

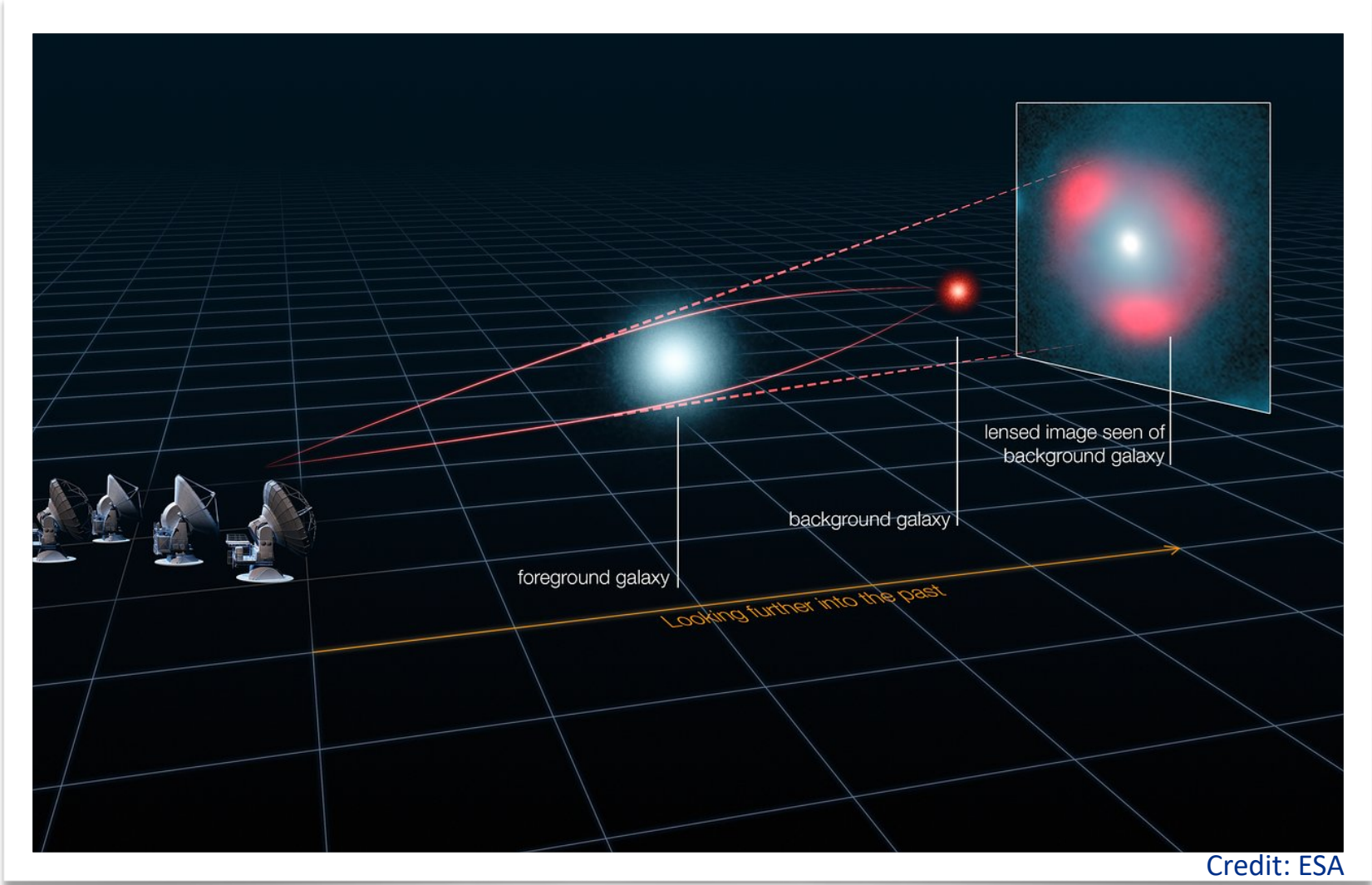
Constraining Dark Energy with Strong Lenses and Machine Learning

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Fermilab

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New Perspectives 2024

Strong Gravitational Lensing



Cosmology from Static Strong Lenses

- Combining the mass measurement within Einstein radius θ_E from lensing and stellar velocity measurements σ_v , the cosmology can be constrained through

distance ratio $\frac{D_{ls}}{D_s}$

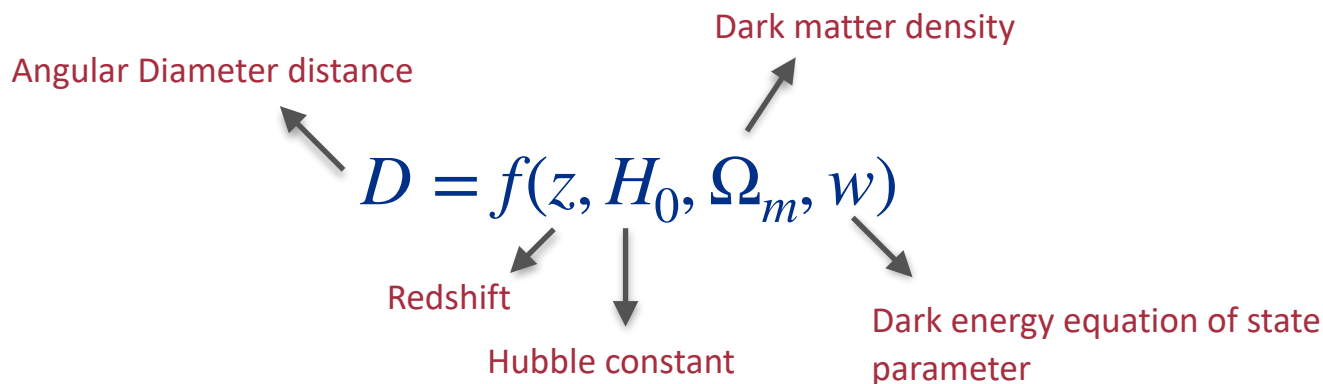
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Einstein Radius Velocity dispersion Distance Ratio

