# Coupling Simulation

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## 1 Coupling Coefficients Matix Construction and Recovery

We consider the following coupling modeling,

$$\boldsymbol{V}' = (\boldsymbol{I} + \boldsymbol{\Xi})\boldsymbol{V}(\boldsymbol{I} + \boldsymbol{\Xi}^\dagger), \tag{1}$$

where $\boldsymbol{V}'$ is the visibility matrix with coupling, $\boldsymbol{V}$ is the visibility matrix without coupling, and $\boldsymbol{\Xi}$ is the coupling coefficients matrix.

We can solve the coupling matrix by Eigen-decomposition of both $\boldsymbol{V}' = \boldsymbol{U}'\boldsymbol{\Lambda}'\boldsymbol{U}'^\dagger$ and $\boldsymbol{V} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^\dagger$, and get

$$\boldsymbol{I} + \boldsymbol{\Xi} = (\boldsymbol{U}'\boldsymbol{\Lambda}'^{\frac{1}{2}}\boldsymbol{K})(\boldsymbol{U}\boldsymbol{\Lambda}^{\frac{1}{2}})^{-1}, \tag{2}$$

where $\boldsymbol{K}$ is an un-determined arbitrary unitary matrix due to degeneracy.

After we have solved the coupling matrix, the coupling-corrected visibility can be obtained as

$$\tilde{\boldsymbol{V}} = (\boldsymbol{I} + \boldsymbol{\Xi})^{-1}\boldsymbol{V}'(\boldsymbol{I} + \boldsymbol{\Xi}^\dagger)^{-1}. \tag{3}$$

Here we want to study how the arbitrary unitary matrix $\boldsymbol{K}$ affacts the recovery of the structure of the coupling coefficients matrix $\boldsymbol{\Xi}$.

We first construct a coupling coefficients matrix $\boldsymbol{\Xi}$ with the following parameterised form:

$$\epsilon_{ij}(\nu) = A_{ij}e^{2\pi i\tau_{ij}\nu + i\phi_{ij}}$$

with parameters $A_{ij}$, $\tau_{ij}$ and $\phi_{ij}$. We set $A_{ij} = 0.1e^{\frac{(i-j)^2}{96}}$, and sampling the phase of $\epsilon_{ij}$ from standard normal distribution. The coupling coefficients matrix $\boldsymbol{\Xi}$ constructed in this way is shown in Figure 1.

We use the real observed calibrated and nighttime mean subtracted visibility matrix $\boldsymbol{V}$ as an approximation of the visibility matrix without couplings, the matrix is shown in Figure 2. We apply the constructed coupling coefficients matrix $\boldsymbol{\Xi}$ to $\boldsymbol{V}$ to obtain a visibility matrix with couplings $\boldsymbol{V}' = (\boldsymbol{I} + \boldsymbol{\Xi})\boldsymbol{V}(\boldsymbol{I} + \boldsymbol{\Xi}^\dagger)$, the result is shown in Figure 3, from which we see effects induced by the couplings samlilar to that present in the real observation data near the diagonal.

We solve the coupling coefficients matrix $\boldsymbol{\Xi}$ according to Eq. 2 with the unknown unitary matrix $\boldsymbol{K} = \boldsymbol{I}$, $\boldsymbol{I} + \boldsymbol{\Xi}_{\text{solved}} = (\boldsymbol{U}'\boldsymbol{\Lambda}'^{\frac{1}{2}})(\boldsymbol{U}\boldsymbol{\Lambda}^{\frac{1}{2}})^{-1}$. We show the solved $\boldsymbol{\Xi}_{\text{solved}}$ in Figure 4, compared with the constructed one in Figure 1, we see the solved $\boldsymbol{\Xi}_{\text{solved}}$ could not resume its original structure.

We show the solved visibility matrix without coupling $\boldsymbol{V}_{\text{solved}} = (\boldsymbol{I} + \boldsymbol{\Xi}_{\text{solved}})^{-1}\boldsymbol{V}'(\boldsymbol{I} + \boldsymbol{\Xi}^\dagger_{\text{solved}})^{-1}$ by applying the solved coupling matrix $\boldsymbol{\Xi}_{\text{solved}}$ in Figure 5. The coupling effects saw in Figure 3 has largely mitigated in the soved visibility matrix as you can see, validating the effectiveness of the coupling modeling and elimination method.

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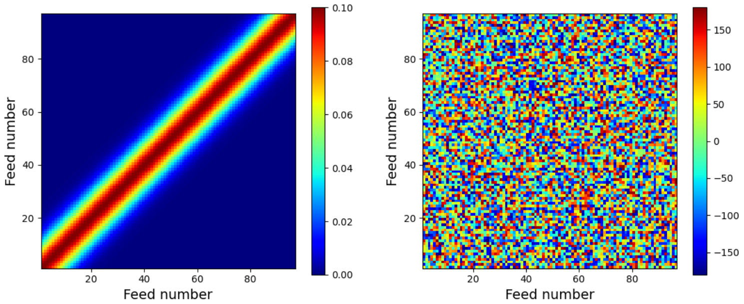


Figure 1: The constructed coupling coefficients matrix $\boldsymbol{\Xi}$. Left is its amplitude, right is its phase.

