

Pion selection studies in the ProtoDUNE-I 2 GeV sample

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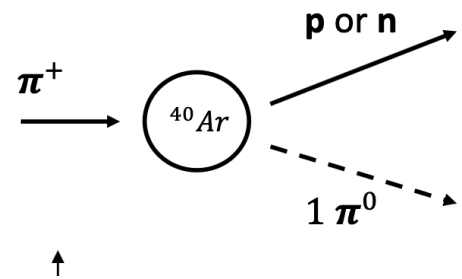
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Hadron Analysis Meeting
May 01st, 2024

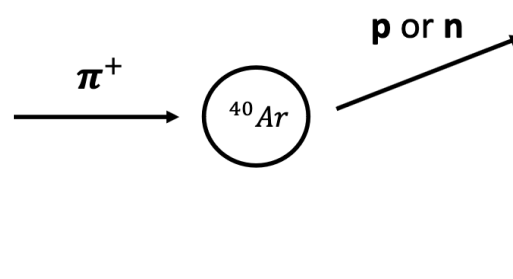


Introduction

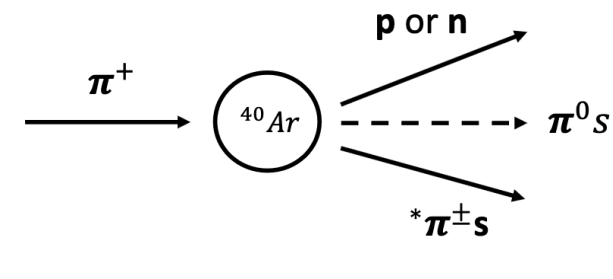
- **Objective** : Studying pion selection in the protoDUNE-1 2 GeV beam sample for a pion absorption cross section measurement at 2 GeV
- Starting with preliminary analysis of the selected 2 GeV pion sample using Kang Yang's framework.
- This talk shows a comparison of the selected pion beams for 1 GeV and 2 GeV using the cuts defined in the framework.
- A Comparison of the percentage of expected different pion interaction topology for 1 GeV and 2 GeV pion beam is also presented.



Pion Charge Exchange



Pion Absorption



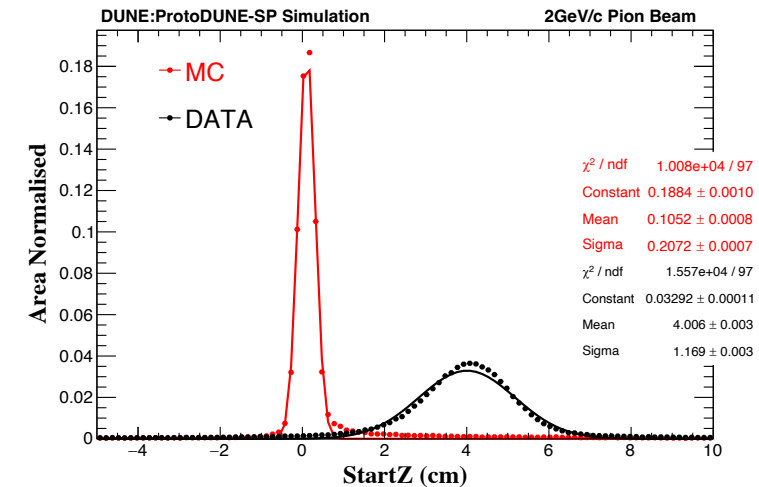
Pion Production

2 GeV Pion Beam Selection

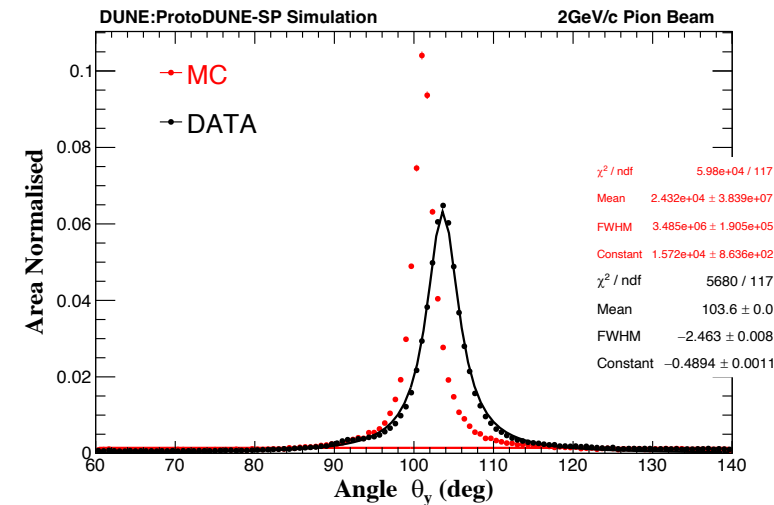
- Cuts used in the analysis in selecting pions :
 1. **Beam PDG Pass**
 2. **Pandora Slice Pass**
 3. **Calorimeter Size**
 4. **Beam Quality**
 5. **Beam End Z Pass**
 6. **Michel Score**
 7. **Proton Cut**
 8. **Beam Scraper Cut**
- Currently use the same cuts as for the 1 GeV case in the 2 GeV analysis.

2 GeV Pion Beam MC issues

- Issues with the 2 GeV **StartZ** position :



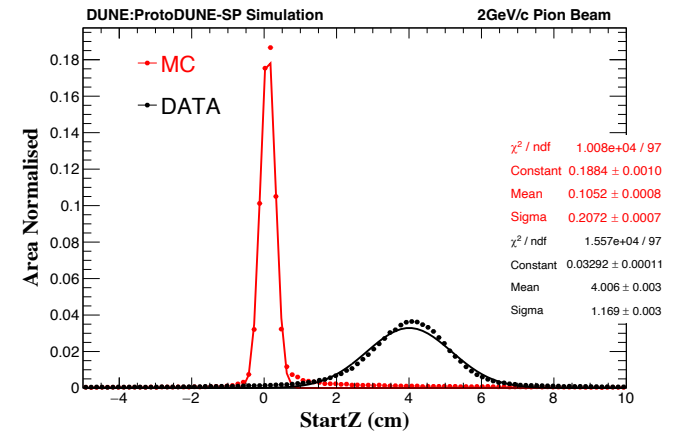
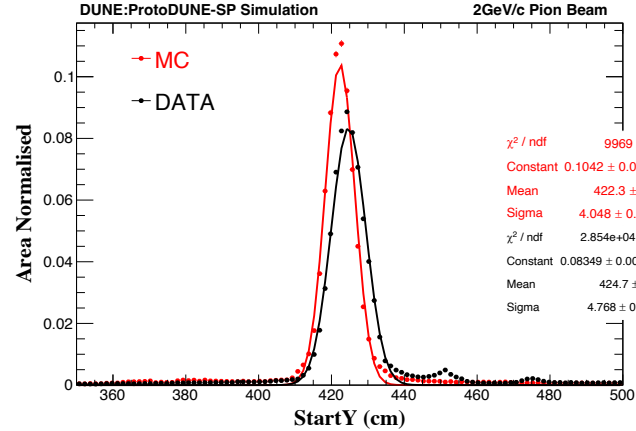
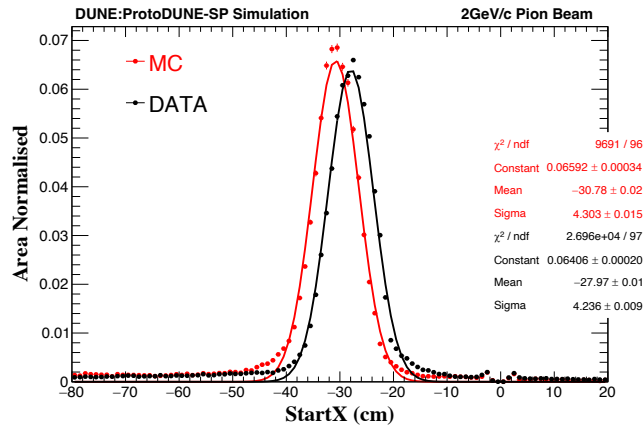
- Issues with the 2 GeV **ThetaY** angle :



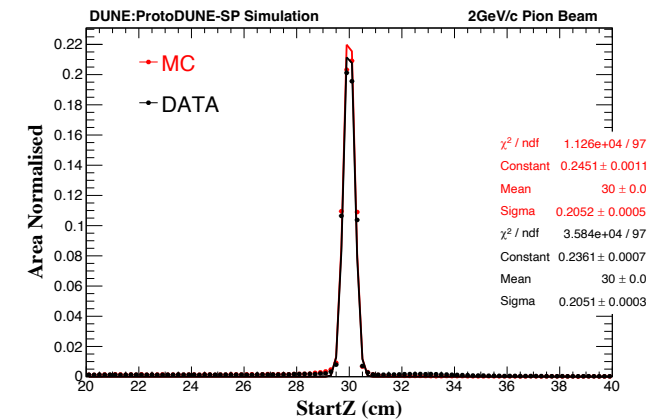
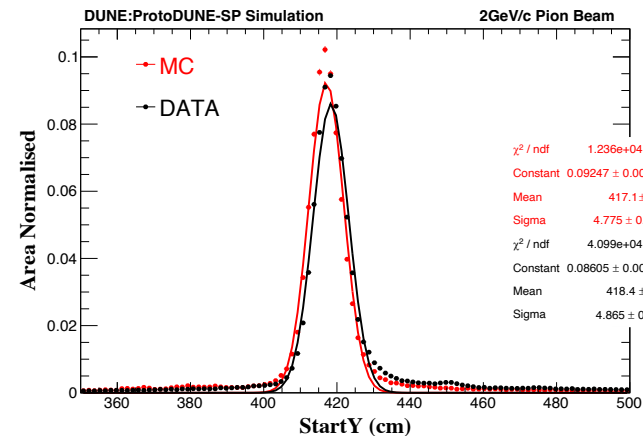
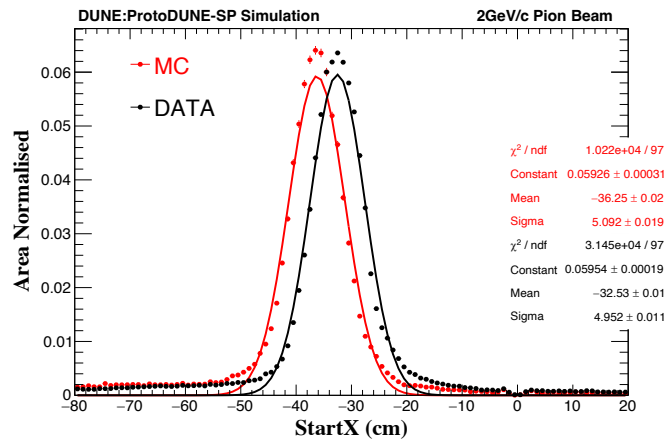
→ Use additional set of cuts to remove the first 30 cm of the tracks (**Cuts proposed by Richie Diurba**)

Reconstructed Beam start position

2 GeV Pion Beam



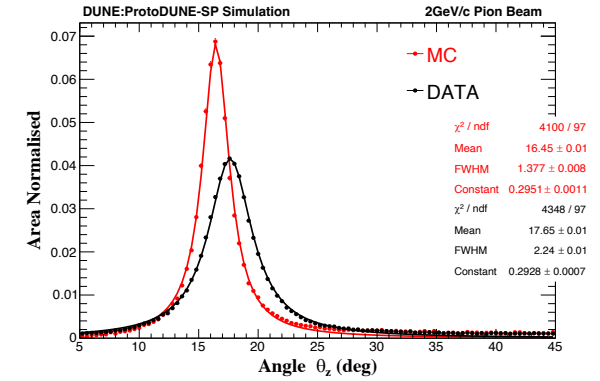
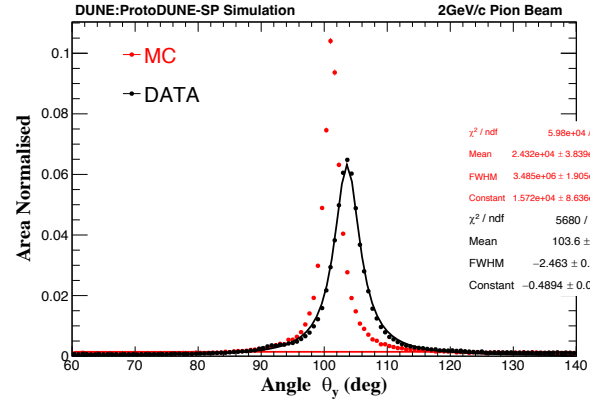
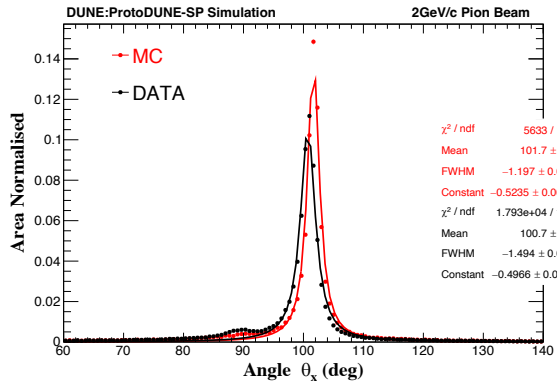
2 GeV Pion Beam with additional cuts



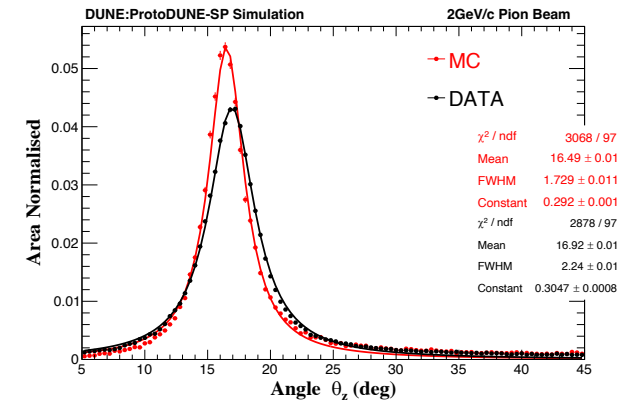
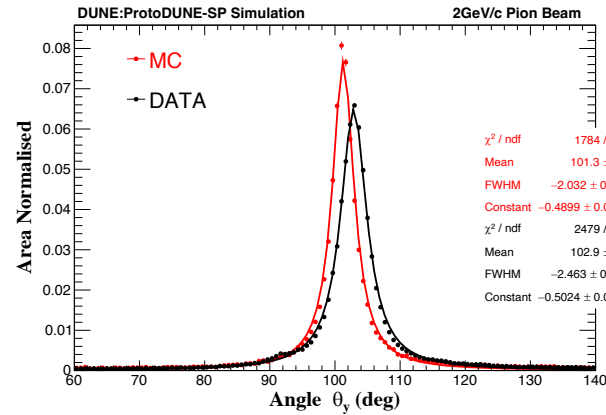
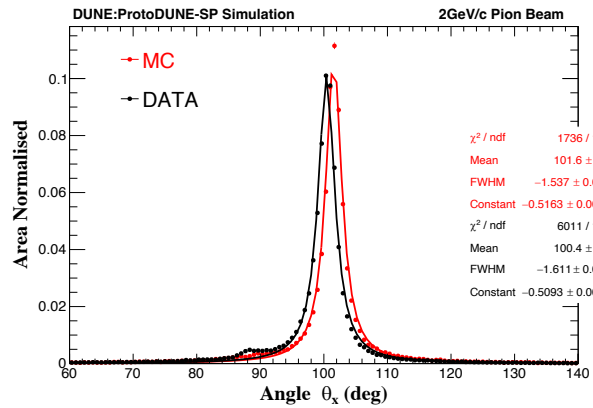
→ Better agreement between MC and Data with the additional set of cuts.

Reconstructed Beam Theta angle

2 GeV Pion Beam



2 GeV Pion Beam with additional cuts

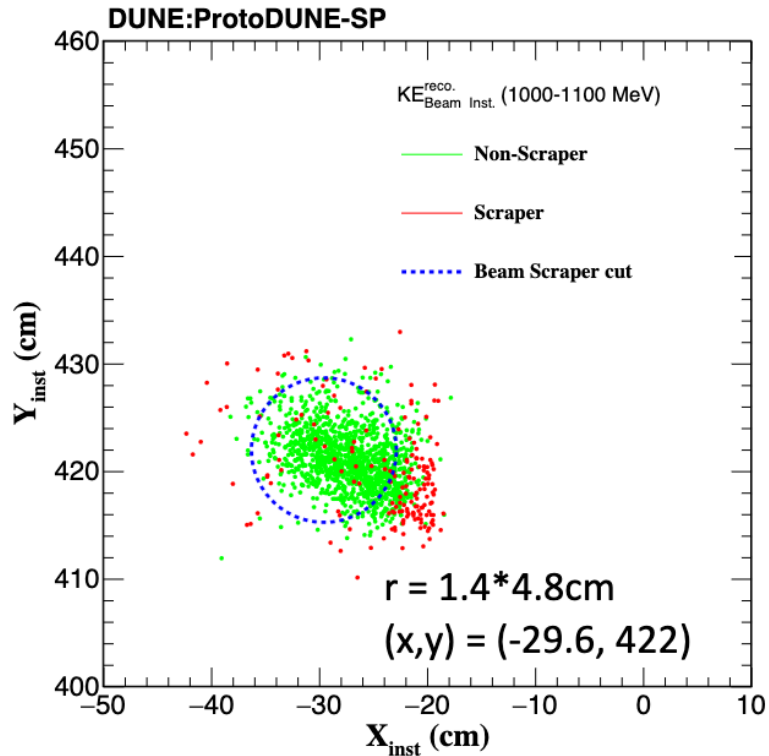


→ The additional set of cuts improved the issues with ThetaY angles.

Cut affected by changes : Beam Scraper Cut

Beam Scraper Cut : requires that the beam particles are aligned with the beam plug.

