The path to precision: theory needs for FCC-ee

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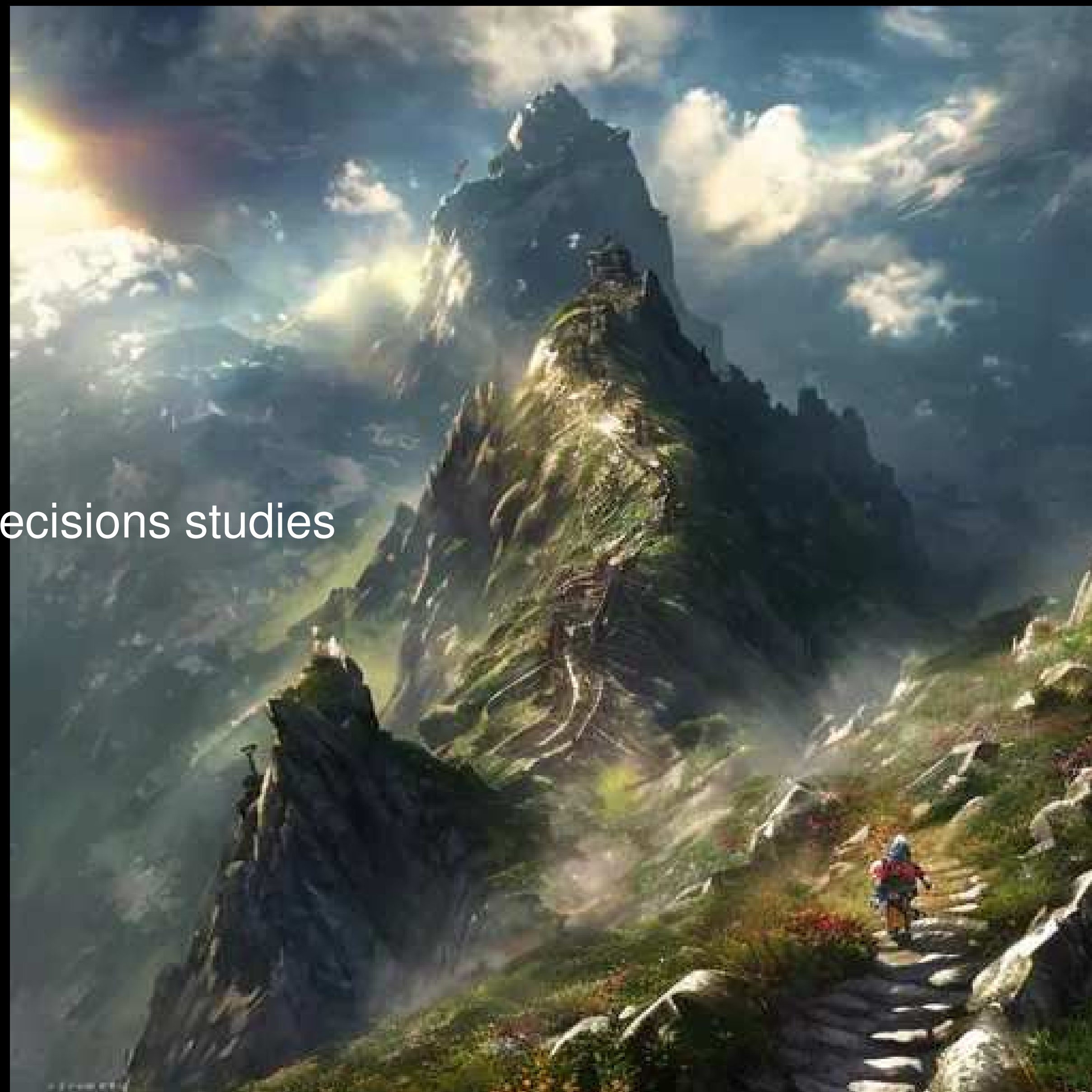


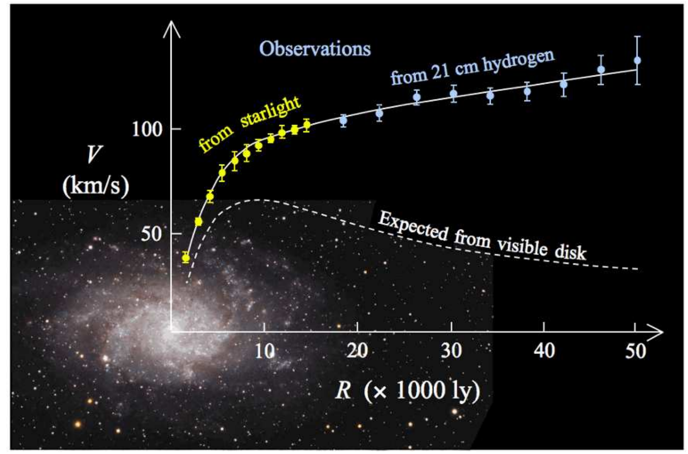
- BSM searches through precisions studies
- Electroweak physics
- Higgs physics
- “Input” parameters
- Theoretical challenges

Why the Standard Model cannot be everything

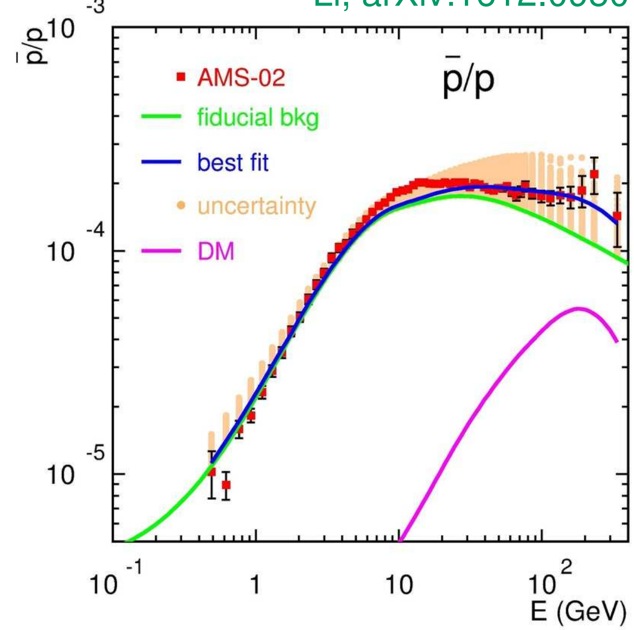
- **Dark matter**
  - only gravitational effects observed
  - massive particles, no electromagn./strong interactions
- **Matter-antimatter asymmetry**

  Sakharov conditions:
  - baryon number violation ✓
  - departure from equilibrium **X**
  - time invariance violation (✓)
- **Unification of forces and/or families**
- **No quantum description of gravity**
- **Hierarchy problem**
  - why is the Higgs mass much smaller than the Planck scale?

Corbelli, Salucci, astro-ph/9909252

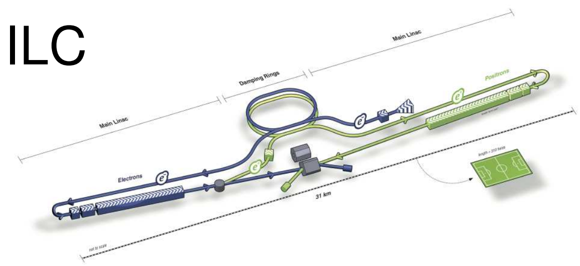


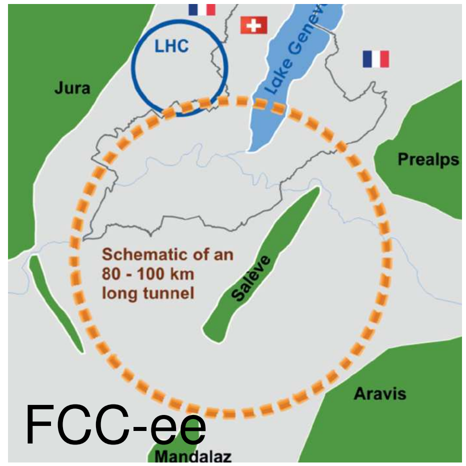
Li, arXiv:1612.09501

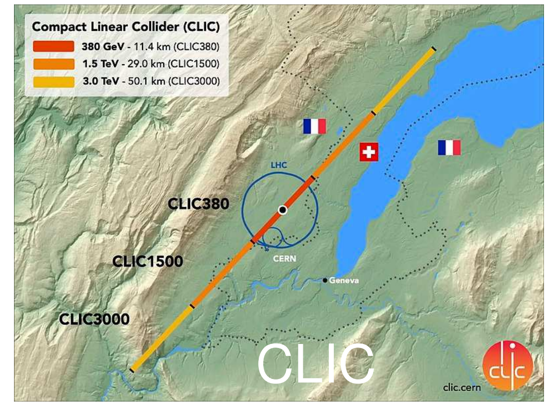


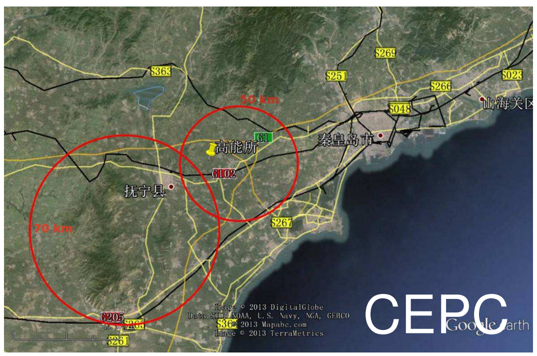
Precision physics with future e^+e^- colliders

ILC









- circular colliders: high-lumi run at $\sqrt{s} \sim M_Z$
- linear colliders: radiative return $e^+e^- \to \gamma Z$

$\sqrt{s}$	M_Z	$2M_W$	240–250 GeV	350–380 GeV
ILC	*100 fb^{-1}*	*500 fb^{-1}*	2 ab^{-1}	200 fb^{-1} (10 pts.)
CLIC	–	–	–	1 ab^{-1}
FCC-ee	150 ab^{-1}	10 ab^{-1} (2 pts.)	5 ab^{-1}	1 ab^{-1} (8 pts.)
CEPC	100 ab^{-1}	6 ab^{-1} (3 pts.)	20 ab^{-1}	1 ab^{-1} ?

Precision physics with future e^+e^- colliders

Exp. precision estimates for electroweak parameters:

	Current exp.	ILC250	CEPC	FCC-ee
M_W [MeV]	11–12	2.4	0.5	0.4
Γ_Z [MeV]	2.3	1.5	0.025	0.025
$R_\ell = \Gamma_\mathrm{Z}^\mathrm{had}/\Gamma_\mathrm{Z}^\ell$ $[10^{-3}]$	25	20	2	1
$R_b = \Gamma_\mathrm{Z}^b/\Gamma_\mathrm{Z}^\mathrm{had}$ $[10^{-5}]$	66	23	4.3	6
$\sin^2\theta^\ell_\mathrm{eff}$ $[10^{-5}]$	13	2	0.3	0.4

Exp. precision estimates for Higgs couplings:

	ILC250	CEPC	FCC-ee
hbb	1.1%	0.7%	0.6%
hcc	2.0%	1.3%	1.3%
$h\tau\tau$	1.0%	0.8%	0.7%
hWW	0.98%	0.73%	0.41%
hZZ	0.22%	0.07%	0.17%
$h\gamma\gamma$	1.4%	1.7%	1.3%
hgg	1.3%	0.9%	0.9%

