



How Matter Matters: The Story of Time Invariance Violation in Neutrino Oscillations

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New Perspectives 2024



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Motivation from the CPT symmetry story



If we follow that CPT is a fundamental symmetry.

CPT

- ✓ Charge conjugation
- ✓ Parity
- ✓ Time reversal

Motivation from the CPT symmetry story



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Charge parity transform (CP) alone is violated in the weak sector.

CPT

+

~~CP~~

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Motivation from the CPT symmetry story



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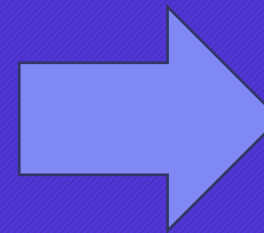
Charge parity transform (CP) alone is violated in the weak sector.

Time reversal transforms alone should also be violated in weak interactions, to preserve such an overall symmetry.

CPT

+

~~CP~~



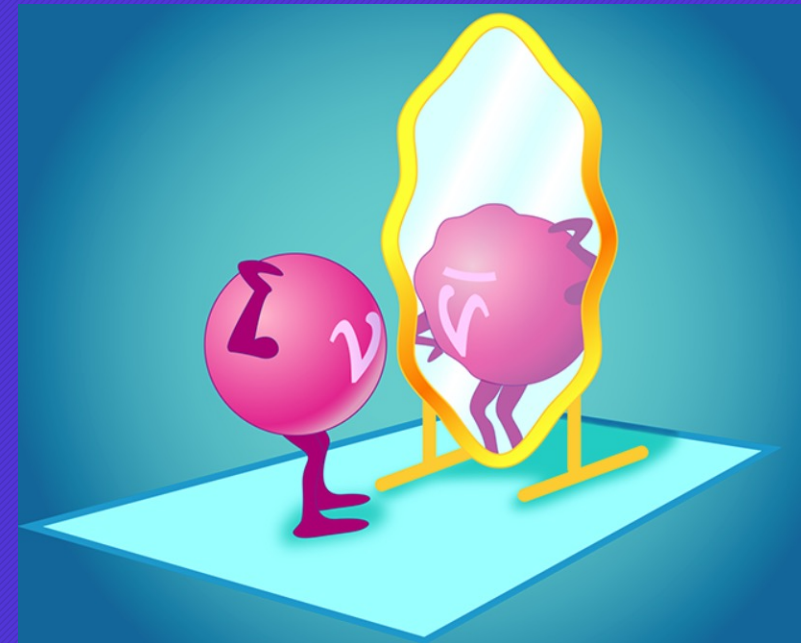
~~T~~

- ✓ Charge conjugation
- ✓ Parity
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Motivation from the CPT symmetry story

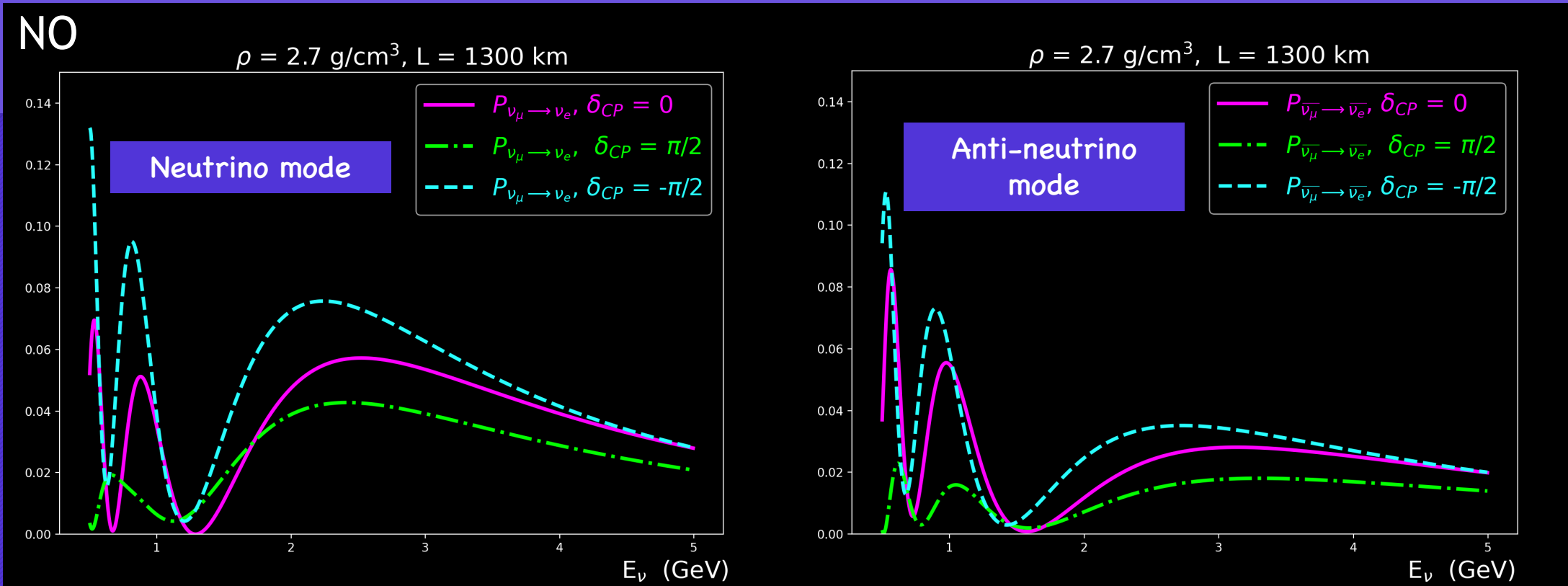


- ✓ Neutrino physics is a well motivated probe for CP violation, but we are limited to an improper test due to living in a matter dominated universe.
- ✓ Let us then consider to what extent time invariance violation occurs within the neutrino sector.
- ✓ Why? New physics may not impact both CP and time reversal in the same way.



Source: <https://physics.aps.org/articles/v15/120>
Credit: APS/Carin Cain

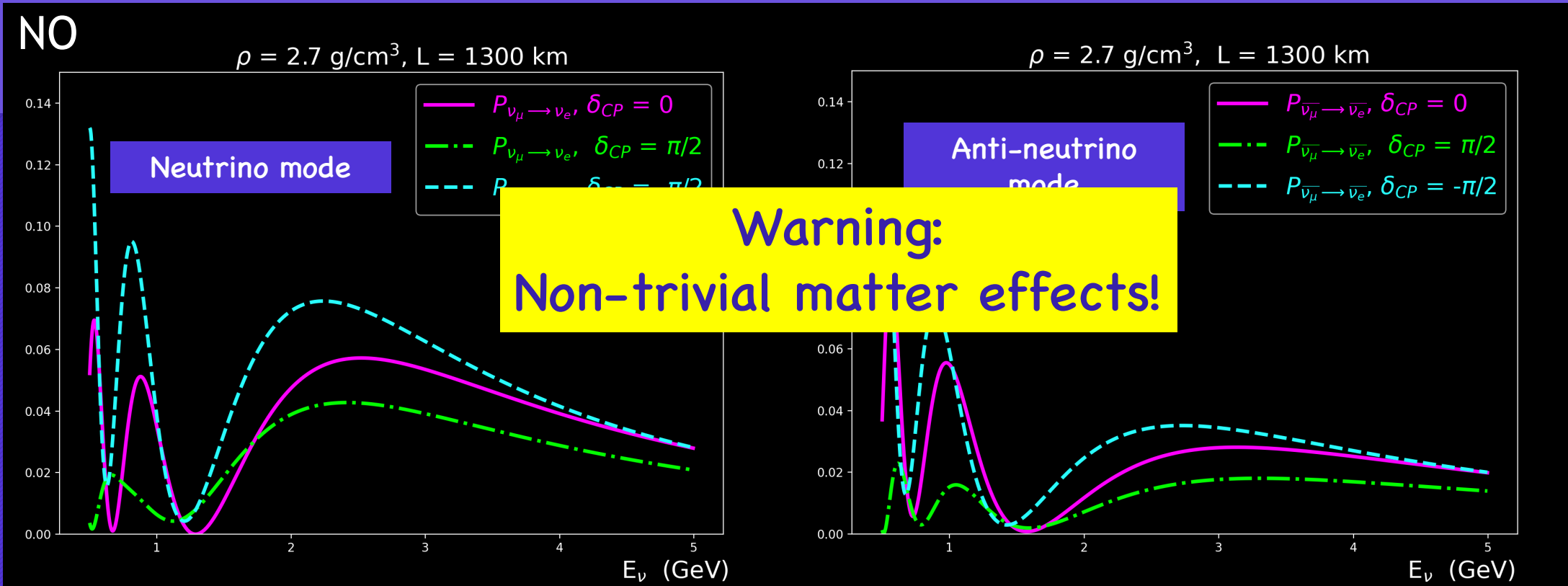
Motivation from experiments story



density \iff Earth's crust

CP conjugate channels are the *most common probes*, as they are more accessible to experiments like long baselines.

Motivation from experiments story



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Motivation from experiments story



What else can
we do?

Motivation from experiments story



Time
Invariance
Tests



Source: (Time-turner) <https://tenor.com/view/time-turner-harry-potter-moving-spinning-gif-16031036>

Motivation from experiments story



Time Invariance Tests

- ✓ Time invariance violation tests give relatively clearer ways to aid in our understanding of how different matter profiles can affect neutrino oscillations

(i.e. distinguishing between intrinsic & induced time invariance violation)



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Motivation from experiments story



Time Invariance Tests

- ✓ Time invariance violation tests give relatively clearer ways to aid in our understanding of how different matter profiles can affect neutrino oscillations

(i.e. distinguishing between intrinsic & induced time invariance violation)

NOTE: time invariance probability tests require comparing

$$P_{\nu_{\mu} \rightarrow \nu_e} \quad \text{vs} \quad P_{\nu_e \rightarrow \nu_{\mu}} \quad \text{(or anti-neutrino versions)}$$

We assume that a new beam capable of producing high energy ν_e 's exists (i.e. muon storage rings).

Recalling 3-Flavor Neutrino Oscillations with charge-current matter effects



$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

(PMNS matrix)

$$H = \frac{1}{2} E_\nu U M^2 U^\dagger + A$$



Sources: <https://neutrino.syr.edu/research/neutrino-oscillations/> & <https://neutrino-history.in2p3.fr/neutrino-oscillation/>

$$M^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix}$$

$$A = \begin{pmatrix} \sqrt{2}G_F N_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

+/- for neutrinos/antineutrinos



Defining time invariance measures

To clarify what “comparison” we are making in looking for time invariance violation effects, we’ve specified two distinct measures:

1. proper time invariance: (aka: true time invariance violation)

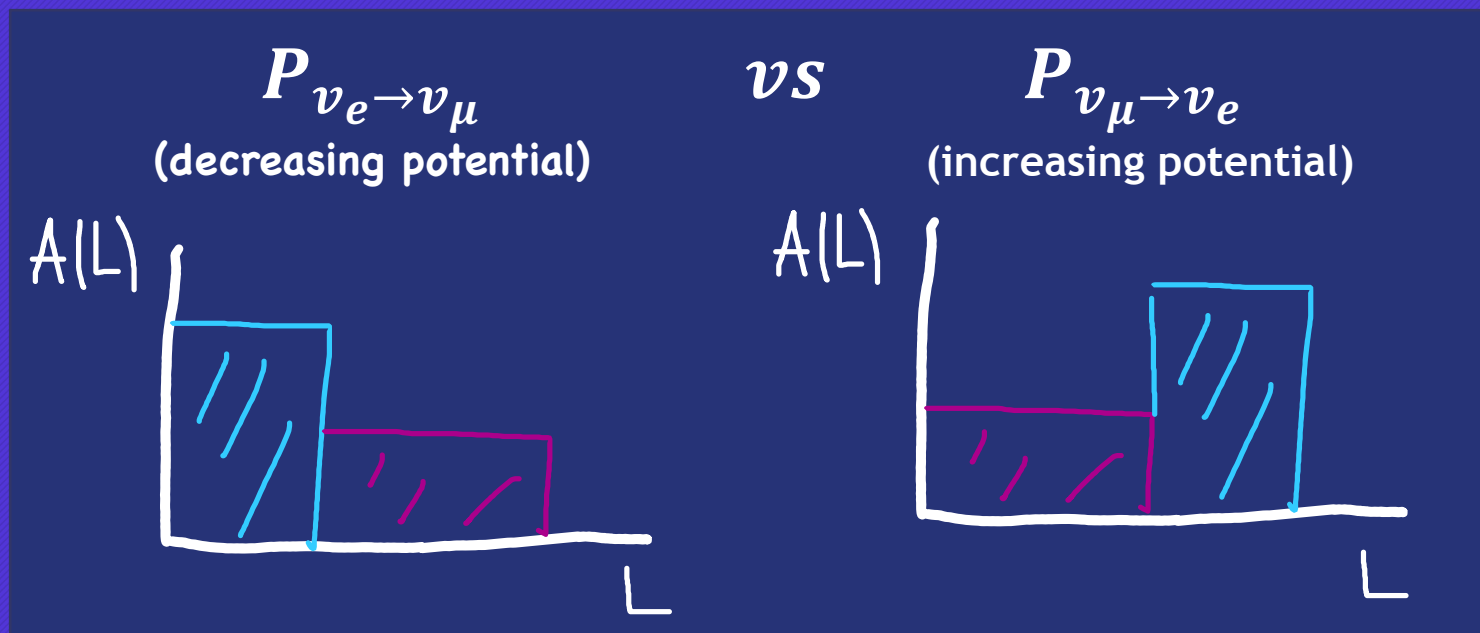
- Not a good observable, but certainly no harder to calculate than CP conjugate channels.
- Requires comparing probabilities with final states exchanged and swapping the detector with source.

Defining time invariance measures



To clarify what "comparison" we are making in looking for time invariance violation effects, we've specified two distinct measures:

1. proper time invariance: (aka: true time invariance violation)



ex: 2-step piece-wise constant matter profiles

$$A_{00} = A(L) = \sqrt{2}G_F N_e$$

N_e changes depending on density

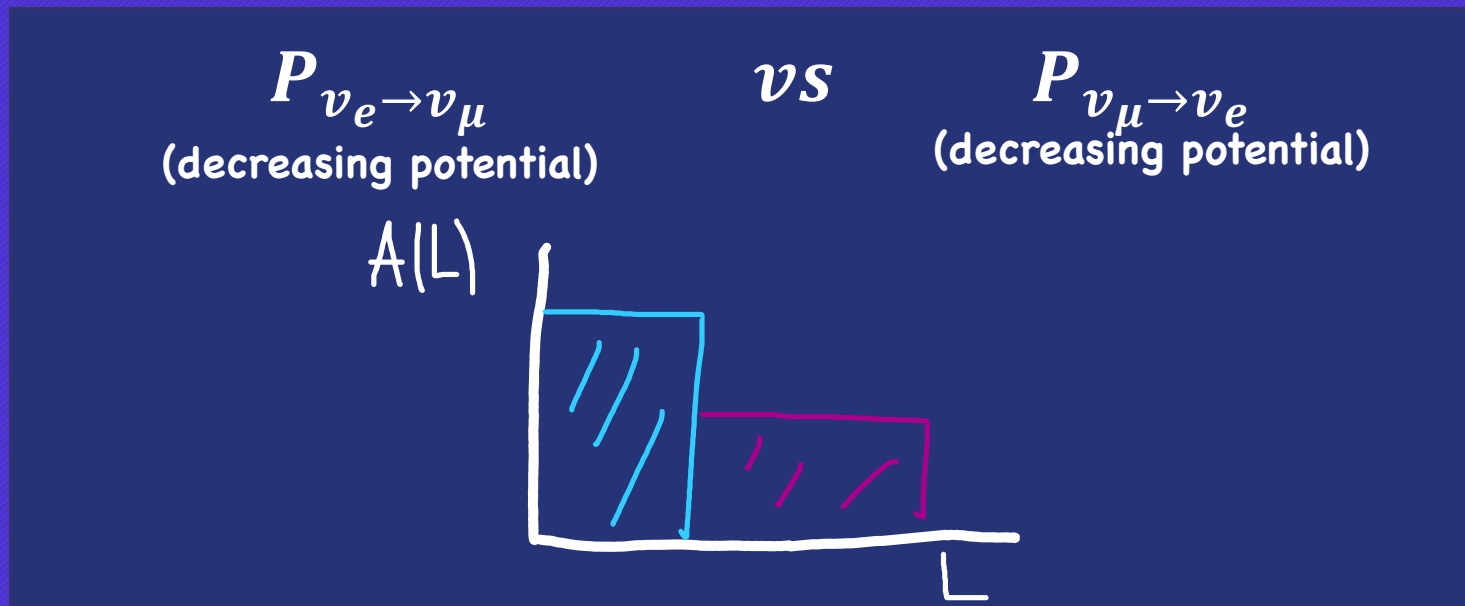


Defining time invariance measures



2. improper time invariance: (next best thing!)

- Compares probabilities with only the final states exchanged.



This is what an experiment can measure!



Modeling Matter Effects for 3-Flavors

✓ For the purposes of our study, we separately two types of matter potential profiles.

