The path to precision: fitting, modeling and uncertainties

A. Freitas
University of Pittsburgh

Precision physics with future e^+e^- colliders

Exp. precision estimates for electroweak parameters:

	Current exp.	ILC250	CEPC	FCC-ee
M_W [MeV]	11–12	2.4	0.5	0.4
Γ_Z [MeV]	2.3	1.5	0.025	0.025
$R_\ell = \Gamma_Z^{\rm had}/\Gamma_Z^\ell$ $[10^{-3}]$	25	20	2	1
$R_b = \Gamma_Z^b/\Gamma_Z^{\rm had}$ $[10^{-5}]$	66	23	4.3	6
$\sin^2\theta^\ell_{\rm eff}$ $[10^{-5}]$	13	2	0.3	0.4

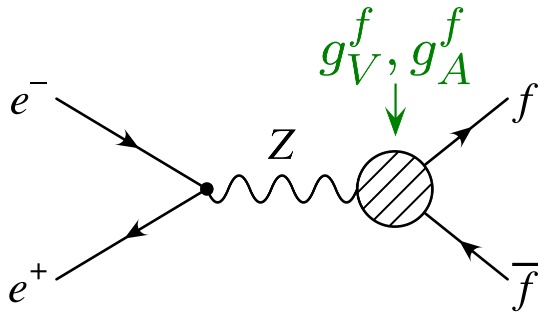
Some issues discussed in the following also relevant for DY @ (HL-)LHC

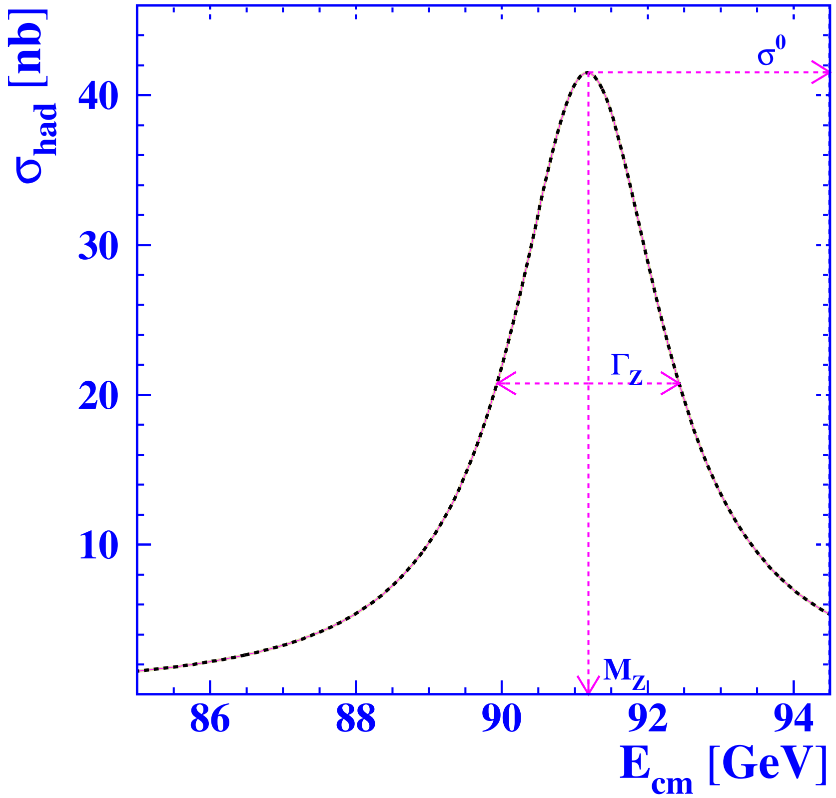
EWPOs: Z cross section and branching fractions

$e^+e^- \to f\bar{f}$ for $E_{\text{CM}} \sim M_Z$:

- Mass M_Z
- Width $\Gamma_Z = \sum_f \Gamma_{ff}$
- Braching ratio $R_f = \Gamma_{ff}/\Gamma_Z$
- $\sigma^0 \approx \frac{12\pi\,\Gamma_{ee}\Gamma_{ff}}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2} = \frac{12\pi}{M_Z^2}R_e R_f$

$$\Gamma_{ff} = C\left[(g_V^f)^2 + (g_A^f)^2\right]$$

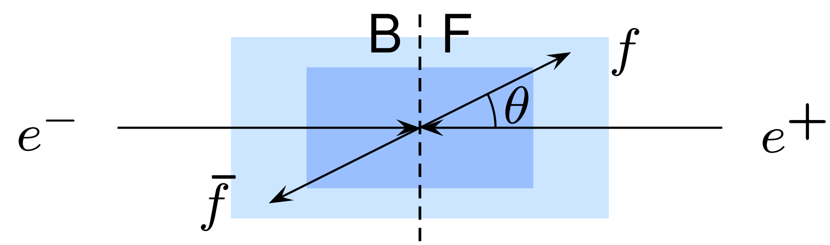




EWPOs: Z-pole asymmetries

Forward-backward asymmetry:

$$A_{\text{FB}} \equiv \frac{\sigma_{\text{F}} - \sigma_{\text{B}}}{\sigma_{\text{F}} + \sigma_{\text{B}}} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f$$



$$\mathcal{A}_f = \frac{2(1 - 4\sin^2\theta^f_{\text{eff}})}{1 + (1 - 4\sin^2\theta^f_{\text{eff}})^2} \qquad \sin^2\theta^f_{\text{eff}} = \frac{1}{4|Q_f|}\left[1 - \text{Re}\,\frac{g^f_V}{g^f_A}\right]$$

Left-right asymmetry:

With polarized e^- beam: $$A_{\text{LR}} \equiv \frac{\sigma_{\text{L}} - \sigma_{\text{R}}}{\sigma_{\text{L}} + \sigma_{\text{R}}} = \mathcal{A}_e$$

Polarization asymmetry:

Average τ pol. in $e^+e^- \to \tau^+\tau^-$: $$\langle \mathcal{P}_\tau \rangle = -\mathcal{A}_\tau$$

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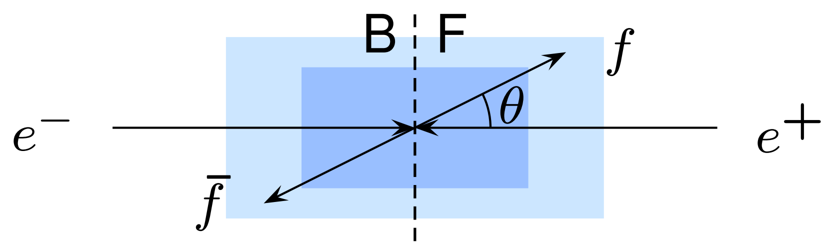

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Polarization asymmetry:

Average τ pol. in $e^+e^- \to \tau^+\tau^-$: $$\langle \mathcal{P}_\tau \rangle = -\mathcal{A}_\tau$$

Decay widths in terms of $\sin^2\theta^f_{\mathrm{eff}}$:

$$\Gamma_{ff} = C\left[(g^f_V)^2 + (g^f_A)^2\right] = C(g^f_A)^2\left[(1 - 4|Q_f|\sin^2\theta^f_{\mathrm{eff}})^2 + \left(\mathrm{Im}\,\frac{g^f_V}{g^f_A}\right)^2\right]$$

Need for theory input

- Comparison of EWPOs with SM to **probe new physics**
  $\longrightarrow$ multi-loop corrections in full SM

- Extraction of EWPOs (**pseudo-observables**) from **real observables**
  $\longrightarrow$ backgrounds (in full SM), QED/QCD, MC tools

- "Other" eletroweak parameters (**"input" parameters**)
  $\longrightarrow$ m_{t}, α_{s}, etc. extracted from other processes

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Pseudo-observables: Γ_{ff}, $\sin^2\theta^f_{\mathrm{eff}}$, g^f_L, g^f_R, ...

Z lineshape

- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s, s') \otimes \sigma_{\text{hard}}(s')$$

Kureav, Fadin '85
Berends, Burgers, v. Neerven '88
Kniehl, Krawczyk, Kühn, Stuart '88
Beenakker, Berends, v. Neerven '89
Bardin et al. '91; Skrzypek '92
Montagna, Nicrosini, Piccinini '97

Soft photons (resummed) + collinear photons

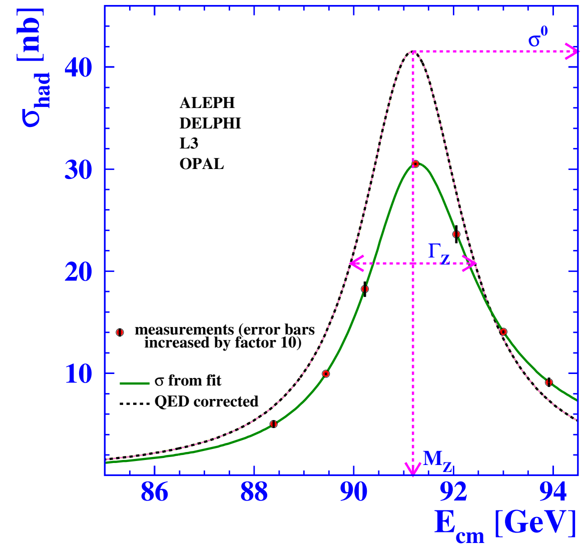
$$\mathcal{R}_{\text{ini}} = \sum_n \left(\frac{\alpha}{\pi}\right)^n \sum_{m=0}^{n} h_{nm} \ln^m\left(\frac{s}{m_e^2}\right)$$

