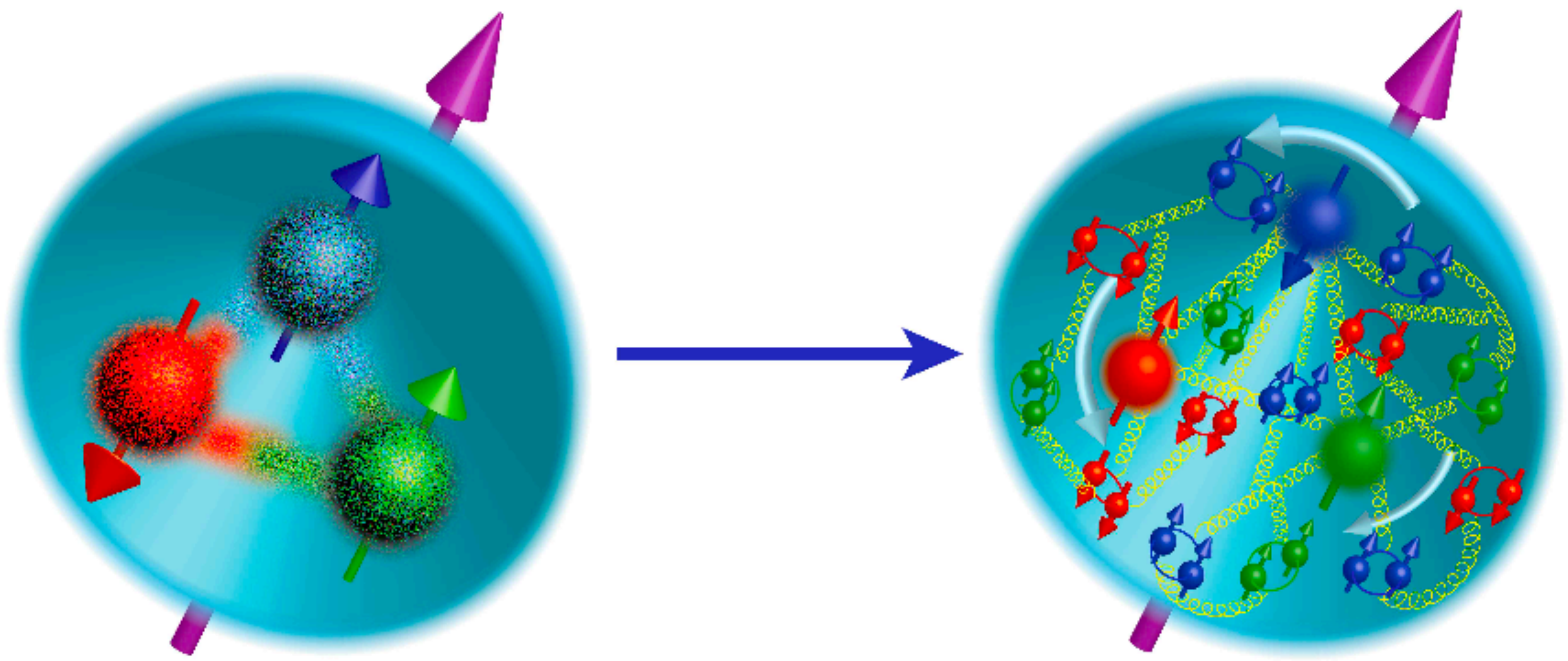
Parton Distributions from Boosted Fields in the Coulomb Gauge

Xiang Gao
Argonne National Laboratory

Theory Seminar @ Fermilab, July 11th, 2024

Argonne NATIONAL LABORATORY

The internal structure of nucleon



- Strong interactions of fast-moving quarks and gluons, described by QCD.

Origin of mass & spin?

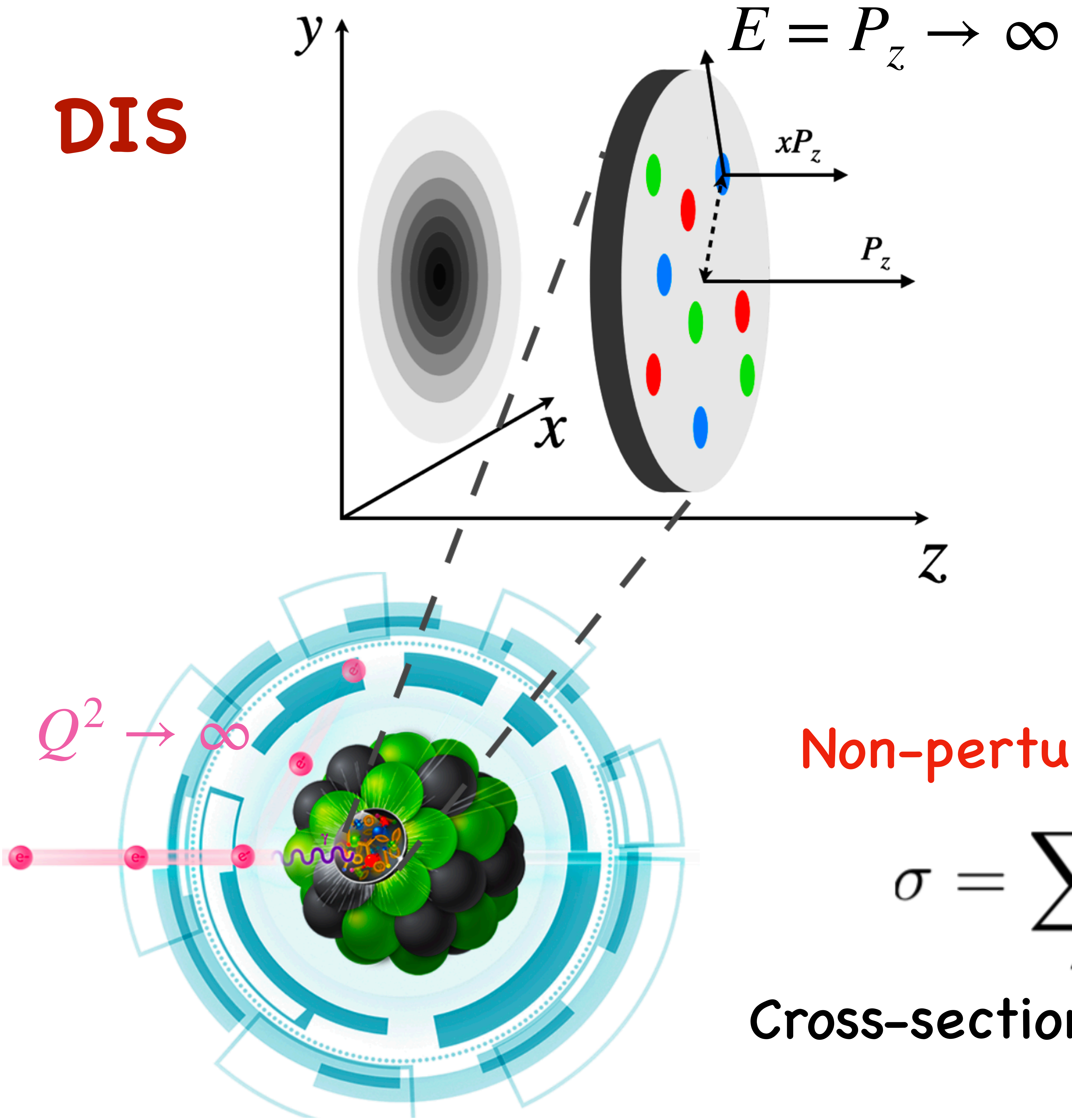
Quark & gluon distributions?

Three-dimensional imaging?

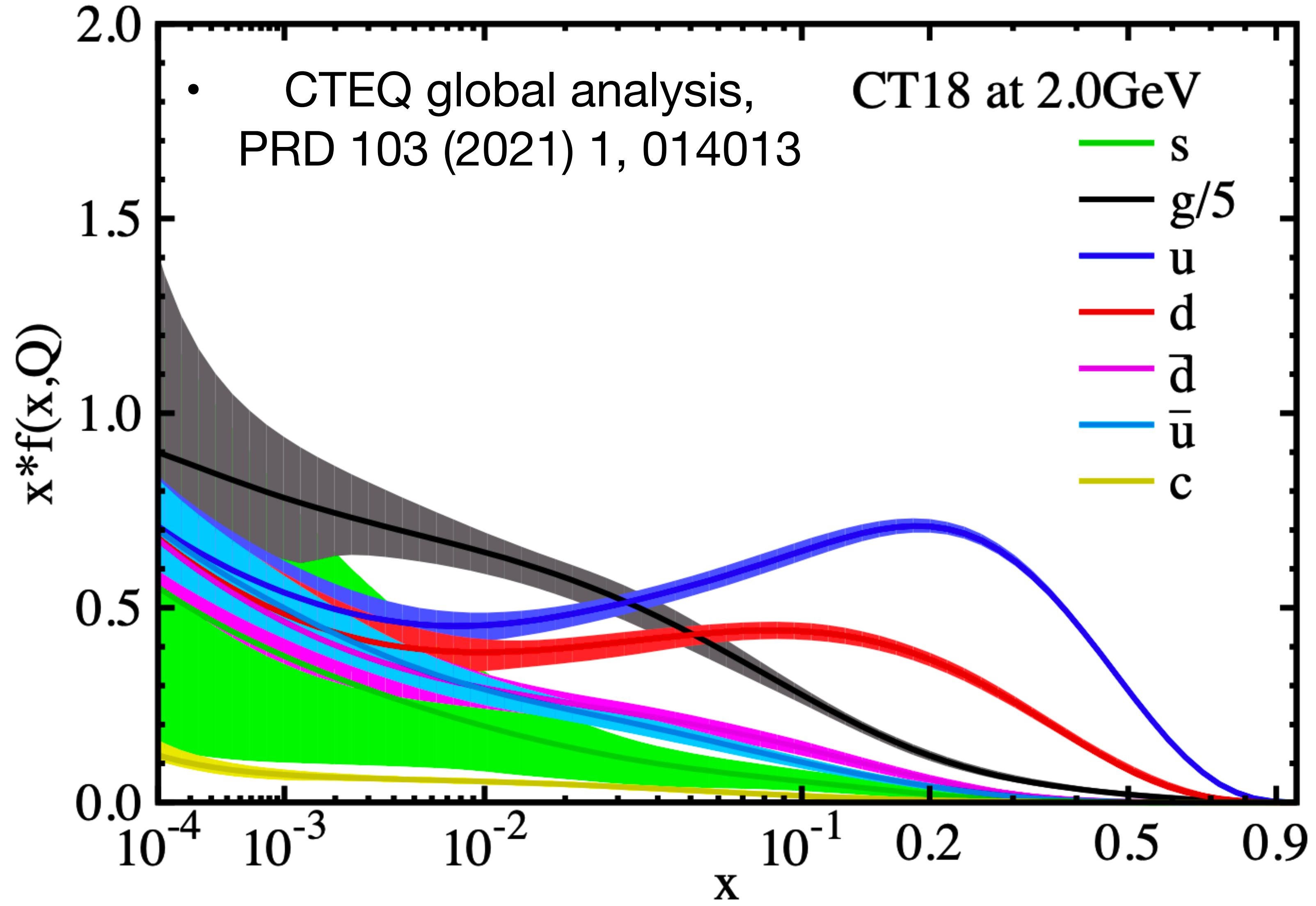
...

3 Nucleon in the eyes of the light — partonic snapshot

DIS



- The possibility to find a parton carrying momentum fraction x.



Non-perturbative parton distributions

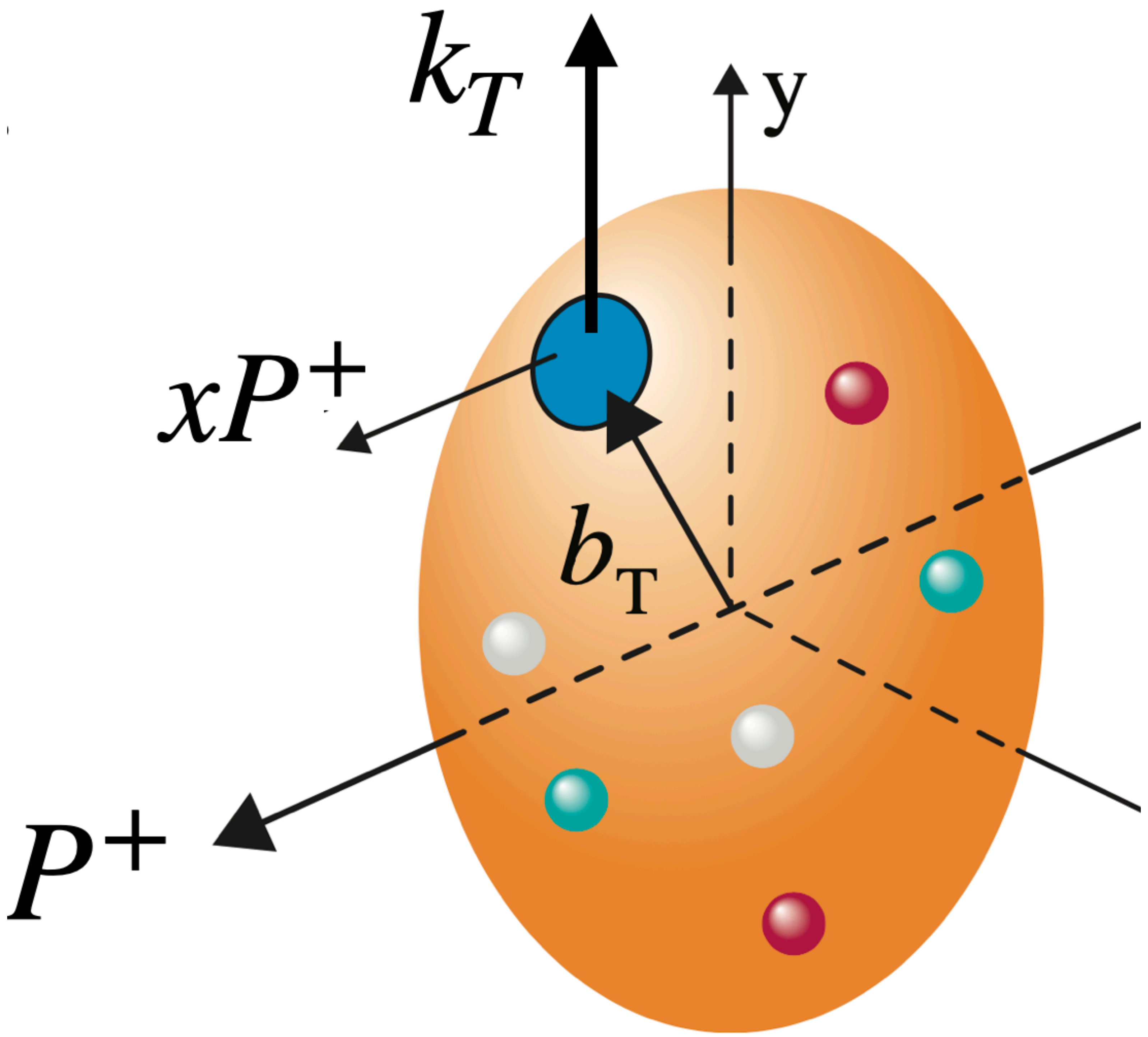
$$\sigma = \sum_i f_i(x, Q^2) \circledast \sigma\{eq_i(xP) \to eq_i(xP+q)\}$$

Cross-section

Perturbative hard process

$\alpha_s(Q^2) \to 0$

The multi-dimensional imaging of nucleon



$W(x, b_T, k_T)$ **5D**

Wigner Distributions

$\int d^2b_T$ ↙ ↘ $\int d^2k_T$

$f(x, k_T)$ — Transverse momentum distributions (**TMDs**)