Symmetric: vacuum or single step constant matter potential profile

Non-symmetric: piece-wise matter potential profiles (increasing or decreasing)



Modeling Matter Effects for 3-Flavors

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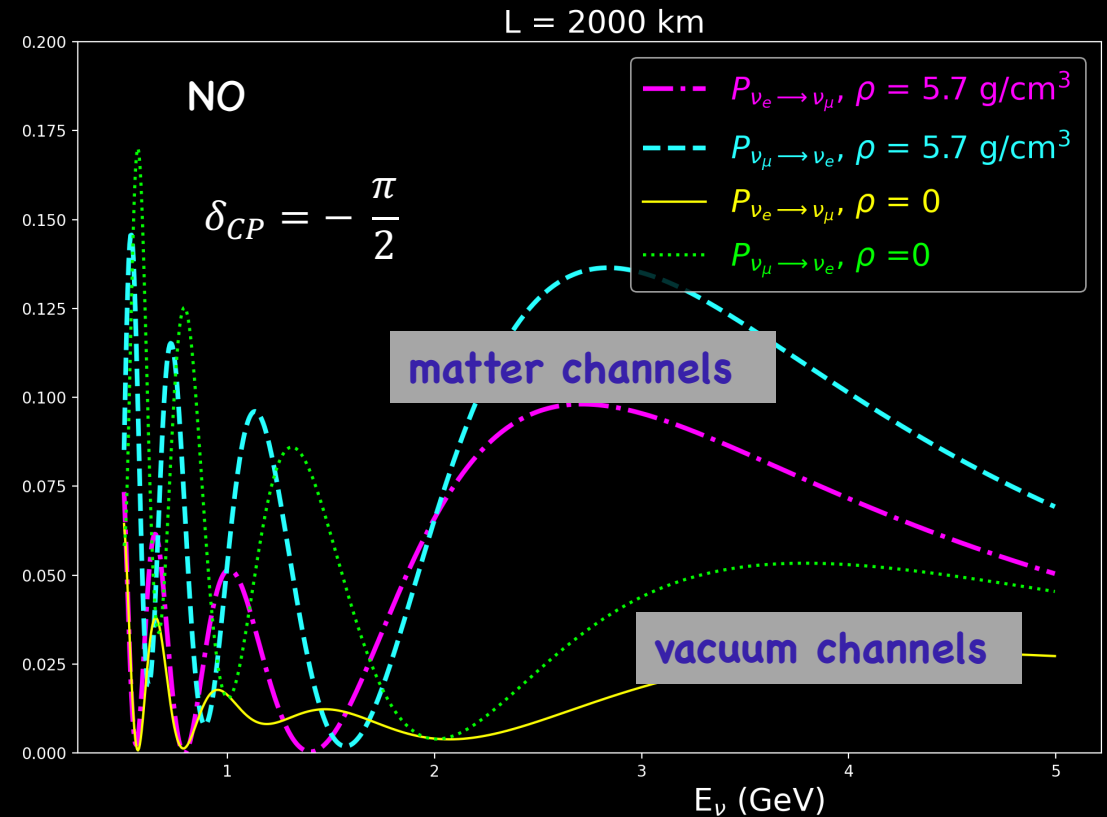
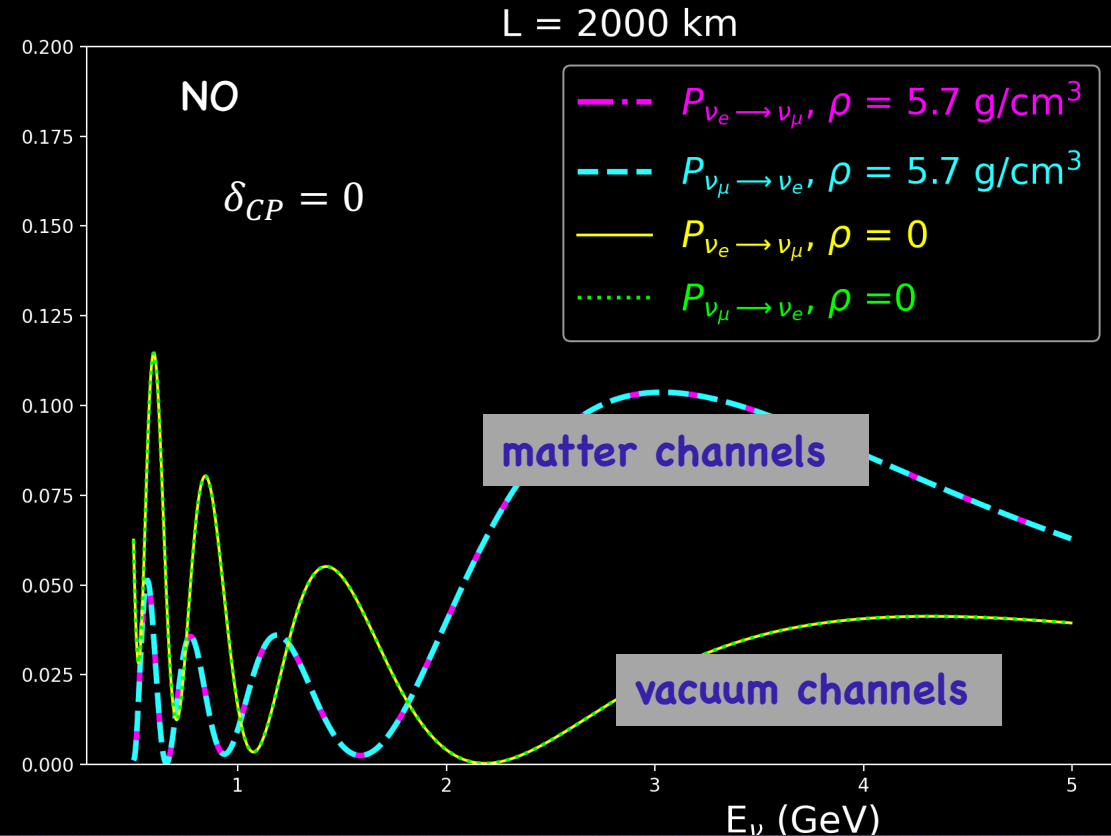
All mixing parameters (apart from δ_{CP}) have been drawn from NuFIT 2024 global fits: (arXiv:2007.14792 & NuFIT 5.3 (2024), www.nu-fit.org)



Disclaimer!

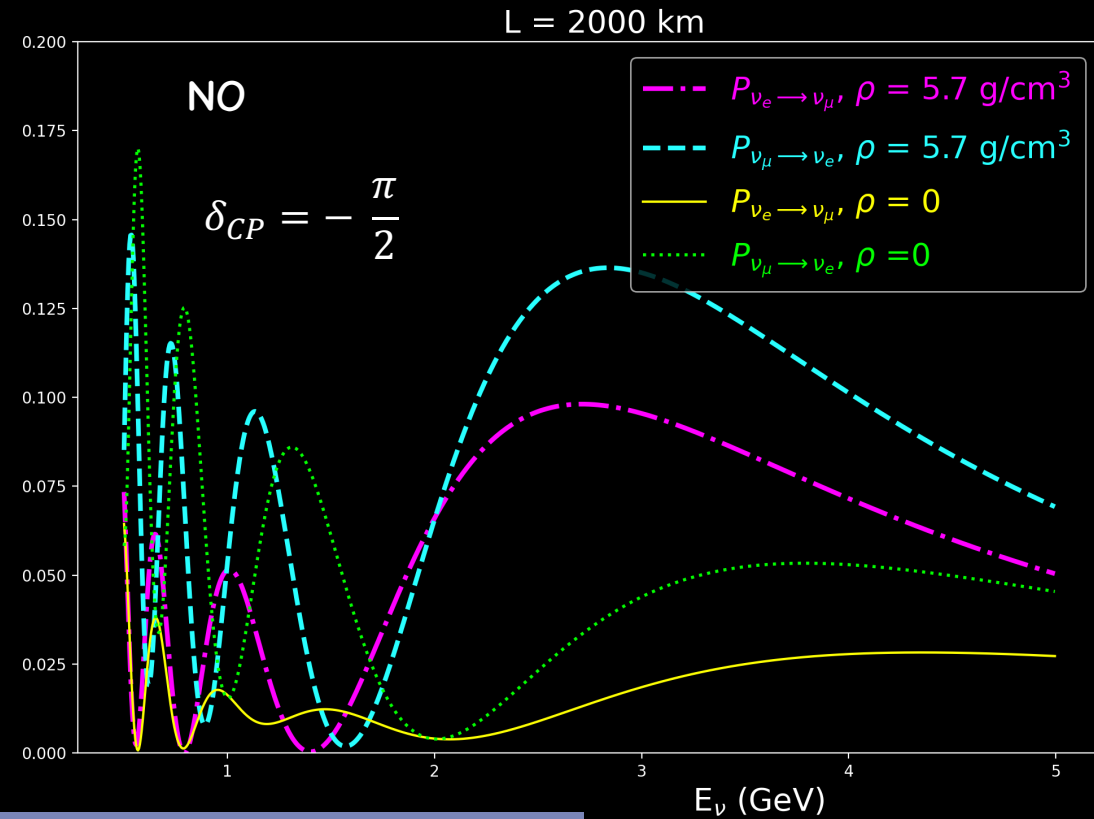
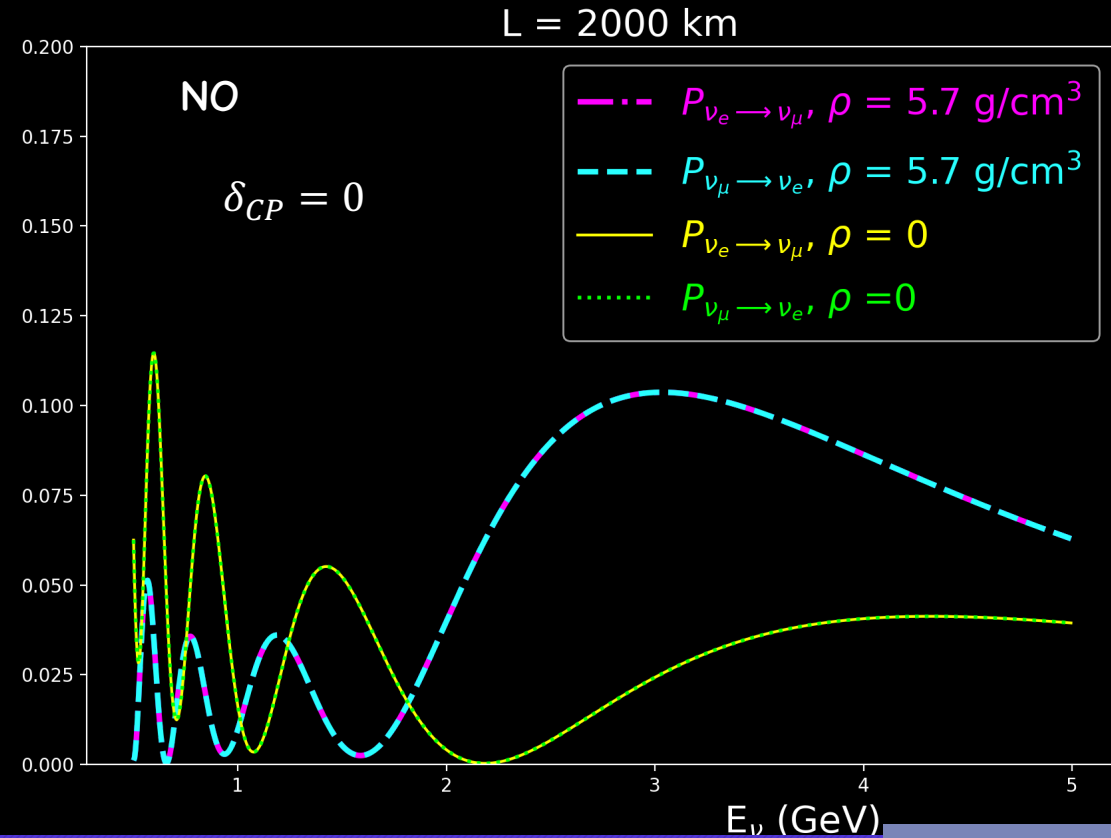
The following preliminary analysis is done with hypothetical/fictitious matter effects, to get a general sense of how intrinsic versus induced time invariance violation behave with matter effects in cases that they have stronger oscillation differences.

Symmetric matter effects for 3-Flavors



Constant matter potential:
 $1.1 \times 10^{-3} \text{ eV}^2/\text{GeV}$
 (5.7 g/cm^3)

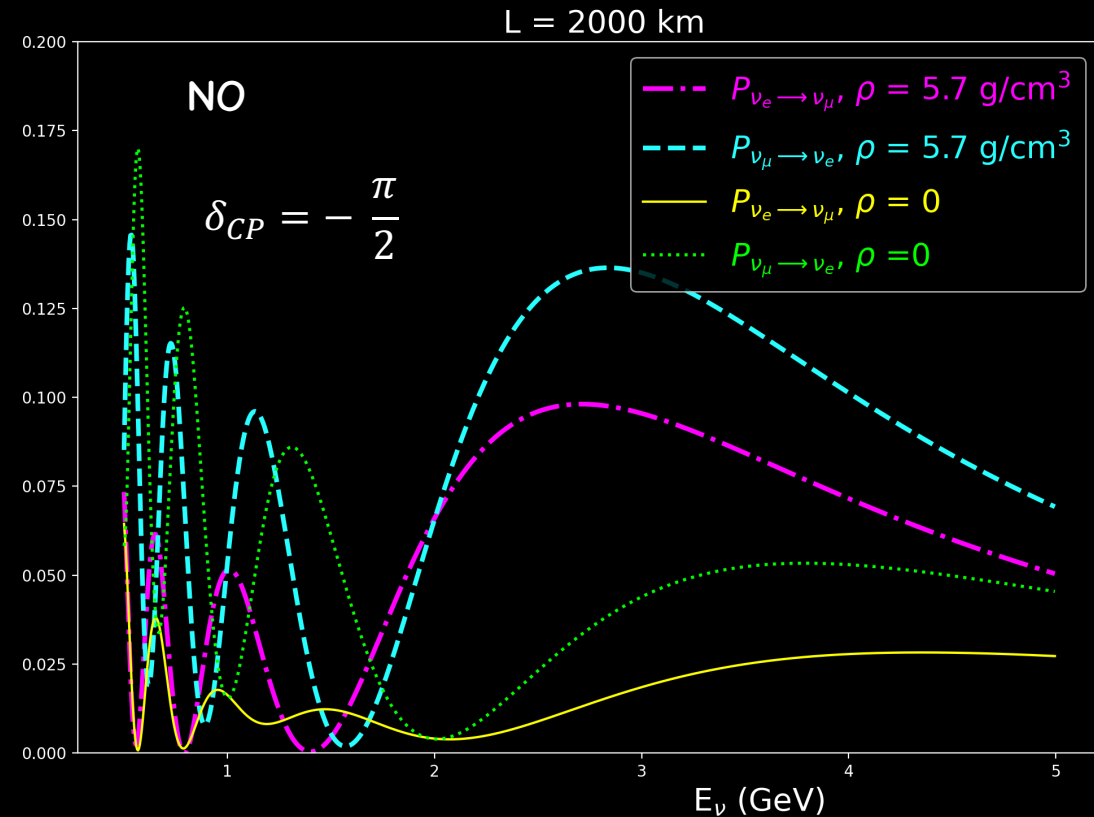
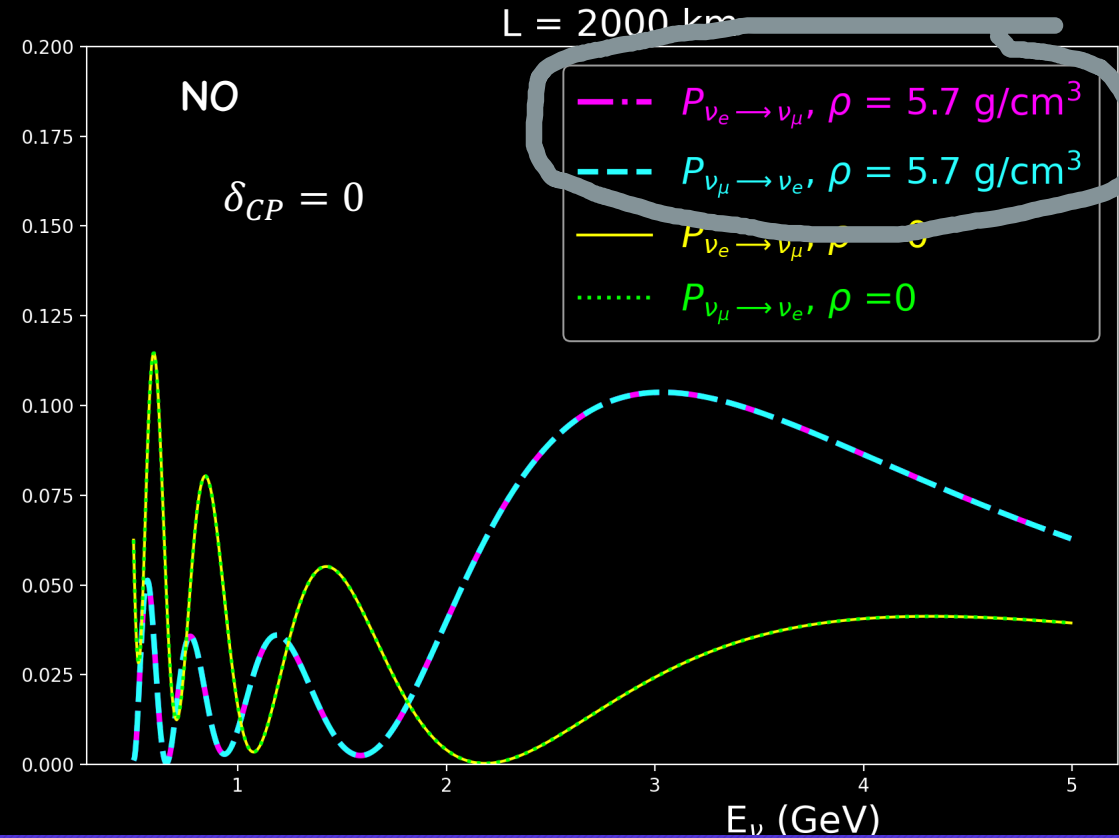
Symmetric matter effects for 3-Flavors