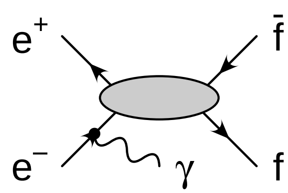
Universal ($m{=}n$) logs known to $n = 6$, also some sub-leading terms

Ablinger, Blümlein, De Freitas, Schönwald '20

Exclusive description: MC tools

LEP EWWG '05





Z lineshape

- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s,s') \otimes \sigma_{\text{hard}}(s')$$

- Subtraction of γ-exchange, γ–Z interference, box contributions:

$$\sigma_{\text{hard}} = \sigma_{\text{Z}} + \sigma_{\gamma} + \sigma_{\gamma\text{Z}} + \sigma_{\text{box}}$$

- Z-pole contribution:

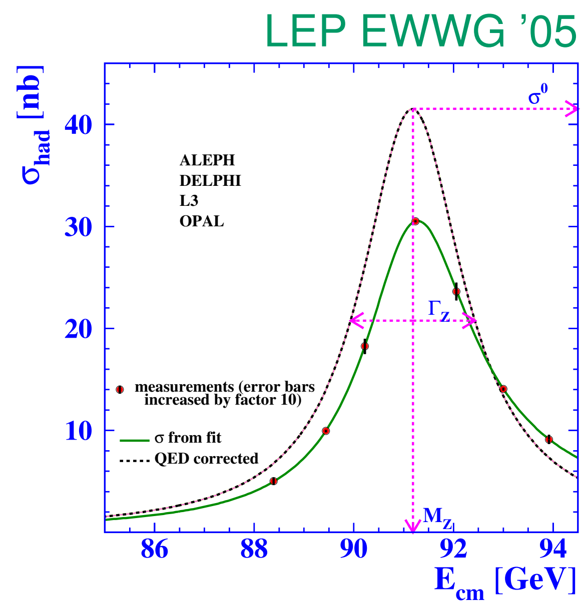
$$\sigma_{\text{Z}} = \frac{R}{(s - \overline{M}_{\text{Z}}^2)^2 + \overline{M}_{\text{Z}}^2 \overline{\Gamma}_{\text{Z}}^2} + \sigma_{\text{non-res}}$$

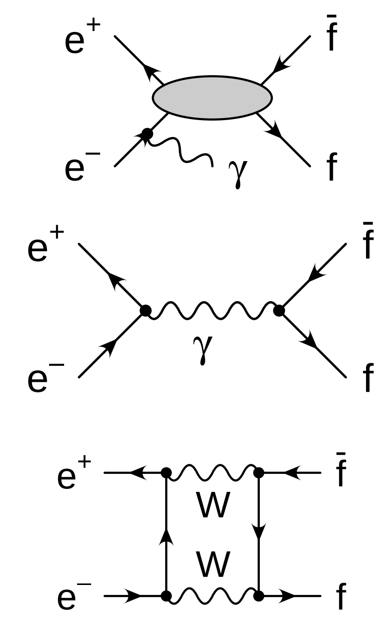
R and $\overline{\Gamma}_{\text{Z}}$ contain dependence on $\sin^2\theta^f_{\text{eff}}$, Γ_{ff}, ...

σ_{γ}, $\sigma_{\gamma\text{Z}}$, σ_{box}, $\sigma_{\text{non-res}}$ known at NLO

$\longrightarrow$ need consistent pole expansion framework

$\longrightarrow$ NNLO likely needed for FCC-ee/CEPC





Z lineshape

- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\rm ini}(s,s') \otimes \sigma_{\rm hard}(s')$$

- Subtraction of γ-exchange, γ–Z interference, box contributions:

$$\sigma_{\rm hard} = \sigma_{\rm Z} + \underbrace{\sigma_\gamma + \sigma_{\gamma \rm Z} + \sigma_{\rm box}}$$

- Z-pole contribution:

computed in SM

$$\sigma_{\rm Z} = \frac{R}{(s-\overline{M}_{\rm Z}^2)^2 + \overline{M}_{\rm Z}^2\overline{\Gamma}_{\rm Z}^2} + \sigma_{\rm non-res}$$

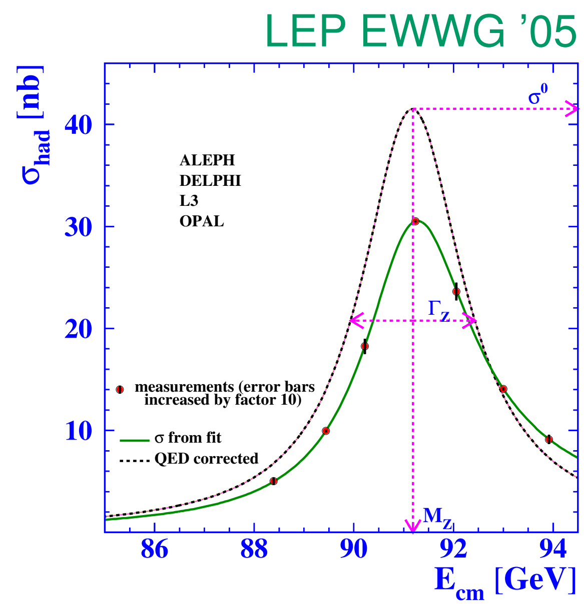
R and $\overline{\Gamma}_{\rm Z}$ contain dependence on $\sin^2\theta^f_{\rm eff}$, Γ_{ff}, ...

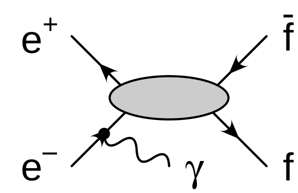
σ_γ, $\sigma_{\gamma \rm Z}$, $\sigma_{\rm box}$, $\sigma_{\rm non-res}$ known at NLO

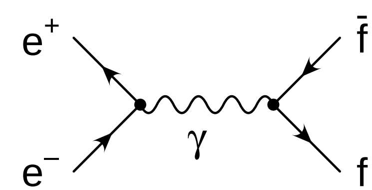
$\longrightarrow$ need consistent pole expansion framework

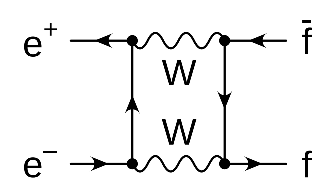
$\longrightarrow$ NNLO likely needed for FCC-ee/CEPC

$\longrightarrow$ possible BSM physics?









Expand amplitude for $e^+e^- \to f\bar{f}$ about **complex pole** $s_0 \equiv \overline{M}_Z^2 + i\overline{M}_Z\overline{\Gamma}_Z$:

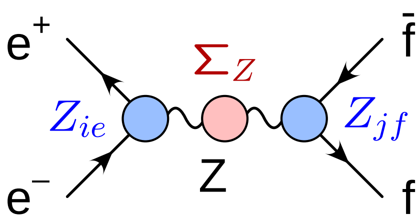
$\to$ All terms are individually gauge-invariant

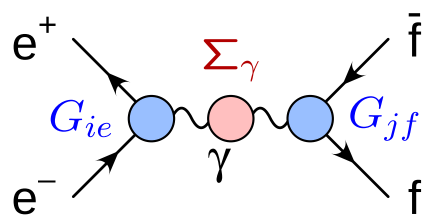
$$\mathcal{M}_{ij} = \frac{R_{ij}}{s - s_0} + S_{ij} + (s - s_0)S'_{ij} + \ldots \qquad (i, j = V, A)$$

$$R_{ij} = \left. \frac{Z_{ie}Z_{jf}}{1 + \Sigma'_Z} \right|_{s=s_0}$$

$$S_{ij} = \left[\frac{Z_{ie}Z'_{jf} + Z'_{ie}Z_{jf}}{1 + \Sigma'_Z} - \frac{Z_{ie}Z_{jf}\Sigma''_Z}{2(1 + \Sigma'_Z)^2} + \frac{G_{ie}G_{jf}}{s + \Sigma_\gamma} + B_{ij} \right]_{s=s_0}$$

$$S'_{ij} = \ldots$$





Pole expansion

Expand amplitude for $e^+e^- \to f\bar{f}$ about **complex pole** $s_0 \equiv \overline{M}_Z^2 + i\overline{M}_Z\overline{\Gamma}_Z$:

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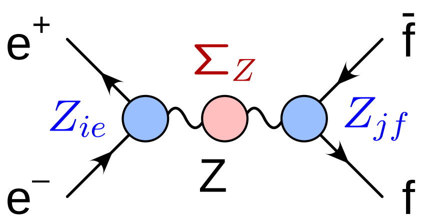
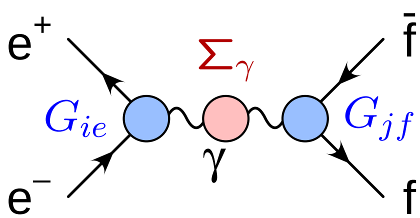
$$\mathcal{M}_{ij} = \frac{R_{ij}}{s - s_0} + S_{ij} + (s - s_0)S'_{ij} + \ldots \qquad (i, j = V, A)$$

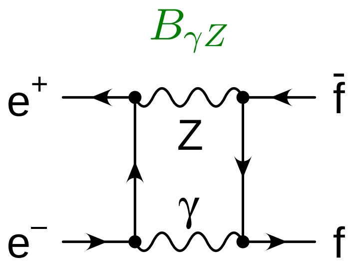
$$R_{ij} = \left.\frac{Z_{ie}Z_{jf}}{1 + \Sigma'_Z}\right|_{s=s_0} + B^R_{\gamma Z,ij} + B^{RL}_{\gamma Z,ij} \ln(1 - \tfrac{s}{s_0})$$

$$S_{ij} = \left[\frac{Z_{ie}Z'_{jf} + Z'_{ie}Z_{jf}}{1 + \Sigma'_Z} - \frac{Z_{ie}Z_{jf}\Sigma''_Z}{2(1 + \Sigma'_Z)^2} + \frac{G_{ie}G_{jf}}{s + \Sigma_\gamma} + B_{ij}\right]_{s=s_0}$$
$$+ B^S_{\gamma Z,ij} + B^{SL}_{\gamma Z,ij} \ln(1 - \tfrac{s}{s_0})$$

$$S'_{ij} = \ldots$$







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$$\mathcal{M}_{ij} = \frac{R_{ij}}{s - s_0} + S_{ij} + (s - s_0)S'_{ij} + \ldots \qquad (i,j = V,A)$$

Current state of art: R @ NNLO + leading higher orders
$S,\ S', \ldots$ @ NLO

For future ee colliders: (at least) one order more!

