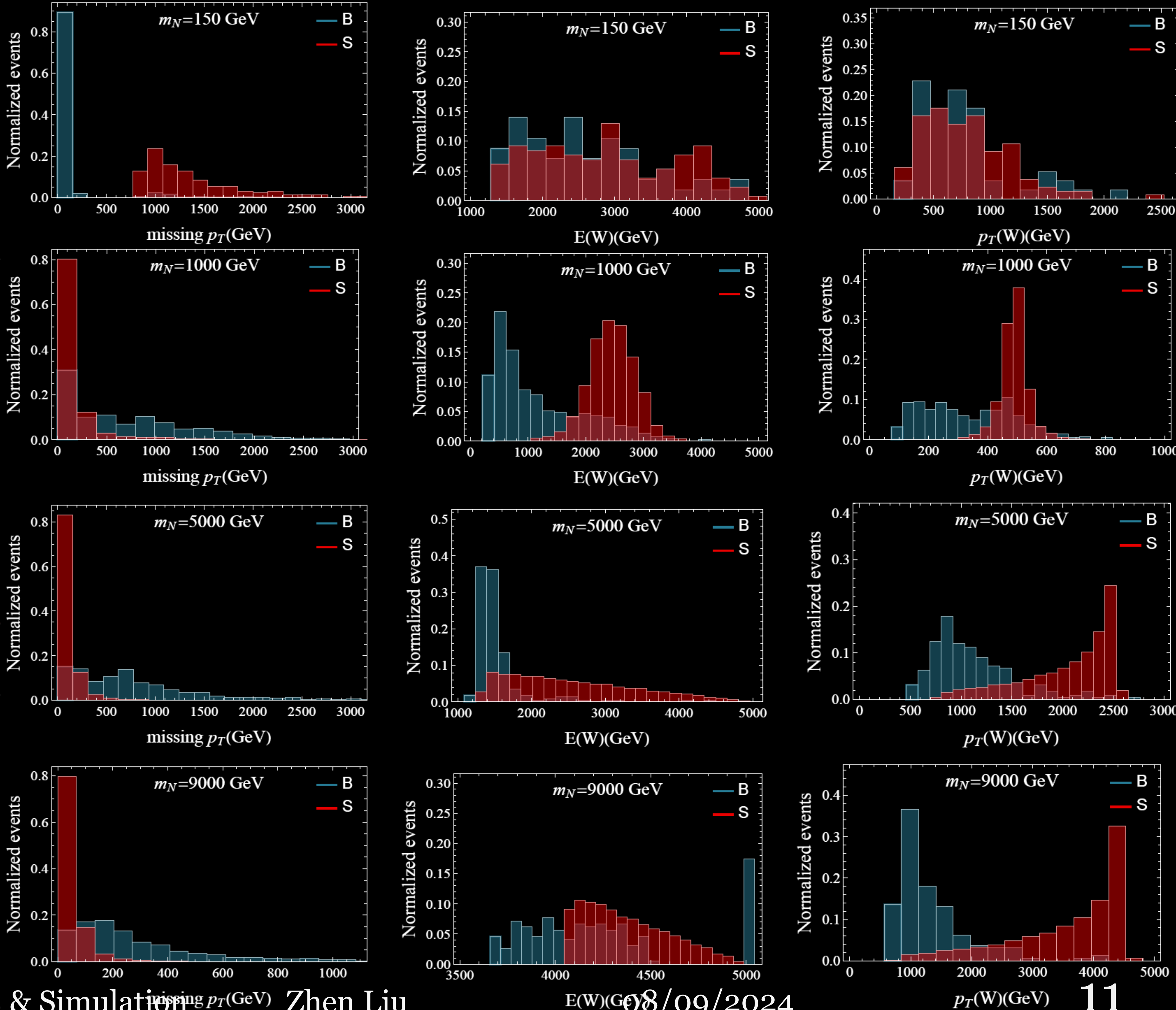
Kinematics



μ flavor
Normalized events
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0
200
400
600
800
1000
1200
1400
missing p_T(GeV)
e flavor
Normalized events
0.00
0.05
0.10
0.15
0
200
400
600
800
1000
1200
1400
missing p_T(GeV)

Kinematics


μ flavor
Normalized events
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0
200
400
600
800
missing p_T(G


m_N=150 GeV
m_N=1000 GeV
m_N=5000 GeV
m_N=9000 GeV
B
S
Normalized events
missing p_T(GeV)
E(W)(GeV)
p_T(W)(GeV)
0.0
0.2
0.4
0.6
0.8
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.1
0.2
0.3
0.4
0.5
0
200
400
500
600
800
1000
1500
2000
2500
3000
3500
4000
4500
5000

Cutflow Analysis

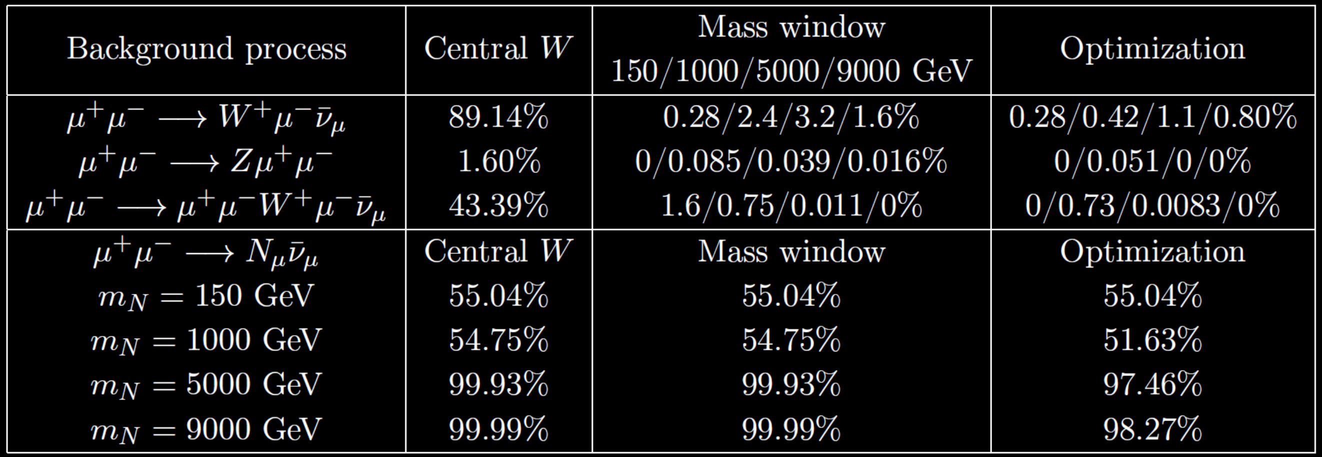
- Pre-selection: require single visible charged lepton
    - $|\eta(\mu)| < 2.5$ and $p_T(\mu) > 20$ GeV

- Central hadronic W selection: require visible on-shell W boson
    - $|\eta(W)| < 2.5$ and $p_T(W) > 20$ GeV

- Mass window: reconstructed mass $m_{W\mu}$ within $m_N \pm 5\% m_N$

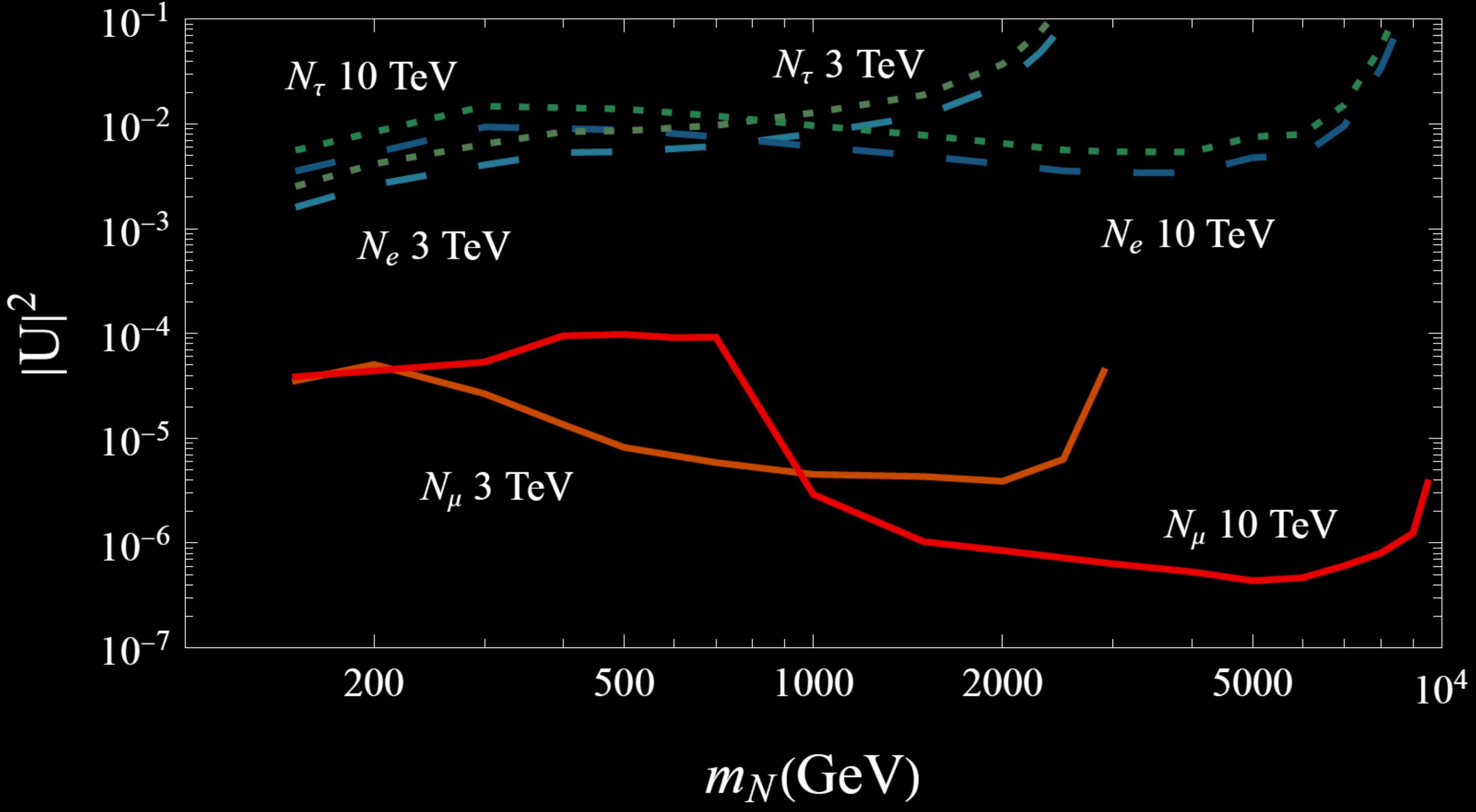
- Optimization cuts:
    - Customized cut on missing p_T, $E(W)$, $p_T(W)$ for each m_N benchmark

$$\mu^+ + \mu^- \rightarrow N_\mu + \bar{\nu}_\mu \rightarrow jj + \mu^- + \bar{\nu}_\mu$$



Background process	Central W	Mass window 150/1000/5000/9000 GeV	Optimization
$\mu^+\mu^- \longrightarrow W^+\mu^-\bar{\nu}_\mu$	89.14%	0.28/2.4/3.2/1.6%	0.28/0.42/1.1/0.80%
$\mu^+\mu^- \longrightarrow Z\mu^+\mu^-$	1.60%	0/0.085/0.039/0.016%	0/0.051/0/0%
$\mu^+\mu^- \longrightarrow \mu^+\mu^-W^+\mu^-\bar{\nu}_\mu$	43.39%	1.6/0.75/0.011/0%	0/0.73/0.0083/0%
$\mu^+\mu^- \longrightarrow N_\mu\bar{\nu}_\mu$	Central W	Mass window	Optimization
$m_N = 150$ GeV	55.04%	55.04%	55.04%
$m_N = 1000$ GeV	54.75%	54.75%	51.63%
$m_N = 5000$ GeV	99.93%	99.93%	97.46%
$m_N = 9000$ GeV	99.99%	99.99%	98.27%

