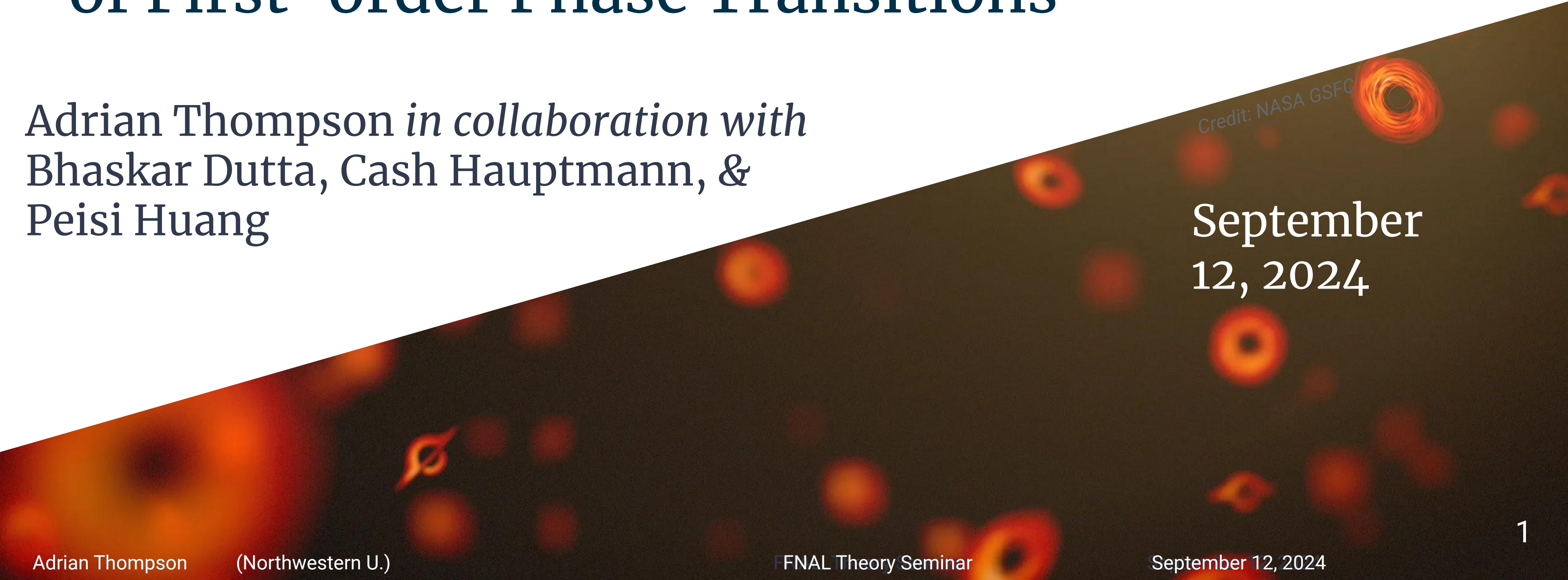
PBH formation and Gravitational Waves as Multi-messenger Signals of First-order Phase Transitions

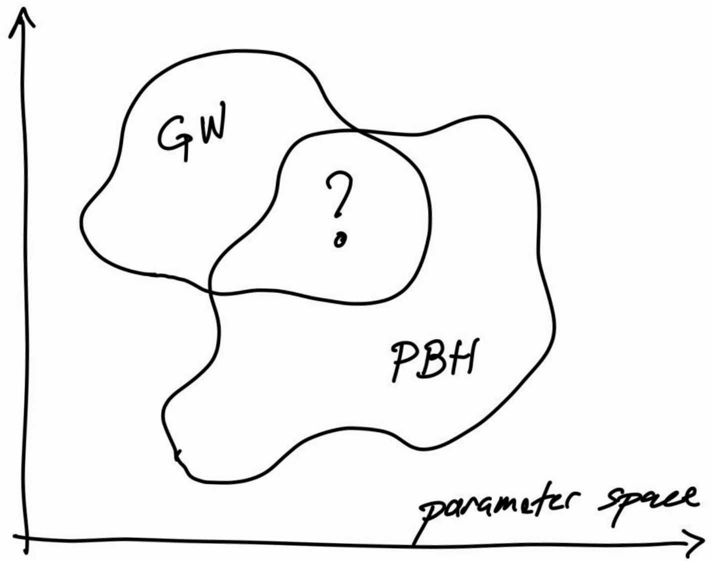
Adrian Thompson *in collaboration with*
Bhaskar Dutta, Cash Hauptmann, &
Peisi Huang

September 12, 2024

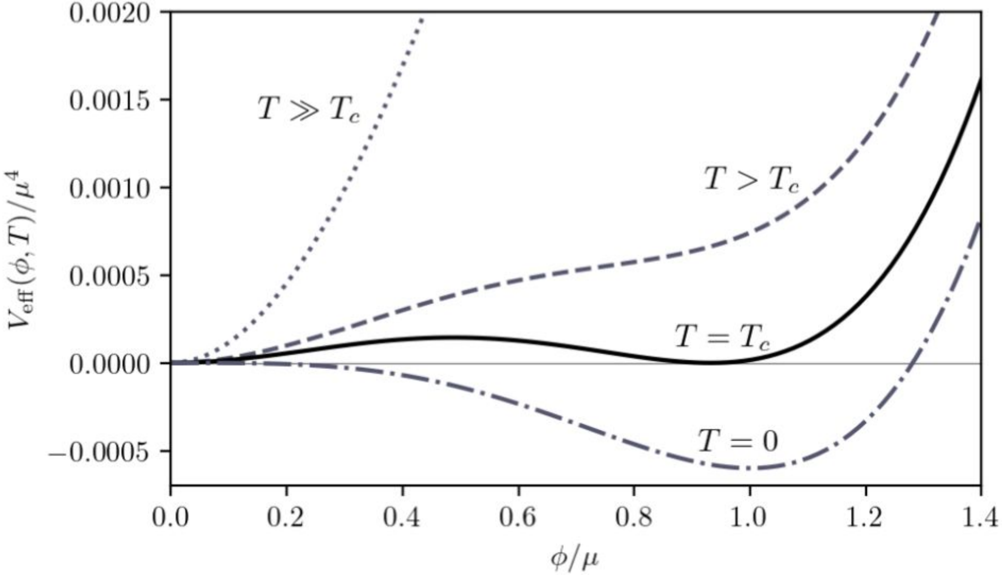


Outline

1. Standard lore
   a. Bubble nucleation
   b. Gravitational Waves
2. A *B-L* model
3. PBH formation mechanisms
4. Multi-messenger parameter space



Scalar Potentials at Finite Temperature



Consider a complex scalar field $\mathbf{\Phi}$, $\phi = |\mathbf{\Phi}|$ with a Higgs-like potential:

$$V(\phi) = -m^2\phi^2 + \frac{\lambda}{4}\phi^4$$

Finite temperature corrections

$$V(\phi, T) = D(T^2 - T_0^2)\phi^2 - (AT + C)\phi^3 + \frac{\lambda}{4}\phi^4$$

[at finite order]

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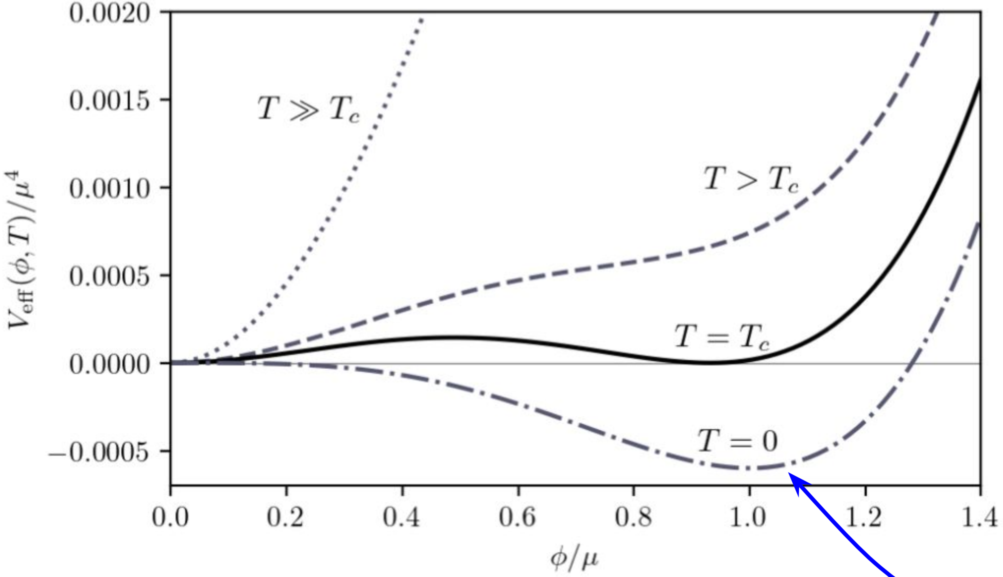

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At T=0, the potential as a VEV = μ

Scalar Potentials at Finite Temperature

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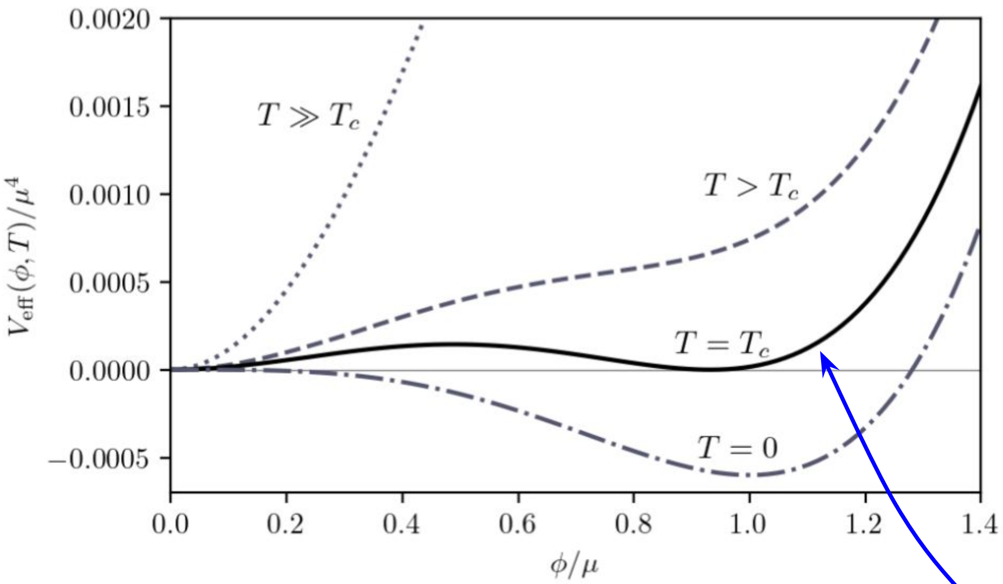

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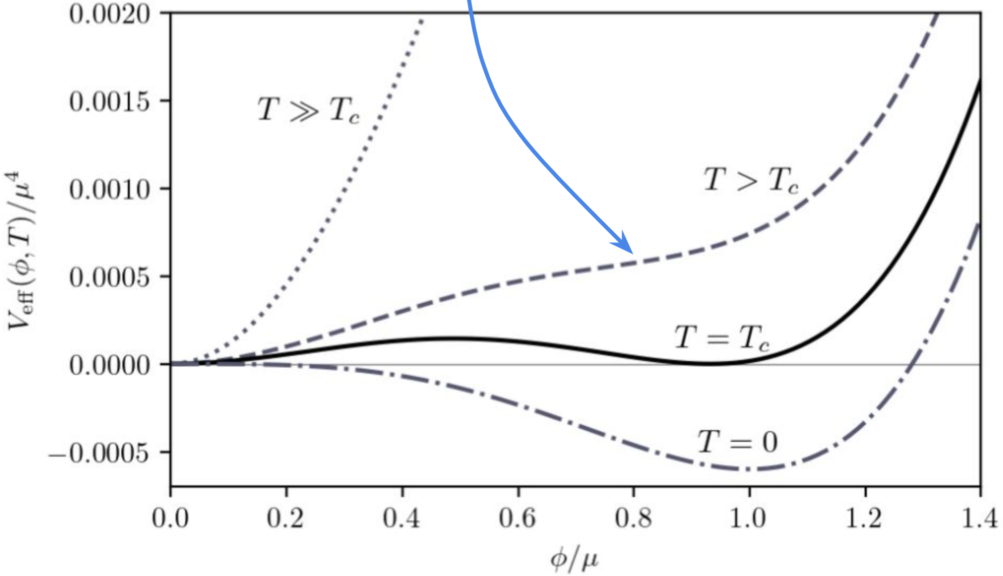
At $T=T_c$, the critical temperature, finite temperature VEV degenerate with $V(\phi=0)$