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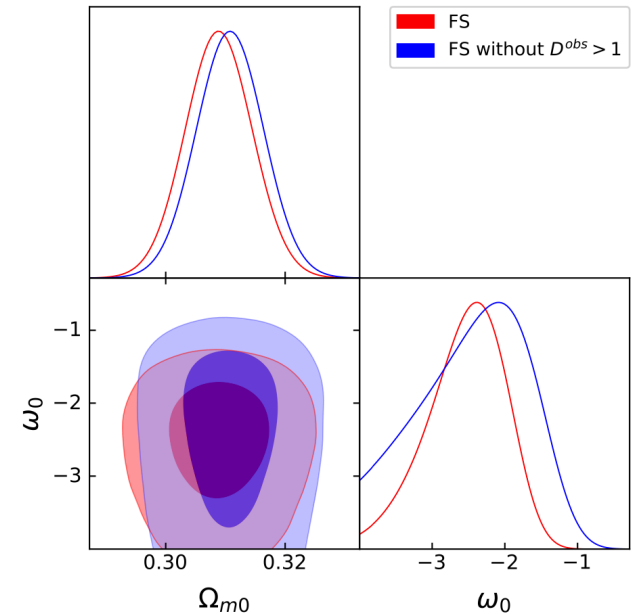
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$$D = f(z, H_0, \Omega_m, w)$$

Constraints from 204 strong lens observations using MCMC



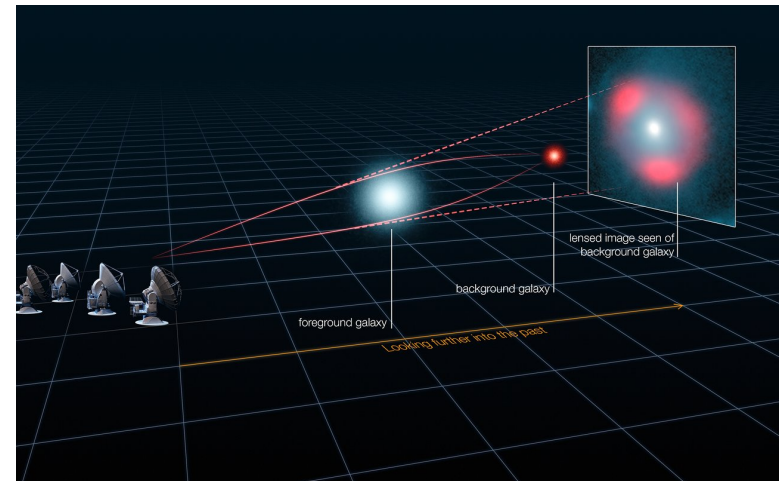
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Challenges for Cosmology Inference

- Likelihood is intractable

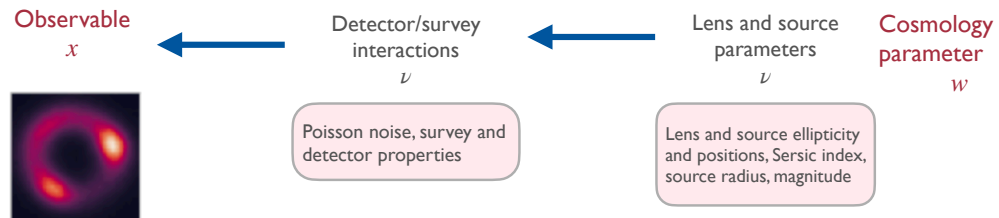
$$p(w | x) = \frac{p(x | w)}{p(x)} p(w)$$

Posterior \swarrow Likelihood \nearrow
 $p(w | x)$ \swarrow Prior \searrow



Intractable Likelihood

$$p(x | w) = \int p(x, \nu | w) d\nu$$

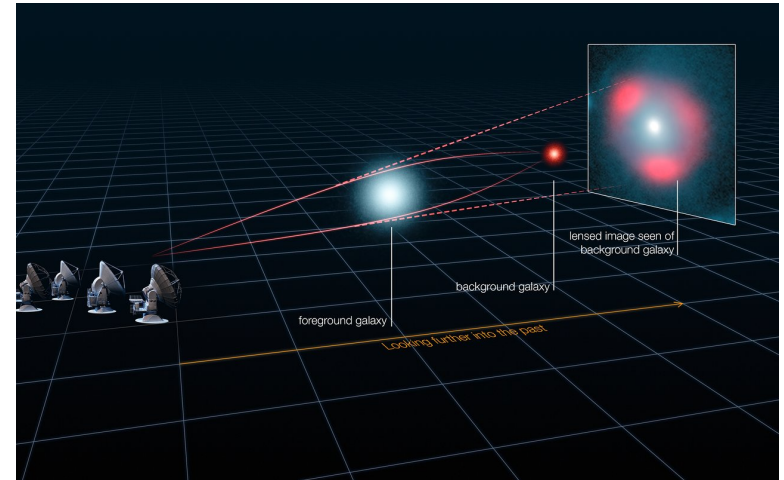


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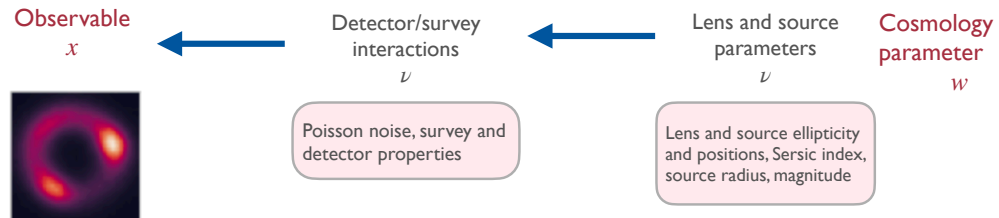
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Intractable Likelihood