1. Symmetric matter potentials *cannot induce time invariance violation.*

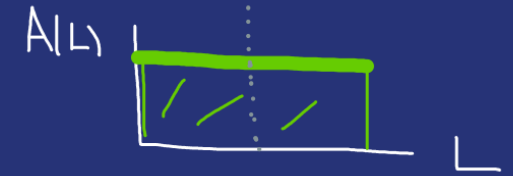
2. If there is intrinsic time invariance from δ_{CP} , then the matter potential simply *changes the degree of the effects.*

Symmetric matter effects for 3-Flavors

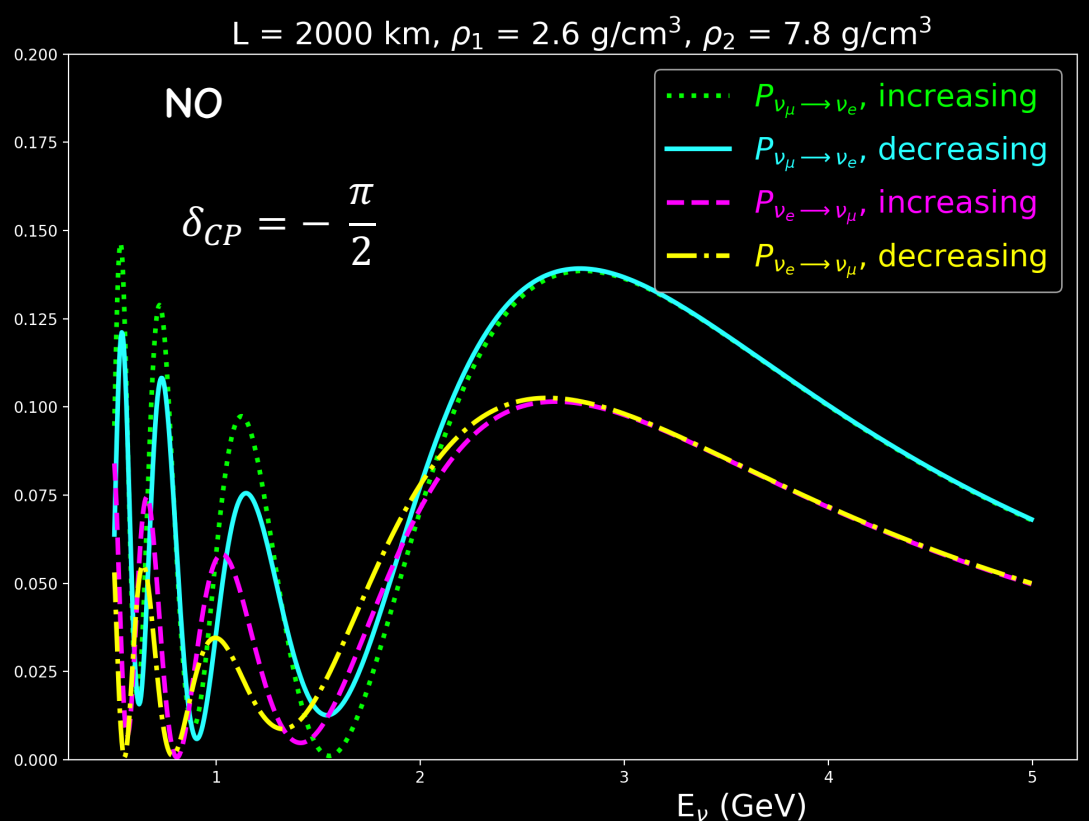
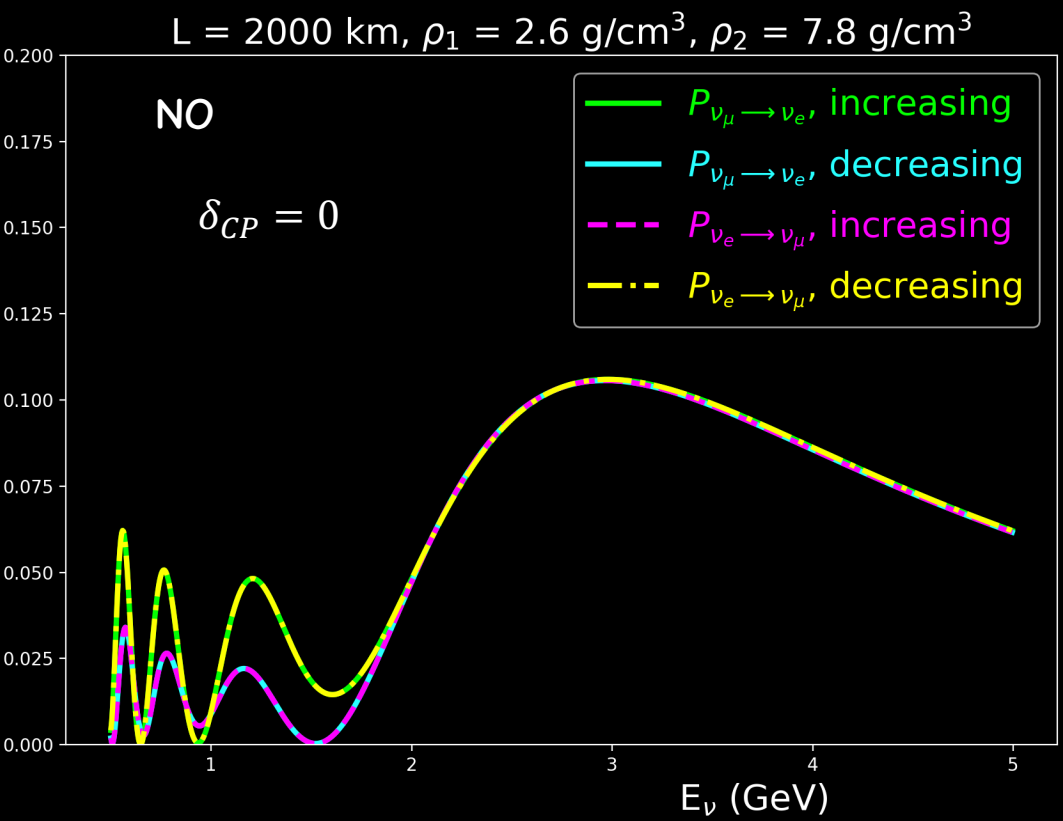


If we exchange final states, same matter potential profile.

3. Improper and proper comparisons are the same if the *matter potential is symmetric*.

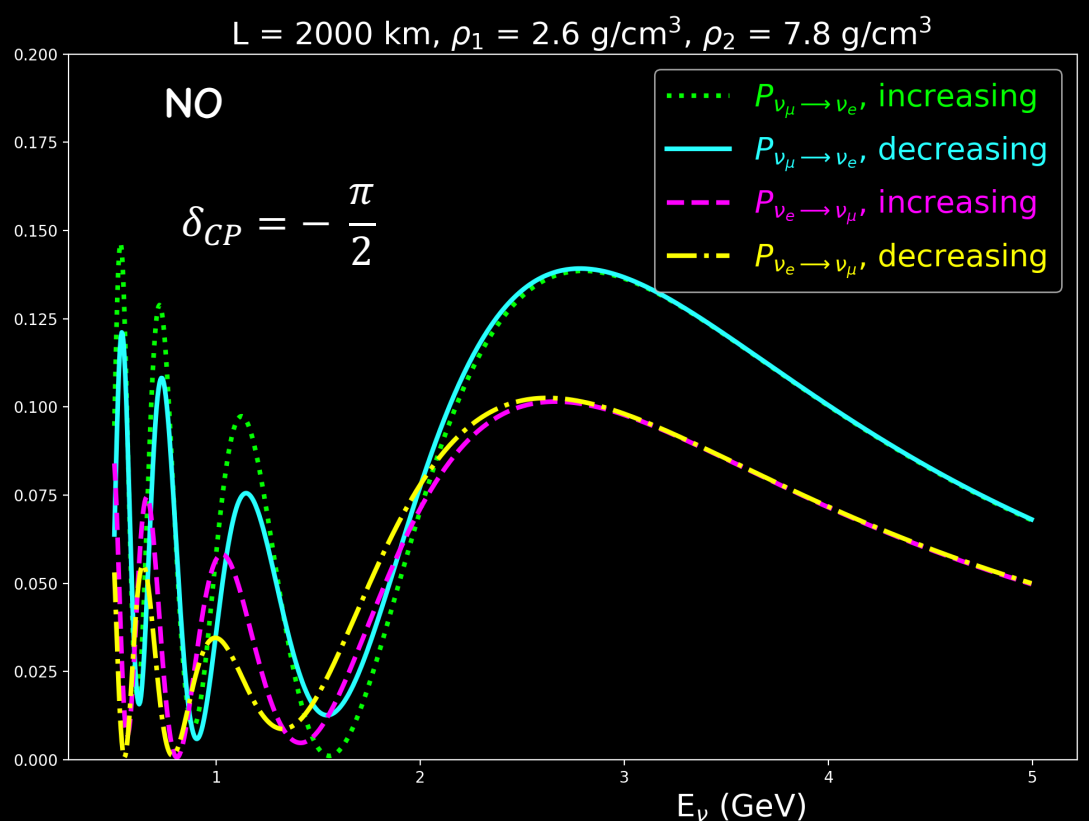
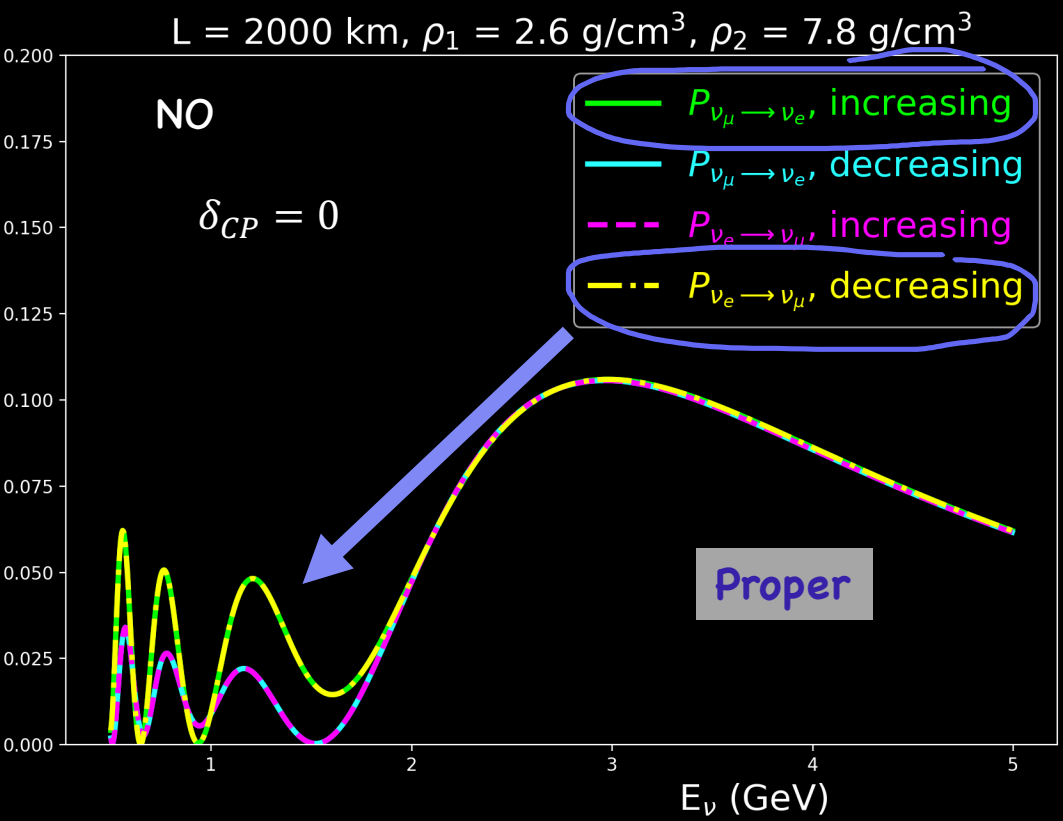


Non-symmetric matter effects for 3-Flavors



Piecewise matter potential (2 steps):
 $A_1 = 5 \times 10^{-4} \text{ eV}^2/\text{GeV}$ (2.6 g/cm^3)
 $A_2 = 1.5 \times 10^{-3} \text{ eV}^2/\text{GeV}$ (7.8 g/cm^3)

Non-symmetric matter effects for 3-Flavors

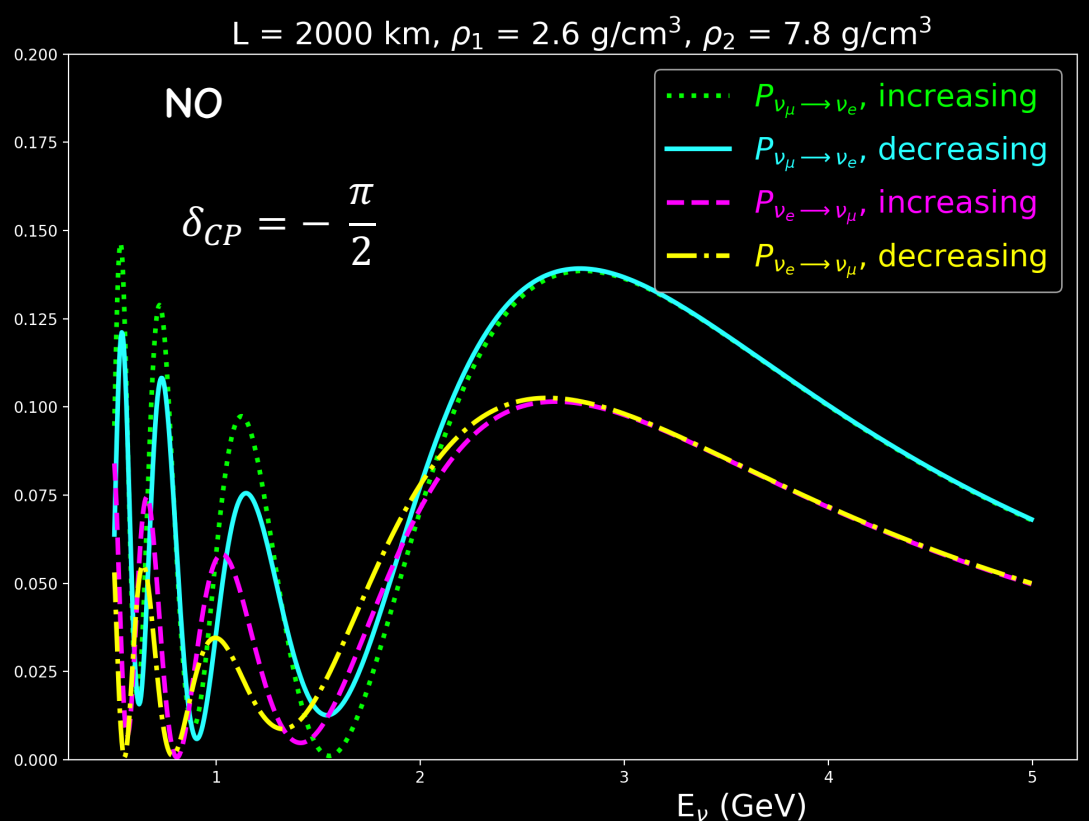
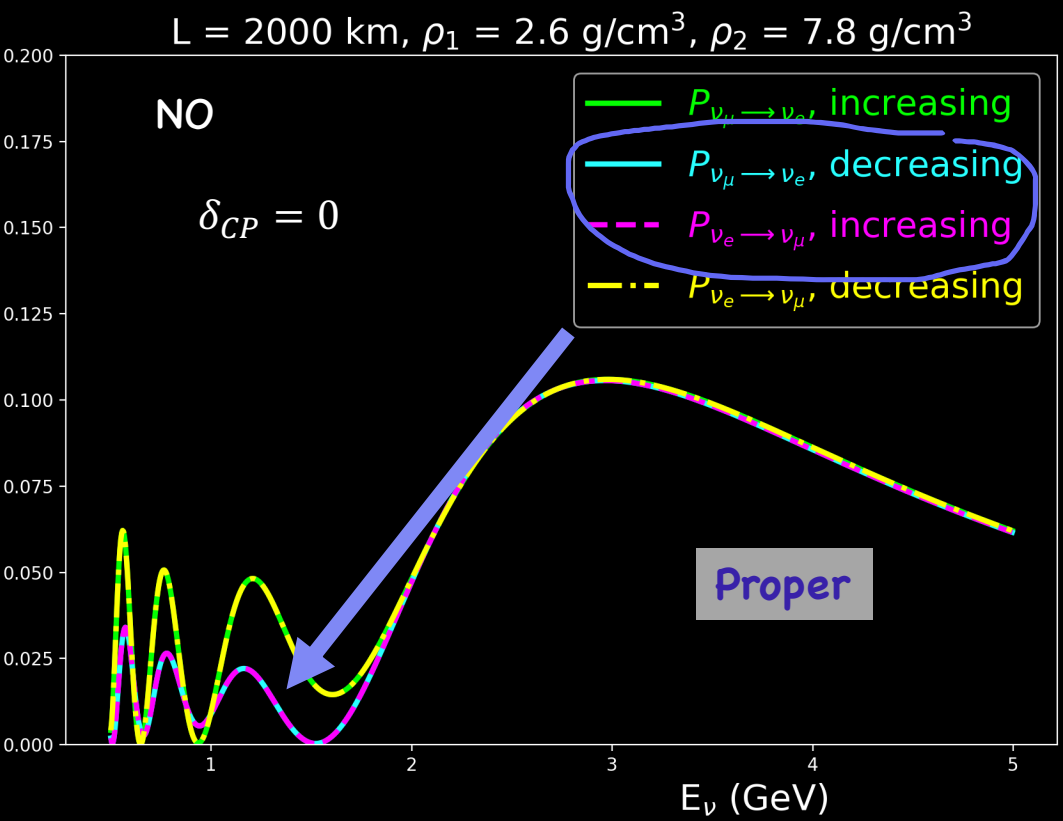