Cross-section: $$\sigma_Z = \frac{R}{(s - \overline{M}_Z^2)^2 + \overline{M}_Z^2\overline{\Gamma}_Z^2} + \sigma_{\text{non-res}}$$

In exp. studies: $$\sigma \sim \frac{1}{(s - M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2}$$

$$\overline{M}_Z = M_Z \Big/ \sqrt{1 + \Gamma_Z^2/M_Z^2} \approx M_Z - 34\text{ MeV}$$
$$\overline{\Gamma}_Z = \Gamma_Z \Big/ \sqrt{1 + \Gamma_Z^2/M_Z^2} \approx \Gamma_Z - 0.9\text{ MeV}$$

Pole expansion

Express R_{ij} in terms of $\sin^2\theta^f_{\rm eff}$ and F^f_A (accurate up to NNLO):

$$\sin^2\theta^f_{\rm eff} = \frac{1}{4|Q_f|}\left[1 - \mathrm{Re}\,\frac{Z_{Vf}}{Z_{Af}}\right]_{s=\overline{M}^2_Z}$$

$$F^f_A = \left[\frac{|Z_{Af}|^2}{1+\mathrm{Re}\,\Sigma'_Z} - \tfrac{1}{2}\overline{M}_Z\overline{\Gamma}_Z|Z_{Af(0)}|^2\,\mathrm{Im}\,\Sigma''_{Z(1)}\right]_{s=\overline{M}^2_Z} + \mathcal{O}(\alpha^3)$$

$$\Rightarrow \quad R_{ij} = \left.\frac{Z_{ie}Z_{jf}}{1+\Sigma'_Z}\right|_{s=s_0}$$

$$= 4I^3_eI^3_f\sqrt{F^e_AF^f_A}\left[Q^e_iQ^f_j + \dots\right], \qquad Q^f_V = 1-4|Q_f|\sin^2\theta^f_{\rm eff}, \quad Q^f_A = 1$$

terms related to
$\mathrm{Im}\, Z_{ie}$, $\mathrm{Im}\, Z_{jf}$, $\mathrm{Im}\,\Sigma_Z$
and the γZ box

(n) = loop order

$$\Gamma_{ff} = \frac{N^f_cM_Z}{12\pi}F^f_A\left[(1-4|Q_f|\sin^2\theta^f_{\rm eff})^2 + \dots\right]$$

"Model-independent" fit

LEP1 era fit parameters: Schael et al. '05

- 9-parameter fit: M_Z, Γ_Z, $\sigma^0_{\text{had}} \approx \frac{12\pi}{M_Z^2}\frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}$, R_e, R_μ, R_τ, A^e_{FB}, A^μ_{FB}, A^τ_{FB}
- 5-parameter fit: M_Z, Γ_Z, $\sigma^0_{\text{had}} \approx \frac{12\pi}{M_Z^2}\frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}$, R_ℓ, A^ℓ_{FB}

Heavy-flavor pseudo-observables ($R_{b,c}$, $A^{b,c}_{\text{FB}}$, ...) handled separately

- Underlying assumption: Can trust SM predictions for

  a) non-resonant contributions (beyond leading Z pole)

  b) Im Z_{if}, Im Σ_Z

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Heavy-flavor pseudo-observables ($R_{b,c}$, $A^{b,c}_{\rm FB}$, ...) handled separately

- Underlying assumption: Can trust SM predictions for
  a) non-resonant contributions (beyond leading Z pole)
  $\longrightarrow$ affected e.g. by Z' bosons
  b) Im Z_{if}, Im Σ_Z
  $\longrightarrow$ affected e.g. by light BSM particles in loops
- Assumption probably ok for LEP1 given constraints from BSM searches
  $\longrightarrow$ doubtful if precision is increased by factor 10-100

"SM Effective Field Theory" (SMEFT)

Extension of SM by **higher-dimensional operators**:

Wilson '69
Weinberg '79

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + \sum_{d=5}^{\infty} \frac{1}{\Lambda^{d-4}} \sum_i c_i \, \mathcal{O}_i^{(d)}$$

- More general description of BSM effects
- Operators ranked by suppression power Λ^{4-d}
- Leading effects typically for $d=6$
- **Assumption:** $E \ll \Lambda$ ($\Lambda \sim$ mass of heavy BSM particles)

SMEFT operators for electroweak precision

Leading dim-6 contribution: $\mathcal{L} = \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i + \mathcal{O}(\Lambda^{-3}) \qquad (\Lambda \gg M_Z)$

$$\mathcal{O}_{\phi 1} = \tfrac{1}{4}|\Phi^\dagger \overleftrightarrow{D}_\mu \Phi|^2 \qquad \alpha\Delta T = -\frac{v^2}{2}\frac{c_{\phi 1}}{\Lambda^2}$$

$$\mathcal{O}_{BW} = \Phi^\dagger B_{\mu\nu} W^{\mu\nu}\Phi \qquad \alpha\Delta S = -e^2 v^2 \frac{c_{BW}}{\Lambda^2}$$

$$O_{LL}^{(3)\mu e} = (\bar{L}_L^\mu \sigma^a \gamma_\mu L_L^\mu)(\bar{L}_L^e \sigma^a \gamma^\mu L_L^e) \qquad \Delta G_F = -\sqrt{2}\frac{c_{LL}^{(3)\mu e}}{\Lambda^2}$$

$$O_R^f = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{f}_R \gamma^\mu f_R) \qquad f = e, \mu\, \tau, b, lq$$

$$O_L^F = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{F}_L \gamma^\mu F_L) \qquad F = \begin{pmatrix}\nu_e \\ e\end{pmatrix}, \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}, \begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}, \begin{pmatrix}u, c \\ d, s\end{pmatrix}, \begin{pmatrix}t \\ b\end{pmatrix}$$

$$O_L^{(3)F} = i(\Phi^\dagger \overleftrightarrow{D}_\mu^a \Phi)(\bar{F}_L \sigma_a \gamma^\mu F_L)$$

More operators than EWPOs $\rightarrow$ need assumptions:

- Family universality, e.g. $c_R^e = c_R^\mu = c_R^\tau$
- U(2)×U(1) flavor symmetry, e.g. $c_R^e = c_R^\mu \neq c_R^\tau$

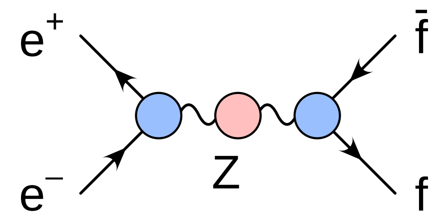
Translation SMEFT ↔ EWPOs

Leading Z pole contribution:

Modified propagators:

e.g. $\mathcal{O}_{\phi 1} = (D_\mu \Phi)^\dagger \Phi \, \Phi^\dagger (D^\mu \Phi)$

$\mathcal{O}_{\rm BW} = \Phi^\dagger B_{\mu\nu} W^{\mu\nu} \Phi$



Modified Z-fermion couplings:

e.g. $O^f = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{f}\gamma^\mu f)$ $\quad f = e, \mu\, \tau, b, \ldots$

Direct correspondence between pseudo-observables and SMEFT ops.:

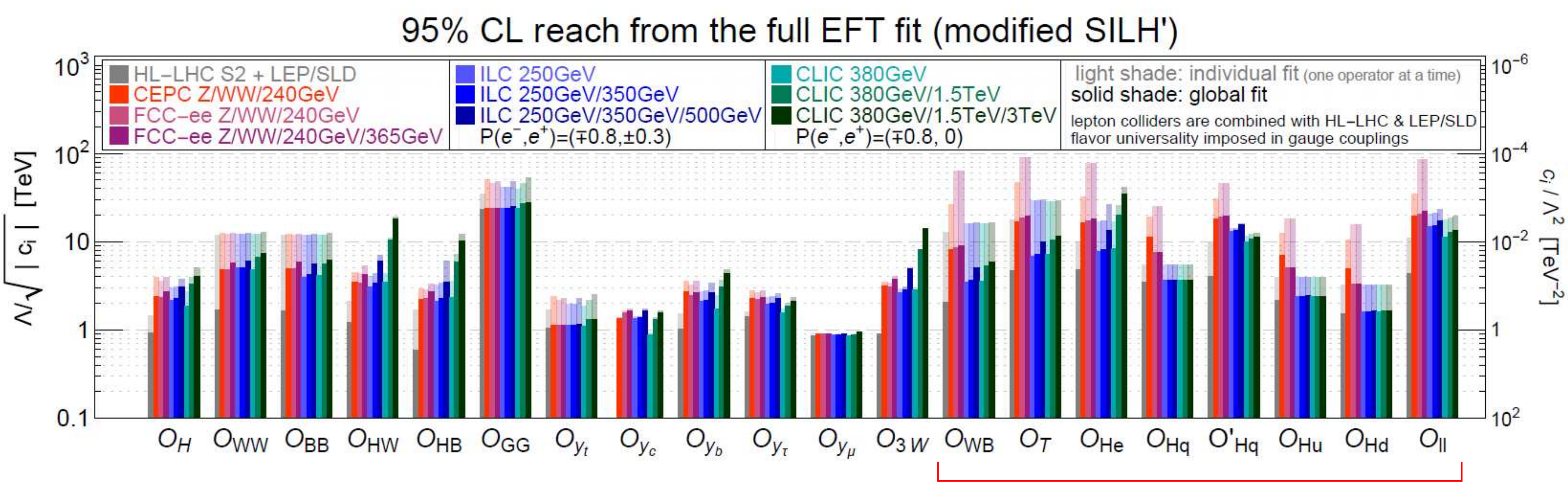
$$\frac{\delta\Gamma_{ff}}{\Gamma_{ff}} = 2\left[\frac{\delta Z_L^f}{Z_L^f} + \frac{\delta Z_R^f}{Z_R^f}\right] \qquad \frac{\delta \sin^2\theta^f_{\rm eff}}{\sin^2\theta^f_{\rm eff}} = \frac{1}{1 - Z_L^f/Z_R^f}\left[\frac{\delta Z_L^f}{Z_L^f} - \frac{\delta Z_R^f}{Z_R^f}\right]$$

$$\delta Z_X^f = \frac{1}{\Lambda^2}\left[I_3^{fX} c_L^{(3)f} - \frac{1}{2}c_X^f - Q_f \frac{sc}{c^2-s^2} c_{WB} - \left(\tfrac{1}{16}c_{\phi 1} - \tfrac{1}{4}c_{LL}^{(3)} + c_L^\ell\right)\left(I_3^{fX} + Q_f \frac{s^2}{c^2-s^2}\right)\right]$$

$$\frac{\delta M_W}{M_W} = \delta Z_L^\nu - \delta Z_L^e - \tfrac{1}{4}c_{LL}^{(3)}$$

SMEFT: future collider reach

Project reach of future colliders to constrain SMEFT operators:

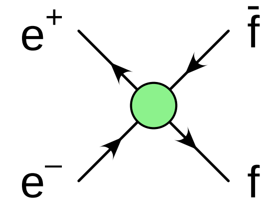


de Blas, Durieux, Grojean, Gu, Paul '19

Translation SMEFT $\leftrightarrow$ EWPOs

Beyond leading Z pole ("background"):

Additional 4-fermion operators



e.g. $O^{(1)}_{ff'} = (\bar{f}\gamma_\mu f)(\bar{f}'\gamma^\mu f')$ $\quad f, f' = e, \mu\,\tau, b, \ldots$

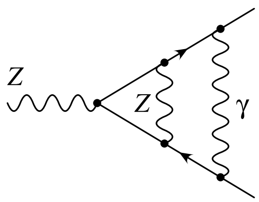
(related via e.o.m. to energy-dependent $Z f f / \gamma f f$ couplings $O^D_f \sim \bar{f}\gamma^\mu F_{\mu\nu} D^\nu f$)

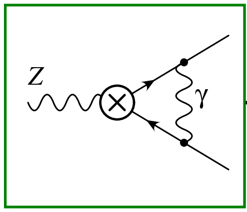
$\longrightarrow$ Model-independent analysis requires additional parameters (or Wilson coeffs.) in non-resonant terms

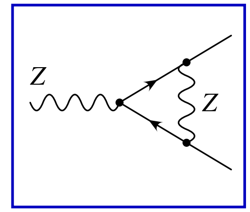
QED radiation

Factorization of massive and QED/QCD FSR:

$$\overline{\Gamma}_f \approx \frac{N_c \overline{M}_Z}{12\pi} \left[\mathcal{R}_V^f F_V^f + \mathcal{R}_A^f F_A^f \right]_{s=\overline{M}_Z^2}$$



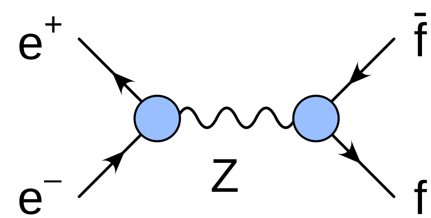
$=$


$+$ finite, with $\otimes =$


$\mathcal{R}_V^f$, $\mathcal{R}_A^f$: Final-state QED/QCD radiation;
known inclusively to $\mathcal{O}(\alpha_s^4)$, $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_s)$ Kataev '92
Chetyrkin, Kühn, Kwiatkowski '96
Baikov, Chetyrkin, Kühn, Rittinger '12

or compute exclusively using MC methods

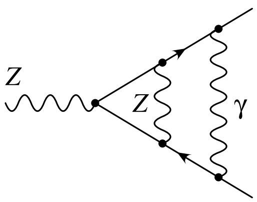
F_V^f, F_A^f: Electroweak corrections



QED radiation

Factorization of massive and QED/QCD FSR:

$$\overline{\Gamma}_f \approx \frac{N_c \overline{M}_Z}{12\pi} \left[\mathcal{R}_V^f F_V^f + \mathcal{R}_A^f F_A^f \right]_{s=\overline{M}_Z^2}$$



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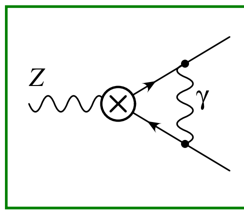
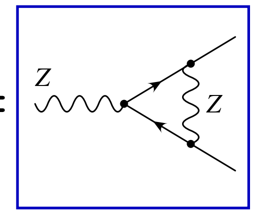


+ finite, with $\otimes$ =



$\mathcal{R}_V^f$, $\mathcal{R}_A^f$: Final-state QED/QCD radiation;

Simulate with Monte Carlo event generator, e.g.

KKMC, Arbuzov, Jadach, Wąs, Ward, Yost '20

SHERPA_YFS, Krauss, Price, Schönherr '22

POWHEG_EW Barzè, Montagna, Nason, Nicrosini, Piccinini '12,13

LL $\left[(\alpha \log \frac{E}{m_e})^n\right]$ + some NLL $\left[\alpha^2 \log \frac{E}{m_e}\right]$

QED radiation

Factorization of massive and QED/QCD FSR:

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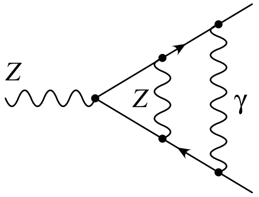

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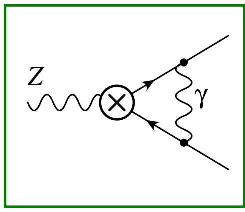
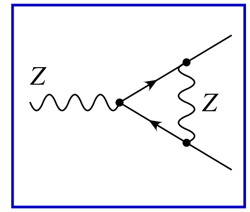
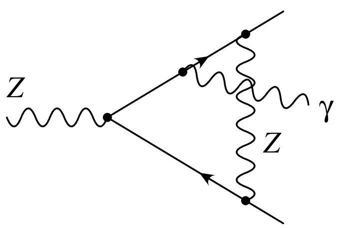


$+$ finite, with $\otimes =$



Additional non-factorizable contributions, e.g.



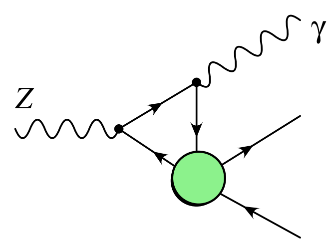
→ Known at $\mathcal{O}(\alpha\alpha_s)$ Czarnecki, Kühn '96
Harlander, Seidensticker, Steinhauser '98

→ Currently not known at $\mathcal{O}(\alpha^2)$ and beyond

→ Small ($< \mathcal{O}(0.1\%)$) but not negligible contribution

→ Additional matching terms in MC simulations

→ Influenced by different SMEFT ops. than Zff vertex



Monte-Carlo for future colliders

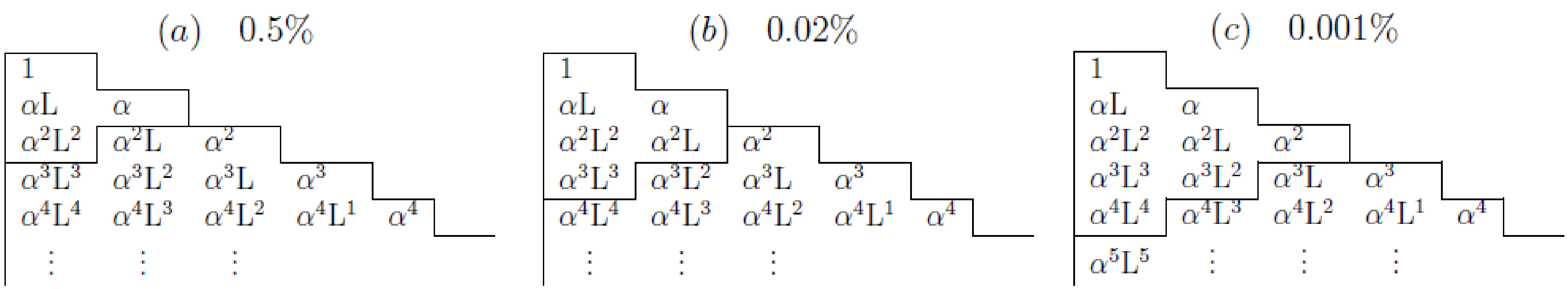
- Current state of art: e.g. `KORALZ, KKMC` → $\mathcal{O}(\alpha^2 L)$ accuracy [$L = \ln(s/m_e^2)$]

Jadach, Ward, ...