Scalar Potentials at Finite Temperature

At $T>Tc$, $<\phi>=0$ and symmetry is restored



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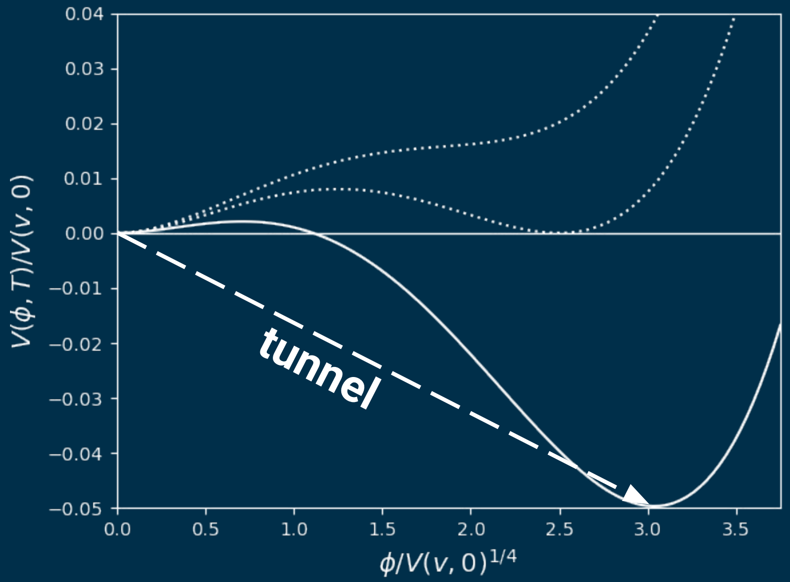
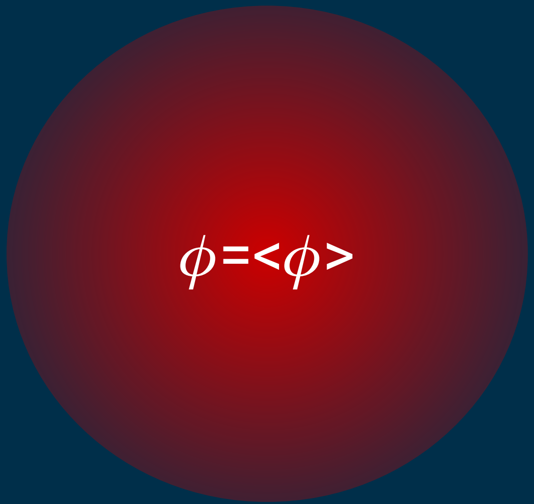
Bubble Nucleation: A very hot cosmos freezing

quantum mechanically **tunnel** through barrier from $<\phi(x)>=0$ to the new minima



$T<T_c$ $\phi=0$



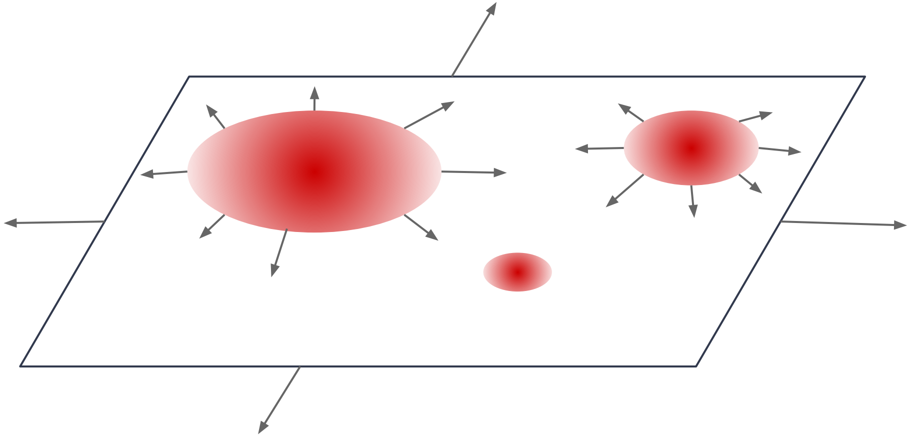
Background Plasma interacting with ϕ

Bubble Nucleation: Theoretical Description

- $S_3(T)$ is the O(3) symmetric bounce action
- Γ(T) is the bubble nucleation tunnelling rate
- The **phase transition happens at temperature T_{PT}** if the tunneling rate can outcompete the Hubble expansion

$$S_3 = 4\pi \int_0^R r^2 dr \left[\frac{1}{2} \left(\frac{d\phi}{dr} \right)^2 + V(\phi(r), T) \right]$$

$$\Gamma(T) = T^4 \left(\frac{S_3}{2\pi T} \right)^{3/2} e^{-S_3(T)/T}$$



Nucleation or Percolation?

- Nucleation temperature T_n: one bubble per Hubble volume
- Percolation temperature $T_p < T_n$: where the false vacuum (FV) fraction is 70%

T_n $$\frac{\Gamma(T)}{H^4(T)} \simeq 1$$

T_p $$g(T_c, T) = \exp\left[-I(T)\right] = 0.7.$$

The effective parameters describing the bubble nucleation

β is the **inverse time of the transition** $\rightarrow$ large beta, fast PT

$$\frac{\beta}{H_{PT}} = T_{PT} \frac{d}{dT}\left(\frac{S_3}{T}\right)\Bigg|_{T_{PT}}$$

See the Diligence paper: Guo, Sinha, Vagie, White [2103.06933] (JHEP)

$$H(T)^2 = \frac{8\pi}{3M_{Pl}^2}(\rho_R(T) + \rho_U(T))$$

Jouget velocity:
Hybrid ←|→ Detonation

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α : the **strength of the transition** includes both the latent heat and potential difference

$$\alpha = \frac{30}{\pi^2 g_* T_{PT}^4}\left(- \Delta V + \frac{1}{4} T \frac{\partial \Delta V}{\partial T}\Big|_{T_{PT}}\right)$$

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v_w is the **bubble wall speed**, and tells us the dynamics of the GWs

$$v_w = \begin{cases} \sqrt{\frac{\Delta V}{\alpha \rho_r}} & \text{for} \quad \sqrt{\frac{\Delta V}{\alpha \rho_r}} < v_J(\alpha) \\ 1 & \text{for} \quad \sqrt{\frac{\Delta V}{\alpha \rho_r}} \geq v_J(\alpha) \end{cases}$$

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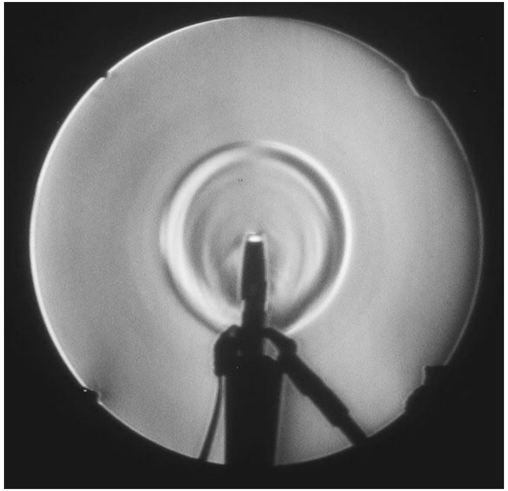
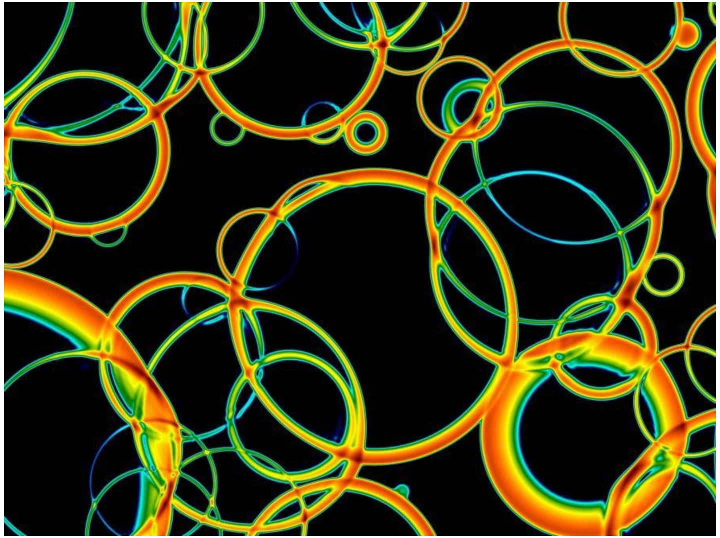
Gravitational Wave Production: Three sources

$$\Omega_{GW} = \Omega_{sw} + \Omega_{col} + \Omega_{turb}$$

Sound Waves

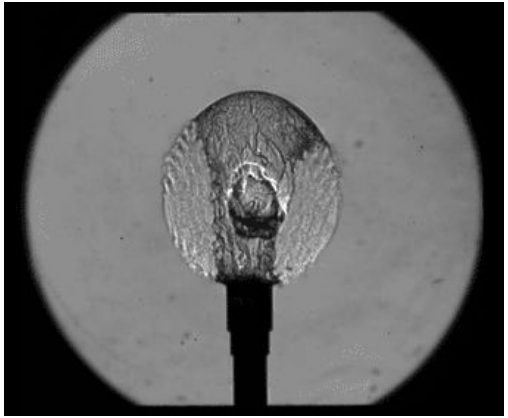


Bubble collisions



David Weir, *Gravitational Waves from Early Universe Phase Transitions*

Turbulence



Gravitational Wave Production

$$\Omega_{\rm GW} = \Omega_{\rm sw} + \Omega_{\rm col} + \Omega_{\rm turb}$$

Example: Sound Wave term

$$h^2\Omega_{\rm sw}(f) = 2.65 \times 10^{-6} \left[\frac{H(T_{\rm PT})}{\beta}\right] \left[\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right]^2 \left[\frac{100}{g_{\rm PT}}\right]^{1/3} v_w \left[\frac{f}{f_{\rm sw}}\right]^3 \left[\frac{7}{4+3(f/f_{\rm sw})^2}\right]^{7/2}$$

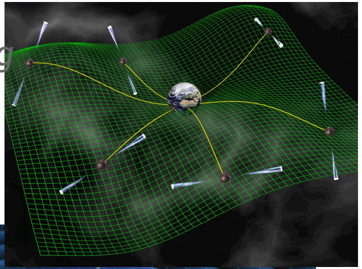
$$f_{\rm sw} = \frac{1.15}{v_w}\left[\frac{\beta}{H(T_{\rm PT})}\right] h_*$$

$$h_* = 1.65 \times 10^{-5}\,{\rm Hz} \left[\frac{T_{\rm PT}}{100\,{\rm GeV}}\right]\left[\frac{g_{\rm PT}}{100}\right]^{1/6}$$

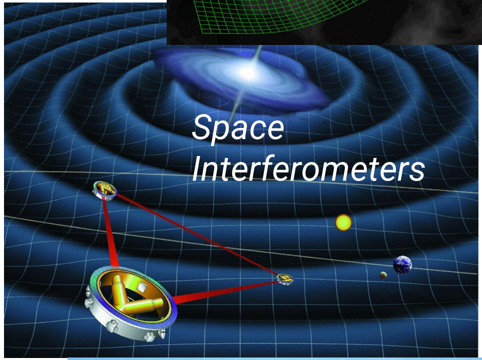
$h^2\Omega$ is the **gravitational strain**, the amount of relative stretching of spacetime

Gravitational Wave Astronomy across Frequency Bands

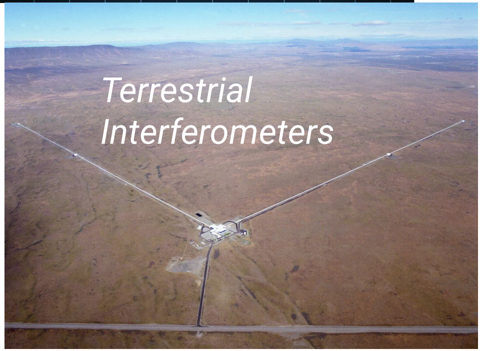
Pulsar Timing Arrays (PTA)



~ nHz range (~10 MeV scale)



~ mHz range (~ GeV scale)



~ Hz range (~TeV scale)

A Conformally Invariant $U(1)_{B-L}$ Model

field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$	g_i
q_L^i	**3**	**2**	$+1/6$	$+1/3$	12
u_R^i	**3**	**1**	$+2/3$	$+1/3$	6
d_R^i	**3**	**1**	$-1/3$	$+1/3$	6
l_L^i	**1**	**2**	$-1/2$	-1	4
ν_R^i	**1**	**1**	0	-1	1
e_R^i	**1**	**1**	-1	-1	2
H	**1**	**2**	$-1/2$	0	1
Φ	**1**	**1**	0	$+2$	1
Z'	**1**	**1**	0	0	3
G	**1**	**1**	0	$+2$	1