1. Non-symmetric matter effects are pairwise degenerate ($\delta_{CP}: 0$)

2. Proper time *remains invariant*, but improper channels are *different*.

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Non-symmetric matter effects for 3-Flavors

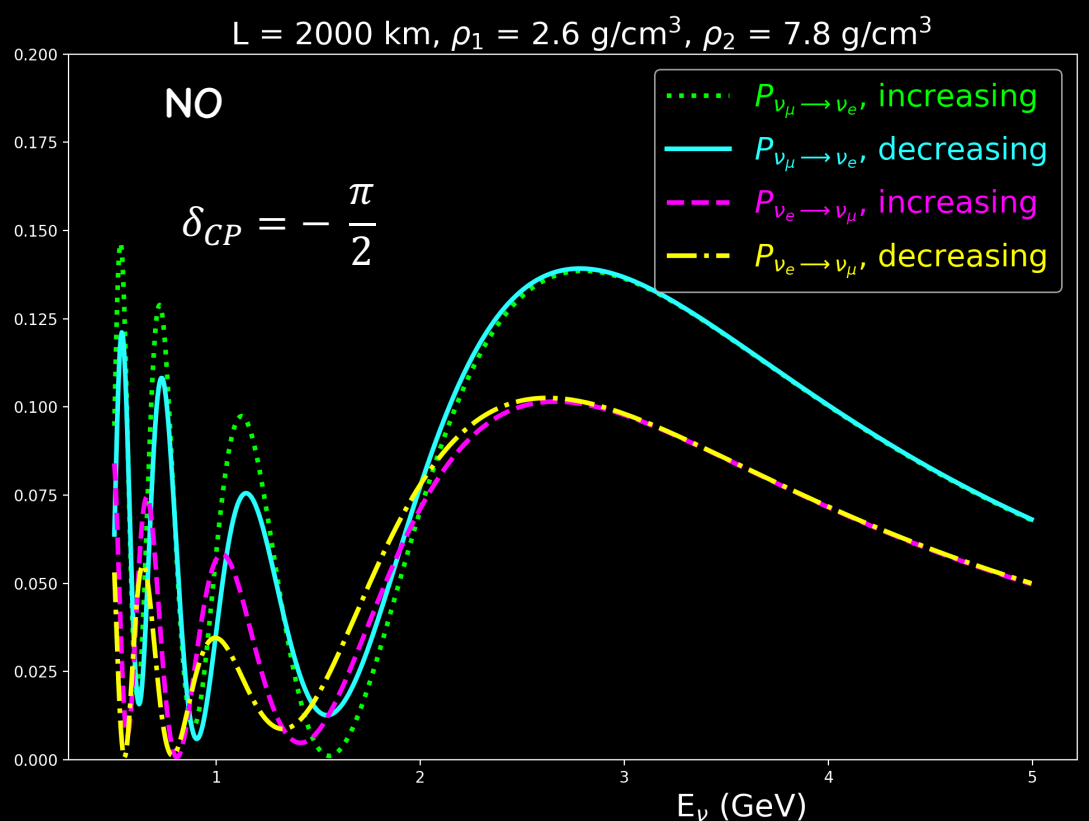
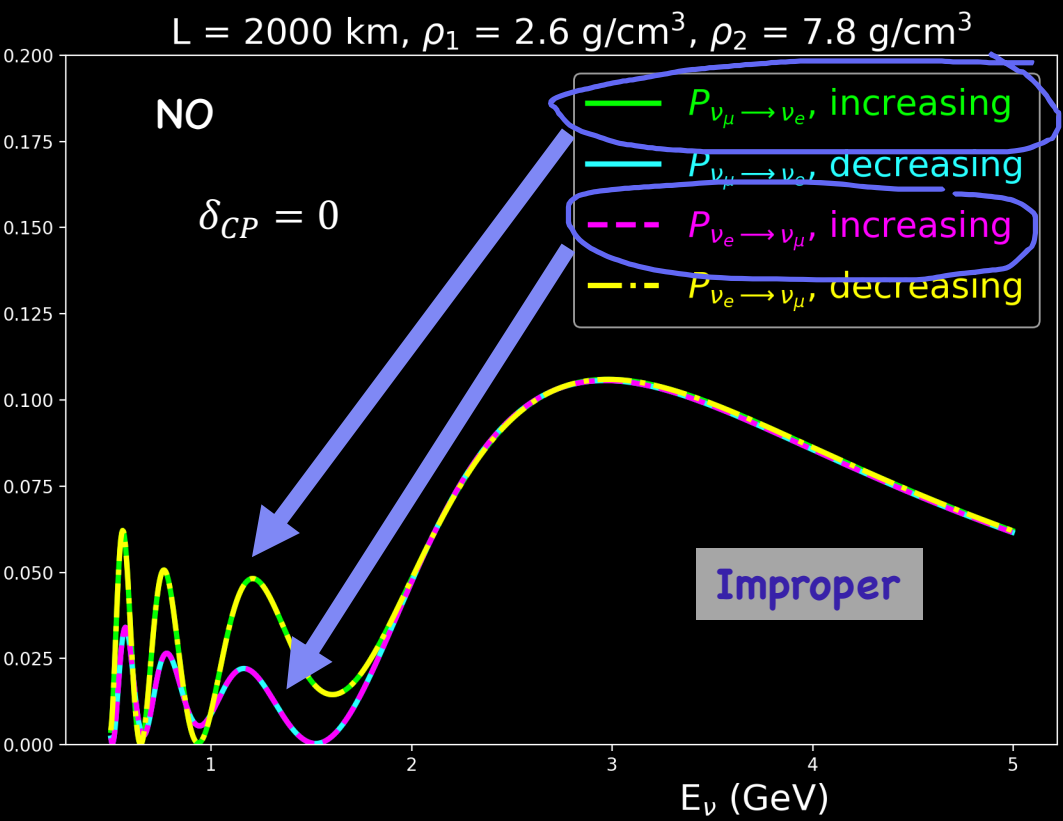


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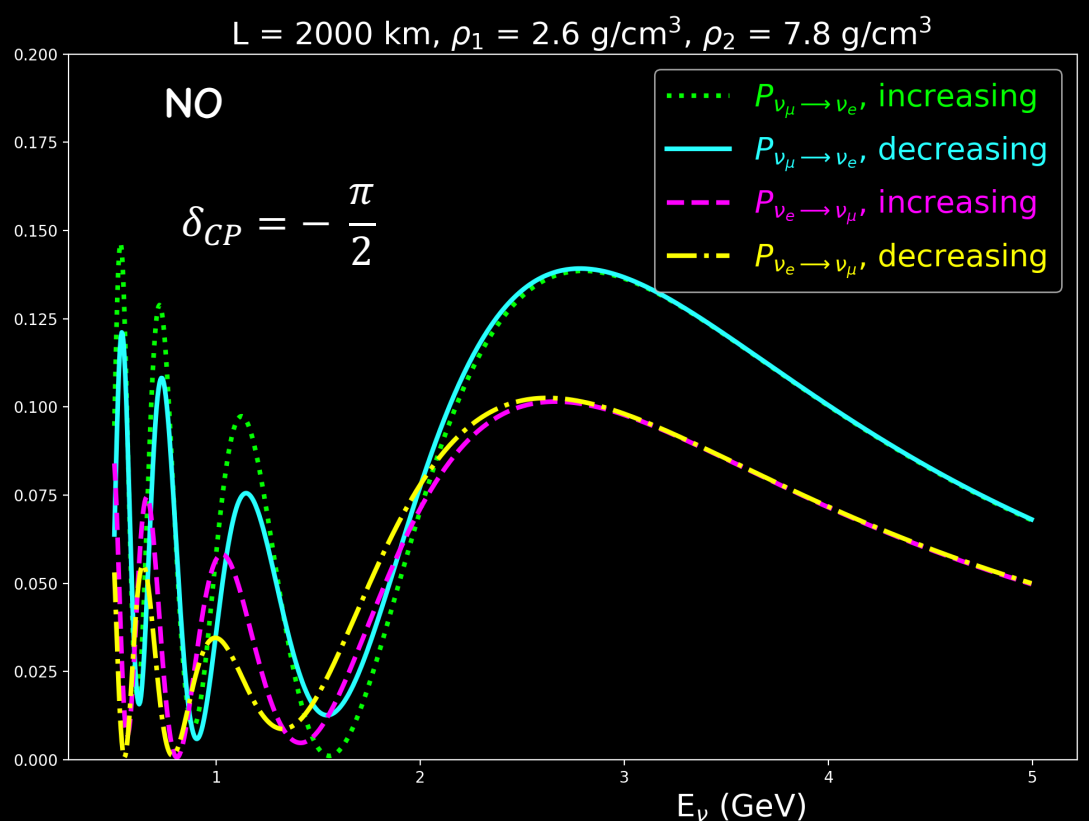
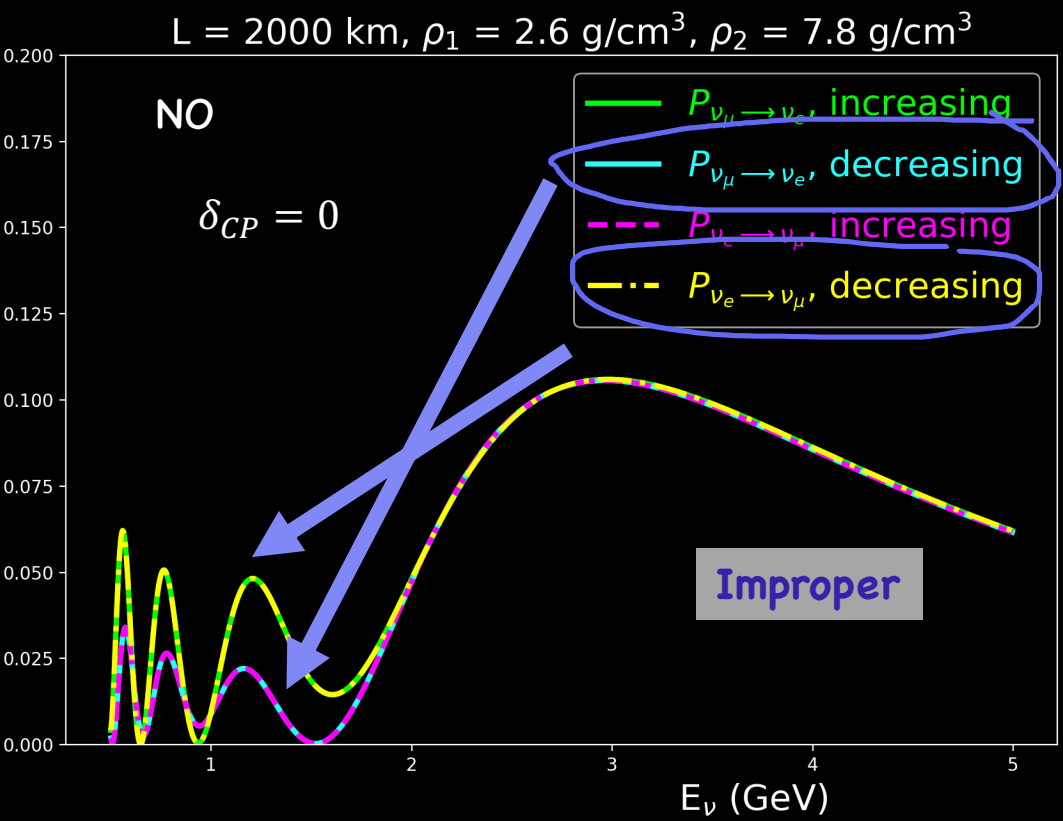


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New Perspectives, 2024

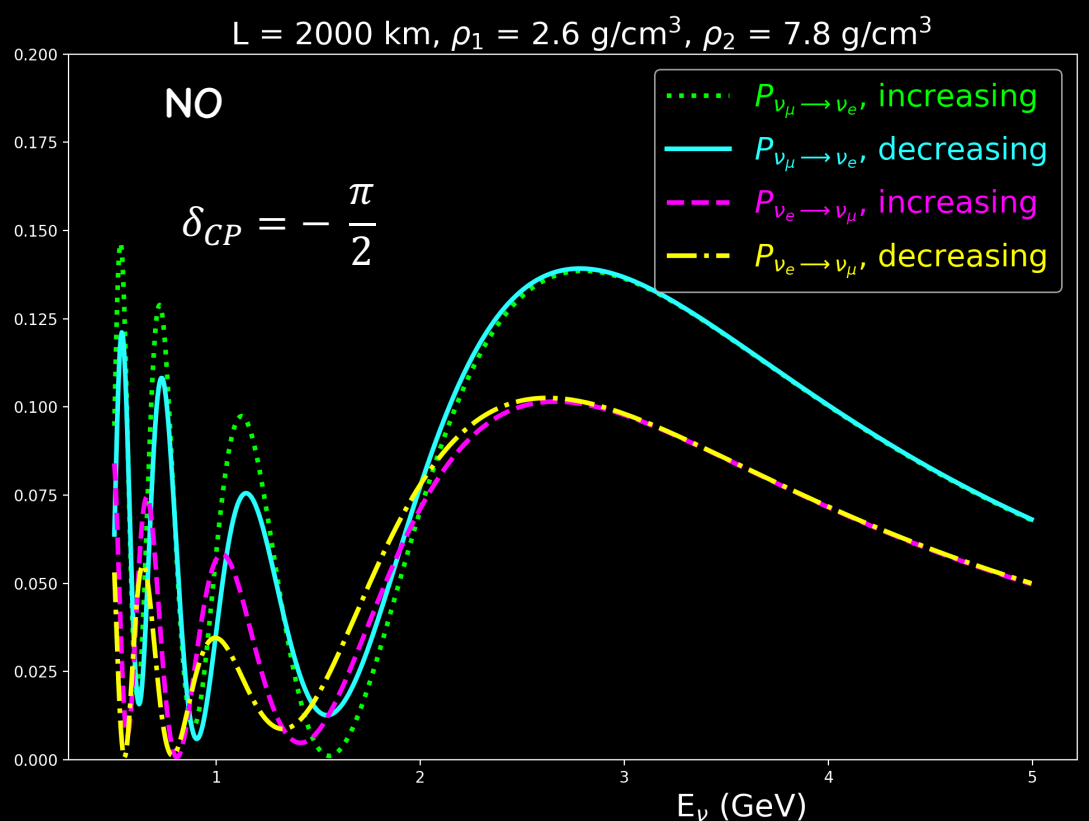
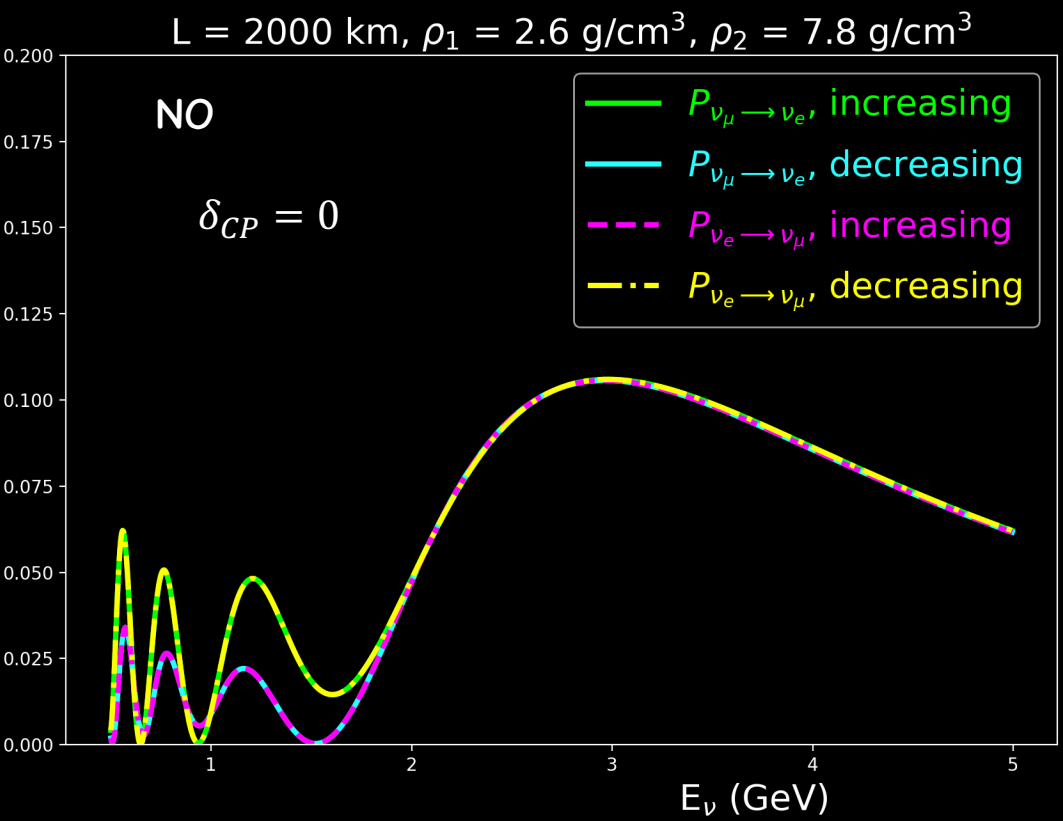
Non-symmetric matter effects for 3-Flavors