- One to two orders improvement needed:

Observable	Where from	Present (LEP)	FCC stat.	FCC syst	Now/FCC
M_Z [MeV]	Z linesh. [28]	$91187.5 \pm 2.1\{0.3\}$	0.005	0.1	3
Γ_Z [MeV]	Z linesh. [28]	$2495.2 \pm 2.1\{0.2\}$	0.008	0.1	2
$R_l^Z = \Gamma_h/\Gamma_l$	$\sigma(M_Z)$ [33]	$20.767 \pm 0.025\{0.012\}$	$6 \cdot 10^{-5}$	$1 \cdot 10^{-3}$	12
$\sigma^0_{\rm had}$[nb]	$\sigma^0_{\rm had}$ [28]	$41.541 \pm 0.037\{0.25\}$	$0.1 \cdot 10^{-3}$	$4 \cdot 10^{-3}$	6
N_ν	$\sigma(M_Z)$ [28]	$2.984 \pm 0.008\{0.006\}$	$5 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	6
$\sin^2\theta_W^{eff} \times 10^5$	$A_{FB}^{lept.}$ [33]	$23099 \pm 53\{28\}$	0.3	0.5	55
$A_{FB,\mu}^{M_Z\pm 3.5\rm GeV}$	$\frac{d\sigma}{d\cos\theta}$ [28]	$\pm 0.020\{0.001\}$	$1.0 \cdot 10^{-5}$	$0.3 \cdot 10^{-5}$	100

Jadach, Skrzypek '19



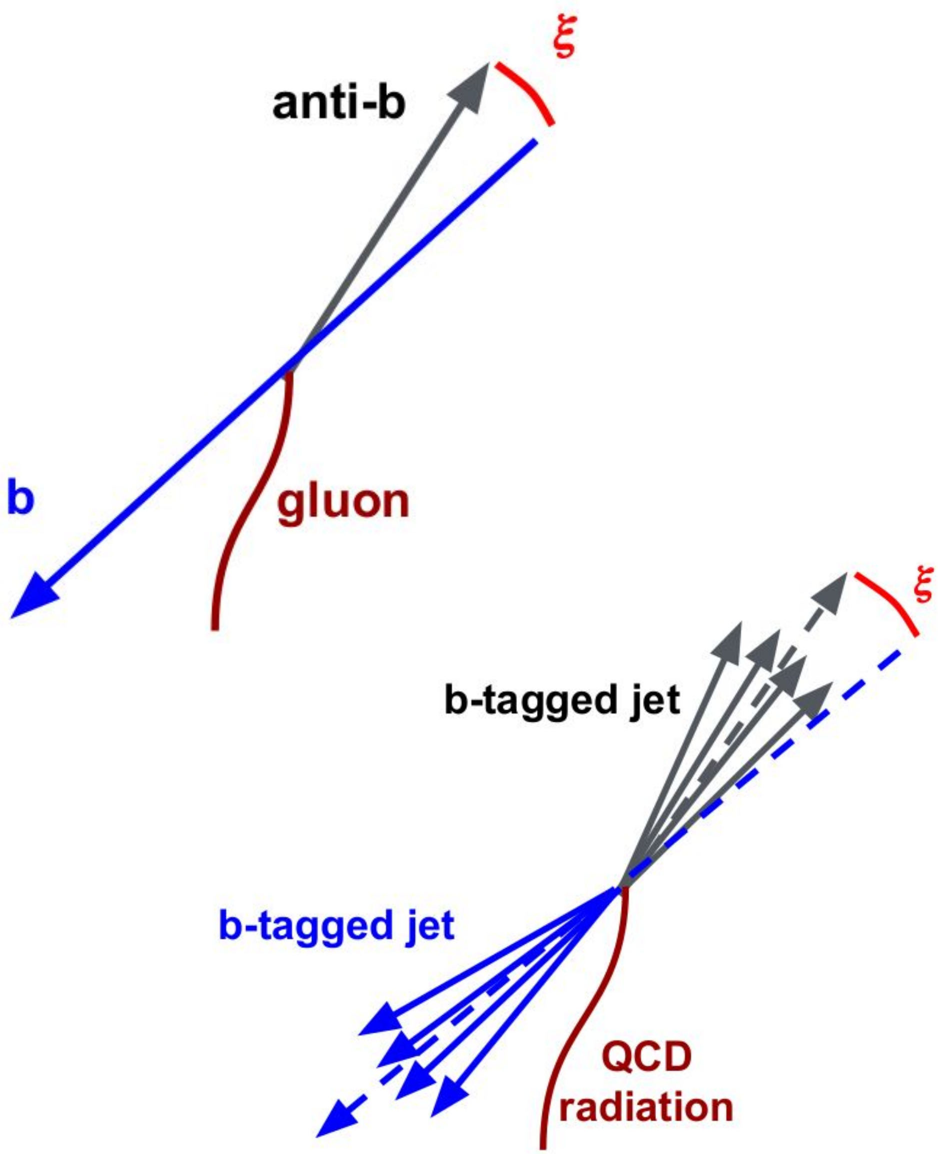
- Need control over
  - multi-γ production
  - hadronization
  - color reconnection
  - $\gamma/g \to f\bar{f}$
  - heavy-flavor correlations
  - ...

Forward-backward asymmetry for heavy quarks

- Measurement of $A_{\mathrm{FB}}^{b,c}$ requires
  - $b/\bar{b}$ ($c/\bar{c}$) discrimination
  - Measurement of b (c) angle
- Mismatch between observed and parton-level b (c) angle due to QCD radiation (requires accurate modeling)
- Contamination from gluon splitting $g \to bb$ ($g \to cc$)
- Impact of hadronization/fragmentation (need more precise models and fragmentation functions)

$A_{FB}(b/c)$

arXiv:2010.08604



- New developments for $A_{FB}(b/c)$: QCD corrections and uncertainties can be reduced significantly using acollinearity (ξ) cuts ⇒ important reduction in systematics, but how much ?
- Further improvements expected from better heavy flavor tagging capabilities and a more accurate measurement of the heavy quark flight direction
- **More sophisticated b/c tagging techniques => minimal charm/light background effects**
- **g->QQ splitting: huge control samples, smaller effect with back-to-back configuration and double tagging**
- Note that all these measurements can be done with exclusive decays. A Tera-Z facility will provide ≈10^8 B^+ exclusive decays

“missing” pieces: gluon fragmentation (1)

- $g \to Q\bar{Q}$ splitting tricky in parton showers

  (no soft enhancement, coll. divergence shielded by masses)

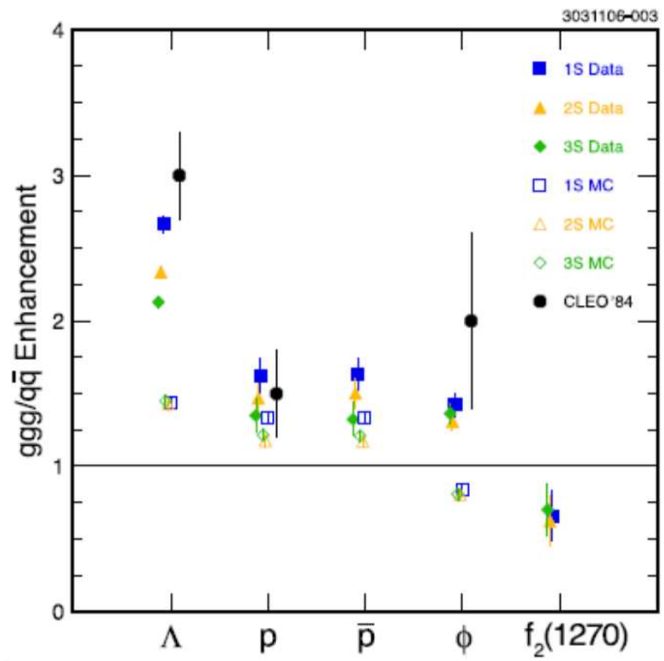
  - HF production is perturbative process
  - analyse $4b$ and $2b2c$ final states
    combine two softest equal flavour HFs into “gluon” and measure the $g \to Q\bar{Q}$ splitting function
    will yield information about shower evolution parameter and correct scale definition for α_S

"missing" pieces: gluon fragmentation (2)

- e^-e^+ (like LEP) dominated by quark jets:
  $\longrightarrow$ questionable handle on details of gluon fragmentation

(examples: enhanced diquark-popping? (leading) baryons? realisation of LPHD in gluons?)

