Systematics: Final list and updated MC statistics

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table of systematics and which method was used

Systematic	NPs	Repeat Data analysis	Toy MC method	Input
MC stat uncertainty	✓	-	-	stat err
upstream energy loss	-	✓	-	$\pm 1\sigma$ fit err
beam momenta mis-modelling	-	✓	-	$\pm 1\sigma$ fit err
shower energy correction	-	✓	-	$\pm 1\sigma$ fit err
track length resolution	-	-	✓	±2.6%
beam momentum resolution	-	-	✓	±2.5%
theory uncertainty	-	-	✓	±20%
fit inaccuracy	-	-	✓	±20%

- each systematic has been implemented
- methods to evaluate each one has been discussed, and results shown prior
- rerun for additional MC stats, compare results

Additional statistics

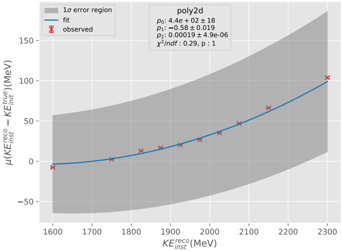
- Number of π beam triggers in Data is 285238
- Originally had 102443 beam π in MC
- Now, MC stat is 481572 4.7 times more
- With the additional MC, analysis was re-ran and systematics updated.

Upstream loss (Again)

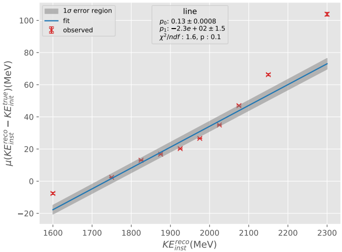
- Used a linear fit to characterise the upstream energy lost by the beam particle prior to entering the TPC.
- with updates MC stats, linear fit no longer holds
- instead, I chose to fit an exponential function:

$$f(x) = p_0 e^{p_1 x} + p_2 \tag{1}$$

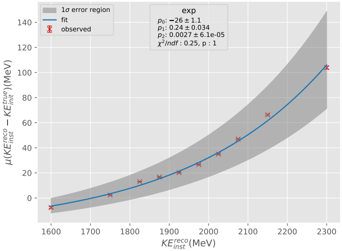
polynomial



linear



exponential



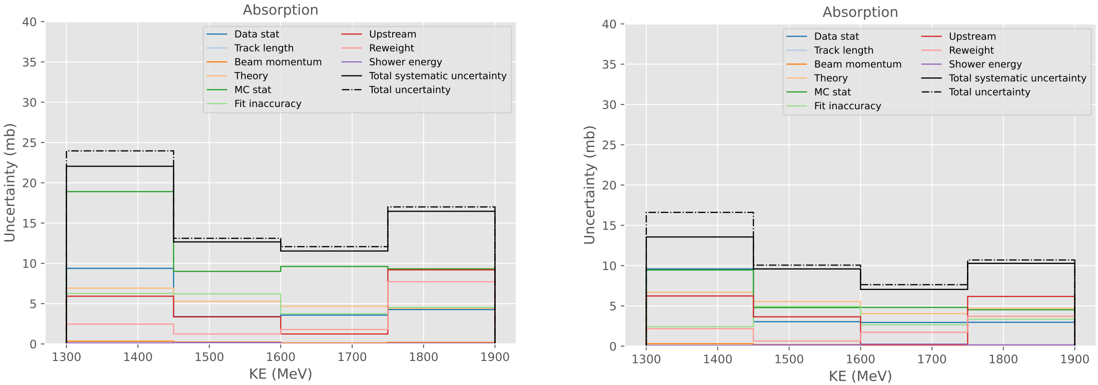
- Uncertainties are smaller than the polynomial fit, χ^2 is comparable

Comparing systematic uncertainties

Absorption

low stat

high stat

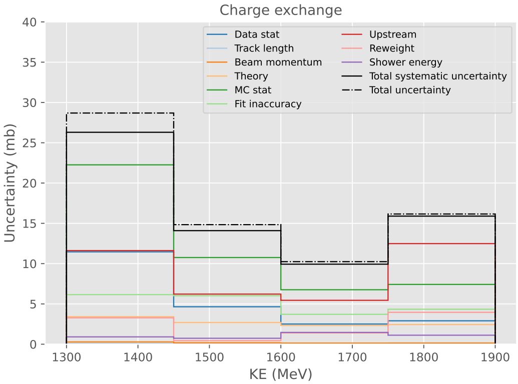


- with low stat MC dominates
- with higher stats, MC and Data stat are comparable.
- top systematics are MC stat, theory and upstream loss

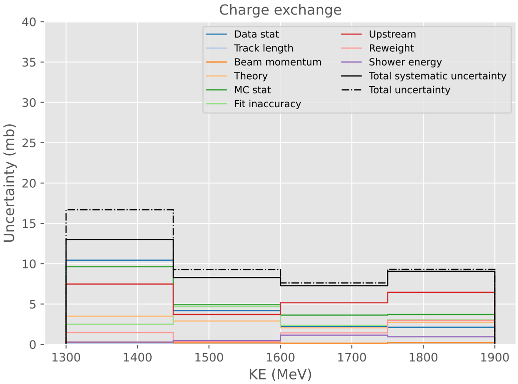
Comparing systematic uncertainties

Charge exchange

low stat



high stat

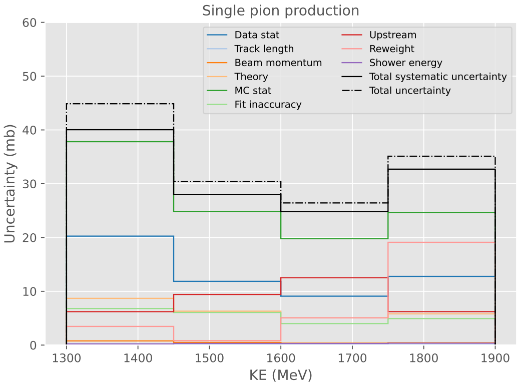


- with low stat MC dominates, and upstream energy uncertainty is large
- with higher stats top systematics are MC stat, theory and upstream energy
- upstream energy and reweight uncertainty decreases, likely due to higher MC stats → fitted values are more precise, and uncertainty decreases.

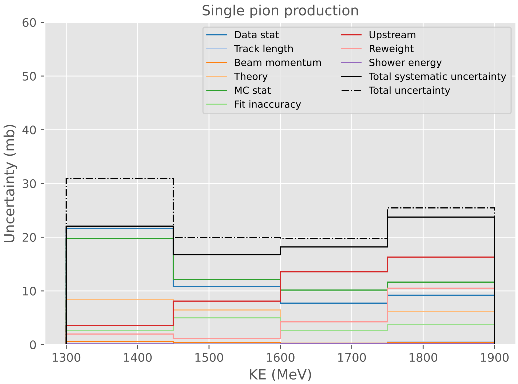
Comparing systematic uncertainties

Single pion production

low stat



high stat

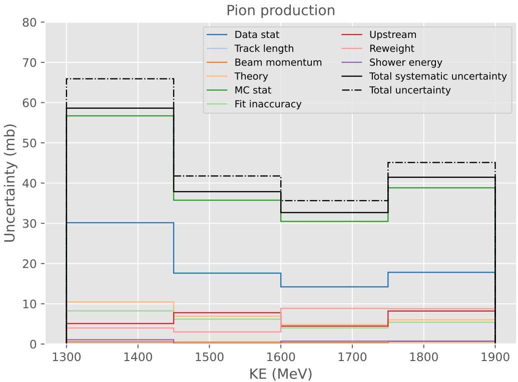


- with low stat MC dominates
- with high stats, upstream energy uncertainty increases in the last two bins

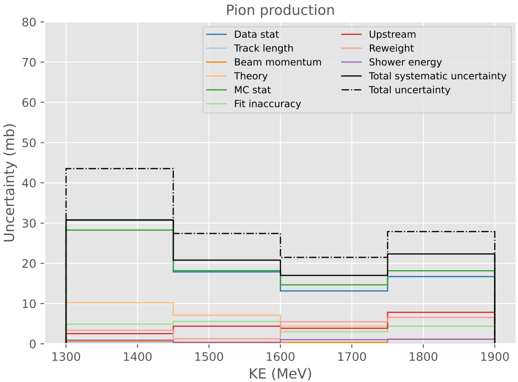
Comparing systematic uncertainties

Pion production

low stat



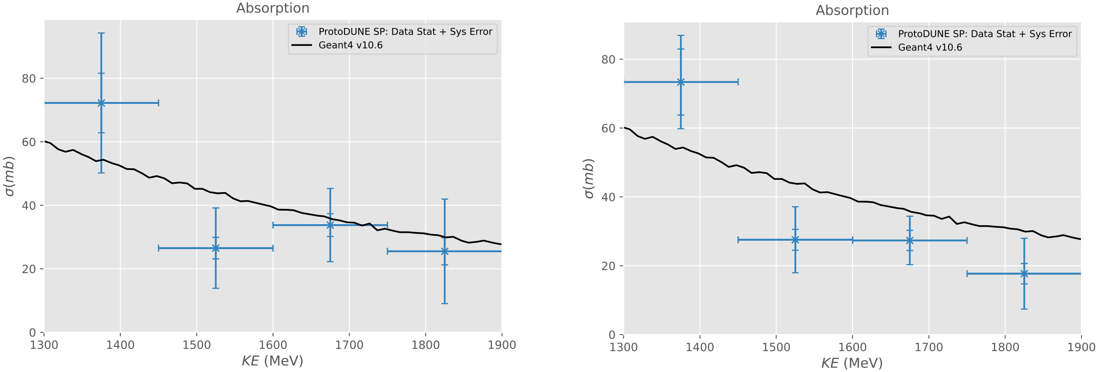
high stat



- For both samples, Data and MC stat uncertainties are the largest
- all other systematics are less significant.