Simulation Based Inference

Challenges for Cosmology Inference

- Likelihood is intractable
- $\mathcal{O}(10^5)$ strong lenses to be discovered from surveys with telescopes such as Rubin Observatory, Euclid, and Roman Space Telescope
- Traditional MCMC methods for inference are computationally prohibitive

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$\mathcal{O}(10^5)$



Machine Learning

SBI : Neural Ratio Estimation (NRE)

- NRE is a classifier neural network to differentiate between the lens image-parameter pairs
 - $(x, w) \sim p(x, w)$ with class label $y = 1$
 - $(x, w) \sim p(x)p(w)$ with class label $y = 0$

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$$r(x|w) = \frac{p(x, w)}{p(x)p(w)} = \frac{p(x|w)}{p(x)}$$

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- The joint likelihood-to-evidence ratio from a population of strong lens observations $\{x\}$:

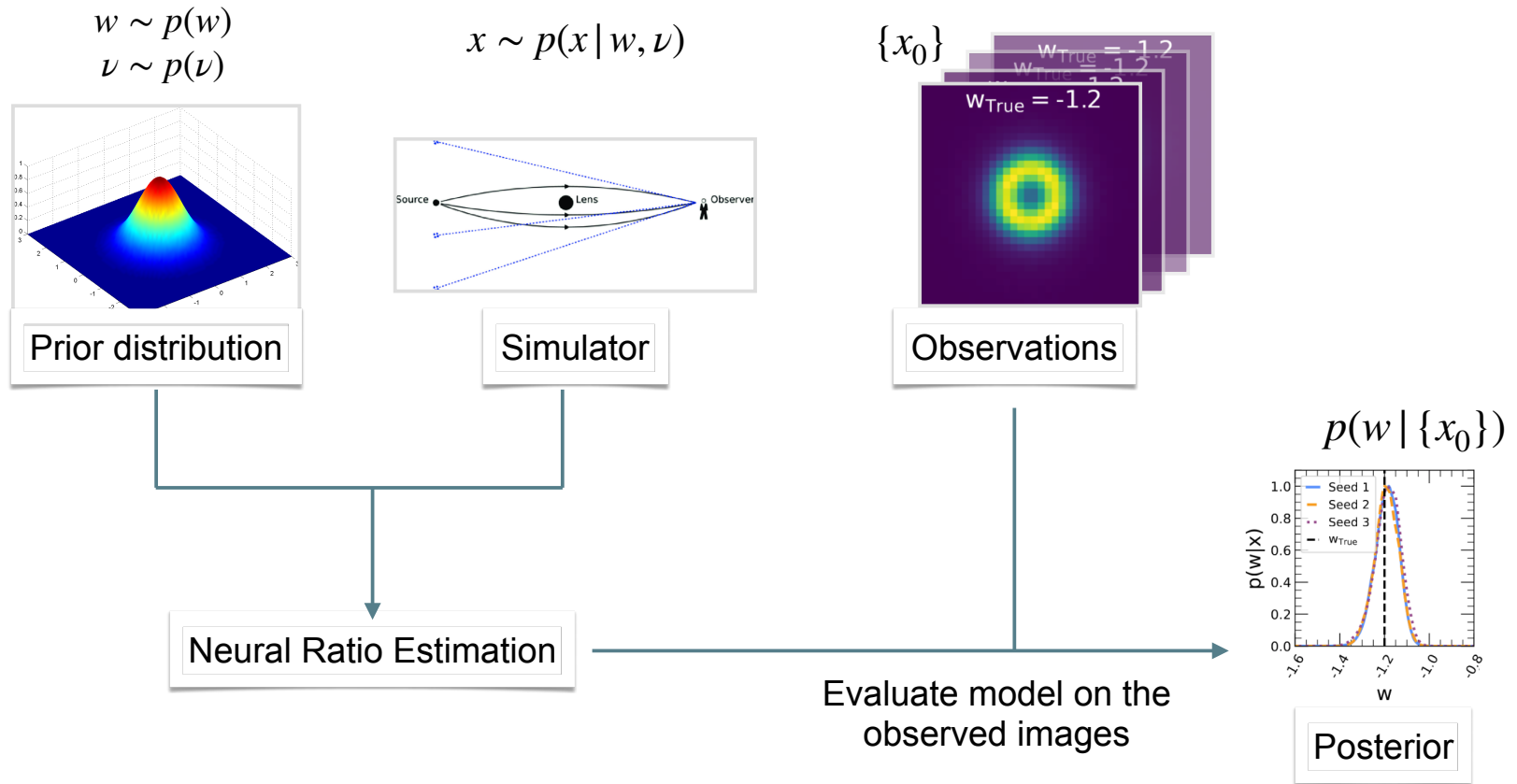
$$r(\{x\} | w) = \prod_i r(x_i | w)$$

Posterior Inference of w

- Using the trained NRE model for posterior inference of w from a population of strong lens images
- Method 1 : MCMC sampling from $r(\{x\} | w) p(w)$
- Method 2 : Analytical calculation