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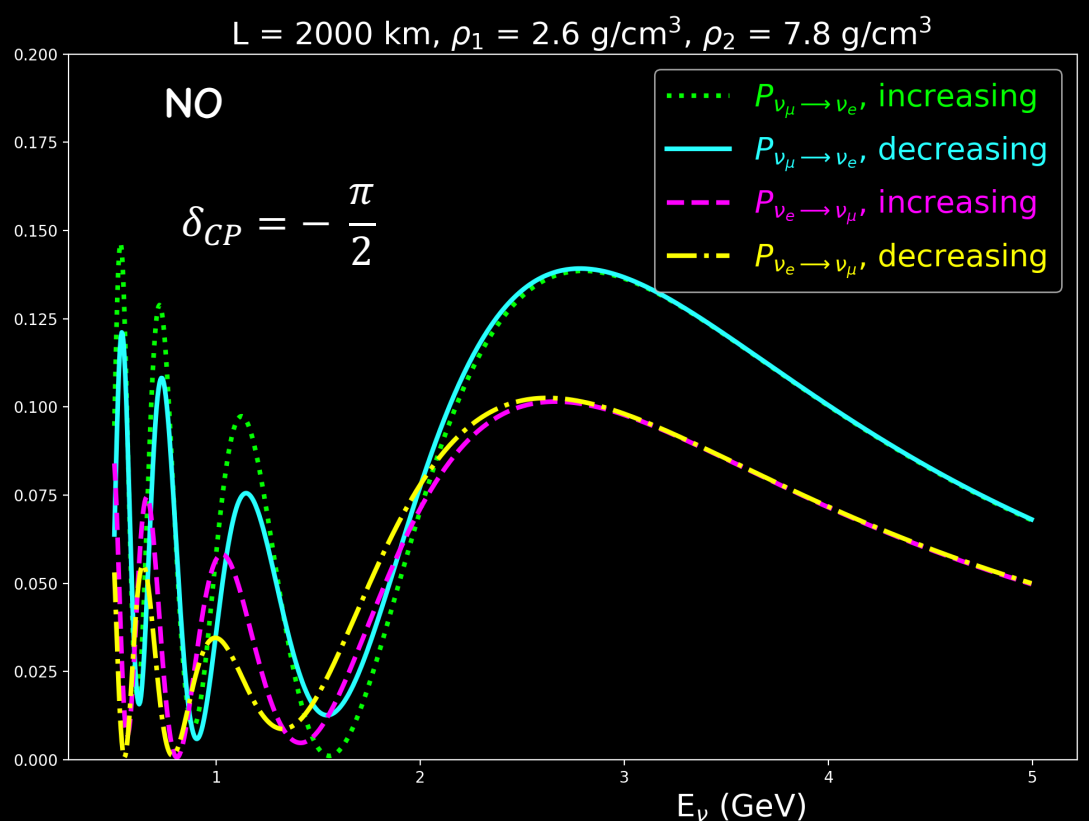
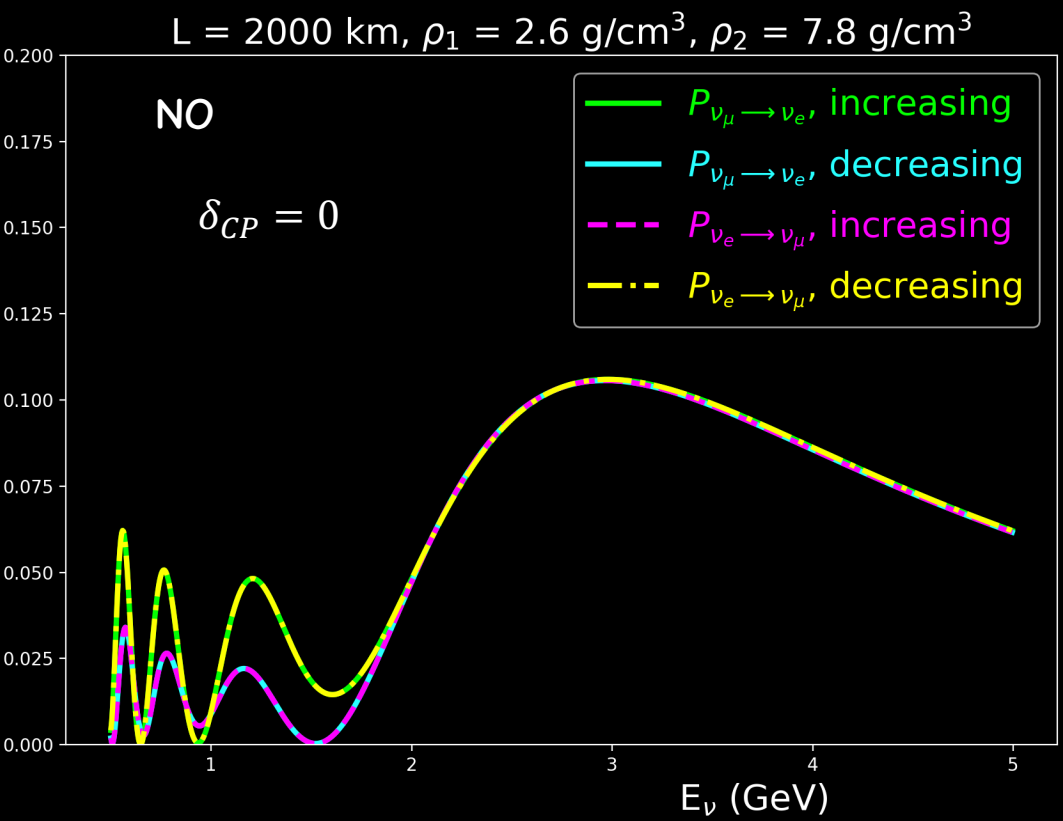


1. Non-symmetric matter effects are pairwise degenerate ($\delta_{CP}: 0$)

2. Proper time *remains invariant*, but improper channels are *different*.

3. No intrinsic time invariance violation but *matter induced time invariance violation occurs*.

Non-symmetric matter effects for 3-Flavors



4. No longer pairwise degenerate if $(\delta_{CP} : -\frac{\pi}{2})$.

5. Matter potential choice changes magnitude of oscillations:
All probabilities are different.

Main Takeaways



- This is an ongoing study into how much **time invariance violation** (if at all) can be observed in neutrino oscillations, and to what extent such observables can be differentiated in simple neutrino matter potential models.
- **Symmetric potentials cannot induce time violation, but if time invariance is intrinsically non-conserved, the matter potential simply changes the degree of the observed effects.**
- **Symmetric potentials provide probes into checking proper time invariance, which could be accessible in the near future with new beams.**
- **Non-symmetric matter potentials can induce improper time violation, while proper time violation is more protected. More on this to come.**
- **Future studies: 2-flavor/3-flavor NSI and more realistic matter profiles just to name a few!**

Thank you!

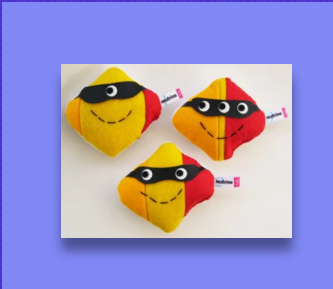


Backups



Symmetric matter effects for 2-Flavors

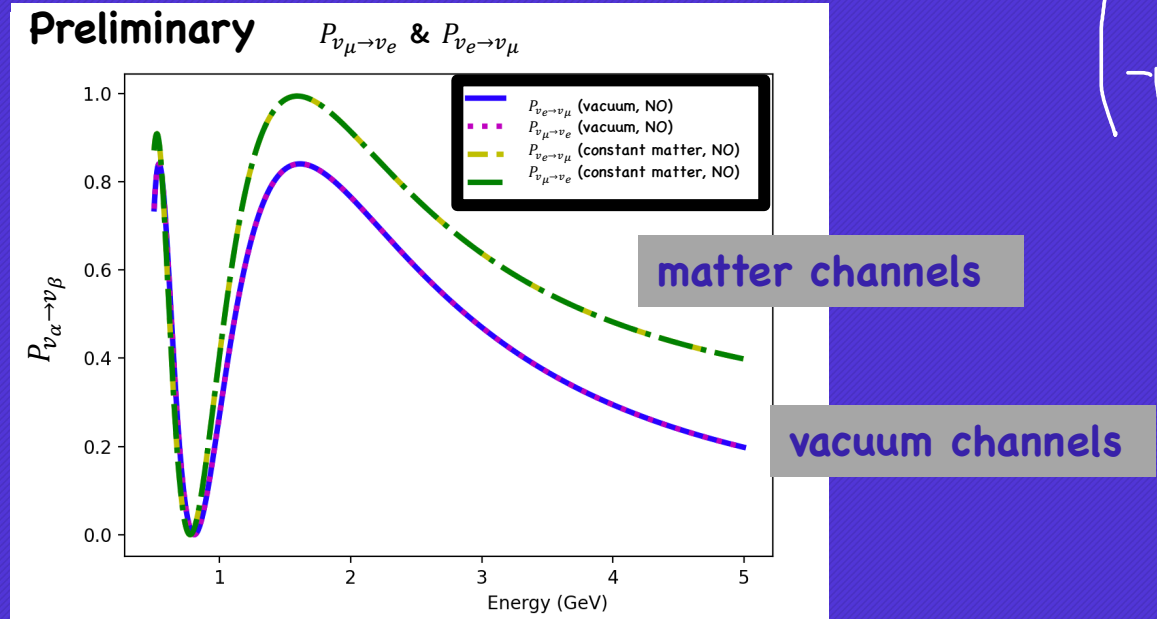
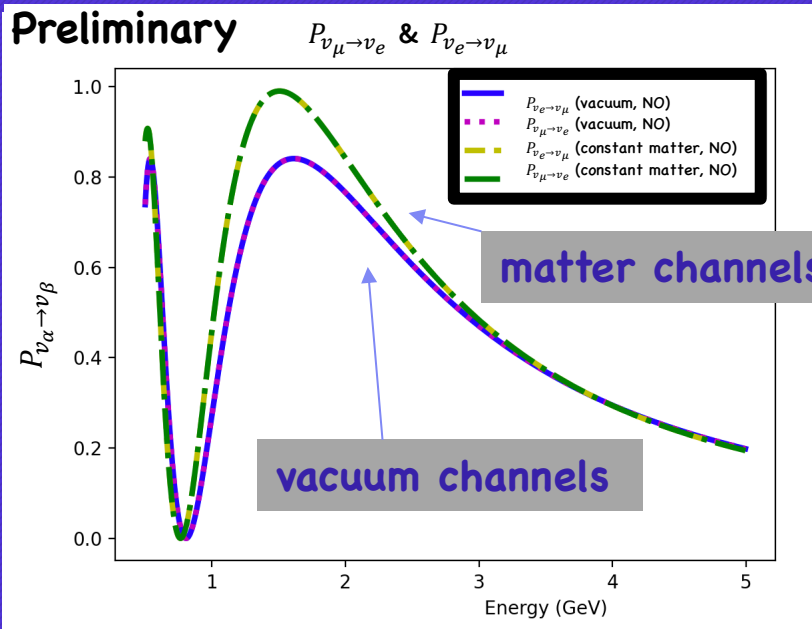
(Normal Ordering, Baseline: 2000 km)



constant matter potential:
 $A \approx 5 \cdot 10^{-4} \text{ eV}^2/\text{GeV}$ (2.6 g/cm³)

NSI + constant matter potential:
 $A \approx 5 \cdot 10^{-4} \text{ eV}^2/\text{GeV}$ (2.6 g/cm³)
 $B \approx i \cdot 2.5 \cdot 10^{-4} \text{ eV}^2/\text{GeV}$ (1.3 g/cm³)

$$\begin{pmatrix} A & iB \\ -iB & 0 \end{pmatrix}$$



Non-symmetric matter effects for 2-Flavors

(Normal Ordering, Baseline: 2000 km)



2 step constant matter potential:

$$A1 \approx 5 \cdot 10^{-4} \text{ eV}^2/\text{GeV} \text{ (2.6 g/cm}^3\text{)}$$

$$A2 \approx 1.5 \cdot 10^{-3} \text{ eV}^2/\text{GeV} \text{ (7.8 g/cm}^3\text{)}$$

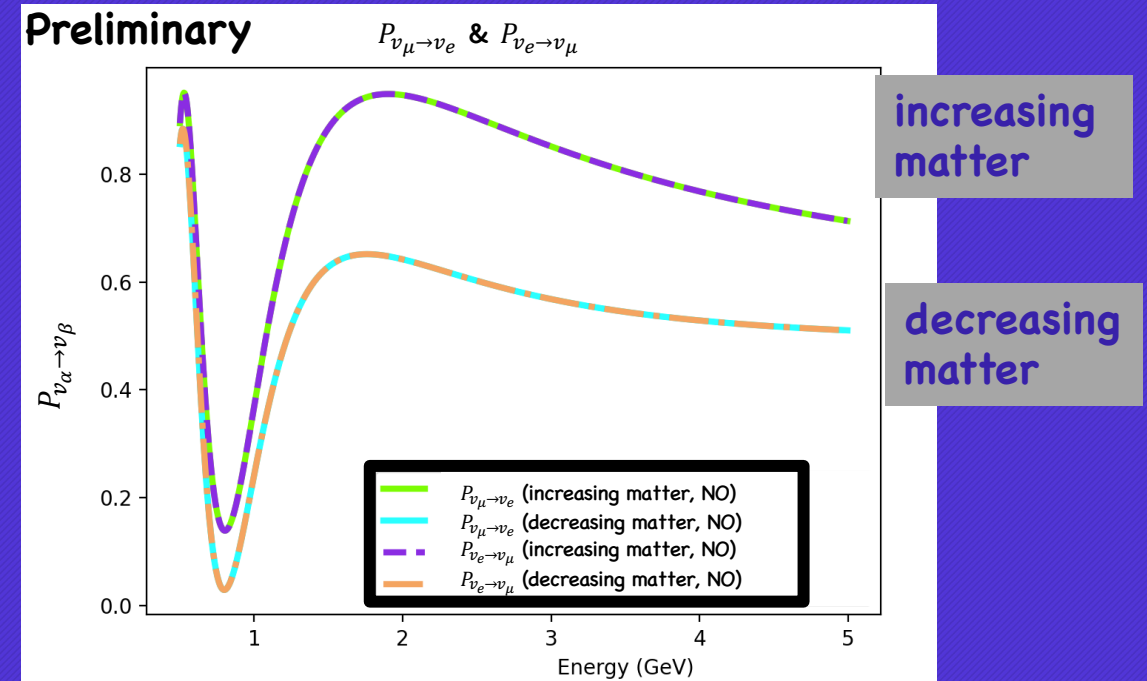
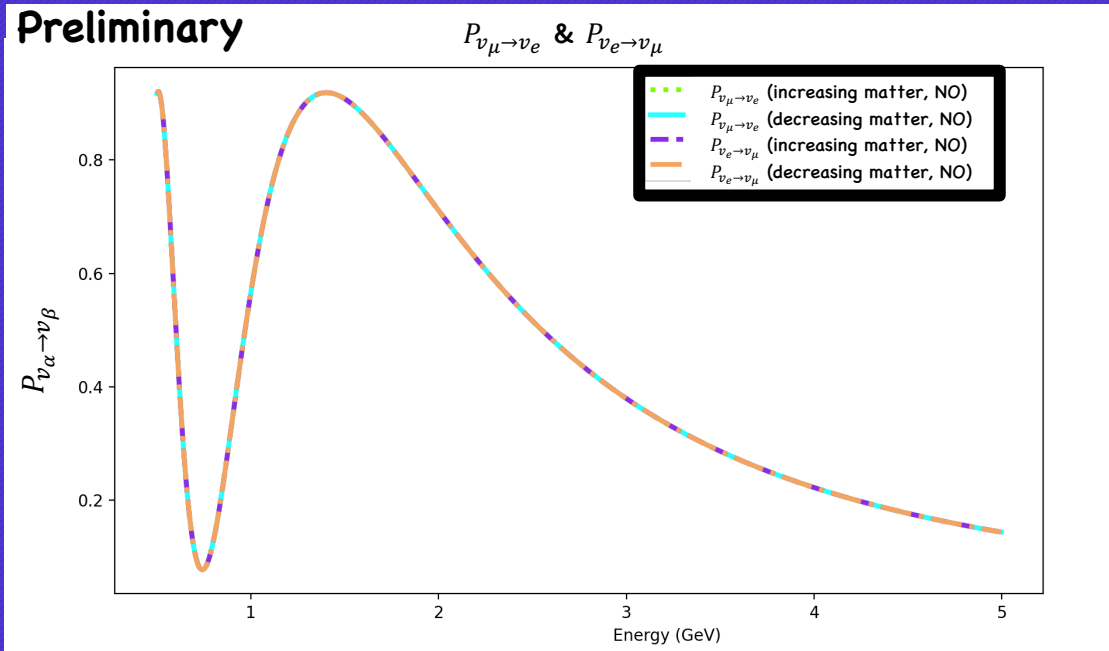
NSI + 2 step constant matter potential:

$$A1 \approx 5 \cdot 10^{-4} \text{ eV}^2/\text{GeV} \text{ (2.6 g/cm}^3\text{)}$$

$$B1 \approx i \cdot 2.5 \cdot 10^{-4} \text{ eV}^2/\text{GeV} \text{ (1.3 g/cm}^3\text{)}$$

$$A2 \approx 1.5 \cdot 10^{-3} \text{ eV}^2/\text{GeV} \text{ (7.8 g/cm}^3\text{)}$$

$$B2 \approx i \cdot 7.5 \cdot 10^{-4} \text{ eV}^2/\text{GeV} \text{ (3.9 g/cm}^3\text{)}$$



increasing matter

decreasing matter

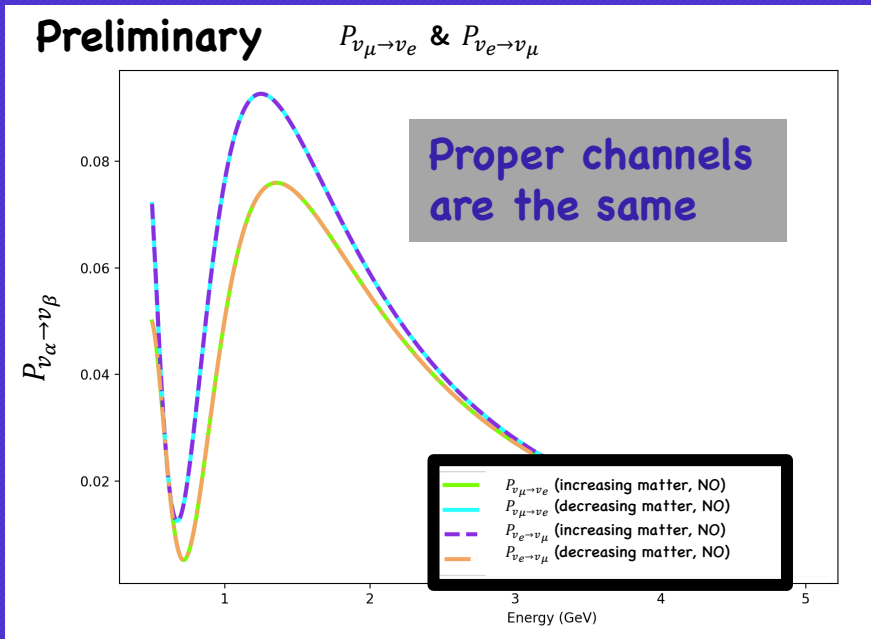
Non-symmetric matter effects for 3-Flavors, 3-Steps (Normal Ordering, Baseline: 2000 km)