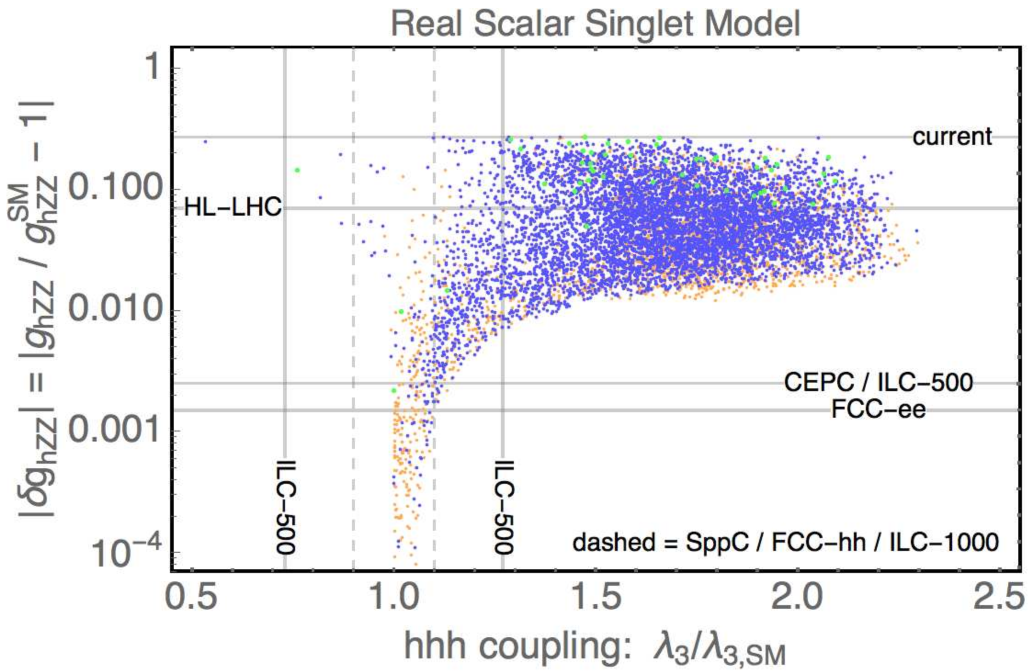
Snowmass ’21/22

Higgs singlet model

- Mixing between scalar singlet S and SM Higgs doublet modifies coupling strength of h(125)

  (dots = motivated by ew. baryogenesis)

  Huang, Long, Wang '16



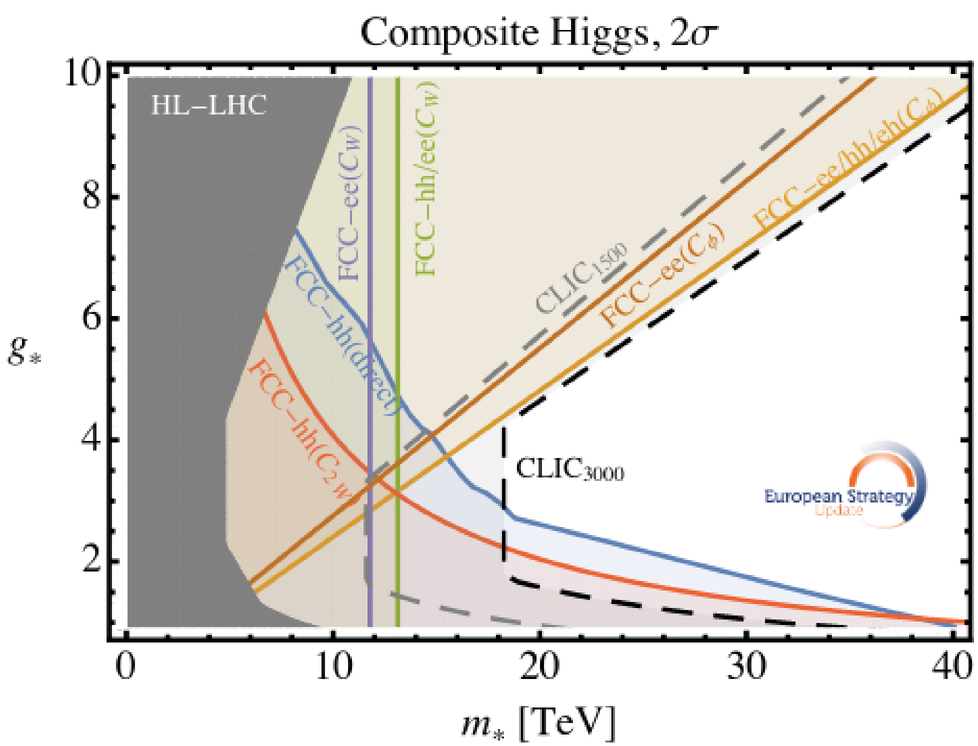
- If $m_S < m_h/2 \rightarrow$ exotic Higgs decays
  Carena, Liu, Wang '19
  Carena et al. '22

Composite Higgs

Higgs boson is bound state of new strongly interating sector

Generates new effective interactions at weak scale:

- $\mathcal{O}_\phi = \frac{g_*^2}{2m_*^2}\left(\partial_\mu |H|^2\right)^2$
  $\rightarrow$ modifies Higgs couplings
- $\mathcal{O}_W = \frac{g}{2m_*^2}(H^\dagger \sigma_a \, i\overleftrightarrow{D}_\mu H) D^\nu W^a_{\mu\nu}$
  $\rightarrow$ also modifies ew. couplings



g_* = strong coupling of composite sector
m_* = mass of heavy composite resonances

Neutrino counting and mixing

Total Z width from line-shape: $\Gamma_Z = 3\Gamma_\ell + \underbrace{\Gamma_{Z\to\text{inv}}}_{N_\nu \Gamma_\nu} + \Gamma_{\text{had}}$

$$N_\nu = \left[\left(\frac{12\pi}{M_Z^2}\frac{R_\ell}{\sigma^0_{\text{had}}}\right)^2 - R_\ell - 3\right]\frac{\Gamma_\ell}{\Gamma_\nu}$$

from measurement

computed in SM

Current data (LEP): $N_\nu = 2.996 \pm 0.007$ — Janot, Jadach '20

FCC-ee: $\pm\, 0.001$

Mixing with sterile neutrino:

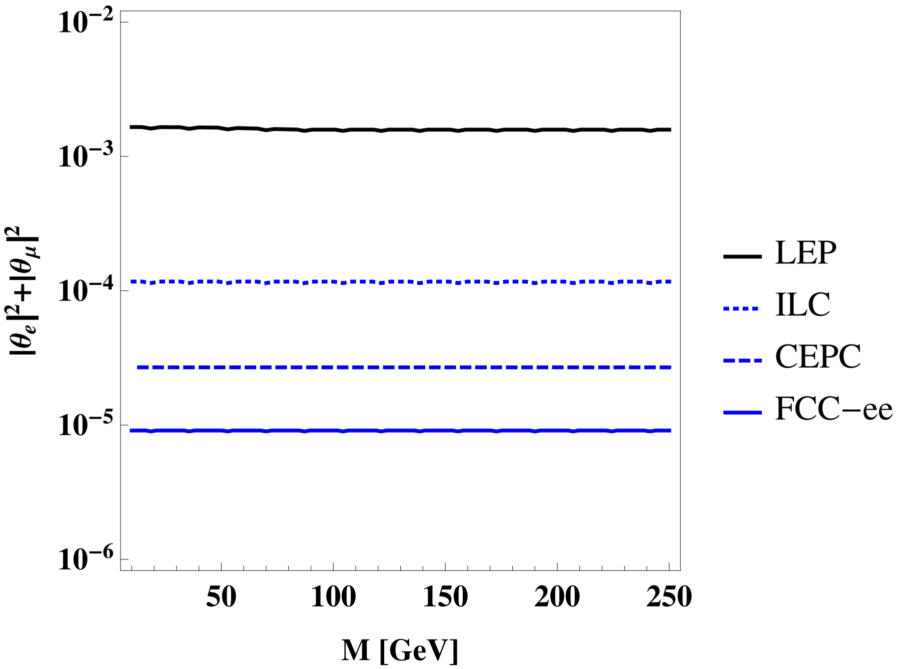
$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8M_W^2}(1+\Delta r)(1-\theta_e^2)(1-\theta_\mu^2)$$

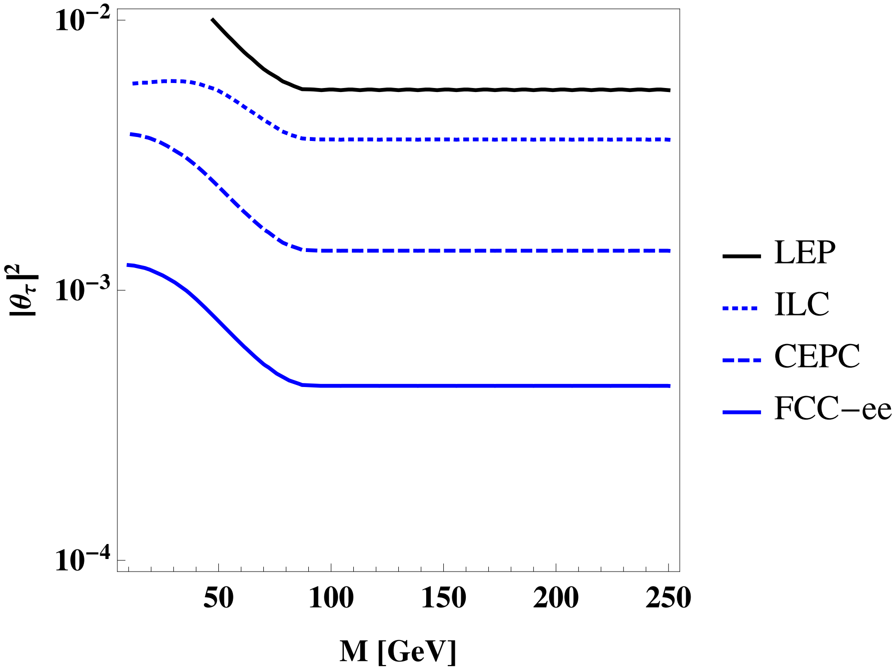
$$\Gamma_{Z\to\text{inv}} = \Gamma_\nu^{\text{SM}}\Big(N_\nu - \sum_{\alpha,\beta}\theta_\alpha\theta_\beta\Big)$$

θ_α: mixing of sterile neutrino with ν_α ($\theta_\alpha \ll 1$)

Neutrino mixing

Estimated sensitivity from electroweak precision tests:





Antusch, Fischer '15

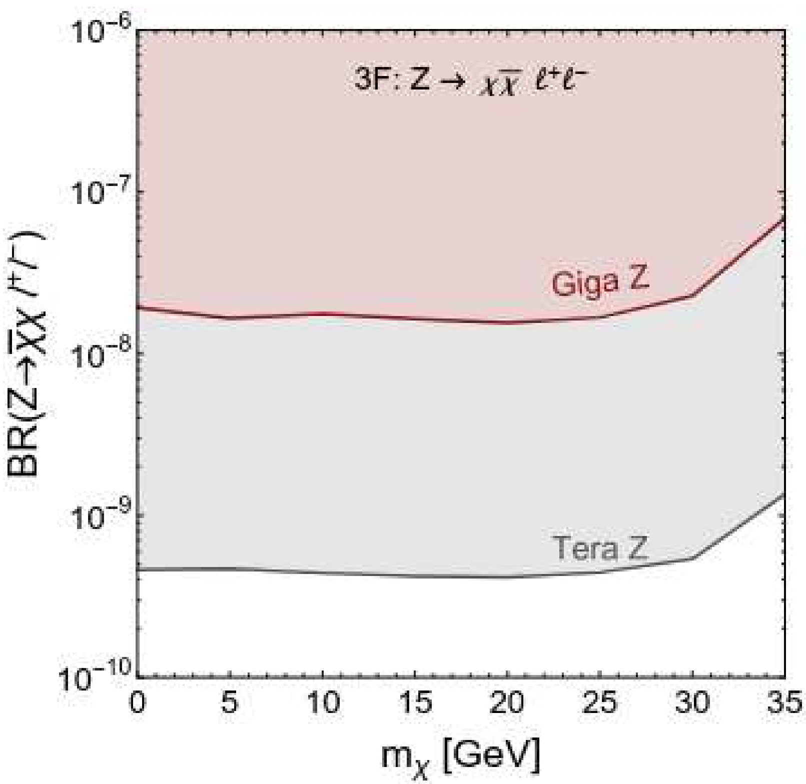
$$\theta_i = \frac{Y_{\nu_i N}}{\sqrt{2}} \frac{v}{M_N} \quad (M_N \gg v)$$

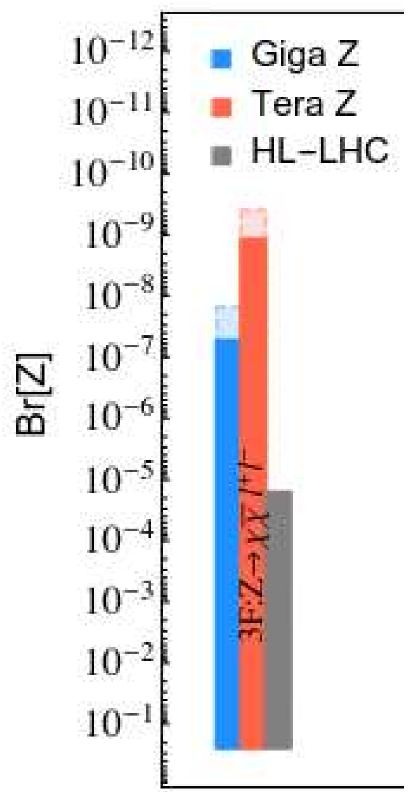
$\rightarrow$ sensitivity to $M_N \lesssim 5$ TeV

Exotic Z decays

Many possibilities in different models:

$Z \to \not{E} + \gamma,\ \not{E} + \gamma\gamma,\ \gamma\gamma\gamma,\ \not{E} + \ell^+\ell^-,\ \not{E} + jj,\ 4j$ Liu, Wang, Wang, Xue '17





requires good understanding of SM background!