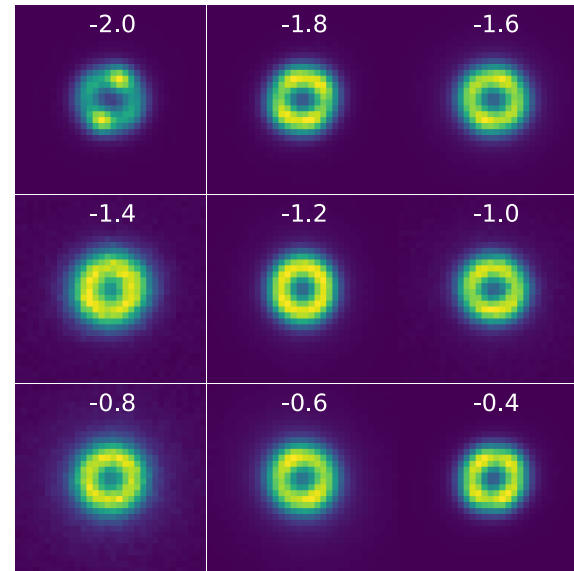
$$p(w | \{x\}) = \frac{p(w) \prod_i r(x_i | w)}{\int dw' p(w') \prod_i r(x_i | w')}$$

Neural Ratio Estimation workflow



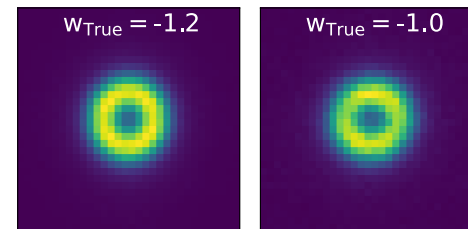
Dataset and Experimental Setup

- We simulate galaxy-galaxy strong lenses using DeepLenstronomy
- DES survey conditions with g -band images
- Image size : 32 x 32 pixels
- Prior: $w \sim \mathcal{U}(-2.0, -0.34)$
 - Training data: 640k images
 - Validation data: 160k images
 - Test data: 2k images
- $w = -1.2, -1.0, -0.8$
 - Test data: 3k images each

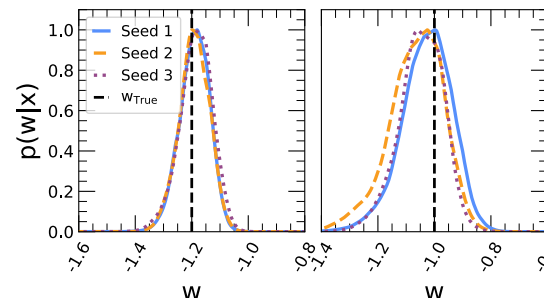


Sample set of images from training data

Fixing $z_l, z_s, \sigma_v, \Omega_m$



Posterior estimated using NRE on test data

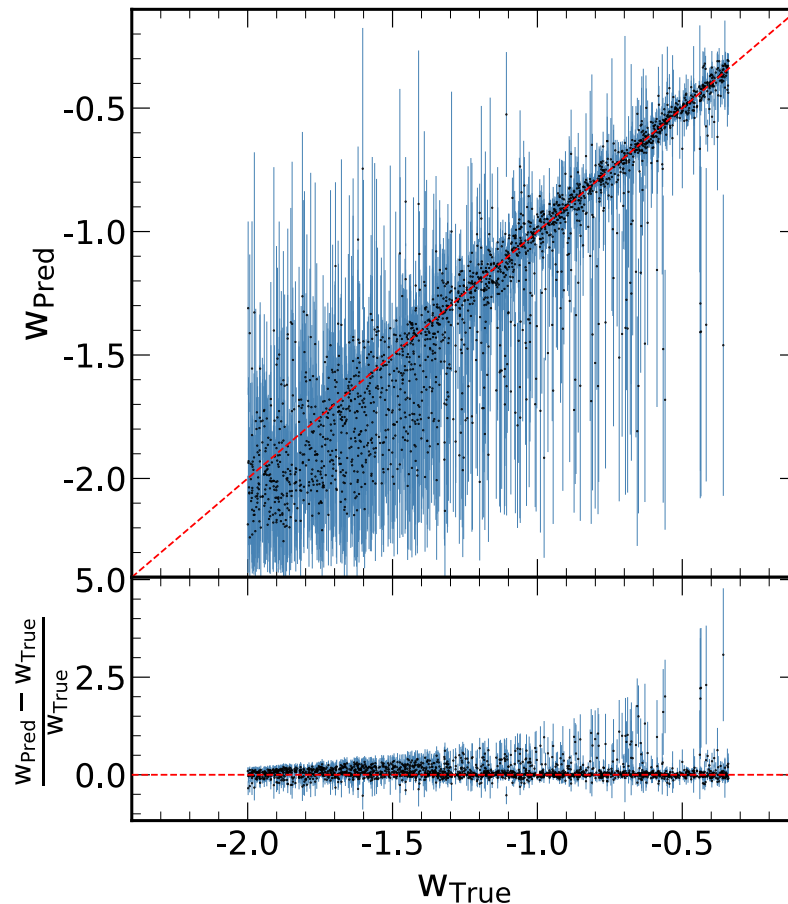


Results: Model Performance

- **Robustness:** Train the model with three different seed initializations
- **Classifier Performance:** The Area under the Receiver Operating Curve (AUC) ~ 0.92
- **Model Calibration:** The posterior coverage plot shows that the model is well calibrated