- A complex scalar Φ with $B - L = 2$
- The gauge boson Z'
- RH neutrino ν_R

Simplifying Assumptions

- Only consider 1 species of ν_R
- Decoupled from SM; $\lambda' << 1$

$$\mathcal{L}_{\text{scalar}} = -\lambda_H (H^\dagger H)^2 - \lambda(\Phi^\dagger \Phi)^2 - \lambda' (\Phi^\dagger \Phi)(H^\dagger H)$$

$$\mathcal{L}_{\text{Yukawa}} = -Y_D^{ij} \overline{\nu_R^i} H^\dagger l_L^j - \frac{1}{2} Y_i \Phi \overline{\nu_R^{ic}} \nu_R^i$$

See e.g. Sher (1989), Meissner & Nicolai (2009), Iso, Okada, Orikasa [0902.4050]

A Conformally Invariant $U(1)_{B-L}$ Model: Radiative Phase Transition

RG running + 1-loop correction-improved potential

$$V_{\text{eff}}(\phi, T) = V_0(\phi) + V_T(\phi, T)$$

$$V_0(\phi) = \frac{1}{4}\lambda(\tau)G(\tau)^4\phi^4$$

Zero temperature piece

$$V_T(\phi, T) = \frac{T^4}{2\pi^2}\sum_j g_j J_j\left(\frac{m_j(\phi)^2}{T^2} + \frac{\Pi_j(T)}{T^2}\right)$$

finite temperature piece

definitions

$$\phi/\sqrt{2} = \text{Re}(\Phi) \qquad \tau \sim \ln\frac{\phi}{\mu}$$

$$G(\tau) \equiv \exp\left[-\int_0^\tau d\tau' \gamma(\tau')\right], \quad \gamma(\tau) \equiv \frac{1}{32\pi^2}\left[Y^2 - 24 g_{B-L}^2\right]$$

See e.g. Sher (1989), Meissner & Nicolai (2009), Iso, Okada, Orikasa [0902.4050]

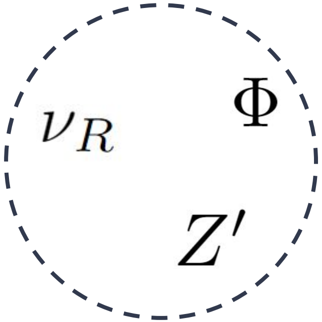
The Simplified Setup

Since we are in the limit decoupled from the SM Higgs, the **free parameters** in this model are

- $\alpha_{B-L} \equiv \frac{g_{B-L}^2}{4\pi}$
- $\alpha_Y \equiv \frac{Y^2}{4\pi}$

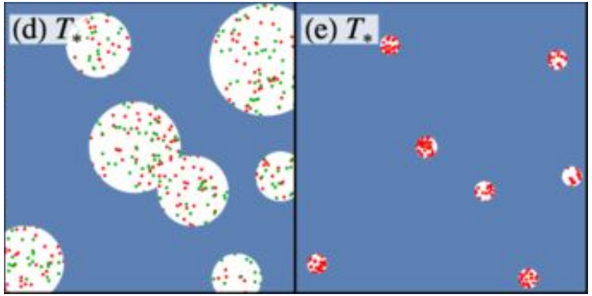
Potential shape parameters

- $\langle \Phi \rangle = \mu$ ← **Sets T_{PT} scale**
- λ and $m_{\nu_R} = \frac{1}{\sqrt{2}} Y \langle \Phi \rangle$ are then determined



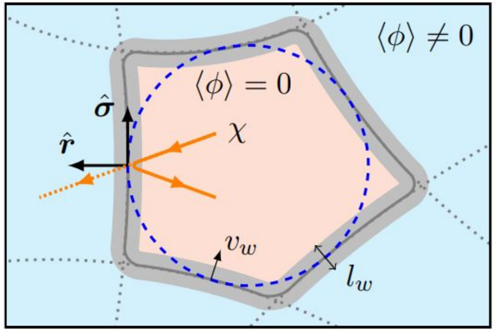
Primordial Black Hole (PBH) Formation Mechanisms

Fermi-balls and soliton collapse



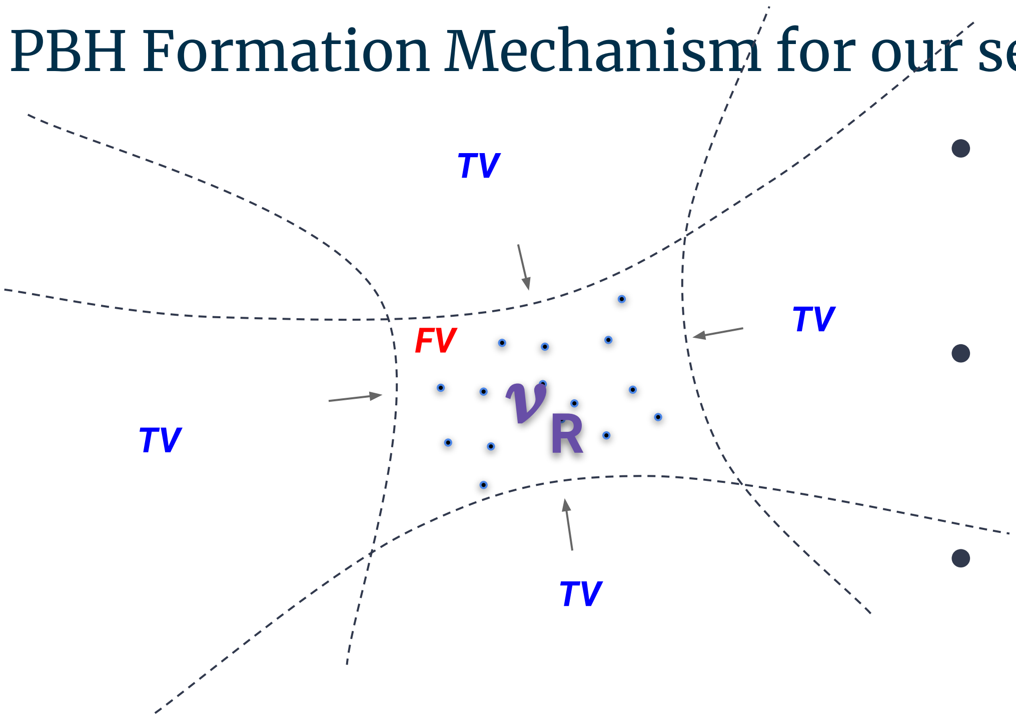
e.g., Hong, Jung, Xie [2008.04430]

False Vacuum Trapping and Collapse



e.g., Baker, Breitbach, Kopp, Mittnacht [2105.07481]

PBH Formation Mechanism for our setup



True Vacuum
$\langle\phi\rangle = v$
$m_{\nu R} \propto Y\langle\phi\rangle > T_{PT}$

False Vacuum
$\langle\phi\rangle = 0$
$m_{\nu R} = 0$

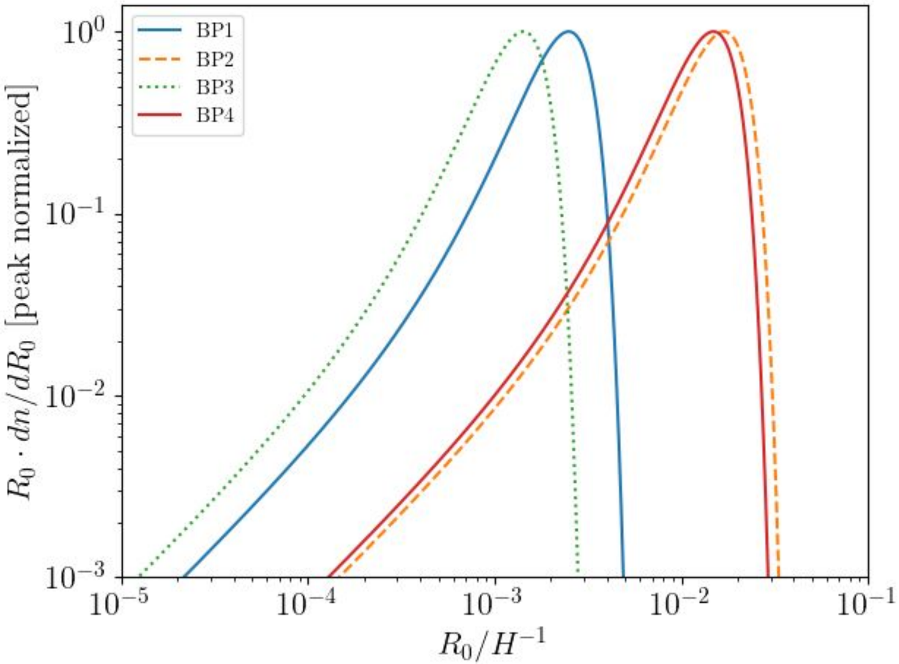
See also:
Lu, Kawana, Xie [2202.03439] PRD 105, 123503

- Similar mechanism to Baker, Breitbach, Kopp, Mittnacht [2105.07481]
- If $m_{\nu R} > T_{PT}$ in the True Vacuum (TV), passage to False Vacuum (FV) is suppressed
- $\rightarrow \nu_R$ **becomes trapped in FV**
  - *Usually* take small Yukawa to protect against $\nu_R\nu_R \rightarrow \phi\phi$, $\nu_R\nu_R \rightarrow \phi$ annihilation
- **FV Collapse, overdense ν_R drives PBH formation**

PBH Formation from False Vacuum Collapse

$$\frac{\mathrm{d}n_{\mathrm{fv}}}{\mathrm{d}R_r^0} \approx \frac{I_*^4\beta^4}{192 v_w^3} e^{(4\beta R_r^0/v_w) - I_* e^{\beta R_r^0/v_w}} \left(1 - e^{-I_* e^{\beta R_r^0/v_w}}\right)$$

See Lu, Kawana, Xie [2202.03439]
PRD 105, 123503



(based on geometric estimator for the FV "spherical" volume distribution)

$$\frac{\mathrm{d}n_{\mathrm{PBH}}}{\mathrm{d}M} = \frac{\mathrm{d}n_{\mathrm{fv}}}{\mathrm{d}R_0}\left(\frac{\mathrm{d}M}{\mathrm{d}R_0}\right)^{-1}$$

$$M \approx \frac{4\pi}{3} R(t_{\mathrm{col}})^3 \rho_c(T_{\mathrm{PT}})$$

(but this is not the end of the story, more on this later…)

PBH Abundance, Evaporation and Hawking Spectra

Use `BlackHawk` for the computation of PBH mass and Hawking spectra
→
Convolve this with the FV fraction distribution to get dn/dM and photon sky

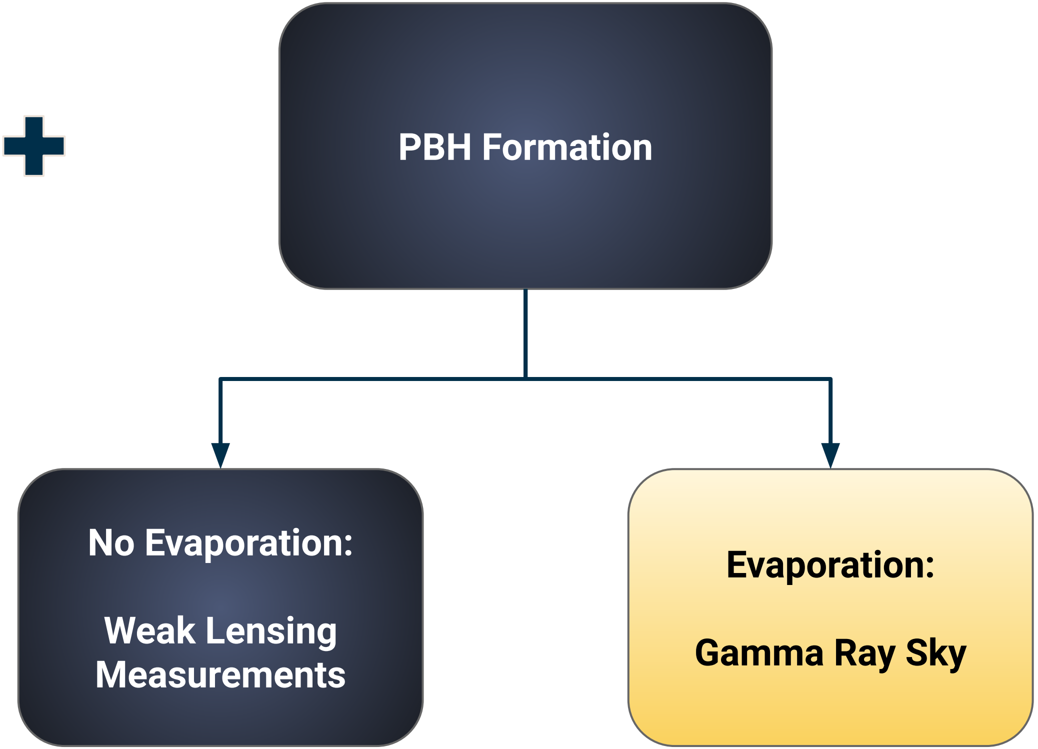
Observatories, past, present and future:

- Gamma-ray sky:
  - Fermi-LAT
  - AMEGO
  - NuStar
  - Chandra
  - COMPTEL
  - …
- Microlensing BH searches:
  - Subaru HSC
  - Roman
  - …

...a Multi-messenger Approach!