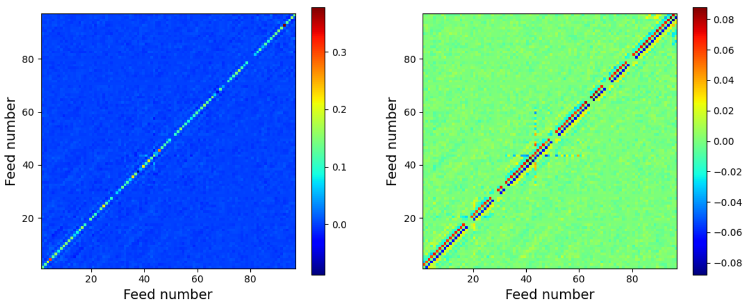


Figure 2: The calibrated and nighttime mean subtracted visibility matrix $\boldsymbol{V}$ as an approximation of the visibility matrix without couplings. Left is its real part, right is its imaginary part.

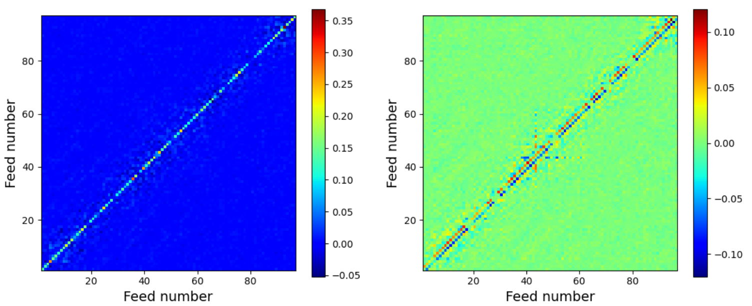


Figure 3: Visibility matrix $\boldsymbol{V}' = (\boldsymbol{I} + \boldsymbol{\Xi})\boldsymbol{V}(\boldsymbol{I} + \boldsymbol{\Xi}^{\dagger})$ with couplings by applying $\boldsymbol{\Xi}$ to $\boldsymbol{V}$. Left is its real part, right is its imaginary part.

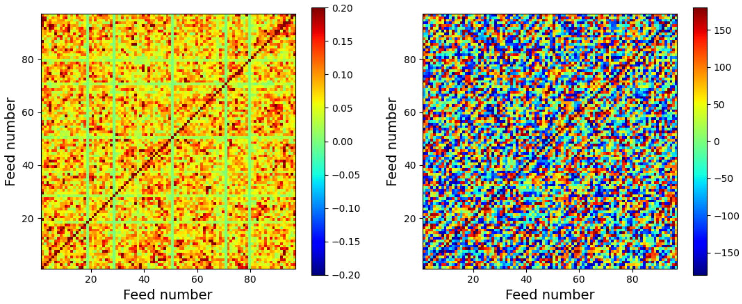


Figure 4: Solved coupling coefficients matrix $\boldsymbol{\Xi}_{\text{solved}} = (\boldsymbol{U}'\boldsymbol{\Lambda}'^{\frac{1}{2}})(\boldsymbol{U}\boldsymbol{\Lambda}^{\frac{1}{2}})^{-1} - \boldsymbol{I}$. Left is its amplitude, right is its phase.

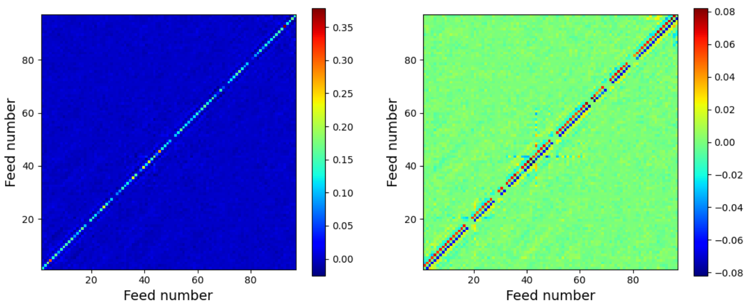


Figure 5: The solved visibility matrix without coupling $\boldsymbol{V}_{\text{solved}} = (\boldsymbol{I} + \boldsymbol{\Xi}_{\text{solved}})^{-1}\boldsymbol{V}'(\boldsymbol{I} + \boldsymbol{\Xi}^{\dagger}_{\text{solved}})^{-1}$ by applying the solved coupling matrix $\boldsymbol{\Xi}_{\text{solved}}$. Left is its real part, right is its imaginary part.