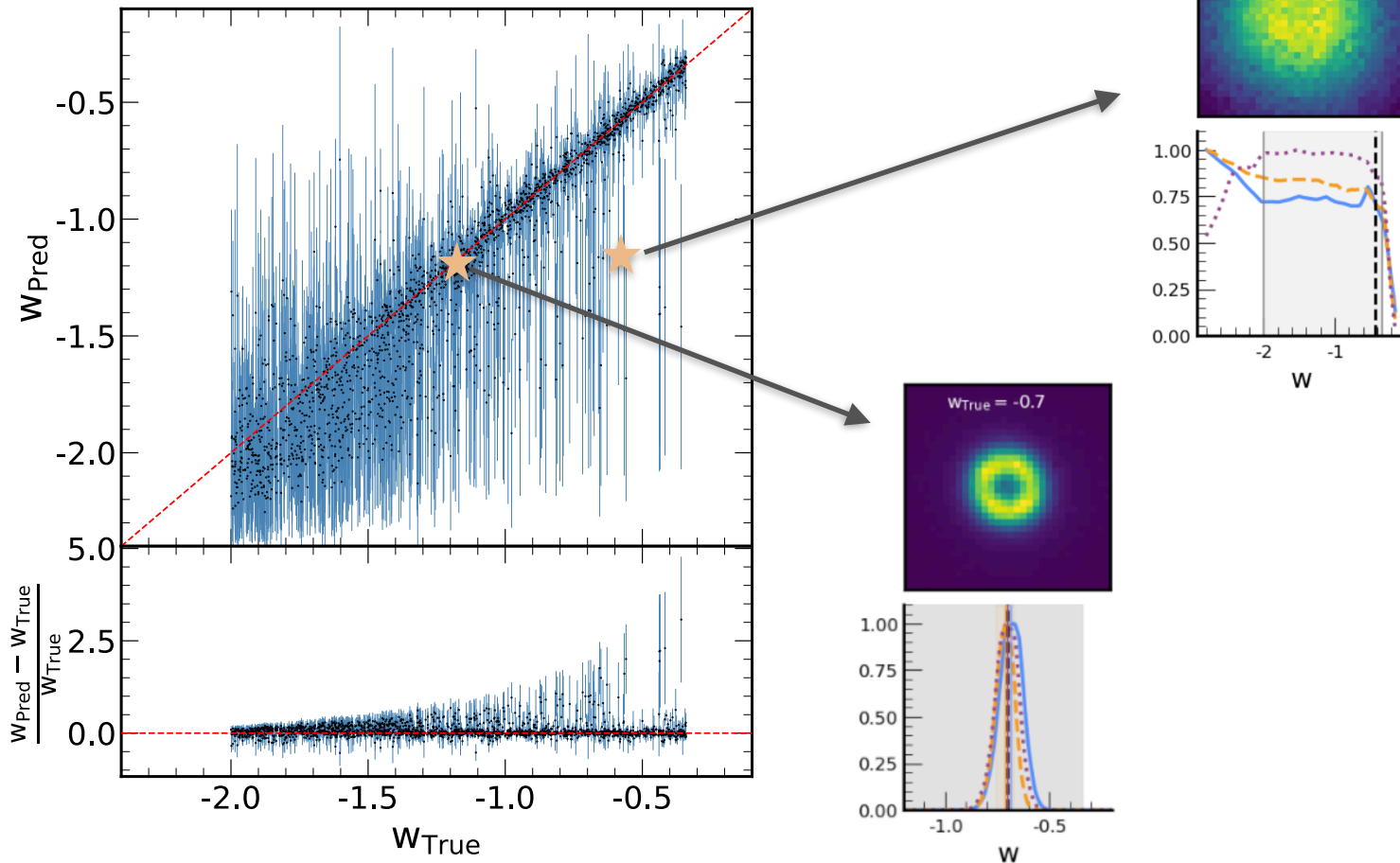
Results: Model Performance

The model can correctly predict w within 1σ for images which have high signal-to-noise ratio



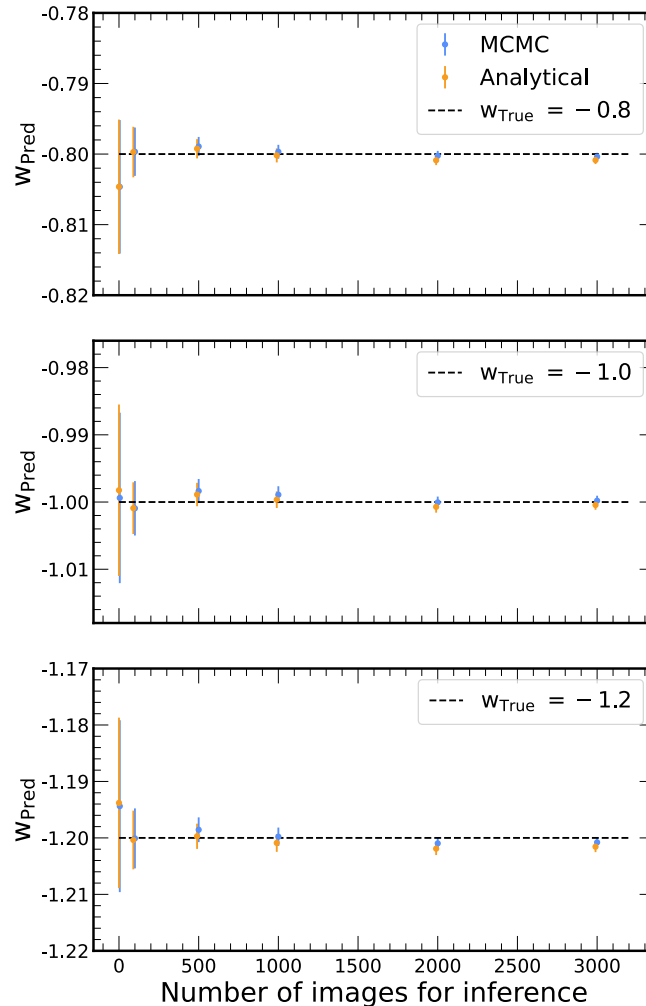
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Results: Population-level w Inference

The posterior is more constrained as the number of observations in the population increase



Summary and Conclusion

- We implemented SBI with NRE for the first time for population-level posterior inference of dark energy equation-of-state parameter from strong lens images.
- Robust and well calibrated model. Provides constraints on w within 1σ .
- The posterior is more constrained with an increasing number of observations in the inference.
- This analysis is crucial for analyzing the thousands of lenses from future surveys.