3 step constant matter potential:

$A1 \approx 5.0 \cdot 10^{-4} \text{ eV}^2/\text{GeV}$ (2.6 g/cm³)

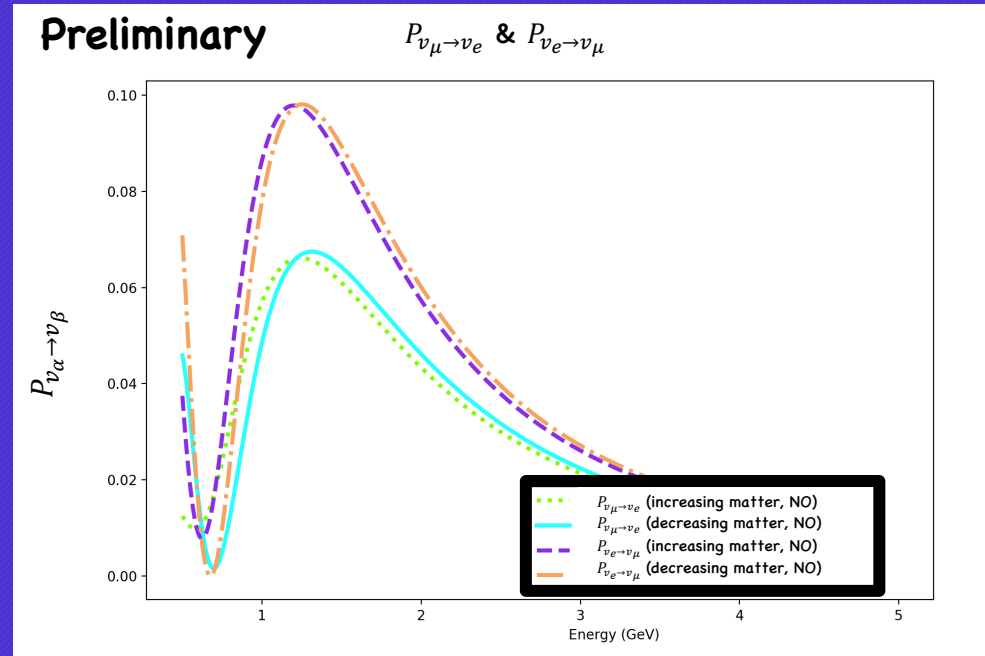
$A2 \approx 1.5 \cdot 10^{-3} \text{ eV}^2/\text{GeV}$ (7.8 g/cm³)

$A3 \approx 3.0 \cdot 10^{-3} \text{ eV}^2/\text{GeV}$ (15.6 g/cm³)



$\delta_{CP} = 0$

$\alpha, \beta \rightarrow [\mu, e]$



$\delta_{CP} = 90$

Recalling 3-Flavor Neutrino Oscillations with charge-current matter effects



We include matter effects explicitly from the following prescription for the probabilities (useful in the context of this study):

$$P_{\nu_{\alpha} \rightarrow \nu_{\beta}} = |\langle \nu_{\beta}(\mathbf{0}) | \nu_{\alpha}(\mathbf{L}) \rangle|^2 \quad \text{where} \quad |\nu_{\alpha}(\mathbf{L}) \rangle = U |\nu_{\alpha}(\mathbf{0}) \rangle$$

$$\text{with } U = e^{-iLH}$$

$$(H = H_{\text{vacuum}} + H_{\text{matter}})$$

Recalling 3-Flavor Neutrino Oscillations with charge-current matter effects



As an example for a Baseline L , let's break L up into 2 steps: L_1 and L_2 , where each evolution $U(L)$ will model different matter potentials A_1 and A_2 respectively.

Recalling 3-Flavor Neutrino Oscillations with charge-current matter effects



As an example for a Baseline L , let's break L up into 2 steps: L_1 and L_2 , where each evolution $U(L)$ will model different matter potentials A_1 and A_2 respectively.

$$U_1 = e^{-iL_1 H_1} \quad \& \quad U_2 = e^{-iL_2 H_2}$$

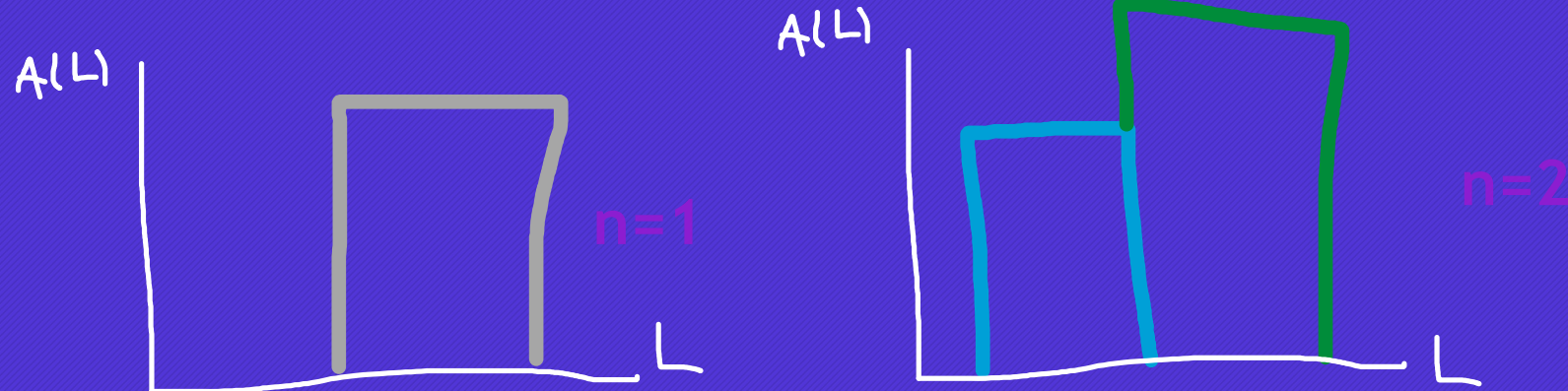
where H_i contain different matter effects in the form:

$$A_i = \sqrt{2}G_F N_e \rightarrow \text{constant}$$

Recalling 3-Flavor Neutrino Oscillations with charge-current matter effects



In principle, one could consider an n multistep matter potential $A = \sum_{i=1}^n A_i$ where each step is itself a constant matter potential (piecewise constant).





Defining time invariance measures

To handle the difficulties of testing for time invariance violation effects in experiments, we've specified two distinct measures:

1. proper time invariance: (aka: true time invariance violation)
 - Not a good observable, but certainly no harder to calculate than CP conjugate channels.
 - Requires comparing probabilities with final states exchanged and swapping the detector with source.

Defining time invariance measures



1. proper time invariance: (ex 2 steps)

(decreasing potential)

(increasing potential)



$$P_{\nu_e \rightarrow \nu_\mu}$$

for $U = U_1 U_2$
(decreasing potential)

vs

$$P_{\nu_\mu \rightarrow \nu_e}$$

for $U = U_2 U_1$
(increasing potential)

Defining time invariance measures



2. improper time invariance: (next best thing!)

- Compares probabilities with only the final states exchanged.

ex 2 steps:
(same potential)

$$P_{\nu_e \rightarrow \nu_\mu}$$
$$U = U_2 U_1$$

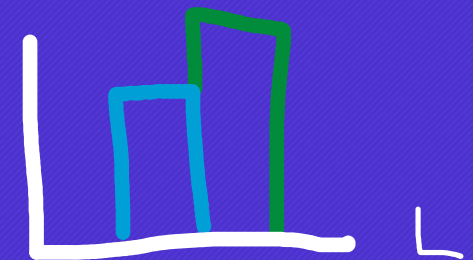
(increasing potential)

vs

$$P_{\nu_\mu \rightarrow \nu_e}$$
$$U = U_2 U_1$$

(increasing potential)

$A(L)$



Defining time invariance measures



2. improper time invariance: (next best thing!)

- Compares probabilities with only the final states exchanged.

Improper time invariance violation has the potential to be observable in experiments.

ex 2 steps:
(same potential)

$$P_{\nu_e \rightarrow \nu_\mu}$$
$$U = U_2 U_1$$

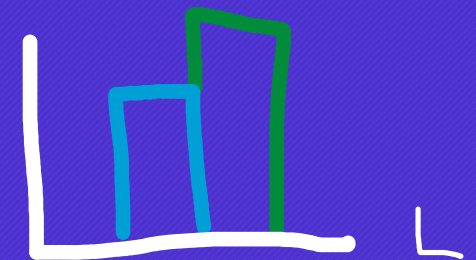
(increasing potential)

vs

$$P_{\nu_\mu \rightarrow \nu_e}$$
$$U = U_2 U_1$$

(increasing potential)

$A(L)$





Modeling Matter Effects for 3-Flavors

- ✓ To make reasonable predictions that includes the role matter effects play in time invariance probes, it's important to keep in mind the characteristics of the matter potentials, as *these influence our results*.
- ✓ For the purposes of our study, we separately considered both **symmetric** and **non-symmetric** matter potential examples.

Modeling Matter Effects for 3-Flavors

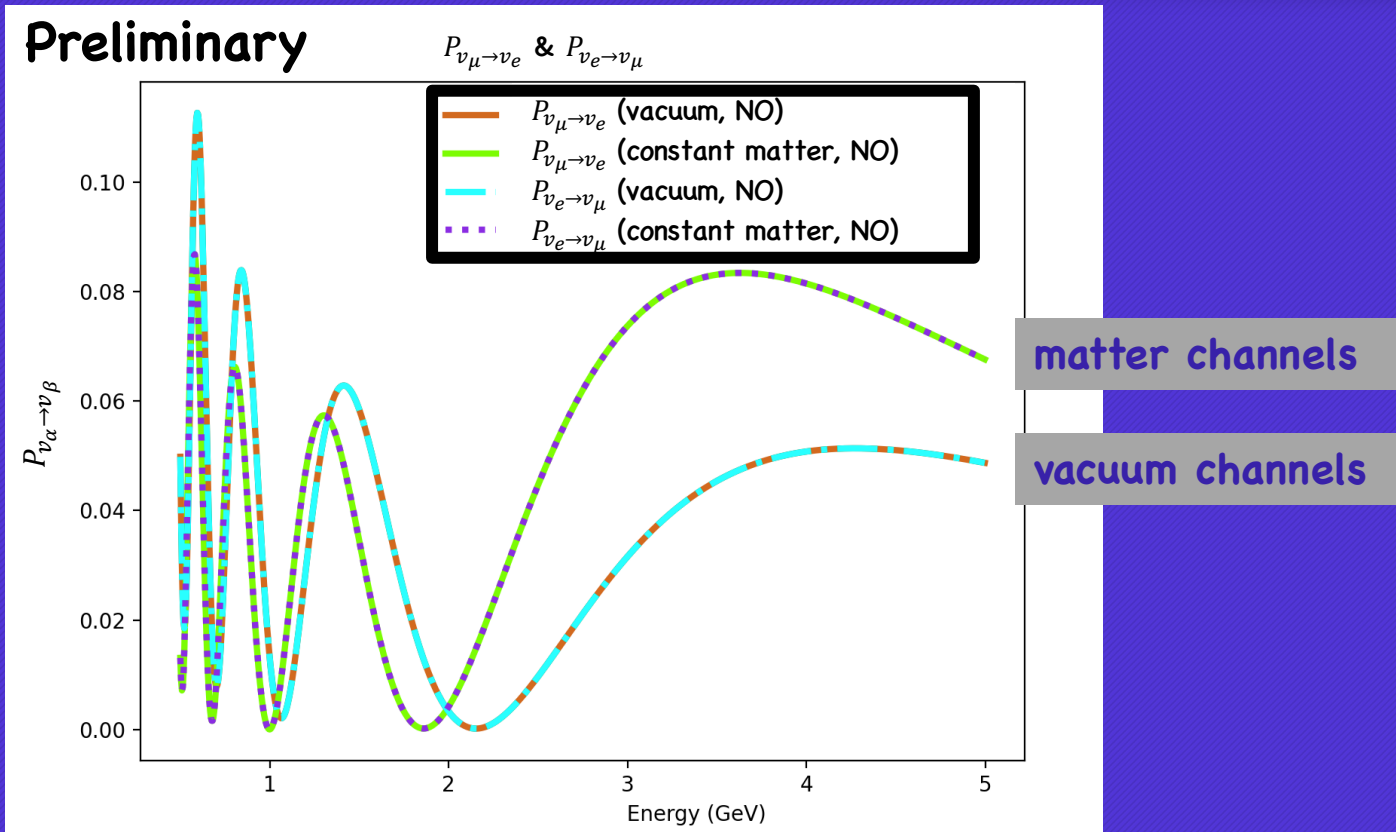


The type of matter potential (symmetric/non-symmetric) provides a quick way to prove if there is matter induced time violation.