- (More information can be found in [ProtoDUNE beam scraper and upstream energy loss, Sungbin Oh](#))



Sungbin Oh

Beam Scarper Cuts (circle definition):

Values used for the 1GeV analysis

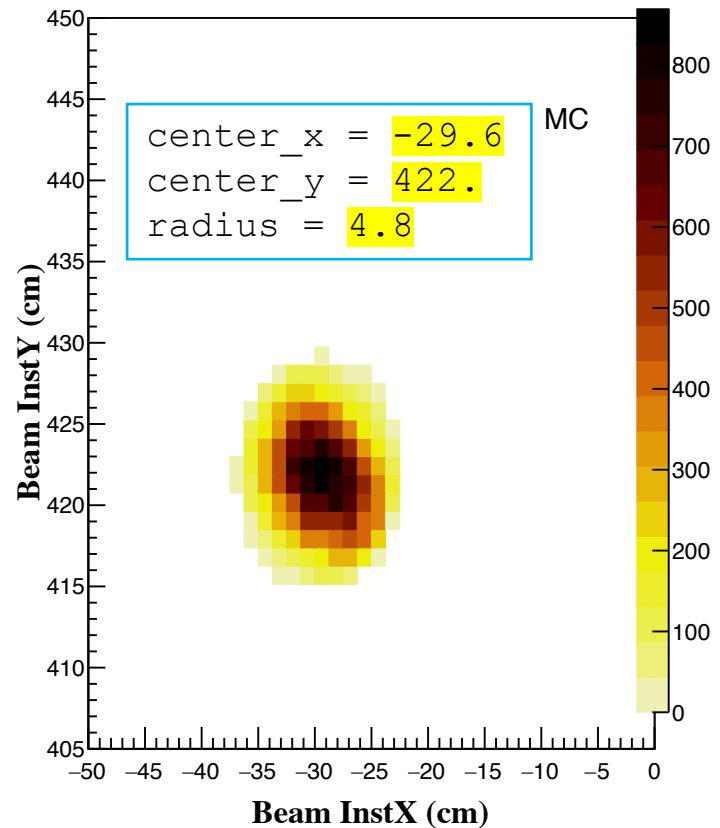
	x (cm)	y (cm)	r (cm)	Multiplicity
MC	-29.6	422	4.8	1.4
Data	-32.16	422	4.8	1.2

[Pion CEX analysis, Kang Yang](#)

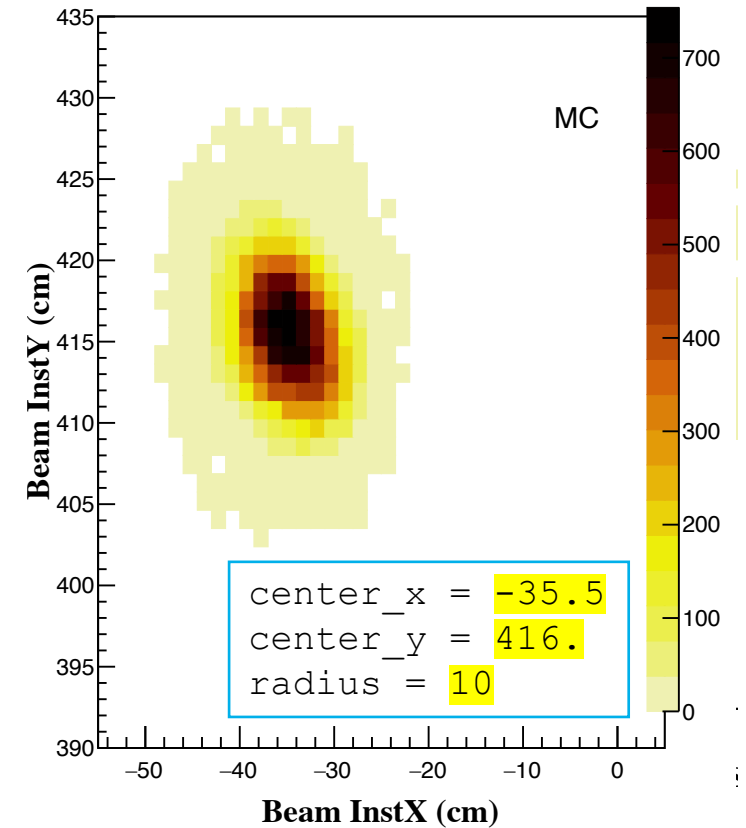
→ Cuts Removing the first 30 cm of the tracks modify the X and Y inst distributions.

Beam Scraper Cut : MC

2 GeV Before cut



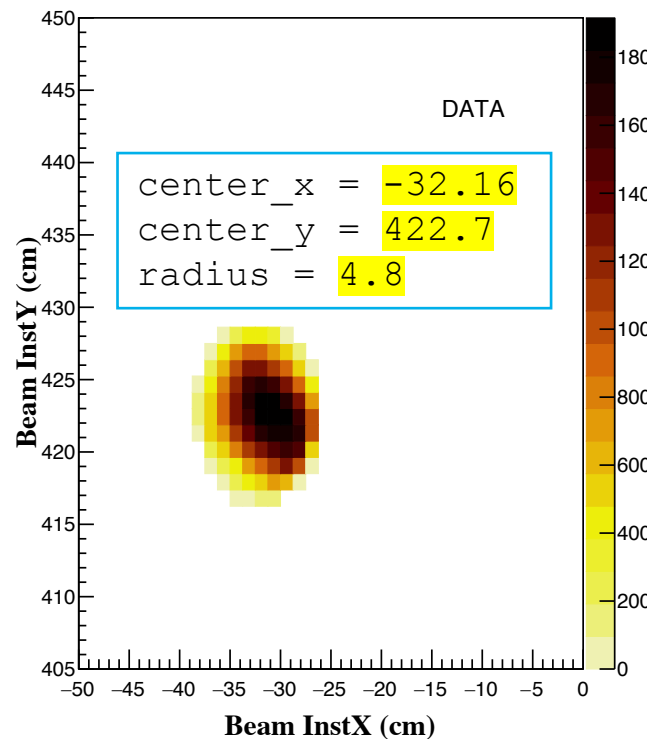
2 GeV After additional cuts



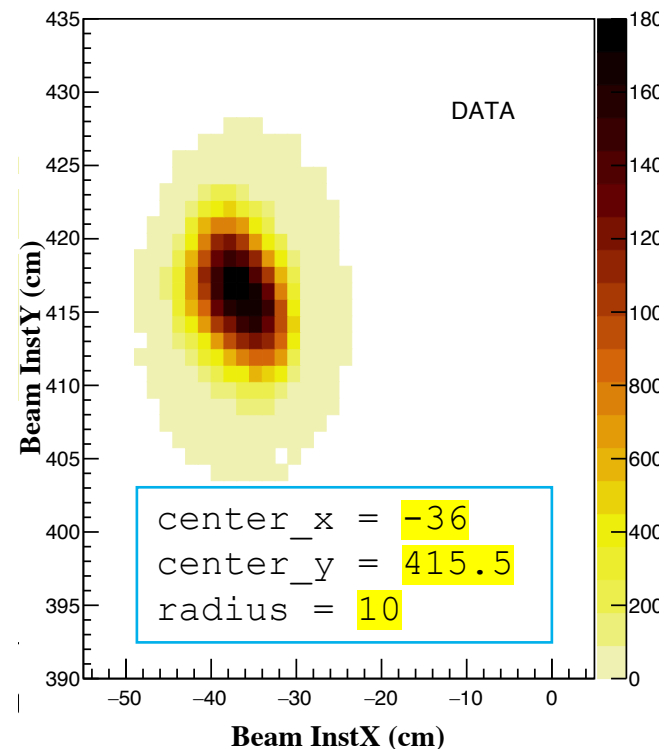
→ The center of the X and Y inst distributions changed after adding the extra cuts.

Beam Scraper Cut : Data

2 GeV Before cut



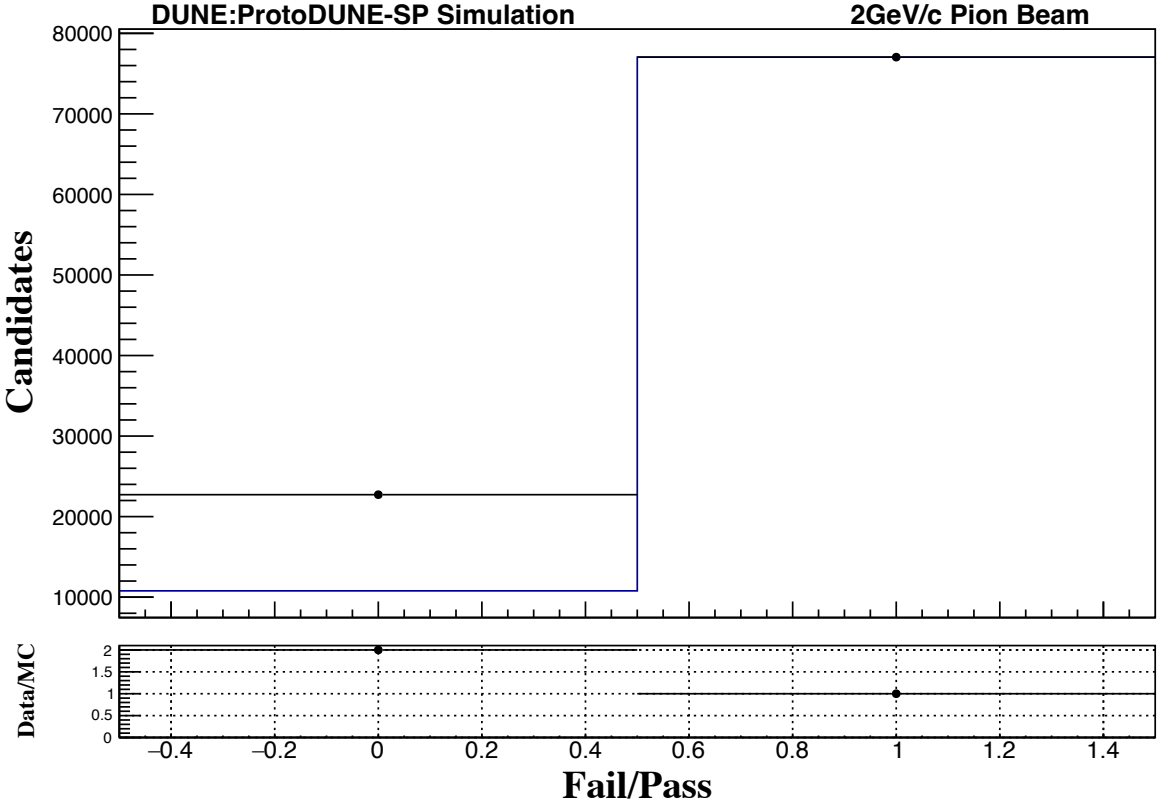
2 GeV After additional cuts



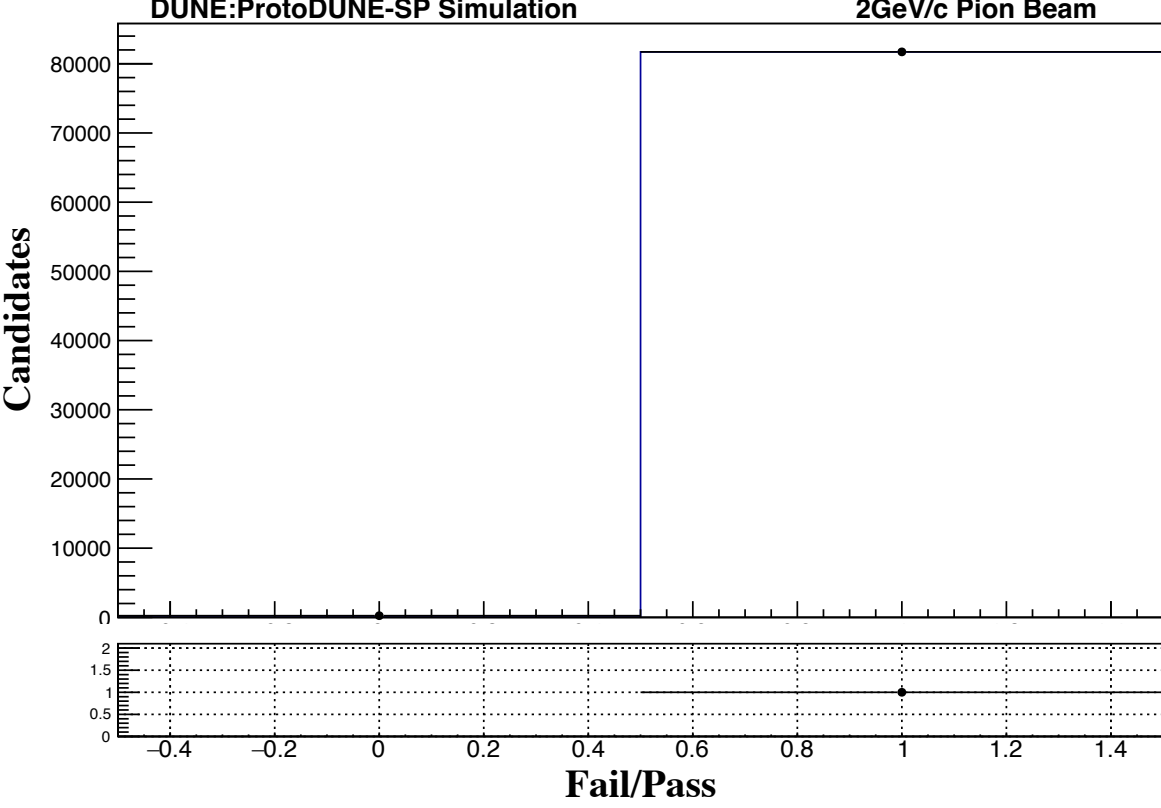
→ The center of the X and Y inst distributions changed after adding the extra cuts.

Beam Scraper Cut : Data

2 GeV Before cut



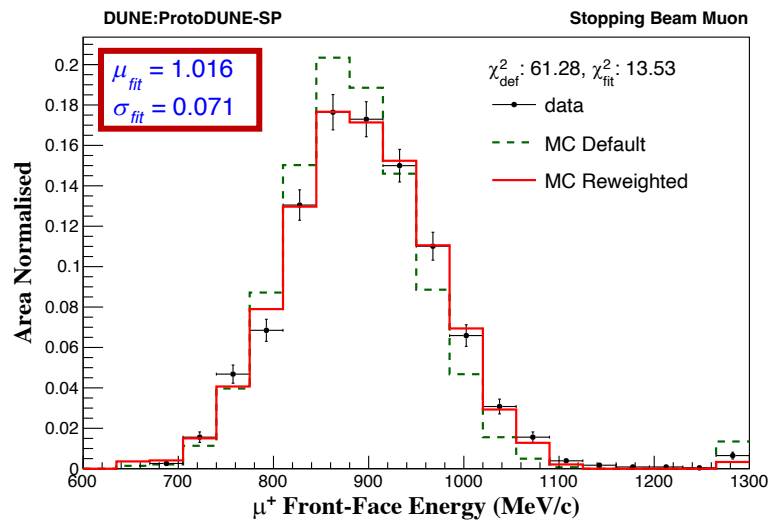
2 GeV After additional cuts



Beam muon reweight

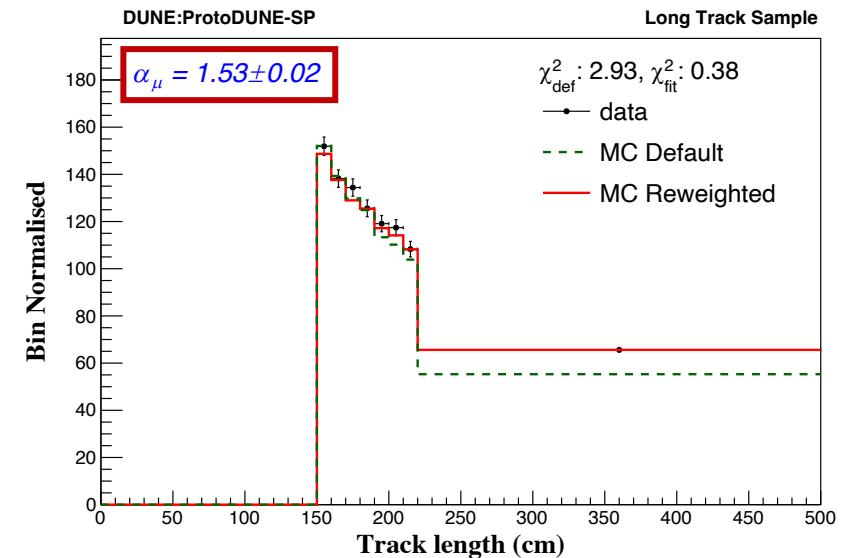
- For the 1GeV analysis, to improve data and Monte Carlo agreement, Muon samples were analyzed.
- Measured energy of muons stopping in the detector can be compared to the expected Bethe-Bloch formula to create a weight used to improve data and MC discrepancy.
- Values used for the **1 GeV analysis** :

1. Weight for momentum distribution (1 GeV)



$$w = \frac{e^{-\frac{(p-\mu_{\text{fit}})^2}{2\sigma_{\text{fit}}^2}}}{e^{-\frac{(p-\mu_{\text{true}})^2}{2\sigma_{\text{true}}^2}}}$$

2. Muon weight (1 GeV)

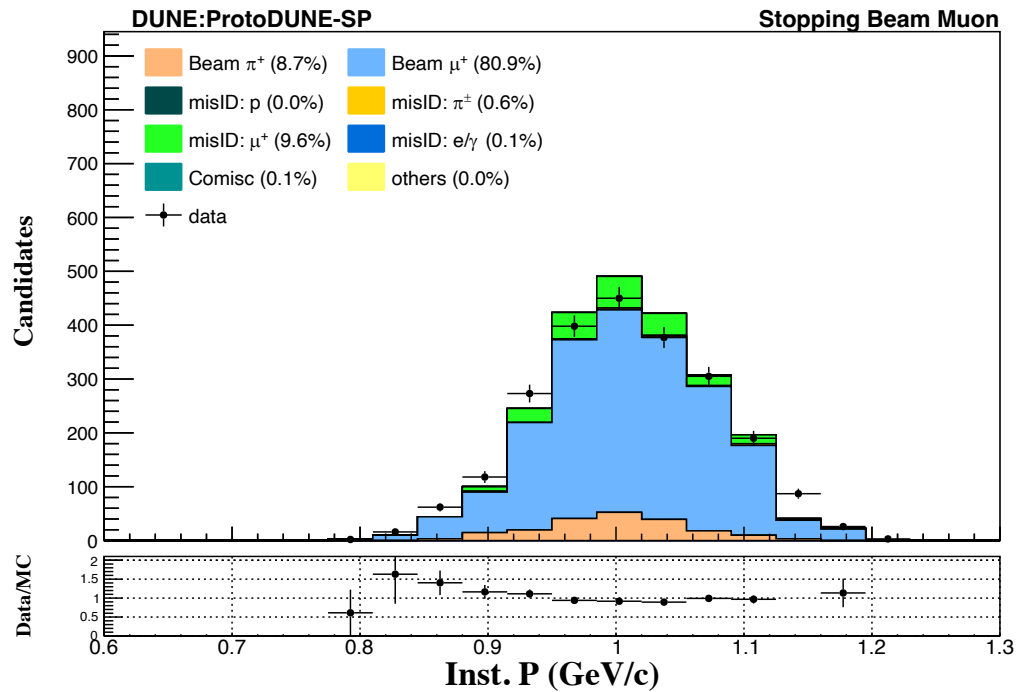


→ I have done the same analysis for the **2 GeV case**.