Extras

Cosmology from Static Strong Lenses

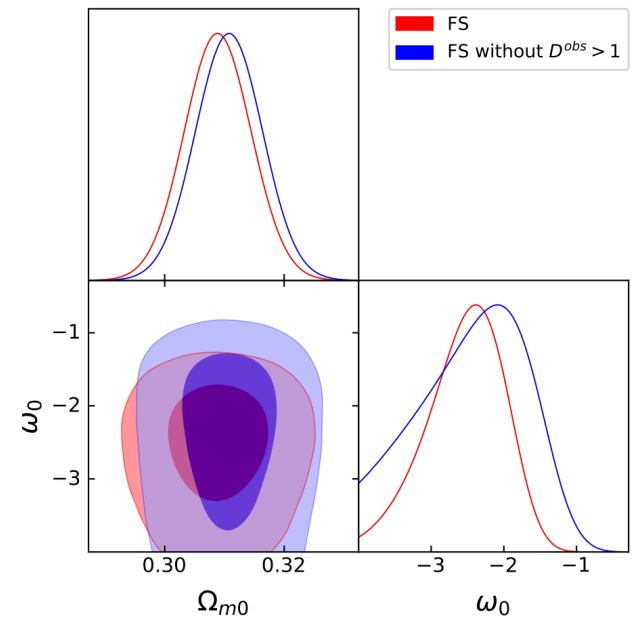
- Dark energy equation-of-state parameter w and Dark Matter density Ω_m constrain through distance ratio $\frac{D_{ls}}{D_s}$ from Einstein radius θ_E and stellar velocity dispersion σ_v

$$\theta_E = 4\pi \left(\frac{\sigma_v}{c} \right)^2 \frac{D_{ls}}{D_s}$$

$$D(z, H_0, \Omega_m, w) = \frac{1}{1+z} \frac{c}{H_0} \int_0^z \frac{dz'}{h(z', \Omega_m, w)}$$

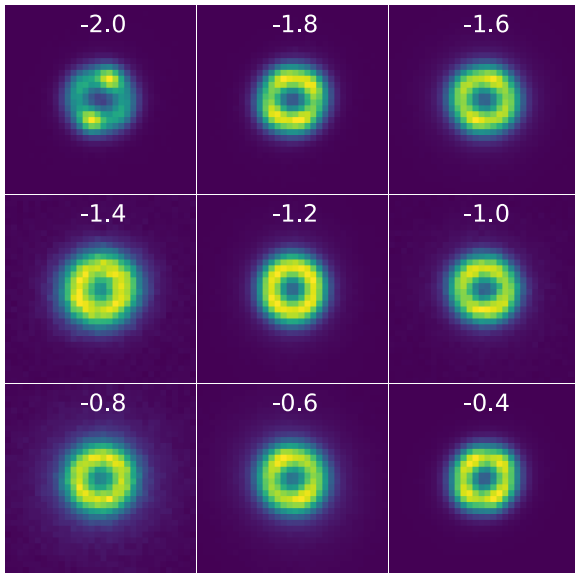
$$h^2(z, \Omega_m, w) = \Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3(1+w)}$$

Constraints from 204 strong lens observations using MCMC

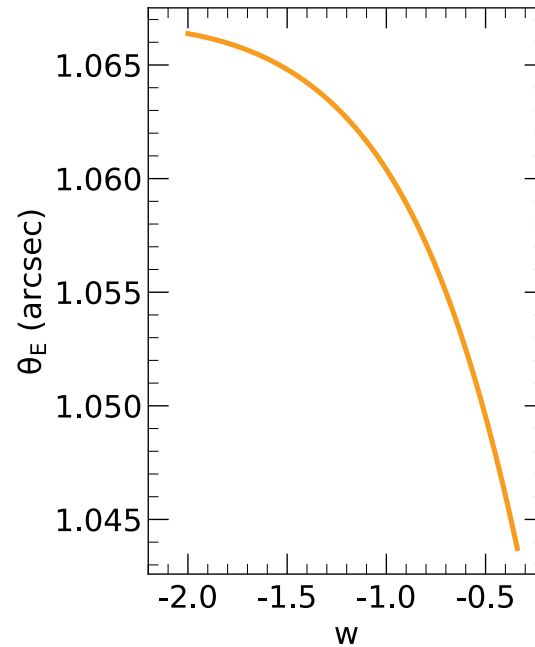


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Sample set of images from training data