$H(x, b_T)$ — Impact parameter distribution

F.T. ⟷

$H(x, \xi, t)$ — Generalized parton distributions (**GPDs**) **3D**

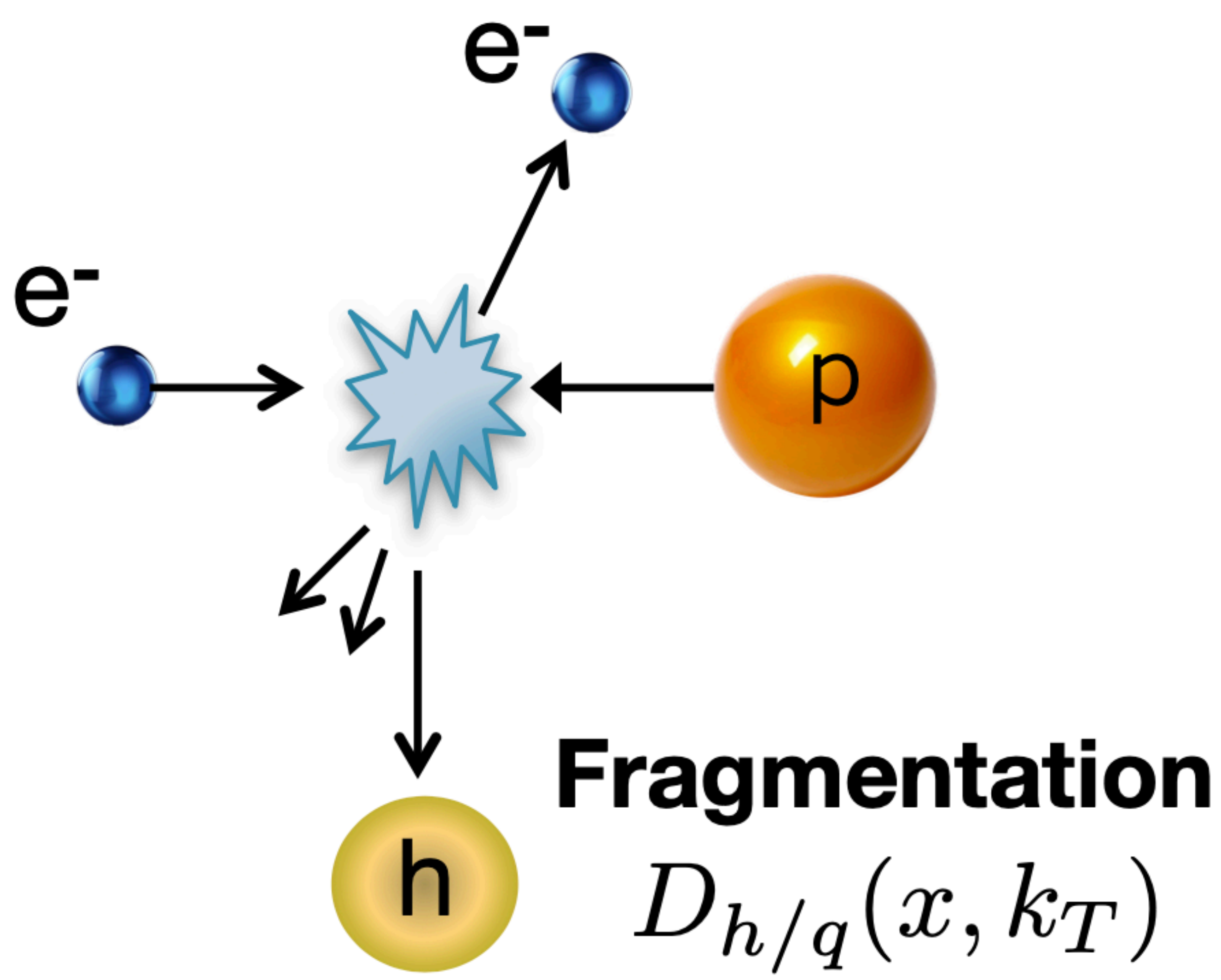
$\int d^2k_T$ ↘ ↙ $\int d^2b_T$

$f(x)$ — Parton distribution functions (**PDFs**) **1D**

5 TMDs from global analyses of experimental data

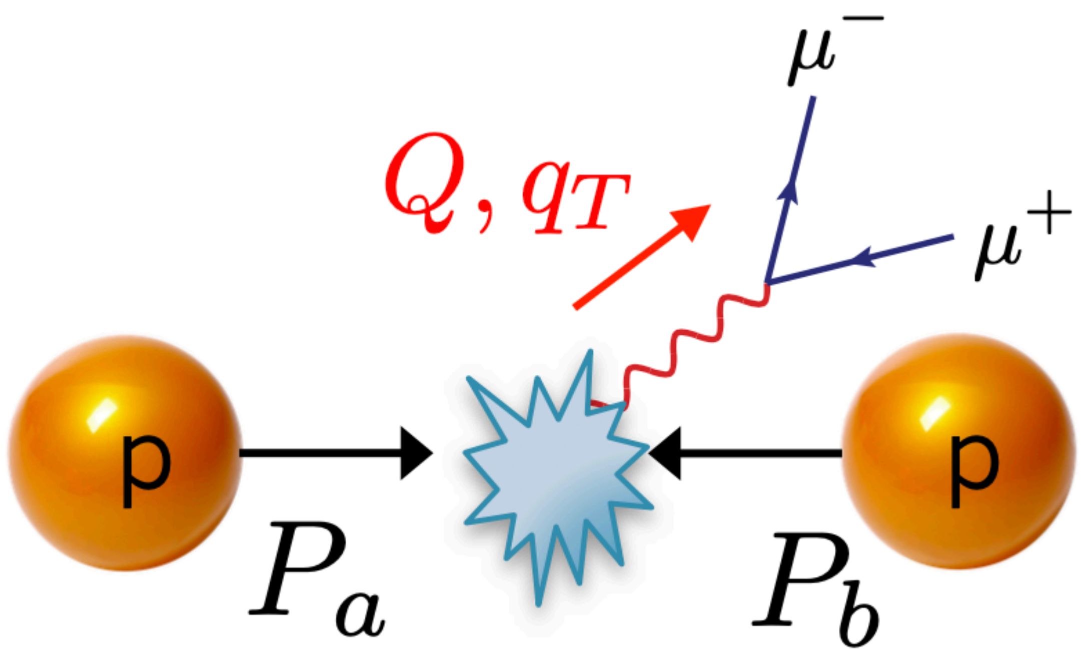
Semi-Inclusive DIS

$$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(x, k_T)$$



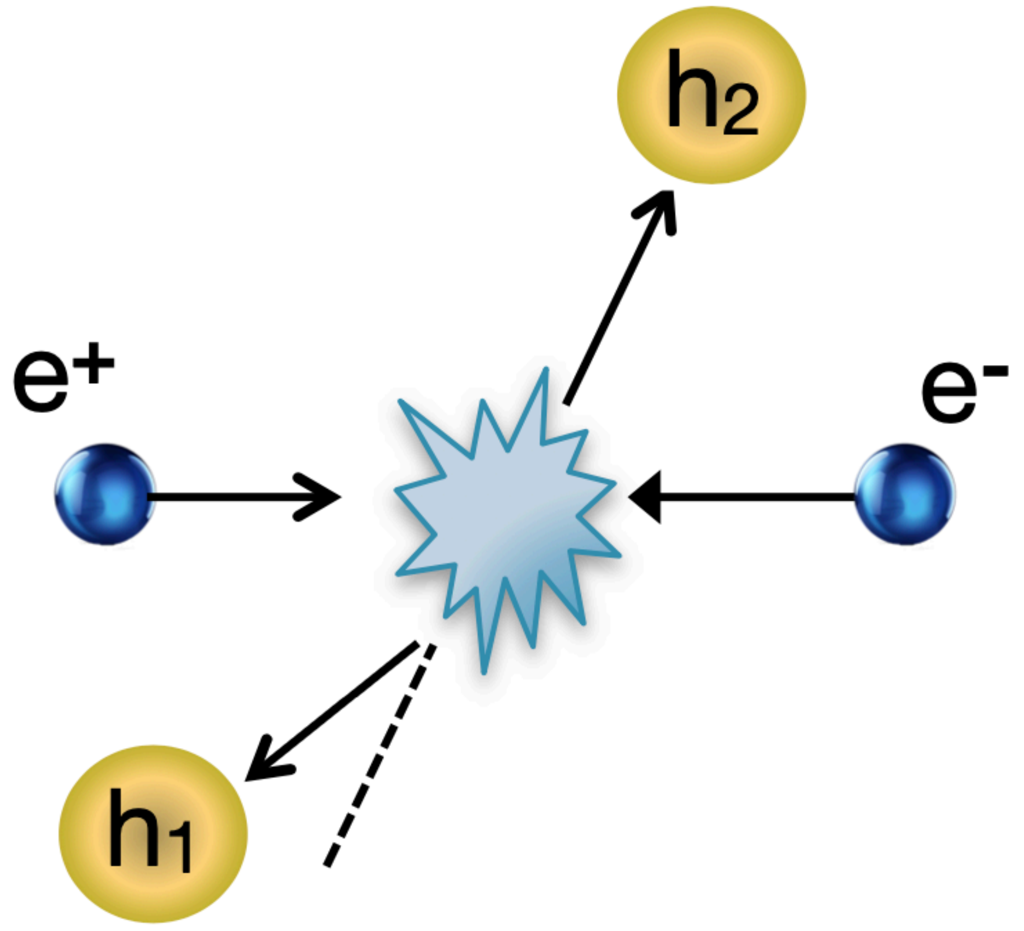
Drell-Yan

$$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$$



Dihadron in e+e-

$$\sigma \sim D_{h_1/q}(x, k_T) D_{h_2/q}(x, k_T)$$



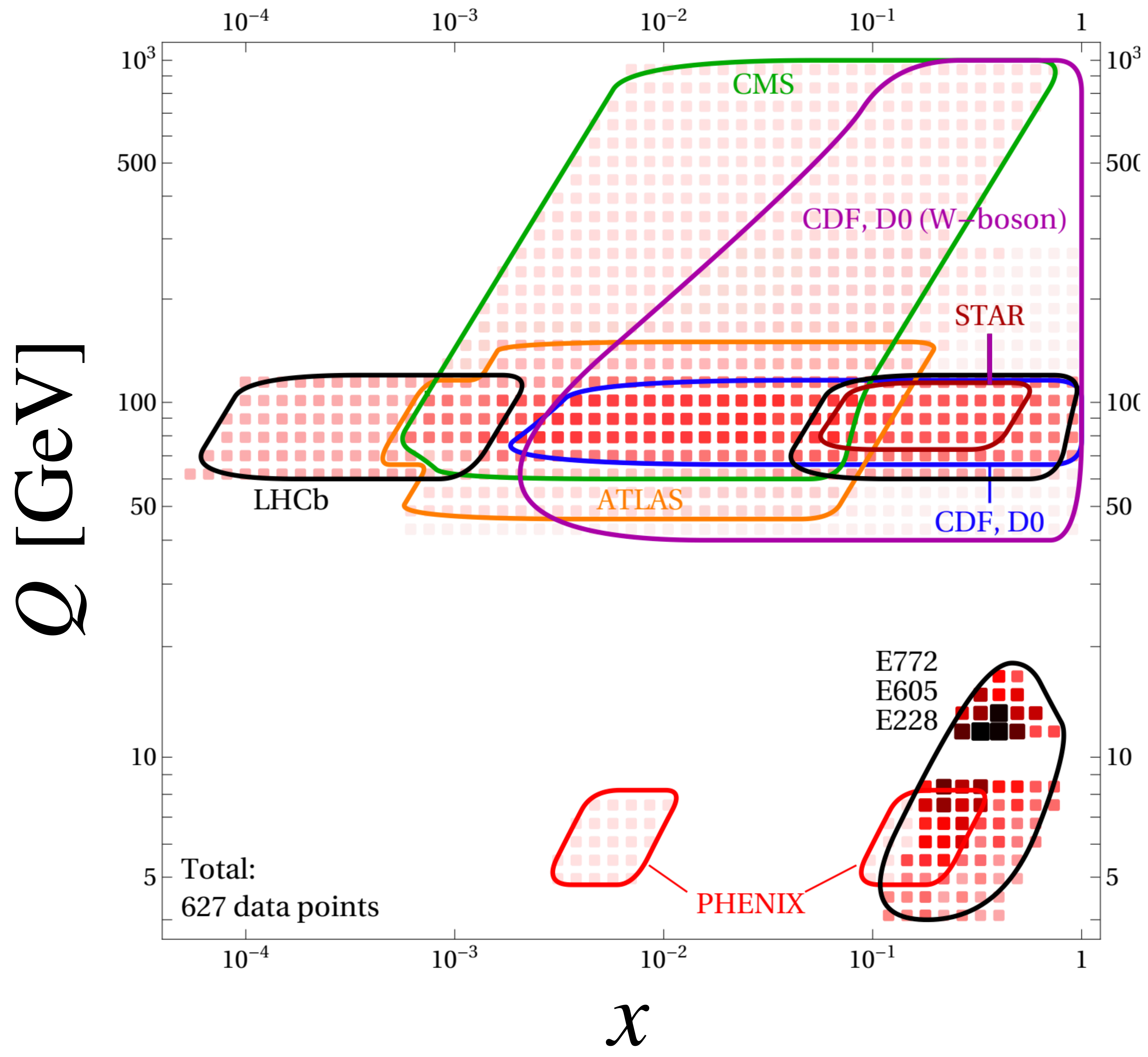
$$\frac{d\sigma_{\mathrm{DY}}}{dQ dY dq_T^2} = H(Q, \mu) \int d^2\vec{b}_T e^{i\vec{q}_T \cdot \vec{b}_T} f_q(x_a, \vec{b}_T, \mu, \zeta_a) \, f_q(x_b, \vec{b}_T, \mu, \zeta_b) [1 + \mathcal{O}(\frac{q_T^2}{Q^2})]$$

Perturbative hard kernels

Nonperturbative TMDs

$$q_T^2 \ll Q^2$$

TMDs from global analyses of experimental data

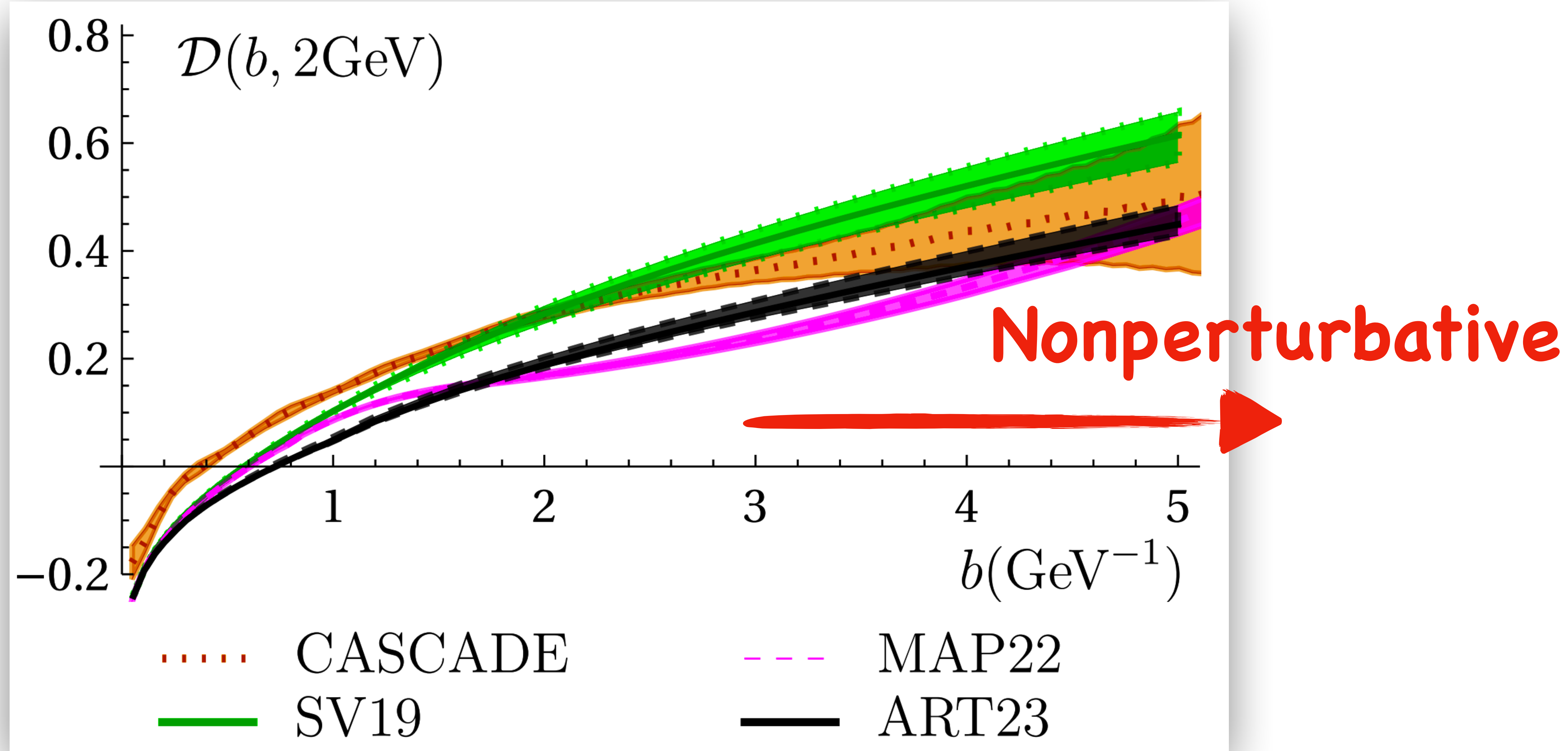


- V. Moos, et. al. (ART23), JHEP 05 (2024) 036

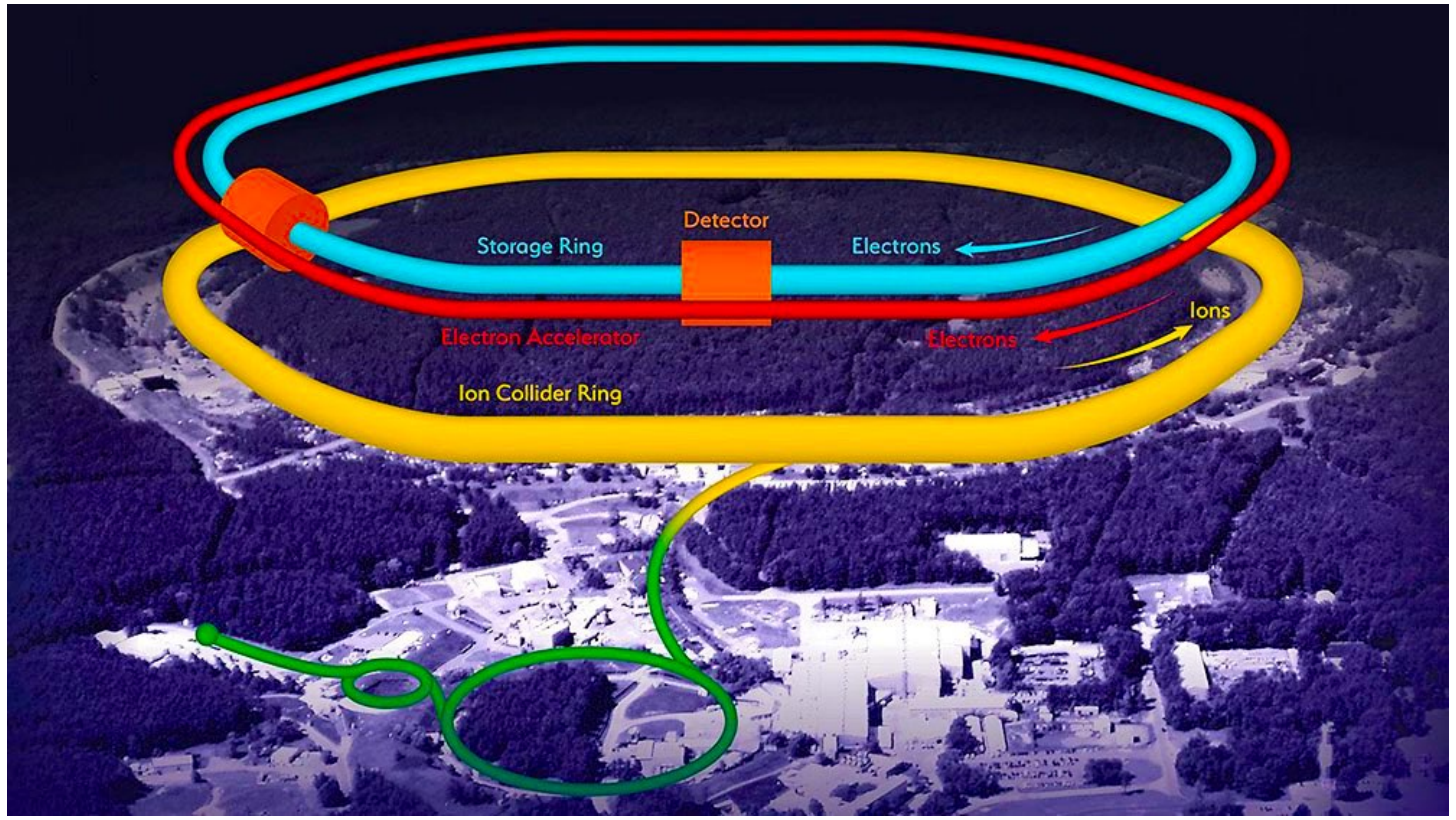
- Relate TMDs at different energy scales

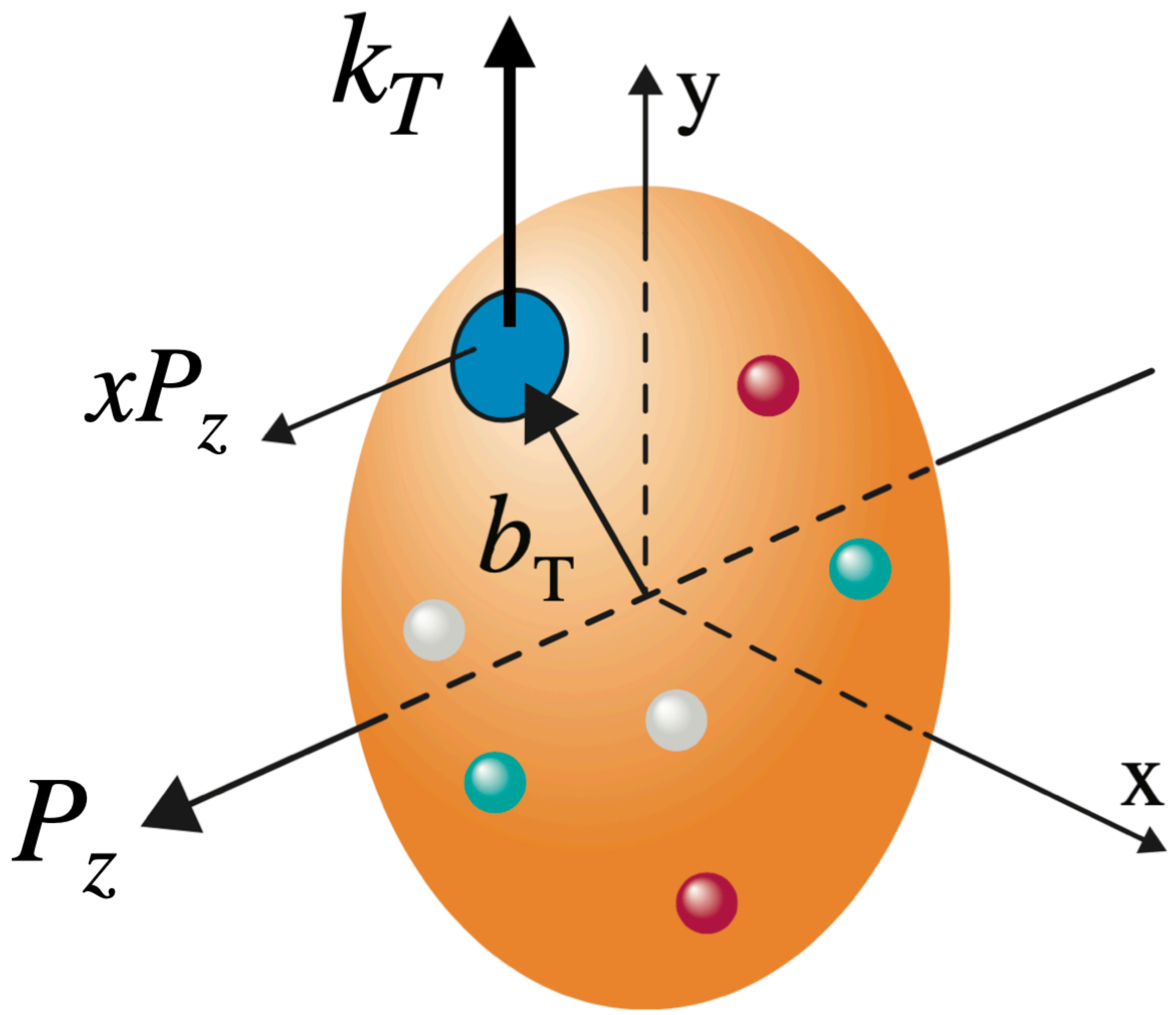
$$\left.\begin{aligned} \mu\frac{d}{d\mu}\ln f_q(x,\vec{b}_T,\mu,\zeta) &= \gamma_\mu^q(\mu,\zeta) \\ \mu\frac{d}{d\zeta}\ln f_q(x,\vec{b}_T,\mu,\zeta) &= \gamma_\zeta^q(\mu,b_T) \end{aligned}\right\}$$

Collins-Soper (rapadity evolution) kernel



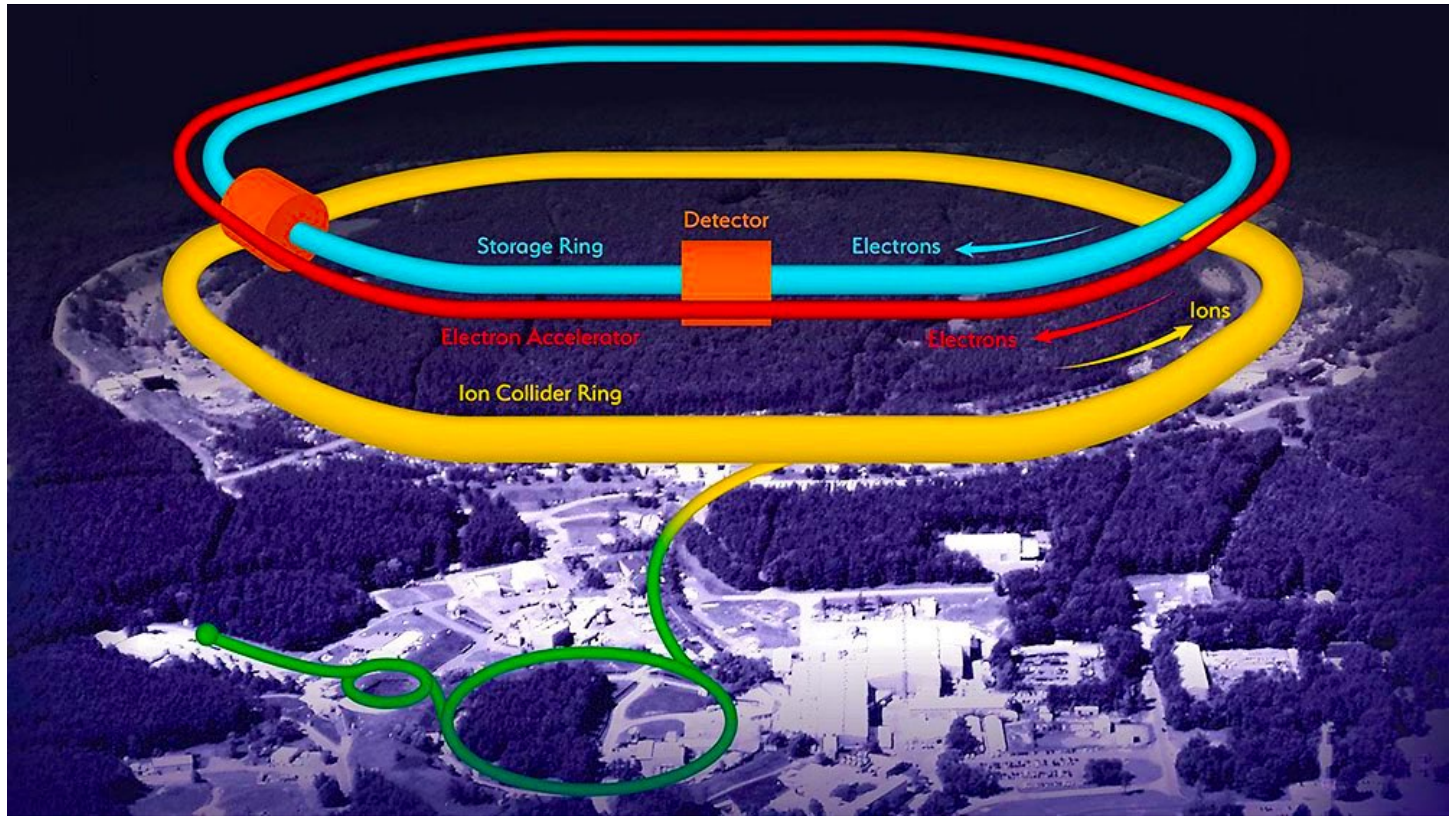
7 Determination of parton distributions

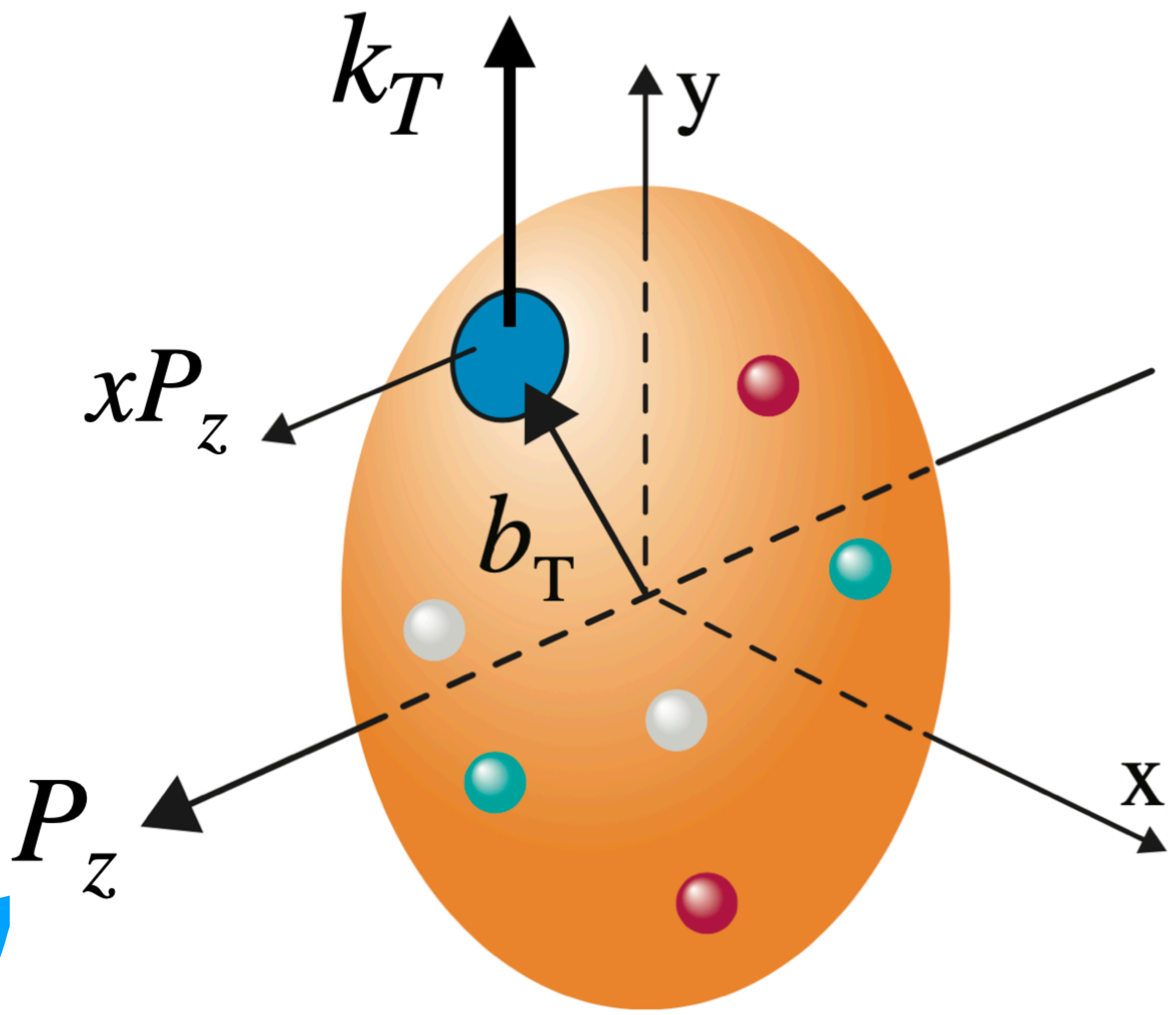


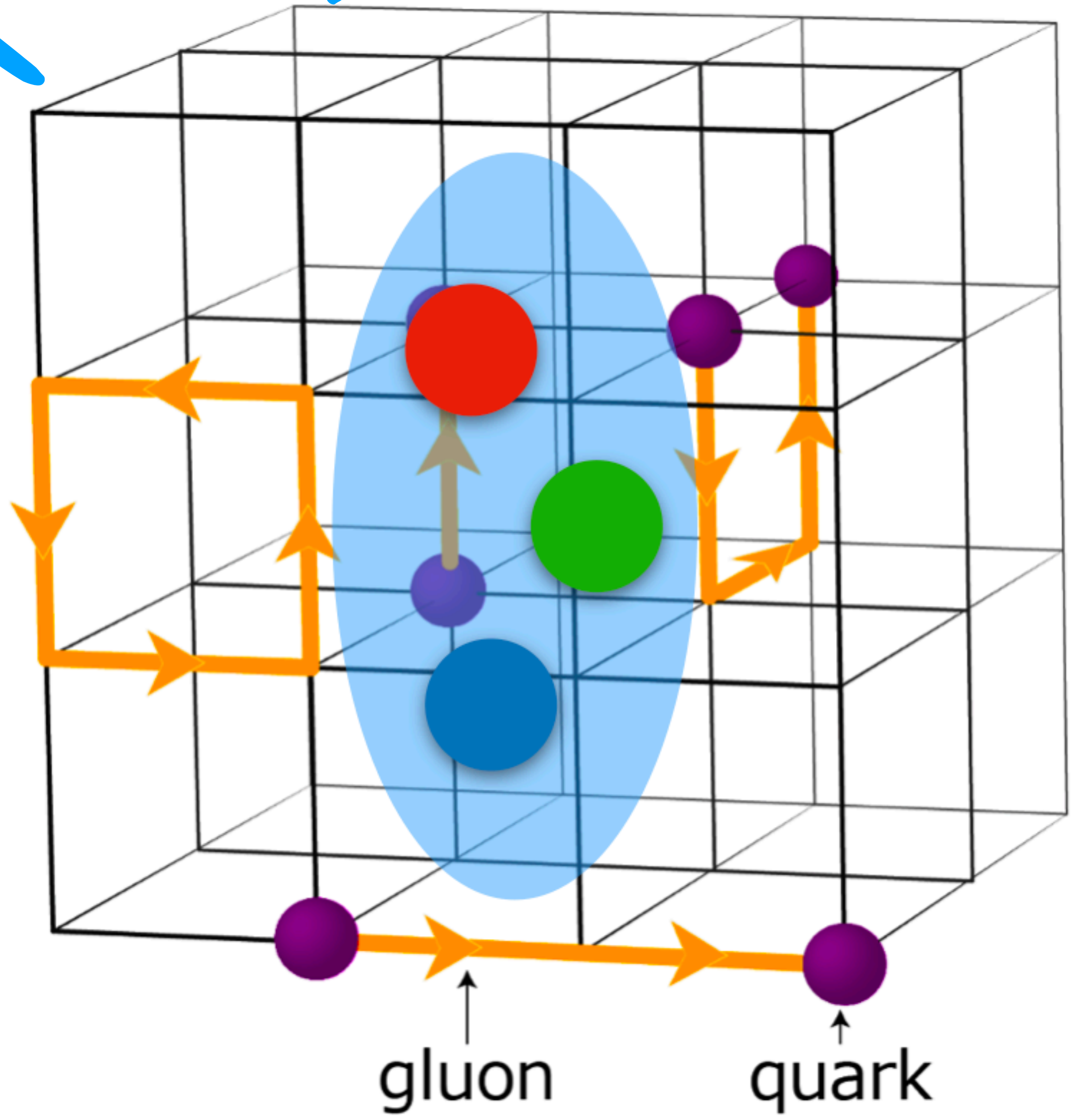


- Global analysis of experimental data with parametrization of CS kernel, x-dependent PDFs, GPDs and TMDs ...