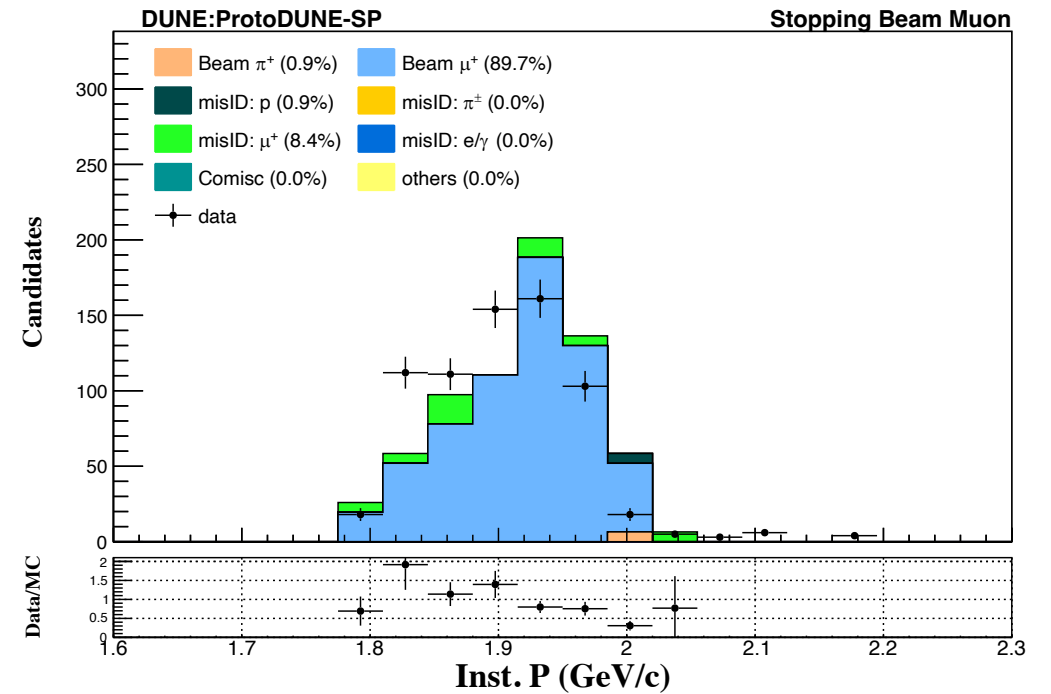
Beam muon reweight

→ Stopping muon Inst. P distribution 1 GeV / 2 GeV Comparison

1 GeV Case



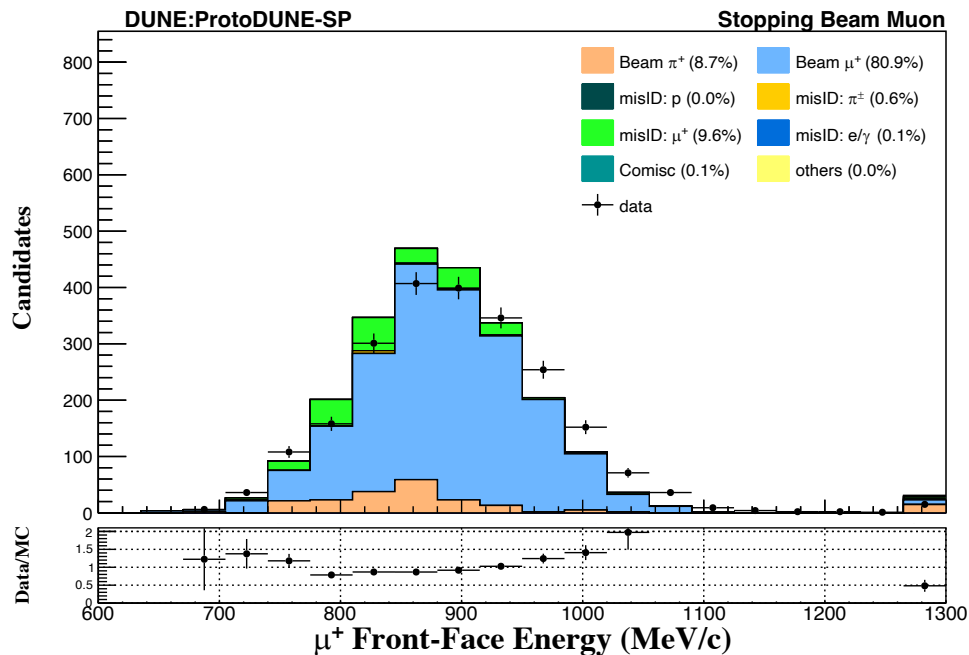
2 GeV Case



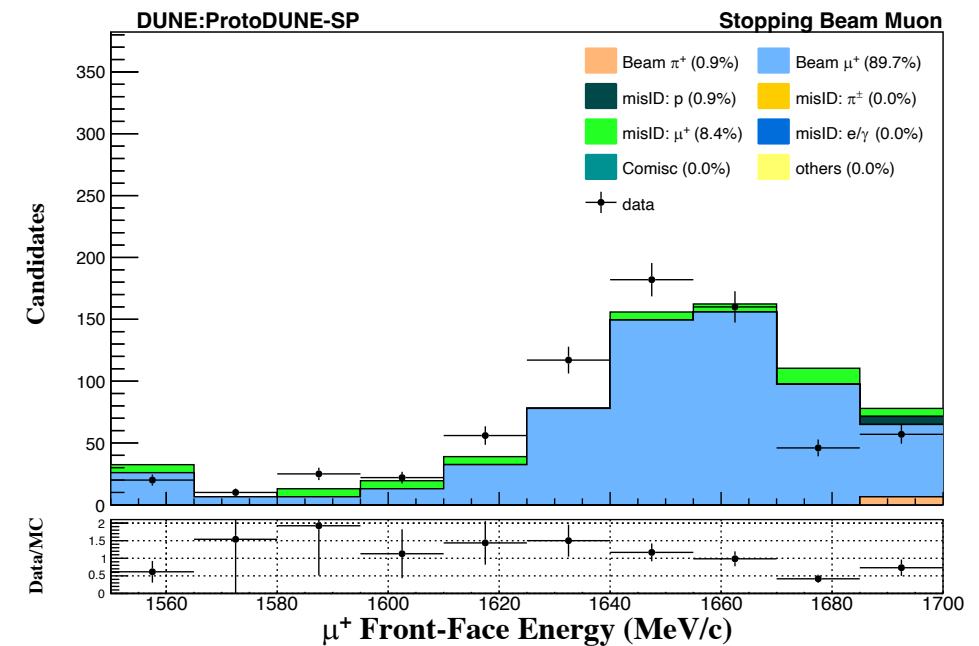
Beam muon reweight

→ Muon front-face energy 1GeV / 2 GeV Comparison

1 GeV Case



2 GeV Case

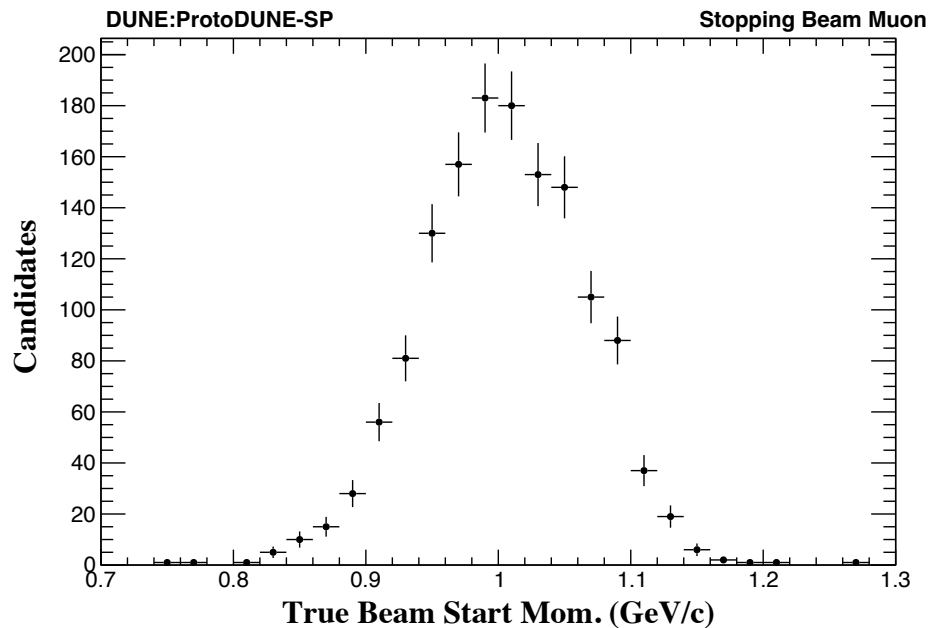


→ The same Energy Loss formula as 1GeV was used for 2GeV (is that still valid?) to calculate the front face energy.

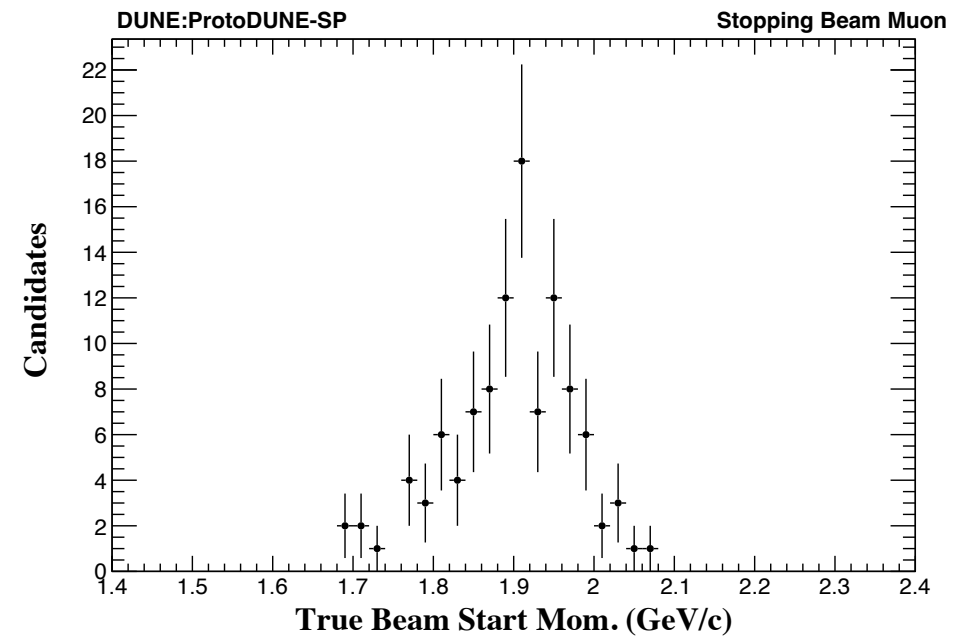
Beam muon reweight

→ Truth matched stopping muon starting momentum 1 GeV / 2 GeV Comparison

1 GeV Case



2 GeV Case

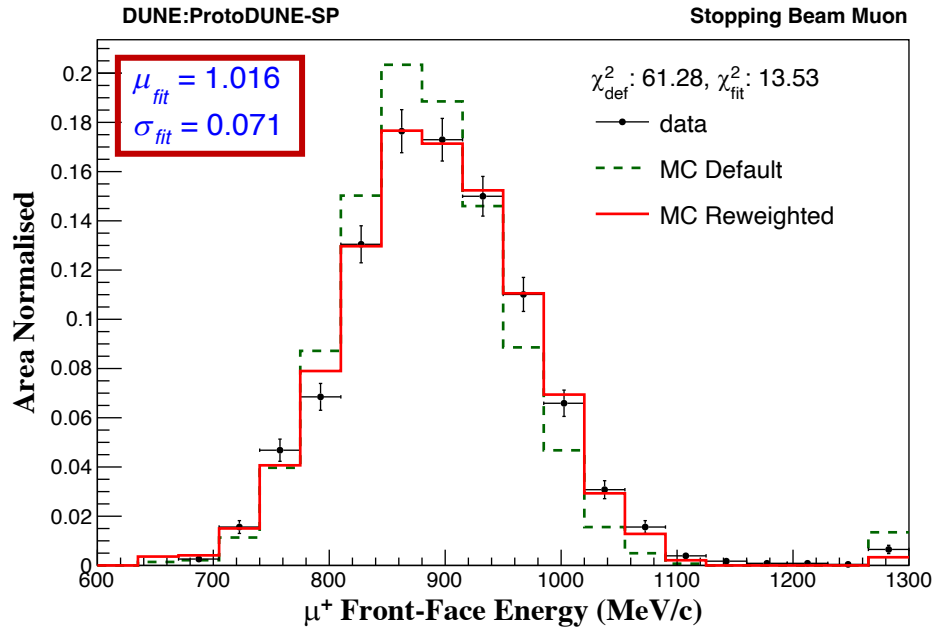


→ Low statistics for the 2 GeV case.

Beam muon reweight

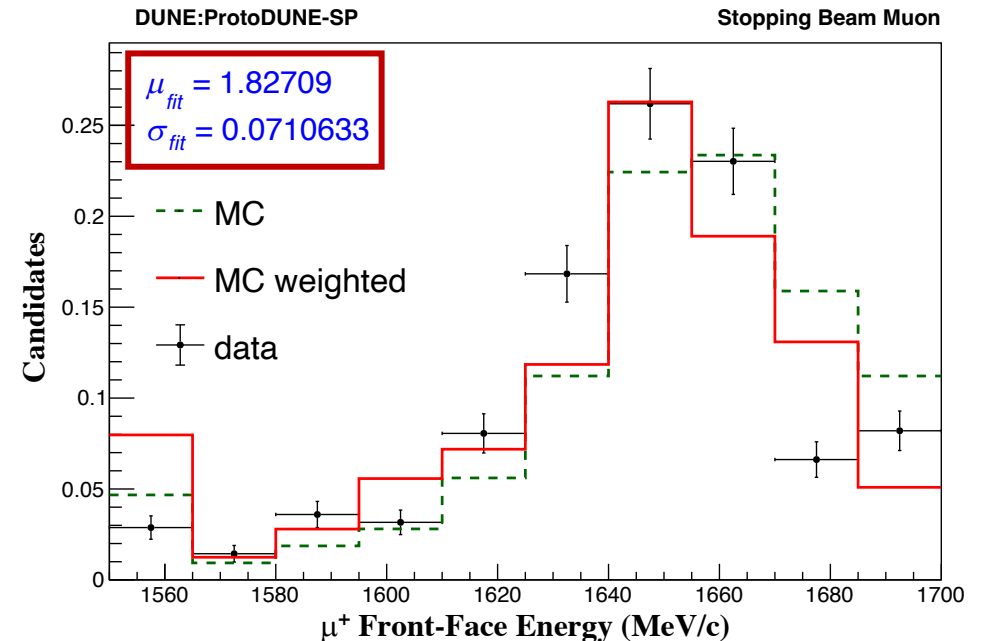
→ Momentum weight 1 GeV / 2 GeV Comparison

1 GeV Case



$$w = \frac{e^{-\frac{(p-\mu_{fit})^2}{2\sigma_{fit}^2}}}{e^{-\frac{(p-\mu_{true})^2}{2\sigma_{true}^2}}}$$

2 GeV Case

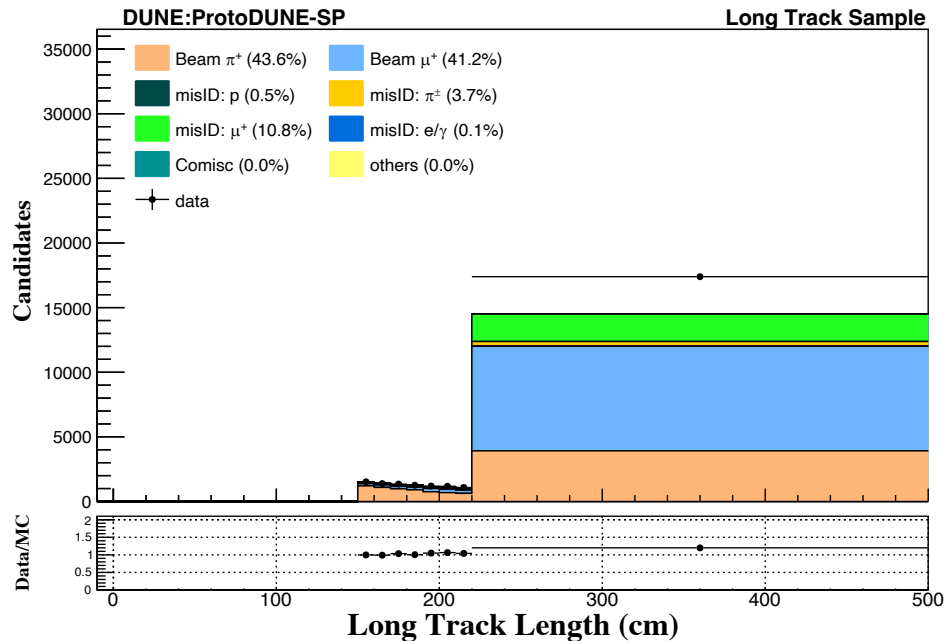


→ Not yet a good agreement between data and MC for the 2 GeV case after reweight (Maybe due to low stats?).