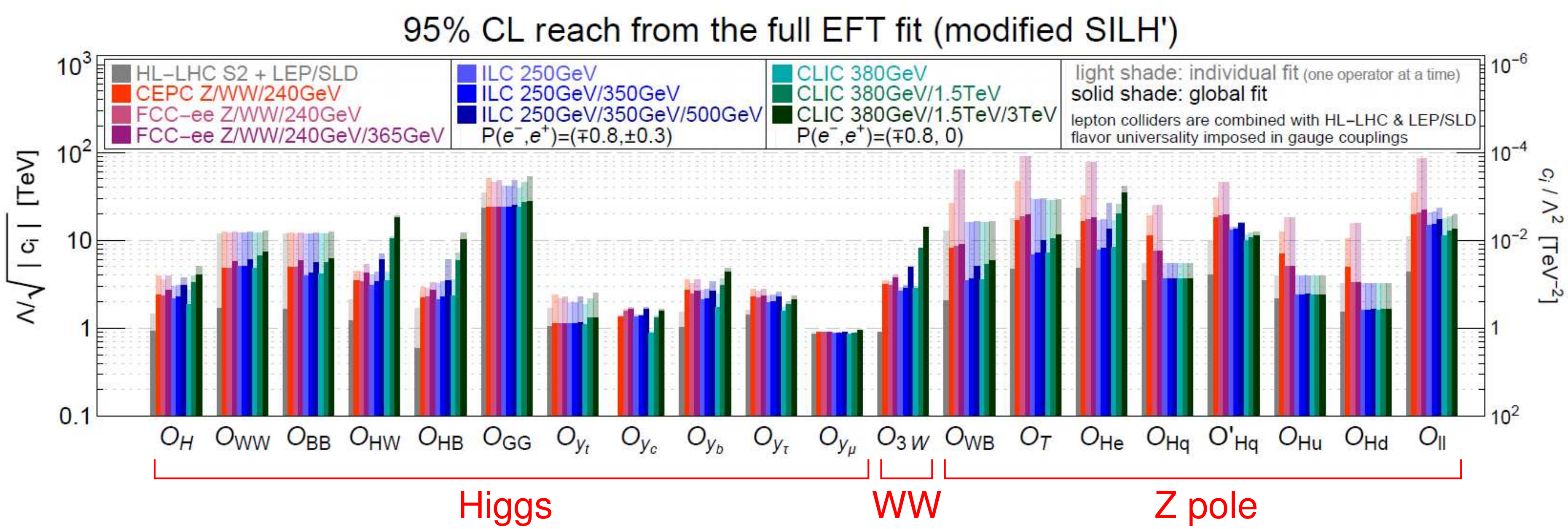
“SM Effective Field Theory” (SMEFT)

- Extension of SM by **higher-dimensional operators**:

Wilson '69
Weinberg '79

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{\Lambda} \ldots + \frac{1}{\Lambda^2} \sum_i c_i \, \mathcal{O}_i^{(6)} + \ldots$$

- SMEFT dim-6 operators provide framework for comparing experiments



de Blas, Durieux, Grojean, Gu, Paul '19

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Snowmass '21/22

Need for theory input

- Comparison of EWPOs / HPOs with SM to **probe new physics**
  $\longrightarrow$ multi-loop corrections in full SM

- Extraction of EWPOs / HPOs (**pseudo-observables**) from **real observables**
  $\longrightarrow$ backgrounds (in full SM), QED/QCD, MC tools

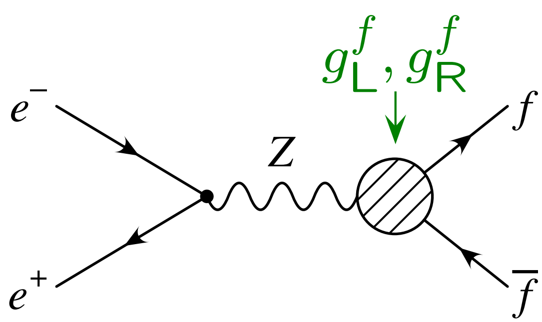
- "Other" eletroweak parameters (**"input" parameters**)
  $\longrightarrow$ m_{t}, α_{s}, etc. extracted from other processes

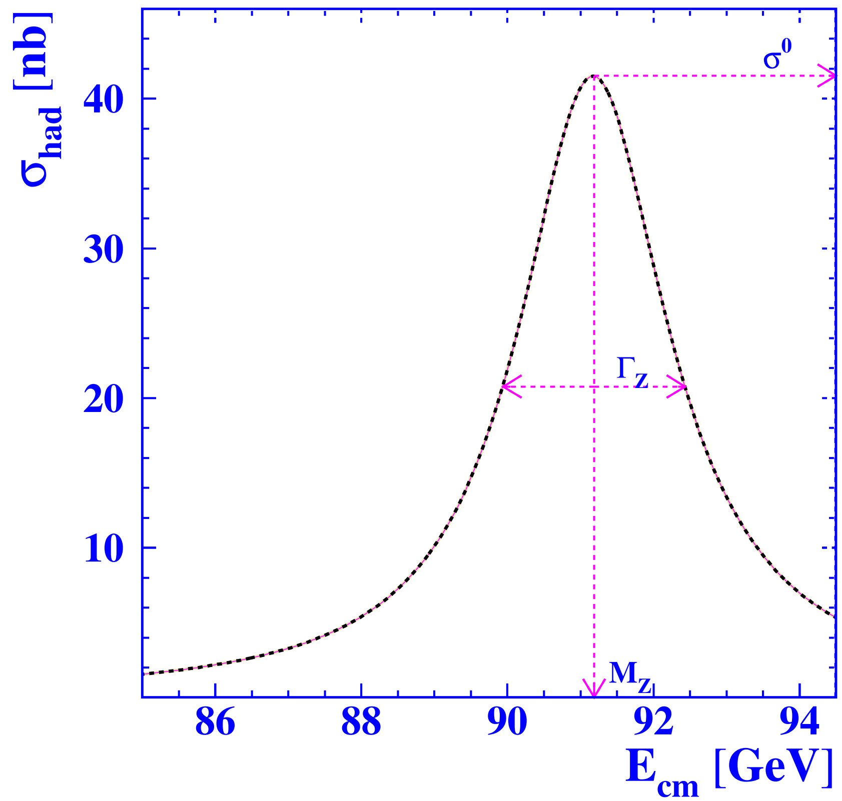
Z cross section and branching fractions

$e^+e^- \to f\bar{f}$ for $E_{\mathrm{CM}} \sim M_{\mathrm{Z}}$:

- Mass M_{Z}
- Width $\Gamma_{\mathrm{Z}} = \sum_f \Gamma_{ff}$
- Braching ratio $R_f = \Gamma_{ff}/\Gamma_{\mathrm{Z}}$
- $\sigma^0 \approx \frac{12\pi\,\Gamma_{ee}\Gamma_{ff}}{(s-M_{\mathrm{Z}}^2)^2+M_{\mathrm{Z}}^2\Gamma_{\mathrm{Z}}^2} = \frac{12\pi}{M_{\mathrm{Z}}^2}R_eR_f$

$$\Gamma_{ff} = C\left[(g_{\mathrm{L}}^f)^2 + (g_{\mathrm{R}}^f)^2\right]$$

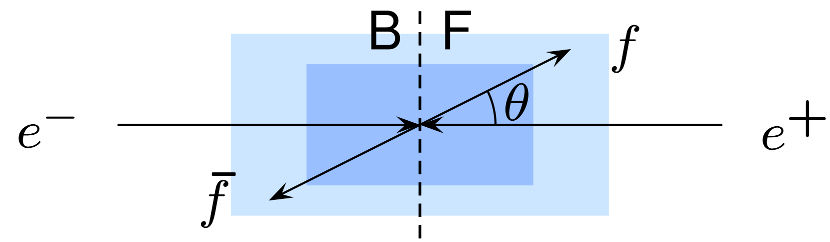




Z-pole asymmetries

Forward-backward asymmetry:

$$A_{\text{FB}} \equiv \frac{\sigma_{\text{F}} - \sigma_{\text{B}}}{\sigma_{\text{F}} + \sigma_{\text{B}}} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f$$



$$\mathcal{A}_f = \frac{2(1-4\sin^2\theta^f_{\text{eff}})}{1+(1-4\sin^2\theta^f_{\text{eff}})^2} \qquad \sin^2\theta^f_{\text{eff}} = \frac{g^f_R}{2|Q_f|(g^f_R - g^f_L)}$$

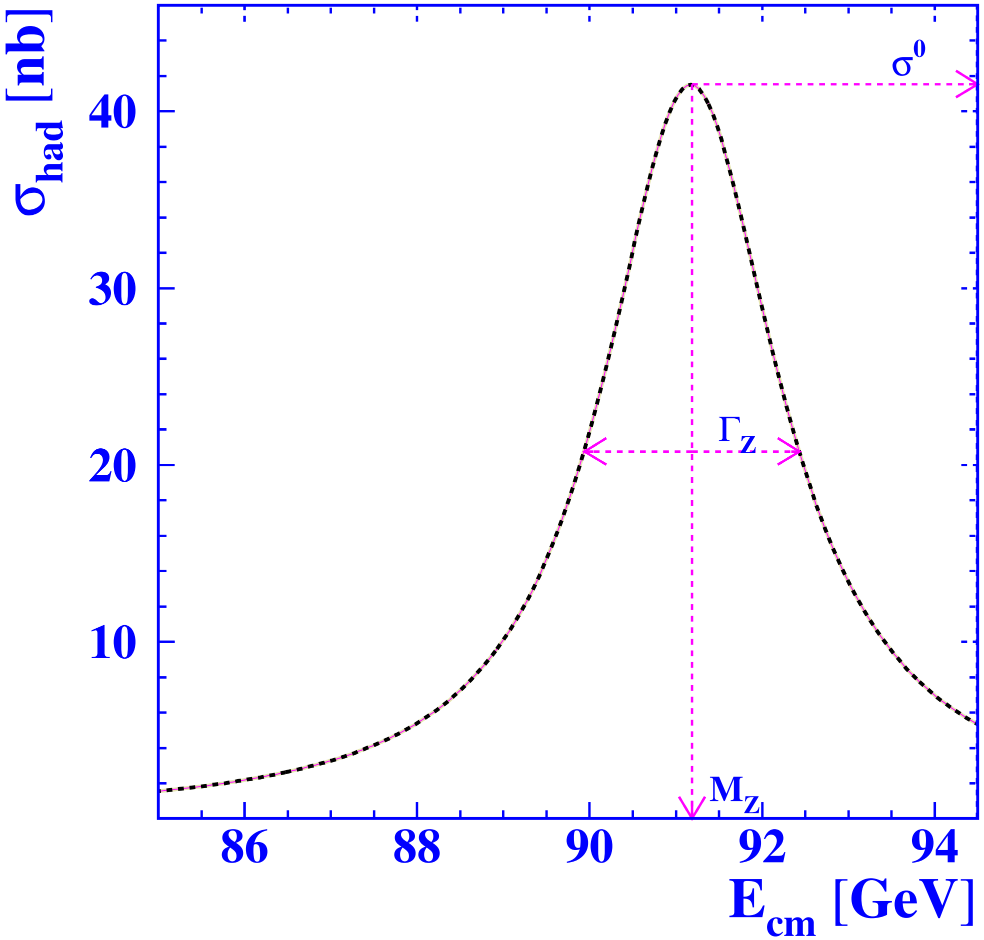
Left-right asymmetry:

With polarized e^- beam: $$A_{\text{LR}} \equiv \frac{\sigma_{\text{L}} - \sigma_{\text{R}}}{\sigma_{\text{L}} + \sigma_{\text{R}}} = \mathcal{A}_e$$