Cross section measurements

Absorption



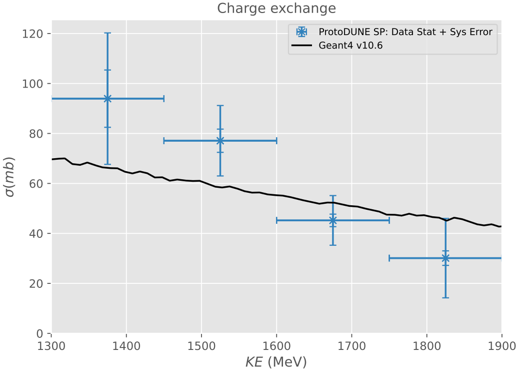
low stat

high stat

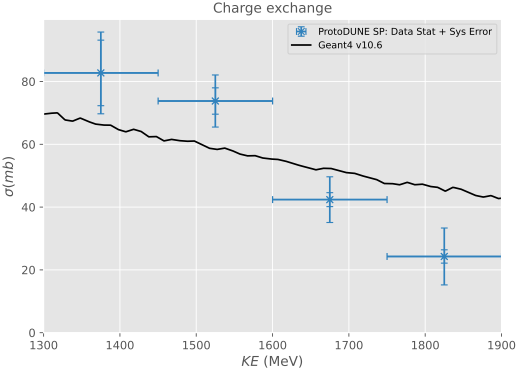
Cross section measurements

Charge exchange

low stat

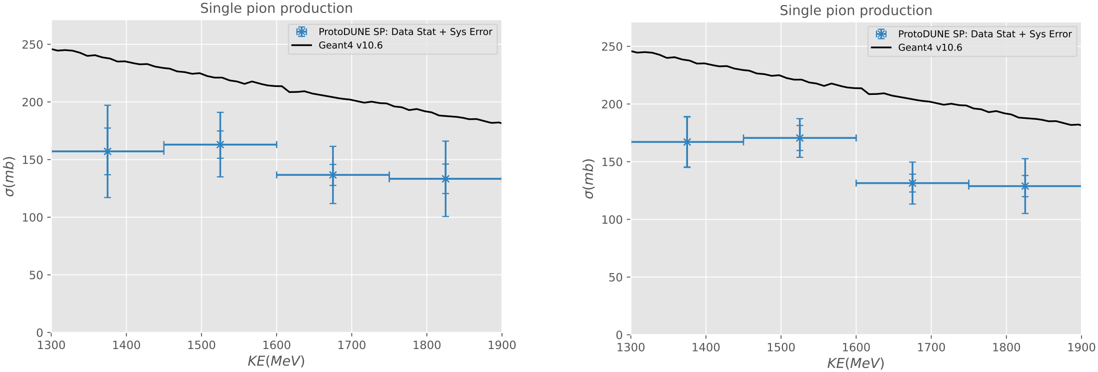


high stat



Cross section measurements

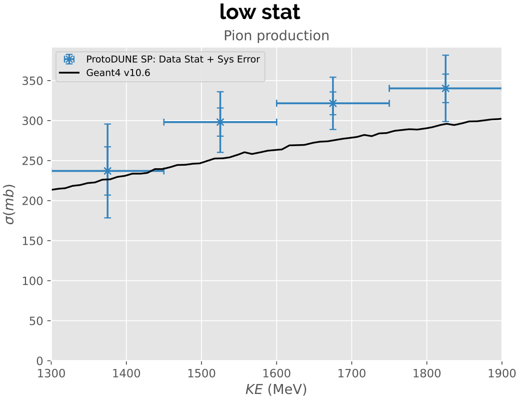
Single pion production

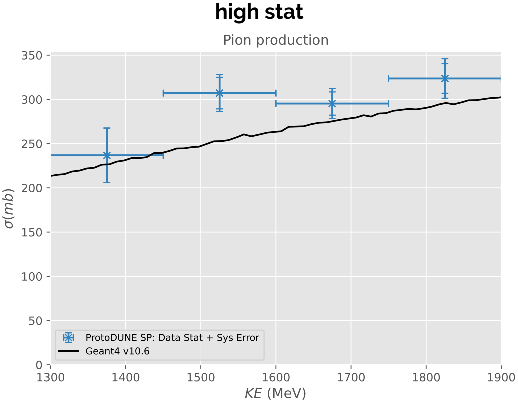

low stat

high stat

Cross section measurements

Pion production





Cross section measurements

absorption

low stat

KE (MeV)	1825	1675	1525	1375
Central value (mb)	25.51	33.78	26.51	72.22
Data stat (mb)	4.27	3.58	3.39	9.38
Track length (mb)	0.02	0.03	0.06	0.12
Beam momentum (mb)	0.19	0.12	0.23	0.34
Theory (mb)	4.42	4.67	5.28	6.93
MC stat (mb)	9.32	9.61	9.00	18.91
Fit inaccuracy (mb)	4.53	3.75	6.22	6.27
Upstream (mb)	9.19	1.23	3.35	5.92
Reweight (mb)	7.72	1.79	1.23	2.46
Shower energy (mb)	0.08	0.03	0.17	0.13
Total systematic uncertainty (mb)	16.46	11.53	12.66	22.05
Total uncertainty (mb)	17.00	12.08	13.11	23.96

high stat

KE (MeV)	1825	1675	1525	1375
Central value (mb)	17.69	27.35	27.54	73.36
Data stat (mb)	2.97	2.94	3.04	9.59
Track length (mb)	0.04	0.04	0.07	0.11
Beam momentum (mb)	0.06	0.14	0.20	0.30
Theory (mb)	4.71	4.04	5.55	6.69
MC stat (mb)	4.52	4.81	4.79	9.45
Fit inaccuracy (mb)	3.33	2.65	4.94	2.43
Upstream (mb)	6.17	0.13	3.65	6.23
Reweight (mb)	3.71	1.73	0.64	2.18
Shower energy (mb)	0.16	0.25	0.16	0.08
Total systematic uncertainty (mb)	10.28	7.05	9.59	13.56
Total uncertainty (mb)	10.70	7.64	10.06	16.60

Cross section measurements

charge_exchange

low stat

KE (MeV)	1825	1675	1525	1375
Central value (mb)	30.11	45.21	77.09	93.93
Data stat (mb)	2.91	2.51	4.65	11.46
Track length (mb)	0.03	0.03	0.06	0.10
Beam momentum (mb)	0.14	0.14	0.18	0.28
Theory (mb)	2.44	2.32	2.69	3.42
MC stat (mb)	7.42	6.76	10.76	22.26
Fit inaccuracy (mb)	4.34	3.71	6.02	6.15
Upstream (mb)	12.49	5.45	6.22	11.62
Reweight (mb)	3.96	1.40	0.44	3.28
Shower energy (mb)	1.12	1.46	0.74	0.91
Total systematic uncertainty (mb)	15.90	9.93	14.09	26.30
Total uncertainty (mb)	16.16	10.24	14.84	28.69

high stat

KE (MeV)	1825	1675	1525	1375
Central value (mb)	24.28	42.37	73.79	82.74
Data stat (mb)	2.12	2.22	4.20	10.44
Track length (mb)	0.03	0.04	0.05	0.10
Beam momentum (mb)	0.21	0.14	0.19	0.26
Theory (mb)	2.74	2.06	2.89	3.49
MC stat (mb)	3.72	3.63	4.93	9.63
Fit inaccuracy (mb)	3.04	2.36	4.69	2.50
Upstream (mb)	6.46	5.16	3.72	7.48
Reweight (mb)	2.98	1.43	0.36	1.48
Shower energy (mb)	0.95	1.14	0.49	0.28
Total systematic uncertainty (mb)	9.06	7.28	8.30	13.02
Total uncertainty (mb)	9.31	7.61	9.30	16.69