Projected sensitivity



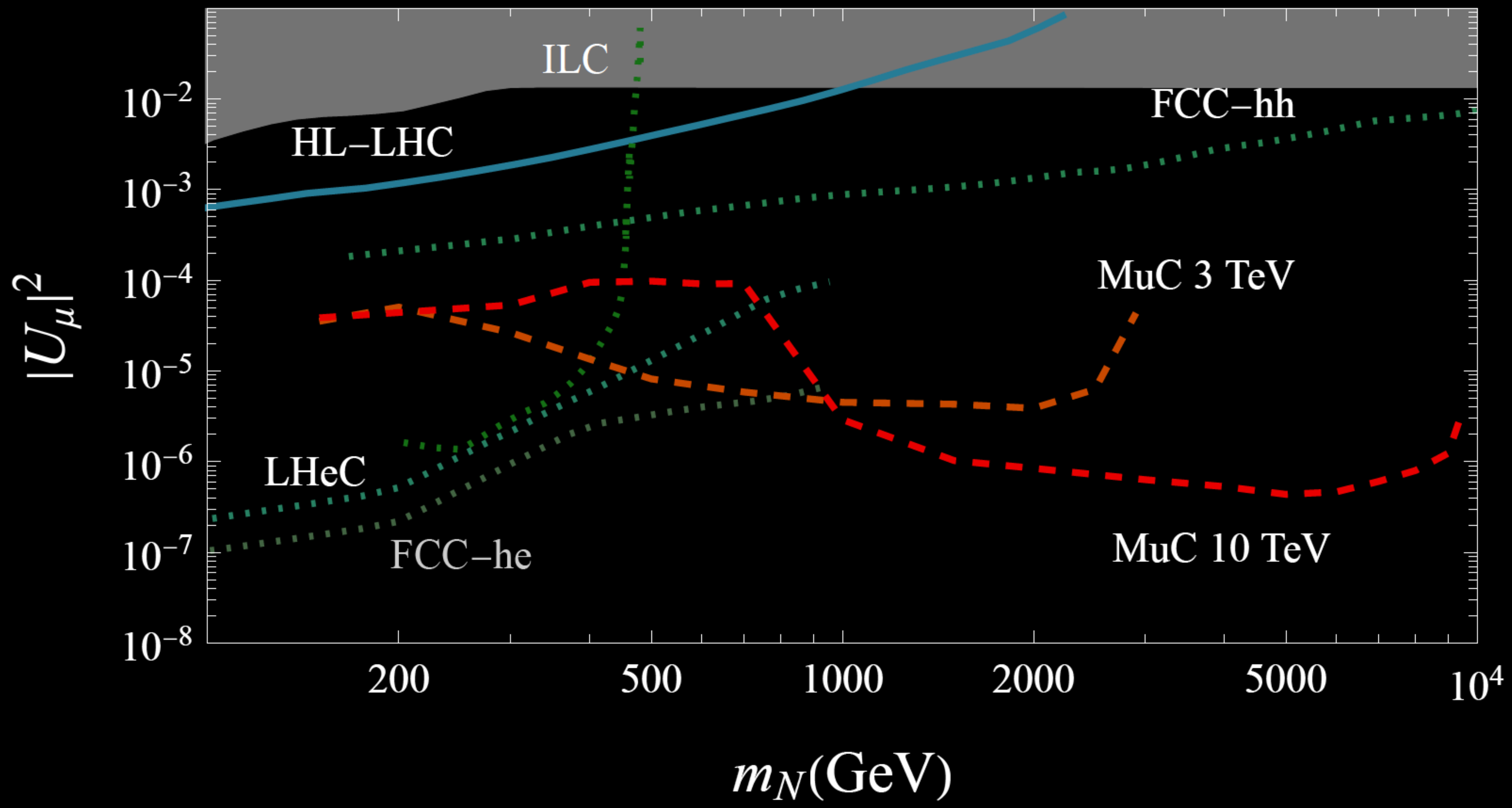
Sensitivity to e and τ flavor is moderate

Muon Collider features the strong direct probe of the μ flavored HNL

10 TeV muon collider can probe the $|U_\mu|^2$ to a few 10^{-7} for TeV scale HNLs.

The VBF background increases for high energy muon colliders and renders the 3 TeV muon collider competitive in sub TeV scale.

Projections w. others



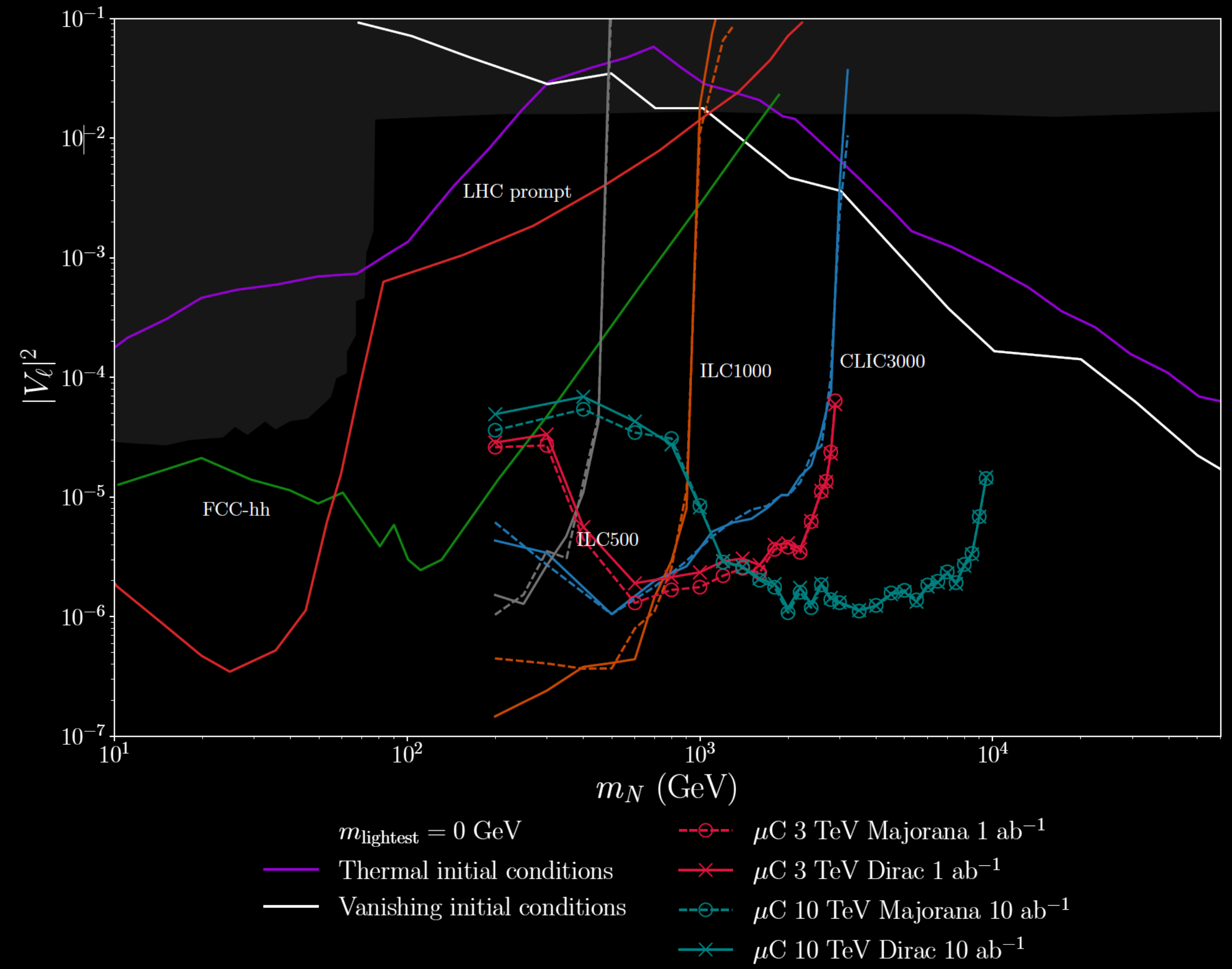
Focusing on the muon-flavored case:

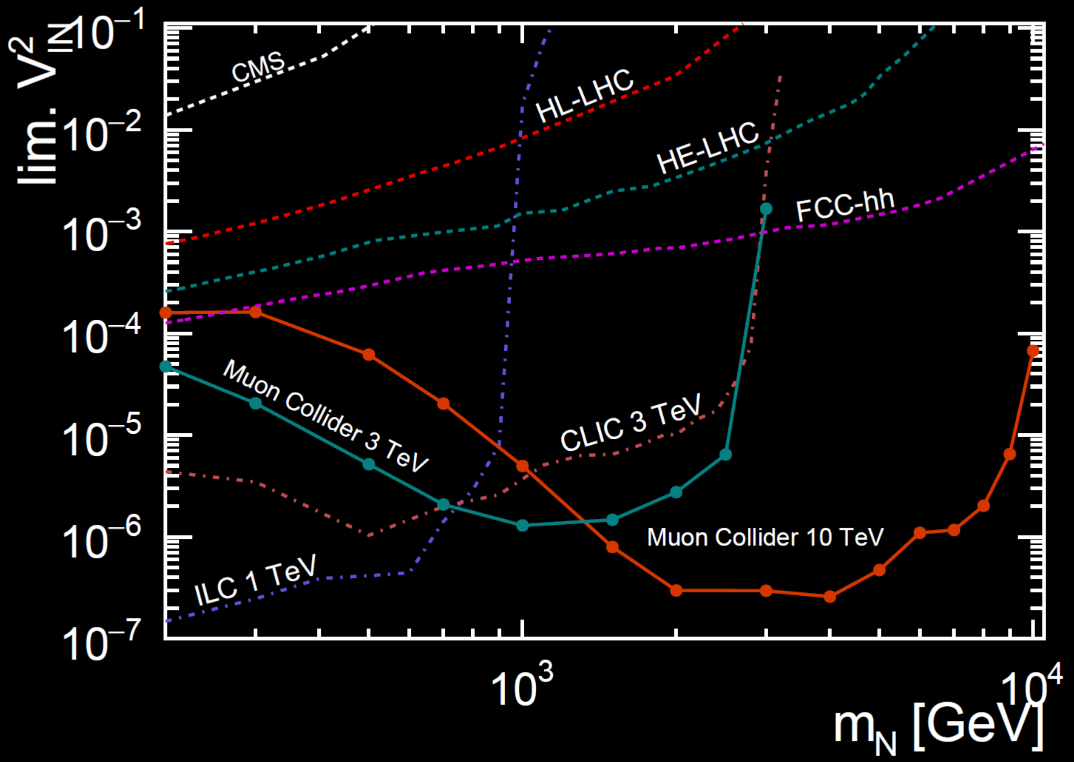
LHC and EWPD probe $O(10^{-3})$

Muon Collider has unique roles in probing the parameter space (thanks to the t-channel enhancement).

In the inverse seesaw setup, $|U_\ell|^2 = \left(\frac{\lambda v}{m_N}\right)^2$, and hence a unitarity limit exist on the upper right corner, overlapping very little with the region of our interests.