Gravitational Wave Signal: f and $h^2\Omega$

+

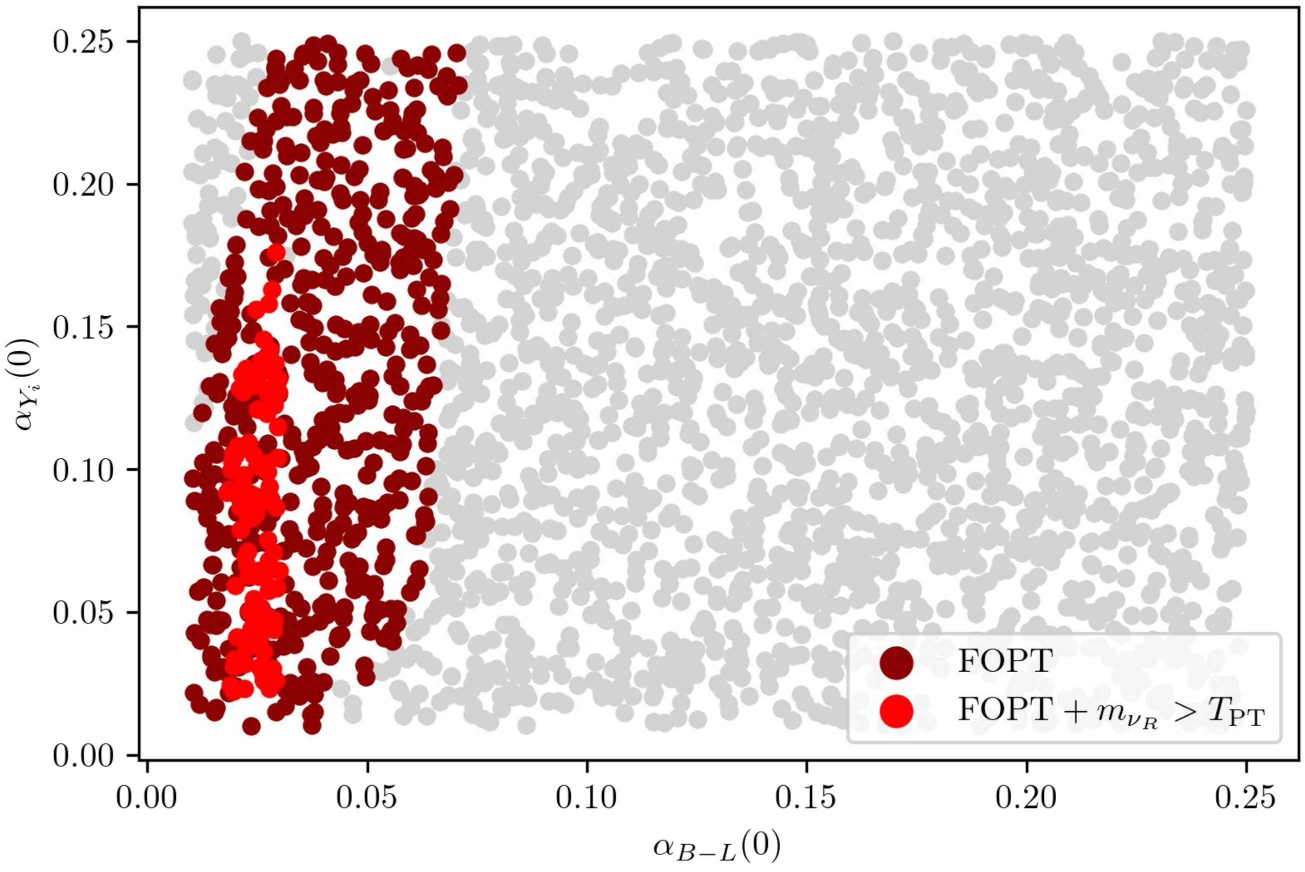


Tools:

- CosmoTransitions for determining bounce action solution
- BlackHawk for PBH mass and Hawking spectra

See also: Marfatia, Tseng [2112.14588] (*JHEP 08 2022*)
+ Marfatia, Tseng [2107.00859] (JHEP 11 2021)

Where does the FOPT happen? Where are the Black Holes?



We scan over the model parameter space and check each point to see if:

- a strong FOPT is supported
- the effective RH neutrino mass is heavy enough to be trapped and form PBH

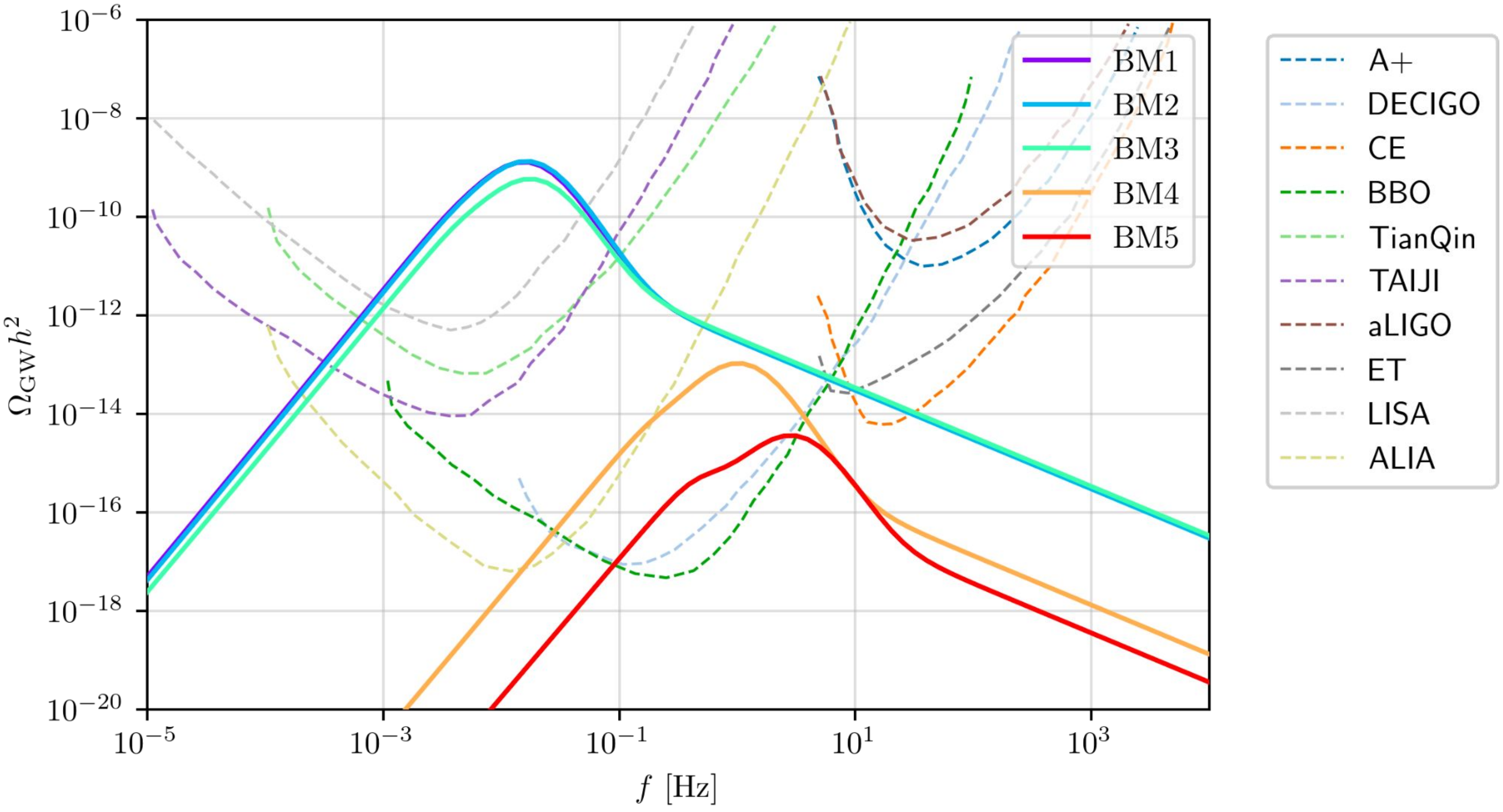
Some benchmark points in model parameter space

	$\alpha_{B-L}(0)$	$\alpha_{Y_i}(0)$	$T_{\text{PT}}/\langle\Phi\rangle$	α	$\beta/H(T_{\text{PT}})$
BM1	1.857×10^{-2}	9.368×10^{-2}	8.694×10^{-2}	7.869×10^{-1}	9.228×10^{1}
BM2	1.998×10^{-2}	1.149×10^{-1}	8.671×10^{-2}	8.194×10^{-1}	9.660×10^{1}
BM3	2.332×10^{-2}	1.503×10^{-1}	1.006×10^{-1}	5.451×10^{-1}	9.021×10^{1}
BM4	3.682×10^{-2}	1.444×10^{-1}	3.075×10^{-1}	4.766×10^{-2}	8.664×10^{2}
BM5	4.507×10^{-2}	1.421×10^{-1}	3.953×10^{-1}	3.231×10^{-2}	1.460×10^{3}