- measurement strategy:
  - "Mercedes star" with two id'd heavy quark jets $\longrightarrow$ third jet is gluon jet
  - jet-shape measurements: sub-jettiness & friends
  - hadron yields inside jet
  - leading hadron identity/x_p
  - di-baryon/di-strange correlations inside jet



Theory calculations: Uncertainties

- To probe new physics, compare EWPOs with SM theory predictions
- Need to take theory error into account:

	Current exp.	Current th.	CEPC	FCC-ee
M_W [MeV]	11–12	4	0.5	0.4
Γ_Z [MeV]	2.3	0.4	0.025	0.025
$R_\ell = \Gamma_Z^\mathrm{had}/\Gamma_Z^\ell$ [10^{-3}]	25	5	2	1
$R_b = \Gamma_Z^b/\Gamma_Z^\mathrm{had}$ [10^{-5}]	66	10	4.3	6
$\sin^2\theta^\ell_\mathrm{eff}$ [10^{-5}]	13	4.5	0.3	0.4

- Theory error estimate is not well defined, ideally $\Delta_\mathrm{th} \ll \Delta_\mathrm{exp}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence

Example: Error estimation for Γ_Z

- Goemetric perturbative series

$\alpha_t = \alpha m_t^2$

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2) \sim 0.20 \text{ MeV}$$

$$\mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.21 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.23 \text{ MeV}$$

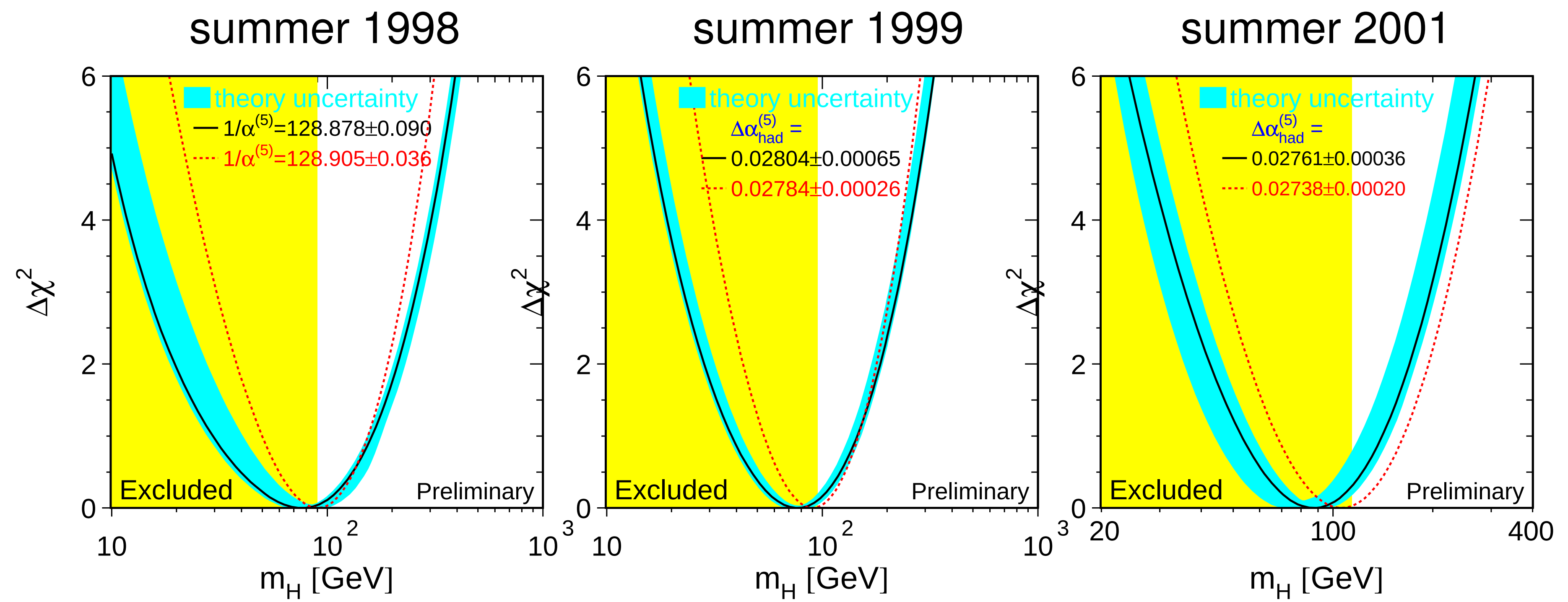
$$\mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s^2) \sim 0.035 \text{ MeV}$$

- Parametric prefactors:

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\alpha n_{lq}}{\pi}\alpha_s^2 \sim 0.29 \text{ MeV}$$

Total: $\delta\Gamma_Z \approx 0.4$ MeV

Uncertainty on the theory uncertainty?



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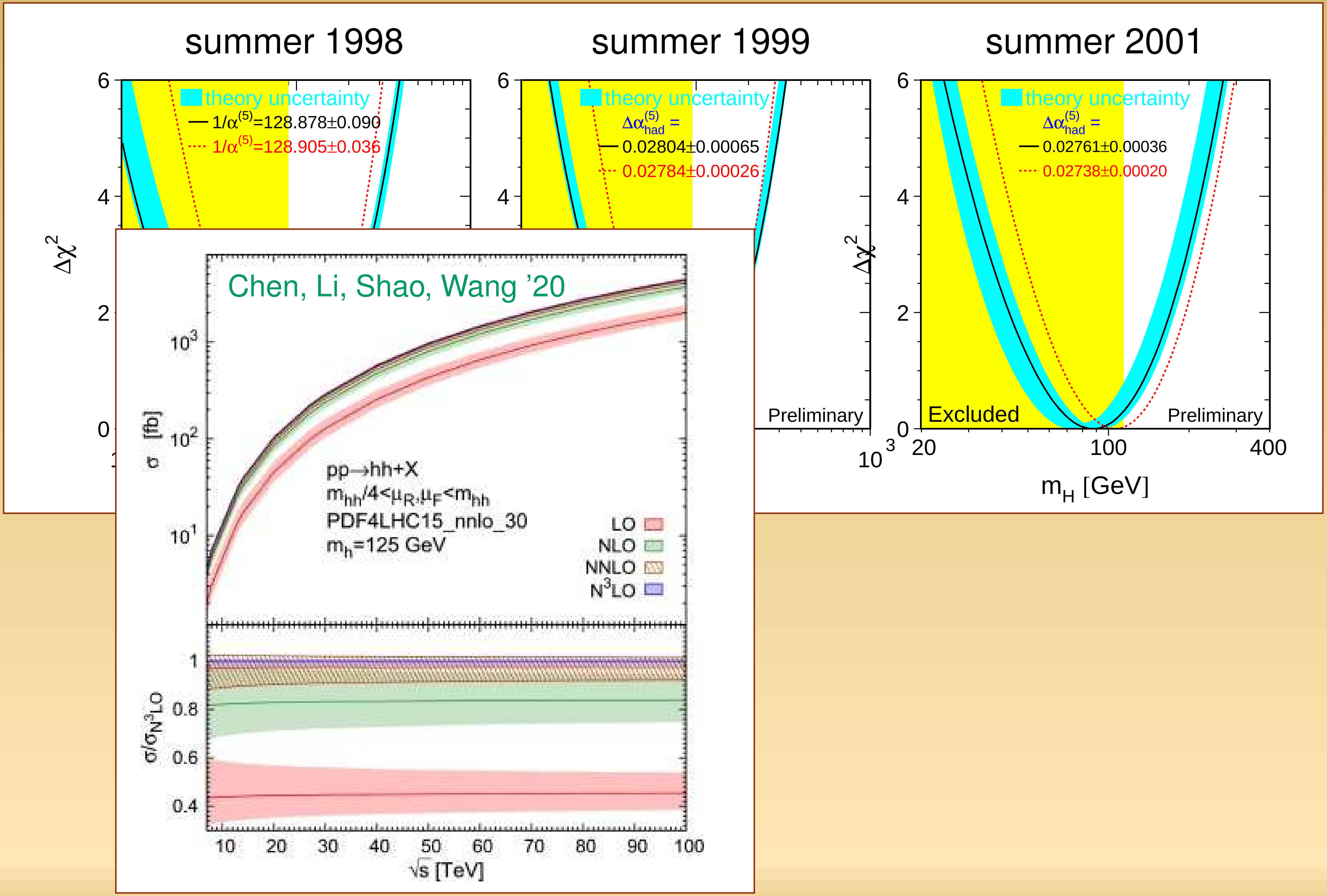

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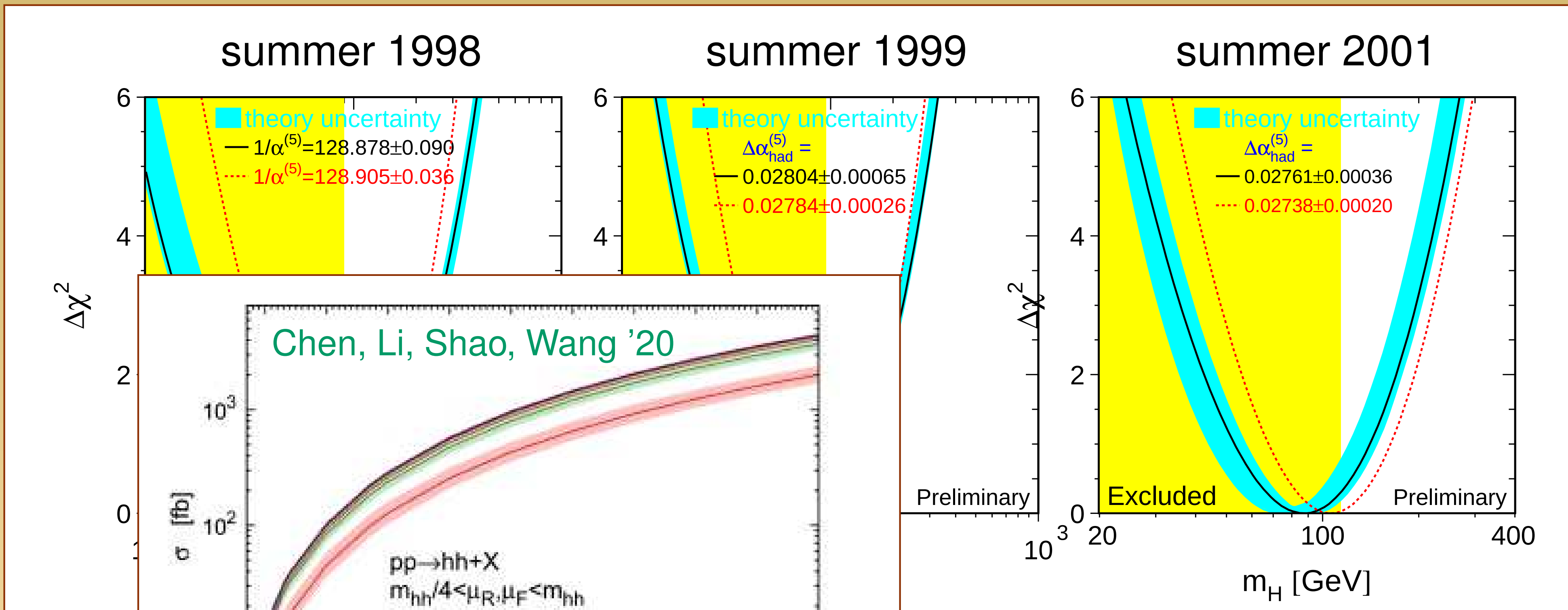