Cross section measurements

single_pion_production

low stat

KE (MeV)	1825	1675	1525	1375
Central value (mb)	133.32	136.68	163.00	157.10
Data stat (mb)	12.77	9.09	11.85	20.26
Track length (mb)	0.09	0.09	0.12	0.22
Beam momentum (mb)	0.43	0.36	0.50	0.77
Theory (mb)	5.80	5.07	6.30	8.69
MC stat (mb)	24.65	19.78	24.85	37.81
Fit inaccuracy (mb)	4.93	4.00	6.07	6.80
Upstream (mb)	6.22	12.52	9.42	6.21
Reweight (mb)	19.11	5.06	0.83	3.47
Shower energy (mb)	0.35	0.30	0.31	0.22
Total systematic uncertainty (mb)	32.71	24.81	28.00	40.04
Total uncertainty (mb)	35.11	26.43	30.41	44.87

high stat

KE (MeV)	1825	1675	1525	1375
Central value (mb)	128.87	131.46	170.56	167.09
Data stat (mb)	9.20	7.72	10.84	21.65
Track length (mb)	0.49	0.08	0.13	0.21
Beam momentum (mb)	0.43	0.26	0.42	0.60
Theory (mb)	6.14	4.32	6.46	8.41
MC stat (mb)	11.64	10.16	12.11	19.80
Fit inaccuracy (mb)	3.77	2.62	5.01	2.62
Upstream (mb)	16.29	13.56	8.10	3.54
Reweight (mb)	10.51	4.25	1.12	1.97
Shower energy (mb)	0.24	0.18	0.09	0.14
Total systematic uncertainty (mb)	23.74	18.19	16.74	22.06
Total uncertainty (mb)	25.46	19.76	19.95	30.91

Cross section measurements

pion_production

low stat

KE (MeV)	1825	1675	1525	1375
Central value (mb)	340.24	321.56	298.14	237.06
Data stat (mb)	17.82	14.22	17.62	30.14
Track length (mb)	0.11	0.08	0.13	0.26
Beam momentum (mb)	0.59	0.35	0.48	0.56
Theory (mb)	6.03	4.85	6.92	10.46
MC stat (mb)	38.82	30.48	35.75	56.71
Fit inaccuracy (mb)	5.40	4.01	6.19	8.23
Upstream (mb)	8.20	4.46	7.78	5.09
Reweight (mb)	8.79	8.86	3.01	3.99
Shower energy (mb)	0.73	0.71	0.10	1.07
Total systematic uncertainty (mb)	41.45	32.67	37.87	58.62
Total uncertainty (mb)	45.12	35.63	41.76	65.92

high stat

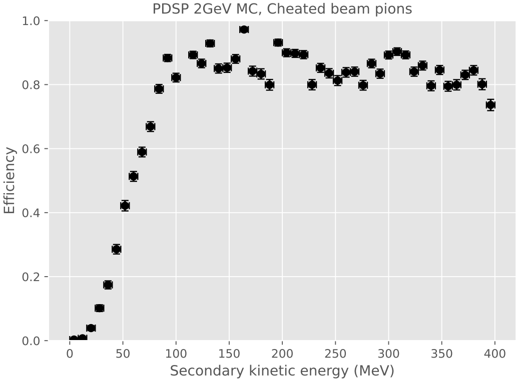
KE (MeV)	1825	1675	1525	1375
Central value (mb)	323.63	295.31	307.11	236.82
Data stat (mb)	16.70	13.13	17.87	30.82
Track length (mb)	0.12	0.10	0.16	0.23
Beam momentum (mb)	1.17	0.36	0.41	0.62
Theory (mb)	6.57	4.39	7.10	10.19
MC stat (mb)	18.16	14.65	18.17	28.26
Fit inaccuracy (mb)	4.38	3.01	5.56	4.89
Upstream (mb)	7.82	3.85	4.38	2.53
Reweight (mb)	6.54	5.48	1.23	3.36
Shower energy (mb)	1.12	1.03	0.28	0.91
Total systematic uncertainty (mb)	22.33	17.00	20.80	30.74
Total uncertainty (mb)	27.88	21.48	27.42	43.53

True $\pi^{\pm}$ KE limit

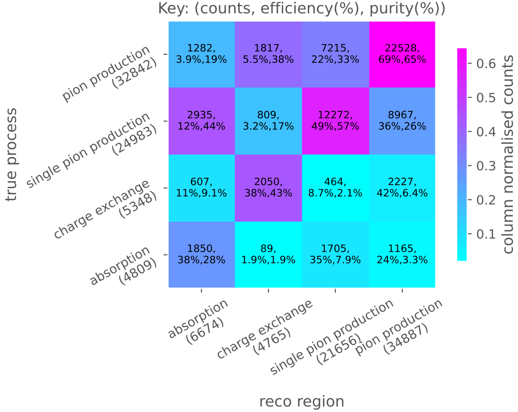
- Pandora struggles to reconstruct low energy $\pi^{\pm}$ in ProtoDUNE

$$\text{efficiency} = \frac{\text{number of true } \pi^{\pm} \text{ reconstructed}}{\text{number of true } \pi^{\pm}} \tag{2}$$

- study only includes secondary pions
- number of true $\pi^{\pm}$ calculated using truth information
- number of true $\pi^{\pm}$ reconstructed is number of reconstructed PFOs backtracked to a $\pi^{\pm}$
- Jake observed similar effect in 1GeV pions, selected true $\pi\pm$ only if starting $KE > 65$ MeV.

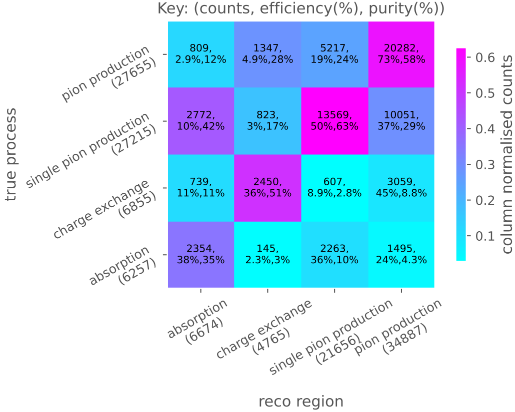


Without true $\pi^{\pm}$ KE limit



true process \ reco region	absorption (6674)	charge exchange (4765)	single pion production (21656)	pion production (34887)
pion production (32842)	1282, 3.9%,19%	1817, 5.5%,38%	7215, 22%,33%	22528, 69%,65%
single pion production (24983)	2935, 12%,44%	809, 3.2%,17%	12272, 49%,57%	8967, 36%,26%
charge exchange (5348)	607, 11%,9.1%	2050, 38%,43%	464, 8.7%,2.1%	2227, 42%,6.4%
absorption (4809)	1850, 38%,28%	89, 1.9%,1.9%	1705, 35%,7.9%	1165, 24%,3.3%

With true $\pi^{\pm}$ KE limit



true process \ reco region	absorption (6674)	charge exchange (4765)	single pion production (21656)	pion production (34887)
pion production (27655)	809, 2.9%,12%	1347, 4.9%,28%	5217, 19%,24%	20282, 73%,58%
single pion production (27215)	2772, 10%,42%	823, 3%,17%	13569, 50%,63%	10051, 37%,29%
charge exchange (6855)	739, 11%,11%	2450, 36%,51%	607, 8.9%,2.8%	3059, 45%,8.8%
absorption (6257)	2354, 38%,35%	145, 2.3%,3%	2263, 36%,10%	1495, 24%,4.3%

- pip background in each region has reduced a little.
- spip background not really affected in the abs region
- more true pip and spip events are classified as abs and cex instead, hence the increased statistics in these true processes

Without true $\pi^{\pm}$ KE limit

μ_{abs}	μ_{cex}	μ_{spip}	μ_{pip}
0.8 ± 0.1	1.0 ± 0.09	0.7 ± 0.05	1.21 ± 0.03

With true $\pi^{\pm}$ KE limit

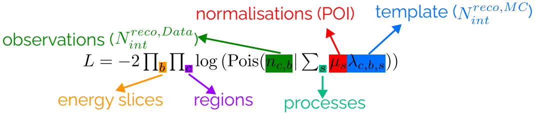
μ_{abs}	μ_{cex}	μ_{spip}	μ_{pip}
0.8 ± 0.1	1.02 ± 0.08	0.76 ± 0.04	1.22 ± 0.04

- spip fit value agrees better with MC, all other fit values are unchanged.
- discrepancy in MC and Data originates from multiple sources

True KE limit Summary

- accounting for low energy $\pi^{\pm}$ reconstruction efficiency when defining true processes slightly improves the appearance of the confusion matrix
- pip background reduces overall, but only slightly
- low energy $\pi^{\pm}$ reconstruction efficiency does not account for the spip background in the abs region
- must be other reconstruction inefficiencies unrelated to low energy $\pi^{\pm}$.