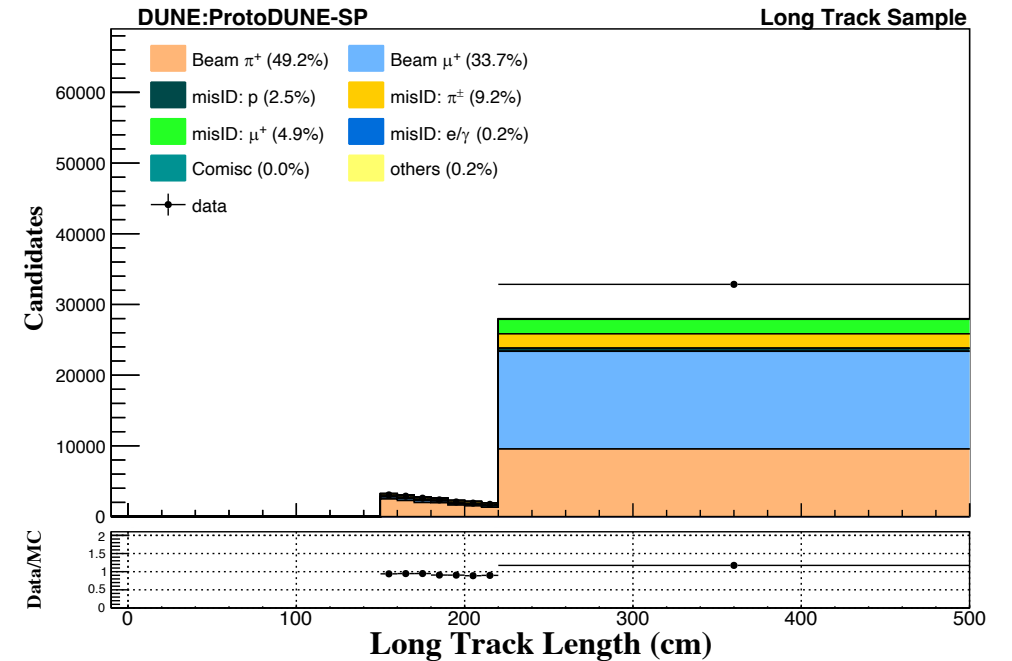
Beam muon reweight – Long tracks

→ Long track length 1 GeV / 2 GeV Comparison

1 GeV Case



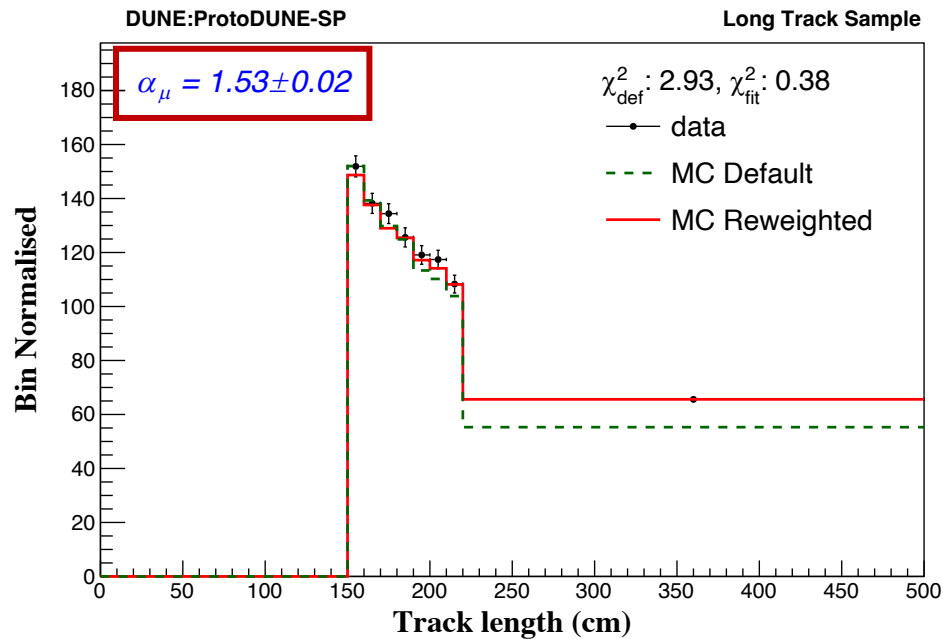
2 GeV Case



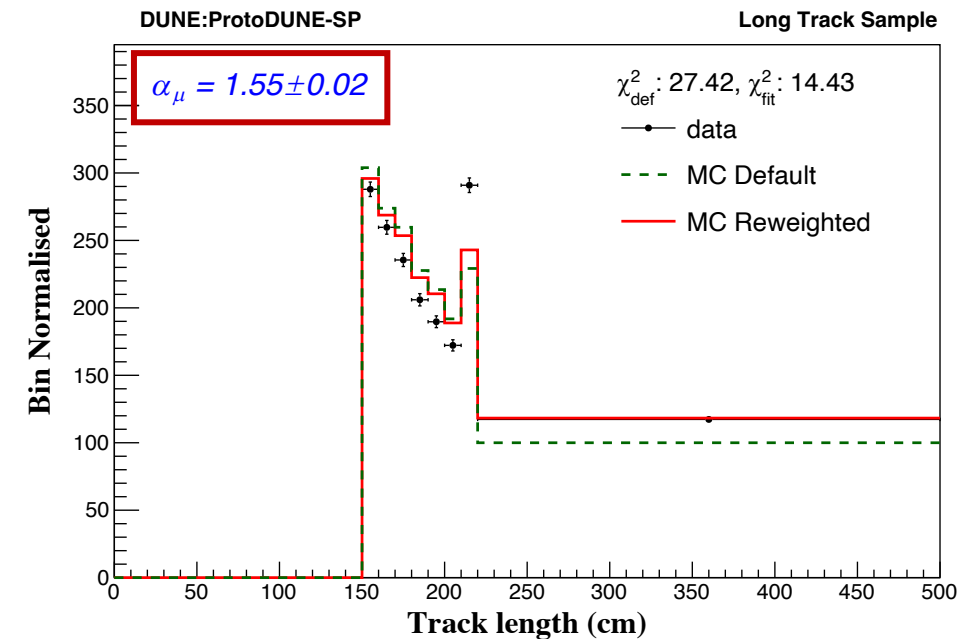
Beam muon reweight

→ Muon weight for long tracks 1 GeV / 2 GeV Comparison

1 GeV Case



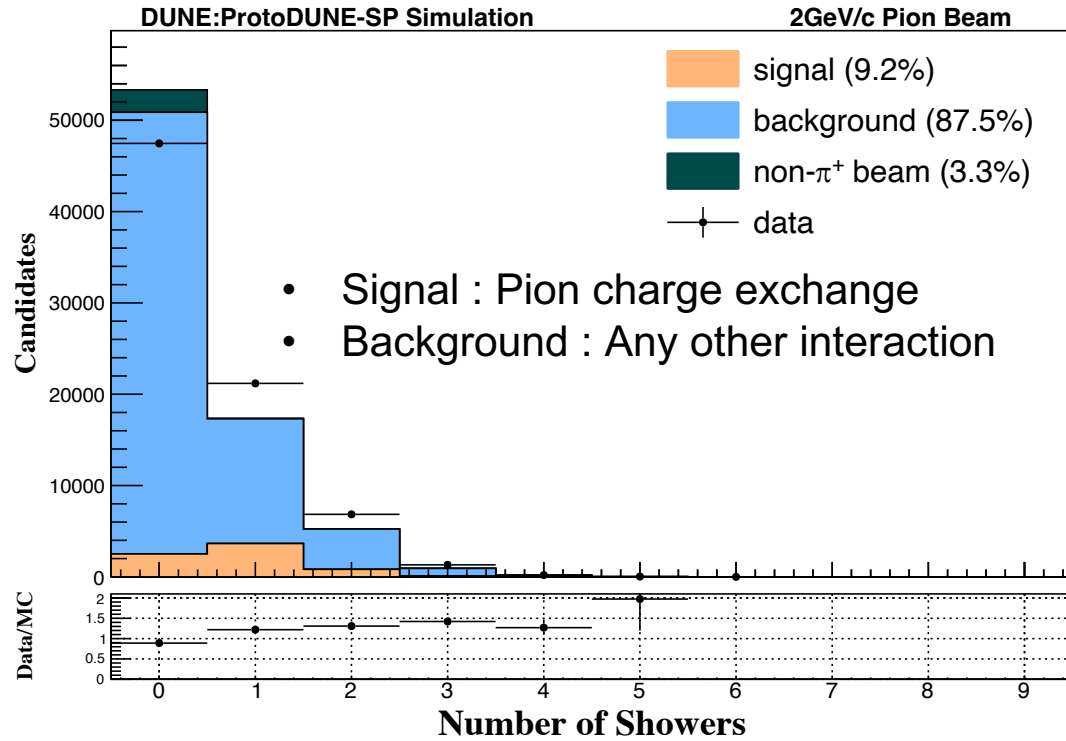
2 GeV Case



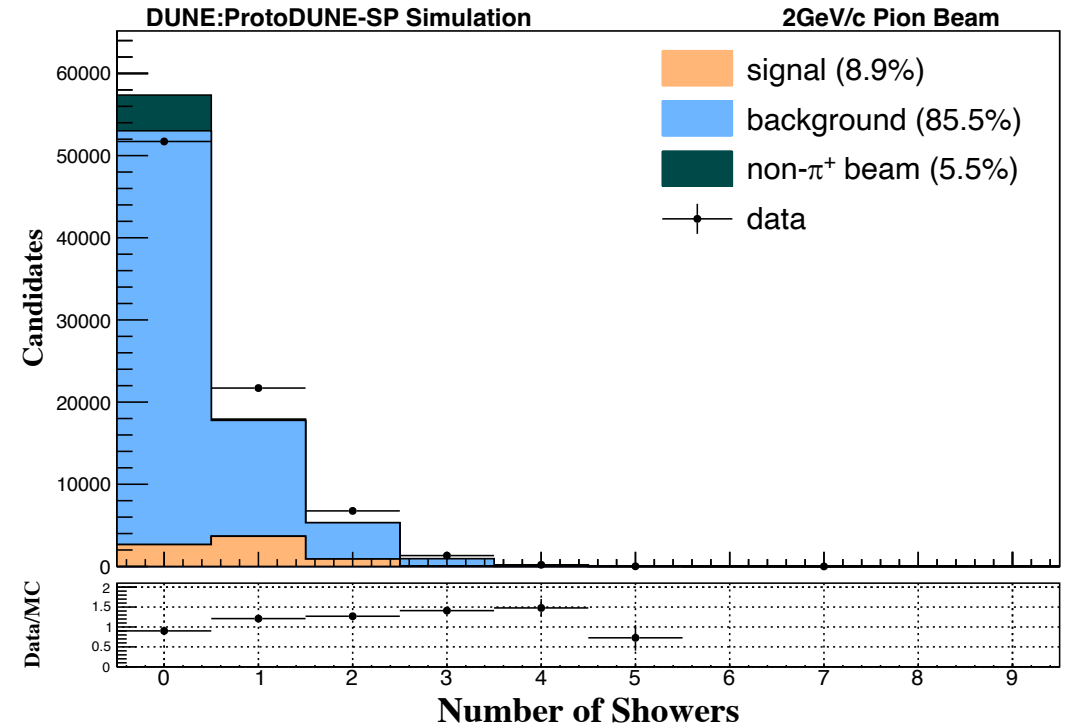
→ A scale-up of 55% is applied to muon tracks exceeding 150 cm in length for the 2 GeV case.

Number of showers

2 GeV Before Cut



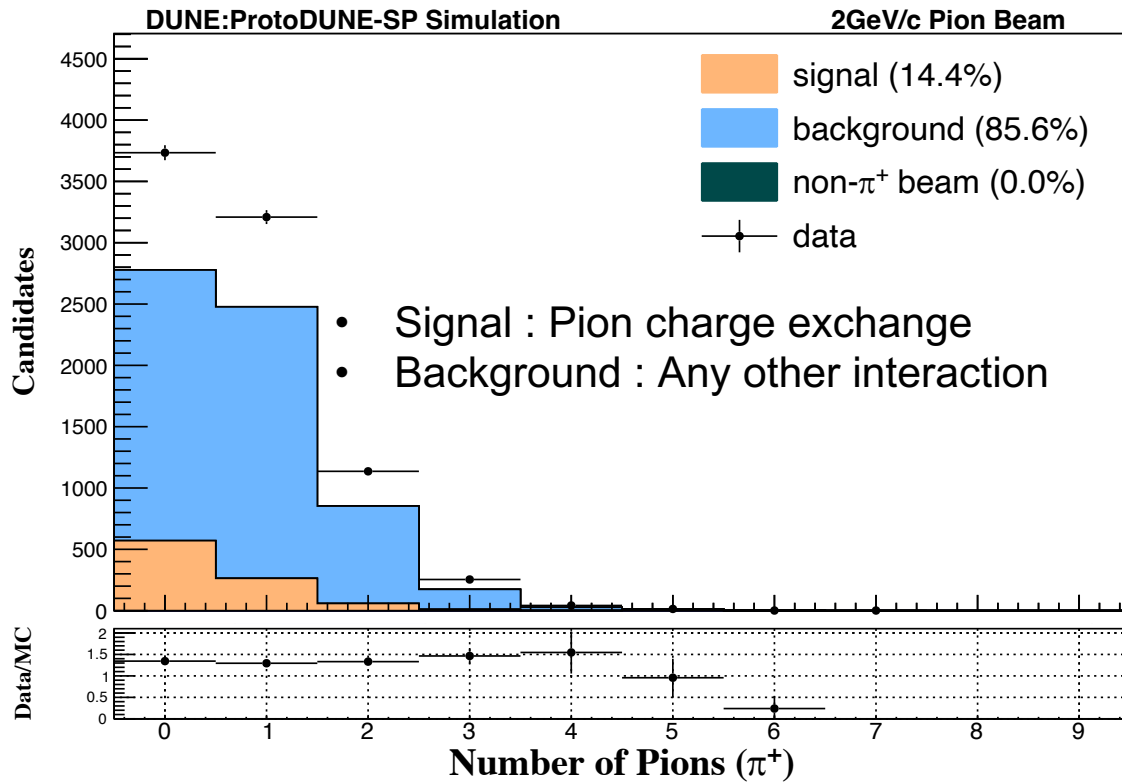
2 GeV After Additional Cuts and Cal weight



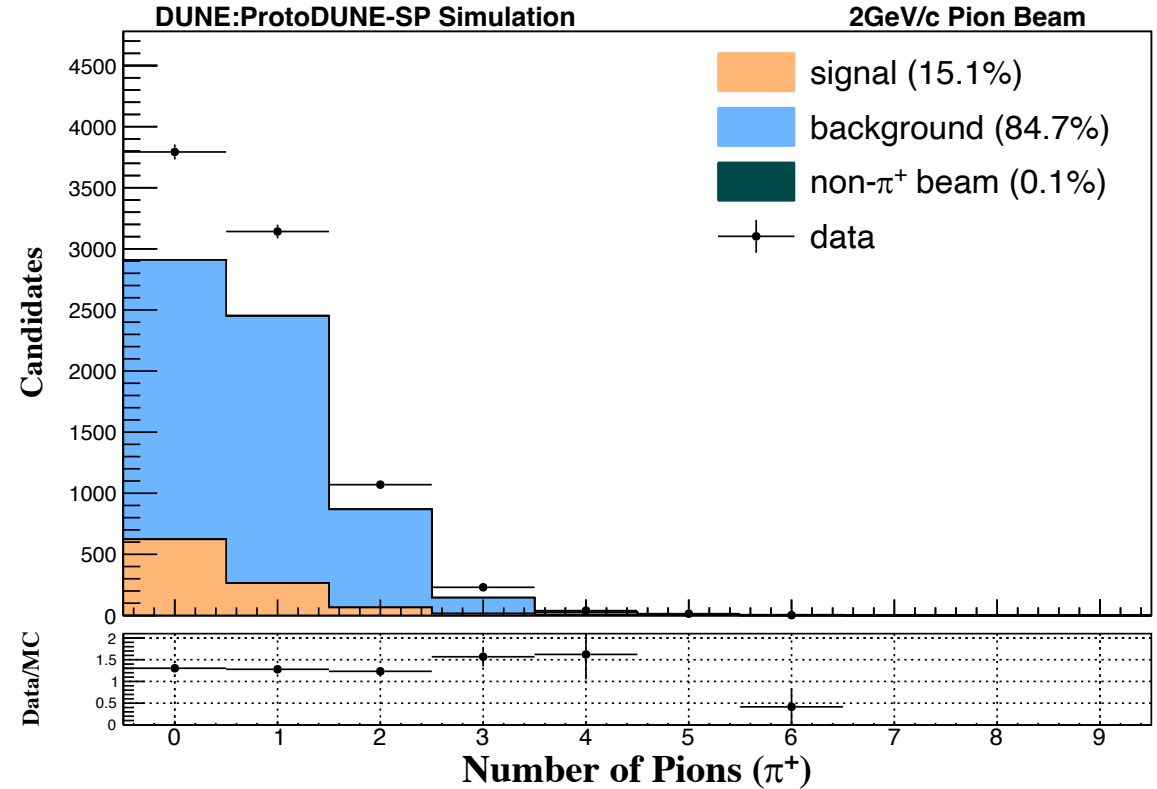
MC	2 Before Cut GeV				2 GeV After Cut			
Cuts	Entries	selected	Percentage	Cumulative	Entries	Selected	Percentage	Cumulative
Nshower	36866	3028	8.21%	2.18%	33985	2667	7.85%	1.92%