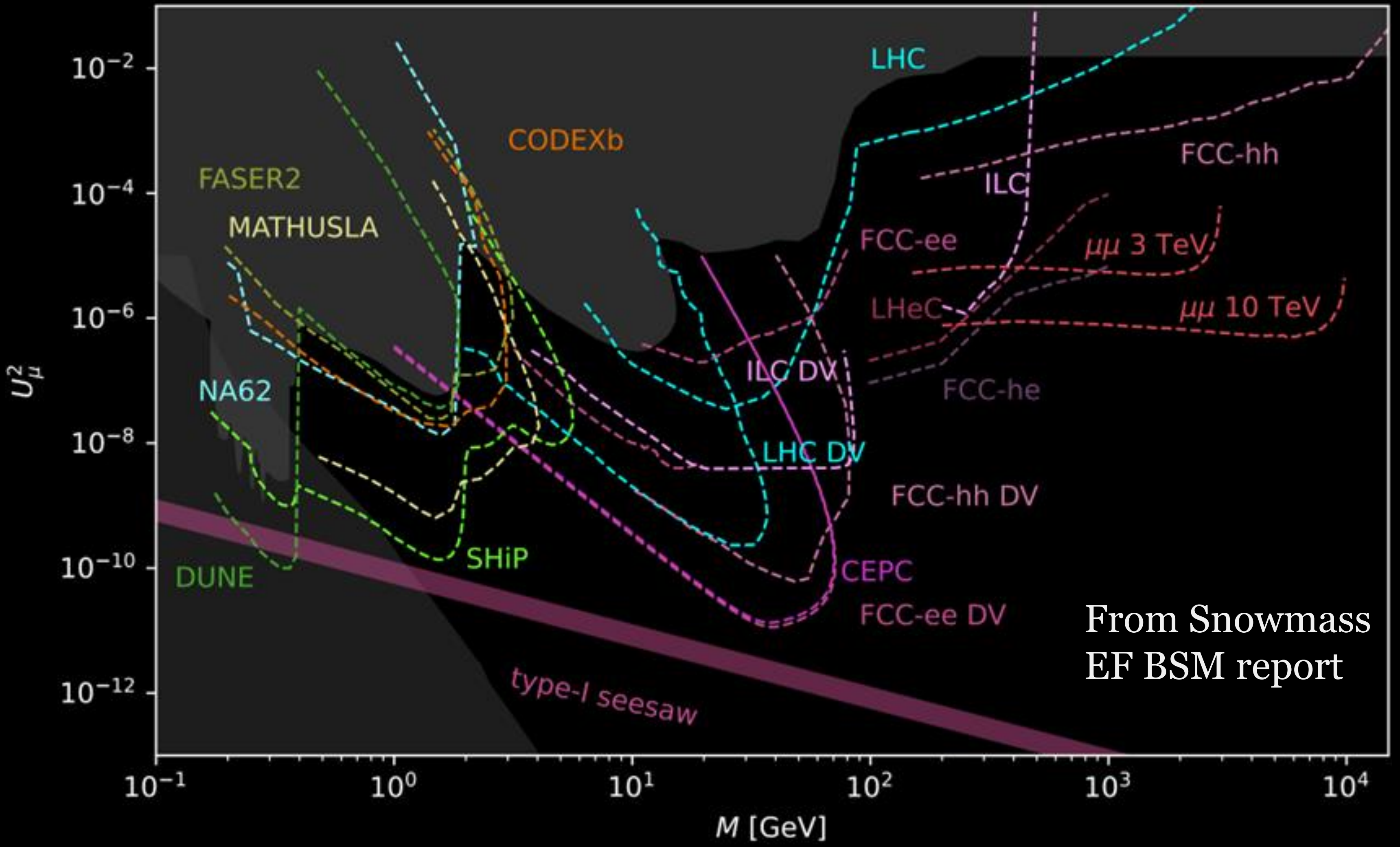
BDT-based projections





T.H. Kwok, L. Li, T. Liu and A. Rock, arXiv:2301.05177
K. Mekała, J. Reuter and A.F. Zarnecki, arXiv:2301.02602

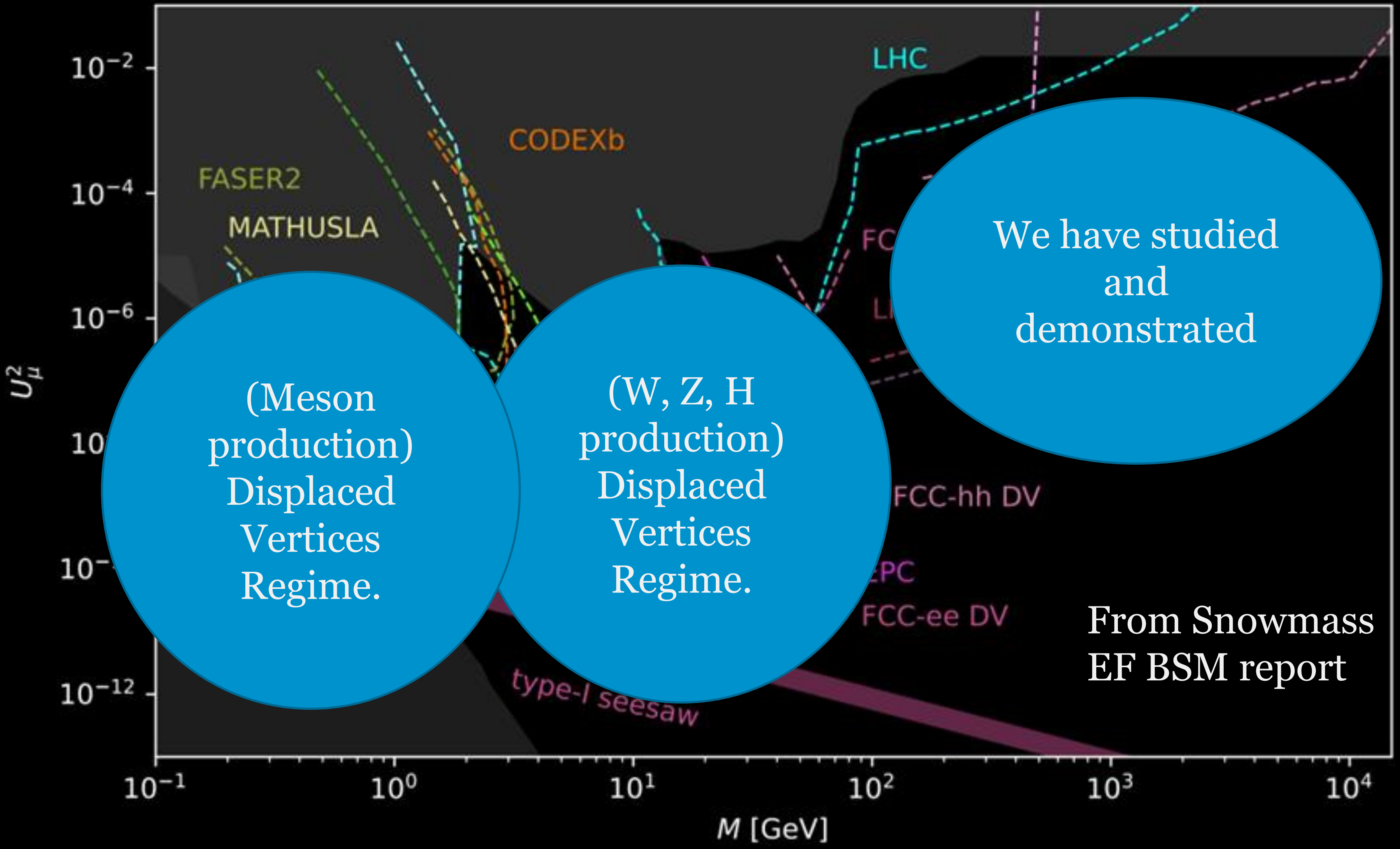
New studies for other regions



From Snowmass EF BSM report

The bottom left “type-I seesaw” represents the most pessimistic seesaw benchmarks. In general multi-generation seesaw, the motivated parameter regions spans over the space above that line, very much like the inverse seesaw spectra.

New studies for other regions



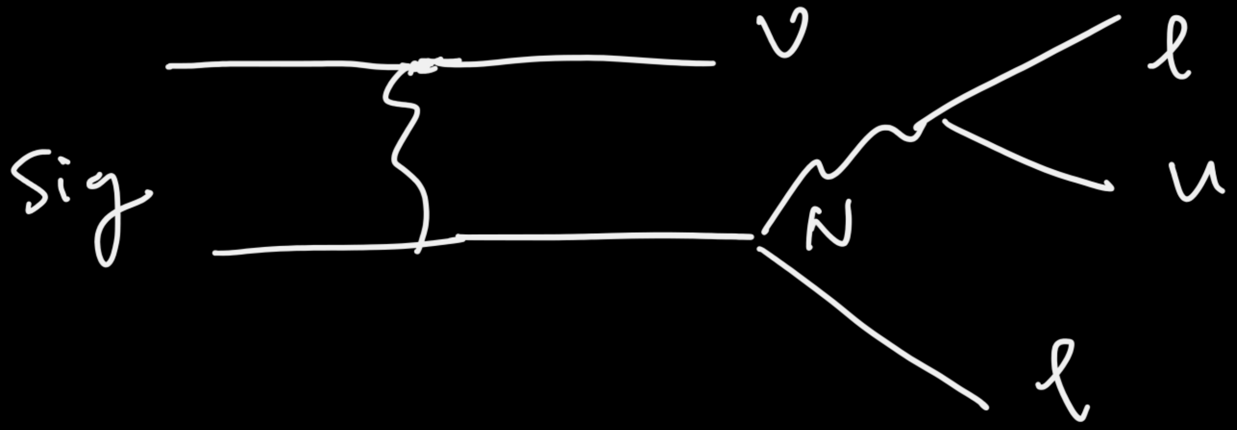
(Meson production) Displaced Vertices Regime.

(W, Z, H production) Displaced Vertices Regime.