Determination of parton distributions



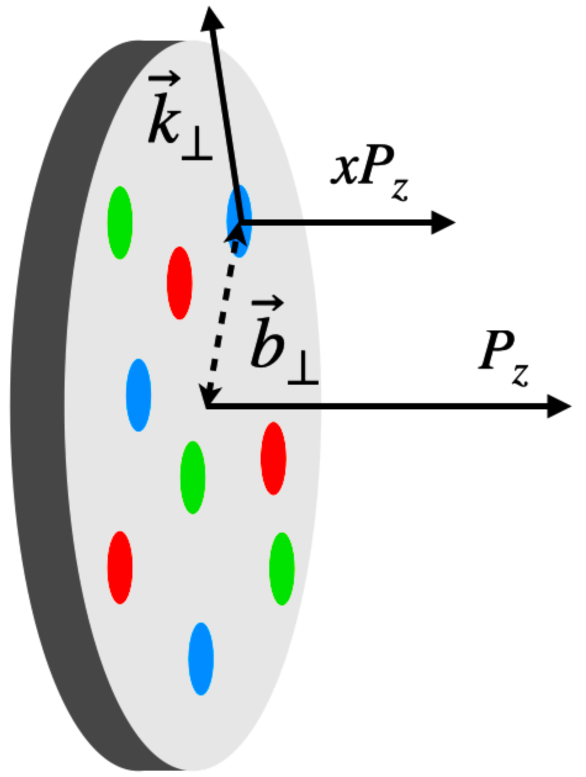


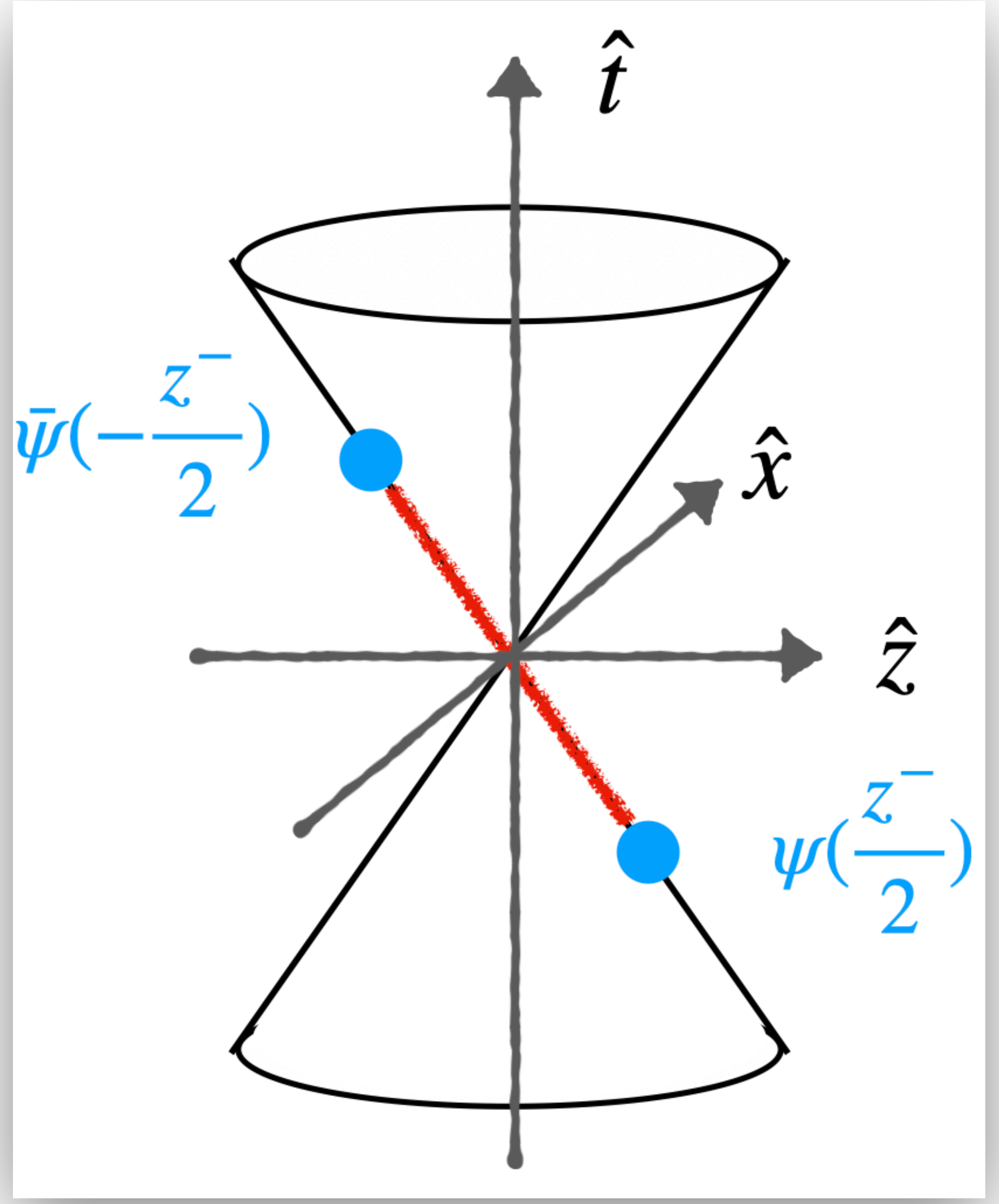


- Global analysis of experimental data with parametrization of CS kernel, x-dependent PDFs, GPDs and TMDs …
- Lattice QCD could provide essential complementary constraints.

9

Parton distributions from lattice QCD simulation





$$f(x,\mu) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle P \,|\, \bar{\psi}(0)\Gamma\mathscr{W}(0,z^-)\psi(z^-) \,|\, P' \rangle$$

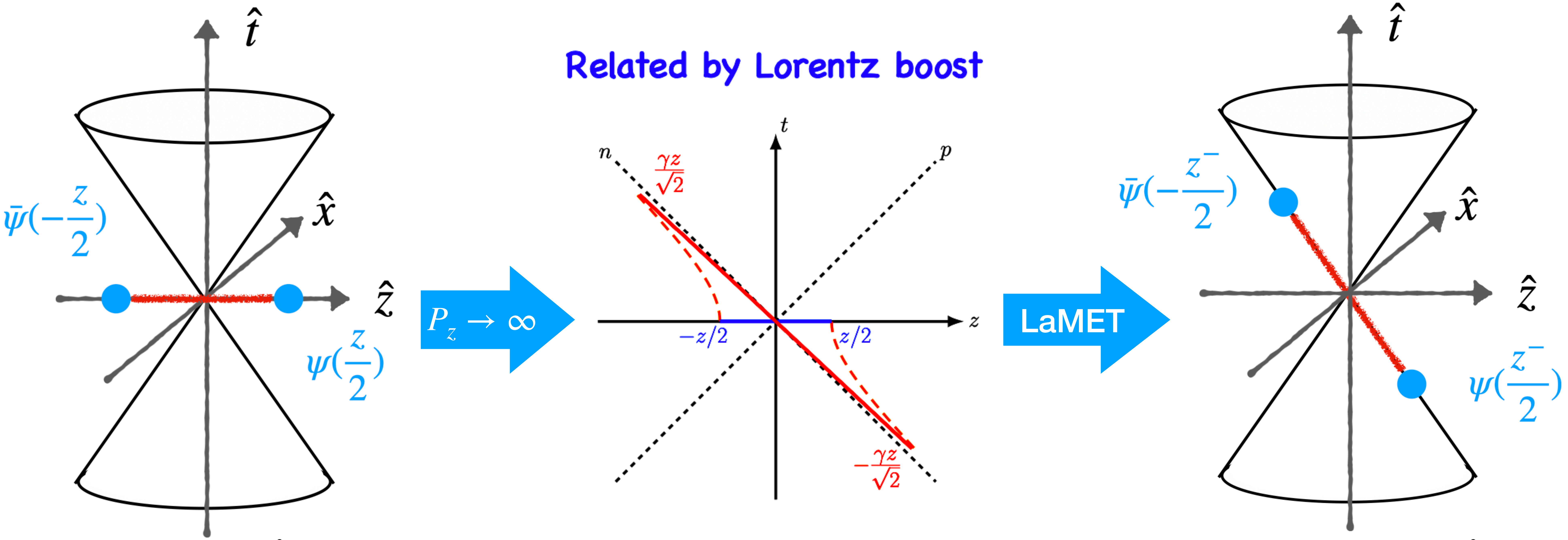
✗ Light-cone correlations: forbidden on Euclidean lattice

10 Large momentum effective theory

The quasi distribution from equal-time correlators,

- X. Ji, PRL 110 (2013); SCPMA57 (2014);

$$\tilde{f}(x, P_z, \mu) = \int \frac{dz}{4\pi} e^{-ixP_z z} \langle P | \bar{\psi}(0) \Gamma \mathscr{W}(0,z) \psi(z) | P \rangle$$



Quasi PDF/GPD

$$\langle P \to \infty | \bar{\psi}(0) \Gamma \mathscr{W}(0,z) \psi(z) | P \to \infty \rangle$$

Light-cone PDF/GPD

$$\langle P | \bar{\psi}(0) \Gamma \mathscr{W}(0,z^-) \psi(z^-) | P \rangle$$

Large momentum effective theory

- X. Ji, PRL 110 (2013); SCPMA57 (2014);
- X. Xiong, X. Ji, et al, 90 PRD (2014);
- Y.-Q. Ma, et al, PRD98 (2018), PRL 120 (2018);
- T. Izubuchi, X. Ji, et al PRD98 (2018).
- X. Ji, Y. Zhao, et al, RMP 93 (2021).

Large P_z expansion of quasi distribution:

Quasi PDF

$$\tilde{f}(x, P_z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) f(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_z^2}\right)$$

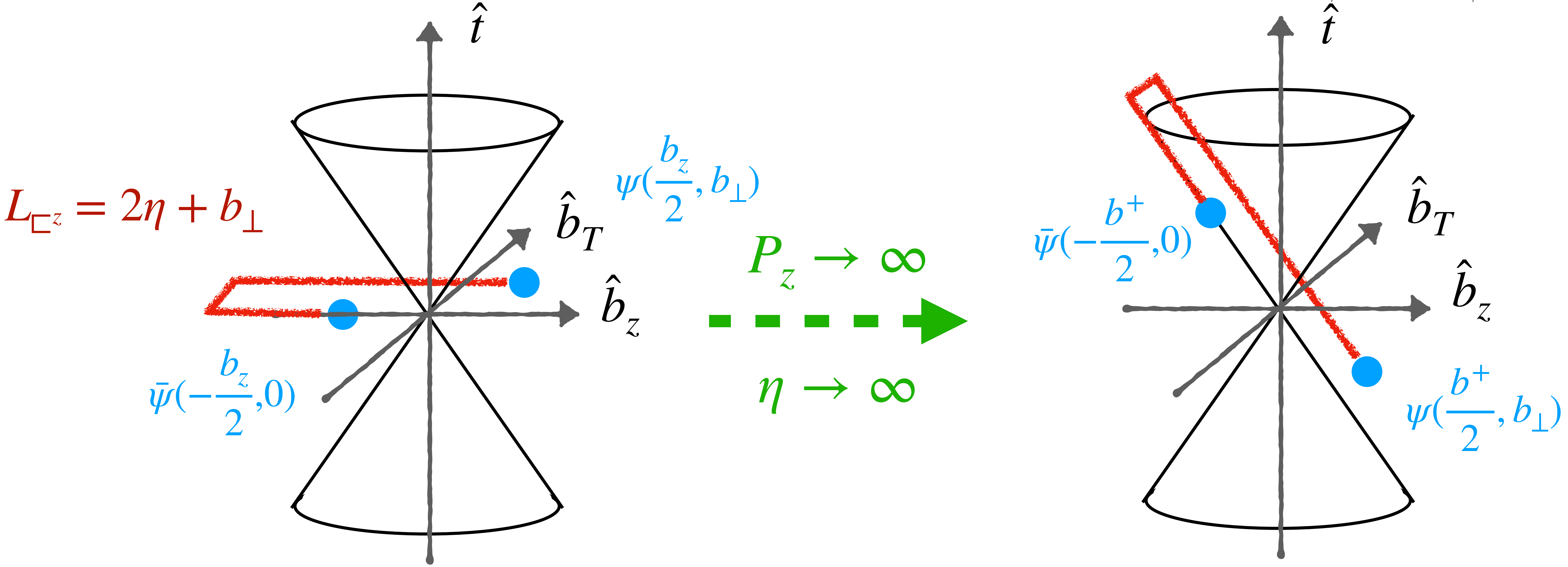
Light-cone PDF

Power corrections

- **Computable** from Lattice QCD with finite $P_z < 1/a$.
- Have **same IR** physics as light-cone PDFs.
- In large P_z limit, Quasi differ from light-cone distribution by $\lim_{P\to\infty} \lim_{a\to 0}$ v.s. $\lim_{a\to 0} \lim_{P\to\infty}$, inducing a **perturbative matching** $C(\frac{x}{y}, \frac{\mu}{yP_z})$.

TMDs from lattice QCD

- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- A. Vladimirov, A. Schäfer Phys.Rev.D 101 (2020), 074517
- I. Stewart, Y. Zhao et al., JHEP 09 (2020) 099
- X. Ji et al., Phys.Rev.D 103 (2021) 7, 074005
- I. Stewart, Y. Zhao et al., JHEP 08 (2022) 084



Quasi TMD Beam function

$$\langle P | \bar{\psi}(\frac{b_z}{2}, b_\perp) \Gamma W_{\sqsupset^z} \psi(-\frac{b_z}{2},0) | P \rangle$$

Light-cone TMD Beam function

$$\langle P | \bar{\psi}(\frac{b^+}{2}, b_\perp) \Gamma W_{\sqsupset^+} \psi(-\frac{b^+}{2},0) | P \rangle$$

TMDs from lattice QCD

Equal-time Quasi TMDs and soft factor from lattice QCD

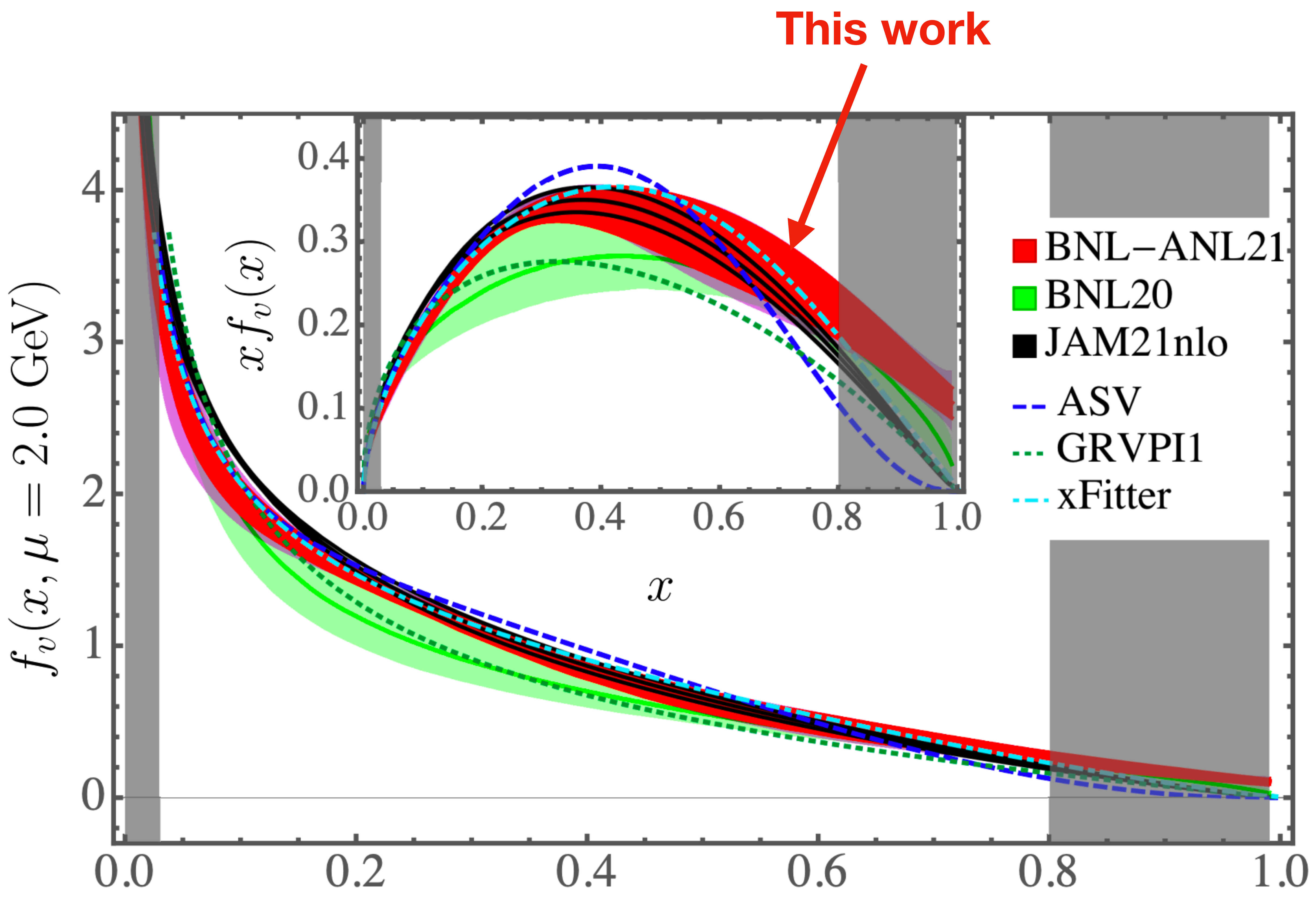
Non-perturbative CS (rapidity evolution) Kernel and physical TMDs

$$\frac{\tilde{f}(x,\vec{b}_T,\mu,P_z)}{\sqrt{S_r(\vec{b}_T,\mu)}} = C(\mu,xP_z)e^{\frac{1}{2}\gamma_\zeta(\mu,b_T)\ln\frac{(2xP_z)^2}{\zeta}} f(x,\vec{b}_T,\mu,\zeta)\{1+\mathcal{O}[\frac{1}{(xP_zb_T)^2},\frac{\Lambda_{\mathrm{QCD}}^2}{(xP_z)^2}]\}$$

Perturbative matching

Power corrections

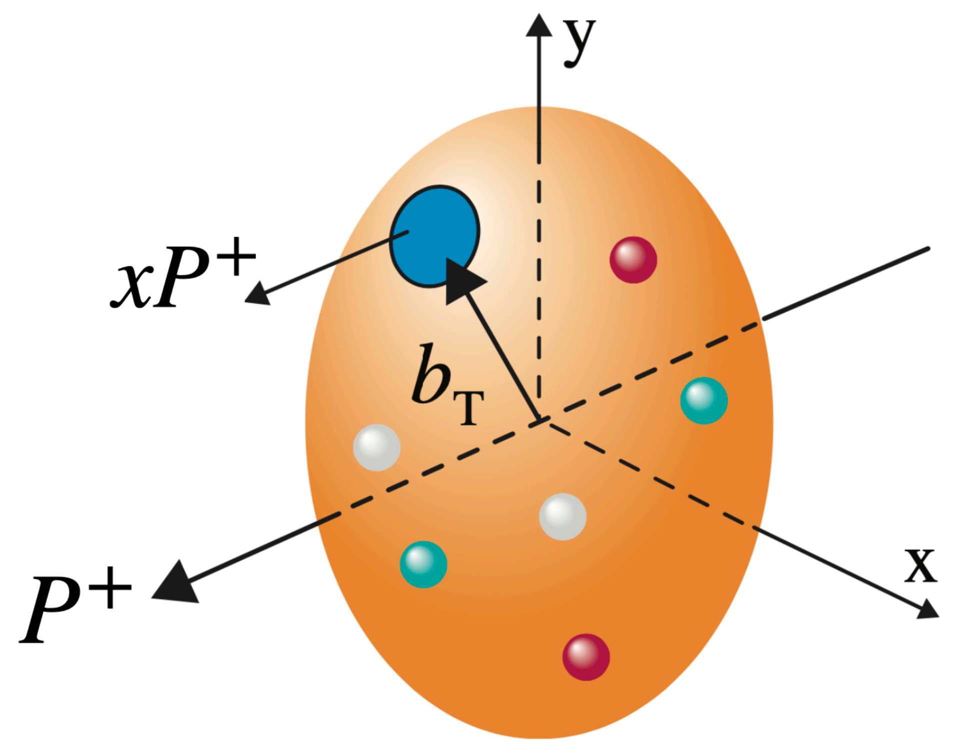
Benchmarking: pion valence quark PDF



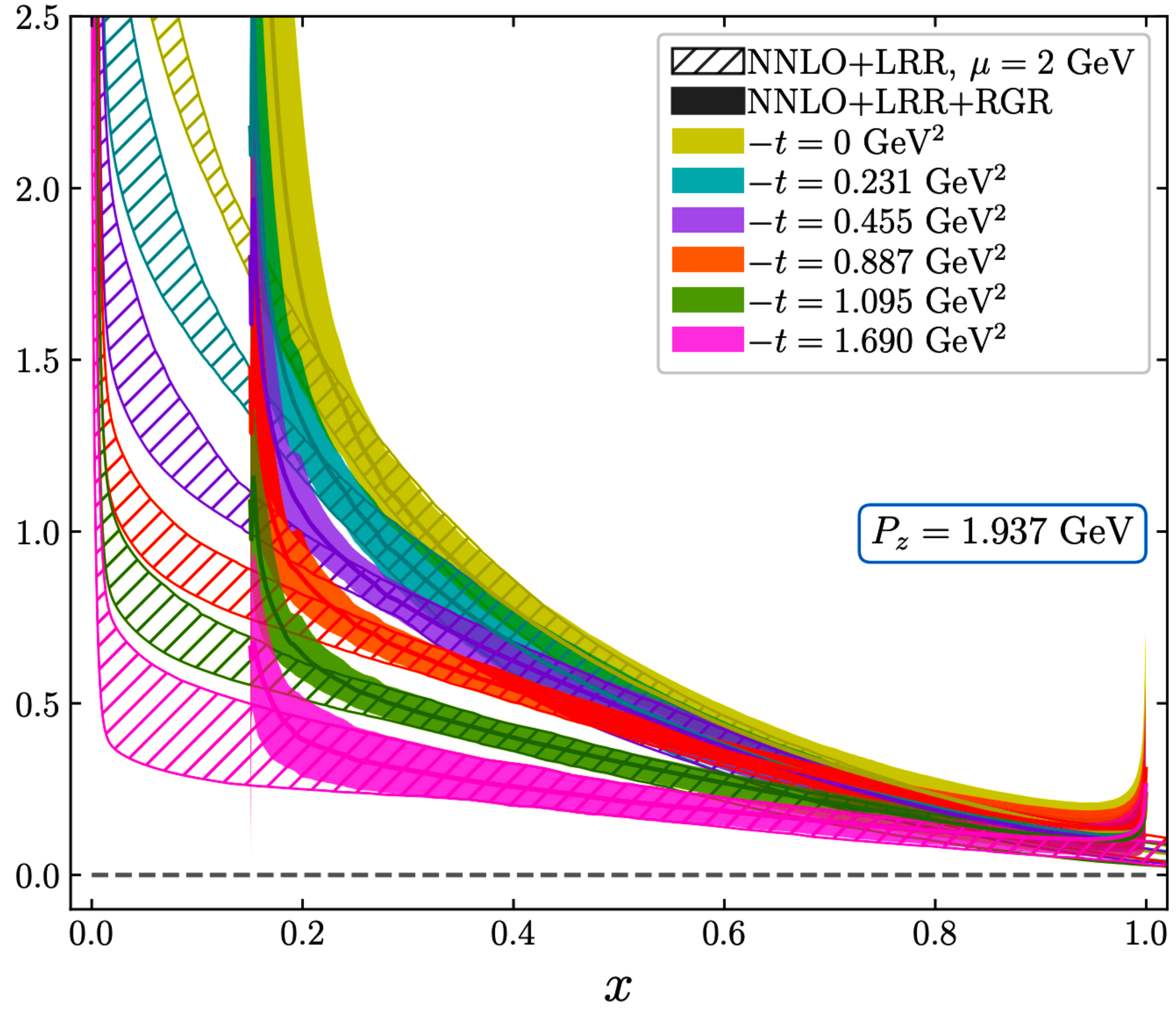
- **XG**. N. Karthik, et al., (BNL), PRD 102 (2020) 9, 094513
- **XG**, Y. Zhao, et al., (BNL+ANL), PRL. 128 (2022), 142003
- **XG**. Y. Zhao., et al., (BNL+ANL), PRD 106 (2022) 11, 114510

- First lattice calculation with:
  - NNLO matching.
  - Continuum limit.
- Independent QCD prediction agree with global analysis of experimental data.

