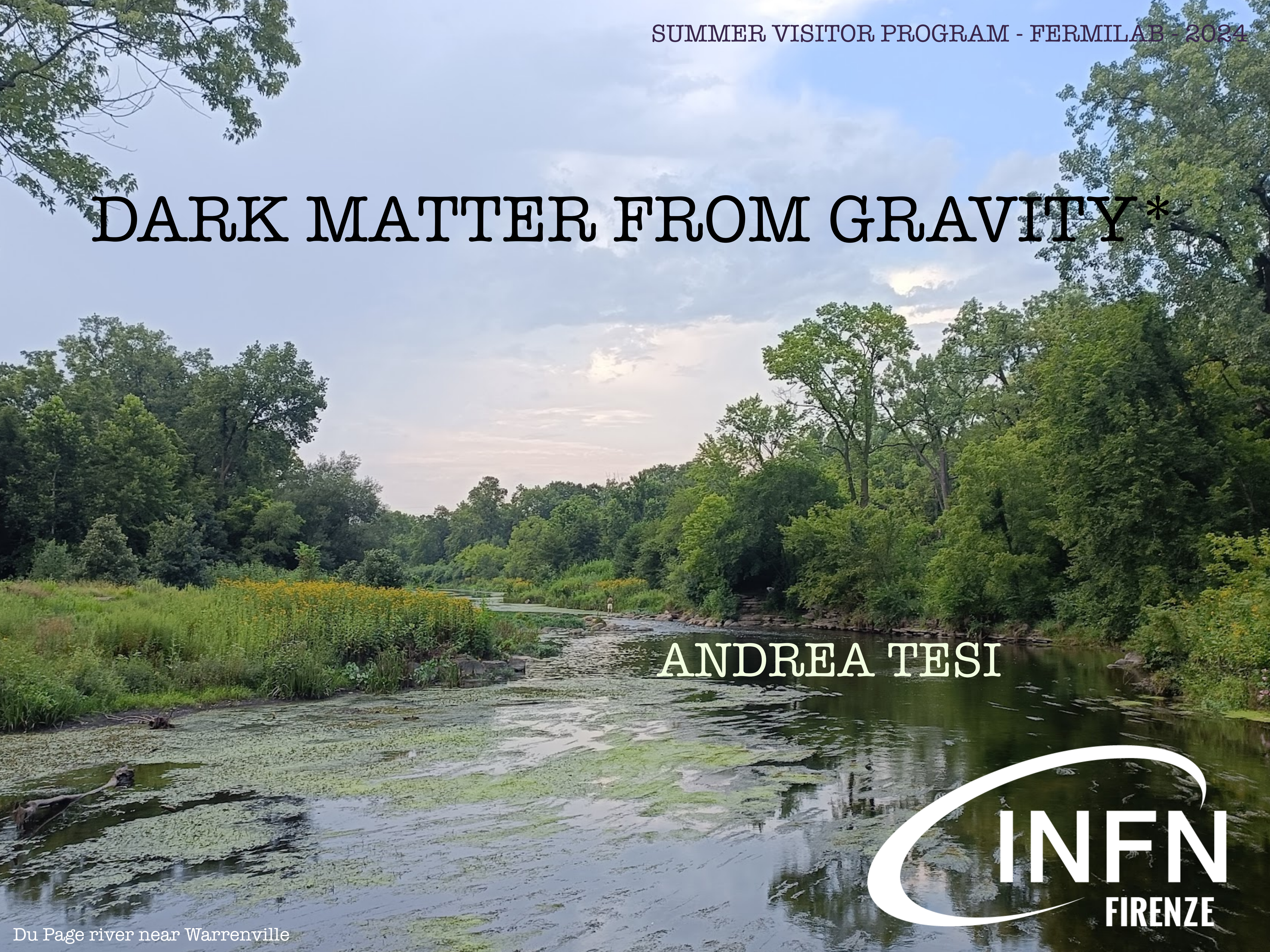
DARK MATTER FROM GRAVITY*

ANDREA TESI

INFN FIRENZE



Du Page river near Warrenville

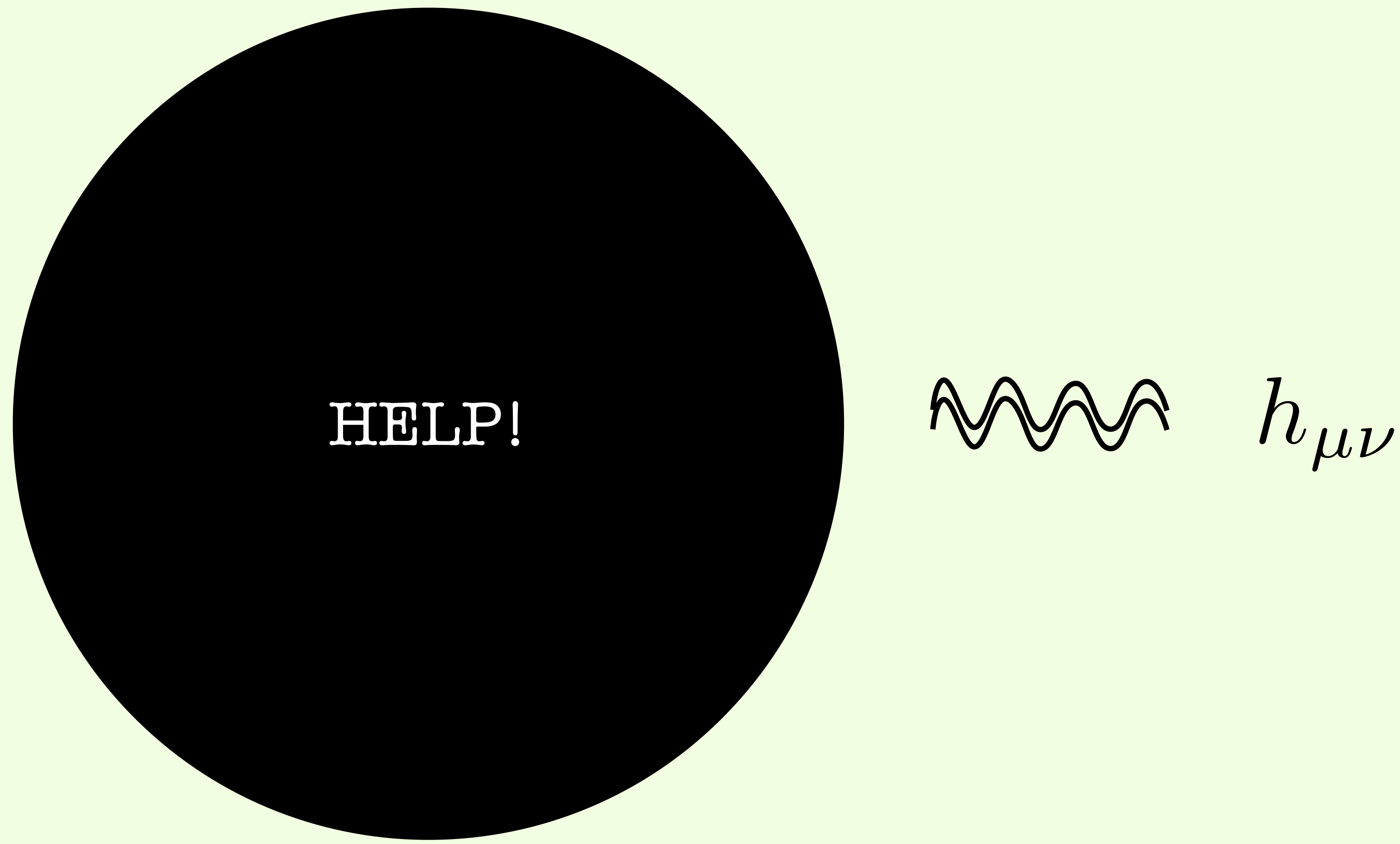
In this talk I’ll try to answer the following question:

How **dark** can be the **DM**?

in recent times this question has often shown up in conversations
with my collaborator **Michele Redi**

I will assume DM is only **gravitationally** coupled to us*

and see how far we can progress



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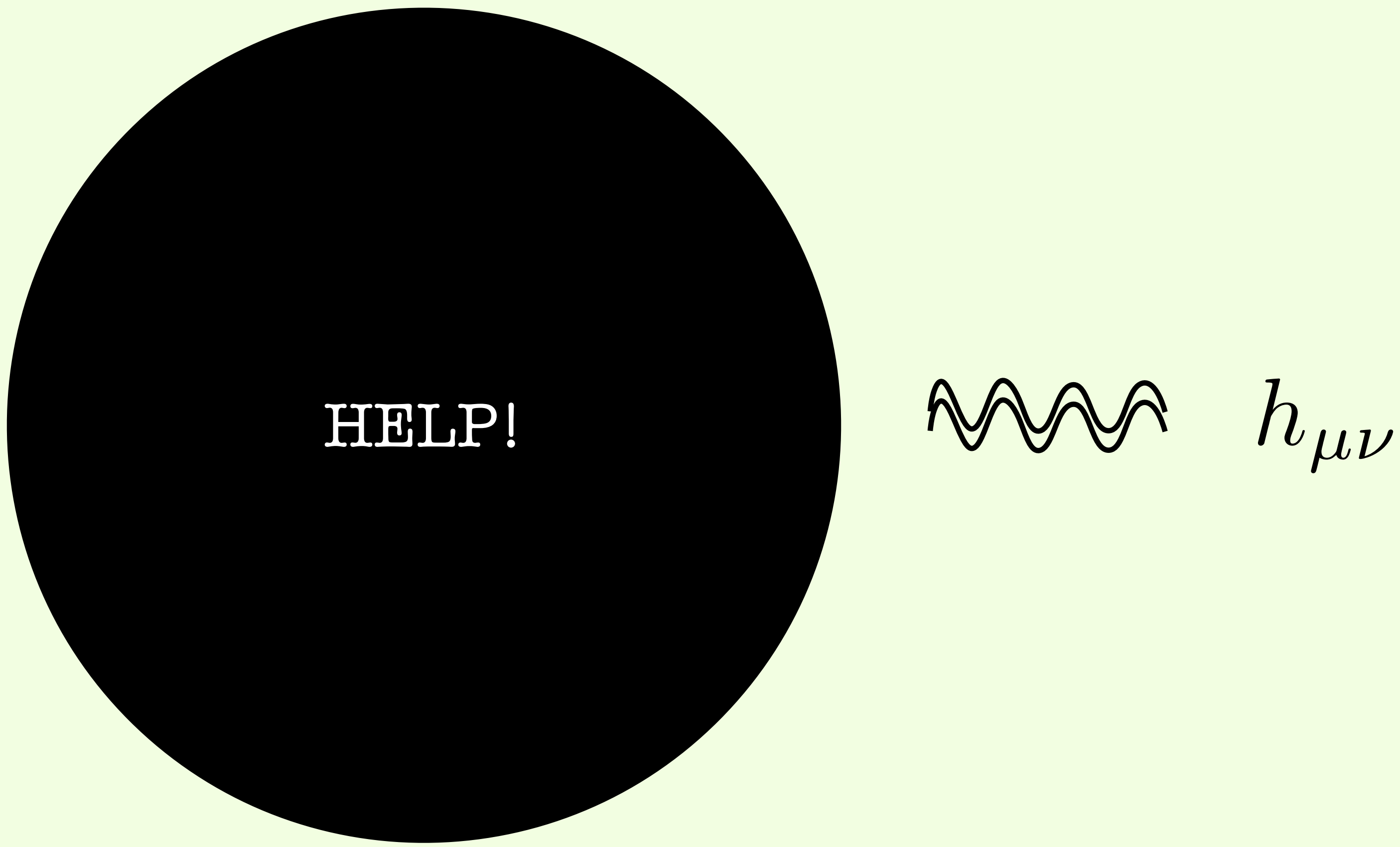

everything become very difficult buying this **assumption**

what about the **abundance**?

*a bit conservative, but it is what data are telling us

A real challenge: initial production

dark sector is **initially empty**, we focus on the '**jump start**' provided by the gravitational coupling to the SM background

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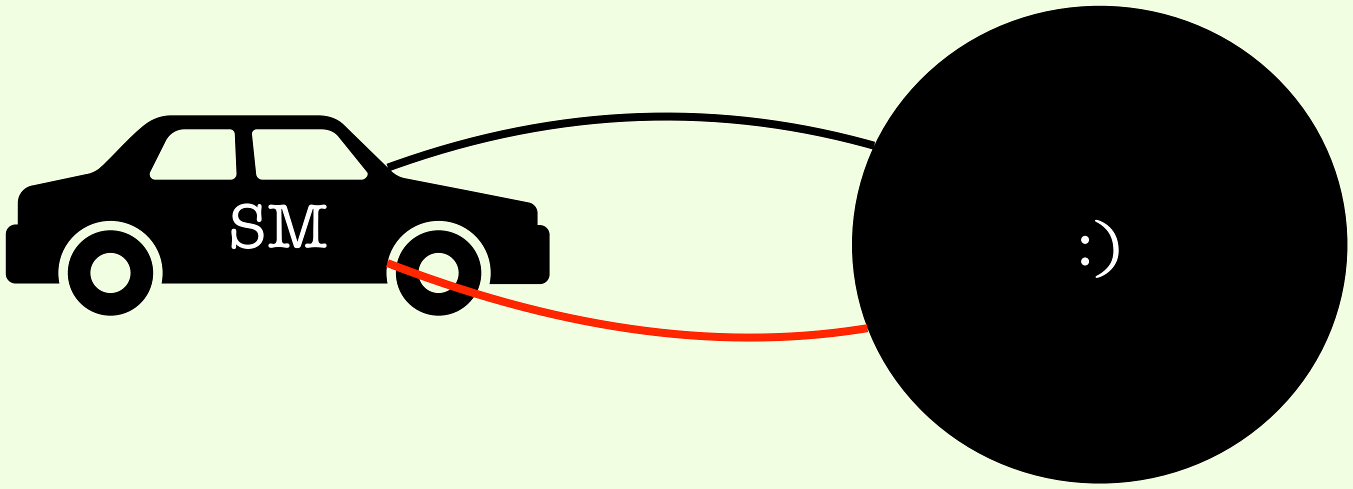