Strong PTs can occur

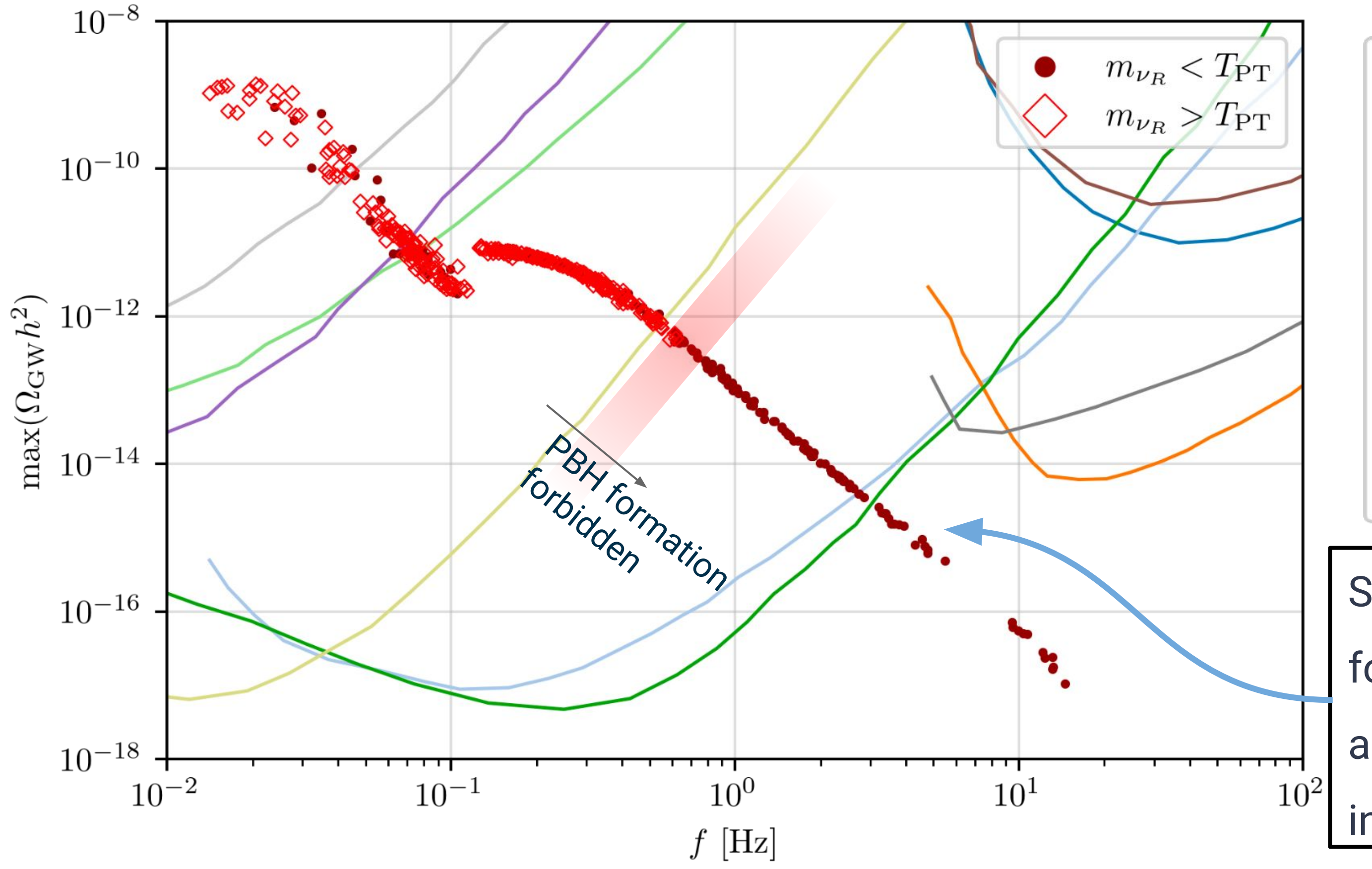
Fast transitions

Some benchmark points: The GW spectrum from B–L



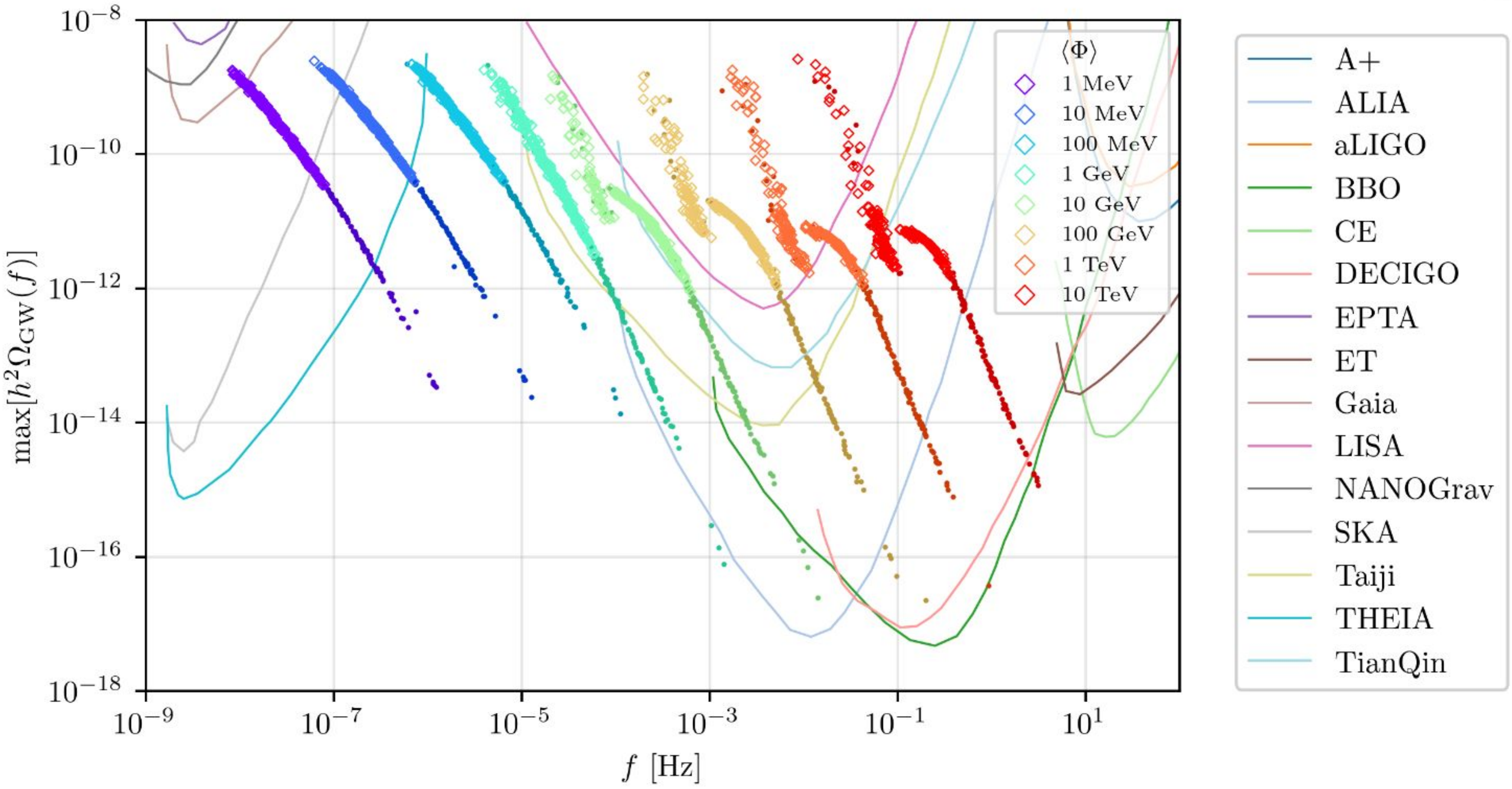
Among those strong FOPTs, where are the PBHs?

$\langle \boldsymbol{\Phi} \rangle$=10 TeV

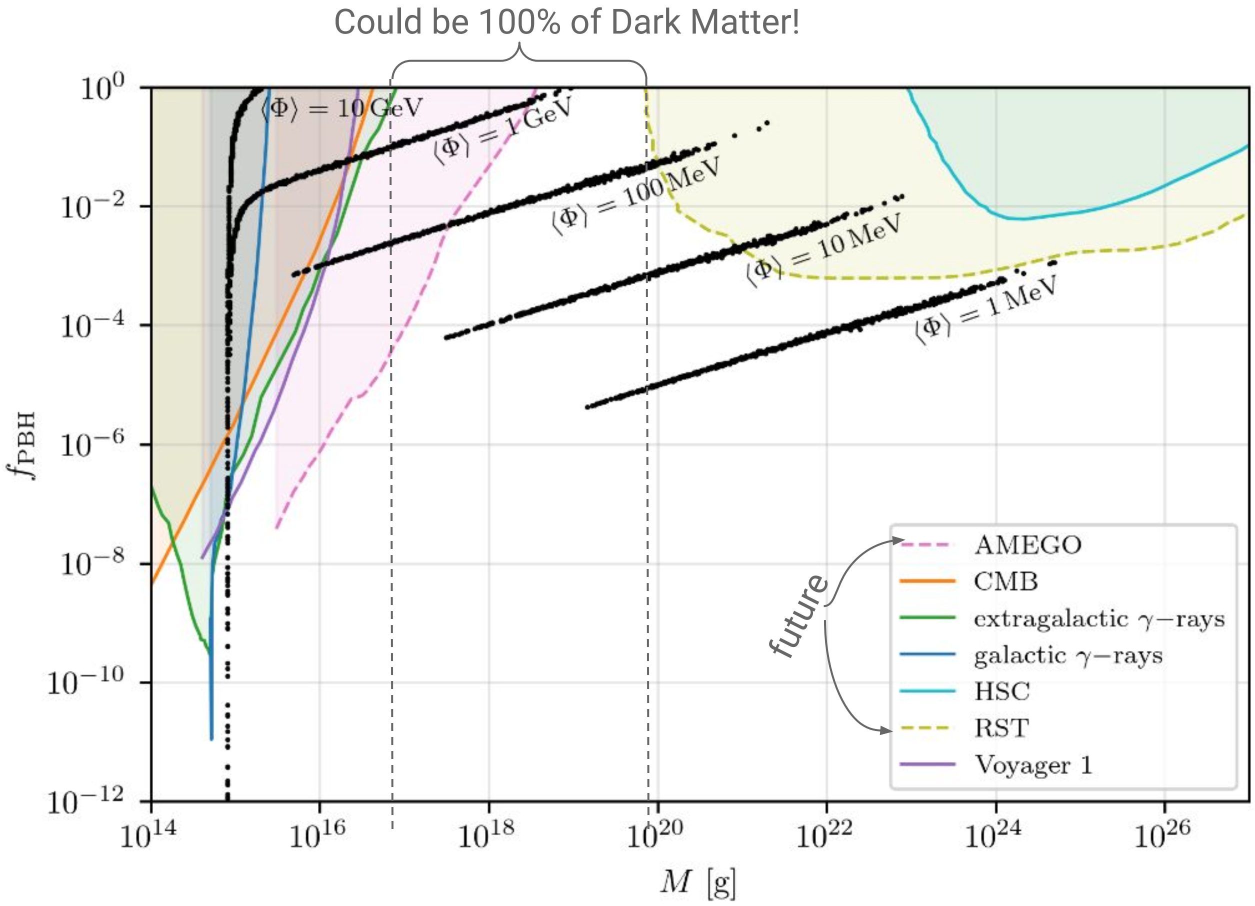


Scanning over $\alpha_{B\text{-}L}$ and α_Y for a fixed VEV produces a well-defined trajectory in the GW phase space

Scan over many VEVs!

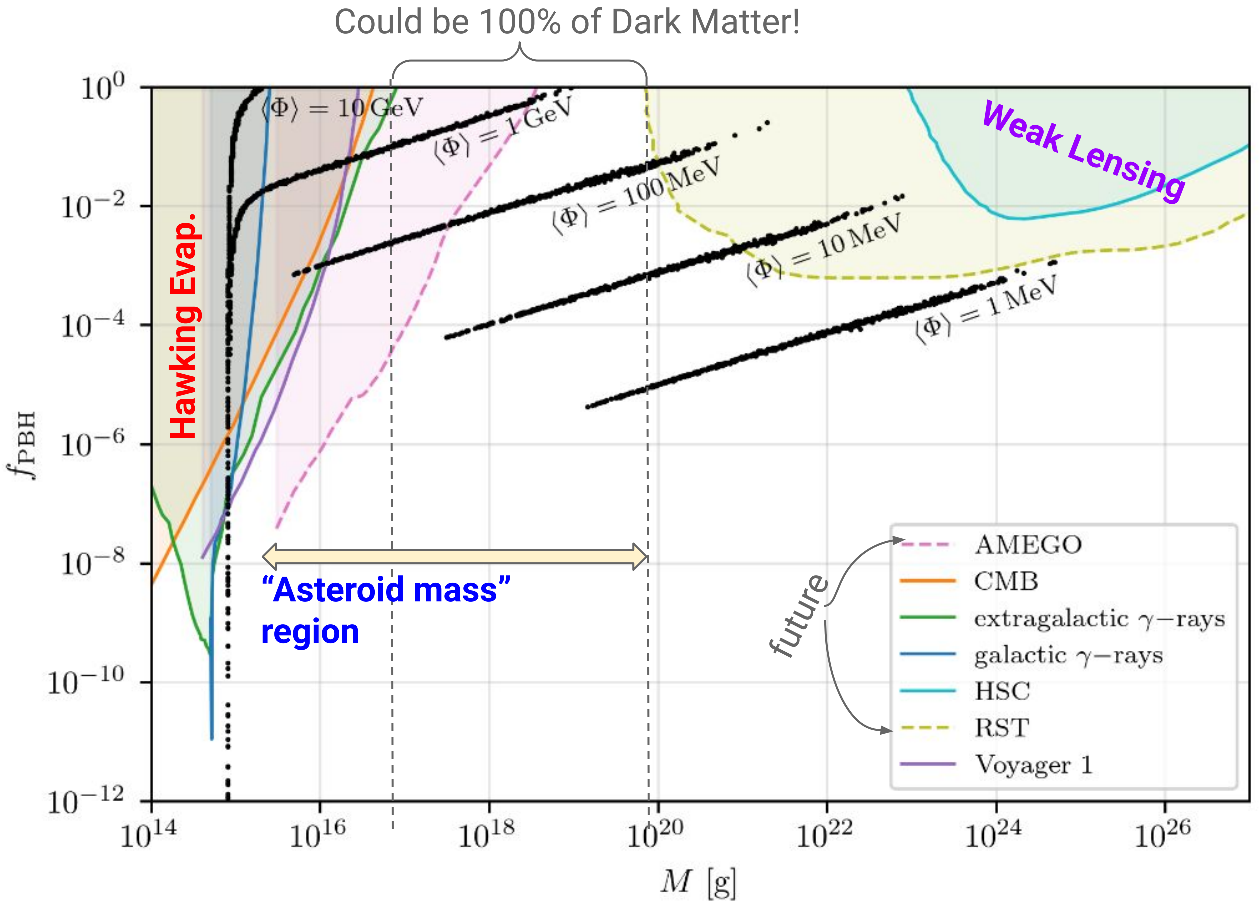


PBH Formation: Scanning over the B–L breaking scale



- We scan over the *B-L* parameters at fixed VEVs
- At ~10^15 g PBH masses, they would all evaporate by today
- For VEVs around 10 GeV, projections for AMEGO telescope's sensitivity can…
- For smaller VEVs, Roman space telescope can discover PBHs from weak lensing effects