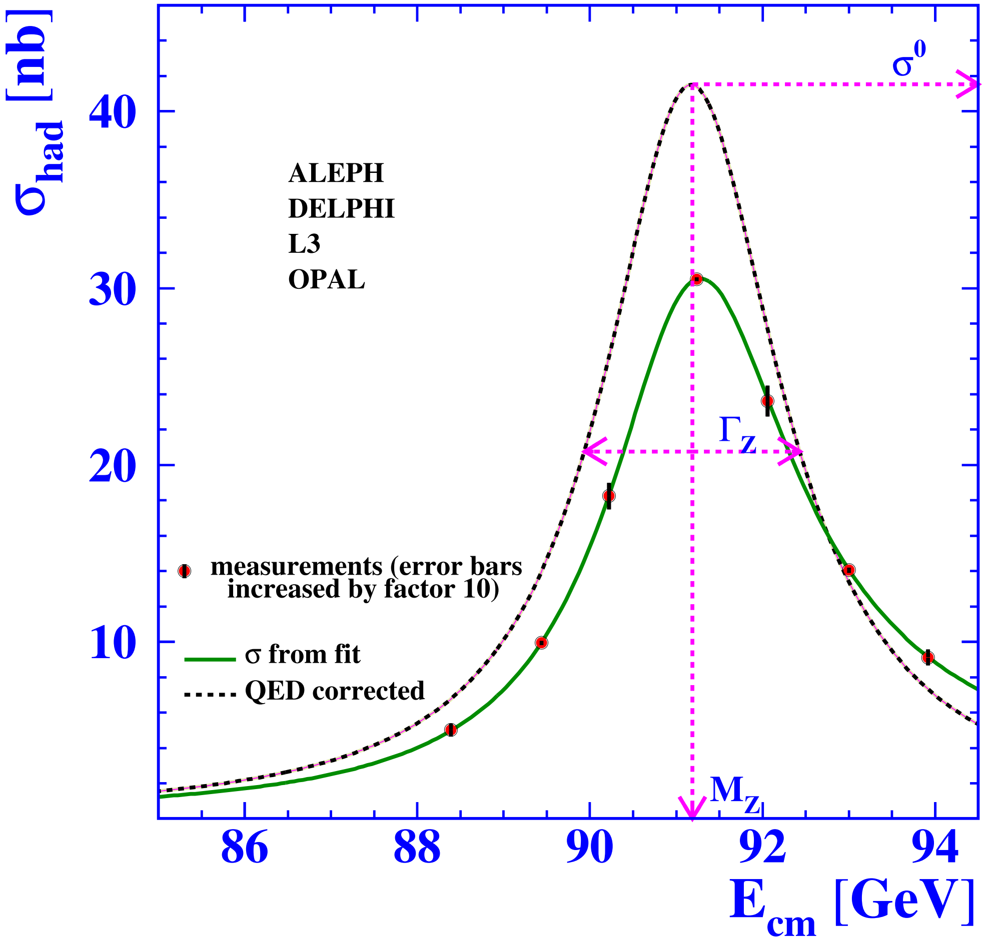
Polarization asymmetry:

Average τ pol. in $e^+e^- \to \tau^+\tau^-$: $$\langle\mathcal{P}_\tau\rangle = -\mathcal{A}_\tau$$

Real observables vs. pseudo-observables



Real observables vs. pseudo-observables



LEP EWWG '05

→ Large effects from initial-state QED radiation

→ Theory input necessary to extract relevant EWPOs ("pseudo-observables"): Γ_{ff}, $\sin^2\theta^f_{\rm eff}$, g^f_L, g^f_R, ...

Z lineshape

- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s, s') \otimes \sigma_{\text{hard}}(s')$$

- Subtraction of γ-exchange, γ–Z interference, box contributions:

$$\sigma_{\text{hard}} = \sigma_{\text{Z}} + \sigma_\gamma + \sigma_{\gamma\text{Z}} + \sigma_{\text{box}}$$

- Z-pole contribution:

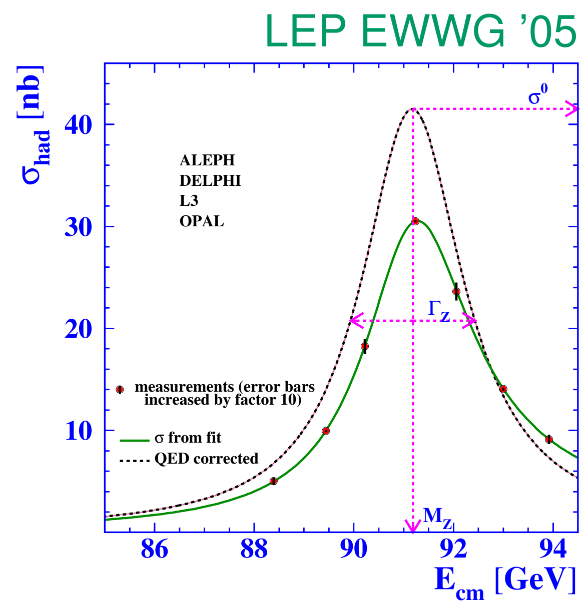
$$\sigma_{\text{Z}} = \frac{R}{(s - \overline{M}_{\text{Z}}^2)^2 + \overline{M}_{\text{Z}}^2 \overline{\Gamma}_{\text{Z}}^2} + \sigma_{\text{non-res}}$$

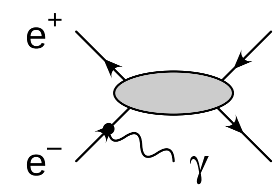
R and $\overline{\Gamma}_{\text{Z}}$ contain dependence on $\sin^2\theta^f_{\text{eff}}$, Γ_{ff}, ...

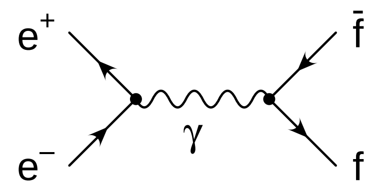
σ_γ, $\sigma_{\gamma\text{Z}}$, σ_{box}, $\sigma_{\text{non-res}}$ known at NLO

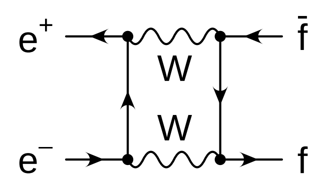
$\longrightarrow$ need consistent pole expansion framework

$\longrightarrow$ NNLO likely needed for FCC-ee/CEPC









Z lineshape

- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s,s') \otimes \sigma_{\text{hard}}(s')$$

- Subtraction of γ-exchange, γ–Z interference, box contributions:

$$\sigma_{\text{hard}} = \sigma_Z + \underbrace{\sigma_\gamma + \sigma_{\gamma Z} + \sigma_{\text{box}}}$$

- Z-pole contribution: computed in SM

$$\sigma_Z = \frac{R}{(s-\overline{M}_Z^2)^2 + \overline{M}_Z^2\overline{\Gamma}_Z^2} + \sigma_{\text{non-res}}$$

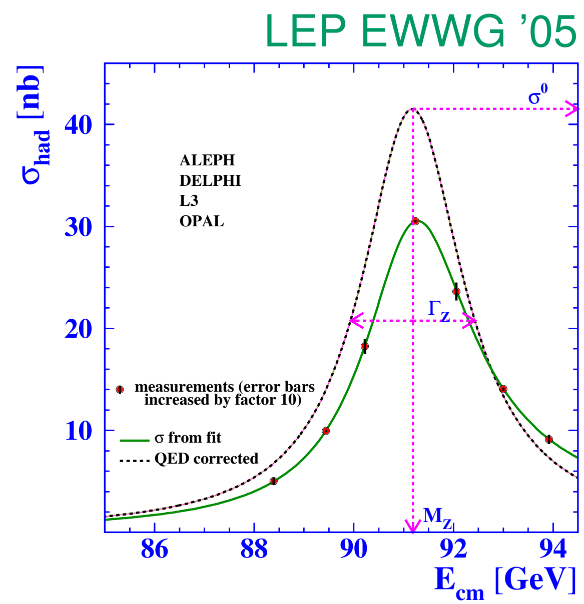
R and $\overline{\Gamma}_Z$ contain dependence on $\sin^2\theta^f_{\text{eff}}$, Γ_{ff}, ...

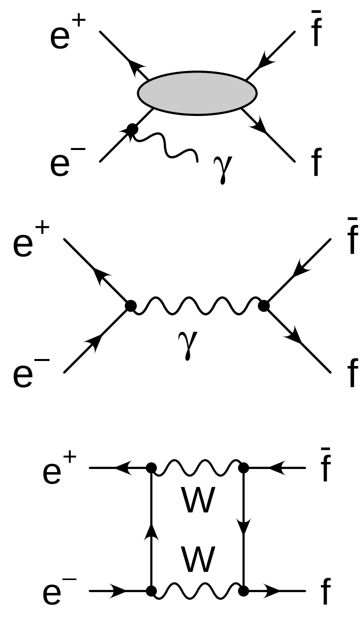
σ_γ, $\sigma_{\gamma Z}$, σ_{box}, $\sigma_{\text{non-res}}$ known at NLO

$\longrightarrow$ need consistent pole expansion framework

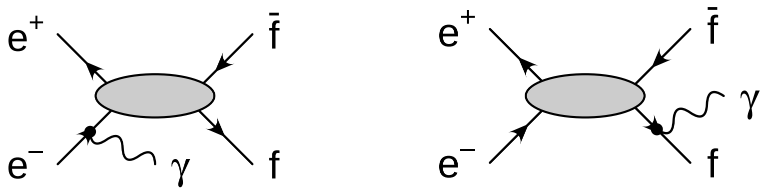
$\longrightarrow$ NNLO likely needed for FCC-ee/CEPC

$\longrightarrow$ possible BSM physics?





QED (QCD) radiation



Final-state QED/QCD radiation known **inclusively** to $\mathcal{O}(\alpha_s^4)$, $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_s)$

Kataev '92; Chetyrkin, Kühn, Kwiatkowski '96; Baikov, Chetyrkin, Kühn, Rittinger '12

Initial-state QED radiation: Soft photons (resummed) + collinear photons

$$\mathcal{R}_{\text{ini}} = \sum_n \left(\frac{\alpha}{\pi}\right)^n \sum_{m=0}^n h_{nm} \ln^m\left(\frac{s}{m_e^2}\right)$$

Universal ($m{=}n$) logs known to $n = 6$, also some sub-leading terms

Ablinger, Blümlein, De Freitas, Schönwald '20

Simulate **exclusively** with Monte Carlo event generator, e.g.

KKMC, Arbuzov, Jadach, Wąs, Ward, Yost '20

SHERPA_YFS, Krauss, Price, Schönherr '22

POWHEG_EW Barzè, Montagna, Nason, Nicrosini, Piccinini '12,13

LL $\left[(\alpha \log \frac{E}{m_e})^n\right]$ + some NLL $\left[\alpha^2 \log \frac{E}{m_e}\right]$

Monte-Carlo for future colliders

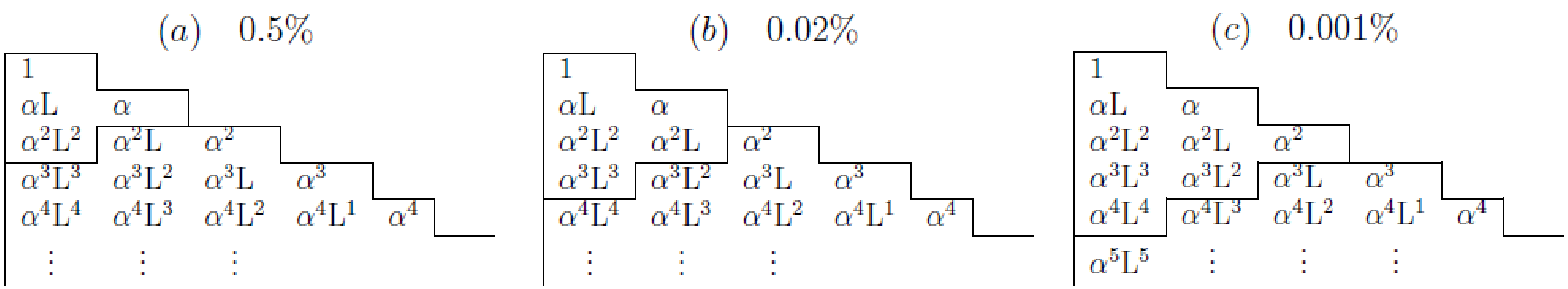
- Current state of art: e.g. `KORALZ, KKMC` $\rightarrow \mathcal{O}(\alpha^2 L)$ accuracy [$L = \ln(s/m_e^2)$]

Jadach, Ward, ...