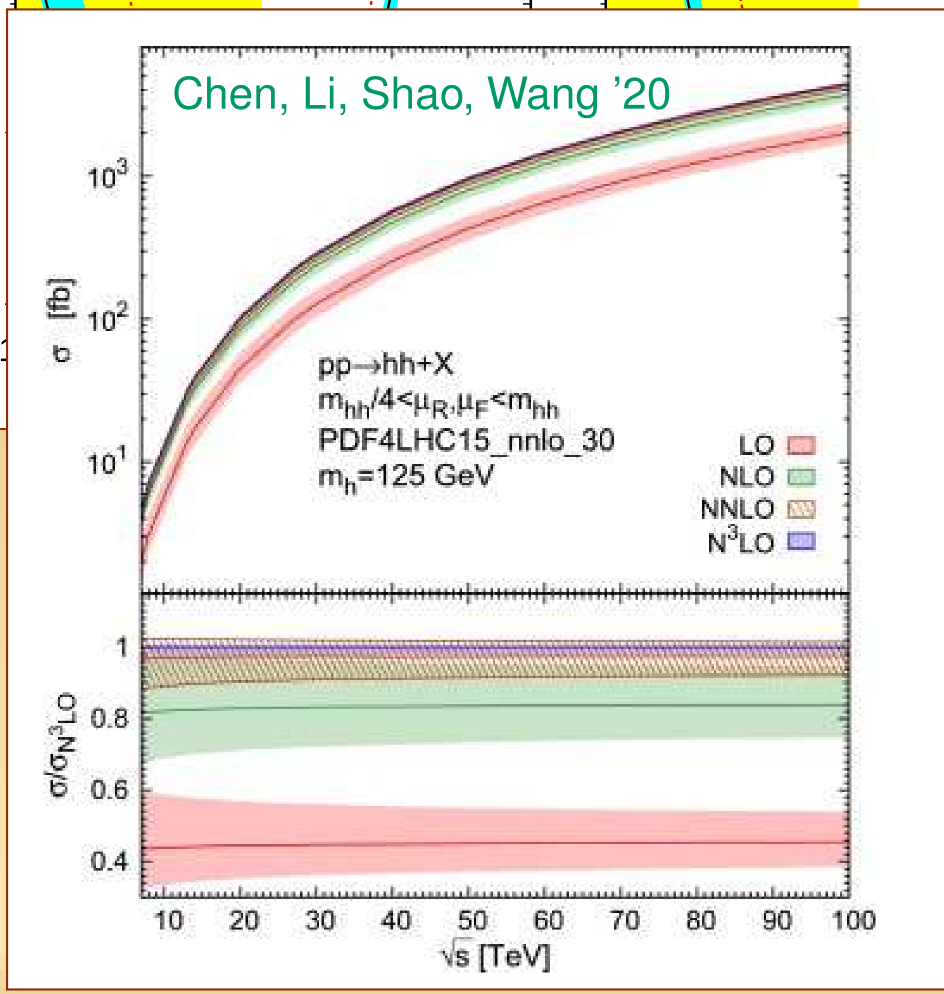

- Theory uncertainties do not have stat. interpretation (except in large-N limit

Summary

- Precision measurements require theory input for **measurements of pseudo-observables** (BRs, widths, masses, cross-sections, ...) and their **SM/BSM interpretation**

- Need for higher-order loop calculations, but also new frameworks for parametrizing cross-sections, matching to Monte-Carlo, and fitting to data

- Many improvements needed for **MC tools**:
  (N)NLL QED/QCD showering, fermions pair production, hadronization, ...

- New ideas for how to estimate and interpret theory uncertainties?

Theory community organization

Organizing theorists is like herding cats

Lessons from LHC:

- Initially the challenge seems daunting
  $\rightarrow$ Les Houches wishlist (2005–09, mostly NLO QCD)

- At some point, critical mass is achieved and progress accelerates

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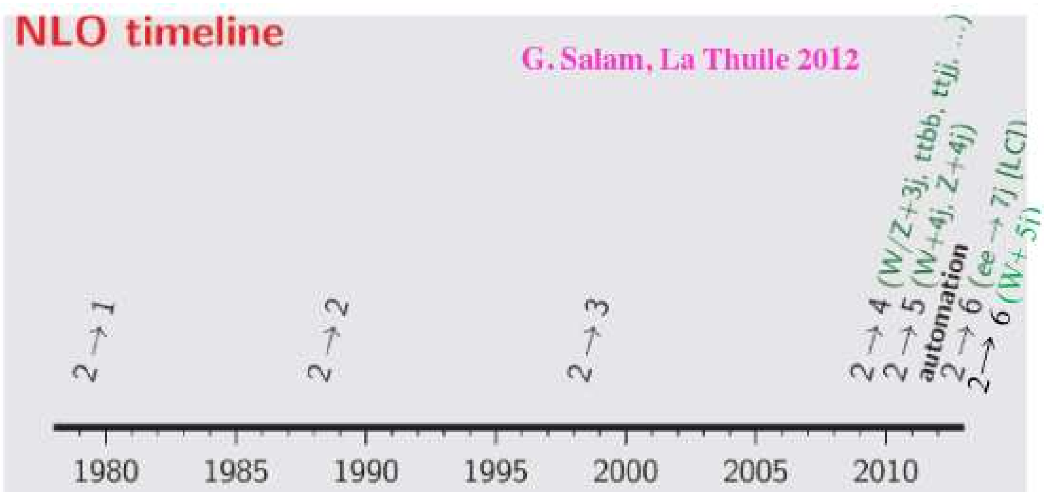
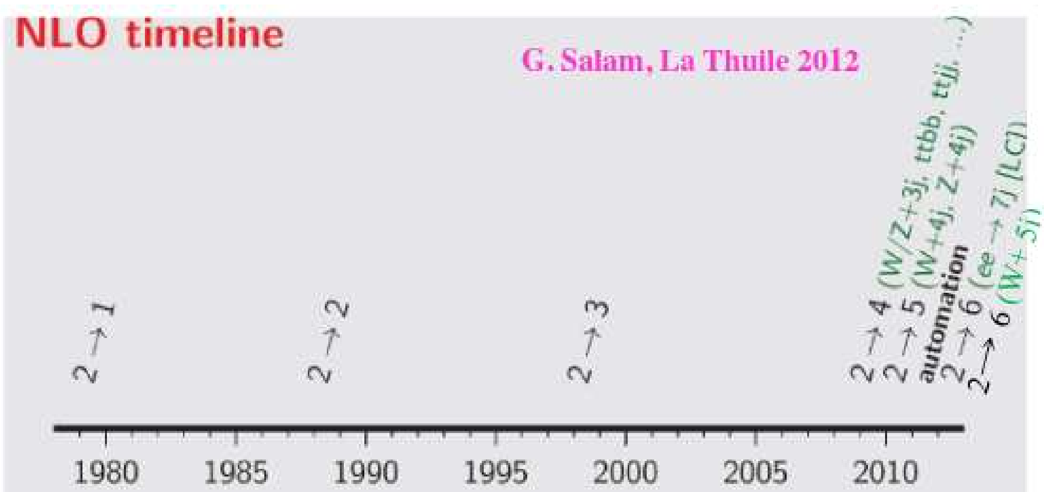