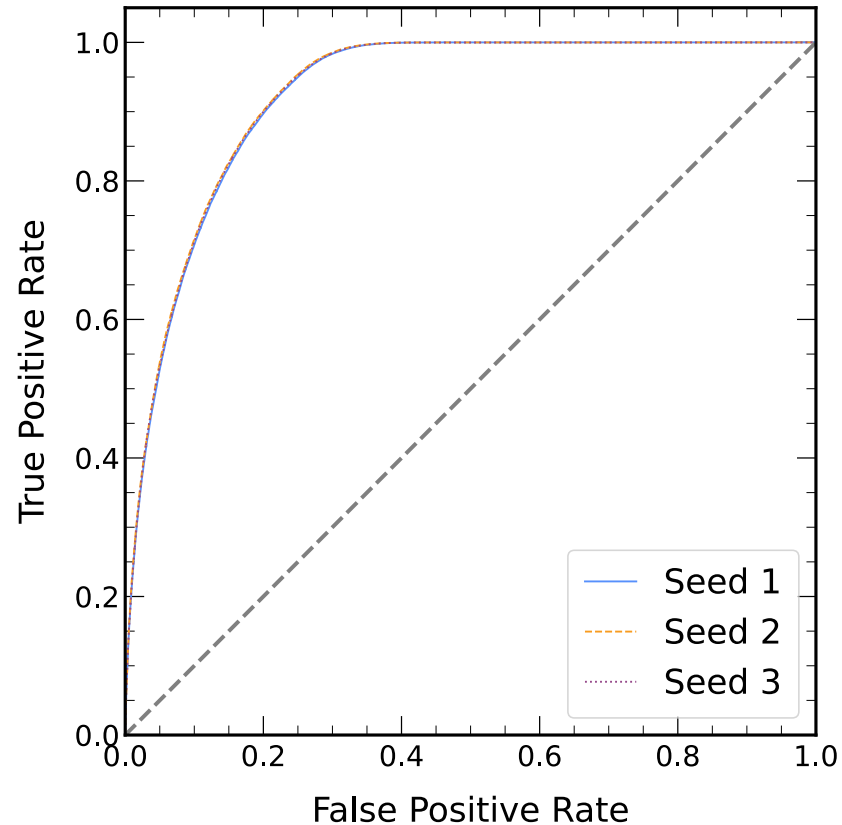


Correlation between Einstein Radius θ_E and w .

The variation in θ_E is larger at high- w

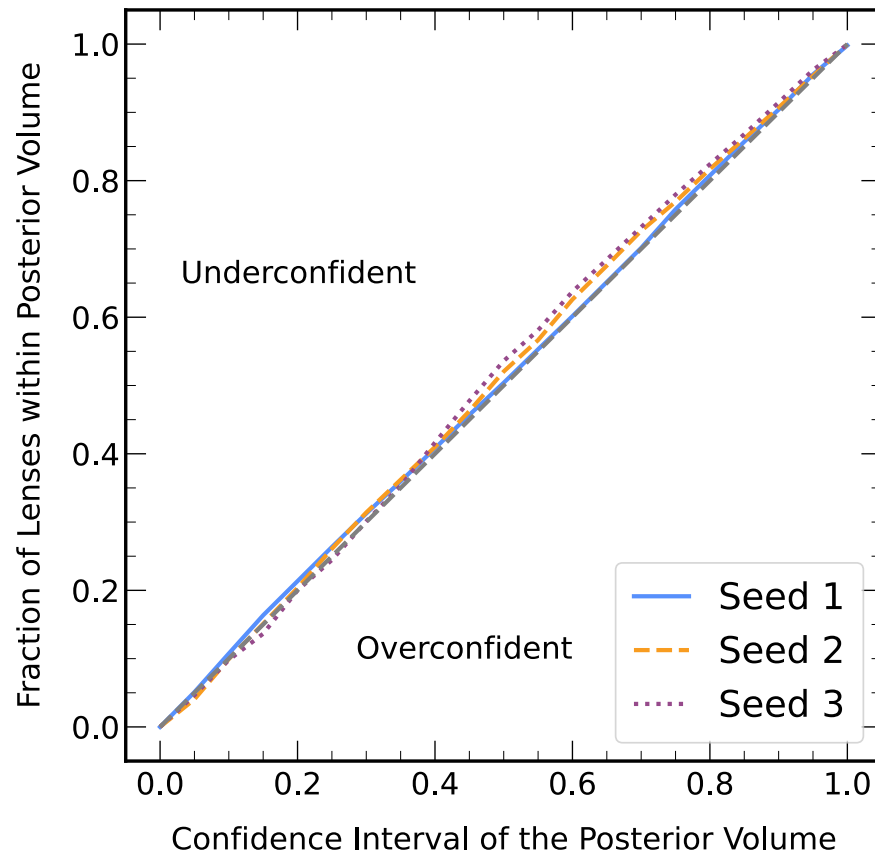
Results: Model Performance

- We run the model training with three different seed initializations to check robustness
- The Area under the Receiver Operating Curve (AUC) ~ 0.92
- Model can differentiate between the two classes $p(x, w)$ and $p(x)p(w)$