Symmetric: vacuum or single constant matter potential

Non-symmetric: piece-wise matter potentials (increasing or decreasing)

Symmetric matter effects for 3-Flavors



$P_{\nu_{\mu \rightarrow \nu_e}}$ & $P_{\nu_{e \rightarrow \nu_{\mu}}}$ channels:

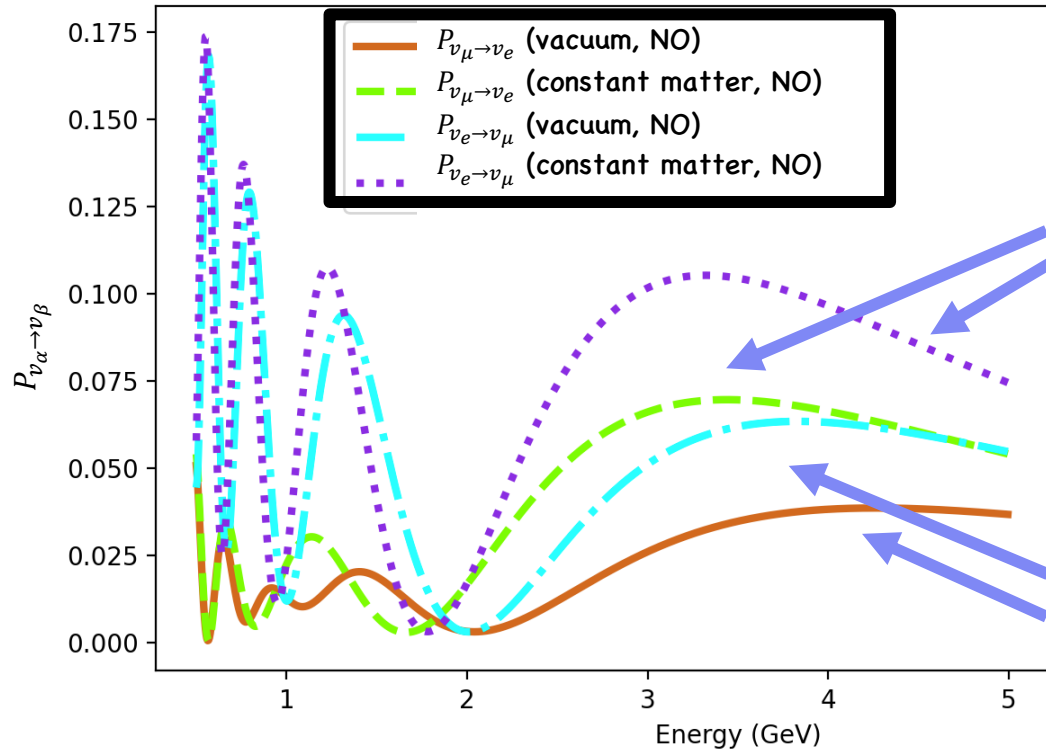
- ✓ Baseline: 2000 km
- ✓ Constant matter potential:
 $5 \times 10^{-4} \text{ eV}^2/\text{GeV}$
(2.6 g/cm^3)
- ✓ $\delta_{CP}: 0$
- ✓ Normal Ordering

Symmetric matter effects for 3-Flavors



Preliminary

$P_{\nu_{\mu} \rightarrow \nu_e}$ & $P_{\nu_e \rightarrow \nu_{\mu}}$



matter channels

vacuum channels

$P_{\nu_{\mu} \rightarrow \nu_e}$ & $P_{\nu_e \rightarrow \nu_{\mu}}$ channels:

✓ Baseline: 2000 km

✓ Constant matter potential:
 $5 \times 10^{-4} \text{ eV}^2/\text{GeV}$
 (2.6 g/cm^3)

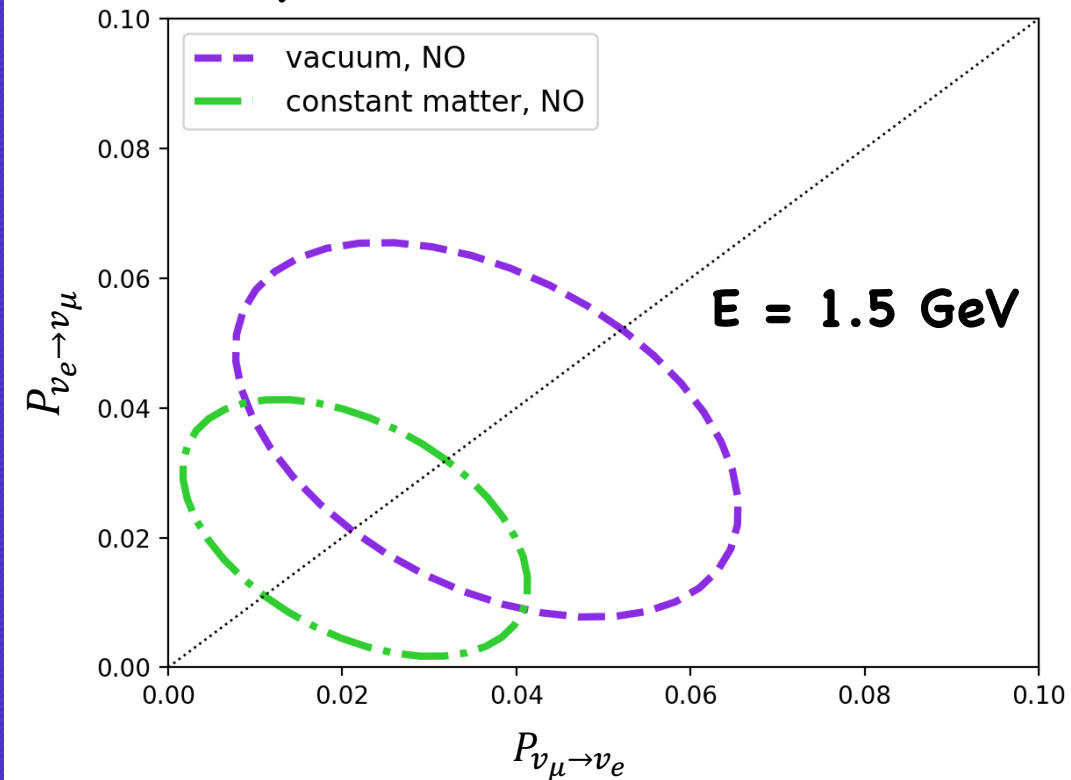
✓ $\delta_{CP} : \frac{\pi}{2}$

✓ Normal Ordering

Symmetric matter effects for 3-Flavors



Preliminary

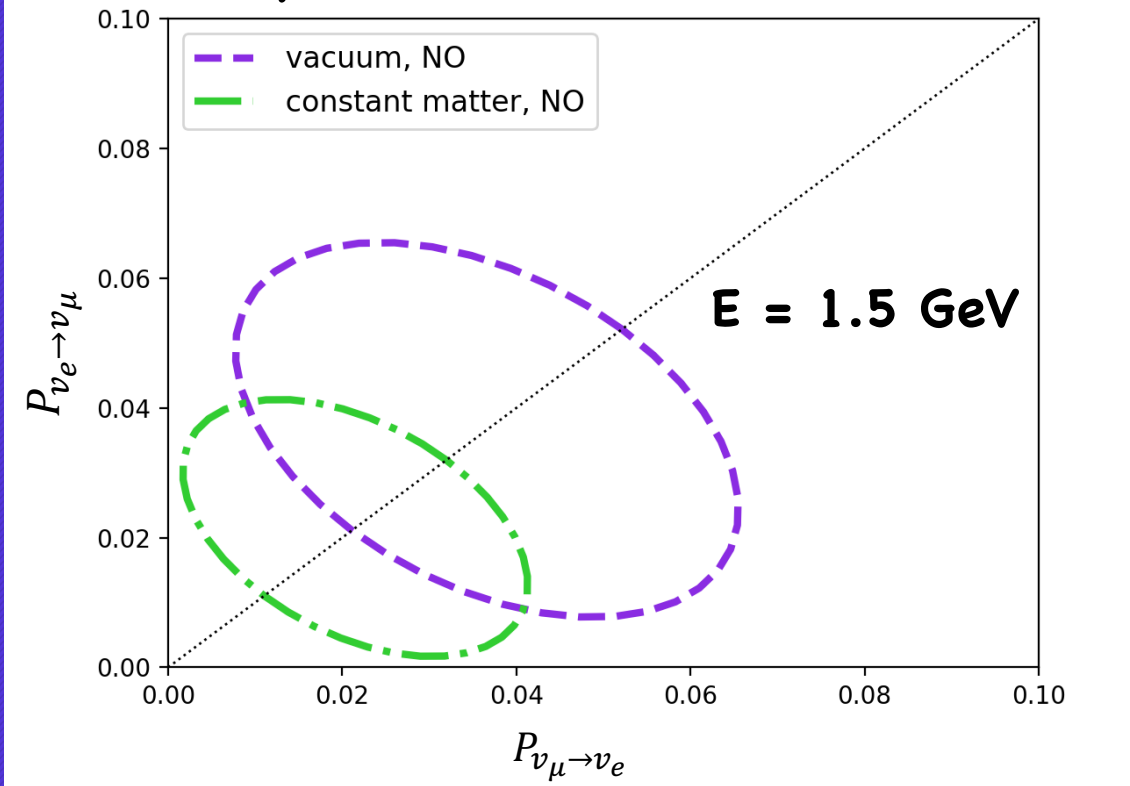


- ✓ Symmetric matter potentials *cannot induce time violation.*
- ✓ If there is intrinsic $P_{\nu_{\mu} \rightarrow \nu_e} \neq P_{\nu_e \rightarrow \nu_{\mu}}$ from δ_{CP} , then the matter potential simply *changes the degree of the observed effects.*

Symmetric matter effects for 3-Flavors



Preliminary



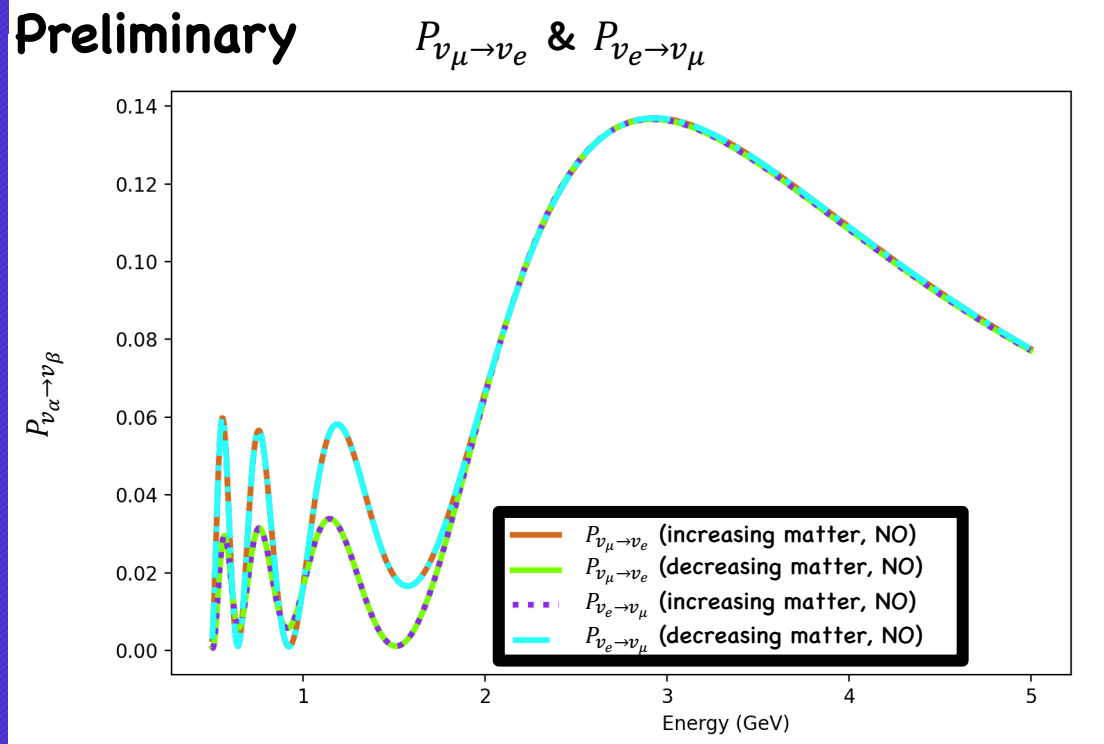
✓ Note: **Improper and proper comparisons are the same if the matter potential is symmetric.**

✓ *Why?*

→ A single constant matter potential is by construction symmetric.

→ We cannot tell the two measures apart if "exchanging source and detector" gives the **same results.**

Non-symmetric matter effects for 3-Flavors



$P_{\nu_{\mu} \rightarrow \nu_e}$ & $P_{\nu_e \rightarrow \nu_{\mu}}$ channels:

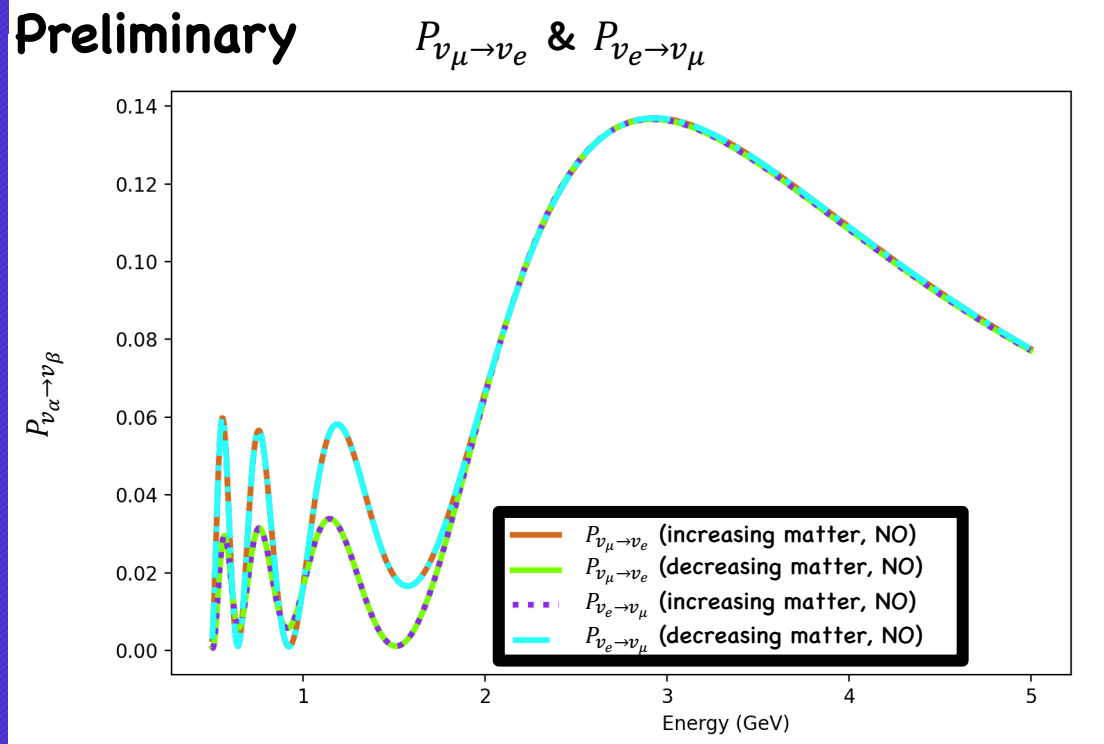
✓ Baseline: 2000 km

✓ Piecewise matter potential (2 steps):
 $A_1 = 5 \times 10^{-4} \text{ eV}^2/\text{GeV}$ (2.6 g/cm³)
 $A_2 = 1.5 \times 10^{-3} \text{ eV}^2/\text{GeV}$ (7.8 g/cm³)

✓ $\delta_{CP} : 0$

✓ Normal Ordering

Non-symmetric matter effects for 3-Flavors



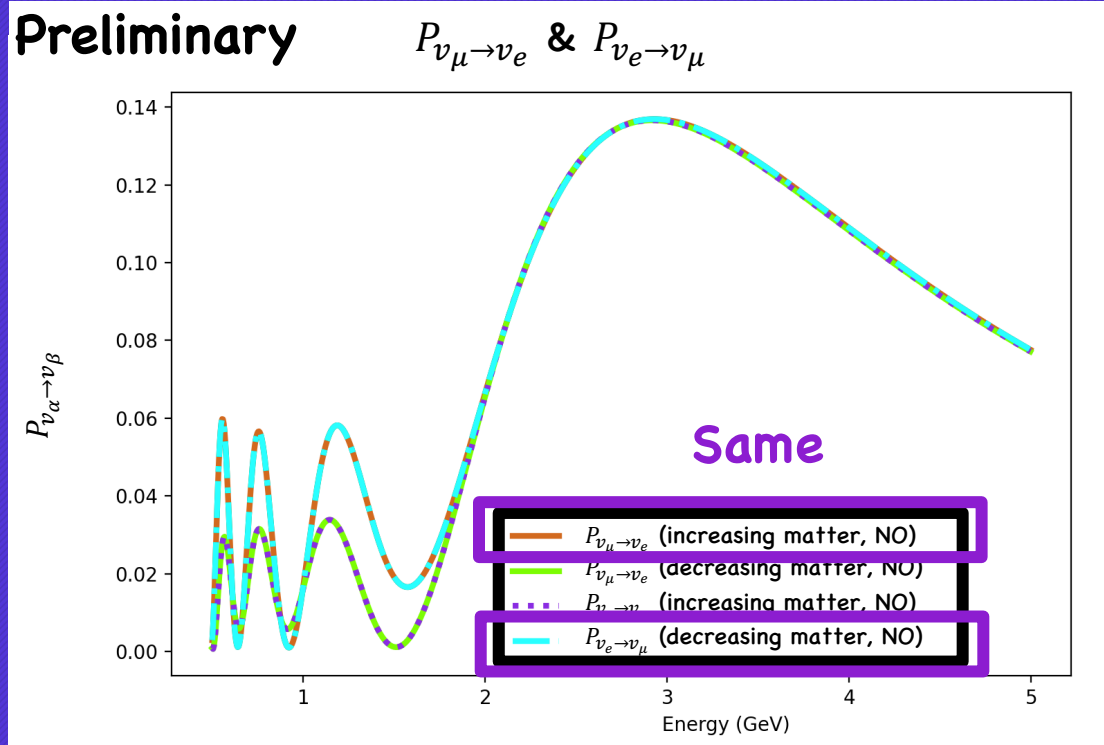
✓ Non-symmetric matter effects are pairwise degenerate ($\delta_{CP}: 0$)

→ Proper time invariance channels hold
→ Improper are different

No intrinsic time invariance violation!

But matter induced time invariance violation can occur.