Pull study recap



normalisations (POI)

template ($N_{int}^{reco,MC}$)

observations ($N_{int}^{reco,Data}$)

$$L = -2\prod_{b}\prod_{c}\log\left(\text{Pois}\left(n_{c,b}\,|\,\sum_{s}\mu_s\lambda_{c,b,s}\right)\right)$$

energy slices

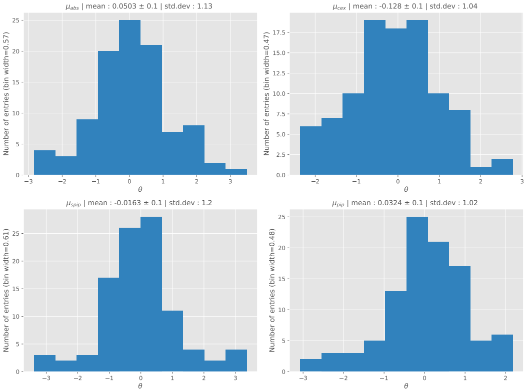
regions

processes

- original presentation
- Fit uses HistFactory likelihood to predict normalisation for each exclusive process in Data
- 4 exclusive cross sections, so four POIs μ_s for $s \in \{abs, cex, spip, pip\}$
- to validate model, pull study was performed

$$\theta_s = \frac{\mu_s^{fit} - \mu_s^{exp}}{\Delta\mu_s^{fit}} \tag{3}$$

- found a bias in the μ_{cex} pulls, pip distribution has slighly asymmetric tails



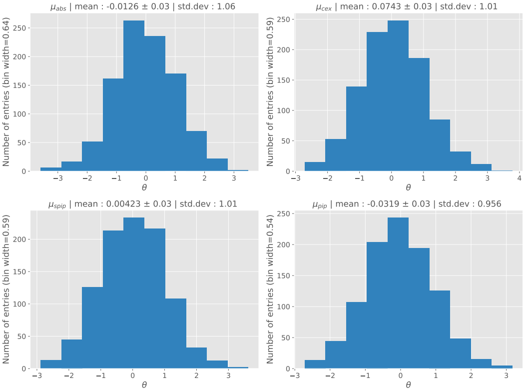
	abs	**cex**	**spip**	**pip**
mean	0.05 ± 0.1	-0.13 ± 0.1	-0.02 ± 0.1	0.032 ± 0.1
st.dev	1.13	1.04	1.2	1.02

Problem with method

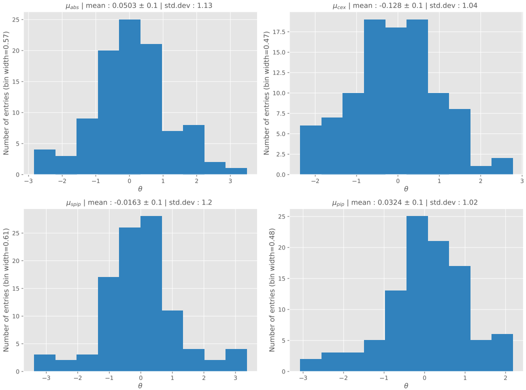
- Initial method of to calculate pulls
  1. Generate Toy template (10 million events)
  2. Generate 100 toy data samples (1 million events)
  3. run fit for each data samples, calculate pulls
- Issue with this method is only one template is used for all data samples, this introduces a slight bias in the model predictions because the template is sampled from large, but finite statistics.
- New method to evaluate pulls
  1. Generate 1 toy template (5 million events) and 1 toy data sample (1 million events)
  2. run fit, calculate pulls
  3. repeat 1000 times
- number of experiments increased to reduce statistical fluctuation.

Comparisons

New



Old



	abs	cex	spip	pip
mean	-0.013 ± 0.03	0.074 ± 0.03	0.0042 ± 0.03	-0.032 ± 0.03
st.dev	1.06	1.01	1.01	0.956

	abs	cex	spip	pip
mean	0.05 ± 0.1	-0.13 ± 0.1	-0.02 ± 0.1	0.032 ± 0.1
st.dev	1.13	1.04	1.2	1.02

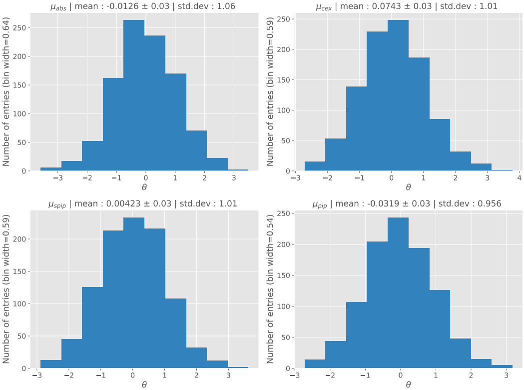
- mean is close to zero in each case, slight bias observations in cex and spip
- Uncertainty is 68% or 1σ, so in each case, the bias is small, and likely due to statistical fluctuation.
- standard deviation of each gaussian is more close to 1
- pull distribution have more gaussian shapes

Disussion

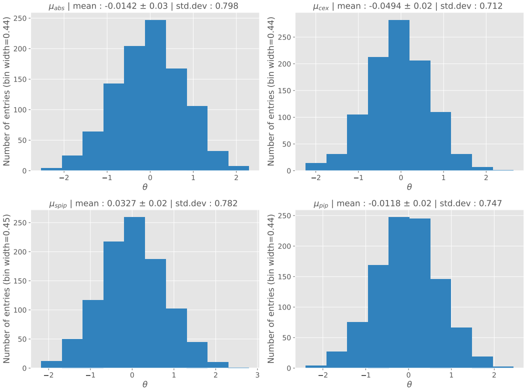
- with new pull method, biases are much smaller, but cex still has some small bias
- Any bias is accounted for by propagating the uncertainty in the model prediction as a systematic.
- higher statistics could be ran, but is much more time consuming
- Note that the model fitted does not include nuisance parameters (NPs), as NPs naturally skew the fit results and uncertainties to account for other systematic effects.
- only NPs used quantify MC stat Uncertainty in the template
- this will increase the fit errors → results in more narrow pull distributions (st.dev decreases)

Comparisons

With NPs



With NPs



	abs	**cex**	**spip**	**pip**
mean	-0.013 ± 0.03	0.074 ± 0.03	0.0042 ± 0.03	-0.032 ± 0.03
st.dev	1.06	1.01	1.01	0.956

	abs	**cex**	**spip**	**pip**
mean	-0.014 ± 0.03	-0.049 ± 0.02	0.033 ± 0.02	-0.012 ± 0.02
st.dev	0.798	0.712	0.782	0.747

- as expected, st.dev for pulls with NPs are smaller, as uncertainty is now fit + MC stat
- means are not impacted in a significant way, biases only decrease slightly.

Backup

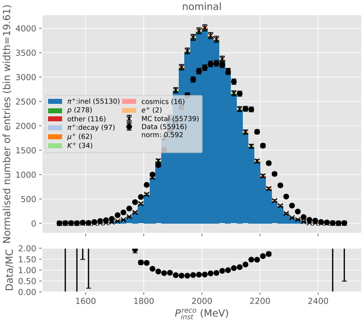
Beam reweight systematic

- Data MC discrepancy in beam profile treated by **weighting MC**
- sideband is beam particle selection, except preselection is inverted so sideband contains events with no secondary PFOs.
- fit to ratio in a sideband sample is done to derive weights for each event

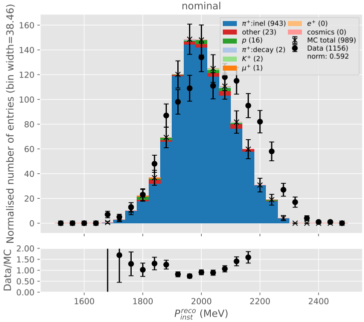
$$r_i = \frac{N_i^{Data}}{N_i^{MC}} \frac{\sum_j N_j^{MC}}{\sum_j N_j^{Data}} \qquad (4)$$

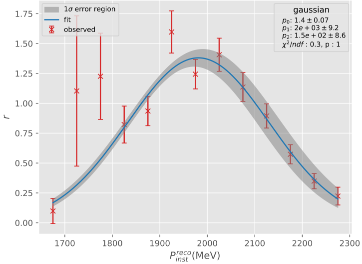
$$w = \frac{1}{r(P_{inst}^{reco}, \{p_i\})}, \qquad (5)$$

Selected beam pions



Sideband





Plots

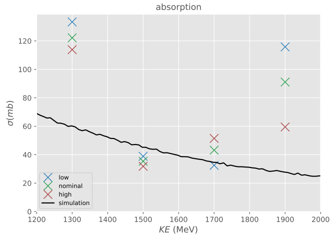
- plots show the nominal central value measurements and the measurements for $p_i \pm \epsilon_{p_i}$
- uncertainty is difference in from the nominal measurement:

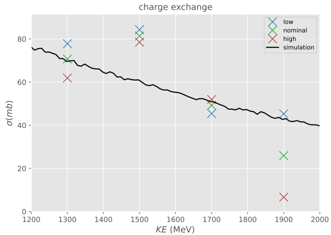
$$\epsilon^{\pm} = xs^{nominal} - xs^{\pm}$$

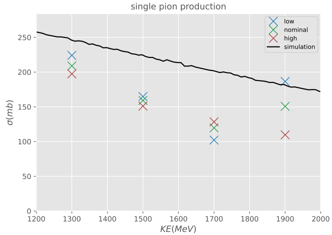
- for a single bin, asymmetric uncertainties are defined as:

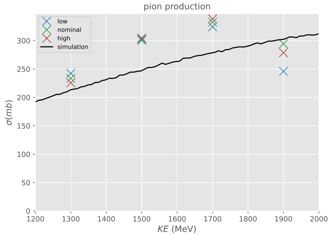
$$\epsilon^{low} \text{ if } \epsilon^{\pm} \geq 0, \;\; \epsilon^{high} \text{ if } \epsilon^{\pm} < 0$$

if both $\epsilon^{\pm} \geq 0$ or $\epsilon^{\pm} < 0$, then take the largest of the two.


absorption


charge exchange


single pion production


pion production

Tables

absorption

KE (MeV)	Total	Data stat	Reweight low	Reweight high
1300	0.23	0.20	0.07	0.09
1500	0.25	0.21	0.10	0.10
1700	0.38	0.21	0.25	0.19
1900	0.60	0.40	0.35	0.27
average	0.37	0.26	0.19	0.16

charge exchange

KE (MeV)	Total	Data stat	Reweight low	Reweight high
1300	0.29	0.24	0.12	0.10
1500	0.11	0.10	0.04	0.04
1700	0.16	0.12	0.08	0.05
1900	1.41	0.94	0.74	0.75
average	0.49	0.35	0.24	0.23

single pion production

KE (MeV)	Total	Data stat	Reweight low	Reweight high
1300	0.20	0.18	0.06	0.07
1500	0.10	0.08	0.05	0.04
1700	0.19	0.09	0.15	0.07
1900	0.44	0.25	0.27	0.24
average	0.23	0.15	0.13	0.10

pion production

KE (MeV)	Total	Data stat	Reweight low	Reweight high
1300	0.18	0.17	0.03	0.04
1500	0.05	0.05	0.00	0.01
1700	0.05	0.04	0.02	0.02
1900	0.22	0.14	0.16	0.00
average	0.12	0.10	0.06	0.02

$$\text{fractional error} = \frac{\epsilon}{xs^{nominal}} \tag{6}$$

- reweighting is the highest in the high 1900 MeV bin, except for pion production.
- reweighting systematic is largest in the charge exchange measurement.

Bckground subtraction

- $c \to$ region, $s \to$ process $b \to$ energy slice
- background subtraction in a region c is

$$N_{c,b} = N^{Data}_{c,b} - \sum_{a} \nu_{c,s} S_{b,s};\ \text{for } c \neq s \tag{7}$$

- $\nu_{c,s}$ is the total number of background counts estimated from the fit
- S_s is the shape of the background, determined using MC:

$$S_{b,s} = \frac{\sum_c N^{MC}_{c,b,s}}{\sum_{c,b} N^{MC}_{c,b,s}} \tag{8}$$

- shape is subject to the nuclear model uncertainty (20%) of the Geant4 cross selections
- propagate this uncertainty through the background subtraction

$$(\Delta N_{c',b})^2 = N^{Data}_{c',b} + \sum_{s'} \left[\left(S_{b,s'}\right)^2 (\Delta \nu_{c',s'})^2 + \frac{(\nu_{c',s'})^2}{\sum_{c,b} N^{MC}_{c,b,s'}} S_{b,s'} (1 + S_{b,s'}) + f^2 S^2_{b,s'} \right] \tag{9}$$

nbvc

- $f = 0.2$

MC stat uncertainty

- MC stat uncertainty accounts for limited MC statistics used to define templates for the fit, and response matrices for the unfolding.
- MC stat uncertainty in the fit can be accounted for using nuisance parameters α_{cb}:

$$L = -2\prod_b \prod_c \left[\log(\mathrm{Pois}(n_{c,b}|\sum_s \mu_s \alpha_{cb} \lambda_{c,b,s})) + \mathrm{Gaus}(\alpha_{c,b}|\gamma_{cb}\delta_{c,b}) \right]$$

  - $\gamma_{c,b} = \sum_s \lambda_{c,b,s} \rightarrow$ total counts in region c, slice b
  - $\delta_{c,b} = \sqrt{\gamma_{c,b}} \rightarrow$ stat uncertainty
  - currently, *pyhf* API only supports using Gaussian constraints for MC stat NPs.
- For unfolding, uncertainty is quantified by a covariance matrix:

$$V = V_{Data} + V_{MC}$$

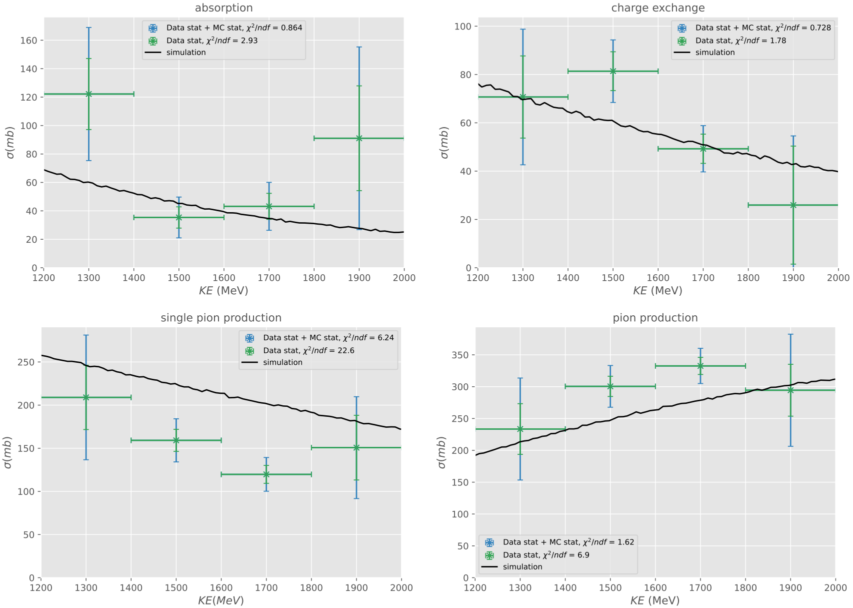
  - $V_{Data} \rightarrow$ covariance of the unfolded distribution (accounts for prior uncertainties, uncertainty for multiple unfolding iterations)
  - $V_{MC} \rightarrow$ covariance of the migration probability i.e. covariance of response matrix. Expressed as the poisson covariance in the *pyunfold* API
  - diagonal component of V is the uncertainty in the unfolded histogram.

How to evaluate the MC stat uncertainty?

- run analysis using fit model with NPs fixed in the fit, and without calculating V_{MC}
- uncertainties in xs will be purely due to Data stat uncertainty.
- run analysis with floating mc stat NPs + V_{MC}, uncertainties in xs are Data + MC stat.

$$\epsilon^2_{\text{MC stat}} = \epsilon^2_{\text{Data stat + MC stat}} - \epsilon^2_{\text{Data stat}} \tag{10}$$

Plots


absorption
Data stat + MC stat, χ^2/ndf = 0.864
Data stat, χ^2/ndf = 2.93
simulation
$\sigma(mb)$
0
20
40
60
80
100
120
140
160
1200
1300
1400
1500
1600
1700
1800
1900
2000
KE (MeV)
charge exchange
Data stat + MC stat, χ^2/ndf = 0.728
Data stat, χ^2/ndf = 1.78
simulation
$\sigma(mb)$
0
20
40
60
80
100
1200
1300
1400
1500
1600
1700
1800
1900
2000
KE (MeV)
single pion production
Data stat + MC stat, χ^2/ndf = 6.24
Data stat, χ^2/ndf = 22.6
simulation
$\sigma(mb)$
0
50
100
150
200
250
1200
1300
1400
1500
1600
1700
1800
1900
2000
$KE(MeV)$
pion production
Data stat + MC stat, χ^2/ndf = 1.62
Data stat, χ^2/ndf = 6.9
simulation
$\sigma(mb)$
0
50
100
150
200
250
300
350
1200
1300
1400
1500
1600
1700
1800
1900
2000
KE (MeV)