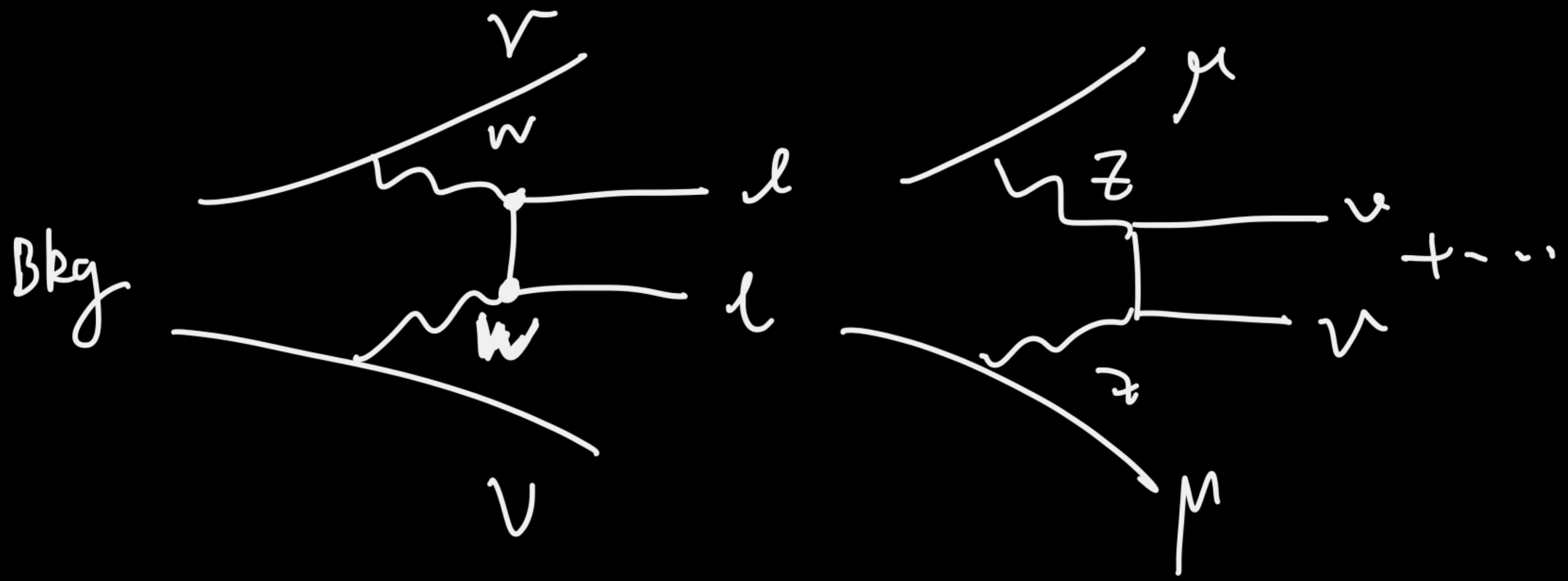
We have studied and demonstrated

From Snowmass EF BSM report

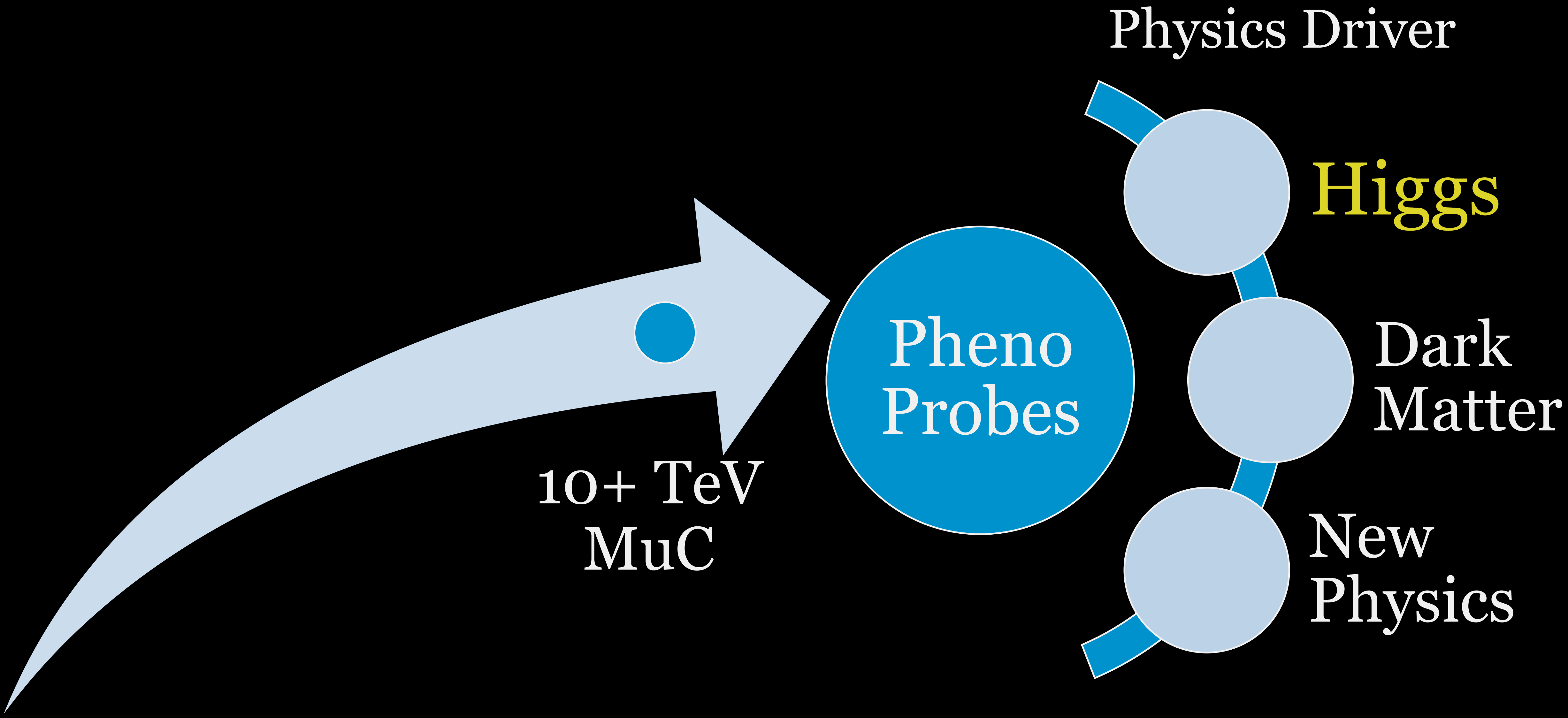
What about leptonic mode of HNL decay?



You will encounter infinity from on-shell intermediate states with EW PDF. This is a fake pole from the quasi-real approximation.



The Muon Shot



Physics Driver

10+ TeV
MuC

Pheno
Probes

Higgs

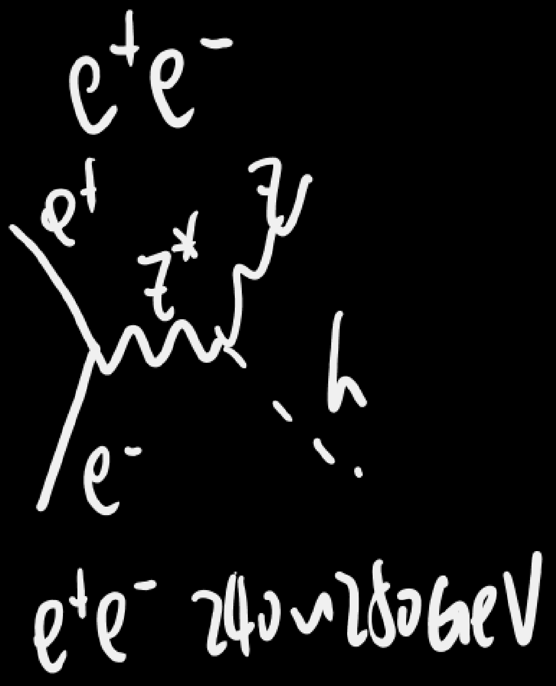
Dark
Matter

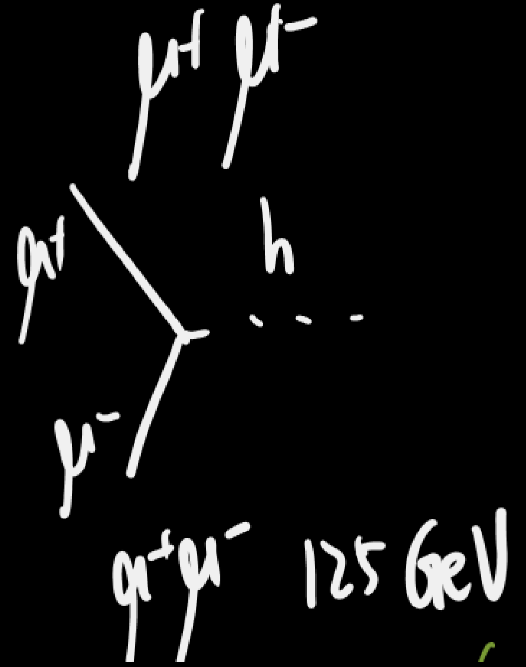
New
Physics

Measurements to be interpreted

Future Higgs factories, e.g., can solve this issue by inclusive Higgs measurement or lineshape scan.

- Inclusive rate: $\sigma(i \to H) = \sum_j \sigma(i \to H \to j) \propto \sum_j \frac{\Gamma_i \Gamma_j}{\Gamma_{tot}} = \Gamma_i$
- Lineshape scan: break the parameterization $\sigma(i \to H \to j) \propto \frac{\Gamma_i \Gamma_j}{\Gamma_{tot}^2}$





Inclusive Higgs rate from ZZ fusion

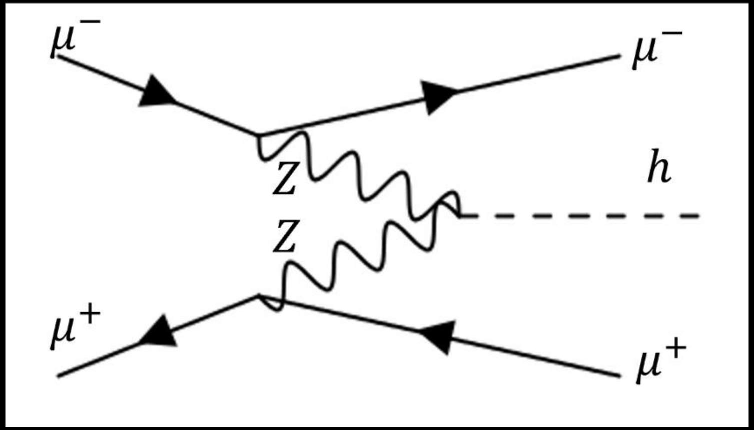
Forward muon coverage: $2.5 < \eta(\mu) < 4, 6, 8$

Peiran Li, Kun-Feng Lyu, ZL, 2401.08756

$$p_h = (\sqrt{s}, 0,0,0) - p_{\mu^+} - p_{\mu^-}$$

$$m_h^2 = \left[(\sqrt{s}, 0,0,0) - p_{\mu^+} - p_{\mu^-}\right]^2$$

Recoil mass of dimuon



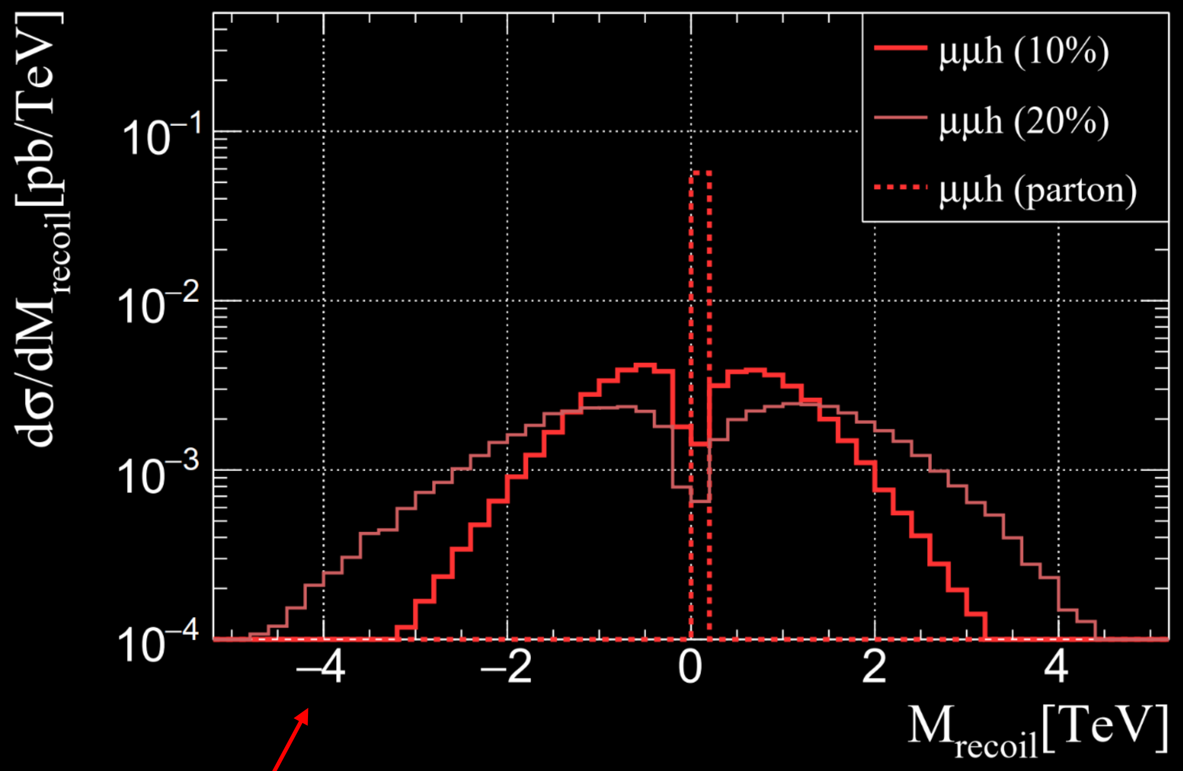
This subleading Higgs production channel, once tagged, does not rely on the detection of Higgs decay channel.

Inclusive rate: $\sigma(i \to H) = \sum_j \sigma(i \to H \to j) \propto \sum_j \frac{\Gamma_i \Gamma_j}{\Gamma_{tot}} = \Gamma_i$

Inclusive Higgs rate from ZZ fusion

Due to the uncertainty of high energy measurement, the smearing effect dominate the recoil mass distribution.

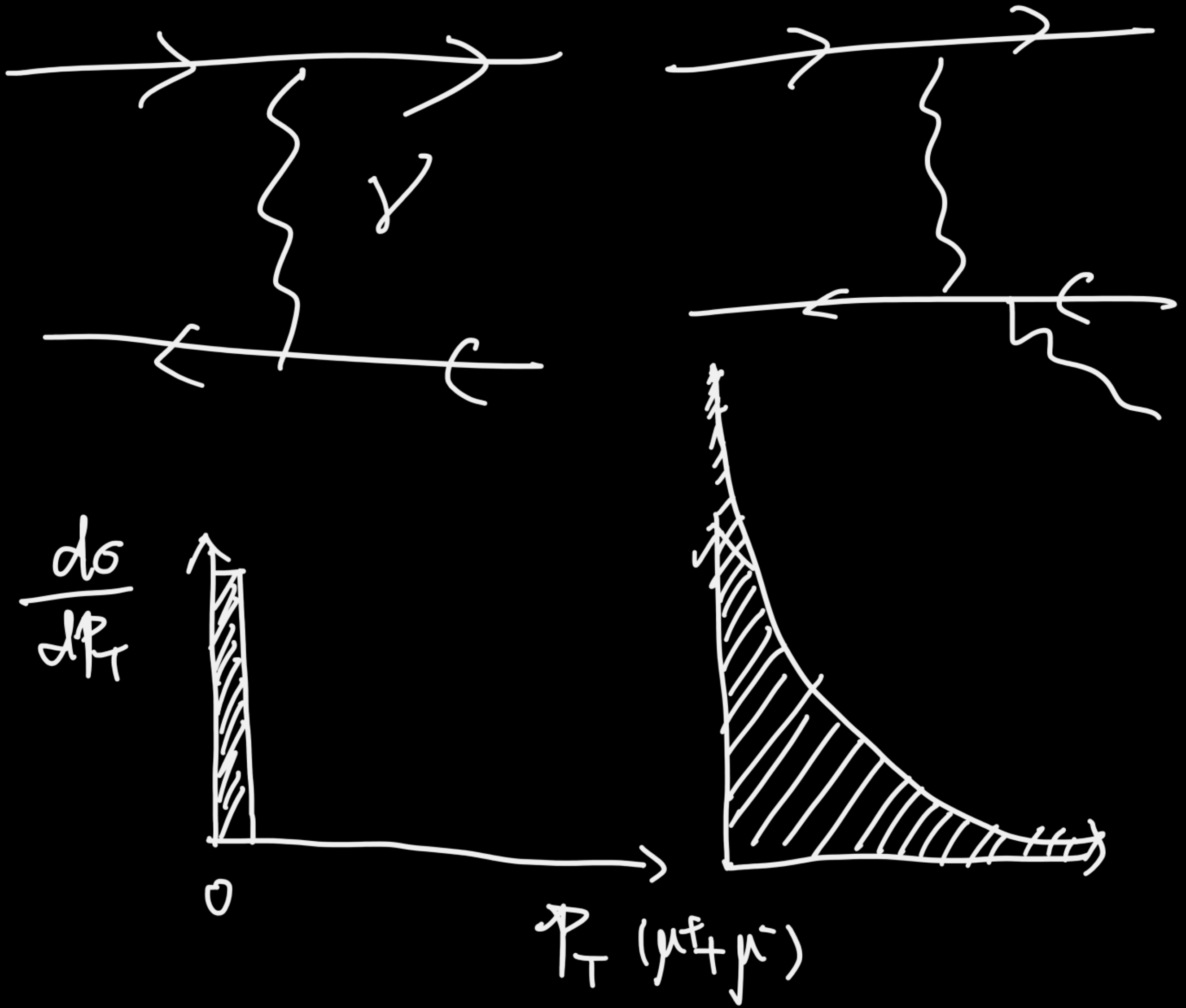
Detector Response/Simulation Matters a lot here!
Completely changes the story.