Number of π^+

2 GeV Before Cut

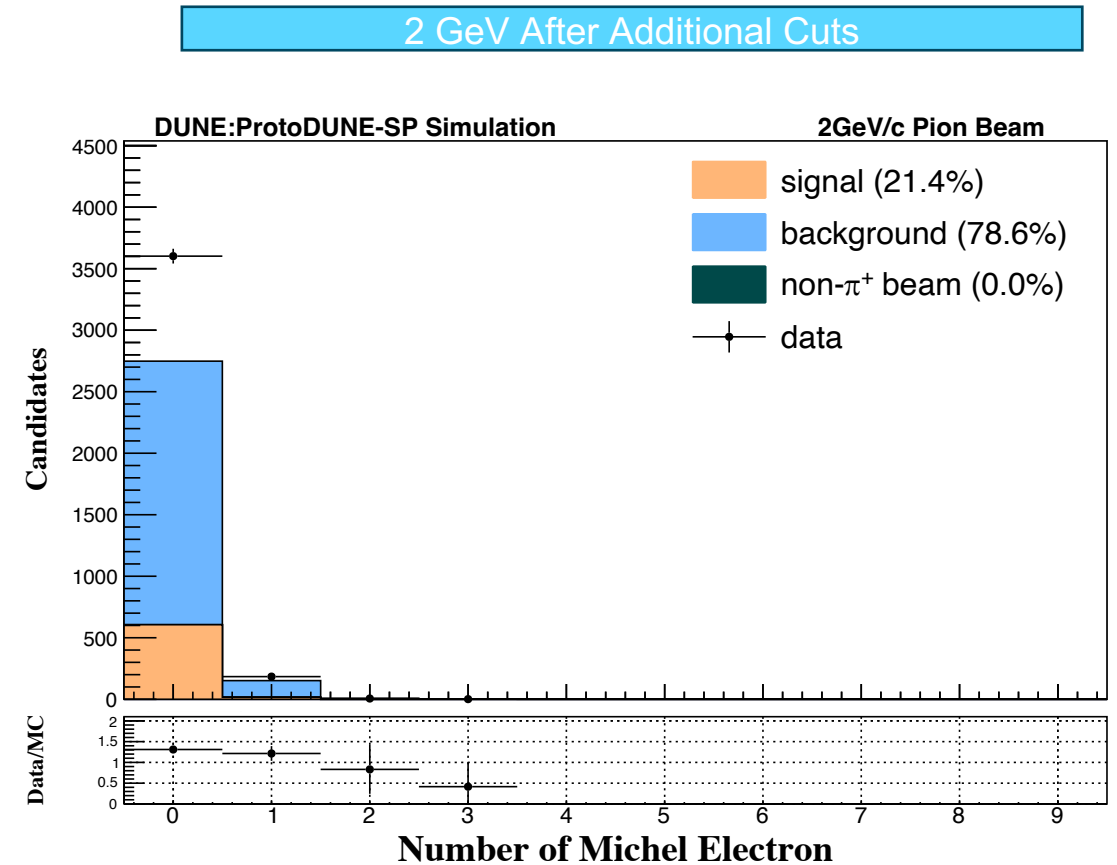
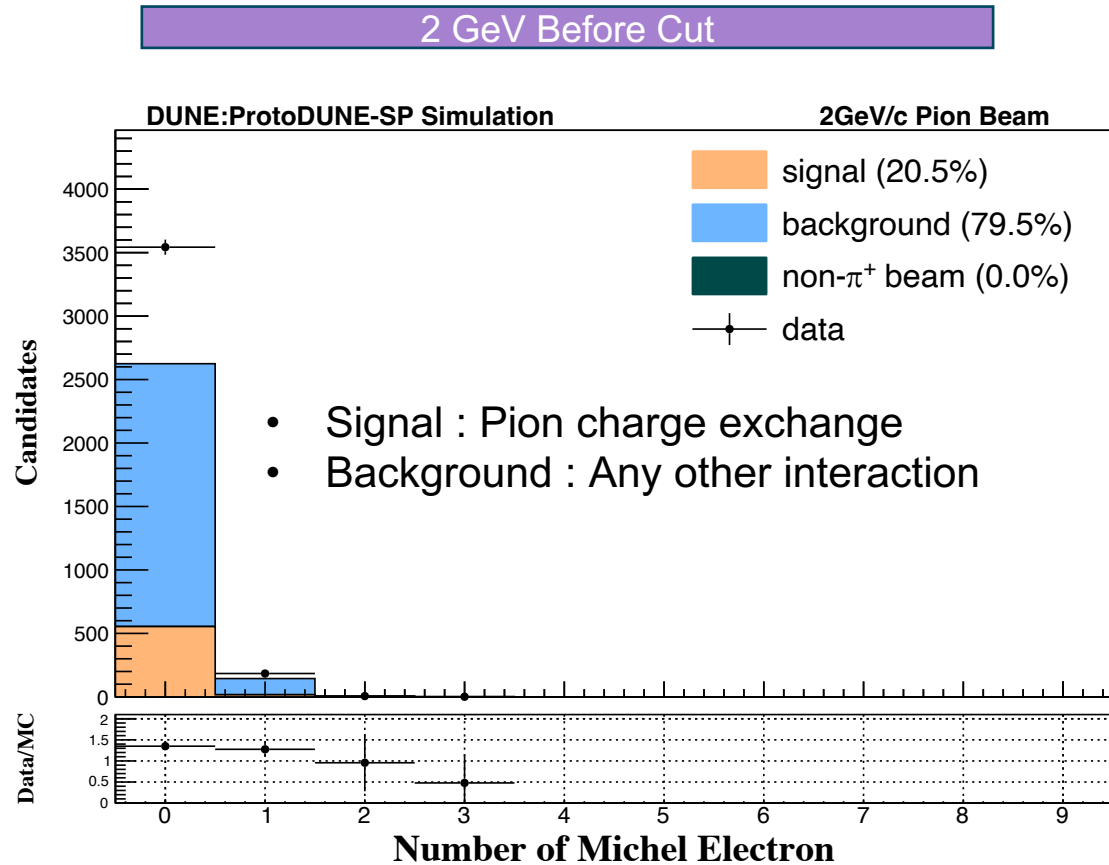


2 GeV After Additional Cuts and Cal weight



MC	2 Before Cut GeV				2 GeV After Cut			
Cuts	Entries	selected	Percentage	Cumulative	Entries	Selected	Percentage	Cumulative
Npiplus	3028	1329	43.89%	0.96%	2667	1210	45.37%	0.87%

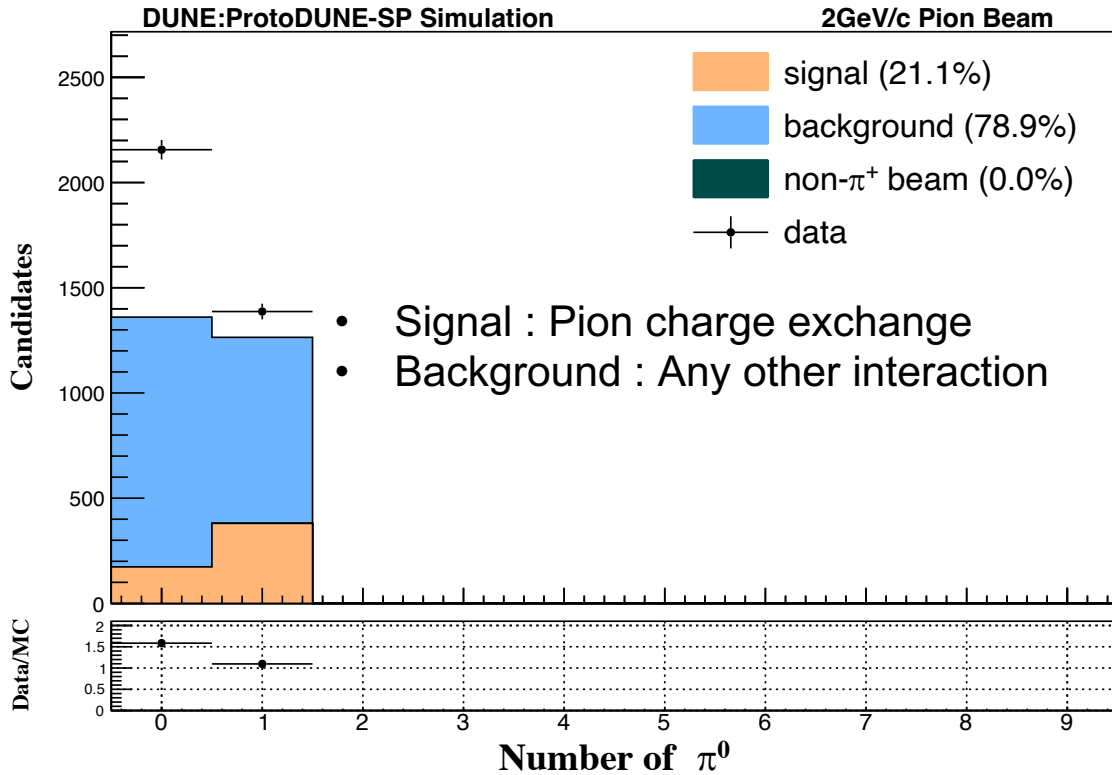
Number of Michel electrons



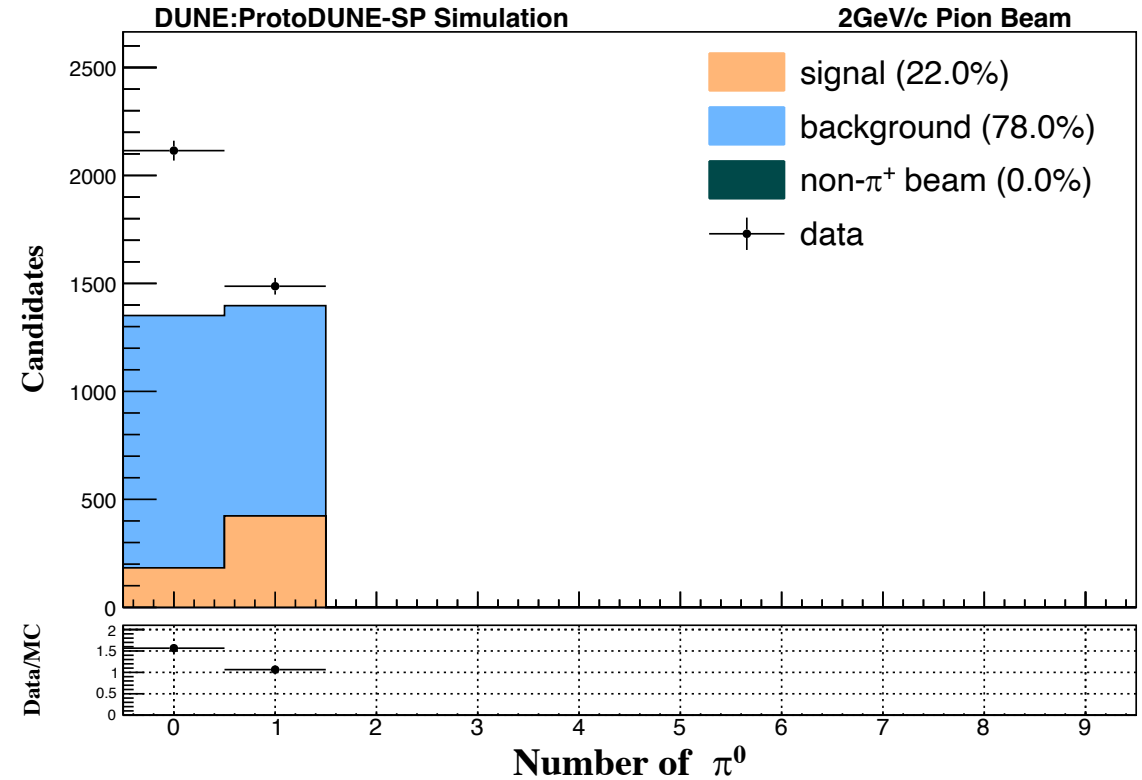
MC	2 Before Cut GeV				2 GeV After Cut			
Cuts	Entries	selected	Percentage	Cumulative	Entries	Selected	Percentage	Cumulative
Nmichel	1329	1256	94.51%	0.90%	1210	1143	94.46%	0.82%

Number of π^0

2 GeV Before Cut



2 GeV After Additional Cuts and Cal weight



MC	2 Before Cut GeV				2 GeV After Cut			
Cuts	Entries	selected	Percentage	Cumulative	Entries	Selected	Percentage	Cumulative
Npi0	1256	605	48.17%	0.43%	1143	581	50.83%	0.42%

Cut summary 2 GeV Before vs After Cut - MC

MC	2 GeV Before Cut				2 GeV After Additional Cuts and Cal Weight			
Cuts	Entries	Selected	percentage	Accumulated	Entries	selected	percentage	Accumulated
Beam PDG	139124	100637	72.34%	72.34%	139124	100637	72.34%	72.34%
Pandora slice	100637	87700	87.14%	63.04%	100637	87700	87.14%	63.04%
Calo size	87700	85864	97.91%	61.72%	87700	85864	97.91%	61.72%
Beam Quality	85864	54106	63.01%	38.89%	85864	45821	53.36%	32.94%
APA3 endZ	54106	43968	81.26%	31.60%	45821	35625	77.75%	25.61%
Michel score	43968	43766	99.54%	31.46%	35625	35471	99.57%	25.50%
Chi2/DOF	43766	42026	96.02%	30.21%	35471	33996	95.84%	24.44%
Beam Scraper	42026	36866	87.72%	26.50%	33996	33985	99.97%	24.43%
Nshower	36866	3028	8.21%	2.18%	33985	2667	7.85%	1.92%
Npiplus	3028	1329	43.89%	0.96%	2667	1210	45.37%	0.87%
Nmichel	1329	1256	94.51%	0.90%	1210	1143	94.46%	0.82%
Npi0	1256	605	48.17%	0.43%	1143	581	50.83%	0.42%

- **2 GeV Before Cut selected pion beam MC : 36866 / 139124 (26.50 %)**
- **2 GeV After Cut selected pion beam MC : 33985 / 139124 (24.43 %)**

Cut summary 2 GeV Before vs After Cut - MC

Starting from Beam Quality :

MC	2 GeV				2 GeV After Additional Cuts and Cal Weight			
Cuts	Entries	Selected	percentage	Accumulated	Entries	selected	percentage	Accumulated
Beam Quality	54106	54106	100.00%	100.00%	85864	85864	100.00%	100.00%
APA3 endZ	54106	43968	81.26%	81.26%	85864	35625	41.49%	41.49%
Michel score	43968	43766	99.54%	80.89%	35625	35471	99.57%	41.31%
Chi2/DOF	43766	42026	96.02%	77.67%	35471	33996	95.84%	39.59%
Beam Scraper	42026	36866	87.72%	68.14%	33996	33985	99.97%	39.58%
Nshower	36866	3028	8.21%	5.60%	33985	2667	7.85%	3.11%
Npiplus	3028	1329	43.89%	2.46%	2667	1210	45.37%	1.41%
Nmichel	1329	1256	94.51%	2.32%	1210	1143	94.46%	1.33%
Npi0	1256	605	48.17%	1.12%	1143	581	50.83%	0.68%

- **2 GeV Before Cut selected pion beam MC : 36866 / 139124 (26.50 %)**
- **2 GeV After Cut selected pion beam MC : 33985 / 139124 (24.43 %)**

Cut summary 2 GeV Before vs After Cut - Data

Data	2 GeV Before Cut				2 GeV After Additional Cuts and Cal Weight			
Cuts	Entries	Selected	percentage	Accumulated	Entries	selected	percentage	Accumulated
Beam PDG	1349399	285238	21.14%	21.14%	1349399	285238	21.14%	21.14%
Pandora slice	285238	240159	84.20%	17.80%	285238	240159	84.20%	17.80%
Calo size	240159	232423	96.78%	17.22%	240159	232423	96.78%	17.22%
Beam Quality	232423	136579	58.76%	10.12%	232423	117710	50.64%	8.72%
APA3 endZ	136579	105037	76.91%	7.78%	117710	86111	73.16%	6.38%
Michel score	105037	104565	99.55%	7.75%	86111	85779	99.61%	6.36%
Chi2/DOF	104565	99767	95.41%	7.39%	85779	81956	95.54%	6.07%
Beam Scraper	99767	77044	77.22%	5.71%	81956	81718	99.71%	6.06%
Nshower	77044	8391	10.89%	0.62%	81718	8290	10.14%	0.61%
Npiplus	8391	3734	44.50%	0.28%	8290	3793	45.75%	0.28%
Nmichel	3734	3543	94.88%	0.26%	3793	3602	94.96%	0.27%
Npi0	3543	1387	39.15%	0.10%	3602	1487	41.28%	0.11%

- **2 GeV Before Cut selected pion beam Data : 77044 / 1349399 (5.71%)**
- **2 GeV After Cut selected pion beam Data : 81956 / 1349399 (6.06%)**

Cut summary 2 GeV Before vs After Cut - Data

Starting from Beam Quality :

Data	2 GeV				2 GeV After Additional Cuts and Cal Weight			
	Entries	Selected	percentage	Accumulated	Entries	selected	percentage	Accumulated
Beam Quality	136579	136579	100.00%	100.00%	117710	117710	100.00%	100.00%
APA3 endZ	136579	105037	76.91%	76.91%	117710	86111	73.16%	73.16%
Michel score	105037	104565	99.55%	76.56%	86111	85779	99.61%	72.87%
Chi2/DOF	104565	99767	95.41%	73.05%	85779	81956	95.54%	69.63%
Beam Scraper	99767	77044	77.22%	56.41%	81956	81718	99.71%	69.42%
Nshower	77044	8391	10.89%	6.14%	81718	8290	10.14%	7.04%
Npiplus	8391	3734	44.50%	2.73%	8290	3793	45.75%	3.22%
Nmichel	3734	3543	94.88%	2.59%	3793	3602	94.96%	3.06%
Npi0	3543	1387	39.15%	1.02%	3602	1487	41.28%	1.26%

- **2 GeV Before Cut selected pion beam Data : 77044 / 1349399 (5.71%)**
- **2 GeV After Cut selected pion beam Data : 81956 / 1349399 (6.06%)**

Different weight for the Cross Section analysis – Extra additional weights

- To better improve agreement between data and MC for the pions interacting energy distribution, extra set of additional weights have been applied for the 1GeV analysis.
- Weights obtained by analyzing Sideband samples.
- Weights used:
 - **WeightPi0Data()**
 - **CallniWeight()**
 - **CalBeamIntWeight();**
 - **CalCEXIntWeight();**
 - **CalCEXPi0KEWeight();**
 - **CalCEXPi0CosThetaWeight();**
 - **CalCEXPi0ThetaWeight();**
- Currently working on running the Sideband studies for the 2GeV samples.

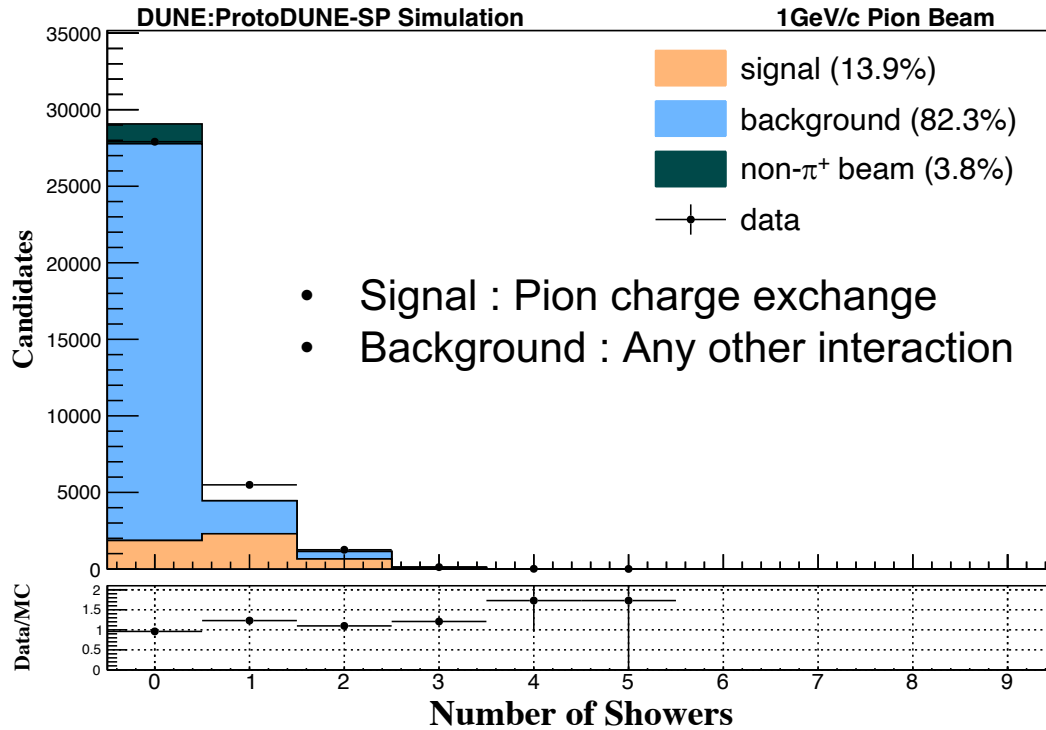
Summary

- Additional cuts to remove the first 30cm have been added to the analysis. The extra cuts improved data and MC agreement for the start Z distribution and helped fix the ThetaY MC distribution issues.
- For the 1GeV analysis, extra weights obtained from the analysis of the stopping muon samples were added to further improve data and MC agreement. The same analysis has been done for the 2GeV analysis, however due to low statistics, the weights obtained do not fully remove the data and MC discrepancy.
- Apart from that calorimetric weight, several other weights obtained from analyzing Sideband samples were also used for the 1GeV analysis. Currently working on re-running the same analysis for the 2GeV samples to get the corresponding weights.