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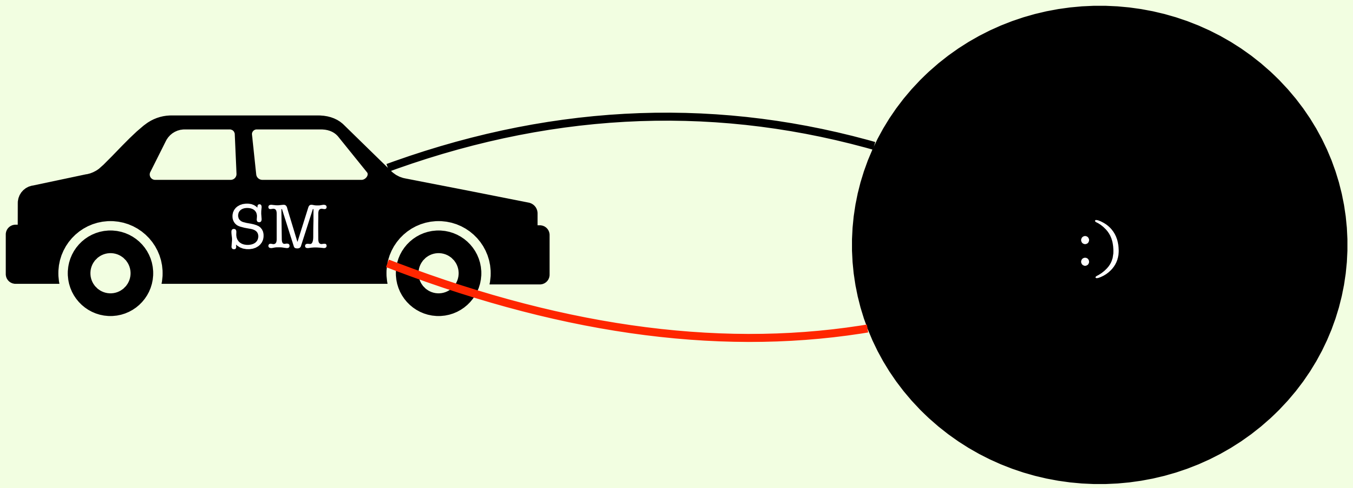

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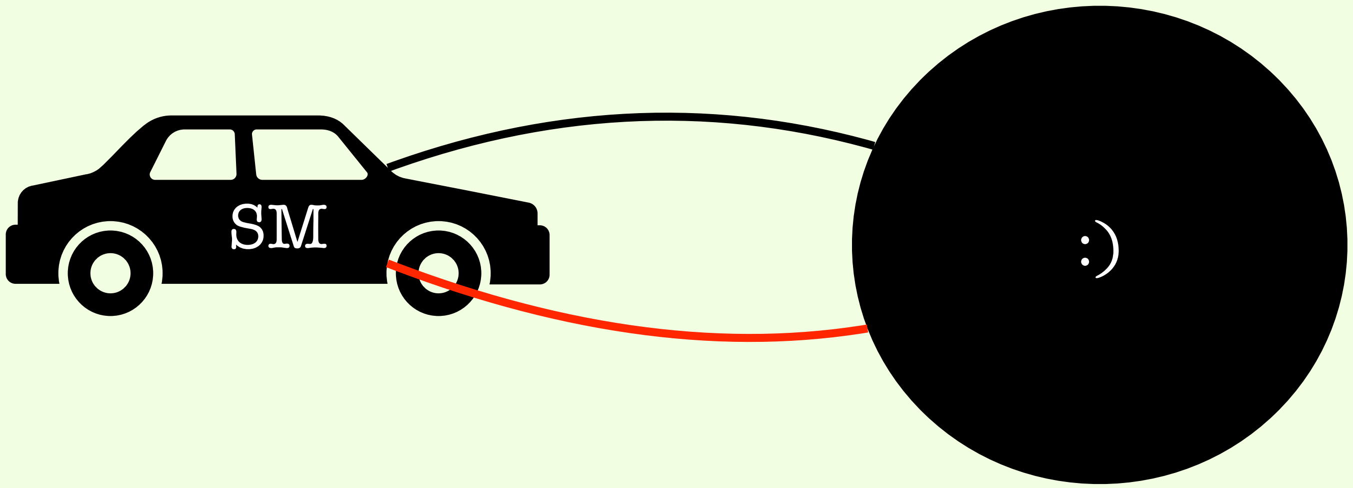

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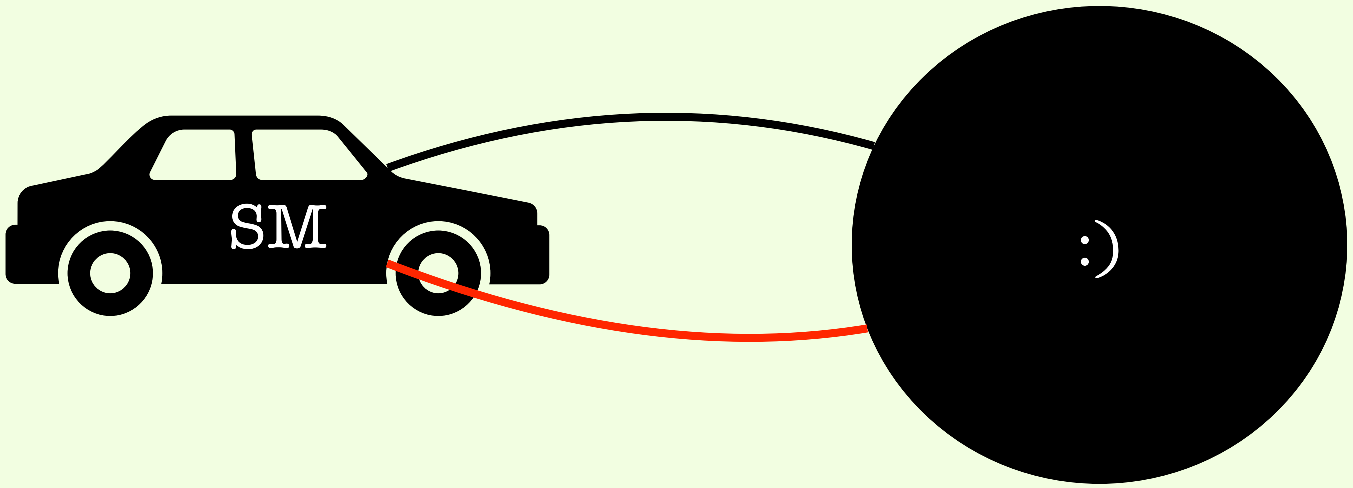

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- Focus on **particle DM**, not discussing PBHs

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two main sources of energy from the SM considered **so far**

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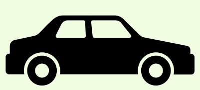

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freeze-in through gravity

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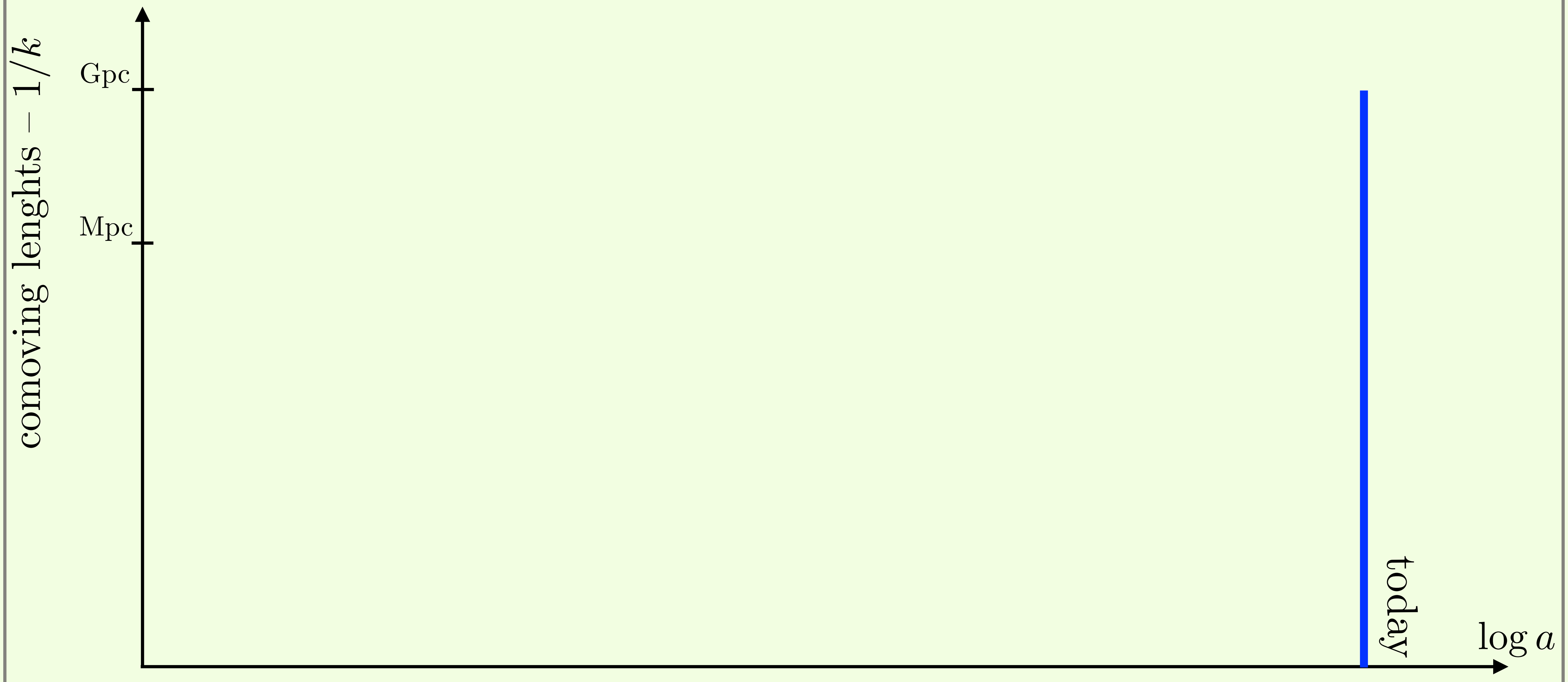
freeze-in through gravity

- The **FRW background** breaks time translation: can this energy non-conservation be used for the dark sector?

$$ds^2 = a^2(\tau)(d\tau^2 - dx^2)$$

particle production in curved space

I will exploit only the cosmological evolution



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don’t want to spoil BBN/REC nor the adiabatic fluctuations

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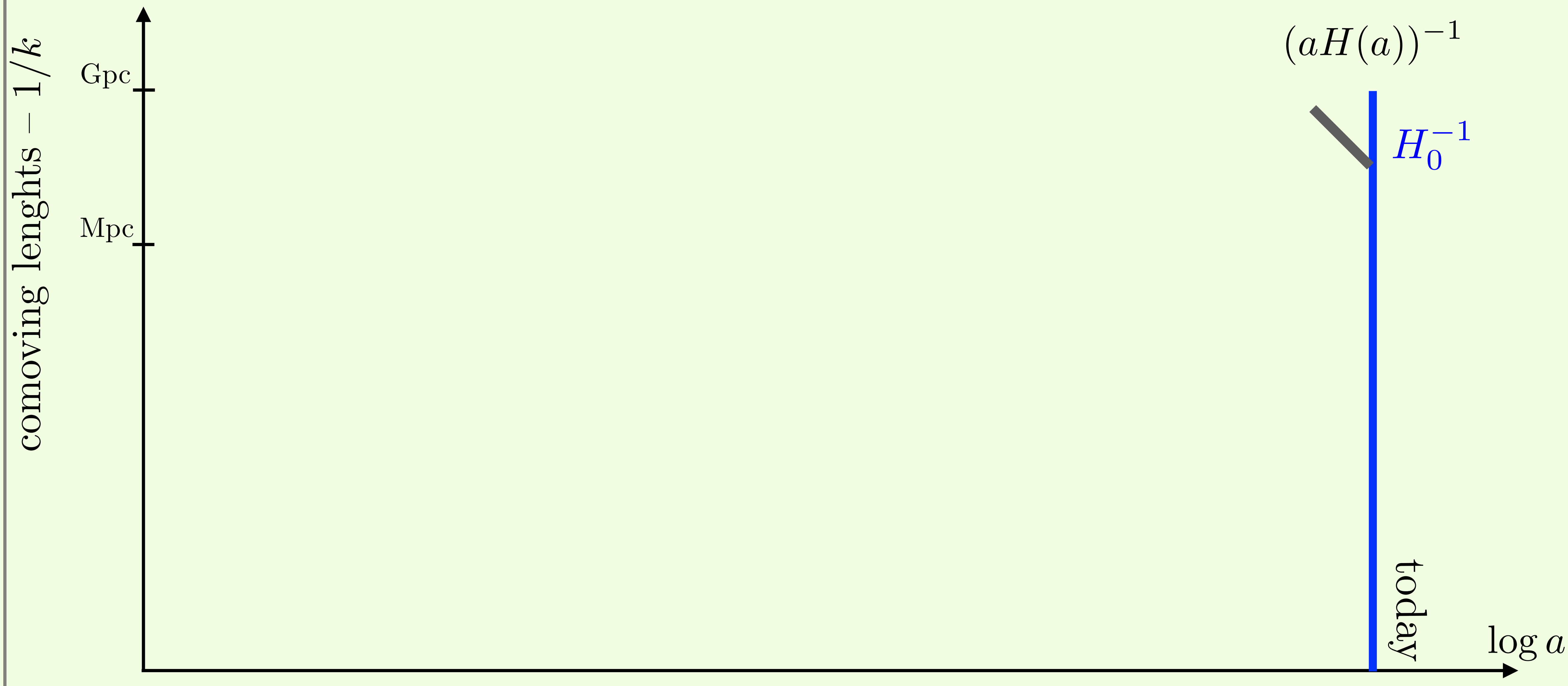