- One to two orders improvement needed:

Observable	Where from	Present (LEP)	FCC stat.	FCC syst	Now/FCC
M_Z [MeV]	Z linesh. [28]	$91187.5 \pm 2.1\{0.3\}$	0.005	0.1	3
Γ_Z [MeV]	Z linesh. [28]	$2495.2 \pm 2.1\{0.2\}$	0.008	0.1	2
$R_l^Z = \Gamma_h/\Gamma_l$	$\sigma(M_Z)$ [33]	$20.767 \pm 0.025\{0.012\}$	$6 \cdot 10^{-5}$	$1 \cdot 10^{-3}$	12
$\sigma^0_{\rm had}$[nb]	$\sigma^0_{\rm had}$ [28]	$41.541 \pm 0.037\{0.25\}$	$0.1 \cdot 10^{-3}$	$4 \cdot 10^{-3}$	6
N_ν	$\sigma(M_Z)$ [28]	$2.984 \pm 0.008\{0.006\}$	$5 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	6
$\sin^2\theta_W^{eff} \times 10^5$	$A_{FB}^{lept.}$ [33]	$23099 \pm 53\{28\}$	0.3	0.5	55
$A_{FB,\mu}^{M_Z \pm 3.5\rm GeV}$	$\frac{d\sigma}{d\cos\theta}$ [28]	$\pm 0.020\{0.001\}$	$1.0 \cdot 10^{-5}$	$0.3 \cdot 10^{-5}$	100

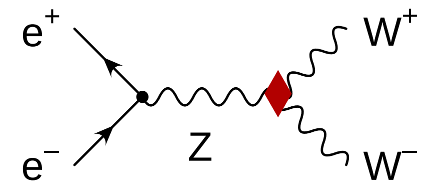
Jadach, Skrzypek '19



- Need control over
  - multi-γ production
  - hadronization
  - color reconnection
  - $\gamma/g \rightarrow f\bar{f}$
  - heavy-flavor correlations
  - ...

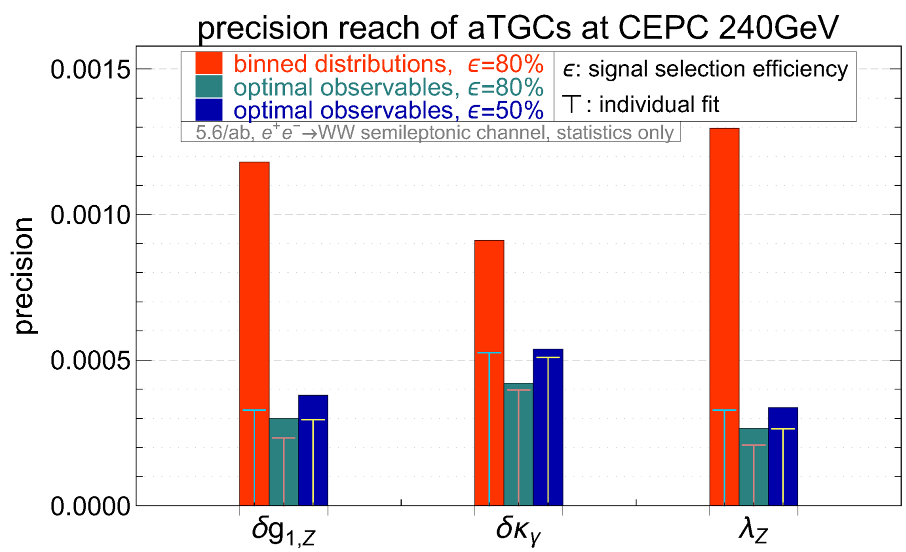
WW production

$e^+e^- \to W^+W^-$ at $\sqrt{s} = 240$ GeV: test of aGC



$\sigma[ee \to WW] \sim 15$ pb $\quad\Rightarrow\quad \mathcal{O}(10^8)$ events

$< 10^{-3}$ precision for aGC

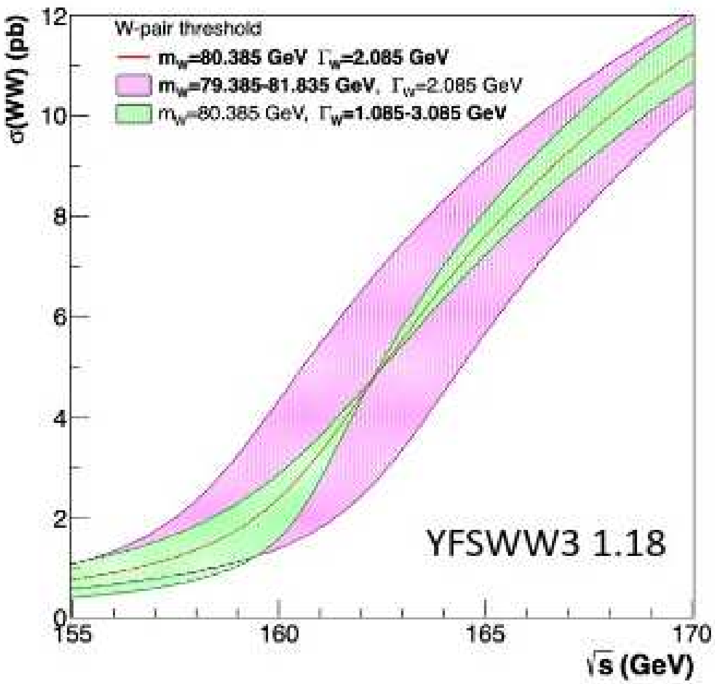


de Blas, Durieux, Grojean, Gu, Paul '19

→ NNLO corrections for full process $e^+e^- \to W^+W^-$ needed (+ partial higher orders)

WW threshold : W mass and width

Scans of possible E_1 E_2 data taking energies and luminosity fractions f (at the E_2 point)



A -minimum of $\Delta\Gamma_W$=0.91 MeV with Δm_W=0.55 MeV taking data at E_1=156.6 GeV E_2=162.4 GeV f=0.25 yields Δm_W=0.47 MeV (as single par)

B- minimum of Δm_W=0.28 MeV $\Delta\Gamma_W$=3.3 MeV with E_1=155.5 GeV E_2=162.4 GeV f=0.95 yields Δm_W=0.28 MeV (as single par)

C- minimum of $\Delta\Gamma_W$=0.96 MeV +Δm_W=0.41 MeV with E_1=157.5 GeV E_2=162.4 GeV f=0.45 yields and Δm_W=0.37 MeV (as single par)

Δm_W, $\Delta\Gamma_W$: error on W mass and width from fitting both
Δm_W: error on W mass from fitting only m_W

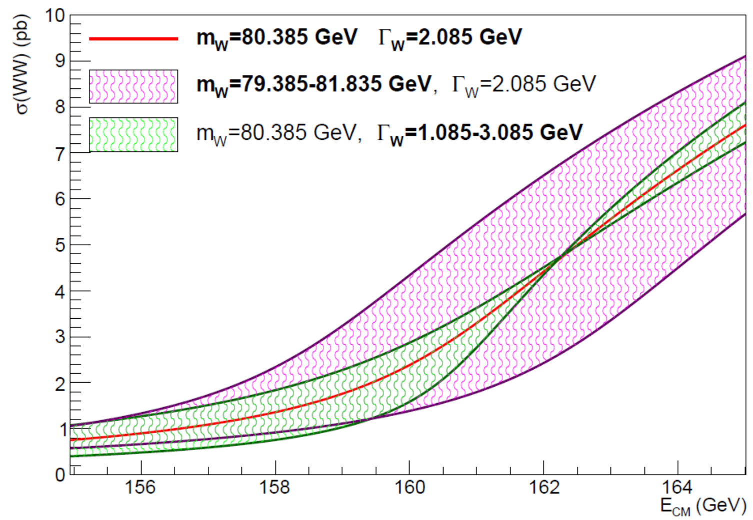
WW threshold

a) Corrections near threshold enhanced by

$1/\beta$ and $\ln\beta$

$$\beta \sim \sqrt{1 - 4\frac{M_W^2 - iM_W\Gamma_W}{s}} \sim \sqrt{\Gamma_W/M_W}$$

b) Non-resonant contributions are important



- Full $\mathcal{O}(\alpha)$ calculation of $e^+e^- \to 4f$

  Denner, Dittmaier, Roth, Wieders '05

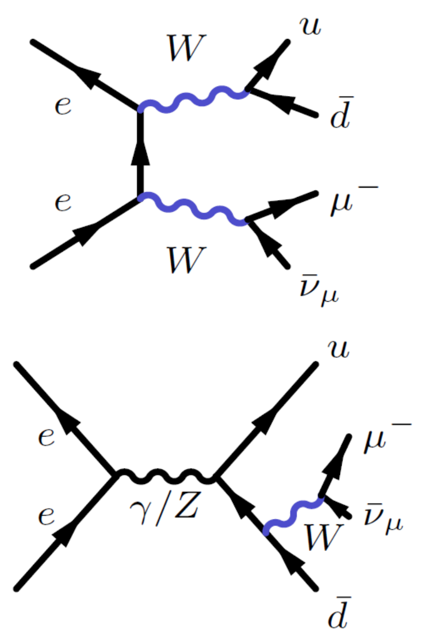
- EFT expansion in $\alpha \sim \Gamma_W/M_W \sim \beta^2$

  Beneke, Falgari, Schwinn, Signer, Zanderighi '07

  - NLO corrections with NNLO Coulomb correction ($\propto 1/\beta^n$): $\delta_{th} M_W \sim 3$ MeV

    Actis, Beneke, Falgari, Schwinn '08

  - Adding NNLO corrections to $ee \to WW$ and $W \to f\bar{f}$ and NNLO ISR: $\delta_{th} M_W \lesssim 0.6$ MeV

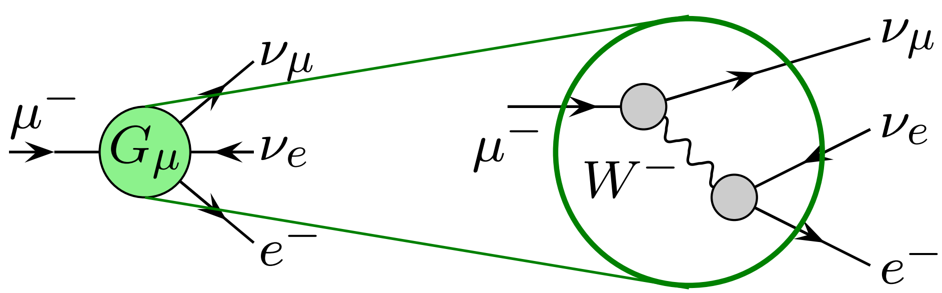


Comparison of EWPOs with theory

- To probe new physics, compare EWPOs with SM theory predictions
- Need to take theory error into account:

	Current exp.	Current th.	CEPC	FCC-ee
M_W [MeV]	11–12	4*	0.5*	0.4*
Γ_Z [MeV]	2.3	0.4	0.025	0.025
$R_\ell = \Gamma_Z^{had}/\Gamma_Z^\ell$ $[10^{-3}]$	25	5	2	1
$R_b = \Gamma_Z^b/\Gamma_Z^{had}$ $[10^{-5}]$	66	10	4.3	6
$\sin^2\theta^\ell_{eff}$ $[10^{-5}]$	13	4.5	0.3	0.4

* computed from G_μ



Theory calculations: Status

- Many seminal works on 1-loop and leading 2-loop corrections

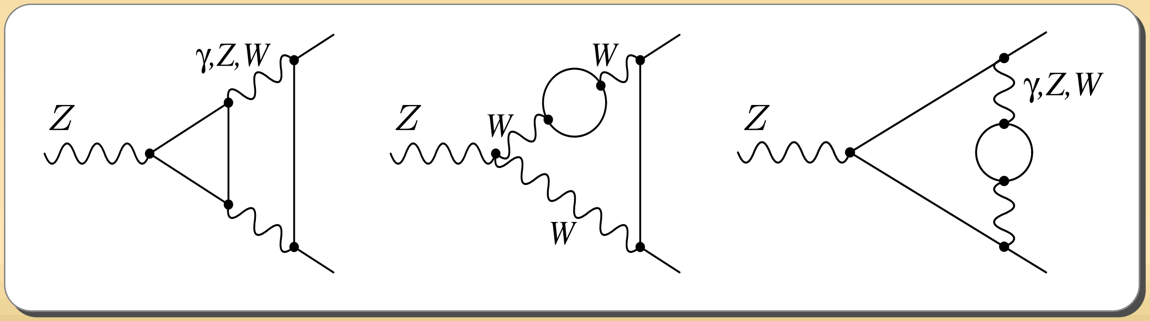
  Veltman, Passarino, Sirlin, Marciano, Bardin, Hollik, Riemann, Degrassi, Kniehl, ...