Tables

absorption

KE (MeV)	Total	Data stat	MC stat
1300	0.38	0.20	0.32
1500	0.41	0.21	0.35
1700	0.39	0.21	0.33
1900	0.71	0.40	0.58
average	0.47	0.26	0.39

charge exchange

KE (MeV)	Total	Data stat	MC stat
1300	0.40	0.24	0.32
1500	0.16	0.10	0.12
1700	0.19	0.12	0.15
1900	1.10	0.94	0.58
average	0.46	0.35	0.29

single pion production

KE (MeV)	Total	Data stat	MC stat
1300	0.35	0.18	0.30
1500	0.16	0.08	0.14
1700	0.16	0.09	0.14
1900	0.39	0.25	0.30
average	0.26	0.15	0.22

pion production

KE (MeV)	Total	Data stat	MC stat
1300	0.34	0.17	0.30
1500	0.11	0.05	0.10
1700	0.08	0.04	0.07
1900	0.30	0.14	0.27
average	0.21	0.10	0.18

- tables show fractional error in the cross section for Data stat and MC stat.

$$\text{fractional error} = \frac{\epsilon}{xs^{nominal}} \tag{11}$$

- for cex, Data stat uncertainty is larger, for abs, spip and pip, MC stat is larger

Upstream correction systematic

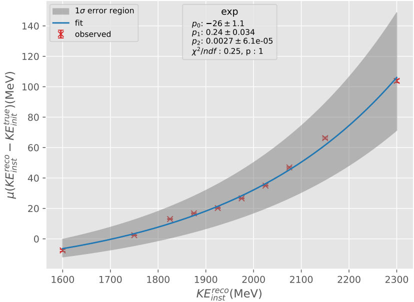
- fit to upstream loss in MC in bins of reco KE inst

$$\Delta E^{MC}_{upstream} = KE^{MC,reco}_{inst} - KE^{MC,true}_{ff} \qquad (12)$$

- from the central values, fit a 2nd order polynomial to obtain an energy dependant upstream loss correction which is applied to **both Data and MC**

$$\Delta E_{upstream} \rightarrow \Delta E_{upstream}(KE^{reco}_{inst}, \{p_i\}) \qquad (13)$$

- to evaluate systematic, shift p_i by $\pm\epsilon_{p_i}$ uncertainties, re-run the analysis, obtain two measurements of the cross section $xs^{\pm}$



p_0	p_1	p_2
97 ± 33	-0.21 ± 0.03	$(8.9 \pm 0.9) \times 10^{-5}$

Plots

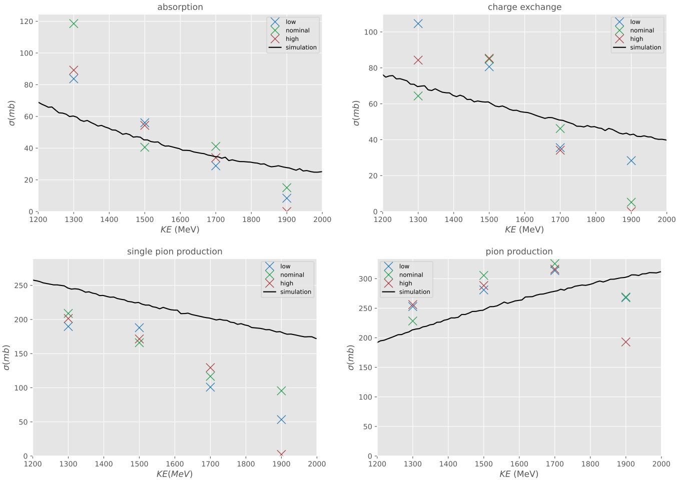
- plots show the nominal central value measurements and the measurements for $p_i \pm \epsilon_{p_i}$
- uncertainty is difference in from the nominal measurement:

$$\epsilon^{\pm} = xs^{nominal} - xs^{\pm}$$

- for a single bin, asymmetric uncertainties are defined as:

$$\epsilon^{low} \text{ if } \epsilon^{\pm} \geq 0, \ \epsilon^{high} \text{ if } \epsilon^{\pm} < 0$$

if both $\epsilon^{\pm} \geq 0$ or $\epsilon^{\pm} < 0$, then take the largest of the two.



Tables

absorption

KE (MeV)	Total	Data stat	Upstream low	Upstream high
1300	0.37	0.22	0.29	-
1500	0.42	0.18	-	0.38
1700	0.36	0.21	0.30	-
1900	1.91	1.63	1.00	-
average	0.77	0.56	0.40	0.10

charge exchange

KE (MeV)	Total	Data stat	Upstream low	Upstream high
1300	0.68	0.27	-	0.63
1500	0.12	0.11	0.05	0.01
1700	0.30	0.14	0.26	-
1900	5.23	2.50	1.00	4.48
average	1.58	0.75	0.33	1.28

single pion production

KE (MeV)	Total	Data stat	Upstream low	Upstream high
1300	0.22	0.20	0.09	-
1500	0.16	0.09	-	0.13
1700	0.20	0.11	0.14	0.11
1900	1.05	0.38	0.98	-
average	0.41	0.20	0.30	0.06

pion production

KE (MeV)	Total	Data stat	Upstream low	Upstream high
1300	0.23	0.20	-	0.12
1500	0.10	0.07	0.08	-
1700	0.06	0.05	0.04	-
1900	0.33	0.16	0.28	-
average	0.18	0.12	0.10	0.03

$$\text{fractional error} = \frac{\epsilon}{xs^{nominal}} \tag{14}$$

- dashed points are when $\epsilon^{\pm} \geq 0$ or $\epsilon^{\pm} < 0$
- uncertainty in the upstream energy loss is largest in the 1900 MeV bin for all measurements
- upstream energy correction is largest for the abs and cex measurement.

Nuisance parameters

- fit model can facilitate nuisance parameters to quantify different systematic effects.
- if a systematic can be expressed as a fractional error on the number of events, it can be incorporated into the model.
- if we allow more than 1 KE bin in the fit model, systematics which can be expressed as a fractional error in the interacting KE
- **benefit:**
    1. implementation is simple, adjust model, re-run fit and subsequent steps
    2. adding nuisance parameters can help the model mitigate the effects on the fit results
- **disadvantages:**
    1. higher number of NPs results in fit being underconstrained, resulting in more unstable fitting.
    2. must rerun pull study and normalisation cross checks
- Current model has takes 4 observations, and has 8 free parameters (4 POIs, 4 NPs), so model is already underconstrained.
- MC stat uncertainty is currently being incorporated using NPs

Repeat data analysis

- analysis performs various corrections by using values extracted from fits
- systematic effect on a specific correction can be determined by changing values used to calculate the correction
- example:
  - upstream energy correction is determined from fit values p_i.
  - fit values have some uncertainty determined by the fit: $p_i \pm \epsilon_i$
  - determine uncertainty in upstream energy correction by re-running analysis for $p_i + \epsilon_i$, and $p_i - \epsilon_i$, obtain σ^{high} and σ^{low}
  - systematic is $(\sigma^{high} - \sigma^{low})/2$
- **benefits:**
  1. no additional changes required to analysis or fit model
  2. does not require rerunning toy studies if the fit results don't impact the region identification
- **disadvantages:**
  1. evaluation of systematics is very simple, does not account for correlations between other effects
  2. magnitude of systematic may to be compatible to the measurement i.e. can't be expressed as a fractional error

MC method

- systematic is evaluated by running multiple pseudo experiments using the toy estimating the effect some systematic has on the measured cross sections. Uncertainties are then expressed as fractional errors and applied to the Data MC measurement.
- **benefits:**
  1. Data MC analysis does not need to be re-run at all
  2. multiple systematic effects can be varied simultaneously (handles correlations between effects)
- **disadvantages:**
  1. depending on the number of pseudo experiments, method may be time consuming
  2. not all systematics can be incorporated e.g. upstream energy correction, beam momentum resolution, selection is not incorporated into the toy.