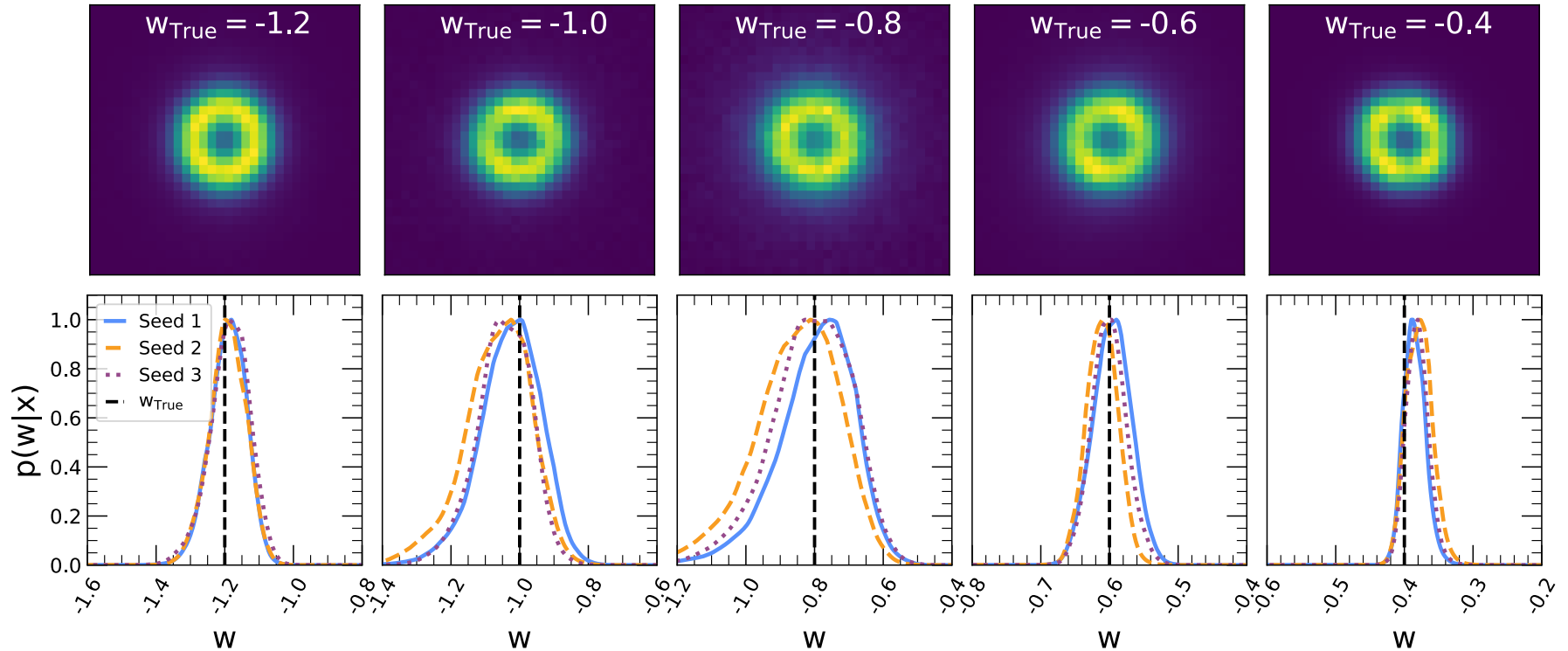


Results: Model Performance

- The posterior coverage plot shows that the model is well calibrated



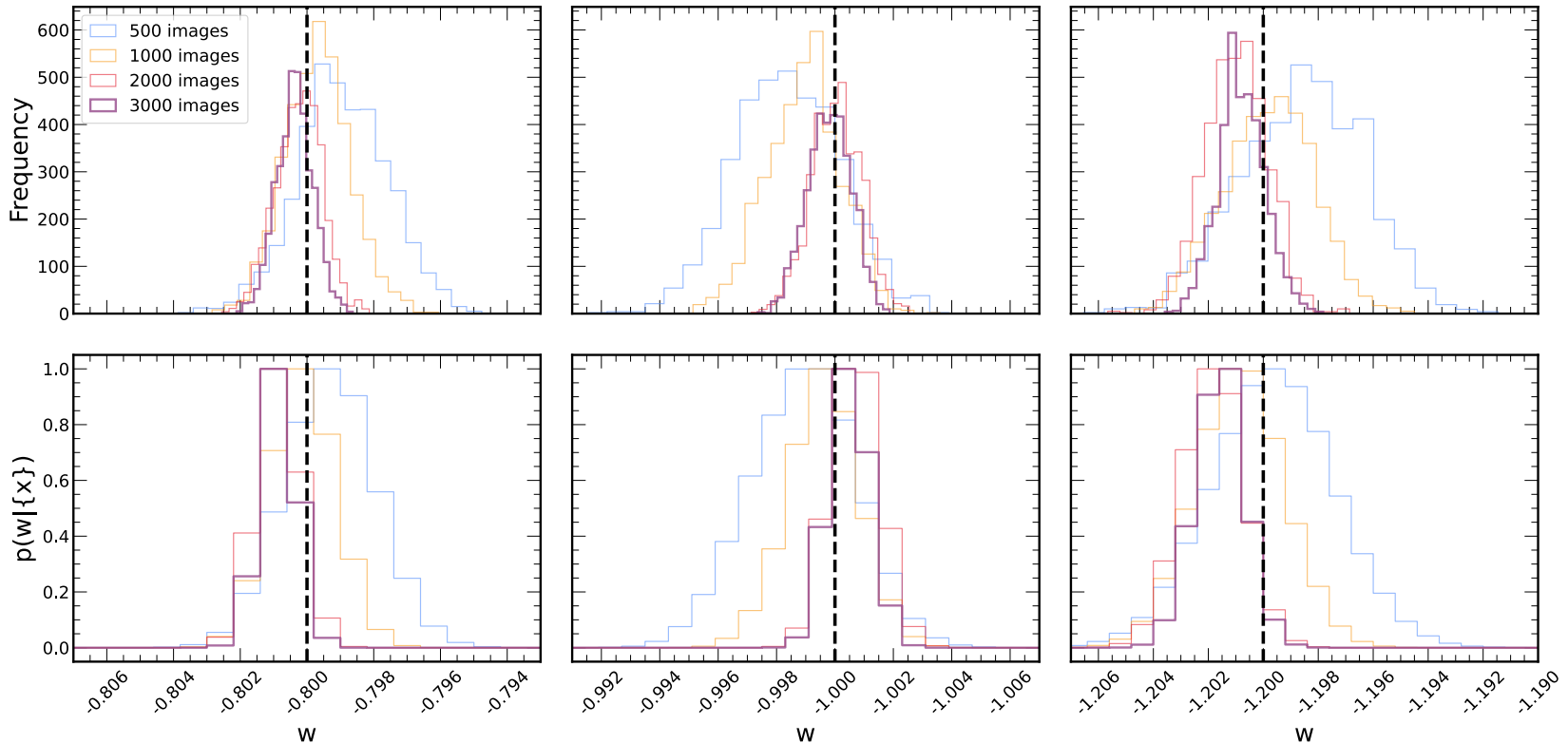
Results



Posterior obtained from single observation using NRE

Results: Population-level w Inference

- The posterior is more constrained as the number of observations in the population increase



The posterior inference $p(w | \{x\})$ from the joint population analysis of 500, 1000, 2000, and 3000 strong lens images using MCMC (Top) and Analytical method (Bottom)