Towards 3D GPDs: 3D imaging of pion



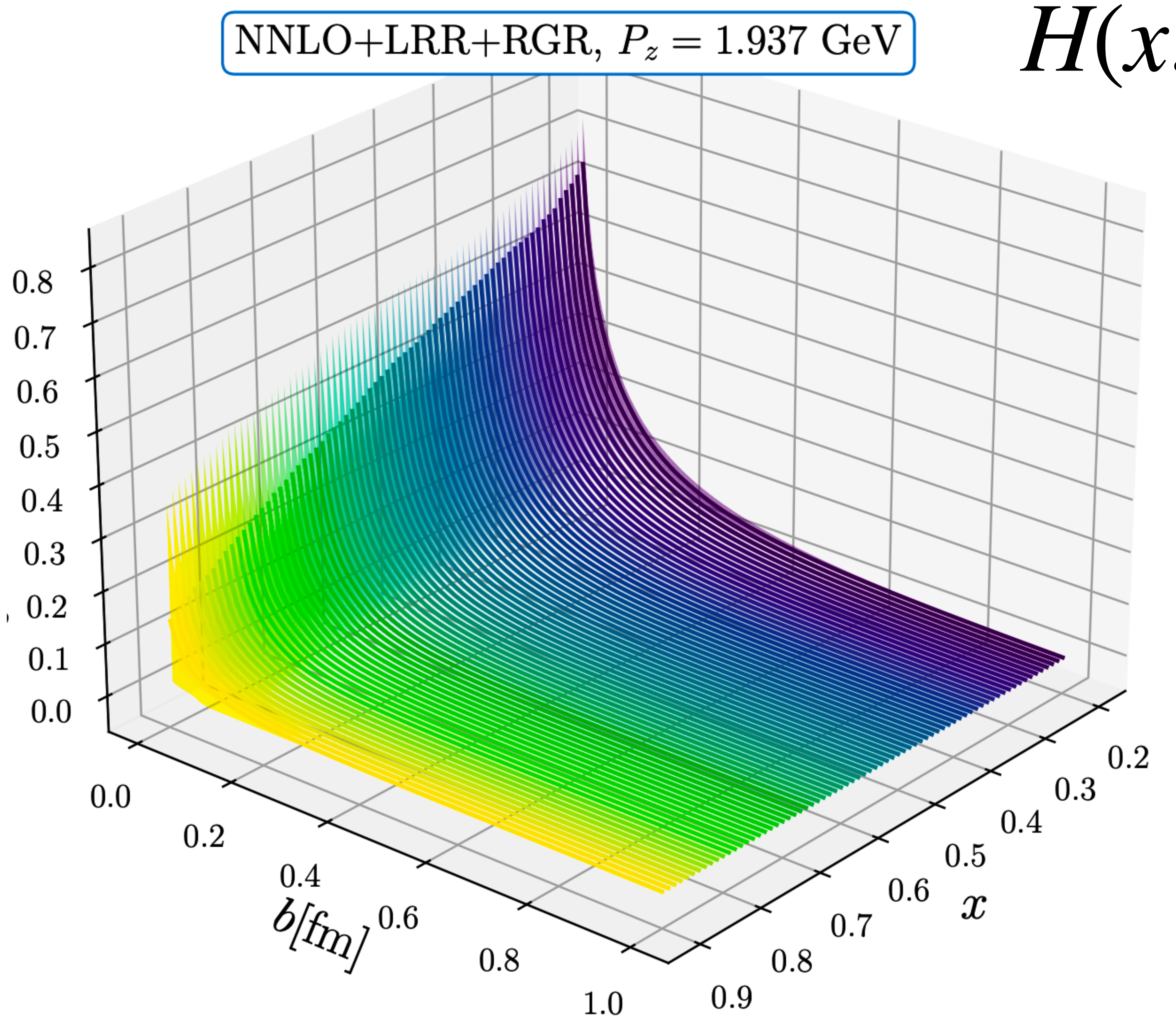
- Pion quark GPD $H(x, \xi = 0, t)$



F.T.

- Impact-parameter distribution

$H(x, b_T)$



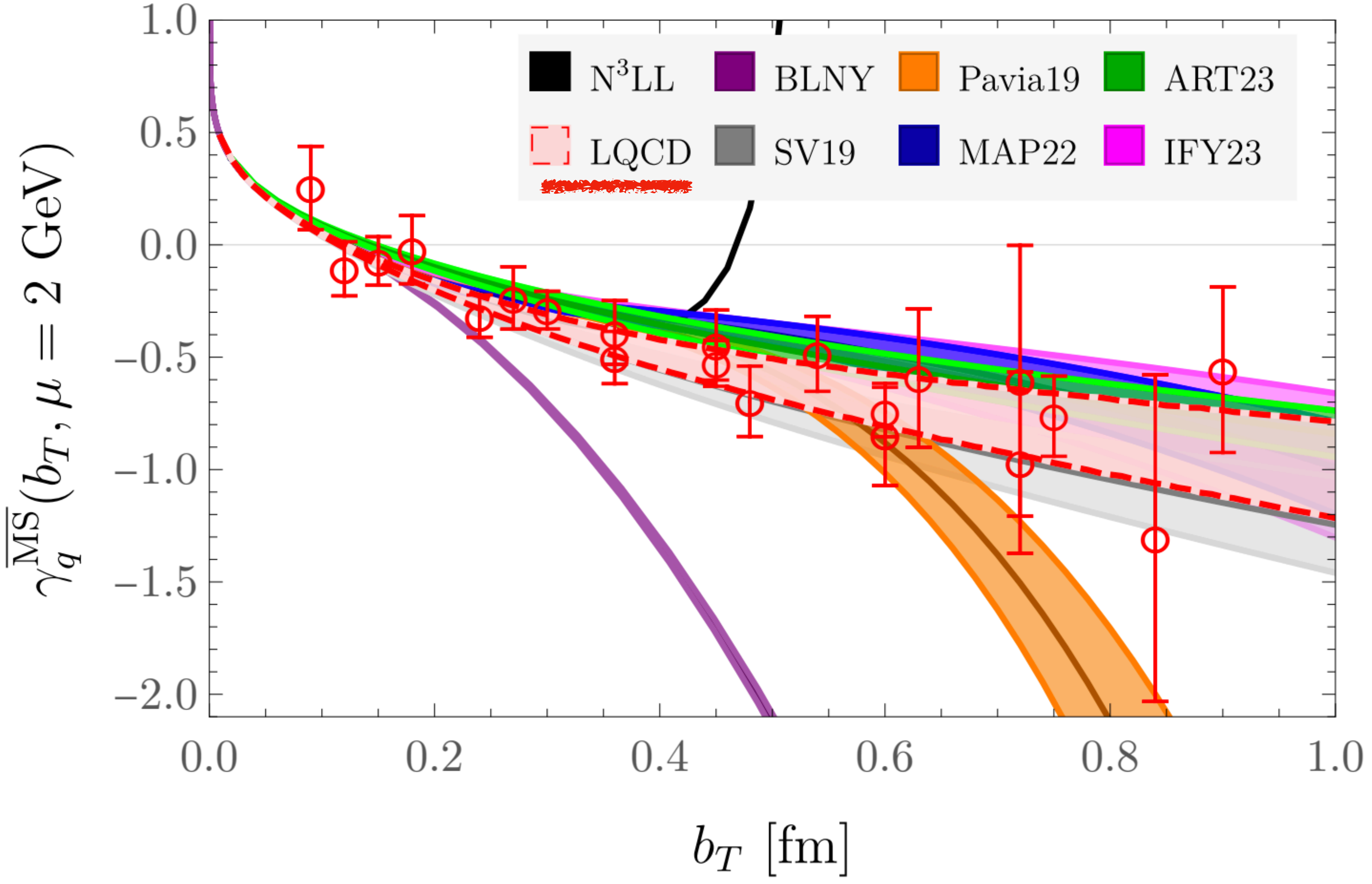
- First lattice calculation with:
  - Multiplie momentum transfer t.
  - NNLO matching.
  - Leading renormalon subtraction (**LRR**) and renormalization group resummation (**RGR**).

• H.-T. Ding, **XG**, Q, Shi, et al., (BNL+ANL), arXiv: 2407.03516

Towards 3D TMDs: the Collins-Soper kernel

Quasi-TMDWF

Collins-Soper kernel



Wilson clover fermion, physical quark masses,
a = 0.09, 0.12, 0.15 fm

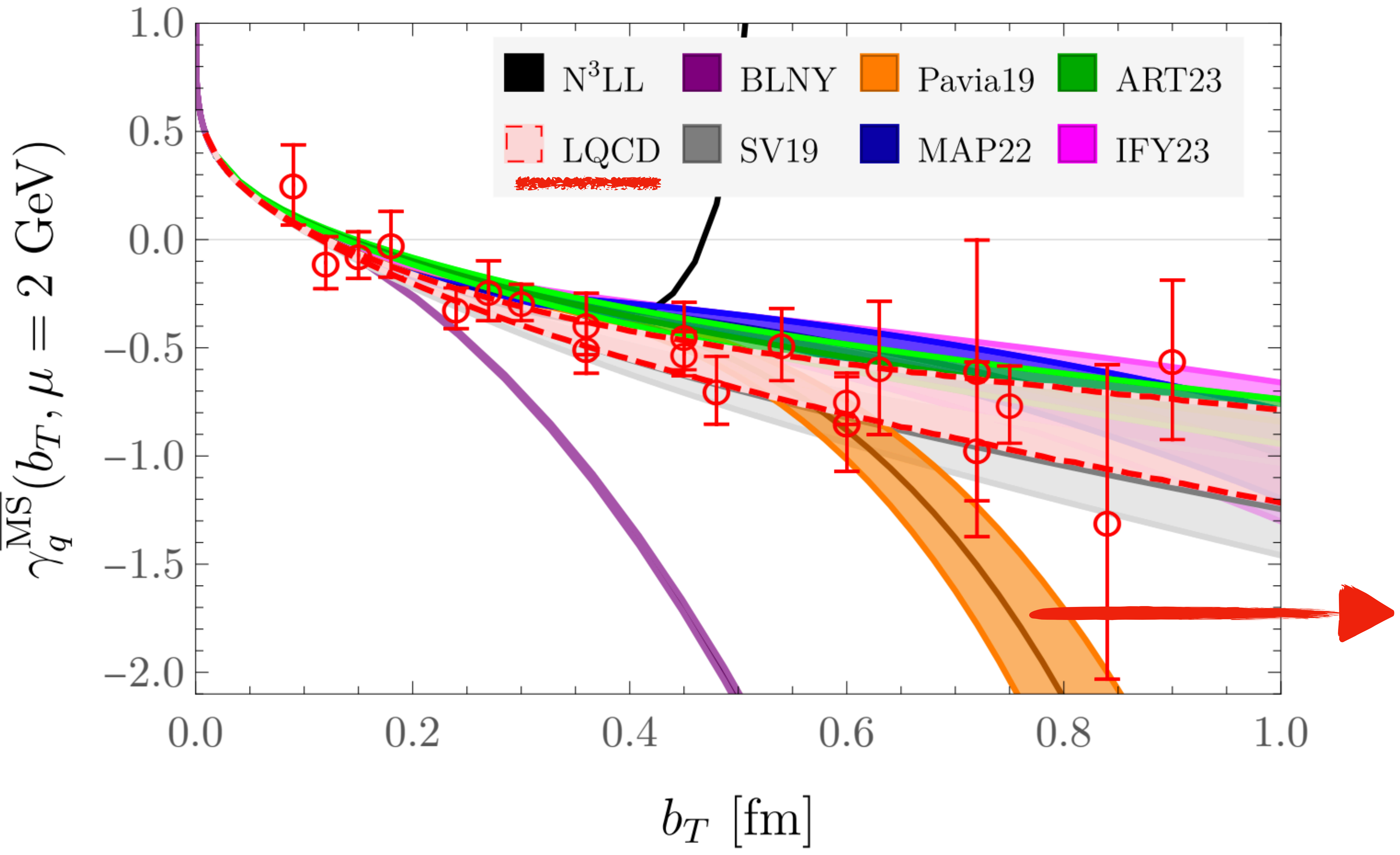
$$\gamma_\zeta(\mu, b_T) = \frac{d}{d \ln P_z} \ln \frac{\tilde{f}(x, \vec{b}_T, \mu, P_z)}{C(\mu, xP_z)}$$

- Three different lattice spacing, physical pion mass.
- Controlled renormalization and Fourier transform.
- Next-to-next-to-leading logarithmic (NNLL) order.

• A. Avkhadiev, P. Shanahan, M. Wagman, Y. Zhao, PRL 132 (2024) 23, 231901

Towards 3D TMDs: the Collins-Soper kernel

Collins-Soper kernel

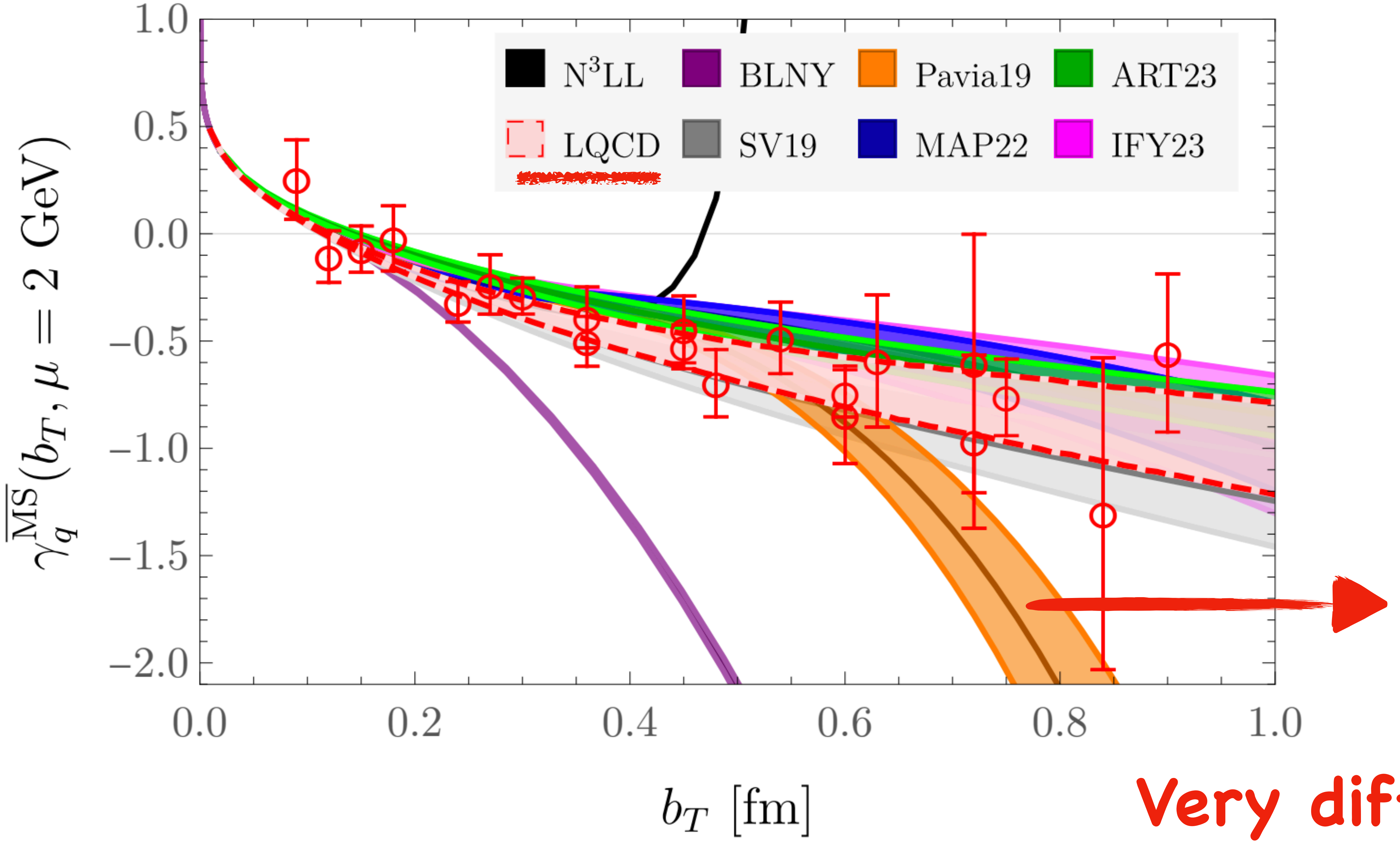


- **Experimental constrain** at deep non-perturbative region is **very limited**: lack of data, model dependence …
- **Can lattice QCD push further?**

• A. Avkhadiev, P. Shanahan, M. Wagman, Y. Zhao, PRL 132 (2024) 23, 231901

Towards 3D TMDs: the Collins-Soper kernel

Collins-Soper kernel

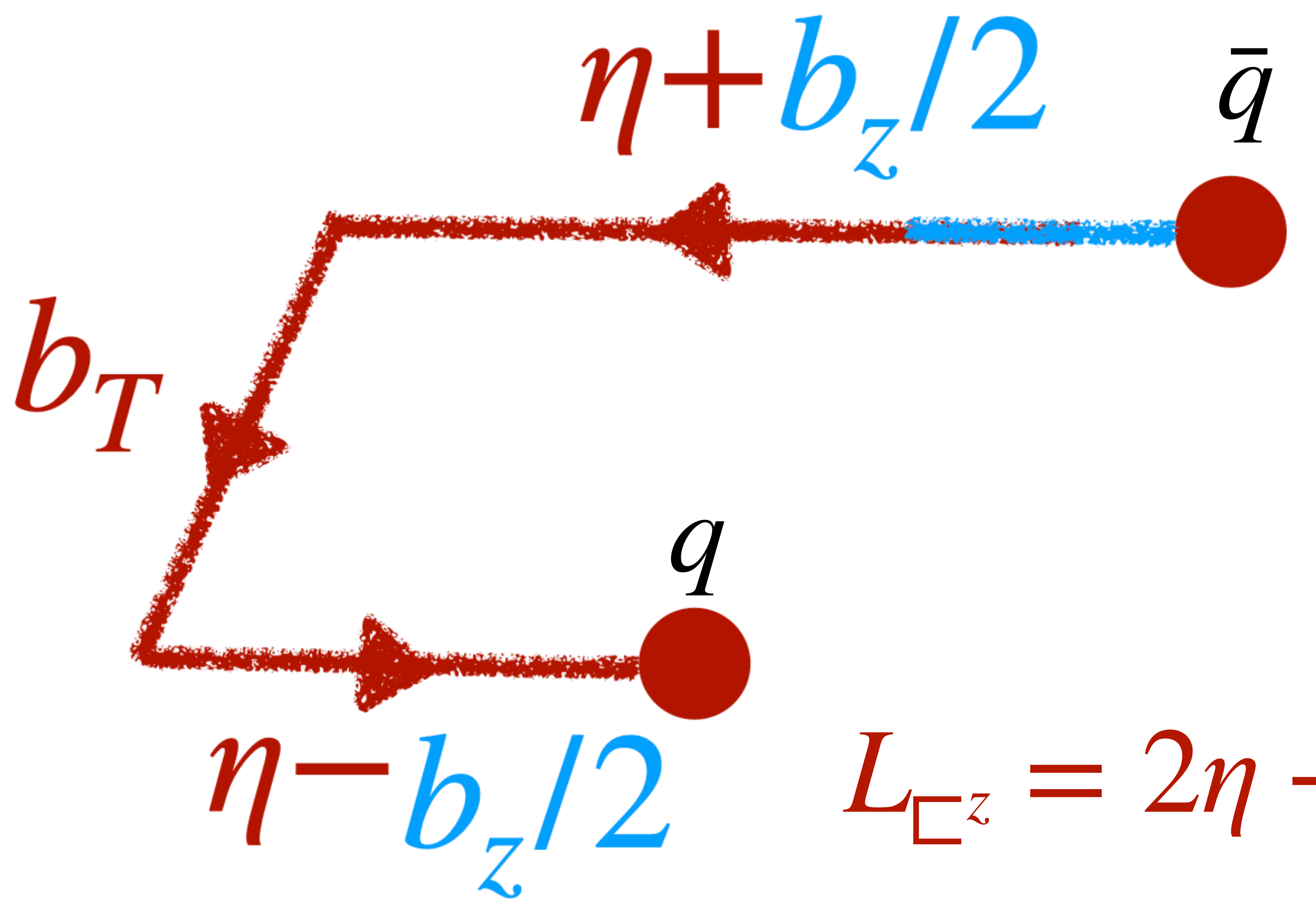


- **Experimental constrain** at deep non-perturbative region is **very limited**: lack of data, model dependence …
- **Can lattice QCD push further?**

Very difficult: errors grow rapidly!