PBH Formation: Scanning over the *B*–*L* breaking scale



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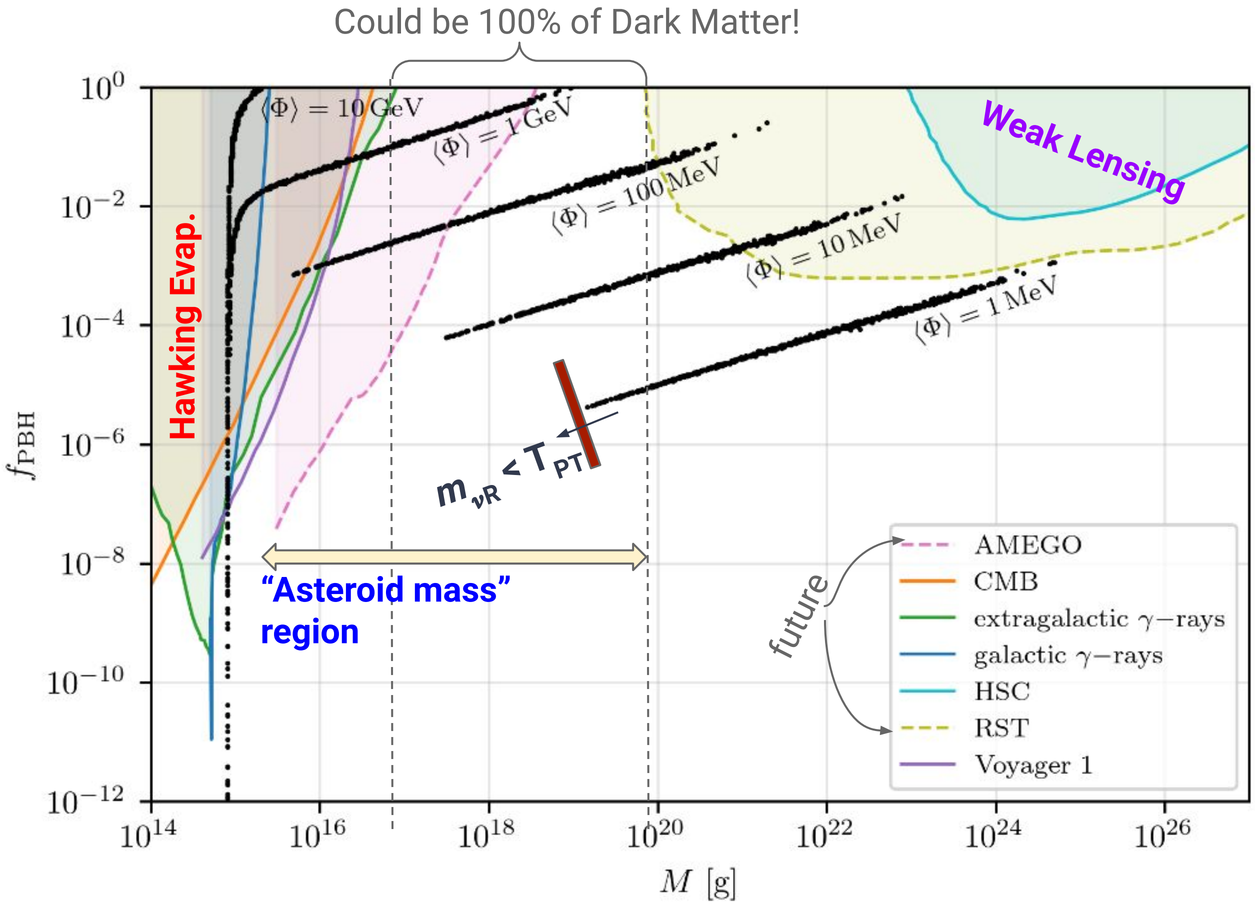

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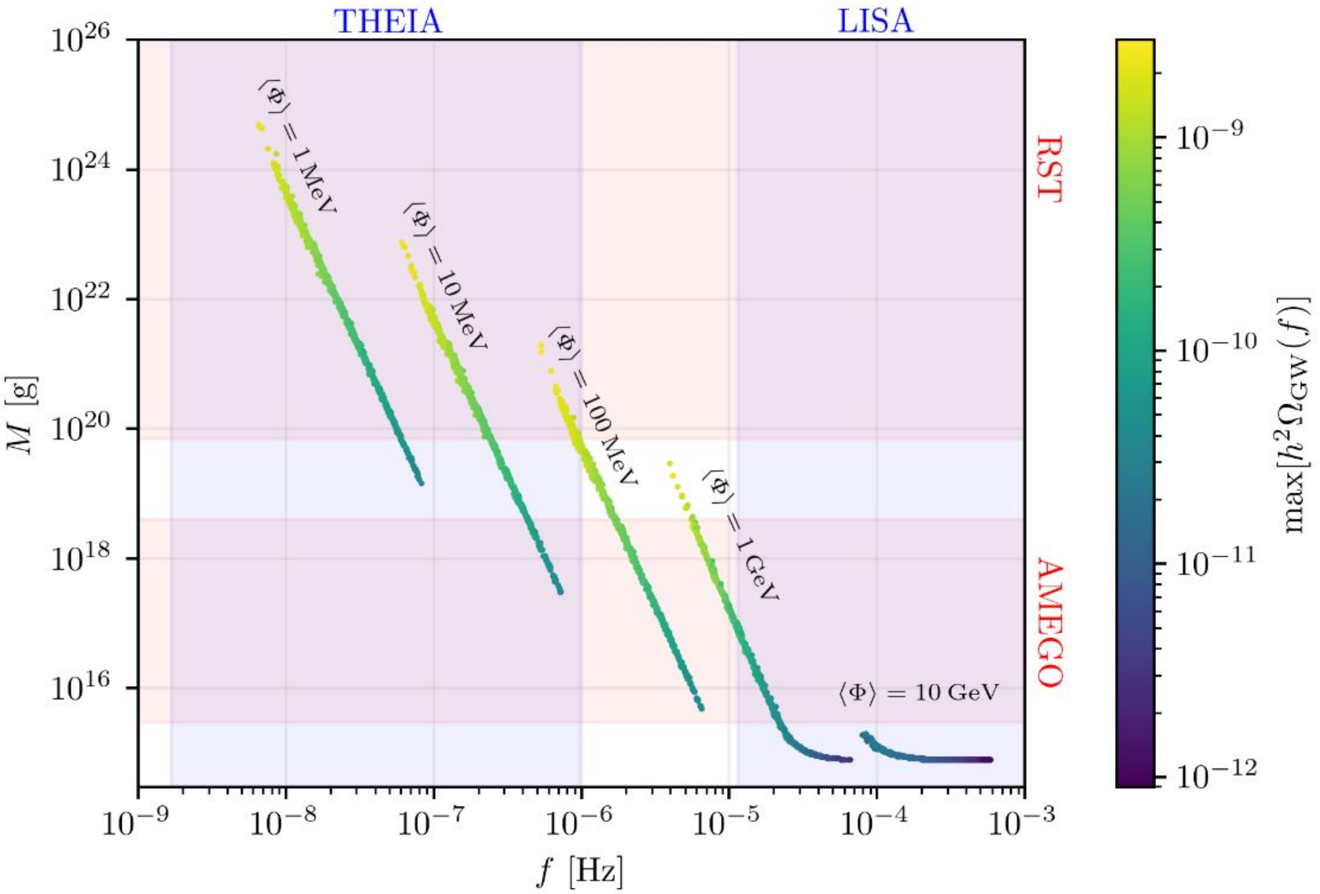
A Bird's Eye View of the Multi-messenger Phase Space



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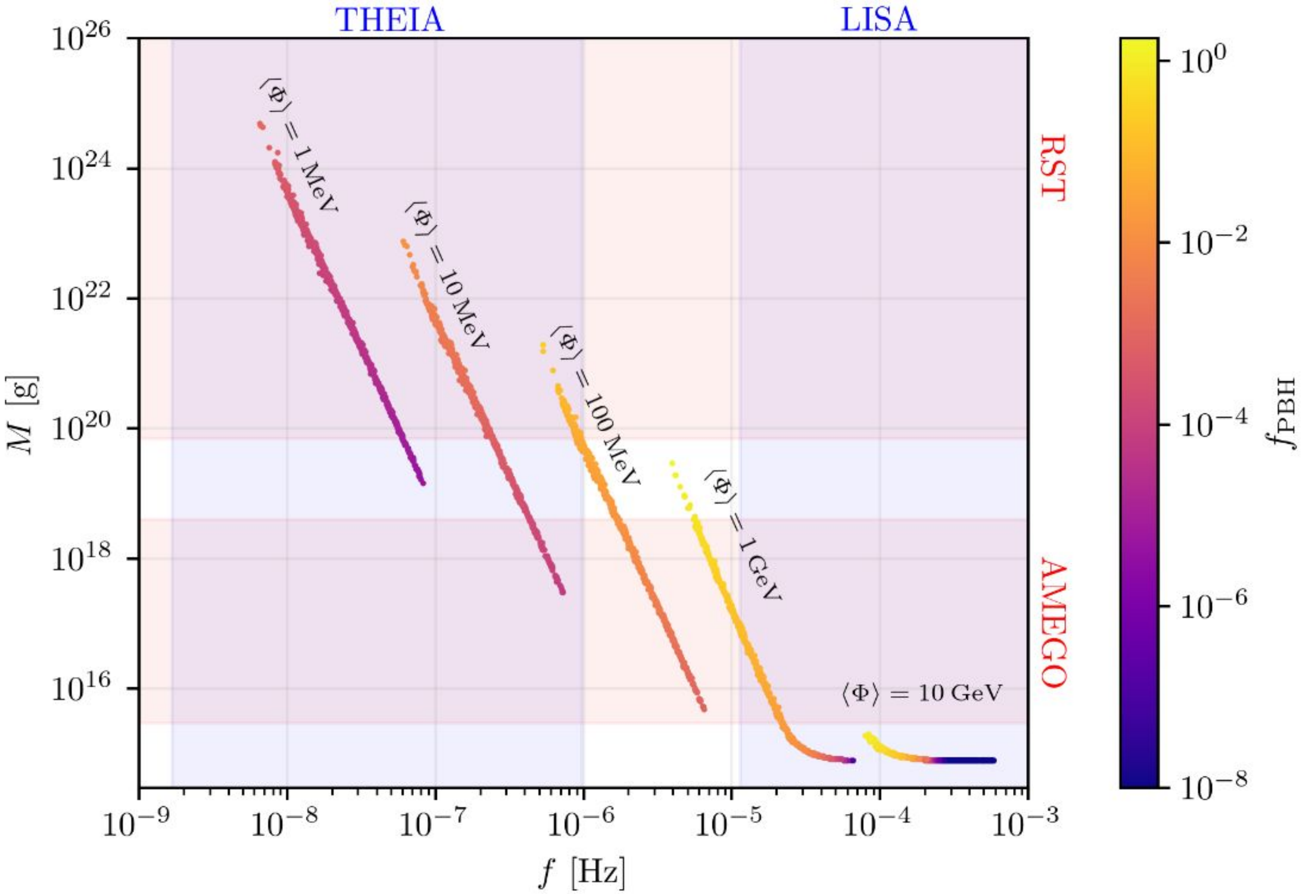


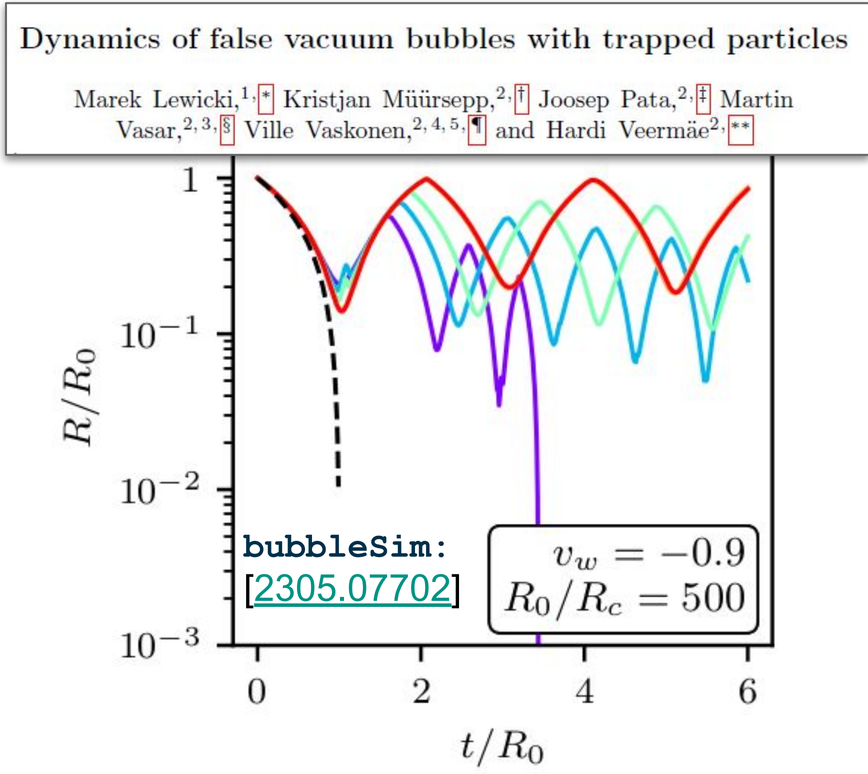
Ongoing work: is PBH formation trivial? (No)

$$M = E_{\text{bubble}} + E_{\text{particles}}(r < R)$$

$$E_{\text{bubble}} = \frac{4\pi}{3} R^3 \Delta V + \frac{4\pi R^2 \sigma}{\sqrt{1 - \dot{R}^2}}$$

$$\ddot{R} + 2\frac{1 - \dot{R}^2}{R} = \frac{(1 - \dot{R}^2)^{3/2}}{\sigma} (-\Delta V + \Delta P)$$