Theory uncertainty

- cross section model uncertainty is ±20%, fit tries to determine normalisation in Data.
- using toys, vary true cross section by ±20%, keep template fixed and re-run analysis.
- uncertainty in normalisation systematic is $\epsilon = \sigma^{meas} - \sigma^{true}$.
- repeat experiment multiple times, obtain average $\bar{\epsilon}$
- convert $\bar{\epsilon}$ to fractional error, apply to data measurements

Background subtraction uncertainty

- background subtraction uses background shapes from MC, thus, also propagate ±20% theory uncertainty through the background subtraction
- For a region c' the background samples are when $c' \neq s'$.
- fit predicts the estimated **counts** of each process in each region $\nu_{c',s'} \pm \Delta\nu_{c',s'}$
- subtract background from N_{int} to get $N_{int,c'}$ **in each energy slice** i.e. we require **shapes** of $N_{int,s'}$
- shape of N_{int} for each process is determined from MC $S_{b,s'} = \frac{\sum_c N^{MC}_{c,b,s'}}{\sum_{c,b} N^{MC}_{c,b,s'}}$
- background subtracted interacting counts in each region is $N_{c',b} \pm \Delta N_{c',b}$ (includes Data stat + MC stat uncertainty):

$$N_{c',b} = N^{Data}_{c',b} - \sum_{s'} \nu_{c',s'} = N^{Data}_{c',b} - \sum_{s'} \nu_{c',s'} S_{b,s'} \tag{15}$$

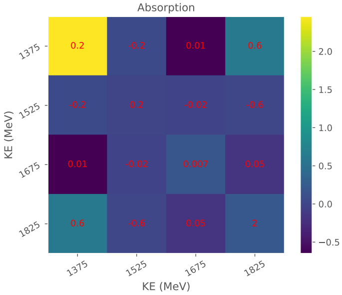
$$(\Delta N_{c',b})^2 = N^{Data}_{c',b} + \sum_{s'} \left[\left(S_{b,s'}\right)^2 (\Delta\nu_{c',s'})^2 + \frac{(\nu_{c',s'})^2}{\sum_{c,b} N^{MC}_{c,b,s'}} S_{b,s'}(1 + S_{b,s'}) + f^2 S^2_{b,s'} \right] \tag{16}$$

- $f = 0.2$
- not really a systematic, need to propagate

Beam momentum systematic

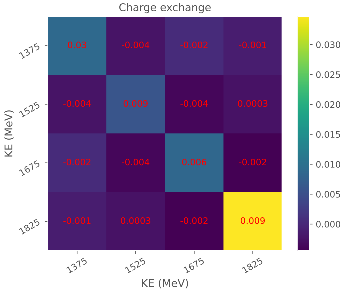
- Beam momentum resolution of 2.5% is on a per particle basis
- Instead of shifting all events by $\pm 2.5\%$, smear the beam momenta by some value *x*.
- *x* is sampled from a gaussian distribution with $\mu = 0$ and $\sigma = 2.5\%$.
- Method to evaluate systematic:
    1. generate toy data sample with high stats
    2. generate toy MC sample with high stats
    3. generate smearing values *x*, apply smearing to toy MC P_{inst}
    4. run analysis, calculate cross section
    5. repeat steps 3-4 *N* times
    6. compute covariance matrices
- Systematic uncertainty is square root of the diagonal elements of the covariance matrices.
- *N* is the number of “experiments”

Covariance matrices



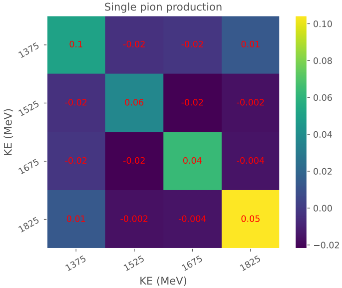
Absorption

KE (MeV)	1375	1525	1675	1825
1375	0.2	-0.2	0.01	0.6
1525	-0.2	0.2	-0.02	-0.6
1675	0.01	-0.02	0.007	0.05
1825	0.6	-0.6	0.05	2



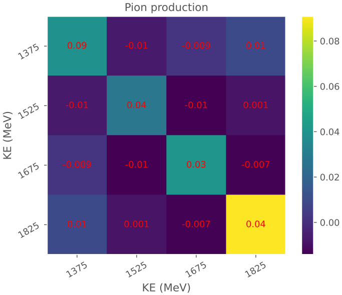
Charge exchange

KE (MeV)	1375	1525	1675	1825
1375	0.03	-0.004	-0.002	-0.001
1525	-0.004	0.009	-0.004	0.0003
1675	-0.002	-0.004	0.006	-0.002
1825	-0.001	0.0003	-0.002	0.009



Single pion production

KE (MeV)	1375	1525	1675	1825
1375	0.1	-0.02	-0.02	0.01
1525	-0.02	0.06	-0.02	-0.002
1675	-0.02	-0.02	0.04	-0.004
1825	0.01	-0.002	-0.004	0.05



Pion production

KE (MeV)	1375	1525	1675	1825
1375	0.09	-0.01	-0.009	0.01
1525	-0.01	0.04	-0.01	0.001
1675	-0.009	-0.01	0.03	-0.007
1825	0.01	0.001	-0.007	0.04

- Colour bar is in units of mb^2.

Tables

absorption

KE (MeV)	Total	Data stat	Beam momentum
1375	0.171	0.170	0.005
1525	0.223	0.223	0.012
1675	0.217	0.217	0.002
1825	1.091	1.081	0.149
average	0.426	0.423	0.042

single pion production

KE (MeV)	Total	Data stat	Beam momentum
1375	0.116	0.116	0.002
1525	0.073	0.073	0.002
1675	0.069	0.069	0.001
1825	0.157	0.157	0.003
average	0.104	0.104	0.002

charge exchange

KE (MeV)	Total	Data stat	Beam momentum
1375	0.156	0.156	0.002
1525	0.109	0.109	0.001
1675	0.119	0.119	0.001
1825	0.339	0.339	0.005
average	0.181	0.181	0.002

pion production

KE (MeV)	Total	Data stat	Beam momentum
1375	0.107	0.107	0.001
1525	0.051	0.051	0.001
1675	0.039	0.039	0.001
1825	0.051	0.051	0.001
average	0.062	0.062	0.001

$$\text{fractional error} = \frac{\epsilon}{xs^{nominal}} \tag{17}$$

- Compared to prior method, systematic on track length resolution is negligible, excluding the 1825 MeV absorption bin.
- absorption events are rare, especially at higher energies, so the number of events in this bin are quite small.

Old Tables

absorption

KE (MeV)	Total	Data stat	Beam momentum low	Beam momentum high
1375	0.46	0.17	0.40	0.16
1525	0.23	0.22	0.00	0.06
1675	0.29	0.22	0.18	0.05
1825	4.28	1.08	0.99	4.03
average	1.32	0.42	0.39	1.07

charge exchange

KE (MeV)	Total	Data stat	Beam momentum low	Beam momentum high
1375	0.25	0.16	0.20	0.00
1525	0.19	0.11	0.16	0.02
1675	0.15	0.12	0.08	0.00
1825	1.54	0.34	0.94	1.17
average	0.53	0.18	0.35	0.30

single pion production

KE (MeV)	Total	Data stat	Beam momentum low	Beam momentum high
1375	0.13	0.12	0.03	0.06
1525	0.27	0.07	0.26	0.00
1675	0.21	0.07	0.20	0.00
1825	1.09	0.16	0.93	0.55
average	0.43	0.10	0.36	0.15

pion production

KE (MeV)	Total	Data stat	Beam momentum low	Beam momentum high
1375	0.11	0.11	0.04	0.00
1525	0.13	0.05	0.09	0.08
1675	0.14	0.04	0.13	0.06
1825	0.10	0.05	0.07	0.03
average	0.12	0.06	0.08	0.04

$$\text{fractional error} = \frac{\epsilon}{xs^{nominal}} \tag{18}$$

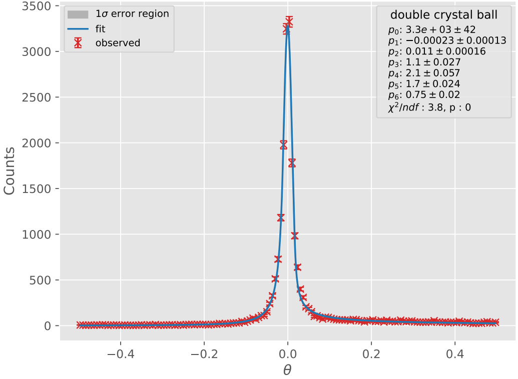
- Beam momentum uncertainty has largest impact in the highest energy bins, except for pip
- for abs and cex, uncertainty in lowest energy bins is also the largest

Track length

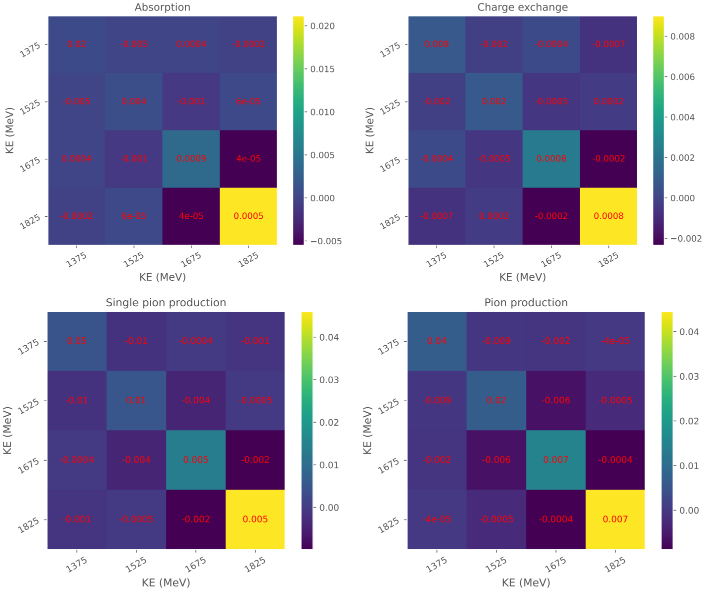
- track length of the beam particle l^{reco} is used to determine KE^{reco}_{int}
- resolution of track length determined using MC
- for each beam particle which passes the beam particle selection, calculate:

$$\theta = \frac{l^{reco} - l^{true}}{l^{reco}} \tag{19}$$

- Double crystal ball function fitted to θ distribution, FWHM is taken to be the beam resolution
- calculated to be 0.026 (2.6%)
- evaluate track length systematic the same way as beam resolution, this time generate smearing for track length.