$\left[(\sqrt{s},0,0,0)-p_{\mu^+}-p_{\mu^-}\right]^2<0$

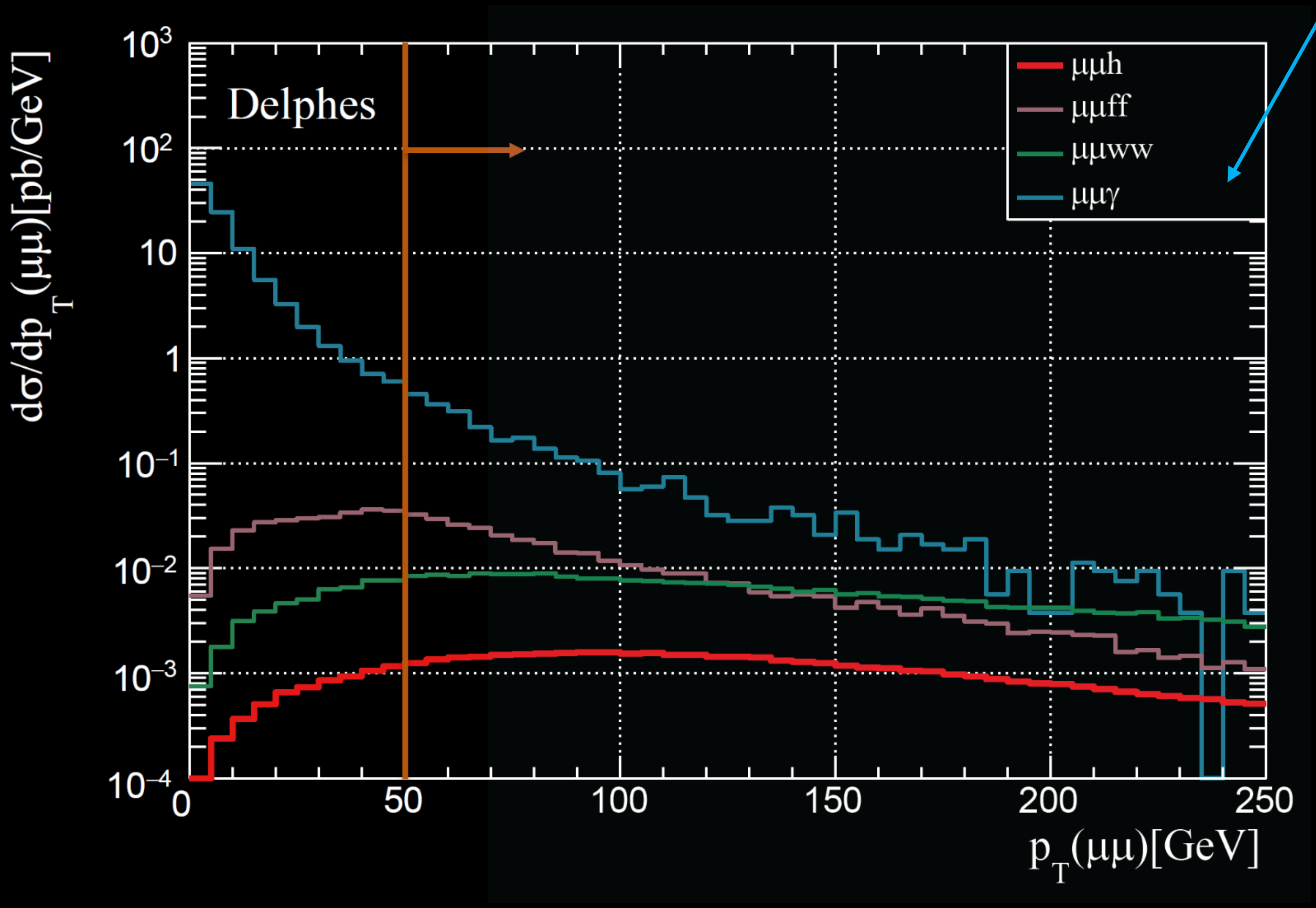
Fast detector simulation using Delphes.