Including the **vacuum density, surface tension**, and **pressure** from particle interaction across the wall can sometimes lead to **bounce solutions where collapse is prevented**



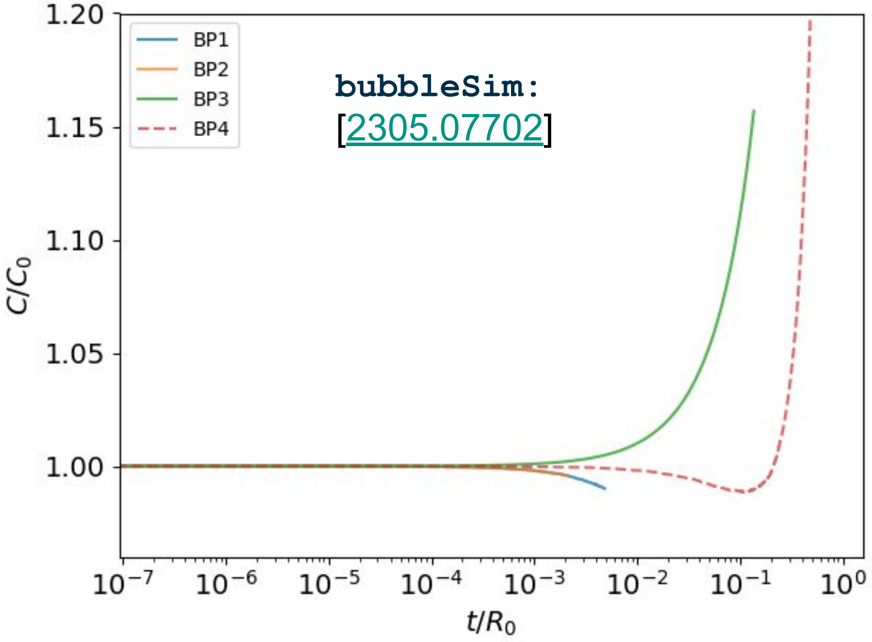
Ongoing work: is PBH formation trivial? (No)

$$M = E_{\rm bubble} + E_{\rm particles}(r < R)$$

$$E_{\rm bubble} = \frac{4\pi}{3} R^3 \Delta V + \frac{4\pi R^2 \sigma}{\sqrt{1 - \dot{R}^2}}$$

$$\ddot{R} + 2\frac{1-\dot{R}^2}{R} = \frac{(1-\dot{R}^2)^{3/2}}{\sigma}(-\Delta V + \Delta P)$$

Including the **vacuum density, surface tension**, and **pressure** from particle interaction across the wall can sometimes lead to **bounce solutions where collapse is prevented**



Outlook: Good

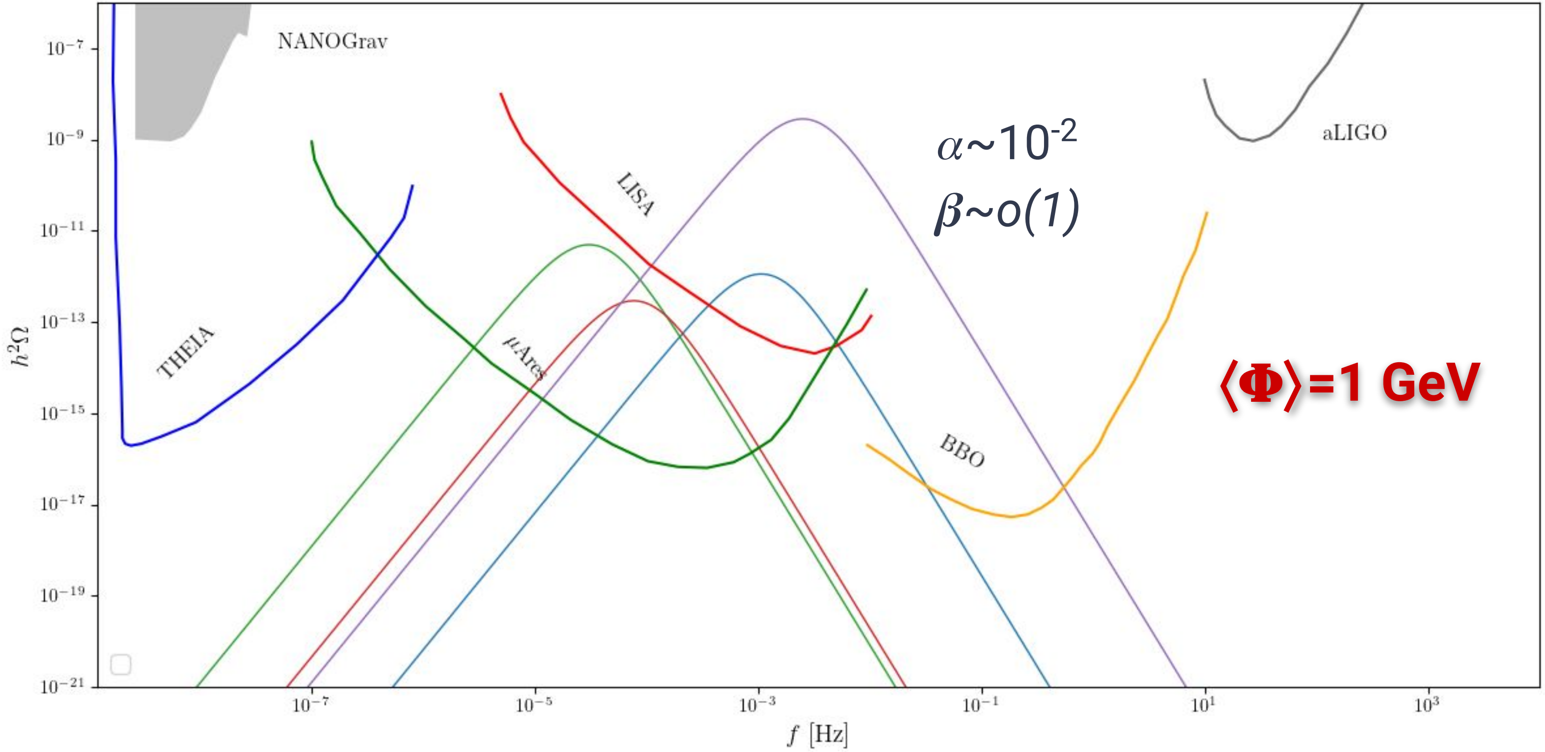
- PTs connect to a broad range of **new physics scales**
- Multi-messenger approach gives a more predictive model space:
  - **Gravitational Waves**
  - **PBH dark matter**
  - **Hawking evaporation**
- Current and upcoming GW observatories, gamma ray telescopes and weak lensing experiments together have **many things to say about new physics in the early cosmos**

Backup Deck

Example GW Spectra: ⟨Φ⟩=1 GeV

$$V(\phi, T) = D(T^2 - T_0^2)\phi^2 - (AT + C)\phi^3 + \frac{\lambda}{4}\phi^4$$

We fix the VEV and numerically scan over *D, A, C,* and λ



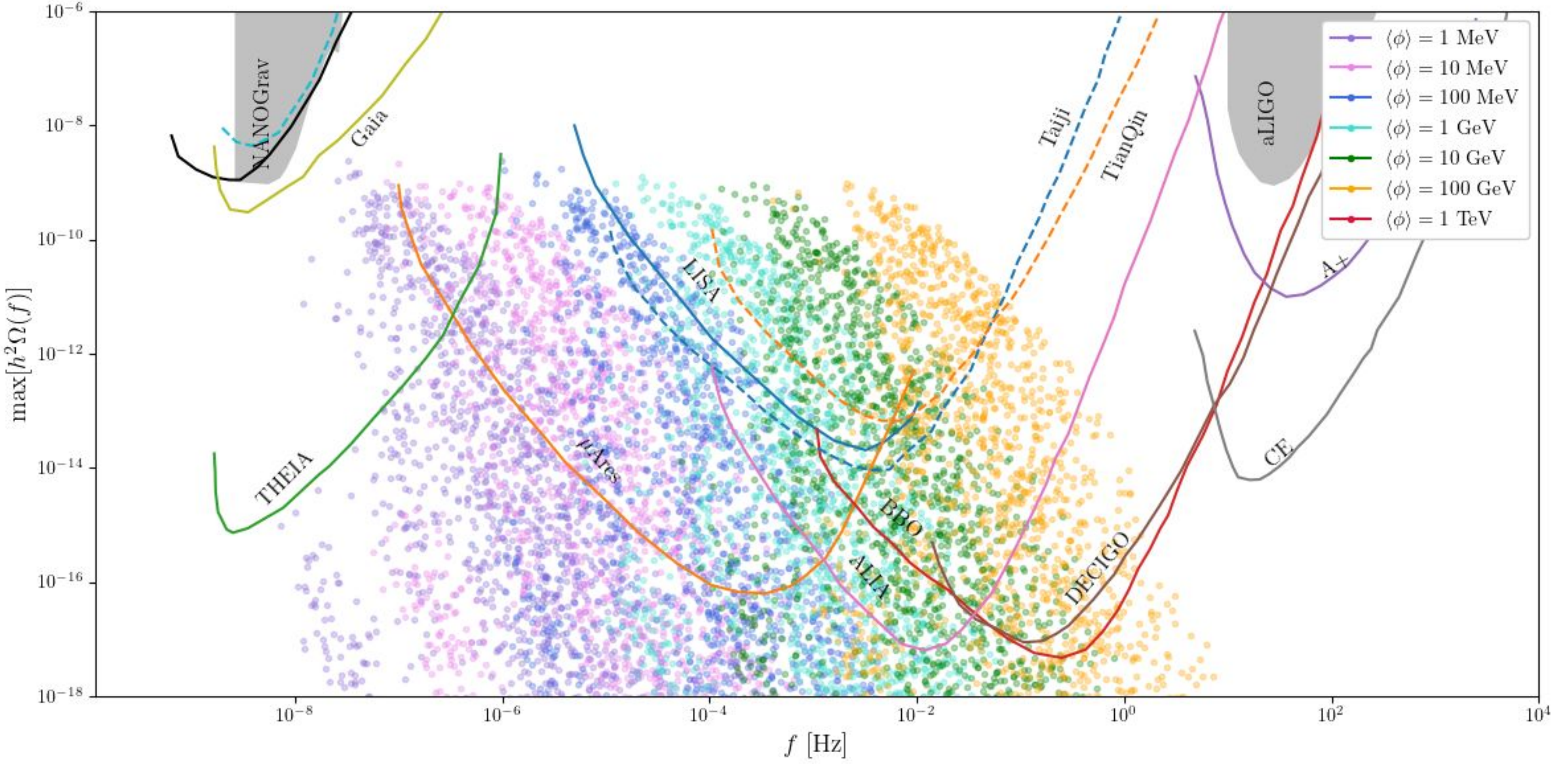
α~10^{-2}

β~*o(1)*

⟨Φ⟩=1 GeV

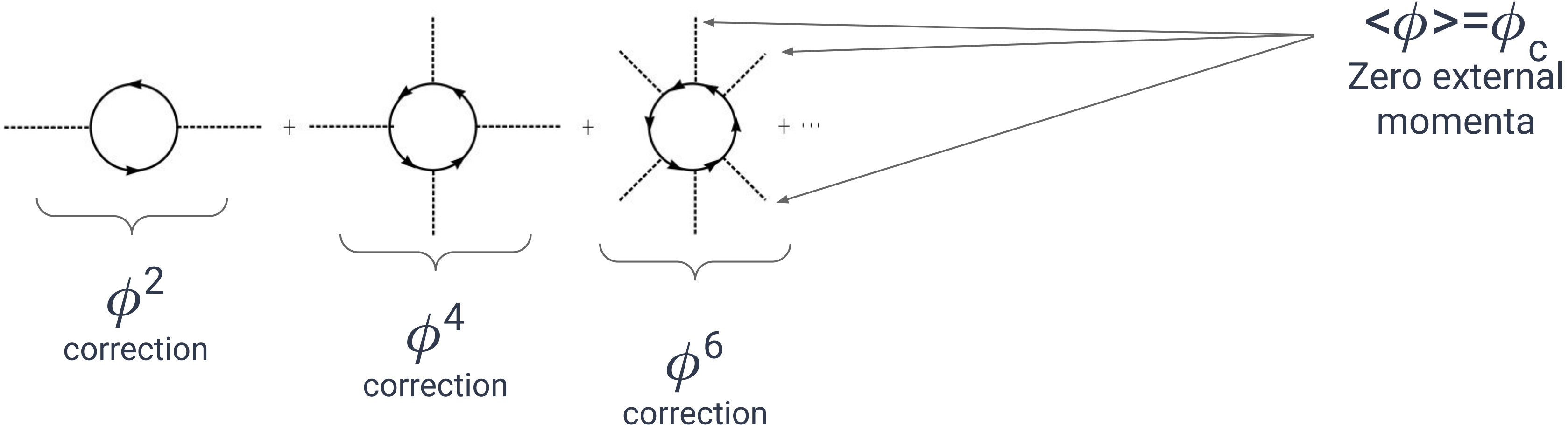
Parameter Scans: GWs from a Generic potential

$$V(\phi, T) = D(T^2 - T_0^2)\phi^2 - (AT + C)\phi^3 + \frac{\lambda}{4}\phi^4$$



How do we calculate the finite temperature effects?