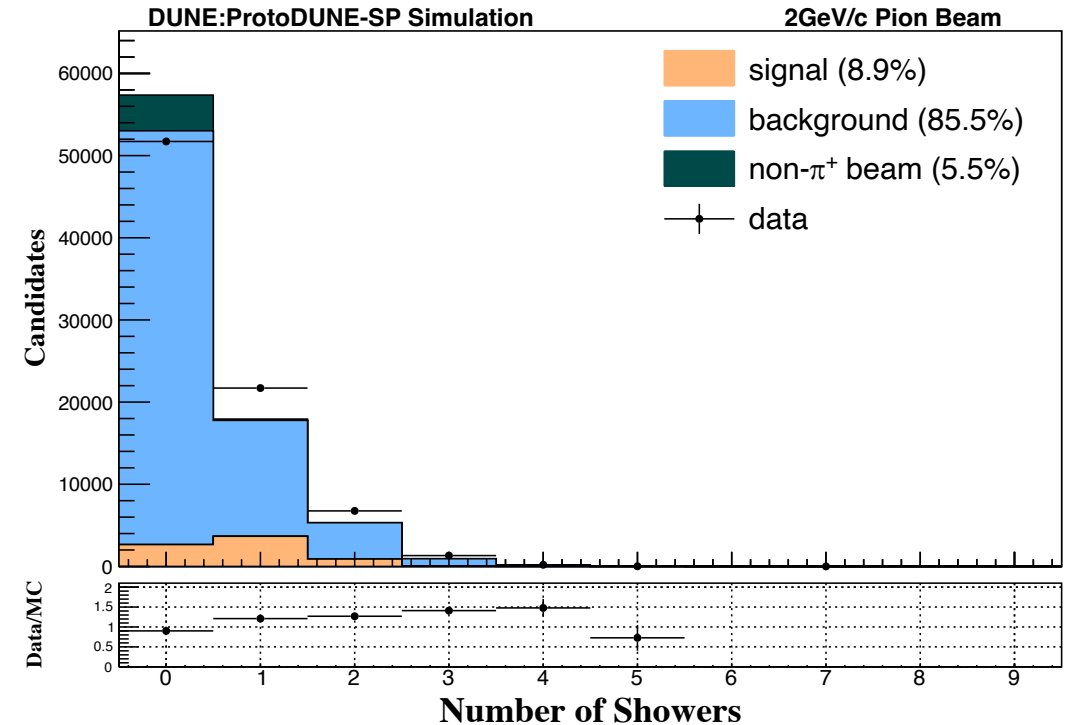
Backup Slides

Number of shower cut

1 GeV Pion Beam



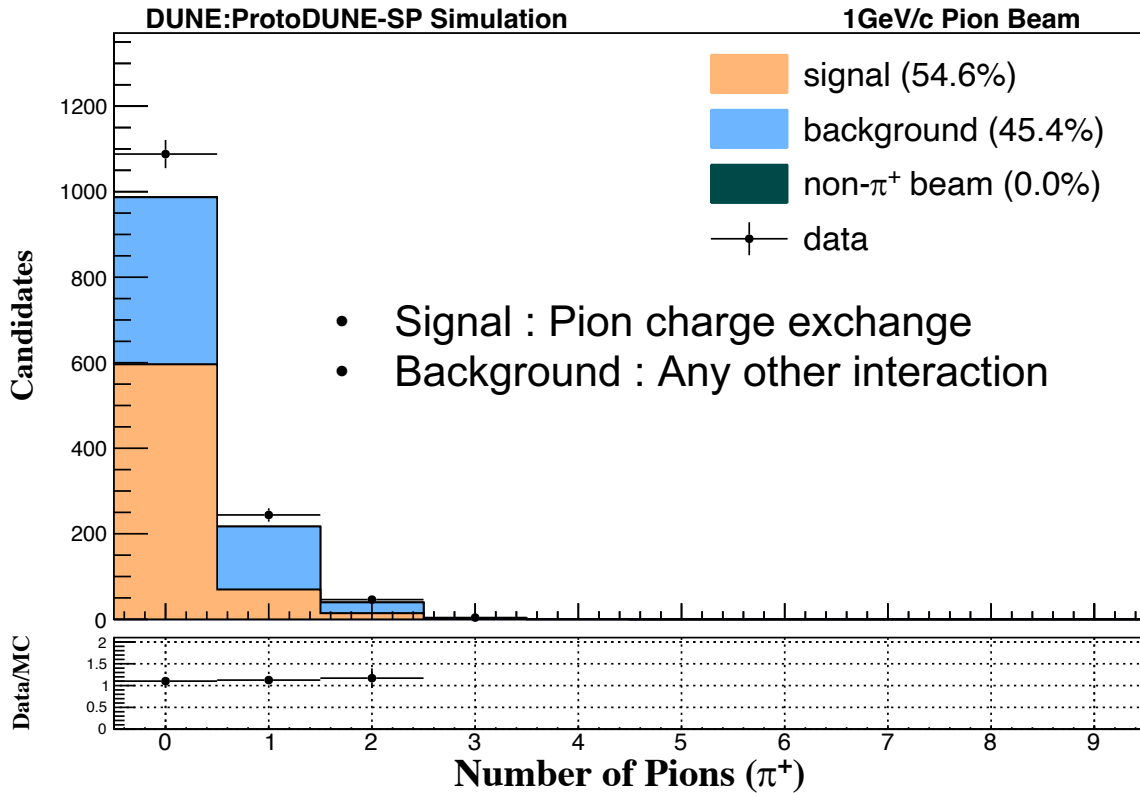
2 GeV Pion Beam



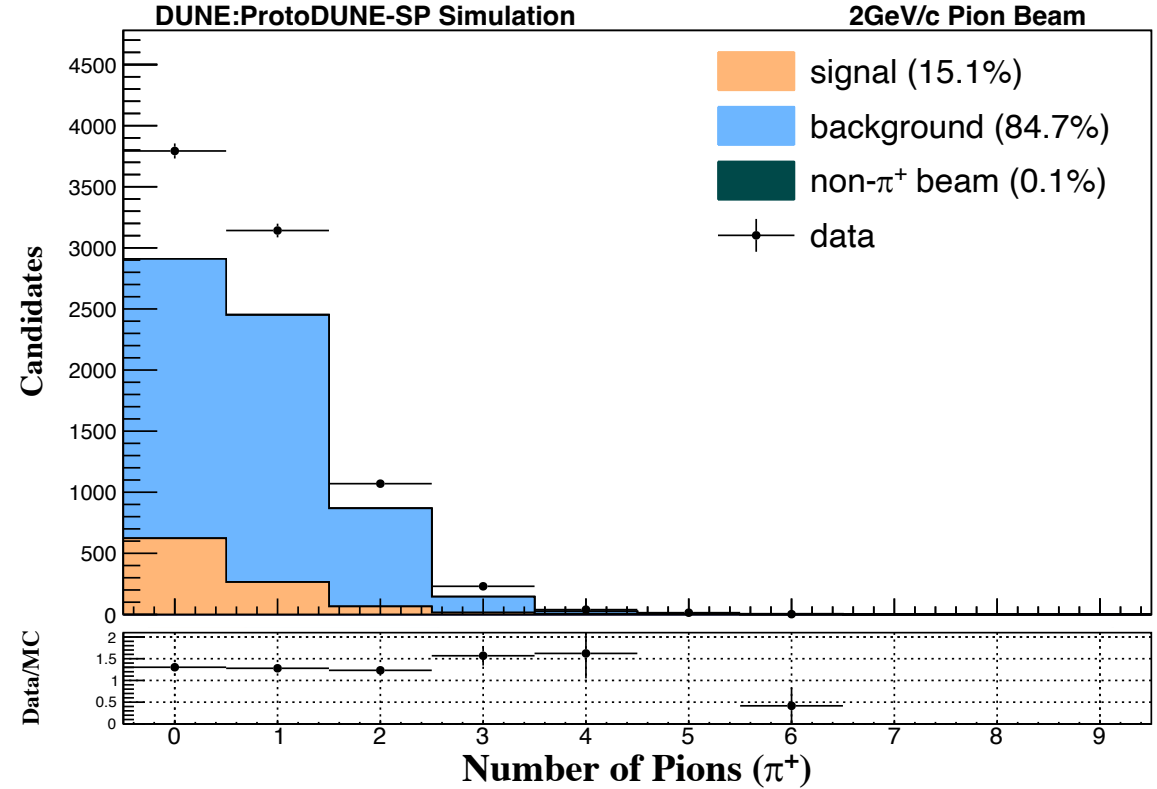
MC	1 GeV				2 GeV			
Cuts	Entries	selected	Percentage	Cumulative	Entries	Selected	Percentage	Cumulative
Nshower	60174	2155	3.58%	0.72%	33985	2667	7.85%	1.92%

Number of π^+ cut

1 GeV Pion Beam



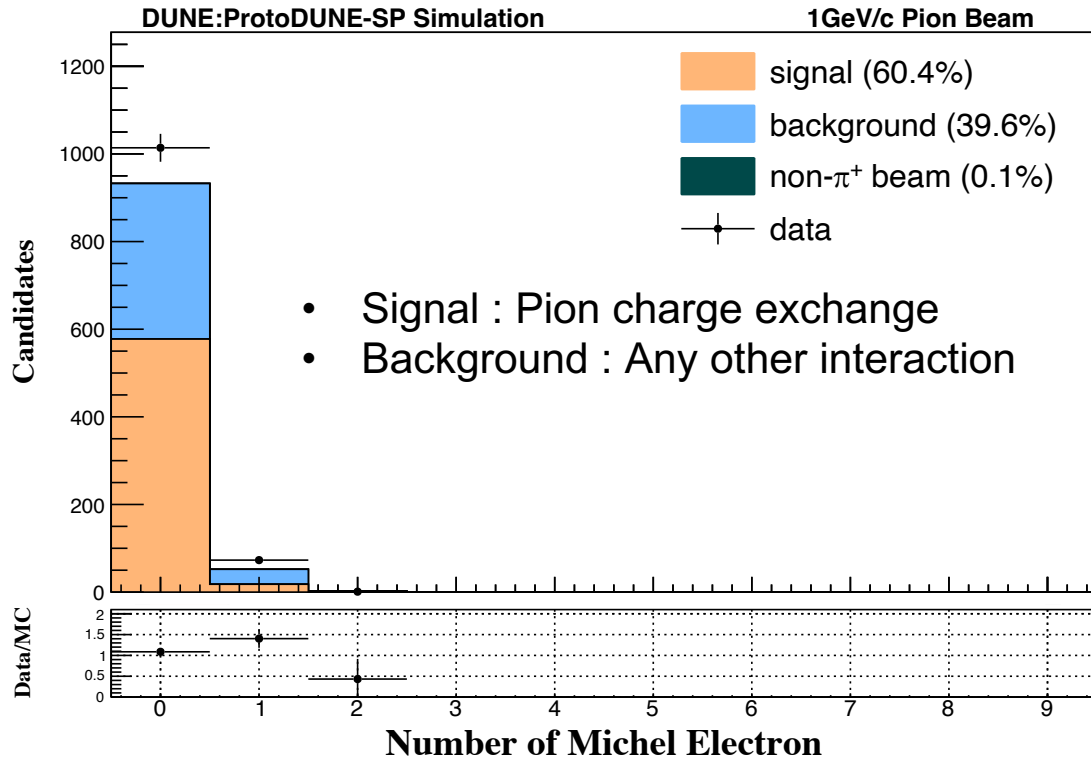
2 GeV Pion Beam



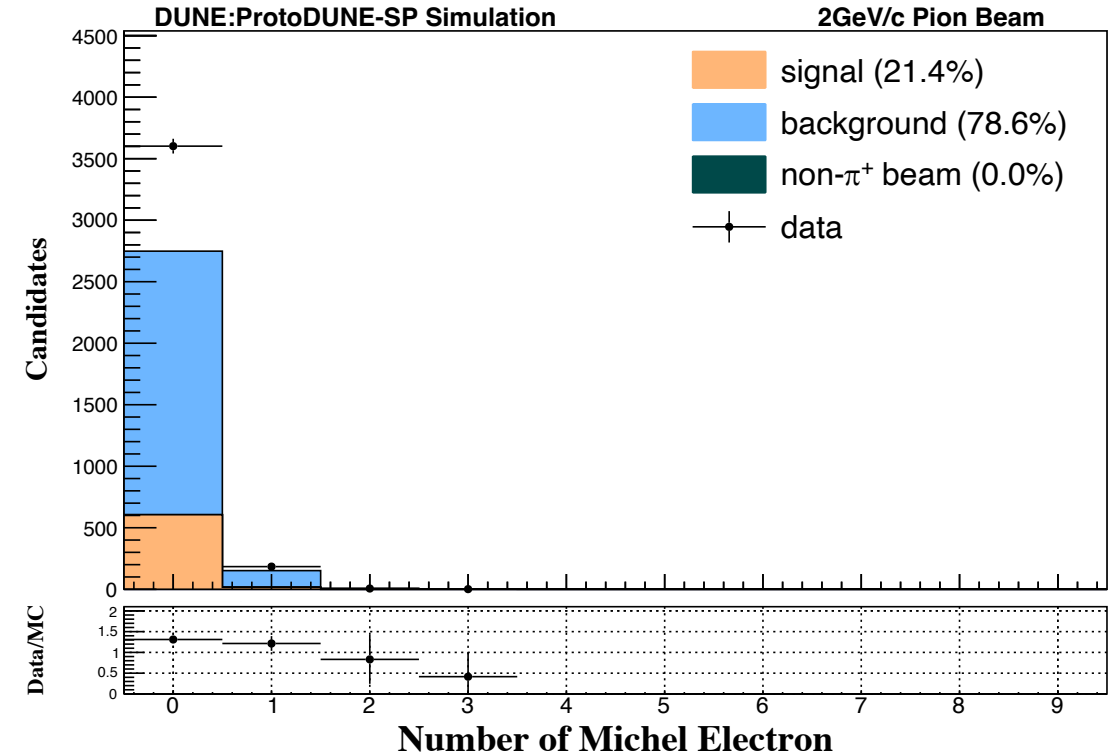
MC	1 GeV				2 GeV			
Cuts	Entries	selected	Percentage	Cumulative	Entries	Selected	Percentage	Cumulative
Npplus	2155	1708	79.26%	0.57%	2667	1210	45.37%	0.87%

Number of Michel cut

1 GeV Pion Beam



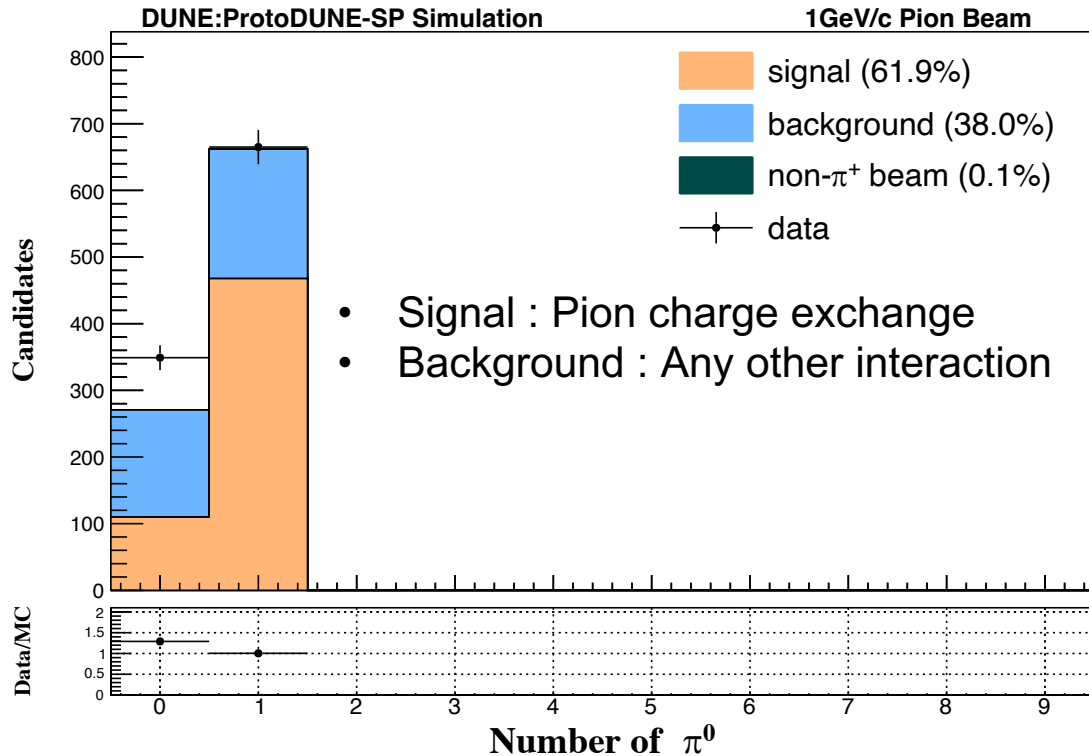
2 GeV Pion Beam



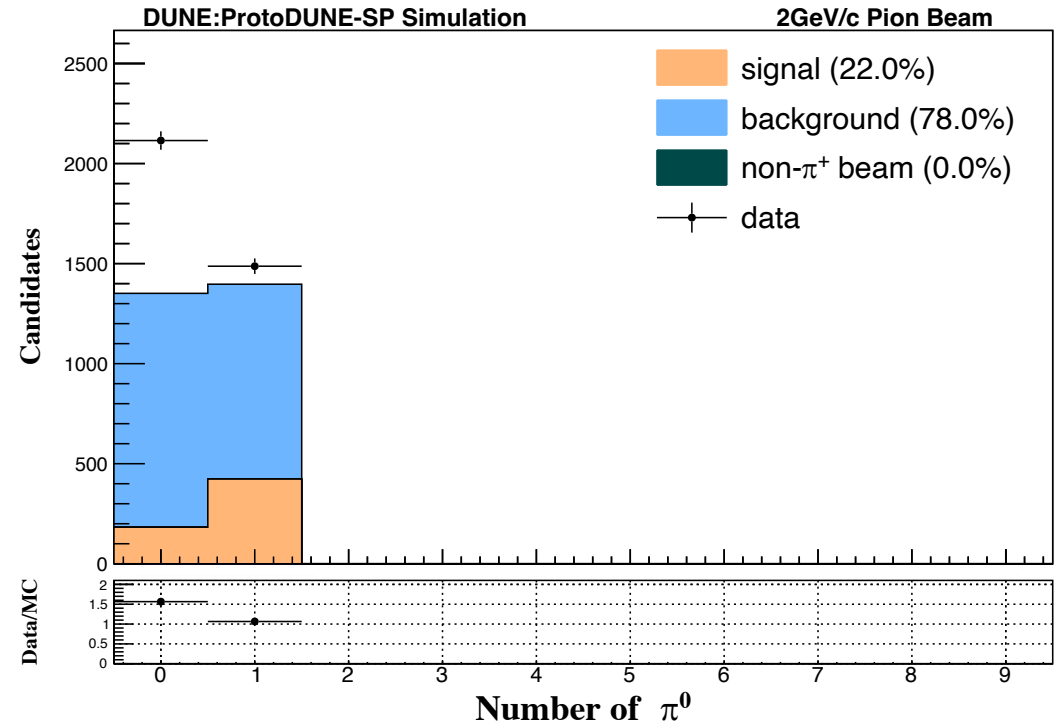
MC	1 GeV				2 GeV			
Cuts	Entries	selected	Percentage	Cumulative	Entries	Selected	Percentage	Cumulative
Michel score	82212	80616	98.06%	27.03%	35625	35471	99.57%	25.50%