Background Simulation

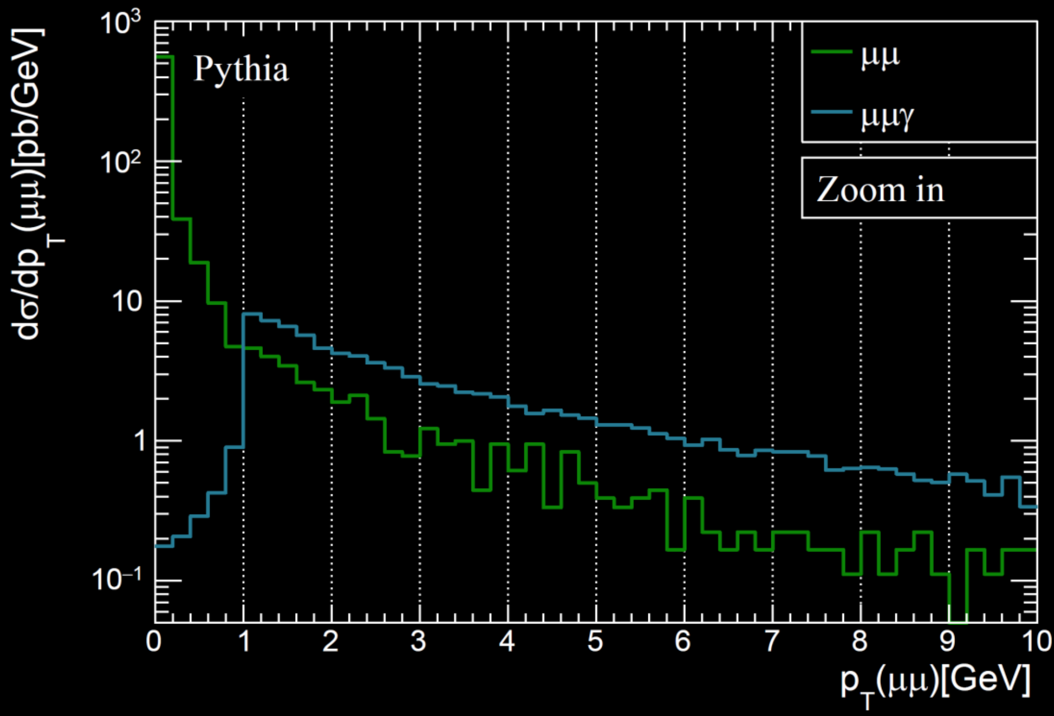


Signal vs. Background $(\sqrt{s} = 10\ \text{TeV})$

Require $p_T(\mu\mu) > 50$ GeV

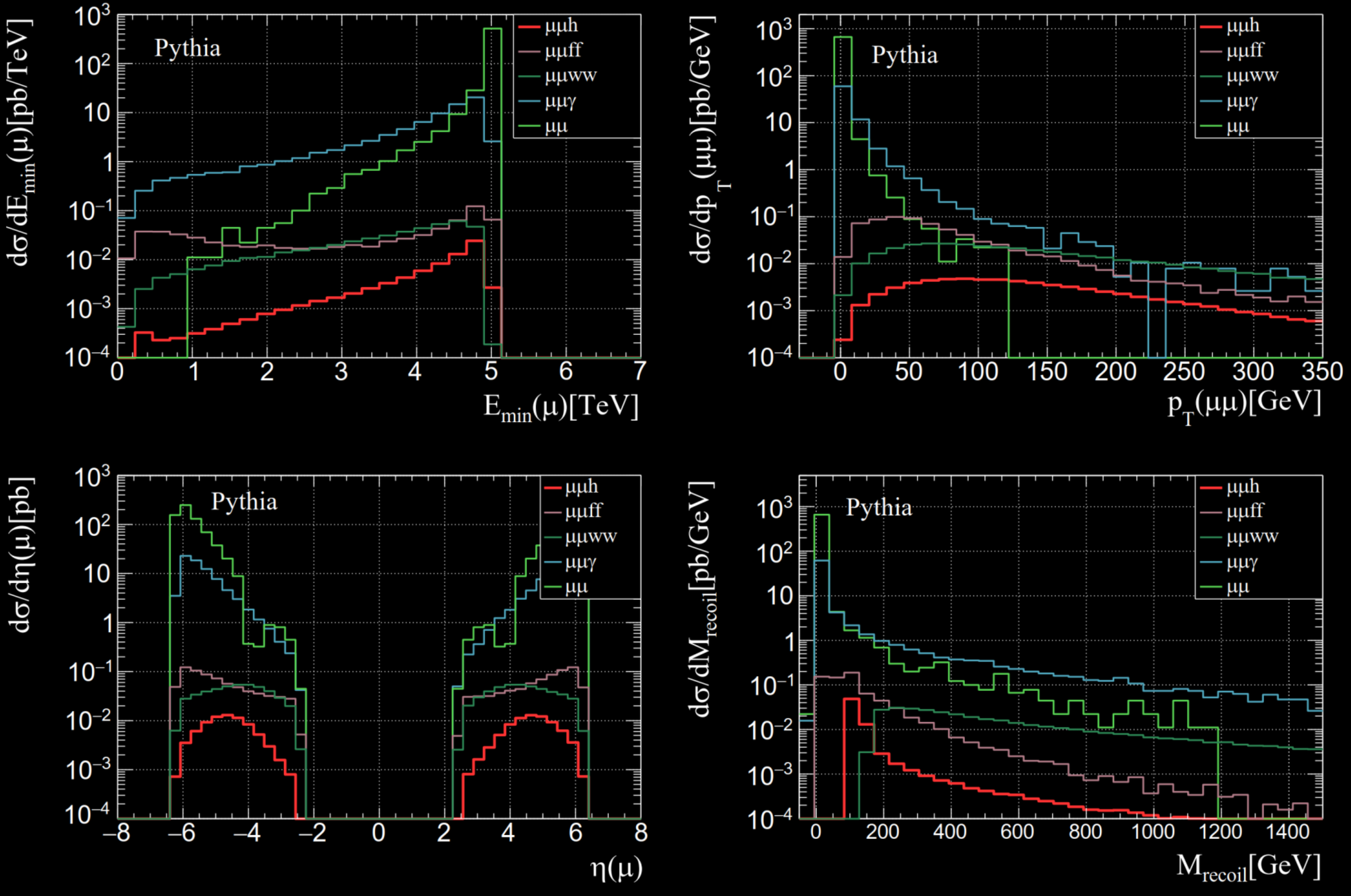


Matching & Merging
$\mu\mu \to \mu\mu$ and $\mu\mu \to \mu\mu\gamma$



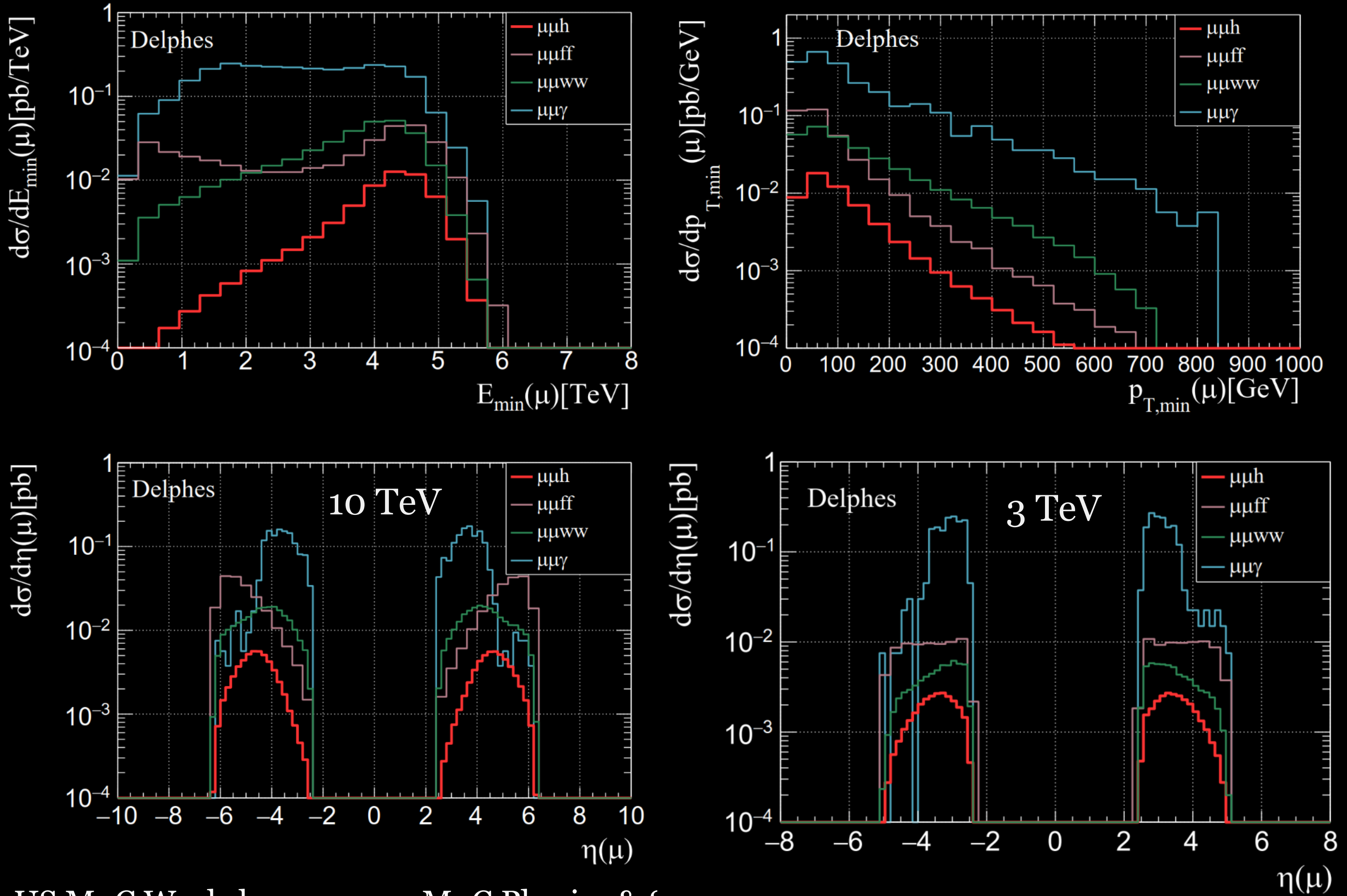
Also see a similar treatment in Higgs invisible study, Ruhdorfer, Salvioni, Wulzer, 2303.14202

Other relevant distributions



For the signal muons, the typical eta is around 5. Dominant background is more forward.

Other relevant distributions (reconstruction)



For the signal muons, the typical eta is around 5. Dominant background is more forward.

Sensitivity

Process	Pre-selection	$p_T(\mu\mu) > 50$ GeV	$E(\mu) > 3000$ GeV & $p_{T,\min}(\mu) < 300$ GeV
$\mu^+\mu^- \to \mu^+\mu^- h$	73.3%	65.7%	56.4% (0.0489 pb)
$\mu^+\mu^- \to \mu^+\mu^- \gamma$	13.1%	0.38%	0.12% (0.906 pb)
$\mu^+\mu^- \to \mu^+\mu^- f\bar{f}$	8.13%	4.69%	2.58% (0.199 pb)
$\mu^+\mu^- \to \mu^+\mu^- W^+W^-$	40.0%	34.9%	22.0% (0.207 pb)

10 TeV

Benchmark	$\lvert\eta(\mu)\rvert < 4$	$\lvert\eta(\mu)\rvert < 6$	$\lvert\eta(\mu)\rvert < 8$
$\Delta\sigma/\sigma$	15%	0.75%	0.74%

Now High Energy Muon Collider is a full-fledged Higgs factory

$\eta(\mu) < 6$

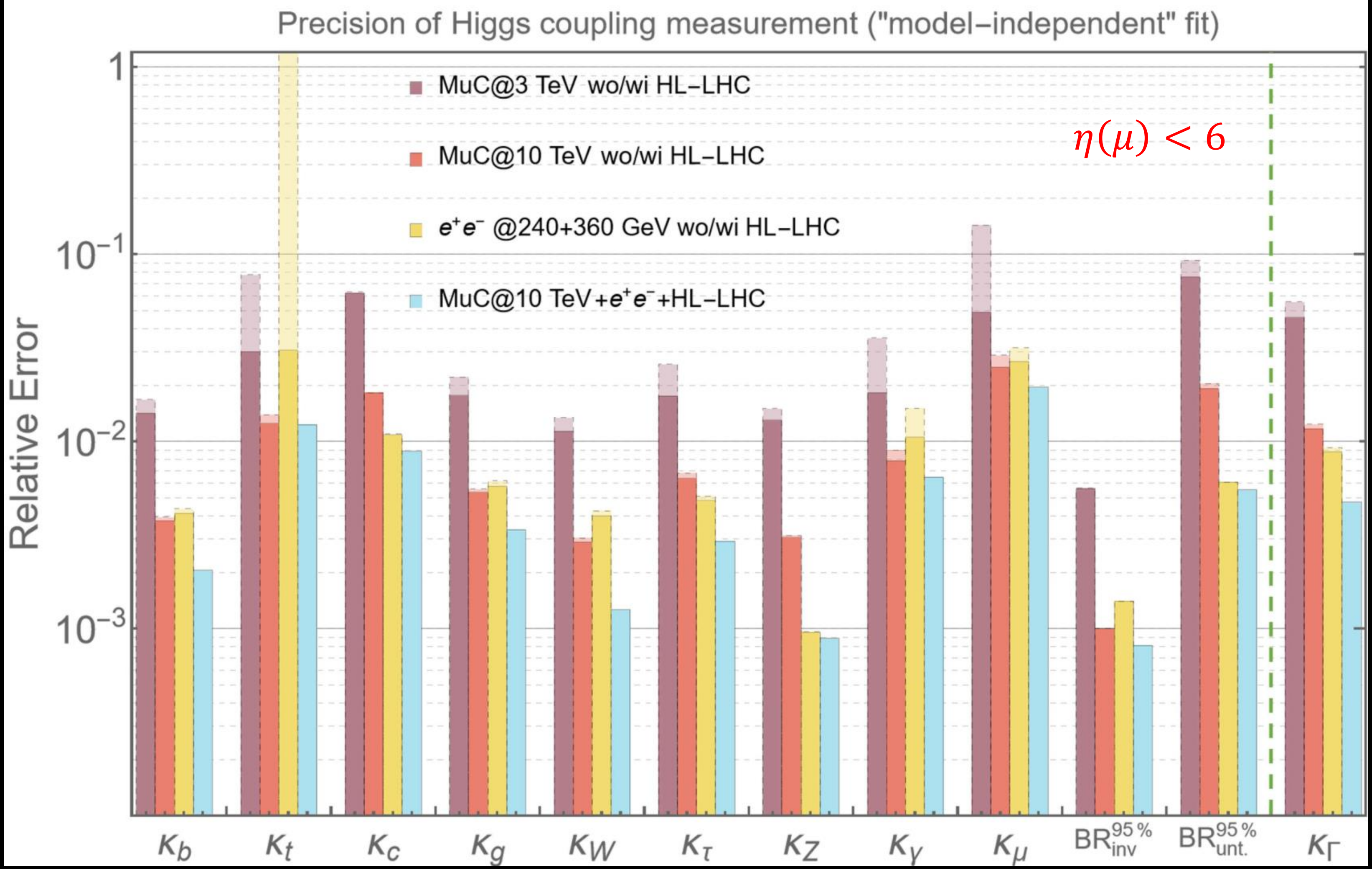
$\mu_{\text{production}}^{\text{decay}}$	μ_{VV}^{tt}	μ_{WW}^{bb}	μ_{WW}^{cc}	μ_{WW}^{gg}	$\mu_{WW}^{\tau\tau}$	μ_{WW}^{WW}	μ_{WW}^{ZZ}	$\mu_{WW}^{\gamma\gamma}$	$\mu_{WW}^{\mu\mu}$
$\Delta\sigma/\sigma(\%)$	2.8	0.22	3.6	0.79	1.1	0.40	3.2	1.7	5.7
$\mu_{\text{production}}^{\text{decay}}$	μ_{ZZ}^{bb}	μ_{ZZ}^{cc}	μ_{ZZ}^{gg}	$\mu_{ZZ}^{\tau\tau}$	μ_{ZZ}^{WW}	μ_{ZZ}^{ZZ}	$\mu_{ZZ}^{\gamma\gamma}$	μ_{ZZ}^{inv}	μ_{ZZ}^{H}
$\Delta\sigma/\sigma(\%)$	0.77	17	3.3	4.8	1.8	11	4.8	0.05	0.75

Requires forward muon

Other inputs used in this study.

- (Exclusive Higgs) M. Forslund and P. Meade. [2203.09425]
- (Invisible Higgs) M. Ruhdorfer, E. Salvioni, A. Wulzer. [2303.14202]
- (Top Yukawa) Z. Liu, K.F. Lyu, I. Mahbub, L.T. Wang. [2308.06323]
- (off-shell Higgs; not used but relevant) M. Forslund and P. Meade [2308.02633]

Now High Energy Muon Collider is a full-fledged Higgs factory



New inclusive Higgs rate result enables a full-fledges Higgs precision.

- With forwarded detection $2.5<\eta(\mu)<6$, the cross-section precision is ~0.75%
- Combining with other studies, we can constraint on Γ_H~2% and Higgs couplings in 0.5% level.

Other inputs used in this study.