Theory community organization

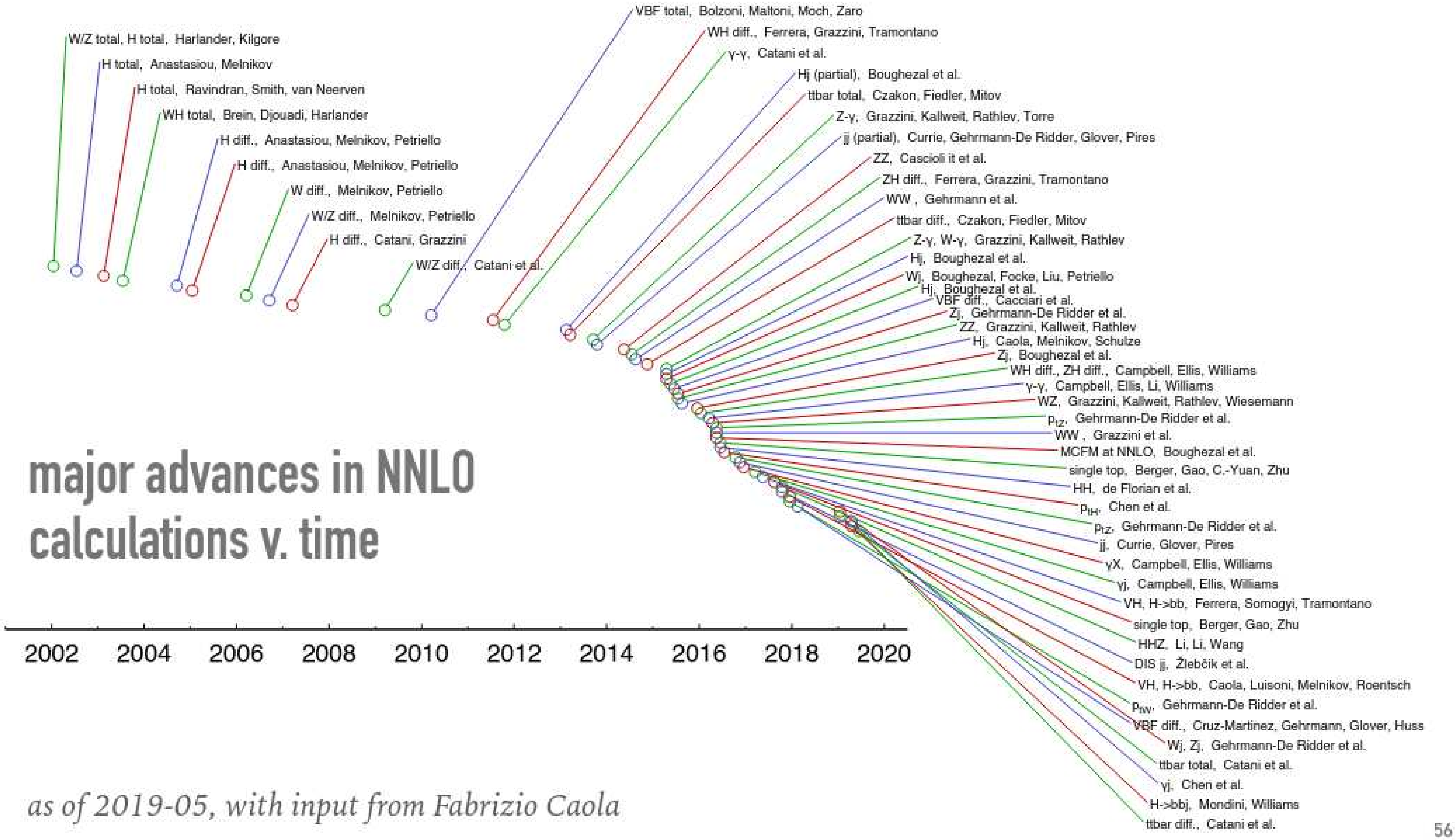
Lessons from LHC:

- Initially the challenge seems daunting
  - → Les Houches wishlist (2005–09, mostly NLO QCD)
  - → Completed in 2012
- At some point, critical mass is achieved and progress accelerates
- Theory expectations for LHC have been scaled up!
  - → Les Houches precision wishlist (2011– , NN(N)LO QCD, NLO EW)



Theory community organization

G. Salam, Erice, 2019



Theory community organization

Lessons from LHC:

- Initially the challenge seems daunting
  $\longrightarrow$ Les Houches wishlist (2005–09, mostly NLO QCD)
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- At some point, critical mass is achieved and progress accelerates
- Theory expectations for LHC have been scaled up!
  $\longrightarrow$ Les Houches precision wishlist (2011– , NN(N)LO QCD, NLO EW)
  $\longrightarrow$ Many items completed, remaining ones in progress

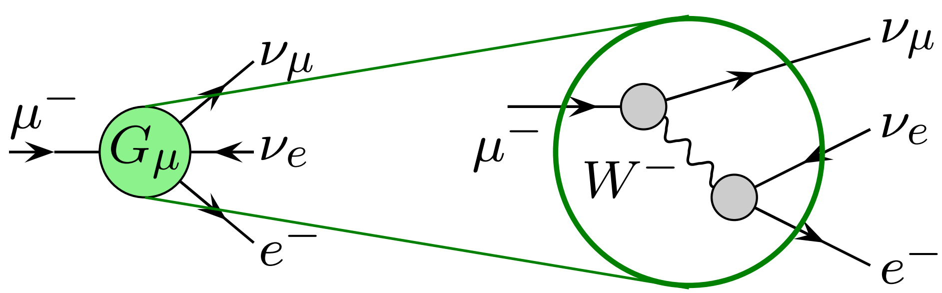
Backup slides

Comparison of EWPOs with theory

- To probe new physics, compare EWPOs with SM theory predictions
- Need to take theory error into account:

	Current exp.	Current th.	CEPC	FCC-ee
M_W* [MeV]	11–12	4	0.5	0.4
Γ_Z [MeV]	2.3	0.4	0.025	0.025
$R_\ell = \Gamma_Z^{had}/\Gamma_Z^\ell$ [10^{-3}]	25	5	2	1
$R_b = \Gamma_Z^b/\Gamma_Z^{had}$ [10^{-5}]	66	10	4.3	6
$\sin^2\theta_{eff}^\ell$ [10^{-5}]	13	4.5	0.3	0.4

* computed from G_μ



Theory calculations: Status

- Many seminal works on 1-loop and leading 2-loop corrections

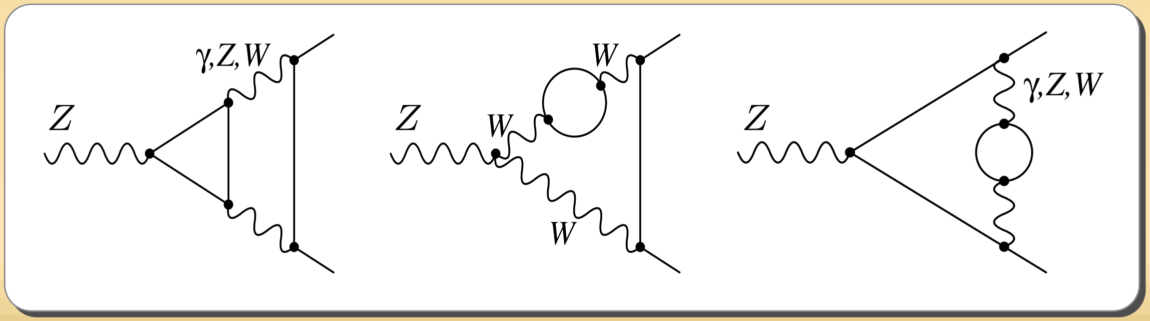
  Veltman, Passarino, Sirlin, Marciano, Bardin, Hollik, Riemann, Degrassi, Kniehl, ...

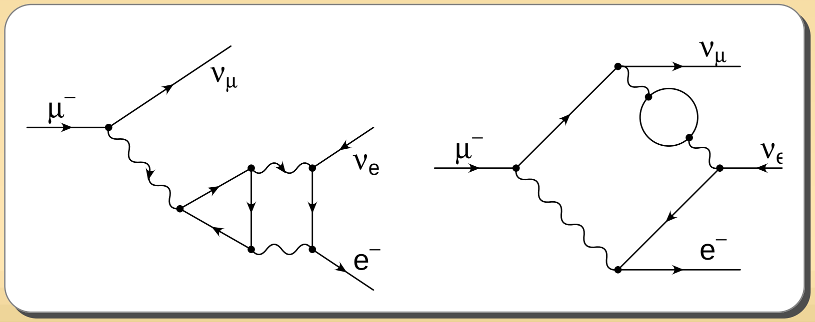
- Full 2-loop results for M_{W}, Z-pole observables

  Freitas, Hollik, Walter, Weiglein '00
  Awramik, Czakon '02
  Onishchenko, Veretin '02
  Awramik, Czakon, Freitas, Weiglein '04
  Awramik, Czakon, Freitas '06
  Hollik, Meier, Uccirati '05,07
  Awramik, Czakon, Freitas, Kniehl '08
  Freitas '14
  Dubovyk, Freitas, Gluza, Riemann, Usovitsch '16,18

- Approximate 3- and 4-loop results (enhanced by Y_{t} and/or N_f)

  Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03
  Boughezal, Tausk, v. d. Bij '05
  Schröder, Steinhauser '05
  Chetyrkin et al. '06
  Boughezal, Czakon '06
  Chen, Freitas '20





Theory calculations: Uncertainty projections

- Estimated impact of future higher-order calculations

Freitas et al. '19

	Current th.	Projected th.†	CEPC	FCC-ee
M_W [MeV]	4	1	0.5	0.4
Γ_Z [MeV]	0.4	0.15	0.025	0.025
$R_\ell = \Gamma_Z^{\rm had}/\Gamma_Z^\ell$ [10^{-3}]	5	1.5	2	1
$R_b = \Gamma_Z^b/\Gamma_Z^{\rm had}$ [10^{-5}]	10	5	4.3	6
$\sin^2\theta^\ell_{\rm eff}$ [10^{-5}]	4.5	1.5	0.3	0.4

† **Theory scenario:** $\mathcal{O}(\alpha\alpha_s^2)$, $\mathcal{O}(N_f\alpha^2\alpha_s)$, $\mathcal{O}(N_f^2\alpha^2\alpha_s)$, leading 4-loop

(N_f^n = at least n closed fermion loops)

Note: Estimates (based on extrapolation of perturb. series and prefactors) are unreliable and only provide a rough guess