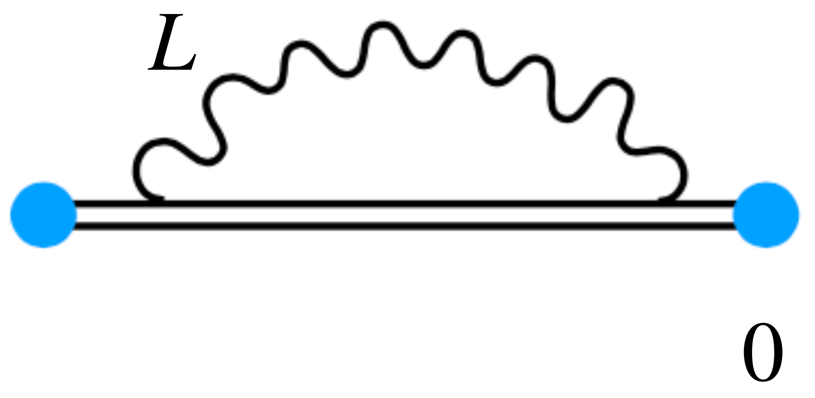
• A. Avkhadiev, P. Shanahan, M. Wagman, Y. Zhao, PRL 132 (2024) 23, 231901

Difficulties in the conventional quasi-TMDs



$$L_{\sqsubset z} = 2\eta + b_T$$

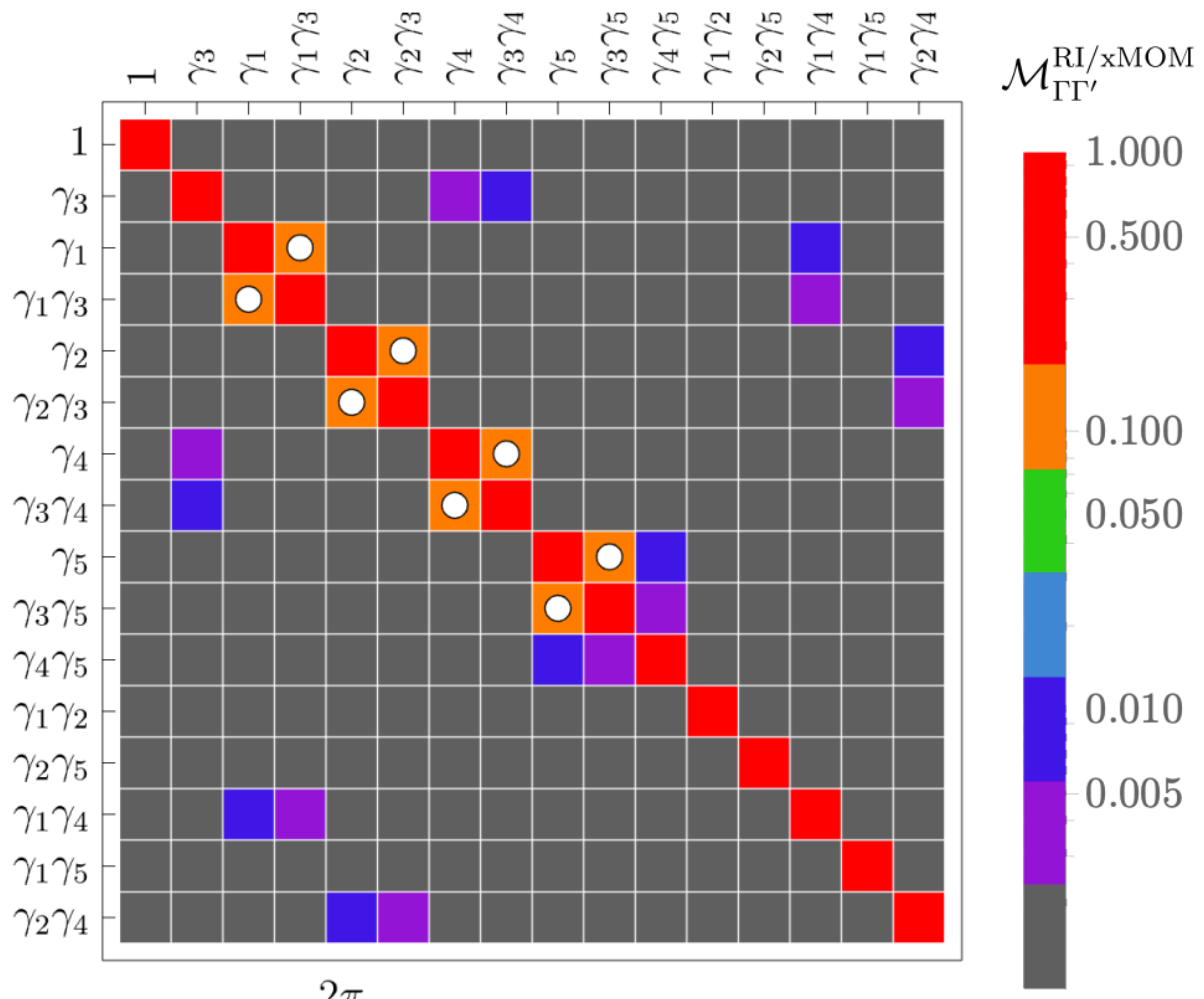
- Exponential decaying signal and complicated renormalization due to the Wilson line artifacts.



$$\sim e^{-\delta m(a) \cdot L_{\sqsubset z}}$$

Linear divergence from Wilson line self energy

Operator mixing



$$p^{\mu}_{\mathrm{R}} = \frac{2\pi}{L} \times (0, 0, 10, 0),\ \xi = 0.36\ \mathrm{fm}$$

$$\langle \Omega \,|\, \overline{\psi}(\frac{b_z}{2}, b_\perp) \Gamma W_{\sqsupset z} \psi(-\frac{b_z}{2}, 0) \,|\, \pi^+, P_z \rangle$$

- A. Avkhadiev, et al., PRD 108 (2023) 11, 114505
- M. Constantinou et al., PRD 99 (2019) 7, 074508

The non-local operator in gauge theory

- The non-local operator in gauge theory:

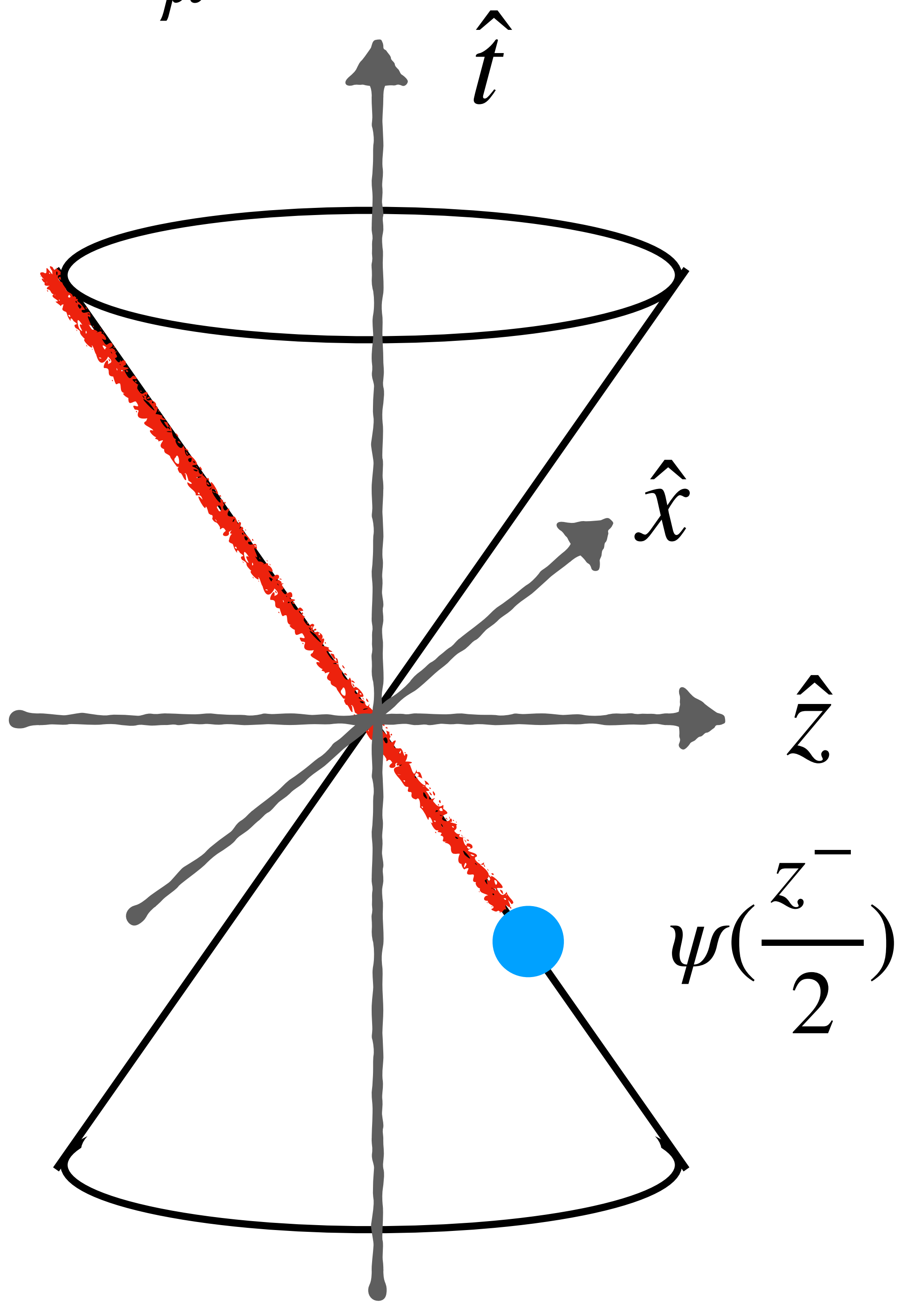
$$\bar{\psi}^*(-\frac{z}{2})\Gamma\psi^*(\frac{z}{2})$$

• P. A. M. Dirac, Can.J.Phys. 33 (1955) 650

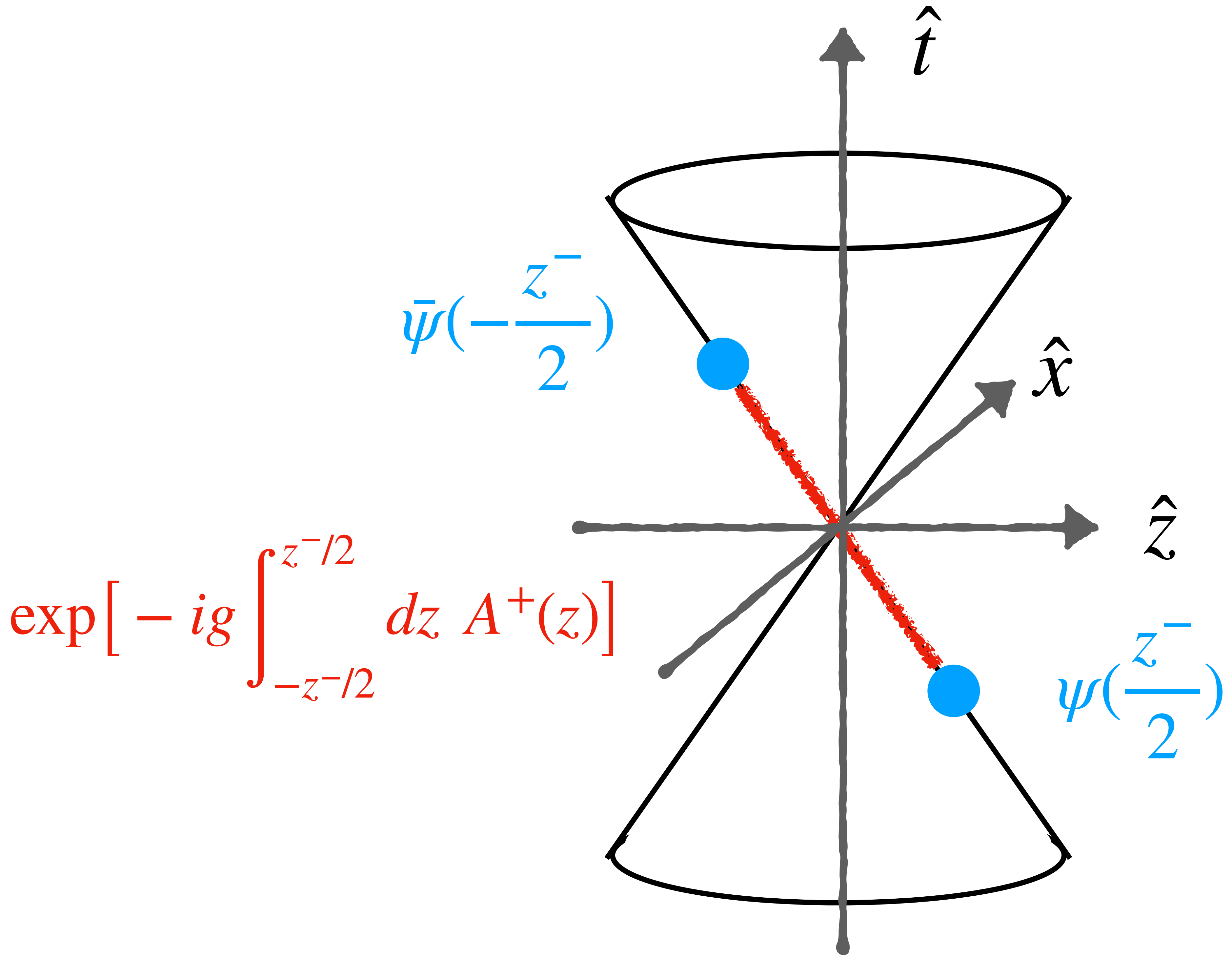
with $\psi^*(z) = \psi(z)e^{iC(z)}$ and $C(z)$ is a linear function of A_μ .

- In DIS, the physical quark $\psi(z)e^{iC(z)}$ represents a gauge-invariant object with a gauge link extended to infinity along the light-cone direction.

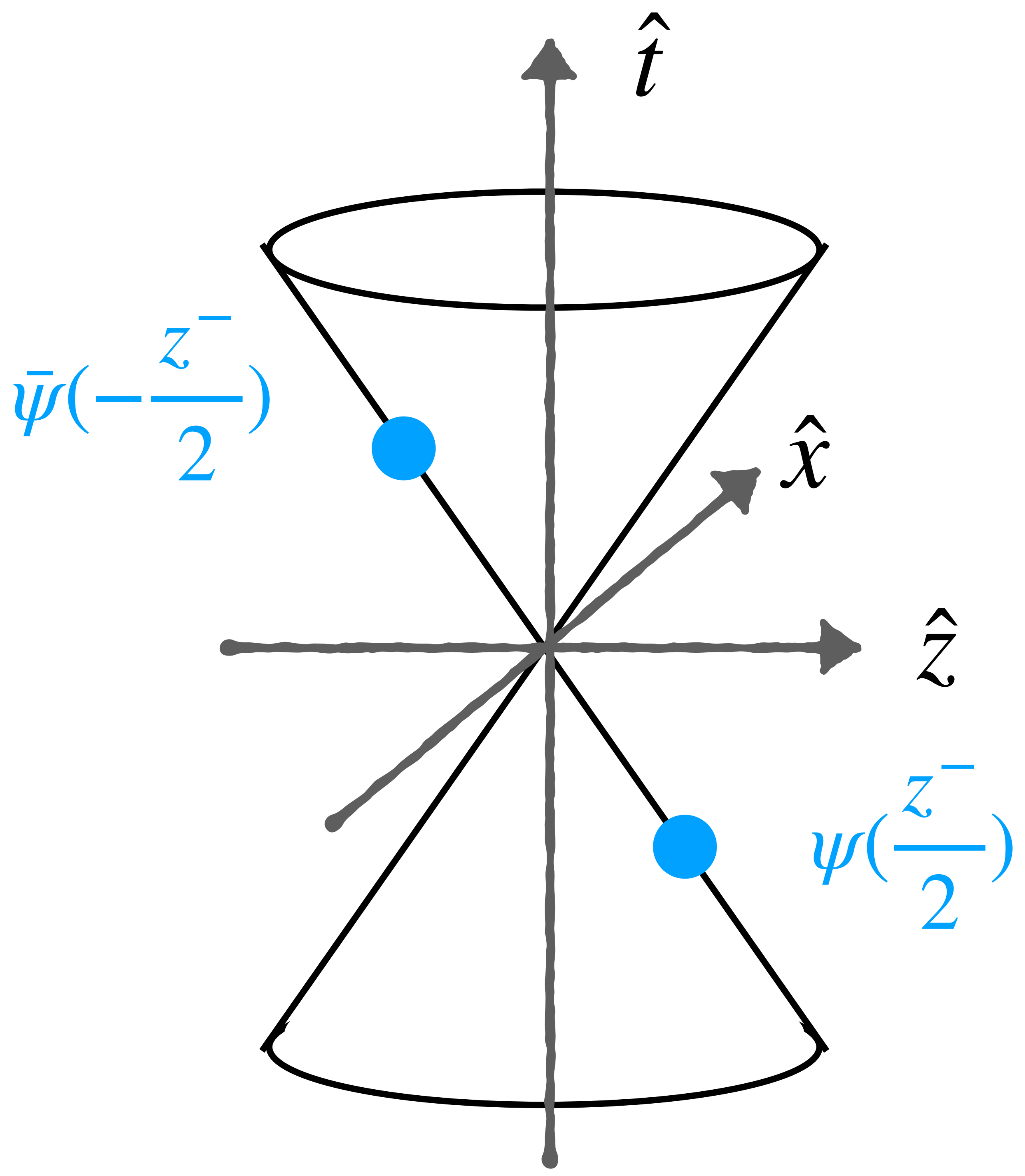
$$\psi^*(z) = \psi(z)e^{\left[-ig\int_{-z^-/2}^{\infty} dz\ A^+(z)\right]}$$



Parton distributions in the light-cone gauge



=



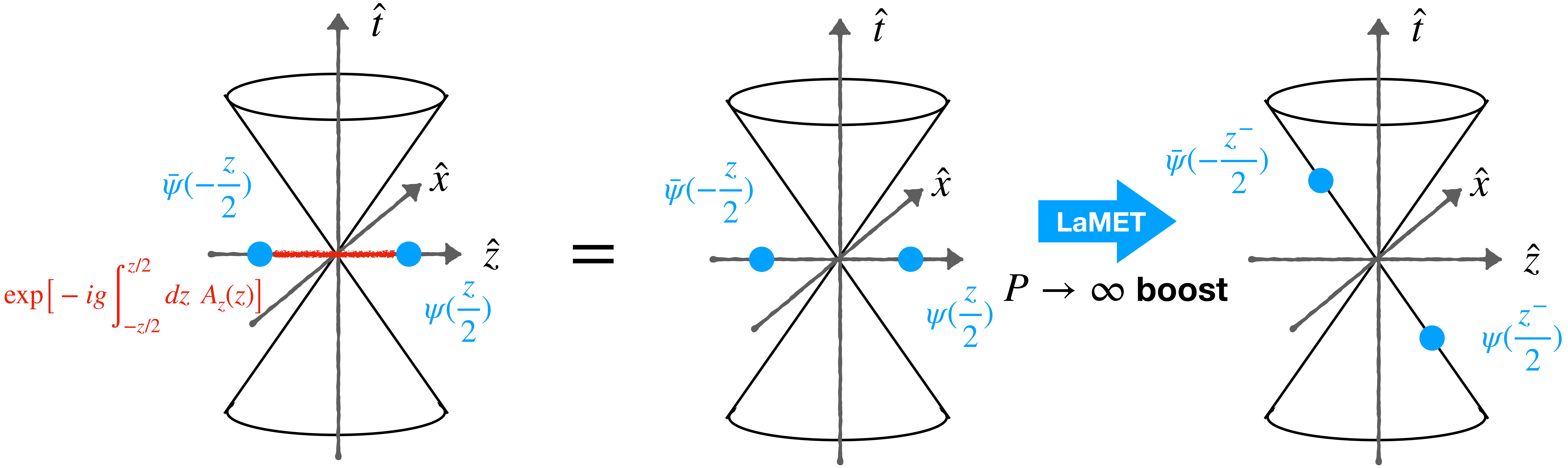
Light-cone PDF/GPD operator

$$\bar{\psi}(-\frac{z^-}{2})\Gamma W(-\frac{z^-}{2},\frac{z^-}{2})\psi(\frac{z^-}{2})$$

Light-cone PDF/GPD operator in light-cone gauge

$$\bar{\psi}(-\frac{z^-}{2})\Gamma\psi(\frac{z^-}{2})\,|_{A^+=0}$$

Parton distributions from axial-gauge LaMET



Gauge invariant (GI) quasi-PDF/GPD

$$\bar{\psi}(-\frac{z}{2})\Gamma W(-\frac{z}{2},\frac{z}{2})\psi(\frac{z}{2})$$

Quasi-PDF/GPD in axial gauge $A_z = 0$

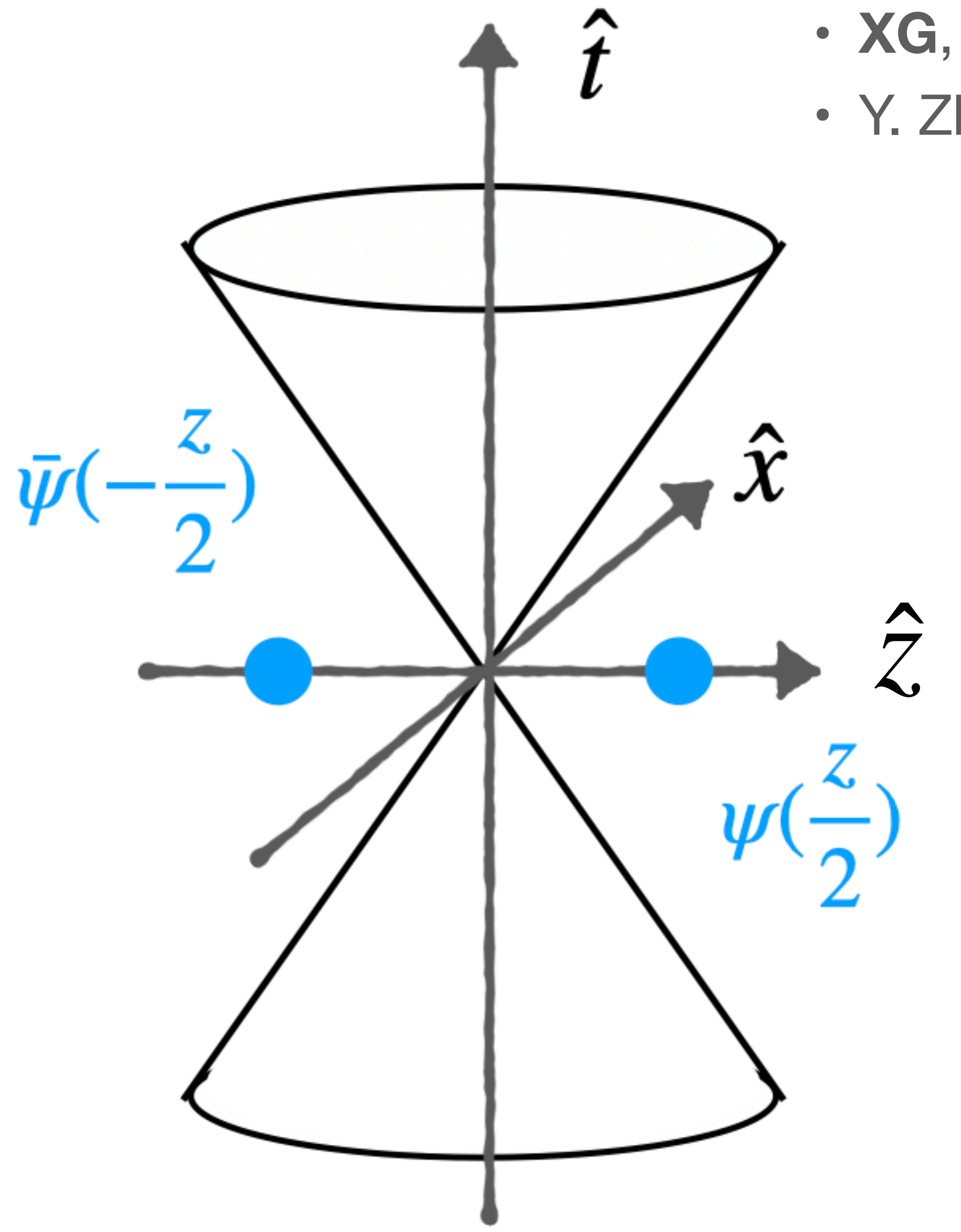
$$\bar{\psi}(-\frac{z}{2})\Gamma\psi(\frac{z}{2})\,|_{A^z=0}$$

Light-cone PDF with $A^+ = 0$

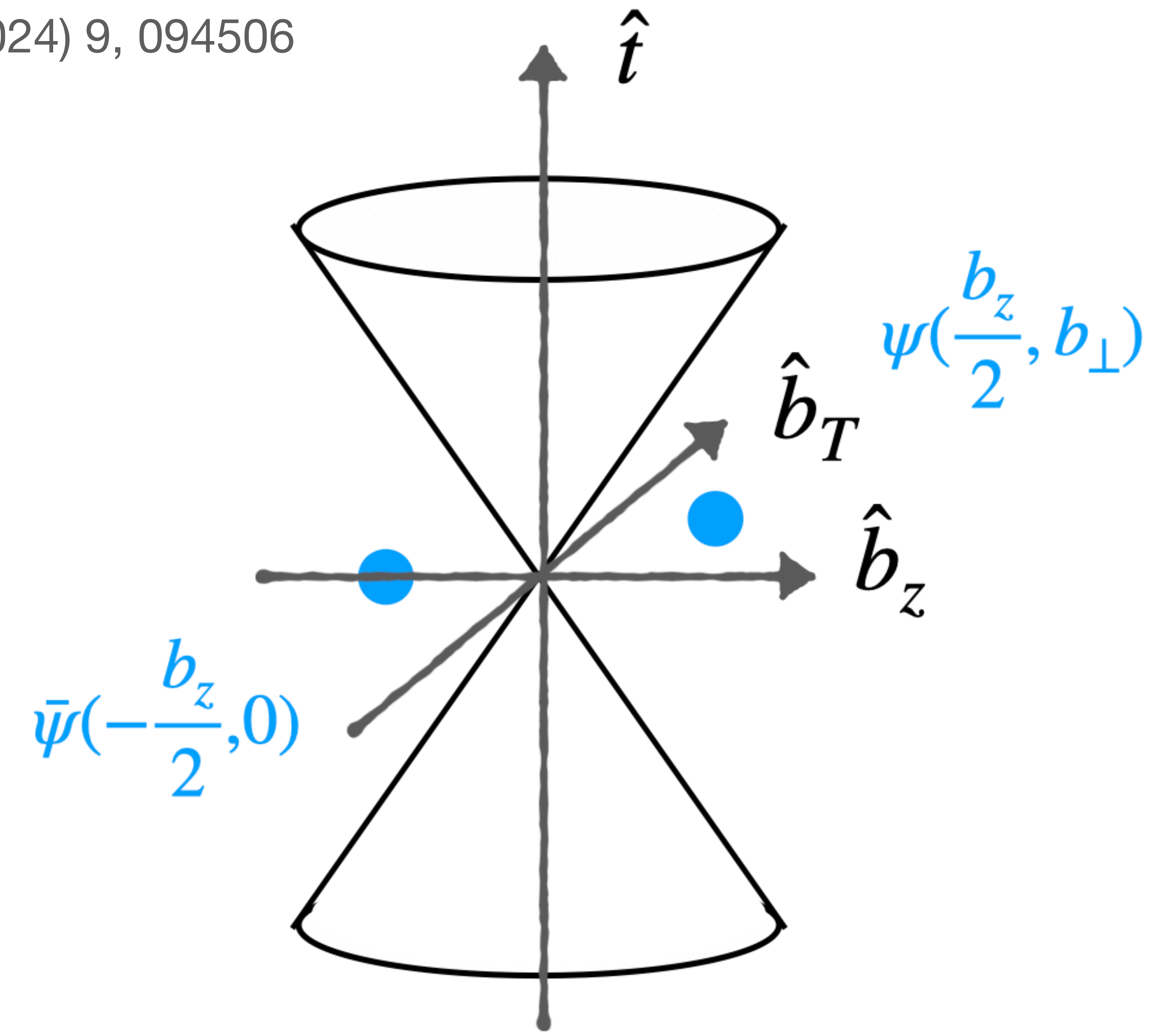
$$\bar{\psi}(-\frac{z^-}{2})\Gamma\psi(\frac{z^-}{2})\,|_{A^+=0}$$