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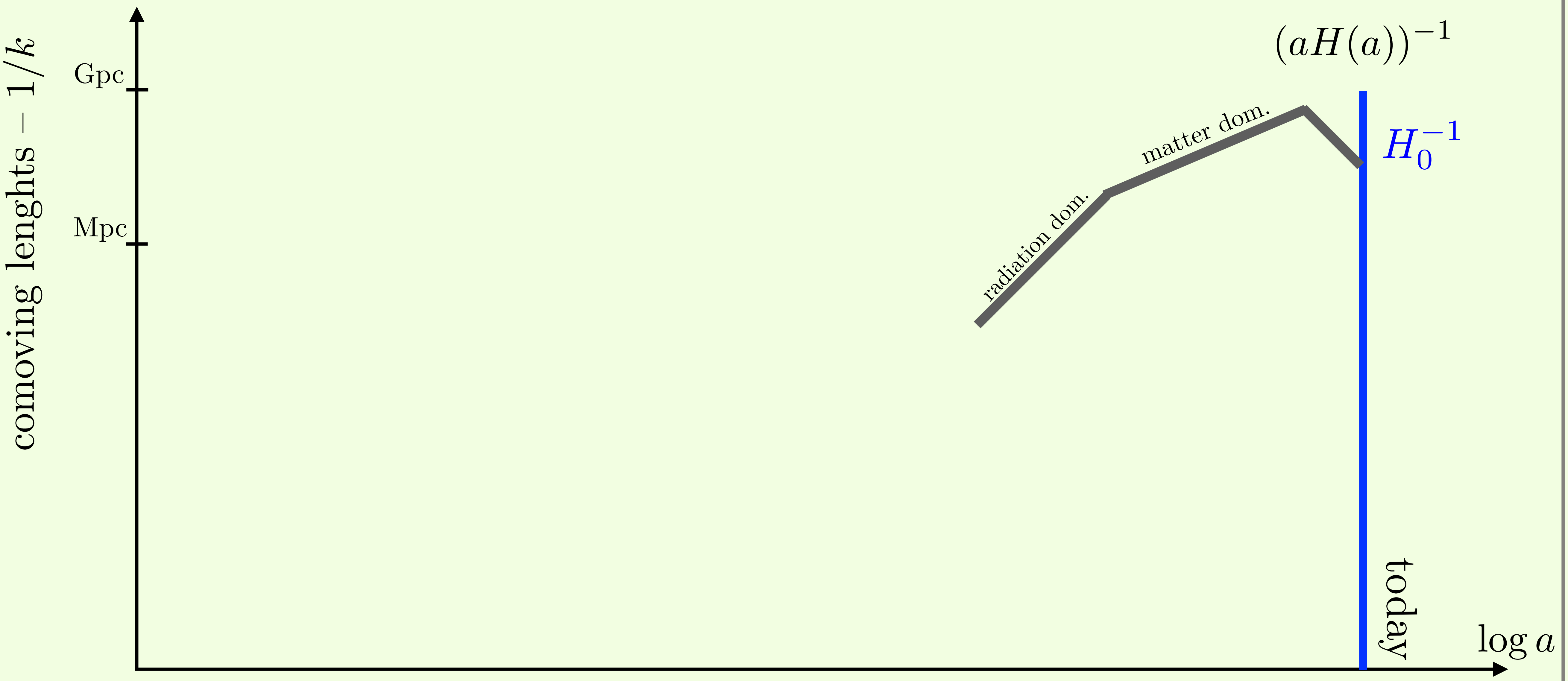

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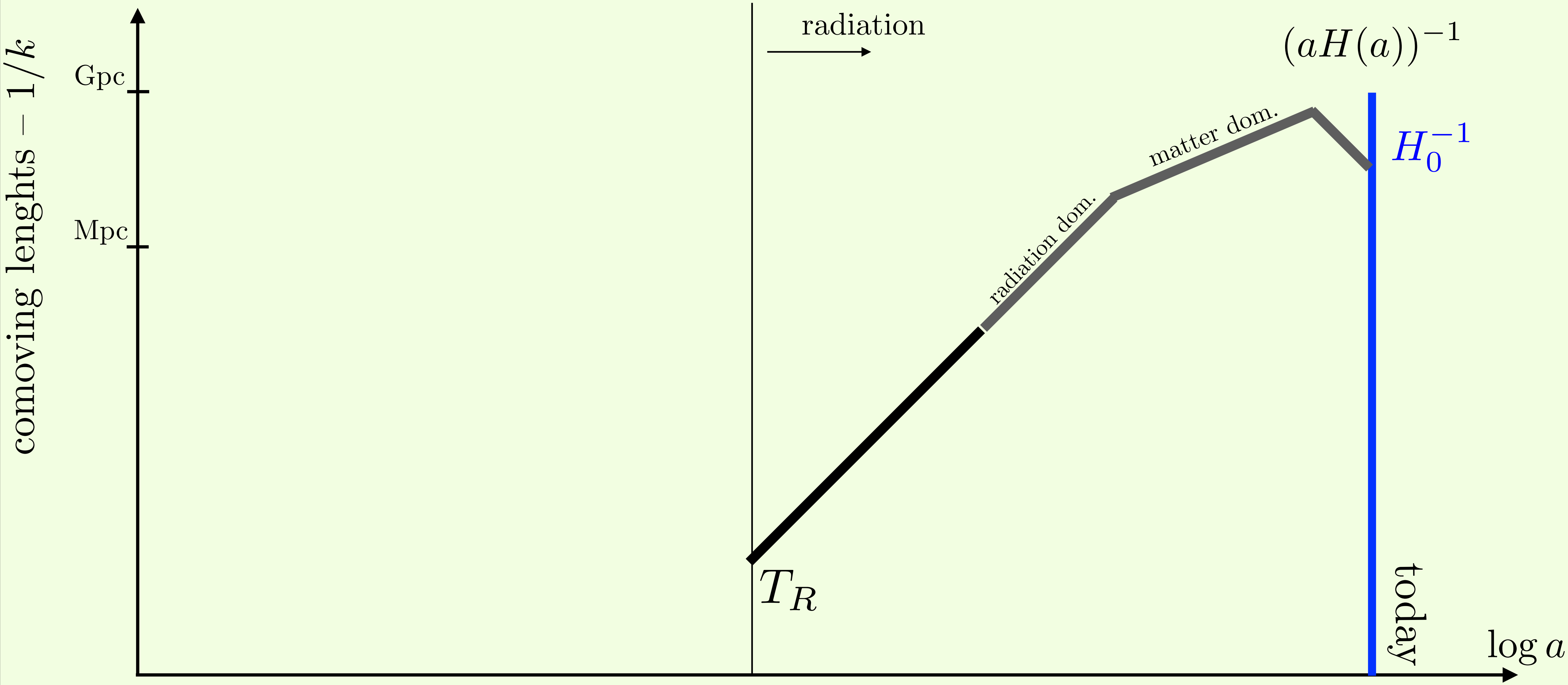

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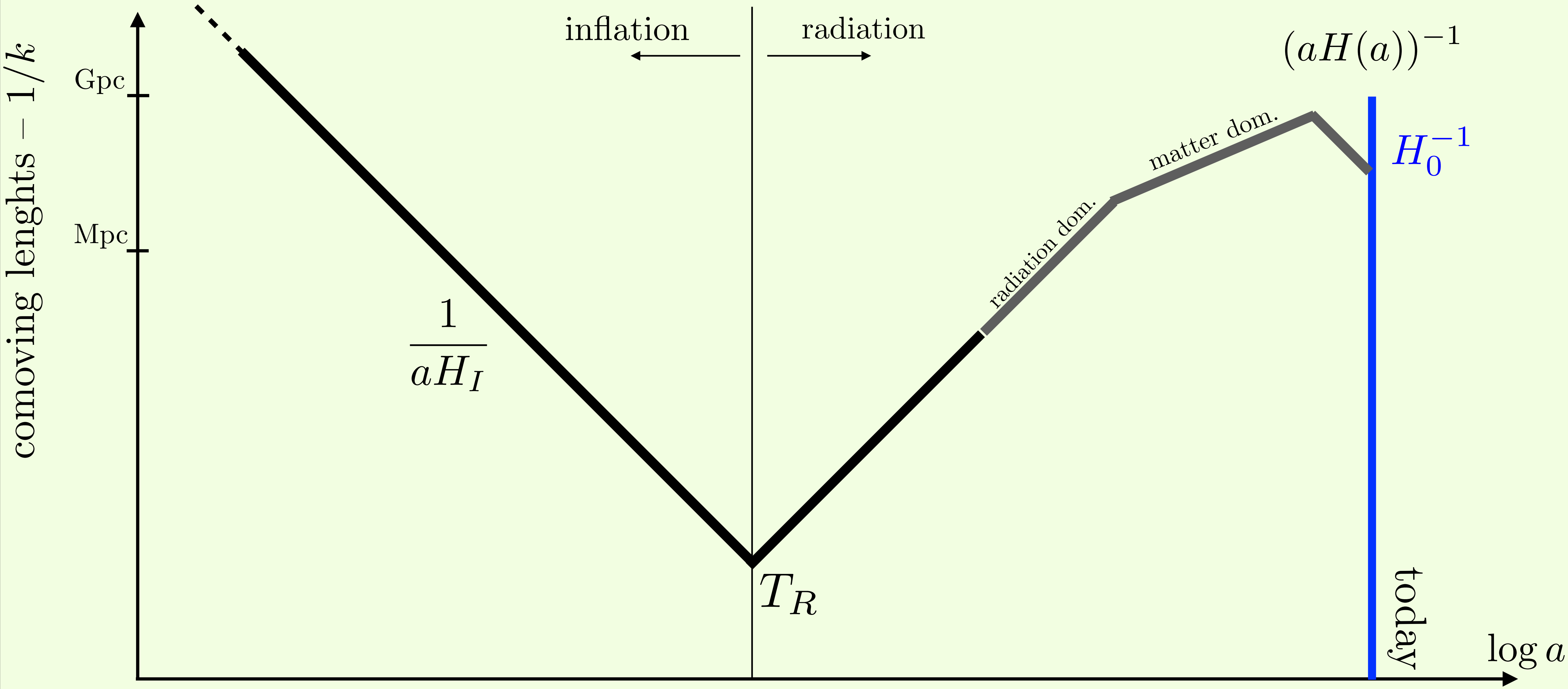

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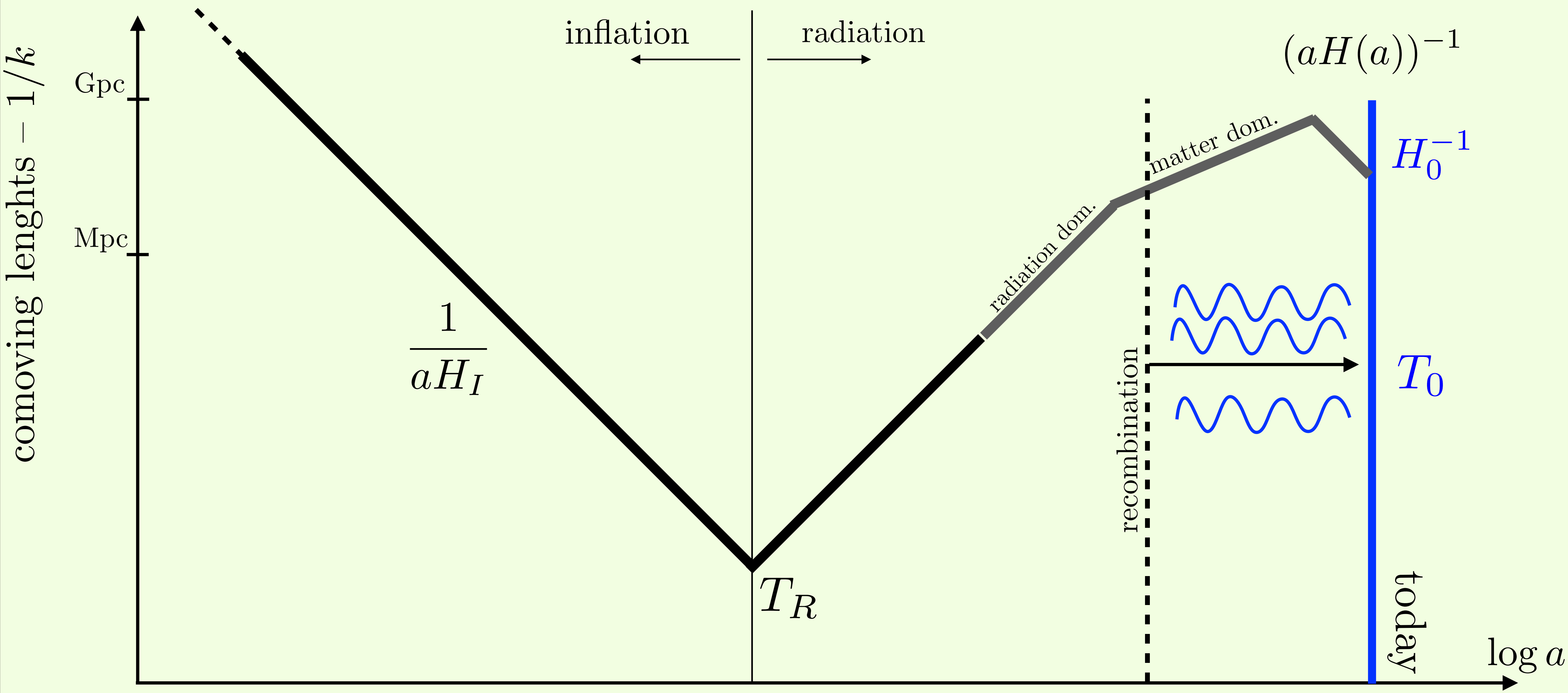