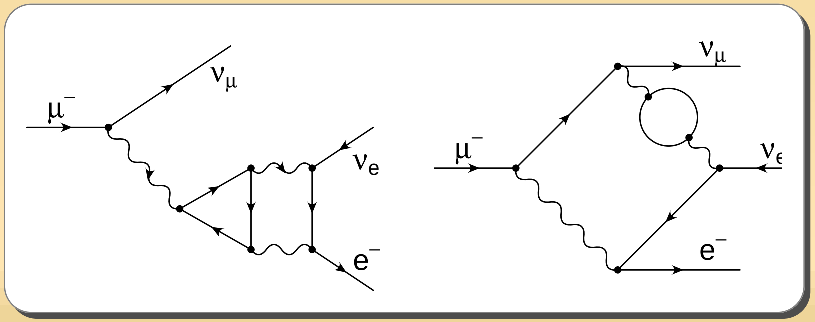
- Full 2-loop results for M_W, Z-pole observables

  Freitas, Hollik, Walter, Weiglein '00
  Awramik, Czakon '02
  Onishchenko, Veretin '02
  Awramik, Czakon, Freitas, Weiglein '04
  Awramik, Czakon, Freitas '06
  Hollik, Meier, Uccirati '05,07
  Awramik, Czakon, Freitas, Kniehl '08
  Freitas '14
  Dubovyk, Freitas, Gluza, Riemann, Usovitsch '16,18

- Approximate 3- and 4-loop results (enhanced by Y_t and/or N_f)

  Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03
  Boughezal, Tausk, v. d. Bij '05
  Schröder, Steinhauser '05
  Chetyrkin et al. '06
  Boughezal, Czakon '06
  Chen, Freitas '20





Theory calculations: Uncertainties

- To probe new physics, compare EWPOs with SM theory predictions
- Need to take theory error into account:

	Current exp.	Current th.	CEPC	FCC-ee
M_W [MeV]	11–12	4	0.5	0.4
Γ_Z [MeV]	2.3	0.4	0.025	0.025
$R_\ell = \Gamma_Z^\mathrm{had}/\Gamma_Z^\ell$ $[10^{-3}]$	25	5	2	1
$R_b = \Gamma_Z^b/\Gamma_Z^\mathrm{had}$ $[10^{-5}]$	66	10	4.3	6
$\sin^2\theta^\ell_\mathrm{eff}$ $[10^{-5}]$	13	4.5	0.3	0.4

- Theory error estimate is not well defined, ideally $\Delta_\mathrm{th} \ll \Delta_\mathrm{exp}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence

Theory calculations: Uncertainty projections

- Estimated impact of future higher-order calculations

Freitas et al. '19

	Current th.	Projected th.†	CEPC	FCC-ee
M_W [MeV]	4	1	0.5	0.4
Γ_Z [MeV]	0.4	0.15	0.025	0.025
$R_\ell = \Gamma_\mathrm{Z}^\mathrm{had}/\Gamma_\mathrm{Z}^\ell$ $[10^{-3}]$	5	1.5	2	1
$R_b = \Gamma_\mathrm{Z}^b/\Gamma_\mathrm{Z}^\mathrm{had}$ $[10^{-5}]$	10	5	4.3	6
$\sin^2\theta^\ell_\mathrm{eff}$ $[10^{-5}]$	4.5	1.5	0.3	0.4

† **Theory scenario:** $\mathcal{O}(\alpha\alpha_\mathrm{s}^2)$, $\mathcal{O}(N_f\alpha^2\alpha_\mathrm{s})$, $\mathcal{O}(N_f^2\alpha^2\alpha_\mathrm{s})$, leading 4-loop

(N_f^n = at least n closed fermion loops)

Note: Estimates (based on extrapolation of perturb. series and prefactors) are unreliable and only provide a rough guess

SM predictions for Higgs decays

Reviews: 1404.0319, 1906.05379

hbb: [CEPC: 1.4%, FCC-ee: 1.2%]

- $\mathcal{O}(\alpha_s^4)$ QCD corrections — Baikov, Chetyrkin, Kühn '05
- $\mathcal{O}(\alpha)$ QED+EW — Dabelstein, Hollik '92; Kniehl '92
- leading $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha\alpha_s)$ for large m_t — Kwiatkowski, Steinhauser '94; Butenschoen, Fugel, Kniehl '07

Current theory error: $\Delta_{th} < 0.4\%$

With full 2-loop: $\Delta_{th} \sim 0.2\%$ (possible with existing methods)

hgg: [CEPC: 1.8%, FCC-ee: 1.8%]

- $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha_s^3)$ (in large m_t-limit) QCD corrections — Baikov, Chetyrkin '06; Schreck, Steinhauser '07
- $\mathcal{O}(\alpha)$ EW — Aglietti, Bonciani, Degrassi, Vicini '04; Degrassi, Maltoni '04

Theory error (dominated by QCD): $\Delta_{th} \approx 3\%$

With $\mathcal{O}(\alpha_s^4)$ in large m_t-limit (4-loop massless QCD diags.): $\Delta_{th} \approx 1\%$

SM predictions for Higgs decays

hWW*/hZZ*: [CEPC: 1.4%, FCC-ee: 0.8%]

- complete $\mathcal{O}(\alpha) + \mathcal{O}(\alpha_s)$ for $h \to 4f$ — Bredenstein, Denner, Dittmaier, Weber '06
- leading $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(\alpha\alpha_s^2)$ for large m_t — Djouadi, Gambino, Kniehl '97; Kniehl, Spira '95; Kniehl, Steinhauser '95; Kniehl, Veretin '12

  $\to$ Small (0.2%) effect

Theory error:

$\Delta_{th,EW} < 0.3\%$, $\Delta_{th,QCD} < 0.5\%$

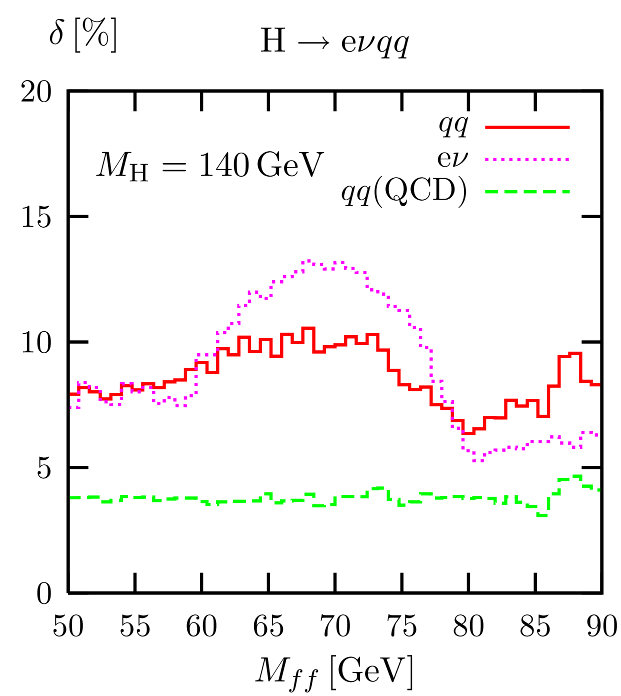
With NNLO final-state QCD corrections:

$\Delta_{th,QCD} < 0.1\%$

Note: Non-trivial effects in distributions

Bredenstein, Denner, Dittmaier, Weber '06

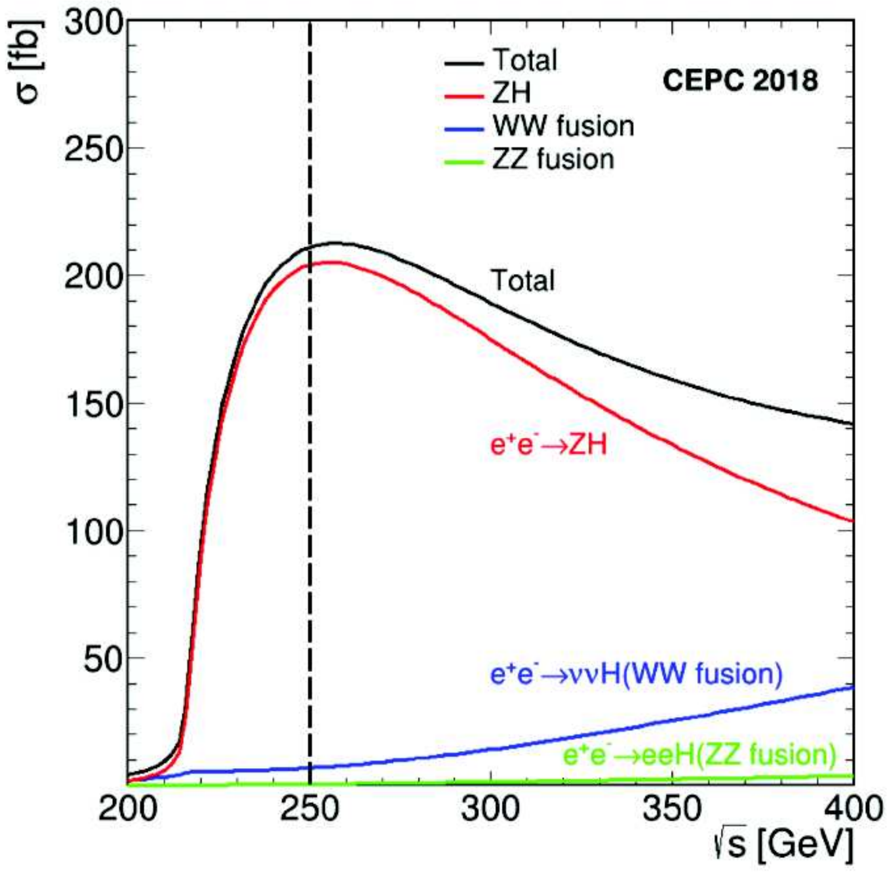
$\to$ Larger theory uncertainty?

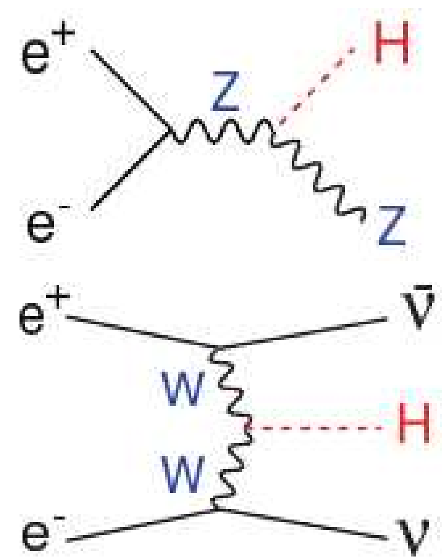


$h\gamma\gamma$, $h\tau\tau$, hcc: Theory uncertainties already subdominant

SM predictions for Higgs production

- hZ production: dominant at $\sqrt{s} \sim 240$ GeV
- WW fusion: sub-dominant but useful for constraining h width Han, Liu, Sayre '13

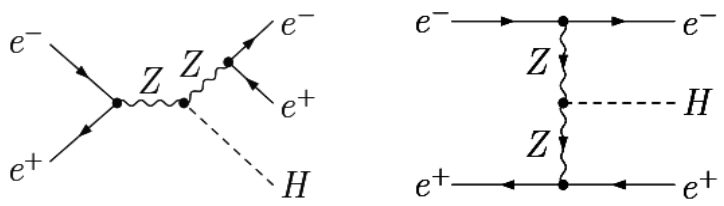




SM predictions for Higgs production

hZ production: [CEPC: 0.15%, FCC-ee: 0.35%]

- $\mathcal{O}(\alpha)$ corr. to hZ production and Z decay — Kniehl '92; Denner, Küblbeck, Mertig, Böhm '92; Consoli, Lo Presti, Maiani '83; Jegerlehner '86; Akhundov, Bardin, Riemann '86
- Technology for $\mathcal{O}(\alpha)$ with off-shell Z-boson available — Boudjema et al. '04; Denner, Dittmaier, Roth, Weber '03