Universality in LaMET

- **XG**, W.-Y. Liu, Y. Zhao, PRD 109 (2024) 9, 094506
- Y. Zhao, arXiv: 2311.01391



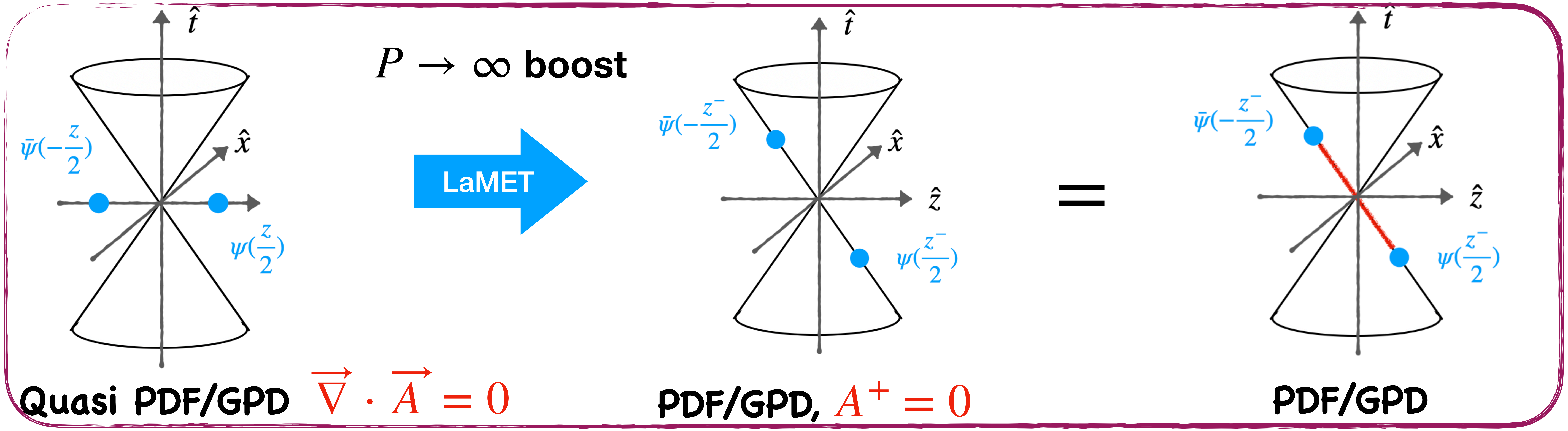
Quasi PDF/GPD

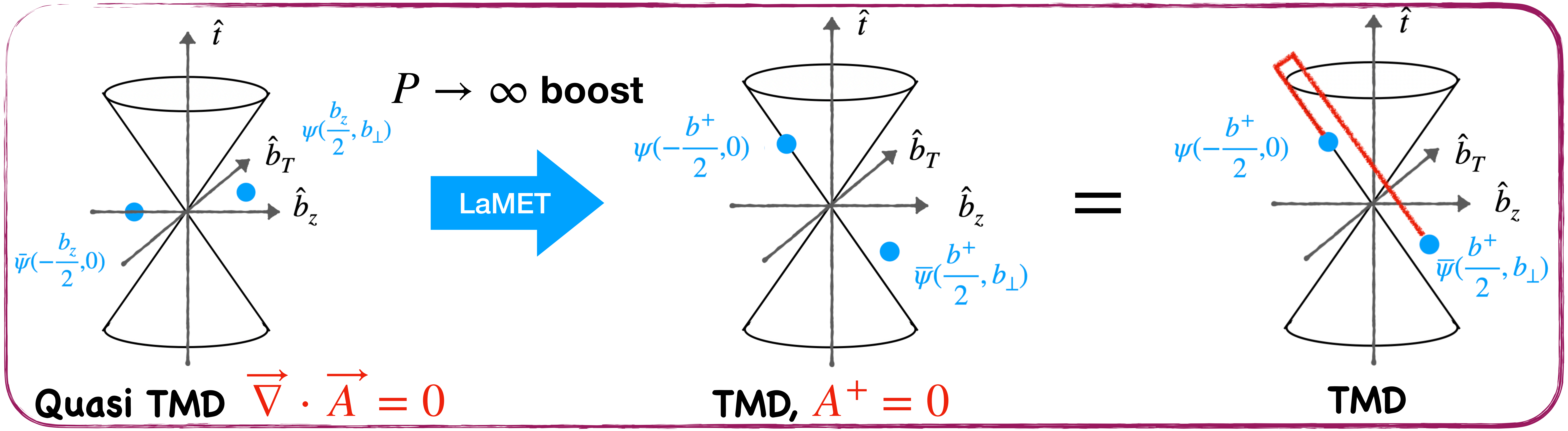


Quasi TMD

- Compute the quasi distributions in the physical gauge condition, e.g., $A_z = 0$, $A_0 = 0$, Coulomb gauge $\vec{\nabla} \cdot \vec{A} = 0$.
- Boost the operator as well as the physical gauge condition to the light-cone $A^+ = 0$ in the $P \to \infty$ limit.

CG quasi distribution without Wilson lines





CG quasi distribution without Wilson lines

- **The quark field in the Coulomb gauge**

$$\psi_C(z) = U_C(z)\psi(z)$$

satisfying,

$$\vec{\nabla} \cdot \left[U_C \vec{A} U_C^{-1} + \frac{i}{g} U_C \vec{\nabla} U_C^{-1} \right] = 0$$

order by order in g, the solution:

$$U_C = \sum_{n=0}^{\infty} \frac{(ig)^n}{n!} \omega_n$$

$$\omega_1 = -\frac{1}{\nabla^2} \vec{\nabla} \cdot \vec{A},$$

$$\omega_2 = \frac{1}{\nabla^2} \left(\vec{\nabla} \cdot (\omega_1^\dagger \vec{\nabla} \omega_1) - [\vec{\nabla} \omega_1, \vec{A}] \right)$$

$$\ldots$$

CG quasi distribution without Wilson lines

- **The quark field in the Coulomb gauge**

$$\psi_C(z) = U_C(z)\psi(z)$$

satisfying,

$$\vec{\nabla}\cdot\left[U_C\vec{A}U_C^{-1} + \frac{i}{g}U_C\vec{\nabla}U_C^{-1}\right] = 0$$

order by order in g, the solution:

$$U_C = \sum_{n=0}^{\infty}\frac{(ig)^n}{n!}\omega_n$$

$$\omega_1 = -\frac{1}{\nabla^2}\vec{\nabla}\cdot\vec{A},$$

$$\omega_2 = \frac{1}{\nabla^2}\left(\vec{\nabla}\cdot(\omega_1^{\dagger}\vec{\nabla}\omega_1) - [\vec{\nabla}\omega_1, \vec{A}]\right)$$

$\ldots$

▸ $P \to \infty$ **limit boost**

$$-\frac{1}{\nabla^2}\vec{\nabla}\cdot\vec{A} = i\int\frac{d^4k}{(2\pi)^4}e^{-ik\cdot z}\frac{1}{k_z^2+k_\perp^2}[k_zA_z(k) + k_\perp A_\perp(k)]$$

$$\approx i\int\frac{d^4k}{(2\pi)^4}e^{-ik\cdot z}\frac{k^+}{(k^+)^2+\epsilon^2}A^+(k)$$

$$= \frac{1}{2}\left[\int_{-\infty^-}^{z^-} + \int_{+\infty^-}^{z^-}\right]d\eta^- A^+ \equiv \frac{1}{\partial^+_{\text{P.V.}}}A^+(z)$$

Principle value prescription (P.V.) averaging over past and future.

Path-ordered integral

$$\frac{\omega_n}{n!} \to \left(\ldots\left(\frac{1}{\partial^+_{\text{P.V.}}}\left(\left(\frac{1}{\partial^+_{\text{P.V.}}}A^+\right)A^+\right)A^+\right)\ldots A^+\right)$$

$$U_C \to \mathscr{P}\exp\left[-ig\int_{z^-}^{\mp\infty^-}dzA^+(z)\right] \equiv W(z^-, \mp\infty^-)$$

Infinite light-cone Wilson link

CG quasi-TMDs without Wilson lines

$$\frac{\tilde{f}_C(x,\vec{b}_T,\mu,P_z)}{\sqrt{S_C(\vec{b}_T,\mu)}} = C(\mu,xP_z)e^{\frac{1}{2}\gamma_\zeta(\mu,b_T)\ln\frac{(2xP_z)^2}{\zeta}} f(x,\vec{b}_T,\mu,\zeta)\{1+\mathcal{O}[\frac{1}{(xP_zb_T)^2},\frac{\Lambda^2_{\rm QCD}}{(xP_z)^2}]\}$$

• Y. Zhao, arXiv: 2311.01391

- The same form of factorization formula as the conventional gauge invariant (GI) case: verified through SCET.
- IR pole cancels in the one-loop calculation, differ only by UV:

$$\tilde{f}_C^{(1)}(x,\vec{b}_T,\mu,P_z,\epsilon_{\rm IR}) - \tilde{f}^{(1)}(x,\vec{b}_T,\mu,\epsilon_{\rm IR}) = -\frac{\alpha_s(\mu)C_F}{2\pi}\left[\frac{1}{2}\ln^2\frac{\mu^2}{4P_z^2}+3\ln\frac{\mu^2}{4p_z^2}+12-\frac{7\pi^2}{12}\right]$$

- Both CG and GI quasi-TMDs fall into the same universality class of LaMET in large P_z limit but with differently: power correction and $C(\mu,xP_z)$.

CG quasi-TMDs without Wilson lines

$$\frac{\tilde{f}_C(x,\vec{b}_T,\mu,P_z)}{\sqrt{S_C(\vec{b}_T,\mu)}} = C(\mu,xP_z)e^{\frac{1}{2}\gamma_\zeta(\mu,b_T)\ln\frac{(2xP_z)^2}{\zeta}}f(x,\vec{b}_T,\mu,\zeta)\{1+\mathcal{O}[\frac{1}{(xP_zb_T)^2},\frac{\Lambda^2_{\rm QCD}}{(xP_z)^2}]\}$$

- Y. Zhao, arXiv: 2311.01391

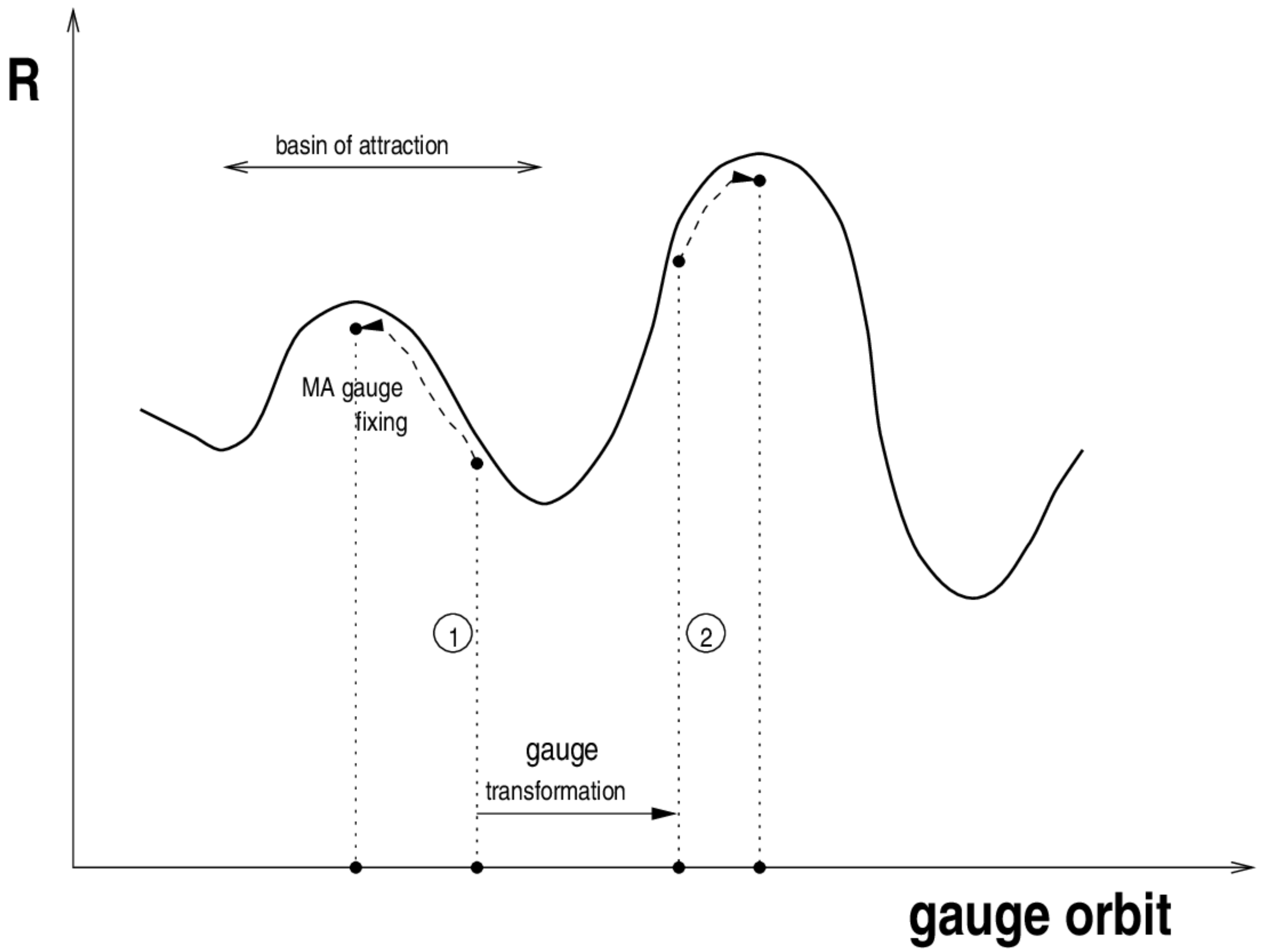
- Gribov copies in the non-perturbative Coulomb gauge fixing?
- Complexity from Wilson line disappear?
- Power corrections under control?

Gauge fixing and Gribov copies

- The Coulomb gauge fixing was done through minimizing the functional,

$$F[U^{\Omega}] = \frac{1}{9V}\sum_{\vec{x}}\sum_{i=1,2,3}\left[-\,\mathrm{Re}\ \mathrm{Tr}\ U_i^{\Omega}(t,\vec{x})\right]$$

- There could be multiple solutions (local minima) for the non-perturbative fields, namely the Gribov copies.



Gauge fixing and Gribov copies

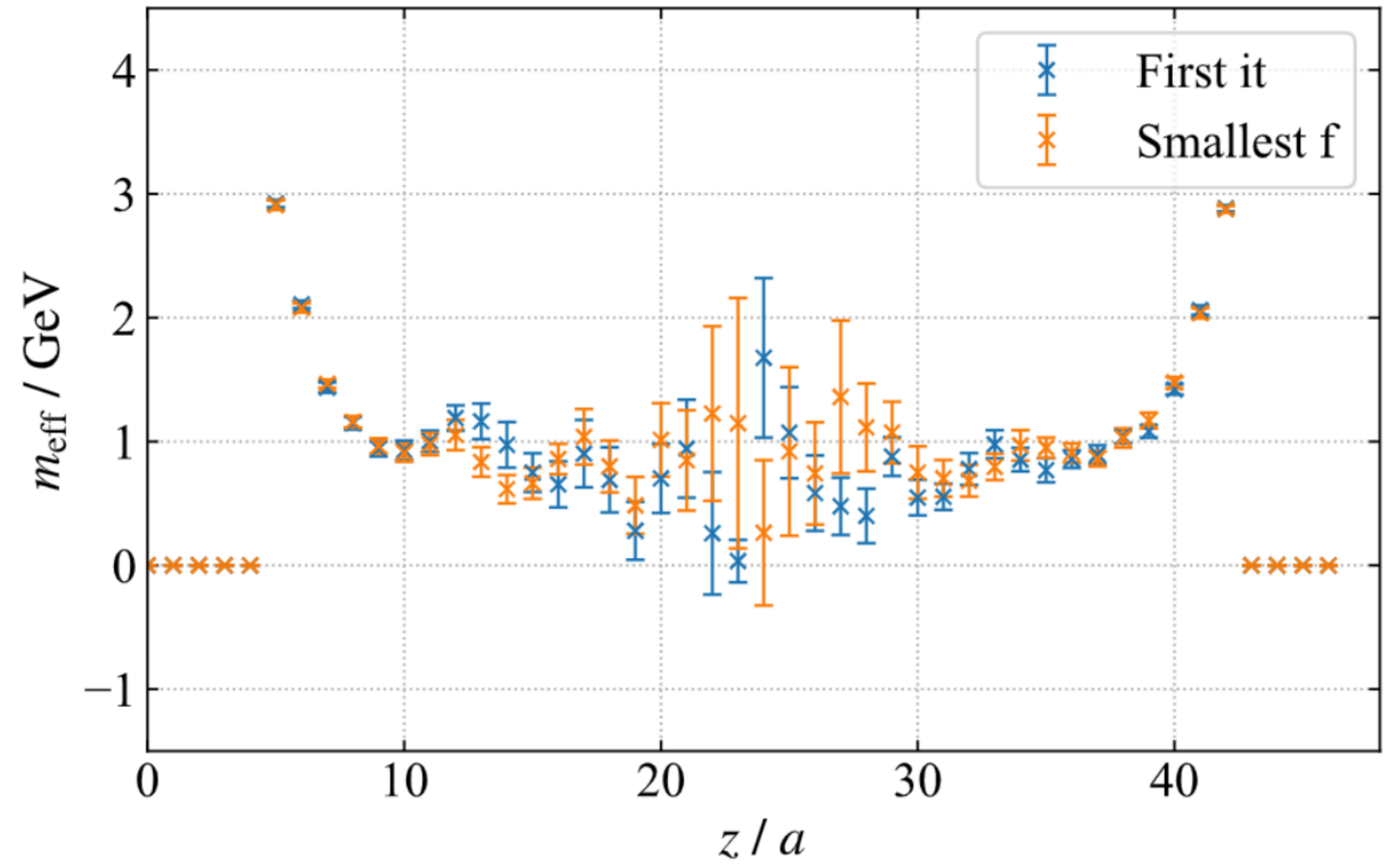
- The copies will behave like statistical noise if the algorithm randomly select the copies, which however won't reduce as the statistics increases.

- To see the significance of the Gribov copies, for each configuration we did gauge fixing multiple times after a random gauge transform.

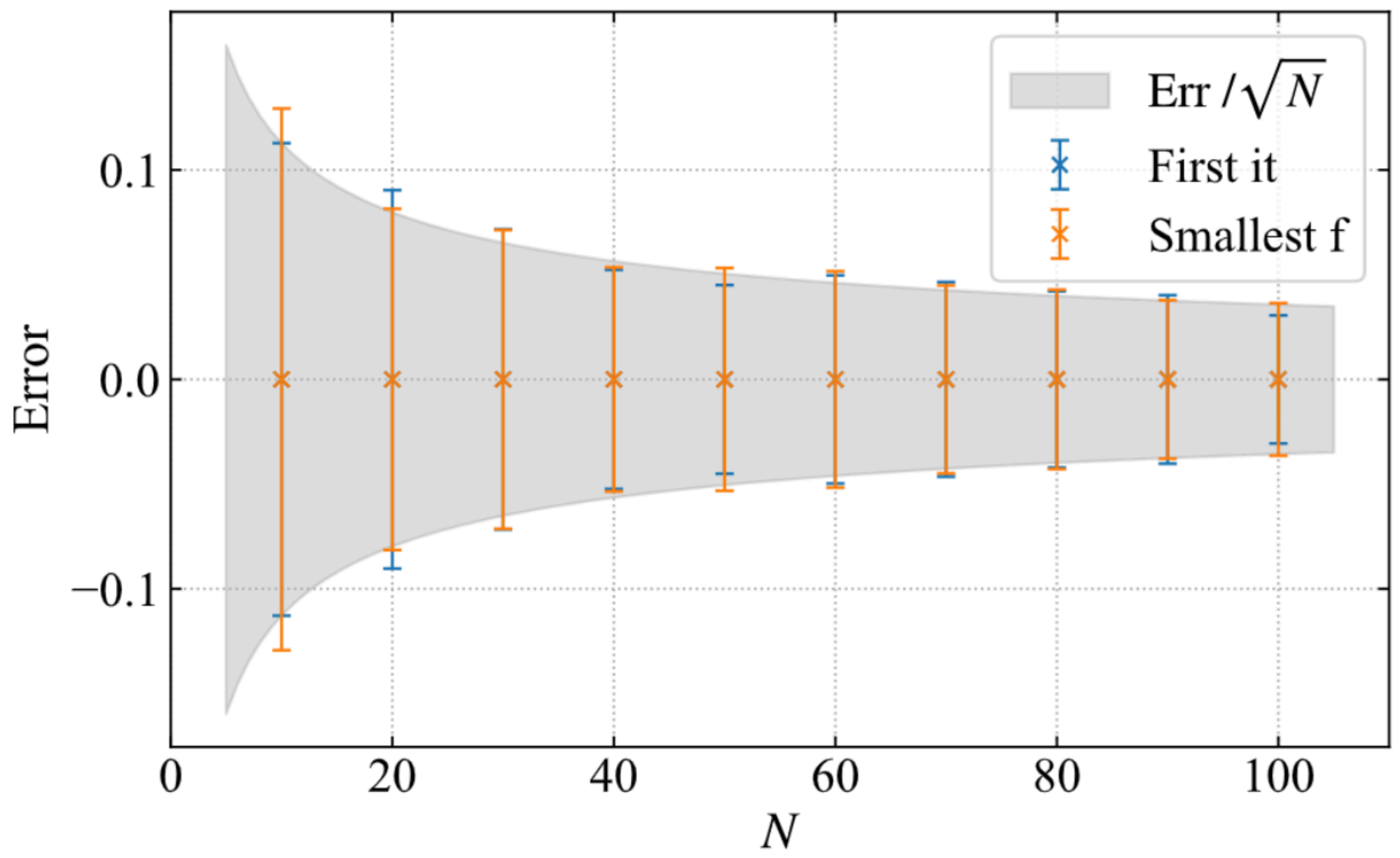
  - "First it": select the first daughter configuration.
  - "Smallest f": select the configuration with the lowest functional value.

The Gribov copies show no significance so far!