- (Exclusive Higgs) M. Forslund and P. Meade. [2203.09425]
- (Invisible Higgs) M. Ruhdorfer, E. Salvioni, A. Wulzer. [2303.14202]
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- (off-shell Higgs; not used but relevant) M. Forslund and P. Meade [2308.02633]

Results and approximate analytics

$$\kappa_\Gamma = \frac{\left(\mu^H_{ZZ}\right)^2}{\mu^{WW}_{WW}} \left(\frac{\mu^{bb}_{WW}}{\mu^{bb}_{ZZ}}\right)^2$$

$$\Delta\kappa_\Gamma = \left[4\left(\Delta\mu^H_{ZZ}\right)^2 + \left(\Delta\mu^{WW}_{WW}\right)^2 + 4(\Delta\mu^{bb}_{WW})^2 + 4(\Delta\mu^{bb}_{ZZ})\right]^{1/2} = 2.2\%$$

$$\kappa^4_W = (\mu^{WW}_{WW})\kappa_\Gamma = (\mu^H_{ZZ})^2 \left(\frac{\mu^{bb}_{WW}}{\mu^{bb}_{ZZ}}\right)^2,$$

$$\Delta\kappa_W = \frac{1}{4}\left[4\left(\Delta\mu^H_{ZZ}\right)^2 + 4(\Delta\mu^{bb}_{WW})^2 + 4(\Delta\mu^{bb}_{ZZ})^2\right]^{1/2} = 0.55\%.$$

$$\kappa^2_b = \frac{\mu^{bb}_{WW}\kappa^2_W}{\mu^{WW}_{WW}} = \frac{\mu^H_{ZZ}(\mu^{bb}_{WW})^2}{\mu^{bb}_{ZZ}\mu^{WW}_{WW}},$$

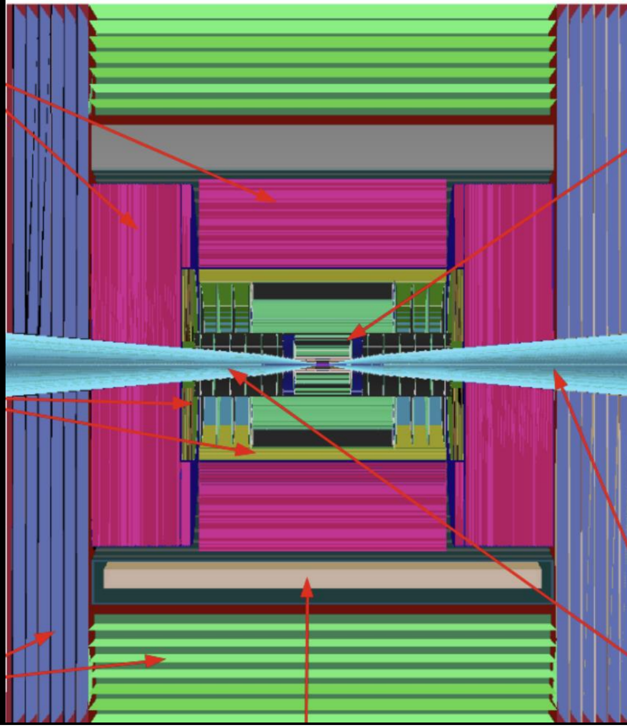
$$\Delta\kappa_b = \frac{1}{2}\left[\left(\Delta\mu^H_{ZZ}\right)^2 + 4(\Delta\mu^{bb}_{WW})^2 + (\Delta\mu^{bb}_{ZZ})^2 + \left(\Delta\mu^{WW}_{WW}\right)\right]^{1/2} = 0.61\%$$

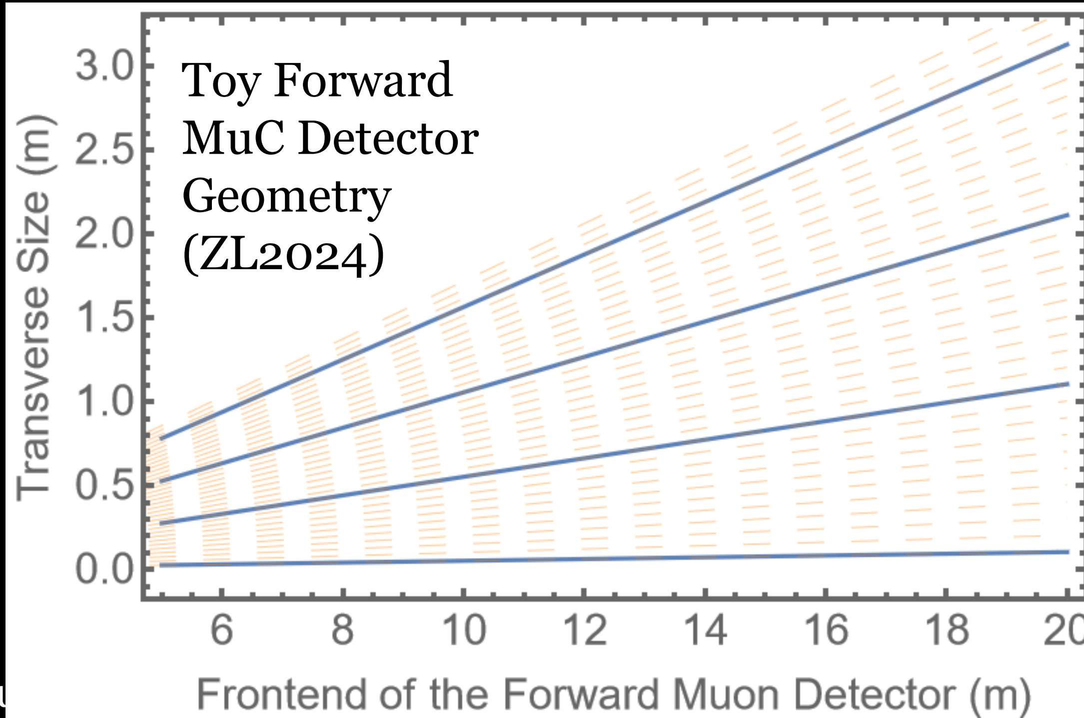
The diverse production and decay measurements at MuC de-correlate many coupling precision, which leads to a good projection in the coupling basis.

	$\lvert\eta(\mu)\rvert < 4$			$\lvert\eta(\mu)\rvert < 6$		
	MuC@10TeV	+HL-LHC	$+e^+e^-$	MuC@10TeV	+HL-LHC	$+e^+e^-$
$\kappa_b(\%)$	+7.5 −0.25	+1.7 −0.24	+0.25 −0.18	+0.56 −0.23	+0.53 −0.23	+0.24 −0.17
$\kappa_t(\%)$	+1.4 −7.1	+1.3 −1.6	+1.3 −1.2	+1.4 −1.4	+1.3 −1.2	+1.3 −1.2
$\kappa_c(\%)$	+7.8 −2.1	+2.6 −2.1	+0.91 −0.91	+1.8 −1.8	+1.8 −1.8	+0.89 −0.89
$\kappa_g(\%)$	+7.5 −0.52	+1.7 −0.50	+0.38 −0.35	+0.67 −0.45	+0.63 −0.44	+0.35 −0.32
$\kappa_W(\%)$	+7.5 −0.15	+1.7 −0.13	+0.17 −0.099	+0.51 −0.10	+0.48 −0.10	+0.16 −0.090
$\kappa_\tau(\%)$	+7.5 −0.62	+1.8 −0.57	+0.33 −0.27	+0.76 −0.56	+0.71 −0.55	+0.32 −0.27
$\kappa_Z(\%)$	+7.3 −1.4	+1.9 −0.93	+0.13 −0.058	+0.37 −0.25	+0.37 −0.25	+0.12 −0.056
$\kappa_\gamma(\%)$	+7.6 −0.83	+1.8 −0.71	+0.66 −0.64	+0.97 −0.82	+0.86 −0.71	+0.65 −0.64
$\kappa_\mu(\%)$	+9.1 −5.0	+3.8 −3.6	+2.3 −2.4	+2.9 −2.9	+2.5 −2.5	+1.9 −2.0
$\mathrm{Br}^{95\%}_{\mathrm{inv}}(\%)$	+0.64 0	+0.63 0	+0.13 0	+0.10 0	+0.10 0	+0.080 0
$\mathrm{Br}^{95\%}_{\mathrm{unt}}(\%)$	+27 0	+6.6 0	+0.57 0	+2.0 0	+1.9 0	+0.54 0
$\kappa_\Gamma(\%)$	+34 −0.45	+6.9 −0.43	+0.69 −0.31	+2.1 −0.41	+1.9 −0.40	+0.65 −0.29

Forward Muon Detector Required!

- Is it feasible?
- We only require to tag Energetic Muons.
- Muons pass through the nozzle regions
- Energy resolution is not important (basically need to separate TeV scale energetic muons from soft muons)
- Angular resolution is not important (~50mrad should be good enough;)
- This is a very strong case for a forward muon detector
- Happy to discuss more and collaborate





Top Yukawa (through Unitarizing Higgs)

- At Large Energies, for $(\pm, \mp)$ the contribution from the γ, Z and t-channel contribution grows as $\mathscr{O}(E^2/m_W^2)$, which cancels off due to gauge invariance

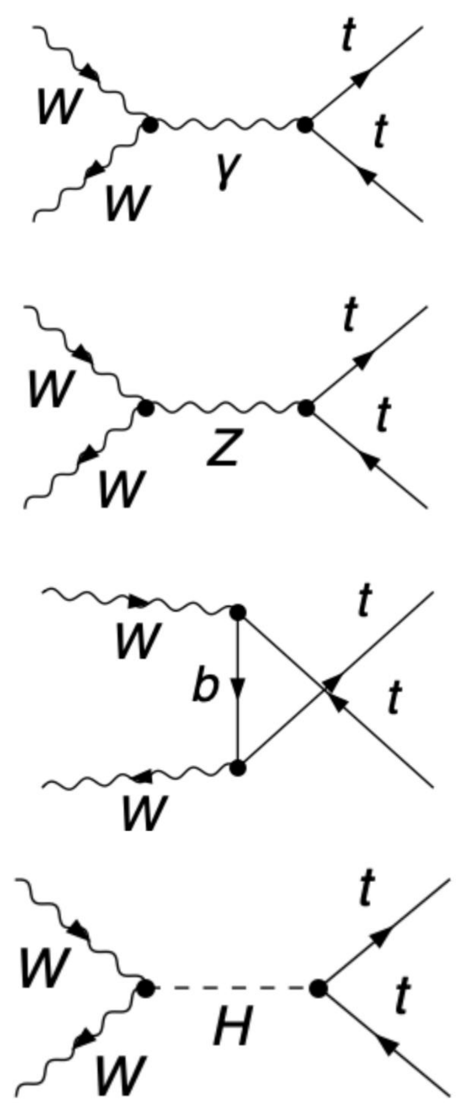
- Contribution from $(\pm, \pm)$ grows as $\mathscr{O}(E/m_W)$

$$\mathscr{M}^{\gamma+Z+b}(W_L^+W_L^- \to t\bar{t}) \;=\; \frac{m_t}{v^2}\sqrt{s} \qquad ;\sqrt{s} >> m_t$$

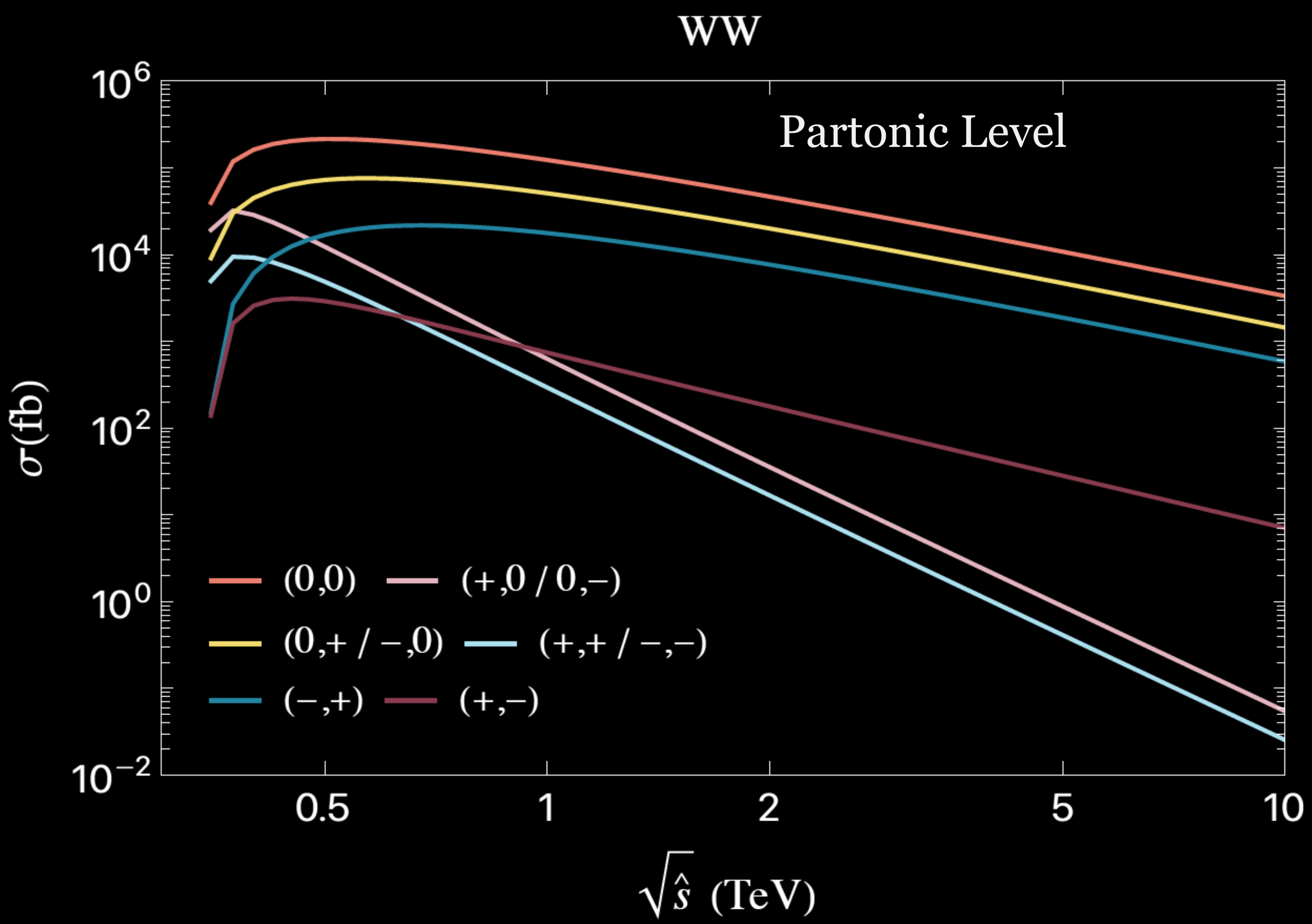
- Higgs channel precisely cancels this growing energy behavior