The answer: loops!

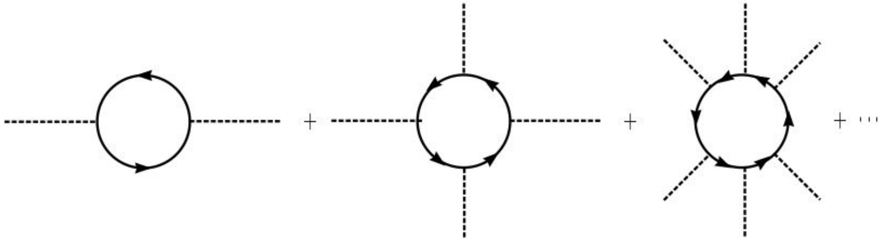


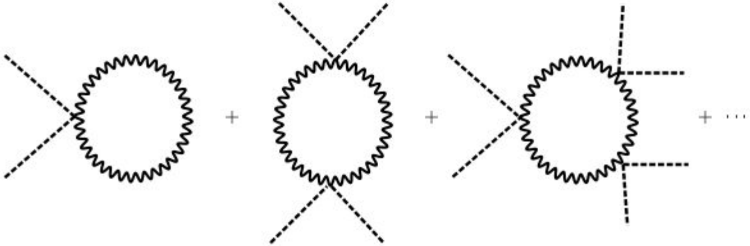
M. Quiros, ICTP Lecture Notes, 1999

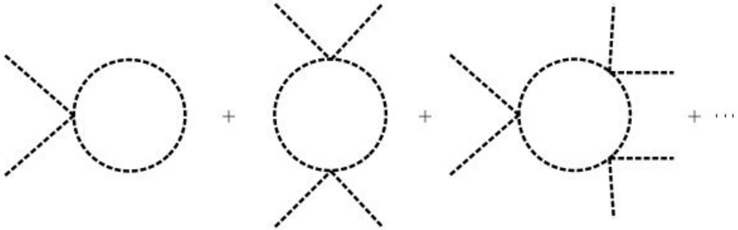
How do we calculate the finite temperature effects?

The answer: loops!

We add up all the corrections from all the field contact that our scalar field interacts with (including itself)



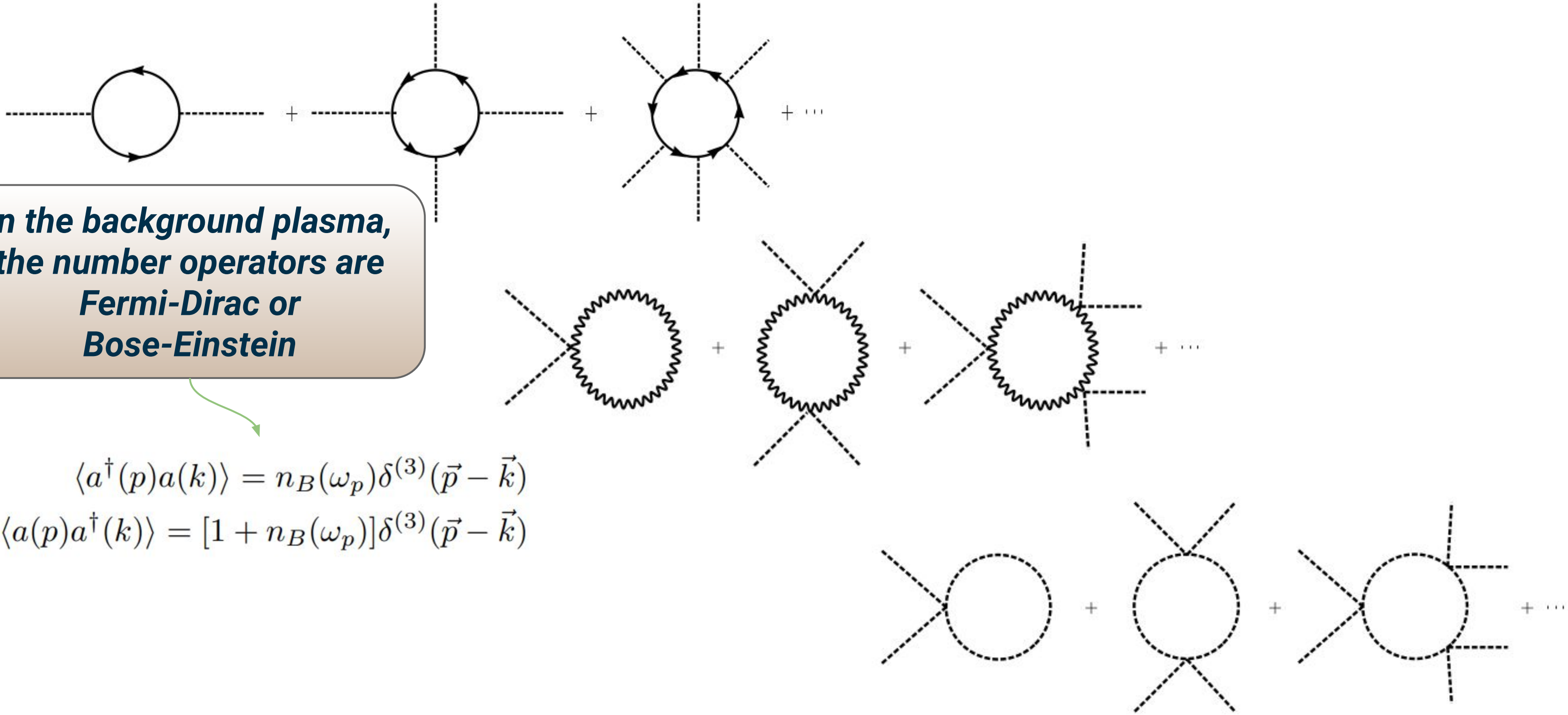




How do we calculate the finite temperature effects?

$\beta = 1/T$

The answer: loops!



In the background plasma, the number operators are Fermi-Dirac or Bose-Einstein

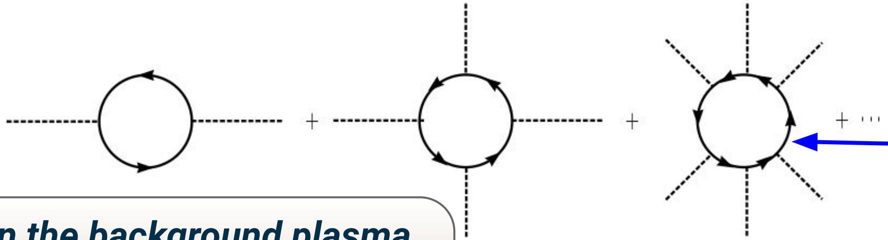
$$\langle a^\dagger(p) a(k) \rangle = n_B(\omega_p) \delta^{(3)}(\vec{p} - \vec{k})$$

$$\langle a(p) a^\dagger(k) \rangle = [1 + n_B(\omega_p)] \delta^{(3)}(\vec{p} - \vec{k})$$

How do we calculate the finite temperature effects?

$\beta = 1/T$

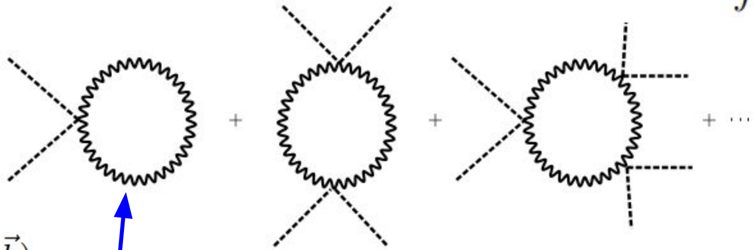
The answer: loops!



$$n_F(\omega) = \frac{1}{e^{\beta\omega} + 1}$$

$$G(\tau, \vec{x}) = \int \frac{d^4p}{(2\pi)^4} \rho(p) e^{i\vec{p}\vec{x}} e^{-\tau p^0} \left[\theta(\tau) + \eta n(p^0)\right]$$

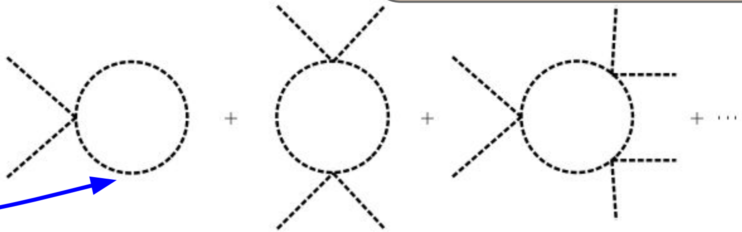
In the background plasma, the number operators are Fermi-Dirac or Bose-Einstein



The propagators are affected by the Bosonic or Fermionic number operators (thermal distributions)

$$\langle a^\dagger(p) a(k) \rangle = n_B(\omega_p) \delta^{(3)}(\vec{p} - \vec{k})$$

$$\langle a(p) a^\dagger(k) \rangle = [1 + n_B(\omega_p)] \delta^{(3)}(\vec{p} - \vec{k})$$



$$n_B(\omega) = \frac{1}{e^{\beta\omega} - 1}$$

Anatomy of a Finite-T Potential: Scalar + massive Dirac fermion

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\mu^2}{2}\phi^2 - \frac{c}{3!}\phi^3 - \frac{\lambda}{4!}\phi^4 + \bar{\chi}(i\partial\!\!\!/ - m_\chi)\chi - g_\chi \phi \bar{\chi}\chi$$

bosonic thermal correction

$$\begin{aligned} V_{\text{eff}}(\phi, T) &= V_{\text{tree}} + V_{1,T}^{\text{ferm.}} + V_{1,T}^{\text{scal.}} + V_{ct} \\ &= \delta\Omega + \delta P \phi + \frac{\mu^2 + \delta\mu}{2}\phi^2 + \frac{c + \delta c}{3!}\phi^3 + \frac{\lambda + \delta\lambda}{4!}\phi^4 \\ &+ \frac{1}{64\pi^2}\mu^4(\phi)\left[\log\left(\mu^2(\phi)\right) - \frac{3}{2}\right] + \frac{T^4}{2\pi^2} J_B[\mu^2(\phi)/T^2] \\ &- \frac{1}{16\pi^2} m_\chi^4(\phi)\left[\log\left(m_\chi^2(\phi)\right) - \frac{3}{2}\right] - \frac{2}{\pi^2} T^4 J_F[m_\chi^2(\phi)/T^2] \end{aligned}$$

Renormalization counter-terms

Scalar 1-loop, T=0 correction

Fermion 1-loop, T=0 correction

Fermion thermal correction

Hawking Spectra from PBH Evaporation: Today's Gamma Ray Sky

$$n_\gamma(E_{\gamma,0}) = \int_{t_{\rm CMB}}^{\min(t_e,t_0)} \mathrm{d}t \int_{E_\gamma-\delta E_\gamma}^{E_\gamma+\delta E_\gamma} \mathrm{d}E \left[\frac{a(t_0)}{a(t)}\right]^3 \frac{\partial^2 n_\gamma^{\rm co}}{\partial t\,\partial E_\gamma}(E)$$
$$\approx \int_{t_{\rm CMB}}^{\min(t_e,t_0)} \mathrm{d}t\, E_\gamma \left[\frac{a(t_0)}{a(t)}\right]^3 \frac{\partial^2 n_\gamma^{\rm co}}{\partial t\,\partial E_\gamma}(E_\gamma)$$
$$\approx E_{\gamma,0} \int_{t_{\rm CMB}}^{\min(t_e,t_0)} \mathrm{d}t \left[\frac{a(t_0)}{a(t)}\right]^4 \frac{\partial^2 n_\gamma^{\rm co}}{\partial t\,\partial E_\gamma}\left(E_{\gamma,0}\left[\frac{a(t_0)}{a(t)}\right]\right).$$

$$4\pi E_{\gamma,0}^2 \frac{\mathrm{d}I_\gamma(E_{\gamma,0})}{\mathrm{d}E_{\gamma,0}} = c\, E_{\gamma,0}^2 \frac{\mathrm{d}n_\gamma}{\mathrm{d}E_{\gamma,0}}(E_{\gamma,0})$$