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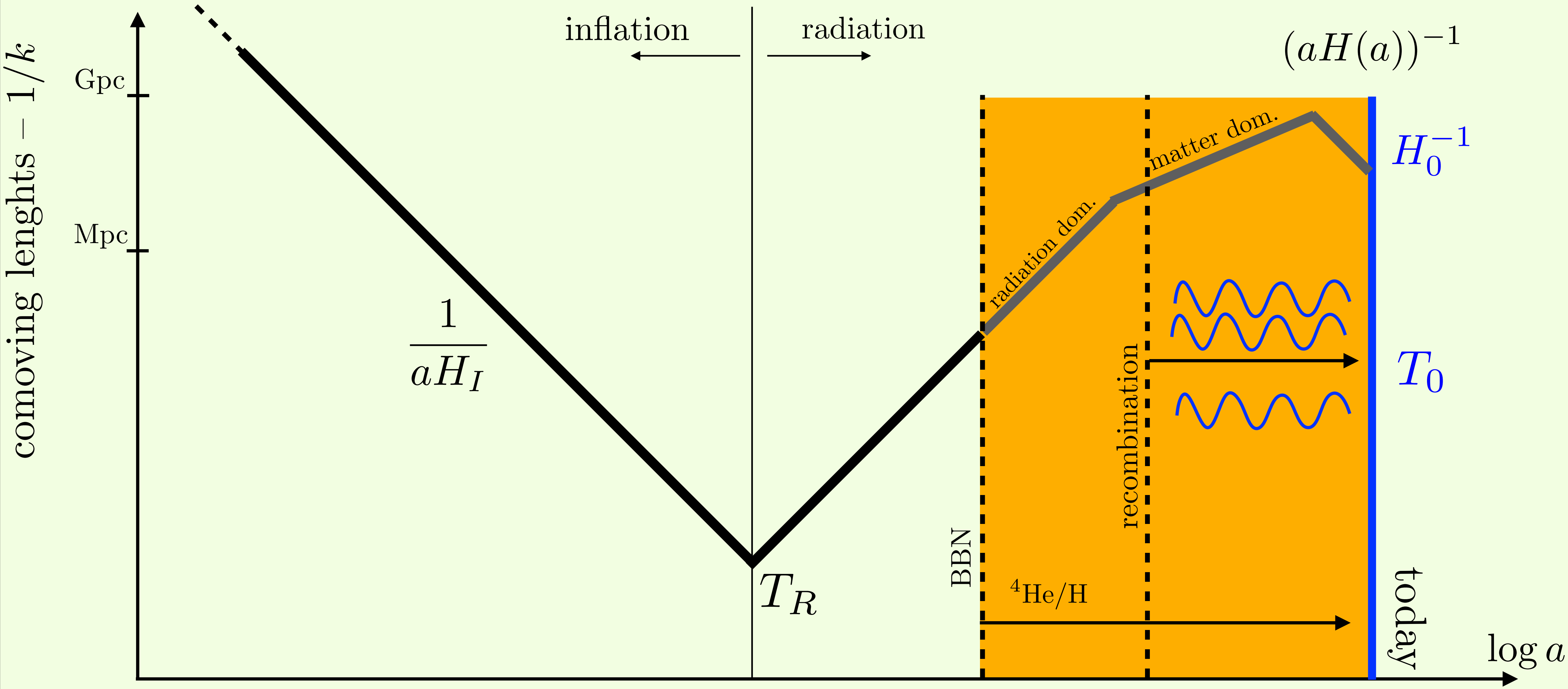

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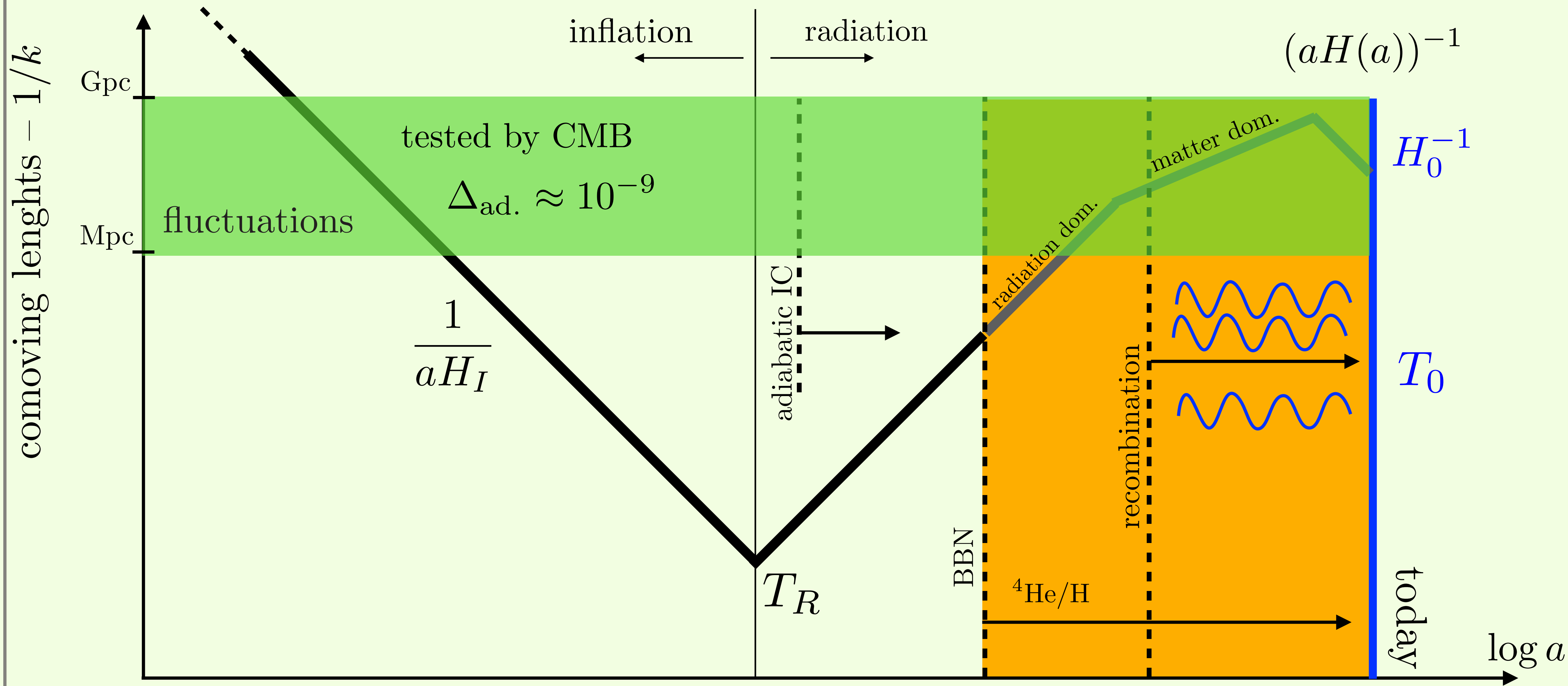

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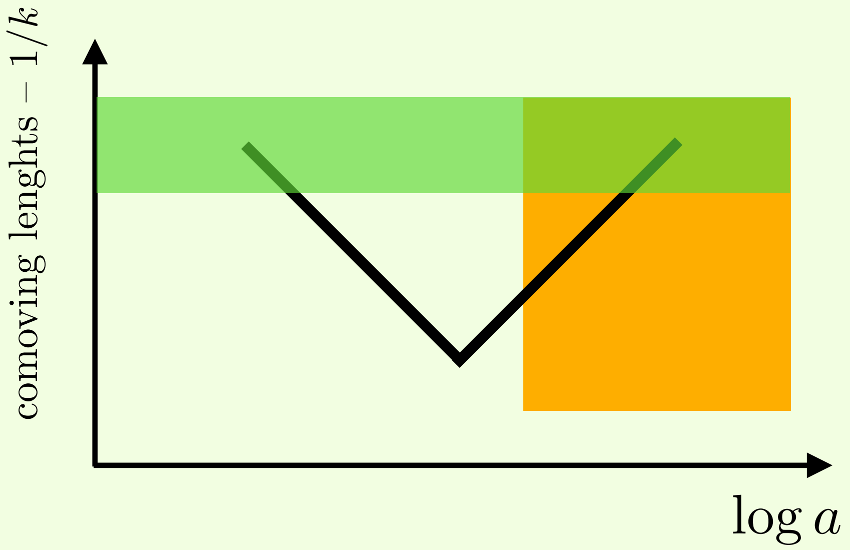
Where/when dark sectors can be produced?



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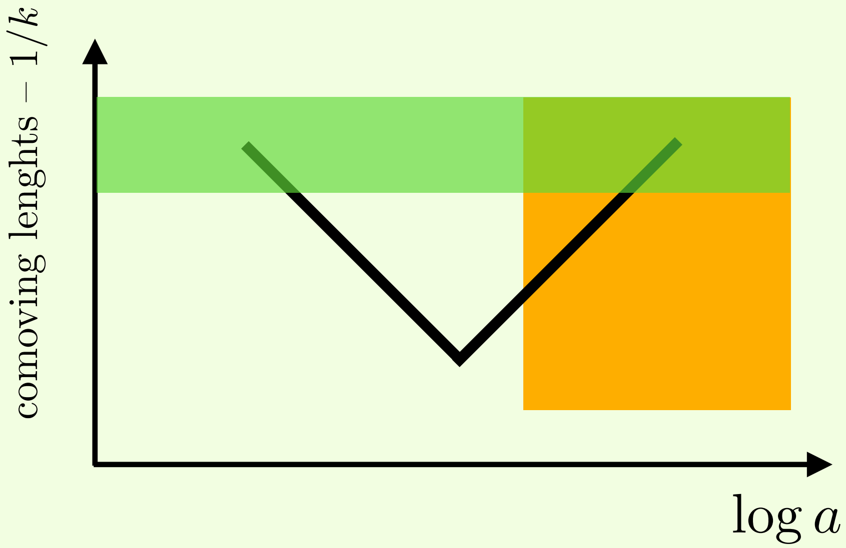

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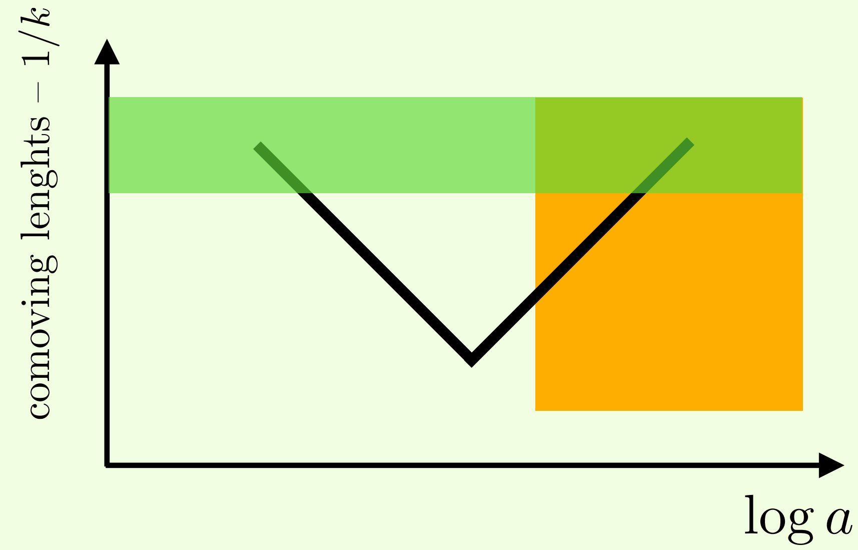

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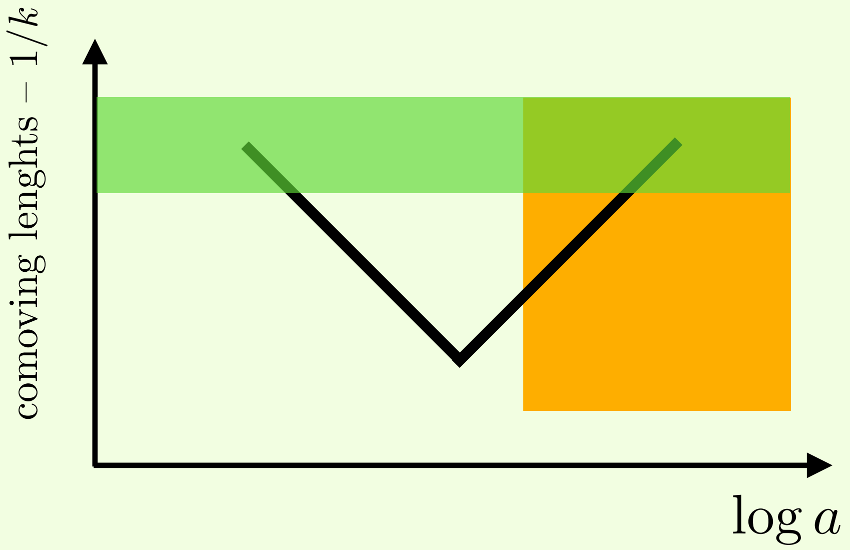

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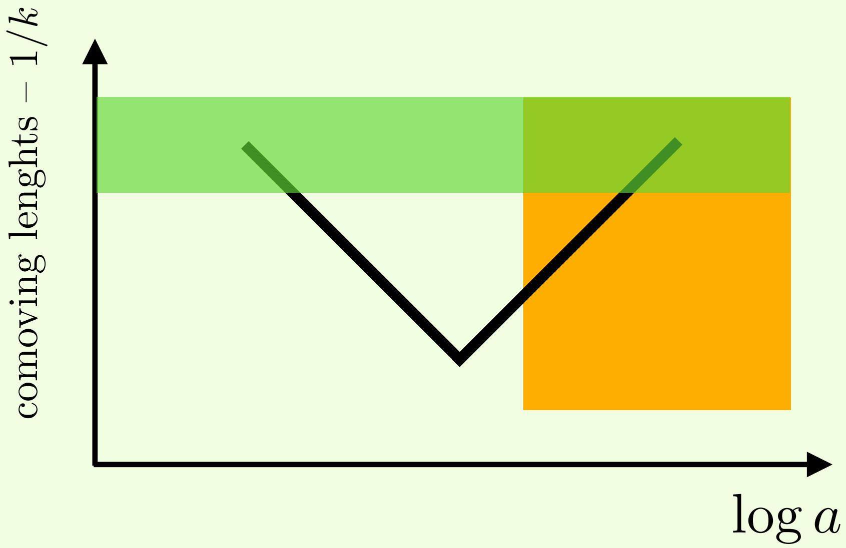

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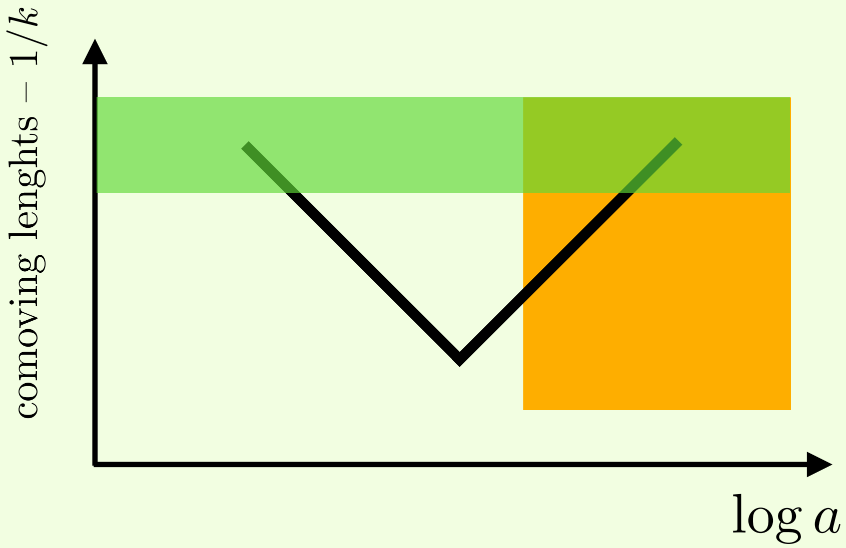

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'jump start' won't give much energy budget —> DM heavy

eventually we are sensitive to the **end of inflation**/start of **reheating**

Hey, look!