- Can be understood from Goldstone boson equivalence theorem

$$\mathcal{M}_{W_L^+W_L^- \to t\bar{t}} = \mathcal{M}_{\phi^+\phi^- \to t\bar{t}}\left[1 + O\left(\frac{m_W^2}{E^2}\right)\right]$$



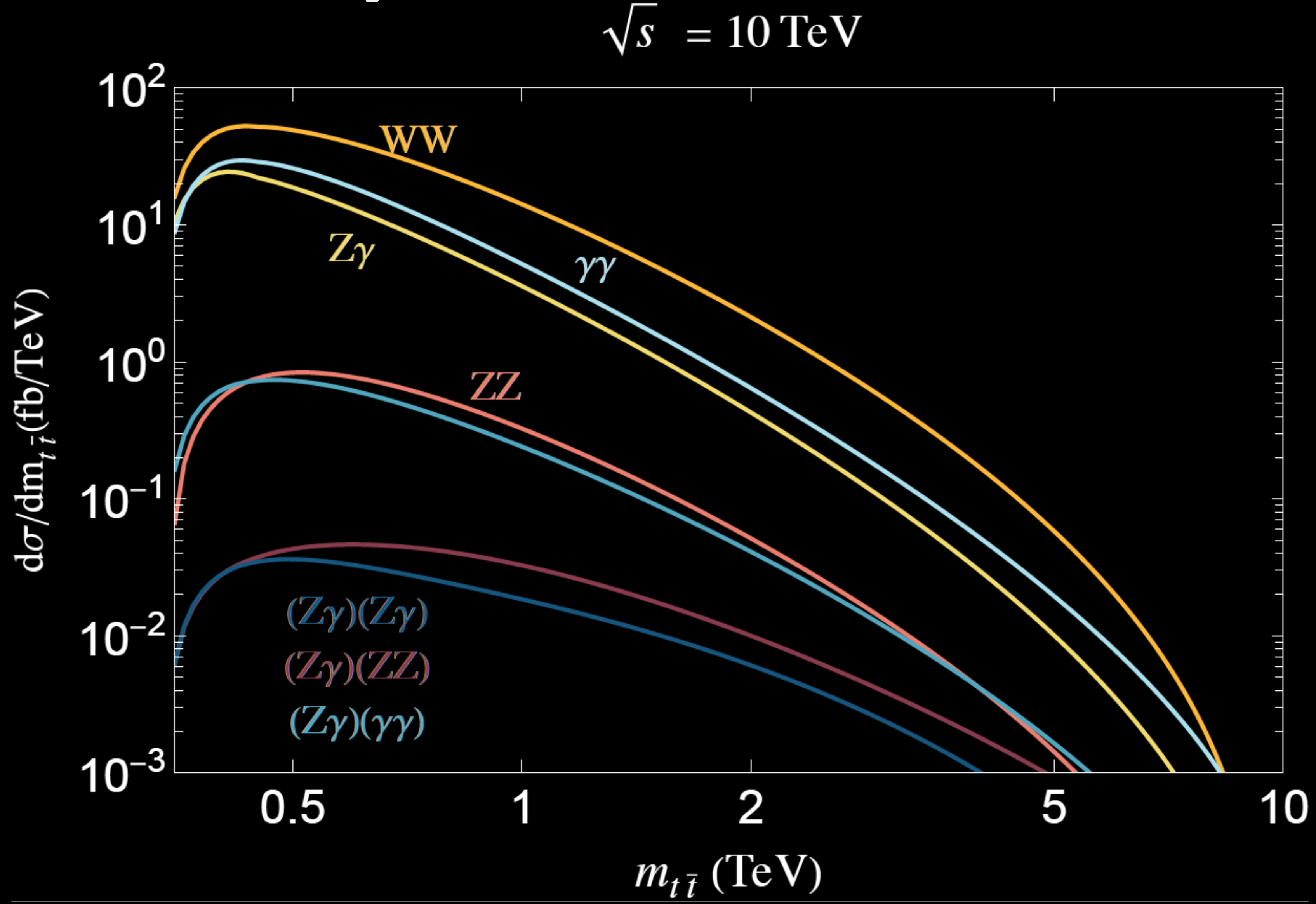
Anatomy of the amplitude and interference



W^+	W^-	$(t,\,\bar{t})$			
		$(+,+)$	$(-,+)$	$(+,-)$	$(-,-)$
0	0	$\hat{s}^{-1}$	$\hat{s}^{0}$	$\hat{s}^{0}$	$\hat{s}^{-1}$
$+$	0	$\hat{s}^{-2}$	$\hat{s}^{-1}$	$\hat{s}^{-1}$	$\hat{s}^{-2}$
$-$	0	$\hat{s}^{-2}$	$\hat{s}^{-1}$	$\hat{s}^{-1}$	$\hat{s}^{0}$
0	$+$	$\hat{s}^{0}$	$\hat{s}^{-1}$	$\hat{s}^{-1}$	$\hat{s}^{-2}$
$+$	$+$	$\hat{s}^{-1}$	$\hat{s}^{-2}$	$\hat{s}^{-2}$	$\hat{s}^{-3}$
$-$	$+$	$\hat{s}^{-1}$	$\hat{s}^{0}$	$\hat{s}^{-2}$	$\hat{s}^{-1}$
0	$-$	$\hat{s}^{-2}$	$\hat{s}^{-1}$	$\hat{s}^{-1}$	$\hat{s}^{-2}$
$+$	$-$	$\hat{s}^{-1}$	$\hat{s}^{0}$	$\hat{s}^{-2}$	$\hat{s}^{-1}$
$-$	$-$	$\hat{s}^{-3}$	$\hat{s}^{-2}$	$\hat{s}^{-2}$	$\hat{s}^{-1}$

Z. Liu, K.F. Lyu, I. Mahbub, L.T. Wang. [2308.06323]
also see discussions in M. Chen, D. Liu, 2212.11067

Study/simulation considerations based on the formulism you take

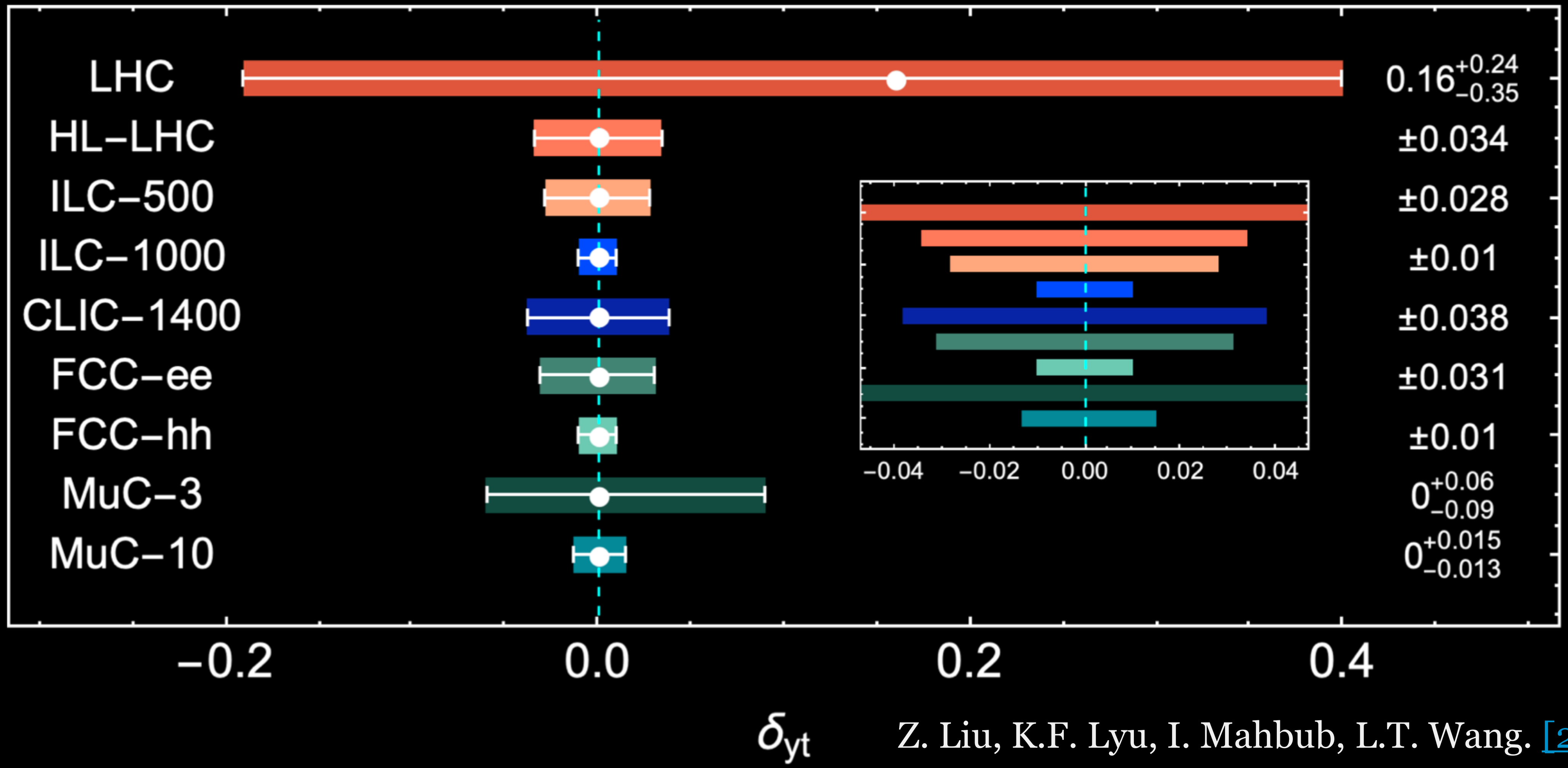


But don't forget pieces beyond this quasi-real approximation

Diagrammatically, the intermediate particles γ and Z shall interfere, or gauge basis, B and W_3 shall interfere.

How large are these contributions?

Projected Top Yukawa precision



Z. Liu, K.F. Lyu, I. Mahbub, L.T. Wang. [2308.06323]

The Muon Shot by us

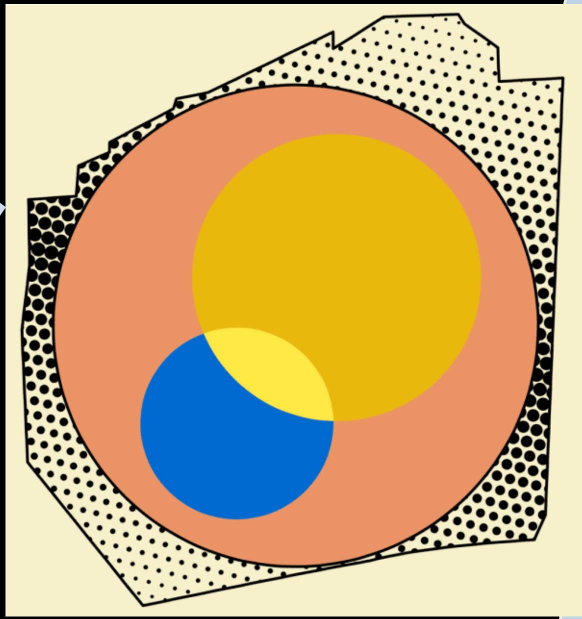
Many key physics needs to be worked out

Your contributions and remarks are critical

Try the simulations and learn the physics on the way

The community is welcoming and growing

10+ TeV MuC



Physics Driver

- Higgs
- Dark Matter
- New Physics