Effective mass of quark propagator



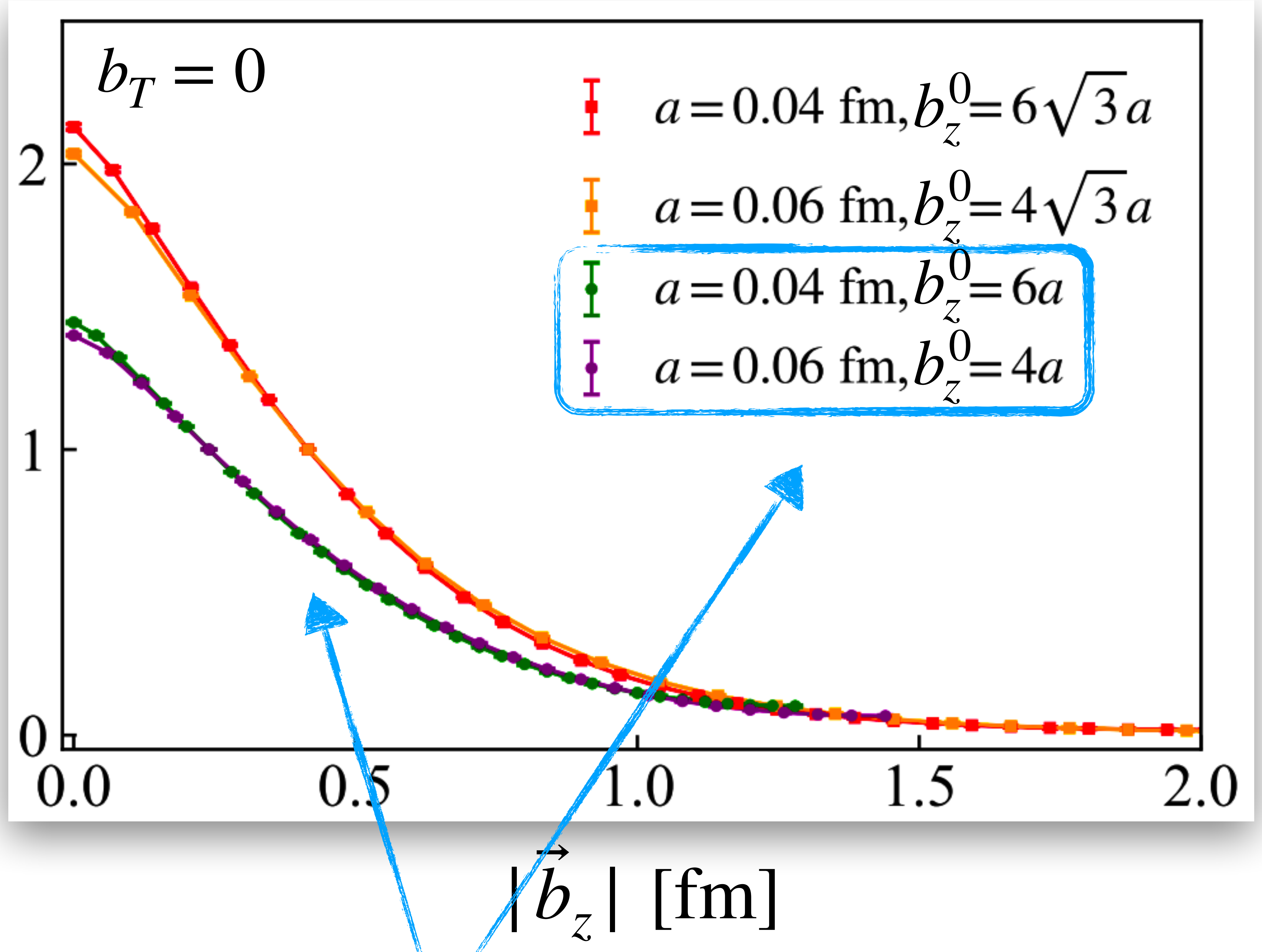
Noise v.s. statisitics for quasi-PDF



• J.-C. He, et al., in preparation…

Simplified renormalization

Renormalized matrix elements



Two lattice spacings: excellent continuum limit!

- No linear divergence: the renormalization is an overall constant.

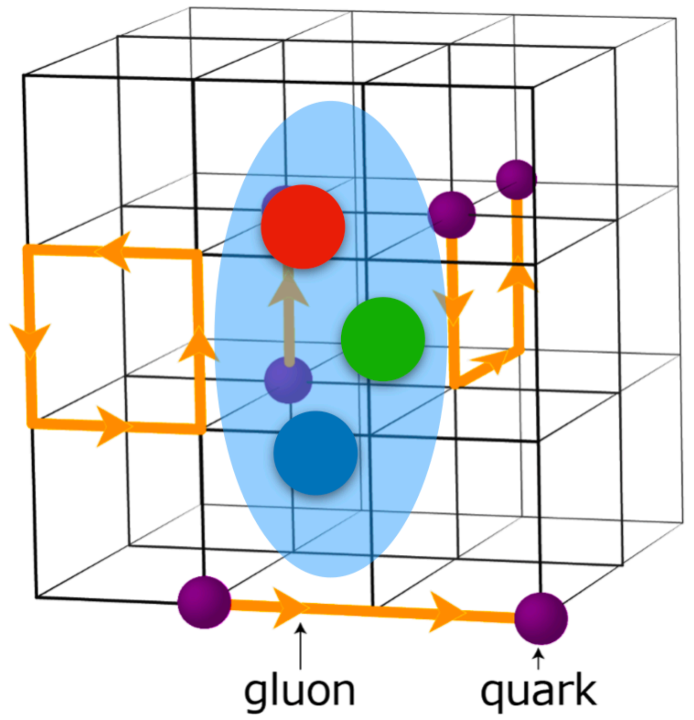
$$[\bar{\psi}(-\frac{\vec{b}}{2})\Gamma\psi(\frac{\vec{b}}{2})]_B = Z_\psi(a)[\bar{\psi}(-\frac{\vec{b}}{2})\Gamma\psi(\frac{\vec{b}}{2})]_R$$

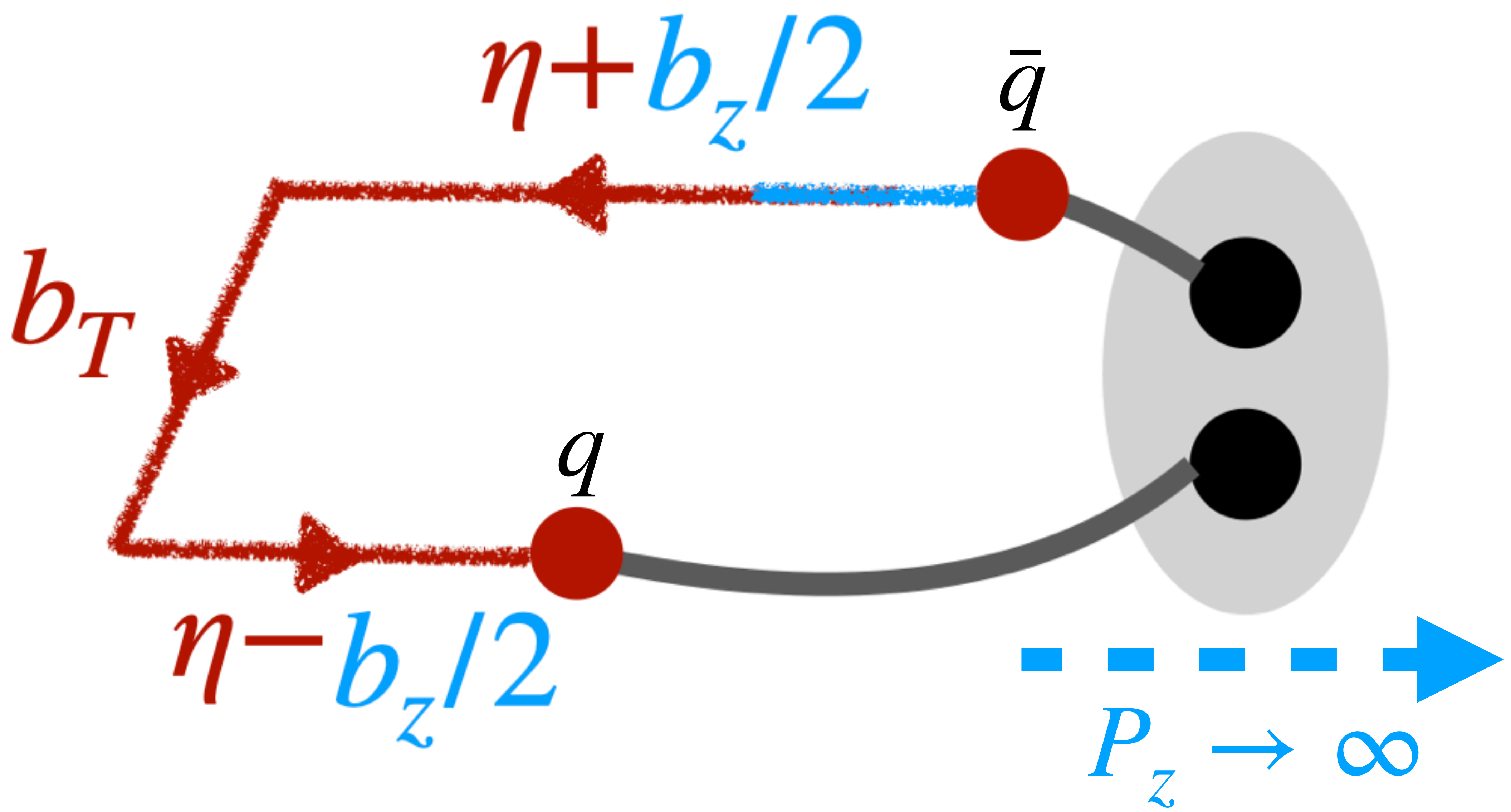
- Matrix elements with any separation $\vec{b}$ can be used to remove the UV divergence.

$$\frac{\tilde{h}^B(b_T, b_z, a)}{\tilde{h}^B(b_T^0, b_z^0, a)} = \frac{\tilde{h}^R(b_T, b_z, \mu)}{\tilde{h}^R(b_T^0, b_z^0, \mu)}$$

• **XG**, W.-Y. Liu, Y. Zhao, PRD 109 (2024) 9, 094506

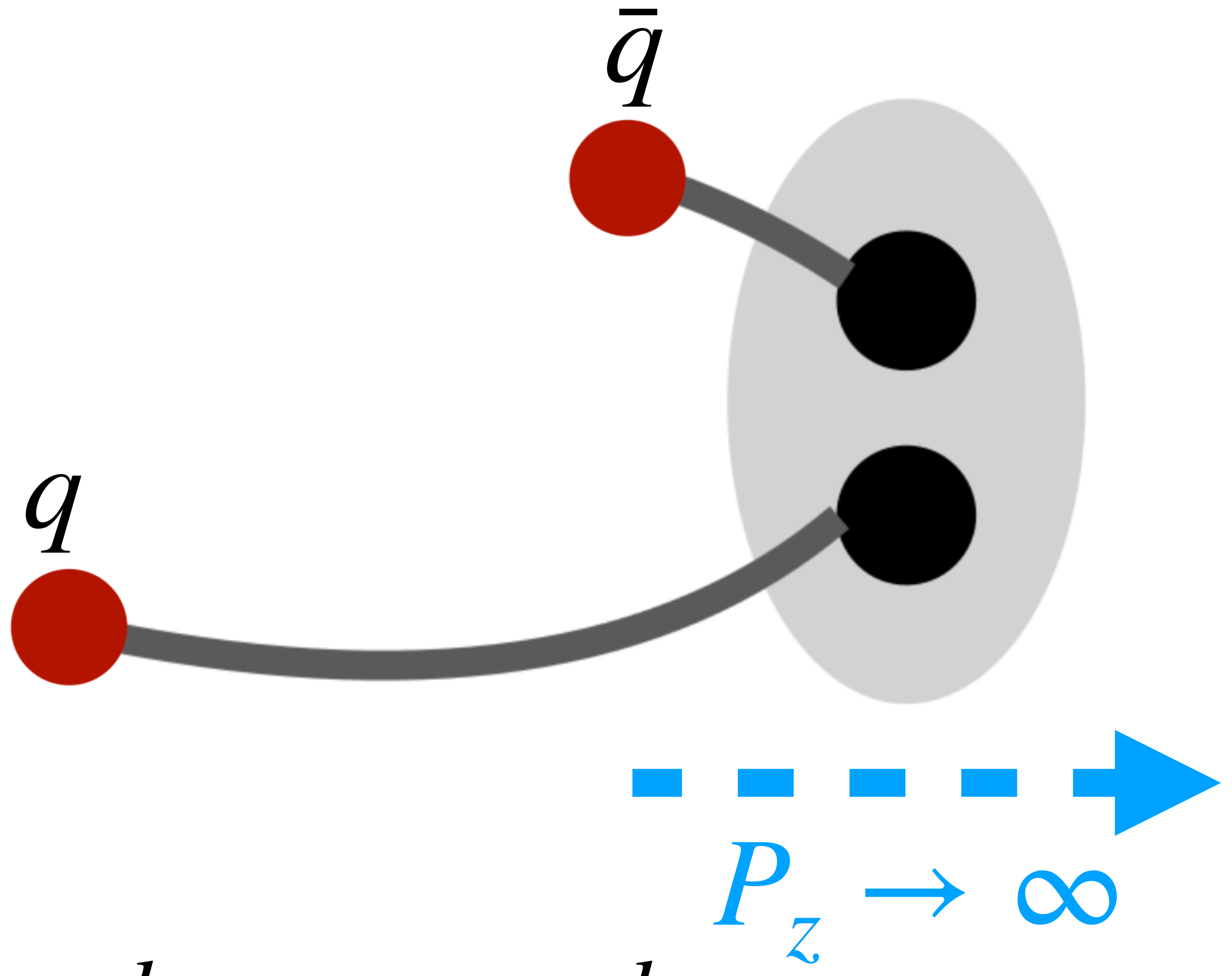
The quasi-TMD wave function from lattice





$$\langle \Omega | \overline{\psi}(\frac{b_z}{2}, b_\perp)\Gamma W_{\sqsubset z}\psi(-\frac{b_z}{2},0) | \pi^+, P_z\rangle$$

Gauge-invariant (GI)
quasi-TMDWF

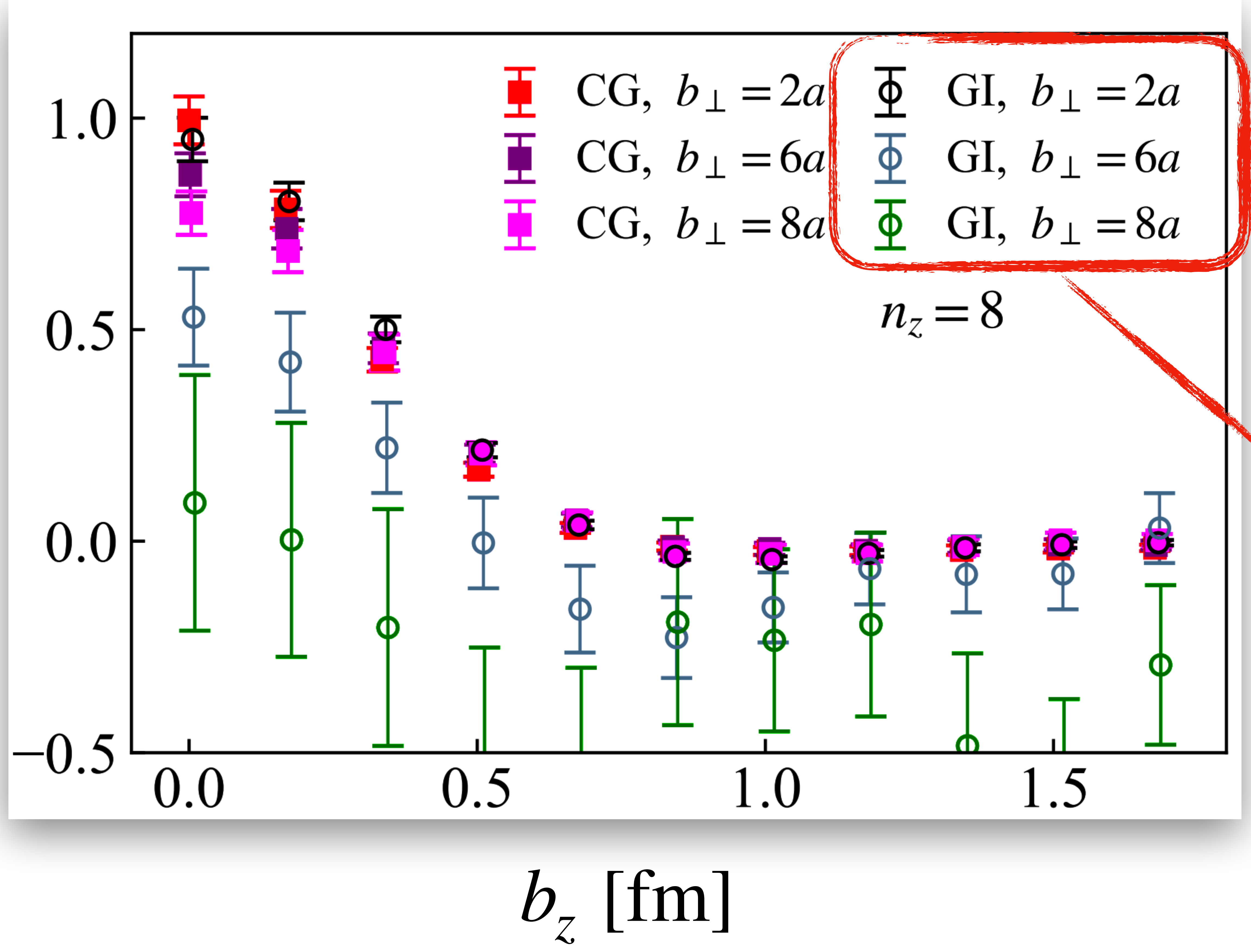


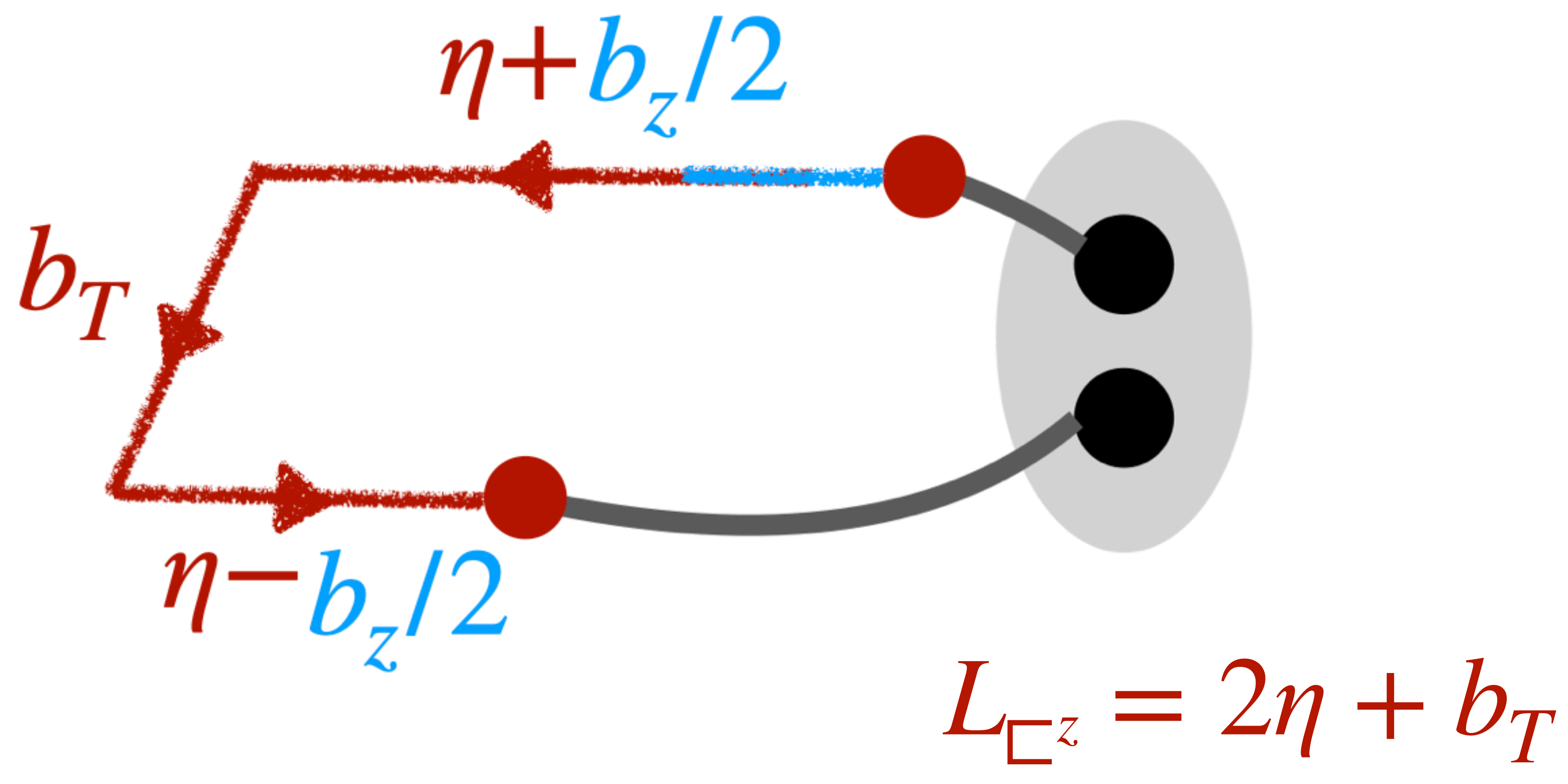
$$\langle \Omega | \overline{\psi}(\frac{b_z}{2}, b_\perp)\Gamma \psi(-\frac{b_z}{2},0) |_{\vec{\nabla}\cdot\vec{A}=0} | \pi^+, P_z\rangle$$

Coulomb gauge (CG)
quasi-TMDWF

CG quasi-TMDs: enhanced long-range precision

Renormalized matrix elements





b_T: $2a \to 6a \to 8a$

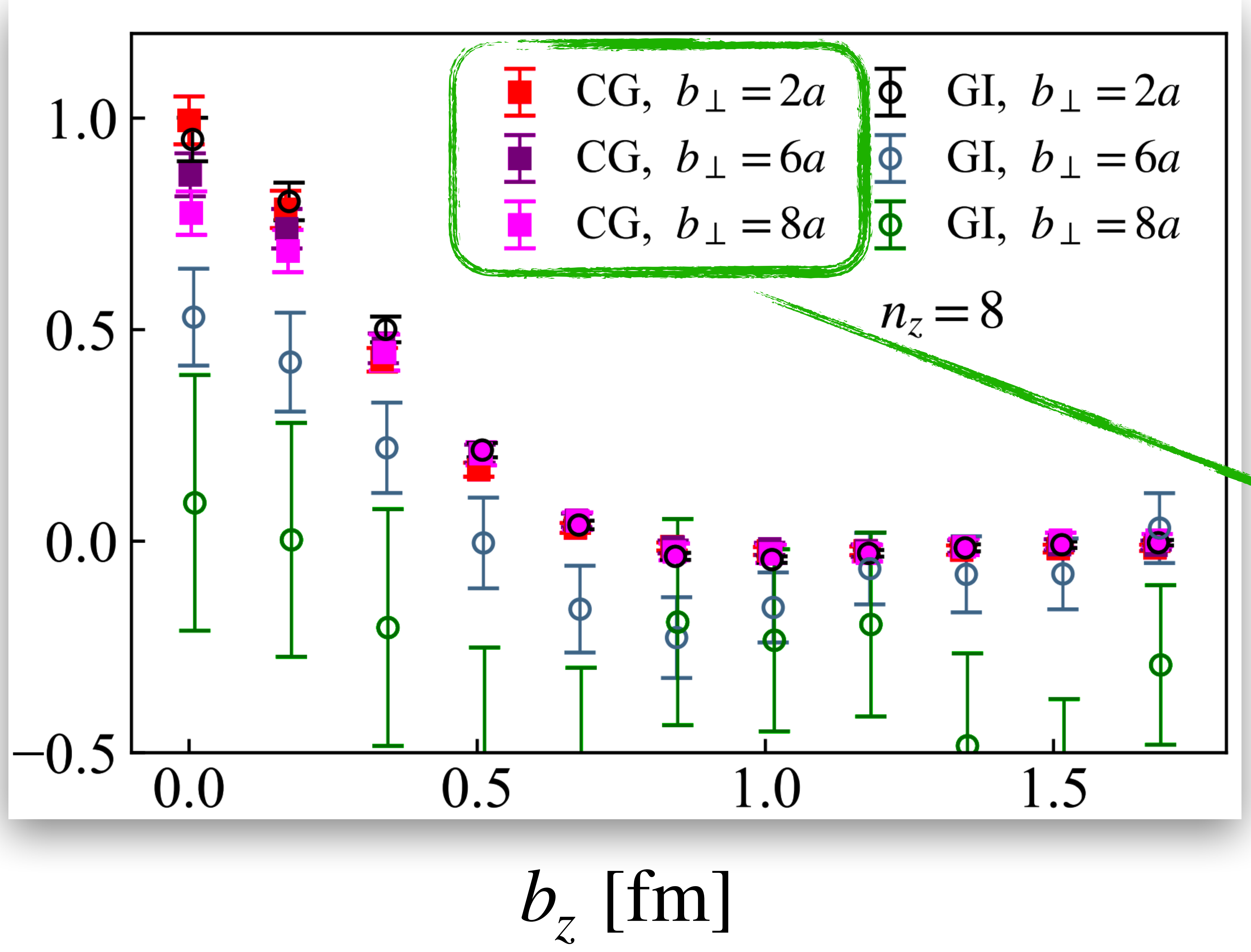
- GI shows rapid signal decay.