I can have renormalizable interactions with the SM!

seclusion can be difficult to achieve
if all terms allowed by symmetry are included

Rules of the game

I am interested in **secluded** and **initially empty** dark sectors

$$\int d^4x\sqrt{-g}\mathcal{L}_{\rm SM+inflaton} + \int d^4x\sqrt{-g}\mathcal{L}_{\rm DM} + M_{\rm Pl}^2\int d^4x\sqrt{-g}R$$

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are there theories like these?

Accidental seclusion

Seclusion is a tough requirement

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how to avoid marginal/relevant coupling to SM?

- Dark sectors with fermion DM
- Dark sectors with gauge symmetries
- Dark sectors with self-interactions

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what is common to these sectors?

Archetype of secluded DM sectors - II

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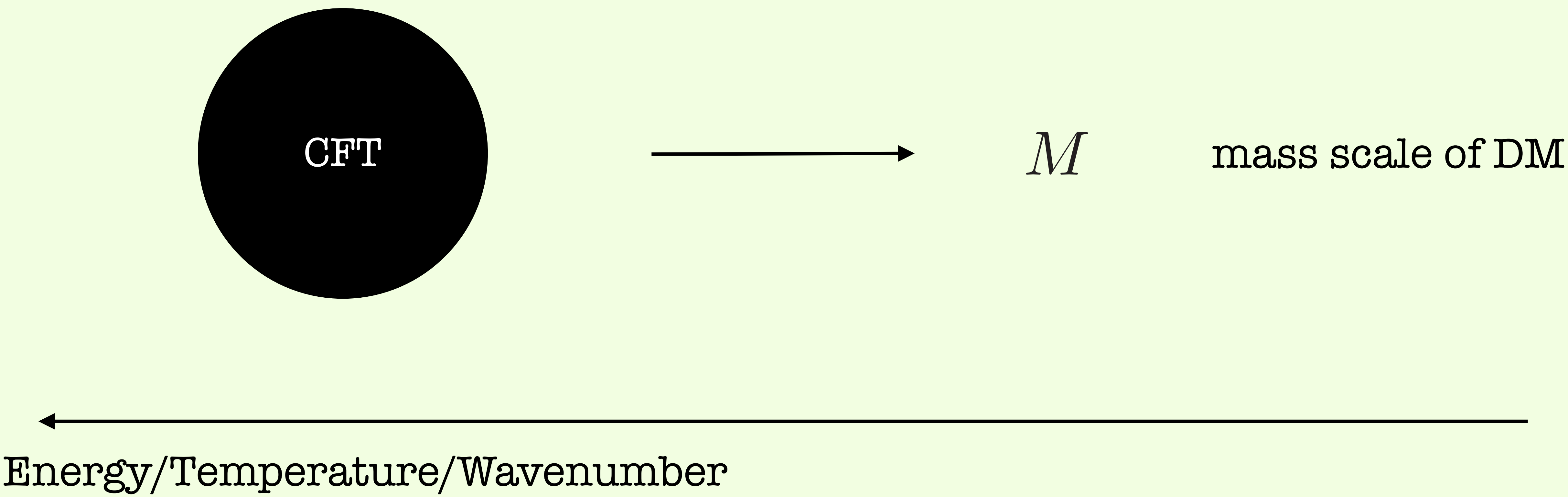

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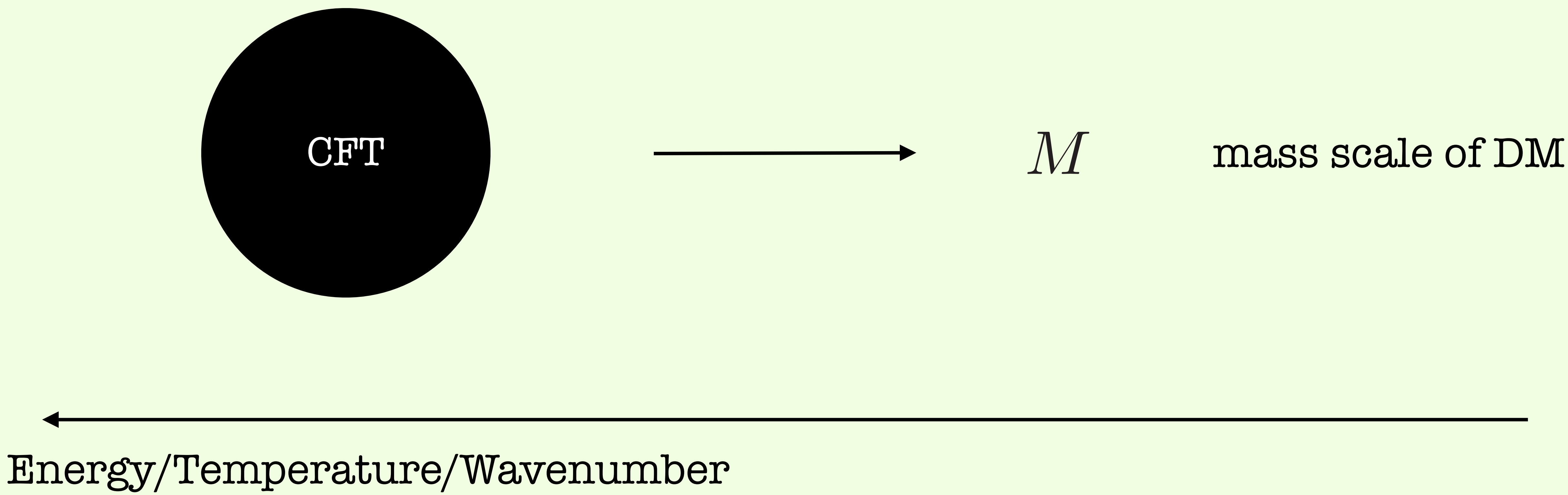

at very high scales production can be insensitive to M

*need to have a symmetry that stabilizes DM or accidental stability

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(production from the **thermal** plasma at work in the **CFT** limit)

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[Garny, Sandora, Sloth '16;
Bernal, Dutra, Mambrini, Olive, Peloso, Pierre ;...
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[Ford ‘76
Chung, Kolb, Riotto ’98
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- Stochastic Gravitational Particle Production

NEW

[Maleknejad ,Kopp '24]

Gravitational freeze-in
(GFI)

with Michele Redi and Hannah Tillim
JHEP 05 (2021) 010

Extracting energy from SM thermal bath

the idea is to extract energy from the SM thermal plasma
via graviton exchange

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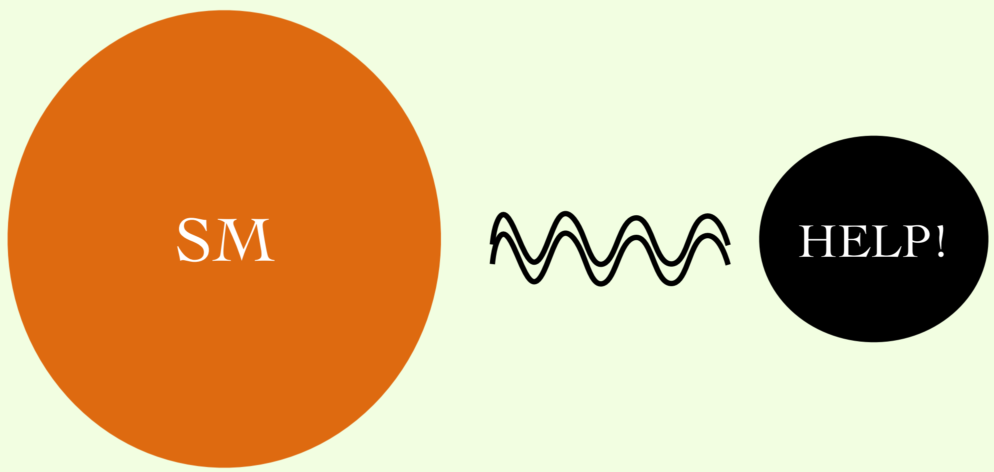
during standard cosmology the universe is reheated at high temperature
graviton mediated annihilations of SM state can produce DM



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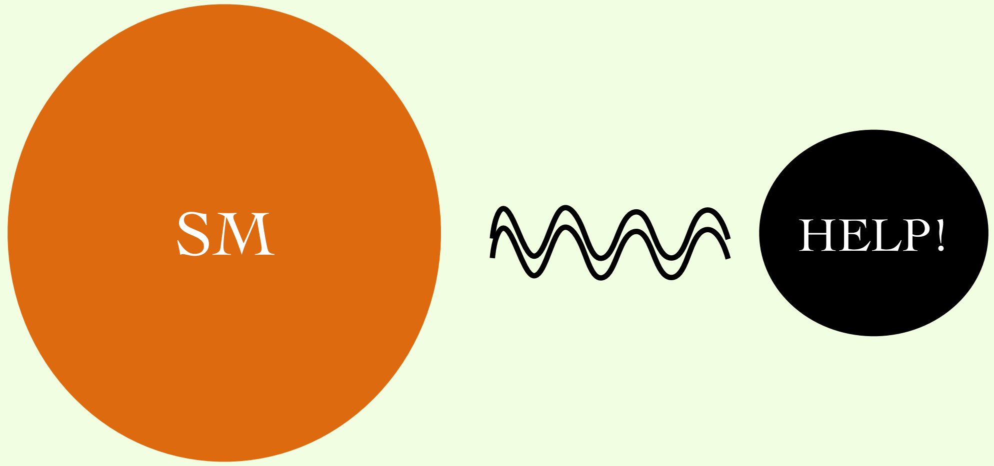
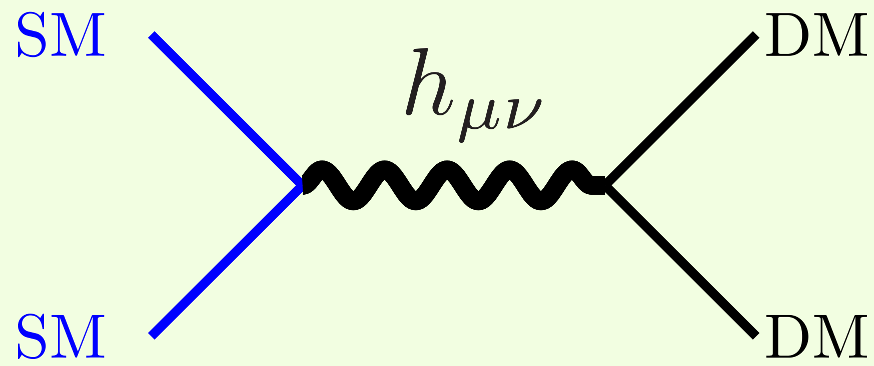


scattering happens at short distances, flat space computation

$$\frac{h_{\mu\nu}}{M_{\rm pl}} T^{\mu\nu}$$

Production insensitive to the mass of DM

Freeze-in type of calculation: thermal cross-section mediated by gravitons



$$\mathcal{A} = \frac{1}{M_{\rm Pl}^2 s}\left(T^{\rm SM}_{\mu\nu} T^{\rm DM}_{\alpha\beta}\eta^{\mu\alpha}\eta^{\nu\beta} - \frac{1}{2} T^{\rm SM} T^{\rm DM}\right)$$

- It is possible to compute explicitly case by case
- Exploiting conformal symmetry, derived **general formula**

Generalized application to relativistic CFTs

by the optical theorem we just need to know

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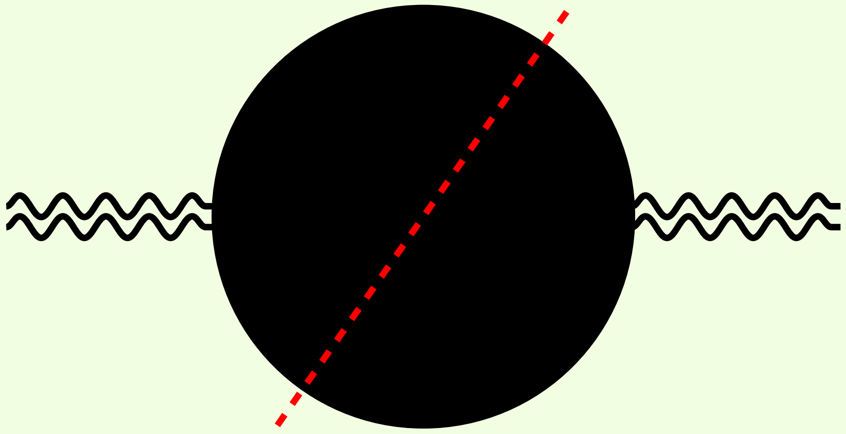
in our case it's just fixed by the conformal symmetries
and we just need the 2-point function of the stress-energy tensor

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(0)\rangle = \frac{c}{4\pi^2} P_{\mu\nu\sigma\rho} \frac{1}{x^8}$$



Thermal cross section

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with this easy to solve the Boltzmann equation

$$\frac{dY_{\rm DM}}{dT} = \frac{\langle \sigma v \rangle s(T)}{HT} (Y_{\rm DM}^2 - Y_{\rm eq}^2) \qquad Y_{\rm DM} \sim T^3 \times (\frac{T_R}{M_{\rm Pl}})^3$$

DM mass from gravitational freeze-in

sharp prediction just based on the mass and central charge

$$M\big|_{\rm DM} \approx \frac{10^6\,{\rm GeV}}{c_{\rm DM}}\left(\frac{10^{15}{\rm GeV}}{T_R}\right)^3$$

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- No visible signals...