Covariance matrices



- Colour bar is in units of mb^2.

Tables

absorption

KE (MeV)	Total	Data stat	Track length
1375	0.170	0.170	0.002
1525	0.223	0.223	0.002
1675	0.217	0.217	0.001
1825	1.081	1.081	0.002
average	0.423	0.423	0.002

charge exchange

KE (MeV)	Total	Data stat	Track length
1375	0.156	0.156	0.001
1525	0.109	0.109	0.001
1675	0.119	0.119	0.001
1825	0.339	0.339	0.001
average	0.181	0.181	0.001

single pion production

KE (MeV)	Total	Data stat	Track length
1375	0.116	0.116	0.001
1525	0.073	0.073	0.001
1675	0.069	0.069	0.001
1825	0.157	0.157	0.001
average	0.104	0.104	0.001

pion production

KE (MeV)	Total	Data stat	Track length
1375	0.107	0.107	0.001
1525	0.051	0.051	0.000
1675	0.039	0.039	0.000
1825	0.051	0.051	0.000
average	0.062	0.062	0.000

$$\text{fractional error} = \frac{\epsilon}{xs^{nominal}} \tag{20}$$

- Compared to prior method, systematic on track length resolution is negligible.

Old Tables

absorption

KE (MeV)	Total	Data stat	Track length low	Track length high
1375	0.17	0.17	0.01	0.03
1525	0.24	0.22	0.07	0.06
1675	0.25	0.22	0.13	0.00
1825	1.16	1.08	0.00	0.41
average	0.46	0.42	0.05	0.13

charge exchange

KE (MeV)	Total	Data stat	Track length low	Track length high
1375	0.19	0.16	0.11	0.00
1525	0.13	0.11	0.03	0.06
1675	0.18	0.12	0.13	0.00
1825	0.48	0.34	0.33	0.00
average	0.24	0.18	0.15	0.01

single pion production

KE (MeV)	Total	Data stat	Track length low	Track length high
1375	0.16	0.12	0.00	0.10
1525	0.10	0.07	0.04	0.05
1675	0.08	0.07	0.04	0.01
1825	0.16	0.16	0.01	0.01
average	0.12	0.10	0.02	0.05

pion production

KE (MeV)	Total	Data stat	Track length low	Track length high
1375	0.12	0.11	0.00	0.04
1525	0.08	0.05	0.04	0.04
1675	0.08	0.04	0.07	0.00
1825	0.07	0.05	0.05	0.00
average	0.09	0.06	0.04	0.02

$$\text{fractional error} = \frac{\epsilon}{xs^{nominal}} \tag{21}$$

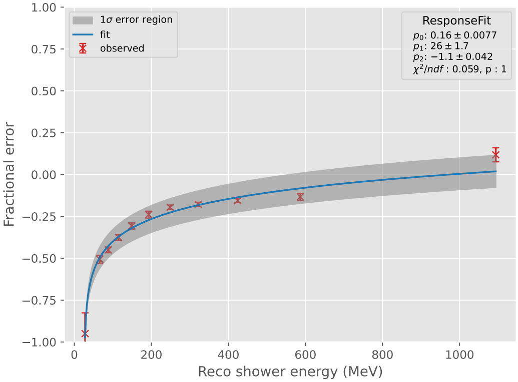
- Compared to P_{inst}^{reco} resolution, l^{reco} systematic is less significant.
- P_{inst}^{reco} is used for reweighting MC, KE_{init} and KE_{int}, so affects much more of the analysis vs l^{reco}.

Shower energy correction

- shower energy is corrected improve π^0 mass reconstruction
- mass used in π^0 selection
- correction is:

$$C(E_{shower}) = p_0 \ln(E_{shower} - p_1) + p_2 \qquad (22)$$

- parameters $p_i, i \in \{0, 1, 2\}$ obtained from fit, and have uncertainties
- vary p_i by $\pm 1\sigma$, re-run analysis with PDSP Data/MC



p_0	p_1	p_2
0.1566 ± 0.008	26 ± 2	-1.073 ± 0.04

Plots

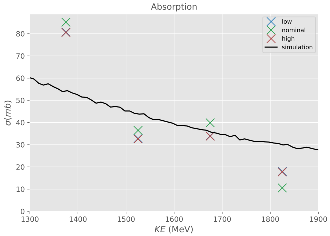
- plots show the nominal central value measurements and the measurements for $p_i \pm \epsilon_{p_i}$
- uncertainty is difference in from the nominal measurement:

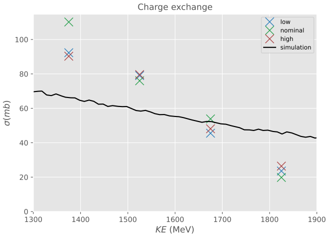
$$\epsilon^{\pm} = xs^{nominal} - xs^{\pm}$$

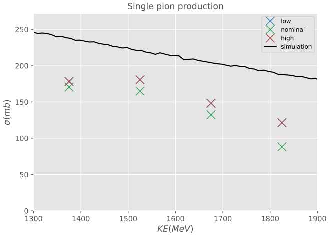
- for a single bin, asymmetric uncertainties are defined as:

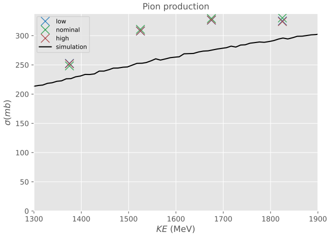
$$\epsilon^{low} \text{ if } \epsilon^{\pm} \geq 0, \ \epsilon^{high} \text{ if } \epsilon^{\pm} < 0$$

if both $\epsilon^{\pm} \geq 0$ or $\epsilon^{\pm} < 0$, then take the largest of the two.









Tables

absorption

KE (MeV)	Total	Data stat	Shower energy low	Shower energy high
1375	0.18	0.17	0.05	0.00
1525	0.25	0.22	0.11	0.00
1675	0.26	0.22	0.15	0.00
1825	1.29	1.08	0.00	0.70
average	0.50	0.42	0.08	0.18

charge exchange

KE (MeV)	Total	Data stat	Shower energy low	Shower energy high
1375	0.24	0.16	0.18	0.00
1525	0.12	0.11	0.00	0.05
1675	0.20	0.12	0.15	0.00
1825	0.48	0.34	0.00	0.33
average	0.26	0.18	0.08	0.10

single pion production

KE (MeV)	Total	Data stat	Shower energy low	Shower energy high
1375	0.12	0.12	0.00	0.05
1525	0.12	0.07	0.00	0.10
1675	0.14	0.07	0.00	0.12
1825	0.41	0.16	0.00	0.38
average	0.20	0.10	0.00	0.16

pion production

KE (MeV)	Total	Data stat	Shower energy low	Shower energy high
1375	0.11	0.11	0.00	0.02
1525	0.05	0.05	0.01	0.00
1675	0.04	0.04	0.01	0.00
1825	0.05	0.05	0.02	0.00
average	0.06	0.06	0.01	0.00

$$\text{fractional error} = \frac{\epsilon}{xs^{nominal}} \tag{23}$$

-

Theory uncertainty

- MC is used for background subtraction and unfolding
- background subtraction and unfolding use number of interactions in energy slices, N_{int}
- N_{int} distribution will depend on Geant4 cross section as a function of KE, which have a 20% theory uncertainty.
- to propagate theory uncertainty use toy MC method:
    1. Generate toy data
    2. Generate toy MC, smear N_{int} distribution by $\pm 20\%$
    3. run analysis, calculate cross sections
    4. repeat step 2 and 3 N times
    5. Calculate covariance of the central values, propagated theory uncertainties are the square root of the diagonal elements
    6. apply to measurement with PDSP Data
- this systematic does not affect the fit, as the total number of events doesn't change, instead systematic due to model inaccuracy should be evaluated (method already discussed here)