- Can be combined with h.o. ISR QED radiation — Greco et al. '17
- $\mathcal{O}(\alpha\alpha_s)$ corrections — Gong et al. '16; Chen, Feng, Jia, Sang '18
- $\mathcal{O}(N_f\alpha^2)$ corrections — Freitas, Song '22 [also see Chen, Guan, He, Li, Liu, Ma '22]

Theory error: $\Delta_{th} \lesssim O(0.3\%)$ (mostly from non-fermionic NNLO)

“Input” parameters

SM predictions for Higgs decays need measured input parameters

Reviews: 1906.05379, 2012.11642

Numerical inpact of input parameter uncertainties:

	$\delta m_t = 0.5$ GeV	$\delta\alpha_s = 0.001$	$\delta(\Delta\alpha) = 10^{-4}$	FCC-ee exp.
M_W [MeV]	3	0.7	2	0.4
$\sin^2\theta^\ell_{eff}$ $[10^{-5}]$	1.5	0.3	3.5	0.4
Γ_Z [MeV]	0.1	0.5	0.1	0.025
$\Gamma[h \to gg]$ [%]	<0.2	3	–	1.8

To keep impact subdominant for FCC-ee precision studies, would need:

$\delta m_t < 50$ MeV

$\delta\alpha_s < 5 \times 10^{-5}$

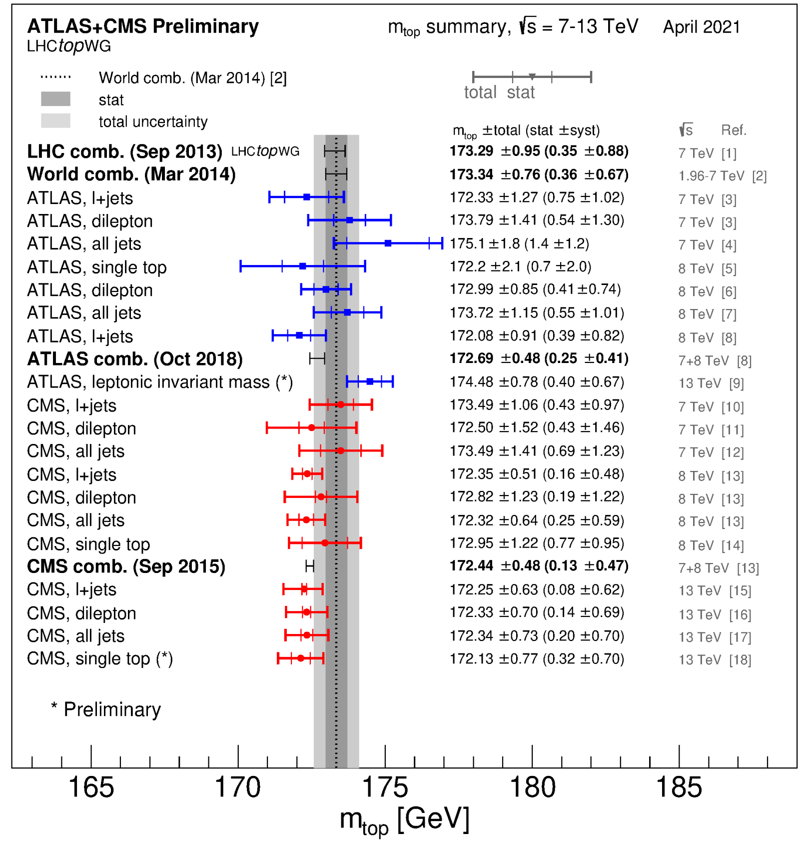
$\delta(\Delta\alpha_s) < 10^{-5}$

- m_{t}: Most precise measurement at LHC: $\delta m_{\mathrm{t}} \sim 0.3$ GeV PDG '24

Theoretical ambiguity in mass def.:

Hoang, Plätzer, Samitz '18

$$m_{\mathrm{t}}^{\mathrm{CB}}(Q_0) - m_{\mathrm{t}}^{\mathrm{pole}}$$
$$= -\tfrac{2}{3}\alpha_{\mathrm{s}}(Q_0)\, Q_0 + \mathcal{O}(\alpha_s^2 Q_0)$$
$$\approx 0.5 \pm 0.2_{\mathrm{pert.}} \pm 0.2_{\mathrm{np.}}\mathrm{GeV}$$

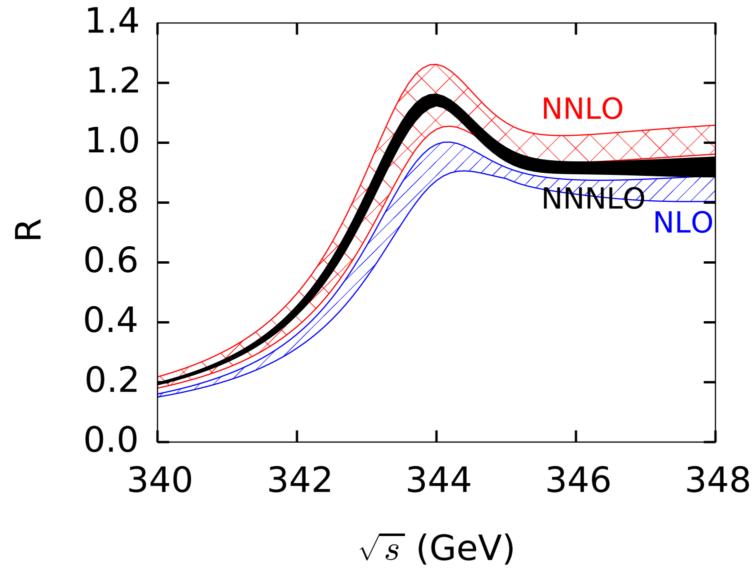


- m_t: Most precise measurement at LHC: $\delta m_t \sim 0.3$ GeV

From $e^+e^- \to t\bar{t}$ at $\sqrt{s} \sim 350$ GeV:

Impact of theory modelling:

$$\delta m_t^{\overline{\text{MS}}} = [\quad]_{\text{exp}} \oplus [50\text{ MeV}]_{\text{QCD}} \oplus [10\text{ MeV}]_{\text{mass def.}} \oplus [70\text{ MeV}]_{\alpha_s} > 100\text{ MeV}$$



Beneke et al. '15

- m_t: Most precise measurement at LHC: $\delta m_t \sim 0.3$ GeV

From $e^+e^- \to t\bar{t}$ at $\sqrt{s} \sim 350$ GeV:

Impact of theory modelling:

$$\delta m_t^{\overline{\text{MS}}} = [\quad]_{\text{exp}} \oplus [50\text{ MeV}]_{\text{QCD}} \oplus [10\text{ MeV}]_{\text{mass def.}} \oplus [70\text{ MeV}]_{\alpha_s}$$

> 100 MeV

future improvements:

$$[20\text{ MeV}]_{\text{exp}} \oplus [30\text{ MeV}]_{\text{QCD}} \oplus [10\text{ MeV}]_{\text{mass def.}} \oplus [15\text{ MeV}]_{\alpha_s}$$

(h.o. resumm., N^{4}LO?) for QCD; ($\delta\alpha_s \lesssim 0.0002$) for α_s

$\lesssim 50$ MeV

Strong coupling

- α_s: d'Enterria, Skands, et al. '15
  - Most precise determination using Lattice QCD:

    $\alpha_s = 0.1184 \pm 0.0006$ HPQCD '10

    $\alpha_s = 0.1185 \pm 0.0008$ ALPHA '17

    $\alpha_s = 0.1179 \pm 0.0015$ Takaura et al. '18

    $\alpha_s = 0.1172 \pm 0.0011$ Zafeiropoulos et al. '19

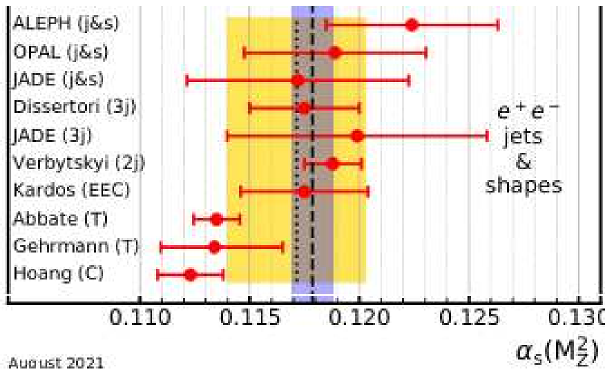
    $\rightarrow$ Difficulty in evaluating systematics

  - e^+e^- event shapes: $\alpha_s \sim 0.113...0.119$

    $\rightarrow$ Large non-pertubative power corrections

    $\rightarrow$ Systematic uncertainties?

PDG '21



  - Hadronic τ decays: $\alpha_s = 0.119 \pm 0.002$ PDG '18

    $\rightarrow$ Non-perturbative uncertainties in OPE and from duality violation

    Pich '14; Boito et al. '15,18

Strong coupling

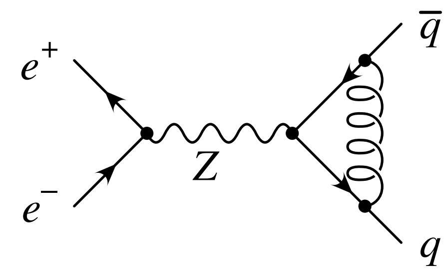
- α_s:
  - Electroweak precision ($R_\ell = \Gamma_Z^{\mathrm{had}}/\Gamma_Z^{\ell}$):
    $\alpha_s = 0.120 \pm 0.003$ PDG '18

    $\longrightarrow$ Negligible non-perturbative QCD effects

    FCC-ee: $\delta R_\ell \sim 0.001$
    $\Rightarrow \quad \delta\alpha_s < 0.0001$

    Theory input: N^{3}LO EW corr. + leading N^{4}LO
    to keep $\delta_{\mathrm{th}} R_\ell \lesssim \delta_{\mathrm{exp}} R_\ell$

    Caviat: R_ℓ could be affected by new physics



- α_s:
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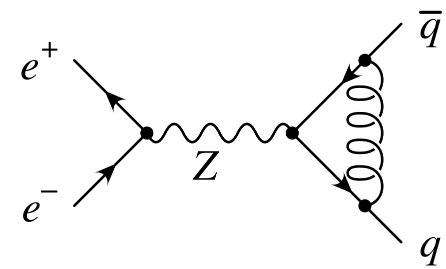
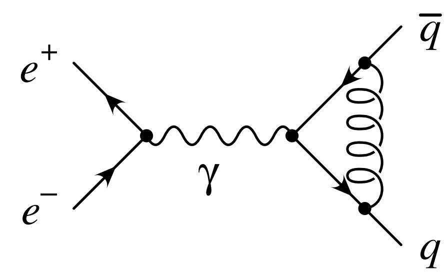


    $\rightarrow$ Negligible non-perturbative QCD effects

    FCC-ee: $\delta R_\ell \sim 0.001$
    $\Rightarrow \quad \delta\alpha_s < 0.0001$

    Caviat: R_ℓ could be affected by new physics

  - $R = \frac{\sigma[ee\to\text{had.}]}{\sigma[ee\to\mu\mu]}$ at lower $\sqrt{s}$



    e.g. CLEO ($\sqrt{s} \sim 9$ GeV): $\alpha_s = 0.110 \pm 0.015$
    Kühn, Steinhauser, Teubner '07