Domain wall fermion, physical quark masses
$64^3 \times 128$, a = 0.084 fm

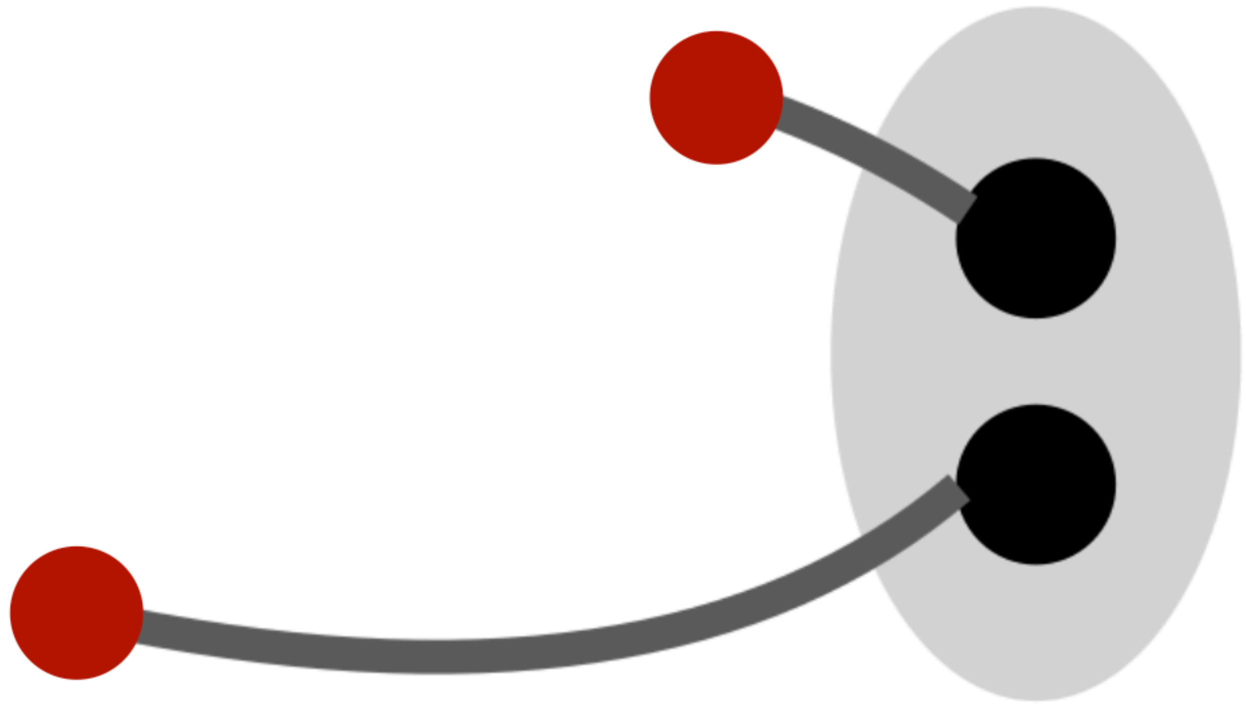
• D. Bollweg, **XG**, S. Mukherjee, Y. Zhao, PLB 852 (2024) 138617

CG quasi-TMDs: enhanced long-range precision

Renormalized matrix elements



Domain wall fermion, physical quark masses
$64^3 \times 128$, a = 0.084 fm

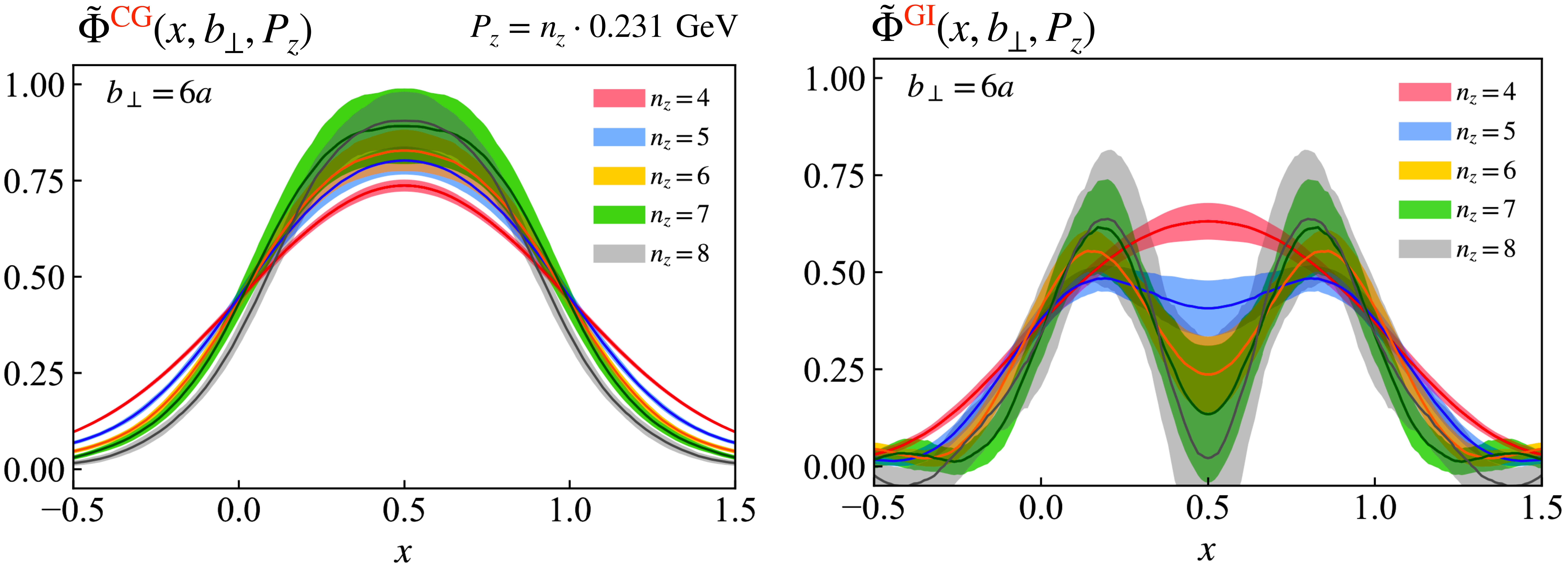


b_T: $2a \rightarrow 6a \rightarrow 8a$

- CG shows much slower signal decay compared to the GI cases.

• D. Bollweg, **XG**, S. Mukherjee, Y. Zhao, PLB 852 (2024) 138617

Quasi-TMD wave functions after F.T.



- The CG quasi-TMD wave functions are more stable and show better signal.

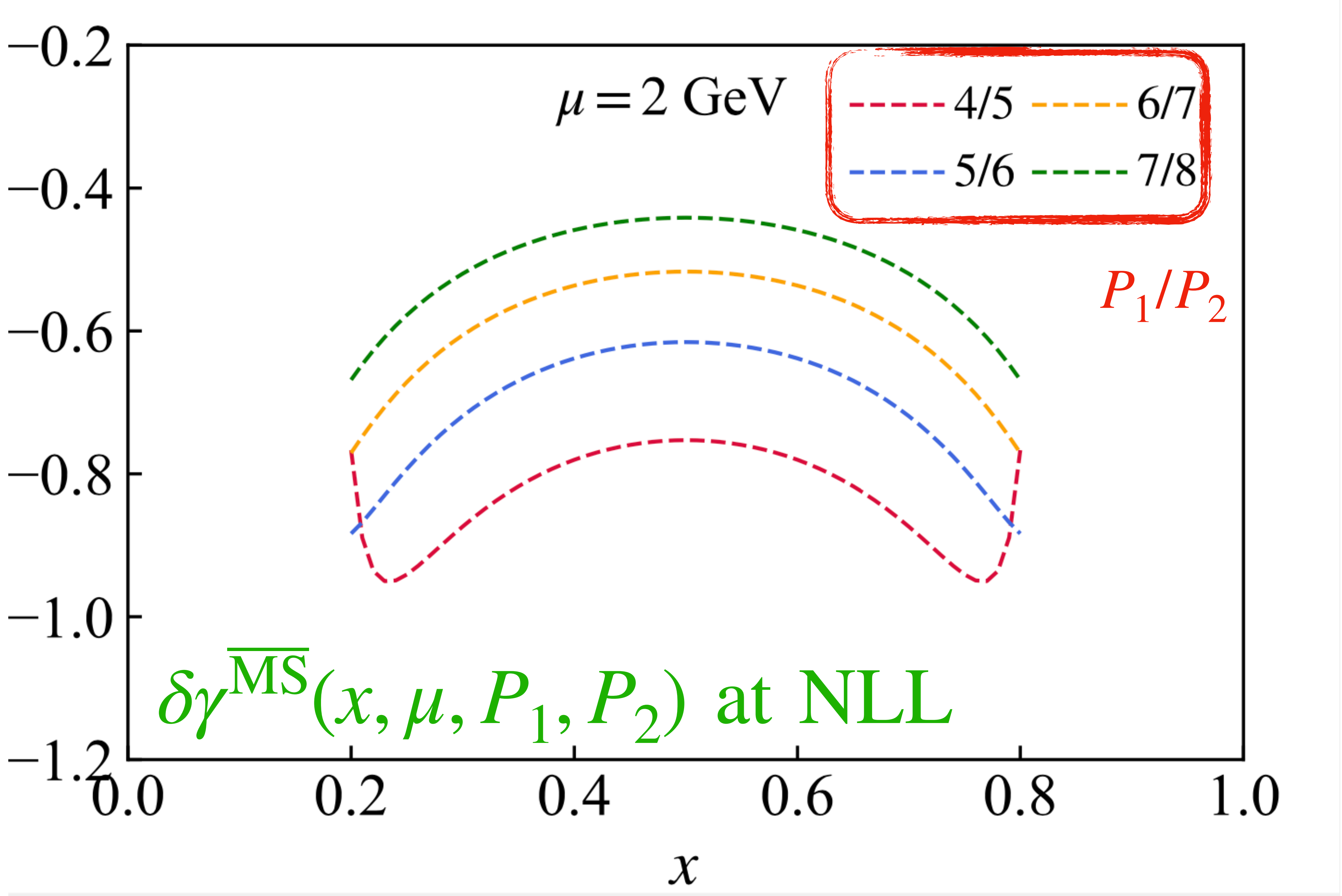
The Collins-Soper kernel from CG quasi-TMDWF

Re-organize the factorization formula into:

Perturbative correction

$$\gamma^{\overline{\mathrm{MS}}}(b_\perp,\mu) = \frac{1}{\ln(P_2/P_1)} \ln\left[\frac{\tilde{\phi}(x,b_\perp,P_2,\mu)}{\tilde{\phi}(x,b_\perp,P_1,\mu)}\right] + \delta\gamma^{\overline{\mathrm{MS}}}(x,\mu,P_1,P_2) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{(xP_z)^2}, \frac{1}{(b_\perp(xP_z))^2}\right)$$

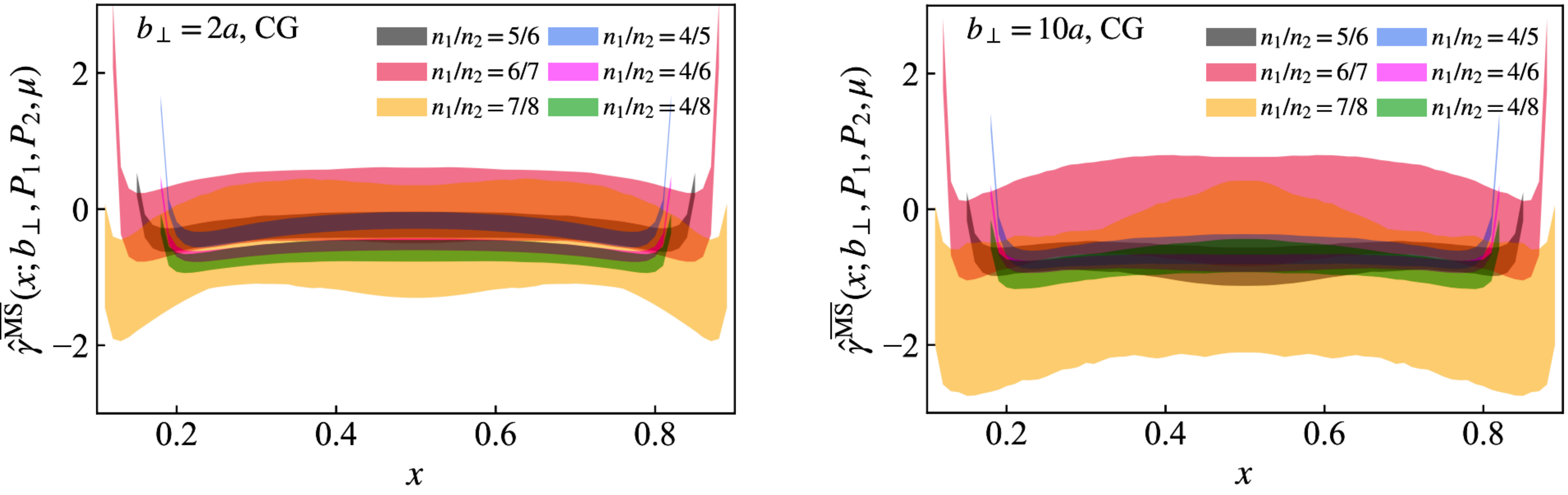
Ratio of quasi-TMDWFs



- The CS kernel $\gamma^{\overline{\mathrm{MS}}}(b_\perp,\mu)$ is **independent (universal) of P_z and x.**
- P_z and x dependence from data should be compensated by the perturbative matching, **up to higher-order and power corrections**.

The CS kernel from NLL matching

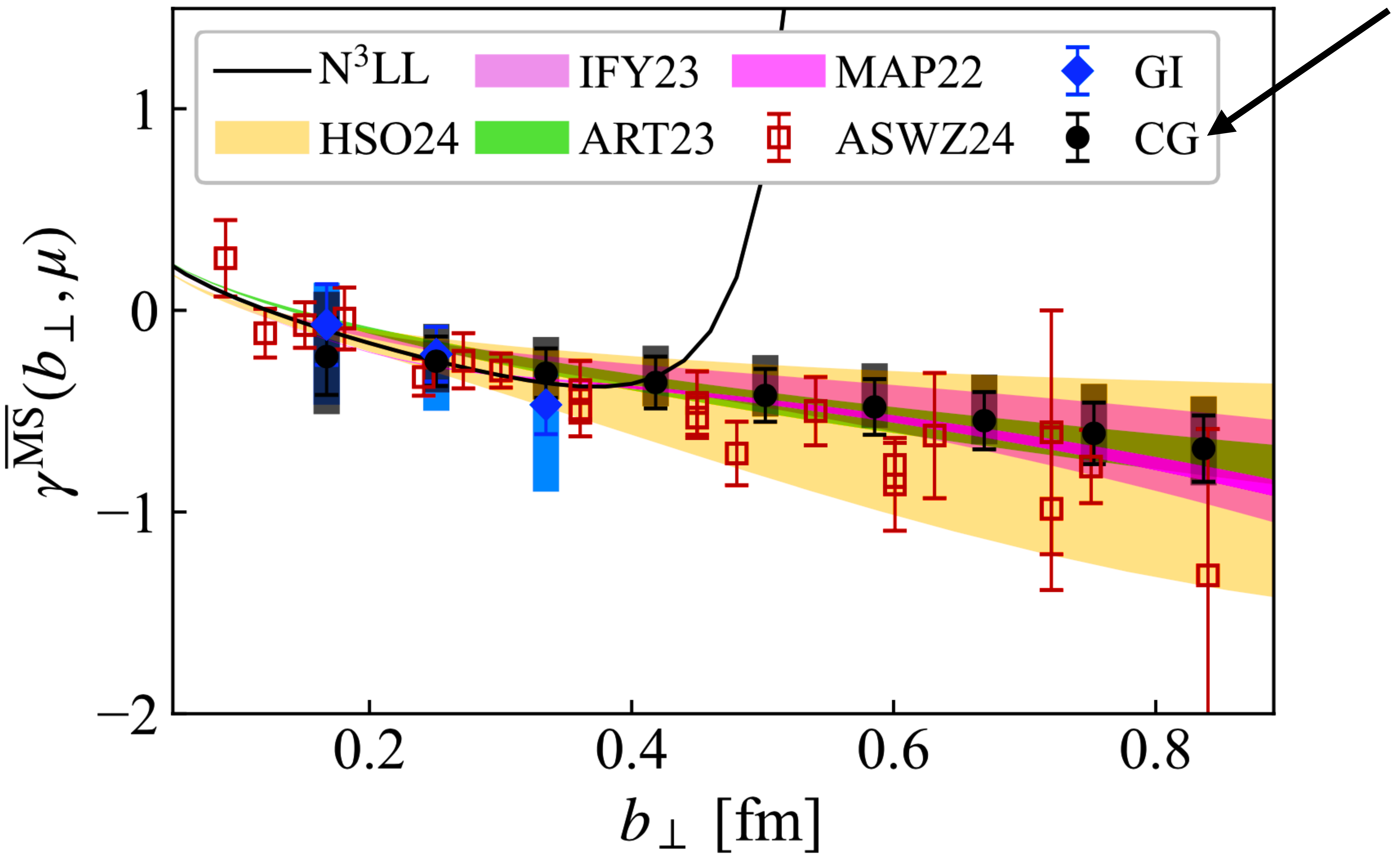
$a = 0.084$ fm, $\mathrm{P_z} = \mathrm{n_z} \cdot 0.23$ GeV



- Small $b_T \sim 0.1$ fm: visible P_z dependence.
- Sizable power corrections.

- Large b_T: no x and P_z dependence.
- Perturbative factorization work well!

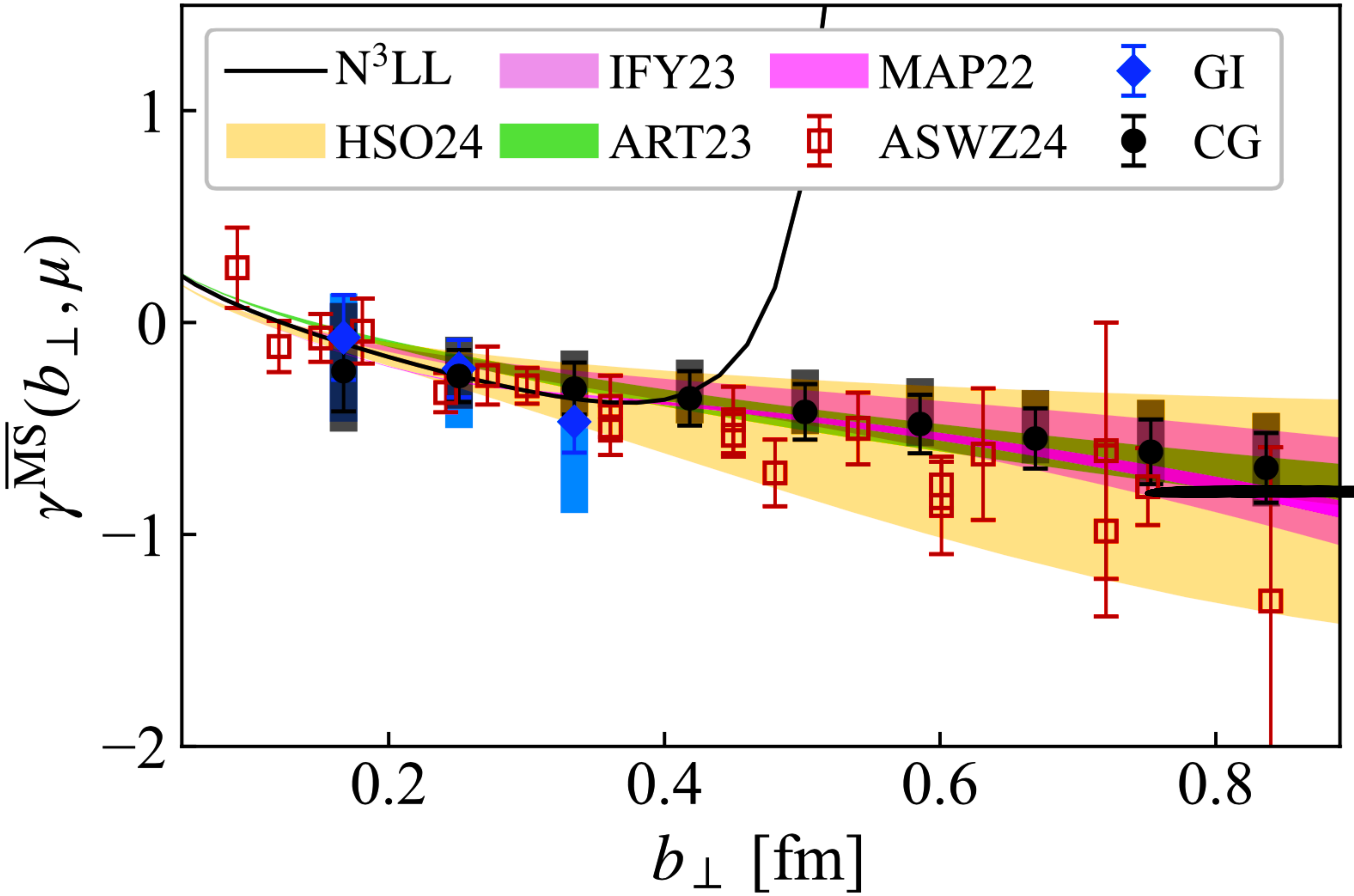
Nonperturbative Collins-Soper kernel

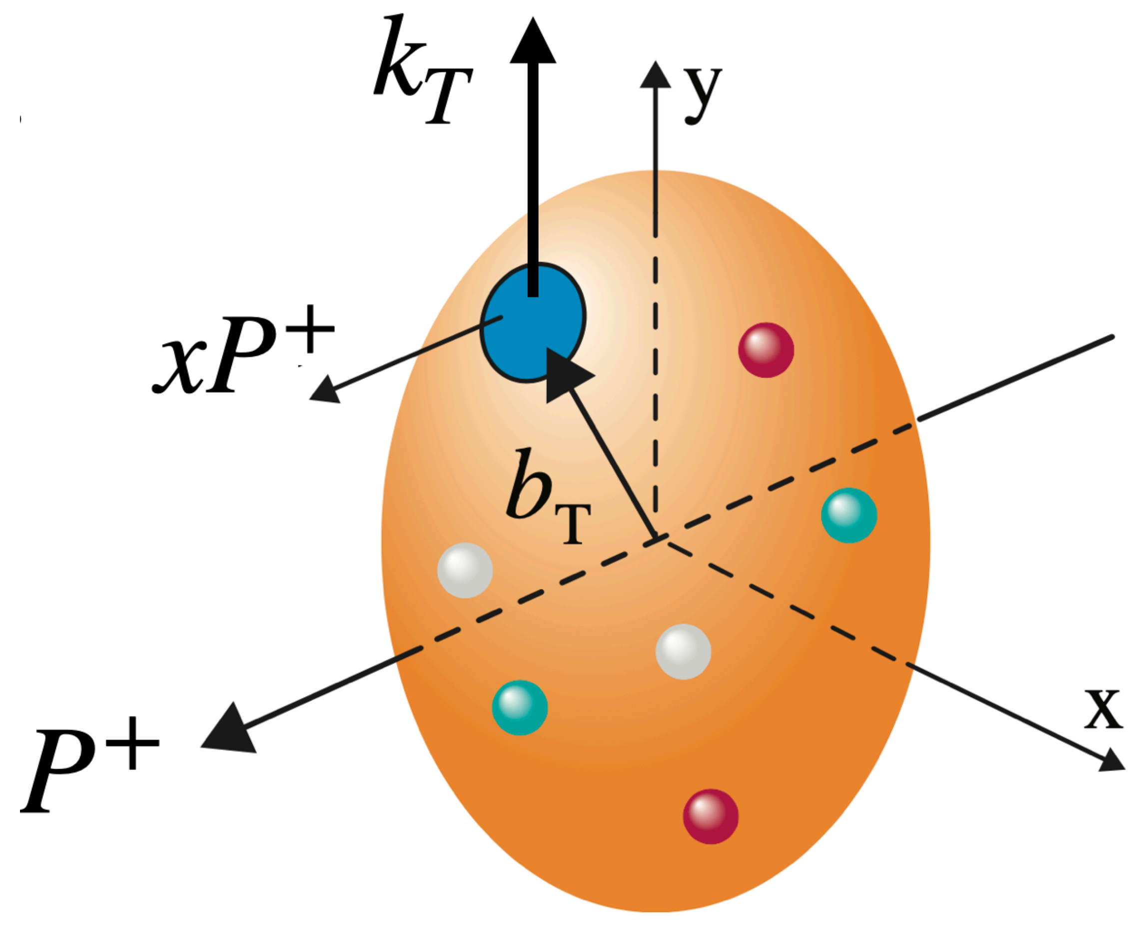


- Our QCD prediction: consistent with recent global fits and lattice results from GI operators.

• D. Bollweg, **XG**, S. Mukherjee, Y. Zhao, PLB 852 (2024) 138617

Nonperturbative Collins-Soper kernel





- CG approach greatly improve the efficiency: broader use in the nonperturbative regime of TMDs!

• D. Bollweg, **XG**, S. Mukherjee, Y. Zhao, PLB 852 (2024) 138617

Summary

- The light-cone parton distributions can be extracted from boosted quasi distributions in the Coulomb gauge, falling into the same universality class of LaMET as the conventional gauge invariant case.
- We extracted the non-perturbative CS kernel from the Coulomb-gauge-fixed quasi-TMD wave functions, appearing to be consistent with recent parametrization of experimental data.
- The CG methods have the advantages of the simplified renormalization and enhanced long-range precision, so that could have broader use in the future particularly in the non-perturbative regime of TMD physics, including the gluons and the Wigner distributions.

Thanks for your attention!