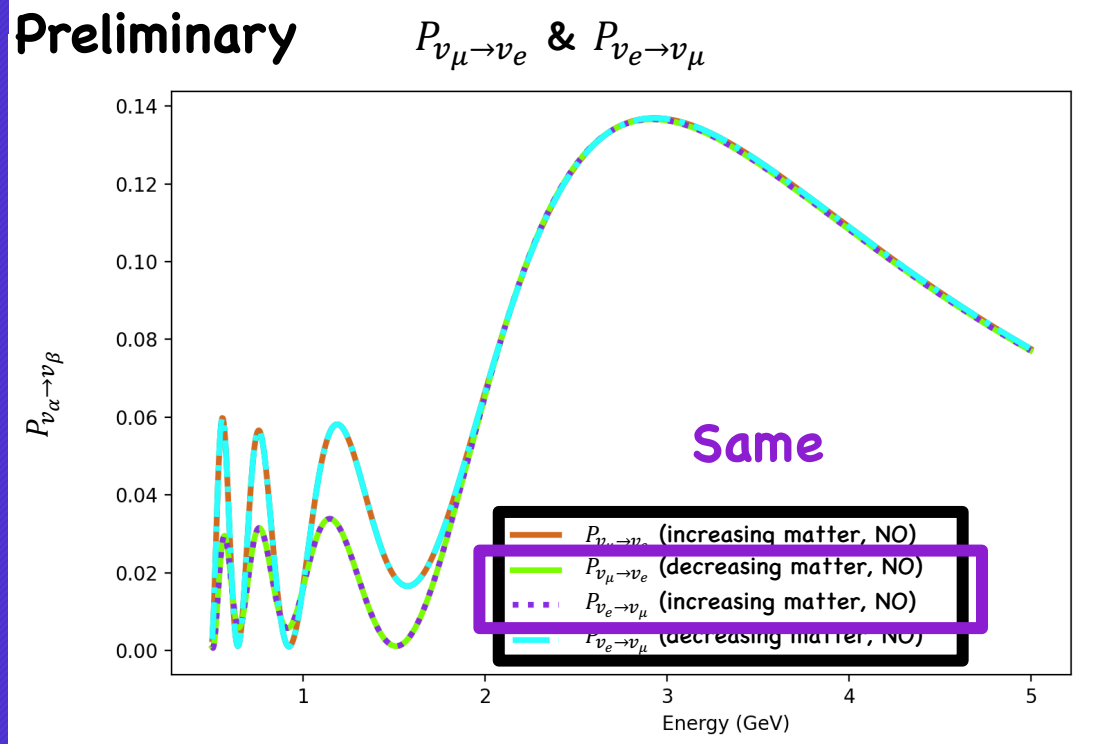
Non-symmetric matter effects for 3-Flavors



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- Proper time invariance channels hold
- Improper are different

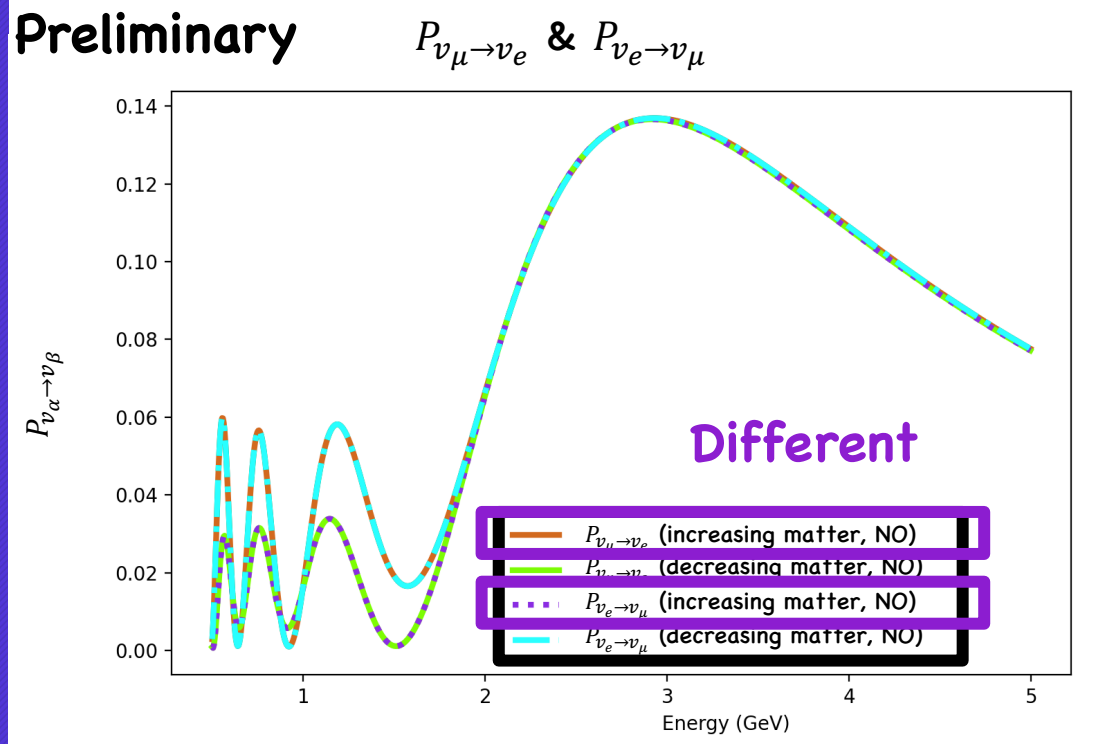
Non-symmetric matter effects for 3-Flavors



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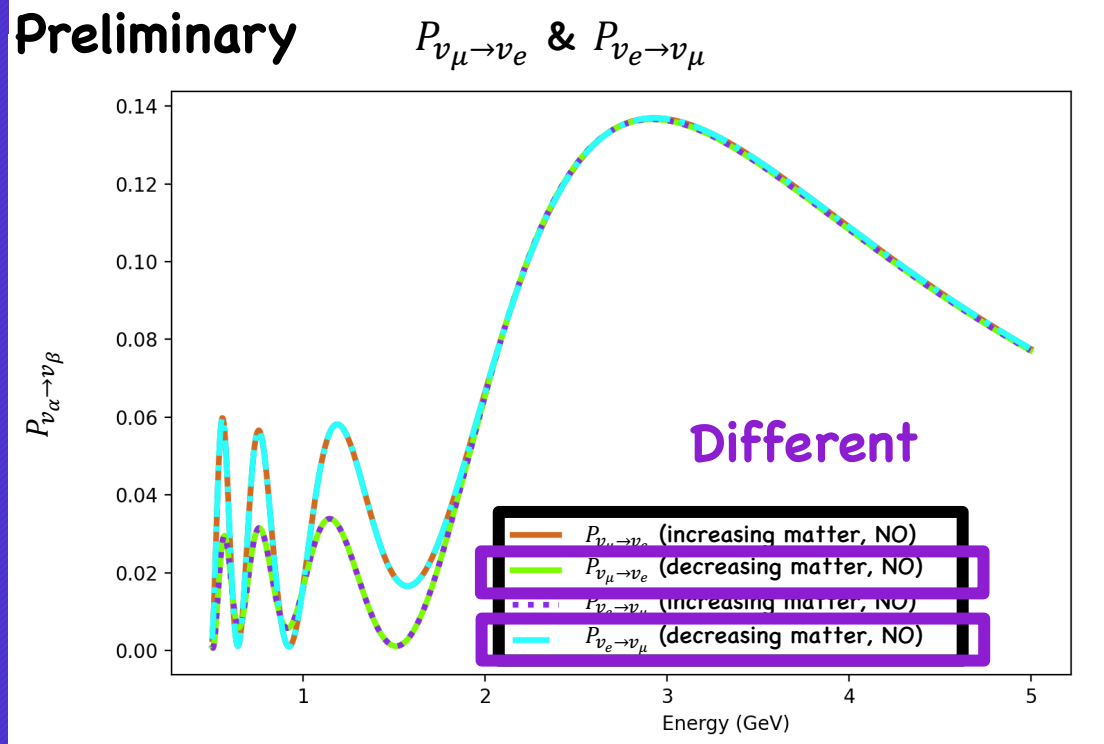
Non-symmetric matter effects for 3-Flavors



✓ Non-symmetric matter effects are pairwise degenerate ($\delta_{CP}: 0$)

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Non-symmetric matter effects for 3-Flavors



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Non-symmetric matter effects for 3-Flavors



$P_{\nu_{\mu} \rightarrow \nu_e}$ & $P_{\nu_e \rightarrow \nu_{\mu}}$ channels:

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✓ Piecewise matter potential (2 steps):

$$A_1 = 5 \times 10^{-4} \text{ eV}^2/\text{GeV}$$

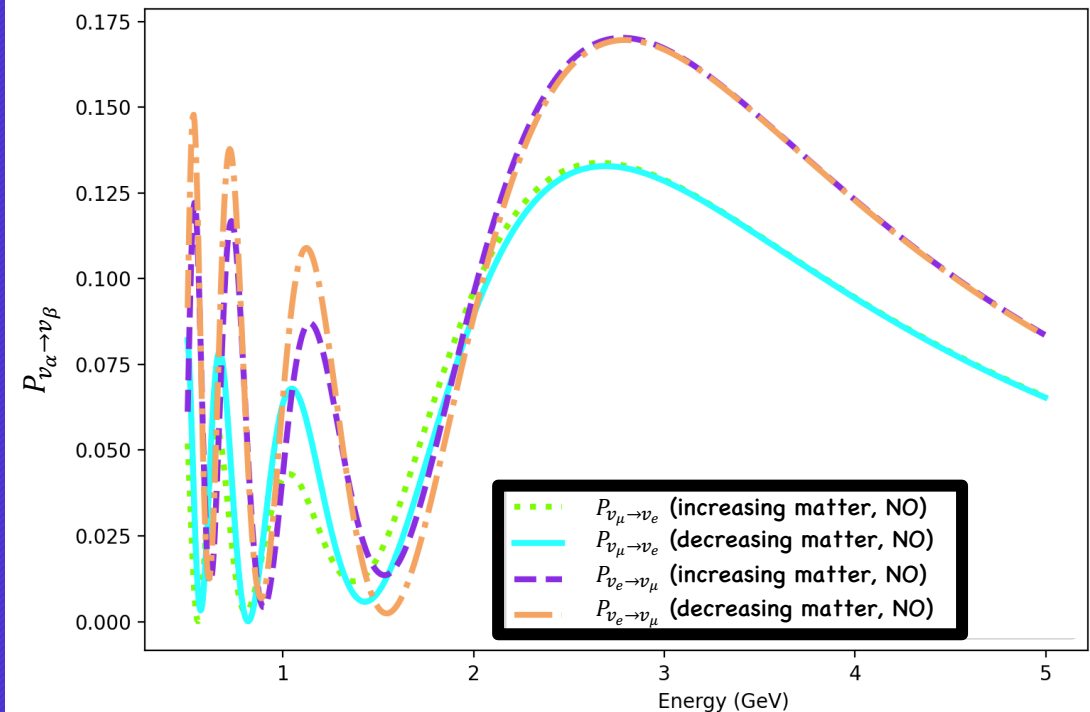
$$A_2 = 1.5 \times 10^{-3} \text{ eV}^2/\text{GeV}$$

✓ $\delta_{CP} : \frac{\pi}{2}$

✓ Normal Ordering

Preliminary

$P_{\nu_{\mu} \rightarrow \nu_e}$ & $P_{\nu_e \rightarrow \nu_{\mu}}$

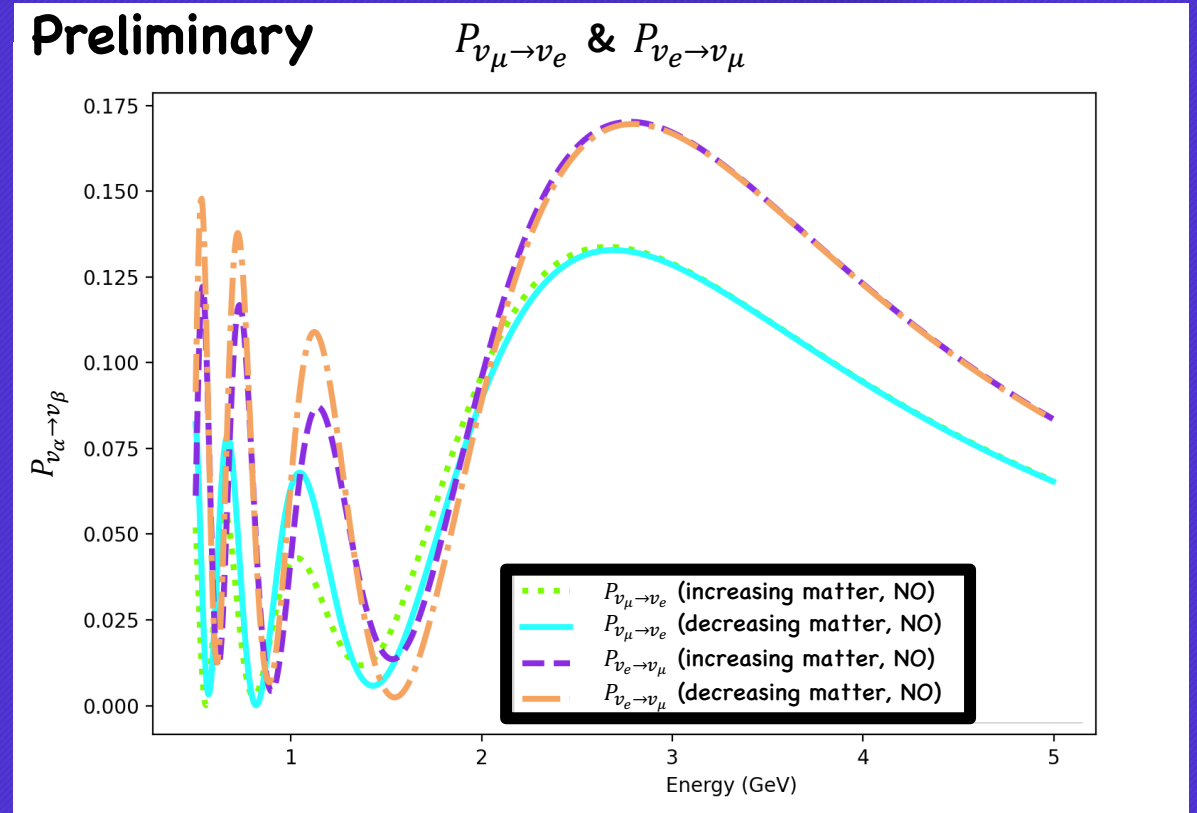


$\alpha, \beta \rightarrow [\mu, e]$

Non-symmetric matter effects for 3-Flavors



- ✓ Non-symmetric matter effects ($\delta_{CP}: \frac{\pi}{2}$)
- ✓ No longer pair-wise degenerate.
(Intrinsic $P_{\nu_{\mu} \rightarrow \nu_e} \neq P_{\nu_e \rightarrow \nu_{\mu}}$)



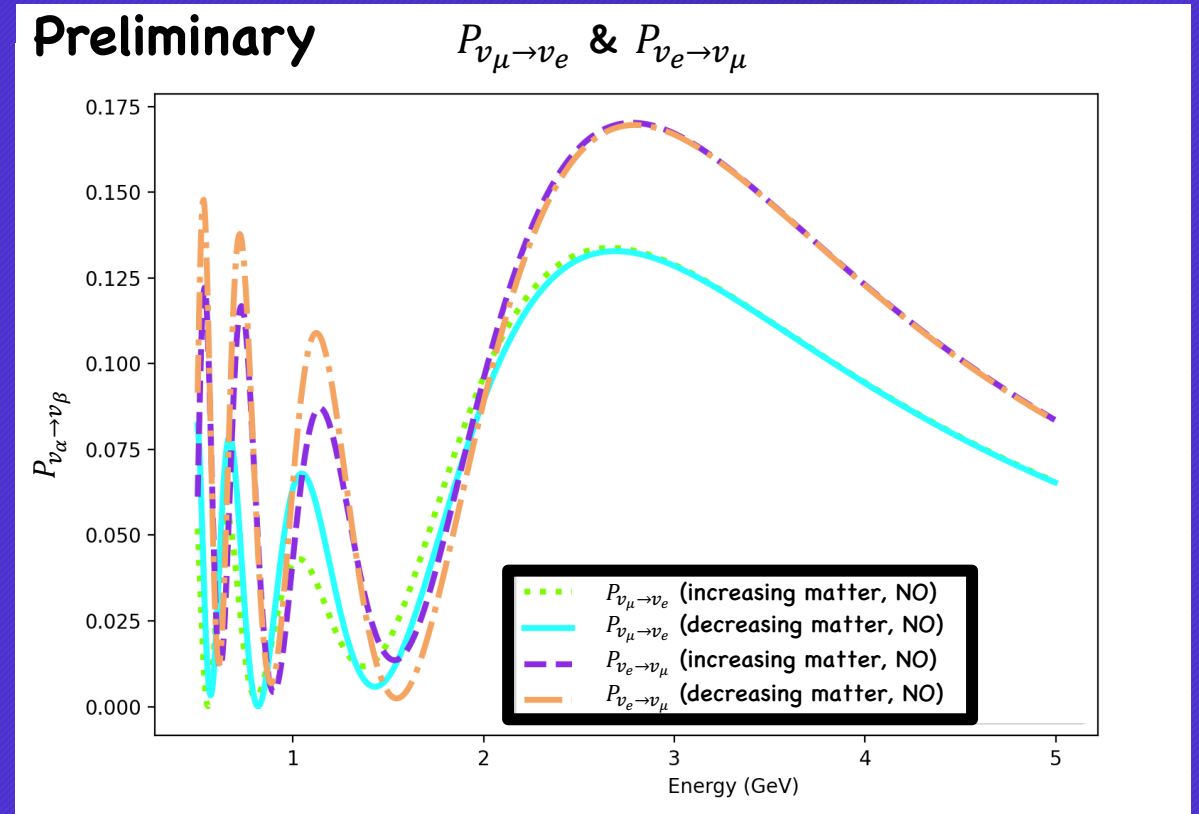
$\alpha, \beta \rightarrow [\mu, e]$

Non-symmetric matter effects for 3-Flavors



- ✓ Non-symmetric matter effects ($\delta_{CP}: \frac{\pi}{2}$)
- ✓ No longer pair-wise degenerate. (Intrinsic $P_{\nu_{\mu} \rightarrow \nu_e} \neq P_{\nu_e \rightarrow \nu_{\mu}}$)
- ✓ Matter potential choice changes magnitude of oscillations.

All probabilities are different.



$\alpha, \beta \rightarrow [\mu, e]$

Future Studies



- ✓ Interesting probes in cases where matter induced time violation occurs and/or realistic models are non-symmetric:
 - Center of the Earth (annihilating dark matter to neutrinos scenario)
 - Geo neutrinos (properties/applications)
- ✓ Next steps include *NSI time invariance* probes applicable to DUNE, (a follow-up to previous work).