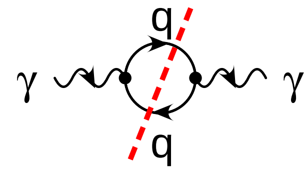
    $\rightarrow$ dominated by s-channel photon, less room for new physics
    $\rightarrow$ QCD still perturbative

    naive scaling to 50 ab^{-1} (BELLE-II): $\delta\alpha_s \sim 0.0001$

Electromagnetic coupling

- $\Delta\alpha \equiv 1 - \frac{\alpha(0)}{\alpha(M_Z)} \approx 0.059 = 0.0315_{\text{lept}} + 0.0276_{\text{had}}$

a) $\Delta\alpha_{\text{had}}$ from $e^+e^- \to$ had. using dispersion relation

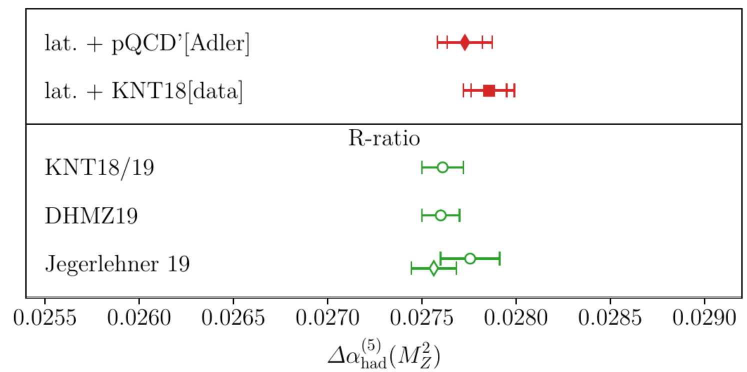


$\to$ Current precision $\sim 10^{-4}$

Davier et al. '19; Jegerlehner '19; Keshavarzi, Nomura, Teubner '19

b) $\Delta\alpha_{\text{had}}$ from Lattice QCD
(challenging but much progress)

Burger et al. '15
Cè et al. '22



Future improvements for methods (a) and (b):

- More precise exp./lattice data
- Full 4-loop pQCD for R-ratio / Adler function (for $|Q^2| \gg \Lambda_{\text{QCD}}$)
- More precise inputs for m_b, m_c, α_s

$\to \delta(\Delta\alpha_{\text{had}}) \lesssim 5 \times 10^{-5}$ likely achievable — Jegerlehner '19

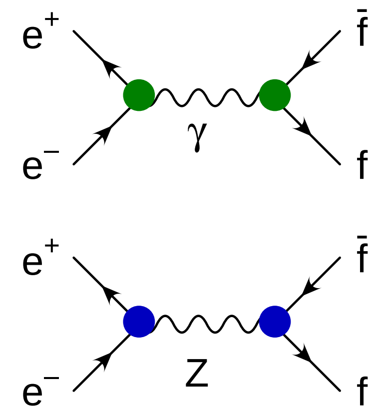
Electromagnetic coupling

- $\Delta\alpha \equiv 1 - \frac{\alpha(0)}{\alpha(M_Z)} \approx 0.059 = 0.0315_{\text{lept}} + 0.0276_{\text{had}}$

c) Direct det. of $\Delta\alpha_{\text{had}}$ from $e^+e^- \to \mu^+\mu^-$ off the Z peak — Janot '15

$$|\mathcal{M}_{ij}|^2 \propto |g_i^\ell|^2 |g_j^\ell|^2 + (s - M_Z^2)\,\alpha(M_Z)|g_{i,j}^\ell|^2 + \ldots$$

↑ determined from Z pole



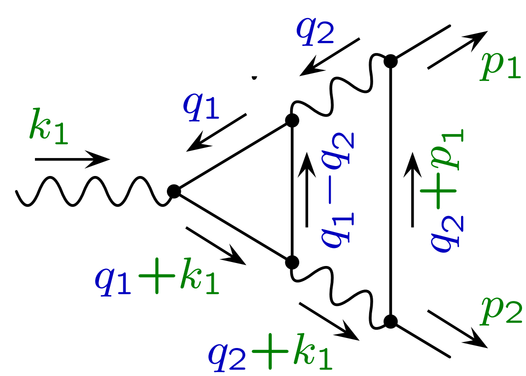
$\rightarrow$ Use $A_{\text{FB}}^{\mu\mu}$ at $\sqrt{s_1} \sim 88$ GeV and $\sqrt{s_2} \sim 95$ GeV to reduce systematics

$\rightarrow$ $\delta(\Delta\alpha_{\text{had}}) \sim 3 \times 10^{-5}$ for $\mathcal{L}_{\text{int}} = 85\ \text{ab}^{-1}$

$\rightarrow$ Requires 2/3-loop corrections for $e^+e^- \to \mu^+\mu^-$

Theoretical challenges

Experimental precision requires inclusion of **multi-loop corrections** in theory



Integrals over loop momenta:

$$\int d^4q_1 d^4q_2 \; f(q_1, q_2, p_1, k_1, ..., m_1, m_2, ...)$$

Challenges:

1. $\mathcal{O}(1000) - \mathcal{O}(10000)$ integrals
2. Individual integrals can be divergent (drop out for physical results)
   $\rightarrow$ Regularization, renormalization
3. Multi-dimensional integrations

General approaches:

- Analytical
- Numerical
- Approximations (expansions), specialized techniques, ...

Analytical calculations

Challenge 1: reduce 1000s of integrals to a small set of *master integrals*

Integration-by-parts (IBP) relations:

1-dim. example: $$\int_{-\infty}^{\infty} dx\, \frac{d}{dx} \underbrace{\frac{1+2x}{1+x^2}}_{f(x)} = f(\infty) - f(-\infty) = 0$$

$$\frac{d}{dx} f(x) = \frac{2}{1+x^2} - \frac{2x(1+2x)}{(1+x^2)^2}$$

$$\Rightarrow \quad \int_{-\infty}^{\infty} dx\, \frac{2x(1+2x)}{(1+x^2)^2} = \int_{-\infty}^{\infty} dx\, \frac{2}{1+x^2}$$

$\rightarrow$ Similar for $\int d^4q$ integrals

- Individual eqs. may contain integrals not in original problem
- Large enough eq. system can be fully solved — Laporta, arXiv:2002.05845
- Public programs: `Reduze, FIRE, LiteRed, KIRA`
  von Manteuffel, Studerus '12; Smirnov '13,14; Lee '13; Maierhoefer, Usovitsch, Uwer '17
- Requires large computing time and memory

Analytical calculations

Challenge 2/3: find solutions for *master integrals*

Many methods, e.g. differential equations or Mellin-Barnes representations

Kotikov, PLB 254, 158 (1991); Remiddi, hep-th/9711188; Smirnov, hep-ph/0111160

→ Complicated functions needed:
Goncharov polylogs, iterated elliptic integrals, hypergeometric functions, ...

$$G(a_1, ..., a_n, x) = \int_0^x \frac{dt_1}{t_1 - a_1} \int_0^{t_1} \frac{dt_2}{t_2 - a_2} \cdots \int_0^{t_{n-1}} \frac{dt_n}{t_n - a_n}$$

$$\Gamma(a_1, ..., a_n, x) = \int_0^x dt_1\, g_{a_1}(t_1, x) \int_0^{t_1} dt_2\, g_{a_2}(t_2, x) \cdots \int_0^{t_{n-1}} dt_n\, g_{a_n}(t_n, x)$$

Numerical integration

Challenge 2: presence of UV/IR divergencies

- Remove through subtraction terms

$$\underbrace{\int d^4q_1 d^4q_2\,(f - f_{\mathsf{sub}})}_{\text{finite}} + \underbrace{\int d^4q_1 d^4q_2\, f_{\mathsf{sub}}}_{\text{solve analytically}}$$

Cvitanovic, Kinoshita ’74
Levine, Park, Roskies ’82
Bauberger ’97
Nagy, Soper ’03
Awramik, Czakon, Freitas ’06
Becker, Reuschle, Weinzierl ’10
Sborlini et al. ’16
...

- Remove through variable transformations:

$$\int d^nq\, f(\vec{q}) = \underbrace{\int dx\, f(x)}_{\text{divergent, solve analytically}} \underbrace{\int d^{n-1}y\, f(y)}_{\text{finite, integrate numerically}}$$

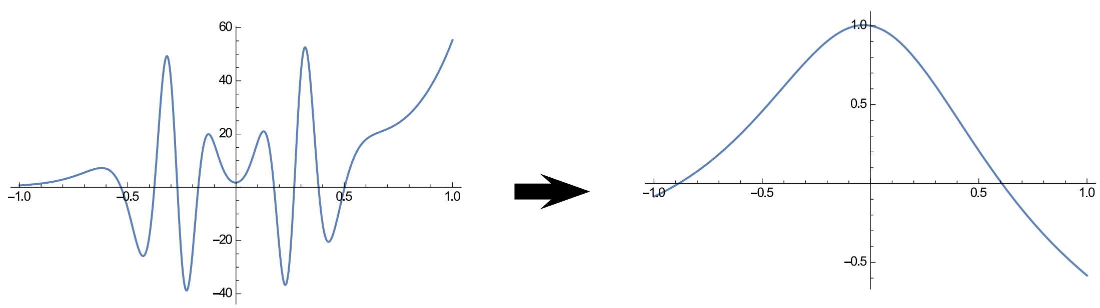
Binoth, Heinrich ’03; Bogner, Weinzierl ’07; Czakon ’06; Smirnov, Smirnov ’09; ...

Challenge 3: stability and convergence

- Integration in momentum space: $4L$ dimensions ($L =$ # of loops)
- Integration in Feynman parameters: $P - 1$ dimensions ($P =$ # of propagators)

$\longrightarrow$ Multi-dim. integrals need large computing resources and converge slowly

- Variable transformations to avoid singularities and peaks



Calculational techniques

Analytical techniques:

- Computational intensive reduction to master integrals (MIs)
- Not fully understood function space of MIs
- Works best for problems with few (no) masses

Numerical techniques:

- Limited precision, slow convergence
- Numerical instabilites, in particule for diagrams with physical cuts
- Works best for problems with many masses

New techniques, e.g.:

- Numerical IBP reduction to MIs, numerical MIs via differential equations (DEs)
  Mandal, Zhao '18, Czakon, Niggetiedt '20
- DEs with respect to auxialiary parameter, $\frac{1}{k_i^2 - m_i^2 + i\epsilon}$ Liu, Ma, Wang '17, Liu, Ma '18,21,22
- Series solutions of DEs Moriello '19, Hidding '20
- Dispersion relations + Feynman parameters Song, Freitas '21, 22

Summary

- **Electroweak and Higgs precision studies** probe physics beyond the Standard Model at **TeV scale** and beyond
- Precision measurements require theory input for **measurements of pseudo-observables** (BRs, widths, masses, cross-sections, ...) and their **SM/BSM interpretation**
- **Future e^+e^- colliders** (FCC-ee, CEPC, ILC) improve precision by 1–2 orders of magnitude
- Theory progress needed both for **fixed-order loop corrections** as well as **MC tools** (shower resummation, hadronization, etc.)
- Improved determinations of **input parameters** require advances in **perturbative** and **non-perturbative** theory tools

Backup slides

Example: Error estimation for Γ_Z

- Goemetric perturbative series

$\alpha_t = \alpha m_t^2$

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2) \sim 0.20 \text{ MeV}$$

$$\mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.21 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.23 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s^2) \sim 0.035 \text{ MeV}$$

- Parametric prefactors:

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\alpha n_{lq}}{\pi} \alpha_s^2 \sim 0.29 \text{ MeV}$$

Total: $\delta\Gamma_Z \approx 0.4$ MeV

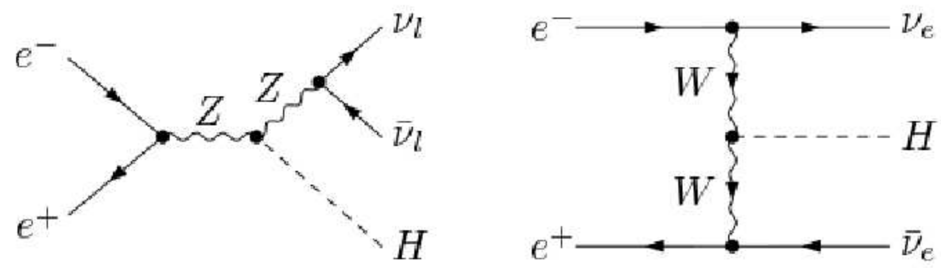
SM predictions for Higgs production

WW fusion:

- $\mathcal{O}(\alpha)$ corrections with h.o. ISR

Belanger et al. '02; Denner, Dittmaier, Roth, Weber '03

Theory error: $\Delta_{\text{th}} \sim O(1\%)$?



Full $\mathcal{O}(\alpha^2)$ calculation for 2→3 process is very challenging

→ Contributions with closed fermion loops maybe feasible

Asymptotic expansions

- Exploit large mass/momentum ratios,
  $e.g.\ M_Z^2/m_t^2 \approx 1/4$
- Evaluate coeff. integrals analytically
- Fast numerical evaluation

→ Public programs:

exp	Harlander, Seidensticker, Steinhauser '97
asy	Pak, Smirnov '10

→ Possible limitations:

- no appropriate mass/momentum ratios
- bad convergence
- impractical if too many mass/mom. scales