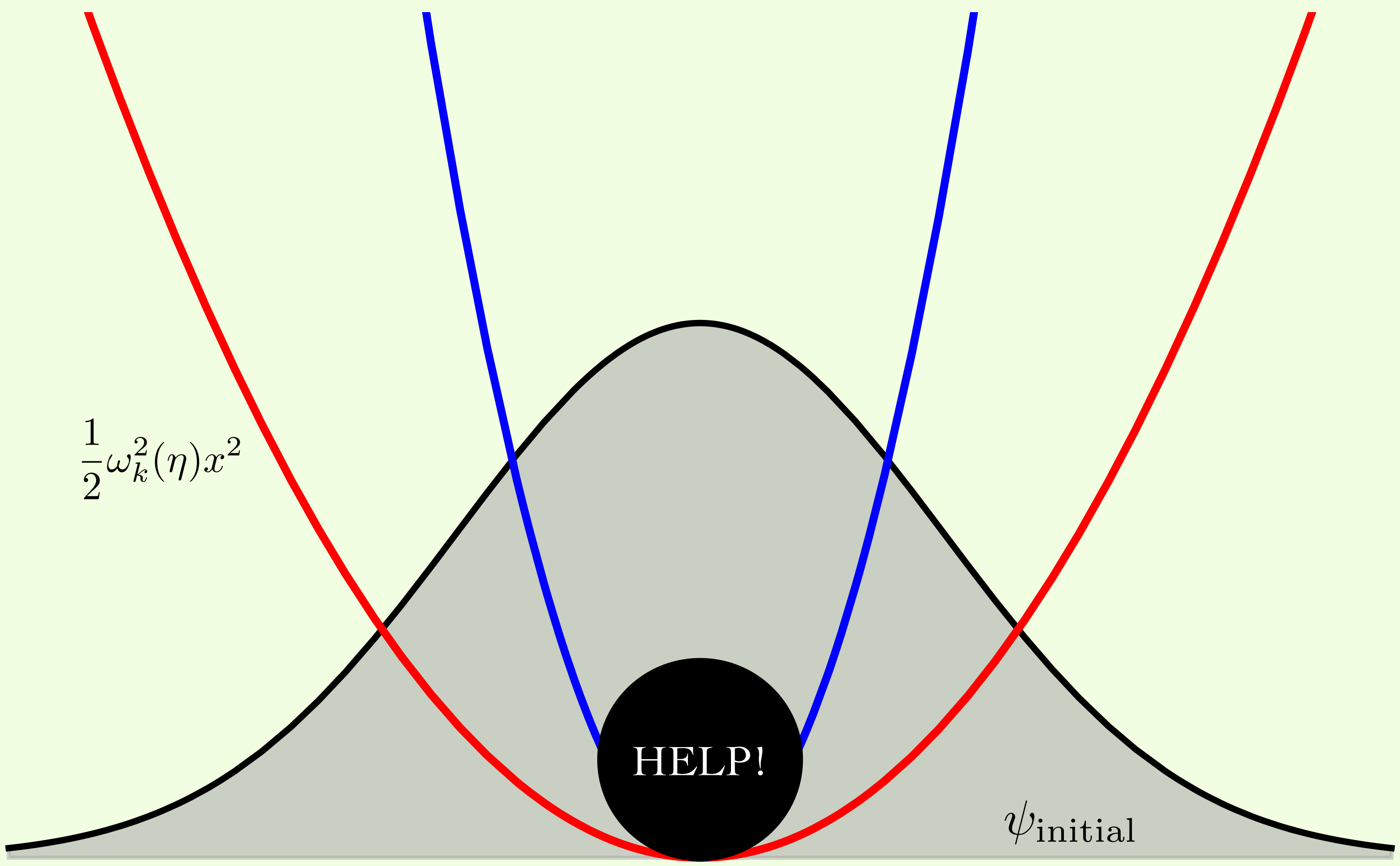
[see also Andrew Long and Rocky Kolb **review** ‘23]

Gravitational Particle Production (GPP)

with Michele Redi
JHEP 01 (2023) 085

Particle Production

need time dependence



initial state has overlap with excited states of new Hamiltonian

Hamiltonian with time dependence

Particle production can be understood in QM

$$\hat{H}(\tau) = \frac{1}{2}\hat{p}^2 + \frac{1}{2}\omega^2(\tau)\hat{x}^2$$

$$\hat{x} = v(\tau)a + v(\tau)^* a^\dagger$$

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The **mode function** satisfies at all times

$$\ddot{v} + \omega^2(\tau)v = 0$$

$$\dot{v}v^* - \dot{v}^* v = -i$$

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$$\dot{v}v^* - \dot{v}^*v = -i$$

for constant frequency, only positive frequency solutions are allowed

$$v_0 = \frac{1}{\sqrt{2\omega}}e^{-i\omega\tau}$$

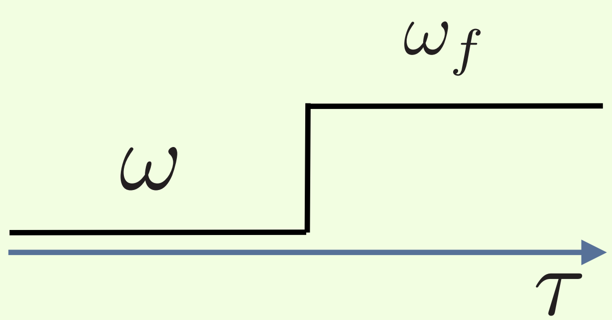
$$a|0\rangle = 0$$

we assume the initial state is the **vacuum** of the **initial** Hamiltonian

Explicit examples (important for later)

Changes of frequency will give particle production

$e^{-i\omega\tau}$



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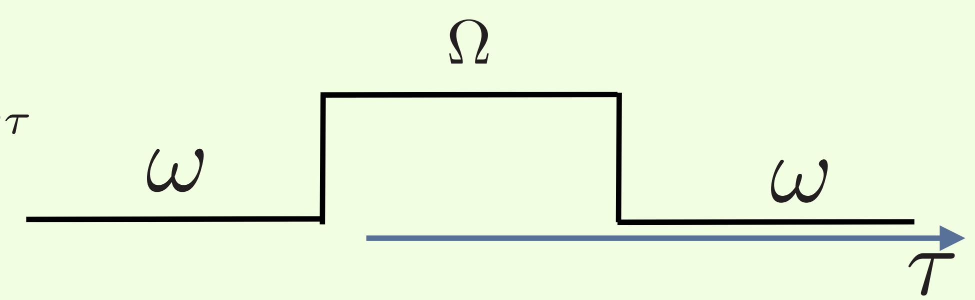

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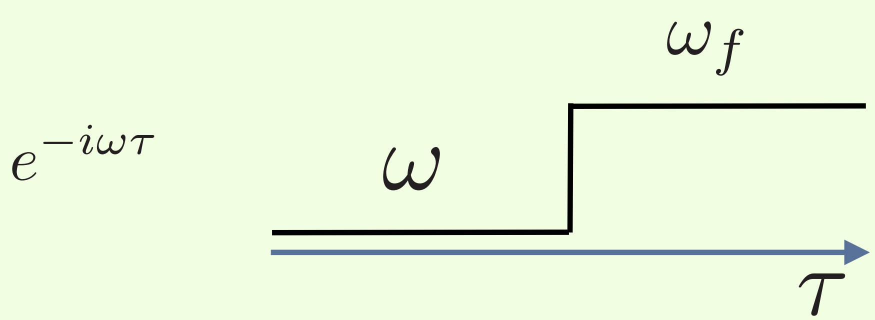

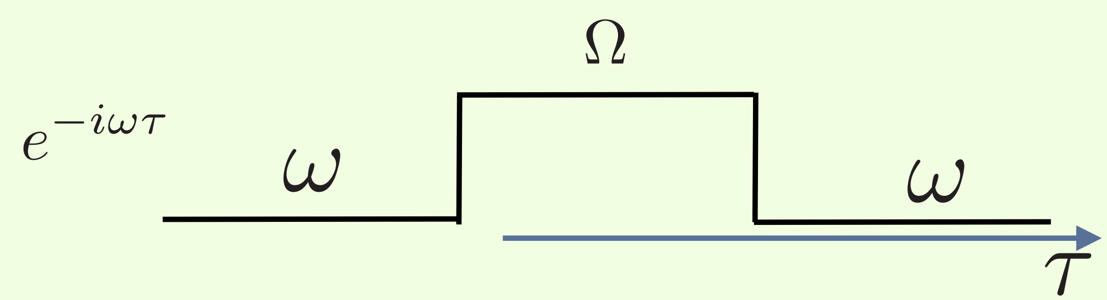

in the far future the solution is

$$v = \frac{\alpha(\tau)}{\sqrt{2\omega(\tau)}} e^{-i\omega(\tau)\tau} + \frac{\beta(\tau)}{\sqrt{2\omega(\tau)}} e^{+i\omega(\tau)\tau} \qquad |\alpha|^2 - |\beta|^2 = 1$$

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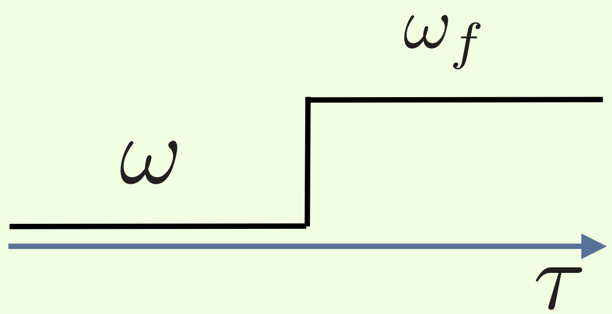

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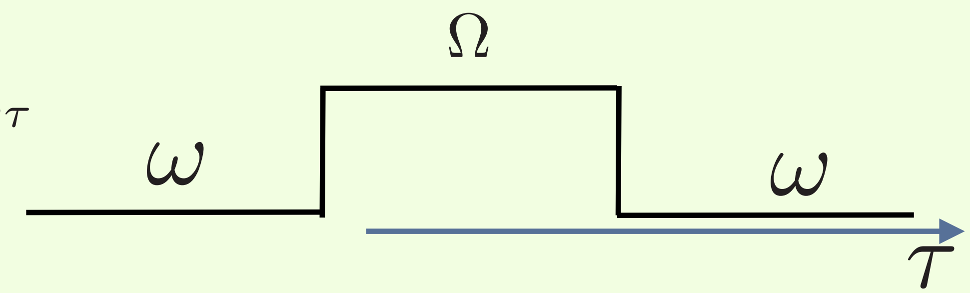

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Bogoliubov coefficient related to the number of particles produced

In terms of the new creation/annihilation operators

$$\text{occupation number} = |\beta|^2$$

Explicit examples (important for later)

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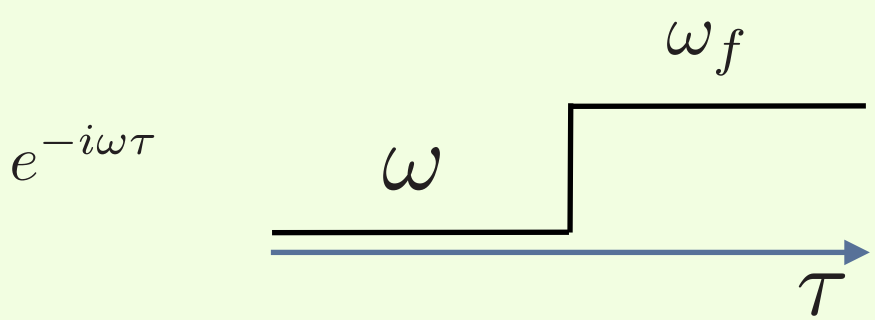
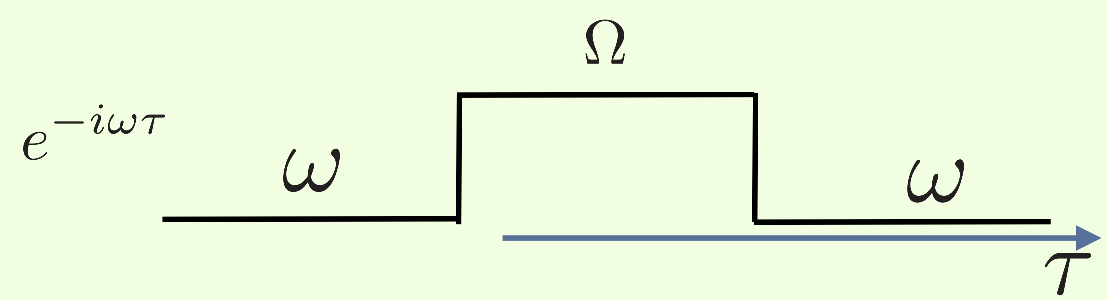


$$|\beta|^2 = \frac{(\omega - \omega_f)^2}{4\omega\omega_f}$$



$$|\beta|^2 = \frac{\left(\omega^2 - \Omega^2\right)^2 \sin^2(\Omega\Delta\tau)}{4\omega^2\Omega^2}$$

[Ford '76]

in QFT slighlty more complicated, but the idea is the same

[at work also for **asymptotically** slow varying frequencies]

Gravitational Particle Production

[Chung, Kolb, Riotto;....;
Long, Kolb]

A consequence of QFT on curved space:
Gravitational Particle Production

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Consider a **massive** field **conformally coupled** to the metric

by **acting** with a Weyl rescaling:

$$\chi(\tau, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} \left[v_k(\tau) e^{+i\vec{k}\cdot\vec{x}} a_k + ... \right] \qquad a_k|0\rangle = 0$$

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$$\partial_\tau^2 v_k + k^2 v_k + a^2(\tau)M^2 v_k = 0$$

we quantize **as in flat space** at **minus infinity**!