Number of π^0

1 GeV Pion Beam



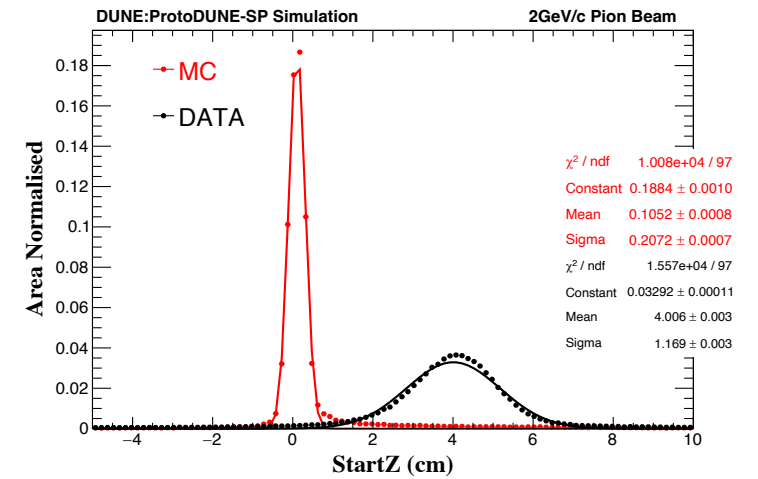
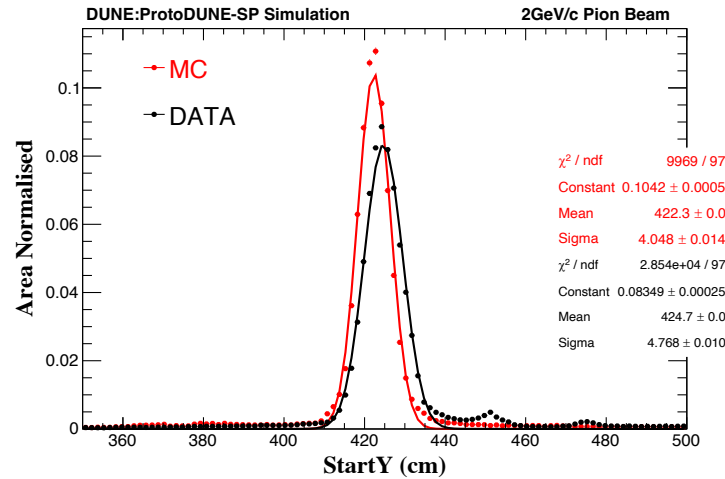
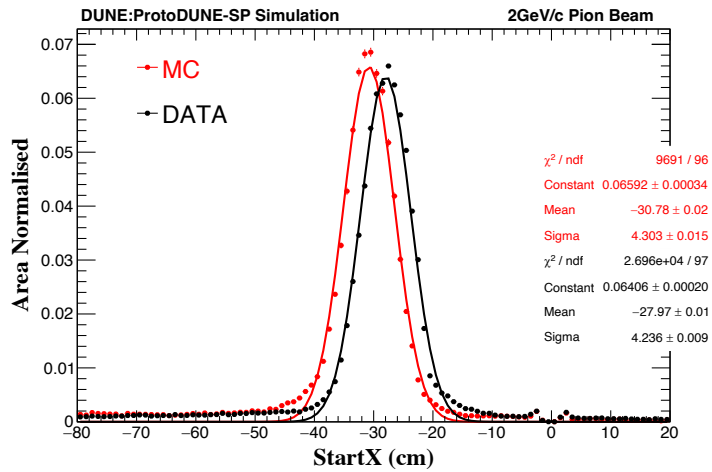
2 GeV Pion Beam



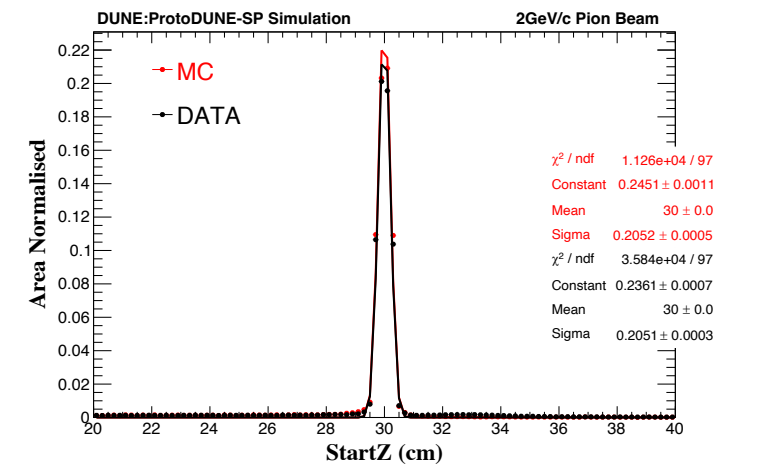
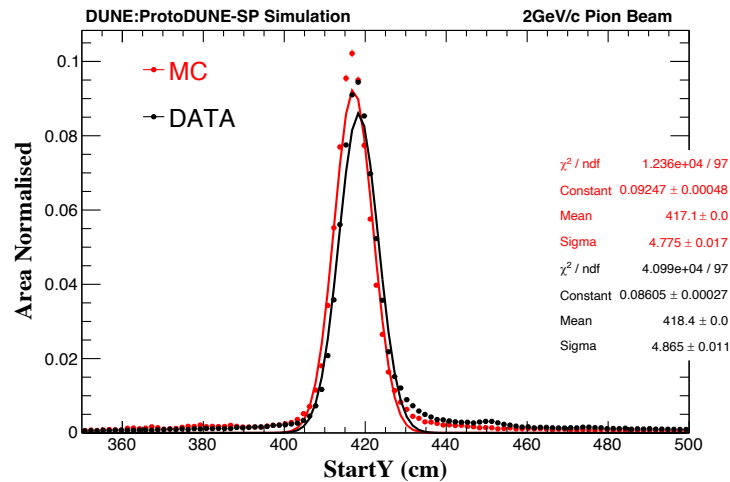
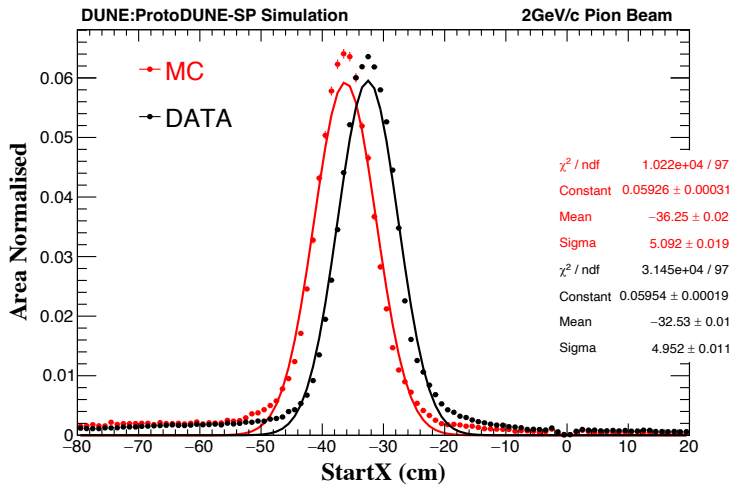
MC	1 GeV				2 GeV			
	Cuts	Entries	selected	Percentage	Cumulative	Entries	Selected	Percentage
Npi0	1614	1146	71.00%	0.38%	1143	581	50.83%	0.42%

Reconstructed Beam Start Position

2 GeV Pion Beam

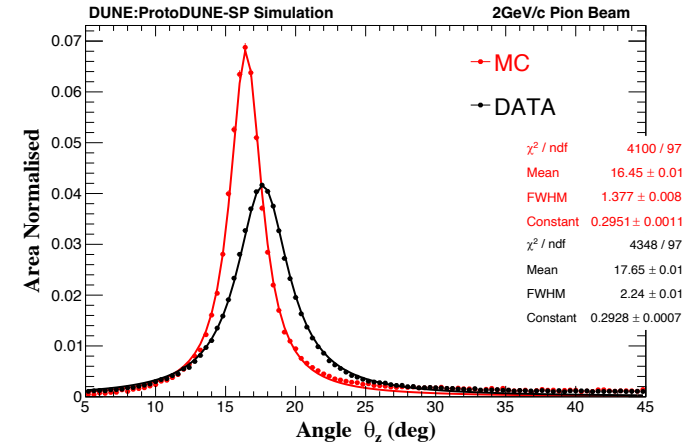
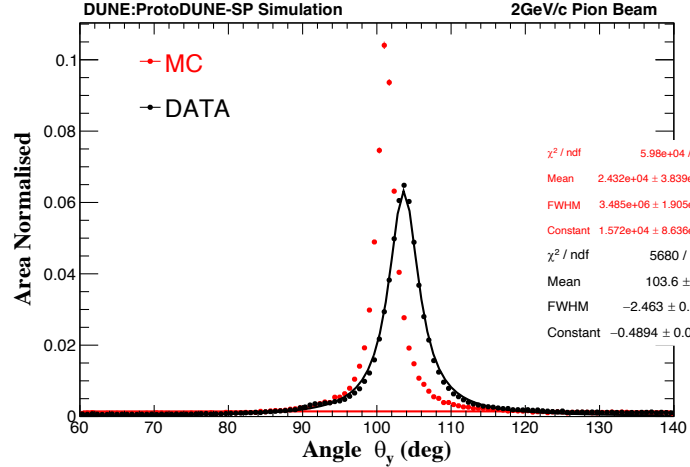
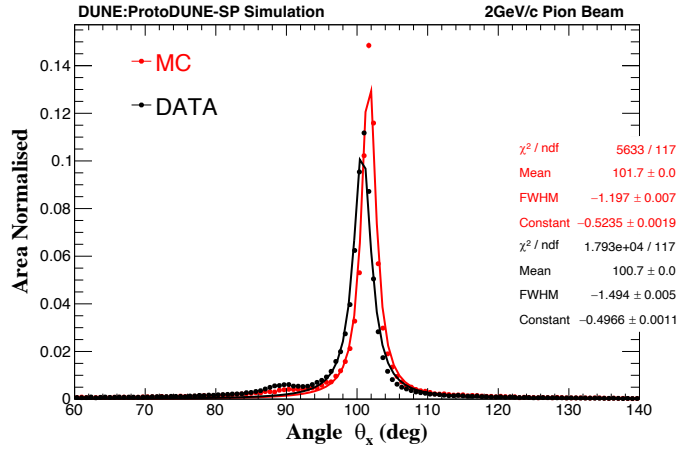


2 GeV Pion Beam with additional cuts

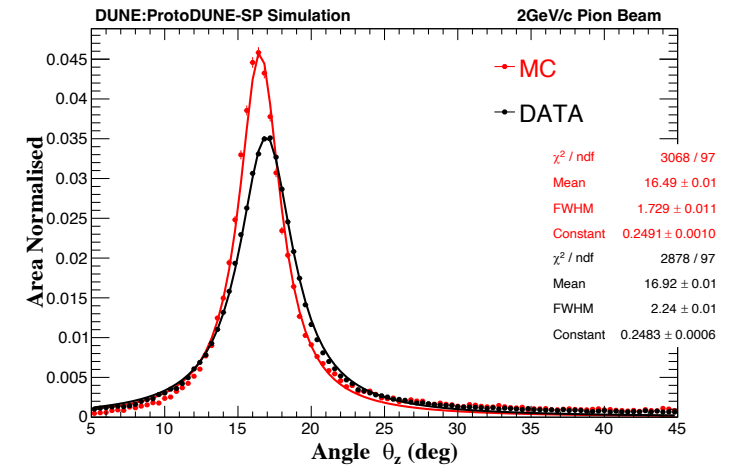
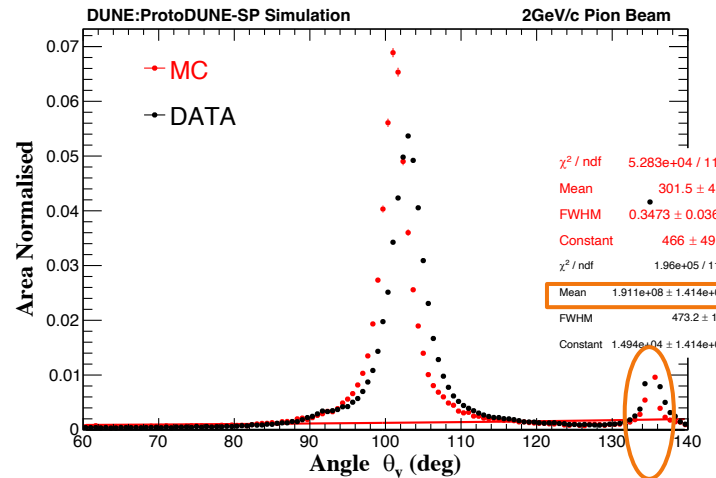
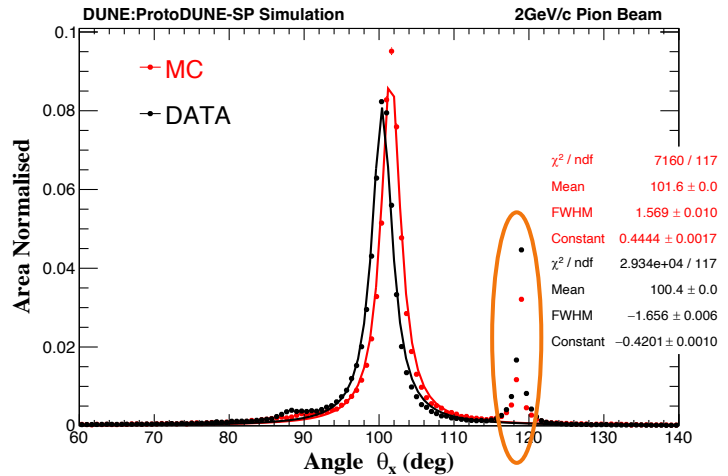