Bunch-Davies type of initial conditions
for k>>aM only positive frequency

GPP needs massive particles

Standard GPP needs **mass** term

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Let us consider a conformal scalar (=1/2 Weyl) with mass M

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[here asymptotically adiabatic evolution]

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The **number** and **energy** densities are computed as follows

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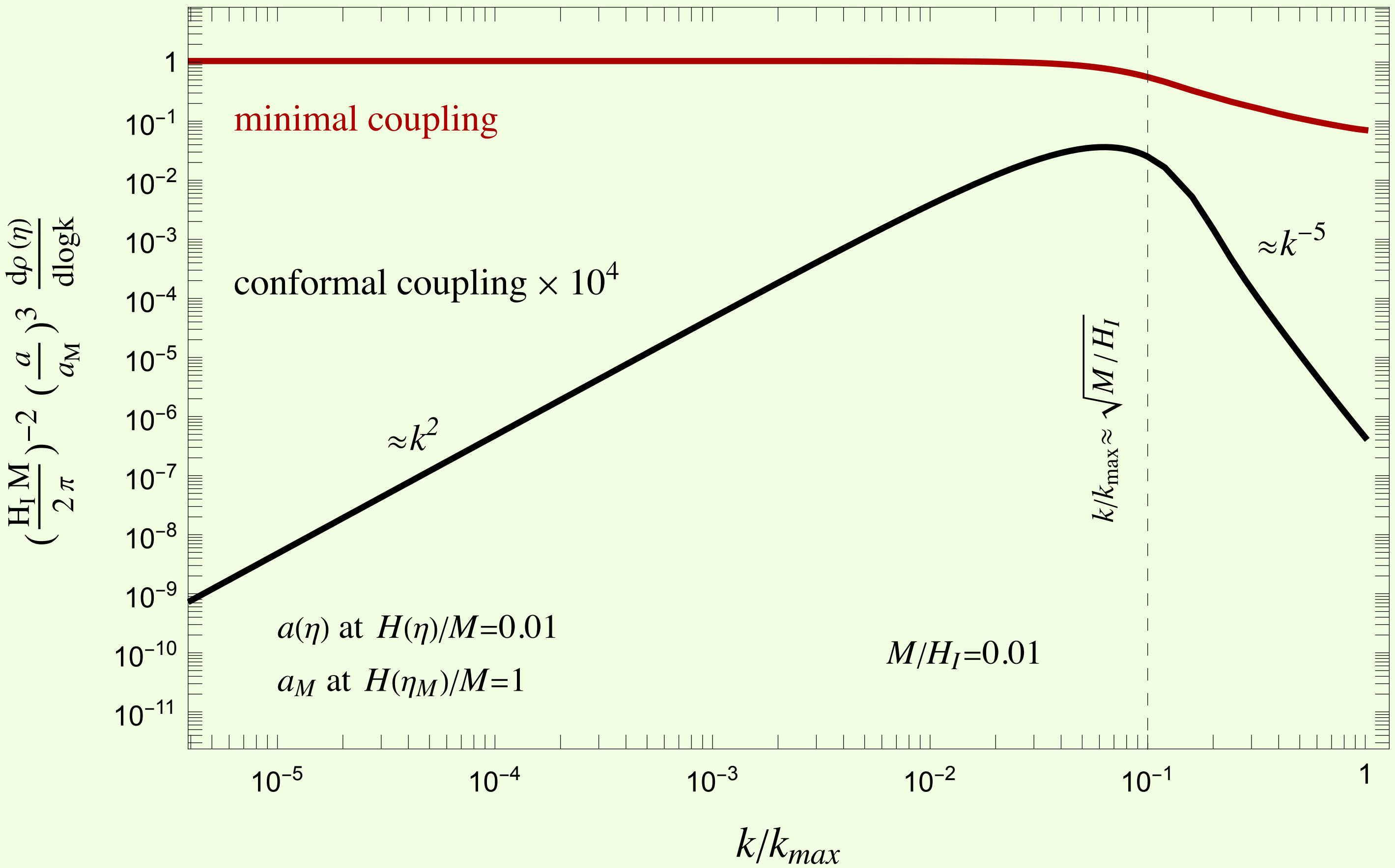

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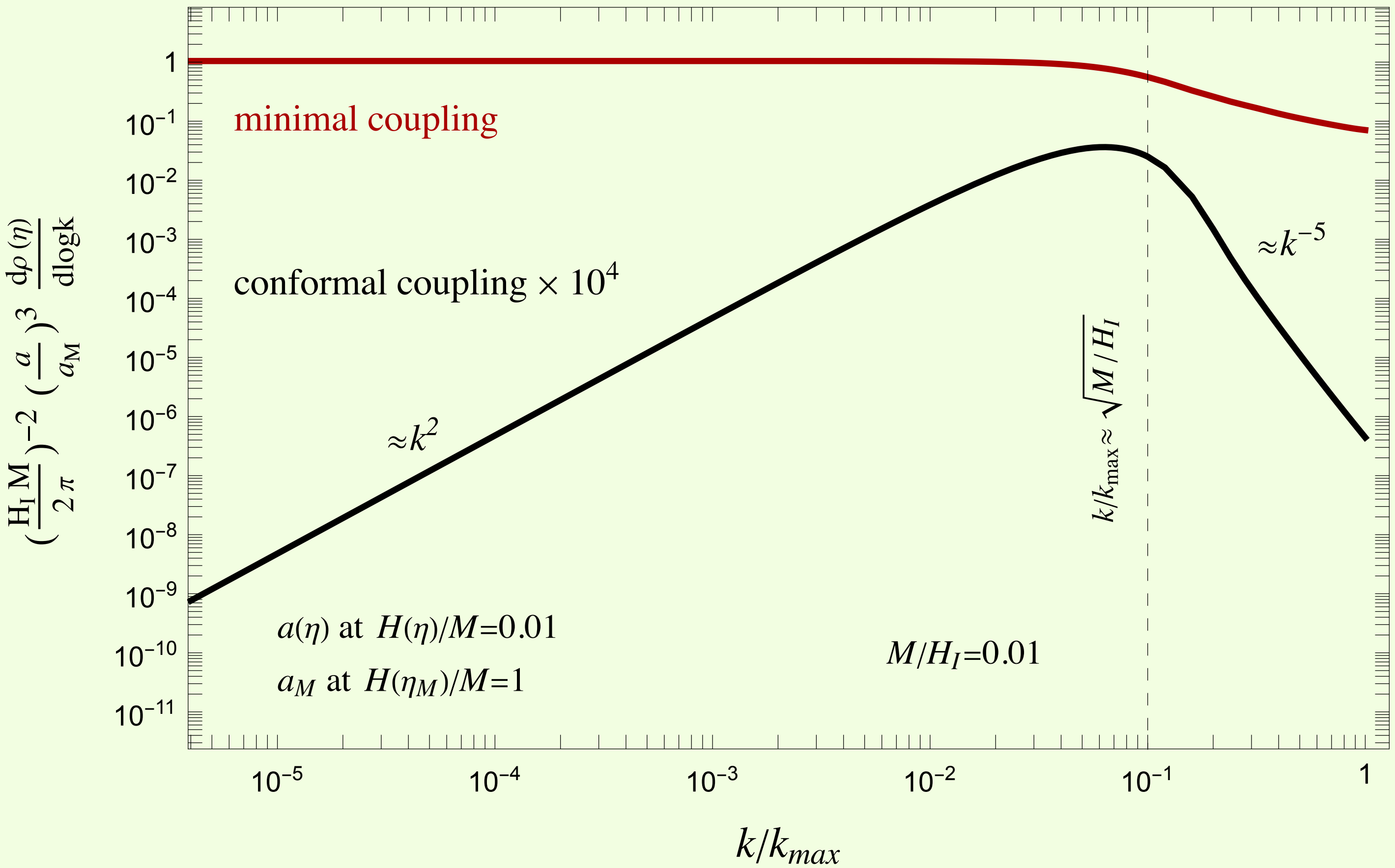

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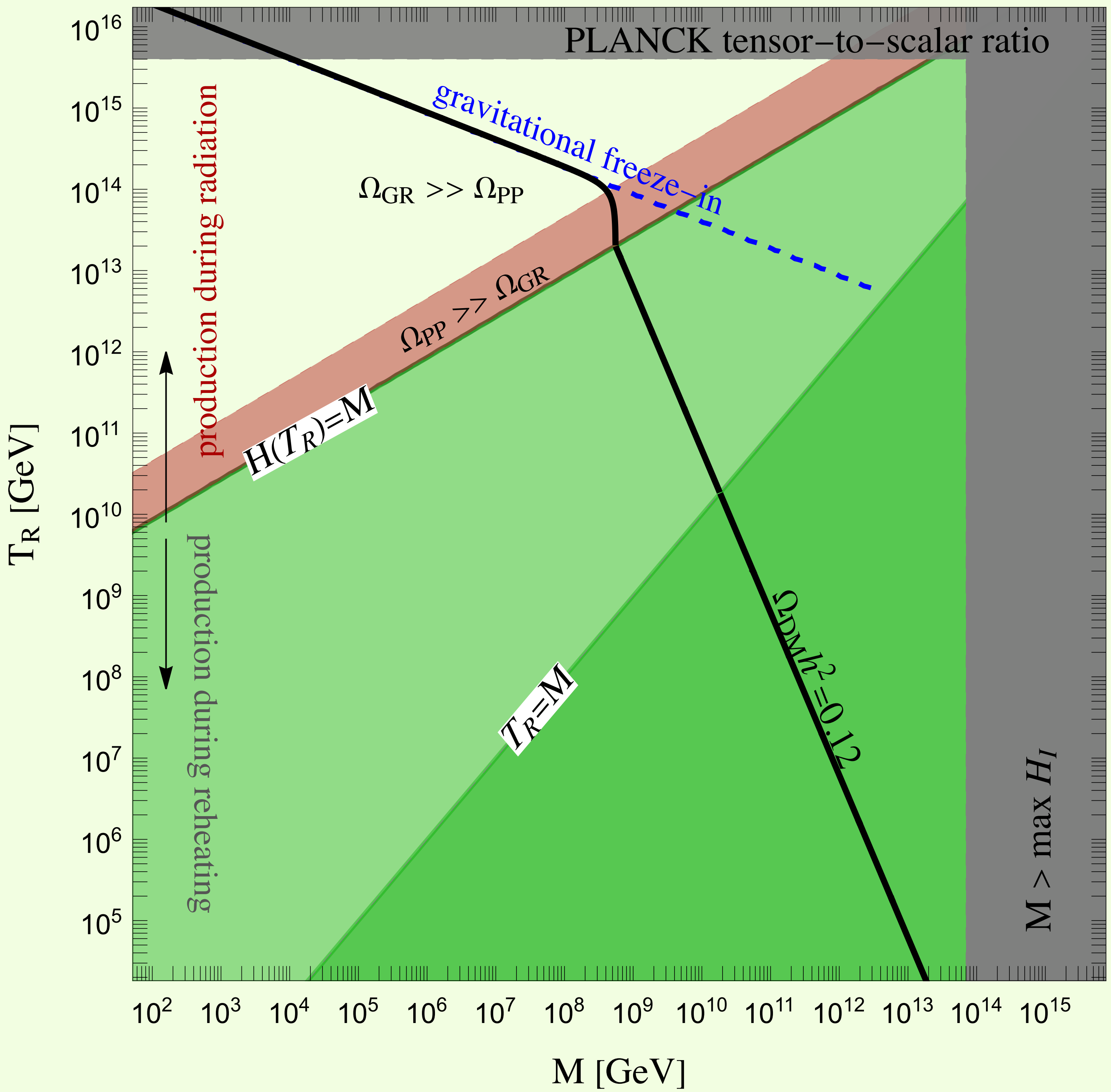
Abundance strongly suppressed by M, Weyl invariance forbids production!

Is there a way out?

DM abundance: GFI + GPP



Stochastic Gravitational Particle Production

NEW

with Raghuveer Garani and Michele Redi
2408.yyyyy + 240x.yyyyy

An obstruction to produce CFTs from inflation

CFTs do **not** see the time dependence of FRW*

Production of conformally coupled matter goes to **zero** with $M \to 0$

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metric **completely disappears** from the action of a Weyl fermion

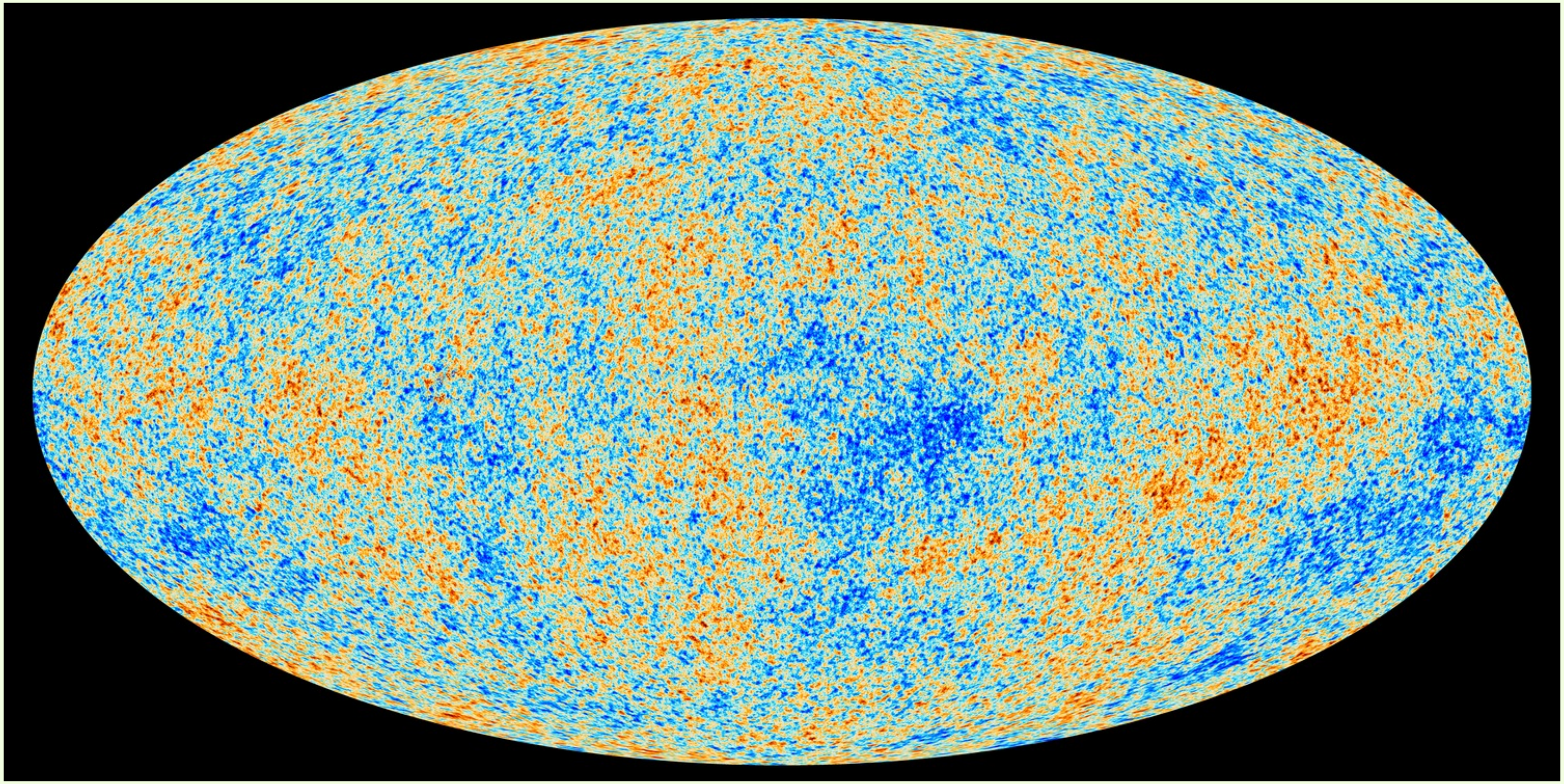
$$\int d^4x \, i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi$$

as in flat space! no GPP!

* no photon production during inflation

Fluctuations breaks Weyl invariance

[PLANCK]



$\Delta_\zeta|_{\rm CMB} \sim 10^{-9}$ generated during inflation

Gravitational fluctuations break Weyl invariance

Dynamical gravity breaks Weyl invariance $M_{\rm Pl}^2 R$

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[for the case of GWs generated at a phase transition,
see **Maleknejad & Kopp '24**]

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In single field inflation,
curvature perturbations constant on super-horizon scales

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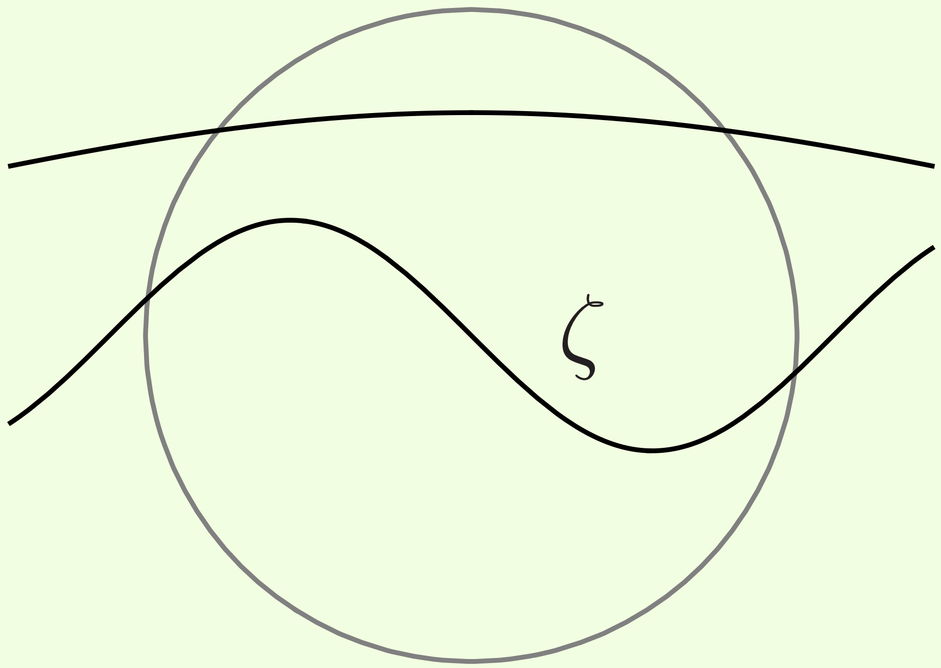

$$\langle \zeta_q \zeta^*_{q'} \rangle = \frac{2\pi^2}{q^3} \delta^{(3)}(\vec{q} - \vec{q}') \, \Delta_\zeta(q)$$

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at shorter scales can be different!

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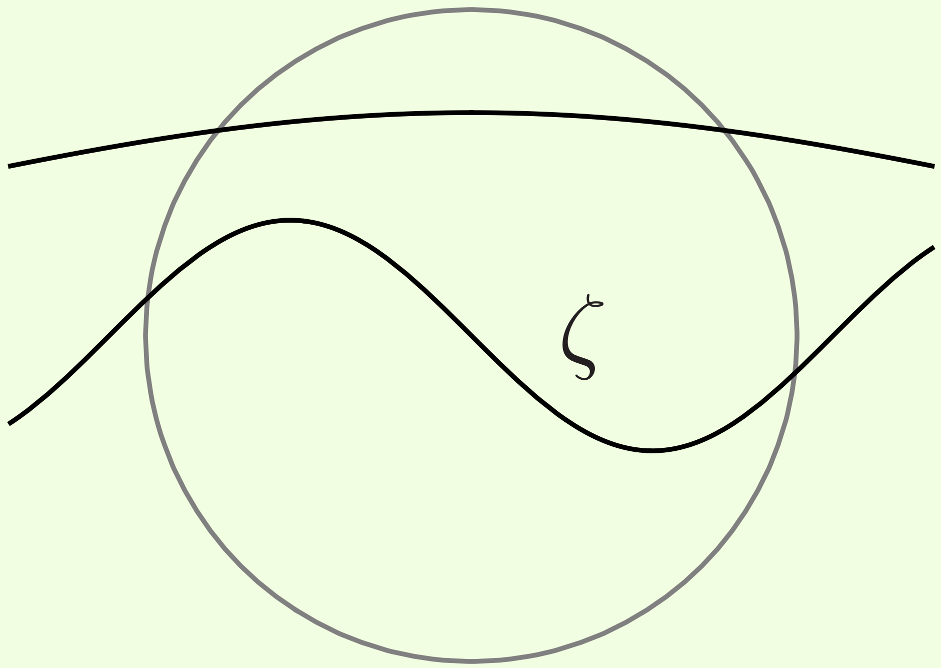

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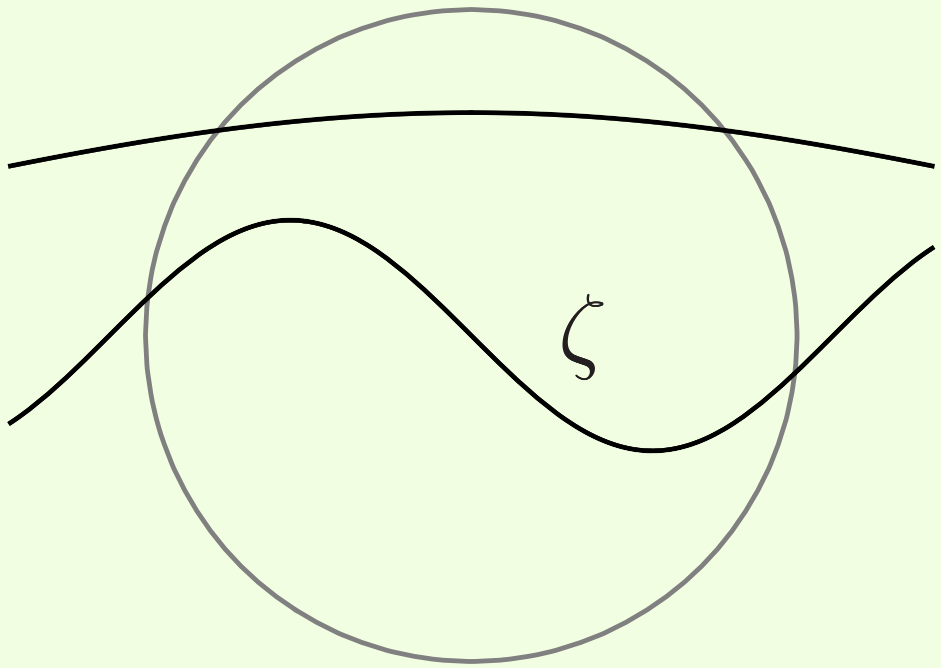

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$\Phi,\ \Psi$ are matched to the super-horizon value of ζ

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in absence of anisotropic stress, we write the following expression*

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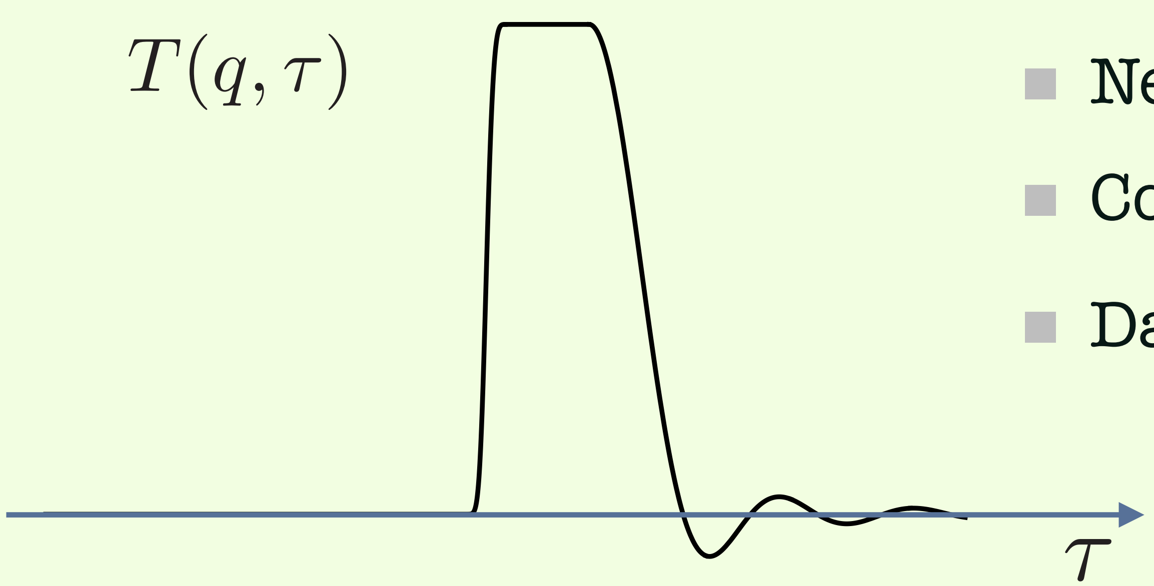

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* Fourier space calculation

A bit more details on massless fermions

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- Technically different from the massive case

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We solve it perturbatively in the source:

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we just **project** onto the Bogoliubov coefficient at **infinity**

*this method can be used instead of the in-in formalism in a few cases
[**Maleknejad & Kopp '24** uses in-in formalism]

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Extracting the coefficient is easy (same frequency)

$$\beta_k(\tau) \sim \int_{-\infty}^{\tau} d\tau' e^{-i(k+\omega)\tau'} \xi^\dagger_{\vec{k},+} J_0 \qquad \vec{\omega} = \vec{k} - \vec{q}$$

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$$\langle |\beta_k|^2 \rangle = \int d\tau \int d\tau' \int \frac{d^3q}{(2\pi)^3} e^{-i(k+\omega)(\tau-\tau')} \times \langle \Psi_{\vec{q}}(\tau) \Psi^*_{\vec{q}}(\tau') \rangle_{\delta} \, \mathcal{K}[k, q, \cos\theta]$$

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The abundance of particles today is

$$\frac{dn}{d\log k} = \frac{k^3}{4\pi^2}\int \frac{d\cos\theta dq}{q}\Delta_\zeta(q)|\mathcal{I}(q,k+\omega)|^2\mathcal{K}[k,q,\cos\theta]$$

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The abundance of particles today is found upon an integral in wavenumber
[all integrals converge, and have a good flat space limit]

DM from curvature perturbations

In the case of **fermion DM**

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- Results depends on the shape/amplitude of power spectrum
- For a peaked spectrum at q* the DM abundance is reproduced:

$$q_* \approx \; 1.23 \times 10^{-7}\mathrm{eV} \left(\frac{10^6\,\mathrm{GeV}}{M}\right)^{\frac{1}{3}} \left(\frac{0.001}{\Delta_\zeta(q_*)}\right)^{\frac{1}{3}} \left(\frac{0.1}{A}\right)^{\frac{1}{3}}$$

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inflationary fluctuations —> particle production

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$$\langle T^0_{\ 0}(\tau)\rangle \sim \int\int \langle hh\rangle \langle T^0_{\ 0}(\tau) T T\rangle$$

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energy density computable by knowing **stress-energy tensor 3-p function** and the power spectrum of all fluctuations h

need more thoughts about this, but looks interesting/general

Recap & Conclusions

DM from gravitationally coupled sectors

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- Stochastic Gravitational Particle Production

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secondary GWs/PBHs
non-trivial inflation needed

Outlook

The end

the possibility that DM is only gravitationally coupled to us
is both concerning and compelling

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Thank you for your attention!

and **thanks** for the **warm hospitality**!