Reconstructed Beam Theta angle

2 GeV Pion Beam

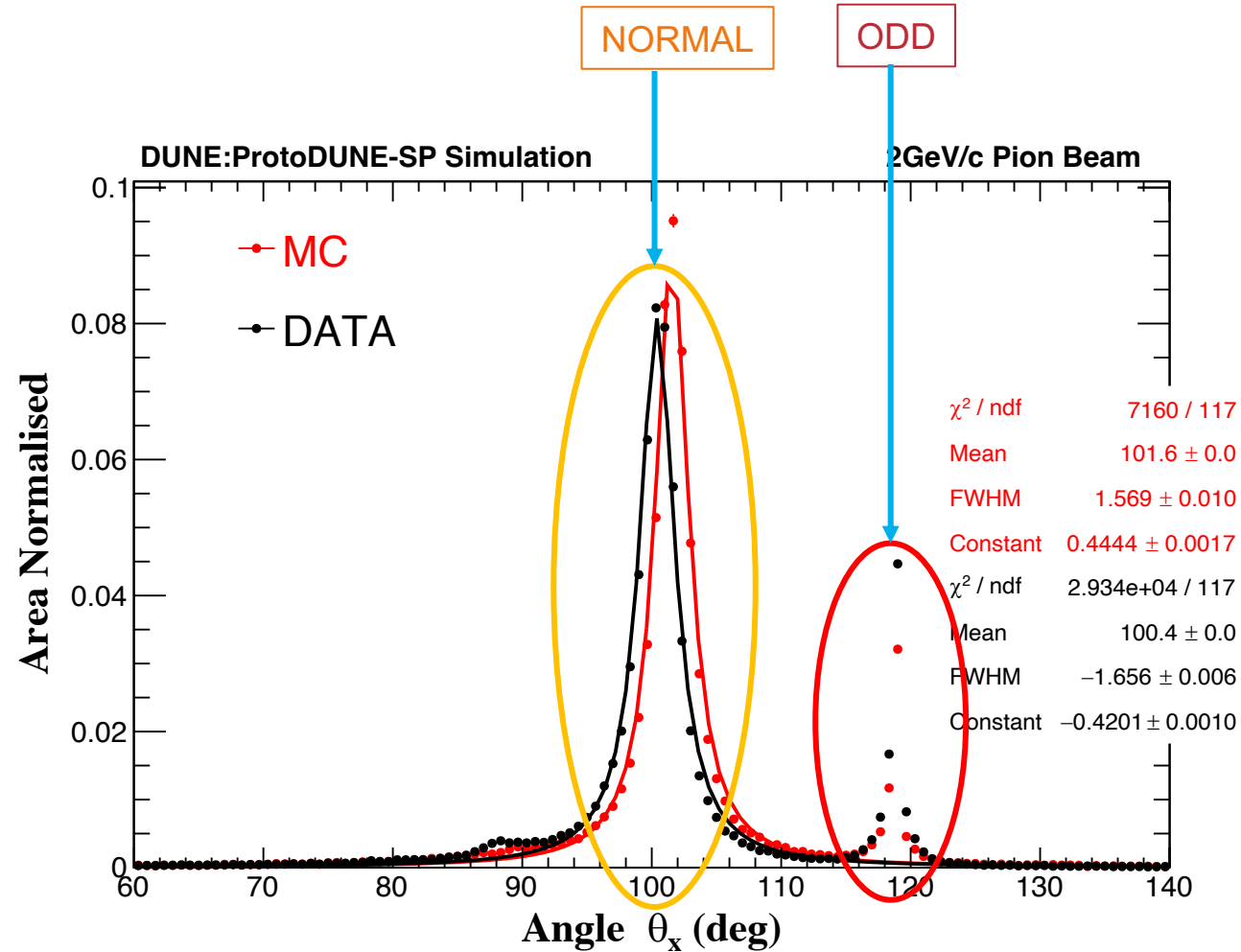


2 GeV Pion Beam additional cuts



Odd events

- ODD events found due to the missing events

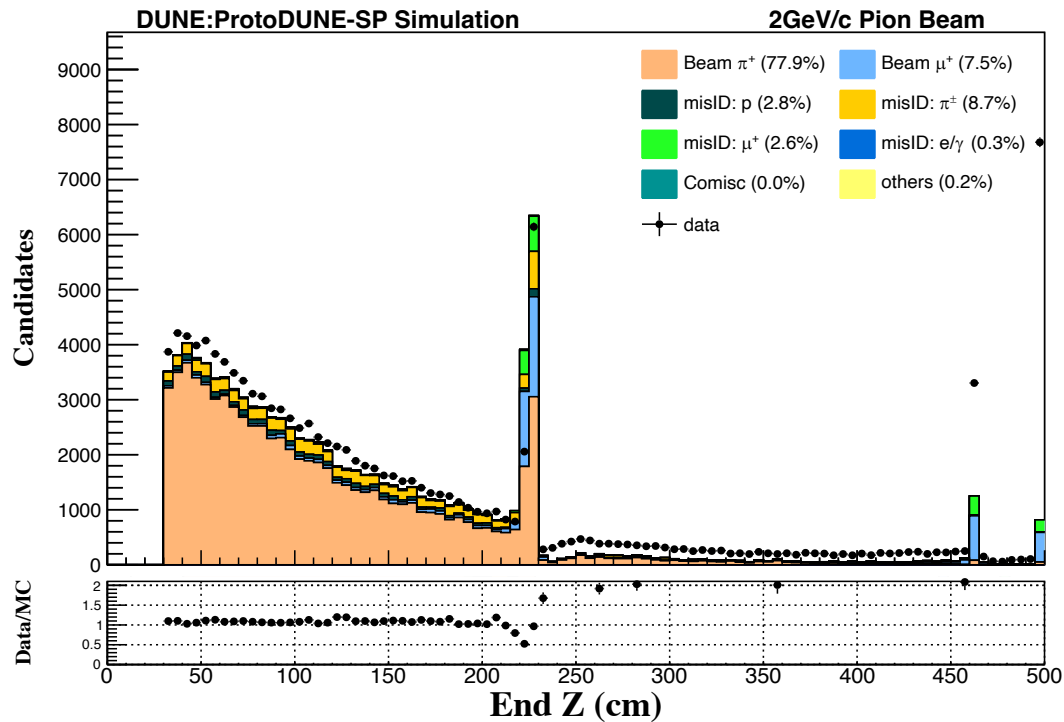


APA3 Cut – 1GeV vs 2GeV

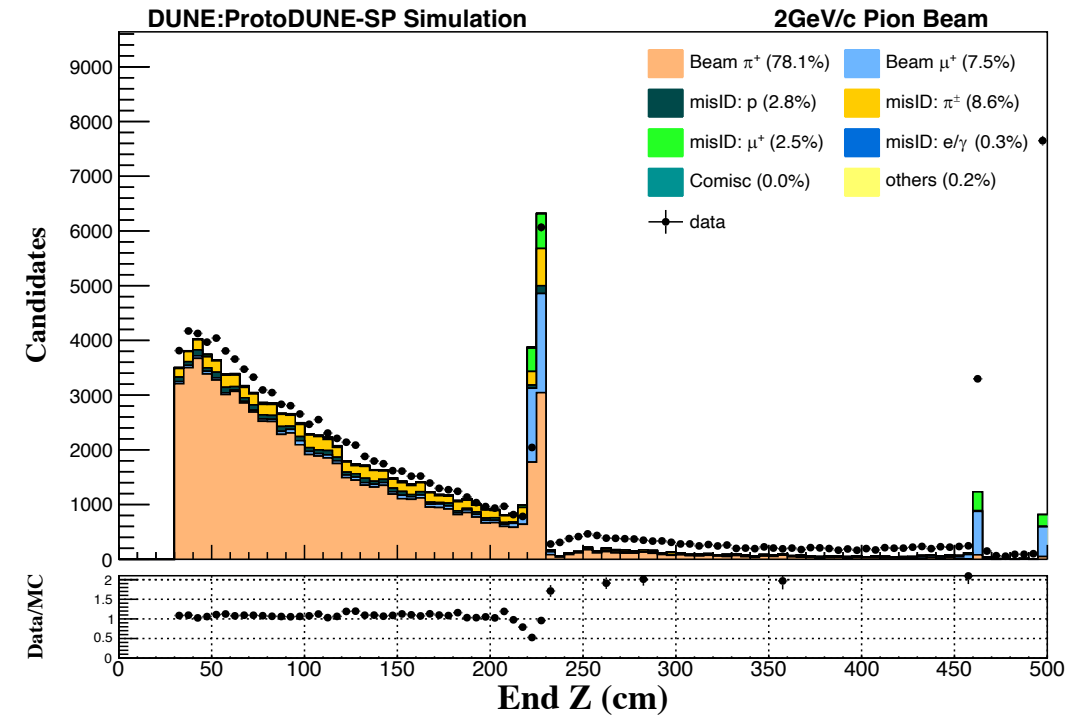
APA3 Cut : requires that the particle interact in the first APA

- Current selection: **beam end Z less than 220 cm.**

2 GeV Before Cut



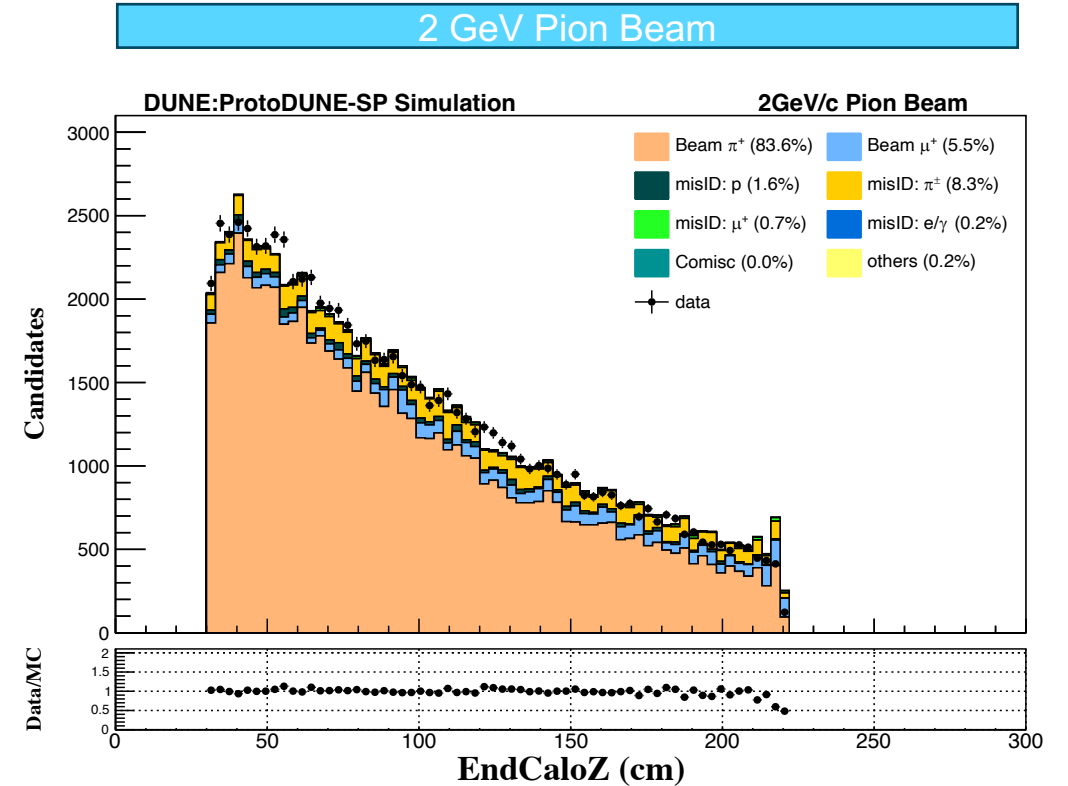
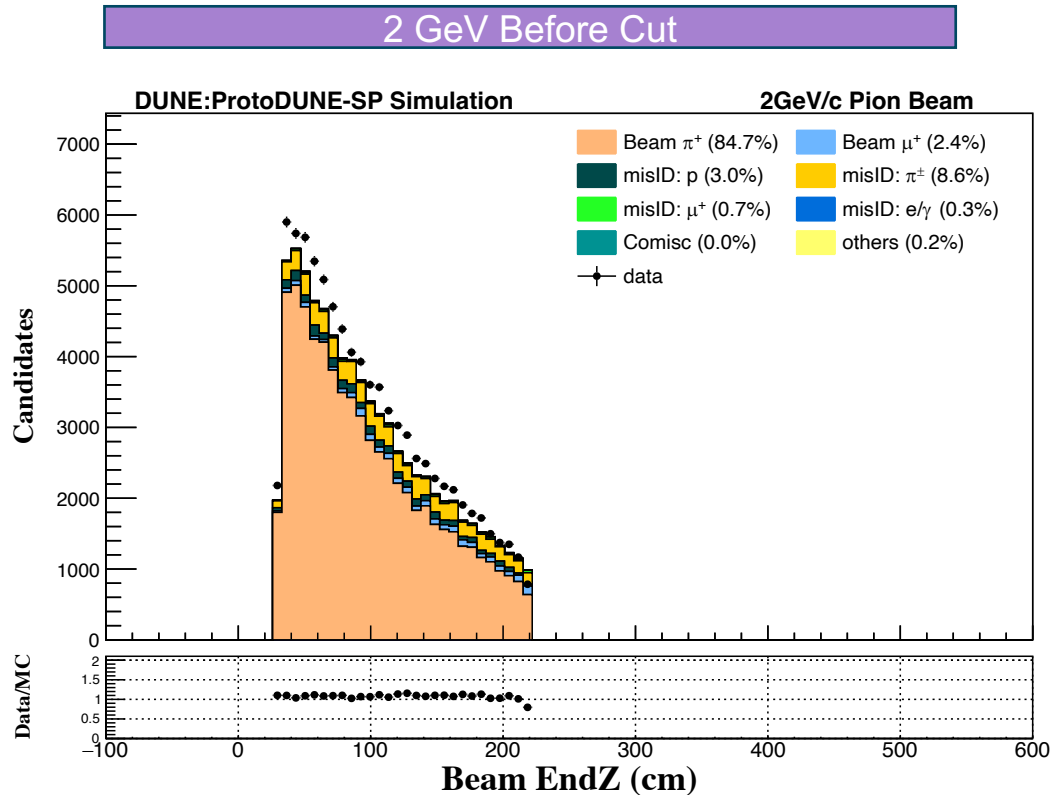
2 GeV Pion Beam Additional Cuts



After APA3 Cut – 1 GeV vs 2 GeV

APA3 Cut : requires that the particle interact in the first APA

- Current selection: **beam end Z less than 220 cm.**



MC	1 GeV				2 GeV			
Cuts	Entries	selected	percentage	Accumulated	Entries	Selected	percentage	Accumulated
APA3 endZ	100824	82212	81.54%	27.57%	45821	35625	77.75%	25.61%

Cut summary 1GeV vs 2GeV - MC

MC	1 GeV				2 GeV			
Cuts	Entries	selected	percentage	Accumulated	Entries	Selected	percentage	Accumulated
Beam PDG	298194	170238	57.09%	57.09%	139124	100637	72.34%	72.34%
Pandora slice	170238	145004	85.18%	48.63%	100637	87700	87.14%	63.04%
Calo size	145004	142362	98.18%	47.74%	87700	85864	97.91%	61.72%
Beam Quality	142362	100824	70.82%	33.81%	85864	45821	53.36%	32.94%
APA3 endZ	100824	82212	81.54%	27.57%	45821	35625	77.75%	25.61%
Michel score	82212	80616	98.06%	27.03%	35625	35471	99.57%	25.50%
Chi2/DOF	80616	76583	95.00%	25.68%	35471	33996	95.84%	24.44%
Beam Scraper	76583	60174	78.57%	20.18%	33996	33985	99.97%	24.43%
Nshower	60174	2155	3.58%	0.72%	33985	2667	7.85%	1.92%
Npiplus	2155	1708	79.26%	0.57%	2667	1210	45.37%	0.87%
Nmichel	1708	1614	94.50%	0.54%	1210	1143	94.46%	0.82%
Npi0	1614	1146	71.00%	0.38%	1143	581	50.83%	0.42%

- **1GeV selected pion beam MC : 60174 / 298194 (20.18%)**
- **2GeV selected pion beam MC : 33985 / 139124 (24.43%)**

Cut summary 1GeV vs 2GeV - Data

Data	1 GeV				2 GeV			
Cuts	Entries	selected	percentage	Accumulated	Entries	Selected	percentage	Accumulated
Beam PDG	1215251	119545	9.84%	9.84%	1349399	285238	21.14%	21.14%
Pandora slice	119545	102882	86.06%	8.47%	285238	240159	84.20%	17.80%
Calo size	102882	100150	97.34%	8.24%	240159	232423	96.78%	17.22%
Beam Quality	100150	70441	70.34%	5.80%	232423	117710	50.64%	8.72%
APA3 endZ	70441	54128	76.84%	4.45%	117710	86111	73.16%	6.38%
Michel score	54128	52872	97.68%	4.35%	86111	85779	99.61%	6.36%
Chi2/DOF	52872	49712	94.02%	4.09%	85779	81956	95.54%	6.07%
Beam Scraper	49712	34783	69.97%	2.86%	81956	81718	99.71%	6.06%
Nshower	34783	1382	3.97%	0.11%	81718	8290	10.14%	0.61%
Npiplus	1382	1088	78.73%	0.09%	8290	3793	45.75%	0.28%
Nmichel	1088	1014	93.20%	0.08%	3793	3602	94.96%	0.27%
Npi0	1014	665	65.58%	0.05%	3602	1487	41.28%	0.11%

- **1GeV selected pion beam Data : 34783 / 1215251 (2.86%)**
- **2GeV selected pion beam Data : 81956 / 1349399 (6.06%)**

Extra Weight 1GeV values

Charged Exchange Pi0 Kinetic Energy Weight

```
double CalCEXPi0KEWeight(const int & binx, const bool IsRange, const bool IsKF){
    double weight = 1.;
    if(IsRange && IsKF){
        double IntWeight[] = {0.0, 0.586469, 0.388012, 0.412948, 0.519354, 0.665045, 0.740563, 0.651374, 0.654317, 0.72544, 0.780647,
                               0.74401, 0.93965, 0.935617, 0.838449, 1.0, 1.0, 0.0, 0.0, 0.0};

        weight *= IntWeight[binx-1];
    }
    else if(IsRange && !IsKF){

        // no weight
        double IntWeight[] = {0.0, 0.0, 0.559718, 0.528286, 0.422081, 0.487519, 0.666412, 0.544518, 0.627939, 0.604775, 0.780329,
                               0.621141, 0.934835, 0.934835, 0.836084, 1.0, 1.0, 0.0, 0.0, 0.0};

        // beam weight
        weight *= IntWeight[binx-1];
    }
    else{
        double IntWeight[] = {0.0, 0.369517, 0.409675, 0.415022, 0.373534, 0.348398, 0.459325, 0.422841, 0.481166, 0.426144, 0.680265,
                               0.633115, 0.933773, 0.909191, 0.747696, 0.9244, 0.864349, 0.0, 1.0, 0.402144};
        weight *= IntWeight[binx-1];
    }
    return weight;
}
```


Extra Weight 1GeV values

Charged Exchange Pi0 Theta Weight

```
double CalCEXPi0ThetaWeight(const int & binx, const bool IsRange, const bool IsKF){
    double weight = 1.;
    if(IsRange && IsKF){
        // pi0 wt 0410
        double IntWeight[] = {0.787775, 0.732404, 0.576011, 0.593378, 0.469896, 0.569643, 0.629882, 0.531519, 0.361952, 0.0};
        // no weight
        //double IntWeight[] = {0.8, 0.765363, 0.644628, 0.689189, 0.574468, 0.7, 0.75, 0.666667, 0.5, 0.0};

        weight *= IntWeight[binx-1];
    }
    else if(IsRange && !IsKF){
        // No KF only beam weight
        double IntWeight[] = {0.720196, 0.714559, 0.549956, 0.647322, 0.560627, 0.743115, 0.657209, 0.765056, 0.0};
        // No KF both weight
        weight *= IntWeight[binx-1];
    }
    else{
        double IntWeight[] = {0.0, 0.0, 0.636412, 0.570768, 0.348301, 0.633125, 0.747292, 0.461547, 0.711167, 0.603851, 0.660517,
            0.747348, 0.859409, 1, 0.742225, 1.0, 1.0, 0.0, 0.0, 0.0};
        weight *= IntWeight[binx-1];
    }
    return weight;
}
```

Extra Weight 1GeV values

```
double CallniWeight(const int & binx, const bool IsRange){
```

```
    double weight = 1.;
    if(IsRange){
        double IntWeight[] = {0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
                               0.0, 0.801123, 0.859177, 0.86919, 0.880718, 0.886977, 0.889779, 0.887032};
        weight *= IntWeight[binx-1];
    }

    else{
        double IntWeight[] = {0.0, 0.0, 0.636412, 0.570768, 0.348301, 0.633125, 0.747292, 0.461547, 0.711167, 0.603851, 0.660517,
                               0.747348, 0.859409, 1, 0.742225, 1.0, 1.0, 0.0, 0.0, 0.0};
        weight *= IntWeight[binx-1];
    }
    return weight;
}
```

Cal Initial Weight

```
double CalBeamIntWeight(const int & binx, const bool IsRange){
```

```
    double weight = 1.;
    if(IsRange){
        double IntWeight[] = {0.439768, 1.0, 0.0, 0.240104, 0.325475, 0.547874, 0.525402, 0.569011, 0.663258, 0.755661, 0.820706, 0.858978,
                               0.886342, 0.901809, 0.920025, 0.931599, 0.945724, 0.945179, 0.943406, 0.938913};
        weight *= IntWeight[binx-1];
    }

    else{
        double IntWeight[] = {0.0, 0.0, 0.636412, 0.570768, 0.348301, 0.633125, 0.747292, 0.461547, 0.711167, 0.603851, 0.660517,
                               0.747348, 0.859409, 1, 0.742225, 1.0, 1.0, 0.0, 0.0, 0.0};
        weight *= IntWeight[binx-1];
    }
    return weight;
}
```